Experimental study of fluid-structure interactions using a soap film channel

Master of Science Thesis

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This document is the final report for the Master Thesis project, submitted in fulfillment of the requirements for the Degree of Master of Science in the subject of Aerospace Engineering on the Flight Performance and Propulsion track at Delft University of Technology. It details the work accomplished during the last seven months and focuses on the experimental study of fluid structure interaction phenomena. I had the opportunity to carry out this project at ONERA (Office national d’études et de recherches aérospatiales), first aerospace research player in France.

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Abstract

Fluid structure interactions are present everywhere, and are of crucial importance in the development of new concepts in the aerospace industry. Their study through numerical approaches such as CFD are of great interest for this sector, and understanding how to control them is key to the evolution of the industry.

In this work, we present an experimental method to generate quasi-2D flows and study fluid structure interactions thanks to the use of a soap film channel and a high-speed video camera. A simple test case is presented, introducing the fluid structure instability affecting a structure composed of a rigid cylinder with a flexible silk filament attached to its rear end. A complete description of the soap film channel is given. Explanations on how to build the apparatus, the required lighting and recording conditions as well as the fluid to be used are also provided. A set of relevant post-processing tools including a vortex shedding frequency analysis method and a position tacking algorithm are given to extract critical information from the experimental video recordings.

A theoretical model was constructed in order to predict the effect of the fluid structure instability on our test case and compare its results to the experimental data obtained with the soap film. The model proved itself to be particularly relevant for the static analysis of the phenomenon, and predicts the stationary instability affecting the average position of the filament with precision. Unlike existing models, it takes into account the action of the stiffness of the filament and provides a better description of the phenomenon. However, its simplistic construction and the strong hypothesis prevent the dynamic results to be efficiently used.

We highlight the advantages and the relevance of the soap film channel to obtain experimental data in quasi-2D conditions, and provide guidance for future researchers to use this technique.
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Introduction

Every deformable or moving structure placed in a flowing fluid will experience energy exchanges with this surrounding flow through induced vibrations and forces. This type of phenomenon that couples the fluid dynamics laws and the structure dynamics is referred to as fluid structure interaction. Such interactions are present everywhere around us, in nature every plant or animal evolving in a flowing environment experiences it. It affects the way fishes swim or the shape of aquatic plants but also how insects fly and human cells, like blood cells, move for instance. In nature, this can benefit such organism as they can harvest energy from the flow and therefore save their own energy reserves. But the fluid structure interaction also impacts man-made constructions like bridges, buildings, ships and aircrafts and can have detrimental effects on them.

Currently, fluid structure interactions and instabilities are limiting the development of innovative concepts in numerous domains such as the building sector, the aerospace [15] [29] and marine industries or the wind power production [18]. Aeroelastic effects are one of the representations of the fluid structure interaction phenomenon, which is preventing aircraft manufacturer to use lighter materials for instance. A better understanding and control of these effects would allow for different perspectives and new design solutions, in a very large spectrum of activities. This is why numerical and empirical models, in two or three dimensions, have been used for decades as a means to physically understand how to predict and control fluid structure instabilities.

The work presented in this report is part of the European project on fluid structure interaction "AEROelastic instabilities and control of FLEXible Structures" also called AEROFLEX. The main challenge tackled by this project is to design and develop new methods to describe aeroelastic instabilities, to control them and use them to reduce drag of large structures, without an increase in the energy cost. In this objective, the French aerospace laboratory ONERA has been asked to investigate on the fluid structure instability control through diverse means such as shape optimization. To this end, a relevant method to experimentally obtain data on various test cases is foreseen. It should enable researchers to compare empirical results with 2D simulations. Therefore, the so-called soap film channel apparatus seems to be the most relevant solution.
In this report, it is proposed to study a particular type of symmetry breaking static instability which is affecting a simple structure composed of a cylinder with a flexible filament. The experimental observation of this fluid structure interaction is made possible thanks to a soap film channel. The protocol to build and use a soap film channel is explained in details as it is a particularly pertinent apparatus to observe this phenomenon. Also, a complete method to help characterize the flow in this experimental device is given, along with useful post-processing tools which are meant to help extract meaningful information from the video recording.

Figure 1.1: Observation of the interaction between a structure (composed of a cylinder and a filament) and the flowing soap film. The bright colors are the result of polychromatic thin film interference effects and allow to visualize the flow patterns.

Then, a theoretical model that aims to predict the effect of the same fluid structure instability is introduced, and experimental results are compared to theoretical ones. All through the report, relevant observations and conclusions are drawn. At the very end, recommendations are made in order to shed light on potential amelioration of both the experimental apparatus and the theoretical model.

The overall goal of this report is to provide sufficient material to answer the following questions:

- How to build and use a soap film channel to study fluid structure interactions?
- What are the relevant parameters to be measured and which methods are to be used to extract relevant information from the experimental results?
- When and why does the symmetry breaking instability occur?
- What parameters can alter this phenomenon?
2 Motivations

In the process of studying and understanding any physical phenomenon, confronting simulation results to experiments is of prime importance hence the need to build apparatus which allow the scientific community to experimentally observe the physical manifestation of divers phenomena. In the case of this particular project, fluid structure interactions are investigated.

ONERA researchers have been setting up and running numerous simulations on various test cases, therefore there is a demand for experimental data. As the top French aerospace laboratory, ONERA has extremely powerful and complete facilities to perform high quality experiments on 3D models, such as the world largest continuous-flow blow-down wind tunnel S1MA in Modane, France.

Figure 2.1: Commercial aircraft model inside the S1MA wind tunnel [1]
Nevertheless, these advanced experimental solutions are obviously not required nor relevant for every project, and more modest setups are regularly used to get experimental data.

Moreover, many calculations and simulations are run in 2D, because of the lower computational power needed to solve them. In some cases the 2D approach can be used in a preliminary state of the project but in others the entire study can rely on 2D simulations, and in both situations this kind of calculation leads to insightful and valuable results. Nevertheless, it is impossible to produce a purely two-dimensional flow in an experiment, but quasi-2D flows can be obtained which under particular circumstance and hypothesis result in a relevant approximation of theoretical two-dimensional flows.

Relatively recent work [37] shows how a two-dimensional flow can be experimentally studied thanks to a so-called soap film channel. Indeed, this apparatus invented by Couder [14] in 1983 relies on a thin film of water and soap solution stretched on a frame to form a flowing channel in which obstacles can be inserted. The film itself has a thickness of few microns. The surfactant molecules (in our case soap) have a hydrophilic head, oriented towards the water, and a hydrophobic tail pointing towards the air [17]. This configuration ensures that water is securely trapped between the two layers of surfactant molecules. The surfactant gives elasticity to the film and prevents major variations in the fluid thickness that would lead to rupture. As the object is inserted through the flowing film, the local velocity of the fluid is changed. Chomaz proved that the local thickness of the film is directly linked to the local velocity [12]. Fortunately, the film thickness field is relatively easy to observe, as variations in the film thickness produce optical thin film interference (see figure 1.1). The optical properties of this kind of soap films allow for the visualization and qualitative interpretation of the flow patterns through high speed video analysis for instance. This rather easy-to-build and low cost visualization techniques allows gathering of experimental data, and give additional insight on a large diversity of phenomena. This is why it was decided to build a soap film channel at ONERA and to test it on simple cases.

2.1. Test case

The fluid structure interaction phenomenon that is proposed to be studied in this report is the instability of a flexible filament in the wake of a cylinder. This physical manifestation of fluid structure interaction was observed by different researchers both for rigid plate [27] [13] and flexible filament [10] cases. In these configurations it was witnessed that for filaments or plates that are short enough compared to the diameter of the cylinder, the appendage was taking an angled equilibrium position, breaking the whole symmetry of the system (figure 2.3) when placed in a flowing fluid.

Therefore, the typical structure we are going to immerse in the quasi-2D flow is composed of a rigid cylinder which has a flexible filament attached at its bottom end. The characteristic parameters and dimensions of the structure are given in chapter 4.
2.1. Test case

Figure 2.2: Rigid cylinder with a flexible filament represented for three values of length $L$. The fluid is flowing from the top to the bottom and the back flow region is represented in blue by a semi ellipse.

When this obstacle is inserted in the soap film, only a quasi-2D cut of it is actually in contact with the fluid. Because of the stream coming onto the structure, a back flow region forms behind the cylinder (represented as a blue semi-ellipse on figure 2.2). Depending on the filament length, the fluid destabilizing action in the back flow region can generate a static instability forcing the filament to rest at an angled equilibrium position. This symmetry breaking instability is the main phenomenon under consideration in this work. In addition, dynamic instabilities such as flapping are also expected.
2.1. Test case

Figure 2.3: Comparison between the model on the left and the experimental observation on the right, showing the symmetry breaking instability.

Numerical two-dimensional models have been developed at ONERA to simulate this phenomenon and try to control it through shape optimization and the first results show that non-intuitive shapes could help control the instabilities. However, before testing these optimized shape, it is necessary to build the soap film channel and to study the instability in the case of a classic cylinder which is aim of the work presented here.

The main idea is to experimentally observe the symmetry breaking happening when the filament attached behind the cylinder is taking an angled equilibrium position. The parameters of interest are the lengths for which the static instability occurs as well as the corresponding angles. To perform these measurements, we are going to insert this structure in the soap film. A high speed camera will capture a two-second movie which corresponds to 8000 frames. Under the appropriate lighting conditions (section 3.3), the flow patterns in the film will also be capture on the video recordings. These movies will then be treated with the relevant post processing tools (chapter 5) to extract useful information.

After validation, the experimental setup will help explore diverse configurations to help
2.1. **Test case**

support numerical model results and investigate new test cases.
Experimental setup

This chapter presents the experimental apparatus we built and used for this work. First we introduce the soap film channel describing the main components and points of interest. Then both the recording and lighting setups are discussed, as well as the solution to be used to form the flowing fluid.

3.1. The soap film channel

Many efforts have been made to generate quasi-two-dimensional flows, in order to maintain the flow structures in a plane, and to keep the vorticity vector normal to the velocity vector, preventing the initial two-dimensional flow to evolve in three dimensions. Different solutions have been created including the soap film channel, in 1983 [14]. Soap films have been used a lot by fluid-dynamics researchers since then [8] [33] [42] [31] [23], as this is one of the best available experimental setups to study two-dimensional flows. Since its invention, the soap film channel has been iteratively modified and has now reached a stable design in the form of the gravity driven vertical soap film introduced by Rutgers [37], offering higher fluid velocity, larger channel size, longer soap film lifetime and lower air friction.

Our vertical soap film channel is in line with Rutgers’s description, and is composed of the following components: a metallic structure, a nylon-wire frame and auxiliary nylon wires, an upper and a lower reservoir, a pump, tubes, an injection nozzle, a valve and a flow meter as represented on figure 3.1.
Figure 3.1: The soap film channel
3.1. The soap film channel

First of all, when building this kind of apparatus one needs to make sure all the component are corrosion resistant, as the properties of the film is extremely sensitive to the quality of the soap water solution. Excluding the metallic frame, the whole setup is made out of plastic materials, such as Nylon for the film frame, PVC for the tubes and polypropylene for the reservoirs. This is supposed to be enough to avoid pollution of the solution, however, it is not recommended to leave the solution to rest for long periods of time (days) in the system [37], therefore one should renew the solution between experimental sessions.

In order to produce efficient and consistent experiments, the fluid injection conditions need to be constant from one session to another, and they should not change with time during the experiment.

