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PRESSURE PROBES IN FREE MOLECULE FLOW

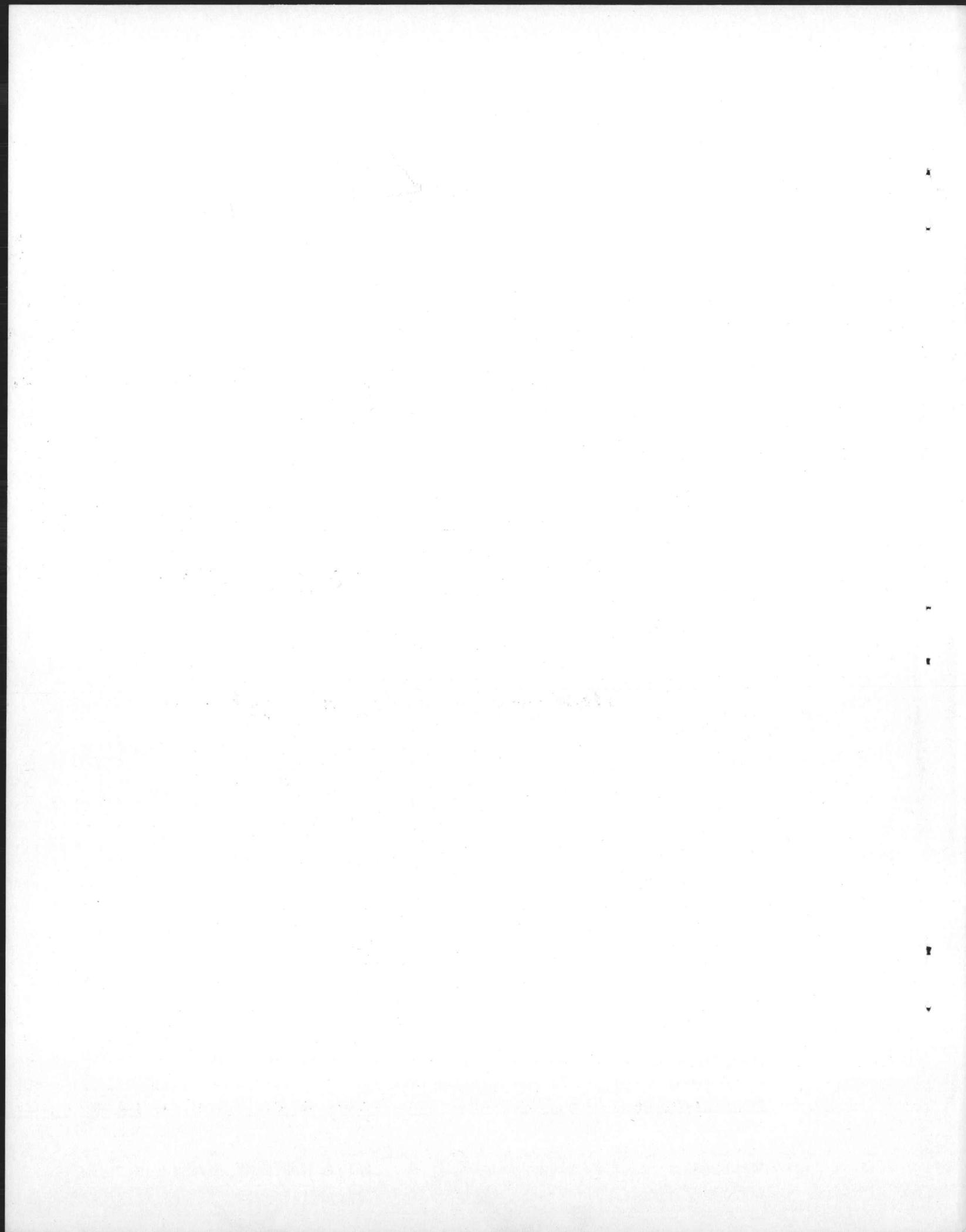
by

K. R. Enkenhus, E. L. Harris and G. N. Patterson



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PRESSURE PROBES IN FREE-MOLECULE FLOW

R. B. EMBERTSON, E. L. MARTIN and G. N. PATTERSON



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Flow on the right of the U.S. Naval Ordnance Laboratory, Silver Spring, Md.

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(ii)

NOTATION

A_i A_{ij}	coefficients of slightly non-Maxwellian distribution function
b	$\tan \epsilon$
c_i	components of random molecular velocity ($i = 1, 2, 3$)
c_m	most probable random molecular speed ($= \sqrt{2RT}$)
C_i	c_i/c_m
D	$2r/l$
f	velocity distribution function
G	function of b
H	function of S and b
Kd	$= \lambda/2r$ Knudsen number based on probe diameter
l	tube length
m	$= D/1+D$
M	Mach number
n	number of molecules per unit volume
p	pressure; p_θ = orifice probe pressure; p_t = long-tube probe pressure
p_1	free-stream static pressure
p_2	orifice-probe pressure, when orifice is in centre of long cylinder
p_3	orifice-probe pressure, when orifice is close to one end of cylinder
p_θ, p_s, p_π	orifice-probe pressure when $\theta = 0^\circ, 90^\circ$ and 180° , respectively
r	tube radius
r_o	orifice radius
q	surface outgassing mass flow rate per unit area
R	gas constant in $p = \int RT$

(iii)

S	molecular speed ratio
T	temperature
T_1, T_0	temperature of gas in the free stream and gauge volume, respectively
u_i	components of mass velocity
W	function of S and D
x_i	Cartesian coordinates
α	Clausing's factor, a function of D
γ	ratio of specific heats
ϵ	angle specifying position of orifice probe relative to plate leading edge
θ	angles between flow direction and normal to orifice
μ	viscosity
ξ_i	components of molecular velocity, $\xi_i = u_i + c_i$
ρ	density
τ	time constant
λ	mean free path $\left(\frac{16}{5} \frac{\mu}{\rho \sqrt{2\pi RT}} \right)$

SUMMARY

When the mean free path becomes a significant fraction of a characteristic dimension of a pressure probe, the usual continuum formulae relating the measured pressure to the free-stream pressure and Mach number are no longer valid. The present paper treats the case where the mean free path is so large compared with the probe diameter that intermolecular collisions may be neglected. This is the condition for free molecule flow.

Theoretical expressions are given for the pressure measured in a flowing gas with an orifice probe and a long-tube pressure probe. Experimental investigations have been conducted using a low-density wind tunnel and a rotating-arm apparatus. Agreement between theory and experiment was quite satisfactory.

Although the reported work was done primarily in connection with the flow of low density gases over models in the laboratory, the results are equally applicable to situations encountered in upper atmospheric research.

ABSTRACT

When the present paper was submitted for publication, the author was in receipt of a letter from the Editor of the Journal of the Royal Society of Medicine, London, in which the Editor stated that the paper was not suitable for publication in the Journal. The author is sorry to hear of this and is sure that the Editor's decision will be based on the fact that the paper is not suitable for publication in the Journal. The author is sure that the Editor's decision will be based on the fact that the paper is not suitable for publication in the Journal.

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I. INTRODUCTION

The interpretation of pressure measurements in a flowing gas under extremely rarefied conditions has been the subject of a number of theoretical and experimental investigations at the Institute of Aerophysics. This paper summarizes the results of the work.

At atmospheric pressure the behaviour of pressure probes, like pitot or impact probes, is well understood. When the density at constant speed is lowered, the value of the Reynolds number decreases and for values of this number below 100 it is necessary to start correcting for viscous effects. For still lower Reynolds numbers these corrections are sizeable and not too well known, so that the interpretation of pressure readings becomes difficult.

However, when the density of the gas is further reduced a situation is reached which again allows precise interpretation. This occurs when the mean free path of the gas particles, which varies inversely with the density, Ref. 1, becomes much larger than the dimension of the probe. The parameter indicating this is the Knudsen number, which is defined as the ratio of the mean free path and a significant linear dimension of the probe. When the gas flow is described by values of Kd much larger than unity, collisions between the gas particles in the main stream and those bouncing off the probe collide on the average only at a considerable distance from the probe and the flow can be determined by neglecting these collisions entirely, i. e. the so-called free molecular conditions apply. Under these conditions it is possible to calculate the probe characteristics by using the kinetic theory of gases.

