Topology Optimization of the Compliant Underactuated Finger with the Focus on Out-of-plane Stiffness

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Faculty of Mechanical, Maritime and Materials Engineering · Delft University of Technology
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Part I

Paper I
AN INVESTIGATION INTO THE EFFECTS OF CURVED BEAM ELEMENTS ON THE OUT-OF-PLANE STIFFNESS PROFILE IN A COMPLIANT LARGE-DISPLACEMENT FINGER

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ABSTRACT

Compliant mechanisms have many advantages over rigid-link mechanisms. However, one of the challenges of compliant mechanisms is the trade-off between a large range of motion and a high out-of-plane stiffness. Furthermore, the out-of-plane stiffness is shown to vary over the range of motion. Especially for large-displacement compliant mechanisms this can be by a significant amount. In this paper the use of curved beam elements in a compliant mechanism is shown to have impact on this trade-off. The influence of curved beam elements on the out-of-plane stiffness over the entire range of motion is presented for simple structures such as a single beam element and double beam elements, as well as a compliant finger. With the use of a genetic algorithm optimization, the difference in performance of a design with only straight beam elements versus one with curved beam elements is highlighted and the effect on the out-of-plane stiffness profile is presented. The optimization with curved beam elements results in solutions with a performance in terms of objective function values that cannot be found by the optimization with only straight beam elements. It is shown that for simple structures the use of curved beam elements has a large influence on the shape of the out-of-plane stiffness profile along the range of motion, while for the compliant finger the influence is mainly in the variables of the out-of-plane stiffness profile.

NOMENCLATURE

- \( c \) Curve of a beam element, [dimensionless]
- \( F_y, F_z \) Force applied on the structure in y- and z-direction respectively, [N]
- \( k_z \) Stiffness of the structure in z-direction, [mm]
- \( p_i \) Coordinates (x,y) of point \( i \), [mm]
- \( \text{RoM} \) Range of motion, [mm]
- \( t_y, t_z \) Thickness of the beam element in y- and z-direction respectively, [mm]
- \( U \) Deflection of the tip of the structure, [mm]
- \( s_{\text{Length}} \) Length of beam element, defined as distance between end point and start point of the beam element, [mm]

1 INTRODUCTION

Due to many advantages over rigid-link mechanisms, such as a reduction of friction and backlash and lower weights and costs, compliant mechanisms are becoming more popular. What distinguishes compliant mechanisms from rigid-link mechanisms is that they use the deflection of their flexible members to transfer motion, force or energy [1]. Their application can vary anywhere from small displacement micro mechanisms, to large displacement macro mechanisms such as a compliant finger [2].

Designing a compliant mechanism also poses some challenges. The stiffness (both bending and torsional) in one of its
directions is inevitably connected the (bending and torsional) stiffness in another direction. A simple beam is already a clear example of this, since changing the thickness in one direction will affect stiffness in all directions. While in some cases this is not a problem and may even be an advantage, it can be problematic when the mechanism should have a low bending stiffness in one direction and a high bending stiffness in the other directions [3], [4]. Furthermore, the bending stiffness in either direction can decrease or increase over the range of motion in the desired direction. Especially for large-displacement mechanisms this change can be considerable, affecting the accuracy of the mechanism [5], [6], [7], [8].

In this research the focus will lie on the out-of-plane stiffness of compliant mechanism. The out-of-plane stiffness is defined as the stiffness in the direction perpendicular to the plane in which the mechanism moves (see Fig. 1).

Research has already been conducted in which methods are provided to increase the out-of-plane stiffness. So far these methods mostly involve simple topology adjustments and size optimization. Simple topology adjustments are for example adding more beam elements [9], [5] or changing the structure from planar to spatial [10]. Often the out-of-plane stiffness of an existing structure is improved by optimizing the size of the separate elements [7], [3], [11], [6], [4]. In an article about elastic averaging Awtar [12] discusses the effect of the degree to which a beam is distributed (as opposed to lumped) on the out-of-plane stiffness. Adjusting the degree of distribution of a beam can be done by shape adjustments and so far this is the only article that refers to this possibility to improve the out-of-plane stiffness. Optimizing the shape and topology of a structure to improve the out-of-plane stiffness has also not been found in any article yet.

One shape adjustment that could prove to be useful for the out-of-plane stiffness properties of large-displacement compliant mechanisms is the use of curved beams in addition to straight beam elements as compliant members. A slightly curved beam can have a load applied in such a way that it becomes straight at some point in the range of motion. Given the fact that a straight beam tends to have a higher out-of-plane stiffness than a curved beam, this may maximize the use of the high out-of-plane stiffness of a straight beam.

As such, the goal of this research is to determine if the use of curved beam elements improves the performance of a complex compliant structure with regard to the out-of-plane stiffness profile across range of motion (henceforth referred to as RoM) in the desired direction, and if so, in what way. For this purpose one compliant structure is chosen and optimized for both only straight beam elements as well as a combination of straight and curved beam elements.

In Section 2 the method used to conduct this research is described. The results and discussion are shown in Section 3 and Section 4 respectively. The conclusions that can be drawn from this research are described in Section 5.

2 METHOD

In this section the definition of a curve as used in this research will be presented first, followed by the compliant structure that was investigated. The final part of the section will contain the method of investigation.

2.1 Curved beam element geometry

The curved beam elements in this research are defined by a cubic Bézier curve, which is readily available in ANSYS as the BSPLIN command and it is more generally applicable than a circular arc, which is also available in ANSYS. The curve is constructed by fitting the Bézier curve to a set of three keypoints: the start and end point of the beam element, and one point inbetween. The distance from this middle point to the straight line between the start and end point given as a percentage of the length between the start and end point defines the curve \( c \), making it a dimensionless number (see Figure 2). So suppose that \( x_{\text{length}} \) is 200 mm, and the maximum height of the curve is 10 mm, then \( c \) is 5%.

2.2 Investigated structure

The complex compliant structure used is a simplified version of the compliant finger presented by Steutel [2]. In order to better
understand the results obtained from the complex structure, two simple structures are also considered: a single beam element and a double beam element structure.

The single beam element consists of one beam element, clamped in at one end and with a tip force $F_y$ applied in the y-direction (in-plane) at the other end (see Fig. 3), as well as a force in z-direction $F_z$ (out-of-plane) to be able to compute the out-of-plane stiffness. The design variables for the single beam element are the curve $c$, the thickness in y-direction $t_y$ (in-plane) and in z-direction $t_z$ (out-of-plane).

The double beam element structure is made up of two beam elements in series. One end of the structure is clamped in while at the other end a tip force is applied in y-direction $F_y$ (in-plane) and also in z-direction $F_z$ (out-of-plane), to obtain the out-of-plane stiffness. The design variables are the curve $c$ of each beam element, their in-plane ($y$) and out-of-plane ($z$) thickness ($t_y$ and $t_z$) and the angle between the two beam elements, as shown in Fig. 3. For both the single beam element and the double beam element structure the deflection $U$ is measured at the tip where the forces are also applied. The material used for the model is Aluminum (a modulus of elasticity set of $70 \text{ GPa}$, a shear modulus of $26 \text{ GPa}$, a Poisson’s ratio of $0.35$ and a yield strength of $345 \text{ MPa}$).

Figure 3 shows the topology of the compliant finger. It consists of one input, where a vertical ($y$-direction) tip force $F_y$ is applied, a ground port, and two contact elements, of which the upper one is the output port. The deflection $U$ is measured at this port and the out-of-plane force $F_z$ ($z$-direction) is used to obtain the out-of-plane stiffness is also applied at this point. The material used for the model is Titanium Grade V (a modulus of elasticity of $113.8 \text{ GPa}$, a Poisson’s ratio of $0.34$ and a yield strength of $827 \text{ MPa}$). The total length of the finger ($y$-direction), from base to tip, is $100 \text{ mm}$ and the maximum width (x-direction) is $30 \text{ mm}$. The thickness in both directions ($t_y$ and $t_z$) as well as the curve $c$ of each individual beam element are design variables. Given the fact that a straight finger can only deflect in one direction, only a positive actuation force is applied to the finger. The situation in which a negative force is applied is not included in this research.

### 2.3 Method of investigation

The method of investigation consists of two parts: the out-of-plane stiffness profile investigation, and the optimization. The out-of-plane stiffness profile shows the out-of-plane stiffness over both the in-plane RoM and the out-of-plane RoM. This part is done to obtain a better understanding of the behavior of the out-of-plane stiffness over both these RoM’s. The optimization is used to show the effect of curved beam elements versus straight beam elements on the performance of the structure.

The models of the structures are created in ANSYS 12.1, using 3D Shells (SHELL181) and a linear material, large deformation analysis. For the other parts of the research such as the data analysis MATLAB is used.

#### 2.3.1 Out-of-plane stiffness profile investigation

The out-of-plane stiffness is computed for a range of forces in both in-plane and out-of-plane directions, by applying forces in both directions and obtaining the corresponding displacement $U$ of the tip of the finger in both directions. In this manner both the out-of-plane stiffness is obtained both as a function of the out-of-plane displacement and as a function of the in-plane displacement. For the single beam element this is done for a number of different curves, while for the double beam element it is done for a number of different curves per beam element as well as the angle between the two beam elements. For the finger the out-of-plane stiffness profile is computed only for a few selected solutions obtained through the optimization. The properties of the investigation are shown in Tab. 2. For the single beam and double beam structure the variation of the out-of-plane stiffness over out-of-plane RoM on different points in the in-plane RoM was also looked into. This was done by computing the difference between the maximum and minimum out-of-plane stiffness as a
TABLE 1. The nodal coordinates of the topology used in this research are given below. The letters and numbers correspond to the points and ports depicted in Fig. 3. The length of the contact elements is 10 mm.

<table>
<thead>
<tr>
<th>Point</th>
<th>x [mm]</th>
<th>y [mm]</th>
<th>Port</th>
<th>x [mm]</th>
<th>y [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>4.5</td>
<td>30</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>b</td>
<td>28.75</td>
<td>30</td>
<td>2</td>
<td>25</td>
<td>0</td>
</tr>
<tr>
<td>c</td>
<td>10.5</td>
<td>70</td>
<td>3 (bottom)</td>
<td>30</td>
<td>40</td>
</tr>
<tr>
<td>d</td>
<td>25</td>
<td>70</td>
<td>3 (top)</td>
<td>30</td>
<td>50</td>
</tr>
<tr>
<td>e</td>
<td>15</td>
<td>100</td>
<td>4 (bottom)</td>
<td>30</td>
<td>80</td>
</tr>
<tr>
<td>f</td>
<td>25</td>
<td>100</td>
<td>4 (top)</td>
<td>30</td>
<td>90</td>
</tr>
</tbody>
</table>

TABLE 2. The dimensions of the beam elements used in the out-of-plane stiffness profile investigations for the single beam element, double beam element structure, and some of the solutions found for the compliant finger are shown below, as well as the range of force and stepsize of the force applied to them. For the investigation of the out-of-plane profile of some of the solutions of the compliant finger only the range of force and stepsize are shown, as the other properties vary per solution.

<table>
<thead>
<tr>
<th>Beam structure</th>
<th>Single</th>
<th>Double beam 1</th>
<th>Double beam 2</th>
<th>Compliant finger</th>
</tr>
</thead>
<tbody>
<tr>
<td>F_y range [N]</td>
<td>0 - 10</td>
<td>0 - 10</td>
<td>0 - 10</td>
<td>0 - 10</td>
</tr>
<tr>
<td>F_y stepsize [N]</td>
<td>0.2</td>
<td>0.5</td>
<td>0.5</td>
<td>0.2</td>
</tr>
<tr>
<td>F_z range [N]</td>
<td>0 - 10</td>
<td>0 - 10</td>
<td>0 - 10</td>
<td>0 - 10</td>
</tr>
<tr>
<td>F_z stepsize [N]</td>
<td>0.2</td>
<td>0.5</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>t_x [mm]</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>t_z [mm]</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>x_length [mm]</td>
<td>100</td>
<td>50</td>
<td>50</td>
<td></td>
</tr>
</tbody>
</table>

2.3.2 Optimization Two different optimizations are done for the compliant finger: one where the beam elements can be curved and one where the beam elements are always straight. The RoM is determined by increasing the force in the in-plane direction until the maximum allowable level of stress (80% of the yield strength) is reached. The out-of-plane stiffness is determined by applying a constant force in out-of-plane direction at each point over the RoM.

For the optimization three different objective functions are used:

1. First of all the in-plane RoM, defined as the length of the path of the output point. The objective is to have this value as high as possible, so the finger can be adaptable to a wide range of different objects. The equation is as follows:

   \[ O_{RoM} = \sum_{i=1}^{n-1} ||p_{i+1} - p_i|| \]  

   where \( n \) is the number of points measured across the in-plane RoM and \( p \) are the \( x \) and \( y \) coordinates of the point at which the output port is located.

2. The second objective function is the out-of-plane stiffness, averaged over the entire in-plane RoM. A high average is desired, to be able to more accurately control the movement of the finger, even when an external load is applied. The objective function is formulated as in Eq. 3:

   \[ O_{mean} = \frac{1}{n} \sum_{i=1}^{n} k_{z,i} \]  

   where \( k_z \) is the out-of-plane stiffness.

3. The third objective function is the deviation from the average out-of-plane stiffness over the in-plane RoM. Ideally this value would be zero, resulting in a constant out-of-plane stiffness. The reason for this is also to be able to more accurately control the finger. The equation is:

   \[ O_{dev} = \frac{1}{n} \sum_{i=1}^{n} (k_{z,i} - \bar{k_z})^2 \]  

   where \( \bar{k_z} \) is the average out-of-plane stiffness over the in-plane RoM.

In order to be able to examine the difference in performance between solutions with and without curved beam elements a Non-dominated Sorting Genetic Algorithm is used (MATLAB uses a variant of NSGA-II, [13]), which results in a set of Pareto solutions instead of only one solution per optimization.

where \( \Delta k_{z,i} \) is written below in equation form:

\[ \Delta k_{z,i} = \frac{\max(k_{z,i}) - \min(k_{z,i})}{\bar{k}_{z,i}} \]  

where \( i \) is the point in the in-plane RoM, and \( \bar{k}_{z,i} \) is the average out-of-plane stiffness over the out-of-plane RoM at point \( i \).
TABLE 3. The possible values of the design variables for each of the 11 beam elements of the compliant finger and the optimization properties of the optimization of the compliant finger, both with the use of curved beam elements and without, are shown below. The first five values are the same for both optimizations.

<table>
<thead>
<tr>
<th></th>
<th>With curves</th>
<th>Without curves</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_y$ range [N]</td>
<td>0 - 10</td>
<td></td>
</tr>
<tr>
<td>$F_y$ stepsize [N]</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>$F_z$ [N]</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>$t_z$ [mm]</td>
<td>7.5, 10, 12.5, 15</td>
<td></td>
</tr>
<tr>
<td>$t_y$ [mm]</td>
<td>0.1, 0.3, 0.5, 0.75</td>
<td></td>
</tr>
<tr>
<td>$c$ [%]</td>
<td>5, 3, 1, 0, none</td>
<td>-1, -2, -3, -5</td>
</tr>
<tr>
<td>generations</td>
<td>15, 13</td>
<td></td>
</tr>
<tr>
<td>population</td>
<td>50, 50</td>
<td></td>
</tr>
</tbody>
</table>

The chromosomes are defined as binary strings, allowing for a predefined discrete set of design variables. This limits the number of possible values per design variable and decreases the necessary size of the population, as well as ensuring the use of a wide range of values for each design variable. Due to the use of a binary string the crossover function had to be restricted to allow only crossover between the bits that define one design variable and not also crossover within the design variables. The design variables together with the optimization properties such as population size for the finger optimization are listed in Tab. 3. The optimization was stopped due to having reached the maximum number of generations. Due to a time restriction one of the optimizations could only run for less than 20 generations, which may not have been enough to search the design space thoroughly.

3 RESULTS

The results of the out-of-plane stiffness profile investigation of the simple structures will be presented first, followed by the results of the research of the compliant finger.

3.1 Results of simple structures

The out-of-plane stiffness profile investigation results in a set of stiffness values that provide insight into how the out-of-plane stiffness varies over both the in-plane and out-of-plane RoM. The results of a wide range of curves ($c$ between -50 and +50 for the single beam structure, and $c$ between -5 and +5, and $\theta$ between -90 and +90 for the double beam structure) show that for both the single beam and double beam structure the out-of-plane stiffness varies very little in out-of-plane direction. The difference between the maximum and minimum out-of-plane stiffness found over the out-of-plane RoM for each point on the in-plane RoM ($\Delta k_z$) was computed according to the equation provided in the Method section. The largest difference found was 18%, in the single beam structure with a $c$ of 5. The average difference, however, was found to be around 3%.

The behavior of the out-of-plane stiffness of the single beam over the in-plane RoM is shown in Fig. 4. It can be seen that the out-of-plane stiffness profile has a Gaussian-like shape, with the out-of-plane stiffness increasing with an increasing deflection for $c$ equal to -5 or lower, and decreasing with an increasing deflection for $c$ equal to 0 or higher. Similarly, for beams with a curve of $c$ lower than -5, the out-of-plane stiffness will go up continuously as the beam deflects in in-plane direction. For both positive and negative $c$ values it is shown that the average out-of-plane decreases as $c$ moves further away from 0 (i.e. as the initial curvature of the beam increases). For values of $c$ between 0 and -5 the out-of-plane stiffness first increases before decreasing again while the beam is deflecting in in-plane direction.

The decrease of out-of-plane stiffness when the beam is
curved can also be shown with the use of two equations. For both equations it is assumed that the beam has a zero in-plane deflection. When the beam is not curved, the well known classical beam theory can be applied to calculate the out-of-plane stiffness for small deflections in the out-of-plane direction:

$$k_z = \left( \frac{L^3}{3EI} \right)^{-1}$$  \hspace{1cm} (5)

where $I$ is the area moment of inertia, $L$ is the length of the beam, and $E$ is the elastic modulus. For the single beam example used in this research this leads to a stiffness of 17.5 N/mm. Through the simulations with ANSYS a stiffness of about 17.3 N/mm was found.

Dahlberg [14] provides the equation to calculate the out-of-plane deflection of a beam curved as a quarter of a circle when an out-of-plane load is applied. The equation is as follows:

$$k_z = \left( \frac{\pi R^3}{4EI} + \left( \frac{3\pi}{4} - 2 \right) \frac{R^3}{GI} \right)^{-1}$$  \hspace{1cm} (6)

where $J$ is the torsional constant, $G$ is the shear modulus and $R$ is the radius of the curvature of the beam. In our example, $R$ is 63.7 mm. This means that the total length of the beam is kept constant at 100 mm. Up to a quarter of a circle the cubic Bézier curve can provide a good approximation of a circular arc. A quarter of a circle corresponds to a curve $c$ of 25. For this curve, a stiffness of 0.85 N/mm was found through ANSYS, which is the same as the value found through the equation.