One of these conditions is the pressure before the valve, ahead of the injection nozzle. Hence, a constant fluid height is to be kept in the upper reservoir. In order to ensure that this is respected, the upper reservoir is composed of two cups: a small inner cup which is connected to the valve and a larger outer cup, as shown on figure 3.1. Fluid gathered in the lower reservoir is brought back to the upper reservoir in the inner cup thanks to the action of a pump (RS PRO M400). The pump debit exceeds the one used to stretch the soap film, therefore, fluid is accumulating in the inner cup until it is full, exceeding fluid then goes in the outer cup which is connected to the lower reservoir. This system keeps the fluid height ahead of the valve constant to the inner cup height. To guarantee that the flow rate at the exit of the nozzle is kept constant, a flow meter (Key Instrument ref MR3L10BVBN) is installed after an on/off valve positioned at the exit of the upper reservoir. With its measure range going from 5mL/mn to 110mL/mn and its precision of 5ml/mn the flow meter allows to finely tune the flow rate and to keep it constant during the experiments. Once the solution is ejected through the nozzle, it can impregnate the nylon wires which will constitute the film frame when put in tension. This nylon frame is composed of two main nylon thread (strong fishing nylon, diameter 0.5mm). Each of these threads are linked to the main metallic frame thanks to two auxiliary wires (thin fishing nylon, diameter 0.1mm) which allow to separate the two main threads from each other and to stretch the nylon frame forming the soap film. It is relevant to use thinner nylon for the auxiliary wires in order to minimize the size of the knot linking them to the main threads, hence to minimize the intrusion inside the film frame. The vertical tension of the nylon frame is ensured by the presence of a 1kg weight hanging at the bottom of the setup, this also makes sure the main nylon threads are in contact when the tension in the auxiliary wires is released. The main nylon threads are directly coming out of the injection nozzle, as per figure 3.2. The shape of the nozzle itself and the fit of the nylon thread inside must be taken care of as they can disturb the flow at the injection point, whereas the objective is to have a flow as smooth as possible.
3.1. The soap film channel

Figure 3.2: The two main nylon threads originating from the nozzle [37]

Figure 3.3: Nozzle on the experimental apparatus
3.2. The soap film

To create the soap film itself, one can proceed according to the following protocol: First the tension on the auxiliary wires is released and the main threads are put in contact thanks to the action of the weight. Then the on/off valve can be opened. It might be necessary to set the flow meter on an higher flow rate compared to the target flow rate during the first seconds, in order to get rid of any air bubbles present in the tubing ahead of the injection nozzle. Once this is done, the operator can set the flow meter on the experimental flow rate. Fluid should now be ejected from the nozzle, running along the main nylon threads until it reaches the bottom limit of the nylon frame, ending up in the lower reservoir. Once the solution is running on the whole length of the main nylon thread, tension can be applied to the auxiliary wire to form the hexagonal nylon frame. Thanks to the solution properties, especially the enhanced elasticity and surface tension provided by the soap, the soap film forms and flows inside the frame.

The nylon frame is divided in three different areas: the expansion zone, the test zone and the contraction zone, see figure 3.4. In the first part, the expansion zone, the two nylon threads are separating from each other at a constant angle, making the soap solution turns into a film. The separation angle is an important parameter as a too large angle, leading to a too short expansion zone, would lead to high thickness gradient in the film, which could cause the film to burst or develop heterogeneously ahead of the test zone. In practice we used an angle between 10 and 15 that provided a smooth transition from the nozzle to the test zone. Once the film has its constant width, it enters the test zone, where the balance between gravity and drag force from air friction give the soap film a constant velocity. This is where the experiment is to be carried out. Exiting the test zone, the films encounters the contraction zone, which has a constant contraction angle, forcing the film to thicken ending up in the lower reservoir. A hydraulic circuit is used to feed back the solution to the upper reservoir.
During the process of using the soap film channel, one of the main side effects is the formation of foam, mostly located in the lower reservoir. Two factors are playing a role in the foam formation. First, the end of the contraction zone is located above the lower reservoir, which implies that the solution takes the form of a small jet to reach the reservoir. The relatively high momentum of the jet can create foam in the reservoir. The second source of foam is due to the action of the recycling tube, which purpose is to send back the exceeding fluid gathered in the outside cup of the double-cup system in the upper reservoir. As soon as the fluid level in this outside cup is higher than the top of the tube, the fluid flows into it by gravity. However, this can generate a suction effect at the entry of the tube that traps air in the flow. This results in foam formation at the end of the tube (in the lower reservoir). In both cases, it is important to ensure the level of fluid in the lower reservoir is high enough to avoid foam being pumped to the inside cup of the upper reservoir.
3.3. Lighting and recording setup

In order to observe the flow patterns created in the soap film by the insertion of any obstacle, different techniques can be used. The Schlieren visualization method [6], the shadowgraph [37] and interferometric techniques are the most commonly used. For our study we use the latter, as it is relatively easy to set up and produce clear images of the flow patterns with interferometric techniques.

This technique consists of the quantitative observation of interference fringes visible directly on the soap film. These fringes are caused by the phase difference between the light rays reflected (or transmitted) by the upper and the lower surface of the film. Indeed, because of the semi-transparency of the film, the two interfaces on each side of the film reflect light, but the part of the initial incoming ray of light reflected on the rear side of the film will have to travel a longer path than the one reflected on the front side, inducing a phase delay.

![Diagram of thin film interference effects](image)

Figure 3.5: Thin film interference effects are produced by the phase difference between the light rays reflected (or transmitted) by the two different interfaces between the air and the soap film.

With polychromatic white light, this phenomenon creates colorful patterns as each different wavelength interacts with the film.
Figure 3.6: Our experimental soap film under white light, in this case a simple cylinder is inserted in the film leading to the observation of Von Karman vortex street behind the obstacle.

If a monochromatic light source is used to light the soap film, the interference will be of two natures: constructive and destructive. Thus, the variation in the fringes contrast will give information on the local film thickness.
Chomaz proved in [12] that the thickness variations of the film are directly slaved to the velocity and pressure fields, considering the only assumption that the velocity is lower than the Marangoni wave speed, which corresponds to the velocity of shock waves in the film.

Rutgers explained that low pressure sodium light, emitting at 595nm would give well-defined interference fringes [37], and it has been verified in numerous cases, [22] [25] [7] for instance. Nevertheless, during the experiments performed for this thesis, a green monochromatic LED light emitting at 535nm (with a very narrow bandwidth) was used and found to be particularly relevant. The LED light source offers different advantages such as an improved longevity and less heat emissions over long periods of recording. Also using a lower wavelength light source is supposed to provide a thinner definition as the thickness variation between two fringes will be lower.
3.4. Soap/water solutions

One of the most critical independent variables of the experiment is the soap water mix used in the soap film channel. Indeed, the type of soap (brand, composition, even the scent) and the water-to-soap ratio can drastically impact the properties of the soap film, in terms of quality, thickness distribution, smoothness or lifetime.

However, the available scientific literature does not provide a lot of information about robust solution to stretch good quality soap films. Most of the authors agree on a volumetric soap-to-water ratio between 1% [37] and 2% [27] [24] [31], but they only refer to "commercial dish-washing detergent". Moreover, as soap is not a comestible good, manufacturers are not bound to explicitly give its composition, which make the comparison between different soaps impossible without testing them. The objective is to get a film that is strong enough not to burst too easily which can be achieved with high soap concentrations or
glycerol addition however this would prevent the film thickness from being uniform, which is also of utmost importance, hence a compromise need to be found. Also, no difference was noticed when using distilled water and tap water so for this study we chose to use tap water.

Different types of soap were tested at the beginning of the research process (see Appendix A), in order to find the most suitable soap solution to perform the experiments in the soap film channel. A test protocol had to be created, to make sure that the comparison is relevant. From each soap, 1% and 2% solution were made. Each solution was tested on different flow rate setup, with different channel widths. The criterion to evaluate the quality of the film was its average lifetime.

As expected, the less additives (color, scent) in the soap the better. Indeed, the solutions made with the "monoprix" soap gave the poorest results and were not good enough to even stretch the film in most of the cases. The most suitable soap was found to be the "Arbre Vert sensitive skins" which gave the most uniform and durable films.

As a general results, independently of the soap type, the 2% solutions seemed to create stronger films, especially at low flow rates. The concentrations is still low enough to guarantee a smooth thickness gradient in the soap film.

Also, it is important to note that during this testing process, the pump which is supposed to keep the level of solution constant in the upper reservoir was not installed, and that the obtained lifetimes might be affected by its action during the experiments.

Also, even if the burst of the soap film is deterministic [3], the film seems to be extremely sensitive to exterior conditions such as air current or vibrations, which can explain some results which were far from the final average and introduced relatively large standard deviations.
Physical properties of the fluid and solid domains for the reference test case

In this chapter all the relevant parameters about the apparatus and the experiments are gathered. Some of them are independent variables such as the size of the obstacle or the flow rate, which can be set directly by the operator, whereas others such as the film viscosity or thickness are dependent parameters, which need to be evaluated as there is direct way to measure them.

Figure 4.1: The system under consideration and the main parameters [32]

4.1. Fluid domain parameter

First of all, the dimension of the film must be known and set in accordance to the need of the experiment. As said earlier, the expansion and contraction zones geometry will impact the film lifetime, but for the experimental session it is the geometry of the test zone which
is of a critical importance. For our work, the test zone was a rectangle 90 cm high and 9 cm wide when the film is stretched in the nylon frame (figure 4.2).

![Figure 4.2: Representation of the soap film test zone with dimensions](image)

### 4.1.1. Velocity and thickness of the film

The velocity and thickness of the film are dependent on two parameters: the geometry of the nylon frame and the flow rate. When the frame has been set to its final geometry with the three different zones, one can set the flow rate. There are no rules for the flow rate and a broad range of values can be used with the soap film, however if the flow rate is too low the film will be more prone to rupture, which occurs when the local thickness of the film becomes too thin to maintain the surface tension. On the other hand, if the flow rate is too high, turbulence will occur and disturb the surface of the film, in this configuration, the film shows ripples and is not planar anymore, hence loosing its characteristic of interest, being quasi-two-dimensional, in the case of our study.
A relevant compromise to get a planar and strong film in the test zone was found setting the flow meter to 30mL/mn.

To determine the velocity of the film, Laser Doppler Velocimetry (LDV) can be used, however, as the film is extremely thin one can have difficulties to focus the laser on the film plane. Particle Image Velocimetry (PIV) is another possibility, nevertheless also because of the low thickness of the film, very thin particles need to be seeded in the solution, and they could get stuck in the hydraulic circuit. This is why the chosen option for this work was simpler and easier to perform. As suggested by Jia in [21], tracking small impurities in the flowing soap film is sufficient to get a relevant approximation of the flow velocity without seeding the fluid with special particles, indeed, after some time, the soap film can capture part of the dust present in the ambient air. These dust particles are visible on the videos recorded with the high speed camera, as they are less reflective than the soap film. These small defects in the film can easily be tracked. Knowing the spatial and temporal frequency of the camera, the velocity can be calculated by dividing the distance traveled by the particles by the time elapsed.
4.1. Fluid domain parameter

Figure 4.4: Dust particle captured in the soap film at t and t+dt, assumed to be moving at the fluid velocity

Thanks to this procedure, the value of the film velocity in the test zone, at a constant flow rate of 30mL/mn is measured at $U = 2.22 m/s$.

