Performance of Micropiles Under axial tensile loading

L.A. Meerdink
Master Thesis - Appendices
June 2013
LIST OF SYMBOLS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>c</td>
<td>cohesion</td>
<td>[kN/m²]</td>
</tr>
<tr>
<td>C</td>
<td>Constant</td>
<td>[-]</td>
</tr>
<tr>
<td>D</td>
<td>Pile diameter</td>
<td>[m]</td>
</tr>
<tr>
<td>E_p</td>
<td>Young’s modulus of the pile</td>
<td>[kN/m²]</td>
</tr>
<tr>
<td>f_3</td>
<td>(corrected) Reduction factor due to the length effect</td>
<td>[-]</td>
</tr>
<tr>
<td>f_3,workingpile</td>
<td>(uncorrected) Reduction factor of the final pile</td>
<td>[-]</td>
</tr>
<tr>
<td>f_3,testpile</td>
<td>(uncorrected) Reduction factor of the testpile</td>
<td>[-]</td>
</tr>
<tr>
<td>G_ave</td>
<td>Average shear modulus of the soil</td>
<td>[kN/m²]</td>
</tr>
<tr>
<td>G_soil</td>
<td>Soil shear modulus</td>
<td>[kN/m²]</td>
</tr>
<tr>
<td>K</td>
<td>Pile compressibility ratio</td>
<td>[-]</td>
</tr>
<tr>
<td>K_0</td>
<td>Neutral soil pressure coefficient</td>
<td>[-]</td>
</tr>
<tr>
<td>K_c creep</td>
<td>Axial stiffness due to creep of the soil</td>
<td>[kN/m]</td>
</tr>
<tr>
<td>K_heave</td>
<td>Axial stiffness due to heave of the soil</td>
<td>[kN/m]</td>
</tr>
<tr>
<td>K_length</td>
<td>Axial stiffness due to lengthening of the pile</td>
<td>[kN/m]</td>
</tr>
<tr>
<td>K_shaft</td>
<td>Axial stiffness due to shaft mobilisation</td>
<td>[kN/m]</td>
</tr>
<tr>
<td>L</td>
<td>Pile length</td>
<td>[m]</td>
</tr>
<tr>
<td>p_o'</td>
<td>Effective overburden stress at the point in question</td>
<td>[kN/m²]</td>
</tr>
<tr>
<td>P_b</td>
<td>Load at the base</td>
<td>[kN]</td>
</tr>
<tr>
<td>r_m</td>
<td>Maximum radius of influence</td>
<td>[m]</td>
</tr>
<tr>
<td>w_b</td>
<td>Displacement of the base</td>
<td>[m]</td>
</tr>
<tr>
<td>R</td>
<td>Pile radius</td>
<td>[m]</td>
</tr>
<tr>
<td>R_f</td>
<td>Reduction factor</td>
<td>[-]</td>
</tr>
<tr>
<td>R_f_rep</td>
<td>Representative value of the bearing capacity</td>
<td>[kN]</td>
</tr>
<tr>
<td>Q_actual</td>
<td>Actual pile capacity</td>
<td>[kN]</td>
</tr>
<tr>
<td>Q_rigid</td>
<td>Ideal capacity of a rigid pile</td>
<td>[kN]</td>
</tr>
<tr>
<td>dP</td>
<td>Micropile initial spring stiffness</td>
<td>[kN/m]</td>
</tr>
<tr>
<td>β</td>
<td>Coefficient from Table 1.2</td>
<td>[-]</td>
</tr>
<tr>
<td>δ_f</td>
<td>Interface friction angle at failure between pile and soil</td>
<td>[degrees]</td>
</tr>
<tr>
<td>δ_h</td>
<td>Boundary displacement</td>
<td>[mm]</td>
</tr>
<tr>
<td>Δσ'_r</td>
<td>Change in the effective stress during loading</td>
<td>[N/mm²]</td>
</tr>
<tr>
<td>Δw_res</td>
<td>Local displacement needed for degradation from peak to residual stress</td>
<td>[mm]</td>
</tr>
<tr>
<td>f</td>
<td>Unit Skin friction</td>
<td>[kN/m²]</td>
</tr>
<tr>
<td>λ_f</td>
<td>FOREVER’s effective reference length</td>
<td>[m]</td>
</tr>
<tr>
<td>λ_m</td>
<td>Misra’s scaling factor</td>
<td>[-]</td>
</tr>
<tr>
<td>λ_r</td>
<td>Randolhs effective reference length</td>
<td>[m]</td>
</tr>
<tr>
<td>λ_r,L_b</td>
<td>Slenderness ratio</td>
<td>[-]</td>
</tr>
<tr>
<td>ν_p</td>
<td>Poisons ratio for the pile</td>
<td>[-]</td>
</tr>
<tr>
<td>ν_s</td>
<td>Poisons ratio for the soil</td>
<td>[-]</td>
</tr>
<tr>
<td>σ_h</td>
<td>Effective horizontal stress</td>
<td>[kN/m²]</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td>Unit</td>
</tr>
<tr>
<td>--------</td>
<td>------------------------------------------</td>
<td>--------</td>
</tr>
<tr>
<td>$\sigma'_{rc}$</td>
<td>Local equilibrium effective stress</td>
<td>[N/mm$^2$]</td>
</tr>
<tr>
<td>$\sigma'_{rf}$</td>
<td>Radial effective stresses on the shaft at failure</td>
<td>[N/mm$^2$]</td>
</tr>
<tr>
<td>$\tau_f$</td>
<td>Ultimate shaft shear stress</td>
<td>[N/mm$^2$]</td>
</tr>
<tr>
<td>$\tau_s$</td>
<td>Ideal shaft friction at any depth</td>
<td>[kN/m$^2$]</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>Base stiffness</td>
<td>[-]</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Degree of strain softening</td>
<td>[-]</td>
</tr>
</tbody>
</table>

Other parameters are given in the list of symbols in the report.
CONTENTS

LIST OF SYMBOLS ................................................................................................................. 2
A1 LITERATURE STUDY ........................................................................................................... 7
  1.1. Axial pile performance ............................................................................................... 7
  1.2. Axial spring stiffness ............................................................................................... 9
  1.3. Maximum bearing capacity .................................................................................... 11
  1.4. Pile head displacement .......................................................................................... 14
A2 LINEAR ELASTIC PILE ANALYSIS ........................................................................... 21
  2.1. FOREVER .................................................................................................................. 21
  2.2. Misra ......................................................................................................................... 23
  2.3. Correlation Misra ($\lambda_m$) and FOREVER ($\lambda_f$) .............................................. 25
A3 STRUCTURAL BEHAVIOUR ............................................................................................ 27
  3.1. Bond hollow reinforcement-grout ........................................................................... 27
  3.2. Horizontal cracking ................................................................................................. 27
  3.3. Combined stiffness ................................................................................................. 32
A4 SOIL BEHAVIOUR ........................................................................................................ 35
  4.1. Shear stress and vertical loading direction ............................................................... 35
  4.2. Softening and the lengthening effect ...................................................................... 42
A5 FINAL MODEL – ANALYTICAL FORMULATION ...................................................... 47
  5.1. Bearing capacity (ULS) ........................................................................................... 47
  5.2. Displacement (SLS) .................................................................................................. 48
  5.3. Micropile’s performance ......................................................................................... 51
A6 FINAL MODEL – EXCEL FILE ................................................................................... 53
A7 MODEL VALIDATION .................................................................................................... 55
REFERENCES ....................................................................................................................... 57
For many years the design of pile foundations was primarily focussed on bearing capacity and based on experience. The pile head displacement is nowadays more important and also more intensively investigated. Different publications discuss the design and execution methods of micropiles. For example, Eurocode 7 [1] gives the general rules about geotechnical design, while NEN 9997 [2] is the Dutch implementation version of this Eurocode. The rules for tension piles of CUR publication 2001-4 [3] are implemented in this norm. Special rules for tension piles of micropiles in EN 14199 [4] are the execution method in CUR publication 236 [5]. Other countries have different norms and regulations. The German DIN 1054 [6] gives rules for ‘piles’ while the ISO 19902 [7] and American API RP 2A-WSD [8] are offshore design norms. Different authors and research projects have investigated the pile behaviour as well Randolph [9, 10] focussed on the offshore piles but discussed the slenderness as an important factor as well. This is useful in micropile design. In France the “Fondations Renforcées Verticalement” [11] was a extensive research project in the ‘90 to the performance of micropiles. Research was done to single and network piles, static and cyclic loading. Many more authors focussed on the calculation of the pile head displacement. This appendix discusses the axial pile performance of micropiles and the previous research on this topic to calculate their load-displacement behaviour. The axial stiffness, displacement and bearing capacity are discussed separately.

