Methods to Reduce Oblique Bending in a Steel Sheet Pile Wall

A 3D numerical simulation

April 1998

J.A.W. Hockx

Cantilever
Horizontal fix
Capping beam
Welded interlock
Methods to Reduce Oblique Bending in a Steel Sheet Pile Wall

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by

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Preface

This report completes my graduation project: ‘Methods to reduce oblique bending in a steel sheet pile wall’, which is the final examination of my study Civil Engineering at the Delft University of Technology.

This graduation project is carried out in co-operation with ISPC profilARBED (Luxembourg) and the Foundation Engineering and Computational Mechanics groups of the Department of Civil Engineering, Delft University of Technology. I am grateful for the provided facilities and expertise which made this project possible.

I would like to express my gratitude to Dr.-Ing. A. Schmitt, Ir. D.A. Kort and Ir. O.M. Heeres for their guidance during the course of this project. Their comments and discussions during this project were of great value. Furthermore, I would like to thank the people who spent their time reading the concept versions of this report.

Delft, April 1998

Johan Hockx
Summary

For over three quarters of a century steel sheet pile walls are applied in geotechnical practice. Sheet pile walls consisting of steel are usually applied when large differences in elevation must be sustained and heavy loads are acting upon the sheet pile wall. This is for example the case with quay walls and construction pits in urban surroundings.

Steel sheet pile walls usually have a wave like cross-section consisting of U- or Z-profiles. The wave like cross-section ensures favorable strength- and stiffness properties. A common used variant is a sheet pile wall composed of so called double U-profiles. These profiles consist of two single U-profiles fixed in the common interlock by welding or crimping. A very specific property of the double U-profile is an asymmetric cross-section which can lead to a rotation of the neutral axis. As a result of this, the sheet pile wall tends to deflect both forward (lateral) and sideways (transverse). Because the double U-profiles are bending in both lateral and transverse direction, this phenomenon is called oblique bending. As a result of oblique bending the strength and lateral stiffness of the sheet pile wall is reduced compared to a continuous sheet pile wall.

Oblique bending goes together with slip in the free interlocks and a transverse deflection of the sheet pile relative to the soil body. In an earlier study, the transverse bearing capacity of the soil (i.e. reduction of the transverse displacement of the sheet pile by the soil) has been evaluated. In that study it is concluded that in case of a high groundwater level, the soil is not able to give a transverse restraint which significantly reduces oblique bending.

Aim of this graduation project is to evaluate four different methods to reduce oblique bending in a steel sheet pile wall. The four methods which have been studied are:
- A steel sheet pile wall without a resulting water pressure.
- A steel sheet pile wall with a fix of the horizontal displacement at the top.
- A steel sheet pile with the sliding interlock welded during the excavation.
- A steel sheet pile wall with a capping beam on top.

The main goal of this study is to determine which method is able to give the highest reduction of oblique bending. Therefore calculations are made with a three dimensional finite element model of a dry excavation of a soil body (a middle-dense compacted sand) in front of a cantilever sheet pile wall consisting of double U-profiles. Furthermore use is made of a perfectly rough (n=1) sheet pile wall which implies no slip of the soil along the axis of the sheet pile. Also the sheet pile is modeled without a resulting water pressure on it. In this way a ‘best case’ model is obtained with which a prediction can be made what best possible result can be obtained by applying some kind of method to reduce oblique bending.

First analytical calculations are made of a simply supported sheet pile with a distributed load, in order to evaluate the influence of the loading conditions on the bending characteristics. The loading conditions in lateral and transverse direction appeared to be of great influence on the strength and stiffness of the sheet pile. The maximum or upper limit strength and stiffness is derived if oblique bending is prevented by fixing the free interlocks (no in plane deformation). Depending on the loading conditions, the stiffness varies from 0.49 to 1 time the maximum stiffness. The strength appeared to vary from 0.59 to 1 time the maximum strength. The lower limit of the strength and stiffness is obtained when no transverse bending moment is activated and the in plane deformation is free. The lower-limit and upper-limit is used in a 2D-finite element computation in order to determine the maximum and minimum lateral displacements.

Furthermore, 3D-finite element computations are made for the four methods to reduce oblique bending. The lateral deflections of the sheet pile wall in these calculations are compared to the upper- and lower-
limit deflections of the 2D calculations. In this way a quantification can be made of the magnitude that a method has on reducing oblique bending.

From the results of the 3D calculations, it appeared that sand is not able to give a transverse restraint which significantly reduces oblique bending under these ‘best case’ condition. A steel sheet pile wall with a fix of the horizontal displacement at the top is reducing oblique bending somewhat more. The stiffness is increased in this case to 0.61 times the maximum stiffness. The strength is increased in this case to 0.70 times the maximum strength.

A capping beam is a concrete casing on the sheet pile head. This concrete casing is installed after the sheet pile is driven into the ground and before the sheet pile is able to deform due to an excavation. When a capping beam is installed on top of a sheet pile wall, it is assumed that the slip in the free interlock at the sheet pile head is impeded. As a result of this impediment, a transverse restraining moment is activated which reduced oblique bending considerably. The stiffness is increased to 0.71 times the maximum stiffness and the strength is increased to 0.72 times the maximum strength.

With every new excavation step the lateral- and transverse displacements increase. As a result of the increasing transverse displacements, the slip in the interlock also increases. From the results of the sheet pile with a capping beam on top, it can be observed that if the slip in the free interlock is impeded the sheet pile acts much stiffer. As a logical follow up, the interlock of the sheet pile is welded after a small excavation step. In this way no further slip can develop in the welded part of the interlock. The procedure for welding of the interlock during excavation is as follows. First an excavation step is made to ensure the interlock is accessible for welding. Because an excavation step is made, there is also some slip in the interlock. When welding of this part of the interlock is completed, the next step can be excavated and is accessible for welding of the interlock etc. From the 3D calculations it followed that welding of the interlock during excavation gave the highest reduction of oblique bending. In this case the stiffness is increased to 0.82 times the maximum stiffness and the strength is increased to 0.77 times the maximum strength.

It is striking to notice that the stiffness of the sheet pile is increasing more than the strength. This is caused by the fact that strength of the sheet pile only depends on the cross-section of the sheet pile where the maximum value of the lateral (= out of plane) bending moment occurs. In this cross-section the maximum stress occurs. The stiffness of the sheet pile depends on the distribution of the transverse (= in plane) restraining moment over the entire sheet pile. The restraining moment in one cross-section is less subjected to variation so the strength of the sheet pile is also.

In order to investigate oblique bending further it is recommended to investigate other types of sheet pile constructions, such as sheet pile walls with anchors, struts or walings, have on reducing oblique bending. It can be found from the results that slip in the interlock is a very important factor for oblique bending. Therefore it is recommended to investigate the influence of friction in the sliding interlock on oblique bending.
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1 Introduction

For over three quarters of a century steel sheet pile walls are applied in geotechnical practice. A sheet pile wall is a row of interlocking, vertical pile segments driven to form an essentially straight wall. Frequently use is made of sheet pile walls for soil and/or water retaining constructions. This means the sheet pile wall must sustain a difference in soil surface elevation or water elevation from one side to the other. This is accomplished by an interaction of the surrounding soil and the vertical pile segments.

Sheet pile walls consisting of steel are usually applied when large differences in elevation must be sustained and heavy loads are acting upon the sheet pile wall. This is for example the case with quay walls. Also in construction pits in urban surroundings, where space is limited and high demands are made with respect to deformations, steel sheet pile walls are often used.

Steel sheet pile walls usually have a wave like cross-section consisting of U- or Z-profiles. The wave like cross-section ensures favorable strength- and stiffness properties. The U- or Z-profiles are generally driven in pairs. Sheet piles joined in pairs are connected by means of welding or crimping the common, middle interlock.

Although double U-profiles are often used with success, sometimes problems arise when the phenomenon oblique bending occurs. As a result of oblique bending the strength and lateral stiffness of the sheet pile wall is reduced compared to a continuous sheet pile wall. This will be explained in the next paragraph.

1.1 What is oblique bending

Oblique bending can occur in steel sheet pile walls which are constructed of double U-profiles. These profiles consist of two single U-profiles fixed in the common interlock by welding or crimping. By connecting two single U-profiles forming one double U-profile, a new profile is created with new properties.

![Diagram of oblique bending](image)

Figure 1: Influence of fixing the interlock on the location of the neutral axis.
One of these properties is an asymmetric cross-section. Due to this asymmetric cross-section, the principal axis of the double U-profile is rotated with regard to the axis of a continuous sheet pile wall. As a consequence, a lateral load acting upon the sheet pile wall (e.g. earth- or water pressure) can result in a rotation of the neutral axis with a lateral (out of plane) deformation and a transverse (in plane) deformation. Because the double U-profiles are bending in the lateral as well as in the transverse direction, this phenomenon is called oblique bending.

In case a sheet pile wall consists of single U-profiles with only free interlocks, a neutral axis is located in every individual U-profile (see Figure 1a). The slip in all interlocks is equal. If oblique bending occurs in a sheet pile wall constructed with double U-profiles, there will be slip only in the free interlocks. As a result the neutral axis will rotate around the fixed interlock of the double U-profile as can be observed in Figure 1b. Because the neutral axis is rotated, the lateral stiffness and strength of the sheet pile wall is reduced, compared to a continuous sheet pile wall consisting of fixed U-profiles (see Figure 1c). In a continuous sheet pile wall all interlocks are fixed, therefore no slip can occur. This configuration of a sheet pile wall has the highest stiffness.

In a steel sheet pile wall consisting of Z-profiles (single or double), no oblique bending occurs (see Figure 1d). This is caused by the fact that the free interlocks are located at the flange of the cross-section. As a consequence, no rotation of the neutral axis occurs. Thus in contrast with double U-profiles, the lateral stiffness and strength is not reduced when Z-profiles are used. Yet, U-profiles are still often used because driving of U-profiles is easier than Z-profiles.

The parameters of the sheet pile for stiffness and strength are usually given by the sheet pile supplier. These values are valid for the beginning of yield in the ultimate fibre of a continuous sheet pile wall with fixed interlocks. With oblique bending an aberration of the normal behaviour is occurring. Even with a small slip in the interlocks, a considerable reduction on the strength and stiffness, in comparison to the values given by the sheet pile suppliers, must be applied.

In the Eurocode (lit[3]), a reduction factor $\beta_D$ is applied at the lateral stiffness of a continuous sheet pile wall with fixed interlocks. With oblique bending an aberration of the normal behaviour is occurring. Even with a small slip in the interlocks, a considerable reduction on the strength and stiffness, in comparison to the values given by the sheet pile suppliers, must be applied.

In order to provide missing knowledge on oblique bending and to determine the reduction factors, an ECSC research project is being carried out. In the report ‘A 3D Numerical Simulation of Oblique Bending in a Steel Sheet Pile Wall.’ by Aukema and Joling (lit[1]), the effect of the soil-shear resistance has been studied. This study has been sponsored by ISPC ProfilARBED (Luxembourg) which is a participant of the ECSC research project. In this report it is concluded that in case of a high groundwater level, the soil is not able to give a transverse restraint which significantly reduces oblique bending. This is caused by the fact that the resulting water pressure represents the main part of the distributed soil and water load at the back of the sheet pile wall. As a follow up, ISPC ProfilARBED sponsored also this study in order to determine what methods can significantly reduce oblique bending in a steel sheet pile wall.
In their study of the soil-shear resistance, Aukema and Joling have used the finite element system DIANA (lit[4]) to generate an appropriate 3D-finite element model in which the soil is modeled elasto-plastic in combination with a correct steel sheet pile mesh. This 3D-finite element model will also be used in this investigation. In this way a comparison can be made between the different effects which influence oblique bending. In this report the effects of fundamentally different methods to reduce oblique bending will be investigated.

1.2 Aim of this project
Aim of this graduation project is to evaluate four different methods to reduce oblique bending in a steel sheet pile wall. This has been done by altering the 3D-finite element model used by Aukema and Joling. The four methods which have been studied are:
• A steel sheet pile wall without a resulting water pressure.
• A steel sheet pile wall with a fix of the horizontal displacement at the top.
• A steel sheet pile with the sliding interlock welded during the excavation.
• A steel sheet pile wall with a capping beam on top.

In this way recommendations can be given on the reduction factors of the stiffness and strength of a steel sheet pile wall.

1.3 The contents of the report
In chapter 2 analytical calculations are made concerning oblique bending in a double U-profile. Based on these formulae, the influence of the loading condition on oblique bending is evaluated. Also reduction factors for the moment of inertia and the section modulus are derived. In chapter 3 a differential equation is derived in order to calculate the shear force in the interlock. Chapter 4 deals with the finite element modeling of the test case. This test case is used to quantify the reduction of oblique bending as a result of the different reduction methods. The modeling of the soil and the sheet pile are also discussed, as well as the dimensions and the boundary conditions of the test case. In chapter 5 the results of the 3D calculations are given. For every method to reduce oblique bending the reduction factors for strength and stiffness are derived and discussed. The typical 3D phenomena that occur in the calculations are discussed in chapter 6. Furthermore in chapter 7 a comparison is made between the calculated 3D results and the normative CUR publication. Finally the general conclusions and recommendations are given in chapter 8.
2 Analytical calculations concerning oblique bending

The main goal in this chapter is to derive general reduction factors for the moment of inertia $R_l$ and the section modulus $R_w$ which take into account the phenomenon oblique bending. These reduction factors can then be applied to the material properties as given by the sheet pile supplier.

In order to derive the reduction factors for oblique bending, the analytical formulae for two directional bending have been studied. Especially the influence of the loading conditions on the bending characteristics of the profile have been evaluated.

2.1 Analytical bending formulae for double U-profile

In Figure 2 all variables for the bending formulae are defined along with the chosen coordinate system. The origin of coordinates is placed at the centre of gravity of the profile. The x-axis coincides with the fixed interlock of the two connected U-profiles which form the double U-profile.

