coastal sediment transport

computation of longshore transport

report on investigation

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1 Flow above a rippled sand bed
LIST OF SYMBOLS

The most important symbols which have been used in the text are listed below. Those variables which have only been used locally have not been included. All symbols are also, in addition to this list, defined when they are used in the text for the first time.

- $a_o$ amplitude of orbital excursion at the bed $L$
- $\dot{a}_o$ product of maximum orbital velocity at bed under wave crest and the time of landward-directed orbital velocity in Van Hijum's wave theory $[28]$ $L$
- $A$ dimensionless parameter in White-Ackers approach $[2]$ $-$
- $b_1$ coefficient in Bhattacharya's concentration profile $[5]$ $-$
- $c$ concentration of suspended sediment $-$
- $C$ dimensionless parameter in White-Ackers approach $[2]$ $-$
- $C_h$ Chézy-roughness coefficient $L^{3/4} T^{-1}$
- $C_p$ dimensionless corrector for particle size in ripple height determination $-$
- $d$ actual water depth (including wave set-up) $L$
- $D_{gr}$ dimensionless grain diameter in White-Ackers approach $[2]$ $-$
- $D_1$ particle diameter of that fraction of the bed material which is exceeded in size by $(100-i)\%$ in weight of the total sample $L$
- $f_w$ Jonsson's wave friction factor $-$
- $g$ gravitational acceleration $L T^{-2}$
- $H$ wave height $L$
- $m$ dimensionless parameter in White-Ackers approach $[2]$ $-$
- $n$ dimensionless parameter in White-Ackers approach $[2]$ $-$
- $p$ porosity $-$
- $P_J$ ratio between the orbital velocity at $z = z'$ and that at the bed, as given by the first order wave theory, after Jonsson $-$
- $r$ bed roughness, given by equation (3.19) $L$
- $Re$ particle Reynolds number $-$
- $S_b(1 - p)$ volume of grains transported in the bed layer/unit width of flow $L^2 T^{-1}$
- $S_t(1 - p)$ volume of grains transported over the total depth/unit width of flow $L^2 T^{-1}$
- $T$ wave period $T^{-1}$
**LIST OF SYMBOLS (continued)**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tr>
<td>$u_o$</td>
<td>orbital velocity at the bed</td>
<td>L T$^{-1}$</td>
</tr>
<tr>
<td>$v$</td>
<td>longshore current velocity</td>
<td>L T$^{-1}$</td>
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<tr>
<td>$v_x$</td>
<td>shear stress velocity</td>
<td>L T$^{-1}$</td>
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<tr>
<td>$w$</td>
<td>particle fall velocity</td>
<td>L T$^{-1}$</td>
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<tr>
<td>$z$</td>
<td>vertical ordinate</td>
<td>L</td>
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<tr>
<td>$\Delta_a$</td>
<td>ripple amplitude</td>
<td>L</td>
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<tr>
<td>$\Delta_r$</td>
<td>ripple height</td>
<td>L</td>
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<tr>
<td>$\Delta_s$</td>
<td>relative apparent density of bed material</td>
<td>-</td>
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<tr>
<td>$\varepsilon$</td>
<td>diffusion coefficient</td>
<td>L$^2$ T$^{-1}$</td>
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<tr>
<td>$\kappa$</td>
<td>von Karman's constant</td>
<td>-</td>
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<tr>
<td>$\lambda$</td>
<td>wave length</td>
<td>L</td>
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<tr>
<td>$\lambda_r$</td>
<td>ripple length</td>
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<tr>
<td>$\tau$</td>
<td>shear stress</td>
<td>ML$^{-1}$ T$^{-2}$</td>
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**Subscripts**
- $c$ refers to current conditions
- $w$ refers to wave conditions
- $wc$ refers to combined current and wave conditions
COMPUTATION OF LONGSHORE TRANSPORT

1 Introduction

1.1 General

As part of the Applied Research Programme of the Department of Public Works (Toegepast Onderzoek Waterstaat) the Coastal Sediment Transport Working Group (Werkgroep Kusttransporten) performed a study into the computation of sediment transport under combined current and wave action. The basic work for this study was done and this report was written by dr.ir. D.H. Swart of the Delft Hydraulics Laboratory.

1.2 Scope of the study

In 1967 Bijker [10] published a method for the computation of sediment transport due to the combined action of currents and waves. This method was a milestone in the computation of sediment transport in the coastal area, since it was the first fundamentally-justified approach opening up the possibility of computing local transports instead of overall transports (as e.g. the formula developed by the Coastal Engineering Research Center [51]). Since then the research into the fundamentals of sediment transport in a wave regime has continued steadily. The Applied Research Programme of the Department of Public Works is a good example thereof. As a result the original Bijker formula has been improved, while retaining its most vital concept, i.e. starting from the bed shear as caused by the combined velocity field of the current and the orbital velocity field.

It seems useful to point out briefly where the present approach deviates from Bijker's method, and why this renewal was initiated. In order to clarify this, Bijker's method is schematized as follows:

a) formulate the bed shear $\tau_{WC}$ under the combined action of waves and currents
b) select an existing formula (Frijlink's [20]) which gives the bed load transport as a function of the bed shear $\tau_c$ under a uniform current
c) substitute $\tau_{WC}$ for $\tau_c$, which gives the bed load transport under the influence of waves and currents
d) add the suspended load by the following procedure:
   d1) assume a bed load transport layer thickness
   d2) determine the average current velocity within this layer
d3) determine the average sand concentration in the bed load layer from c), d1) and d2), substitute this value in the Einstein-Rouse relation for the vertical distribution of sand concentration, and multiply by the respective vertical velocity distribution to get the suspended load.

In using the above approach for actual computations, a weak point appeared to be d1), as the resulting suspended load very sensitively depends on the assumed bed load layer thickness. Of course this is quite important in those cases where the suspended load plays a dominating part in the total load. Moreover, this distinction itself between bed load and suspended load is of a somewhat arbitrary nature, leaving much to be desired as far as a quantitative physical understanding is concerned of the processes very close to the bed.

Therefore, in the present approach τ_{wc} as derived sub a) will be substituted for τ_c directly in a total load formula rather than in a bed load formula. This procedure leaves all possibilities open of making any distinction in terms of bed load and suspended load afterwards, if necessary.

In Chapter 2 the choice of a total load formula will be made, whereafter a suitable description of the vertical distribution of suspended sediment will be chosen in Chapter 3. Finally, in Chapter 4 the derivation of the bed load will be made in terms of the total load.

1.3 Conclusions

The most important conclusions of the investigation described in this report may be summarized as follows:

1) The total load formula of White and Ackers [1], [2], [57] was chosen as basis for the computation of sediment load under combined current and wave action. This choice is based on the good comparison with both laboratory and field uniform flow data of the White-Ackers formula over a wide range of boundary conditions, and on its favourable comparison with laboratory data for combined current and wave action.

2) A formula in which the diffusivity of solid particles has been assumed to increase linearly with increasing distance from the bed, yielded the best
comparison between the computed and measured vertical distribution of suspended sediment for combined current and wave action over a wide range of boundary conditions in both the laboratory and the field.

3) The bedform is the most important single parameter which determines the magnitude of the computed bed and total load.

4) The bedform characteristics for oscillatory flow can be determined uniquely in terms of the bed material size, the orbital excursion of the water particles at the bed and the local length of the surface waves.
2 Total sediment load

2.1 General

Due to the fact that the present knowledge of the phenomena occurring in the nearshore coastal area does not permit the computation of the longshore transport of sediment directly from the internal mechanism (water and sediment movement), an approximation technique has to be used for the time being.

Bijker [10] suggested the use of a sediment transport formula, derived for uniform current flow only, in which the bed shear stress terms have been adapted to allow for the increase in shear stress due to wave action. He concluded that transport rates for current only and for combined current and wave action can be represented by one formula, when the shear stress for uniform flow is written as the resultant shear stress over the whole wave period for combined current and wave action. The Bijker-formula, derived in this manner, has been used with success since 1967, provided that the roughness height is chosen with care. It has, however, been observed that the formula yields too high sediment transport rates outside the breaker zone, especially for fine sediment ($D_{50} < 200 \times 10^{-6}$ m). This effect can partly be compensated for by adapting the coefficients in the formula.

Bijker used the Frijlink-formula for bed load as basis for his derivation. Frijlink [20], on the other hand, based his formula on the formulae of Kalinske [34], Meyer-Peter and Mueller [40] and Einstein [16], [17]. In an appraisal of the available methods to compute sediment transport for uniform flow, White, Milli and Crabbe [58] have shown, by comparing the formulae with a wide variety of flume and field data, that:

1) The Kalinske-formula for bed load overestimates transport rates for flume data while transport rates for field and lightweight sediment data are underestimated. Superimposed on these systematic trends is a large scatter of the mean errors.

2) The Meyer-Peter and Mueller-formula for bed load predicts better the transport rates for fine sediments than for coarse sediments, even though fine sediments are more likely to travel in suspension. As the formula is claimed to be a bed load formula and is based on transport rates measured for coarse sediments, this is a surprising result. Main reason for the discrepancies seems to lie in the Meyer-Peter and Mueller-description of initial movement of bed material.
3) The Einstein-formula for bed load, which is based on probability concepts, underestimates transport rates for particle sizes smaller than approximately $300 \times 10^{-6}$ m. The extent of this underestimation increases with decreasing particle size. In the range $300 \times 10^{-6}$ m $< D_{50} < 1600 \times 10^{-6}$ m the formula yields better agreement with measurements. For $D_{50} > 1600 \times 10^{-6}$ m the transport rates are overestimated.

The Frijlink-formula, now, which is based on the above-mentioned three methods, cannot be expected to offer the best possible prediction of the sediment transport rates.

In the light of the foregoing discussion it was decided, rather than to adapt the Frijlink-formula to the new insights, to make an evaluation of the available methods for the prediction of sediment transport for uniform flow conditions, and to choose that method which gives the best agreement with the available uniform flow data over the complete range of particle sizes and flow conditions, i.e. also in the vicinity of beginning of movement of bed material. The formula which is chosen should be easily adaptable to incorporate the effect of the increase in shear stress due to wave action. In order to obtain some sort of quantitative check on the resulting formula, which will apply to combined current and wave action, it will be used to predict the transport rates as measured in the model tests performed by Bijker [10].

The first part of the study, viz. the choice of the best formula for uniform flow conditions, was performed by De Haan [15], as part of his graduation study at the Delft University of Technology. Following a quantitative evaluation of the available formulae, De Haan chose three formulae for comparison with the available flume and field data, viz.:

1) the Engelund-Hansen-formula for total load (2 versions) [19]
2) the White-Ackers-formula for total load [1], [2], [57]
3) the Frijlink-formula for bed load [20], as a reference for (1) and (2).

After completing the evaluation of one-third of the available 4000 flume and field data, De Haan concluded (private communication on 14th November, 1975) that the White-Ackers formula yields the best fit with the data, followed by Engelund-Hansen and Bijker-Frijlink. The mean relative error, which is defined as the ratio between the error in the prediction and the measured transport rate, amounted to 40 %, 60 % and 70 % for the above-mentioned three approaches. The data evaluated up to this moment comprise both flume and field data.
Although De Haan's evaluation of the data had not been completed, it is also known from previous evaluations of transport formulae (see for instance, White, Milli and Crabbe [58], Cole et al [22], Task Committee for Preparation of Sediment Manual [50]), that both the Engelund-Hansen and White-Ackers formulae yield good results over a wide range of boundary conditions. The report of White [58] shows that, although the two formulae yield comparable results, the White-Ackers formula gives slightly better predictions of the transport rates over the full range of boundary conditions. This is especially the case in the vicinity of the beginning of movement of bed material. In this range the Engelund-Hansen formula overestimates the transport rates. If the final formula is also to be used for the prediction of transport rates under laboratory conditions, the Engelund-Hansen formula will seemingly have to be ruled out, as it does not include a beginning of movement criterion.

However, all three the above-mentioned methods will be adapted to include the effect of the increased shear stress due to wave action and thereafter compared with both Bijker's data and his original formula. From this comparison the necessity of using a formula with a beginning of movement criterion can then be judged.

De Haan's comparison of the three methods with the available flume and field data for uniform flow conditions is of importance for an overall picture of the behaviour of the chosen formula over a wide range of boundary conditions. Knowing the behaviour of the formula for such a wide range of boundary conditions, facilitates a better use of the formula for combined current and wave action under all conditions.

2.2 Procedure

The transport rate data described by Bijker in [10] were obtained by measuring the amount of sediment caught in a narrow sand trap. It is not known whether the measured transport rates are total loads or bed loads, however, the measured quantities will always contain the bed load plus some fraction of the suspended load.

In order to facilitate a proper comparison of the results of all three the methods with the available test results, the vertical distribution of transported sediment must be known. In the case of the total load formulae of Engelund-Hansen and White-Ackers, the known concentration distribution can then be used
to assess the predicted amount of sediment travelling as bed load, while it can be used to obtain the comparable predicted total load in the case of the Frijlink bed-load formula. For the present the well-known Einstein-Rouse formula \([17], [42]\) for the vertical distribution of sediment will be used for the sake of comparison; in Chapter 3 the vertical distribution of suspended sediment will be discussed in more detail. Reason for this choice is the fact that Bijker \([11]\) also used this method for the determination of the total load.

The formulae for the prediction of the rate of sediment movement under uniform flow condition can all be written in the following form:

\[
\bar{c}_c = \frac{S(1-p)}{v d} = F(v, v_{hc}, d, r, D_e, D_{90}, \Delta_s) \tag{2.1}
\]

where:
\[
\bar{c}_c \quad = \text{mean concentration (in vertical sense) by volume of the transported sediment in a uniform flow; if the formula under consideration is a bed-load formula, } \bar{c}_c \text{ is the ratio between the bed load (} S_b (1-p) \text{) and the total fluid flux per unit width of flow (} = vd) \text{; for a total-load formula it is a mean concentration over the whole depth}
\]
\[
S(1-p) = \text{volume of the transported grains per unit width of flow}
\]
\[p \quad = \text{porosity of the bed, i.e. the ratio between the volume of voids and the total volume of voids plus solids in the bed}
\]
\[v \quad = \text{mean current velocity}
\]
\[d \quad = \text{mean flow depth}
\]
\[F() \quad = \text{function of ( )}
\]
\[v_{hc} \quad = \text{shear velocity} = (g \, d \, I)^{\frac{1}{2}} \tag{2.2}
\]
\[g \quad = \text{gravitational acceleration}
\]
\[I \quad = \text{energy gradient}
\]
\[r \quad = \text{equivalent Nikuradse roughness of the bed (see Section 3.4 for a discussion of } r \text{)}
\]
\[D_e \quad = \text{equivalent particle diameter}
\]
\[D_{90} \quad = \text{particle diameter of that fraction of the bed material which is exceeded in size by 10% in weight of the total sample}
\]
\[\Delta_s \quad = \text{relative apparent density of bed material} = (\rho_s - \rho_w)/\rho_w
\]
\[\rho_s \quad = \text{density of bed material}
\]
\[\rho_w \quad = \text{density of water.}
\]
Bijker's concept [10] is based on the assumption that equation (2.1) remains valid for sediment transport under combined current and wave action. The only difference is that the shear velocity \( v_{ WC} \) for uniform flow is replaced by the increased resultant shear velocity \( v_{ WNC} \) for combined current and wave action. Bijker has shown that the increase in the resultant bed shear, averaged over one wave period, equals

\[
\frac{\tau_{ WC}}{\tau_c} = 1 + \frac{1}{2} \left( \xi \frac{u_o}{v} \right)^2 \hspace{1cm} (2.3)
\]

and thus

\[
\frac{v_{ WNC}}{v_{ WC}} = \left( 1 + \frac{1}{2} \left( \xi \frac{u_o}{v} \right)^2 \right)^{\frac{1}{4}} \hspace{1cm} (2.4)
\]

where:

\( \tau_{ WC} \) = resultant average bed shear due to combined current and wave action

\( \tau_c \) = bed shear due to uniform current flow

\( u_o \) = maximum value of the wave orbital velocity at the bed.

\[
uo = \frac{\pi H}{T \sinh \left( \frac{2n \lambda}{\lambda} \right)} \hspace{1cm} (in \ first\ order\ wave\ theory) \hspace{1cm} (2.5)
\]

\( H \) = local wave height

\( T \) = wave period

\( \lambda \) = local wave length

\( \xi \) = parameter, dependent on the bed roughness and water depth

Bijker found \( \xi \) to be equal to

\[
\xi = \frac{p_B \kappa C_h}{g^{\frac{1}{2}}} \hspace{1cm} (2.6)
\]

where:

\( \kappa \) = von Karman constant

\( C_h \) = Chézy roughness coefficient = 18 \log 12 \ d/r \hspace{1cm} (2.7)

\( p_B \) = constant = 0.45

Basis for equation (2.6) is that the maximum orbital velocity \( u_{oz'} \) at a distance \( z' = er/33 \) above the mean bed elevation stands in a constant ratio to the fully developed maximum orbital velocity \( u_o \), viz.
\[ u_{oz} = p_B^* u_0 \]  

(2.8)

In reality, \( p_B \) is not a constant, but a function of the bedform. Assuming \( p_B \) to be a function of the bed roughness \( r \), Swart [49] used the wave friction factor \( f_w \) for rough turbulent flow, as used by Jonsson [33], to show that

\[ p_J = \left( \frac{f_w}{2k^2} \right)^{1/4} \]  

(2.9)

where:

- \( p_J \) = the variable value of \( p_B \)
- \( f_w \) = Jonsson's wave friction factor for rough turbulent flow
  \[ = \exp \left( -5.98 + 5.21 \left( \frac{a_o}{r} \right)^{-0.19} \right) \text{ for } \frac{a_o}{r} > 1.57 \]
  \[ = 0.30 \text{ for } \frac{a_o}{r} < 1.57 \]
- \( a_o \) = the orbital amplitude at the bed, i.e. one half of the total orbital excursion at the bed
  \[ = \frac{T u_o}{2\pi} \text{ (in first order wave theory)} \]  

(2.10)

Combination of equations (2.6) and (2.9) yields:

\[ \xi_J = C_h \left( \frac{f_w}{2g} \right)^{1/4} \]  

(2.12)

where:

- \( \xi_J \) = the value of \( \xi \) in which \( p_B \) has been assumed a function of the bed roughness.