Since the mean free path in the atmosphere is about 10 cm at an altitude of 100 km, and increases by a factor of about 4 every 10 km increase in altitude, it is clear that in soundings of the atmosphere above approximately 120 km free molecular conditions will be valid. This fact was recognized early in the history of rocket research of the atmosphere as shown in Refs. 2 and 3. Free molecular probes have been successfully used in the past years in rocket flights.

The work at the Institute of Aerophysics concerned a detailed study of probes under free molecular and near free molecular conditions in the laboratory, where the flow conditions could be controlled and measured by conventional methods. In this way it has been possible to substantiate the expected correctness of the free-molecular theory for sufficiently large numbers of the Knudsen number. In addition a practical criterion was established for the minimum value of the Knudsen number required for free molecular conditions.

Two aspects of the work are evident. The results have an important practical application to the measurement of flow speed in rare-

fied gas flows as they occur in upper atmospheric flight and as they can be produced in the laboratory. In addition the experiments provide a verification of the concepts of the kinetic theory of gases in the particular region of gas dynamics under investigation.

II. PRESSURE PROBES

A typical system for measuring pressures in a rarefied gas consists of a sensing element enclosed in a gauge volume which is connected by a length of tubing to a probe. The two types of pressure probes which will be considered are shown schematically in Fig. 1.

The orifice probe consists of a tube with a hole in its side; the axis of the tube is normal to the flow direction. The diameter of the hole must be large compared with the thickness of the tube wall. In free molecule flow the measured pressure is influenced by the geometry of the probe only through the angle θ between the outward normal to the orifice and the flow direction; $\theta = 0$ corresponds to the orifice facing into the flow. (See Fig. 2).

The long-tube type of pressure probe consists of a tube of length ℓ and internal radius r which points into the flow. For this type of probe the measured pressure depends on the ratio $2r/\ell$.

A typical orifice probe was constructed by drilling a small hole 0.008 inches in diameter in aluminum foil 0.00035 inches thick, and then gluing the foil over a larger hole in the wall of a No. 18 gauge stainless steel hypodermic tube. The tube had an inside diameter of 0.033 inches and an outside diameter of 0.049 inches. The long-tube probe was made from tube of similar size, externally chamfered at the open end. Construction details are given in Refs. 4 and 5.

III. THEORY

3.1 Velocity Distribution Functions

The expressions given below relate the measured pressures (P_θ and P_t) in the gauge volume to the quantities existing in the flow. The equilibrium pressure in the gauge depends on a balance between the number of molecules entering and leaving the probe. According to the kinetic theory of gases the computation involves the mean velocity in the external flow and the densities and distributions of random molecular velocities both inside and outside the probe. The velocity distribution function f is defined in the following way. If n is the number of molecules per unit volume, and ξ_i ($i = 1, 2, 3$) the components of molecular velocity, then the quantity $n f d\xi_1 d\xi_2 d\xi_3$ is the number of molecules per unit volume which have velocities in the range ξ_i to $\xi_i + d\xi_i$. Both a Maxwellian and a slightly non-Maxwellian distribution function will be used (Ref. 1). The Maxwellian f has the form,

(3)

$$f_0 = \left(\frac{1}{\pi c_m^2} \right)^{\frac{3}{2}} e^{- (c_1^2 + c_2^2 + c_3^2)} \quad (1)$$

which describes the distribution of random molecular velocities when the flow is isentropic. The slightly non-Maxwellian distribution function may be written

$$f = f_0 \left[1 + A_i C_i \left(1 - \frac{2}{5} C_j C_j \right) + A_{ij} C_i C_j \right] \quad (2)$$

where the summation convention for repeated indices is used, and according to Ref. 1

$$A_i = \frac{15\mu}{4p} \sqrt{\frac{2R}{T}} \frac{\partial T}{\partial x_i} \quad (3)$$

$$A_{ij} = - \frac{2\mu}{p} e_{ij} \quad (4)$$

where

$$e_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{1}{3} \frac{\partial u_k}{\partial x_k} \delta_{ij} \quad (5)$$

Equation (2) describes the distribution of random molecular velocities in a viscous, heat conducting flow.

The expressions given below for the gauge volume pressure refer to steady-state conditions. Intermolecular collisions are neglected in the calculations, that is, free molecule flow exists. Molecules which enter the orifice are assumed to lose their mass motion after striking the internal wall of the tube and to take up a Maxwellian distribution function corresponding to the wall temperature. The effects introduced by the time response of the probe, outgassing from the internal walls of the gauge, and thermal transpiration will be treated in Sec. 3.8.

3.2 Orifice Probe, Maxwellian Distribution Functions

When the molecules in the free stream have a Maxwellian velocity distribution function (see Eq. 1), theoretical calculations show that the pressure in the gauge volume is related to the free-stream pressure, temperature and velocity by the relation given in Ref. 6.

$$\frac{p_0}{p_1} \sqrt{\frac{T_1}{T_0}} = X(S \cos \theta) \quad (6)$$

(4)

where
$$\chi(y) = e^{-y^2} + y\sqrt{\pi}(1 + \operatorname{erf}y) \quad (7)$$

and
$$\operatorname{erf}y = \frac{2}{\sqrt{\pi}} \int_0^y e^{-x^2} dx \quad (8)$$

The flow direction is along the x_1 -axis, that is,

$$S = \frac{u_1}{c_m}, \quad u_2 = u_3 = 0 \quad (9)$$

Measured values of the orifice probe pressure when $\theta = 0^\circ, 90^\circ$ and 180° are given the symbols p_0, p_s and p_π , respectively*. Then

$$\frac{p_0}{p_1} \sqrt{\frac{T_1}{T_0}} = e^{-S^2} + S\sqrt{\pi}(1 + \operatorname{erf}S) \quad (10)$$

$$\frac{p_s}{p_1} \sqrt{\frac{T_1}{T_0}} = 1 \quad (11)$$

$$\frac{p_\pi}{p_1} \sqrt{\frac{T_1}{T_0}} = e^{-S^2} - S\sqrt{\pi}(1 - \operatorname{erf}S) \quad (12)$$

3.3 Orifice Probe, Slightly Non-Maxwellian Distribution Function

The corresponding expressions for the pressure indicated by an orifice probe when the fluid possesses a distribution function as in Eq. 2 is given in Ref. 6 as follows

$$\frac{p_0}{p_1} \sqrt{\frac{T_1}{T_0}} = \left[\left(1 + \frac{A_1}{5} S \cos^2 \theta - \frac{A_2}{5} S \sin \theta \cos \theta + \frac{A_{11}}{2} \cos^2 \theta + \frac{A_{22}}{2} \sin^2 \theta - A_{12} \sin \theta \cos \theta \right) e^{-S^2 \cos^2 \theta} + \sqrt{\pi} S \cos \theta (1 + \operatorname{erf} S \cos \theta) \right] \quad (13)$$

With the orifice orientated in the three directions of principal interest, Eq. 13 gives

$$\frac{p_0}{p_1} \sqrt{\frac{T_1}{T_0}} = \left(1 + \frac{5A_1}{5} + \frac{A_{11}}{2} \right) e^{-S^2} + S\sqrt{\pi}(1 + \operatorname{erf}S) \quad (14)$$

* It is found experimentally that various orientations of orifice have no appreciable effect on the temperature inside the gauge volume. Hence is assumed to be independent of θ .