The double beam element investigation shows a similar result, but with a few differences (see Fig. 5). As seen with the single beam element, each curve corresponds to a part of the profile. However, now the curve does not influences only the shape of the profile. It also has an effect on the magnitude of the stiffness profile, meaning that the out-of-plane stiffness profiles of the different curves do not connect seamlessly anymore as they did in the single beam element structure. The results also show that while the curve of the first (clamped-in) beam element determines both the part of the profile it corresponds with and the magnitude of the out-of-plane stiffness profile, the second beam element only influences the magnitude. The angle between the two beam elements behaves the same as the curve of the first beam element, influencing both the shape of its out-of-plane profile as well as the magnitude (see Figure 6).

### 3.2 Results of compliant finger

The optimization of the compliant finger for both cases (with the possibility of using curved beam elements and without) resulted in a set of 141 Pareto solutions, of which 67 solutions use curved beam elements and 74 solutions have only straight beam elements. The genetic algorithm in MATLAB only provides a fraction of the Pareto solutions, which are used as elite solutions. As such it is possible to have more Pareto solutions over many generations than the population size used. Of the 67 Pareto solutions found in the optimization where curved beam elements were allowed, the vast majority (50) had no or only one straight beam element, and no solution had more than four straight beam elements. The out-of-plane stiffness profile is shown for three different curves for the first beam element. The y-axis the out-of-plane stiffness is shown. The in-plane deflection is shown both on the x-axis and as color in the graph, to clarify how the stiffness changes depending on the curve. A schematic view of the double beam element structure is given at the top of each graph. Note that the shape of the total out-of-plane stiffness is still Gaussian-like, but that the graphs of the different curves do not connect seamlessly anymore.

![Figure 5](image1.png)

**FIGURE 5.** The out-of-plane stiffness profile over the in-plane RoM is shown for three different curves for the first beam element. On the y-axis the out-of-plane stiffness is shown. The in-plane deflection is shown both on the x-axis and as color in the graph, to clarify how the stiffness changes depending on the curve. A schematic view of the double beam element structure is given at the top of each graph.

![Figure 6](image2.png)

**FIGURE 6.** The out-of-plane stiffness profile over the in-plane RoM is shown for three different angles. The curve $c$ of both beam elements is zero in this example. On the y-axis the out-of-plane stiffness is shown. The in-plane deflection is shown both on the x-axis and as color in the graph, to clarify how the stiffness changes depending on the curve. A schematic view of the double beam element structure is given at the top of each graph.
FIGURE 7. This graph shows the resulting Pareto solutions as dots and their performances regarding the three objective functions, with the average out-of-plane stiffness on the y-axis, the deviation from the average on the x-axis, and the RoM depicted as the color of the solution. The stiffness profiles of the solutions highlighted by a black circle and number are shown in Fig. 9. Note the vertical line of red dots on the y-axis, indicating solutions with a low RoM, and the line of dark blue dots starting in the lower left corner, moving up to the middle right part, indicating solutions with a high RoM.

Figure 7 shows the relationship between the three objective functions. There is a vertical region of red dots, which indicates that those solutions have a low in-plane RoM (which is undesirable). The location of the dots shows that these solutions mostly have a low deviation from the average out-of-plane stiffness. The average out-of-plane stiffness, however, varies per solution, ranging from a very high average stiffness to a low one. There is also a clear region of dark blue dots, starting in the lower left corner and moving up to the middle right part of the figure. These blue dots have a high RoM and their locations indicate that these solutions usually have a relatively low average out-of-plane stiffness and that the deviation from the average can vary between very high and very low.

The comparison between the solutions found for the optimization of the finger with curved beam elements and the one without curved beam elements can be seen in Fig. 8. The solutions with the highest average out-of-plane stiffness are those without curved beam elements. However, the solutions with the highest RoM are in general with curved beam elements.

For a few solutions the out-of-plane stiffness profile has been computed (see Fig. 9). The results show that the out-of-plane stiffness as a function of the out-of-plane deflection can still be assumed to be constant. The shape of the profiles is rather similar for the different solutions. What changes is mostly the deflection at which the maximum out-of-plane stiffness is reached, the stiffness at zero deflection, and the last stiffness measured (which is either at the point where the maximum allowable stress level is reached or where the maximum force is applied, as listed in Tab. 2).

4 DISCUSSION

Regarding the comparison between the performances (in terms of objective function values) of the solutions with curved beam elements and those without (see Fig. 8), two observations can be made. First of all, the optimization without curved beam elements was shown to provide the solutions with the highest average stiffness. This is similar to what was found in the double beam element example, where the straight double beam elements also had the highest out-of-plane stiffness, regardless of the angle between the beam elements.

Secondly, the solutions with curved beam elements tend to have a high RoM and show a very clear trade-off between the average out-of-plane stiffness, deviation from the average, and

FIGURE 8. The Pareto solutions found for the optimization of the finger with and without the use of curved beam elements are shown in respectively red and blue dots. The average out-of-plane stiffness is shown on the y-axis and the deviation from the average on the x-axis. The corresponding RoM can be found in Fig 7. Note that the solutions without curved beam elements (blue) tend to have a higher average out-of-plane stiffness, a slightly lower deviation from average, but also a lower RoM than the solutions with curved beam elements (red).
FIGURE 9. The out-of-plane stiffness profile along with an image of the deformed and undeformed structure of six of the solutions is provided here. The out-of-plane stiffness is depicted on the y-axis. To make the graph more clear the deflection in x-direction is shown both on the x-axis and as color. In Fig. 7 the performance of these solutions in regard to the objective functions are shown. Solution 1, 2, 4 and 5 have curved beam elements, while solution 3 and 6 do not. Note how the profiles all have a similar shape. Also note how solution 5 and 6 have almost the same performance while having clearly different profiles. The dimensions of these topologies are given in Tab. 4.

RoM. Performances such as that of solution number 2 in Fig. 7 and Fig. 9, with both a relatively good RoM and a good average out-of-plane stiffness, were not found for solutions without curved beam elements. This suggests the added value of the use of curved beam elements in a complex structure like the compliant finger. It should be noted however, that curved beam elements do have a longer cord length than straight beam elements. This may have some influence on the RoM as well, regardless of whether the beam element is curved or not.

The results of the single beam structure, double beam structure and compliant finger optimization show that how much curvature is good curvature depends on the topology and as such a general optimum curvature was not found.

Some observations can also be made with regards to the out-of-plane stiffness profiles shown in Fig. 9. As mentioned in Section 3.2 the deflection at which the maximum out-of-plane stiffness is attained varies per design. Solutions 1, 2 and 5 reach their maximum level at a deflection of 3 to 4 mm, while the other solutions 3, 4 and 6 reach it at a deflection smaller than 2 mm. One thing that sets the designs of these solutions apart is that the beam element directly connected to the ground of the group of solutions 1, 2 and 5 has a negative curve, while the others are straight. The deflection of various solutions over the in-plane RoM also show that this particular beam element has a large influence on the performance, both in terms of the RoM as well as when the maximum stress level is reached. This also matches the results of the double beam element structure, in which it was shown that it is in large part the curve of the beam element directly clamped in that determines both the magnitude and the shape of the out-of-plane stiffness profile. As such, it is likely that the deflection at which the stiffness peak is reached is influenced by the curve of this beam element. The peak itself is most likely caused by the fact that at zero deflection the force applied in out-of-plane direction is not in line with the beam element (which would only cause a bending torque), but on the left next to the beam element, meaning that it is creating a torque on the beam element that causes it to twist. As the finger deflects more, this force becomes in line with the beam element, thus the torque it creates decreases and with it the out-of-plane stiffness increases. Once past this peak the force is again not in line with the beam element but on the right next to the beam element and thus causing it to twist again. The thickness $t_y$ and $t_z$ of the beam elements of these structures does not show any relation to the deflection at which the peak in out-of-plane stiffness is reached. It has to be noted that curving this beam element too far could result in buckling and bi-stability, which was not taken into account in this research.

Another point of interest are the profiles of solution 5 (curved beam elements) and 6 (only straight beam elements). Both have nearly the same performance in terms of objective function values, yet their profiles are rather different. Although they reach the same maximum stiffness, solution 5 reaches it at a later point than solution 6. Solution 5 starts with a lower out-of-plane stiffness, but it never drops to a stiffness of 10 N/mm, unlike solution 6. In terms of design variable values they differ both by the curve of their beam elements (solution 6 has only
TABLE 4. The dimensions responding to the 6 topologies shown in Fig. 9 are provided below. The beams are named after the points they connect, which can be seen in Fig. 3. Beam 1a for example connects port 1 with point a. The contact elements always have the same dimensions: $c = 0$, $t_r = 3$ mm, $t_s = 10$ mm.

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Table: The dimensions responding to the 6 topologies shown in Fig. 9 are provided below. The beams are named after the points they connect, which can be seen in Fig. 3. Beam 1a for example connects port 1 with point a. The contact elements always have the same dimensions: $c = 0$, $t_r = 3$ mm, $t_s = 10$ mm.

straight beam elements) and by the thickness $t_r$ and $t_s$. As such it does not indicate that the use of curves alone brings about such a change in profile. It does, however, point out the possibility of this optimization with curved beam elements to result in solutions that have similar performances and yet a large difference in out-of-plane stiffness profile. Which of these solutions is considered to be better depends on the purpose of the application, whether the stiffness in the first few millimeters is most important, or the lowest stiffness over the entire RoM.

A few recommendations or improvements for future work are also in order. First of all, although it was shown that the use of curved beam elements does influence the out-of-plane stiffness profile, the general shape of the profile did not change. However, the profiles obtained for three different structures - the single and double beam element and the compliant finger - vary greatly. Therefore, it is recommended to look into the effect of a change of topology combined with the use of curved beam elements on the out-of-plane stiffness profile.

Secondly, two separate optimizations were done in this research: one with curved beam elements, and one without. For the one with curved beam elements only very few Pareto solutions with straight beam elements were found, while the combined set of Pareto solutions showed that there were also plenty of solutions with only straight beam elements. In order to ensure that these solutions could also be found with only one optimization the chance of choosing $c$ equal to 0 (straight) would need to become larger. One way to do this would be by adding more than one option of zero to the possible values of $c$ as shown in Tab. 3. A larger population is also recommended.

Thirdly, the current calculation of the deviation from the average out-of-plane stiffness is based on the absolute deviation, instead of on the deviation relative to the average. As such, having a very high out-of-plane stiffness may result in a high deviation, while relatively speaking it could still be quite a low deviation. Rephrasing the objective function so that it computes the relative deviation may provide more insight in the trade-off between the out-of-plane stiffness profile and the RoM.

Last but not least, other methods of computing the optimum curvature for the performance of a specific topology could also be investigated. One possible method could be using the curvature of beams found when deforming a structure with straight beams. For example the curvature found when applying a negative force on the structure could be superimposed on the original structure. When a positive force is then applied to this structure this could result in the beams straightening out.

5 CONCLUSION

In this research it was shown that the use of curved beam elements does affect the performance (in terms of objective function values) of a chosen complex compliant structure with regard to the out-of-plane stiffness profile across the RoM, by both providing performances that could not be obtained with only straight beam elements and by changing variables of the out-of-plane stiffness profile that were also not seen in solutions with only straight beam elements.

In the optimization of a compliant finger the use of curved beam elements allows for solutions with performances that are not found in an optimization with only straight beam elements, and vice versa as well. Solutions with straight beam elements tend to have a high average out-of-plane stiffness, a low deviation from this average, and a low RoM, while solutions with curved beam elements tend towards a high RoM and a trade-off between the average out-of-plane stiffness and the deviation from this average.
For a single beam element and double beam element structure the total out-of-plane stiffness was shown to have a Gaussian-like shape over the in-plane RoM. For both structures the curve determined which part of this profile corresponded to the out-of-plane stiffness of the beam element in question. In the double beam element structure the curve also influenced the magnitude of the out-of-plane stiffness profile. For the compliant finger the use of curves did not show much influence on shape of the out-of-plane stiffness profile, but mostly on the variables of the profile.

In the compliant finger the deflection at which the maximum out-of-plane stiffness was obtained shows a tendency to depend largely on the curve of the beam element directly connected to the ground. Structures for which this beam element had a positive curve reached the maximum stiffness at a later point than structures for which this beam element was straight.

The effect of using curved beam elements was clearly shown in two solutions with the same performance in terms of RoM, average out-of-plane stiffness and the deviation from this average, but with a different out-of-plane stiffness profile. The difference between these structures was that one solution was obtained through optimization with the possibility of curved beam elements while the other had only the option of straight beam elements. The deflection at which the stiffness peaked was higher for the structure with curved beam elements and while the stiffness at zero deflection was lower, the lowest stiffness over the entire RoM was still higher than the lowest stiffness obtained from the structure with only straight beam elements.

In the end the choice of which solution is better - with or without curved beam elements - depends largely on the application it is meant for. What is clear though, is that using curved beams allows for a larger range of possibilities when it comes to out-of-plane stiffness and how it behaves over the range of motion.

REFERENCES
Part II

Paper II
Abstract—For large-displacement mechanisms, such as underactuated fingers, the out-of-plane stiffness can pose problems, especially as it tends to vary over the in-plane range of motion. In this paper a method is presented to design compliant underactuated fingers using topology optimization with a focus on obtaining a desired out-of-plane stiffness profile. Load path representation is used in combination with a non-dominated sorting genetic algorithm. Aside from the thickness of beam elements, the curve of a beam element is also employed as design variable. A set of objective functions is used to evaluate the behavior of the out-of-plane stiffness over the range of motion. The resulting Pareto solutions provide insight into the trade-off between the different objective functions on contact force and out-of-plane stiffness and also show the capability of this method to design compliant underactuated fingers. Using objective functions on out-of-plane stiffness results in solutions with a higher out-of-plane stiffness than when these objective functions were not used, while the solutions decrease little in performance in terms of contact force. The resulting shape of the out-of-plane stiffness over the in-plane range of motion is shown to vary per solution. It is also shown that using curved beam elements can have a positive effect on the out-of-plane stiffness. Experiments on an underactuated finger prototype verified the simulation results.

I. INTRODUCTION

Unlike rigid-link mechanisms, compliant mechanisms make use of deflection of their flexible members to transfer or transform motion, force or energy. Using this flexibility to enable motion instead of using conventional joints has several advantages such as reduction of friction, backlash, weight and costs [1]. The applications of compliant mechanisms range from high precision micro mechanisms to everyday macro mechanisms such as binder clips and shampoo lids.

One of these applications is the underactuated finger. A mechanism that is underactuated has fewer actuators than degrees of freedom (DoFs). In a mechanical finger this can lead to self-adaptability which allows the finger to automatically adjust itself to the shape of the object it is in contact with. This happens without the use of sensors or control systems [2] (see Fig. 1).

A. Underactuated Fingers

In the past many underactuated fingers have already been designed using rigid links, using either tendons or linkages [2],[3],[4],[5],[6],[7]. In the past ten years also a number of papers have been published on underactuated fingers using lumped compliance [8],[9],[10]. A mechanism is considered to have lumped compliance if only a small portion of the mechanism can undergo elastic deformation (for example with the use of notch hinges) [1]. Of these lumped compliant underactuated fingers Carrozza [8] and Dollar [10],[11] both use tendons for actuation and the structure of the fingers has the shape of a human finger, replacing the joints by compliant hinges. Doria uses a more complicated structure and instead of tendons uses a system similar to the linkage system used by Laliberte [2] where the input force is used to ‘push’ the mechanism rather than pull it.

Distributed compliance means that elastic deformation is no longer concentrated in only small sections of the mechanism, but can occur over a large part of the mechanism. So far only two underactuated fingers were designed using this principle [12],[13]. The finger designed by Steutel [12] is based on the general topology of existing compliant underactuated fingers. It is quite flexible and able to adjust itself to various objects. However, despite having a high contact force to actuation force ratio of 0.75, the contact forces are very low (the maximum is 0.8N). The finger designed by Petkovic [13] is also based on existing gripper models. Similar to the finger by Steutel, it is very flexible, but has a higher contact force (between 4 and 10N). However, it is not clear what the actuation force is. The paper also states that due to the high flexibility the finger is not able to hold heavy items.

Two things are worth noting on the designs of these two distributed compliant underactuated fingers. The first is the low contact force. The second is the fact that both of the designs were based on existing underactuated fingers, whether rigid-body or lumped compliant. In an article on elastic averaging Awtar shows that the degree to which a beam is distributed (as opposed to lumped) causes the mechanism to behave differently [14]. As such, methods that can make use of this difference in behavior (for example topology optimization or the building block approach) are more suited to design a distributed compliant mechanism.

B. Out-of-plane stiffness

In compliant mechanisms the stiffness (both bending and torsional) in one of its directions is inevitably connected to the (bending and torsional) stiffness in another direction. While in some cases this may not be a problem and even desired, it can be problematic when one direction should have a low stiffness while the other should have a high stiffness. Furthermore, the bending stiffness in either direction can decrease or increase over the range of motion in the desired direction. Especially for large-displacement mechanisms this change can be considerable, affecting the accuracy of the mechanism [15],[16],[17],[17]. Only one ([10]) of the papers on compliant underactuated fingers has mentioned the out-of-plane stiffness of its mechanism.

In this research the focus will lie on the in-plane behavior and the out-of-plane behavior of the compliant underactuated finger. The out-of-plane direction is defined as the direction...
perpendicular to the plane in which the mechanism moves (see Fig. 2).

Much research has already been done on how to increase the out-of-plane stiffness of a mechanism. Most of these methods focus on simple topology adjustments (such as adding more beam elements) [18],[15], or size optimization of the mechanism [17],[19],[20],[16],[21]. Papers in which topology optimization was used to influence the out-of-plane behavior of a mechanism were not found.

In a previous paper we showed that using curved beam elements can influence the out-of-plane stiffness of a complex compliant structure [22]. In this paper this research is extended by combining it with topology optimization.

To summarize, at this moment only two distributed compliant underactuated fingers have been designed, both based on existing rigid-body or lumped compliant underactuated fingers. Aside from one mechanism, the out-of-plane stiffness was not investigated for any of the compliant underactuated fingers.

The goal of this research is to obtain designs of compliant underactuated finger with a desired out-of-plane stiffness profile by using topology optimization. The out-of-plane stiffness profile is defined as the out-of-plane stiffness over the in-plane range of motion (RoM). It is desired to have this as high and as constant over the RoM as possible, to create a reliable finger that can be controlled accurately.

First the method proposed for the design of such fingers will be explained in Section II. This method will then be used to generate designs of compliant underactuated fingers and one of these will be fabricated to verify the results through an experiment. Both the results of the method and the experiment will be shown in Section III. This is followed by a discussion on these results in Section IV. The conclusions that can be drawn from this research are presented in Section V.

II. METHOD

The method that has been developed to design the compliant underactuated finger consists of several parts. Each of these will be explained briefly below. More information on the ANSYS model and the experiment conducted to verify the results is also given.

A. Finger design

The compliant finger has one input port, one ground port, and two output ports. Aside from this, 6 interconnecting points are added in a 3x2 grid. The object to make contact with is shaped as a quarter of a cylinder, with a radius of $3L/\pi$ where $L$ is the maximum length of the finger (see Fig. 3). The intended maximum height of the finger is 100mm and the maximum width 30mm. In order to allow fabrication through rapid prototyping, the finger is scaled up by a factor 3, resulting in a new maximum height and width of 300mm and 90mm respectively. This is done for both the analysis and the prototype, to allow for the comparison of these two.