1.1. Axial pile performance

The performance of a micropile is given by the load-displacement graph. This gives the displacement of the pile head under a certain load. The load-displacement graph of tests in Saint-Remy on type II micropiles under compression are given in Figure 1.1. This form of the load-displacement graph is general for similar executed piles. In Figure 1.2 the load-displacement graph is given for different micropile tests under tension in Leeuwarden. In this project two different execution methods are applied: high frequency driven (VF, type E) and double tube inner drilled (GEWI, type A) micropiles. There is a difference between the behaviour in the ‘GEWI’ and ‘VF’ piles: the GEWI micropiles failed under the test load, while the VF micropiles could still bear more load without causing extreme displacement of the pile head.

Load-displacement graphs are used in the design of foundations: when the load of a construction on top of a pile is known, the displacement of the pile head can be derived. The relation between the two is called the axial spring stiffness of the pile and the load-deformation graph indicates a non-linear behaviour of this stiffness: the displacement...
increases fast when loading a pile over 80% of its maximum capacity. Because displacements should be within acceptable limits, the axial spring stiffness of micropiles is important in the design. Different systems can be assumed to cause this non-linear behaviour of the pile stiffness.

At first, the development of the shear stress is important in the design of micropiles [9]. With their long length, slender micropiles are assumed to be compressible. The difference in displacements on pile top (closest to soil level) and pile tip (deepest in the soil) can be substantial. While the soil-pile shear stress on the bottom of the pile will start developing shear stress, the top part can already have developed the maximum shear stress or exceeded this limit (see Figure 1.3). Depending on the soil conditions, softening may occur and therefore the residual shear stress can be sometimes about 50% of the maximum shear stress when having high displacements (Figure 1.4). The softening process can lower the maximum bearing capacity, and therefore ought to be taken into account as the lengthening effect. This softening is investigated in chapter 6 and appendix A4.2.

Second, grout and steel can have big differences in properties of in interaction and therefore in their behaviour. The brittle material grout can resist a lot of compression but only small tension stresses. Steel can handle big compression as well as tension stresses. From this difference in material their interaction is important. Under high loads, two possible interactions on the steel-grout interface are assumed, debonding and (ideal) bonding. When the deformations in the steel and grout differ, the grout can debond from the steel. In this case, the maximum bearing capacity lowers and the deformations increase as the stiffness of that part of the pile consists only of steel. Assuming this bond is perfect (ideal), the stress in the grout body can be too high. Cracking is expected, lowering the pile stiffness and increasing deformations as result. The fact that these possible failure modes occur in the soil makes it difficult for research, so not a lot of data is present about the steel/grout interface and the behaviour of grout. The interaction and behaviour of grout is investigated in chapter 5.

Furthermore some authors write that there is a difference in shaft capacity for pull of push loading situations, use different soil models and claim that reinforcement hasn’t a constant Young’s modulus. These topics together with the grout and softening are investigated in chapter 5.
1.2. **Axial spring stiffness**

The axial spring stiffness of micropiles gives the relation between the load on top and the corresponding pile head displacement (Eq. 1). Due to the non-linear behaviour, the pile spring stiffness is not a constant value. The value can be calculated from the load-displacement graph.

\[ K_{pile} = \frac{P_0}{u_{head}} \]  

Eq. 1

- **\( K_{pile} \)**: Axial spring stiffness of the pile  
  - Unit: [kN/m]

- **\( P_0 \)**: Micropile load  
  - Unit: [kN]

- **\( u_{head} \)**: Displacement of the pile head  
  - Unit: [m]

The bearing capacity and the displacement can be calculated separately and use Eq. 1 to obtain the axial spring stiffness of the micropile. CUR 236 used this method and divides the axial pile stiffness (or displacements) in a lengthening and pile tip displacement-part. On the other hand Misra [12] and Randolph [13] give direct calculation methods to obtain the spring stiffness.

**CUR 236 [5]**

In the CUR 236 the axial pile stiffness of the pile depends on the lengthening of the pile, mobilisation of the shaft friction, creep and heave of the soil (Eq. 2). For the lengthening it is assumed that the shaft development is equal over the full length of the pile. It is advised to only use the EA of the reinforcement, since the added value of the grout for the stiffness is not clear. The shaft mobilisation is related to the pile type, soil, actual load on top and maximum capacity. Creep and heave will be considered as location-dependent and valid for all constructions in that area. Therefore these factors will not be taken into this calculation.

\[ \frac{1}{K_{pile}} = \frac{1}{K_{length}} + \frac{1}{K_{shaft}} + \frac{1}{K_{creep}} + \frac{1}{K_{heave}} \]  

Eq. 2

\[ K_{length} = \frac{(EA)_{pile}}{L_{eff}} \]  

Eq. 3

\[ L_{eff} = L_{free} + 0.5L_b \]  

Eq. 4

\[ K_{shaft} = C \cdot R_{t,rep} \]  

Eq. 5

- **\( K_{length} \)**: Axial stiffness due to lengthening of the pile  
  - Unit: [kN/m]

- **\( K_{shaft} \)**: Axial stiffness due to shaft mobilisation  
  - Unit: [kN/m]

- **\( K_{creep} \)**: Axial stiffness due to creep of the soil  
  - Unit: [kN/m]

- **\( K_{heave} \)**: Axial stiffness due to heave of the soil  
  - Unit: [kN/m]

- **\( L_{eff} \)**: Effective length  
  - Unit: [-]

- **\( L_b \)**: Bond length  
  - Unit: [m]

- **\( (EA)_{pile} \)**: Stiffness of a pile  
  - Unit: [kN]

- **\( C \)**: Constant, from table 8.1 of CUR 236. Depends on pile type.  
  - Between 120 and 170.

- **\( R_{t,rep} \)**: Representative value of the bearing capacity  
  - Unit: [kN]

The axial stiffness of the shaft mobilisation may also be taken related to the load and displacement corresponding to Figure 1.5.
The axial spring stiffness used in designs has to be defined; CUR 236 uses the stiffness $k_{50}$, the stiffness at 50% of the maximum capacity ($R_t$) to determine the pile tip displacement. This seems reasonable: due to safety factors (in the bearing capacity as well as the load on top) the final (representative) load on the micropile will be about 50% of the maximum (design) capacity $R_{td}$ of the micropile. Due to use of safety factors in the design, the most interesting zone of the development of the axial stiffness is in general between 25 and 75% of the maximum capacity $R_t$.

