Predicting the Shear Capacity of Reinforced Concrete Slabs subjected to Concentrated Loads close to Supports with the Modified Bond Model

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Summary

The shear problem is typically studied by testing small, heavily reinforced, slender beams subjected to concentrated loads, resulting in a beam shear failure, or by testing slab-column connections, resulting in a punching shear failure. Slabs subjected to concentrated loads close to supports, as occurring when truck loads are placed on slab bridges, are much less studied. For this purpose, the Bond Model for concentric punching shear was studied at first. Then, modifications were made, resulting in the Modified Bond Model. The Modified Bond Model takes into account the enhanced capacity resulting from the direct strut that forms between the load and the support. Moreover, the Modified Bond Model is able to deal with moment changes between the support and the span, as occurs near continuous supports, and can take into account the reduction in capacity when the load is placed near to the edge. The resulting Modified Bond Model is compared to the results of experiments that were carried out at the Stevin laboratory. As compared to the Eurocodes (NEN-EN 1992-1-1:2005) and the ACI code (ACI 318-11), the Modified Bond Model leads to a better prediction.

Keywords: arching action; bond; compressive strut; continuous supports; edge loading; punching shear; shear; slabs; torsion.

1. Introduction

The shear problem is typically addressed by studying two well-defined cases: 1) the one-way shear capacity of beams (beam shear), and 2) the two-way shear capacity of slabs (punching shear). One-way shear in beams is most often studied on small, heavily reinforced, slender beams, tested in four point bending [1, 2], resulting in the semi-empirical expressions as given in NEN-EN 1992-1-1:2005 [3] and ACI 318-11 [4] for the beam shear capacity. Two-way shear in slabs is studied on slab-column specimens [5]. These experiments form the basis of the semi-empirical punching shear provisions as given in NEN-EN 1992-1-1:2005 and ACI 318-11.

Besides these two standard cases of the shear problem that have been widely studied over the past decades [6], other loading cases, often at the intersection of beam shear and punching shear, arise in practice. An example is the shear capacity of existing reinforced concrete solid slab bridges, subjected to concentrated live loads when located close to the support, resulting in large shear stresses at the support [7]. The available code provisions are not fully suitable to determine the shear capacity of slabs subjected to concentrated loads close to supports, a problem at the intersection between one-way and two-way shear.

To describe the behaviour of reinforced concrete slabs subjected to concentrated loads close to the support, a new model is proposed. This model is a combination of load-bearing quadrants and strips, and is based on the Bond Model [8, 9]. The resulting Modified Bond Model can be considered a mechanical model, in which the concept of a limiting one-way shear stress is incorporated. Where most beam shear and punching shear models make a strict distinction between these two modes of failure, the Bond Model considers the shear-carrying behaviour as an action of two-way quadrants and one-way strips. As such, it is the most suitable model for the considered case that is a combination of one-way shear and two-way shear.

2. Bond Model for concentric punching shear

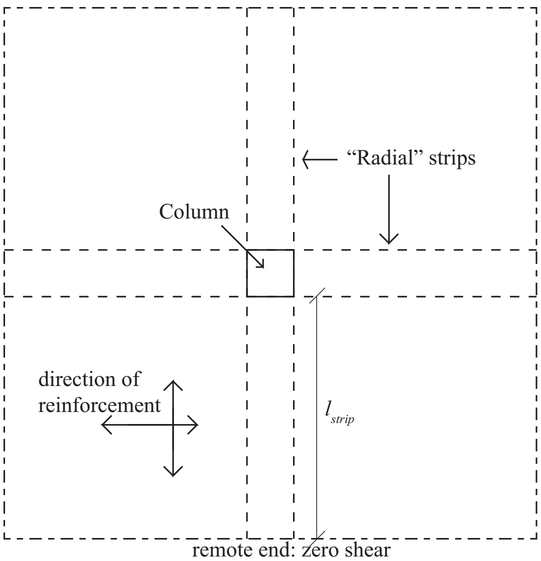

Fig. 1: Layout of strips

The Bond Model for concentric punching shear [8, 9] is a mechanical model for slab-column connections that explains the load transfer between plate and column by combining radial arching action and the concept of a critical shear stress, as used for beam shear. Shear, V, (moment gradient) results where the magnitude of the force T or lever arm z varies along the length of the member. As such, shear is carried by a combination of beam action and arching action:

$$V = \frac{d(Tz)}{dx} = \frac{d(T)}{dx}z + \frac{d(z)}{dx}T \qquad (1)$$

For slabs, arching action, expressed by the radial compression strut, is the dominant mechanism in the radial direction. It is assumed that the load is distributed in the radial directions from the column by arching.