B. Load Path Representation

In this paper load path representation is used to parametrize the design domain [23],[24], which is needed for the topology optimization. Similar to the popular ground structure parameterization [1] it uses interconnecting points (IP). How these points are connected to each other and to the different ports determines the topology of the design.

The advantages of this representation over the ground structure parameterization are that it cannot result in disconnected designs, in which one of the ports is not connected to the rest of the design. It also needs fewer design variables.
The main concept of this method is that the possible paths (set of IP’s) to go from one port to the other - for example from input to output - are provided as design variables. So instead of having the existence of each possible beam elements as a design variable, only the paths between the ports are design variables.

The other design variables specific to the beam elements, such as the thickness, are defined per path (e.g. input-output, ground-output), instead of per beam element. It can happen that two paths overlap partially, by for example both using a beam element between the points IP1 and IP2. This beam element is then assigned two different values for its properties. In her paper [24], Lu suggests randomly picking one of these values to overwrite the other. In this paper, however, the values are averaged to find the final value in order to maintain reproducibility.

In this research all possible paths to go from one port to the other were allowed to be chosen. Since the grid is rather small (3x2) this was possible.

C. Use of curved beam elements

In a previous paper [22] it was shown that curved beam elements can be used to manipulate the out-of-plane stiffness behavior of a mechanism. For this reason the curvature of the beam elements is added as a design variable of the topology optimization.

The curved beam elements are defined by a cubic Bézier curve, which is readily available in ANSYS as the BSPLIN command and it is more generally applicable than a circular arc, which is also available in ANSYS. The curve is constructed by fitting the Bézier curve to a set of three keypoints: the start and end point of the beam element, and one point inbetween. The distance from this middle point to the straight line between the start and end point given as a percentage of the length between the start and end point defines the curve \( c \), making it a dimensionless number (see Figure 4). So suppose that \( x_{\text{length}} \) is 200 mm, and the maximum height of the curve is 10 mm, then \( c \) is 5%. This definition also implies that with an increasing curve \( c \), the absolute length of the beam element also increases.

Previous research [22] also showed that the average out-of-plane stiffness of a beam element is the highest with a curve of \( c \) between 5 and -5. As such this range is used for the design of the compliant finger.

With the possible locations of the interconnecting points (IP’s) in the topology optimization it is possible that two beam elements meet at the interconnecting points under a sharp angle. For straight beam elements this poses no problem. However, it is possible that these two beam elements cross when either one has a large curve \( c \). To avoid this, the curve design variable is defined as a percentage of the maximum curve \( c \) that these beam elements can have without crossing. If for example for a beam element the curve design variable is 50% and the maximum curve \( c \) before it crosses with another beam element is 5, then the resulting curve \( c \) of this beam element will be 50% of this \( c \) of 5, thus a \( c \) of 2.5.

D. Optimization

1) Objective functions: A total of four objective functions are used for this research:

- The first objective function is the out-of-plane stiffness, averaged over the entire in-plane RoM. This objective function was also used in [22]. A high average value is desired, to be able to more accurately control the movement of the finger, even when an external load is applied. The objective function is formulated as in Eq. 1:

\[
O_{\text{mean}} = \frac{1}{n} \sum_{i=1}^{n} k_{z,i}
\]

where \( k_z \) is the out-of-plane stiffness and \( n \) is the number of points measured across the in-plane RoM.

- The second objective function is meant to result in an out-of-plane stiffness as constant as possible. In previous research this was done by minimizing the deviation from the average out-of-plane stiffness across the in-plane RoM. A low deviation, however, does not guarantee a low difference between the minimum and maximum out-of-plane stiffness. As such, for this research the objective function is modified to minimize the difference between the minimum and maximum out-of-plane stiffness found across the in-plane RoM. It is desired to have this as low as possible. The corresponding equation is shown in Eq. 2:

\[
O_{\text{const}} = |\max(k_z) - \min(k_z)|
\]

- The third objective function is meant to quantify the performance of the finger in terms of contact force distribution over the two contact elements. It is desired to have force isotropy - equal forces on both contact elements - to avoid damage on the object [25]. Instead of using the contact force for the objective function, the difference between the final distance to the center of the object of both contact elements relative to the difference between the initial distance is used. Due to the use of spring elements instead of contact elements, this assumption that an equal distance means equal contact forces can be made. If the contact forces were used, a design that does not even come close to the object may be favored over a design that makes contact with one of the contact elements, since the latter one has a relatively very high force on one contact element compared to the other. The equation used for this objective function is
The thickness in z-direction ($t_z$) is chosen for the entire structure, whereas the thickness in y-direction ($t_y$) can vary per beam. The IP’s can vary from their original position. The amount that they can vary is given as a design variable. The input port is located at the center (0,0). The length of a contact element is set to 30 mm.

given in Eq. 3 and it is desired to reach a value as low as possible for this objective function:

$$O_{disct} = \frac{|d_{n,1} - d_{n,2}|}{|d_{1,1} - d_{1,2}|}$$  (3)

where $d_{n,i}$ is the distance from the contact element $i$ to the center of the object at either the maximum actuation force or the maximum allowable stress level, and $d_{1,i}$ is the distance from the contact element $i$ to the center of the object at zero displacement.

- The fourth objective function maximizes the contact force to actuation force ratio. Eq. 4 shows the equation used:

$$O_{cf/of} = \frac{F_{c,1} + F_{c,2}}{F_a}$$  (4)

where $F_{c,i}$ is the contact force on contact element $i$, and $F_a$ is the actuation force.

2) Optimization method: For this multi-objective problem a Non-dominated Sorting Genetic Algorithm is used (MATLAB uses a variant of NSGA-II, [26]), which results in a set of Pareto solutions instead of only one solution per optimization.

The chromosomes are defined as binary strings, allowing for a predefined discrete set of design variables. This limits the number of possible values per design variable and decreases the necessary size of the population, as well as ensuring the use of a wide range of values for each design variable. Due to the use of a binary string the crossover function has to be restricted to allow only crossover between the bits that define one design variable and not also crossover within the design variables. A tournament selection operator is used, together with a uniform crossover operator and a uniform mutation operator. The design variables together with the optimization properties such as population size for the optimization are listed in Tab. I. The first generation is created randomly. The optimization was stopped due to having reached the maximum number of generations.

<table>
<thead>
<tr>
<th>Design variable</th>
<th>Possible values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_y$ [mm]</td>
<td>1; 2; 3; 4</td>
</tr>
<tr>
<td>$t_z$ [mm]</td>
<td>10; 20</td>
</tr>
<tr>
<td>Ground port (x,y) [mm]</td>
<td>(75.0, 0); (90.0, 0)</td>
</tr>
<tr>
<td>Output port 1 (x,y) [mm]</td>
<td>(90, 155); (90, 180)</td>
</tr>
<tr>
<td>Output port 2 (x,y) [mm]</td>
<td>(90, 225); (90, 270)</td>
</tr>
<tr>
<td>Difference IP’s (x,y) [mm]</td>
<td>(±7.5, 0); (0, ±12); (±15, ±24)</td>
</tr>
</tbody>
</table>

Paths Input port to Output port 1
Paths Input port to Output port 2
Paths Ground port to Output port 1
Paths Ground port to Output port 2

TABLE I. THE DESIGN VARIABLES AND OPTIMIZATION PROPERTIES.

The process of computing the fitness values of the objective functions necessary for the genetic algorithm consists of a number of steps. These are shown in Fig. 5.

**Check output connection**: The way load path representation is implemented allows for the possibility of having a contact element connected to the rest of the design at only one end, thus having it not fully connected to the rest of the design. This is possible since the paths leading to that output could all be using the same interconnecting point before connecting to the output. The result is undesirable since it does not allow any control over the behavior of the contact element through actuation. As such designs that have this property are given poor fitness values.

**Check for no crossing beams**: One issue that is inherent to both the ground structure and the load path representation is the possibility of the crossing of beams in designs. If this happens, one of the paths is changed so that the beam elements do not cross anymore. For example, if IP1 is connected to IP4 (IP1-IP4), and IP2 is connected to IP3 (IP2-IP3), the first connection could be changed to go through IP3 (so IP1-IP3-IP4). In some cases such as the connections with the outputs the paths cannot be changed, in which case the design is penalized by giving it poor fitness values.

**Check flexibility design**: After running a simulation to compute the out-of-plane stiffness (OOPS) the flexibility of the design is checked to see whether it can reach the object. If it cannot, time is saved and the third and fourth objective functions are calculated based on the displacement information obtained from the out-of-plane stiffness model run.

**F. ANSYS model**

ANSYS 12.1 is used to do the FEM analysis. For the compliant finger the shell element SHELL181 is used. The shell is placed in the y-z plane. A finger consists of about 2500 elements. A linear actuation force is applied at the input, which is constrained to have a degree of freedom in the y-direction. The two output ports are used to measure the displacement in the three directions and the contact forces.

The contact is modeled using non-linear spring elements (COMBIN39) that are only allowed to move in the x-y plane. The springs are attached at the center of the cylindrical object and at the center of the respective contact element of the finger, located at the output ports. The stiffness of the spring is defined to be 0.001 N/mm until the length of spring is equal to the radius of the object. At this point it is considered to be in contact with the object and the stiffness is raised to 10 N/mm.

For the out-of-plane stiffness computation the spring elements are removed. Over the in-plane RoM the position of the second output is measured for each force step applied in y-direction. This is done for two situations: 1) no force in z-direction and 2) a force of 1N applied in z-direction on the second output. This allows for computation of the out-of-plane stiffness over the RoM.

The material used for the simulation was Acrylic (a Young’s Modulus of 2450 MPa, a Poisson Ratio of 0.375, a tensile strength of 62 MPa). Although Acrylic is a brittle material,
Fig. 5. This flowchart depicts the steps that are taken to obtain the fitness values of the objective functions. Some of these steps (step 2, 3, and 7) are explained in more detail in Section II.

**G. Experiment**

In order to verify the results obtained through ANSYS one of the designs is fabricated. The material used was Acrylic and the fabrication method was laser cutting. The experiments conducted on the prototype are in line with the simulation done with FEM: one experiment focuses on the obtaining the out-of-plane stiffness over the in-plane RoM, while the other focuses on the contact with the object.

For both experiments a linear actuator is used, to which a loadcell is connected, which is in turn connected to the input of the finger. A camera is placed high above the finger to record the movement of both the input port and the two output ports (see Fig. 6 and 7).

For the experiment on the contact behavior, two loadcells are placed at the location at which the finger comes in contact with the object, to record the two contact forces (see Fig. 6). For the out-of-plane stiffness the displacement of the second output is investigated when a load of 0.98N is placed on it, as well as when no load is applied. A camera is used to measure the displacement of the second output in z-direction (see Fig. 7).

**III. Results**

The results section is divided into three parts: some general remarks, the Pareto solutions resulting from the optimization, and the results of the experiment.

**A. General**

In the first generation about 63% of the solutions had one or more outputs that were only connected to the rest of the
design on one end and that were therefore penalized. Over the generations this decreased to 0.05% in the final generation, with about 14% over all generations.

The number of solutions that had issues with beam elements crossing that could not be solved and thus were penalized was only about 0.5% over the 20 generations, and about 1% in the first generation.

In total 218 Pareto solutions were found. However, when inspecting the results, it was found that for 74 of them beam elements had bent to the extent that they were crossing each other. In practice this would lead to contact between beam elements. In ANSYS however, this results in the crossing of these beam elements. This is something that does not stop ANSYS from converging, but does make the results infeasible. Aside from this 56 Pareto solutions also showed strong signs of buckling (many also had crossing beam elements). Although this causes ANSYS to not converge, the results are still taken into account by the GA. As such these also had to be discarded. This resulted in a final set of 124 Pareto solutions.

Another issue that was encountered were designs in which the connection between the Input and Ground ports and either one of the Output ports is only one interconnecting point (these points are from here on referred to as beam element hubs, an example can be seen in solution 2 in Fig. 9). This issue occurred at 54 of the 124 Pareto solutions. These solutions were considered undesirable, since the point of contact cannot be controlled if it can only be influenced through one point.

B. Pareto solutions

Figure 8 shows a plot of the Pareto solutions. Some general remarks on this figure can be made. The higher the out-of-plane stiffness becomes, the larger the difference between the minimum and maximum out-of-plane stiffness also tends to become. This can be seen at solutions with a low average out-of-plane stiffness (a dark blue color in the figure), which tend to have a low difference between minimum and maximum out-of-plane stiffness (a small circle in the figure). A high average out-of-plane stiffness (yellow to red) results in such a high stiffness that the finger does not make contact with the object, which is why these solutions are located at the bottom right in the figure. These solutions that have a high average out-of-plane stiffness but do not make contact with the object also have a smaller difference between the minimum and maximum out-of-plane stiffness, than solutions with the same high average out-of-plane stiffness that do make contact with the object. This can easily be explained by the fact that a larger deflection means that the structure will be more susceptible to twist, which means a larger drop in out-of-plane stiffness.

To determine the effect of adding objective functions on out-of-plane stiffness, the Pareto solutions of only objective functions 3 and 4 were also looked at. This resulted in 7 Pareto solutions, which can be recognized in Fig. 8 by being marked with a red hexagon. The average values of the two out-of-plane stiffness objective functions of both the complete set of Pareto solutions, and this subset of seven, can be found in Tab. II.

For five solutions, as numbered in Fig. 8 the out-of-plane stiffness profiles have been computed. These profiles show
Pareto solutions of obj: 1 through 4 Only 3 and 4
Average value obj. 1 0.983 N/mm 0.187 N/mm
Average value obj. 2 0.085 N/mm 0.052 N/mm

TABLE II. The average values of objective functions 1 and 2 (which are the average out-of-plane stiffness and the difference between the minimum and maximum value of out-of-plane stiffness) of all Pareto solutions and only the solutions found for objective functions 3 and 4.

<table>
<thead>
<tr>
<th>Port</th>
<th>Location (x,y) [mm]</th>
<th>Beam element t_z [mm]</th>
<th>curve c [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>(0,0)</td>
<td>I-1</td>
<td>1.5</td>
</tr>
<tr>
<td>G</td>
<td>(90,0)</td>
<td>G-2</td>
<td>3</td>
</tr>
<tr>
<td>O1</td>
<td>(90, 135)</td>
<td>1.3</td>
<td>15</td>
</tr>
<tr>
<td>O2</td>
<td>(90, 225)</td>
<td>1.3</td>
<td>15</td>
</tr>
<tr>
<td>1</td>
<td>(22.5,75)</td>
<td>2.4</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>(90.99)</td>
<td>3.4</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>(0, 204)</td>
<td>3.5</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>(90, 204)</td>
<td>4-O1</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>(67.5, 270)</td>
<td>4-O2</td>
<td>1.5</td>
</tr>
</tbody>
</table>

TABLE III. The locations of the interconnecting points and ports of solution 1 are given, as well as the properties of the beam elements. The thickness in Z-direction, t_z, is 10mm for all beams. The length of the contact elements on the output ports is 10mm, and their thickness in Y-direction is also 10mm.

C. Experimental results

A prototype was made of solution 1 (see Fig. 10, Fig. 11 and Tab. III). The simulation showed that both contact forces were relatively high, and the out-of-plane stiffness relatively low, which made it easier to measure the contact forces and out-of-plane stiffness.

The fabrication was done with lasercutting. The prototype showed warping around the x-axis, most likely due to the heat emitted by the laser. This resulted in a difference of about 6 mm in z-direction between the middle of the prototype (x = 135 mm) and its two extremities (x = 0 mm and x = 270 mm). For this reason the out-of-plane stiffness experiment was done with the prototype placed as shown in Fig. 7 where it is bending downwards due to the warping, and also turned upside down, so it is bending upwards due to the warping.

Fig. 12 shows the actuation force applied as a function of the displacement of the input port for the out-of-plane stiffness situation. Both the experiment and ANSYS show a nearly constant in-plane stiffness, which is about 0.058 N/mm in ANSYS, and about 0.04 N/mm for the prototype (70% lower). Fig 13 depicts the out-of-plane stiffness for both. What can be
Fig. 10. The prototype fabricated to verify the simulation results.

Fig. 11. A schematic view of the structure of solution 1. The letters stand for the ports, while the numbers depict the interconnecting points. The location of these points and the properties of the beam elements are shown in Tab. III.

seen here is that not only is the out-of-plane stiffness much lower for the prototype than for the simulation, the shape is also different. The out-of-plane stiffness of the simulation first increases before decreasing, while that of the prototype only decreases. The average out-of-plane stiffness of the simulation is 0.167 N/mm, while that of the prototype is 0.695 N/mm (bending upwards) and 0.496 (bending downwards). The average out-of-plane stiffness is thus about 60% lower for the prototype than for the simulation.

In Fig. 14 one can see the actuation force as a function of the displacement of the input port for the contact situation. Both the prototype and the simulation show a sudden increase in stiffness when the first contact point touches the object, and then another increase when the second contact point touches the object. The overall stiffness of the prototype is again lower than that of the simulation. The simulation has a stiffness of about 0.071 N/mm before the first contact, and about 0.408 N/mm after, while the prototype has a stiffness of 0.047 N/mm before the first contact and 0.267 N/mm after. This means that that the stiffness of the prototype is about 65% of that of the simulation, but the increase in stiffness due to contact is the same (570%). Unlike the simulation, the stiffness of the prototype starts to decrease at a certain point and even almost reaches the zero stiffness point. The maximum actuation force applied is 14.75 N for the simulation, and 12.5 for the prototype.

Fig. 15 shows the contact force of both contact points as a function of the displacement of the input port. For both the prototype and the simulation the contact force on the first output port increases linearly until a certain point, after which it levels off. The simulation continues to a force of 6.3N, while the prototype staggers at a force of 4N. At the second plot, showing the second contact point, it can be seen that the contact force increases dramatically once contact is made
Fig. 14. The actuation force as a function of the displacement of the input port, for both the simulation and the prototype.

Fig. 15. The contact forces as a function of the displacement of the input port, for both the simulation and the prototype.

for the simulation, and then is abruptly halted due to having reached the maximum allowable level of stress at 5N. The prototype on the other hand shows a contact force that keeps increasing but seems to reach a steady state at about 6N.

The deformed state of the compliant underactuated finger when the maximum actuation force is applied is shown in Fig. 16. As can be seen there, the beam elements of the prototype seem to be more flexible and are more curved at full deflection than those of the simulation.

IV. DISCUSSION

A. Load path representation

Several remarks can be made on issues that arose due to using load path representation.

The number of solutions that were penalized due to badly connected outputs was very high in the first generation, which has quite an impact on the gene pool. Although the number of solutions with this issue decreases rapidly over the generations, having this many penalized solutions should be avoided. One possible solution could be to view the two connection points to each output as separate output ports for the load path representation, thus ensuring that there will always be a connection with both endings.