**Misra [12]**

Misra gives a direct formulation for the initial stiffness of the micropile, depending on the local soil conditions ($k_\tau$), length ($L_b$), stiffness of bond ($EA_{pile}$) and free ($EA_{steel}$) length. In his examples, the EA of the pile consists of the contribution of the steel and grout.

$$\frac{dP}{du} = \frac{P_u L}{(EA)_p} \cdot \frac{\lambda_m (EA)_p (EA)_s}{(EA)_s L_p \coth \lambda_m + (EA)_p L_{free} \lambda_m}$$  \hspace{1cm} \text{Eq. 6}

$$\lambda_m = \sqrt{\frac{k_c \pi D_p L_p^2}{(EA)_p}}$$  \hspace{1cm} \text{Eq. 7}

<table>
<thead>
<tr>
<th>$\frac{dP}{du}$</th>
<th>Micropile initial spring stiffness [kN/m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_c$</td>
<td>Soil spring stiffness [kN/m$^3$]</td>
</tr>
<tr>
<td>$\lambda_m$</td>
<td>Misra’s scaling factor [-]</td>
</tr>
<tr>
<td>$P_u$</td>
<td>Micropile ultimate pull out capacity [kN]</td>
</tr>
<tr>
<td>$L_b$</td>
<td>Bond length of the pile [m]</td>
</tr>
<tr>
<td>$L_{free}$</td>
<td>Free / debonded length of the pile [m]</td>
</tr>
<tr>
<td>$D_p$</td>
<td>Diameter of the pile [m]</td>
</tr>
<tr>
<td>$EA_{pile}$</td>
<td>Stiffness of the pile [kN]</td>
</tr>
<tr>
<td>$EA_{steel}$</td>
<td>Stiffness of the steel [kN]</td>
</tr>
</tbody>
</table>
Randolph [13]

Randolph includes the stiffness due to the bearing capacity of the pile tip in the formulation. $EA_p$ is given as the cross section rigidity of the pile. The equation is valid for single piles in a homogeneous layer.

$$K_{pile} = EA_p \lambda_r \frac{\Omega + \tanh(\lambda_r L_b)}{1 + \Omega \tanh(\lambda_r L_b)} \tag{Eq. 8}$$

$$\Omega = \frac{P_b}{w_b EA_p \lambda_r} \tag{Eq. 9}$$

$$\lambda_r L_b = \sqrt{\frac{k_c \pi D_p L_b}{EA_p}} \tag{Eq. 10}$$

<table>
<thead>
<tr>
<th>$K_{pile}$</th>
<th>Micropile spring stiffness [kN/m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_r$</td>
<td>Randolphs effective reference length [m]</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>Base stiffness [-]</td>
</tr>
<tr>
<td>$w_b$</td>
<td>Displacement of the base [m]</td>
</tr>
<tr>
<td>$P_b$</td>
<td>Load at the base [kN]</td>
</tr>
<tr>
<td>$\lambda_r L_b$</td>
<td>Slenderess ratio [-]</td>
</tr>
</tbody>
</table>

1.3. Maximum bearing capacity

The bearing capacity of a pile is determined by the shaft area and created shear stress at the shaft (Eq. 11). When piles are placed in groups or tension loads are assumed, factors can be used to take these aspects in the calculation. The Dutch [2,3,5] calculation method for the design of the maximum bearing capacity including the safety factors is given in Eq. 12. The method is based on the cone tip resistance measured by Cone Penetration Tests. Safe design is implemented by the use a maximum for the cone resistance $q_{c,z,exc}$ and by using factors $\xi, \gamma_s, \gamma_t$ in material and load.

$$R_t = A_{shaft} \cdot \tau_{max} \tag{Eq. 11}$$

$$R_{t,d} = \int_{0}^{L_b} \frac{\pi \cdot D_g \cdot f_1 \cdot f_2 \cdot f_3 \cdot \alpha_t \cdot q_{c,z,exc} \cdot \xi_{m,n} \cdot \gamma_{s,t} \cdot \gamma_{m,var,qc}}{Y_s \cdot Y_{m,var,qc}} d_z \tag{Eq. 12}$$

| $R_{t,d}$ | Design value of bearing capacity [MN] |
| $L_b$ | Length grout body for calculating capacity (bond length) [m] |
| $D_g$ | Diameter of the grout [m] |
| $f_1$ | factor for the effect of compressing, is with sand and soil compressing piles (only for pile groups) [-] |
| $f_2$ | factor for lowering the effective stress by the tension force, only for sands (only for pile groups) [-] |
| $f_3$ | Lengthening factor [-] |
| $\alpha_t$ | shaft friction coefficient that takes the influence of the installation of the pile into account (tables 6.1 and 6.2 in CUR 236) [-] |
| $q_{c,z,exc}$ | cone resistance at depth z, taking into account the possibility of an over-consolidation and an excavation [MPa] |
| $\xi_{m,n}$; $\gamma_s$; $\gamma_{m,var,qc}$ | Material and load factors for safe design [-] |

The area of the shaft is more or less defined; the determination of the shaft friction can be done in various ways. Therefore the determination of the maximum shear stress will be discussed below. The maximum shear stress in soil will differ with the depth: the
horizontal stresses in the soil will increase with increasing depth. This can be calculated by the soil volumetric weight and other soil properties or the resistance measured with cone penetration tests.

1.3.1. Theory

Different failure criteria are developed to determine the maximum possible shear stress. Physics states that friction is created by a stress and friction coefficient. In soil mechanics this maximum value is known as the Mohr-Coulomb failure criterion (Eq. 13). By knowing the horizontal stresses in the soil (via the vertical stresses and the neutral soil pressure coefficient (relation horizontal/vertical stresses)) and the cohesion the maximum shear stress can be defined. The appropriate value of $K_0$ will depend on the in-situ earth pressure coefficient, the method of installation of the pile and the initial density of the sand. In clean sand, the cohesion can be assumed as 0 and $K_0$ will be about 0.6 to 0.9. A graph is developed to obtain the factor $K_0$ [14].

\[
\tau_f = c + \sigma'_h \tan \delta \quad \text{Eq. 13}
\]

\[
\tau_f = c + K_0 \sigma'_v \tan \delta \quad \text{Eq. 14}
\]

\[
K_0 \approx 1 - \sin \phi \quad \text{Eq. 15}
\]

$c$: cohesion [kN/m$^2$]

$\sigma'_h$: Effective horizontal stress [kN/m$^2$]

$\delta$: Friction coefficient between the two materials [degrees]

$K_0$: neutral soil pressure coefficient [-]

![Figure 1.6 - Creating shear stress](image)](image)

![Figure 1.7 – Method to determine the $K_0$](image)

Other failure criteria in soil mechanics are Tresca, Drucker-Prager and Von Mises. All methods will need soil specific parameters. These can be obtained by lab tests directly, or indirect by using representative values for soil types, such as NEN6740 table 1 [16]. The soil type can be determined by a cone penetration test in combination with borings.

1.3.2. Direct empirical

The German DIN1054 gives a general, conservative way of determining this shaft friction: the kind of soil and pile and execution type determines its value. This brings the representative value of the shaft capacity, multiplying by a safety factor brings the design value. The values from Table 1.1 are valid for piles in tension or compression, in case no pile tests on location are done.
Table 1.1 - Characteristic values for the shaft friction with Verpressten Micropiles (with no pile tests, valid for tension and compression [6]

<table>
<thead>
<tr>
<th>Bodenart</th>
<th>$\sigma_{\text{ax}}$ [MN/m²]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mittel- und Großbäue</td>
<td>0.20</td>
</tr>
<tr>
<td>Staub und Kiesland</td>
<td>0.15</td>
</tr>
<tr>
<td>Bindiger Boden</td>
<td>0.10</td>
</tr>
</tbody>
</table>

Offshore design norms ISO19902 [7] and API RP [8] are basically for long, big, open steel pipes. Despite this difference with micropiles, the method to calculate the skin friction can still be used for tension micropiles. In these norms/regulations the shaft friction is related to the undrained shear strength (cohesive soils, alpha method) or the overburden pressure (cohesionless soils, beta method). For in cohesionless soils they now recommend methods based on CPT’s. These are not discussed because they are specially for open ended offshore driven piles. The beta method is to determine the skin friction is Eq. 16,

$$ f = \beta p_0' $$

Eq. 16

- $f$ Unit Skin friction $[kN/m^2]$
- $\beta$ Coefficient from Table 1.2
- $p_0'$ Effective overburden stress at the point in question $[kN/m^2]$

The coefficient $\beta$ is based on the soil classification and relative density (Table 1.2). It has a limit to represent maximum shear stress. These values are for open-ended piles, for full displacement piles these values are 25% higher.