In the Bond Model, arching is represented by four strips branching out from the column, parallel to the reinforcement, Fig. 1. These strips separate the column from the slab quadrants. The length of the strips, l_{strip}, is determined from the column to a remote end, a position of zero shear. The strips are loaded in shear on their side faces only and are described as cantilever beams as shown in Fig. 2. These cantilevers have negative and positive moment capacities of M_{neg} and M_{pos} that can be combined into M_s, the total flexural capacity of the strip. At the side of the column, the axial load $P_{AS,1}$ is acting. The length l_w is the loaded length of the strip, and w the uniformly distributed load. The loading term w is an estimate of the shear that can be delivered by the adjacent quadrant of the slab to one side face of the strip. For a strip with two side faces, the total uniformly distributed load on the strip is $2w$. Using force and moment equilibrium of the cantilever strip (Fig. 2) results in the following expressions for the total flexural capacity M_s and the concentrated load $P_{AS,1}$:

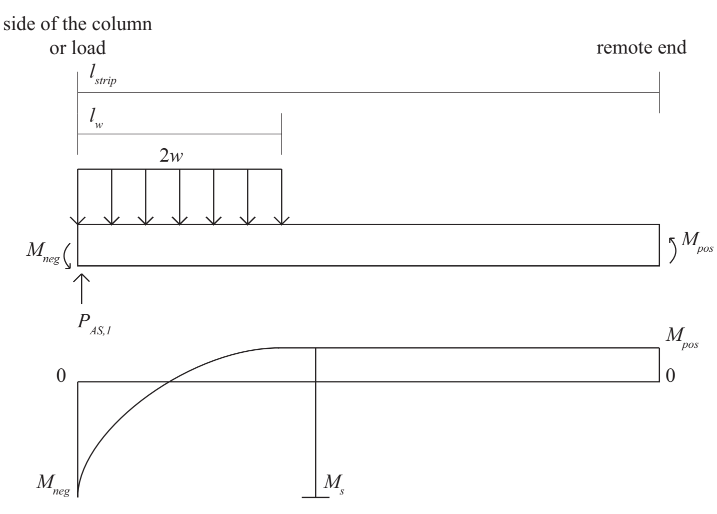

Fig. 2: Equilibrium of radial strip

$$M_s = \frac{2wl_w^2}{2} \qquad (2)$$

$$P_{AS,1} = 2wl_w \qquad (3)$$

Solving Eq. (2) for the unknown loaded length l_w and substituting this into Eq. (3) results in:

$$P_{AS,1} = 2\sqrt{M_s w} \qquad (4)$$

To find the maximum column axial load P_{AS}, the capacity of all four strips can be summed:

$$P_{AS} = 8\sqrt{M_s w} \qquad (5)$$

The maximum value of the loading term w can be found based on the equivalence between the maximum value of beam action shear and a limiting nominal one-way shear stress as prescribed by the codes. Using the one-way shear capacity from ACI 318-

11, empirically defined as the inclined cracking load [10], was found to lead to the best results [9]. When the maximum value of the loading term is limited by beam action shear, it is expressed as follows:

$$w_{ACI} = 0{,}1667 d \sqrt{f_{ck}} \quad (6)$$

In Eq. (6), w_{ACI} is given in [kN/m] with f_{ck} in [MPa] and d in [mm].