The constitutive relations for two directional bending in a matrix form are (see also den Hartog, lit[6] and Verruijt, lit[10]):

\[
\begin{bmatrix}
M_y(x) \\
M_z(x)
\end{bmatrix} = E
\begin{bmatrix}
I_{yy} & I_{yz} \\
I_{zy} & I_{zz}
\end{bmatrix}

\begin{bmatrix}
\kappa_y(x) \\
\kappa_z(x)
\end{bmatrix}
\]

The moments of inertia with respect to y and z axes and the product of inertia with respect to the y and z axes are defined by the integrals:

\[
I_{yy} = \int_A y^2 dA
\]

\[
I_{zz} = \int_A z^2 dA
\]

\[
I_{yz} = I_{zy} = \int_A yz dA
\]

and the curvature by:

\[
\kappa_y(x) = -\frac{d^2 w_y(x)}{dx^2}
\]

\[
\kappa_z(x) = -\frac{d^2 w_z(x)}{dx^2}
\]

Inversion of this matrix gives the general equations for bending:

\[
\begin{bmatrix}
\kappa_y(x) \\
\kappa_z(x)
\end{bmatrix} = \frac{1}{E(I_{yy}I_{zz} - I_{yz}I_{zy})}
\begin{bmatrix}
I_{zz} & -I_{yz} \\
-I_{zy} & I_{yy}
\end{bmatrix}

\begin{bmatrix}
M_y(x) \\
M_z(x)
\end{bmatrix}
\]

The neutral axis of the cross-section is the line of zero stress and divides the cross-section in a part with tensile stresses and a part with compressive stresses. The angle $\beta(x)$ that the neutral axis forms with the y-axis, see Figure 3, is defined by the relation:
\[
\tan \beta(x) = -\frac{\kappa_y(x)}{\kappa_z(x)}
\]

Substituting this relation in the constitutive relation gives the following equation:

\[
\tan \beta(x) = \frac{-I_{z y} M_y(x) + I_{z z} M_z(x)}{-I_{y z} M_y(x) + I_{y y} M_z(x)}
\]

In this way the rotation of the neutral axis can be determined if the geometrical properties of the profile and the applied load are known. The rotation of the neutral axis differs for every specific point along the x-axis of the profile. Depending upon the magnitudes and directions of the bending moments, the angle \( \beta \) may vary from -90° to +90°.

![Diagram of neutral axis and bending moments](image)

**Figure 3:** Definition of the angles \( \alpha \), \( \beta \) and \( \gamma \).

The transverse (in plane) bending moment \( M_y(x) \) and the lateral (out of plane) bending moment \( M_z(x) \) are components of the resultant moment vector \( \mathbf{M} \). This vector forms an angle \( \gamma(x) \) with the z-axis (see Figure 3). Therefore \( \tan \gamma(x) \) is defined as the ratio:

\[
\tan \gamma(x) = -\frac{M_y(x)}{M_z(x)}
\]

The direction to which the vector \( \mathbf{M} \) is pointing, is the area where tension stresses occur. This means that a positive moment \( M_z \) produces compression (negative stresses) on the left-hand side of the cross-section and tension (positive stresses) on the right-hand side. Similarly, a positive moment \( M_y \) produces compression on the bottom and tension at the top, see also Figure 2.

Using Mohr’s circle, we can derive the principal axes for the double U-profile and the angle \( \alpha \). This is the angle between the chosen coordinate system (\( y \), \( z \)), and the coordinate system of the principal axes (\( y^\ast \), \( z^\ast \)). In Figure 4 the principal axes of the double U-profile are determined.
The $y^*$-axis is the principal direction yielding the maximum moment of inertia $I_1$. This is therefore the strongest axis of the profile. The $z^*$-axis is the principal direction yielding the minimum moment of inertia $I_2$ and is consequently the weakest axis of the profile. From Figure 4 also follows:

$$\tan(2\alpha) = \frac{I_{yx}}{\frac{1}{2}(I_{yy} - I_{zz})}$$

Now we have defined three different angles:
- $\alpha$ = angle of rotation of principal directions
  This angle only depends on the geometry of the profile.
- $\beta$ = angle of rotation of the neutral axis
  This angle depends on both geometrical properties and the loading conditions.
- $\gamma$ = angle of rotation of resultant moment vector $\mathbf{M}$ (= load angle)
  This angle only depends on the applied loads.

2.2 Material properties of modeled double PU8 profile
The double U-profile used in this study is the double PU8 profile from Profil Arbed (lit[7]). This profile has been simplified in order to obtain a workable 3D sheet pile model (lit[1]). As a consequence, the stiffness parameters of the modeled profile differ somewhat from the parameters which can be calculated from the geometrical dimensions of the real profile as given by the sheet pile supplier.

There are two different stiffness properties:
1) Stiffness given by the supplier of the sheet pile.
2) Stiffness that has been calculated from the geometry of the modeled sheet pile.

The different stiffness properties for the double PU8 profile are given in table 1.
Table 1: Two different stiffness properties for the double PU8 profile.

<table>
<thead>
<tr>
<th>Supplier</th>
<th>$I_{zz}$</th>
<th>$I_{yy}$</th>
<th>$I_{yz}$</th>
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<tr>
<td>Supplier</td>
<td>1.3953-10^4 m^4</td>
<td>1.83100-10^3 m^4</td>
<td>-3.3986-10^4 m^4</td>
</tr>
<tr>
<td>Modeled pile</td>
<td>1.3953-10^4 m^4</td>
<td>1.83143-10^3 m^4</td>
<td>-3.5990-10^4 m^4</td>
</tr>
</tbody>
</table>

The stiffness properties of the modeled sheet pile have been used in the analytical calculations.

Note that the moment of inertia $I_{yz}$ is negative in this case because the cross-section of the profile is only present in the negative second and fourth quadrant of the yz-plane (see Figure 3). Thus $I_{yz}$ is only negative because of the chosen coordinate system.

2.3 Determination of influence of loading conditions

It is interesting to see what the relation is between the angles $\alpha$, $\beta$ and the load angle $\gamma$. With this relation we can see which is the effect of the loading conditions on the rotation of the neutral axis. The rotation of the neutral axis is of great importance in order to derive the reduction factor $R_w$ for the section modulus.

In subjoined figure for different loading conditions ($\tan \gamma$), the value for the rotation of the neutral axis ($\tan \beta$) is given. Also the angle of the principal axes ($\tan \alpha$) is pictured.

![Figure 5: Influence of loading condition (tan $\gamma$) on the rotation of the neutral axis (tan $\beta$) for a double PU8 profile.](image)

It can be observed that the transverse bending moment rotates the neutral axis back to the axis of a continuous sheet pile wall. There are three significant points (A, B and C) in this figure, in which different phenomena play an important role. These three points will be discussed hereafter.

- **(A) Oblique bending fully impeded**

If no oblique bending occurs, the angle of rotation $\beta(x)$ of the neutral axis must be zero in every point along the x-axis. This is only possible when:

$$\tan \beta(x) = \frac{-I_{yy} M_y(x) + I_{yz} M_z(x)}{-I_{yy} M_y(x) + I_{yz} M_z(x)} = 0 \Rightarrow M_y(x) = \frac{I_{yz}}{I_{yy}} M_z(x) = -2.58 \cdot M_z(x)$$

So in order to fully impede oblique bending, a transverse bending moment $M_y(x)$ is necessary which is a
factor $I_y/I_z = -2.58$ bigger than the lateral bending moment $M_z(x)$ in every cross-section of the profile, see also lit[8]. Note that this value only applies for this particular profile.

![Figure 6: Situation in case oblique bending is fully impeded.](image)

- **(B) Free oblique bending**

If free oblique bending is occurring, no transverse moment $M_y(x)$ is working on the sheet pile. Thus the load angle $\gamma(x)$ is zero. In this case the rotation of the neutral axis is:

$$\tan \beta(x) = \frac{I_{yy}}{I_{yy}} = -0.1965 \quad \Rightarrow \quad \beta(x) = -11.17^\circ$$

So if free oblique bending occurs, the rotation of the neutral axis is equal for every cross-section along the axis of the profile. The rotation of the neutral axis $\beta$ is somewhat smaller than the angle of the principal directions $\alpha$.

![Figure 7: Situation of free oblique bending.](image)

- **(C) Neutral axis is perpendicular to the resultant moment vector $M$**

It can be found from Figure 8 that this occurs when:

$$\tan \gamma(x) = \tan \beta(x) \quad \Rightarrow \quad M_y(x) = -\tan \left( \frac{1}{2} \arctan \left( \frac{I_{yy}}{\frac{1}{2}(I_{yy} - I_{xx})} \right) \right) M_z(x) = 0.204 \cdot M_z(x)$$

Figure 8 shows that if $\beta = \gamma$, automatically $\gamma = \alpha$ applies. So if the neutral axis is perpendicular to the loading vector $M$, the direction of this vector has to correspond with one of the principal directions.
Figure 8: Situation when neutral axis is perpendicular to the resultant moment vector $M$.

From Figure 5 and the three significant points in this figure it can be concluded that the loading conditions of the profile have a great influence on the rotation of the neutral axis. By altering the ratio of the transverse and lateral bending moment from $-\infty$ to $+\infty$, in theory every rotation angle of the neutral axis is possible. In the next paragraph the influence of the loading conditions on the reduction factors for the moment of inertia and the section modulus is derived.

2.4 Derivation of reduction factor for moment of inertia $I$

When a rotation of the neutral axis occurs, this is an indication for oblique bending. However, the magnitude of this rotation is not an indication for the reduction factor $R_I$ of the moment of inertia. In order to derive the reduction factor $R_I$ for the moment of inertia, the maximum out of plane displacement $w_z$ at the top of the sheet pile has to be compared for one and two directional bending (= oblique bending).

- **General deflection curve for one directional bending**

In case of one directional bending the following general equation is applicable:

$$\kappa_z(x) = \frac{M_z(x)}{EI_{zz}}$$

The general deflection curve is obtained by a double integration of this equation.

$$w_z(x) = -\frac{1}{EI_{zz}} \int \int M_z(x) \, dx^2 + C_1x + C_2$$

Herein, $C_1$ and $C_2$ are constants of integration for the slope of the sheet pile and for the deflection, respectively.

- **General deflection curve for two directional bending**

In case oblique bending occurs, the following general equation for bending is applicable:

$$\kappa_z(x) = \frac{1}{E(I_{yy}I_{zz} - I_{yx}I_{zy})} \left[ -I_{yx} M_y(x) + I_{yy} M_z(x) \right]$$

Via double integration of this equation, we get the general deflection curve $w_z(x)$.

$$w_z(x) = -\int \int \kappa_z(x) \, dx^2 = -\frac{1}{E(I_{yy}I_{zz} - I_{yx}I_{zy})} \left[ -I_{yx} \int \int M_y(x) \, dx^2 + I_{yy} \int \int M_z(x) \, dx^2 \right] + C_3x + C_4$$
Again, $C_3$ and $C_4$ are constants of integration for the slope of the sheet pile and for the deflection, respectively. With the ratio:

$$R_y = -\int \int M_y(x) dx^2 \int \int M_z(x) dx^2$$

the formula for the general deflection curve becomes:

$$w_x(x) = -\frac{I_{yy} R_y + I_{yy}}{E(I_{yy} I_{zz} - I_{yy} I_{yy})} \int \int M_y(x) dx^2 + C_3 x + C_4 = -\frac{1}{E I_{zz}} \int \int M_z(x) dx^2 + C_3 x + C_4$$

in which $I_{zz}^*$ is the apparent moment of inertia of the sheet pile defined by:

$$I_{zz}^* = \frac{I_{yy} I_{zz} - I_{yy} I_{yy}}{I_{yy} R_y + I_{yy}}$$

The apparent moment of inertia $I_{zz}^*$ is defined as the moment of inertia that must be applied in a 2D calculation in order to get the same maximum displacement at the top of the sheet pile as in a 3D calculation. Thus by using the apparent moment of inertia, the effects that occur in a 3D calculation can be taken into account in a 2D calculation.

Normally, the moment of inertia is a material property and only depends on the geometry of the cross-section. But in case of the apparent moment of inertia, it also depends on the loading condition.

The general deflection curve for two directional bending has to be compared with the general deflection curve for one directional bending to derive the reduction factor $R_l$ for the moment of inertia. It is assumed that the boundary conditions at the beginning of the sheet pile ($x = 0$) are equal for both one and two directional bending. This implies that the displacement and the slope of the sheet pile wall at $x = 0$ are equal thus $C_1 = C_3$ and $C_2 = C_4$.

In case of a cantilever sheet pile wall, the deflection and the slope of the sheet pile wall at $x = 0$ are zero, see Figure 9. Thus we obtain:

$$\phi_y(0) = 0 \Rightarrow C_1 = C_3 = 0$$

$$w_x(0) = 0 \Rightarrow C_2 = C_4 = 0$$

Because the displacement $w_x(x = l)$ at the end of the sheet pile wall is compared for one- and two directional bending, the double integral from the beginning ($x = 0$) to the end ($x = l$) of the sheet pile wall has to be taken. The reduction factor $R_l$ for the moment of inertia is defined as the ratio of the apparent moment of inertia $I_{zz}^*$ and the full moment of inertia $I_{zz}$ of a continuous sheet pile wall (highest stiffness).
The reduction factor \( R_l \) depends on the distribution of the transverse- and lateral bending moments over the sheet pile. For different loading conditions, the reduction factor \( R_l \) has been calculated. The results are given in Figure 10.

It can be observed that the reduction factor \( R_l \) for the moment of inertia is equal to 1 (= full moment of inertia) if the factor \( R_y = -I_{yy}/I_{zz} = 2.58 \). This is the upper limit moment of inertia. If there is no transverse bending moment \( M_y(x) \) available to reduce oblique bending, a situation of unrestrained oblique bending occurs. In this case the factor \( R_y = 0 \) and the reduction factor \( R_l \) for the moment of inertia is 0.49. This is the lower limit moment of inertia.

The reduction factor \( R_l \) depends on the double integral of the bending moments \( M_y(x) \) and \( M_z(x) \). This implies that the reduction factor \( R_l \) depends on the distribution of the bending moments \( M_y(x) \) and \( M_z(x) \) from the bottom of the sheet pile \( (x = 0) \) up to the top of the sheet pile \( (x = l) \).