(5)

$$\frac{p_s}{p_i} \sqrt{\frac{T_i}{T_0}} = 1 + \frac{A_{22}}{2} \quad (15)$$

$$\frac{p_\pi}{p_i} \sqrt{\frac{T_i}{T_0}} = \left(1 + \frac{SA_1}{5} + \frac{A_{11}}{2}\right) e^{-s^2} - s\sqrt{\pi} (1 - \text{erf } s) \quad (16)$$

3.4 Orifice Probe, Any Given Distribution Function

An expression may be derived for the pressure indicated by an orifice probe for any given distribution function which exists in the neighbourhood of the probe if n , the number density, is also given. An example of this is described in Sec. 4.2 where a comparison is made of experimental and theoretical values of impact pressure indicated by an orifice probe less than one mean free path from the leading edge of a flat plate.

3.5 Long-Tube Pressure Probe

Theoretical calculations for the pressure indicated by a long-tube probe as shown in Fig. 1(b) are based on the assumption that the distribution of random molecular velocities in the free stream is Maxwellian. Molecules which enter the probe and strike the walls are assumed to reflect diffusely. That is, the direction at which a molecule is emitted from the wall is independent of that at which it strikes and the probability of emission in a particular direction is proportional to the cosine of the angle between the normal to the surface and the direction of emission. The calculated value of p_t is found in Ref. 7 in the form

$$\frac{p_t}{p_i} \sqrt{\frac{T_i}{T_0}} = \frac{W(s, D)}{W(0, D)} \quad (17)$$

$$W(s, D) = \chi(s) - e^{-s^2} \left\{ \alpha \psi(D) + [1 - 2\alpha] S(D) \right\} - \frac{4s}{\sqrt{\pi}} \left\{ \alpha \eta(s, D) + [1 - 2\alpha] \omega(s, D) \right\} \quad (18)$$

where

$$D = \frac{2r}{L}$$

$$\psi(D) = \frac{2}{D^2} \left\{ \sqrt{1 + D^2} - 1 \right\}$$

$$S(D) = \frac{2}{3D^2} \left\{ (1 + D^2)^{3/2} - D^3 - 1 \right\}$$

$$\eta(S, D) = \frac{1}{D} \int_0^1 dY \int_0^\beta (1 + \operatorname{erf} S \cos \phi) \cos \phi e^{-S^2 \sin^2 \phi} d\phi \quad (6)$$

$$\beta = \arctan D \sqrt{1 - Y^2}$$

$$\omega(S, D) = \int_0^1 \eta(S, \frac{D}{X}) dX$$

α = a function of D only, evaluated by Clausing in Ref. 8.

3.6 Short-Tube Pressure Probes

The formula given in Sec. 3.5 is valid for any S and D but the calculation is difficult except for small S and D (that is, low speeds and long tubes). Hence an approximate formula valid for short tubes ($L/2r < 0.1$) is given here (Refs. 5 and 9).

$$\frac{p_t}{p_i} \sqrt{\frac{T_i}{T_0}} = \frac{\chi(S) - \frac{1}{D}(1 + \operatorname{erf} S)}{1 - \frac{1}{D}} \quad (20)$$

3.7 Application of Formulae to Determination of Molecular Speed Ratio

The molecular speed ratio S may be found from the ratio of two measured orifice probe pressures when the distribution function is Maxwellian. Division of Eq. 10 by Eq. 11 gives,

$$\frac{p_0}{p_s} = \chi(S) \quad (21)$$

Thus a graph of $\chi(S)$ vs S gives S from the ratio of the two measured pressures, p_0 and p_s .

If the distribution function is slightly non-Maxwellian then Eqs. 12 and 13 should be replaced by Eqs. 14 and 15. It was found in Refs. 4 and 5 that in a nozzle flow the A-coefficients are sufficiently small to enable Eq. 21 to be used for speed ratio determination. Anywhere that f is far removed from Maxwellian it may be necessary to use the more general Eqs. 14 and 15.

3.8 Outgassing, Time Response and Thermal Transpiration

Figure 1 illustrates a typical system for measuring pressures in a rarefied gas. When the pressure of the external gas changes to a new value, there is a time lag before the gas in the gauge volume reaches a new equilibrium. For a gas sufficiently rarefied so that the internal flow in the probe is in the free molecule regime, it may be shown that in response to an instantaneous change in external pressure, the pressure in the gauge

reaches its new equilibrium value exponentially. If the time constant is defined as the time required for the pressure to reach $(1 - 1/e)$ or 63.3% of the total change, then in the free molecule flow regime this time constant is independent of pressure, and depends only on the dimensions of the tube and orifice and on the associated volume, as indicated in Ref. 10. At low pressures the rate at which adsorbed gas is given off from the walls of the gauge becomes important in determining the pressure distribution in the gauge. This effect is described in Refs. 10 and 11. This gas, which is evolved at a mass flow rate q per unit area flows from the gauge out the probe and produces a pressure drop along the probe which subjects the gauge to a higher pressure than that which would exist without outgassing. This pressure difference is a function of q and the geometry of the gauge. The outgassing rate depends strongly on the temperature and pressure history of the surface and its cleanliness; for this reason it is impossible to specify a number or graph for q except under very limited conditions. Hence, for accurate pressure readings the gauge is designed so that the pressure error due to outgassing is small.

Under steady-state conditions the pressure in the gauge volume due to outgassing exceeds the pressure which would exist without outgassing by an amount. (see Ref. 10).

$$\Delta p = q \sqrt{\frac{gRT_0}{32\pi}} \left[\frac{8A}{3r_0^2} + \frac{Al}{r^3} + \frac{16\pi r l}{3r_0^2} + \frac{\pi l^2}{r^2} \right] \quad (22)$$

When the connecting line between gauge volume and probe consists of tubing of a different diameter than the probe, then for a given probe and gauge geometry there is an optimum size of connecting line which will result in a minimum outgassing pressure drop. The expression for the optimum line size has been verified experimentally in Ref. 11.

The time constant, τ , for the pressure measuring system in Fig. 1(a), was found in Ref. 10 to be

$$\tau = \frac{3}{\sqrt{32\pi RT_0}} \left[\frac{Vl}{r^3} + \frac{8V}{3r_0^2} + \frac{8\pi r^2 l}{3r_0^2} + \frac{\pi l^2}{3r} \right] \quad (23)$$

In the case of a probe without an orifice, such as the long tube (Fig. 1(b)) the two terms involving r_0 in Eqs. 22 and 23, which arise from the pressure drop across the orifice, are omitted.

It is found that in most practical applications the last two terms of Eqs. 22 and 23 may be neglected. Then Δp becomes proportional to τ .

That is

$$\Delta p = qRT_0 \frac{A}{V} \tau \quad (24)$$

In the measurement of pressure it is necessary to be able to correct for the effects of Δp and τ . By waiting a time of at least 5τ following a pressure change the influence of a finite time constant on the pressure reading may be eliminated. It may be expected that in the region of free molecule flow where all intermolecular collisions are neglected that a correction for the pressure error in the gauge volume due to outgassing may be applied by subtracting an amount which depends only on the outgassing rate and gauge geometry. This is so since for free molecule flow molecular movement depends only on the molecule surface interactions and is not influenced by the presence of other molecules. Experiments have shown this to be the case - that is, the pressure error due to outgassing is independent of the measured pressure. Hence the method used to correct for outgassing is to find Δp when the system external to the gauge is at high vacuum. This amount Δp is subtracted from the pressure which is measured later. Applications of this procedure are given in Refs. 4, 5 and 9. The time interval between these two measurements should be small enough to prevent a significant change in q , the outgassing rate.

Another effect which must be considered is thermal transpiration which occurs between the gauge volume and probe. With an orifice probe, for example, as in Sec. 3.2, theoretical calculations actually give the relation

$$\frac{p_0'}{p_i} \sqrt{\frac{T_1}{T'}} = \chi(S \cos \theta) \quad (25)$$

where p_0' and T' are the pressure and temperature of the gas just inside the orifice opening. However, along the connecting tube between the orifice and gauge volume, the thermal transpiration formula (see Ref. 1) holds relating the measured quantities p_0 and T_0 to the primed quantities in Eq. 25. That is,

$$\frac{p_0}{p_0'} \sqrt{\frac{T'}{T_0}} = 1 \quad (26)$$

Multiplication of Eqs. 25 and 26 gives

$$\frac{p_0}{p_i} \sqrt{\frac{T_1}{T_0}} = \chi(S \cos \theta) \quad (27)$$

which is the form given in Eq. 6. The expression for p_t in Eq. 17 may be handled similarly. Thus the only temperatures which influence the measured pressures are T_1 and T_0 . This has been verified experimentally.