Figure 4.5: Free stream velocity measurements, and average free stream velocity

Following the determination of the flow velocity, a value for the thickness can be calculating using the mass flow conservation law.
4.1. Fluid domain parameter

Considering that the cross section of the film in the test zone is a rectangle of dimensions \( W \times h \) with \( W \) the width (set to 9cm) and \( h \) the thickness, taking a constant flow rate of \( Q = 30mL/mn \) the thickness is given by the following relation:

\[
h = \frac{Q}{U W},
\]

which leads to the value of \( h = 2.53 \mu m \). This result is in line with the expected soap film thickness [12].

4.1.2. Determination of the film viscosity

The viscosity of the soap/water solution as it is in its three dimensional form cannot be taken as the viscosity of the soap film. The soap film is composed of a thin layer of solution in between of two surfactante interfaces with the surrounding air.

Trapeznikov [40] proposes that the viscosity of a soap film is simply the combination of the viscosity \( \nu_b \) of the bulk solution and the viscosity \( \nu_{surf} \) of the air-liquid interfaces composed of the surfactant layer.

\[
\nu = \frac{2}{h} \nu_{surf} + \nu_b,
\]

with \( h \) the film thickness.

Actual measurement of the film viscosity is far from being easy. Martin and Wu [30] created an experimental setup based on the Zimmer viscometer able to measure the viscosity of a fluid film. However, it requires specific equipment and resources which are not easily accessible. Comparing their results with the Trapeznikov formula, they found that the term \( \nu_{surf} \) was dominant and dictated the behavior of the soap film. Nonetheless, Rutgers arrived to the opposite conclusion [36], meaning that in their case the term \( \nu_b \) was dominant, which led him to the conclusion that the surfactant layer viscosity had a minor role. A potential explanation for this difference is that Martin and Wu used additional glycerol to increase the overall viscosity of the film. By doing this with the objective of changing \( \nu \), it is likely that they made \( \nu_{surf} \) rise and become prevalent over \( \nu_b \).

However, in the absence of a direct measurement solution, there is another method that has proved to be relevant by Jia [21] and which has been used to give an equivalent viscosity of the flowing soap film, based on fitting curves which link the Strouhal and Reynolds numbers. The principle is to get the Strouhal number from observations of the vortex shedding frequencies generated by the insertion of a simple cylinder in the soap film. The Strouhal number can then be related to a Reynolds number choosing one Strouhal/Reynolds relationship equations (figure 4.6). As the velocity and characteristic dimension of the system are know, the viscosity value can be calculated.
For instance, the Roshko model [34], originally introduced for 3D vortex shedding has been used in several papers such as [16], [21], but this relation is not specific to quasi two-dimensional flows.

More recent models are available, such as the one presented in [35] and used in [41] which has been developed using a flowing soap film channel and which is supposed to be more up-to-date and accurate considering the specific conditions of this kind of experiments. This specific relation, specially dedicated to quasi two-dimensional flowing films is given by:

$$S_t = \frac{1}{5.12 + \frac{313}{Re}}$$  \hfill (4.3)
4.2. Solid parameter estimation

As it can be seen on figure 4.6 this last Re-St law gives lower Strouhal numbers compared to the other models for the same Reynolds numbers. Moreover, Roushan states that the asymptotic Strouhal number (for large Reynolds numbers) in his law is lower by approximately 10% compared to the typical 3D measurements which is around 0.2 [35]. In our experimental measurements (see chapter 6), the asymptotic Strouhal number is found to be just below 0.18. Hence, only Roushan’s law is relevant for us, as other relations from figure 4.6 would lead to a maximum Reynolds number of 150 which is less realistic looking at the flow structures observed in our experiments as well as high viscosity values that do not correspond to the one given in the literature (see table 4.1).

We found an equivalent viscosity of $2.07 \times 10^{-5} \, m^2/s$.

It is relevant to compare our experimental results with those found in the literature, presented in table 4.1.

<table>
<thead>
<tr>
<th>Paper</th>
<th>Kinematic viscosity $\nu (m^2/s)$</th>
<th>Flow velocity (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Araya15 [7]</td>
<td>$7 \times 10^{-6}$</td>
<td>2.2</td>
</tr>
<tr>
<td>Auliel17 [6]</td>
<td>$5.5 \times 10^{-6}$</td>
<td>0.8 to 5</td>
</tr>
<tr>
<td>Jia08 [22]</td>
<td>$2 \times 10^{-5}$</td>
<td>1.3 to 1.5</td>
</tr>
<tr>
<td>Lacis14 [27]</td>
<td>$5.9 \times 10^{-6}$</td>
<td>1.9</td>
</tr>
</tbody>
</table>

Table 4.1: Fluid parameters

It is stated in Auliel’s article [6] that he used the Trapeznikov relation (equation 4.2) to evaluate the film viscosity, and it seems that Araya [7] and Lacis [27] did the same as their values are close to the water viscosity. However, without none of them explain how they extract the surface viscosity needed to obtain their result. Without a proper way to do it, we chose to follow the vortex shedding observation methods and ended up with results comparable to Jia’s [22].

### 4.2. Solid parameter estimation

About the solid structure, the characteristic length is the main parameter, be it the diameter of the cylinder $D$ or the length of the plate/filament $L$. Because of the practical limitations in terms of channel width (generally between 2cm [11] and 20cm [31]) and to keep the Reynolds number relatively low, these parameters are bounded.

In the majority of recent paper dealing with the introduction of obstacles in soap film, the obstacles characteristic dimension is between 0.5 and 1cm. In our case, to make the Reynolds number lower the characteristic cylinder we use has a diameter $D = 3.2 \, mm$.

Moreover, the stiffness of the flexible part (filament) which reacts to the fluid forces is of great importance for the dynamic of the system. The filament we are using is the exact same filament as in the work of Ugis Lacis [27]. The critical parameter to evaluate is the bending stiffness coefficient $K$ which is often used in beam structure theory and given by:

$$K = EI,$$  \hspace{1cm} (4.4)
Looking in the literature material [27] [22] at different values of silk filament bending stiffness for which the geometry is known, an approximation of the value of the Young’s modulus of our filament is chosen to be $E = 2$ GPa. In our case the diameter of the filament is $d = 0.08mm$. 

with

$$I = \frac{\pi d^3}{64}$$

(4.5)
Post processing tools for experimental data analysis

This chapter focuses on the post treatment codes and tools which are used to extract information from the experimental video recordings. First a POD and DMD algorithm that aims to extract relevant frequencies is presented, then a filament tracking method to get the position of the filament is introduced.

5.1. POD and DMD for modal analysis

In the objective to make the post treatment of the recorded videos taken during the experiments easier and more time efficient, implementation of numerical solutions were required. Dynamic modal analysis is almost exclusively used on numerical simulation, as it often relies on the system matrix to generate simulated flow field and study their modes. However, when using experimental data the system matrix is not given, and only observations of the physical phenomenon are available. In the present study case, the information about the vortex shedding frequency is meant to be extracted from video files, or sequences of snapshots. Fortunately, different methods have been developed and can be used to accomplish this objective, and two have been found useful and relevant.

5.1.1. POD

The POD method - for Proper Orthogonal Decomposition- is often used in the study of turbulent phenomenon since its introduction by Lumley [28] in 1967. It relies on a statistical treatment of the signal, which highlights coherence and structure, and results in its dynamical characterization.

The POD principle is similar to that of other decomposition methods, projecting the function to be studied on a basis composed of modes, reducing the study to a set of scalars which describe the function of interest. Nevertheless, the POD has the particularity of providing an orthogonal and optimal basis of modes.
Let us take a spatial domain $\Omega$ of $\mathbb{R}$ with a space variable $x$ in $\Omega$ and $v(t)$ a function (e.g. representing the velocity field) in $V$ a Hilbert space. The POD objective is to find the orthogonal basis which give the best average approximation of $v$:

$$\frac{\langle (v(t), \Phi)^2 \rangle}{(\Phi, \Phi)} = \max_{\Phi \in V^*} \frac{\langle (v(t), \phi)^2 \rangle}{(\phi, \phi)}$$

(5.1)

In our case, we use a discrete POD, with $M$ values of $v(t_i), i = 1..M$. With this configuration, the maximization problem becomes:

$$\frac{\langle (v(t), \Phi)^2 \rangle}{(\Phi, \Phi)} = \max_{\Phi \in V^*} \frac{1}{M} \sum_{i=1}^{M} \frac{(v(t_i), \phi)^2}{(\phi, \phi)}$$

(5.2)

If we define the linear operator $R$ as follows:

$$R : V \rightarrow V$$

$$\Phi \rightarrow R\Phi = \frac{1}{M} \sum_{i=1}^{M} (v(t_i), \phi) v(t_i)$$

The solution of equation 5.2 is the largest eigenvalue $\lambda$ such that:

$$R\Phi = \lambda \Phi.$$  \hspace{1cm} (5.3)

$v$ can then be decomposed onto this orthogonal basis made of the eigenvectors:

$$v(x, t) = \sum_i a_i(t) \phi_i(x),$$  \hspace{1cm} (5.4)

where $a_i$ are the temporal coefficients.

Assuming that the eigenvalues $\lambda$ are organized in descending order, let us write $v$ on the truncated basis composed of the $\Phi_{i=1..N}$:

$$\hat{v}(x, t) = \sum_{i=1}^{N} a_i(t) \phi_i(x),$$  \hspace{1cm} (5.5)

Because of the optimal character of the basis, the energy captured by the first $N$ elements of the POD basis will be larger than the energy captured by all the remaining $\phi$. Hence, only a limited number of eigenvectors are necessary to capture the principal modes of the studied phenomenon.

### 5.1.2. Snapshots method

Finding the eigenvectors of the operator $R$ can be costly in terms of computational power. To bypass this issue, a method based on sequential snapshots was introduced by Sirovitch [39].
The idea is not to focus on the research of $\phi_i$ but to look for the coefficients $A_k = (A^1_k, ..., A^M_k) \in \mathbb{R}^M$ for $k = 1, \ldots, M$ such that:

$$\phi_k(x) = \sum_{i=1}^{M} A^i_k \nu(x, t_i)$$  

(5.6)

The problem is now taking the following form:

$$\sum_{j=1}^{M} \frac{1}{M} (\nu(t_i), \nu(t_j)) A^i_k A_k^j = \lambda_k A^i_k \ for \ i = 1..M$$  

(5.7)

where $\lambda_k$ corresponds to the eigenvalue associated to the POD mode $\phi_k$. The POD basis is then obtained using equation 5.6, and the temporal coefficients by projecting the $\nu$ field onto the POD basis:

$$a_i(t) = (\nu(t), \phi_i).$$  

(5.8)

This method allows to drastically reduce the problem dimensions when compared to the problem given in equation 5.3.

### 5.1.3. SVD method

In practice, one can use a singular value decomposition to obtain the POD of a series of $M$ snapshots of the field $\nu(x, t)$ [20]. Each snapshot being represented as a column vector of length $X$, the matrix $W$ of size ($X \times M$) can be created by gathering all the $M$ snapshots.

Using the singular value decomposition (SVD) on $W$, the following equation is obtained:

$$W = \Phi \Sigma A^*$$  

(5.9)

where $()^*$ is a conjugate transpose matrix operation.

The POD eigenvalues are then found to be given by:

$$\lambda = diag(\Sigma)^2 / (X - 1),$$  

(5.10)

and the $A^i_k$ coefficients correspond to the $M \times M$ coefficients of matrix $A$.