2.5 Derivation of reduction factor for section modulus \( W \)

In contrast with the reduction factor \( R_l \) for the moment of inertia, the reduction factor \( R_W \) for the section modulus \( W \) only depends on the bending moments in one particular point along the sheet pile. So if we want to calculate the reduction factor \( R_W \) of the section modulus in point \( x = a \), we only need to know the bending moments \( M_y(a) \) and \( M_z(a) \) in point \( x = a \).

With the section modulus, the stresses in a cross-section of the sheet pile due to bending moments can be calculated. By comparing the occurring maximum stress in a cross-section for one- and two directional bending, we can calculate the reduction factor \( R_W \) for the section modulus.
• **Maximum stress in cross-section for two directional bending**

In order to obtain the stresses in a cross-section of the sheet pile, the bending moments $M_x$ and $M_z$ have to be converted to bending moments $M_1$ and $M_2$ working in the principal directions $y^*$ and $z^*$ of the profile (Figure 11).

$$M_1 = M_y \cos \alpha + M_z \sin \alpha$$
$$M_2 = -M_y \sin \alpha + M_z \cos \alpha$$

![Figure 11: Situation of a cross-section in two directional bending.](image)

The maximum stress in the cross-section for two directional bending can be found with the equation:

$$\sigma_{\text{max},yz} = \frac{M_1 y^*}{I_1} + \frac{M_2 z^*}{I_2}$$

in which $y^*$ and $z^*$ are the coordinates of the ultimate fibre with respect to the neutral axis, see Figure 11. Furthermore, the $y^*$-axis is the principal direction yielding the maximum moment of inertia $I_1$. The $z^*$-axis is the principal direction yielding the minimum moment of inertia $I_2$.

• **Maximum stress in cross-section for one directional bending**

In case of one directional bending, there is no rotation of the neutral axis. The maximum stress can be found with:

$$\sigma_{\text{max},z} = \frac{M_z}{W_{\text{ref}}} = \frac{M_z h}{I_{zz}}$$

Herein is $h$ the height of one PU8 profile. This is the distance of the ultimate fibre with respect to the neutral axis. In this fibre the maximum stress occurs. $W_{\text{ref}}$ is the section modulus of a continuous sheet pile wall (= highest strength).

When two directional bending (= oblique bending) occurs, the stresses in the cross-section are bigger than when only one directional bending occurs due to the rotation of the neutral axis. This of course with equal loading conditions. By applying a reduction factor $R_w$ to the reference section modulus $W_{\text{ref}}$ for one directional bending, we are able to calculate the stresses for two directional bending for different loading conditions. To obtain this reduction factor, the maximum stress in case of one directional bending $\sigma_{\text{max},z}$ with a reduction factor $R_w$ for the section modulus has to be equal to the maximum stress $\sigma_{\text{max},yz}$ in case of two directional bending.
\[ \sigma_{\text{max,}r} = \sigma_{\text{max,}yz} \quad \Rightarrow \quad \frac{M_y h}{I_{\text{ref}} R_w} = \frac{M_{1,y}''}{I_1} + \frac{M_{2,z}''}{I_2} \]

\[ R_w = \frac{\sigma_{\text{max,}r}}{\sigma_{\text{max,}yz}} = \frac{M_y h}{I_{zz}} \frac{I_2}{M_{1,y}''} + \frac{M_{2,z}''}{I_2} \]

For different loading conditions (\( \tan \gamma \)), the reduction factor for the section modulus has been calculated. The results are shown in Figure 12.

![Figure 12: Influence of the loading condition \( \tan \gamma \) on the reduction factor \( R_w \).](image)

The kink shown by the graph of \( R_w \), is caused by the shift of the point were the maximum stress occurs. For \( \tan \gamma \geq 0.189 \) the maximum stress occurs in the corner of the flange (point 1), see Figure 11. For \( \tan \gamma \leq 0.189 \) the maximum stress occurs in the free interlock (point 2). It can be observed that the reduction factor \( R_w \) for the section modulus is equal to 1 (= reference section modulus) if \( \tan \gamma = -I_{yz}/I_{zz} = 2.58 \). If there is no transverse bending moment \( M_y \) available to reduce oblique bending, a situation of unrestrained oblique bending occurs. In this case \( \tan \gamma = 0 \) and the reduction factor \( R_w \) for the section modulus is 0.59. This is the lower limit section modulus.

The reduction factor \( R_w \) depends on the ratio of the bending moments \( M_y \) and \( M_z \). This implies that only the bending moments \( M_y \) and \( M_z \) in a cross-section need to be known in order to derive the reduction factor for the section modulus. This is a result of the direct relation between the stresses in a cross-section and the bending moments.

### 2.6 Maximum admissible bending moment in sheet pile

The reduction factor \( R_w \) for the section modulus depends on the loading conditions. In the worst case, of unrestrained oblique bending for a double PU8 profile with free interlocks, the reduction factor \( R_w \) is 0.59 (see Figure 12). So the section modulus in this worst case scenario becomes \( W = R_w W_{\text{ref}} \). Where:

\[ W_{\text{ref}} = \frac{I_{zz}}{h} = \frac{13953 \cdot 10^{-4}}{0.14} = 9.9664 \cdot 10^{-4} \text{ kNm/12 m} \]
In order to ensure the strains in the sheet pile stay elastic, the applied moment on the sheet pile has to be restricted. When a high steel grade is used, the admissible stress in the sheet pile is about 375 N/mm². The maximum admissible moment corresponding with this maximum stress is then:

\[ M_{z,\text{max}} = f_y \cdot R_w \cdot W_{\text{ref}} = 375 \cdot 10^3 \text{ N/mm}^2 \cdot 0.59 \cdot 9.9664 \cdot 10^{-4} \text{ m}^3 / 1.2 \text{ m} = 221 \text{ kNm/1.2 m} \]

2.7 Evaluation

In this chapter the general reduction factors for the moment of inertia \( R_I \) and the section modulus \( R_w \) are derived which take into account the phenomenon oblique bending. Depending on the loading conditions, the reduction factor \( R_I \) for the modeled double PU8 profile varies from 0.49 to 1. The reduction factor \( R_w \) varies from 0.59 to 1.

The reduction factor \( R_I \) depends on the bending moments \( M_y(x) \) and \( M_z(x) \) along the entire sheet pile (from top to bottom). In contrast with the reduction factor \( R_I \), the reduction factor \( R_w \) only depends on the ratio of the bending moments \( M_y \) and \( M_z \) in a cross-section. In order to obtain a deeper understanding of the derivation of the reduction factors two examples are worked out. In annex B an example of a cantilever sheet pile is given with equal loading conditions in both transverse and lateral direction. In annex C an example is given of the same sheet pile but now with different loading conditions in transverse and lateral direction. In order to derive the reduction factors \( R_I \) and \( R_w \) for these examples, use is made of the theory presented in this chapter.

From these examples it can be observed that the reduction factor \( R_w \) for the section modulus \( W \), can be derived directly at the cross-section of the sheet pile where the maximum value of the lateral bending moment \( M_z \) occurs. Because the stresses in a cross-section are directly related to the bending moments, only the bending moments \( M_y \) and \( M_z \) in this cross-section need to be known in order to derive the reduction factor for the section modulus. Furthermore it can be found from the examples that the rotation of the neutral axis is only constant along the longitudinal axis of a sheet pile if the loading conditions in the lateral and transverse direction are congruent. This means that the shape of the bending moments \( M_y(x) \) and \( M_z(x) \) must be equal.
3 Calculation of shear force in the interlock using a differential equation

In this chapter several analytical calculations concerning oblique bending in double U-profiles are made. Emphasis is put on modeling the shear force in the sliding interlocks. The results of the analytical solutions are evaluated by 3D-finite element computations.

3.1 General deflection equation for a simply supported double U-profile

In this paragraph the deflection formulae for a simply supported sheet pile (one double PU8 profile) with a uniformly distributed load are derived. Use is made of the coordinate system as given in Figure 13.

The sheet pile has a length \( l \) of 9 m. The uniformly distributed load \( q_z \) produces a bending moment \( M_z \) and an out of plane displacement \( w_z \). Because free oblique bending is occurring in this case, there is also an in plane displacement \( w_y \). It can be found from the general formulae for two directional bending that the equation for the deflection curve in this particular case is:

\[
M_z = \frac{1}{12} q_z b \left[ x^4 - 2lx^3 + l^3 x \right]
\]

\[
w_z(x) = \frac{I_{yz}}{E(I_{yy}I_{zz} - I_{yz}I_{xy})} \frac{1}{12} q_z b \left[ x^4 - 2lx^3 + l^3 x \right]
\]

\[
w_y(x) = -\frac{I_{xy}}{E(I_{yy}I_{zz} - I_{yz}I_{xy})} \frac{1}{12} q_z b \left[ x^4 - 2lx^3 + l^3 x \right]
\]

The stiffness properties used in these formulae are the ones for a modeled sheet pile as given in table 1. In order to impede the in plane displacement \( w_y \), an additional bending moment \( M_y \) is required. This in plane bending moment can be provided by a shear force \( t \) which works along both free interlocks, see Figure 13. In the following paragraph an expression will be presented where the shear force \( t \) is coupled to the in plane displacement \( w_y \).
3.2 Derivation of the differential equation for shear force

To schematise the sheet pile, an element with length $dx$ has been taken and analysed as shown in Figure 14. All bending moments and the distributed shear force are drawn positive in this figure.

![Figure 14: Element of sheet pile with acting forces.](image)

The shear force $t$ on the top and bottom face of the element can be schematised as a distributed moment $m_y$.

- **distributed moment**: $m_y = 2bt$
- **moment equilibrium**: $-M_y + m_y \, dx + M_y + dM_y = 0 \Rightarrow \frac{dM_y}{dx} = -m_y$
- **constitutive equation**: $M_y = -EI_y \frac{d^2w_y}{dx^2}$

From the moment equilibrium and the constitutive equation we obtain the general differential equation of the shear force:

$$EI_y \frac{d^3w_y}{dx^3} = m_y = 2bt$$

Because the profile of the double U-profile has an asymmetric cross-section and therefore a product of inertia $I_{yz}$, the differential equation becomes:

$$\frac{E(I_{yy}I_{zz} - I_{yz}I_{zy})}{I_{zz}} \frac{d^3w_y}{dx^3} = m_y = 2bt$$

3.3 Reduction of oblique bending by a linear distributed shear force

The equation for the in plane deflection curve $w_y(x)$ of the sheet pile, produced by the uniformly distributed load $q_z$ is:

$$w_y(x) = -\frac{I_{yz}}{E(I_{yy}I_{zz} - I_{yz}I_{zy})} \frac{1}{12} q_z b \left[ x^4 - 2lx^3 + l^3 x \right]$$

If we substitute this deflection curve in the differential equation derived in the previous paragraph, we obtain a distributed shear force $t_q(x)$. This shear force can be seen as a load acting on the sheet pile in the transverse y-direction.

$$\frac{E(I_{yy}I_{zz} - I_{yz}I_{zy})}{I_{zz}} \frac{d^3w_y(x)}{dx^3} = 2bt_q(x)$$
\[-\frac{I_{yz}}{I_{zz}} q_z b [2x - l] = 2bt_q(x) \quad \Rightarrow \quad t_q(x) = -\frac{I_{yz}}{I_{zz}} q_z \left[ x - \frac{l}{2} \right] \]

In order to impede the in plane deformation, a distributed shear force \( t(x) \) is needed that is opposite to the shear force \( t_q(x) \) caused by the distributed load. In this way, the deflections caused by the distributed load \( q_z \) are neutralised and no in plane deformations occur.

\[ t_q(x) = -t(x) \quad \Rightarrow \quad t(x) = \frac{I_{yz}}{I_{zz}} q_z \left[ x - \frac{l}{2} \right] \]

Thus in order to fully impede oblique bending of a simply supported sheet pile with uniformly distributed load \( q_z \), the sheet pile has to be loaded by a linear distributed shear force \( t(x) \) as depicted in Figure 15.

\[ \frac{I_{yz}}{I_{zz}} q_z \left[ x - \frac{l}{2} \right] \]

Figure 15: Distribution of shear force \( t(x) \) in the free interlock.

From this figure it can be observed that the shear force has a maximum value at both ends of the sheet pile and decreases to zero in the middle. The shear force is positive on the left half of the sheet pile and negative on the right half. By integrating the formula for the shear force, we obtain the in plane bending moment \( M_y(x) \) that is needed to impede oblique bending.

\[ M_y(x) = \int \frac{I_{yz}}{I_{zz}} q_z \left[ x - \frac{l}{2} \right] dx = \frac{I_{yz}}{I_{zz}} q_z \left[ \frac{1}{2} x^2 - \frac{1}{2} lx \right] + C_1 \]

The boundary condition at the support is \( M_y(0) = 0 \), which implies that \( C_1 = 0 \). Thus the equation for the in plane bending moment \( M_y(x) \) is:

\[ M_y(x) = \frac{I_{yz}}{I_{zz}} q_z \left[ x^2 - lx \right] = \frac{I_{yz}}{I_{zz}} M_z(x) \]

This is exactly the same formula as derived in the previous chapter. This has been tested in the 3D model of the sheet pile. The results are presented in the following table.
Table 2: Shear force needed to impede oblique bending.

<table>
<thead>
<tr>
<th></th>
<th>( t(x=0) ) [kN/m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analytical</td>
<td>116.07</td>
</tr>
<tr>
<td>Numerical</td>
<td>116.73</td>
</tr>
</tbody>
</table>

The analytical and numerical solution differ only 0.6%. So this theory corresponds with the values which were found numerically.

3.4 Reduction of oblique bending by springs in the free interlock

By placing springs in the free sliding interlock of the sheet pile, a distributed shear force \( t_{spring}(x) \) is simulated. The springs have a constant spring stiffness \( k \) and can only deform in the x direction. The distributed shear force only exists if there is a displacement of the sheet pile, as shown in Figure 16. A displacement of the sheet pile results in a slip \( u(x) \) in the sliding interlock. Because the springs are only mobilised if they are deformed in axial direction, a certain displacement of the sheet pile is necessary.

