

Magnetovibrational coupling in small cantilevers

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A nanomagnetomechanical system consisting of a cantilever and a thin magnetic film is predicted to display magnetovibrational modes, which should enable applications for sensors and actuators. The “polaritonic” modes can be detected by line splittings in ferromagnetic resonance spectra. © 2003 American Institute of Physics. [DOI: 10.1063/1.1603338]

Two unconventional roads into the realm of nanoscale devices may lead to useful results in the not too distant future, *viz.* magnetoelectronics¹ and nanoelectromechanical systems (NEMS).² In magnetoelectronics, the spin degree of freedom of the electron is employed for new or extended functionalities, which can be used, e.g., in magnetic random-access memories,³ whereas advanced sensing and actuating have been realized by mechanical systems, though yet limited to the micrometer scale [microelectromechanical systems (MEMS)].⁴ With the notable exception of mechanically detected ferromagnetic resonance (FMR),⁵ these two fields have surprisingly little overlap. Here we demonstrate that a combination of a conventional nanomechanical structure, namely an elastic bar clamped on one side and freely vibrating on the other (“cantilever”), displays new physics when combined with a film of ferromagnetic material deposited on top of it. We find theoretically that the lowest vibrational modes of the cantilever are mixed with the coherent motion of the magnetic order parameter by the magnetic anisotropy. We can translate information carried by rf (microwave) fields at certain frequencies effectively into mechanical motion and *vice versa*. The present case study leads us to believe that nanoscale magnetomechanical systems deserve more attention.

Some time ago Benda⁶ predicted that FMR implies a mechanical torque. Observation of its effect on the mechanical motion requires small samples, however. Experimental proof of mechanical detection of FMR (as well as electron paramagnetic resonance) has therefore been obtained only recently.⁵ In these experiments the torque is detected by a static deflection of a cantilever in the presence of dc and ac magnetic fields. Here we study a similar system (sketched in Fig. 1) but concentrate on the high frequency regime, in which the magnetic and mechanical motion turn out to be coupled. Traditionally time scales of FMR [usually at gigahertz frequencies] and mechanical oscillations [in MEMS typically kilohertz] are different and this mode mixing cannot be observed. More recently, NEMS oscillate at megahertz frequencies or even GHz,⁷ which is accessible by FMR. The predicted effects can be measured for example by calorimetric techniques⁸ or, indeed, mechanically.⁵

We consider a small dielectric cantilever with a single-

domain ferromagnetic layer deposited on its far end (see Fig. 1). A constant external field \mathbf{H}_0 is applied along the y axis as well as an oscillating field H_x along the x axis. The effective field \mathbf{H}_{eff} felt by the magnetization consists of \mathbf{H}_0 as well as the crystal anisotropy and the demagnetizing fields. The strains are localized in the mechanical link of the cantilever between the ferromagnetic film at one end and the other end that is fixed. The lattice of the ferromagnet then oscillates without internal mechanical strains, but crystalline and form anisotropies couple the magnetic order parameter to the torsional mode of the cantilever (the longitudinal mode does not cause any internal magnetization dynamics).

The magnetization \mathbf{M} of the ferromagnet precesses around an effective magnetic field \mathbf{H}_{eff} according to the Landau–Lifshitz–Gilbert equation⁹

$$\frac{d\mathbf{M}}{dt} = -\gamma\mathbf{M}\times\mathbf{H}_{\text{eff}} + \alpha\mathbf{M}\times\frac{d\mathbf{M}}{dt}, \quad (1)$$

where γ denotes the gyromagnetic ratio. The phenomenological Gilbert constant is typically $\alpha \leq 0.01$. It will be disregarded in the following for simplicity, which is valid when the mechanical damping dominates. To first order, the deviations from the equilibrium magnetization in the y direction lie in the x - z plane: $\mathbf{M} = m_x\mathbf{x} + M_s\mathbf{y} + m_z\mathbf{z}$, where M_s is the saturation magnetic moment. Without loss of generality we may take the crystalline anisotropy field contribution H_A to be uniaxial and directed along the y axis of the system at rest. H_A rigidly follows the vibration of the crystal lattice described by the torsion angle φ (see Fig. 1). For small φ , the effective field oscillates in the y - z plane: $\mathbf{H}_{\text{eff}} = (H_A + H_0)\mathbf{y} + (H_A\varphi + \nu M_s\varphi - \nu m_z)\mathbf{z}$, where ν describes the demagnetizing dipolar field ($\nu \approx 4\pi$ for our geometry). Note that the coupling does not rely on a strong magnetocrystalline field