The use of \( \xi_J \) from equation (2.12) is preferred to that of equation (2.6). Consequently, the resultant shear velocity for combined current and wave action becomes:

\[ v_{\text{wbc}} = v_{\text{wc}} \left[ 1 + \frac{1}{2} \left( \xi_J \frac{u_0}{v} \right)^2 \right]^{1/2} \text{ (from equation (2.4))} \]  

(2.13)

The Bijker approach can now be summarized as follows:

\[ \frac{c_{\text{wc}}}{v} = \frac{S_x}{v d} = F (v, v_{wbc}, d, r, D_e, D_90, A_s) \]  

(2.14)

where:

- \( S_x \) = volume of sediment transported in the longshore direction/unit width of longshore flow (\( S_x \) includes the voids).
Bijker [11] used the Einstein-Rouse solution for the diffusion equation for suspended sediment to describe the vertical distribution of sediment concentration. The resulting relationship between the total load and the bed load reads as follows:

\[
S_{xt} = S_{xb} \left[ 1 + 1.83 \{ I_1 \ln \left( \frac{33d}{r} \right) + I_2 \} \right] \tag{2.15}
\]

where:

- \( S_{xt} \) = the total volume of sediment transported in longshore direction/unit width of longshore flow
- \( S_{xb} \) = the sediment transported in longshore direction in the bed layer/unit width of longshore flow; for equation (2.15) the bed layer is assumed by Bijker to have a thickness equal to the bed roughness.
- \( I_1 \) and \( I_2 \) are integrals, given by Einstein [17] as:

\[
I_1 = 0.216 \left( \frac{r}{d} \right) \frac{e_1}{e_{11}} \int_{1}^{1} \left( \frac{1-z/d}{z/d} \right)^{e_1} d \left( \frac{z}{d} \right) \tag{2.16}
\]

\[
I_2 = 0.216 \left( \frac{r}{d} \right) \frac{e_1}{e_{11}} \int_{1}^{1} \ln \left( \frac{z}{d} \right) d \left( \frac{z}{d} \right) \tag{2.17}
\]

where:

\[
e_1 = \frac{w}{\kappa} \nu_{w/c} \tag{2.18}
\]

\( w \) = fall velocity of grains in still water.

For sand particle sizes ranging between approximately 60x10\(^{-6}\) m and 6000x10\(^{-6}\) m and a temperature \( T = 20^\circ \) C, \( w \) can be approximated by:

\[
\log \left( \frac{1}{w} \right) = 0.447 \left( \log D_{50} \right)^2 + 1.961 \log D_{50} + 2.736 \text{ (see Figure 1 and [8])} \tag{2.19}
\]

where:

- \( D_{50} \) = median particle diameter.

The use of equations (2.14) ... (2.19) will make it possible to split the transport rates as predicted by Frijlink-Bijker, Engelund-Hansen and White-Ackers respectively, in terms of bed load and suspended load.
2.3 Data

The data used in the present study to evaluate the behaviour of the different formulae when applied to combined current and wave action under model conditions, were collected by Bijker [10], as part of a study regarding the scales of movable-bed coastal models.

The tests were performed in a basin, which was 27 m long and 17 m wide. On one of the longer sides a wave generator was installed, while a morphological slope of 1:7 was constructed on the opposite side in order to avoid reflection and to dissipate the energy of the waves. A constant current velocity was created in the horizontal part of the model by a system of canals and sluices. Three basic types of tests were performed, viz.:
1) with current action only,
2) with waves perpendicular to the current, and
3) with waves at an angle with the current.
The latter two sets of data will be used in the present study.

Bed material with $D_m = 250 \times 10^{-6}$ m and $D_{90} = 340 \times 10^{-6}$ m was used. The sediment transport resulting from the resultant flow conditions was measured in a narrow sand trap, which had a length of 1.5 m perpendicular to the current direction and a width of 0.15 m in the current direction. To evaluate the effect of the width of the trap on the amount of trapped sand, a few tests were performed with a wide sand trap, being 0.93 m x 0.93 m. The wide trap was divided into 81 squares. The results of these tests indicated that the amount of sediment caught in the narrow trap was certainly not the total load. The mean ratio between the volume of sediment per unit width of flow as determined from the wide and narrow traps for comparable tests, was about 3.2. However, it was observed that the amount of trapped sand in a row of squares, parallel to the current direction and situated on the side of the wave generator, was about 50% higher than that in a row of squares on the landward side of the basin. This can be contributed to the orbital motion, which is not purely symmetrical. As the suspended sediment was mostly concentrated in a layer close to the bed, it is to be expected that not only the pure bed load, but also some fraction of the suspended load was caught in the narrow trap.
The hydraulic parameters, i.e. the wave height $H$, period $T$, water depth $d$, current velocity $v$ and energy gradient $I$, were all measured directly in the model. The bed roughness $r$ was computed from the Chézy-formula and the observed energy gradient, in the case of the tests with current only. In principle, Bijker applied the roughness computed in this manner to the tests with combined current and wave action. However, as the roughness values for certain groups of tests did not vary too much, the total number of tests was subdivided into three groups, for each of which a different average roughness was calculated and used in the further evaluation:

1) perpendicular wave attack with $T = 1.57 \text{ s}; \ r = 0.003 \text{ m}$
2) perpendicular wave attack with $T = 0.68 \text{ s}; \ r = 0.003$ and $0.029 \text{ m}$
3) oblique wave attack with both $T = 0.68$ and $2 \text{ s}; r = 0.009 \text{ m}$

Bijker furthermore concluded from his tests that the alignment of the ripples had no significant influence on the roughness values. For a summary of the data used in the present study, see Table 1.

2.4 Frijlink-Bijker formula

As stated in Section 2.1, Frijlink [20] based his bed load formula for use in the Dutch rivers on the bed load formulae of Kalinske [34], Meyer-Peter and Mueller [40] and Einstein [16], [17]. The first two equations are of the deterministic type, while the Einstein approach is probabilistic. Only the Meyer-Peter and Mueller equation contains a beginning of movement criterion.

The original Frijlink-formula, written in the form of equation (2.1) reads as follows:

$$\frac{C_c}{C} = S_b[F](1-p) \frac{v}{d} = 5(1-p) \frac{(\mu g D_{50}^2)^{1/4}}{C_h} \exp\left(-0.27 \frac{\Delta_s D_{50}^2}{\mu v^2} \frac{C_h}{C_{D_{90}}} \right)$$

(2.20)

where:

$S_b[F]$ = bed load, as given by Frijlink

$\mu$ = ripple factor, being that part of the bed shear which is not used to overcome bed resistance

$$\mu = \frac{C_h}{C_{D_{90}}}^{3/2}$$

(2.21)

$C_{D_{90}} = 18 \log (12d/D_{90})$

(2.22)
Bijker [10] argued that the ripple factor \( \mu \) should only be operational in the stirring up of the sediment, and not in its transportation. For combined current and wave action he consequently changed equation (2.20) to read

\[
\frac{S_{\text{xb}}[\text{B-F, l}]}{v_{d}} (1-p) = b(1-p) \left( \frac{D_{50}^2}{d C_{h}} \right)^{\frac{1}{2}} \exp \left( -0.27 \frac{\Delta_{S} D_{50}^2}{\mu v_{\text{wsc}}^2} \right) \tag{2.23}
\]

where:

\( S_{\text{xb}}[\text{B-F, l}] \) = bed load transportation rate in longshore direction/unit width of longshore flow, as given by Bijker-Frijlink

\( b \) = a constant

\( v_{\text{wsc}} \) = resultant average bed shear as computed from equation (2.4)

Inspection of equations (2.20) and (2.23) reveals that:

\[
b = 5\mu^{\frac{1}{4}} \tag{2.24}
\]

As \( \mu < 1 \), it follows that

\[
b < 5 \tag{2.25}
\]

For Bijker the value of \( b \) was not of real importance, as his main objective was to derive scales for movable bed coastal models. In order to allow a comparison between the transport rates given by the initial Bijker formula (equation (2.23)) and those given by the equations derived from the methods of Engelund-Hansen and White-Ackers, the average value of \( b = 5\mu^{\frac{1}{4}} \) for all the tests of Bijker was used, i.e.

\[
b = 5\mu^{\frac{1}{4}} = 3.66 \tag{2.26}
\]

The total load according to the Bijker procedure is obtained by combining equations (2.15) and (2.23):

\[
S_{\text{xt}}[\text{B-F, l}] = S_{\text{xb}}[\text{B-F, l}] \left[ 1 + 1.83 \frac{l_{1}}{l_{2}} \ln \left( \frac{33d}{r} \right) + I_{2} \right] \tag{2.27}
\]

The computed transport rates are plotted against the measured transport rates in Figures 2 and 3, the lines drawn through the data correspond to \( S_{\text{computed}} = k_{1} S_{\text{measured}} \), in which \( k_{1} \) is a constant.
As described in Section 2.2, the use of equation (2.13) for the computation of \( v_{\text{wfc}} \) is preferable. Accordingly, the Bijker transports were also computed by using equation (2.13) instead of equation (2.4). The corresponding bed and total loads were denoted with \( S_{xb}[B-F,2] \) and \( S_{xt}[B-F,2] \) and are plotted in Figures 4 and 5 against the measured transport capacities.

A study of Figures 2 ... 5 reveals a few clear trends:

1) The Bijker-Frijlink method yields transport capacities which are clearly higher than the measured transport rates; even the computed bed load is on the average between 5 (for \( \xi \)) and 10 (for \( \xi_J \)) times higher than the measured transport. This tendency has also been observed by Bijker himself [10]; in his curve-fitting the best value of the factor \( b \) was 0.74, which is approximately one-fifth of the expected value of 3.66 (see equation (2.26)).

2) There is a tendency for \( S_{xt[B-F]} / S_{\text{measured}} \) to increase for decreasing values of \( S_{\text{measured}} \), i.e. the transport rate is overestimated for low transport capacities. This is characteristic for a formula without a beginning of movement criterion.

3) As will be shown in Section 2.6 the scatter in the data of Figures 4 and 5 (in which \( \xi_J \) was used) is slightly bigger than that of Figures 2 and 3. The reason for this seems to be that the value of the bed roughness \( r \) has a bigger influence on the computed transport rate when \( \xi_J \) is used, than when \( \xi \) is used. As can be seen from Table 1, the bed roughness was taken a constant for each series of constant wave period and direction; this will of course not be exactly true, resulting in an additional scatter in the data.

4) No systematic differences can be found between the data for perpendicular wave attack and those for oblique wave attack. This would seem to indicate that the choice of the resultant bed shear instead of, for instance, the average of the longshore component of the bed shear, is substantiated for the Frijlink-Bijker approach. If the use of the resultant bed shear had led to an overestimation of the effect of the wave-effect, that must have been visible in the plots.

2.5 Engelund-Hansen formula

Engelund and Hansen [19] derived a formula for the total volume of sediment transport per unit time, by making the following general assumptions:

1) The derivation was concentrated on the case where the bed was covered by dunes.
2) Sand grains are eroded from the upstream side of the dunes and deposited on the lee side.
3) The migration velocity of the moving particles is assumed proportional to the shear velocity.
4) A simple energy balance is made between the energy necessary to elevate the eroded sediment over a height equal to the dune height $\Delta_d$ and the work done by drag forces on the moving particles in the same time interval.
5) Alluvial streams tend to adjust their bed roughness in such a manner that

\[
(2g/C_h^2)(\lambda_D/\Delta_D) = \text{constant} = 0.47
\]  

where $\lambda_D$ and $\Delta_D$ denote the length and height of the dunes, respectively.

Equation (2.28) is based on bedforms of the dune type.

Introducing an empirical relationship between the total shear and the effective shear which acts directly on the dune crests, they arrive at an equation, which can be rewritten in the form of equation (2.1) to read:

\[
\frac{S_t}{C_C} = \frac{S_t[E-H,1]}{v_d} = 0.05 (1-p) \frac{C_h v_{4}^{4}}{g^{5/2} \Delta_{s}^{2} v_{50d}}
\]  

(2.29)

where:

$S_t[E-H,1] = \text{total load, as given by Engelund-Hansen's 1st approach.}$

The coefficient 0.05 in equation (2.29) was derived from prototype data of Fort Collins in the USA; the original plot in [19] shows that the form of equation (2.29) fits the trend of the data quite well over the full range of the data. This is a remarkable result, since the data comprise cases with dunes, transition, standing waves, anti-dunes and chute- and pool flow. White et al [58] also come to the conclusion, that although equation (2.29) is most probably the simplest transport formula ever proposed, it is in general one of the most accurate. Its one shortcoming is the errors which are introduced in the prediction of low transport rates.

Combined with equation (2.13), equation (2.29) yields the following prediction for the longshore transport rate:

\[
\frac{S_{xt}}{C_C} = \frac{S_{xt}[E-H,1]}{v_d} = 0.05 (1-p) \frac{C_h v_{4}^{4}}{g^{5/2} \Delta_{s}^{2} v_{50d}}
\]  

(2.30)

Combination of equations (2.15) and (2.30) furthermore yields the bed load.
The computed transport rates (bed and total load) are plotted against the measured transport rates in Figures 6 and 7.

Engelund and Hansen [19] also reasoned that it may be more logical to assume the migration velocity of the moving particles proportional to the effective shear velocity, and not the total shear velocity, as stated in assumption (3). The corresponding transport equation, which must be seen as an alternative to equation (2.29), reads:

$$\frac{S_c}{v \cdot d} = \frac{0.0385 (1-p) v^{4}_{\text{HMC}}}{(\Delta g)^{5/2} D_{50}^{3/2} v \cdot d} (v^4 + 0.15 C_h^{4/2} h^{-1} D_{50}^{2})^{1/3}$$

(2.31)

where:

$S_c = \text{total load, as given by Engelund-Hansen's 2nd approach}$

The coefficient 0.0385 was again determined from the Fort Collins data. Equation (2.31) follows the trend of the data as good as equation (2.29) over the full range.