Solutions with beam element hubs exist because it is possible for paths leading to one output to share the same interconnecting point which then also happens to be the last point they have in common with the other paths. To solve this one could define a permanent connection between the outputs. In any case, the number of solutions that show formation of beam element hubs is high enough to discourage penalization.

The high percentage of solutions in the first generation that had more than half of their beam elements curved (70%) can partly be caused by the approach taken to handle overlapping characteristics of the beam elements. As mentioned in Section II paths can use the same beam elements, causing these beam elements to for example be assigned two different curves. To solve this, these different values are averaged to determine the value that will be used. As such, even if half of the paths have a curve of 0, when averaged out, the beam elements will still be curved.

The set of interconnecting points that was used had influence on the solutions that were found. The current set of possible locations of the interconnecting points covers the design domain well, and using a different set of interconnecting points by merely changing these locations should result in similar solutions. However, increasing the number of interconnecting points can have a larger impact. It could for example lead to beam elements that are S-shaped instead of C-shaped, as they are now, by having interconnecting points that only connect
two beam elements. Most likely this will have an effect on the in-plane behavior.

B. Out-of-plane stiffness and topology optimization

Table II shows that for the compliant underactuated finger optimizing without considering the out-of-plane stiffness would result in a set of solutions with on average an out-of-plane stiffness over 5 times as low as the average out of plane stiffness of solutions found when using out-of-plane stiffness objective functions.

Some solutions worth discussing in more detail are number 1, 2, and 3, as shown in Fig. 8. Number 1 and 2 are members of the Pareto set of objective functions 3 and 4, while number 3 isn’t. Tab. IV shows a comparison of all four fitness values of these three. What this shows is that the fitness values of the objective functions related to out-of-plane stiffness are much better for solution 3 than for solutions 1 and 2 - the average out-of-plane stiffness of solution 3 is respectively 7 and 17 times as high as those of solutions 1 and 2. The value of the third objective function seems much worse for solution 3, being 3 to 30 times as high as the objective function values of solution 1 and 2. However, this is because the first contact element touches the object, while the second one is about 3mm away from it. If the actual location of the object the finger trying to touch varies a bit, this difference can be overcome. The fourth objective - contact force to actuation force ratio - is a bit worse for solution 3.

This comparison shows that it is possible to find solutions that are comparable in performance, but have a higher out-of-plane stiffness. As such, this goes to show that topology optimization can be effective for designing a mechanism with desired out-of-plane stiffness characteristics.

C. Out-of-plane stiffness profiles

Concerning the out-of-plane stiffness profiles shown in Fig. 9 some remarks can be made. The shape of the profiles are consistent with those shown in the previous research [22], all appearing to be parts of a Gaussian-like shape. In both the previous research and this research it was found that changing the topology, shape and size of a structure in 2D has effect on the out-of-plane stiffness profile, by altering a part of this Gaussian-like shape. In previous research the same optimization was done on a compliant underactuated finger, with the exception that the topology was fixed instead of a design variable. The resulting out-of-plane stiffness profiles varied to some degree, but they all showed similar behavior of first increasing and then decreasing. The results in Fig. 9 show that when changing the topology the behavior also changes greatly, varying from only increasing for solution 5 to only decreasing for solution 4. However, it still has little effect on this fundamental Gaussian-like shape. One cause for this may be the structure of the finger in z-direction. In the x-y plane (in which it moves) the structure is quite complex. When observing the finger along the out-of-plane direction, though, it is still a simple beam structure. As such, it is to be expected that it continues to behave as one. If it is desired to fundamentally change this Gaussian-like shape of the out-of-plane stiffness, it is recommended to view it as a 3D structure and investigate the effect of changing the topology or shape in z-direction.

D. Using curved beam elements

For two solutions the simulation was done with the curve of all beam elements set to 0. The resulting fitness values relative to those of the original structure are shown in Tab. V. For solution 1 removing the curves actually resulted in better out-of-plane stiffness characteristics. However, it became less flexible, resulting in both lower contact forces and a worse distribution of these forces. Unlike the first solution, removing the curves of solution 3 worsened the out-of-plane stiffness characteristics, while the contact force distribution and the contact force to actuation force ratio also became worse. How the use of curves affects the out-of-plane stiffness characteristics depends on the structure. As shown above, this effect can be positive and for this reason using curved beam elements in topology optimization is considered a success.

E. Objective functions

The fact that in general the difference between minimum and maximum out-of-plane stiffness increases as the average out-of-plane stiffness increases implies that the stiffness of solutions with a low average is more constant over the RoM than that of solutions with a high average. However, the actual deflection of the structure contradicts this (see Tab. VI). According to the previous equation for the second objective function (Eq. 2), solution 3 is by far the worst. When looking at the deflection 1N causes though, solution 3 is clearly the most constant. To correct this, the following modification of objective function 2 is recommended:

<table>
<thead>
<tr>
<th>Solution</th>
<th>Obj. 1</th>
<th>Obj. 2</th>
<th>Obj. 3</th>
<th>Obj. 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.1652</td>
<td>0.0834</td>
<td>0.0629</td>
<td>0.8071</td>
</tr>
<tr>
<td>2</td>
<td>0.0698</td>
<td>0.0031</td>
<td>0.0258</td>
<td>1.0200</td>
</tr>
<tr>
<td>3</td>
<td>1.1834</td>
<td>0.4021</td>
<td>0.0090</td>
<td>0.7584</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Solution</th>
<th>Obj. 1</th>
<th>Obj. 2</th>
<th>Obj. 3</th>
<th>Obj. 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original</td>
<td>0.1624</td>
<td>0.0384</td>
<td>0.0029</td>
<td>0.8017</td>
</tr>
<tr>
<td>No curves</td>
<td>0.1801</td>
<td>0.0441</td>
<td>0.0051</td>
<td>0.7284</td>
</tr>
<tr>
<td>Relative to original</td>
<td>1.10</td>
<td>1.15</td>
<td>1.76</td>
<td>0.91</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Solution</th>
<th>Obj. 1</th>
<th>Obj. 2</th>
<th>Obj. 3</th>
<th>Obj. 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original</td>
<td>1.1834</td>
<td>0.4021</td>
<td>0.0090</td>
<td>0.7584</td>
</tr>
<tr>
<td>No curves</td>
<td>1.1602</td>
<td>0.3221</td>
<td>0.1801</td>
<td>0.5967</td>
</tr>
<tr>
<td>Relative to original</td>
<td>0.98</td>
<td>0.80</td>
<td>2.02</td>
<td>0.79</td>
</tr>
<tr>
<td>Solution</td>
<td>Change in stiffness [N/mm]</td>
<td>Change in deflection [mm/N]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>----------</td>
<td>---------------------------</td>
<td>-----------------------------</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.0384</td>
<td>1.397</td>
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<tr>
<td>2</td>
<td>0.0031</td>
<td>1.089</td>
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<td></td>
</tr>
<tr>
<td>3</td>
<td>0.4021</td>
<td>0.346</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

TABLE VI. THE VALUES OBTAINED FOR SOLUTIONS 1 TO 3, WHEN LOOKING AT BOTH THE DIFFERENCE BETWEEN THE MAXIMUM AND MINIMUM OUT-OF-PLANE STIFFNESS, AND THE DIFFERENCE BETWEEN THE MAXIMUM AND MINIMUM DISPLACEMENT CAUSED BY 1N IN OUT-OF-PLANE DIRECTION.

\[ O_{\text{const}} = \left| \frac{1}{\min(k_z)} - \frac{1}{\max(k_z)} \right| \] (5)

From the seven Pareto solutions of objective function 3 and 4 a few things can also be said about the objectives functions themselves. The solutions tend to have a very low value for objective function 3 (less than 0.05), while the value of objective function 4 (contact force to actuation force ratio) varies greatly (from 0.3 to almost 1.2). This goes to show that objective function 3 leads to two types of solutions. 1) Those that have beam element hubs near the outputs which makes them very flexible. This allows them to deform enough to touch the object, but with the result of having a low contact force. 2) Those that are much stiffer and have a design that allows them to easily make contact, which results in a higher contact force to actuation force ratio. This design could have a beam with a high stiffness that is connected to the ground - thus effectively creating a pivot point at a convenient location - and having the contact elements at a correct location in relation to the pivot point.

The fact that there are only seven solutions also may seem to show little trade-off between these two objectives. However, many of the complete set of Pareto solutions don’t reach the object with one or both contact elements. As such, the group from which Pareto solutions of objective functions 3 and 4 can be chosen is already rather small.

F. Experiment

The results of the prototype and the simulation are rather similar, aside from the lower stiffness of the prototype. This low stiffness can be seen in all results: the in-plane stiffness, out-of-plane stiffness, and the shape of the finger at the last deformation. One possible explanation could be a lower Young’s Modulus of the Acrylic sheet than expected.

The difference in average out-of-plane stiffness between the prototype and the simulation can also be explained by a lower Young’s Modulus. As shown in the two graphs of solution 3 in Fig. 9, the in-plane deflection at which the maximum out-of-plane stiffness (the peak) is obtained can change depending on the curve of the beam elements. As such, it is possible that due to in-plane warping of the beam elements the peak shifted. The material nonlinearity could also have contributed to this difference between simulation and prototype.

Taking the difference in stiffness between prototype and simulation into account, the contact forces found with the experiment do differ from those found with the simulation. With the prototype the second contact element actually reached a higher value than the first, contradictory to the simulation. However, the contact force distribution was still very much alike, with a division over the first and second contact point of 40%/60% for the prototype and 55%/45% for the simulation. The contact force to actuation force ratio was also nearly the same: 0.77 for the simulation, and 0.79 for the prototype.

V. Conclusion

A method to design compliant underactuated fingers with the use of topology optimization was proposed, with the focus on obtaining a desired out-of-plane stiffness profile. Implementation of the method resulted in a large set of compliant underactuated fingers that showed the potential of the method to create compliant underactuated fingers with a good performance in terms of contact force and out-of-plane stiffness.

Load path representation was found to be a suitable design domain parameterization for the design of compliant underactuated fingers. The representation did result in many solutions with poorly connected outputs and many solutions in which the Input and Ground port were connected to the Output ports through only one point. However, by predefining essential connections these issues can be avoided.

Topology optimization with objective functions on out-of-plane stiffness can be used successfully to obtain solutions with a high out-of-plane stiffness. It was shown that without the objective functions on out-of-plane stiffness the solutions had on average a much lower out-of-plane stiffness than those for which this objective function was taken into account. The performance of the solutions with and without these objective functions was similar in terms of the contact force behavior.

The out-of-plane stiffness profile of the various solutions obtained in this research had a Gaussian-like shape. This is in accordance with the out-of-plane stiffness profiles of the simple beam structures investigated in previous research.

As such it was shown that the topology optimization with the current set of design variables has little effect on the fundamental shapes of the out-of-plane stiffness profiles.

A prototype of one of the solutions verified the simulation results. The contact forces were similar to those of the simulation as well as the in-plane behavior. The absolute height of the in-plane stiffness and the average out-of-plane stiffness was found to be lower for the prototype than for the simulation, which could be the result of a lower Young’s Modulus of the prototype material than expected. The shape of the out-of-plane stiffness across the in-plane range of motion of the prototype did not contain a peak, in contrast to that of the simulation. This can be contributed to material nonlinearity and warping of the prototype during fabrication.

REFERENCES


Part III

Appendix
In Part IV a literature review is given on methods to improve the off-axis stiffness of large displacement compliant mechanisms. This gives the reader valuable background information on the out-of-plane stiffness behavior of large displacement compliant mechanisms, an important part of this research. This appendix provides more information on two other important aspects of the research: underactuated fingers, and the load path representation, which is the design domain parameterization that was used.

A.1 Underactuated fingers

A mechanism that is underactuated has fewer actuators than DoFs. In a mechanical finger this can lead to an adaptability which allows the finger to automatically adjust itself to the shape of the object it is in contact with. This happens without the use of sensors or complex control systems [13]. In the past a number of underactuated fingers have been designed, either with rigid links, lumped compliance, or distributed compliance. To give the reader a sense of what has been done so far a selection of these will be described below.

A.1.1 Rigid-body underactuated fingers

In his paper Laliberte [13] conducted a literature review that showed that underactuated fingers can be divided into two groups: those that use linkages, and those that are tendon-driven. He continues his paper with a description of the underactuated finger he designed. The finger has three DoF and one actuator. The dimensions of the finger are varied to optimize its performance, which is measured by the direction of the contact forces, the distribution of the contact forces over the phalanges, and the location of the equilibrium point. The contact forces are not accurately known and the actuation force is also not given.

Crisman [3] designed a tendon-based finger with three DoF and one motor. The hand
Figure A.1: The prosthetic tendon-driven underactuated compliant hand by Carrozza.

was shown to be able to grasp several items successfully. No data on the actuation force nor the resulting contact forces was given, although it was said that the objects weighed up to 2.4 kg.

Similar to Crisman, Hirose [11] also designed a tendon-driven underactuated finger. In this case the finger, called a soft gripper, consisted of 10 DoF actuated with only one single motor. An experiment showed that the finger can grip an object with uniform pressure. However, no information was given on the exact height of this pressure. The actuation force applied on the tendon is about 100 N.

Shimojima [18] designed a gripper with three DoF and one actuator using linkages. By varying dimensions of the gripper the contact forces were optimized for various object shapes. Depending on the dimensions, the contact forces could vary between 0.5 N and 6 N, with an input torque of about 0.98 Nm.

Another underactuated hand using linkages, the TBM hand, was designed by Dechev [7]. He designed the fingers (four finger plus a thumb) based on the notion that a finger should be able to curl as they flex. The four fingers had three DoF, while the thumb had only two. One actuator was used for the entire hand. Per finger 3 N of force was necessary, which resulted in a contact force on the object varying between 0 and 4 N.

A.1.2 Underactuated fingers using lumped compliance

In the past ten years also a number of papers on underactuated fingers using lumped compliance have been published. A mechanism is considered to have lumped compliance if only a small portion of the mechanism can undergo elastic deformation (for example with the use of notch hinges) [12].

Carrozza [2] designed a tendon-driven compliant underactuated finger. The finger is based on the shape of a human finger (see Fig. A.1). The joints, however, have been replaced by compliant notch hinges located on the upper half of the finger. In the lower half of the finger the tendon is located that is used to actuate the finger, running from the tip of the finger down to the hand. Unfortunately it is unclear how much actuation force is needed and what the resulting contact force is. No mention of its out-of-plane behavior is made either.

Doria [10] designed a compliant underactuated finger with five DoF and a single actuator (see Fig. A.2). The structure of the finger was obtained by optimizing three different structures based on the contact force properties, the deformation of the joints, and the contact force to actuation force ratio. The required actuation force of the final structure was shown to vary between 10 N and 1110 N, depending on the material use. No infor-
A.1 Underactuated fingers

Dollar [8] designed a tendon-based compliant grasper with two DoF and one actuators. The compliant parts between the phalanges are compliant flexures said to be designed to be compliant in the in-plane direction and stiff in the out-of-plane direction. An experiment was done to determine the out-of-plane stiffness of the finger, which showed that the finger has an out-of-plane stiffness of 0.014 N/mm. The SDM hand developed using these fingers [9] could produce 30N of contact force with approximately 300N of actuator force. The hand itself was shown to be very robust and fully functional after impact and other large loads.

A.1.3 Underactuated fingers using distributed compliance

Distributed compliance means that the entire mechanism can undergo elastic deformation. So far only few underactuated fingers were designed using this principle.

Lan [14] designed two compliant fingers (one for object handling and one for snap-fit applications) based on nonlinear constrained minimization. The finger used for object handling is in essence just a flexible beam and cannot grab an object. It is mainly used to show how the method of nonlinear constrained minimization can provide accurate results while being much faster than other methods such as FEM. Whether or not this method would also work on more complicated structures is unclear.

Steutel [19] designed an underactuated distributed compliant finger with three DoF and a single actuator. The structure of the finger was based on the general topology of existing compliant underactuated fingers and then modeled using the Pseudo-Rigid Body approach. The inner connection point of the finger to the hand is grounded and on the outer connection point a linear vertical force is applied which pushes the finger to flex (see Fig. A.3). A semi-manual optimization was conducted for the dimensions of the finger. An experiment on the final finger showed that the contact forces on the three phalanges were 0.79 N, 0.37N and 0.32N (from proximal to distal), for an actuation force of 2.17N. This results in a high contact force to actuation force ratio of 0.75. No mention of the out-of-plane stiffness behavior has been made.
Another distributed compliant underactuated finger was designed by Petkovic [17] (see Fig. A.4). The design was based on existing gripper models. Instead of measuring the contact force, Petkovic measured the contact pressure and contact area. Converting this back to force, the contact force varied between about 4N and 10N. No data is provided on either the actuation force applied, or the out-of-plane stiffness behavior of the gripper.

### A.2 Load path representation

There are several methods that can be used to represent the topology of a compliant mechanism. The two best known methods are the continuous material density parameterization, and the full or partial ground structure parameterization [12].

The continuous material density parameterization divides the design domain into a grid of elements. Through optimization it is determined which of these elements exist in the final topology. This has as effect that the resulting topology can have an odd shape which may be difficult to manufacture.

The ground structure parameterization defines a grid of points in the design domain which can be connected by beam elements, which makes it easier to manufacture. Which points are connected by beams and which aren’t, is determined by the optimization. In the full
A.2 Load path representation

ground structure parameterization a point can be directly connected to any other point, whereas in the partial ground structure parameterization a point can only be directly connected to a point in its direct neighborhood. This results in fewer design variables when using the partial ground structure parameterization than when using the full ground structure parameterization. However, even so the amount of design variables needed are rather high, since for each possible connection it has to be specified whether or not the beam exists, and quite often the thickness per beam in one direction is also a design variable. Furthermore, this method often results in 'disconnected designs', where one of the ports is not connected to the other ports.

Lu [15],[16] developed a method named the 'load path representation'. This method is based on the ground structure approach but requires fewer design variables and also cannot lead to disconnected designs. In the load path representation a set of 'interconnecting points' is defined, which can be similar to that used in the ground structure approach. For each pair of ports that have to be connected - for example input-output, or ground-output - a set of possible "paths" is defined. These are possible connections to go from one of the ports to the other. In the case of input-output, one of the paths could be "input - interconnecting point 1 - interconnecting point 2 - output". Through optimization it is then determined which path is chosen for the final topology. This method of defining the topology by paths instead of individual beams ensures connectivity and at the same time also decreases the number of design variables.
At the beginning of this research a number of so-called "simple design problems" (abbreviated to SDP) were conducted. The purpose of these SDP's is to already test, determine and understand some parts of the final optimization of the compliant underactuated finger before performing the actual optimization. For example, the objective functions for the out-of-plane stiffness were validated, it was investigated how to implement the contact model in Ansys, and the effect of the curved beam elements on the out-of-plane stiffness was looked into. In the following sections each SDP will be explained in detail.

B.1 SDP1: Single beam investigation

The purpose of the single beam investigation was to gain a better understanding of how exactly the curve of a beam influences the out-of-plane stiffness over the entire range of motion in both in-plane and out-of-plane direction of that beam. This was needed to see both if the option of having curved beams in the optimization could provide better results as well as which values of "curvedness" would be valuable.