### 5.1.4. DMD

The Dynamic Mode Decomposition was first introduced by Peter Schmid [38] in the objective of bringing a new method to extract flow fields, through the analysis of numerically-generated images or experimental data (similarly to the snapshot POD).

Firstly, the data describing the flow field must take the shape of a temporal series of observation or snapshots. Each snapshot $\nu_t$ for $t = 1..N$ is gathering information about the flow.
field at the time $t$. One can build a matrix that gathers all the instantaneous snapshots in a time sequential order as follows:

$$V_1^N = \{v_1, v_2, v_3, ..., v_N\}, \quad (5.11)$$

where $V_1^N$ represents a series of snapshots from time $t = 1$ to $t = N$. The time step $\Delta t$ between $v_t$ and $v_{t+1}$ is assumed to be constant.

The core assumption of this method is to assume the existence of a linear evolution operator $A$ which embodies the evolution of the flow field between two snapshots, and therefore links $v_t$ and $v_{t+1}$ with the following relation:

$$v_{t+1} = Av_t. \quad (5.12)$$

For a linear phenomenon, this mapping can be assumed to be the same on the full timescale of the observation, from $t = 1$ to $t = N$, and can be used to rewrite the full set of snapshot from equation 5.11:

$$V_1^N = \{v_1, Av_1, A^2 v_1, ..., A^{N-1} v_1\}, \quad (5.13)$$

or

$$V_1^N = \{v_1, Av_1, Av_2, ..., Av_{N-1}\}, \quad (5.14)$$

The characteristics of the operator $A$ such as its eigenvalues and eigenvectors are related to the temporal dynamic of the flow field.

If a sufficient number of snapshots are taken and used in this analysis, it is fair to assume that the vectors representing the snapshots in the matrix $V_1^N$ are linearly dependent, so one can write:

$$v_N = a_1 v_1 + a_2 v_2 + a_3 v_3 + ... + a_{N-1} v_{N-1}. \quad (5.15)$$

In practice, the following matrix relation is used:

$$v_N = V_1^{N-1} a + r, \quad (5.16)$$

where $a^T = \{a_1, a_2, a_3, ..., a_{N-1}\}$ and $r$ is the residual matrix, which is null when relation 5.18 is verified. In practice when working with experimental data the operator $A$ cannot entirely describe all the behaviors of the flow field leading to the existence of this residual matrix $r$.

Hence let us write:

$$A\{v_1, v_2, ..., v_{N-1}\} = \{v_2, v_3, ..., v_N\} = \{v_2, v_2, v_3, ..., V_1^{N-1} a\} + r \quad (5.17)$$

which leads to:

$$AV_1^{N-1} = V_2^N = V_1^{N-1} S + r, \quad (5.18)$$

introducing the companion matrix $S$ that shows that from $V_1^{N-1}$ to $V_2^N$ only the last column of $V_2^N$ is changing.
5.1. POD and DMD for modal analysis

\[ S = \begin{pmatrix}
0 & a_1 \\
1 & 0 & a_2 \\
& \ddots & \ddots & \ddots \\
& 1 & 0 & a_{N-2} \\
& & 1 & a_{N-1}
\end{pmatrix} \]  

(5.19)

As the matrix \( S \) and \( A \) are similar, if \( r \) is null they share the same eigenvalues. Moreover, the only unknowns in \( S \) are the \( \{a_1, a_2, a_3, \ldots, a_{N-1}\} \) coefficients that link the very last snapshot of the series to the others, which can be determined by minimizing the norm of the difference vector:

\[ a_{\text{optimal}} = \min_a |v_N - \sum_{i=1}^{N-1} a_i v_i| \]  

(5.20)

5.1.5. The SVD approach

The companion matrix approach explained above is often used as a type of Arnoldi method, and even if it is correct in a mathematical point of view, it finds its limits when used with experimental data which show noise and uncertainties. Alternatively, instead of computing the \( S \) matrix by searching for the \( \{a_1, a_2, a_3, \ldots, a_{N-1}\} \) a Singular Value Decomposition can be used on \( V_1^{N-1} \):

\[ V_1^{N-1} \equiv U \Sigma W^*, \]  

(5.21)

In this configuration, the matrix \( U \) contains the POD modes of the data set in \( V_1^{N-1} \) (see section 5.1.3). Then, using this decomposition in relation 5.18 one can write the the projection on the POD basis of the linear operator \( A \) as follows:

\[ U^* A U = U^* V_2^N W \Sigma^{-1} \equiv \hat{S} \]  

(5.22)

Once again, as matrix \( A \) and \( \hat{S} \) are linked through a similarity transform, they share the same eigenvalues \( \lambda_i \) such that \( \hat{S} y_i = \lambda_i y_i \). Moreover the DMD modes are given by:

\[ \Psi_i = U y_i \]  

(5.23)

This methods is more robust and adapted to the analysis of experimental data.

5.1.6. Numerical implementation

For this thesis work, it was decided to couple both Proper Orthogonal Decomposition and Dynamic Mode Decomposition in a hybrid POD/DMD method [19].

5.1.7. Validation and Results

In order to validate the POD/DMD code, we fed it snapshots of DNS that was simulating vortex shedding behind a cylinder which is corresponding to our experimental setup.
Figure 5.1: From left to right: raw DNS simulation snapshot, DNS snapshot with grey colormap to match experimental video recordings, DMD analysis of a sequence of grey DNS snapshots.

The idea is to compare the POD/DMD output vortex shedding frequencies with the values found in the DNS log at different Reynolds numbers. For each Reynolds number four series of DNS snapshots are analyzed.
5.2. Filament tracking

We can observe that the performances of the POD/DMD code are good enough to use it as a way to extract the vortex shedding frequency from the experimental video recordings.

5.2. Filament tracking

In the aim to study the behavior of a filament attached behind different obstacles and placed in the soap film, a simple tracking tool was created. The objective is to record and save information about the instantaneous position of the filament on each frame that composes the video recording resulting from the experiments.

Because of the lighting configuration and the materials used in the experiments, the filament is seen as a dark object on the video footage, whereas the film is reflecting light and is a lot brighter. To track the filament, all the frames are converted into a gray scale image which is translated into a matrix. Each element of this matrix represents a pixel and has a value between 0 (pure black) and 1 (pure white). For example, the following matrix is
5.2. Filament tracking

describing the image shown in figure 5.3:

\[
\begin{bmatrix}
0.667 & 0.958 & 0 & 0.291 & 0.853 \\
0.916 & 0.167 & 0.250 & 0.541 & 0.625 \\
0.125 & 0.208 & 0.500 & 0.791 & 0.875 \\
0.375 & 0.458 & 0.750 & 0.833 & 0.833 \\
0.416 & 0.708 & 1 & 0.041 & 0.333
\end{bmatrix}
\]

Figure 5.3: Gray scale pixels forming a 5 × 5 image

Thus, a simple threshold will efficiently sort the dark pixels which represent the filament and the light ones that correspond to the fluid. With this methods, the coordinates of each pixel darker than the threshold are saved and this process is repeated for every frame.
5.2. Filament tracking

By targeting the center of the cylinder on the first frame, it is then easy to extract information about the position of the filament or points of interest such as the extremity of the filament by selecting the points that are the farthest away from the cylinder. The number of points to detect is adjustable to get an average position of the filament tip.

The dark area which can be seen on the very top part of figure 5.4 is caused by the reflection of the camera objective on the soap film and cannot be avoided. Nevertheless, when studying the behavior of the filament, each frame can be cropped at the height of the cylinder to prevent the detection of this area by the program.

Figure 5.4: Filament detection with the threshold method, the filament tip is marked with in red
Determination of the experimental conditions

To guarantee useful results, deep knowledge of the setup is required prior to the experiments. The fact that the physical properties of the soap film are not directly measurable make this characterization process even more difficult. The active variables, which we can directly control, are the flow rate $Q$, the film width $W$ and the soap concentration. In order to perform reliable and reproducible experiments, information about the film viscosity $\nu$ and its thickness $h$ are mandatory. A good approximation of the last one can be given by using the mass flow conservation law, assuming a constant thickness and flow speed in a slice perpendicular to the film and the flow. However, the viscosity value remains to be found.

In the following chapter, is detailed a complete method introduced in chapter 4, on how to experimentally determine an equivalent dynamic viscosity for a flowing soap film through the observation of the vortex shedding created by the insertion of cylinders thought the film.

If a cylinder is introduced in the soap film perpendicularly to the flow, a periodic pattern of vortices will be observed behind the obstacle, because of the unstable separation of the fluid around the object. The resulting Von Karman vortex street can help us characterize the flow as the vortex shedding characteristics such as its frequency are dependent on the Reynolds number.
Figure 6.1: Observation of the vortex Sheding behind a cylinder of diameter 3.2mm inserted in the soap film. Photograph taken under polychromatic white light creating thin film interferences which allow to visualize the flow patterns
6.1. Protocol

The idea of this methods lies in the Strouhal-Reynolds number relationship established in [35], using a similar flowing soap film channel. Cylindrical obstacles with various diameters are introduced in the soap film, creating vortex shedding. This phenomenon can then be recorded at high frequency with a high speed camera. These video recordings can then be analyzed with POD and DMD methods (see section 5.1.3) to extract useful information such as the vortex shedding frequency and the Strouhal number. Using the previously mentioned Strouhal-Reynolds number relationship the Reynolds number is obtained, and a linear regression of the experimental Reynolds number as a function of the diameter times the velocity gives the equivalent dynamic viscosity of the film.

6.2. Acquisition, measurements and results

For the acquisition process, the flowing soap film setup is used with a constant flow rate of \( Q = 30 \text{ mL/min} \) and a width of \( W = 9 \text{ cm} \). The solution in the reservoir is composed of 2% in volume of commercial dish-washing soap and 98% of tap water at ambient temperature. The estimated film thickness is: \( 2.5 \text{ m} \).

- Free stream velocity measurement: Six independent video recordings of the flowing film without any obstacle are taken. From each of this recording, four samples are extracted and analyzed to find the film velocity. In order to reach this value, small particles (mostly dust and air bubbles, as explain in section 4.1.1) are tracked on the recorded footage. Knowing the frame rate and the spatial scaling on the video recording, the free stream velocity can be extracted.

![Figure 6.2: Free stream velocity measurements, and average free stream velocity](image)
Even if the standard deviation on the measurements of $U$ featured on figure 6.2 is of the order of 0.1 m/s, the 24 measured values of $U$ give a robust average free stream velocity of 2.227 m/s.

- Then, different objects are inserted through the film, and vortex shedding can be observed as shown on figures 6.3 and 6.4.

Figure 6.3: Observation of the vortex Shedding with different obstacles $D=1.5$ $D=3$ and $D=4$ mm

Figure 6.4: Observation of the vortex Shedding with different obstacles $D=5$ $D=6$ and $D=8$ mm
In order to access the vortex shedding frequency corresponding to each cylinder, each recording is analyzed with POD and DMD methods. To guarantee that the vortex shedding velocity and wavelength (hence the frequency) is constant, the analysis is focusing on an area of the recording located at a minimum distance of 10 times the diameter of the object, as advised in [35]:

![Figure 6.5: Vortex shedding wavelength and velocity evolution with distance to the obstacle [35]](image)

For each obstacle, two different two-second recordings shot at 4000Hz are used. From these two recordings, fifty different samples are analyzed.