Table 1.2 - Design parameters for cohesionless siliceous soils (open ended piles). [8]

<table>
<thead>
<tr>
<th>Relative density</th>
<th>Soil classification</th>
<th>Skin friction factor $\beta$</th>
<th>Limiting unit skin friction values $f$ [kPa (psi/ft²)]</th>
<th>Limiting unit end bearing values $f$ [kPa (psi/ft²)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very loose</td>
<td>Sand</td>
<td>Not applicable</td>
<td>Not applicable</td>
<td>Not applicable</td>
</tr>
<tr>
<td>Loose</td>
<td>Sand</td>
<td>0.29</td>
<td>67 (1,4)</td>
<td>12</td>
</tr>
<tr>
<td>Loose</td>
<td>Sand-silt&lt;sup&gt;0&lt;/sup&gt;</td>
<td>Not applicable</td>
<td>Not applicable</td>
<td>Not applicable</td>
</tr>
<tr>
<td>Medium dense</td>
<td>Silt</td>
<td>0.37</td>
<td>81 (1,7)</td>
<td>20</td>
</tr>
<tr>
<td>Dense</td>
<td>Silt</td>
<td>0.46</td>
<td>96 (2,0)</td>
<td>40</td>
</tr>
<tr>
<td>Very dense</td>
<td>Sand-silt&lt;sup&gt;0&lt;/sup&gt;</td>
<td>0.56</td>
<td>115 (2,4)</td>
<td>50</td>
</tr>
<tr>
<td>Dense</td>
<td>Sand</td>
<td>0.56</td>
<td>115 (2,4)</td>
<td>50</td>
</tr>
</tbody>
</table>

Al design limit from the API is that friction fatigue is not taken into account and also the overconsolidation not is implemented. The shaft friction and shaft area together define the representative value of the bearing capacity. For design values, the representative
value has to be divided by a factor (between 1.5 and 2.0) depending on the loading conditions

1.3.3. Cone Penetration Tests
Dutch calculation methods are based on the cone tip resistance measured by Cone Penetration Tests. Following the Dutch norms ([2][3][5] and previous norms) a simplification of the maximum shear stress on depth \( z \) is given by:

\[
\tau_{z} = \alpha_{t} \cdot q_{z,ext}
\]

Eq. 17

Shaft friction coefficient \( \alpha_{t} \) is an empirical specified value to relate the stress in the soil to the shear stresses that will develop on the shaft. This value \( \alpha_{t} \) is different for each type of (execution of) micropiles and has to be tested on the location. In Table 1.3 expected values are given.

<table>
<thead>
<tr>
<th>Type anchorpile</th>
<th>Maximum usable ( q_{c} ) [MPa]</th>
<th>Calculation diameter pile shaft [mm]</th>
<th>Range Diameter shaft [mm]</th>
<th>Lower boundary Factor ( \alpha_{t} ) (no in-situ tests)</th>
<th>Expected value Factor ( \alpha_{t} ) (with in-situ tests)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>20</td>
<td>D boretube + 20</td>
<td>180-200</td>
<td>0,011</td>
<td>0,017</td>
</tr>
<tr>
<td>B</td>
<td>20</td>
<td>D boretube + 20</td>
<td>180-200</td>
<td>0,011</td>
<td>0,017</td>
</tr>
<tr>
<td>C</td>
<td>20</td>
<td>D borepoint + 20</td>
<td>180-380</td>
<td>0,008</td>
<td>0,012</td>
</tr>
<tr>
<td>D</td>
<td>15</td>
<td>D blade</td>
<td>180-350</td>
<td>0,008</td>
<td>0,012</td>
</tr>
<tr>
<td>E</td>
<td>15</td>
<td>D tube</td>
<td>Ca 200</td>
<td>0,008</td>
<td>-</td>
</tr>
</tbody>
</table>

1.4. Pile head displacement
Several methods have been developed to analyse the response of axially loaded piles. Regardless the model used, the displacement of the pile head will be developed by the pile tip displacement and the lengthening of the pile (Eq. 18). Assuming a rigid pile, the pile will totally be pulled upwards. The displacement of the pile head is than only depending of the pile tip displacement. When a pile with a low elasticity is considered, the lengthening of the pile gives the displacement of the pile head. Something between is expected to be happening with micropiles: the pile deforms vertically but will also move in total.

\[
u_{\text{head}} = u_{\text{length}} + u_{\text{tip}} \quad \text{Eq. 18}
\]

\( u_{\text{head}} \) Displacement of the pile head [m]
\( u_{\text{length}} \) Lengthening of the pile [m]
\( u_{\text{tip}} \) Displacement of the pile tip [m]

The CUR 236 and DIN describe a direct method to obtain the pile head displacement. The calculation method of the CUR follows the determination of the axial pile stiffness described in 1.2. The DIN gives a relation based on the load. However, the displacement of the total pile has to be developed by shear stress and vice versa. Displacement is therefore depending on the actual value of the shear stress. And, as discussed before, micropiles are assumed to be compressible piles, with the shear stress is in a different state along the pile length.

Misra [12] and FOREVER [11] use an analytical method based on the development of shear stresses for single piles embedded in a homogeneous soil. But the stresses in the soil will vary with depth. For a Gibson soil, with increasing horizontal stress at greater depth, analytical calculation methods are also possible [13]. The methods suggested above are quite easy. To be able to implement the inhomogeneity of the soil, and the more realistic
non-linear shear stress-strain behaviour of soil, numerical methods are used. Calculations can be done by Finite Element Method (Yap [17]), Boundary Element Method and load-transfer methods (Randolph and Wroth [18] and Van Dalen [19]). A closer look is given to the load-transfer methods.

1.4.1. Load-transfer functions

When using the load-transfer method, the pile is divided in elements as can be seen in Figure 1.8. On each element the soil is acting, given by a spring representing a load-transfer function. When a compression load is assumed, also a vertical spring on the last element is added, the pile tip spring. The use of the load-transfer methods is popular, because different load-transfer functions can be used to model the soil and also soil-inhomogeneity can be implemented.

Many load-transfer functions can be taken to represent the soil. The first implementations of the load-transfer method were based on empirical data from instrumented piles. Theoretical load-transfer functions are now more used. In Table 1.4 some examples of load-transfer functions are given. When using the linear elastic-perfectly plastic load-transfer function in combination with a pile in homogeneous soil, the pile head displacement can be determined analytical. For the other methods iteration is needed. The calculation method using load-transfer functions is given in chapter 4. The two types of linear load-transfer curves will be discussed in detail and compared.