B.1.1 Method

The curve in both this example and in the rest of the research is defined as a cubic Bézier curve. Ansys, the FEM analysis tool used, provides two commands that can create curved lines: a cubic Bézier curve, and a circular arc. Both are equally easy to use, but the Bézier curve offers the option to easily adapt the code and change the simple curve to a more complex shape, for instance an S-shape. For this reason the choice fell upon the cubic Bézier curve.

The curve is constructed by fitting the Bézier curve to a set of three keypoints: the start and end point of the beam element, and one point inbetween. The distance from this middle point to the straight line between the start and end point given as a percentage of the length between the start and end point defines the curve \( c \), making it a dimensionless
Figure B.1: A schematic view of the single beam structure is provided. \( x_{\text{length}} \) is the difference between the end point and the start point of the beam element. \( c \) is the curve of the beam element presented as a percentage of \( x_{\text{length}} \). Only the curve is a design variable in the single beam investigation.

Table B.1: The dimensions of the beam elements used in the out-of-plane stiffness profile investigations for the single beam element and double beam element structure are shown below, as well as the range of force and step size of the force applied to them.

<table>
<thead>
<tr>
<th>Beam structure</th>
<th>Single</th>
<th>Double</th>
</tr>
</thead>
<tbody>
<tr>
<td>( F_y ) range [N]</td>
<td>0 - 10</td>
<td>0 - 10</td>
</tr>
<tr>
<td>( F_y ) stepsize [N]</td>
<td>0.2</td>
<td>0.5</td>
</tr>
<tr>
<td>( F_z ) range [N]</td>
<td>0 - 10</td>
<td>0 - 10</td>
</tr>
<tr>
<td>( F_z ) stepsize [N]</td>
<td>0.2</td>
<td>0.5</td>
</tr>
<tr>
<td>( t_y ) [mm]</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>( t_z ) [mm]</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>( x_{\text{length}} ) [mm]</td>
<td>100</td>
<td>50</td>
</tr>
</tbody>
</table>

number (see Figure B.1). So suppose that \( x_{\text{length}} \) is 200 mm, and the maximum height of the curve is 10 mm, then \( c \) is 5%.

The material used to test the single beam is aluminum (the elastic modulus is 70,000 MPa, the Poisson ratio 0.35, and the yield strength is 345 MPa). The Von Mises stress in the single beam was not allowed to pass 80% of the yield strength. The ANSYS model uses SHELL181 elements, with roughly 300 elements.

The model is shown in Fig. B.1. The total length of the beam was set to 100 mm, the in-plane thickness to 1 mm, and the out-of-plane thickness 10 mm. For a range of curves both the out-of-plane stiffness as well as the in-plane stiffness was computed. The stiffness was computed by slowly increasing the in-plane force \( F_y \) starting at 0 N. For each step in \( F_y \) a range of forces in out-of-plane direction was applied, starting at 0 N as well. This resulted in a grid of forces applied in both directions and the corresponding displacements in both directions, measured at the location at which the force was also applied. Fig B.2 shows an example of one of these grids. The dimensions of the beam as well as the range of curves and forces applied are provided in Tab. B.1.
Figure B.2: The data collected for a single beam structure with a curve $c$ of 0 is shown. On the x-axis the displacement in the y-direction is shown ($U_y$), which is also shown in color with blue being no displacement and red being the maximum displacement. On the y-axis the displacement in z-direction is shown ($U_z$) and on the z-axis the out-of-plane stiffness is shown ($K_z$). The data shows how the out-of-plane stiffness drops as the displacement in y-direction becomes larger, while the out-of-plane stiffness hardly varies with the displacement in z-direction.

B.1.2 Results

The analysis of the results provided two interesting insights regarding the out-of-plane stiffness of the beam. Both will be explained below.

Out-of-plane stiffness in out-of-plane direction

The results of a wide range of curves show that the out-of-plane stiffness varies very little in out-of-plane direction, with a maximum deviation of 15% from the average out-of-plane stiffness over the out-of-plane RoM. Fig. B.2 shows an example of the data points collected in out-of-plane direction for one of the curves tested. This result allows the assumption that a good estimation of the out-of-plane stiffness can be made by using the displacement caused by applying one set force, for example 1 N.

Out-of-plane stiffness in in-plane direction

The behavior of the out-of-plane stiffness of the single beam over the in-plane RoM is shown in Fig. B.3. It can be seen that the out-of-plane stiffness profile has a Gaussian-like shape, with the out-of-plane stiffness increasing with an increasing deflection for $c$ equal to -5 or lower, and decreasing with an increasing deflection for $c$ equal to 0 or higher.
The single beam element investigation shows that there is one Gaussian-like out-of-plane stiffness profile and that each curve of a beam element corresponds to a part of this profile. This does not hold for only the curves shown in Fig. B.3. Curves of \( c \) between -30 and +30 were also investigated and were shown to have the same properties. This Gaussian-like curve shows that for beams with a curve of \( c \) higher than 0 the out-of-plane stiffness will go down continuously as the beam deflects in in-plane direction. Similarly, for beams with a curve of \( c \) lower than -5, the out-of-plane stiffness will go up continuously as the beam deflects in in-plane direction. For both positive and negative \( c \) values it is shown that the average out-of-plane decreases as \( c \) moves further away from 0 (i.e. as the initial curvature of the beam increases). For values of \( c \) between 0 and -5 the out-of-plane stiffness first increases before decreasing again while the beam is deflecting in in-plane direction. The decrease of out-of-plane stiffness when the beam is curved can also be shown with the use of two equations. For both equations it is assumed that the beam has a zero in-plane deflection. When the beam is not curved, the well known classical beam theory can be applied to calculate the out-of-plane stiffness for small deflections in the out-of-plane direction:

\[
k_z = \left( \frac{L^3}{3EI} \right)^{-1}
\]

where \( I \) is the area moment of inertia, \( L \) is the length of the beam, and \( E \) is the elastic modulus. For the single beam example used in this research this leads to a stiffness of 17.5 N/mm. Through the simulations with ANSYS a stiffness of about 17.3 N/mm was found.

Dahlberg [4] provides the equation to calculate the out-of-plane deflection of a beam curved as a quarter of a circle when an out-of-plane load is applied. The equation is as follows:
B.2 SDP 2: Single beam optimization

The design variables of the single beam optimization (SDP 2) and their possible values are shown in Table B.2.

<table>
<thead>
<tr>
<th>Design variable</th>
<th>Possible values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_y$ [mm]</td>
<td>1, 1.4, 1.8, 2.2, 2.6, 3.0, 3.4, 3.8</td>
</tr>
<tr>
<td>$t_z$ [mm]</td>
<td>7, 8, 9, 10, 12, 13, 14, 15</td>
</tr>
<tr>
<td>curve $c$ [-]</td>
<td>10, 5, 2, 0, -1, -2, -5, -10</td>
</tr>
</tbody>
</table>

Table B.2: The design variables of the single beam optimization (SDP 2) and their possible values.

\[
k_z = \left( \frac{\pi R^3}{4EI} + \left( \frac{3\pi}{4} - 2 \right) \frac{R^3}{GJ} \right)^{-1}
\]

where $J$ is the torsional constant, $G$ is the shear modulus and $R$ is the radius of the curvature of the beam. In our example, $R$ is 63.7 mm. This means that the total length of the beam is kept constant at 100 mm. Up to a quarter of a circle the cubic Bézier curve can provide a good approximation of a circular arc. A quarter of a circle corresponds to a curve $c$ of 25. For this curve, a stiffness of 0.85 N/mm was found through ANSYS, which is the same as the value found through the equation. The results indicate that the use of curved beams may positively affect the out-of-plane stiffness. It also shows that although beams with a curve $c$ higher than 5 or lower than -5 may have a more constant out-of-plane stiffness over the in-plane RoM, the average out-of-plane stiffness is significantly lower.

B.2 SDP 2: Single beam optimization

The purpose of the single beam optimization was to optimize the single beam to a set of objective functions which would also be needed for the final optimization. This would allow us to test the objective functions and gain a better understanding of how the design variables and the objective functions relate and where the trade-off’s lie.

B.2.1 Method

The same single beam structure is used for this SDP as for the previous one (see Fig. B.1). The design variables are the thickness in $y$-direction ($t_y$), the thickness in $z$-direction ($t_z$) and the curve $c$. The values these design variables can take are shown in Tab. B.2. In $y$-direction a force of 20N is applied in steps of 0.5N. At each step the displacement in $x$-, $y$-, and $z$-direction is recorded without a force applied in $z$-direction, as well as with a force of 1N applied.

For the optimization three different objective functions are used:

1. First of all the in-plane RoM, defined as the length of the path of the output point. The objective is to have this value as high as possible, so the finger can be adaptable to a wide range of different objects. The equation is as follows:

\[
O_{RoM} = \sum_{i=1}^{n-1} ||p_{i+1} - p_i||
\]
where \( n \) is the number of points measured across the in-plane RoM and \( p \) are the \( x \) and \( y \) coordinates of the output port, which is also where the actuation force was applied.

2. The second objective function is the out-of-plane stiffness, averaged over the entire in-plane RoM. A high average is desired, to be able to more accurately control the movement of the finger, even when an external load is applied. The objective function is formulated as in Eq. B.4:

\[
O_{\text{mean}} = \frac{1}{n} \sum_{i=1}^{n} k_{z,i}
\]

(B.4)

where \( k_z \) is the out-of-plane stiffness.

3. The third objective function is the deviation from the average out-of-plane stiffness over the in-plane RoM. Ideally this value would be zero, resulting in a constant out-of-plane stiffness. The reason for this is also to be able to more accurately control the finger over the in-plane RoM. The equation is:

\[
O_{\text{dev}} = \frac{1}{n} \sum_{i=1}^{n} (k_{z,i} - \bar{k}_z)^2
\]

(B.5)

where \( \bar{k}_z \) is the average out-of-plane stiffness over the in-plane RoM.

In order to be able to examine the difference in performance between solutions with and without curved beam elements a Non-dominated Sorting Genetic Algorithm is used (MATLAB uses a variant of NSGA-II), which results in a set of Pareto solutions instead of only one solution per optimization. The same ANSYS model is used as for SDP 1, with roughly 300 SHELL181 elements. A population of 50 and 20 generations were used. The size of the binary chromosome was 9 bits.

B.2.2 Results

The resulting set of Pareto solutions are shown in Fig. B.4. Solutions with a high out-of-plane stiffness tend to have a low RoM. This is to be expected since the stiffness in both in-plane and out-of-plane direction are connected up to a certain extent. Since the out-of-plane stiffness has been shown to vary with the in-plane deflection, a low RoM also means that a smaller variation in out-of-plane stiffness can be found.

This group of high out-of-plane stiffness, low RoM and low deviation from the average out-of-plane stiffness can be recognized by the blue dots in the upper left corner. As mentioned before, the range of design variable curve \( c \) varies from -10 to +10. However, of this group only has curves \( c \) of -2 and -1. With the information we obtained through the single beam investigation this makes perfect sense, since the Gaussian-like shape shown in Fig. B.3 shows a peak in out-of-plane stiffness around those values. The thickness in \( z \)-direction of this group consists of the largest values allowed. The average out-of-plane stiffness tends to become lower for a decreasing thickness in \( y \)-direction. This can be explained with the fact that the thinner a beam becomes, the lower its torsional stiffness.
B.3 SDP 3: Double beam investigation

Figure B.4: The resulting 18 Pareto solutions. On the y-axis the out-of-plane stiffness of the single beam is shown and on the x-axis the deviation of the out-of-plane stiffness from the average out-of-plane stiffness. The in-plane range of motion (RoM) is portrayed as the color of the dots, with blue signifying a low RoM and red a high RoM.

also becomes. Thus, when curved it will twist more, resulting in a lower out-of-plane stiffness.

At the bottom part of Fig. B.4 the second group of solutions can be seen. The characteristics are a low average out-of-plane stiffness and a high RoM (orange to dark red dots). The deviation from the average out-of-plane stiffness varies from very high, to very low. In this group only low values of in-plane thickness are used, which increases the RoM. The values of the curves $c$ of these solutions vary between -5 and -1, with one example of 10. This particular solution also has a very low thickness in y- and z-direction. As such, it has the lowest out-of-plane stiffness of all these solutions and one of the highest RoMs. The deviation from the average out-of-plane stiffness is also very low, since this average is almost zero.

Aside from a better understanding of the trade-off between the different objective functions and the effect of curved beams, more information on the range of the design variable values was also obtained. It was concluded that a curve $c$ between -5 and 5 would be sufficient, while maintaining the same range of thickness in y- and z-direction.

B.3 SDP 3: Double beam investigation

After having looked into the behavior of a single curved beam, the next step is to understand if and how using curved elements effects the out-of-plane stiffness in a more complex system. For this reason a double beam system was used, of which the curve $c$ of
Figure B.5: A schematic view of the double beam structure is provided. $x_{\text{length}}$ is the difference between the end point and the start point of the beam element. This length is the same for both beam elements. $c$ is the curve of the beam element presented as a percentage of $x_{\text{length}}$. The curve of both beam elements and the angle between them are design variables in this investigation.

each individual beam element can vary, as well as the angle between the beam elements.

B.3.1 Method

The schematic view of the double beam structure is shown in Fig. B.5. The angle between the beam elements varies from -90 degrees to +90 degrees. The dimension of the double beam structure as well as the forces and range of curves $c$ applied are shown in Tab. B.1.

The material used to test the double beam structure is also aluminum (the elastic modulus is 70,000 MPa, the Poisson ratio 0.35, and the yield strength is 345 MPa). The Von Mises stress in the single beam is not allowed to pass 80% of the yield strength. The ANSYS model uses SHELL181 elements, with roughly 600 elements.

Similar to the single beam investigation, both the out-of-plane stiffness as well as the in-plane stiffness is computed for a range of curves and angles. The stiffness is computed by slowly increasing the in-plane force $F_y$ starting at 0 N. For each step in $F_y$ a range of forces in out-of-plane direction is applied, starting at 0 N as well.

B.3.2 Results

The double beam element investigation shows a similar result, but with a few differences (see Fig. B.6). As seen with the single beam element, each curve corresponds to a part of the profile. However, now the curve does not influences only the shape of the profile. It also has an effect on the magnitude of the stiffness profile, meaning that the out-of-plane stiffness profiles of the different curves do not connect seamlessly anymore as they did in the single beam element structure. The results also show that while the curve of the first (clamped-in) beam element determines both the part of the profile it corresponds with and the magnitude of the out-of-plane stiffness profile, the second beam element only influences the magnitude (see Fig. B.6). A larger moment due to the out-of-plane force is applied on the first beam element than on the second beam element. As such it makes sense that the curve of the first beam element has a larger influence on the out-of-plane stiffness over the RoM. The angle between the two beam elements behaves the same as
B.4 SDP 4: Contact model optimization

Since contact will need to be modeled for the compliant underactuated finger, it was decided to first test it on a simplified contact model. A better sense of the contact possibilities in ANSYS and their issues would be obtained, and it would also provide an opportunity to test objective functions related to contact.

B.4.1 Method

A simple beam model with two contact elements (output ports) and a varying thickness is used (see Fig. B.8). The beam is clamped in on one end and a force is applied on the other end perpendicular to the beam. The design variables are the location of the first contact element in horizontal direction, the thickness of the contact elements in y-direction, the thickness of the entire structure in z-direction, and the location of the points connecting the beam elements with the contact elements and the ground (see Fig. B.8 and Tab. B.3).
The out-of-plane stiffness profile over the in-plane RoM is shown for three different angles and three different sets of curves: the curve $c$ of both beam elements is -5 in the upper row, 0 in the middle row, and 5 in the bottom. On the y-axis the out-of-plane stiffness is shown. The in-plane deflection is shown both on the x-axis and as color in the graph, to clarify how the stiffness changes depending on the curve. A schematic view of the double beam element structure is given at the top of each graph. Note how the angle also influences the shape of the out-of-plane stiffness, resulting in a similar Gaussian-like shape seen in SDP1.

The beam structure is modeled as a solid (PLANE42) upon which a contact surface is placed (CONTA172). The object was also modeled as a solid (PLANE42) and the contact elements (TARGE169) correspond to those on the beam structure.

Three objective functions are used:

1. The first objective function is meant to minimize the contact force before the grasp is complete. The though behind this is that if a high contact force is needed to adjust the finger to make full contact with the object this may push the object away or damage it. The equation used is shown below:

$$ O_{cf, pregrasp} = \max(F_{c,1, pregrasp}) + \max(F_{c,2, pregrasp}) \quad (B.6) $$

where $F_{c,i, pregrasp}$ is the vector of contact forces on contact element $i$ before the grasp is complete. The grasp is considered to be complete when the change in displacement of the contact elements is below a certain threshold.
A schematic view of the model used for SDP4. It shows a simple design of a beam with two contact elements. The location of the first contact element is determined by $L_{c1}$. The thickness of the beam is varied at the points at which it connects to the contact elements and the ground. The thickness of the contact elements in $y$-direction is also varied, as well as the thickness of the entire structure in $z$-direction.

**Table B.3**: The design variables used for SDP4 with the corresponding possible values. The possible values for $Y_{k1}$ through $Y_{k4}$ are the same.

<table>
<thead>
<tr>
<th>Design variable</th>
<th>Possible values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_{c1}$ [mm]</td>
<td>0.5, 0.57, 0.63, 0.7</td>
</tr>
<tr>
<td>$Y_{ki}$ [mm]</td>
<td>0.5, 0.7, 0.9, 1.1</td>
</tr>
<tr>
<td>$t_c$ [mm]</td>
<td>2, 2.7, 3.4, 4</td>
</tr>
<tr>
<td>$t_z$ [mm]</td>
<td>7, 9, 11, 13</td>
</tr>
</tbody>
</table>

2. The second objective function is aimed to maximize the contact force to actuation force ratio:

$$O_{cf,af} = \frac{F_{c,1} + F_{c,2}}{F_a}$$  \hspace{1cm} (B.7)

where $F_a$ is the maximum actuation force applied, and $F_{c,i}$ is the contact force on contact element $i$ at the maximum actuation force.

3. The third objective function is meant to optimize the resultant contact force direction, by having it point as much as possible toward the palm of the hand. The angle of resultant contact force vector relative to the ground is calculated in such a way that it is -90 degrees when it is pointed completely to the ground and 0 degrees when it is pointed towards the fingers that are supposed to be opposite of it. If it is pointed anywhere outside of this 90 degree region a high fitness value is given as a penalization, since this will cause the finger to lose grip on the object.

A population of 20 and 15 generations were used. The size of the binary chromosome was 14 bits.
Figure B.9: An image of the Ansys model of SDP4. The beam with the two contact elements are shown, as well as the object. The object consists of two separate surfaces, which are part of a circular object. Only modeling the part of this surface that will make contact reduces computation time and effort.