First the POD (Proper Orthogonal Decomposition, spatially orthogonal) analysis returns the frequency of the dominant orthogonal mode corresponding to the dominant flow structures. This frequency is then used as a target frequency, to run a DMD analysis (Dynamic Mode Decomposition, temporally orthogonal) to find the objective vortex shedding frequency.
Two different sets of object were used, one made of micro drill with diameters from 0.8 mm to 3 mm and the other from divers cylindrical object with diameters between 1.5 and 6.3 mm. To ensure the relevance of the results two different recording sessions were organized. For each session, a new freshly made solution was used. The results are shown on figure 6.7:
The relation between the diameter of the object and the vortex shedding frequency can then be approximated by a rational function as follows:

\[ f = \frac{411}{0.1987 + D \times 10^3} \quad (6.1) \]
Figure 6.8: Rational approximation of the f(D) relation. The experimental frequencies are plotted with the standard deviation coming from the analysis of the 50 samples for each diameter.

- From these experimental frequencies and the rational law, the experimental Strouhal numbers and a St(D) law can be derived:
Figure 6.9: The red curve is obtained by multiplying the rational relation found earlier by \( \frac{D}{U} \). The blue dots are found by taking each discreet frequencies given by the POD DMD analysis and multiplying them by \( \frac{D}{U} \).

- Finally the Reynolds number is computed by inverting Roushan’s formula (equation 4.3) [35]:

\[
Re = \frac{313}{\frac{1}{St} - 5.12} \tag{6.2}
\]
As it can be seen on figure 6.10 the relation between the diameter of the obstacle and the apparent Reynolds number is not linear. The main hypothesis to explain this is that for larger object ($D > 4\text{mm}$) a blockage effect is preventing the velocity profile of the flow from being almost uniform in the test area of the film. Also this could modify the thickness of the film in the test region and therefore alter the local viscosity value.
Figure 6.11: Reynolds numbers, as before both values from the experimental frequencies and the approximated ones are plotted.

The final value of $v = 2.07 \times 10^{-5}$ of the apparent viscosity of the film is found thanks to a linear regression of the Reynolds/diameter curve for Reynolds number between 200 and 500.

It is important to notice how the process of calculating the Strouhal number from the experimental results to finally get to the Reynolds numbers is magnifying the final standard deviations on the Reynolds numbers. Even if the deviation on the experimental frequency (figure 6.8) gets amplified because of the formulas used to compute the Reynolds numbers, the accuracy of the experiment and the fitting quality of the rational approximation of the diameter/frequency relation provide a relevant value of the film apparent viscosity for obstacles with diameter lower than 4mm.
Analytical model and comparison with the experiment

In this chapter, it is proposed to build a model that aims to describe the symmetry breaking phenomenon experimentally observed (see chapter 2). To fulfill this objective, a preexisting rigid and static model [27] is used as a base to construct our model, first by adding flexibility to the static case and then by constructing a dynamic model.

7.1. Preexisting rigid plate static model

The inverted pendulum like instability that is ruling the behavior of a cylinder with a rigid plate fixed at its rear end placed in a flowing fluid was studied by U. Lacis [27]. They experimentally shown evidence of this phenomenon thanks to a flowing soap film channel and developed a theoretical model (figure 7.1) to understand its mechanisms and try to predict its impact.

Figure 7.1: Inverted pendulum like model to describe the fluid structure interaction phenomenon [27]
• a) Representation of the inverted pendulum problem, because of gravity, the symmetrical position becomes an unstable equilibrium.

• b) Representation of the fluid structure interaction problem, the cylinder is free to rotate around its axis. The back flow region is materialized by the grey area. To reach a steady state the plate has to be exposed to the outside of the back flow region.

• c) The forces acting on the structure are of two natures: stabilizing outside the back flow region, tending to bring back the structure to a symmetrical equilibrium, and destabilizing inside the back flow region, breaking the symmetry of the system.

Their model is said to be usable for Re=45-12000, and predicts a bifurcation point corresponding to the symmetry breaking when the ratio \( L/D \) between the plate length and the cylinder diameter reaches its critical value.

![Figure 7.2: Deflection angle as a function of the L/D ratio as predicted by U. Lacis model](image)

When \( L/D \) is lower than the critical value, the destabilizing force acting on the plate inside the back flow region is dominant and set the system into an angled equilibrium (zone 1 on figure 7.2).

Even though they have shown the relevance of this model to describe the rigid plate case, its prediction for the flexible filament case were not as accurate.
7.2. Flexible filament static analysis

The system we are focusing on is composed of a rigid cylinder with a flexible filament attached at its rear-end (figure 7.3). The addition of the impact of the filament bending stiffness is modifying the problem, therefore an updated approach is necessary.

![Figure 7.3: Color photograph of the experimental structure inserted in the soap film, lighted with white light. The fluid is flowing from the top to the bottom](image)

7.2.1. Model geometry definition

The system geometry from figure 7.3 is modeled as shown on figure 7.4. The main difference with the previous model (figure 7.1) is that the filament hinge point has been shifted to its junction point with the cylinder.
The intersection between the soap film plan and the structure is represented by a disc of diameter $D$ and a straight segment of length $L$ (as the curvature of the filament is neglected) forming an angle $\phi$ with the vertical axis. From experimental observations and using the literature [27], the back flow region is modeled as an ellipse of which the center is at a vertical distance $y_0$ of the circle center. Thus, the attachment points of the back flow region are set to form an angle $\theta_0 = 55$ with the vertical axis. The semi-ellipse representing the back flow region can be defined using its two semi-axes $s_1$ and $s_2$, or its polar coordinates $r$ and $\Gamma$. The equilibrium deflection angle is given by the angle between the filament straight vertical position and the segment linking the hinge point to the tip of the filament.

The length $B$ between the filament connection point to the intersection of the filament
and the back flow region boundary is representing the length of filament inside the back flow region. The value of $B$ can be determined by a moment equilibrium problem and is assumed to be known (see section 7.2.3). The unknowns of this geometrical problem are: the deflection angle $\phi$, and the polar coordinates of the ellipse $r$ and $\Gamma$. To get the deflection angle from the value of $B$, we need to express $\phi$ as a function of $B$.

The additional parameters are $y_0 = \frac{D}{2} \cos \theta_0$, $s_1 = \frac{D}{2} \sin \theta_0$ and $s_2 = B_{max} + \frac{D}{2} - y_0$. The value of $B_{max}$ is set to 2.6 according to Lacis’s observations [27].

The geometric definition of the ellipse can be written as:

$$r^2 \sin^2 \Gamma + r^2 \cos^2 \Gamma = 1 \quad (7.1)$$

Using the sine rule we have:

$$\frac{r}{\sin \phi} = \frac{B}{\sin \Gamma}, \quad \sin \Gamma = \frac{B \sin \phi}{r} \quad (7.2)$$

Now the cosine rule is used to get the last equation of the system:

$$r^2 = B^2 + \left( \frac{D}{2} - y_0 \right)^2 + 2B \left( \frac{D}{2} - y_0 \right) \cos \phi \quad (7.3)$$

Let us remove the unknown $\Gamma$ by introducing relation 7.2 into 7.1:

$$\frac{B^2 \sin^2 \phi}{s_1^2} + \frac{r^2 (1 - \sin^2 \Gamma)}{s_2^2} = 1 \quad (7.4)$$

$$r^2 s_1^2 + \sin^2 \phi B^2 (s_2^2 - s_1^2) = s_1^2 s_2^2$$

Thanks to relation 7.3 we can write the last equation as:

$$\left[ B^2 + \left( \frac{D}{2} - y_0 \right)^2 + 2B \left( \frac{D}{2} - y_0 \right) \cos \phi \right] s_1^2 + \sin^2 \phi B^2 (s_2^2 - s_1^2) - s_1^2 s_2^2 = 0 \quad (7.5)$$

$$B^2 (s_2^2 - s_1^2) \cos^2 \phi - 2B \left( \frac{D}{2} - y_0 \right) s_1^2 \cos \phi + s_1^2 s_2^2 - s_2^2 B^2 - s_1^2 \left( \frac{D}{2} - y_0 \right)^2 = 0$$

Hence we have:

$$\phi = \arccos \left[ -\frac{b}{2a} + \frac{\sqrt{b^2 - 4ac}}{2a} \right] \quad (7.6)$$
With
\[
\begin{aligned}
a &= B^2(s_2^2 - s_1^2) \\
b &= -2B\left(\frac{D}{2} - y_0\right)s_1^2 \\
c &= s_1^2s_2^2 - s_2^2B^2 - s_1^2\left(\frac{D}{2} - y_0\right)^2
\end{aligned}
\] (7.7)

Only one of the two potential solutions of the quadratic equation 7.5 is used to find \(\phi\) as the second would return angles close to 180 which is not allowed because of the cylinder presence.

Now that we can relate \(\phi\) to \(B\), let us set up the static problem to get the latter value.

### 7.2.2. Definition of the forces acting on the system

All the forces can be projected on the filament axis \((X_f, Y_f)\) and the filament is supposed to be unable to extend or contract. Moreover, forces along \(X_f\) have no effect on the moment at the hinge point. Therefore all the described forces are projected on \(Y_f\).

![Sketch of the problem with the addition of the spring to model the restoring force. The filament is allowed to rotate in the fluid plan. The fluid is flowing from the top to the bottom along the X axis. Only the half of the cylinder is represented. We introduce a new set of axis \((X_f, Y_f)\) which is found after applying a rotation of \(\phi\) to the fix axis \((X, Y)\).](image-url)
7.2. Flexible filament static analysis

To take into account the filament stiffness, we use simple beam theory \[2\] to introduce a normal restoring load distribution of the type \[ f_r(l) = -K l \sin \phi \] for \( l \) from 0 to \( L \) the length of the filament (triangular load distribution). In this model, \( \phi \) represents the filament deflection angle from the vertical position, and its value is between -90° and 90°. This distributed load can be modeled as a resulting punctual force acting on the filament at:

\[
\frac{\int_0^L K l \sin \phi \, dl}{\int_0^L K l \sin \phi \, dl} = \frac{2}{3} L. \tag{7.8}
\]

The restoring force tends to bring back the filament in the symmetrical position. The more the filament is angled from the position \( \phi = 0 \) the larger the force. This force is modeling the action of the filament stiffness that is fighting the bending. It is independent of the back flow region geometry, but it is linked to \( L \) and \( \phi \), and is referred as \( F_r \):

\[
F_r = -\frac{1}{2} K L \sin(\phi), \tag{7.9}
\]

where \( K \) is the equivalent spring constant \[2\] of the filament acting at \( 2/3 \) \( L \) given by:

\[
K = \frac{6EI}{(2/3 L)^2 (3L - 2/3 L)}, \tag{7.10}
\]

with \( E \) the Young's modulus of the filament and \( I \) the second area moment:

\[
I = \frac{\pi d^4}{64}, \tag{7.11}
\]

where \( d \) is the filament diameter.

For the fluid forces, we use the same approach as Lacis \[27\] and consider a free steady stream acting on the structure. The characteristic flow velocity is assumed to be \( U \) outside the back flow region and \(-\sqrt{\gamma} U \) inside (figure 7.5). The filament is allowed to rotate on the axis perpendicular to the plan of the film, at the hinge point where its meeting the cylinder.