IV. EXPERIMENTAL VERIFICATION

4.1 Rotation of Orifice Probes

From Eq. 6 at a given value of S the pressure indicated by an orifice probe is a function of the angle θ . The pressure varies from a maximum at $\theta = 0^\circ$ to a minimum at $\theta = 180^\circ$. Figure 3 gives a polar plot of theoretical and experimental p_θ/p_1 vs. θ at a Mach number of 1.98 in air ($S = 1.67$). The Knudsen numbers based on the outside diameter of the orifice probes (Kd) varied from 0.119 to 1.44. The dashed lines show the theoretical pressure ratios for both continuum and free molecule flow. The flow through the nozzle was isentropic and the static pressure (p_1) and temperature (T_1) were known from the nozzle calibration. As may be seen from Fig. 3 the Knudsen numbers were not large enough to achieve fully developed free molecule flow. However, the experimental curves approach the theoretical free molecule flow curve as Kd increases.

Figure 4 shows another comparison of theoretical and experimental curves of orifice pressure vs. angle of rotation θ performed at a Knudsen number of 6.8 and a Mach number of 0.86 ($S = 0.72$). These measurements were made at a higher Kd than those of Fig. 3 and the agreement between theory and experiment is seen to be satisfactory.

Some further experiments were conducted to assess the importance of the length of the probe cylinder. The idea here was that although the Knudsen number based on diameter was large, the length dimension is so much larger that it corresponds to a much smaller Knudsen number. To bring out any such effect of the long lengths if existing, an arrangement was used in which the orifice was located very close to one end of the cylindrical probe as shown in Fig. 5. The orifice pressure was first measured with this probe by itself and afterwards the measurement was repeated with a dummy cylinder in line with and close to the first one. This resulted in a situation with the orifice effectively located in the centre of a probe virtually infinite in length in both directions. Any effect due to the long length should then be doubled for the latter case. The experimental results are given in Fig. 6. The quantity plotted is the ratio of the pressure for the orifice probe alone and that for the probe with added length. It is seen from the curves that for the Knudsen numbers considered the effect of probe length represent only a small correction, and at Knudsen numbers above approximately 10, the effects at subsonic Mach numbers will be negligible. For higher Mach numbers the effects may be somewhat accentuated.

4.2 Orifice Probe Near the Leading Edge of a Flat Plate

The molecules in the region less than one mean free path from the leading edge of a thin flat plate may be separated into two groups. The first group is composed of those molecules which have come from the region upstream of the plate and possess the distribution function of this

flow. The molecules of the second group are those which have struck the plate and are emitted after a diffuse reflection from it. Thus for an orifice probe with the outward normal to the hole pointing into the flow the distribution function may be assumed to be known and theoretical orifice probe pressures can be calculated as a function of the position of the orifice relative to the plate leading edge.

In Ref. 5 the calculated impact pressure in the region less than one mean free path downstream of the plate leading edge under the above assumptions is shown to be

$$\frac{p_0}{p_1} \sqrt{\frac{T_1}{T_0}} = \frac{\chi(S)}{2} + \frac{G(b)}{2} + H(S, b) \quad (28)$$

where

$$G(b) = 1 - \frac{1}{\sqrt{1 + \frac{1}{b^2}}} \quad (29)$$

$$b = \tan \epsilon \quad (30)$$

and

$$H(S, b) = \int_0^{\infty} \Lambda e^{-\frac{(\Lambda - S)^2}{b^2}} \operatorname{erf} b \Lambda d\Lambda \quad (31)$$

S is the molecular speed ratio upstream of the plate leading edge.

Experimental measurements were made to verify Eq. 28 in a subsonic nozzle in the low-density wind tunnel. Figure 7 shows a comparison of measured and theoretical orifice probe pressures. The difference between theory and experiment may be attributed to longitudinal and transverse gradients in the flow. The best agreement is obtained for the slope of the curves near the wall. These results demonstrate experimentally the existence of a free-molecule-flow region less than one mean free path from the leading edge of a flat plate and also illustrate the usefulness of orifice probes in regions where the velocity distribution function is considered to be known.

4.3 Long-Tube Impact Probe

The results of experimental studies on the pressure indicated by a long impact probe in free molecule flow are plotted in Fig. 8. Air was used in all experiments. Two of the studies performed in the low-density wind tunnel, are described in Refs. 2 and 5, while another set of results, presented in Ref. 9, was obtained using a rotating-arm apparatus.

The Knudsen numbers (ratio of mean free path to outside diameter of the tube) used in the rotating-arm experiments were approxi-

mately 20, and the value of D was 0.2. S was found from the rotational speed of the drive motor. The theoretical curve in Fig. 8 is for $D = 0.04$, but calculations showed a negligible shift in the curve due to changes in D from 0 to 0.2 for values of the speed ratio less than one. It is seen that the experimental values of the pressure obtained using the rotating-arm apparatus agree very well with those predicted by theory.

In the low-density wind tunnel tests a free molecule static probe was used to measure the quantity $p_1 \sqrt{T_0/T_1}$. This was essentially an orifice probe with $\theta = 90^\circ$. Thus the results taken from Ref. 5 are the ratio of two measured pressures. The value of D was 0.04. The speed ratio was found by using an orifice-type impact probe and the Knudsen numbers based on tube outside diameter were between 9 and 20. Agreement between theory and experiment is quite good.

The points taken from Table 10 of Ref. 4 where D was 0.04 were calculated assuming that the flow from the stagnation chamber was adiabatic. The Knudsen number was approximately 8.

4.4 Short-Tube Impact Probe

A few experimental measurements were made in Ref. 5 with short tubes to check the validity of Eq. 20. The probes were constructed by drilling a 0.008 inch diameter hole in the wall of No. 18 gauge tube and honing the tube until the desired thickness was reached. These probes had a D of 4 and 2. A large uncertainty existed in these values of D because of the small dimensions involved. The results are presented in Fig. 9. The abscissa is $D/D+1$ and the ordinate is the impact pressure obtained for a tube of diameter length ratio D divided by the impact pressure obtained for an orifice probe. Five different speed ratios were used and the short straight lines on the left of the graph give theoretical values of $(P_t)_{D=0}/p_0$. Also on the graph are given the theoretical slopes at $D/D+1 = 1$ for the five speed ratios. These slopes may be found from the relation (see Eq. 20)

$$\lim_{D \rightarrow \infty} \left\{ \frac{\partial}{\partial \left(\frac{D}{D+1} \right)} \left[\frac{\chi - \frac{1}{D} (1 + \text{erf } s)}{\frac{1 - \frac{1}{D}}{\chi}} \right] \right\} = \frac{(1 + \text{erf } s) - \chi(s)}{\chi(s)} \quad (32)$$

It is seen that reasonable agreement was obtained between experimental and theoretical slopes at $D/D+1 = 1.0$ (i. e., $D = \infty$) thus justifying the use of Eq. 19 for short tubes.

V. CONCLUSIONS

Theoretical expressions have been derived for the pressure measured by orifice probes and long-tube type probes in free molecule flow. The probes may be used to determine the flow speed in a rarefied gas stream. Good agreement was obtained between theory and experiment in tests conducted in a low-density wind tunnel and a rotating-arm apparatus when the conditions for free molecule flow were fulfilled. The measured time response, outgassing and thermal transpiration effects were found to follow theoretical predictions.

A probe placed in a stream under free-molecule-flow conditions (i. e., with the molecular mean free path greatly exceeding the probe diameter) is affected by factors which have no influence in continuum flow. First, thermal transpiration causes the ratio of free stream static temperature to gauge volume temperature to enter the expression for the measured pressure. Second, the reading of a long-tube type probe becomes dependent on probe geometry, namely, the diameter to length ratio. Geometrical effects may be eliminated by using the orifice-type probe. Furthermore, by taking the ratio of pressure readings with the orifice facing into the stream and at right angles to it, the molecule speed ratio may be computed without further measurements.