Table B.4: The values of the design variables of the three Pareto solutions of SDP4, together with their fitness values

<table>
<thead>
<tr>
<th>Solution</th>
<th>Obj. 1</th>
<th>Obj. 2</th>
<th>Obj. 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[N]</td>
<td>[-]</td>
<td>[°]</td>
</tr>
<tr>
<td>1</td>
<td>6.631</td>
<td>2.097</td>
<td>-37.93</td>
</tr>
<tr>
<td>2</td>
<td>6.863</td>
<td>2.082</td>
<td>-39.31</td>
</tr>
<tr>
<td>3</td>
<td>10.19</td>
<td>2.019</td>
<td>-38.78</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>(L_{c1})</th>
<th>(Y_{k1})</th>
<th>(Y_{k2})</th>
<th>(Y_{k3})</th>
<th>(Y_{k4})</th>
<th>(t_c)</th>
<th>(t_z)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[mm]</td>
<td>[mm]</td>
<td>[mm]</td>
<td>[mm]</td>
<td>[mm]</td>
<td>[mm]</td>
<td>[mm]</td>
</tr>
<tr>
<td>1</td>
<td>0.7</td>
<td>0.9</td>
<td>0.5</td>
<td>0.5</td>
<td>2</td>
<td>11</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.63</td>
<td>0.9</td>
<td>0.5</td>
<td>0.5</td>
<td>2</td>
<td>11</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.7</td>
<td>0.9</td>
<td>0.5</td>
<td>0.5</td>
<td>2</td>
<td>7</td>
<td></td>
</tr>
</tbody>
</table>

B.5 Results

The results show three Pareto solutions when using the three objective functions. The values of the design variables of these solutions are shown in Tab. B.4, together with their fitness values. Fig. B.9 shows the Ansys model of the structure with the object.

When investigating this further by optimizing the structure with the focus only on objective function 2 and 3, it was found that this results in only one Pareto solution. What this tells us is that for this particular structure there is no trade-off between objective function 2 and 3. For this particular design problem this makes sense. The input force is always applied perpendicular to the beam elements, which means in the same direction as the contact force. As such, for the contact force to actuation force ratio (objective function 2) it is best to have the two contact elements as close as possible to each other. The distribution of the contact forces influences the fitness value of objective function 3 (optimizing the resultant contact force direction). However, the location of the contact element does so as well, and the more the contact elements are located towards the end of the beam, the better the fitness value becomes. As such, both objective functions 2 and 3 benefit from a high \(L_{c1}\) and there is no trade-off for this design problem. For a more
complex problem this will probably not be the case.
Appendix C

Genetic Algorithm

In this appendix it is explained why a non-dominated sorting genetic algorithm was used, which operators were used and why, and what the stopping criteria is.

C.1 Multiobjective genetic algorithm

There are two often used ways to deal with multiple objective functions in an optimization problem. The first one is combining them into one function, for example by multiplying them, dividing them, or by applying weights and adding them up. The advantage of this method is that it makes the optimization less complex, since there is only one objective function. The disadvantage is that by combining the objective functions into one, the designer is applying a bias on which objective function is considered to be more important.

The second option is to not combine the different objective functions, but to search for the Pareto front. This front is made up of Pareto solutions. These are optimal solutions in the sense that it is not possible to find a solution that has a higher value for one of the objective functions without worsening the value of one of the other objective functions. They’re also often referred to as non-dominated solutions. The advantages of this method over the first one are that the solutions found are not biased by the designer and that the designer is given more insight into the trade-off between the different objective functions and the corresponding solutions.

When looking for Pareto solutions evolutionary algorithms are often favored, as they use a population of solutions in each iteration, rather than only a single solution [5]. One of the most popular evolutionary algorithms is the NSGA-II: Non-dominated Sorting Genetic Algorithm [6]. This algorithm uses a so-called elite-preservation strategy, which means that some of the best solutions will continue to the next generation untouched. This has been shown to speed up the performance of the genetic algorithm and it also avoids the loss of good solutions. In MATLAB the multi-objective genetic algorithm function, `gamultiobj`, is based on the NSGA-II.
C.2 Binary chromosome

In this research the chromosomes are binary and not continuous. This means that for each design variable a set of possible values is defined, corresponding to a binary value. There are several reasons to choose a binary chromosome string explained in [5]. Reeves (also explained in [5]) provided one of the more intuitive reasons. The initial population should be large enough to have every possible value of each gene present, which is needed to ensure that the genetic algorithm can create any possible solution. The study concluded that the higher the cardinality of the alphabet used (the binary alphabet has a cardinality of 2, while the English alphabet has a cardinality of 26), the larger the minimum population size needed is.

C.3 Selection operator

The selection operator determines which solutions of one generation will be used as parents for the next generation. There are two common methods that MATLAB also provides [5]: proportionate selection and tournament selection. Each of these methods will be explained in more detail below.

In proportionate selection the solutions are scaled based on their fitness values. One example is the "Roulette wheel", where each solution is assigned a portion of the wheel, depending on what their fitness value is. The wheel is then spun N times, to select the parents. This means that the better the fitness value compared to the other solutions, the higher the chance is that that solution is picked. One of the issues with this selection operator is the scaling. For example, if there is one solution with a much better fitness value than the other solution, this solution will end up dominating the parent pool.

In tournament selection tournaments are played between two solutions. The one with the better fitness values is used as a parent. Between all the solutions selected to be parents another tournament is played. The "winning" solution is added as a parent again. This means that each solution will in the parent pool 0, 1 or 2 times. This selection operator is commonly used for NSGA-II and according to Deb [5] it has been shown that "tournament selection has better or equivalent convergence and computational time complexity properties when compared to any other reproduction operator that exists in literature". In this research this selection operator is used.

C.4 Crossover operator

The crossover method creates children by recombining the parent chromosomes. This allows for new, hopefully better, solutions to be found. There are several crossover methods that can be found in literature. However, only three common methods are recommended by MATLAB when using binary chromosomes [1]: single point, two point, and uniform (in MATLAB referred to as `scattered`) crossover operators. Hence, only these three will be explained further.

- The single point crossover operator choses one random point on the chromosome string and swaps the parts of the two parent chromosomes on the right of this point.
The two point crossover operator chooses two random points and swaps the part inbetween these two points.

The uniform crossover operator chooses per allele or bit whether to use the value of parent 1 or parent 2 for the child chromosome.

It is unclear which crossover operator is better suited for this problem. In MATLAB the uniform crossover operator is the default operator and this was also used for this research. The crossover operator was restricted to allow only crossover between the bits that define one design variable and not also crossover within the design variables.

C.5 Mutation operator

The mutation operator adds a mutation to the set of children chromosomes, in order to increase diversity. In MATLAB two mutation operators are used:

- The Gaussian mutation operator, which add a random number with a mean of 0 to each bit or allele.
- The uniform mutation operator. This operator first selects a random group of bits or alleles in each chromosome to mutate. It then replaces the allele by a random number selected uniformly from the range of that entry.

For binary chromosomes MATLAB recommends using a uniform mutation operator, and as such this was used for this research.

C.6 Stopping criteria

For the multi-objective genetic algorithm MATLAB uses a stopping criteria based on the average spread of the Pareto front over a predefined number of generations, and one based on the maximum number of generations. For this research the genetic algorithm was set to stop when the number of generations reached 20, or when the average spread of the Pareto front over 5 generations varied less than 0.001.
Appendix D

Topology Optimization Flowchart

In the objective function file of the genetic algorithm in MATLAB several steps are taken to compute the objective function of the design provided by the genetic algorithm. The steps will be explained further below. The numbers of the steps correspond to the numbers shown in the flowchart in Fig. D.1.

D.1 Getting the design variables

The genetic algorithm is set to use binary chromosomes. As such, these have to be transformed into the corresponding design variables first. The design variables and their possible values are shown in Tab. D.1.

The location of an interconnecting point (IP) is defined as the ”original” location plus a certain deviation. This deviation has multiple possible values and thus makes the interconnecting points design variables as well. This is visualized in Fig. D.2 and the original locations are provided in Tab. D.2. The genetic algorithm is used to optimize the offset of the base location of the 6 interconnecting points.

D.2 Checking the output connection

The way load path representation is implemented allows for the possibility of having an output port connected to the rest of the design at only one end, thus having it not fully connected to the rest of the design (see Fig. D.3). This is possible since the paths leading to that output can all be using the same interconnecting point before connecting to the output. The result is undesirable since it does not allow any control over the behavior of the output port through actuation. As such designs that have this property are given bad fitness values.
Figure D.1: This flowchart depicts the steps that are taken to obtain the fitness values of the objective functions.
D.2 Checking the output connection

Figure D.2: The possible values of all the interconnecting points, the ground port, and the two output ports. A gray square is placed around the possible locations of each of the six interconnecting points. The output ports each have two possible locations: the upper two (light) green lines show those for the second output port, while the bottom two (dark) green lines show those for the first output port.
Table D.1: The design variables and optimization properties. The thickness in z-direction ($t_z$) is chosen for the entire structure, whereas the thickness in y-direction ($t_y$) can vary per beam. The interconnecting points can vary from their original position. The amount that they can vary is given as a design variable. The input port is located at the center (0,0). The length of an output port is set to 30 mm.

<table>
<thead>
<tr>
<th>Interconnecting point</th>
<th>(x, y) [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(15, 75)</td>
</tr>
<tr>
<td>2</td>
<td>(75, 75)</td>
</tr>
<tr>
<td>3</td>
<td>(15, 180)</td>
</tr>
<tr>
<td>4</td>
<td>(75, 180)</td>
</tr>
<tr>
<td>5</td>
<td>(15, 270)</td>
</tr>
<tr>
<td>6</td>
<td>(75, 270)</td>
</tr>
</tbody>
</table>

Table D.2: The original positions of the interconnecting points are given here. The deviation of these original points is a design variable of the optimization.
D.3 Checking the crossing of beams

One issue that is inherent to both the ground structure and the load path representation is the possibility of the crossing of beams in designs. If this happens, one of the paths is changed so that the beam elements do not cross anymore (see the image on the left in Fig. D.4). For example, if IP1 is connected to IP4 (IP1-IP4), and IP2 is connected to IP3 (IP2-IP3), the first connection could be changed to go through IP3 (so IP1-IP3-IP4). In some cases such as the connections with the outputs the paths cannot be changed, in which case the design is penalized by giving it bad fitness values.

In this same step the maximum allowable curve of a beam element is also calculated, in order to prevent the crossing of beams due to being curved (see Fig. D.4 for an example of this crossing of beams).

D.4 Writing Ansys code

Using MATLAB a file is created that contains the locations of the keypoints of the structure, as well as the information about the lines, areas, mesh density, stepsize for the force, and so on. This information is needed to run both the code for the out-of-plane stiffness as well as that for contact.

D.5 Running Ansys out-of-plane stiffness code

The ANSYS APDL code is then run to compute the out-of-plane stiffness for this specific structure. A .txt file is written with information on the displacement of both output ports, needed to compute the out-of-plane stiffness and estimate the flexibility of the structure. The corresponding Von Mises stress is also collected.
D.6 Computing out-of-plane stiffness fitness values

With the data collected from the .txt file the fitness values of the out-of-plane objective functions are calculated. The out-of-plane stiffness is calculated as the force applied in z-direction (1N) divided by the difference in displacement in z-direction caused by that force.

D.7 Checking flexibility design

The largest displacement of the first output port shows if the structure would be able to make contact with the object. If the distance between the output port and the center of the object is smaller than the radius of the object it is considered to be flexible enough to make contact. If it isn’t flexible enough the fitness values for the contact objective functions are computed based on the displacement obtained with the out-of-plane stiffness .txt file.

D.8 Running Ansys contact code and computing the contact fitness values

If the structure is flexible enough, the contact model is run. Information is obtained on the displacement of both output ports and the contact forces on both the ports, as well as the corresponding Von Mises stress. With this information the fitness values for the contact objective functions are computed.
Choosing Paths

There are two possible ways to obtain the set of paths needed for the load path representation. The first one is allowing the designer to choose a set of paths that he or she thinks is likely to provide a large variety of well performing topologies, based on experience and intuition. The second possibility is determining all possible paths and not exerting any influence as the designer on which paths will or won’t be used. The advantages and disadvantages of both methods will be discussed briefly, after which will be shown how the method chosen for this optimization problem is applied.

E.1 Choosing a set of paths or allowing all possible paths

The main disadvantage of allowing all possible paths to be chosen (and the main advantage of choosing the set of paths by yourself) is the required computation time and the high number of paths. For small design problems this is manageable. As the number of interconnecting points increases, so does the computation time and number of paths, only at a much higher pace. When using a binary chromosome, a large number of paths to choose from means a large number of design variables. The result is that the population size will also have to increase, in order to obtain a good approximation of the Pareto front.

When allowing the designer to choose a set of paths based on experience and intuition, the chance of finding an unexpected or surprising topology that performs well is smaller. For a large number of interconnecting points the decrease in computation time and the smaller chromosome may be worth this, but for a small number of interconnecting points it is recommended to allow for all possible paths to be chosen. As such this method is used for the design problem at hand, which has only 6 interconnecting points.

E.2 Computing the set of paths

The process of computing the set of paths is explained in this section. Each point, both interconnecting points and ports, is assigned a number. A list of all possible combinations
of these numbers is then formed. Each combination is then checked for a number of criteria to determine whether it is a viable path. This results in a set of paths for each input-output port combination. For a visualization of these criteria, see Tab. E.1 and E.2.

On the right an example of a compliant finger is shown, which will be used in the following figures to visualize the different criteria used to determine whether a path is considered viable or not.

**Criteria 1:** Each point can appear only once in a path, to avoid unnecessary loops.

**Criteria 2:** The interconnecting point connected to one of the ports has to be in the neighborhood of that port, to avoid the crossing of beams.

**Table E.1:** Part I: A visualization of the different criteria used to determine whether or not a path can be used in the topology optimization.
Criteria 3: A point can only connect directly to another point in its neighborhood, also to avoid the crossing of beams.

Criteria 4: A vertical line in the right-most column is not allowed. The location of the interconnecting points in both in-plane directions is a design variable, meaning that it’s subject to change and can lead to an interconnecting point close to the output ports. If there is a vertical beam between the interconnecting points right next to the output port, this will most likely result in contact between that beam and the contact element, when the finger is deformed.

Criteria 5: The path cannot cross itself.

Table E.2: Part II: A visualization of the different criteria used to determine whether or not a path can be used in the topology optimization.
Appendix F

Ansys

For this research two models were used: one to compute the out-of-plane stiffness, and one to compute the contact forces. Some parts of these models require more information, which is given below.

F.1 Contact model

Since the compliant finger is only expected to move in the in-plane direction when making contact, this model did not need to be 3D. As such it was decided to use a 2D model, which also decreases the computation time. A simple design problem was used to model contact in 2D in ANSYS (described in Appendix B.4). The object to make contact with was modeled as a solid (PLANE42) upon which a contact surface was placed (TARGE169), corresponding to the contact surface on the beam structure (CONTA172). It was found to be challenging to get ANSYS to converge when making contact with the second contact element, until the right set of keyoptions for the contact element was found through trial and error. In the contact guide by ANSYS and on several presentations by ANSYS on the web, recommendations are made of which keyoptions to use in which cases. Below the combination of keyoptions are given that were found to work for this 2D model:

- Keyoption 2 (contact algorithm) is 1 (Penalty function)
- Keyoption 6 (contact stiffness variation) is 2 (make an aggressive refinement to the allowable stiffness range)
- Keyoption 9 (effect of initial penetration or gap) is 4 (include offset only (exclude initial geometrical penetration or gap), but with ramped effects)
- Keyoption 10 (contact stiffness update) is 2 (each iteration based on current mean stress of underlying elements (pair based))
- Keyoption 12 (behavior of contact surface) is 0 (standard)
Figure F.1: An example of a badly meshed intersection of beam elements for the 2D model. In the middle, indicated by a red arrow, a quadrilateral element is shown with one very sharp angle, which will give inaccurate results.

For the compliant underactuated finger the model had to be completely adaptable to each solution, since not only the size and shape were varied, but the topology as well. This resulted in difficulties with meshing at the locations where the beam elements intersected. Several methods were applied in an attempt to resolve this: using both quadrilateral and triangular elements; using only one of these elements; having the mesh in the beams be mapped or free; refining the mesh of these intersections; and changing the way in which these beams collided. However, despite these attempts, each topology showed multiple elements with a very sharp angle (see Fig. F.1 for an example), resulting in an unreliable analysis. As such, it was decided to use the 3D model, that used shell elements and had no issues with the intersection of the beam elements, as this was modeled as a line (see Fig. F.2).

The next step was to model the contact for the 3D model. The contact elements CONTA173 and TARGE170 were used. It was again extremely challenging to get Ansys to converge, this time even for the first contact element. As such, the problem was approached from another angle and nonlinear springs were used to approximate contact.

Two nonlinear springs (COMBIN39) were used, one for each output port. The springs were connected to the center of the object and the middle of each output port. The force-deflection curve was defined as such that the stiffness was 0.001 N/mm until the length of the spring was equal to the radius of the object. After this, the stiffness increased to 10 N/mm. The nonlinear springs have shown to converge more easily than contact elements for the compliant underactuated finger. The only recommendation for future use is the use of a nonlinear spring at each end of a contact elements (thus resulting in four nonlinear springs for this research). With only one spring per contact element, the angle of the contact element in relation to the object is not controlled. With one spring at each end it is, resulting in a more accurate contact modeling.
F.2 Element choice for 3D Model

For the computation of the out-of-plane stiffness a 3D model was needed. For this computation shell elements were used. Two other alternatives are beam elements, and solid elements.

The disadvantage of beam elements is that it only takes into account bending and axial stresses and not shear stress as well. The compliant underactuated finger will be loaded in both in-plane and out-of-plane direction, forcing it to twist. This is expected to cause significant shear stresses and as such a beam element is no longer an option.

Using solid elements for the 3D model would result in the same issues encountered when using solid elements for the 2D model: bad meshing at the intersection of the beam elements.

As such it is preferred to use shell elements (SHELL181) for this research.

F.3 Sensitivity analysis

In order to determine the size of the elements and the number of substeps a sensitivity analysis was done. For two designs and for both the 3D model on out-of-plane stiffness and the 3D model on contact various element sizes and numbers of substeps were tested. The resulting maximum Von Mises stress was compared to determine which element size and what number of substeps would lead to an accurate result. In general it holds that
the smaller the size of the element and the larger the number of substeps is, the longer the run takes and the more accurate the result becomes.