3. Development of Modified Bond Model

3.1 Concentrated loads close to the support

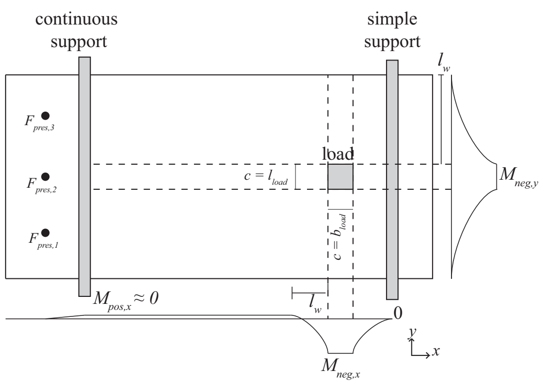

Fig. 3: Sketch of application of Bond Model to slabs under concentrated loads close to supports.

To apply the Bond Model to the case of slabs subjected to concentrated loads close to supports, it is necessary to take into account direct load transfer. For one-way slabs, the different properties in the span direction and in the transverse direction need to be taken into account. The four cantilevering strips branching out from the load can be studied together to sketch the assumed moment distribution, Fig. 3. In Fig. 3, the geometry and layout from the slab shear experiments carried out at Delft University of Technology [11] are used.

For the x-direction strip between the concentrated load and the support, direct load transfer between the load and the support is taken into account. Regan described the punching capacity of slabs under concentrated loads close to supports by considering the 4 sides of the punching perimeter separately [12]. To take into account the beneficial influence of direct load transfer, the capacity of the side of the punching perimeter at the support was enhanced with a factor $2d_x / a_v$, in which d_x is the effective depth to the reinforcement in the x-direction and a_v is the face-to-face distance between the load and the support. Similarly, it is proposed to increase the capacity of the strip between the load and the support by enhancing the capacity with $2d_x / a_v$ for $0{,}5d_x < a_v < 2d_x$ and 4 for $a_v \leq 0{,}5d_x$.

3.2 Loads close to the continuous support

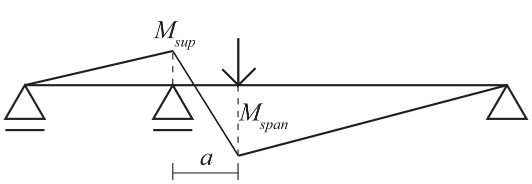

Fig. 4: Illustration of moments at a continuous support

A first extension of the Bond Model deals with loads applied close to the continuous support, in which the positive moment reinforcement can increase the total flexural capacity. As the negative and positive moment capacities are not activated in the same cross-sections of the strips and because yielding of the compression reinforcement is not assumed in the model, the following expression is proposed to take into account the effect of the positive moment reinforcement: $M_s = M_{neg} + \lambda_{moment} M_{pos}$. The factor λ_{moment} ranges from 0 (for simply supported edges) to 1 (for fully restrained cases) and equals: $\lambda_{moment} = M_{sup} / M_{span} \leq 1$ (Fig. 4).

When the concentrated load is placed close to a continuous support, two quadrants experience the change in moment from hogging moment M_{sup} to sagging moment M_{span}. As a result, the combined effect of the top and bottom reinforcement should be taken into account on the three strips that border these two quadrants: the two y-direction strips as well as the x-direction strip between the load and the support.

4. Study of edge effect

4.1 Extreme cases

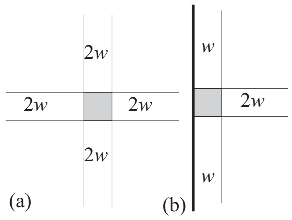

Fig. 5: Strips and loads for: (a) load in middle of slab, or (b) load at the edge

When loads are placed close to the edge of the slab, the edge effect plays a role. Before studying the case of concentrated loads close to the free edge of the slab, two extreme cases are considered: the case where the load is in the middle of the width and no edge effect is present and the case where the load is placed right at the edge and only 3 strips can be used.

When no edge effect is present, the load and strips are as shown in Fig. 5a. All strips are loaded with $2w$ and the capacity of each strip is $P_{AS,1}$ as given by Eq. (4).

The second case is the case in which the load is placed right at the edge, as shown in Fig. 5b. For this case, only 3 strips can be used and only 2 quadrants result. As a result, the strips in the y-direction are loaded with w and the strip in the x-direction is loaded with $2w$. The case of loading with $2w$ gives a capacity $P_{AS,1}$ from Eq. (4). For the strips in the y-direction a load of w is placed over a loaded length of l_w as sketched in Fig. 6.