**Table 1.4 – Load-transfer functions.**

<table>
<thead>
<tr>
<th>Load-transfer function</th>
<th>Equation</th>
</tr>
</thead>
</table>
| API / Misra / FOREVER – linear elastic-perfectly plastic | \( \tau(z) = \tau_{\text{max}} \frac{u}{u_0} \text{ for } u < u_0 \)  
\( \tau(z) = \tau_{\text{max}} \text{ for } u > u_0 \) |
| Randolph and Wroth (1978) – linear | \( z_s = \frac{r_0 \tau_0}{G} \ln \left( \frac{r_m}{r_0} \right) \) |
| Kraft et al (1981) – hyperbolic | \( z_s = \frac{r_0 \tau_0}{G} \left[ \ln \left( \frac{r_m - \beta}{r_0 - \beta} \right) + \frac{\beta (r_m - r_0)}{(r_m - \beta)(r_0 - \beta)} \right] \) in which \( \beta = \frac{r_0 \tau_0 R_f}{\tau_{\text{max}}} \) |
| NEN 9997 – Hyperbolic soil model | \( u_{\text{tip}} = \frac{2 - r_f}{2 \cdot k_{s0}} \cdot \frac{\tau}{\tau_0} \) |

Figure 1.8 – Spring model for load-transfer
Linear elastic perfectly plastic: API, FOREVER

The API uses a linear relation between shear stress and local displacement. For sand they assume that with 2.5 mm local displacement the maximum shear stress has developed.

![Figure 1.9 – API load-transfer function [8]](image1)

FOREVER also uses a linear elastic-perfectly plastic model, as simple mode to predict the soils behaviour. The local displacement at which the maximum shear stress has developed is not discussed.

![Figure 1.10 - Soil shear boundary layer with a linear elastic-perfectly plastic model](image2)

![Figure 1.11 - Comparison Hyperbolic Soil and Mohr-Coulomb (linear) model](image3)

A schematisation of the linear elastic-perfectly plastic is given in Figure 1.10. The soil spring stiffness $k_z$ can be calculated by Eq. 19:

$$k_z = \frac{\tau(z)}{u(z)} = \frac{\tau_{\text{max}}}{u_0}$$  \hspace{1cm} Eq. 19

The local displacement $u_0$ at which point the maximum shear stress is reached can be determined by direct shear tests or a triaxial test. The maximum shear stress is discussed before. It can also be expressed as a soil spring stiffness in a one-dimensional representation:

$$k_q = \frac{\pi D \tau(z)}{u(z)} = \frac{\pi D \tau_{\text{max}}}{u_0}$$  \hspace{1cm} Eq. 20
Linear elastic: Randolph and Worth

Randolph and Worth assume that soil deformations around a pile shaft can be idealized by concentric cylinders in shear. So, the displacement of the soil due to the pile axial load is mostly vertical and radial displacements are negligible. This assumption corresponds with FEM analysis. So, the stress-strain relation that Randolph and Worth assume is linear as well, but other variables are used. The shear stress (Eq. 21) depends on the displacement $u$, soil shear modulus $G_{\text{soil}}$, diameter of the pile $D_p$, and the area of influence $r_m$. The Winkler spring constant $k$ can therefore be determined by eq.30. In case of floating piles $\delta$ is about 1.5. For piles with end-bearing this is higher, varying with slenderness ration $L/d$ and stiffness ratio $E_p/G_{\text{soil}}$, given Eq. 24.

$$\tau = \frac{2G_{\text{soil}}u}{\zeta d_p} \quad \text{Eq. 21}$$

$$\zeta = \ln \left( \frac{2r_m}{d_p} \right) \quad \text{Eq. 22}$$

$$k_q = \frac{\pi d \tau}{u} = \frac{2\pi G_{\text{soil}}}{\zeta} = \delta G_{\text{soil}} \quad \text{Eq. 23}$$

$$\delta = \frac{k_q}{G_s} \approx 1,3 \left( \frac{2(1+\nu)G_{\text{soil}}}{E_p} \right)^{1/40} \left[ 1 + 7 \left( \frac{L}{d_p} \right)^{0.6} \right] \quad \text{Eq. 24}$$

$$G_{\text{soil}} = \frac{E_{\text{soil}}}{2(1+\nu)} \quad \text{Eq. 25}$$

$\tau$ Shear stress at pile wall [kN/m²]

$G_{\text{soil}}$ Soil shear modulus [kN/m²]

$r_m$ Maximum radius of influence [m]

$u$ Local displacement [m]

Comparison between Randolph and Dutch practise

While Randolph uses the Elasticity modulus of soil, in the Netherlands cone penetration tests are used to obtain the Winkler springs (soil spring stiffness). The formulas of Randolph discussed above are used. Assuming dense sand, the following parameters are assumed for a ‘floating’ pile and calculated Winkler springs:

<table>
<thead>
<tr>
<th>$E_{\text{soil}}$</th>
<th>45000 kPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_p$</td>
<td>0.25</td>
</tr>
<tr>
<td>$d_p$</td>
<td>0.2 m</td>
</tr>
</tbody>
</table>

Despite CUR 236 does not use the load-transfer system, the soil spring stiffness (Winkler spring) $k$, can still be determined of the method. For the shaft mobilisation 50% values are used, as explained in paragraph 1.2. At 50% of the maximum capacity the corresponding displacement is 4 mm. This equation can also be reduced to the local situation and local displacement. It is then assumed that at 50% of the maximum shear stress 0,004 m local displacement is developed.

$$k_c = \frac{50\% \text{ of maximum shear stress}}{0.004 \text{ m local displacement}} = 125 \cdot \tau_{\text{max}} \quad \text{Eq. 26}$$

$\tau_{\text{max}}$ Maximum shear stress at pile wall [kN/m²]

$k_c$ Soil spring stiffness [kN/m³]
Assuming the same sand and pile as the previous example, but then calculated on the ‘Dutch way’, the Winkler spring can be defined, using:

\[ E_s = 3 \cdot q_c \quad \text{Eq. 27} \]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( q_c )</td>
<td>15000</td>
<td>kPa</td>
</tr>
<tr>
<td>( \sigma_0 )</td>
<td>1.2</td>
<td>%</td>
</tr>
<tr>
<td>( d_0 )</td>
<td>0.2</td>
<td>m</td>
</tr>
<tr>
<td>( u_0 )</td>
<td>0.004</td>
<td>m</td>
</tr>
</tbody>
</table>

\[ q_c = 15000 \text{ kPa} \]
\[ \sigma_0 = 1.2 \% \]
\[ d_0 = 0.2 \text{ m} \]
\[ u_0 = 0.004 \text{ m} \]

It can be concluded that the methods from Randolph and Wroth and CUR 236 assume about the same value for the Winkler springs.

1.4.2. **Lengthening of the pile**

The elongation of the micropile under tensile loading depends on the normal stress in the pile. The normal force in the pile follows from the load on top \( P_0 \) and shear stress that is already developed (Eq. 28). This can be seen from measurements in Figure 1.12. Using Hooke’s law (Eq. 29) and the stress definition (Eq. 30) the lengthening can be calculated: the strain on each location \( z \) of the pile depends on the pile stiffness and the normal force on that location. Enumerating all strains gives the lengthening of the micropile.

\[ N(z) = P_0 - \pi D \tau \cdot dz \quad \text{Eq. 28} \]

\[ \sigma = E \epsilon \quad \text{Eq. 29} \]

\[ \sigma = \frac{N}{A} \quad \text{Eq. 30} \]

\[ \epsilon(z) = \frac{N(z)}{EA} \quad \text{Eq. 31} \]

\[ u_{\text{length}} = \int_0^L \epsilon = \int_0^L \frac{N(z)}{EA_{\text{pile}}} \quad \text{Eq. 32} \]

\( \sigma \) Stress \([\text{kN/m}^2]\)
\( E \) Elasticity \([\text{kN/m}^2]\)
\( \epsilon \) (Local) deformation \([\text{m/m}]\)
\( EA_{\text{pile}} \) Stiffness of the pile, in this case only of the steel \([\text{kN}]\)
\( L \) Total length of the pile \([\text{m}]\)
\( N(z) \) Normal force in depth \( z \) \([\text{kN}]\)

The axial force in the pile depends on the development of the shaft friction and is therefore not constant. Assuming a constant value for stiffness \( EA \) of the pile, the strain will therefore not be a constant value, as also can be seen from Figure 1.12. The axial force in the pile can be concluded as an important factor for the pile lengthening.