Because a certain displacement is necessary to mobilise the springs, it is evident that the springs in the interlock never can impede oblique bending fully. Only when \( k = \infty \), no oblique bending occurs. In this case of \( k = \infty \), a fixed interlock by welding or crimping is modeled.

![Figure 16: Principle of springs in the free interlock.](image)

From the geometry of Figure 16, we obtain the following relationship between the rotation \( \varphi_2(x) \) and the slip in interlock \( u(x) \).

\[
\begin{align*}
\varphi_2(x) &= \frac{dw_y(x)}{dx} \\
\varphi_1(x) &= \frac{-u(x)}{2b} \\
u(x) &= -2b \frac{dw_y(x)}{dx}
\end{align*}
\]

The distributed shear force \( t_{spring}(x) \) acting on the sheet pile produced by the springs, can be found by introducing the distributed spring stiffness \( k \) [N/m²]:

\[
t_{spring}(x) = -ku(x) = 2bk \frac{dw_y(x)}{dx} \quad \text{[\%/m]}
\]

Like in paragraph 3.3, we can assume the distributed shear force \( t_d(x) \) caused by the distributed load \( q_x \), as a load acting on the sheet pile. The shear force \( t_{spring}(x) \) as derived above is the phenomenon which opposes this load. So we can write for the differential equation of the shear force:
Substituting the expressions for the shear forces $t_q(x)$ and $t_{spring}(x)$ yields:

$$\frac{E(I_{yy}I_{zz} - I_{yz}I_{yz})}{I_{zz}} \frac{d^3 w_y(x)}{dx^3} = 2b\left[ t_q(x) + t_{spring}(x) \right]$$

If we solve this equation for $w_y$ in $x$, we get the following function:

$$w_y(x) = -\frac{A}{C} x - \frac{B}{2C} x^2 - \frac{B}{C^2} e^{\sqrt{G}x} - \frac{ACl + \frac{1}{2} BCl^2 + B - Be^{\sqrt{G}l}}{C^2\left(e^{\sqrt{G}l} - e^{-\sqrt{G}l}\right)} \left[e^{-\sqrt{G}x} - e^{\sqrt{G}x}\right]$$

with:

$$A = \frac{blq_z I_{yz}}{E(I_{yy}I_{zz} - I_{yz}I_{yz})}, \quad B = \frac{-2bq_z I_{yz}}{E(I_{yy}I_{zz} - I_{yz}I_{yz})}, \quad C = \frac{4b^2 k l_{zz}}{E(I_{yy}I_{zz} - I_{yz}I_{yz})}$$

For the derivation of this function, see annex A.

If the distributed spring stiffness $k = 0$, free oblique bending occurs with an in plane deflection curve $w_y(x)$ as given in paragraph 3.3. If $k = \infty$ the in plane deflection $w_y$ of the sheet pile is zero. In this case, as mentioned before, no oblique bending will occur. With the use of the analytical formula for the in plane deflection with springs in the interlock, in an iterative way the distributed spring stiffness $k_{ref}$ has been found where the in plane deflection at mid span ($x = l/2 - l$) is half the value of the deflection in the situation of free oblique bending $w_y(l/2)_{free}$. This has been done in order to determine the influence of the spring stiffness on the in plane deflection. The reference distributed spring stiffness $k_{ref}$ turned out to be $1.5959 \times 10^7$ N/m². This is very high stiffness.

For different spring stiffnesses $k_{spring}$, the corresponding deflection $w_y(l/2)$ at mid span has been determined. This is done analytically as well as numerically.

![Figure 17: The transverse displacement $w_y$ at mid span as a function of the spring stiffness $k_{spring}$](image)

From Figure 17 it can be found that the in plane displacement is decreasing rapidly for values up to 4 to
6 times the reference value for the spring stiffness. After this, the displacements are not decreasing very fast. Furthermore it can be noticed the spring stiffness $k_{spring}$ has to go to infinity to completely impede the in plane displacement at mid span (modeling of fixed interlock).

Also it can be observed that the numerical solution with 10 springs is closer to the analytical solution than the numerical solution with 20 springs.
4 Finite element modeling

In order to study the phenomenon oblique bending of a steel sheet pile wall in combination with the interaction of the soil, a 3D test case has been designed by Aukema and Joling (lit[1]). This test case consists of an excavation of a soil body (sand) in front of a cantilever steel sheet pile wall. The finite element computations have been carried out, using the finite element package DIANA.

4.1 Dimensions of model

In order to obtain a good model, the dimensions of the soil body are such that the boundaries of the model have no effect on the behaviour of the sheet pile wall and vice versa. Thus if the boundaries would have been chosen further away from the sheet pile wall, the behaviour of the sheet pile wall would not have changed. In Figure 18, the dimensions are given of the problem that is analysed.

All calculations in this report are made without a resulting water pressure on the sheet pile. The effective soil stresses are therefore bigger than with a resulting water pressure. This might have a positive effect on the transverse bearing capacity of the soil, i.e. the soil reduces the transverse deformation of the sheet pile. Furthermore a perfectly rough sheet pile ($\delta = \phi$) is modeled. This implies equal displacements in all three directions for adjacent soil and sheet pile nodes. In this way the soil is attached to the sheet pile so that no slip can occur between the sheet pile and the soil body.

By using a perfectly rough sheet pile wall with no resulting water pressure on it, a ‘best case’ model is made. With the use of this ‘best case’ model, a prediction can be made of what is the best possible result that can be obtained by applying some kind of method to reduce oblique bending.
### 4.2 Boundary conditions of model

The boundary conditions of the model are presented in Figure 19. The soil body at the front (= left) and back (= right) plane of the model is able to slide up- or downwards in axial direction. In lateral and transverse direction the soil body is supported. At the bottom plane the soil body is supported in all three directions. Furthermore a distributed load is present on top of the soil body.

*Figure 19: Boundary conditions of the model.*

It is assumed that the sheet pile wall has an infinite length in transverse direction. In this way the effects at the end of the sheet pile wall are neglected, which allows to assume a periodic behaviour. One period takes up the width of one double U-profile. Because of simplicity of the finite element model, a volume between the fixed interlock is chosen. In Figure 20 the modeled periodic sheet pile volume is depicted with the applied boundary conditions.

*Figure 20: Equal boundary conditions on both sides of the periodic volume to model an infinitely long sheet pile wall.*

The boundary conditions at both ends of the sheet pile are equal. This implies that the displacements $u$, $v$, $w$ and rotation $\theta$ are equal. For further boundary conditions, reference is made to Aukema and Joling, lit[1].
4.3 Modeling the soil
The applied material properties for the sand are:

- Young’s modulus: \( E = 20 \text{ MPa} \)
- Poisson’s ratio: \( \nu = 0.33 \)
- Density (dry): \( \gamma = 18 \text{ kN/m}^3 \)
- Friction angle: \( \phi = 35^\circ \)
- Dilatancy angle: \( \psi = 5^\circ \)
- Cohesion: \( c = 1 \text{ kPa} \)

These material properties are realistic values for middle-dense compacted sand (see also lit[9]). As mentioned before, all calculations have been done without a resulting water pressure.

For modeling sands in the deviatoric plane, various possibilities exist. The oldest way of modeling sands is the Mohr-Coulomb criterion, which has proven to be an accurate description of triaxial tests on sands. Unfortunately, in a finite element framework the stress paths are not known in advance and certainly not constant as with triaxial tests. Implementation of the Mohr-Coulomb criterion in a fully three-dimensional continuum poses difficulties which adheres to the multisurface character of the Mohr-Coulomb yield criterion. These problems can be avoided using a model with a continuous yield surface. Therefore the van Eekelen model has been used (lit[5]). With this relatively simple model accurate predictions of the behaviour of sands can be obtained.

4.4 Modeling the sheet pile
The sheet pile used in this study is a simplified version of the PU8 profile from Profil Arbed (lit[7]). This profile is simplified in order to obtain a workable 3D sheet pile model. Simplification of the profile is necessary because the soil elements must follow the shape of the sheet pile. A well fitting model shape of the sheet pile would result in too many small adjacent soil elements. The modeled sheet pile consists of eight shell elements with variable thickness in order to obtain elastic properties similar to that of the PU8 profile. The modeled sheet pile is depicted in Figure 21.

![Figure 21: Dimensions of the modeled cross-section of the double PU8 profile.](image)

The material properties of the simplified sheet pile model are already given in paragraph 2.2. The elastic parameters for the sheet pile are:

- Young’s modulus: \( E = 2.1 \times 10^{11} \text{ N/m}^2 \)
- Poisson’s ratio: \( \nu = 0.3 \)

Furthermore it is assumed that the sheet pile is behaving fully elastic during all calculations. In order to ensure no extreme stresses will occur, the maximum admissible bending moment in the sheet pile of 221 kNm/1.2 m as derived in paragraph 2.6 will not be exceeded.

4.5 Finite element mesh
The soil mass is modeled with twenty-noded brick elements. Use is made of eight-noded shell elements to model the sheet pile. In subjoined figure, the used meshes of the soil mass and the sheet pile are depicted.
The excavation depth in front of the sheet pile is divided in excavation steps of 0.9 m each. With every excavation step a row of soil elements with a height of 0.9 m is removed. From Figure 22 it can be noticed the soil elements are smaller near the sheet pile. This is done because we are interested in the interaction of the sheet pile wall with the surrounding soil. By using small elements, a more accurate model obtained in this area.

4.6 Calculation of bending moments and displacements
The bending moments $M_y$ and $M_z$ are calculated by integrating all normal stresses $\sigma_{xx}$ over the cross-section $A$ of the sheet pile at a certain point, multiplied by the distance $y^*$ and $z^*$ respectively (see also Figure 23). Again, the origin of coordinates is situated in the fixed interlock of the double U-profile.

$$M_y = \int_A \sigma_{xx} y^* dA$$

$$M_z = \int_A \sigma_{xx} z^* dA$$

Because all stresses over the cross-section of the double PU8 profile are taken, a bending moment is obtained that acts per width $2b (= 1.2 \ m)$. Therefore all bending moments are presented as [kNm/1.2m].

Also the definition of the bending moments and rotations is given here. The moments shown in Figure 24 are drawn positive. So due to a positive bending moment $M_y$, there will be tension (positive stresses) along the positive $z$-axis and compression (negative stresses) along the negative $z$-axis. Similarly, a positive bending moment $M_z$ produces tension along the positive $y$-axis and compression along the negative $y$-axis.
Figure 24: Definition of positive bending moments and rotations.

A positive rotation $\varphi_y$ is defined as a rotation around the y-axis from the z-axis towards the x-axis. The other rotations are found in a similar way.

The displacements of the sheet pile which will be presented in the next paragraphs, are derived by taking the average displacements of all eighteen nodes in a cross-section of the sheet pile.
5 Calculations of methods to reduce oblique bending

In this chapter the results of the 3D calculations performed with DIANA are presented. For every method to reduce oblique bending, the bending moments, the displacements and the interlock slip are presented.

The lateral displacements calculated with the 3D model are compared with the results from the 2D model. This 2D model is built up the same way as the side view of the 3D model (the 2D model is a ‘slice’ from the 3D model). In this 2D model there are two limiting cases to model the moment of inertia of the sheet pile wall:

- A situation in which it is assumed that oblique bending is unrestrained. This means that there is no transverse bending moment \( M_y \) available to reduce oblique bending. Hence the smallest reduction factor \( R_I \) for the moment of inertia must be used, as pictured in Figure 10. The apparent moment of inertia \( I_{zz}^* \) that must be applied in the 2D calculation is therefore the lower limit moment of inertia.

\[
I_{zz}^* = R_I I_{zz} = \frac{(I_{yy} I_{zz} - I_{yz}^2)}{I_{yy}}
\]

- A situation in which it is assumed that oblique bending in the steel sheet pile wall is fully impeded and hence the full moment of inertia of a continuous sheet pile wall is used. The reduction factor \( R_I \) is therefore equal to one. The apparent moment of inertia \( I_{zz}^* \) that must be applied in the 2D calculation is equal to the full moment of inertia and is therefore the upper limit moment of inertia.

\[
I_{zz}^* = R_I I_{zz} = I_{zz}
\]

In order to justify the assumption that these 2D calculations are the upper- and lower limit behaviour of the 3D calculations, the lateral deflection of the 3D calculation with fixed interlocks (full moment of inertia) should be the same as the lateral deflection of the upper limit 2D calculation. This assumption appeared to be correct within an accuracy of 5%.

5.1 Cantilever sheet pile wall with free interlocks

A cantilever sheet pile wall derives its support solely through interaction with the surrounding soil in which it is driven. This implies that no additional measures are taken to stabilise the sheet pile wall.

5.1.1 Results of 3D calculation

In the presentation of the bending moments of all 3D calculations, reference is made to a sheet pile wall with fixed interlocks. In this way a comparison can be made between the transverse bending moment \( M_y \) provided by the method to reduce oblique bending and the lateral moment that is necessary to impede oblique bending.

In Figure 25 the distribution of the bending moments \( M_y(x) \) and \( M_z(x) \) over the sheet pile are given. The maximum lateral bending moment \( M_z \) peaks at a slightly lower level than the excavation depth.
The transverse bending moment $M_y(x)$ for a cantilever sheet pile wall with fixed interlocks (= oblique bending impeded) is also depicted in Figure 25. It can be observed that the transverse bending moment $M_y(x)$ in case of a cantilever wall with fixed interlocks is by far greater than the one with free interlocks. This indicates that oblique bending is only reduced for a small part. The lateral bending moment $M_z(x)$ for a cantilever wall with free interlocks and a cantilever wall with fixed interlocks turned out to be exactly the same.