Horizontal equilibrium for the strips at the edge gives:

$$P_{edge} = wl_w \quad (7)$$

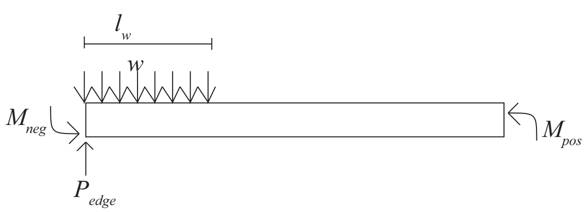

Fig. 6: Loading on slab strips in y-direction equals w when load is placed at the edge of the width

Moment equilibrium gives:

$$M_s = \frac{wl_w^2}{2} \quad (8)$$

so that:

$$l_w = \sqrt{\frac{2M_s}{w}} \quad (9)$$

Substituting the value for l_w into the expression of P_{edge} from Eq. (7) gives the capacity of the y-direction strips as:

$$P_{edge} = \sqrt{2M_s w} \quad (10)$$

4.2 Describing the edge effect for the loaded length

To describe the edge effect, the loaded length of the x-direction strip between the load and the edge is studied. When all strips are loaded with $2w$, as shown in Fig. 5a, the loaded length will be:

$$l_w = \sqrt{\frac{M_s}{w}} \quad (11)$$

For loads close to the edge, the distance between the edge and the face of the load, l_{edge} (Fig. 7a) is expressed as (with b_r the distance between the centre of the load and the edge and l_{load} the length of the load):

$$l_{edge} = b_r - \frac{l_{load}}{2} \quad (12)$$

If l_w from Eq. (11) is larger than l_{edge} from Eq. (12), the model would be assuming a loaded length of the strip that is longer than what is physically possible. Therefore, for those cases the edge effect needs to be taken into account: the loaded length l_w needs to be limited to the edge length l_{edge}. The capacity of the strip between the edge and the load then becomes:

$$P_{edge} = 2 l_{edge} w_{ACI,y} \quad (13)$$ if the edge effect is present, and

$$P_y = 2 l_w w_{ACI,y} \quad (14)$$ if no edge effect is present.

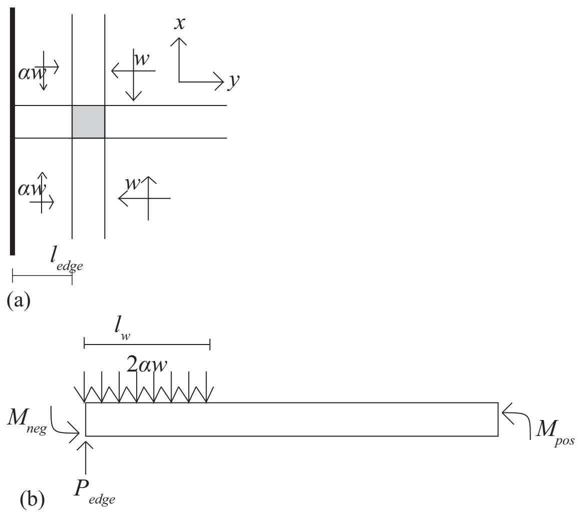

Fig. 7: Assuming a uniform influence of the edge effect: (a) strips and quadrants, (b) resulting loading on strip

The previous considerations did not take into account the fact that not the full load w will be transferred to the strip between the load and the edge when the edge effect is present. Let's now assume that only a fraction αw with $\alpha < 1$ can be carried off in the quadrants instead of w. The value of α can be expressed as:

$$\alpha = \frac{l_{edge}}{l_w} \text{ for } l_{edge} \leq l_w. \quad (15)$$

This situation is sketched in Fig. 7a. As a result, a load of $2\alpha w$ instead of $2w$ acts on the y-direction strip between the load and the support, as shown in Fig. 7b. The resulting capacity of this strip is then:

$$P_{edge} = 2\alpha w_{ACI,y} l_w. \quad (16)$$

Note that Eq. (16) is valid for the case in which an edge effect is present as well as the case for which no edge effect is present through the use of the factor α.