Combined with equation (2.13), the following longshore transport rate results:

$$\frac{S_{xt} [E-H,2]}{v \cdot d} = \frac{0.0385 (1-p) v^{4}_{\text{HMC}}}{(\Delta g)^{5/2} D_{50}^{3/2} v \cdot d} (v^4 + 0.15 C_h^{4/2} h^{-1} D_{50}^{2})^{1/3}$$

(2.32)

The bed load was again predicted by using equation (2.15). The computed transport rates are plotted against the measured transport rates in Figures 8 and 9.

It should be pointed out that the particle migration velocity has its mean direction coinciding with the longshore direction, and is, according to the assumption (3) of Engelund-Hansen, proportional to the shear velocity and was not increased according to equation (2.13), for obvious reasons.

A study of Figures 6 ... 9 reveals that:

1) Both Engelund-Hansen formulae yield transport capacities which are higher than the measured transport rates. The extent of the overestimation is, however, not quite as much as for the Bijker-Frijlink equations.

2) There exists a strong tendency for $S_{\text{xt}} [E-H]/S_{\text{measured}}$ to increase with decreasing values of $S_{\text{measured}}$. As has been stated in Section 2.4, this is characteristic for a formula without a beginning of movement criterion. A comparison of Figures 2 ... 5 with Figures 6 ... 9 shows that the tendency
can be more clearly discerned for the Engelund-Hansen formulae. This can most probably be contributed to the form of the empirical relationship between the total shear and the effective shear, wherein the effective shear is found to be proportional to the square of the total shear.

3) The scatter in the data on all four Figures (6 ... 9) is considerable.

4) The computed transport rates overestimate the transport capacities for oblique wave attack by a bigger margin than for the data with wave attack at right angles to the main current. This means that, in any case for the Engelund-Hansen type of equation, the use of the resultant shear stress tends to overestimate the transport rates for oblique wave attack. Again the reason must seemingly be sought in the form of the empirical relationship between the total and the effective bed shear, which leads to an increase in the sediment load, due to the presence of waves, of the fourth power of the result in equation (2.13).

2.6 White-Ackers formula

The general function of White and Ackers [1], [2], [57] is one of the most recent formulae for the evaluation of the total sediment transport rate. It is based on physical considerations and on dimensional analysis. The basic assumptions made for the theoretical derivation are as follows [2]:

1) The transportation rates of coarse and fine sediments are treated separately. Seeing that the two approaches yield comparable types of equations, they are finally combined to yield one general function.

2) A coarse sediment (D > 2500 x 10^{-6} m) is considered to be transported mainly as a bed process. If bed features exist, it is assumed that the effective shear stress bears a similar relationship to mean current velocity as with a plain grain-textured surface at rest.

3) A fine sediment (40 x 10^{-6} m < D < 100 x 10^{-6} m) is considered to be transported within the body of the flow, where it is suspended by the turbulence.

4) Sediment mobility is described by the ratio of the appropriate shear force on a unit area of the bed to the immersed weight of a layer of grains.

5) Dimensionless expressions for sediment transport were based on the stream power concept; in the case of coarse sediments using the product of net grain shear and current velocity as the power per unit area of the bed, and for fine sediments using the total stream power.

6) The useful work done in sediment transport in the two cases takes account of the different modes of transport assumed; the hypothesis is made that
the efficiency is dependent on the sediment mobility.

7) All empirical parameters necessary for the determination of sediment transport in the transitional range between fine and coarse sediments are related to a dimensionless grain diameter.

Using the above-mentioned working hypothesis, a basic form of the sediment transport equation resulted. Empirical parameters in this equation were determined from flume data in a wide range of boundary conditions. The final form of the equation is as follows:

\[
\frac{C}{c} = \frac{S_t[W-A,1]}{v \cdot d} = \frac{D_{35}}{d} \left( \frac{v}{v_{wc}} \right)^n \frac{C}{A_m} \left( \frac{C}{c} - A \right)^m
\]  

(2.33)

where:

\[S_t[W-A,1] = \text{total load, as given by White and Ackers, when using } D_{35} \text{ as effective grain size} \]

\[n = 1 - 0.2432 \ln \left( D_{gr} \right) \quad \text{(2.34)} \]

\[m = \frac{9.66}{D_{gr}} + 1.34 \quad \text{(2.35)} \]

\[A = \frac{0.23}{D_{gr}} + 0.14 \quad \text{(2.36)} \]

\[C = \exp \left\{ (2.86 \ln(D_{gr}) - 0.4343 (\ln(D_{gr}))^2 - 8.128 \right\} \quad \text{(2.37)} \]

\[D_{gr} = \text{dimensionless particle diameter} \]

\[= \left( \frac{g \Delta_s}{\nu^2} \right)^{1/3} \quad \text{(2.38)} \]

\[\quad = \frac{D_{35}}{v^2} \]

As the values of the parameters \(n, m, A\) and \(C\) were determined empirically from a large amount of data, the use of the kinematic viscosity \(\nu\) in equation (2.38) leads to an automatic inclusion of the effect of the temperature \(T\) on the transport rates, even though the \(\nu\) entered equation (2.38) by means of a dimensional analysis. In some other formulae for the determination of transport rates, such as that of Colby [12] a special graph is given to determine a correction factor for the effect of temperature on the computed transport rates. Empirically the value of \(\nu\) for \(0 < T < 30^\circ \text{C}\) was found to be:
\[ \nu = 1.792 \times 10^{-6} \exp (-0.042 \ T^{0.87}) \]  
with \( \nu \) in m\(^2\)/s and \( T \) in °C.  
\[ F_c = \text{sediment mobility} \]  
\[ = \frac{\nu^{1-n} \ n}{C_D \ g^{1/2} (A_s D_{35})^{1/2}} \]  
\[ C_D = \text{particle Chézy-coefficient} \]  
\[ = 18 \log \left( \frac{10d}{D_{35}} \right) \]  

In the above set of equations (2.33) ... (2.40) one of the conclusions of White and Ackers, namely that the effective particle diameter, necessary to schematize the graded sediment, is \( D_{35} \), i.e. the particle diameter exceeded in size by 65% in weight of the total sample, has already been incorporated.

The evaluation of White et al. [58] has shown that the equation (2.33) gives good results consistently over the full range of the data tested, which included a wide range of prototype conditions. For about 70% of the data the ratio of the computed to measured transport rate fell between \( \frac{1}{2} \) and 2, which is a higher percentage than for any of the other 17 methods evaluated.

The value of \( n \) facilitates the transition from fine to coarse material. When \( D_{35} = 1 \), i.e. approximately when \( D_{35} = 40 \times 10^{-6} \) m, \( n = 1 \). On the other hand, when \( D_{35} = 60 \), i.e. approximately when \( D_{35} = 2500 \times 10^{-6} \) m, \( n = 0 \). The parameter \( A \) is actually the critical value of the sediment mobility \( F_c \), above which sediment movement starts taking place. The value of \( A \) compares favourably with other incipient motion relationships (see White and Ackers [2] for a comparison).

When the shear velocity terms in equation (2.33) are increased according to equation (2.13), the following longshore transport rate results:

\[ S_{xt} [v-A,1] (1-p) = \frac{D_{35}}{d} \left( \frac{v}{v_{HWC}} \right)^n \frac{C}{A^m} \left( F_{wc-A} \right)^m \]  

where:
\[ S_{xt} [v-A,1] = \text{total longshore transport capacity/unit width of longshore current, according to White-Ackers} \]  
\[ F_{wc} = \text{mobility number for combined current and wave action} \]  
\[ = \left( \frac{v_{HWC}}{v_{HWC}} \right)^n F_c \]  

(2.42)
As before the bed load was calculated by combining equation (2.15) with the 
equation (2.41) for the total load. The results of the computations are 
plotted against the measured transport rates in Figures 10 and 11.

A personal communication with White has revealed that the latest development 
for well-graded sediments is to compute the sediment load for each size 
fraction separately, and to add these together. In order to evaluate the 
significance of the choice of $D_{35}$ as representative particle diameter, the 
sediment transport according to equation (2.41) was also computed for each 
size fraction separately, and added afterwards. Equation (2.41) then reads:

$$S_{xt} \left[ \frac{w-A}{2} \right] (1-p) = \frac{N}{d} \sum_{i=1}^{N} \frac{D_i}{d} \left( \frac{v}{v_{w}} \right) \frac{n_i}{m_i} \frac{C_i}{A_i} \left( F_{wci} - A_i \right)^{m_i}$$  (2.43)

where:

- $N$ = number of size fractions
- subscript $i$ = index denoting different size fractions

The transport rates computed from equations (2.43) and (2.15) are shown in 
Figures 12 and 13.

A study of Figures 10 ... 13 reveals the following:

1) The comparison between the computed and measured transport rates is remark-
ably good. For equation (2.41) 57% and 41% of the data has a ratio 
$S_{x} \frac{W-A}{S_{measured}}$ which lies between $\frac{1}{2}$ and 2 for bed load and total load 
respectively. For the computations for separate size fractions the result 
is 51% and 32% respectively. Keeping in mind that the measured transport 
must contain the bed load plus some fraction of the suspended load, this 
is a remarkably good result.

2) No clear tendency exists for $S_{x} \frac{[W-A]}{S_{measured}}$ to increase with decreasing 
values of $S_{measured}$, as was the case with the other formulae (Figures 2 ... 
9). The trend in the computed transport rates follows rather close to that 
for the measured transport rates.

3) Relative to the results of the other formulae (shown in Figures 2 ... 9) 
the scatter for the White-Ackers formula can be called small.

4) There is only a very slight overestimation of transport rates for perpen-
dicular wave attack.

Figure 14 shows that in case of bed load for 59% of the perpendicular wave 
attack data and for 47% of the oblique wave attack data $S_{computed} > S_{measured}$.
in case of total load these figures are 91 % and 60 % respectively.
These figures are for practical purposes equal. Consequently the choice of
the mean resultant bed shear also holds good for the White-Ackers approach,
and it can be stated that the choice of resultant shear is in general justi-
fied.

As can be seen from Figure 15, the use of the various size fractions for the
computation of the transport rates, leads to transports, which are consistent-
ly lower than those computed by using $D_{35}$. The extent of the difference between
the two approaches clearly increases for decreasing values of $S_{\text{computed}}$. At
about $5 \times 10^{-6}$ m$^3$/m/s the two transport rates appear to become equal. The
reason for this increasing difference would seem to be that the chosen
representative diameter $D_{35}$ is too low for low transport rates and a floating
diameter should be chosen.

The effect of the sediment gradation can however, be incorporated into the
1$^{\text{st}}$ approach, by defining a relationship of the type:

$$S_{xt}[W-A,2] = A (S_{xt}[W-A,1])^B \quad (\text{for } S_{xt}[W-A,1] < 5 \times 10^{-6} \text{ m}^3/\text{m/s})$$

(2.44)

where $A$ and $B$ have to be determined empirically from figures of type of Figure
15. For the time being no correction will be applied.

2.7 Evaluation and conclusions

A visual evaluation of the Figures 2 ... 13 reveals clear tendencies, the most
important being that the use of a theory containing a beginning of movement
criterion is essential. If this is not done, laboratory data will be over-
estimated consistently. The extent of the overestimation will increase as the
bed shear decreases towards the critical shear stress, necessary for initiation
of sediment movement.

Although the above-mentioned fact will lead directly towards the choice of the
White-Ackers approach for the determination of transport rates, based on a
visual observation only, a numerical evaluation was also made, in order to
obtain a more sound base for the choice of the best formula. The criteria
chosen to evaluate the different approaches numerically, are:

1) the extent of the scatter in the data
2) the quantitative prediction of transport rates
3) the trend in the data.

re. 1) As measure for the scatter in the data was chosen the root mean square error, defined by

\[
\sigma' = \text{Error}_{\log \text{rms}} = \left[ \frac{1}{N_u} \sum_{i=1}^{N_u} (\log (S_{\text{computed}}) - \log (S_{\text{measured}}))^2 \right]^{1/2}
\]

(2.45)

where:

\( N_u \) = number of tests evaluated

The case with the smallest root mean square error has the least scatter. The result of this evaluation is summarized below.

<table>
<thead>
<tr>
<th>Formula</th>
<th>( \sigma' ) (bed load)</th>
<th>( \sigma' ) (total load)</th>
<th>( \sigma' ) (mean)</th>
</tr>
</thead>
<tbody>
<tr>
<td>White-Ackers (1)</td>
<td>0.435</td>
<td>0.409</td>
<td>0.422</td>
</tr>
<tr>
<td>Bijkers-Frijlink (1)</td>
<td>0.491</td>
<td>0.479</td>
<td>0.485</td>
</tr>
<tr>
<td>Bijkers-Frijlink (2)</td>
<td>0.525</td>
<td>0.517</td>
<td>0.521</td>
</tr>
<tr>
<td>Engelund-Hansen (1)</td>
<td>0.641</td>
<td>0.626</td>
<td>0.634</td>
</tr>
<tr>
<td>Engelund-Hansen (2)</td>
<td>0.653</td>
<td>0.635</td>
<td>0.644</td>
</tr>
</tbody>
</table>

From the above figures it can be seen that the White-Ackers approach shows the smallest scatter, followed by the Bijkers-Frijlink approach and the Engelund-Hansen approach. The reason for the large scatter in the latter approach must be sought in the fact that the use of the resultant shear stress seems to overestimate the increase in bed shear for cases with perpendicular wave attack. It should be stressed that the White-Ackers (2) equation was not evaluated separately, as it is in reality no separate approach (see Figure 15).

re. 2) If it is assumed that the measured and computed transport rates follow the same trend, the data can be represented by an equation of the form:

\[
S_{\text{computed}} = K_l S_{\text{measured}}
\]

(2.46)

\( K_l \) is then a measure of the overestimation of the transport rates by each separate formula. The smallest value of \( K_l \) corresponds to the best quantitative prediction (as long as \( K_l > 1 \)). In sequence of best quantitative prediction, the result of the evaluation is as follows:
<table>
<thead>
<tr>
<th>Formula</th>
<th>( K_1 ) (bed load)</th>
<th>( K_1 ) (total load)</th>
</tr>
</thead>
<tbody>
<tr>
<td>White-Ackers (1)</td>
<td>1.128</td>
<td>2.295</td>
</tr>
<tr>
<td>Engelund-Hansen (1)</td>
<td>4.115</td>
<td>8.123</td>
</tr>
<tr>
<td>Engelund-Hansen (2)</td>
<td>5.339</td>
<td>10.537</td>
</tr>
<tr>
<td>Bijker-Frijlink (1)</td>
<td>5.609</td>
<td>10.395</td>
</tr>
<tr>
<td>Bijker-Frijlink (2)</td>
<td>9.980</td>
<td>20.000</td>
</tr>
</tbody>
</table>

Except for the White-Ackers approach, all other formulae completely overestimate the transport rates. As has been stated earlier, this result is logical as these equations do not include a beginning of movement criterion in their final formulation.

re. 3) If it is realized that the trend in the computed transports is different from that in the measured transports, the relationship between the measured and computed rates can be written as:

\[
S_{\text{computed}} = K_2 \left( S_{\text{measured}} \right)^{K_3}
\]

\( K_3 \) can now be used to evaluate the trend in the data. Larger values of \( |K_3 - 1| \) points to a trend which deviates more from the desired trend \((S_{\text{computed}} = K_1 S_{\text{measured}})\). The result of the evaluation is as follows:

<table>
<thead>
<tr>
<th>Formula</th>
<th>( K_3 ) (bed load)</th>
<th>( K_3 ) (total load)</th>
</tr>
</thead>
<tbody>
<tr>
<td>White-Ackers (1)</td>
<td>1.107</td>
<td>1.134</td>
</tr>
<tr>
<td>Bijker-Frijlink (1)</td>
<td>1.503</td>
<td>1.295</td>
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<tr>
<td>Bijker-Frijlink (2)</td>
<td>1.539</td>
<td>1.734</td>
</tr>
<tr>
<td>Engelund-Hansen (1)</td>
<td>1.692</td>
<td>2.126</td>
</tr>
<tr>
<td>Engelund-Hansen (2)</td>
<td>2.059</td>
<td>1.817</td>
</tr>
</tbody>
</table>

The figures above confirm the visual observation, viz. that the computed White-Ackers transports follow the trend in the data quite well, whereas the Bijker-Frijlink and Engelund-Hansen formulae deviate considerably from the line \( S_{\text{computed}} = K_1 S_{\text{measured}} \).