In case of free oblique bending, no transverse bending moment $M_y$ develops. In this case of a cantilever wall, some mechanism is providing a moment $M_y$ which reduces oblique bending. There are three mechanisms which can provide a transverse bending moment in the sheet pile:

- transverse bearing capacity of the soil
- distributed shear force in the interlock by friction
- distributed shear force in the interlock by fixing (parts of) the free interlock

The free interlock of the sheet pile is modeled in such a way that no friction can develop here. This means the slip in the interlock is not impeded in any way so that no distributed shear force can develop in the interlock. In this calculation no parts of the free interlock are fixed. Therefore the transverse bearing capacity of the soil is the only mechanism that can provide the occurring transverse bending moment $M_y$. This moment however, is very small. To impede oblique bending fully, $M_y(x)$ should be $-2.58$ times $M_z(x)$ in every point along the sheet pile axis as can be observed in Figure 25.

It is observed that the largest slip appeared at the top of the sheet pile and is only about 1.5 mm for this excavation depth. Because the magnitude of the slip is very small, the interlock slip is in practice not perceptible. Because there is a slip in the interlock, there also will be a transverse displacement of the sheet pile alongside the lateral displacement (= oblique bending).

The lateral displacements of the sheet pile wall which have been calculated for the three-dimensional case as well for both two-dimensional limit cases are displayed in Figure 26.
Both lateral and transverse displacements in the sheet pile are negative which means the sheet pile is respectively bending towards the front- and left side, see also Figure 18.

Aukema and Joling (lit[1]) found that the soil with a resulting water pressure on the sheet pile wall was not able to give a transverse restraint which significantly reduces oblique bending. The results of this 3D calculation show that without a resulting water pressure on the sheet pile wall, there is some transverse bearing capacity of the soil. However this transverse bearing capacity is still rather small. The lateral displacement at the top of the sheet pile wall calculated with the three-dimensional model is still very close to the two-dimensional lower limit case.

Although the slip at the top of the sheet pile is very small (1.6 mm), the lateral displacement at the top increased with 68% compared to a continuous sheet pile wall with fixed interlocks.

In order to obtain the apparent moment of inertia which can be applied in a 2D calculation, the reduction factor $R_I$ for the moment of inertia must be derived. In theory, the reduction factor $R_I$ can be derived from the transverse and lateral bending moments of the 3D calculation using the theory presented in paragraph 2.4. However, in order to do so the double integral of both transverse and lateral bending moments must be calculated. In practice, these calculated double integrals were not accurate enough which resulted in wrong reduction factors $R_I$ for the moment of inertia. As a result, the reduction factor $R_I$ which can be found with Figure 10 is not applicable when results from the 3D calculations are used.

In order to derive the reduction factor $R_I$, a 1D calculation is made with the spring model MSHEET. This has been done for two reasons.

- Results from MSHEET can be obtained quickly compared to results from the 2D model in DIANA.
- The spring model MSHEET is often used in Dutch design practice of sheet pile walls.

The procedure for obtaining the reduction factor is as follows. First the deflection for a cantilever sheet pile wall with fixed interlocks (= full moment of inertia) is calculated. Then, in an iterative way, the moment of inertia of the sheet pile wall is reduced until the deflection at the top is equal to the deflection as calculated with the 3D model. The applied reduction on the moment of inertia is therefore the reduction factor $R_I$ for the moment of inertia. In this case of a cantilever wall with free interlocks, the reduction factor for the moment of inertia turned out to be $R_I = 0.53$.

The reduction factor for the section modulus also depends on the loading condition of the sheet pile wall. But now we only have to look at the cross-section of the maximum lateral bending moment $M_y$ and corresponding transverse bending moment $M_x$. This is because the stresses in the cross-section are directly related to the bending moments. In paragraph 2.5 the reduction factor for the section modulus $R_{w_p}$ is given as a function of the bending moments $M_y$ and $M_x$. From Figure 25 and Figure 12 it can be
observed that:

\[
\begin{align*}
M_z &= 50 \text{kNm} \\
M_y &= -14 \text{kNm}
\end{align*}
\]

\[
\frac{M_y}{M_z} = 0.28 \quad \Rightarrow \quad R_w = 0.64
\]

The reduction factor for the section modulus is somewhat increased, compared to the situation of unrestrained oblique bending. In this situation the reduction factor for the section modulus is \(R_w = 0.59\).

5.2 Cantilever sheet pile wall with impeded horizontal displacement at the top

One way to reduce the oblique bending in a sheet pile wall is to impede the transverse displacements at the top of the sheet pile. Because only the transverse displacement at the top is impeded, slip in the free interlocks is still possible.

5.2.1 Results of 3D calculation

For an excavation depth of 3.6 m the bending moments \(M_y(x)\) and \(M_z(x)\) are presented in Figure 27.

![Figure 27: Bending moments \(M_y(x)\) and \(M_z(x)\) and interlock slip \(u(x)\) after a 3.6 m excavation for a sheet pile wall with impeded horizontal displacement at the top.](image)

The transverse bending moment \(M_y(x)\) initiated by the transverse impediment is greater than the transverse bending moment produced by the transverse bearing capacity of the soil (see Figure 25). But still the transverse bending moment \(M_y(x)\) initiated by the transverse impediment is much smaller than the \(M_y(x)\) which is needed to impede oblique bending. The slip in the interlock however, is reduced by almost 50\% compared to the slip in the cantilever wall with free interlocks.

For an excavation depth of 3.6 m the lateral displacements \(w_z(x)\) and the transverse displacements \(w_y(x)\) are presented in Figure 28.
Figure 28: Displacements curves \( w_z(x) \) and \( w_y(x) \) for an excavation of 3.6 for a sheet pile wall with impeded horizontal displacement at the top.

It can be noted that the transverse displacement \( w_y(x) \) of the sheet pile is now positive as a result of the horizontal impediment at the top. The horizontal impediment at the top is forcing the whole sheet pile to the right.

In order to derive the reduction factor \( R_i \) for the moment of inertia, again a 1D calculation is made with MSHEET. In the case of a cantilever wall with a fix of the transverse displacement at the top, the reduction factor for the moment of inertia \( R_i = 0.61 \).

The reduction factor for the section modulus is determined by the maximum lateral bending moment \( M_z \) and corresponding transverse bending moment \( M_y \). From Figure 27 and Figure 12 it can be found that:

\[
\begin{align*}
M_z &= 50 \text{kNm} \\
M_y &= -43 \text{kNm}
\end{align*}
\]

\[
\frac{M_y}{M_z} = 0.86 \quad \Rightarrow \quad R_w = 0.70
\]

Because the transverse bending moment \( M_y \) in the cross-section of the maximum lateral bending moment \( M_z \) increased, the reduction factor for the section modulus increased also.

5.3 Cantilever sheet pile wall with a capping beam on top

A capping beam is a concrete casing on the sheet pile head. This concrete casing is installed after the sheet pile is driven into the ground and before the sheet pile is able to deform due to an excavation. When a capping beam is installed on top of a sheet pile wall, it is assumed that the slip in the interlock at the sheet pile head is impeded. For this reason the sheet pile head must be embedded adequately in the capping beam in order to absorb the shear forces which occur due to this restriction.

5.3.1 Modeling of capping beam

The capping beam is modeled with the use of 'tyings'. This is a kind of 'virtual modeling', because no use is made of real elements to model the capping beam. Only the restrictions that a capping beam imposes upon the sheet pile have been implemented. Thus the slip in the interlock at the sheet pile head is impeded.

The rotation \( \varphi_z \) around the z-axis is only impeded on top of the sheet pile (see Figure 29). This is done by tying the interlock nodes of the upper sheet pile element in the x-direction. In this way no slip will occur between the two top interlock nodes. The rest of the upper sheet pile nodes are tied in the y- and z-direction. By this, all relative displacements of the top nodes in the z- and y-direction are zero for adjacent interlock nodes.
The profile isn't able to deform internally. Only all nodes of the top extremity of the sheet pile as a whole can translate in the z and y direction. So the only degree of freedom for rotation at the top of the sheet pile is the rotation \( \varphi_z \) around the y-axis.

### 5.3.2 Results of 3D calculation

For an excavation depth of 3.6 m the bending moments \( M_y(x) \) and \( M_z(x) \) are presented in Figure 30. If we compare the lateral bending moment \( M_z(x) \) after a 3.6 m excavation with the corresponding result from the cantilever wall (Figure 25), it is noticed that this bending moment is exactly the same. The installation of the capping beam has therefore no effect on the lateral load upon the face of the sheet pile.

Because the rotation \( \varphi_z \) of the sheet pile is prevented, an additional transverse bending moment \( M_y \) has developed at the top of the sheet pile. This bending moment must be absorbed by the capping beam. The transverse bending moment \( M_y(x) \) initiated by the capping beam is by far greater than that produced by the transverse bearing capacity of the soil (see Figure 25). But still the transverse bending moment \( M_y(x) \) initiated by the capping beam is smaller than the \( M_y(x) \) which is needed to impede
oblique bending.

It is striking to see that the transverse bending moment $M_y(x)$ in the capping beam and the top part of the sheet pile is equal to the lateral bending moment $M_z(x)$ at the excavation depth. At the point in the sheet pile where the maximum lateral bending moment $M_z(x)$ appears, the slip in the interlock is zero (see Figure 30).

For an excavation depth of 3.6 m the displacements $w_y(x)$ and $w_z(x)$ are presented in Figure 31.

![Displacement Curves](image)

Figure 31: Displacements curves $w_y(x)$ and $w_z(x)$ for an excavation of 3.6 for a sheet pile wall with a capping beam on the sheet pile head.

The lateral displacement at the top of the sheet pile wall with a capping beam is now closer to the 2D upper limit of a continuous sheet pile wall with fixed interlocks. The lateral displacement is now only 31% bigger. So by installing a capping beam, a significant reduction of the lateral displacements is achieved.

In order to derive the reduction factor $R_i$, again a 1D calculation is made with MSHEET equal to that for the previous methods. In the case of a cantilever wall with a capping beam on the sheet pile head, the reduction factor for the moment of inertia $R_i = 0.71$.

The reduction factor for the section modulus again is determined by the maximum lateral bending moment $M_z$ and corresponding transverse bending moment $M_y$. From Figure 30 and Figure 12 it can be found that:

$$\begin{align*}
M_z &= 50 \text{kNm} \\
M_y &= -50 \text{kNm}
\end{align*}$$

$$\frac{M_y}{M_z} = 1.00 \quad \Rightarrow \quad R_w = 0.72$$

The reduction factor $R_i$ for the moment of inertia increased with 0.1 compared to the situation with an impediment of the horizontal displacement at the top. However, the reduction factor $R_w$ for the section modulus only increased with 0.02. This can be explained by the fact that the reduction factor $R_i$ for the moment of inertia depends on the distribution of the bending moments over the whole sheet pile. On the contrary, the reduction factor $R_w$ for the section modulus depends on the transverse bending moment $M_y$ in the cross-section of the maximum lateral bending moment $M_z$. The transverse bending moment did not increased much in this cross-section and therefore the reduction factor did not either.

The slip in the interlock in the adjacent nodes on top of the sheet pile is impeded by the capping beam. This means that the maximum slip in the interlock is reduced compared to that of the cantilever wall (see Figure 32). Furthermore the place of maximum slip is shifted downwards.
During the excavation of the soil in front of the sheet pile wall, the place of zero slip shifted downwards. This is caused by the increase of lateral load upon the sheet pile with every new excavation step. For deeper excavations, the slip at the bottom of the sheet pile isn’t zero any more.

5.4 Cantilever sheet pile wall with welded interlock during excavation
From the results of the sheet pile with a capping beam on top, it can be observed that if the slip in the free interlock is impeded the sheet pile acts much stiffer. As a logical follow up, the interlock of the sheet pile is welded after every excavation step of 0.9 m. In this way no further slip can develop in the welded part of the interlock.

5.4.1 Modeling of the weld
With every new excavation step of 0.9 m, the lateral- and transverse displacements increase. As a result of the increasing transverse displacements, the slip in the interlock also increases, see Figure 33.

---

Figure 32: Slip in the interlock for the excavation depths 2.7 m, 3.6 m and 4.5 m for a sheet pile wall with a capping beam on the sheet pile head.

Figure 33: Slip in the interlock due to in plane (= transverse) displacement of sheet pile.
The procedure for welding of the interlock during excavation is as follows. First an excavation step of 0.9 m (= the height of one soil element) is made to ensure the interlock is accessible for welding. Because an excavation step is made, there is also some slip in the interlock. When welding of this part of the interlock is completed, the next step of 0.9 m can be excavated and is accessible for welding of the interlock etc.

At first an attempt was made to model the weld in the interlock by means of 'tyings'. In this case, the corresponding nodes of two adjacent interlock elements have to be tied in the axial direction in order to prevent further sliding. However, DIANA didn't support the tying of two nodes which already have different displacements. Therefore the weld in the interlock is modeled with a simple truss element between two adjacent nodes of the sliding interlock, see Figure 34. The truss element can only deform in the axial direction, there is neither bending nor shear deformation.

![Figure 34: Model of the weld in the interlock.](image)

The stiffness of this truss element is much greater than that of the sheet pile. This is necessary to ensure the axial deformation of the truss element is insignificant compared to the slip in the interlock. On the other hand, the difference in stiffness between the truss element and the sheet pile elements cannot be too great in order to prevent inaccurate solutions. After several tests, the following material properties of the truss element met the desired requirements:

- Young's modulus: $E = 1.0 \times 10^{14} \text{ N/m}^2$ ($\approx 480$ times stiffer than a sheet pile element)
- Cross-section: $A = 0.25 \text{ m}^2$

5.4.2 Results of 3D calculation
For an excavation depth of 4.5 m the bending moments $M_y(x)$ and $M_z(x)$ are presented in Figure 35. Again the lateral bending is equal to the previous cases. The transverse bending moment of course is different, due to the application of the truss elements. In contrast with the capping beam, the weld in the interlock is not initiating a transverse bending moment at the top of the sheet pile but rather in the middle. Also the maximum value of this moment is bigger than with the capping beam.
Because an excavation is made with a depth of 4.5 m, there was only space for four truss elements of 0.9 m which reached a depth of 3.6 m. It can be observed that over this depth of 3.6 m the transverse bending moment $M_y(x)$ is almost equal to the transverse bending moment which is necessary to impede oblique bending. This is an indication that oblique bending is significantly reduced.