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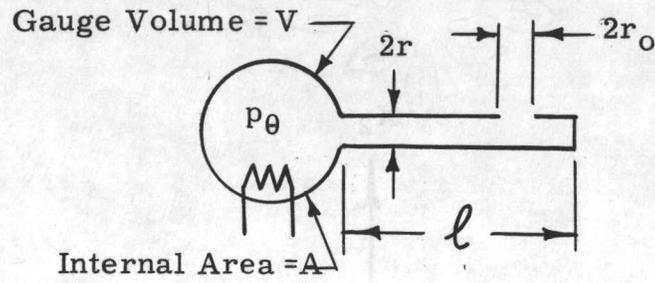


FIG. 1(a) ORIFICE PROBE

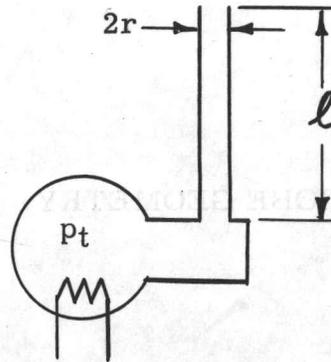
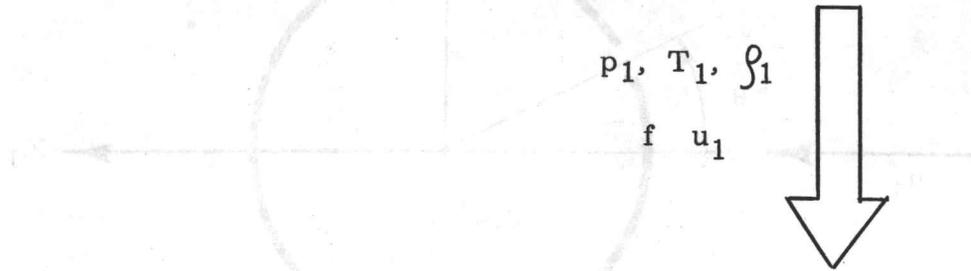


FIG 1(b) LONG-TUBE PROBE

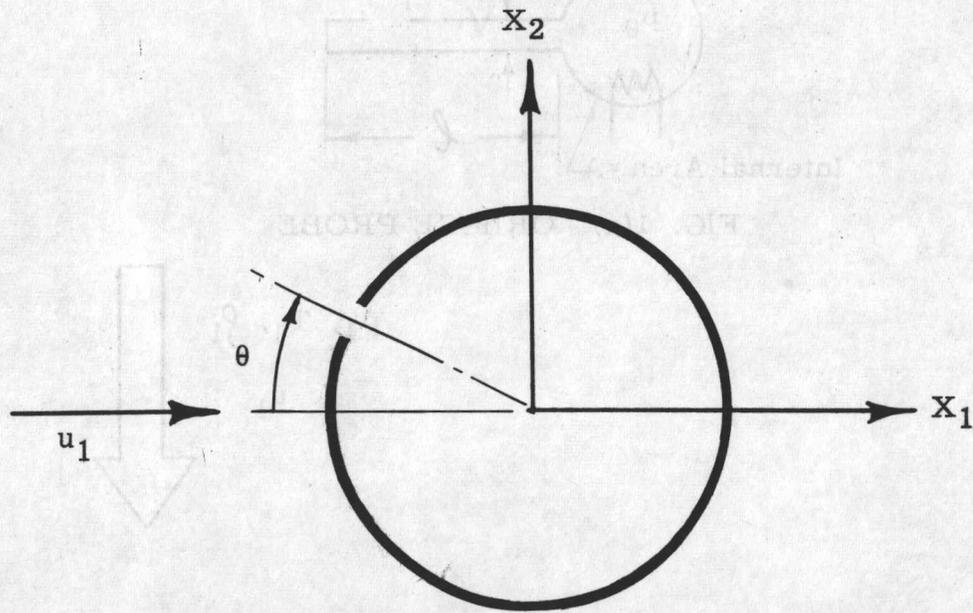


FIG. 2 ORIFICE-PROBE GEOMETRY

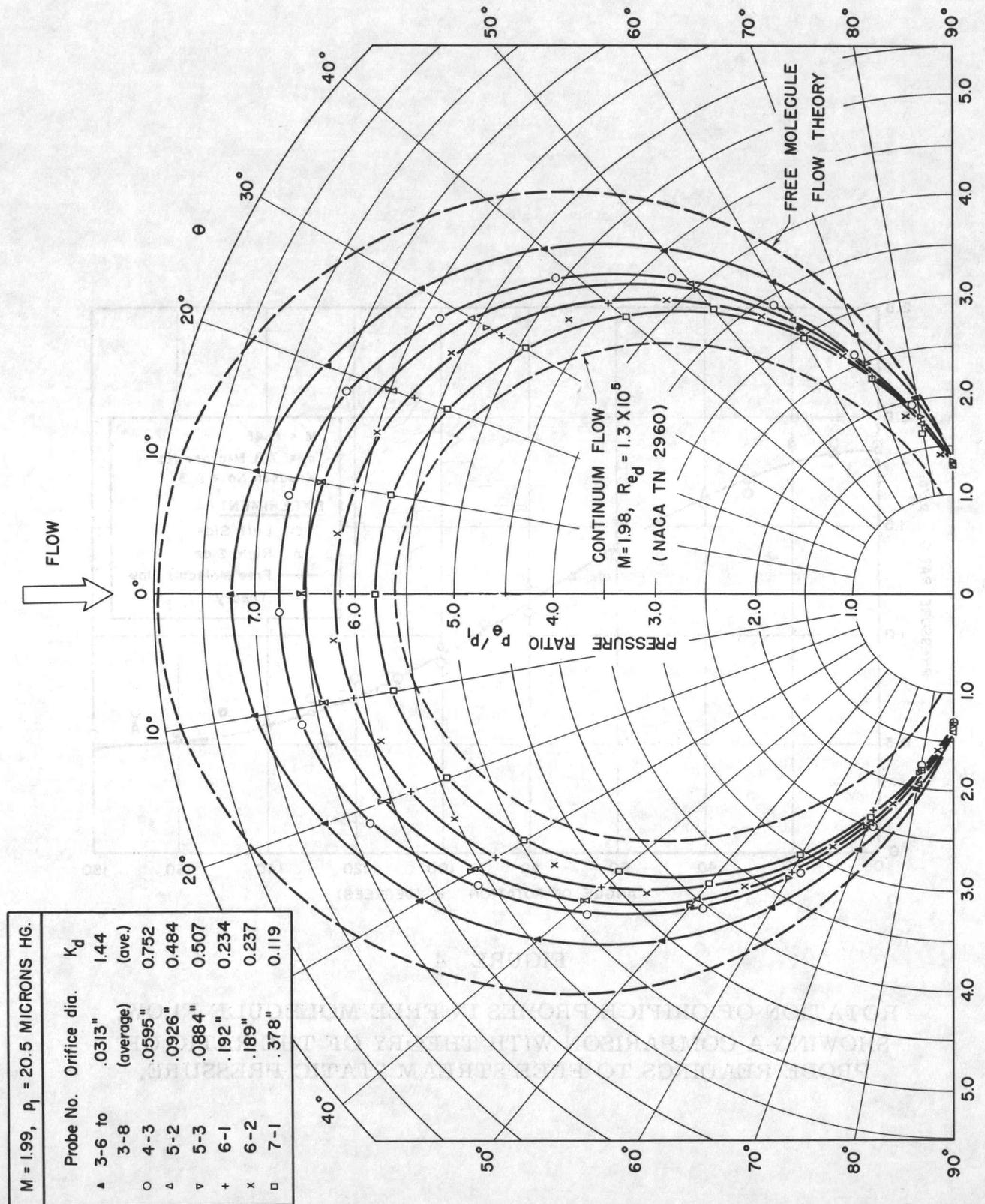


FIGURE 3

RATIO OF PRESSURE READING TO FREE STREAM STATIC PRESSURE OBTAINED BY ROTATING ORIFICE PROBES OF VARIOUS DIAMETERS IN MACH 2 JET FOR $0^\circ \leq \theta \leq 90^\circ$.