Concluding, the result of the evaluation of the behaviour of the formulae of Bijker-Frijlink, Engelund-Hansen and White-Ackers points unanimously towards the choice of the White-Ackers approach to determine the transport rate due to combined current and wave action. For normal beach sand it is sufficient to
choose $D_{35}$ as the representative grain size. For well graded sediment it might be necessary to compute the transport rate for each size fraction separately, and to add the transports afterwards. More research into this aspect of the problem is necessary.
3 Sediment concentration

3.1 General

The relationship between the total sediment load and the sediment being transported as bed load was derived by assuming the Rouse-solution for the sediment concentration for uniform flow to apply to combined current and wave flow conditions. It has been proven that the Rouse description of sediment concentration distribution agrees well with both laboratory and field uniform flow data. However, the agreement is one in form only, because the $e_1$-values giving the best fit are not equal to the theoretical $e_1$-values according to equation (2.18). The most important reasons for this inequality are:

1) in the equations leading towards the Rouse-solution, it has been assumed that the concentration $c$ is small with respect to unity, while this is not the case near the bed;

2) the fall velocity in a fluid containing sediment in suspension is not the same as that of a single particle in still water; in fact, the fall velocity is a function of both the sediment concentration and the water movement;

3) the von Karman $\kappa$-value, i.e. the universal constant in the logarithmic velocity law, has a value of 0.384 in open channel flow without sediment; in the presence of sediment the $\kappa$-values are found to vary, with a tendency for $\kappa$ to decrease with increasing suspended load.

In general the experimentally determined values of $e_1$ are smaller than the theoretical values. This implies that if the theoretical values of $e_1$ are used the resultant sediment transport rates will be too low. It is thus to be expected that application of the Rouse-solution to combined wave and current flow will yield sediment concentration profiles which are too meagre.

Kennedy and Locher [37] made a study of available theoretical models to determine the dimensionless vertical distribution of suspended sediment under wave action. The most promising theories were compared with data collected in a flume with a horizontal bed. Their conclusions are rather enlightening:

1) Of the four theoretical models compared with the experimental data, none could be discarded due to lack of conformity with the data.

2) This positive correlation between theory and data can be attributed to the fact that the sediment concentration distribution is not very sensitive to the variation over the vertical of the diffusion coefficient.
3) The conformity, or the lack of it, between the form of the predicted and measured concentration distributions cannot be used to infer the nature of the transfer mechanisms. 
Das [14] came to a similar conclusion in a later review.

Application of Rouse’s solution for the vertical concentration distribution complicates the computation of suspended load, due to the integrals in equations (2.16) and (2.17), which can not be integrated directly. This fact, together with the conclusions drawn by Kennedy and Locher [37], prompted a search for a simplified relationship for the vertical distribution of sediment concentration. A choice of two of the theoretical models presented by Kennedy and Locher [37] was made after a careful study of all available data on sediment concentration under combined current and wave action. These two theoretical approaches, together with the Rouse-solution was then compared with the available data on sediment concentration under combined current and wave action. The data, taken from literature, comprise both model and prototype cases. In total 86 concentration profiles were obtained, in which the bed material was sand and for which the boundary conditions were known. The range of the data was wide enough to cover the normal area of application of transport computations under waves and current action.

3.2 Theoretical considerations

Solid particles are kept in suspension due to turbulence, the effect of which on the particles may be assumed to be analogous to the diffusion - dispersion process. Although such a model does not account adequately for all influences, it has been found to explain satisfactorily many suspension problems.

If the mean concentration, averaged over a time longer than the wave period is considered, and steady state conditions and a uniform turbulence distribution, with negligible gradients in horizontal direction are assumed for this mean concentration, the diffusion relationship for the equilibrium exchange of solid particles becomes:

$$ \overline{w_c} z + \varepsilon_s \frac{\partial \overline{c}}{\partial z} = 0 $$

(3.1)

where:

- \( \overline{c} \) = average sediment concentration over a time \( t \gg T \), at a height \( z \) above the bed
\( \epsilon_s \) = diffusivity of the solid particles.

The solution of the differential equation (3.1) can be written as:

\[
\frac{\bar{c}_z}{c_a} = \exp \left[ -w \int_a^z \frac{1}{\epsilon_s} \, dz \right]
\]

(3.2)

where:
\( \bar{c}_a \) = average sediment concentration over a time \( t >> T \), at a reference height \( z=a \) above the bed.

As can be seen from equation (3.2), the form of the relative concentration of suspended sediment is determined to a large extent by the choice of the diffusivity \( \epsilon_s \) of solid particles. Rouse obtained his solution by following the procedure outlined below.

In a two-dimensional channel, the relative shear stress distribution is given by:

\[
\frac{\tau_z}{\tau_c} = \frac{d-z}{d}
\]

(3.3)

where:
\( \tau_z \) = shear stress at a height \( z \) above the bed.

For a logarithmic velocity distribution it follows that:

\[
\frac{dv}{dz} = \frac{(\tau_c/\rho_w)\frac{1}{2}}{\kappa z} = \frac{v_c}{\kappa z}
\]

(3.4)

The shear stress \( \tau_z \) can be written as:

\[
\tau_z = \rho_w \epsilon_m \frac{dv}{dz}
\]

(3.5)

where:
\( \epsilon_m \) = diffusivity of momentum.

The concept of an analogy between the process of mass and momentum transfer is known as the Reynolds analogy. Under the assumption that the kinematic viscosity \( \nu \) is much smaller than the diffusivity of momentum \( \epsilon_m \), and that the molecular diffusivity is much smaller than the turbulent diffusivity, the
Reynolds analogy is valid if the mechanisms, which control both the mass and the momentum transfer, are identical. The ratio \( \frac{e_w}{e_m} \) between the mass diffusivity \( e_w \) and the momentum diffusivity \( e_m \) might vary from one flow environment to another, due to difference in the processes of transfer of mass and momentum. However, it will be assumed that the Reynolds analogy is valid, and thus \( e_w = e_m \).

Combination of equations (3.3) ... (3.5) will then yield:

\[
e_w = e_m = \kappa \nu \frac{z}{d} (d-z) \quad (3.6)
\]

If the solid particles in suspension follow the motion of the fluid particles, an equality such as \( e_w = e_s \) will exist. In general, \( e_w \) may differ from \( e_s \) due to inertial and gravitational effects. \( e_s/e_w \) may be considered to depend on \( w/\sqrt{(v')^2} \) and \( (w D_{50} \rho_w/\mu) \) in which \( v' \) is the velocity fluctuation and \( \mu \) is the dynamic viscosity of the fluid, (see [48]). In general, however, a relationship can be written, such that

\[
e_s = \beta \frac{e_w}{e_m} = \beta \frac{e_m}{e_m} \quad (due \ to \ the \ Reynolds \ analogy) \quad (3.7)
\]

where:

\( \beta \) = a factor of proportionality.

For practical purposes, the value of \( \beta \) can be assumed equal to unity for fine particles. For coarser particles some authors (Brush et al [9], Matyukhin et al [39], and Majumdar et al [38]) found \( \beta < 1 \), although also values \( \beta > 1 \) have been reported, e.g. Coleman [13]. Rouse assumed \( \beta = 1 \), and thus

\[
e_s = e_w = e_m = \kappa \nu \frac{z}{d} (d-z) \quad (3.8)
\]

Bijker [11] used equation (3.8) in the present application to wave and current conditions, and substituted \( \nu \) by \( \nu_{K_C} \) from equation (2.3).

The resulting solution to equation (3.2) is then:

\[
\frac{e}{e} = \left( \frac{d-z}{z} \cdot \frac{a}{d-a} \right) \quad (3.9)
\]

with
\[ e_1 = \frac{w}{\kappa \nu_{NWC}} \]  

This solution will be called the Rouse-Bijker solution in the further comparison.

The variation of \( \varepsilon_s \) with \( z \) is, however, open to considerable question in the case of combined current and wave action. In general it can be stated that [37]:

\[ \frac{\varepsilon_s}{\varepsilon_a} = (\frac{z}{a})^\alpha \]  

where:
\( \varepsilon_a \) = diffusivity at \( z = a \) above the bed = \( \varepsilon_s(a) \).

A visual study of all the available vertical concentration distributions showed that most of the distributions conform to a large extent with the solution of equation (3.2), when either \( \alpha = 1 \) or \( \alpha = 0 \) in equation (3.10). As these two solutions can be obtained easily, they were chosen as possible alternatives for equation (3.9).

When \( \alpha = 0 \), i.e. with a constant diffusivity \( \varepsilon_s \) over the depth, the solution to equation (3.2) is:

\[ \frac{c_z}{c_a} = \exp \left\{-\frac{w}{\varepsilon_a} (z-a)\right\} \]  

(3.12)

Coleman [13] found a similar type of solution for the outer part of the flow in uniform flow conditions; for brevity's sake equation (3.12) will in the following be called the Coleman-solution. The value of \( \varepsilon_a \) in equation (3.12) still remains to be determined in terms of hydraulic conditions.

When \( \alpha = 1 \), the diffusivity varies linearly with depth, and has a maximum value at the water surface. Although this seems improbable, the major portion of the wave and current data points towards a relationship of this type. Zagustin [59] found a similar result for uniform flow. The solution to equation (3.2) is:

\[ \frac{c_z}{c_a} = (\frac{z}{a})^{-\omega a/\varepsilon_a} \]  

(3.13)
Bhattacharya [5] correlated his results to an equation of similar form, viz.

\[
\frac{-c_z}{c_a} = \frac{-w_T/\varepsilon_w}{\varepsilon_w} \frac{d}{a}
\]

(3.14)

in which \(\varepsilon_w\) is a dimensionless parameter. From equations (3.13) and (3.14) it follows that:

\[
\varepsilon_w = \frac{\varepsilon_T}{a/d}
\]

(3.15)

This last approach will in the following be called the Bhattacharya-solution.

3.3 Data

The three approaches described in Section 3.2 were compared with the results of measurements regarding the relative sediment concentration distribution, performed in both the laboratory and the field. Only cases with sand as bed material were considered. The data are listed in Table III.

Only in the last decade an appreciable advance has been made in the development of measuring devices for the fluctuating sediment concentration in a wave field. The data reported herein were collected in the years 1958-1974, and it is thus to be expected that the results from the earlier measurements will not be as reliable as the more recent measurements. In Table IV a summary is given of the variety of measuring devices, used by the various investigators. The absolute value of the concentration will most probably not be accurately measured with, for instance, a bamboo sampler, however, it is felt that the relative concentration will be accurate to a higher degree than the absolute concentration. On the other hand, even the most modern equipment is not above suspicion. It is known [55] that the Iowa Sediment Concentration Measuring System, which was used by, for instance, Bhattacharya [5], was not adapted to allow for the effect of the water velocity on the sediment concentration. After the measurements of Bhattacharya this adaptation was applied to the I.S.C.M.S. The giving of a relative accuracy to any specific instrument (or technique) will thus constitute a difficult problem. Accordingly it was decided not to give a different weight to the results from the measurements with the different measuring devices.

The roughness values \(r\) for the 86 different cases tabulated in Table III were unknown. They were, however, computed by using the procedure which will be
described in Section 3.4.

The range in the different variables, as listed in Table III, is as follows:

\[ 0.027 \, \text{m} \leq d \leq 16 \, \text{m} \]
\[ 0.017 \, \text{m} \leq H \leq 2.06 \, \text{m} \]
\[ 0.89 \, \text{s} \leq T \leq 11.2 \, \text{s} \]
\[ 0.012 \leq H/\lambda \leq 0.107 \]
\[ 80 \times 10^{-6} \, \text{m} \leq D_{50} \leq 250 \times 10^{-6} \, \text{m} \]

3.4 Determination of bed roughness

For the computation of \( v_{\text{HVC}} \) in equation (3.10), the value of the Nikuradse bed roughness \( r \) is needed. The roughness \( r \) is related to the bed form, i.e. the height \( \Delta_r \) and the length \( \lambda_r \) of the ripples.

In uniform flow applications the value of \( r \) is determined from the energy gradient or:

\[ r = 12 \, d \frac{V}{18 \sqrt{d \, T}} \quad \text{(Chézy-equation)} \quad (3.16) \]

Bijker [11] chose for \( r \) a value of equal to one half the ripple height, i.e.

\[ r = \frac{1}{2} \Delta_r \quad (3.17) \]

Assumption (3.17) is based on the tests performed by Van Breugel [7] in a wind tunnel, with fixed ripples profiled on the side of the tunnel. Shinohara and Tsubaki [46] found that the ratio between the roughness \( r \) and the bed form height \( \Delta_r \) is a function of the bed form steepness \( \Delta_r/\lambda_r \), i.e.:

\[ \frac{r}{\Delta_r} = f \left( \frac{r}{\lambda_r} \right) \quad (3.18) \]

The bed form steepness quoted by Shinohara and Tsubaki ranges between 0.012 and 0.09, which indicates bed form steepnesses which are lower than those encountered in the coastal environment. In the reported range of \( \Delta_r/\lambda_r \) the ratio \( r/\Delta_r \) increases from approximately 0.65 to 2.0.
It is, of course, logical to assume a relationship such as equation (3.18), as the flow will be able to follow the bed to a greater extent for flat ripples (low values of $\Delta_r/\lambda_r$). On the other hand, if the steepness becomes too big, the flow will pass over it, and the relative bed roughness $r/\Delta_r$ will tend to decrease again with a further increase in the steepness $\Delta_r/\lambda_r$. The variation of relative roughness described above, i.e. an initial increase followed by a decrease of relative roughness, when the bedform steepness is increasing, is reported in literature by Johnson [32] (for rectangular artificial roughness) and by the Delft Hydraulics Laboratory [53] for strips.

Tests performed in a flume in the Delft Hydraulics Laboratory [6], in which both the energy slope and the bed form characteristics were measured, could be reduced to a similar form as the data of Shinohara and Tsubaki. The range was not increased. Measurements in a coastal morphological model, at a time when only current flow was set in, yielded some results for bed form steepnesses of 0.085 and 0.235 respectively [54]. The corresponding values of the relative roughness were 2.3 and 6.9. Jonsson [33] lists a value of $r/\Delta_r = 4$ for a corresponding bed form steepness of 0.15. It is thus clear that in uniform flow as well as combined wave and current conditions the bed roughness can be a multiple of the ripple height $\Delta_r$.

All the available data (in total 30 points) were plotted in the format as suggested by Shinohara and Tsubaki (see Figure 16). The range of steepnesses which are applicable to the coastal area is

$$0.05 \leq \frac{\Delta_r}{\lambda_r} \leq 0.20$$

In this range the relationship between the relative roughness $r/\Delta_r$ and the bed form steepness $\Delta_r/\lambda_r$ can be closely approximated by:

$$\frac{r}{\Delta_r} = 25 \left( \frac{\Delta_r}{\lambda_r} \right)$$

(3.19)

Equation (3.19) does not solve the problem of the determination of the bed roughness $r$, it just shifts it. To be able to use equation (3.19) it is necessary to know the ripple geometry.

Although the flow conditions above the bed are more complicated in the case of a combined current and wave action, the resulting bed form is better organised than in the case of uniform flow conditions, due to the effect of
the oscillating movement at the bed.

Mogridge and Kamphuis [41] found, after tests in an oscillating water tunnel, that both the bed form height $\Delta_r$ and length $\lambda_r$ are related to the wave orbital excursion $d_o$ at the bed, where $d_o$ was measured from film strips, made during the tests. The data for sand as bed material fit nearly perfectly lines of the form:

$$\log \left( \frac{\lambda_r}{D_{50}} \right) = a_1 \log \left( \frac{d_o}{D_{50}} \right) + b_1 \tag{3.20}$$

and

$$\log \left( \frac{\Delta_r}{D_{50}} \right) = a_2 \log \left( \frac{d_o}{D_{50}} \right) + b_2 \tag{3.21}$$

where:

- $a_1, a_2, b_1, b_2$ are constants
- $d_o$ = orbital excursion at the bed
- $a_o = 2a_o + a_\xi$ \tag{3.22}
- $a_o$ = orbital amplitude at the bed
- $a_\xi = \frac{H}{2 \sinh \frac{2\pi d}{\lambda}}$ (first order wave theory) \tag{3.23}
- $a_\xi$ = the net distance moved by the fluid in the direction of wave propagation, directly outside the boundary layer, during one wave period due to mass transport

$D_{50} = 360 \times 10^{-6}$ m for all their data.