Fig. F.3 shows the results of the sensitivity analysis for one of the solutions tested on the 3D model. The maximum Von Mises stress obtained was compared to the Von Mises stress obtained from the most accurate run. For example, when looking at the number of elements needed on the short side of the beam, 20 elements was the highest number of elements tested. The results of the other number of elements were compared to this stress level. Remaining within a 5% difference of this maximum was considered acceptable. As such, it was chosen to use 5 substeps per 2N, 10 elements on the short side of the beam, and elements with a length of 3 mm on the long side of the beam.
Figure F.3: On these graphs the resulting Von Mises stress is shown compared to the Von Mises stress of the most accurate result. This most accurate result in marked with a green dot, while the result that was chosen for the compliant underactuated finger is marked by a red dot. The first graph shows the number of elements on the short side of the beam. Using half the number of elements (10 instead of 20) results in less than 5% change in maximum Von Mises stress. The second graph shows the size of the elements on the long side of the beam. Doubling the size of the elements (3 instead of 1.5) also results in a change of less than 5% change in maximum Von Mises stress. The third graph shows the number of substeps taken for a loadstep of 2N. The results show that using 5 substeps instead of 100 only results in a change of less than 2% of the maximum Von Mises stress.
Appendix G

Experiment

The experiment conducted on the prototype consisted of two parts: one to determine the contact behavior of the mechanism, and the other to determine the out-of-plane stiffness behavior. Both will be explained in this section with the use of photo’s.
Figure G.1: The experiment setup for the measurement of the contact behavior. There are two loadcells to measure the contact force on each output port and one loadcell, located between the input port and the actuator, to measure the input force. A linear actuator is used.
Figure G.2: The upper image shows the two loadcells used to measure the contact force. The middle image shows the actuation and the loadcell between the actuation and the input port. As can be seen the input port was attached to a gliding system to enable linear movement only. The actuator is attached to the mechanism with the use of a long thin screw, to allow for a large enough range of movement for the actuator. The bottom image shows the camera used to record the movement. This was only used as a reference and not as a sensor.
Figure G.3: The experiment setup for the measurement of the out-of-plane stiffness. There is a linear actuator, and one loadcell located between the input port and the actuator, to measure the input force. The camera at the top, which was also used in the contact experiment, is used to record the displacement of the input port. The camera at the bottom is used to make a high resolution photo of the location of the second output port in z-direction.
Figure G.4: The upper image shows how a 100g weight was attached to the second output port. At the edge of the output port a black line is drawn which can be seen both with and without weight. This point is photographed by the camera shown in the middle image. A ruler is placed next to it, as a reference for how much the structure deflects due to the weight. The deflection of the structure is then obtained from the photo through image processing. An example of the photo is shown in the bottom image.
Appendix H

Examples of compliant underactuated fingers

Below eight examples of the structures obtained through topology optimization are given, to show the variation in solutions. Solutions f, g and h show the beam element hubs that were mentioned in the paper in Part II.
Examples of compliant underactuated fingers

Figure H.1
Figure H.2
Examples of compliant underactuated fingers


Part IV

Literature review
Abstract

A high off-axis stiffness to motion stiffness ratio is often desirable in compliant mechanisms. Achieving this is a challenge, in particular for large displacement mechanisms. There exist numerous methods to improve the off-axis stiffness (both bending and torsional) of large-displacement compliant mechanisms. Also, there exist many compliant joints of which it is claimed that they have a high off-axis stiffness. In order to enable a well-founded choice of joint and/or method and to show which methods can still be developed, a literature research is conducted. The main methods used are found to be (1) basic topology adjustments and calculations and (2) size optimization. Other methods that bear promise but have not or not often been applied are also presented. Information regarding the size properties and stiffness properties of each mechanism is provided. A direct comparison between mechanisms and between methods was not possible, due to insufficient information and due to the fact that the improvement obtained depends as much on the method as on the mechanism used. The results indicate a trade-off between off-axis stiffness and range of motion and also that the off-axis stiffness can vary greatly over the entire range of motion.

1 Introduction

Unlike rigid-link mechanisms, compliant mechanisms make use of deflection of their flexible members to transfer or transform motion, force or energy. Using this flexibility to enable motion instead of using conventional joints has several advantages such as reduction of friction, backlash, weight and costs [1]. Even so the design of compliant mechanisms brings along a number of challenges. In conventional joints it is possible to have negligible stiffness in the direction of desired motion, while having high stiffness in the other directions. In compliant mechanisms these two aspects are linked up to a certain degree and it is difficult to increase one without affecting the other. Thus there will always be a trade-off between the stiffness in the free direction (henceforth referred to as 'DoF stiffness') and the stiffness in the other directions - the off-axis stiffness. Figure 1 illustrates the DoF direction and the off-axis directions for a cross-flexural hinge. Similarly there is also a trade-off between the DoF stiffness and the range of motion: by increasing the DoF stiffness, you will reach the maximum allowable stress level at a smaller deflection, thus decreasing the range of motion. Given the fact that a change in DoF stiffness affects the off-axis stiffness and vice versa, a change in off-axis stiffness can also cause a change in the range of motion. Another challenge is loss of off-axis stiffness as the displacement becomes greater. Especially with large-displacement compliant mechanisms this can pose a problem, as there can be a considerable change in off-axis stiffness over the large range of motion, affecting the accuracy of the mechanism.

The performance of a compliant mechanism (for example in terms of precision) can be improved by increasing the off-axis stiffness to DoF stiffness ratio. Many researchers have realized this and have taken this into consideration when designing a mechanism. A brief overview will follow next.

Some simply convert a rigid-body model to a compliant one by replacing the conventional joints with compliant joints that have a high off-axis stiffness. Quennouelle [2], for example, used cross-type revolute hinges to create a compliant Tripteron. These hinges are designed by Trease...
and have a high off-axis to DoF stiffness ratio. Hafez [4] designed a binary modular robotic device and chose to use compliant cross-flexural hinges as joints. One of the arguments for this was that these bearings have a higher out-of-plane stiffness than other flexural bearings. However, no reference or explanation is given to support this statement. As a last example, Wood [5] designed a micro-robot using fiber-reinforced composites. He acknowledged low off-axis stiffness to be one of the problems one is faced with when using compliant joints and proposed a solution for both small-displacement and large-displacement joints. For the latter a cross-flexural hinge was recommended, although again no explanation or reference was given to support this suggestion.

Other researchers increase the off-axis stiffness of their already existing compliant mechanism by adjusting its topology, shape or size. Cannon [6] changed his planar compliant end-effector to a spatial one, in order to increase the lateral stiffness. Mackay [7] on the other hand used optimization to adjust the size of the different parts of his large-displacement linear-motion compliant mechanism to achieve this.

As such there are two issues. First of all, no collection exists of articles that provide information on the stiffness properties of compliant joints, making it difficult and time-consuming to support the choice of compliant joint with the use of references. Secondly, many different methods of increasing the off-axis stiffness have been proposed. As yet no comparison has been made to show how they perform relative to each other and where there is still room for improvement or for the development of new methods.

The purpose of this literature review is to give a clear overview of the different ways found in literature to increase the off-axis stiffness of large-displacement compliant mechanisms. It should 1) provide a helping hand to those who want to incorporate a compliant joint with high off-axis stiffness into their design, by supplying them with a collection of joints and their stiffness values and 2) facilitate those who wish to increase the off-axis stiffness of their compliant mechanism or wish to develop new ways to do so, by presenting the existing methods and by providing a comparison between them.

The method employed to search for and categorize articles is presented in Section 2. The results are shown in Section 3 and discussed in Section 4. The literature review finishes with a conclusion in Section 5.

2 Method

2.1 Search method

To ensure that the literature search is structured and thorough a search plan first needs to be made. The task is to ‘find and compare ways to increase the off-axis stiffness in large-displacement compliant mechanisms’. The constraints are determined to be ‘large-displacement’ and ‘compliant mechanism’. The key concept is ‘increasing the off-axis stiffness’. With this information the search terms can be defined, which are listed in Table 1. Different combinations of these terms are used to look for articles in ScienceVerse and Google Scholar.

The articles are filtered out by first reading the abstract to determine whether it is relevant to the topic. If it is, the introduction is examined to determine whether it is useful for the literature review and to look for references that may be useful. Both articles that explicitly state that the off-axis stiffness is improved and articles that only suggest this are considered to be valuable.

2.2 Classification

The mechanisms in the articles will be judged according to a set of criteria. Each criterion is briefly explained below.

Methods of improvement The articles will also be categorized according to the general idea behind their solution for improving off-axis stiffness. This could involve a simple adjustment in the topology, such as adding another beam to the structure, or it could be an optimization with the off-axis stiffness as the objective function. In the next section the existing methods will be elaborated.

Improved versus suggested A distinction is made between articles that state that their method has a positive effect on the off-axis stiffness and articles that only suggest ways of doing this.

Structural form adjusted The structural form of a compliant mechanism can be expressed in terms of topology, shape and size [1]. The topology of a structure is determined by the way in which the different elements of the structure are connected to each other. It could be adjusted by, for example, adding another beam or creating a hole somewhere in the structure. An adjustment to the shape would be realized by changing the shapes of the separate elements of the structure. This could mean changing a straight beam into a curved one. The size can be adjusted by, of course, changing the size of the elements.

Characteristics Three characteristics will be looked for in the mechanism discussed in the article:

- **Translational/rotational** This depends on the type of Degrees of Freedom the mechanism has. If it has both translational and rotational DoF the mechanism will be marked as both translational and rotational.
- **Distributed/lumped** If only a small portion of the mechanism can undergo elastic deformation (for example with the use of notch hinges) the mechanism is
<table>
<thead>
<tr>
<th>Key concepts and constraints</th>
<th>Search terms</th>
</tr>
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<tbody>
<tr>
<td>compliant mechanism</td>
<td>- compliant, flexure, monolithic</td>
</tr>
<tr>
<td>large-displacement</td>
<td>- mechanism, system, structure, component, design, joint</td>
</tr>
<tr>
<td>off-axis stiffness</td>
<td>- large-displacement, large-deflection</td>
</tr>
<tr>
<td></td>
<td>- off-axis, out-of-plane, off-plane, lateral, transverse, supporting</td>
</tr>
<tr>
<td></td>
<td>- stiffness, compliance, loading</td>
</tr>
</tbody>
</table>

Table 1: The search terms used to find articles

considered to have lumped compliance [1]. Otherwise we speak of distributed compliance.

- **Planar/spatial** The mechanism is considered to be planar if it moves in one plane and if it is possible to attach it to a mechanism that also moves in the same plane. For example, Cannon [8] has designed a rotational joint, which in itself is planar. However, it cannot be used in another planar mechanism and therefore it is considered to be spatial.

**Size and Range of Motion** When provided the size of the mechanism and the range of motion (RoM) are also noted down.

**Stiffness** In this article stiffness will signify bending, torsional and axial stiffness. If given both the stiffness of the improved mechanism and that of the original version are obtained. Attention is also devoted to the point at which the stiffness was measured or computed and how this was done. The latter could be, for example, achieved through Finite Element Analysis or through an experiment. The stiffness itself could be acquired at the point of zero DoF displacement, over the entire RoM, or at some specific point in the RoM.

**Travel metric** As mentioned in the introduction there is a trade-off between the RoM and the stiffness of the mechanism. In order to be able to say something about the trade-off made in each mechanism, the RoM must be comparable. Smith [9] and Mackay [7] both recognized this and used a 'travel metric' in their articles, in which both the RoM and the size was taken into account. This metric was intended for planar, translational mechanisms, but can easily be adjusted to fit a spatial translational mechanism:

\[
\tau_{tr} = \frac{\delta}{\sqrt{x^2 + y^2 + z^2}}
\]

where \(\delta\) is the range of motion and \(x, y\) and \(z\) are the length, width and depth of the mechanism. For rotational mechanisms we propose a metric that is slightly easier, as degrees is already dimensionless. A simple equation ensures that it is comparable to the travel metric for translations in terms of scaling:

\[
\tau_{rot} = \frac{\theta}{2\pi}
\]

where \(\theta\) is the range of motion in radian.

3 Results

3.1 Methods of improvement

The different ways of improving the off-axis stiffness of a large-displacement compliant mechanism that have been used will be discussed here. For example, this could be making a basic adjustment by adding a third beam, as done by Trease [3] (see Figure 2, C), optimizing the distance between two bistable beams (Chen [10], Figure 2, D) or designing a contact-aided compliant mechanism (Cannon [8], Figure 2, E). The main properties of the articles are listed in Table 2, while the stiffness properties can be found in Table 3.

![Figure 2: A. Trease’s rotational joint [3], B. Goldfarb’s split-tube joint [11], C. Trease’s translational joint [3], D. Chen’s zero-stiffness end-effector [10], E. Cannon’s contact-aided compliant rotational joint [8], F. Folkersma’s five-leaf flexural joint [12], G. Mackay’s X-Bob [7], H. Tang’s prismatic joint [13].](image-url)
### 3.1.1 Basic adjustments and calculations

This method includes mechanisms where the off-axis stiffness or off-axis to DoF stiffness ratio has been improved by making basic adjustments to an existing structure. For the basic adjustments, such as adding an extra beam or combining two joints, either no or only simple formulae were given to justify the change.

Trease [3] compares different compliant joints - both translational and rotational - on 5 different criteria, such as the range of motion, the axis drift and the off-axis stiffness. He then presents the design of two compliant joints (Figure 2, A and C) that perform better on overall than the joints used for comparison. The first joint presented is a translational leaf spring joint (CT). The off-axis to DoF stiffness ratio has been improved by adding a third beam. This addition is justified with the use of linear beam theory. The joint can be planar, but also spatial, in which case two joints are placed at 90 degrees to each other.

The second joint is a rotational cross-type joint (CR) that can be seen as two cruciform hinges in parallel. Using the stiffness equations obtained in previous research on cruciform hinges it is shown that this increases the off-axis stiffness and that by making these joints smaller the off-axis to DoF stiffness ratio decreases.

Cannon [6] designed a compliant end-effector for microscribing. The end-effector is a folded-beam linear-motion mechanism. In order to increase the lateral stiffness three segments of the mechanism are placed at an angle of 120 degrees to each other, instead of keeping the mechanism planar.

Goldfarb [11] introduces the split-tube flexure (Figure 2, B). A tube has a high stiffness in all directions. The split ensures a low torsional stiffness while the stiffness in the other directions remains almost unchanged.

For a large-stroke planar 2-DoF positioning stage Folkersma [12] used cross-flexural joints (Figure 2, F). The off-axis stiffness was improved by using five leaf springs instead of two.

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**Table 2: Overview of the characteristics of the mechanisms discussed in the articles.**

<table>
<thead>
<tr>
<th>Improved Type</th>
<th>Size</th>
<th>RoM [mm or degrees]</th>
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<tbody>
<tr>
<td>Adjusted Type</td>
<td>Size</td>
<td></td>
</tr>
<tr>
<td>Topology</td>
<td>Size</td>
<td>Rotational</td>
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<tr>
<td>Basic adjustments and calculations</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Trease (CT)</td>
<td>x</td>
<td>x</td>
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<tr>
<td>Trease (CR)</td>
<td>x</td>
<td>x</td>
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<tr>
<td>Cannon</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Goldfarb</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Folkersma (hinge)</td>
<td>x</td>
<td>x</td>
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<tr>
<td>Optimization</td>
<td></td>
<td></td>
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<tr>
<td>Mackay</td>
<td>x</td>
<td>x</td>
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<tr>
<td>Chen</td>
<td>x</td>
<td>x</td>
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<tr>
<td>Smith</td>
<td>x</td>
<td>x</td>
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<tr>
<td>Wiersma</td>
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<td>Tang</td>
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<td>Tian</td>
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<tr>
<td>Zhou</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>CCM</td>
<td>Cannon (CCM)</td>
<td>x</td>
</tr>
<tr>
<td></td>
<td>Jeanneu</td>
<td>x</td>
</tr>
<tr>
<td>ECD</td>
<td>Folkersma (mech)</td>
<td>x</td>
</tr>
<tr>
<td>Elastic averaging</td>
<td>Awtar</td>
<td>x</td>
</tr>
<tr>
<td>Basic adjustments and calculations</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chen (lin. guide)</td>
<td>x</td>
<td>x</td>
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<tr>
<td>Chen (spatial)</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Optimization</td>
<td>Quennouelle</td>
<td>x</td>
</tr>
<tr>
<td>FACT method</td>
<td>Hopkins</td>
<td>x</td>
</tr>
</tbody>
</table>

* Data provided for the large X-Bob
  1 Data provided for the five-leaf cross-flexural hinge

* Estimated with graph or figure
3.1.2 Optimization

Another way to improve the off-axis stiffness of a compliant mechanism is by optimization with the off-axis stiffness included in the cost function. This includes both manual optimization and optimization with the use of an algorithm.

Smith [9] introduces a large-displacement linear-motion micro mechanism and optimizes the performance with a cost function based on two metrics. One metric is the ratio between the transverse stiffness and the DoF stiffness and the other is a so-called travel metric, in which the displacement and size of the mechanism are taken into account. A genetic algorithm is used for the optimization and the parameters are the size of different elements of the mechanism.

Mackay [7] also uses metrics to compare and/or optimize large-displacement linear-motion micro mechanisms: the travel metric used by Smith, a transverse stiffness metric (similar to that of Smith) and a torsional stiffness metric. Both of the stiffness matrices are based on the ratio of off-axis stiffness to DoF stiffness.

Chen [10] presents an adjustable constant-force end-effector. The constant-force is achieved by creating a zero-stiffness mechanism in which a linear spring is combined with bistable beams (Figure 2, D). He suggests three ways to improve the off-axis stiffness: adding a linear guide, placing the segments in a symmetric spatial configuration (as previously recommended by Cannon [6]) or adjusting the space between the bistable beams. Only the latter was investigated in his paper. Using FEA to simulate the off-axis stiffness for different spacing is shown, allowing for a manual optimization.

Wiersma [14] optimizes four different flexures using a Nelder-Mead algorithm, with the objective to maximize the first unwanted eigenfrequency. The parameters are the size and thickness of the flexure, together with the loadcase orientation.

Tang [13] introduces a prismatic joint consisting of rigid links and notch hinges (Figure 2, H). In order to minimize the parasitic motion of the joint, the notch hinges are optimized to have an off-axis to DoF stiffness ratio as high as possible, by changing the size parameters of the hinges. A gradient projection method is used.

Tian [15] presents the closed-form compliance equations - both of in-plane and out-of-plane compliances - of filleted V-shaped flexure hinges. As stated in the article, these equations could be used to predict and optimize the performance of the hinge. They can be guidelines to find an optimal trade-off between large-displacement and high motion accuracy.

Zhou [16] designed a linear guided compliant mechanism. Using a single beam building block he constructed the mechanism in eight different ways. The stiffness of each mechanism was analyzed with a FEA model. This enables manual optimization to find the combination that has the highest off-axis to DoF stiffness ratio.

Quennouelle [2] presents a kineto-static model of compliant parallel mechanisms. To demonstrate the model he uses a compliant version of the Tripteron. One of the matrices that can be constructed with the use of the model is the compliance matrix. This matrix gives the relation between the application of an external wrench on the end-effector and the change in its pose. Although not suggested in the article, this information should be usable for the optimization of the mechanism, in order to increase the off-axis stiffness and/or the off-axis to DoF stiffness ratio.

3.1.3 Contact-aided Compliant Mechanisms (CCM)

Cannon [8] designed a compliant contact-aided revolute joint that consists of two parts (Figure 2, E). The outer part of the joint has multiple flexures that have a free end in the middle of the joint. These free ends are constrained by the gauge pin. When turning the gauge pin the flexures make contact with a rigid surface of the outer part that forces them to have a constant radius of curvature. This contact increases the deflection of the flexure and at the same time improves the ability to bear lateral loads and as such increases the resistance to buckling.