As every object inserted in a flow with an angle, the filament experiences pressure forces. For the part of the filament of length \( B(\phi) \) that is inside the back flow region, this pressure force is destabilizing and is denoted as \( F_n^+ \) on figure 7.6. The magnitude of this force is given by:

\[
F_n^+ = 2 \sin(\phi) \tilde{A} \rho_f h B(\phi) \gamma U^2. \tag{7.12}
\]

Outside the back flow region, on the filament portion of length \( (L - B(\phi)) \), the fluid force is stabilizing and called \( F_n^- \):

\[
F_n^- = -2 \sin(\phi) \tilde{A} \rho_f h (L - B(\phi)) U^2. \tag{7.13}
\]
In these equations, \( h \) is the film thickness, \( \rho_f \) is the fluid density and \( \tilde{A} \) is a force calibration factor.

It is also assumed that these three forces are the only one involved in the behavior of the system, and therefore the weight of the filament is neglected. Figure 7.6 shows the system with all the previously introduced forces acting:

![Figure 7.6: Schematic of the forces acting on the system](image)

**7.2.3. Determination of the equilibrium position**

Finding the equilibrium angle \( \phi_s \) is equivalent to find the value of \( B \) that makes the total torque at the hinge point of the filament null. Let us write the moment equilibrium at the hinge point:

\[
T = F_n^+ \left( \frac{B(\phi)}{2} \right) + F_n \left( \frac{B(\phi)}{2} + \frac{L}{2} \right) + F_r \left( \frac{2L}{3} \right)
\]

(7.14)

Which can be rewritten as
\[ T = \sin(\phi) \left[ (1 + \gamma)B^2 - \left(1 + \frac{K}{3}\right) \right. \]
\[ \frac{\tilde{A} \rho_f U^2 h L^2}{\tilde{A} \rho_f U^2 h} = \sin(\phi) \left[ (1 + \gamma)\hat{B}^2 - \left(1 + \frac{K}{3}\right) \right. \]
\[ \frac{\tilde{A} \rho_f U^2 h \hat{L}^2}{\tilde{A} \rho_f U^2 h D^2} \]

introducing the dimensionless parameters \( \hat{B} \) and \( \hat{L} \).

There are two possibilities for the equilibrium to occur. If \( \phi = 0 \) the equilibrium is reached which is the symmetrical case found to be unstable under particular circumstances. If \( \phi = \phi_s \neq 0 \), the term between square brackets in equation 7.15 must be null:

\[ (1 + \gamma)\hat{B}^2 - \left(1 + \frac{K}{3\tilde{A} \rho_f U^2 h}\right)\hat{L}^2 = 0, \]  

(7.16)

which leads to

\[ \hat{B}(\phi_s) = \sqrt{\frac{1 + \frac{K}{3\tilde{A} \rho_f U^2 h}}{1 + \gamma} \hat{L}} \]  

(7.17)

Thanks to equation 7.6, it is possible to get the value of \( \phi_s \) corresponding to the \( \hat{B} \) that cancels the moment at the hinge point.

### 7.2.4. Model predictions and stability analysis

Now that we can compute \( B \) thanks to equation 7.17, and that we know how to geometrically find \( \phi_s \) as a function of \( B \) (relation 7.6), we are able to model the static equilibrium positions of the system for different \( L/D \).
7.2. Flexible filament static analysis

The models parameters have been set as follows: $\sqrt{\frac{V}{v}} = 0.9$ which represent the ratio between the norm of the velocity inside and outside the back flow region, and $\hat{A} = AD/d$ with $A = 0.094$, in accordance with Lacis's advice [27]. To scale the forces in the back flow region, this factor is directly linked to the drag force coefficient of the cylinder of diameter $D$ in the flowing film.

As previously observed with the rigid model, after a critical value of $\frac{L}{D} = 3.6$ the position $\phi = 0$ is stable and is the only equilibrium allowed (zone 3 on figure 7.7). Indeed, as the length $L$ of the filament is increasing, the portion outside the back flow region is generating a larger stabilizing force. To balance it, the filament portion of length $B$ has to increase and therefore the angle $\phi$ has to decrease. However, $B$ cannot go higher than the value $B_{max}$ which corresponds to $\phi = 0$.

Before this limit, for the filament to find a stable equilibrium position, it must drift to an angle $\phi \neq 0$, on one side or the other (zone 2). As a matter of fact, for a fixed $L/D < 3.6$ the only way to compensate for the destabilizing force $F^+_n$ is to increase $\phi$ in order to make the portion of filament outside the back flow region grow, hence reducing $F^+_n$ and rising $F^-_n$. 

Figure 7.7: Angular equilibrium position predicted by the flexible model. The figure is plotted for $E = 2 \times 10^5 Pa$
We also see that there is another critical value of $L/D = 0.65$ for which the filament goes from his most angled position to the straight position equilibrium defining a new set of lengths for which the symmetry of the system is respected (zone 1). This is caused by the action of the restoring force $F_r$ due to the stiffness of the filament. Naturally, because of its dependency in $d^4/L^3$, the equivalent spring constant will grow when $L$ become small, tending towards infinity when $L$ approaches 0.

![Figure 7.8: Torque evolution with $L/D$](image)

As it can be observed on figure 7.8, when $\sin \phi = 0$ all the moments are null (equation 7.15), but at $L/D = 0.65$ the change in position is inducing a high destabilizing fluid moment. Indeed, as the filament is short, the portion outside the back flow region does not provide much of a restoring moment, however the stiffness of the filament is balancing the destabilizing moment and allows it to reach an equilibrium position. As this restoring moment is becoming weaker with larger values of $L/D$ the angles $\phi$ has to decrease as discussed earlier.

To understand why the change in angular position takes this particular shape at low $L/D$, it is interesting to study the stability of the position $\phi = 0$. By taking the derivative with respect to $\phi$ of the total torque equation 7.15 at $\phi = 0$ we get:

$$\left. \frac{dT}{d\phi} \right|_{\phi=0} = (1 + \gamma) \hat{B}(0)^2 - \left(1 + \frac{K}{3\rho_f U^2 D^2 h}\right) \hat{L}^2$$

(7.18)
which leads to the following stability condition for the position $\phi = 0$ to be stable:

$$
(1 + \gamma) \hat{B}(0)^2 - \left(1 + \frac{K}{3 \hat{A} \hat{D}^2 U^2 h} \right) \hat{L}^2 < 0
$$

(7.19)

The graphic representation of relation 7.18 as a function of $L/D$ is presented on figure 7.9, and shows there are two values of $L/D$ that cancel the relation:

![Graph showing stability analysis](image)

**Figure 7.9**: Stability analysis around the position $\phi = 0$ for different values of $L/D$

- In the region where $L/D$ is larger than 3.6, the position $\phi = 0$ is stable due to the stabilizing fluid forces acting on the part of the filament outside the back flow region.

- Between 0.65 and 3.6, the symmetrical position in unstable and any disturbance from this equilibrium position will make the filament drift toward its $\phi_s$ configuration.

- Before 0.65, the position $\phi = 0$ becomes stable again because of the stiffness of the filament. For these lengths, if the filament is getting pushed from this position, the restoring force will bring it back to the symmetrical case. Right after the change in the sign of relation 7.18 any small disturbance will lead the filament to leave its vertical position.
7.2. Flexible filament static analysis

7.2.5. Comparison with experimental data and the rigid model

We performed experiments using different lengths of silk filament using a soap film channel of width 9cm. The fluid density is assumed to be equal to the density of water. The experimental parameters are the following:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value66636</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>d</td>
<td>0.08</td>
<td>mm</td>
</tr>
<tr>
<td>D</td>
<td>3.2</td>
<td>mm</td>
</tr>
<tr>
<td>h</td>
<td>2.5</td>
<td>( \mu m )</td>
</tr>
<tr>
<td>E</td>
<td>2</td>
<td>GPa</td>
</tr>
<tr>
<td>U</td>
<td>2.2</td>
<td>m.s(^{-1})</td>
</tr>
<tr>
<td>( \rho_f )</td>
<td>1000</td>
<td>Kg.m(^{-3})</td>
</tr>
</tbody>
</table>

Table 7.1: Experimental parameters

As predicted by the flexible model we observe three different behaviors: the symmetrical equilibrium at low \( L/D \), the symmetry breaking for filament lengths between the critical values, and the straight position for higher \( L/D \).

Figure 7.10: Equilibrium position for \( L/D \) equal to 0.6, 1.1, 3.6

Thanks to the filament tracking algorithm (see section 5.2) we are able to extract the values of the time average angle of the filament and to superimpose those values with the both the rigid and flexible model as shown on figure 7.11:
7.2. Flexible filament static analysis

The first observation that can be made is that except for the configurations where the stiffness of the filament is dominant (for $L/D < 0.65$), the rigid plate model [27] gives a good prediction of the system behavior. At $L/D < 0.65$, the flexible model predicts the symmetry breaking accurately. However the peak seems to happen earlier than predicted, and the discontinuous change from the symmetrical regime to the oriented one at $L/D < 0.65$ appears to be gradual in the experimental recordings. It is important to know that because of its implementation, the filament tracking algorithm has a precision of about $3^\circ$. This precision also varies with the length of the filament, indeed as length of the filament is decreasing, the relative size of a pixel is increasing, hence some precision is lost.

Nevertheless, the experimental data fits the flexible model predictions adequately for most values of $L/D$ and verifies the impact of the stiffness of the filament on the equilibrium positions.

Figure 7.11: Comparison between experimental observation and model prediction of the equilibrium angle $\phi$ for different values of $L/D$. The rigid model representation is obtain by taking $K = 0$ to cancel the restoring moment.
When comparing the stability condition from the two models (figure 7.12) we can see that there is only one change of sign in the rigid model, thus only one bifurcation.

![Graph showing stability condition with rigid and flexible models](image)

**Figure 7.12:** Comparison of the stability around the position $\phi = 0$ for the rigid and flexible models

The stability condition plot once again shows that the effect of the bending stiffness of the filament is extremely limited to small lengths (0 to $3 \times D$). Independently of the configuration of the system, the moment caused by the filament stiffness will always have a restoring action, which is stabilizing, hence for a given length $L/D$ the stability condition value will always be lower for the flexible case when compared to the rigid configuration.

### 7.3. Flexible filament dynamic analysis

The static model gives us significant information about the symmetry breaking mechanism and the shift in the stability of the $\phi = 0$ equilibrium position.

The next step to capture more details about the system behavior is to study its dynamic, which allows to observe the transient state.
7.3.1. Dynamic model

Starting from Lacis’s model, a new velocity profile inside the back flow region is introduced to make it a bit more realistic. Indeed, the previous hypothesis was that inside the bubble, the value of the characteristic velocity was $\sqrt{\gamma}U$, independently of the position of the filament. We propose to enhance this approach by making $\gamma$ dependent on the location inside the back flow region. It is chosen to consider this change of $\gamma$ along the $X$ axis, the simplest way to do this is to notice that the velocity needs to be null at the contact point between the bubble and the cylinder (the hinge point), and at the border of the bubble (at $B_{\text{max}}$). $\gamma$ is then described by a parabolic law, and reaches a maximum of 1 at $B_{\text{max}}/2$. As the destabilizing force acts at a point located at $B/2$ on the filament the value of gamma will be $\gamma(B/2)$ and the characteristic velocity $\sqrt{\gamma(B/2)U}$.

This could not be done previously with a static (non iterative) model, as the value of $B$ has to be known to compute the characteristic velocity in the back flow region.