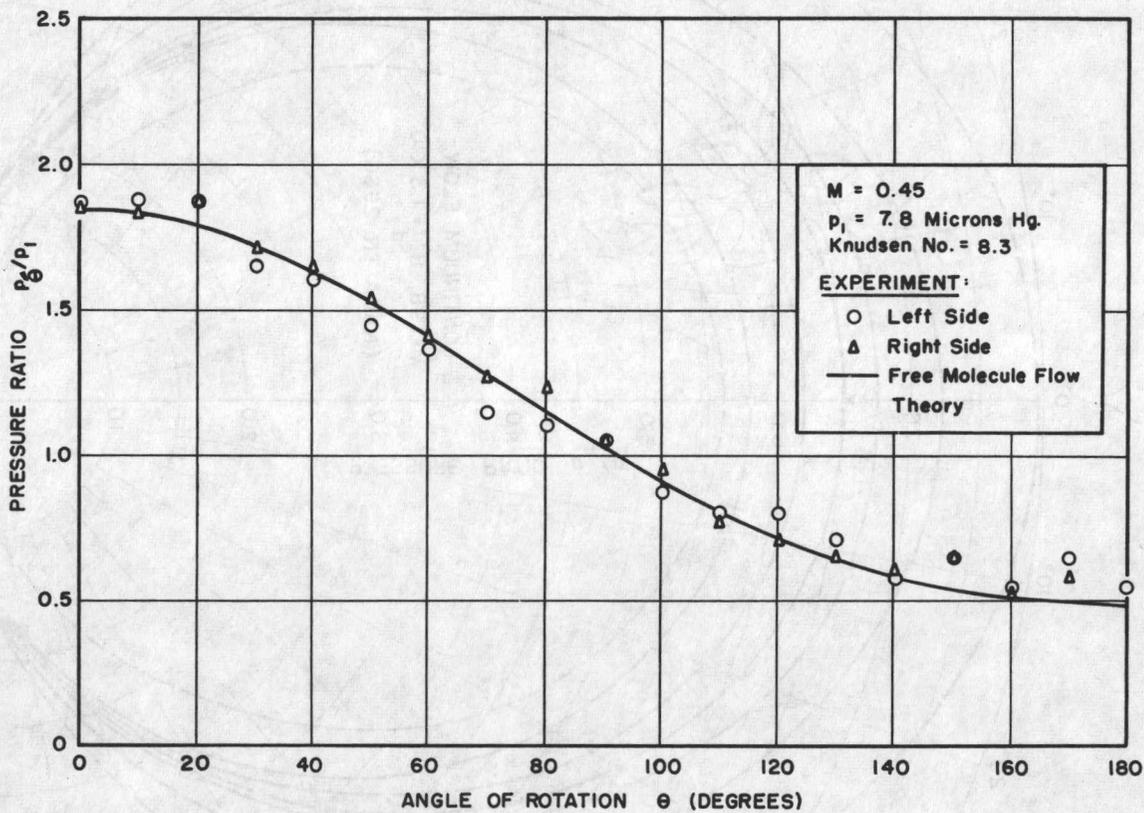


FIGURE 4

ROTATION OF ORIFICE PROBES IN FREE MOLECULE FLOW,
 SHOWING A COMPARISON WITH THEORY OF THE RATIO OF
 PROBE READINGS TO FREE STREAM STATIC PRESSURE.

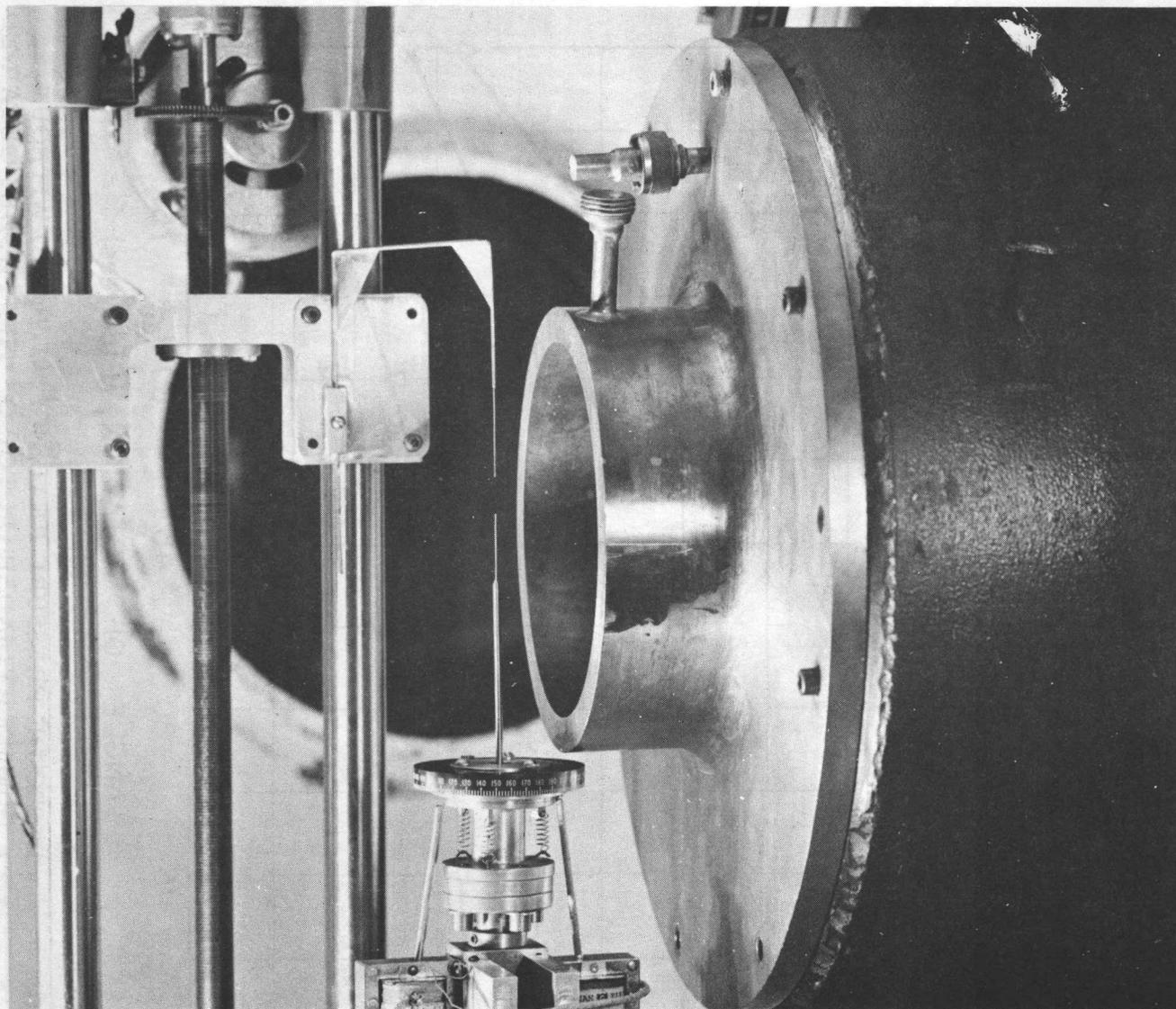


FIGURE 5

ARRANGEMENT FOR DETERMINING THE EFFECT OF THE CYLINDER LENGTH ON A PRESSURE READING, USING A SUBSONIC NOZZLE. AN ORIFICE PROBE IS PLACED IN THE FLOW CENTER, AND A DUMMY CYLINDER ABOVE THE PROBE.