4.3 Influence of the torsional moment

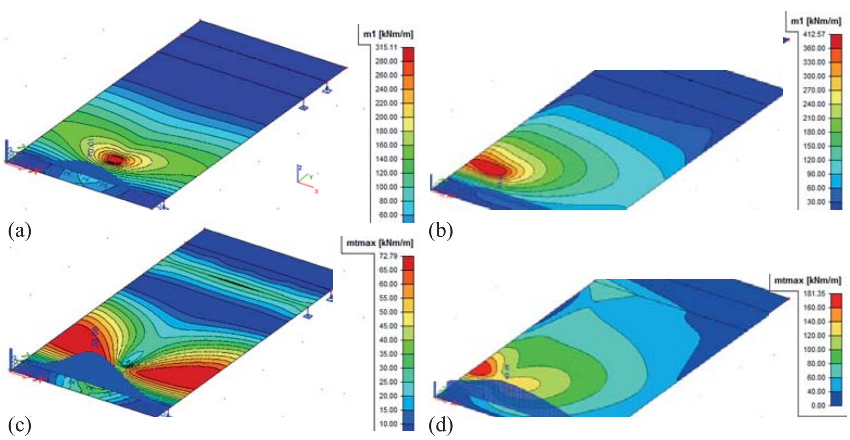

Fig. 8: Bending moments (a) in S1T1, (b) in S4T1, torsional moments (c) in S1T1, (d) in S4T1.

Torsion was neglected in the original Bond Model, as the influence of the torsional moments is small when loads are placed on infinitely large slabs. However, when the load is placed close to the edge of a slab, torsional distress influences the capacity. As a result of the influence of torsion, a smaller capacity than predicted by the (Modified) Bond Model is found.

The magnitude and influence of the torsional moment is studied based on experiments S1T1 and S4T1, as carried out in the Stevin Laboratory of Delft University of Technology [11]. In S1T1, a single concentrated load is placed on a reinforced concrete slab in the middle of the width. In S4T1, the load is placed close to the edge of the slab. The concrete compressive strength of slab S1 was $f_{c,meas}$ = 35,8MPa and for S4, $f_{c,meas}$ = 50,5MPa. Both slabs have a longitudinal reinforcement ratio of ρ_l = 0,996%, yet a different transverse reinforcement ratio: ρ_t = 0,132% for S1 and ρ_t = 0,182% for S4. All other material and geometric properties are the same for the two experiments. Failure occurs in S1T1 for a concentrated load of 954kN, which corresponds to a shear force at the support of 799kN. In S4T1, failure occurs for a concentrated load of 1160kN, corresponding to a shear force at the support of 964kN.

To study the influence of the torsional moment, linear finite element models are used. The resulting principal bending moments for S1T1 are shown in Fig. 8a and for S4T1 in Fig. 8b. The principal bending moments are determined as:

$$m_1 = \frac{1}{2}\left(m_x + m_y + \sqrt{(m_x - m_y)^2 + 4m_{xy}^2}\right) \quad (17)$$

The torsional moments for S1T1 and S4T1 are shown in Fig. 8c and in Fig. 8d, respectively. The torque moments are determined as:

$$m_{t\max} = \frac{\sqrt{(m_x - m_y)^2 + 4m_{xy}^2}}{2} \quad (18)$$

Table 1: Results of bending and torsional moment at location of load

Experiment	m_1	m_{tmax}	m_{tmax}/m_1
S1T1	315 kNm/m	20 kNm/m	6%
S4T1	412,57 kNm/m	181,35 kNm/m	44%

The resulting bending and torsional moments at the location of the load, around which the cruciform shape resulting from the strips of the Modified Bond Model is applied, are given in Table 1. Table 1 shows that for the experiment with the load at the edge, the influence of the torsional moment is significantly larger than for the experiment with the load in the middle of the width.

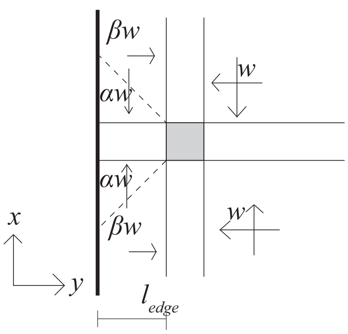

Fig. 9: Proposed design method for taking into account the influence of the torsional moment.