![Figure 7.13: Evolution along the vertical axis of the coefficient that shapes the fluid velocity profile inside the back flow region, $B_{\text{max}}$ being equal to 2.6D.](image)

Using the exact same representation in terms of geometry as in the static model, which is shown on figure 7.4, we can write $B$ - the length of the filament portion inside the back flow region - as a function of $\cos \phi$: 
7.3. Flexible filament dynamic analysis

\[ B = -\frac{b}{2a} + \frac{\sqrt{b^2 - 4ac}}{2a} \]  
(7.20)

With

\[
\begin{align*}
    a &= (s_2^2 - s_1^2) \cos^2 \phi - s_2^2 \\
    b &= (2y_0 - D)s_1^2 \cos \phi \\
    c &= (\frac{D}{2} - y_0)^2 s_1^2 - s_1^2 s_2^2
\end{align*}
\]  
(7.21)

Let us now focus on the total torque about the pivot point, where the filament is linked to the cylinder. The exact same forces are considered, acting at the exact same position as in the static model (see section 7.2.3).

However, it is now needed to take into account the velocity of the filament as it is not static anymore. Hence, the filament is experiencing a velocity that is the addition of the fluid velocity \( U_{fl} \) (which is equal to \( U \) outside the back flow region and \( -\sqrt{\gamma}U \) inside, see figure 7.14) and the opposite of its own velocity. Let us find the upstream apparent velocity felt by the filament:

![Figure 7.14: Axis definition and fluid velocity](image)

- On \((X, Y)\) the fluid velocity is \((U_{fl}, 0)\)
• On the filament, the velocity of a point at a length $l$ from the hinge point on $(X_f, Y_f)$ is $(0, l \dot{\phi})$, which gives $(-l \dot{\phi} \sin(\phi), l \dot{\phi} \cos(\phi))$ on $(X, Y)$, where $\dot{\phi}$ stands for derivative with respect to time $\frac{d\phi}{dt}$.

• Thus, the total velocity $U_f$ seen by the filament point is $(U_f + l \dot{\phi} \sin(\phi), -l \dot{\phi} \cos(\phi))$ on $(X, Y)$. In practice the first component of the total velocity vector is about two orders of magnitude higher than the second, hence only the $X$ total velocity component will be considered.

• Then, the fluid forces take the same shape as in section 7.2.3: $F = 2 \tilde{A} \rho_f U_f^2 h l$ for the point at a distance $l$ from the origin. After projecting these forces on the $Y_f$ axis, we get:

$$
F^+_n = 2 \tilde{A} \rho_f h B \left( -\sqrt{U} + \frac{B}{2} \dot{\phi} \sin(\phi) \right)^2 \sin(\phi)
$$

$$
F^-_n = -2 \tilde{A} \rho_f h (L-B) \left( U + \frac{L+B}{2} \dot{\phi} \sin(\phi) \right)^2 \sin(\phi)
$$

Moreover, a drag force needs to be introduced as the result of the filament moving in the fluid. This force, projected on $Y_f$ is given by:

$$
F_d = -2 \tilde{A} \rho_f h L \left( \frac{L}{2} \dot{\phi} \right)^2,
$$

and is found by calculating the fluid force acting on the filament when the latter is in motion in a static fluid. As for the restoring force $F_r$, the distributed load is represented as a concentrated load acting at $2L/3$. Obviously its orientation (thus its sign when projected) is dependent on the sign of $\dot{\phi}$ as it going against the filament motion.

When $\dot{\phi} = 0$, the static forces as describe in section 7.2.3 are retrieved. Obviously, the restoring force $F_r$ linked to the stiffness of the filament is not changed, we can now write the equation of motion of the filament as follows:

$$
I_0 \ddot{\phi} = F_n^+ \left[ \frac{B}{2} \right] + F_n^- \left[ \frac{B}{2} + \frac{L}{2} \right] + F_r \left[ \frac{2L}{3} \right] + F_d \left[ \frac{2L}{3} \right],
$$

where $\ddot{\phi}$ stands for the second derivative with respect to time $\frac{d^2 \phi}{dt^2}$.

We introduce $I_0$ the moment of inertia of the filament about its tip as:

$$
I_0 = \frac{1}{3} ML^3,
$$

with $M$ the linear density of the silk filament. An actual measurement of some properties (including the linear density) of a similar silk filament was performed by Jia [21], giving a value of $M = 1.3 \times 10^{-5} \text{kg/m}$ which is the value we use in this model.
To solve this ordinary differential equation, we use the ode45 function from Matlab [4]. The time spans from 0 to 10 seconds with steps of 0.0001 seconds. The relative tolerance is set to $10^{-12}$ in the solver options to get accurate results. The initial point is set to $(\phi, \dot{\phi}) = (10^{-3}, 0)$.

First, we can have a look at the dynamic model prediction for $t$ large enough that the transitional state effect can be neglected. By plotting the time average value of $\phi$, we can compare the dynamic and static models, as well as the experimental data.

![Graph showing dynamic and static model results](image)

Figure 7.15: Time average position of the filament as predicted by the dynamic model, compared with the static model results

Figure 7.15 shows that the dynamic model results converge toward values slightly different than the static predictions. The most noticeable difference with the static model is the continuity of the first transition at $L/D = 0.65$ due to the introduction of the updated velocity profile inside the back flow region. Its parabolic shape also lightly alters the values of the equilibrium angle $\phi_s$ around $0.65 < L/D < 1.5$ and $2.5 < L/D < 3.8$, indeed if the dynamic model is run with $\sqrt{f} = 0.9$ everywhere in the bubble as it was the case in the static analysis, the two curves are overlapping.
As the dynamic results are close to the static ones, the same general conclusion stays valid when comparing the model results to the experimental data. Nevertheless, the introduction of the parabolic velocity profile eliminates the discontinuity at $L/D = 0.65$ and provides predictions which are closer to the experimental results, which can be clearly seen for experimental point for $0.5 < L/D < 0.9$ despite some data scattering after 0.9. This makes the dynamic model more relevant for our work, even when only the static equilibrium is to be studied.

### 7.3.2. Dynamic results

Now that we used the static result to verify that the dynamic model was giving coherent results, we can interpret the dynamic results. To do this we first focus on the angular position $\phi(t)$ and velocity $\dot{\phi}(t)$ given when solving the ordinary differential equation 7.24. In the following figures the angular position of the filament with time and the phase portrait $(\phi, \dot{\phi})$ of the system are plotted for three different $L/D$ ratios: 0.6, 1.1, 4, which are characteristic of the three different experimental observations.
• $L/D = 0.6$ the symmetric position $\phi = 0$ is stable

In this configuration the symmetrical equilibrium $\phi = 0$ is stable and the filament is oscillating around this position. As the perturbation is small and the equilibrium sta-
7.3. Flexible filament dynamic analysis

...ble, the angular velocity remains under $1/s$, leading to really low damping. The transitional state is very long compared to the experimental recording time, and the system is oscillating around its equilibrium position with a very small amplitude, which could not be seen on the video footage.

Considering the above mentioned result, let us try with a larger perturbation. The length of the filament is kept at $L/D = 0.6$ but now the initial condition are set further from the equilibrium $\phi = 0$, in this case $(\phi, \dot{\phi}) = (3, 0)$. The damping action is stronger:

![Figure 7.18: Evolution of the angular position and phase portraits for $L/D$ equal 0.6 given by the dynamic model for $\dot{\phi}(0) = 3$](image)
The system is behaving like a standard damped oscillator, and is converging toward the symmetrical equilibrium position. After 4 seconds, the oscillations amplitude is $10^{-2}$ which is not noticeable on the video recordings.
• $L/D = 1.1$ the symmetric position $\phi = (0)$ is unstable and there are two potential angled stable equilibrium, one on each side

![Image](image_url)

**Figure 7.19:** Experimental snapshot, evolution of the angular position and phase portraits for $L/D$ equal 1.1 given by the dynamic model for $\dot{\phi}(0) = 1 \times 10^{-3}$

Given that the length of the filament is equals to $1.1 \times D$ the system is in the symme-
try breaking region and the filament is expected to take an angled average position. The model describes this phenomenon and unlike the previous case even for a small perturbation the transitional state is clear, as the filament is oscillating around its equilibrium position for the first few periods. During the first period, where the filament is going away from the unstable $\phi = 0$ position to get to its equilibrium at a $25.1$, the maximum angular velocity of $1.7 \times 10^4 / s$ is reached. This provides a significant damping effect caused by both the change in the characteristic fluid velocity seen by the filament and the drag force generated by its movements, which explains the short transitional state. When the steady state is reached (after 0.04 seconds according to the model) the stable equilibrium position is reached. The model predicts a static equilibrium, whereas during the experiments oscillations were observed.
7.3. Flexible filament dynamic analysis

- \( L/D = 4 \) the symmetric position \( \phi = 0 \) is stable

At \( L/D = 4 \) the system is just after the second bifurcation point and the position \( \phi = 0 \) is now stable again. For the same reasons as in the \( L/D = 0.6 \) case, the transition
state is very long. However, with a larger portion of the filament outside the back flow region, undergoing stabilizing fluid forces, the convergence toward $\phi = 0$ seems to be slightly faster. In practice, the cases with long length of filament (after $L/D = 4$ are prone to flapping (see figure 7.21), sometime with relatively large amplitude of the order of 10. It shows a first limitation to our model as this effect is not predicted.

Figure 7.21: The filament is flapping around its stationary equilibrium position $\phi = 0$, here for $L/D = 8.35$

As for the first case, let us see the impact of a larger perturbation by setting the initial condition to $\phi = 3$: 
As it could be expected the result is close to what was found for $L/D = 0.6$ as the filament is oscillating around $\phi = 0$, losing energy on each oscillation. Once again the experimental results diverge from the dynamic model outputs as flapping was observed.
7.4. Frequency analysis

The results of the dynamic calculations allow us to extract the position of the filament at each time step. Therefore a simple fast Fourier transform (FFT) can give us the frequency of the filament oscillations as predicted by the model. By doing this for different length of filament, we obtain figure 7.23:

As before, three zones are clearly visible and delimited by the same value of \( L/D = 0.65 \) and 3.6.

Thanks to the filament tracking code (see section 5.2) we can apply the same procedure to the experimental footage. First the position of the filament tip on each frame is extracted:
When the time evolution of the filament position is known, a FTT algorithm gives us the frequency spectrum of the filament displacements:
7.4. Frequency analysis

The POD analysis (section 5.1.1) can also be used to confirm the FFT results.

Figure 7.25: Frequency spectrum of the filament trajectory for $L/D = 3.09$

We can observe that the experimental frequency peaks given by this method are not partic-
ularly clear, especially in the low frequencies. In some cases as in figure 7.25 the first peak value can still be extracted but the secondary peaks are more problematic. Several explanations can be given, from the recording and lighting conditions, to the noise introduced by the filament tracking code.

For larger $L/D$, when the symmetric position is stable, the peaks are clearer as it can be seen of figure 7.27:

![Graph showing frequency spectrum of the filament trajectory for $L/D = 6.7$](image-url)

Figure 7.27: Frequency spectrum of the filament trajectory for $L/D = 6.7$
This can be due to the fact that the filament is flapping with a larger amplitude as it gets longer, hence the oscillations are clearer on the video recording.

We can compare these experimental frequencies to theoretical ones. When plotting only the principal peak for each experimental recordings we obtain figure 7.29:
As observed on the experimental results, the dominant experimental frequencies seem to be low frequencies which are not predicted by the model, especially in the range where $L/D$ is between 0 and 3.