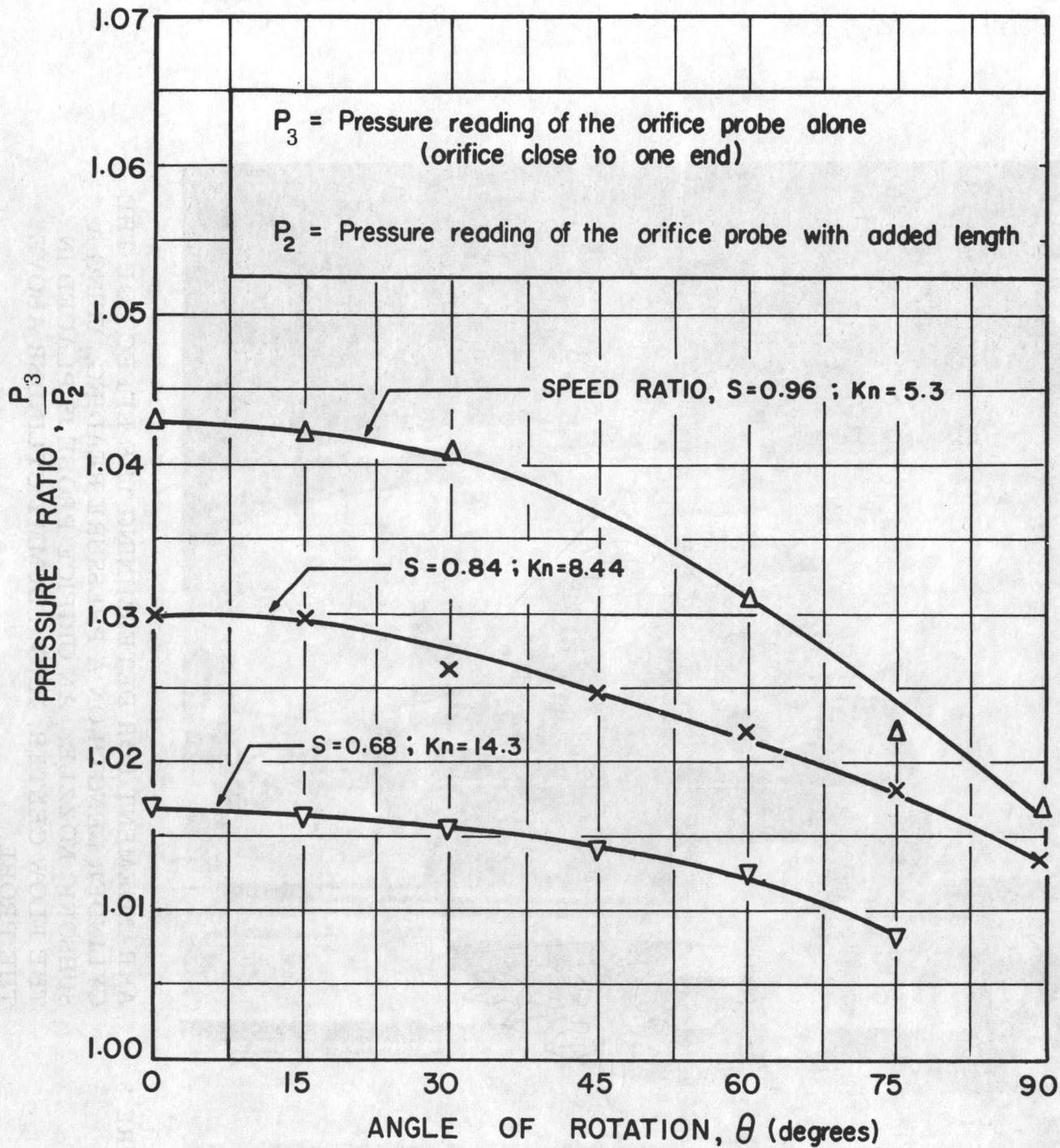


FIG. 6 RATIO OF PRESSURE READINGS WITH AND WITHOUT A DUMMY CYLINDER ON TOP OF THE PROBE OBTAINED BY ROTATING 0.049" DIAMETER ORIFICE PROBE IN SUBSONIC NOZZLE JET.