The slip in the upper half of the sheet pile is almost zero. Due to the application of a truss element after every excavation step of 0.9 m, the slip is not able to fully develop here. The slip in the lower half of the sheet pile has the same shape as the previous two cases. Because a truss element only connects the top and bottom node of two sheet pile elements (see Figure 34), the displacement of the middle node is bigger. This explains the irregular distribution of the interlock slip in the area where truss elements are applied.

For an excavation depth of 4.5 m the displacements $w_y(x)$ and $w_z(x)$ are presented in Figure 36.

It is striking to notice that the transverse displacement $w_y$ at the sheet pile top is positive in this case, whereas in all other cases the transverse displacement are negative. The explanation for this is that hardly any positive slip could be developed in the sheet pile due to the application of the truss elements. Furthermore a negative transverse displacement causes a positive slip (see Figure 16). In case of the
cantilever wall and the wall with the capping beam, the maximum positive slip in the sheet pile is bigger than the maximum negative slip. So a resulting negative transverse displacement follows.

The lateral displacement at the top of the sheet pile wall with a capping beam is now almost equal to the 2D upper limit of a continuous sheet pile wall with fixed interlocks. The lateral displacement is now only 17% bigger compared to a continuous sheet pile wall. So welding of the interlock almost completely impedes oblique bending.

The reduction factor $R_I$ for the moment of inertia is determined with the use of MSHEET and has the value $R_I = 0.82$.

The reduction factor for the section modulus again is determined by the maximum lateral bending moment $M_z$ and corresponding transverse bending moment $M_y$. From Figure 35 and Figure 12 it can be found that:

$$\begin{align*}
M_z &= 100 \text{kNm} \\
M_y &= -147 \text{kNm}
\end{align*}$$

$$\frac{M_y}{M_z} = 1.47 \Rightarrow R_W = 0.77$$

The reduction factor $R_W$ has increased again on account of the increased transverse bending moment $M_y$.

5.5 Overview of results

In Figure 37, the transverse bending moments and lateral displacements of all methods to reduce oblique bending are presented.

If we compare the distribution of the transverse bending moments over the sheet pile, it is clear that a bigger, negative transverse bending moment is reducing oblique bending more. The maximum displacement at the top, and therefore also the reduction factor $R_I$, depends on the double integral of the bending moments $M_z(x)$ and $M_y(x)$ as mentioned in paragraph 2.4. Therefore the shape of the transverse bending moment distribution has little influence on the maximum displacement.

The reduction factor $R_W$ for the section modulus $W$ only depends on the maximum lateral bending moment $M_z$ along with the transverse bending moment $M_y$ present in that particular cross-section. Therefore the shape of the transverse bending moment distribution has a great influence on the reduction factor for the section modulus. In table 3 all derived reduction factors are presented.
### Table 3: Reduction factors for oblique bending

<table>
<thead>
<tr>
<th></th>
<th>( R_f )</th>
<th>( R_w )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Free oblique bending ((M_e = 0))</td>
<td>0.59</td>
<td>0.49</td>
</tr>
<tr>
<td>Cantilever (free interlock)</td>
<td>0.64</td>
<td>0.53</td>
</tr>
<tr>
<td>Horizontal fix</td>
<td>0.70</td>
<td>0.61</td>
</tr>
<tr>
<td>Capping beam</td>
<td>0.72</td>
<td>0.71</td>
</tr>
<tr>
<td>Welded interlock</td>
<td>0.77</td>
<td>0.82</td>
</tr>
<tr>
<td>No oblique bending</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

In the following figure, a sketch of the deformed sheet pile for the four analysed cases is given. Note that this is a sketch, therefore the displacements are not proportional.

![Sketch of deformed sheet pile](image)

**Figure 38:** Sketch of the deformed sheet pile in transverse direction for the four calculated cases.

If we compare the sheet pile with the horizontal fix at the top with the cantilever sheet pile it is observed that the whole sheet pile has moved because of the horizontal fix. However, a slip in the free interlock still occurs. As mentioned before, installation of a capping beam impedes a rotation at the top of the sheet pile. As a result no slip can develop at this point. Because little slip can develop in the upper half of the sheet pile with the welded interlock, the sheet pile is nearly straight in this area.

In Figure 39 and Figure 40 the plots of the 3D sheet pile after a 3.6 m excavation are depicted for respectively a cantilever sheet pile wall and a cantilever sheet pile wall with an horizontal fix at the top. These plots show the deformed mesh (scaled up 50 times) along with the axial stresses \( \sigma_{xx} \) due to the bending moments. It can be observed the stress in the free (i.e. middle) interlock is not zero. This clearly indicates oblique bending because the neutral axis (zero stress) is rotated. The maximum stresses in the sheet pile with an horizontal fix at the top are smaller than the stresses in the cantilever sheet pile. This is a result of the transverse bending moment \( M_y(x) \). Also it can be noticed from Figure 40, the sheet pile deforms internally at the top due to the horizontal restraint.

In Figure 41 and Figure 42 the plots of the 3D sheet pile after a 4.5 m excavation are shown for respectively a cantilever sheet pile wall with a capping beam on top and a sheet pile wall with a welded interlock during excavation. These plots show the deformed mesh (scaled up 25 times) along with the axial stresses \( \sigma_{xx} \) due to the bending moments. It can be observed from Figure 40 that the stresses in the free (i.e. middle) interlock in the top half of the sheet pile are considerably big. This is caused by the
transverse bending moment $M_y(x)$ initiated by the capping beam.

In Figure 43 the vertical (axial) soil stresses $\sigma_{x}$ and horizontal (lateral) soil stresses $\sigma_{z}$ for a cantilever sheet pile wall after a 3.6 m excavation are pictured. As a result of the excavation, the vertical soil stresses in front of the sheet pile wall increases. Directly behind the sheet pile wall the soil caves in which results in an unloading of the vertical stresses. It can be observed that there is a distributed load of 10 kN/m² present at the top of the soil body. Behind the sheet pile wall there is an unloading of the soil which results in a decrease of the horizontal soil stresses. In contrast with this, the horizontal soil stresses in front of the sheet pile wall are rather high. Both effect are caused by the deflection of the sheet pile wall.

In Figure 44 the transverse displacement contours $w_y$ and lateral displacement contours $w_z$ are depicted on the deformed mesh of a cantilever sheet pile wall after a 3.6 m excavation. The deformed mesh is scaled up 10 times. As can be noticed the transverse displacements $w_y$ only occur directly behind the sheet pile wall. The lateral displacements $w_z$ cover a much larger area of the mesh. Especially at the toe of the sheet pile there is some increase of the lateral displacements.

In Figure 45 the vertical (axial) soil stresses $\sigma_{x}$ and horizontal (lateral) soil stresses $\sigma_{z}$ for a cantilever sheet pile wall with a capping beam on top after a 4.5 m excavation are pictured. Here the same effects occur as with the cantilever sheet pile wall. Because the excavation depth is now 4.5 m the soil stresses are a little bigger.

In Figure 46 the transverse displacement contours $w_y$ and lateral displacement contours $w_z$ are depicted on the deformed mesh of a cantilever sheet pile wall with a capping beam on top after a 4.5 m excavation. The deformed mesh is scaled up 5 times. As a result of installation of the capping beam the displacements cover a much smaller area of the mesh than without a capping beam.

In all plots of the soil stresses, the three columns of elements at the right of the mesh always have horizontal contour lines. This implies no variation in stresses occurs. From this it can be concluded that these elements are not necessary to model the sheet pile in the sand. However this only applies with these excavation depth up to 4.5 m. For deeper excavation, the three columns can be useful because with deeper excavations larger stresses are generated.
Figure 39: Deformed meshes of a cantilever sheet pile wall with free interlocks with tensile- (positive) and compressive (negative) stresses $\sigma_{xx}$ [N/mm$^2$] due to the occurring bending moments $M_y(x)$ and $M_z(x)$ after an excavation of 3.6 m. (a) Front view of the sheet pile with transverse deformation. (b) 3D-view with both transverse- and lateral deformations (deformations scaled up 50 times).

Figure 40: Deformed meshes of a cantilever sheet pile wall with an horizontal fix at the top with tensile- (positive) and compressive (negative) stresses $\sigma_{xx}$ [N/mm$^2$] due to the occurring bending moments $M_y(x)$ and $M_z(x)$ after an excavation of 3.6 m. (a) Front view of the sheet pile with transverse deformation. (b) 3D-view with both transverse- and lateral deformations (deformations scaled up 50 times).
Figure 41: Deformed meshes of a cantilever sheet pile wall with a capping beam on top with tensile-(positive) and compressive (negative) stresses $\sigma_{xx}$ [N/mm$^2$] due to the occurring bending moments $M_y(x)$ and $M_z(x)$ after an excavation of 4.5 m. (a) Front view of the sheet pile with transverse deformation. (b) 3D-view with both transverse- and lateral deformations (deformations scaled up 25 times).

Figure 42: Deformed meshes of a cantilever sheet pile wall with welded interlocks during excavation with tensile- (positive) and compressive (negative) stresses $\sigma_{xx}$ [N/mm$^2$] due to the occurring bending moments $M_y(x)$ and $M_z(x)$ after an excavation of 4.5 m. (a) Front view of the sheet pile with transverse deformation. (b) 3D-view with both transverse- and lateral deformations (deformations scaled up 25 times).
Figure 43: The soil stresses for a cantilever sheet pile wall after a 3.6 m excavation. (a) The vertical (axial) soil stresses $\sigma_{xx}$ [kN/m$^2$]. (b) The horizontal (lateral) soil stresses $\sigma_{zz}$ [kN/m$^2$].
Figure 44: The displacement contours of a cantilever sheet pile wall after a 3.6 m excavation. (a) The transverse displacements $w_y$ [mm]. (b) The lateral displacements $w_z$ [mm].
Figure 45: The soil stresses for a cantilever sheet pile wall with a capping beam on top after a 4.5 m excavation. (a) The vertical (axial) soil stresses $\sigma_{xx}$ [kN/m$^2$]. (b) The horizontal (lateral) soil stresses $\sigma_{zz}$ [kN/m$^2$].
Figure 46: The displacement contours of a cantilever sheet pile wall with a capping beam on top after a 4.5 m excavation. (a) The transverse displacements $w_y$ [mm]. (b) The lateral displacements $w_z$ [mm].
6 Different phenomena which occur in 3D calculations

In this chapter three subjects will be treated that are typical 3D phenomena and can only be found by using 3D calculations.

6.1 Accuracy of displacements in calculations

In order to check the displacements from the 3D calculations performed with DIANA, different formulae for calculation of the bending moments have been compared.

There are four possibilities of calculating the transverse bending moment \( M_y(x) \) in the sheet pile.
1. Integration of the stresses \( \sigma_{xy} \) in a cross section of the sheet pile
2. Use of \( \kappa_y(x) \) and \( \kappa_z(x) \)
3. Use of \( M_z(x) \) and \( \kappa_z(x) \)
4. Use of \( M_y(x) \) and \( \kappa_y(x) \)

Being analogous to the transverse bending moment \( M_y(x) \), the lateral bending moment \( M_z(x) \) can also be calculated by these four methods. The formulae can easily be derived by substitution of the basic bending formulae for two directional bending.

\[
M_y(x) = EI_{yy} \kappa_y(x) + EI_{yz} \kappa_z(x)
\]
\[
M_y(x) = \frac{I_{yz}}{I_{zz}} M_z(x) + \frac{E(I_{yy} I_{zz} - I_{yz} I_{zy})}{I_{zz}} \kappa_y(x)
\]
\[
M_y(x) = \left[ M_z(x) - \frac{E(I_{yy} I_{zz} - I_{yz} I_{zy})}{I_{yy}} \kappa_z(x) \right] \frac{I_{yy}}{I_{yz}}
\]
\[
M_z(x) = EI_{yz} \kappa_y(x) + EI_{zz} \kappa_z(x)
\]
\[
M_z(x) = \frac{I_{yy}}{I_{yy}} M_y(x) + \frac{E(I_{yy} I_{zz} - I_{yz} I_{zy})}{I_{yy}} \kappa_y(x)
\]
\[
M_z(x) = \left[ M_y(x) - \frac{E(I_{yy} I_{zz} - I_{yz} I_{zy})}{I_{zz}} \kappa_z(x) \right] \frac{I_{zz}}{I_{yy}}
\]

In order to check the accuracy of the 3D results, all four possibilities of calculating the bending moment are pictured in Figure 47. The curvatures \( \kappa_y(x) \) and \( \kappa_z(x) \) are derived by a second order derivation of respectively the transverse displacement \( w_y(x) \) and the lateral displacement \( w_z(x) \).

\[
\kappa_y(x) = -\frac{\partial^2 w_y(x)}{\partial x^2}, \quad \kappa_z = -\frac{\partial^2 w_z(x)}{\partial x^2}
\]

These displacements are taken from the 3D calculations. For the moments \( M_y(x) \) and \( M_z(x) \) in the formulae, the 'real' moments are taken which have been calculated by integration of the stresses over the cross section \( A \) of the profile.

\[
M_y(x) = \int_A \sigma_{xy}(x) y dA, \quad M_z(x) = \int_A \sigma_{xz}(x) z dA
\]

If all data (stresses and displacements) from the 3D calculations would be accurate and the sheet pile would obey Bernoulli's hypothesis, all four methods of calculating the bending moment should be equal. The hypothesis of Bernoulli implies that a cross section of the profile is not able to deform...
Figure 47: Comparison of four methods to calculate the bending moments $M_y(x)$ and $M_z(x)$.

In Figure 47 the bending moments $M_y(x)$ and $M_z(x)$ for a cantilever wall with a capping beam on top after an excavation of 5.4 m are depicted. From this figure it follows that as soon as use is made of the transverse curvature $\kappa_y(x)$, the derivation of the moment is not corresponding with the 'real' moment. This moment is found by integration of the stresses in a cross section of the profile. The calculated moment corresponds best with the 'real' moment when use is made of the lateral curvature $\kappa_z(x)$.