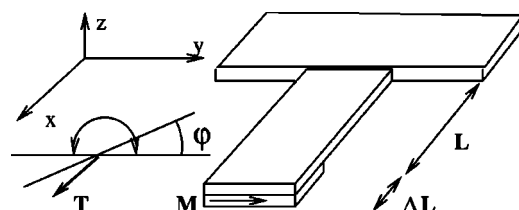


FIG. 1. A nanomagnetomechanical cantilever supporting magnetovibrational modes. On a dielectric substrate (such as Si) a single-domain ferromagnetic film is deposited at the free end.

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H_A , since the surface forces of demagnetizing currents also provide a restoring torque. For a thin ferromagnetic layer the latter dominates and the magnetization does not precess, but oscillates like a pendulum in the x - y plane due to the oscillating field in the z direction $\mathbf{H}_{\text{eff}} = \nu M_s \varphi \mathbf{z}$.

The torsional motion of the part of the cantilever that is not covered by the ferromagnet can be found applying the variational principle to the total elastic energy¹⁰

$$E_{\text{el}} = \frac{1}{2} \int_0^L C \tau^2 dx,$$

where $\tau = \partial\varphi/\partial x$ and C is an elastic constant defined by the shape and material of the cantilever ($C = \frac{1}{3}\mu da^3$ for a plate with thickness a much smaller than width d , $a \ll d$, μ is the Lamé constant). $T = C\tau(x)$ is the torque flowing through the cantilever at point x . The integration is taken from the clamping point $x=0$ until the cantilever endpoint $x=L$. The equation of motion reads

$$C \frac{\partial^2 \varphi}{\partial x^2} = \rho I \frac{\partial^2 \varphi}{\partial t^2} + 2\beta \rho I \frac{\partial \varphi}{\partial t}, \quad (2)$$

where $I = \int (z^2 + y^2) dz dy \approx ad^3/12$ is the moment of inertia of the cross section about its center of mass, ρ is the mass density, and β is a phenomenological damping constant related to the quality factor Q at the resonance frequency ω_e as $Q = \omega_e/(2\beta)$. The oscillating solution has the form $\varphi = \sin(kx)[A_1 \sin(\omega t) + A_2 \cos(\omega t)]$, where $k = \omega/c$ is the wave number, $c = c_t 2h/d$ and $c_t = \sqrt{\mu/\rho}$ is the transverse velocity of sound. The free constants A_1 and A_2 depend on the initial conditions. The clamping boundary condition $\varphi|_{x=0} = 0$ at the clamping point is already fulfilled, and the boundary condition at the end $x=L$ is discussed in the following.

The conservation law for the mechanical angular momentum \mathbf{V}^{el} for $x \in \{0, L\}$ (without magnetic overlayer) $d\mathbf{V}^{\text{el}}(x)/dt = \mathbf{T}(x)$, where the torque \mathbf{T} has been defined before, is modified by the coupling to the magnet in a region $x \in \{L, L + \Delta L\}$ (ΔL is the length of the cantilever covered by the magnetic layer) as

$$\frac{d}{dt} \left[\mathbf{V}^{\text{el}}(x) + \left(-\frac{\hbar}{2\mu_B} \right) \mathbf{M}(x) \right] = \mathbf{T}(x) + \mathbf{T}_{\text{field}}, \quad (3)$$

where $\mathbf{T}_{\text{field}} = (\hbar/2\mu_B) \gamma \mathbf{M} \times \mathbf{H}_0$ (μ_B is Bohr magneton). For small amplitudes, $V_x^{\text{el}} = N(d/dt)\varphi$ and $T_x = -K\varphi$, where N and K are geometry and material specific constants. Assuming $\Delta L \ll L$, internal strains in the magnetic section $x \in \{L, L + \Delta L\}$ are disregarded. The magnetovibrational coupling may then be treated as a boundary condition to the mechanical problem, which is expressed as the torque $C\tau|_{x=L}$ exerted by the magnetization on the cantilever

$$C\tau|_{x=L} = -\frac{\hbar}{2\mu_B} \gamma \mathbf{M} \times (\mathbf{H}_{\text{eff}} - \mathbf{H}_0)|_x - N \frac{d^2}{dt^2} \varphi. \quad (4)$$