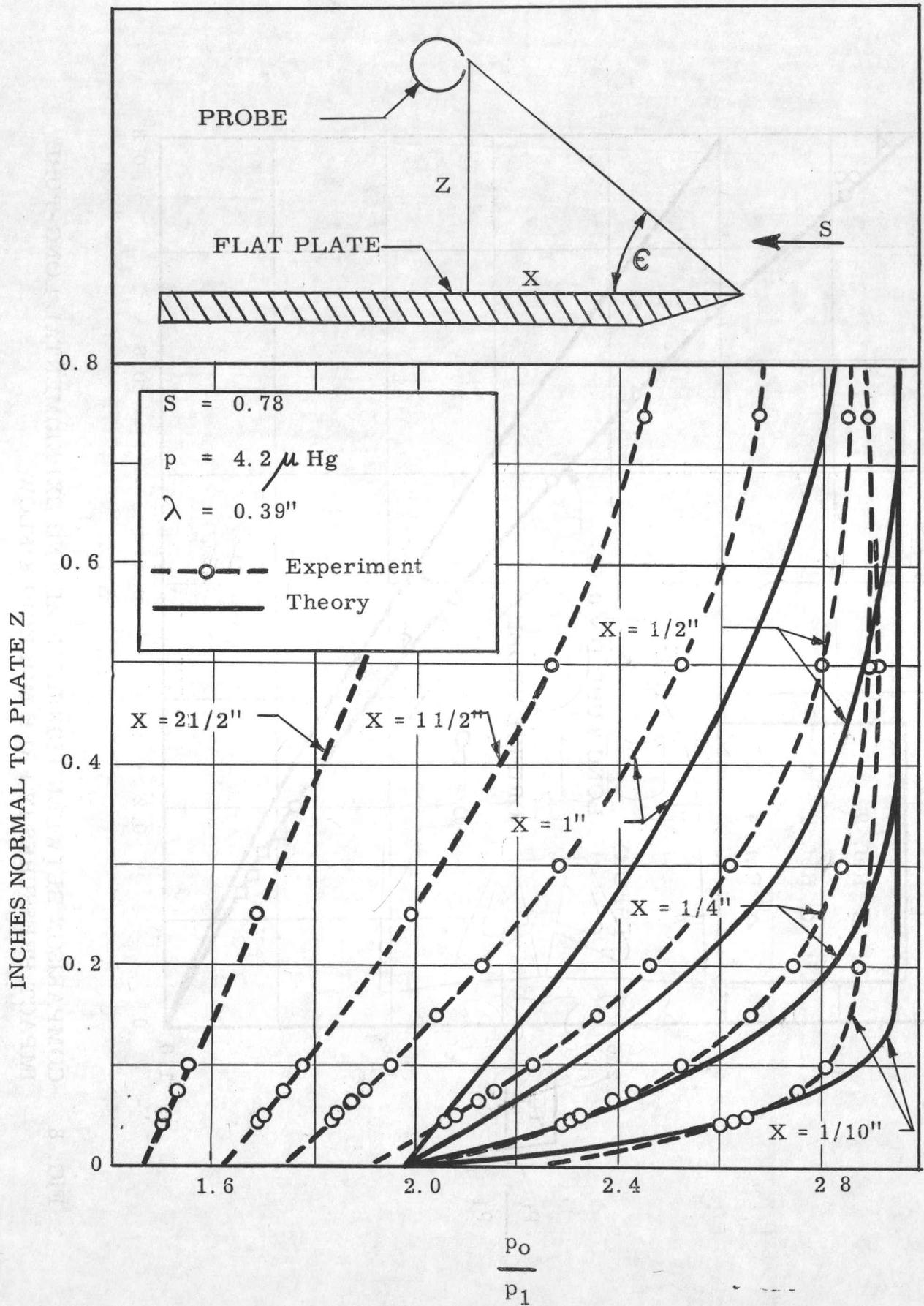


FIG. 7 ORIFICE-PROBE PRESSURE PROFILES AT VARIOUS DISTANCES FROM THE LEADING EDGE OF A FLAT PLATE

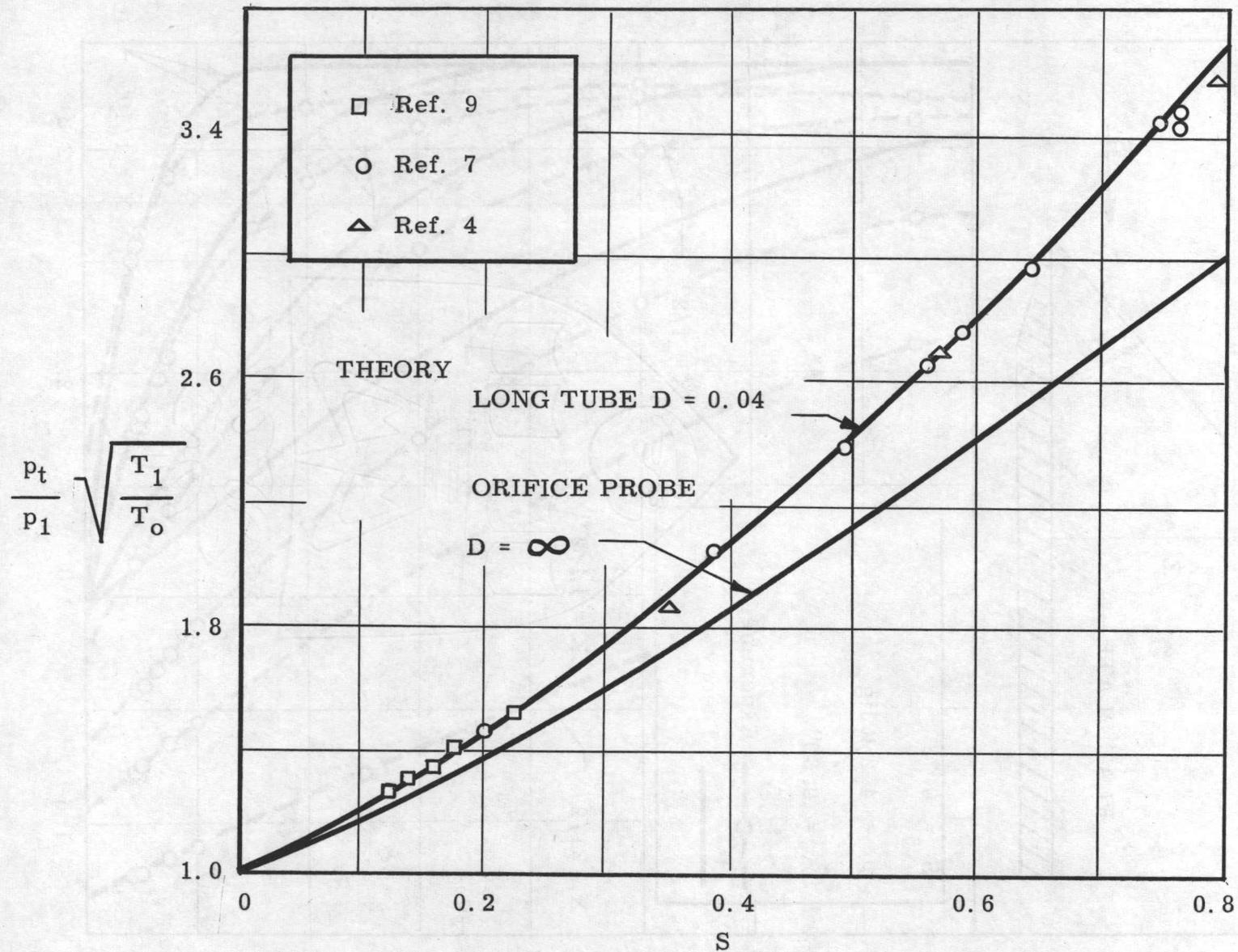


FIG. 8 COMPARISON BETWEEN THEORETICAL AND EXPERIMENTAL LONG-TUBE IMPACT PRESSURES IN FREE MOLECULE FLOW

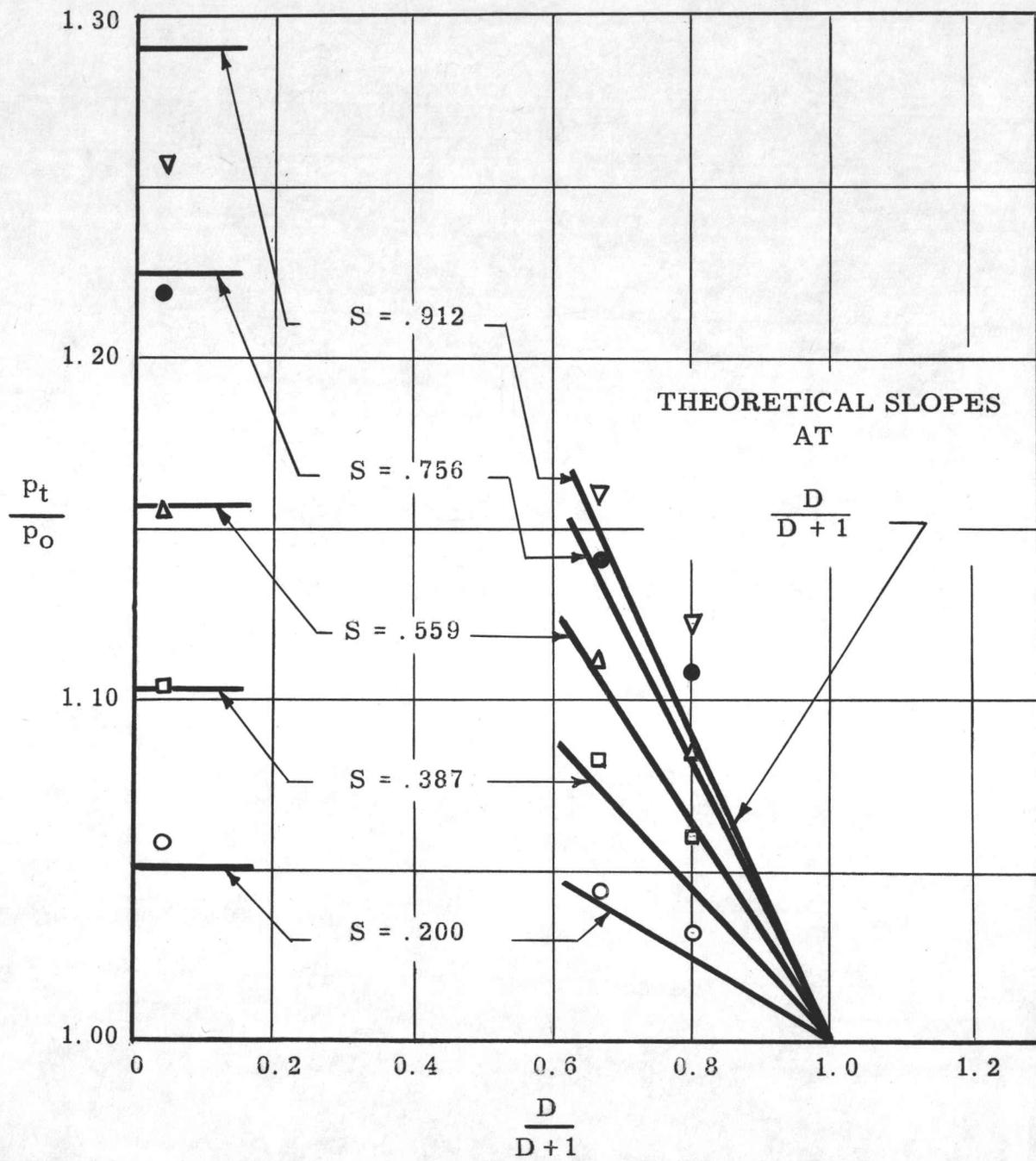


FIG. 9 RATIO OF LONG-TUBE TO ORIFICE-PROBE IMPACT PRESSURE VS $D/D+1$