In order to extract more information from these experimental results, the three principal peaks of each sample are taken into account. By doing this, we can highlight modes that were not immediately visible in the spectrum because of the low frequencies dominance:
Figure 7.30 shows that even if the results are closer to the model prediction, a significant difference remains. This highlight the fact that our model is performing well for the static case as it predicts the angled position of the filament with precision, but is not as prefer-ment in the dynamic analysis. This is due to the lack of dynamic in the fluid part, as one of the main hypothesis is that the velocity field in the fluid is constant which is obviously not realistic. As a matter of fact, in this interaction, only the action of the fluid on the filament is taken into account, whereas in reality the filament movements also have an effect on the fluid. Moreover the back flow region geometry is dynamic as well as vortices are shed. This last point explains the lack of oscillations and flapping in the steady state for the model results.
Conclusion and recommendations

In this final chapter, the conclusions about the results of the reported work are gathered, as well as recommendations and pieces of advice which should help any member of the scientific community to build and use a soap film channel to recreate this experiment or explore new configurations.

8.1. Conclusion

The first conclusion to be drawn is that the soap film channel is indeed a truly practical, relatively easy to build, and inexpensive way to experimentally create quasi two-dimensional flows in a research environment.

As a matter of fact, the available literature offers insightful descriptions on how to build it, especially Rutgers's paper [37] which gives a good overview and tackles the different aspects of this technique (building the channel, making the solution, choosing a visualization method). Hopefully, some points that were harder to find in the scientific papers such as the film apparent viscosity determination have been covered in this report. A complete protocol to evaluate the apparent viscosity of the soap film was created and documented in chapter 6).

Using the soap film channel on our test case which includes a structure made of a rigid cylinder with a flexible silk filament attached to its bottom end, we successfully highlighted the effects of the static instability that makes the filament take an angled average position because of the unbalance between the fluid forces inside and outside the back flow region for certain lengths. Thanks to the appropriate post-processing tools described in chapter 5, a map of the experimental equilibrium positions and the corresponding filament lengths was created to emphasize the static instability effects on the structure.

The theoretical model we built and used in chapter 7 efficiently introduced the flexibility of the filament and a new description of the velocity in the back flow region compared to preexisting models [27]. We saw that when the filament gets short enough, its diameter $d$ is not negligible against its length $L$ anymore, the restoring moment caused by its stiffness becomes comparable to the fluid moment. Furthermore, as the filament gets shorter and closer to the cylinder, the fluid velocity in the back flow region is decreasing, hence the destabilizing fluid force magnitude is diminished. The combination of these two effects
makes the restoring action compensate the fluid moment and restore symmetry to the system. This leads to the apparition of a new stable theoretical symmetrical equilibrium zone for short filaments which was experimentally observed using the soap film channel. Nevertheless, the limitations of this model were shown when the dynamic frequency predictions did not match the experimental data.

The soap film channel along with the lighting and recording setup coupled to the post processing tools were well suited for the study of the symmetry breaking instability, and provided consistent experimental results that ultimately adequately fitted our simple theoretical model. We highlighted the importance of the ratio between the length of the filament and the cylinder diameter in the magnitude of the deviation, as the cylinder geometry directly affects the back flow region shape which is responsible for the instability. This suggests that the soap film channel would be extremely pertinent in order to experimentally study the control of such instabilities through shape-optimization of the cylinder in future research.

8.2. Recommendations for future research

Building and using the soap film channel is relatively simple and straightforward, but to get the most pertinent results it is important to pay attention to fine details. Following are some pieces of advice that could serve future researchers using this type of apparatus.

- One of the main difficulties was to find the solution that would give the most durable film. The soap film is supposed to be able to flow for several minutes, even hours according to some sources in the literature [9] whereas our soap film average lifespan was of one to two minutes. This could be simply due to the soap we were using, as we observed significant differences in the film quality from one soap to another (see Appendix A). The task of choosing the right component for the solution is made harder by the fact that very little information is given in the literature except the generic terms "commercial dish washing soap" and that manufacturers are not bounded to write the composition on their product. Nevertheless, we noticed that the less additive (scent, coloring agent) in the soap the better. Even if the life span of our film was of the order of the minute, it gave us sufficient time to gather quality recordings. Moreover, we achieved to build a soap film with a very homogeneous thickness, as it can be seen by the uniformity of the colorful background on the film on figure 1.1 (the larger the fringes the smoother the film).

- It was also noticed that the film quality was decreasing with time, probably due to dust being captured by the flowing film as it is accumulating in the solution. Also, the temperature of the solution seems to impact the lifetime of the film as it was found to be particularly hard to maintain a soap film during warm summer days. For these reasons we recommend renewing the solution as often as needed to maintain the reproducibility and consistency of the experiments, and to place the apparatus in a location where dust and air currents are minimal.
8.2. Recommendations for future research

- Observation of the fluid pattern linked to the pressure and velocity fields were not of prime order for the study of the filament deviation. However, it was absolutely necessary for the film apparent viscosity estimation based on the vortex shedding observation. The soap film optical properties allow researcher to easily observe those pattern thanks to optical interference. Spectacular and colorful images can be observed under white light but for the study of vortex shedding monochromatic light which gives contrasted images with constructive and destructive interference effects are more suitable. We used green LEDs reflected on a white screen, which gave satisfying results. Nevertheless, many researchers [37] [27] [16] [5] [26] seem to use yellow light (low pressure sodium light), to this point it is undetermined if it would have given us more contrasted or clearer images than the green LEDs.

The post-processing codes and the theoretical model were strongly oriented toward the study of the particular problem of the symmetry breaking instability. To further explore this kind of phenomenon, we identified the following recommendations.

- The characterization and determination of the film properties such as its viscosity can be notably difficult, and the method we adopted in chapter 6 gives a practical solution to this issue. However, it is strongly dependent on the chosen Re-St relation. In our case, the theoretical models are not directly Reynolds dependent and only an approximation of the viscosity was needed but in other cases where the Reynolds number is a critical parameter it would be relevant to further investigate this apparent viscosity value - by comparing experimental results to DNS simulation for instance - to lower the uncertainty.

- The theoretical model we introduced generates static results which are describing the experimental observations with a satisfying accuracy. The dynamic model verified the static predictions, however the lack of dynamic description of the fluid part can explain the differences between the experimental filament oscillation frequencies and the model results. Taking into account the dynamic of the fluid part and the change in the back flow region geometry with time would represent a significant enhancement of the theoretical model and has great chances of making the dynamic results more relevant.
**Nomenclature**

**Experimental conditions**

- \( E \) Filament Young's modulus.
- \( I \) Filament moment of area.
- \( K \) Filament equivalent spring constant.
- \( Q \) Soap film channel flow rate.
- \( U \) Free stream velocity.
- \( W \) Soap film channel width.
- \( \nu \) Soap film dynamic viscosity.
- \( \rho_f \) Fluid density.
- \( h \) Soap film thickness.

**Geometric parameters**

- \( B \) Length of the portion of the filament inside the back flow region.
- \( D \) Cylinder diameter.
- \( L \) Filament length.
- \( \Gamma \) Ellipse radial second coordinate.
- \( \phi \) Deflection angle of the filament.
- \( \theta_0 \) Back flow bubble attachment point angle.
- \( d \) Filament diameter.
- \( r \) Ellipse radial first coordinate.
- \( s_1 \) Ellipse semi-minor axe.
- \( s_2 \) Ellipse semi-major axe.
- \( y_0 \) Distance between the center of the cylinder and the center of the ellipse.

**Theoretical model**

- \( F_n^+ \) Destabilizing fluid force.
- \( F_n^- \) Stabilizing fluid force.
- \( F_d \) Filament drag force.
- \( F_r \) Restoring force.
- \( T \) Total torque at the filament hinge point.
- \( \tilde{A} \) Force scaling factor.
- \( \gamma \) Velocity scaling factor.
- \( \phi_s \) Equilibrium deflection angle of the filament.
- \( f_r \) Distributed restoring load.
Bibliography


[34] Anatol Roshko. On the development of turbulent wakes from vortex streets. 1191, 01 1954.


Soap comparison

In the aim to make the use of a soap film channel easier for future researchers, this appendix presents the main conclusions of a commercial dish-washing comparative study carried out at an early stage of the project. Three different soaps are tested, using 0.99% and 1.96% soap to water solutions. In all the cases a 9cm wide film is stretched, and two different flow rates are used 100mL/mn and 30mL/mn.

The following protocol was used:

1. 20mL of soap is mixed in 2L of tap water (corresponding to 0.99% soap to water)
2. The solution is transferred to the upper reservoir
3. The valve is opened on the "high debit" position (100mL/mn)
4. Five independent measurement are performed
   • The film is stretched and the clock is started
   • The clock is stopped when the film breaks
5. The valve is opened on the "low debit" position (30mL/mn)
6. Five independent measurement are performed
7. Both upper and lower reservoirs are cleaned and dried
8. 40mL of soap is mixed in 2L of tap water (corresponding to 1.96% soap to water)
9. Steps 2 to 7 are repeated.
10. Some soap is saved and left at rest for 20 hours in an open container. Solutions are made with this soap and tested by repeating the whole protocol
A.0.1. Results

Three different kinds of soap were assessed, all found in the french supermarket "Monoprix". For each of them the average lifetime of the film is reported in the following tables:

- Monoprix Orange (orange scent and coloring agent)

<table>
<thead>
<tr>
<th>Monoprix Orange</th>
<th>High debit</th>
<th>Low debit</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.99%</td>
<td>0s</td>
<td>0s</td>
</tr>
<tr>
<td>1.96%</td>
<td>0s</td>
<td>0s</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Monoprix Orange After 20h</th>
<th>High debit</th>
<th>Low debit</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.99%</td>
<td>21s</td>
<td>0s</td>
</tr>
<tr>
<td>1.96%</td>
<td>6s</td>
<td>0s</td>
</tr>
</tbody>
</table>

- Arbre Vert Citron (lemon scent, no coloring agent)

<table>
<thead>
<tr>
<th>Arbre Vert Citron</th>
<th>High debit</th>
<th>Low debit</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.99%</td>
<td>46s</td>
<td>39s</td>
</tr>
<tr>
<td>1.96%</td>
<td>1mn31s</td>
<td>42s</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Arbre Vert Citron After 20h</th>
<th>High debit</th>
<th>Low debit</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.99%</td>
<td>1mn02s</td>
<td>41s</td>
</tr>
<tr>
<td>1.96%</td>
<td>3mn21s</td>
<td>57s</td>
</tr>
</tbody>
</table>
- **Arbre Vert Peaux Sensibles** (Sensitiv skin, no scent, no coloring agent)

<table>
<thead>
<tr>
<th>Arbre Vert Citron</th>
<th>High debit</th>
<th>Low debit</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.99%</td>
<td>3mn42s</td>
<td>43s</td>
</tr>
<tr>
<td>1.96%</td>
<td>5mn02s</td>
<td>2mn17s</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Arbre Vert Citron After 20h</th>
<th>High debit</th>
<th>Low debit</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.99%</td>
<td>2mn53s</td>
<td>1mn04s</td>
</tr>
<tr>
<td>1.96%</td>
<td>4mn41s</td>
<td>3mn12s</td>
</tr>
</tbody>
</table>

Without a doubt, considering the experiment conditions, the best option is the "Arbre Vert Peaux Sensibles" soap, which we used for all the solutions made for this work. It was found that the film was stronger when made with soap that had left to rest in contact with ambient air for at least 20 hours.