To take the influence of the torsional moment into account in the Modified Bond Model, a simplified method is proposed which does not require the use of finite element models. This method is sketched in Fig. 9.

As discussed in the previous section, the influence of the edge effect can be taken into account by using the reduction factor α. For the method proposed in Fig. 9, the edge effect is working on the y-direction strip between the load and the edge. The capacity of the strip is then P_{edge} as given in Eq. (16). The influence of torsion is taken into account by using the factor $\beta \leq 1$. Because the influence of torsion is related to the edge effect, this influence is assumed to act only in the quadrants between the load and the edge. Moreover, it is assumed that this influence acts only in the y-direction, the weaker direction in which a lower amount of reinforcement is provided. In other words, due to torsion, it is assumed that the capacity of the x-direction strips, which carry the larger part of the load in one-way slabs, is reduced. As a result, the x-direction strips are loaded with $(1 + \beta)w$. The capacity of the x-direction strips is then:

$$P_{sup} = \frac{2d_l}{a_v}\sqrt{2(1+\beta)M_{s,x}w_{ACI,x}} \quad (19)$$

$$P_x = \sqrt{2(1+\beta)M_{neg,x}w_{ACI,x}} \quad (20)$$

In Eqs. (19) and (20) the following symbols are used:

P_{sup} = the capacity of the strip between the load and the support
P_x = the capacity of the x-direction strip from the load towards the span of the slab
d_l = effective depth to the longitudinal reinforcement
a_v = face-to-face distance between the load and the support
$M_{s,x}$ = $M_{neg,x} + \lambda_{moment}M_{pos,x}$ = the moment capacity, as explained in §3.2
$M_{neg,x}$ = the hogging moment capacity of the x-direction reinforcement
$M_{pos,x}$ = the sagging moment capacity of the x-direction reinforcement
$w_{ACI,x} = 0,1667 d_l \sqrt{f_c'}$ with f_c' in [MPa].

The value of the factor β, which takes torsion into account, can be expressed based on the resulting bending moment m_1 and the resulting torsional moment m_{tmax}. Since the goal of the Modified Bond Model is to provide a design method which can be used without the need of finite element programs, a simplification is to use $\beta = 0$ for loading cases where the maximum bending moment and maximum torsional moment in a slab coincide, such as the case of loads close to the edge, and to use $\beta = 1$ when the maximum bending moment coincides with a small torsional moment, such as for loading in the middle of the slab width.