For different loading conditions (cantilever wall, fixed interlock, welded interlock) the same comparison is made and the same conclusions can be drawn.

The displacement $w_y(x)$ has been calculated from the 3D results by taking the average displacement of the 18 nodes of the cross section of the modeled profile. If we use the displacements $w_y(x)$ of the node which corresponds with the fixed interlock, the results derived with $w_y(x)$ are somewhat better, but still not satisfactory. The difference between these two values can be explained by the occurrence of internal deformation of the sheet pile. This means that not all nodes of the sheet pile have got the same transverse displacement.

Conclusions:
- The transverse displacement $w_y(x)$ is not accurate.
- The lateral displacement $w_z(x)$ is accurate.
- The moments calculated by integration of stresses are satisfactory.
- The sheet pile obeys Bernoulli's hypothesis quite well, when the lateral displacement $w_z(x)$ is disregarded.
6.2 Calculation of normal stresses in cross-section
In order to check the stresses that occur in the sheet pile, the normal stresses that were found numerically from the 3D calculations are compared with the stresses calculated from the bending moments.

\[ \sigma_{xx} \text{ [N/mm}^2\text{]} = \begin{cases} \text{Analytical} & \text{(dotted line)} \\ \text{Numerical} & \text{(solid line)} \end{cases} \]

Figure 48: Distribution of normal stresses in a cross-section due to bending moments \( M_x \) and \( M_y \).

In Figure 48, the normal stresses across the cross-section of the PU8 profile are given for a transverse bending moment \( M_x \) of -20.58 kNm and a lateral bending moment \( M_y \) of 49.35 kNm. This is the load on a cantilever wall at an excavation depth of 4.5 m.

Figure 48 shows that the neutral axis intersects the profile at the point where the stress is zero (this is the definition of the neutral axis). For both numerical and analytical calculations, this intersection point is equal. Because the neutral axis intersects the fixed interlock which is the centre of gravity, it can be concluded that there is no normal force acting upon the sheet pile. Thus all normal stresses are initiated by bending moments.

It can also be found that the stress distribution across the cross-section is equal for both numerical and analytical calculations. The stresses are distributed linear across the cross-section, so this again is an indication that the modeled sheet pile is obeying Bernoulli's hypothesis.

6.3 Calculation of the interlock slip
By introducing an equation which couples the lateral displacement \( w_i(x) \) of the sheet pile to the slip \( u(x) \) in the free interlock, the slip in the interlock can be calculated, using the transverse and lateral bending moments. The interlock slip \( u(x) \) is defined positive if a negative rotation \( \phi(x) \) is working on the sheet pile as can be observed in Figure 49.
Figure 49: Definition of variables for calculation of interlock slip.

The origin of coordinates is placed at the bottom of the sheet pile wall. There are different ways to calculate the slip in the interlock. From the figure presented above, the following formula can be derived for the interlock slip \( u(x) \).

\[
\begin{align*}
- \varphi_z(x) &= - \frac{dw_y(x)}{dx} \\
- \varphi_z(x) &= \frac{u(x)}{2b}
\end{align*}
\]

\[
u(x) = -2b \frac{dw_y(x)}{dx}
\]

In this way, the interlock slip depends on the transverse displacement \( w(x) \) of the sheet pile. In paragraph 6.1 it was showed that the results of the 3D calculation for the values of \( w(x) \) did not turned out to be accurate. Another way of calculating the interlock slip, and avoid the use of the transverse displacement \( w(x) \), is possible when use is made of the transverse curvature \( \kappa_y(x) \).

\[
\begin{align*}
\kappa_y(x) &= \frac{d \varphi_z(x)}{dx} \\
\kappa_y(x) &= \frac{1}{2b} \int_0^x \kappa_y(x) dx + C
\end{align*}
\]

In this equation, \( C \) is a constant of integration. The transverse curvature can be expressed in \( M_y(x) \) and \( M_z(x) \), see also paragraph 6.1.

\[
\kappa_y(x) = \frac{I_{zz} M_y(x) - I_{xy} M_z(x)}{E(I_{yy} I_{zz} - I_{yzz} I_{yy})} = \frac{I_{zz}}{E(I_{yy} I_{zz} - I_{yzz} I_{yy})} M_y(x) - \frac{I_{yz}}{E(I_{yy} I_{zz} - I_{yzz} I_{yy})} M_z(x)
\]

Substitution of this equation in the one for the interlock slip \( u(x) \) gives:

\[
u(x) = -\frac{2b I_{zz}}{E(I_{yy} I_{zz} - I_{yzz} I_{yy})} \int_0^x M_y(x) dx + \frac{2b I_{yz}}{E(I_{yy} I_{zz} - I_{yzz} I_{yy})} \int_0^x M_z(x) dx + C
\]

This expression is the general formula for calculation of the interlock slip. In this equation, the interlock
slip \( u(x) \) only depends on the lateral and transverse bending moments \( M_y(x) \) and \( M_z(x) \). In this way we bypass the use of the transverse displacements \( w_y(x) \), which appeared not to be accurate enough.

If we want to find the \( x \) coordinate in the sheet pile where the slip in the interlock is zero, the following equation must be solved for \( x \).

\[
u(x) = -\frac{2bl_{zz}}{E(I_{yy}I_{zz} - I_{zy}I_{zy})} \int_0^x M_y(x) \, dx + \frac{2bl_{yz}}{E(I_{yy}I_{zz} - I_{zy}I_{zy})} \int_0^x M_z(x) \, dx + C = 0
\]

\[
\int_0^x M_y(x) \, dx = \frac{I_{yz}}{I_{zz}} \int_0^x M_z(x) \, dx + C
\]

It is very important to see that not the values of the bending moments \( M_y(x) \) and \( M_z(x) \) at a certain height of the sheet pile determine the slip in the interlock. Yet the integral of these bending moments over the sheet pile from one point to another particular point determine the slip.

**Calculation of interlock slip for cantilever wall (3.6 m excavation)**

To solve the integral for the interlock slip, a boundary condition is necessary. From the 3D calculations we know the slip at the top of the sheet pile; \( u(l) = 1.617 \cdot 10^{-3} \) m. If we substitute this boundary condition in the general formula for the interlock slip, we obtain:

\[
u(l) = -\frac{2bl_{zz}}{E(I_{yy}I_{zz} - I_{zy}I_{zy})} \int_0^l M_y(x) \, dx + \frac{2bl_{yz}}{E(I_{yy}I_{zz} - I_{zy}I_{zy})} \int_0^l M_z(x) \, dx + C = 1.617 \cdot 10^{-3} m
\]

\[
5.26 \cdot 10^{-4} - 2.072 \cdot 10^{-3} + C = 1.617 \cdot 10^{-3} \quad \Rightarrow \quad C = 3.163 \cdot 10^{-3} m
\]

Substituting this result in the general equation for the interlock slip gives the formula for slip in the interlock for this particular case.

\[
u(x) = -\frac{2bl_{zz}}{E(I_{yy}I_{zz} - I_{zy}I_{zy})} \int_0^x M_y(x) \, dx + \frac{2bl_{yz}}{E(I_{yy}I_{zz} - I_{zy}I_{zy})} \int_0^x M_z(x) \, dx + 3.163 \cdot 10^{-3}
\]

![Slip in interlock](image)

**Figure 50:** Slip in the interlock for a cantilever wall after a 3.6 m excavation.

From Figure 50 can be concluded that the formula for the interlock slip corresponds very well with the
Calculation of interlock slip for sheet pile wall with capping beam (3.6 m excavation)
In the case of a sheet pile wall with a capping beam, the interlock slip at the top of the sheet pile is zero. So the boundary condition at the top of the sheet pile is: \( u(l) = 0 \). When this boundary condition is substituted in the general formula, the following equation can be derived for the slip in the interlock.

\[
u(x) = -\frac{2bI_{xx}}{E(I_{yy}I_{zz} - I_{zz}I_{yy})} \int_{0}^{x} M_y(x) dx + \frac{2bI_{yy}}{E(I_{yy}I_{zz} - I_{zz}I_{yy})} \int_{0}^{x} M_z(x) dx + 1.28 \cdot 10^{-4}\]

Figure 51: Slip in interlock for a sheet pile wall with a capping beam after a 3.6 m excavation.

The figure above shows that in this case the results from the analytical formula and the 3D calculations differ somewhat. Although the shape of the curves is corresponding quite well. Only at the top of the sheet pile the curve for the analytical formula is steeper.
7 Comparison of reduction factors with CUR-publication

In this chapter a comparison is made between the numerical results of the reduction factors as derived in chapter 5 and the Dutch CUR-publication 166 (lit[2]). This publication is an important publication concerning oblique bending. Furthermore some remarks are made concerning the reduction factors that can be applied for the different methods to reduce oblique bending.

In the CUR-publication 166 guide-lines are given concerning the application of reduction factors at the moment of inertia and section modulus of single and double U-profiles. These reduction factors are based on various experiments and publications. The value of the reduction factors depend on the following four points:

1) The amount of elements of which the steel sheet pile wall is constructed (one, two or more U-profiles).
2) The number of support levels like anchors and struts.
3) The treatment of the 'free' interlocks on site. If after driving the sheet pile, at least at one place the slip in the 'free' interlock is impeded, the 'free' interlock is regarded as 'locked'.
4) The type of soil in which the sheet pile is constructed. A distinction is made here between 'sand' and 'no sand'. With 'sand' is meant fine-grained soils (sand, silt) and with 'no sand' is meant peat, clay, gravel and also water. Also a distinction is made between the soil type at the upper side and down side (around the toe) of the sheet pile wall.

In CUR-publication 166, together with the table of reduction factors, a list of special cases and exceptions is given in order to derive a thoughtful and correct usage of the table by the design engineer. Some of these exceptions, relevant to the derivation of the reduction factors for the four studied methods to reduce oblique bending, are listed below.

- If there is sand above the groundwater level, both reduction factors $R_w$ and $R_l$ should be increased with 0.1.
- If the height over width ratio of the used U-profile (h/b) is smaller than 0.2, both reduction factors should be reduced with 0.1.
- The reduction factors increase when after driving the sheet pile the 'free' interlock is 'locked' (see also point 3 above). This positive effect of a 'locked' interlock is only applicable if the toe of the sheet pile is driven in sand.

In table 4 the reduction factors as presented in the CUR-publication 166 are given.
Table 4: Reduction factors to be applied at the parameters of a continuous wall with fixed interlocks, CUR-publication 166.

<table>
<thead>
<tr>
<th>Sheet pile elements (amount of U-profiles)</th>
<th>Amount of support levels locked = 1 free = 0</th>
<th>Soil type (top/toe)</th>
<th>Reduction factor</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>s/s, n/s, n/n</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>--</td>
<td>all</td>
<td>0.6, 0.35</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>s/s</td>
<td>0.8, 0.7</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>n/s</td>
<td>0.7, 0.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>n/n</td>
<td>0.7, 0.6</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>s/s</td>
<td>0.9, 0.8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>n/s</td>
<td>0.8, 0.7</td>
</tr>
<tr>
<td></td>
<td></td>
<td>n/n</td>
<td>0.8, 0.7</td>
</tr>
<tr>
<td>&gt;=2</td>
<td>0</td>
<td>s/s</td>
<td>0.9, 0.8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>n/s</td>
<td>0.8, 0.7</td>
</tr>
<tr>
<td></td>
<td></td>
<td>n/n</td>
<td>0.8, 0.7</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>s/s</td>
<td>1.0, 0.9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>n/s</td>
<td>0.9, 0.8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>n/n</td>
<td>0.9, 0.8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>s/s</td>
<td>1.0, 1.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>n/s</td>
<td>1.0, 1.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>n/n</td>
<td>0.9, 0.8</td>
</tr>
<tr>
<td>3</td>
<td>--</td>
<td>0 and 1</td>
<td>1.0, 1.0</td>
</tr>
<tr>
<td>4</td>
<td>--</td>
<td>0 and 1</td>
<td>1.0, 1.0</td>
</tr>
</tbody>
</table>

For the four methods that have been simulated, the reduction factor $R_{W}$ for the section modulus $W$ and the reduction factor $R_{I}$ for the moment of inertia $I$ have been determined in accordance with CUR-publication 166. The values are given in table 5 together with the values derived with the 3D calculations.

Table 5: Reduction factors for oblique bending.

<table>
<thead>
<tr>
<th>Method:</th>
<th>3D calculation</th>
<th>CUR-publication</th>
</tr>
</thead>
<tbody>
<tr>
<td>Free oblique bending ($M_s = 0$)</td>
<td>0.59, 0.49</td>
<td>---, ---</td>
</tr>
<tr>
<td>Cantilever (free interlock)</td>
<td>0.64, 0.53</td>
<td>0.9, 0.8</td>
</tr>
<tr>
<td>Horizontal fix</td>
<td>0.70, 0.61</td>
<td>0.9, 0.8</td>
</tr>
<tr>
<td>Capping beam</td>
<td>0.72, 0.71</td>
<td>1.0, 0.9</td>
</tr>
<tr>
<td>Welded interlock</td>
<td>0.77, 0.82</td>
<td>1.0, 0.9</td>
</tr>
<tr>
<td>No oblique bending</td>
<td>1, 1</td>
<td>1, 1</td>
</tr>
</tbody>
</table>

It can be observed from table 5 the reduction factor $R_{I}$ for the moment of inertia is always smaller than the reduction factor $R_{W}$ for the section modulus in CUR-publication 166. $R_{I}$ is always smaller than $R_{W}$ when the rotation of the neutral axis is constant over the entire length of the sheet pile. If the rotation of the neutral axis varies over the entire length of the sheet pile, $R_{I}$ can also be bigger than $R_{W}$. An
example of this is given in annex B and C. From this follows that in CUR-publication 166, the implicit assumption is made that the rotation of the neutral axis is constant over the entire length of the sheet pile.