By equating this with the substitution of φ in τ , $C\tau|_{x=L} = Ck \cos(kL)[A_1 \sin(\omega t) + A_2 \cos(\omega t)] = Ck\varphi \cot(kL)$, we obtain in frequency space

$$Ck\varphi \cot(kL) = \frac{\hbar}{2\mu_B} (-i\omega m_x + \gamma H_0 m_z) - \omega^2 N \varphi \quad (5)$$

$$\approx -C\varphi \frac{L}{2c^2} (\omega^2 + 2i\beta\omega - \omega_e^2), \quad (6)$$

where in the second line the cot has been expanded close to the resonance frequency $\omega_e = c\pi(1/2 + s)/L$ (s is an integer, here we concentrate mainly on coupling to the first harmonic and $s=0$). For negligible damping of the magnetization dynamics, the external rf field H_x does not create any torque along the x axis. N scales like $\Delta L/L$ and we therefore disregard the term proportional to N (as long as $N \ll CLd^2/2c^2$ it only slightly shifts the resonant frequency ω_e).

The magnetic susceptibility $\chi_\omega = (m_x/H_x)_\omega$ describes the linear response of the magnetization m_x to a (weak) rf magnetic field H_x at frequency ω and can be found after writing the Landau–Lifshitz equation in frequency space using Eq. (5):

$$\frac{\chi_\omega}{\gamma^2 M_s V} = \left\{ \frac{\omega^2 - \omega_m^2}{(H_A + H_0 + \nu M_s)} + \frac{\omega^2 G L \tan(kL)}{2kc^2(H_A + \nu M_s + H_0[1 - GL \tan(kL)/2kc^2])} \right\}^{-1}, \quad (7)$$

where $G = \hbar M_s V \gamma (H_A + \nu M_s) c^2 / (\mu_B CL)$ is the magnetovibrational coupling constant (V is the volume of the magnet) and the unperturbed resonance frequency $\omega_m^2 = \gamma(H_A + \nu M_s) \gamma H_A$. The imaginary part of χ_ω is proportional to the FMR absorption signal. In the absence of the external field H_0 the resonance frequencies in the vicinity of ω_e can be found after expanding $\cot(kL)$ as in Eq. (6):

$$\omega_{1(2)} = \sqrt{\frac{1}{2}(\omega_e^2 + \omega_m^2 + G \pm \{[(\omega_e + \omega_m)^2 + G][(\omega_e - \omega_m)^2 + G]\}^{1/2})^{1/2}}. \quad (8)$$

When the external field H_0 is smaller than the demagnetizing field, Eq. (8) holds with $\omega_m^2 = \gamma^2(H_A + H_0 + \nu M_s)(H_A + H_0)$. This fact can be used to tune the FMR frequency in order to match the elastic frequency. In this limit and $H_0 A / (M_s \beta \omega_e) \ll 1$, Eq. (7) simplifies to

$$\chi_\omega \approx \frac{\gamma^2 \nu M_s V (H_A + \nu M_s)}{\omega^2 - \omega_m^2 + \omega^2 G L \tan(kL) / (2kc^2)}. \quad (9)$$

Its imaginary part corresponding to the rf absorption is plotted in Fig. 2 for $G > 0$ (which is the case in our setup). We observe a typical anticrossing behavior between an optically active and nonactive mode, with level repulsion and transfer of oscillator strength. The intrinsic damping of the mechanical system (for MEMS Q factors can reach 10^4 , which quickly deteriorate with decreasing size, however) imposes an extra damping on the magnetization dynamics, which close to the mode crossing may dominate the intrinsic damping due to a small Gilbert constant α disregarded here.