By combining the result of equations (3.20) and (3.21) with the results obtained for light weight materials, they arrived at a set of design curves, reproduced in Figure 17.

In Figure 18 the Mogridge and Kamphuis-data on ripple height are plotted together with data obtained by Van Hijum [29] in a wave flume of the Delft Hydraulics Laboratory. The values of $d_o$ for the tests of Van Hijum were computed from first order wave theory. The effect of mass transport was neglected. As can be seen from Figure 18, the data of Van Hijum follow a different trend than the Mogridge and Kamphuis-data. When the product $(u_{ov} t_{crest})$ is substituted for $d_o$ for the tests of Van Hijum, where $u_{ov}$ = the maximum orbital velocity at the bed according to a wave theory proposed by Van Hijum [28], and $t_{crest}$ = the time during which the orbital velocity is directed in the same direction as the wave celerity, and the Mogridge and Kamphuis-data
are increased by a factor

\[ \frac{u_0 T/2}{d_0} = \frac{\pi}{2}, \]

both the trend in the data and the order of magnitude of the prediction coincide. Van Hijum [28] gives the values of \( u_{ov} \) and \( t_{crest} \) as:

\[ u_{ov} = \frac{2\pi}{T} \frac{H(1-A_o)}{\sinh k(d+H(1-A_o))} \]  
(3.24)

\[ t_{crest} = \beta_o T \]  
(3.25)

\[ \hat{A}_o = u_{ov} t_{crest} \]  
(3.26)

where:

\[ A_o = \frac{1}{(\pi P)^{1/2}} \left[ 1 - \frac{1}{8P} + \frac{1}{128P^2} \right] \]  
(3.27)

\[ \beta_o = \frac{1}{90} \arccos \left( A_o^{2P} \right) \]  
(3.28)

\[ P = \frac{Q}{13.6} \quad \text{for } Q \geq 34.8 \]
\[ = 1 + \frac{Q}{22.3} \quad \text{for } Q < 34.8 \]  
(3.29)

\[ Q = \left( \frac{H}{d} \right) \left( \frac{\lambda}{d} \right)^2 \]  
(3.30)

\( H \) and \( \lambda \) are computed from first order wave theory. Figure 19 shows the importance of working with actual orbital motion at the bed and not with that according to the first order wave theory.

Van Hijum [29] found the following relationship for the ripple length:

\[ \frac{u_{ov} \beta_o T D^{1/2}}{\lambda \lambda_{90}} = \frac{1}{30} \]  
(3.31)

Ripple measurements performed in the Delft Hydraulics Laboratory [56] during a study of dune erosion, led to the conclusion that both the ripple height and length are practically independent of the water depth \( d \), for a specific set of wave and bed material characteristics. This result is in contradiction to the trend given in Figure 19.
Inman [30] made an extensive study of ripple dimensions under prototype conditions in water depths ranging between 1 m and 52 m. The most important conclusions of this study are:

1) ripples are present on the bed for $0.1 \text{ m/s} < u_o < 1.0 \text{ m/s}$;
2) the ripple length $\lambda_r$ is related to the median particle diameter $D_{50}$ and the orbital excursion $d_o$, and increases with an increase in both $D_{50}$ and $d_o$;
3) the ripple steepness $\Delta_r/\lambda_r$ increases with increasing particle diameter, and decreases with increasing velocity $u_o$.

Inman calculated the orbital motion at the bed from $1^{st}$ order wave theory while in deep water, and from the solitary wave theory in the nearshore region. When his data are plotted (in Figure 19), it becomes apparent that the flattening-off of the ripple dimensions for large orbital velocities is not predicted by equation (3.21). The data of Inman show a similar tendency as Figure 17. It was, however, not possible to verify the design curves, presented in Figure 17, by means of the data in Figure 19.

Ripple measurements on a beach profile by Scott [44], performed in a laboratory flume, lead basically towards the same conclusions as found by Inman. Homma and Horikawa [25] plotted the data of Inman and Scott along with data of their own in a form as shown in Figure 20. Especially the good correlation for constant values of $D_{50}$, obtained in Figure 20b, can be attributed, at least partially, to a spurious correlation, as $(d_o/\lambda_r)$ is plotted against $(\pi d_o^2/\nu T)$.

A careful study of all above-mentioned data and conclusions, led eventually to a decision to plot the data in a form as shown in Figures 21 and 22. The relationship, shown in Figure 21 for the ripple height, can be expressed as follows:

$$\frac{\Delta_r}{a_o} = A_r \left( \frac{a_o}{(D_{50}/\lambda)^{1/3}} \right)^{-1.56}$$  \hspace{1cm} (3.32)

in which:

$A_r$ = a particle corrector, determined by the particle diameter and the ratio $D_{50}/\lambda$. 
A set of curves is given in Figure 23 for the determination of the value of \( A_r \) for a given set of boundary conditions. The curves are valid for a temperature \( T = 20^\circ \text{C} \). Analytically \( A_r \) can be expressed as follows:

\[
\begin{align*}
A_r &= C_p \left( \frac{\lambda}{D_{50}} \right)^{0.05} \left( \frac{R_e}{R_{e_\infty}} \right)^{-0.5} \\
R_e &= \frac{w(D_{50} - D_o)}{\nu} \\
R_{e_\infty} &= \frac{w(D_{50} + D_\infty)}{\nu} \\
C_p &= 1.475 \exp \left\{ 0.645 - 9.2 \times 10^{-4} \left( \frac{\lambda}{D_{50}} \right)^{0.5} \right\} \\
&\quad - (0.009 \left( R_e \right) + 0.2) \exp \left\{ -0.008 \left( R_e \right) \right\}
\end{align*}
\]  

(3.33) (3.34) (3.35) (3.36)

where:

\[
R_e = \frac{wD_{50}}{\nu} = \text{particle Reynolds number}
\]

\[D_o = 50 \times 10^{-6} \text{ m}
\]

\[D_\infty = 500 \times 10^{-6} \text{ m}
\]

Equation (3.32) predicts the ripple heights in the range of the data with a mean error of 31 % (see Figure 24).

In Figure 25 a comparison is made between ripple heights predicted by equation (3.32) for a selection of the data of Scott [44] and Inman [30], which was not used for the determination of equation (3.32), and the measured ripple heights. As can be seen, the correlation is reasonably good.

It is not pretended that equation (3.32) yields the final solution for the prediction of the ripple height in a case with combined current and wave action. It does, however, clearly give a trend for a variation in the bed material size and hydraulic conditions respectively, which is in reasonable accordance with experimental evidence.

The ripple steepness can be found accurately enough for practical application from Figure 22, viz.:

\[
\frac{\Delta r}{\lambda r} = 0.31 \left( \frac{\Delta r}{a_o} \right)^{0.22}
\]

(3.37)
A combination of equations (3.19), (3.32) and (3.37) will now yield the value of the bed roughness \( r \) for areas with dominating wave action.

In areas where the longshore current velocity \( v \) is not negligible, when compared with the orbital velocity \( u_o \), it is possible that current ripples (or dunes) develop with their crests aligned in the onshore-offshore direction. Bijk [10] finds in his tests that three types of ripples can occur, viz.:

1) ripples having their crests normal to the direction of the main longshore current, called current-induced ripples;
2) ripples having their crests normal to the direction of wave propagation, called wave-induced ripples;
3) ripples forming a cross pattern or alternating normal to the direction of the main current and wave propagation, called combination-type ripples.

From the above-mentioned tests of Bijk he concluded:

1) current ripples occur for values of \( u_o / v_{MC} < 6 \)
2) wave ripples occur for values of \( u_o / v_{MC} > 20 \)
3) combination-type ripples occur for \( 6 < u_o / v_{MC} < 20 \).

For all the observations used in the determination of equation (3.32) the value of \( u_o > 20 v_{MC} \). In order to determine the ripple height for cases in which \( u_o < 6 v_{MC} \), the following approximation is suggested:

For a plane bed the resistance to the flow is given by

\[
\tau_c = \tau'_c
\]  

(3.38)

where:

\( \tau'_c \) = shear stress due to grain roughness under uniform flow conditions; it is often referred to as the surface drag.

For a bed with bedforms (either ripples or dunes) superimposed, the resistance to the flow is given by

\[
\tau_c = \tau'_c + \tau''_c
\]  

(3.39)

where:

\( \tau''_c \) = additional shear stress due to the bedforms, frequently referred to as the form drag.
Equation (3.39) can be rewritten as:
\[
v_{hc}^2 = (v'_{hc})^2 + (v''_{hc})^2
\]  
(3.40)

where:
\[
v'_{hc} = \frac{1}{\gamma}v/C'_h = \text{shear velocity due to grain roughness}
\]  
(3.41)
\[
C'_h = 18 \log \left(12 \frac{d}{D_{65}}\right) = \text{Chézy-coefficient for flow over a plane bed;}
\]  
(3.42)
\[
D_{65} \text{ was chosen as representative diameter after Einstein}
\]
\[
v''_{hc} = \frac{1}{\gamma}v/C''_h = \text{shear velocity due to bedforms}
\]  
(3.43)
\[
C''_h = \text{a Chézy-coefficient due to the existence of bedforms.}
\]
\[
C''_h \text{ is unknown, and should be found to solve the problem of the roughness values for uniform flow conditions.}
\]

Substitution of equations (3.41) and (3.43) into equation (3.40) yields eventually:
\[
\left(\frac{1}{C'_h}\right)^2 = \left(\frac{1}{C'_h}\right)^2 + \left(\frac{1}{C''_h}\right)^2
\]  
(3.44)

It has been observed [23] that bedforms change if the rates of sediment transport change. It is therefore likely that a certain relationship will exist between the flow resistance due to bedforms and the total sediment transport. Einstein [17], [18] suggested that:
\[
\frac{v}{v_{hc}} = F_1(\psi_{35})
\]  
(3.45)

where:
\[
\psi_{35} = \Delta_s \frac{D_{35}(C'_h)^2}{v^2}
\]  
(3.46)

Equation (3.45) was extended by Shen [45], who suggested that the particle Reynolds number \( R_e = \omega D_{50}/v \) should also enter the relationship, i.e.
\[
\frac{v}{v_{hc}} = F_2(\psi_{35}, R_e)
\]  
(3.47)
Both flume and field data were correlated to equation (3.47), finally yielding for \( R_e \leq 100 \), i.e. for \( D_{50} \leq 850 \times 10^{-6} \) m:

\[
\frac{v''_{NC}}{v} = \log_{10} \left\{ 1.07 \left( \psi_{35} R_e^{-\frac{1}{4}} \right)^{0.11} \right\}
\]  

(3.48)

Combination of equations (3.43) and (3.48) yields:

\[
\left( \frac{1}{C_h} \right) = g^{-\frac{1}{3}} \log_{10} \left\{ 1.07 \left( \psi_{35} R_e^{-\frac{1}{4}} \right)^{0.11} \right\}
\]

(3.49)

As both terms on the right hand side of equation (3.44) are now known in terms of the hydraulic conditions and the grain size, it is possible to solve for \( C_h \). The bed roughness \( r \) can be found from:

\[
C_h = 18 \log_{10} \left( \frac{12d}{r} \right)
\]

It should be remembered that this approach is limited to \( u_o/v_{NC} < 6 \). As \( v_{NC} \) is in itself a function of \( r \), an iteration will be necessary.

If \( 6 < u_o/v_{NC} < 20 \) it is suggested to determine the bed roughness as the highest value found from (3.19) and (3.50).

As will be shown in Section 4.3, the ratio between the bed load and the suspended load is a function of the relative roughness \( r/\Delta_r \), consequently it will also be necessary to know the bedform length \( \lambda_r \), before equation (3.19) can be applied. In the case of \( u_o/v_{NC} < 6 \), it is suggested to compute \( \lambda_r \) from the following approximate equation (3.50) which can be found from a study of data given by Anderson [3]:

\[
\frac{\lambda_r}{d} = \exp \left\{ 3.12 \left( \frac{v}{(gd)^{\frac{1}{3}}} \right) - 1.28 \right\}
\]  

(3.50)

The curve given by equation (3.50) is shown in Figure 26b, together with the data used for its determination. Equation (3.50) also shows a reasonable comparison with the data given by Kennedy [35] (Figure 26a).

It should be stated here that the above analysis of the bedform geometry under essentially uniform flow conditions should only be regarded as a first, rather pragmatic, approximation. Further work, e.g. in separating dune and ripple conditions, is urgently needed. For a further discussion see Section 4.4.
3.5 Discussion of results

The three approaches outlined in Section 3.2 were written in the form given below, whereafter the coefficients \( e_1 \), \( m_1 \) and \( b_1 \) were determined by a non-linear least-square computer programma.

1) Rouse-Bijker

\[
\frac{c - c_a}{c_a} = \left( \frac{z}{d} \right) \left( \frac{a}{d} \right) e_1
\]

(3.51)

2) Coleman

\[
\frac{c - c_a}{c_a} = \exp \left\{ -m_1 \left( \frac{z}{d} - \frac{a}{d} \right) \right\}, \text{ with } m_1 = \frac{wd}{e_a}
\]

(3.52)

3) Bhattacharya

\[
\frac{c - c_a}{c_a} = \left( \frac{z}{d} \cdot \frac{d}{a} \right)^{-b_1}, \text{ with } b_1 = \frac{wa}{c_a}
\]

(3.53)

The computed values of \( e_1 \), \( m_1 \) and \( b_1 \) are listed in Table III along with the boundary conditions of each of 86 profiles.

In order to enable the evaluation of the goodness of fit of the three approaches the following procedure was adopted:

1) A root-mean-square error \( \varepsilon_{\text{rms}} \) was computed from the summation of the squares of the differences between the computed and measured values of the logarithms of the relative concentrations

\[
\varepsilon_{\text{rms}} = \left[ \frac{\sum_{i=1}^{Nu} \left( \log \left( \frac{c}{c_a} \right)_{\text{computed}} - \log \left( \frac{c}{c_a} \right)_{\text{measured}} \right)^2}{Nu} \right]^\frac{1}{2}
\]

(3.54)

where:

- \( Nu \) = number of measurements in a concentration profile.

The logarithms were chosen as a result of the large variation in \( c/c_a \)-values, while both small and large values are of importance.
2) A relative error \( \varepsilon_r \) was defined, such that

\[
\varepsilon_r = 10^\varepsilon_{\text{rms}}
\]  \hspace{1cm} (3.55)

\( \varepsilon_r \) is in actual fact the ratio between the mean value of the data plus its scatter (computed from \( \varepsilon_{\text{rms}} \)) and the mean value of the data itself, in every point along the line of \( c/c_a \) vs \( z/d \) (i.e. \( c_{\text{mean}} + c_{\text{scatter}} / c_{\text{mean}} \)). If \( \varepsilon_r = 1 \), the data exhibit no scatter.

The relative errors are listed in Table V. A study of this table reveals that the Bhattacharya-method yields the smallest relative error, if averaged over all 86 profiles, second-best is the Rouse-Bijker-method while the Coleman-method had the biggest mean relative error. It should, however, be stressed, that the differences between the methods are small. The present results are not sufficient to disregard any one approach as yielding inferior results; in fact, the study was crowned by the same success, as reported earlier by Kennedy and Locher [37] and Das [14]. The results of the present study are, however, based on a comparison of a larger number of concentration profiles.

A detailed study of the relative errors, listed in Table V, furthermore reveals that the Bhattacharya-method had the smallest relative error for 50 % of the profiles while the corresponding figures for the Rouse-Bijker and Coleman-approaches are 22 % and 28 % respectively. In Figure 27 two examples are given of concentration profiles for which Bhattacharya gives the best fit. As stated above, 50 % of the profiles were of this type. In Figures 28 and 29 two examples each are given of concentration profiles for which Coleman and Rouse-Bijker respectively yield the best fit; 28 % and 22 % respectively of the profiles were of these types.