Jeanneau [17] presents a compliant rolling joint, which consists of two parts of which the surfaces roll without slipping on each other. The two parts are attached by thin strips that enable this behavior. Jeanneau claims that the joint has a very low deformation in off-axis direction, provided that the joint is wide enough and the loading in those directions is limited.

3.1.4 Exact Constraint Design (ECD)

As mentioned in the paragraph on Basic Adjustments and Calculations (3.1.1) Folkersma [12] designed a large stroke planar 2-DoF positioning stage. He improved the mechanism in two ways: by changing the topology of the crossflexural hinge, and by making the mechanism an exact constraint design. When overconstraints exist in a design a small misalignment can lead to high internal stresses. These in turn can result in loss of off-axis stiffness. Folkersma handled this situation by releasing DoF’s of flexure’s at specific locations, for example by making a beam torsional compliant.

3.1.5 Elastic Averaging

While exact constraint design (ECD) is widely used for rigid-body models, Awtar [18] argues that it should be examined critically before applying it to compliant mechanisms. Elastic averaging is the ability of distributed compliant mechanisms to tolerate small imperfections (e.g. small misalignments) without it leading to binding. A condition is given that determines whether elastic averaging can take place in the beam in question. Awtar shows that elastic averaging will play a part in mechanisms that have at
least a certain degree of distribution and not for lumped-compliance configurations, in which case ECD would be useful. An example is given of how adding a third beam to a simple beam flexure - which basically means making it overconstrained - can actually help increase the off-axis to DoF stiffness when elastic averaging can take place. Unfortunately this improvement is not shown through FEA, an experiment or equations.

### 3.1.6 FACT Method

Hopkins [19],[20] introduces the design principle Freedom And Constraint Topology that can be used to synthesize complex flexures based on the desired Degrees of Freedom and Degrees of Constraint. It also can provide information on where to place additional constraints, which could for instance improve the stiffness in the constrained directions. However, it is also noted that this would also increase the stiffness in the free directions.

### 3.2 Improved stiffness

In this part the results related to the stiffness of the mechanisms will be presented. This involves how the stiffness (both DoF and off-axis) was measured and how the method of improving the off-axis stiffness was validated.

#### 3.2.1 Measuring the stiffness

Folkersma [12], Zhou [16], Wiersma [14] and Mackay were the only ones to examine the off-axis stiffness over the entire range of motion. Folkersma computed the stiffness of the five-leaf cross-flexural joint with the use of a FEA model. Zhou also used a FEA model to compute the stiffness and range of motion of the eight different configurations of his linear guided mechanism. Instead of looking at off-axis stiffness, Wiersma examined the first unwanted eigenfrequency, again using a FEA model.

Mackay [7] presented three different demonstrations of the metrics in his paper. The first shows the stiffness metrics plotted against the travel metric for three different mechanisms and for varying in-plane thickness. The second is simply an example implementation of the metrics, for the three different mechanisms. Since the size and RoM was given for this demonstration, one of the mechanisms (the Large X-Bob, Figure 2, G) is shown in Table 2. The value for the transverse stiffness metric is given for this mechanism although not included in Table 3. The third demonstration is a design example in which the X-Bob is optimized, also shown in the article published by Smith [9]. All the demonstrations are done with the use of FEA and the off-axis stiffness was examined over the entire RoM, as it is necessary for the computation of the stiffness metrics. Cannon [6] and Smith [9] looked at the stiffness when the mechanism was displaced by specified distance. Cannon applied a lateral force at the tip of the mechanism for zero DoF displacement and for 0.762 mm displacement as well. He also recorded the lateral force - lateral displacement graph. Smith also used a FEA model to compute the stiffness. The off-axis stiffness was measured at full DoF displacement.

The DoF stiffness value of Chen [10] was computed with a FEA model and measured through experiments. The DoF stiffness was given over the range of motion, while the off-axis stiffness (computed with a FEA model) was given over the range of displacement in the off-axis direction and only for zero DoF displacement.

Folkersma [12] used for his planar 2-DoF positioning stage both a FEA model and an experimental setup to find the first two natural eigenfrequencies and the third eigenfrequency, which is the first unwanted one. He also noted that the unwanted eigenfrequency drops when the mechanism is deflected further.

As an example of the resulting matrices of the kineto-static model Quennouelle [2] provided the compliance matrix for the compliant version of the Tripteron. This compliance matrix is computed for zero DoF displacement and was not validated.

For both of the joints Trease [3] introduced a FEA model was used to find the off-axis stiffness values for zero DoF displacement. To be able to compare torsional stiffness to translational stiffness he used the moment arm of the mechanism to convert one to the other.

Goldfarb [11] fabricated his split-tube joint and verified the stiffness characteristics through experiments, although no information was given as to how exactly it was carried out. Tian [15] computed the stiffness for filleted V-shaped flexure hinge with the equations presented and compared the stiffness in one of the off-axis direction found with the equations to that found through FEA. Unfortunately it was not mentioned at which point in the range of motion the off-axis stiffness was computed. He provided graphs that show both the DoF and off-axis stiffness for different parameter values. The stiffness found in one point in the graph (with geometric parameters $\beta = 0.750$ and $\theta = 90$) is shown in Table 3, as an example.

Awtar [18] used a FEA model and an experimental setup but only provided the DoF force-displacement graphs. Tang [13] validated the stiffness he had calculated through an experiment. However, he only calculated and tested the DoF stiffness.

Cannon (CCM) [8] made both a FEA model and a prototype. However, only for the torsional stiffness a torque-rotation graph is provided.

#### 3.2.2 Validation of the methods

In this part the method is discussed by which the authors validate whether their method of improvement is successful. This can be a comparison to another comparable mecha-
Table 3: Overview of the stiffness properties of the mechanisms discussed in the articles. Aside from that of Smith all the stiffness values are for a zero DoF displacement.

nism for instance, or to the original, unimproved version of the mechanism.

Smith [9] and Goldfarb [11] validated their improvements through a comparison to similar mechanisms. Smith compared the optimized X-Bob mechanism to two other similar mechanisms: a folded beam and an X-Bob mechanism with different parameters [21]. Both of these mechanisms scored high on one metric and low on the other. The optimized X-Bob did not have the best travel metric, nor the best stiffness metric. Instead it had better a trade-off between these two. The split-tube joint by Goldfarb was compared to a conventional leaf hinge. The hinge and split-tube joint were designed to have the same rotational stiffness and axial stiffness, the ability to withstand a minimum axial load and they were constructed from the same stainless steel alloy. In some articles the validation was done through comparison with the unimproved version of the mechanism or joint. Folkersma [12] modeled both the five-leaf cross-flexural joint as well as its more conventional version - the two-leaf cross-flexural joint - on which his version was based and compared both of them for the entire range of motion. Wiersma [14] investigated four different flexure hinges, although he only compared the optimized design with the previous version for the five-flexure cross hinge. As such this flexure is used in Table 2 and Table 3. Awtar [18] provided the resulting DoF force-displacement graph for several parameter sets, enabling a comparison between a mechanism where elastic averaging can take place and one where it cannot.

Unfortunately a good number of articles also did not provide any validation. Trease [3] compared the two new joints (CR and CT) to the joints they’re based on. In the case of the translational joint this is a two-beam flexure instead of a three-beam flexure and the comparison is based solely on the analytic equations of axial and transverse stiffness. The rotational joint, based on the cruciform hinge, is said to ‘twice the axial, bending, and torsional stiffness’ and ‘8 times the bending-rotational stiffness’. Unfortunately no explanation is given as to where these numbers come from. Cannon [6] and Tang [13] did not compare the stiffness results of their improved mechanism to that of a previous version nor to that of a comparable mechanism. Similarly Folkersma [12] did not compare the resulting eigenfrequencies of his planar 2-DoF positioning stage to that of a similar mechanism. Cannon (CCM) [8] also did not compare his compliant contact-aided bearing to similar mechanism. He did mention that this contact-aided design is over eight times more resistant to buckling than the fixed-free option, although no reference or proof was given to support this. Mackay [7] compared the stiffness metrics and travel metric of three different mechanisms and for varying in-plane
thickness as well. However, he did not provide the actual stiffness values of the mechanisms.

For Chen [10], Tian [15] and Zhou [16] it is difficult to separate the method of improvement from the validation and is therefore shown as ‘?’ in Table 3. All three of them use manual optimization in order to improve the off-axis stiffness. The outcome is a set of results for different parameter values (e.g. beam spacing in the article of Chen) from which then the parameter value with the best off-axis stiffness is chosen. However, this set of results can also be seen as a validation that changing these parameter values indeed has a positive effect on the off-axis stiffness.

4 Discussion

As mentioned before the goal of this literature review is to facilitate the selection and development of methods to improve the off-axis stiffness by providing a clear overview and comparison of the existing methods found in literature.

An important thing to note when it comes to comparing the methods is that no conclusions can be drawn from the comparison between the stiffness of one mechanism and that of similar mechanism, as the choice of mechanism to compare it with influences the results. Weighing an improved mechanism against its older version is useful when only the size or shape has been adjusted. However, when the topology of a mechanism is adjusted, it is similar to sizing it up to a different mechanism. As such it is very difficult to make a good comparison between the different methods of improvement, since the stiffness improvement is as much a function of the method as it is of the mechanism used.

Taking this into account, the advice when choosing a method to improve the off-axis stiffness of a specific mechanism is to consider several possible methods and especially the methods that were used for comparable mechanisms. To get an idea of how well the mechanism performs in terms of off-axis stiffness after improvement, one can then compare the stiffness results to that of other comparable mechanisms.

In order to do this - choosing a method and comparing the results - certain information is needed: the resulting stiffness in both the direction of motion and other directions and over the RoM would be needed, as well as the size and RoM of the mechanism. The latter is important given the existing trade-off between stiffness and RoM. In Figure 3, which allows a visual comparison between the stiffness of the improved mechanisms, this trade-off is also noticeable for the rotational mechanisms. Since there are only three mechanisms that provide enough information on the stiffness and the travel metric, no real conclusion can be drawn from this observation. Even so, it does point toward the trade-off between off-axis stiffness and RoM. This effect is important to take into account when improving the off-axis stiffness. Mackay [7] does this by plotting the transverse and torsional stiffness metrics against the travel metric. Similarly Smith [9] set out to improve the trade-off between off-axis stiffness and range of motion, instead of solely focusing on the off-axis stiffness. Taking the stiffness over the RoM into account is especially important when the purpose is to improve the off-axis stiffness of a mechanism or to provide joints that could be used and claim to have good off-axis stiffness properties. In the graph provided by Folkersma [12] it is shown that for the two-leaf and five-leaf cross-flexural hinge the off-axis stiffness in $x$ and $y$ direction becomes almost 100 times as small over the range of 20 degrees. Wiersma [14] on the other hand showed a decrease of a third to two third in the first unwanted eigenfrequency over the range of 20 degrees. Assuming a constant mass this would mean a stiffness almost twice to ten times as low. Zhou [16], on the other hand, shows that while for some configurations the off-axis stiffness may drop, for others it can also increase. The conclusion that can be drawn from these examples is that although it is very likely that the off-axis stiffness will drop over the range of motion, one should be very cautious with making assumptions about how much it will change.

Unfortunately many provide little to no information on the stiffness of the improved mechanism, let alone the stiffness of the unimproved version. Furthermore, in a number of papers either the size or the range of motion is missing and no estimation can be made based on a picture either, as it is not clear whether the model in the picture is scaled accurately or not. In Figure 3 and 4 it can also be noticed quite quickly how few stiffness results are given for translational mechanisms compared the results provided for rotational mechanisms. It is also striking that few have considered the change of off-axis stiffness over the entire range of motion. Less than half of the articles mentioned looked at the off-axis stiffness at a non-static point and only two-thirds of those examined the off-axis stiffness over the entire RoM.

In order to compare and evaluate the mechanisms, aside from information on the size and RoM, a more extensive list of stiffness (both off-axis and DoF) is needed. As such it is recommended to measure or compute both the DoF stiffness and the off-axis stiffness in as many directions as possible and over the entire RoM. If for some reason this is not feasible, the advice is to show (1) the DoF stiffness, since this puts the off-axis stiffness into perspective, and (2) the off-axis stiffness at the point of zero displacement and at maximum displacement, since this gives a more accurate image of the performance of the mechanism than only providing the stiffness at the point of zero displacement.

If the stiffness is known, the next issue is how to compare the rotational and translational stiffness. In order to be able to compare these two Trease divided the rotational off-axis stiffness by the moment arm squared, which revealed that the off-axis stiffness (when translated to translational stiffness) is still about three times as large as the translational DoF stiffness, despite the fact that the torsional off-axis
stiffness of the translational joint of Trease [3] seems to be smaller than the (translational) DoF stiffness in Figure 3. This method of comparing stiffness in translational and rotational direction makes sense, but one should be careful when choosing the moment arm. For example, it is slightly strange that Trease used a moment arm in the x-direction to convert the torsional stiffness around the y-axis. A better comparison can be made between the DoF stiffness and the off-axis stiffness with the same dimensions - torsional in case of a rotational DoF, lateral in case of a translational DoF. Table 4 shows the smallest proportion between DoF stiffness and the comparable off-axis stiffness, for the articles that have provided enough information.

Some remarks can be made regarding the methods presented. The two main methods of improving the off-axis stiffness are shown to be basic adjustments and calculations and optimization. All the basic adjustments and calculations bring about a change in topology, whereas almost all articles that use optimization cause a change in size. Only the use of elastic averaging (Awtar [18]) can be seen as a shape adjustment and even this depends on how an adjustment in distribution is executed. Although the use of contact-aided compliant mechanisms, ECD and elastic averaging all sound promising, their ability to improve the off-axis stiffness still needs to be proven. Cannon [8] created a contact-aided compliant rotational joint but did not provide any results regarding the stiffness. Folkersma [12] made a compliant exact constraint design and though he did give the resulting eigenfrequencies, there was no information on the eigenfrequencies of the original mechanism or of a similar mechanism. Awtar briefly mentions how elastic averaging can improve the off-axis stiffness, but there is yet to be an article that applies this to an actual compliant design. There are also other methods that may prove to be useful. The use of anisotropic materials could be named as one. Noll [22] used it in his paper to design a three-axis rotational flexure with low DoF stiffness and a high (off-axis) axial stiffness by using fiber joints. Although the article was not included in the results due to lack of relevant data, the idea behind the mechanism is worth looking into. Another example is taking the off-axis stiffness into account in the conceptual phase through topology optimization. Mackay [7] who used optimization to adjust the size, also noted that the largest opportunities to make a significant change in off-axis stiffness may lie in the conceptual phase.

Some of the articles also ask for a bit of extra attention. Folkersma and Wiersma [14] both used cross-axial hinges: Folkersma compared a two-leaf to a five-leaf flexure hinge (Figure 3), while Wiersma improved a three-leaf and a five-leaf flexure hinge (Figure 4). Wiersma shows that in terms of the eigenfrequency corresponding to an (off-axis) rotation around the y-axis the three-leaf and five-leaf perform equally well. Folkersma, on the other hand, presents a graph in which the five-leaf is shown to perform significantly better than the two-leaf version in terms of torsional stiffness around the y-axis. This difference in results is likely to be caused by the asymmetry in the two-leaf which is eliminated in both the three-leaf and five-leaf version. The stiffness values collected from the graphs provided by Tian [15] has been added to the graphs in this review, although it

![Graph showing compliant mechanisms and stiffness ratios](image_url)

**Figure 3:** The compliant mechanisms of which the stiffness was provided. The final stiffness of each one is given and the mechanism are ranked by their travel metric value. The translational mechanisms are on the left side, while the rotational mechanisms are on the right side. The mechanisms in bold are joints.

<table>
<thead>
<tr>
<th>Type of mechanism</th>
<th>Author</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotational</td>
<td>Trease</td>
<td>216</td>
</tr>
<tr>
<td></td>
<td>Goldfarb</td>
<td>8343</td>
</tr>
<tr>
<td></td>
<td>Folkersma</td>
<td>10000</td>
</tr>
<tr>
<td>Translational</td>
<td>Smith</td>
<td>67</td>
</tr>
<tr>
<td></td>
<td>Zhou</td>
<td>367</td>
</tr>
<tr>
<td></td>
<td>Cannon</td>
<td>1390</td>
</tr>
</tbody>
</table>

Table 4: Overview of the ratios of the smallest comparable off-axis stiffness and the DoF stiffness.
should be kept in mind that these only serve as an example and not as the best values or best ratio Tian has to offer. It should also be said that the closed-form compliance equations presented by Tian do not depend on the displacement of the joint. For large-displacement this is not realistic and even though the article states that the equations could be used for large-deformation, it seems to indicate the equations are actually meant for small-displacement only. Figure 5 shows the stiffness ratio between the improved mechanism and either the original mechanism or a different, comparable mechanism. In general the improved stiffness is around one to ten times as large. There are some points to take notice of. First of all the comparison of Smith’s X-Bob to the X-Bob of Hubbard and to the folded beam [9]. It may seem as though Hubbard did a better job than Smith. However, the travel metric of Hubbard is 0.096, whereas that of Smith is 0.141. Similarly, the folded beam had an even better travel metric of 0.173. The large improvement of Goldfarb is also quite notable. However, the comparison was done with a different mechanism, which makes the result not as significant.

![Figure 4: The compliant mechanisms of which the stiffness was provided. The four flexural hinges optimized by Wiersma and the 2-DoF positioning stage by Folkersma are shown. The mechanisms in bold are joints.](image)

**5 Conclusion**

Comparison between the existing methods was thwarted by two issues. First of all by insufficient information on the (stiffness) properties of the mechanisms and secondly by the choice of mechanisms. When comparing methods based on the stiffness improvement they produce, the improvement found will depend as much on the method that was chosen as on the mechanism that one is trying to improve.

However, the overview provided helps by presenting the existing methods and by showing which methods were used for mechanisms comparable to that of the reader. The performance of the mechanism can be evaluated by comparing it to the DoF stiffness to off-axis stiffness ratio of other comparable mechanisms. Where possible a comparison was already done between the DoF stiffness and the off-axis stiffness of the same type (i.e. lateral, torsional).

From the overview it can be seen that the most commonly used methods to improve the off-axis stiffness are found to be size optimization and basic calculations and adjustments of the topology. Methods that have been used but need to be developed and tested further are contact-aided compliant mechanisms, elastic averaging and exact constraint design. Topology optimization and anisotropic materials were suggested as possible methods that have not been implemented yet for the purpose of increasing the off-axis stiffness.

The stiffness results indicate a trade-off between the range of motion and the off-axis stiffness, which was also found in some of the articles. It was also noted that few consider the change in stiffness over the entire range of motion. The few who did show that the change can vary greatly between mechanisms.

For future work on how to improve the off-axis stiffness it is recommended that studies present - aside from the RoM and size - a more complete set of stiffness results, by looking at as many off-axis directions as possible as well as how the stiffness varies over the range of motion. If measuring over the entire range of motion is not possible, this should at least be done at zero deflection and maximum deflection.

**References**