There is some discussion about the \( EA \) of the pile. The CUR 236 advise to be sure and take only the \( EA \) of steel into account. FOREVER only takes the reinforcement into account. Misra uses in his examples the \( EA \) of both reinforcement and grout in the bond length. The bond length may be taken as the difference between drillhole depth and the casing depth. A closer look to the behaviour of grout and therefore its influence on the pile stiffness will be investigated in chapter 5.
In their calculation of pile displacement, CUR 236 simplifies the displacement due to lengthening by assuming a linear decrease of the normal force:

$$u_{\text{length}} = \frac{P_0 \cdot L_{\text{eff}}}{(EA)_{\text{pile}}}$$

Eq. 33

$L_{\text{eff}}$ effective length [kN/m]

$P_0$ Load on top of the pile [kN]

FOREVER only takes the groutbody-length into account and takes therefore 0.5$L_b$ as effective length. CUR 236 adds also the length of the free anchor to this: $L_{\text{free}} + 0.5L_b$.

### 1.4.3. Pile tip displacement

Many methods have been developed to determine the displacement of the pile tip when the load is under compression. End bearing capacity is then also developed. Tension loading doesn’t always immediately cause an upwards movement of the pile tip, this is related to the axial stiffness of the pile. Although, this is theory, it is difficult to measure in practice. The pile tip displacement follows from the shear stress and load-transfer function, when using the load-transfer method.

In the Dutch CUR 236 does not use load-transfer functions but gives for tension piles a standard value and the Hyperbolic Soil (HS) model (as in PLAXIS implemented) which can be used to predict the upward movement of the tip (i.e. the mobilisation of the shaft). The standard value (table 8.1 CUR236) is about 3.0-4.0mm, depending on the micropile type and if tests are performed on location. In the HS model, the displacement of the pile tip is related to the ratio: the load on top in relation to the ultimate representative load ($R_t$).

$$u_{\text{tip}} = \frac{2 - r_f \cdot P_0}{2 \cdot k_{50} / F_a}$$

Eq. 34

$r_f$ Failure ratio ($R_t/F_a$) = 0.8 (this value is taken from the original formula in [CUR 236])

$k_{50}$ Stiffness at 50% mobilisation of maximum shear stress [kN/m]

$P_0$ Load on top [kN]

$F_a = R_t/r_f$, asymptote of the function [kN/m²]
No method, but only a maximum value is given by the French investigation to micropiles in the research program ‘FOREVER’: the limit pile tip displacement is 5 mm. This is comparable with the Dutch norm when looking to the used curves to obtain the shaft and base mobilisation. Piles will be due to safety be loaded to 50-60% of their maximum bearing capacity (SLS). A displacement of 5 mm corresponds with 52% of the maximum bearing capacity (following eq 24 and corresponding Figure 1.5). A comparable graph is given for compression piles. 5 mm displacement corresponds to 65% of the maximum bearing capacity of compression piles.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_c$</td>
<td>Bearing capacity, including $\xi$</td>
<td>[kN/m²]</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Factor for uncertainty</td>
<td>[-]</td>
</tr>
<tr>
<td>$u_{50}$</td>
<td>Interface displacement at 50% of the shear stress</td>
<td>[m]</td>
</tr>
</tbody>
</table>
Misra, [12] and FOREVER[11] refer both their calculation methods to the pile analysis of Scott. Both methods are based on the equilibrium, strain and a linear stress-strain relation but there final formulas are presented different. Most important is therefore the different use of the parameter $\lambda$. FOREVER calls this $\lambda_f$ the effective reference length and implements the stiffness of the pile $EA$, linear development of the shear stress $k$, and the diameter of the pile $D_p$ in this. Misra defines this $\lambda_m$ the scaling factor and takes except for the previous parameters the bond length $L_b$ into account as well. In this appendix both methods are mathematical derived.

### 2.1. FOREVER

Equations F4 (second order differential equation) and F6 (development of the axial force along the pile) have to be proven.

<table>
<thead>
<tr>
<th>F1</th>
<th>equilibrium</th>
<th>$dN = \pi D_p \cdot \tau \cdot (dz)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>re-writing F1</td>
<td>$\frac{dN}{dz} = \pi D_p \cdot \tau$</td>
</tr>
<tr>
<td>F3</td>
<td>linear shear stress</td>
<td>$\tau(z) = k \cdot u(z)$</td>
</tr>
<tr>
<td>F2a</td>
<td>strain</td>
<td>$\varepsilon = \frac{dy}{dz}$</td>
</tr>
<tr>
<td>F2b</td>
<td>strain</td>
<td>$\varepsilon = \frac{N(z)}{EA}$</td>
</tr>
<tr>
<td>2</td>
<td>differentiate F3 to $z$</td>
<td>$\frac{d\tau}{dz} = k \frac{dy}{dz}$</td>
</tr>
<tr>
<td>3</td>
<td>combine eq. 2 &amp; F2a</td>
<td>$\frac{d\tau}{dz} = k \cdot \varepsilon$</td>
</tr>
<tr>
<td>4</td>
<td>combine eq. 3 &amp; F2b</td>
<td>$\frac{d\tau}{dz} = k \cdot \frac{N(z)}{EA}$</td>
</tr>
<tr>
<td>5</td>
<td>differentiate eq. 1 to $z$</td>
<td>$\frac{d^2N}{dz^2} = \pi D_p \cdot \frac{d\tau}{dz}$</td>
</tr>
<tr>
<td>6</td>
<td>combine eq. 5 &amp; 4</td>
<td>$\frac{d^2N}{dz^2} = \pi D_p \cdot \frac{k}{\tau} \cdot \frac{N(z)}{EA}$</td>
</tr>
<tr>
<td>7</td>
<td>re-writing eq. 6</td>
<td>$\frac{d^2N}{dz^2} - \frac{\pi D_p \cdot k}{\lambda_f^2} \cdot \frac{N(z)}{EA} = 0$</td>
</tr>
<tr>
<td>F4</td>
<td>definition [FOREVER, pp 31]</td>
<td>$\frac{d^2N}{dz^2} - \frac{N(z)}{\lambda_f^2} = 0$</td>
</tr>
<tr>
<td>F5</td>
<td>definition [FOREVER, pp 31]</td>
<td>( \lambda_f = \sqrt{\frac{EA}{k_c \pi D_p^2}} )</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>8</td>
<td>combine F4 &amp; F5</td>
<td>( \frac{d^2N}{dz^2} - \frac{N}{\lambda_f^2} = 0 )</td>
</tr>
<tr>
<td>9</td>
<td>re-writing eq. 8</td>
<td>( \frac{d^2N}{dz^2} - \frac{k_c \pi D_p^2}{EA} N = 0 )</td>
</tr>
<tr>
<td>F4</td>
<td>continue with eq. F4</td>
<td>( \frac{d^2N}{dz^2} - \frac{N}{\lambda_f^2} = 0 )</td>
</tr>
<tr>
<td>10</td>
<td>standard solution 2nd order diff (choice)</td>
<td>( N = A e^{\lambda_f z} + B e^{-\lambda_f z} )</td>
</tr>
<tr>
<td>F5a</td>
<td>boundary condition a</td>
<td>( P = P_0 ) if ( z = 0 )</td>
</tr>
<tr>
<td>F5b</td>
<td>boundary condition b</td>
<td>( P = 0 ) if ( z = L_0 )</td>
</tr>
<tr>
<td>13</td>
<td>combine eq. 12 and F5a</td>
<td>( P_0 = A e^{\lambda_f z} + B e^{-\lambda_f z} )</td>
</tr>
<tr>
<td>14</td>
<td>combine eq. 12 and F5b</td>
<td>( 0 = A e^{\lambda_f z} + B e^{-\lambda_f z} )</td>
</tr>
<tr>
<td>15</td>
<td>re-writing eq. 14</td>
<td>( 0 = A e^0 + B e^0 )</td>
</tr>
<tr>
<td>16</td>
<td>re-writing eq. 15</td>
<td>( B = -A )</td>
</tr>
<tr>
<td>17</td>
<td>combine eq. 16 in 13</td>
<td>( P_0 = A e^{\lambda_f z} - A e^{-\lambda_f z} )</td>
</tr>
<tr>
<td>18</td>
<td>re-writing eq. 17</td>
<td>( P_0 = A e^{\lambda_f z} - A e^{-\lambda_f z} )</td>
</tr>
<tr>
<td>19</td>
<td>re-writing eq. 18</td>
<td>( A = \frac{P_0}{(e^{\lambda_f z} - e^{-\lambda_f z})} )</td>
</tr>
<tr>
<td>20</td>
<td>Combine eq. 16 and 19</td>
<td>( B = \frac{-P_0}{(e^{\lambda_f z} - e^{-\lambda_f z})} )</td>
</tr>
<tr>
<td>21</td>
<td>combine eq. 19, 20 and 10</td>
<td>( N = \frac{-P_0}{(e^{\lambda_f z} - e^{-\lambda_f z})} e^{\lambda_f z} - \frac{-P_0}{(e^{\lambda_f z} - e^{-\lambda_f z})} e^{-\lambda_f z} )</td>
</tr>
<tr>
<td>22</td>
<td>re-writing eq. 21</td>
<td>( N = \left( \frac{P_0}{(e^{\lambda_f z} - e^{-\lambda_f z})} \right) \left( \frac{e^{\lambda_f z} - e^{-\lambda_f z}}{\lambda_f} \right) )</td>
</tr>
</tbody>
</table>
23 re-writing eq. 22
\[ N = P_0 \frac{e^{\frac{-L_b - z}{\lambda_f}} - e^{\frac{-L_b}{\lambda_f}}}{e^{\frac{L_b}{\lambda_f}} - e^{\frac{-L_b}{\lambda_f}}} \]