The magnetovibrational coupling in our cantilever is observable by FMR, but the signal of nanoscale magnets is small. It might therefore be preferable to detect the resonance by the static deflection of the same cantilever due to an ad-

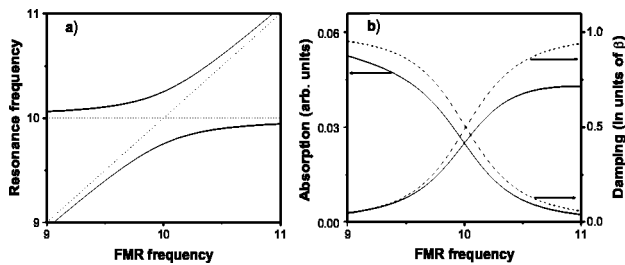


FIG. 2. (a) Dependence of the resonance frequency of the coupled motion on the FMR frequency ω_m of the uncoupled magnetization and (b) the corresponding oscillator strength of each resonance in arb. units plotted by full lines and its width plotted by dashed lines ($\omega_e=10$ MHz, $\Delta\omega\sim 500$ kHz, $\alpha=0$).

ditional constant magnetic field \mathbf{H}_T along the z axis, as described by Lohndorf *et al.*¹¹ \mathbf{H}_T creates a torque $\gamma M_y H_T \mathbf{x}$, whose modulation at the FMR conditions should be detectable.

Since the vibrational frequencies of state-of-the-art artificial structures are relatively low, the use of soft ferromagnets (such as permalloy) is advantageous. The magnetic mode frequencies are then determined by shape anisotropies. The FMR frequency and the mechanical resonance frequency should not differ by much more than $\Delta\omega\sim\sqrt{G}$ for a pronounced effect. A Si cantilever with $a\times d\times L=(1\times 5\times 50)$ μm ($C=10^{-13}$ N m²) has a torsional resonance frequency of the order of $\omega_e=10$ MHz. Taking our ferromagnetic layer of dimensions $a_1\times d\times \Delta L=50$ nm $\times 5$ $\mu\text{m}\times 5$ μm (thickness, width, and length), then $\sqrt{G}\sim 100$ kHz, meaning that we should tune the magnetic resonance to $\omega_e\pm 100$ kHz to observe the ‘‘polariton.’’ The necessary rf field H_x depends on the elastic and magnetization viscous dampings. At low frequencies additional sources of damping complicate measurements¹² and the coherent motion of the magnetization can be hindered by domain formation. Coupling to higher resonance modes¹³ or structuring of the ferromagnet may help to carry out measurements. Downscaling the device size is most favorable, since the mechanical resonance frequency increases inversely proportional to the scale coefficient, whereas the coupling constant scales as the second power of the inverse coefficient.

An actual observation of the predicted splittings would give information about, e.g., the magnetic moment of the film and the broadening would yield the quality factor of the elastic motion. From a technological point of view the tunable damping due to the magnetovibrational coupling might be interesting for optimizing switching speeds. A ferromagnet effectively absorbs microwaves and turns them into a precessing magnetization, which via the magnetovibrational coupling can be transformed into a coherent mechanical motion. On the other hand, the ferromagnet may interact with

the mechanical motion, to cause a magnetization precession, which in turn emits polarized microwaves. The emission in the coupled regime is more energy efficient in comparison with a fixed magnetic dipole emission in the case of small Gilbert constant but relatively low mechanical quality factor. This might be interesting for, e.g., on-chip communication applications.

The present device can be interpreted as a MASER source in which the microwave cavity is replaced by the mechanical resonator. After submission of the present manuscript (cond-mat/0303114), Bargatin and Roukes (cond-mat/0304605) worked out very similar ideas for nuclear spin systems. The magnetization reversal of the ferromagnetic tip for parameters mentioned above can be described by semiclassical Bloch–Maxwell laser equations¹⁴ with coupling constant $g\sim 10$ s⁻¹ and relaxation rate $\Gamma\sim\omega_m\alpha$, as will be explained in more detail elsewhere.

To summarize, we have calculated the magnetic susceptibility of a system with magnetovibrational coupling by magnetocrystalline fields or via surface forces of demagnetizing currents. A condition for effective energy transfer from an external rf magnetic field into mechanical motion and *vice versa* has been established. FMR spectra are predicted to split close to the resonance, and to strongly depend on the mechanical damping. The predicted effects should be observable with existing technology, but a further reduction of system size would strongly enhance them.

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