From the evaluation above, it appears that the Bhattacharya-method yields good results over a wide range of boundary conditions (see Section 3.3 for the boundary conditions). This fact, together with the knowledge that an equation of the form of equation (3.53) will lead to a much simpler relationship for the determination of the bed load, once the total load is known from the White-Ackers-approach, prompted the choice of the Bhattacharya-approach (equation (3.13)) for the description of the vertical distribution of suspended sediment.
The theoretical form of the term \((wa/e_a) = b_1\) in equation (3.13) is unknown; to determine which variables will be of importance for the determination of \(b_1\), a comparison will be made between the formulae of Rouse-Bijker and Bhattacharya. It will be assumed that both formulae will yield the same mean concentration over the depth in the optimal case, where both \(e_1\) and \(b_1\) have been determined correctly.

For Bhattacharya it then follows that:

\[
\frac{c}{c_a} = \frac{1}{d-a} \int_a^d \frac{c}{c_a} \, dz = \frac{1}{d-a} \int_a^d \left( \frac{z}{a} \right)^{-b_1} \, dz
\]

(3.56)

If \(b_1 \neq 1\), the solution of equation (3.56) is:

\[
\frac{c}{c_a} = \left( \frac{a}{d-a} \right) \left( \frac{1}{1-b_1} \right) \left( \frac{z}{a} \right)^{-b_1+1} \int_a^d \left( \frac{a}{d} \right)^{b_1-1} \left\{ \frac{a}{d} \right\}
\]

(3.57)

For \(b_1 = 1\), the solution of equation (3.56) is:

\[
\frac{c}{c_a} = \frac{a}{d-a} \ln \left( \frac{d}{a} \right)
\]

(3.58)

The solution is more complicated in the case of Rouse-Bijker:

\[
\frac{c}{c_a} = \frac{d}{d-a} \int_{a/d}^1 \left[ \frac{(1-z/d)}{(z/d)} \left( \frac{a}{d} \right)^{e_1} \right] \frac{d}{d} \left( z/d \right)
\]

(3.59)

The solution to equation (3.59) can be written as:

\[
\frac{c}{c_a} = \left( \frac{d}{d-a} \right) \left( \frac{a/d}{1-a/d} \right) \left( \frac{E(e_1)-1}{E(e_1)-e_1} \right) \sum_{n=0}^{\infty} \left( \frac{1-a/d}{e_1-n} \right) \left( \frac{e_1-n}{e_1-n-1} \right) \\
+ \frac{e_1}{E(e_1)-e_1} \int_{a/d}^1 \left( \frac{1-z/d}{z/d} \right)^{e_1-E(e_1)} \, d(z/d)
\]

(3.60)

where:

\[
\sum_{n=0}^{\infty} \left( \frac{m-e_1}{e_1-n} \right) = -e_1 (1-e_1) (2-e_1) \ldots \ldots (n-e_1)
\]

(3.61)

\(E(e_1)\) = the highest integer for which \((E(e_1)-e_1)\) is still positive \((E(2.1) = 2)\).
If $e_1-E(e_1) \ll 1$ and $a/d << 1$ the remaining integral in equation (3.60) can be approximated by:

$$\int \frac{(1-z/d)}{z/d} \frac{e-E(e_1)}{d(z/d)} \frac{z}{d} \approx \frac{(e_1-E(e_1))\pi}{\sin ((e_1-E(e_1))\pi)}$$

For all other cases the integral will have to be solved numerically. However, a comparison of equation (3.57) (or (3.58)) and (3.60) leads to the conclusion that:

$$b_1 = F(e_1, \frac{a}{d})$$

The logical next step will now be to assume that for $e_1$ can be substituted its theoretical value $(\nu/\kappa v_{wrc})$. A study of Figure 30, however, reveals that this is, in any case for the present data, not the case. There are various reasons for this discrepancy between theory and data. Except for those listed in Section 3.1, the following possible reasons should also be mentioned:

1) The data might be incomplete, the existence of an ocean current might not have been mentioned in the case of a prototype situation.

2) In the prototype the hydraulic conditions change continuously, while the observations of hydraulic conditions are frequently done only in the day-light hours, or are done visually (wave heights, for instance). It is accordingly possible that the given hydraulic boundary conditions are not representative for those yielding the sediment suspension.

3) The given grain size of the bed material might not be representative for the material in suspension.

4) The ripple height might not have been predicted correctly, either due to deficiencies in the predictive formulae, or due to incorrect hydraulic boundary conditions. The ripples can also have been formed by a previous set of hydraulic conditions.

In river flow problems there exists some correlation between the theoretical and observed values of $e_1$. This correlation can be approximated by:

$$e_{ld} = \frac{5}{6} \left( \frac{w}{\kappa v_{w}} \right)$$

(see [23], Figure 8.5)

where:

$e_{ld}$ = observed value of $e_1$

$\kappa$ = value of the von Karman constant, computed from the observed velocity distribution; $\kappa < 0.384$. 

\( \kappa \) decreases for increasing sediment concentration, viz.

\[ \kappa = 0.384 \left( \frac{1 + \Delta \frac{c_m}{c_o}}{1 + 2.5 \frac{c_m}{c_o}} \right) \]  

(3.65)

where:

- \( c_m \) = mean concentration, averaged over the depth
- \( c_o \) = mean concentration at the bed.

The highest concentration observed in the 86 concentration profiles listed in Table III, is 1.8 \% by volume. The effect of the sediment concentration on the value of \( \kappa \) will in general be negligible in the coastal environment for normal sediments.

In order to keep the effect of possible error in the given boundary conditions on the determination of the values of \( b_1 \) to a minimum, it was decided to use the observed values of both \( e_1 \) and \( b_1 \), to determine the relationship in equation (3.63). In first instance a selection of 51 of the available \( b_1 \)-values were made, for which the correlation of the form of equation (3.63) was made. The relationship was found to be:

\[ b_1 = 1.25 \frac{e_{1d}}{(R/d)} + 0.0152 \]  

(3.66)

The data are given in Figure 31.

Afterwards, equation (3.66) was used to determine the theoretical \( b_1 \)-value for the remaining distributions of suspended sediment. The result of this computation is shown in Figure 32. It is clear that the correspondence between computed and measured \( b_1 \)-values is exceptionally good.

Of the original 86 distributions of suspended sediment which were available 12 were discarded before the above-mentioned evaluation leading to Figures 31 and 32. The reason for not using these points, is that only 3 or 4 measurements of the concentration were available over the water depth, with as a consequence that the resulting \( b_1 \) - (and \( e_1 \)-) values had a large standard deviation.

Based on the river flow experience, as given by equation (3.64), equation (3.66) will be written as:
\[ b_1 = 1.25 \left( \frac{5}{6} \left( \frac{w}{\kappa v_{\text{MWC}}} \right)^{\frac{1}{6}} \right) \left( \frac{r}{d} \right) \]

\[ 0.96 \left( \frac{w}{\kappa v_{\text{MWC}}} \right)^{\frac{1}{6}} \left( \frac{r}{d} \right) = 1.05 \left( \frac{w}{\kappa v_{\text{MWC}}} \right)^{\frac{1}{6}} \left( \frac{r}{d} \right) \]  

(3.67)

where:
\[ \kappa = 0.384 \]

\[ v_{\text{MWC}} = \text{shear stress velocity under combined current and wave action given by equation (2.13)}. \]

Equation (3.67) is suggested for the prediction of the value of \( b_1 \) under marine conditions. It can also be used in tidal estuaries, where the wave height has diminished to zero, in which case \( v_{\text{MWC}} = v_{\text{HC}} \).
4 Bed load transport

4.1 General

The procedure adopted in the present report for the build-up of the computation of longshore transport is, as has been stated earlier, the inverse of that initially proposed by Bijker [10]. Whereas Bijker used the bed load as basis for the transport equations, the total load is used as starting point in the present study. In Section 4.3 the bed load will be calculated in terms of the total load, by using the description of the vertical distribution of suspended sediment, found in Chapter 3, to give the best correlation with the available data. In order to do this, a decision must firstly be made regarding the thickness of the layer, in which the sediment transport will take place as bed load transport. This will be done in Section 4.2.

Finally a discussion of the obtained result will be given in Section 4.4, with special reference to the tests of Bijker [10].

4.2 Bed layer thickness

The choice of a bed layer thickness is complicated by the fact that no clear division can be observed between the sediment transported as bed load and that transported as suspended load. In fact, no clear definition can be given of the bed load, and thus also not of the bed layer thickness. In the present study bed load will be defined as that part of the transported sediment of which particles can be exchanged easily with those on the bed. In the case of non-uniform conditions the transport in this layer of readily-exchangeable particles will be adapted nearly simultaneously to a change in the boundary conditions.

Observations have learned that sediment transport in small-scale models can be classified into a few clearly different modes, viz.:

1) Sediment which is being moved close to the bed, either saltating or sliding.
2) During the passing of the wave crest, when relatively large landward-directed orbital velocities occur, small vortices (containing sand in suspension) are formed on the downstream (landward) side of the ripples in a zone with its upper limit a distance equal to approximately the ripple amplitude above the seabed. When the wave through passes, on the other hand,
the seaward-directed velocity, which is mostly lower than the landward-directed velocity under the crest, but which occurs during a longer period of time, carries away the vortices in seaward direction. In a turbulent boundary layer above the above-mentioned vortices, with a thickness of a few ripple heights, the vortices are diffused.

3) Above the turbulent boundary layer the sediment transport can be characterized as convection transport, i.e. it can be assumed that the sediment particles are transported with the water velocity. Inside the breaker zone this might be an oversimplification of the actual phenomenon, partly due to the unknown turbulence pattern and partly due to the presence of trapped air.

The sediment transport in the prototype can be classified into the same modes, with this difference that the vortex layer described in 2) above, will be smaller than in the model, when measured relative to the water depth. As the sediment contained in the vortices can be easily exchanged with that on the bed, it seems physically justified to choose as the bed layer the layer between the top of the vortex dissipation layer, described under 2) above.

A careful study of the concentration profiles for small-scale model conditions reveals that close to the bed a tendency exists in some of the profiles for the concentration to show a smaller variation with depth than predicted by an equation of the type of equation (3.53), where \( b_1 \) is a constant over the full water depth (see for instance Figure 28b). The same tendency should exist inside the vortices, where the variation with depth of the concentration can be expected to be smaller than in the area above the vortex layer.

Computations show that for the 11 concentration profiles, measured in the model, in which the deviating tendency can be observed, the \( r \)-values as computed from equations (3.19) and (3.32) are of the same order of magnitude as the area in which the deviation occurs. In fact

\[
\frac{z_{dt}}{r} = 1.12
\]

(4.1)

with a standard deviation of \( z_{dt}/r \) of 71%.

\( z_{dt} \) = thickness of the layer in which the deviating tendency in the concentration can be observed.
Hulsbergen [27] reports an analysis of some frames out of a movie picture, made of the suspended sediment in motion above a rippled sand bed. As these pictures show quite clearly the vortices mentioned above, they are reproduced in this report. Furthermore, as the ripple dimensions are known from the photographs, it is possible to apply equation (3.19) to determine the relative roughness \( r/A_R \). The ripple steepness is 0.177, consequently the relative roughness \( r/A_R = 4.42 \), measured relative to the mean bed level. As can be seen in Photograph 1, the \( r \)-value has the same order of magnitude as the top of the vortices.

Hulsbergen analysed 12 photographs of the type shown in Photograph 1, which were taken at regular intervals during one wave period \((T = 1.49 \text{ s})\), with the aid of a photo-scanner, in order to determine the relative concentration. The average values of the relative concentrations obtained in this manner are given in Figure 33. It can be seen that the concentration profile deviates slightly from the straight line according to equation (3.53) (on a double-logarithmic plot) in the region close to the bed. The area in which the deviating tendency is observed, has a size which is comparable to the bed roughness \( r \).

It should be mentioned at this stage that it is possible that this deviating tendency can be the result of an inaccurate determination of high sediment concentrations. However, even if this should be the case, it is not of importance in the present investigation. The most important conclusion from the information presented herein is, that the height above the bed of the top of the vortex layer is of the same order of magnitude as the roughness value computed from equations (3.19), (3.32) and (3.37). As a first approximation a layer with a thickness equal to the bed roughness \( r \) (computed from the last-mentioned equations) will be chosen as the bed layer.

4.3 Vertical distribution of sediment transport

Bijker [11] assumed the sediment concentration to be a constant inside his bed layer, which had a thickness equal to one half of the ripple height \((r = \frac{1}{2}A_R)\). In the present study the bed roughness \( r \) is found to vary between 2 and 5 times the ripple height. In order to obtain an indication of the variation of the concentration inside the bed layer \((z < r)\) the top of the bed layer was drawn in on the Figure 27 ... 29. Keeping in mind that the concentration profile of the type shown in Figure 27 was observed in 50% of the
cases, it must be concluded that in general equation (3.53) also applies
within the bed layer. No measurements of sediment concentration lower than
the ripple crest are available. It seems, however, that the concentration in
the area between the ripple crest and the fictitious bed will not follow
equation (3.53). Under certain circumstances it is even possible that the
concentration might decrease with decreasing distance from the bed, for
instance, when the point under consideration falls below a vortex filled
with sediment. Consequently, it was decided to assume the concentration
to have a constant value equal to $c_{z_a}$ (i.e. the value of $c$ at $z = z_a$), according
to equation (3.53) in the area below the ripple crest. The concentration
profile will thus be schematically represented as given in Figure 34.

The variation with depth of the longshore current velocity $v_z$ will be assumed
to be logarithmic, as for uniform current flow. Furthermore it will be assumed
that the sediment particle velocity will be equal to the water velocity; the
sediment transport $S$ between any two elevations $z_1$ and $z_2$ can then be found
from:

$$S = \int_{z_1}^{z_2} v_z c_z \, dz$$  \hspace{1cm} (4.2)

where:

$$v_z = \frac{\nu c}{k} \ln \frac{30.2z}{r}$$ \hspace{1cm} (4.3)

$$c_z = c_r \left(\frac{z}{r}\right)^{-b_1}$$ \hspace{1cm} (4.4)

An exception to the above-mentioned approach will be made inside the bed
layer, i.e. for $z < r$. As has been shown in Section 4.2, the water movement
in this area is rather complex, and is built up mainly of vortices. The
physical meaning of an integral such as equation (4.2) is then not clear. For
this reason the transport in the bed layer will be computed from

$$S_b = \bar{v}_b \bar{c}_b \, r$$ \hspace{1cm} (4.5)

where:

$S_b$ = sediment transport in the bed layer/unit width of flow

$\bar{v}_b$ = mean velocity over the depth in the bed layer

$\bar{c}_b$ = mean concentration over the depth in the bed layer.
Bed load

The mean velocity in the bed layer, \( \bar{v}_b \), can be computed from equation (4.3):

\[
\bar{v}_b = \frac{1}{r} \left\{ \int_r^0 v_z \, dz + \frac{1}{2} \nu \frac{e_r}{30.2} \left( \frac{e_r}{30.2} \right) \right\} \tag{4.6}
\]

where:

\[
\frac{\nu \, e_r}{30.2} = \frac{\nu \, \kappa_c}{\kappa} \quad \text{from equation (4.3)} \tag{4.7}
\]

Integration of equation (4.6) yields:

\[
\bar{v}_b = 6.27 \, \nu \, \kappa_c \tag{4.8}
\]

when \( \kappa = 0.384 \) is substituted in the solution.

The mean concentration \( \bar{c}_b \) in the bed layer can be computed from equation (4.4):

\[
\bar{c}_b = \frac{1}{r} \left[ c_r \left( \frac{r}{\Delta_a} \right)^{-b_1} \int_a^r \left( \frac{z}{r} \right)^{-b_1} \, dz + c_{\Delta_a} \Delta_a \right] \tag{4.9}
\]

where:

\( c_r \) and \( c_{\Delta_a} \) are the values of \( c_z \) at \( z = r \) and \( z = \Delta_a = \frac{1}{2} \Delta_r \) respectively.