F6 definition
[FOREVER, pp 31]
\[ N(z) = P_0 \frac{\sinh \left( \frac{L_b - z}{\lambda_f} \right)}{\sinh \left( \frac{L_b}{\lambda_f} \right)} \]

RR definition
sinus hyperbolic
\[ \sinh(x) = \frac{e^x - e^{-x}}{2} \]

24 re-writing F6
\[ N(z) = \frac{N_f \cdot \left( e^{\frac{(L_b - z)}{\lambda_f}} - e^{\frac{-L_b}{\lambda_f}} \right)}{2} = \frac{N_f \cdot \left( e^{\frac{L_b}{\lambda_f}} - e^{\frac{-L_b}{\lambda_f}} \right)}{2} \]

23 \[\Rightarrow \] 24 23=34

When FOREVER had chosen the z-as positive in upward direction instead of downwards, it had the boundary conditions \( P=P_0 \) if \( z=L \), and \( P=0 \) if \( z=0 \). Then the formula would be more simple:

F6, but with different z-axis
\[ N(z) = P_0 \frac{\sinh \left( \frac{z}{\lambda_f} \right)}{\sinh \left( \frac{L_b}{\lambda_f} \right)} \]

2.2. Misra
Equations M2 (definition of the shear friction) and M3 (second order differential equation) have to be proven. Misra uses a normalised length in the calculation method. This suggests that the length of the pile is important in the development of the displacement along the pile. Finally Misra gives a formulation for the development of the displacement along the length of the pile, M5. This equation isn’t proven, but can be analysed on the same method used at FOREVER.

1 Hookes law
\[ \sigma = E \varepsilon \]

2 strain
\[ \varepsilon = \frac{\Delta l}{l} = \frac{du}{dz} = \frac{dy}{dz} = \frac{N(z)}{EA} \]

3 stress definition
\[ \sigma = \frac{N}{A} \]

4 re-writing eq. 3
\[ N = \sigma A \]

5 combine eq. 1 & 4
\[ N = E \varepsilon A \]

6 for every element is valid (eq.2 & 5)
\[ dN = E \frac{du}{dz} - \frac{\pi D^3}{4} \frac{d}{dz} (du + dz) \]

7 equilibrium
\[ dN = \pi D \cdot \tau \cdot (du + dz) \]
du is relatively small: neglect

\[ dN = \pi D_p \cdot \tau \cdot (dz) \]

combine eq. 8 & 6

\[ \pi D_p \cdot \tau \cdot (dz) = \frac{E}{d_z^2} \frac{du}{\pi D^2} \]

10 π and D can be scored out and re-writing eq. 9

\[ \tau = \frac{E}{d_z^2} \frac{d^2 u}{4} \]

11 q=shear stress / m length of the pile

\[ q = \tau \cdot \pi D_p \]

combine eq. 10 & 11

\[ q(z) = \tau(z) \cdot \pi D_p = \frac{E}{d_z^2} \frac{d^2 u}{4} D \cdot \pi D_p = \frac{E}{d_z^2} \frac{d^2 u}{A_p} \]

M2 definition [Misra eq. 2]

\[ q(z) = \frac{EA}{d_z^2} \]

12 12 = M2

13 linear shear stress

\[ \tau(z) = k_z u(z) \]

combine eq. 13 & 10

\[ \frac{E}{d_z^2} \frac{d^2 u}{4} D = k_z u(z) \]

re-write eq. 14

\[ \frac{d^2 u}{d_z^2} - \frac{4k_z}{ED} u(z) = 0 \]

divide eq. 15 by D

\[ \frac{d^2 u}{D_p d_z^2} - \frac{4k_z}{ED} u(z) = 0 \]

17 area circle

\[ A_p = \frac{\pi D_p^2}{4} \]

18 re-write eq. 16

\[ D_p^2 = \frac{4A_p}{\pi} \]

19 combine eq. 18 & 16

\[ \frac{d^2 u}{D_p d_z^2} - \frac{4k_z}{E4A} u(z) = 0 \]

20 Eq. 19 * D and re-write

\[ \frac{d^2 u}{d_z^2} - \frac{k_z \pi D_p}{E4A} u(z) = 0 \]

M3 definition [Misra eq. 3]

\[ \frac{d^2 u}{d \xi^2} - \lambda^2 u(\xi) = 0 \]

M4 definition [Misra eq. 4]

\[ \lambda^2 = \frac{k_z \pi D_p L_b^2}{E4A} \]

21 combine M4 and M3

\[ \frac{d^2 u}{d \xi^2} - \frac{k_z \pi D_p L_b^2}{E4A} u(\xi) = 0 \]

MT definition [Misra, from text]

\[ \xi = \frac{x}{L_b} \]

22 re-write MT

\[ x = L_b \xi \]

23 differentiate eq. 22

\[ dx = L_b \cdot d \xi \]

24 differentiate eq. 23

\[ dx^2 = L_b \cdot d \xi^2 \]
25 re-write eq.24

\[ dS = \frac{dx}{L_0} \]

26 combine MT, eq. 24 and 21

\[ \frac{d^2 u}{dx^2} - \frac{k_r \pi D_p L_0^2}{EA L_0} u(x) = 0 \]

27 score out L in eq. 26

\[ \frac{d^2 u}{dx^2} - \frac{k_r \pi D_p}{EA} u(x) = 0 \]

28 definition total displacement

\[ u = \int_0^L \varepsilon \]

29 Combining eq. 2 & 28

\[ \int_0^L N \]

\[ u = \frac{0}{EA} \]

M5 definition [Misra eq. 5]

\[ u(\xi) = \frac{P_0 L_0}{EA \lambda_m} \frac{\cosh \lambda_m \xi}{\sinh \lambda_m} \]

Misra also defined the transition point \( z_l \) in homogenous soil. At this point the shear stress and displacement are equal. This formulation given in M6 will not be proven.