According to the table in CUR-publication 166, a reduction factor $R_w = 0.8$ must be applied in the case of a cantilever wall in sand. However, this reduction factor has to be increase with 0.1 to $R_w = 0.9$ because there is sand above the groundwater level. The height over width ratio ($h/b$) of the used U-profile is 0.23 (=14/60). When this value is smaller than 0.2, both reduction factors should be reduced with 0.1. The value of these height over width ratios are very close to each other. Therefore a value of $R_w = 0.8$ for the reduction factor also has to be considered.

The reduction factors given in the CUR-publication 166, are values that were found in practice. This means that friction in the interlock is incorporated in these reduction factors. In the 3D numerical simulations, there is no friction in the interlock. Therefore the reduction factors cannot be compared with each other. The transverse bending moment is solely produced by the measures taken to reduce oblique bending (e.g. welding of the interlock, installation of capping beam) and/or the transverse bearing capacity of the soil.

The reduction factors for the cantilever wall and the cantilever wall with an horizontal fix in the transverse direction have equal values according to the CUR-publication. This is because no separate category is made for a transverse fix of the displacements. The reduction factors for the capping beam and the welded interlock during excavation are also equal according to the CUR-publication.
8 Conclusions and recommendations

Conclusions regarding calculations

- A simulation is made of a dry excavation in front of an infinitely long cantilever sheet pile wall. This simulation is made without a resulting water pressure on the sheet pile. The sheet pile is assumed to be perfectly rough ($\delta = \phi$) which means the soil (a middle-dense compacted sand) is attached to the sheet pile. In this configuration the sand is not able to give a transverse restraint which significantly reduces oblique bending. The conditions of no resulting water pressure, a perfectly rough sheet pile and the use of sand are 'best case' conditions. So also with other conditions no transverse restraint is to be expected.

- Because the soil is not able to significantly reduce oblique bending, the influence of three different construction details on oblique bending have been investigated. These three construction details are:
  - A steel sheet pile wall with a fix of the horizontal displacement at the top.
  - A steel sheet pile wall with a capping beam on top.
  - A steel sheet pile with the sliding interlock welded during the excavation.
When free oblique bending occurs (no transverse restraint), the reduction factor for the lateral stiffness is 0.49. From the simulations it followed that welding of the interlock during excavation gives the highest reduction of oblique bending. In this case the reduction factor for the lateral stiffness increased to 0.82.

- The installation of a capping beam on the sheet pile head is probably the most practical way of reducing oblique bending. In this case the reduction factor for the lateral stiffness increases to 0.71. This is less than the reduction factor as derived for welding of the interlock. However, welding of the interlock takes a lot of effort whereas installation of a capping beam is relatively simple.

- If there is sand above the groundwater level, both reduction factors $R_w$ and $R_l$ should be increased with 0.1 according to CUR-publication 166. This increase of the reduction factors is not supported by the simulations carried out in this study.

Conclusions regarding theory of oblique bending

- The rotation of the neutral axis is only constant along the longitudinal axis of a sheet pile if the loading conditions in the lateral and transverse direction are congruent. This means that the shape of the bending moments $M_y(x)$ and $M_z(x)$ must be equal. Only when the rotation of the neutral axis along the longitudinal axis of a sheet pile is constant, the rotation of the neutral axis can be determined directly from the lateral and transverse displacements of the sheet pile. In practice, the bending moments $M_y(x)$ and $M_z(x)$ are not congruent, therefore the rotation of the neutral axis cannot be determined directly from the lateral and transverse displacements.

- The reduction factor $R_w$ for the section modulus $W$, can be derived directly at the cross-section of the sheet pile where the maximum value of the lateral bending moment $M_y$ occurs. In this cross-section the maximum stress occurs. Because the stresses in a cross-section are directly related to the bending moments, only the bending moments $M_y$ and $M_z$ in this cross-section need to be known in order to derive the reduction factor for the section modulus.

- In CUR-publication 166, the reduction factor $R_l$ for the moment of inertia is always smaller than the reduction factor $R_w$ for the section modulus. $R_l$ is always smaller than $R_w$ when the rotation of the neutral axis is constant over the entire length of the sheet pile. If the rotation of the neutral axis varies over the entire length of the sheet pile, $R_l$ can also be bigger than $R_w$. From this follows that in CUR-publication 166, the implicit assumption is made that the rotation of the neutral axis is constant over the entire length of the sheet pile.

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Recommendations

- A further investigation of what influence other types of sheet pile constructions, such as sheet pile walls with anchors, struts or walings, have on reducing oblique bending.

- In order to investigate the influence of friction in the sliding interlock on oblique bending, it is recommended to model a sheet pile with friction in the sliding interlocks. For instance, this friction can be modeled with springs.
References


Annex A: Analytical solution of deflection \( w_y(x) \) with springs in the interlock

In order to solve the differential equation

\[
\frac{E(I_{yy}I_{zz} - I_{zz}I_{yy})}{I_{zz}} \frac{d^3w_y(x)}{dx^3} = 2b \left[ -\frac{I_{yy}}{I_{zz}} q_1 \left( x - \frac{1}{2} l \right) + 2bk \frac{dw_y(x)}{dx} \right]
\]

we have to simplify the expression by introducing three constants

\[
\frac{d^3w_y(x)}{dx^3} = A + Bx + C \frac{dw_y(x)}{dx}
\]

with:

\[
A = \frac{bI_{yy}q_1}{E(I_{yy}I_{zz} - I_{zz}I_{yy})}, \quad B = -\frac{2bI_{yy}q_1}{E(I_{yy}I_{zz} - I_{zz}I_{yy})}, \quad C = \frac{4b^2kI_{zz}}{E(I_{yy}I_{zz} - I_{zz}I_{yy})}
\]

We assume that the particular solution is a polynomial of the second degree:

\[
w_y(x)_{part} = K + Lx + Mx^2
\]

It follows that:

\[
\frac{dw_y(x)}{dx} = L + 2Mx \quad \frac{d^2w_y(x)}{dx^2} = 2M \quad \frac{d^3w_y(x)}{dx^3} = 0
\]

Substituting these expressions for \( w_y(x)_{part} \) and its derivatives in the differential equation and collecting terms, we obtain:

\[
0 = A + Bx + C \left[ L + 2Mx \right]
\]

Equating coefficients of like powers of \( x \), we find the expressions for \( L \) and \( M \).

\[
0 = Bx + 2CMx \quad \Rightarrow \quad M = -\frac{B}{2C}
\]

\[
0 = A + CL \quad \Rightarrow \quad L = -\frac{A}{C}
\]

Hence a particular solution of the differential equation is:

\[
w_y(x)_{part} = -\frac{A}{C} x - \frac{B}{2C} x^2
\]

We assume that \( w_y(x)_{hom} = e^{rx} \), and it then follows that \( r \) must be a root of the characteristic equation:

\[
r^2 = C
\]

Thus the possible values for \( r \) are \( r_1 = \sqrt{C} \) and \( r_2 = -\sqrt{C} \). The homogenous solution of the differential equation is then:
The general solution is the sum of the particular and the homogeneous solution and becomes:

\[ w_y(x) = \frac{A}{C} x - \frac{B}{2C} x^2 + C_1 e^{\sqrt{C}x} + C_3 e^{-\sqrt{C}x} \]

For this problem of a simply supported sheet pile with a uniformly distributed load, the following boundary conditions apply:

\[ w_y(0) = 0 \quad \Rightarrow \quad C_1 = -C_2 - C_3 \]
\[ M_y(0) = 0 \quad \Rightarrow \quad C_2 = \frac{B}{C^2} - C_3 \quad \Rightarrow \quad C_1 = -\frac{B}{C^2} \]
\[ w_y(L) = 0 \quad \Rightarrow \quad C_3 = -\frac{ACl + \frac{1}{2} BCl^2 + B - Be^{\sqrt{C}L}}{C^2 \left[ e^{\sqrt{C}L} - e^{-\sqrt{C}L} \right]} \]

Thus the solution for this particular case is:

\[ w_y(x) = -\frac{A}{C} x - \frac{B}{2C} x^2 - \frac{B}{C^2} e^{\sqrt{C}x} + C_3 \left[ e^{-\sqrt{C}x} - e^{\sqrt{C}x} \right] \]
Annex B: Example of cantilever sheet pile with equal loading conditions in y- and z-direction

The deflection formulae for a cantilever sheet pile with a uniformly distributed load $q_y$ in the z-direction and $q_z$ in the y-direction are:

$$w_y(x) = -\frac{I_{yy}}{E(I_{yy}I_{zz} - I_{yz}I_{zy})} \frac{1}{24} q_y \left[ x^4 - 4lx^3 + 6l^2x^2 \right] + \frac{I_{zz}}{E(I_{yy}I_{zz} - I_{yz}I_{zy})} \frac{1}{24} q_z \left[ x^4 - 4lx^3 + 6l^2x^2 \right]$$

$$w_z(x) = \frac{I_{yy}}{E(I_{yy}I_{zz} - I_{yz}I_{zy})} \frac{1}{24} q_y \left[ x^4 - 4lx^3 + 6l^2x^2 \right] - \frac{I_{zz}}{E(I_{yy}I_{zz} - I_{yz}I_{zy})} \frac{1}{24} q_z \left[ x^4 - 4lx^3 + 6l^2x^2 \right]$$

The formulae for the bending moments $M_y(x)$ and $M_z(x)$ produced by the loads $q_y$ and $q_z$ respectively are:

$$M_y(x) = q_y (-x^2 + 2lx - l^2)$$

$$M_z(x) = q_z (-x^2 + 2lx - l^2)$$

From earlier results it was found that oblique bending is fully impeded when $q_y(x) = -2.58 - q_z(x)$. Because in this case no rotation of the neutral axis occurs and no transverse deflections, this case will not be dealt with. We will concentrate on a reduction of oblique bending at the end of the cantilever sheet pile with 50%.

When free oblique bending occurs, the transverse deflection $w_y$ at the end ($x = l$) of the cantilever pile is:

$$w_{y,\text{free}}(l) = -\frac{I_{yy}}{E(I_{yy}I_{zz} - I_{yz}I_{zy})} \frac{1}{8} q_z l^4$$

If we want to halve this transverse deflection, the transverse load $q_y(l)$ that is needed can be found by:

$$w_y(l) = \frac{1}{2} w_{y,\text{free}}(l) = -\frac{I_{yy}}{E(I_{yy}I_{zz} - I_{yz}I_{zy})} \frac{1}{16} q_z l^4$$

$$q_y = \frac{1}{2} \frac{I_{yy}}{I_{zz}} q_z$$

When this transverse load is applied in the y-direction together with the lateral load $q_z$ in the z-direction. The following deflection curves and bending moments can be derived.
From this figure it can be seen that the deflection in every cross-section of the sheet pile is reduced by 50%. This applies for deflections in the z-direction as well in the y-direction. The reduction factors can be derived with the theory presented in chapter 2. In this case the reduction factors are: $R_z = 0.66$ and $R_{yw} = 0.76$. Because the rotation of the neutral axis $\beta$ is constant over the entire length of the sheet pile, the reduction factor $R_z$ is smaller than the reduction factor $R_{yw}$.

In the following figure, the load angle $\gamma$ and the angle of rotation of the neutral axis $\beta$ are depicted. Because the distribution of the bending moments $M_z(x)$ and $M_y(x)$ over the sheet pile are congruent (equal in shape), the load angle $\gamma$ is constant along the sheet pile. The angle of rotation of the neutral axis $\beta$ is also constant in case of congruent bending moments.
Annex C: Example of cantilever sheet pile with different loading conditions in y- and z-direction

The deflection formulae for a cantilever sheet pile with a uniformly distributed load $q_z$ in the z-direction and a point load $F_y$ in the y-direction at the end of the pile is:

$$ w_y(x) = -\frac{I_{yz}}{E(I_{yy}I_{zz} - I_{yz}^2)} \frac{1}{24} q_z \left[ x^4 - 4lx^3 + 6l^2x^2 \right] - \frac{I_{zz}}{E(I_{yy}I_{zz} - I_{yz}^2)} \frac{1}{6} F_y \left[ x^3 - 3x^2l \right] $$

$$ w_z(x) = \frac{I_{yy}}{E(I_{yy}I_{zz} - I_{yz}^2)} \frac{1}{24} q_z \left[ x^4 - 4lx^3 + 6l^2x^2 \right] + \frac{I_{yz}}{E(I_{yy}I_{zz} - I_{yz}^2)} \frac{1}{6} F_y \left[ x^3 - 3x^2l \right] $$

In order to impede oblique bending at the end ($x = l$) of the cantilever pile, the transverse deflection $w_y$ has to be zero in this point, yielding:

$$ w_y(l) = 0 \quad \Rightarrow \quad F_y = \frac{3}{8} \frac{I_{yz}}{I_{zz}} q_z l $$

which makes the transverse bending moment $M_y(x)$:

$$ M_y(x) = F_y (x - l) \quad \text{or} \quad M_y(x) = \frac{3}{8} \frac{I_{yz}}{I_{zz}} q_z l (x - l) $$

When the point load $F_y$ is applied in the y-direction together with the lateral load $q_z$ in the z-direction. The following deflection curves and bending moments can be derived.
Because now two different kinds of loads are applied on the sheet pile, a distributed load $q_z$ and a point load $F_z$, the deflections are not reduced in the same amount in every cross-section. For example, there is still a deflection $w_y$ in the middle of the sheet pile. Again, the reduction factors can be derived with the theory presented in chapter 2. In this case the reduction factors are, $R_l = 1$ and $R_w = 0.86$. This is an example of a situation were the rotation of the neutral axis $\beta$ varies over the length of the sheet pile and the reduction factor $R_l$ is bigger than the reduction factor $R_w$.

In the following figure, the load angle $\gamma$ and the angle of rotation of the neutral axis $\beta$ are depicted. Because the distribution of the bending moments $M_y(x)$ and $M_z(x)$ over the sheet pile are not congruent, the load angle $\gamma$ varies along the sheet pile axis. As a consequence, the angle of rotation of the neutral axis $\beta$ also varies. This figure also shows that oblique bending does occur when $\beta = 0^\circ$. 