5. Comparison to experimental results

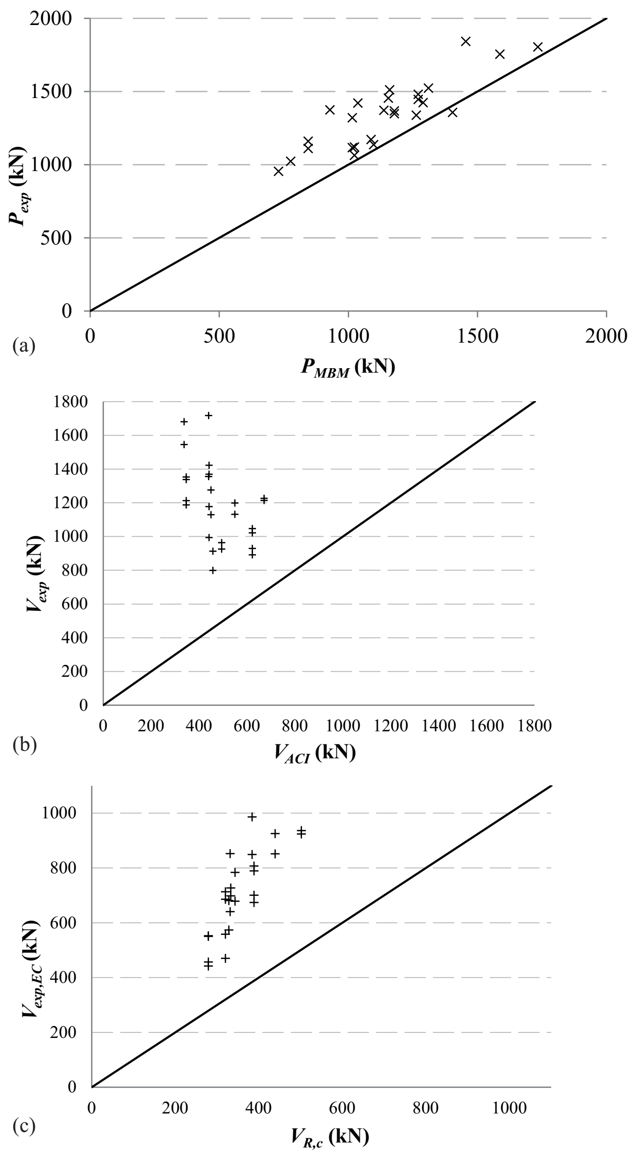

Fig. 10: Comparison between experiments and predicted values according to: (a) Modified Bond Model, (b) shear capacity from ACI 318-11, (c) shear capacity from EN 1992-1-1:2005

To study the performance of the Modified Bond Model, all results of the experiments on slabs S1 to S10, tested at Delft University of Technology [11] are compared to the results obtained with the Modified Bond Model. The maximum concentrated load in the experiment, P_{exp}, is compared to the maximum capacity obtained with the Modified Bond Model, P_{MBM}. Mean values are used for the material properties. The evaluation showed an average ratio $P_{exp}/P_{MBM} = 1{,}19$ with a standard deviation of 0,13 and a coefficient of variation of 11%. Considering that the problem under study is a shear problem with a large number of parameters that have been varied in the experiments, the statistical results are excellent. The results are shown graphically in Fig. 10a.

Next, these experimental results are compared to the shear capacities according to ACI 318-11 [4] and NEN-EN 1992-1-1:2005 [3], to evaluate the performance of the Modified Bond Model as compared to existing code provisions. Note that the code provisions predict a maximum sectional shear force, while the Modified Bond Model predicts a maximum concentrated load. The values of the experimental sectional shear force V_{exp} compared to the shear capacity according to ACI 318-11, V_{ACI}, are shown in Fig. 10b. The reduced sectional shear force $V_{exp,EC}$ is compared to the shear capacity according to NEN-EN 1992-1-1:2005, $V_{R,c}$, in Fig. 10c. The reduced sectional shear force $V_{exp,EC}$ takes into account a reduction of the contribution of the loads close to the support ($0{,}5d_x \leq a_v \leq 2d_x$) to the shear force by $\beta = a_v/2d_x$ as prescribed by NEN-EN 1992-1-1:2005 §6.2.2 (6). The ratio of V_{exp}/V_{ACI} has an average value of 2,67, with a standard deviation of 1,00 and a coefficient of variation of 37%. For $V_{exp,EC}/V_{R,c}$ the average value is 1,99, with a standard deviation of 0,27 and a coefficient of variation of 13%. The statistical results show that the Modified Bond Model gives a better prediction of the experiments. The 45^{o} line in Fig. 10a, b, and c indicates experimental shear capacities that are exactly as predicted by the method under consideration. When using ACI 318-11 and NEN-EN 1992-1-1:2005 to compare to the experimental results, an increasing conservatism with increasing shear capacities is seen. The Modified Bond Model, on the other hand, shows more consistent results, as the cloud of test results lies above and parallel to the 45° line.

6. Summary and conclusions

A model for the shear capacity of reinforced concrete slabs under concentrated loads close to supports is proposed: the Modified Bond Model. This model is based on the Bond Model for concentric punching shear and takes direct load transfer between the load and the support into account. It is applicable to slabs with different amounts of reinforcement in the *x*- and *y*-direction and with a concentrated load close to the support. For continuous supports, the effect of the positive moment reinforcement is taken into account. For loads close to the edge, the edge effect and the influence of torsion are studied and a simplified method is proposed.The experimental results indicate that the Modified Bond Model is an improvement as compared to the code methods for slabs under concentrated loads close to supports.

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