For the solution of equation (4.9) two different cases can be discerned, viz.:

When \( b_1 = 1 \):

\[
\bar{c}_b = c_r \ln \left( \frac{e_r}{\Delta_a} \right) \tag{4.10}
\]

\[
= K^{(1)}_b c_r \tag{4.11}
\]

where:

\( K^{(1)}_b = \ln \left( \frac{e_r}{\Delta_a} \right) \) = the factor by which the mean concentration \( \bar{c}_b \) in the bed layer exceeds \( c_r \), for the case when \( b_1 = 1 \). \tag{4.12}

When \( b_1 \neq 1 \):

\[
\bar{c}_b = \frac{c_r}{1-b_1} \left[ 1-b_1 \left( \frac{\Delta_a}{r} \right)^{1-b_1} \right] \tag{4.13}
\]
\[ K_b^{(2)} = K_b^{(2)} c_r \]  \hspace{1cm} (4.14)

where:

\[ K_b^{(2)} = \frac{1}{1-b_1} \left[ 1-b_1 \left( \frac{\Delta_a}{r} \right)^{1-b_1} \right] \]  \hspace{1cm} (4.15)

In general it can thus be written that:

\[ c_b = K_b c_r \]  \hspace{1cm} (4.16)

where:

\[ K_b = \ln \left( \frac{e}{r} \right) \text{ for } b_1 = 1 \]  \hspace{1cm} (4.12)

and

\[ K_b = \frac{1}{1-b_1} \left\{ 1-b_1 \left( \frac{\Delta_a}{r} \right)^{1-b_1} \right\} \text{ for } b \neq 1 \]  \hspace{1cm} (4.15)

\[ K_b \] is always bigger than unity.

Combination of equations (4.5), (4.8) and (4.16) now yields the bed load in terms of the sediment concentration \( c_r \) at the top of the bed layer, viz.:

\[ S_b = \bar{v}_b \bar{c}_b r \]
\[ = 6.27 K_b \bar{v}_w c_r r \]  \hspace{1cm} (4.17)

Comparison of this result with the expression for \( S_b \) found by Bijker, shows that it differs with a factor \( K_b \), due to the difference in the assumption on the variation of \( c_z \) inside the bed layer.

**Suspended load**

As stated above, the suspended load can be found from equation (4.2), i.e.:

\[ S_s = \int_r^d v_z c_z dz \]
\[ = \frac{v_w}{K} c_r \int_r^d \ln \left( \frac{30.2z_r}{r} \right) \left( \frac{z}{r} \right)^{-b_1} \]  \hspace{1cm} (4.18)

Again two cases can be discerned, viz.: \( b_1 = 1 \) and \( b_1 \neq 1 \).
For \( b_1 = 1 \) the solution of equation (4.18) after integration is:

\[
S_s = 1.3 \, \nu_{HC} \, c_r \, r \left[ \ln \left( \frac{912 d}{r} \right) \ln \left( \frac{d}{r} \right) \right]
\]

where:

\[
\frac{1}{2k} = 1.3.
\]

For \( b_1 \neq 1 \) the solution becomes:

\[
S_s = 2.6 \, \nu_{HC} \, c_r \, r \,(1-b_1)^2 \left[ (1-b_1) \left( \frac{d}{r} \right)^{1-b_1} \ln \left( \frac{30.2d}{r} \right) - 3.4 \right]
\]

\[
+ \left[ 1 - \left( \frac{d}{r} \right)^{1-b_1} \right] \] \quad (4.20)

The above-mentioned equations (4.19) and (4.20) can be written in general as:

\[
S_s = 6.27 \, K_s \, \nu_{HC} \, c_r \, r \quad (4.21)
\]

where:

\[
K_s = 0.205 \ln \left( \frac{912 d}{r} \right) \ln \left( \frac{d}{r} \right) \quad \text{for} \quad b_1 = 1 \quad (4.22)
\]

and

\[
K_s = 0.41 \,(1-b_1)^{-2} \left[ (1-b_1) \left( \frac{d}{r} \right)^{1-b_1} \ln \left( \frac{30.2d}{r} \right) - 3.4 \right]
\]

\[
+ \left[ 1 - \left( \frac{d}{r} \right)^{1-b_1} \right] \quad \text{for} \quad b_1 \neq 1 \quad (4.23)
\]

Combination of equations (4.17) and (4.21) yields the total load:

\[
S_t = S_b + S_s
\]

\[
= 6.27 \, \nu_{HC} \, c_r \, r \,(K_b + K_s) \quad (4.24)
\]

**Ratio bed load/total load**

In the present exercise the total load is known from equation (2.41) (or (2.43)) and the bed load is wanted. The bed load can be obtained by combining equations (4.17) and (4.24), viz.:
\[ S_b = \left( \frac{K_b}{K_b + K_s} \right) S_t \]  

(4.25)

\( K_b \) and \( K_s \) can be directly computed from equations (4.12), (4.15) and (4.22), (4.23) respectively.

4.4 Discussion

The values of \( b_1 \), \( \Delta_r \) and \( r \), and finally of \( S_{xb} \), were computed with the aid of the foregoing relationships, for the tests performed by Bijker [10]. The total loads as computed with White-Ackers (method 2) were used to determine the bed load. The results of these computations are given in Table VI. The mean ratio of \( (S_{xb}/S_t) \), as computed with the aid of equation (4.25), is 0.85, whereas it was 0.49 for the computations reported in Chapter 2, when the Bijker-approach for the computation of suspended load was used. This means that the amount of sediment in suspension as computed with the new method is roughly 3 times less than that computed with the old procedure. The most important reason for this decrease is the fact that the bed layer thickness used in the new approach is bigger than that used before.

The ripple heights, as computed with the aid of equation (3.32) and Figure 23 (or equation (3.43)), vary between 0.009 m and 0.028 m for the tests of Bijker. These computed ripple heights are realistic, when compared with the photographs of ripple patterns in Bijker's report [10], as well as with experience in coastal models. As a result of the relationship in equation (3.19) the roughness height \( r \) is between two and five times bigger than the ripple height. The roughness heights computed in this manner for Bijker's tests with combined current and wave action are on the average much bigger than the values of \( r \), computed by Bijker for the tests with current action only. These last-mentioned roughnesses ranged between 0.00013 m and 0.043 m. Due to the fact that the computed ripple heights have realistic values, and that the data leading to equation (3.19) are rather convincing, the roughness heights as computed with equation (3.32) and Figure 23, or with equation (3.43), were used. As the roughness heights used by Bijker to determine the limiting values of \( u_o/\nu_{MC} \) for the different modes of ripple formation, were lower than those from the equations in this report, it has been decided to adapt the limiting values accordingly. The criteria which are to be used are:

1) current ripples occur for \( u_o/\nu_{MC} < 3 \)
2) wave ripples occur for \( u_o/\nu_{MC} > 10 \)
3) combination-type ripples occur for \( 3 \leq u_o/\nu_{MC} \leq 10 \).
The ratio between all computed bed loads and all measured loads for Bijker's tests is 1.44, which means that the computed bed load overestimates the amount of sediment in the sand trap by about 40%. A certain amount of overestimation can be explained as follows.

The sand trap had a width of 0.15 m, while the current velocity in the reported tests \( v > 0.2 \) m/s. Accordingly, the mean velocity in the bed layer \( \bar{v}_b \) is usually more than 0.1 m/s. Since it is assumed that the sediment particles are moving with the water velocity in the bed layer, the time of passing over the sand trap will be \( t_s < 1.5 \) s. The particle fall velocity \( w = 0.025 \) m/s, with as a consequence that the particles can travel a vertical distance of approximately \( wt_s = 0.038 \) m during time \( t_s \). This implies that in cases where \( r > wt_s = 0.038 \) m only part of the sediment in the bed layer can settle in the sand trap.
5 Application of the results

5.1 General

The results derived in the foregoing chapters can be used to compute both bed load and total load transport rates. In order to allow transport computations to be performed efficiently all the steps that will have to be performed to arrive at these transport rates, will be listed systematically below, together with a summary of the appropriate formulae. Firstly, the formulae leading to the computation of the total load, will be given, thereafter the bed load will be treated.

In the case of irregular waves it is proposed that the wave height used for the transport computations is the root-mean-square value, whereas the wave height for a ripple height computation is the significant value. The representative wave height for a transport computation will actually be determined by the local water movement and the water depth, and will vary with these conditions. For a further study of this aspect more data will be necessary than are available at this moment.

5.2 Computation of total load

Bedform characteristics

(1) Calculate the ripple height \( \Delta_r \) due to wave motion from equation (3.32)

\[
\frac{\Delta_r}{\Delta_o} = A_r \left( \frac{\Delta_o}{(D_{50})^\frac{1}{3}} \right)^{1.56}
\]

where:

\[
\Delta_o = u_{ov} t_{crest}
\]

(equation (3.26))

(5.1)

\[
u_{ov} = \frac{2\pi}{T} \frac{H(1-A_o)}{\sinh \{k(d+H(1-A_o))\}}
\]

(equation (3.24))

(5.2)

\[t_{crest} = \beta T\]

(equation (3.25))

(5.3)

\[A_o = \frac{1}{(\pi P)^\frac{3}{2}} \left\{ 1 - \frac{1}{8P} + \frac{1}{128P^2} \right\}
\]

(equation (3.27))

(5.4)
\[ \beta = \frac{1}{90} \arccos \left( A_o \frac{1}{2F} \right) \quad \text{(equation (3.28))} \quad (5.6) \]

\[ P = \frac{Q}{13.6} \quad \text{for } Q \geq 34.8 \quad \text{(equation (3.29))} \quad (5.7) \]

\[ = 1 + \frac{Q}{22.3} \quad \text{for } Q < 34.8 \]

\[ Q = \left( \frac{H}{d} \right) \left( \frac{\lambda}{d} \right)^2 \quad \text{(equation (3.30))} \quad (5.8) \]

\[ k = \frac{2\pi}{\lambda} \quad \text{(5.9)} \]

H and \( \lambda \) are computed from first order wave theory.

\( A_r \) can be determined either from the design curves in Figure 23, or from the following equations:

\[ A_r = \frac{C_p R e_o^{0.05}}{R e_{\infty}^{0.5}} \quad \text{(equation (3.33))} \quad (5.10) \]

where:

\[ C_p = 1.475 \exp \left\{ 0.645 - 9.2 \times 10^{-4} \left( \frac{\lambda}{D_{50}} \right)^{0.5} - (0.009 R e + 0.2) \exp(-0.008 R e) \right\} \quad \text{(equation (3.36))} \quad (5.11) \]

\[ R e = \frac{w D_{50}}{\nu} \quad \text{(5.12)} \]

\[ \log \left( \frac{1}{w} \right) = 0.447 \left( \log D_{50} \right)^2 + 1.961 \log D_{50} + 2.736 \quad \text{(equation (2.19))} \quad (5.13) \]

\[ \nu = 1.792 \times 10^{-6} \exp(-0.042 T^{0.87}) \quad \text{(for } 0 < T < 30^\circ \text{C)} \quad (5.14) \]

\[ R e_o = \frac{w(D_{50} - D_o)}{\nu} \quad \text{(equation (3.34))} \quad (5.15) \]

\[ R e_{\infty} = \frac{w(D_{50} + D_o)}{\nu} \quad \text{(equation (3.35))} \quad (5.16) \]

\[ D_o = 50 \times 10^{-6} \text{ m} \]

\[ D_{\infty} = 500 \times 10^{-6} \text{ m} \]
(2) Calculate the ripple steepness due to wave motion from equation (3.37):

\[
\frac{\Delta r}{\lambda r} = 0.31 \left( \frac{a_o}{a_o} \right)^{0.22}
\]  

(5.17)

(3) Calculate the bed roughness \( r \) from equation (3.19):

\[
r = 25 \Delta_r \left( \frac{\Delta r}{\lambda r} \right)
\]  

(5.18)

(4) Calculate the Chézy-roughness coefficient \( C_h \) and the wave friction factor \( f_w \) from the value of \( r \) given by equation (5.18)

\[
C_h = 18 \log \left( \frac{12d}{r} \right)
\]  

(5.19)

\[
f_w = \exp \left( -5.98 + 5.21 \left( \frac{a_o}{r} \right)^{-0.19} \right) \quad \text{for} \quad \frac{a_o}{r} > 1.57
\]

\[
= 0.30 \quad \text{for} \quad \frac{a_o}{r} < 1.57
\]

(equation (2.10))

(5.20)

where:

\[
a_o = \frac{H}{2 \sinh (kd)}
\]  

(5.21)

(5) The roughness calculated above is valid for \( u_o/v_{MC} > 10 \). In order to check if this is the case, compute \( u_o/v_{MC} \):

\[
u_o/v_{MC} = \left( \frac{\pi H}{T \sinh (kd)} \right) \left( \frac{C_h}{v g} \right)
\]  

(5.22)

If \( u_o/v_{MC} < 10 \), the \( r \)-value computed above is not necessarily valid. When the longshore current \( v \) is predominating \( r \) is calculated from:

\[
\left( \frac{1}{C_h'} \right)^2 = \left( \frac{1}{C_h} \right)^2 + \left( \frac{1}{C_h^*} \right)^2
\]  

(equation (3.44))

(5.23)

where \( C_h^* \), given by equation (5.19), is unknown because \( r \) is unknown.

\[
C_h' = 18 \log \left( \frac{12d}{D_{65}} \right)
\]  

(equation (3.42))

(5.24)

\[
\frac{1}{C_h^*} = g^{-\frac{1}{2}} \log_{10} \left( 1.07 \left( \psi_{35} R_e^{-0.11} \right) \right)
\]  

(equation (3.49))

(5.25)

\[
\psi_{35} = \Delta_s \left( \frac{D_{35} (C_h')^2}{v^2} \right)
\]  

(equation (3.46))

(5.26)
The current-induced ripple length for \( u_o / v_{\text{HC}} < 10 \) is given by

\[
\lambda_r = d \exp \left[ 3.12 \left\{ \frac{v}{(gd)^{1/3}} \right\} - 1.28 \right] \quad \text{(equation (3.50))} \quad (5.27)
\]

The value of \( \Delta_r \) can then be computed from equation (5.18).

For \( 3 < u_o / v_{\text{HC}} < 10 \), that value of \( r \), as computed from equations (5.1) ... (5.18) or equations (5.23) ... (5.26) respectively, must be chosen, which has the highest value. For \( u_o / v_{\text{HC}} < 3 \) the value of \( r \) from equations (5.23) ... (5.26) must be chosen. The values of \( C_h \) and \( f_w \) must be recalculated from equation (5.19) and (5.20) with the new \( r \)-value.

**Shear stress**

(6) Calculate \( \xi_J \) from equation (2.12):

\[
\xi_J = C_h \left( \frac{f_w}{2g} \right)^{1/3} \quad (5.28)
\]

(7) Calculate the shear-stress due to combined current and wave action from equation (2.13):

\[
v_{\text{HWC}} = v_g \left\{ 1 + \frac{1}{2} \left( \frac{u_o}{v} \right)^2 \right\}^{1/3} \quad (5.29)
\]

where:

\[
u_o = \frac{\pi h}{T \sinh (kd)} \quad (5.30)
\]

**Particle diameter**

(8) If the transport rate is to be calculated with a representative particle diameter, the \( D_{35} \) should be chosen as the representative diameter. The alternative is to compute the transport rate for each size fraction separately, by using the \( D_{50i} \) of the separate fractions, and to add the transport rates together afterwards. In the following it will be assumed that \( D_{35} \) is used; if that is not the case \( D_{50i} \) must be substituted for \( D_{35} \) and the addition done afterwards.
(9) Calculate a dimensionless particle diameter \( D_{gr} \) from equation (2.38):

\[
D_{gr} = \left( \frac{g \Delta_s}{v^2} \right)^{1/3} D_{35}
\]  

(5.31)

(10) Calculate the particle-dependent coefficients \( n, m, A \) and \( C \) from equations (2.34) ... (2.37):

\[
n = 1 - 0.2432 \ln \left( D_{gr} \right)
\]  

(5.32)

\[
m = \frac{9.66}{D_{gr}} + 1.34
\]  

(5.33)

\[
A = \frac{0.23}{D_{gr}} + 0.14
\]  

(5.34)

\[
C = \exp \left\{ 2.86 \ln \left( D_{gr} \right) - 0.4343 \left( \ln \left( D_{gr} \right) \right)^2 - 8.128 \right\}
\]  

(5.35)

**Total load**

(11) Calculate the sediment mobility \( F_{wc} \) from equations (2.39) and (2.42):

\[
F_{wc} = \frac{v^1 - \text{n} \left( \frac{v_{HWC}}{C_D} \right)^{1/2}}{g^{1-n} (\Delta_s D_{35})^{1/2}}
\]  

(5.36)

where:

\[
C_D = 18 \log \left( \frac{10d}{D_{35}} \right)
\]  

(equation (2.40))  

(5.37)

(12) The total sediment load/unit width of longshore current can now be computed from equation (2.41):

\[
S_{xt} = \left( \frac{1}{1-p} \right) v D_{35} \left( \frac{C_h}{g^2} \right)^n \left( 1 + \frac{u_{avg}}{c_{avg}} \right)^{-n/2} \frac{C}{A^m} \left( F_{wc} - A \right)^m
\]  

(5.38)

where: \( 1/(1-p) \) can be taken equal to 1.45. This corresponds to a porosity \( p = 0.31 \). In river flow porosities ranging between 0.35 and 0.40 are common, however, as the porosity is a function of the gradation of the sediment as well as of the pressure-fluctuation above the bed, it is felt that a porosity lower than 0.35 can be chosen for the coastal environment; \( p = 0.31 \) is based on measurements performed under laboratory conditions.
5.3 Computation of bed load

Sediment suspension

(1) The vertical distribution of suspended sediment is given by:
\[
\frac{c_z}{c_r} = \left(\frac{z}{r}\right)^{b_1}
\]
(equation (3.13)) \hspace{1cm} (5.39)

The value of power \(b_1\), which determines the vertical gradient in the suspension vertical, is found from:
\[
b_1 = 1.05 \left(\frac{w}{w_{wsc}}\right)^{0.96} \left(\frac{r}{d}\right)^{0.013} \left(\frac{w}{w_{wsc}}\right)
\]
(equation (3.67)) \hspace{1cm} (5.40)

Bed layer

(2) The bed load is assumed to take place in a layer equal to the roughness height \(r\) above the fictitious bed. The roughness \(r\) is computed as shown in Section 5.2.

(3) The transported sediment in this bed layer can be determined in terms of the water velocity and sediment concentration in the bed layer, viz.:
\[
S_{xb} = K_b (6.27 v_{wsc} c_r r)
\]
(equation (4.17)) \hspace{1cm} (5.41)

where:
\[
c_r = \text{the sediment concentration at the upper limit of the roughness layer; } c_r \text{ is unknown.}
\]

\[
K_b = \ln \left(\frac{er}{\Delta_a}\right) \hspace{1cm} \text{for } b_1 = 1
\]
(equation (4.12)) \hspace{1cm} (5.42)

\[
= \left(\frac{1}{1-b_1}\right) \left\{1-b_1 \left(\frac{\Delta_a}{r}\right)^{1-b_1}\right\} \hspace{1cm} \text{for } b_1 \neq 1
\]
(equation (4.15)) \hspace{1cm} (5.43)

\[
\Delta_a = \frac{1}{b_1} \Delta_r
\]
(5.44)

\(\Delta_r\) = the height of the bedform (ripples), measured from the fictitious bed, which lies at the midheight of the ripples.

\(K_b\) is always bigger than unity.
Suspended load

(4) The amount of sediment transported in suspension above the bed layer can also be related to the conditions at the upper limit of the bed layer, viz.:

\[ S_{xs} = K_s (6.27 \frac{v}{\mu_c} c_r r) \]  \hspace{1cm} \text{equation (4.21)} \hspace{1cm} (5.45)

where:

\[ K_s = 0.205 \ln \left( \frac{912d}{r} \right) \ln \left( \frac{d}{r} \right) \]  \hspace{1cm} \text{for } b \_1 = 1 \hspace{1cm} \text{equation (4.22)} \hspace{1cm} (5.46)

\[ = 0.41 (1-b_1)^{-2} \left[ (1-b_1) \left( \frac{d}{r} \right)^{1-b_1} \ln \left( \frac{30.2d}{r} \right) - 3.4 \right] \]

\[ + \left[ 1 - \left( \frac{d}{r} \right)^{1-b_1} \right] \]  \hspace{1cm} \text{for } b \_1 \neq 1 \\text{equation (4.23)} \hspace{1cm} (5.47)

Total load

(5) The total load is the sum of equations (5.41) and (5.45), i.e.

\[ S_{xt} = S_{xb} + S_{xs} = (K_b + K_s)(6.27 \frac{v}{\mu_c} c_r r) \]  \hspace{1cm} \text{equation (4.24)} \hspace{1cm} (5.48)

Bed load

(6) The bed load is obtained in terms of the total load from equations (5.41) and (5.48), viz.: 

\[ S_{xb} = \left( \frac{K_b}{K_b + K_s} \right) S_{xt} \]  \hspace{1cm} \text{equation (4.25)} \hspace{1cm} (5.49)

where:

\[ S_{xt} \] is given by equation (5.38).
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**Note:**
1. In the case of wave spectra the significant wave height was used.
2. In the case of wave spectra the period at the peak of the energy spectrum was used.

**Table III**
Boundary conditions - sediment concentration data
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Table IV Summary of measuring devices for sediment concentration
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Table V Sediment concentration: comparison of relative errors
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Table V Sediment concentration: comparison of relative errors (continuation)
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Table V Sediment concentration: comparison of relative errors (continuation)
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<td>0.878</td>
<td>4.80</td>
</tr>
<tr>
<td>335</td>
<td>0.34</td>
<td>0.0220</td>
<td>0.072</td>
<td>1.407</td>
<td>0.720</td>
<td>97.9</td>
</tr>
<tr>
<td>335'</td>
<td>0.34</td>
<td>0.0220</td>
<td>0.072</td>
<td>1.407</td>
<td>0.720</td>
<td>97.9</td>
</tr>
<tr>
<td>341</td>
<td>0.20</td>
<td>0.0188</td>
<td>0.091</td>
<td>1.986</td>
<td>0.955</td>
<td>7.37</td>
</tr>
<tr>
<td>341'</td>
<td>0.20</td>
<td>0.0188</td>
<td>0.091</td>
<td>1.973</td>
<td>0.954</td>
<td>7.92</td>
</tr>
<tr>
<td>342</td>
<td>0.20</td>
<td>0.0136</td>
<td>0.041</td>
<td>1.941</td>
<td>0.882</td>
<td>68.4</td>
</tr>
<tr>
<td>342'</td>
<td>0.20</td>
<td>0.014</td>
<td>0.041</td>
<td>1.941</td>
<td>0.882</td>
<td>68.4</td>
</tr>
<tr>
<td>344</td>
<td>0.27</td>
<td>0.0252</td>
<td>0.130</td>
<td>2.133</td>
<td>0.971</td>
<td>0.66</td>
</tr>
</tbody>
</table>

Table VI Computed bed load with Bhattacharya-method
PARTICLE FALL VELOCITY IN STILL WATER AT 20°C

DELFt HYDRAULICS LABORATORY
1. perpendicular wave attack (\( \phi = 0^\circ \))
2. oblique wave attack (\( \phi = 15^\circ \))
3. line A is best fit of the type
   \[ S_{\text{computed}} = K_1 S_{\text{measured}} \]
1. Perpendicular wave attack ($\theta = 0^\circ$)
2. Oblique wave attack ($\theta = 15^\circ$)
3. Line A is the best fit of the type
   $S_{\text{computed}} = K_1 S_{\text{measured}}$

$S_{\text{measured}}$ in $\text{m}^3/\text{m}\cdot\text{s}$

$S_{\text{xt}} [\text{B-F, 1}]$ in $\text{m}^3/\text{m}\cdot\text{s}$

$K_1 = 1$

$K_1 = 10.395$

BUKER - FRULINK TOTAL LOAD COMPUTATION,
METHOD 1

DELFt HYDRAULICS LABORATORY

R 968 FIG. 3
BUKER - FRULINK BED LOAD COMPUTATION, METHOD 2
DELFt HYDRAULICS LABORATORY

**Diagram:**

- 1 x perpendicular wave attack (\( \Phi = 0^\circ \))
- 2 o oblique wave attack (\( \Phi = 15^\circ \))
- 3 line (A) is best fit of the type
  \[ S_{\text{computed}} = K_1 S_{\text{measured}} \]

**Axes:**
- \( S_{\text{measured}} \) in \( \text{m}^3/\text{m/s} \)
- \( S_{x_b} \) in \( \text{m}^3/\text{m/s} \)

**Parameters:**
- \( K_1 = 1 \)
- \( K_1 = 9.980 \)
1. perpendicular wave attack (θ = 0°)
2. oblique wave attack (θ = 15°)
3. line (A) is best fit of the type
   \[ S_{\text{computed}} = K_1 S_{\text{measured}} \]
1 x perpendicular wave attack (\( \Phi = 0^\circ \))
2 o oblique wave attack (\( \Phi = 15^\circ \))
3 line (A) is best fit of the type
\[ S_{\text{computed}} = K_1 S_{\text{measured}} \]
1. x perpendicular wave attack (Φ = 0°)
2. o oblique wave attack (Φ = 15°)
3. line (A) is best fit of the type

\[ S_{\text{computed}} = K_1 S_{\text{measured}} \]
1. Perpendicular wave attack ($\theta = 0^\circ$)
2. Oblique wave attack ($\theta = 15^\circ$)
3. Line A is the best fit of the type $S_{\text{computed}} = K_1 S_{\text{measured}}$

WHITE - ACKERS BED LOAD COMPUTATION,
METHOD 1
DELFT HYDRAULICS LABORATORY
R 968 FIG. 10
1 x perpendicular wave attack ($\phi = 0^\circ$)
2 o oblique wave attack ($\phi = 15^\circ$)
3 line A is best fit of the type $S_{\text{computed}} = K_1 S_{\text{measured}}$

$S_{\text{measured}}$ in $m^3/m/s$ vs $S_{xt}$ [W-A, 1] in $m^3/m/s$
1 x perpendicular wave attack (θ = 0°)
2 x oblique wave attack (θ = 15°)
3 line (A) is best fit of the type

S_{computed} = K_1 S_{measured}

K_1 = 0.913
K_1 = 1

WHITE - ACKERS BED LOAD COMPUTATION,
METHOD 2

DELFt HYDRAULICS LABORATORY
1. x perpendicular wave attack (Φ = 0°)
2. o oblique wave attack (Φ = 15°)
3. line (A) is best fit of the type
   \[ S_{\text{computed}} = K_1 S_{\text{measured}} \]
### Distribution of Computed Load
For White-Ackers, Method 1

<table>
<thead>
<tr>
<th></th>
<th>Bed Load</th>
<th>Total Load</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>K₁</strong></td>
<td><img src="#" alt="Graph" /></td>
<td><img src="#" alt="Graph" /></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Size Interval</th>
<th>Bed Load</th>
<th>Total Load</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{8}$</td>
<td><img src="#" alt="Graph" /></td>
<td><img src="#" alt="Graph" /></td>
</tr>
<tr>
<td>$\frac{1}{4}$</td>
<td><img src="#" alt="Graph" /></td>
<td><img src="#" alt="Graph" /></td>
</tr>
<tr>
<td>$\frac{1}{2}$</td>
<td><img src="#" alt="Graph" /></td>
<td><img src="#" alt="Graph" /></td>
</tr>
<tr>
<td>$\frac{3}{4}$</td>
<td><img src="#" alt="Graph" /></td>
<td><img src="#" alt="Graph" /></td>
</tr>
<tr>
<td>$\frac{5}{8}$</td>
<td><img src="#" alt="Graph" /></td>
<td><img src="#" alt="Graph" /></td>
</tr>
<tr>
<td>$\frac{7}{8}$</td>
<td><img src="#" alt="Graph" /></td>
<td><img src="#" alt="Graph" /></td>
</tr>
</tbody>
</table>

**Remark:** $K₁ = \frac{S_{\text{computed}}}{S_{\text{measured}}}$

**Delft Hydraulics Laboratory**

**R 968**  **Fig. 14**

- **All Data**: 37 observations
- **Perpendicular Waves**: 22 observations
- **Oblique Waves**: 15 observations
Determination of Relative Roughness

DELTFT HYDRAULICS LABORATORY

R 968 FIG. 16
MOGRIDGE - KAMPHUIS DESIGN CURVES FOR BEDFORM

DELFT HYDRAULICS LABORATORY

R 968 FIG.17
\[
\frac{\Delta r}{\lambda_r} = 0.31 \left( \frac{\Delta r}{\delta_0} \right)^{0.22}
\]

- \text{Van Hijum } D_{50} = 250 \times 10^{-6} \text{ m}
- \text{Van Hijum } D_{50} = 480 \times 10^{-6} \text{ m}
- \text{Inman } 100 \times 10^{-6} \text{ m} < D_{50} < 300 \times 10^{-6} \text{ m}
- \text{Inman } D_{50} > 300 \times 10^{-6} \text{ m}
1. Computations performed with the aid of equations (3.32) ... (3.36)

2. o Inman [30] \( 81 \times 10^{-6} \text{ m} \leq D_{50} \leq 525 \times 10^{-6} \text{ m} 

- Van Hijum [29] \( D_{50} = 250 \times 10^{-6} \text{ m} 

x Van Hijum [29] \( D_{50} = 480 \times 10^{-6} \text{ m} 

COMPARISON OF MEASURED AND PREDICTED RIPPLE HEIGHTS

DELFt HYDRAULICS LABORATORY R 986 FIG. 24
Remarks: 1) computations performed with the aid of equations (3.32) ...(3.36)
2) x Inman [36] $118 \times 10^{-6}$ m $\leq D_{50} \leq 927 \times 10^{-6}$ m
o Scott [44] $D_{50} = 310 \times 10^{-6}$ m
● Horikawa et al [26] $D_{50} = 200 \times 10^{-6}$ m
D Delft Hydraulics Lab. [55] $D_{50} = 190 \times 10^{-6}$ m

RIPPLE HEIGHT PREDICTION FOR OTHER DATA

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RIPPLE LENGTH: CURRENT FLOW DATA

DELFIT HYDRAULICS LABORATORY

R 968 FIG.26
CONCENTRATION PROFILES OF BHATTACHARYA - TYPE

DELFt HYDRAULICS LABORATORY R 968 FIG. 27
CONCENTRATION PROFILES OF ROUSE / BUKER - TYPE

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R 968 FIG. 29
COMPARISON OF $\varepsilon_{\text{data}}$ WITH $\frac{W}{KV_{*\text{WC}}}$

DELFT HYDRAULICS LABORATORY

R 968 FIG. 30

- Bhattacharya [5] profiles 1..23
- Delft Hydraulics Lab. [55] 24
- Horikawa et al [26] 27,28
- Shinohara et al [47] 29.46
- Homma et al [24] 47.48
- Homma et al [25] 49.52
- Fukushima et al [21] profiles 53..61
- Homma et al [25] 62
- Jensen et al [31] 72
- Sato et al [43] 73.79
- Delft Hydraulics Lab. [56] 25, 26
- 80..86
Remarks:
   • Delft Hydraulics Lab [55] △ Homma et al [25]
   ✦ Shinohara et al [47] ▼ Jensen et al [31]
   + Homma et al [25] ○ Sato et al [43]
   ✗ Delft Hydraulics Lab [56]

2) $(b_1)_{\text{computed}}$ was calculated from equation (3.66)
Remarks:
1) ○ Bhattacharya [5]
   ● Horikawa et al [24]
   × Shinohara et al [47]
   ⊗ Homma et al [24]
   ● Sato et al [43]
   ★ Delft Hydraulics Lab. [56]
   ⊘ Basinski et al [4]

2) $(b_1)_{\text{computed}}$ was calculated from equation (3.66)

PREDICTION OF $b_1$ FOR OTHER DATA

DELFt HYDRAULICS LABORATORY
Direction of wave propagation

a) time = 0
computed roughness height
fictitious bed

b) time ≈ T/4
computed roughness height
fictitious bed

c) time ≈ T/2
computed roughness height
Vicarious bed

d) time ≈ 3T/4
computed roughness height
fictitious bed

Flow above a rippled sand bed [27]  Photograph 1