M6 definition [Misra eq. 24]

\[ (\xi_0 - 1) - \frac{\tanh \lambda_m \xi}{\lambda_m} + \frac{P_0}{R} = 0 \]

### 2.3. Correlation Misra (\( \lambda_m \)) and FOREVER (\( \lambda_f \))

Misra defines its scaling factor:

\[ \lambda_m = \sqrt{\frac{k_r \pi D_p L_0^2}{EA}} \quad \lambda_m^2 = \frac{k_r \pi D_p L_0^2}{EA} \]

FOREVER defines its effective reference length:

\[ \lambda_f = \sqrt{\frac{EA}{k_r \pi D_p}} \quad \lambda_f^2 = \frac{EA}{k \pi D} \]

Combining by using the length:

\[ \frac{L_0^2}{\lambda_m^3} = \frac{L_0^2}{\lambda_f^3} = \frac{EA}{k_r \pi D_p} = \lambda_f' \]
In the report the grout cracking behaviour of massive (GEWI) reinforcement is discussed. Hollow reinforcement will behave different in combination with grout.

3.1. Bond hollow reinforcement-grout
The bond between the reinforcement and grout can be calculated using results of a pull out test. In Eq. 35a the general formulation is given [20], in Eq. 35b the bond stress-displacement relation for general massive reinforcement [21] and in Eq. 35 the bond stress-displacement relation for hollow tubes based on a TITAN pull out test [22].

\[
\tau_b = a \cdot f_{cm,cube} U_r^{b} \quad \text{Eq. 35a}
\]

\[
\tau_b = 0,38 f_{cm,cube} U_r^{0,18} \quad \text{Eq. 35b Mass}
\]

\[
\tau_b = 0,43 f_{cm,cube} U_r^{0,3} \quad \text{Eq. 35c Hol}
\]

As can be seen in Figure 3.1 this equation is valid up to a displacement of 0,7 mm and therefore limited to a crack width of 1,4 mm. Eq. 35c is with these constants only valid for the bond of TITAN tubes in combination with this grout. The formula given in Eq. 35b will be taken as valid for GEWI reinforcement. The type of reinforcement and its bond strength will have its influence on the development of cracks.

![Figure 3.1 – Development of the bond between the TITAN tube and grout, using the adapted formula.](image)

3.2. Horizontal cracking
In the calculation to obtain the cracking behaviour of micropiles it is assumed that the micropile behave as a reinforced concrete element. The generalized load-deformation graph of a reinforced concrete element loaded in tension in axial direction is given in Figure 3.2. Three phases can be distinguished: stiff and bonded (I), cracking (II) and totally cracked (III).
In the first branch the stiffness consists of the grout and steel together. At a certain point
(1), the strain in the cross section is higher than the tensile failure strain of grout: \( \varepsilon_{\text{cr}} \) is
reached. Then the grout cracks, reducing the tensile stress (and therefore strain) in the
grout in this cross section. At the location of the crack the stiffness in the cross section is
only determined by the steel and has corresponding deformations. With increasing load
the strain will exceed the tensile failure strain and new cracks can develop (cracking in
branch 2). This happens until all cracks are developed (\( \varepsilon_{\text{fcr}} \), point 2). In the third branch of
the graph all the cracks are fully developed. Therefore the number of cracks is assumed to
be constant. The stress in the steel can increase until the yield strength of the steel is
reached (corresponding to \( \varepsilon_{\text{sy}} \)). It can be seen that the stiffness of the fully developed
cracked element exceeds the stiffness of only the reinforcement. There is still a
contribution of the concrete between the cracks to the stiffness of the pile; this is called
tension stiffening (\( \Delta \varepsilon_{\text{ts}} \)). It is assumed that the tension stiffening has a constant value
throughout the stage of the fully developed crack pattern.

One difference between a ‘normal’ reinforced concrete element and a micropile is in the
load-transfer. While for a reinforced element the full load is transferred to the next
element and the axial force is over the length the same, in micropiles the load is
transferred to the soil resulting in a decrease in axial force. As a consequence the cracking
will start in the upper part of the pile: the axial force and therefore tensile stresses in the
grout are the highest here. With increasing load the cracking will continue to the pile tip.
Due to the development of the stresses the cracks are assumed to appear at almost equal
spacing: in this case a fully developed crack pattern is obtained. This is the opposite of
what would happen in case of imposed deformation (temperature) where the crack spacing would be irregular.

The calculation method (Eq. 36-Eq. 46) explained below is following [20,21] and is for ‘a
congrete element loaded in tension’, normally explained as columns and beams.
Micropiles have for example very bad and uncontrolled concrete (B15 instead of B35)
and a high reinforcement ratio (10% instead of maximum 4% in columns). Therefore it is
already known that with a high reinforcement ratio the tension stiffness might be very
low.
Normal crack force and tension stiffening

The influence of the steel has to be taken into account when calculating the normal force at which the grout starts to crack. The relation between the properties of grout and steel is then important. The crack force depends on the steel area compared with the grout area, therefore a difference is found between the crack force in hollow and massive bars. In Eq. 36 the calculation method is given.

\[ N_{cr} = \varepsilon_{e,cr} E_c A_c (1 + \alpha_c \rho) \quad \text{Eq. 36} \]

\[ \alpha_c = \frac{E_s}{E_c} \quad \text{Eq. 37} \]

\[ \rho = \frac{A_s}{A_g} \quad \text{Eq. 38} \]

- \( \alpha_c \): ratio of E-moduli
- \( \rho \): reinforcement ratio
- \( E_c A_c \): combined stiffness grout and steel [kN]
- \( N_{cr} \): Normal force at which the grout starts to crack in the pile [N]

Starting with the crack force, the strain at which the crack pattern has fully developed is given by (empirical) Eq. 39. The long term factor is assumed to have its maximum of 1,3.

\[ \varepsilon_{\text{floc,longterm}} = \gamma_o \cdot (60 + 2,4 \sigma_{s,cr}) \cdot 10^{-6} \quad \text{Eq. 39} \]

\[ \sigma_{s,cr} = N_{cr} / A_s \quad \text{Eq. 40} \]

- \( \varepsilon_{\text{floc}} \): Strain in grout at fully development crack pattern [m/m]
- \( \sigma_{s,cr} \): Stress in the steel directly after cracking [N/mm²]
- \( \gamma_o \): Long term factor, will be taken as 1,3 (maximum value) [mm²/N]

Crack width and distance

The crack width \( w_{mv} \) and the transition length \( l_{cr} \) can be calculated by Eq. 44 and Eq. 43 respectively, assuming mean and maximum values in the fully developed crack phase. The crack width and distance depends on the bond between the reinforcement and the grout as well as the stresses in the steal. The calculation will therefore depend on the type of reinforcement: massive or hollow bars. The basic calculation in Eq. 41a and b is corrected using Eq. 35b for massive reinforcement (Mas) and using Eq. 35c for hollow bars (Hol).

The crack width depends on the actual stresses; the stresses in a crack have to be taken by the steel and the pile will deform on this location by the steel elasticity. This is a linear relation. Due to corrosion protection-regulations the crack width has a limit. For reinforced concrete elements this limit is 0,1 mm, for micropile is it assumed to be 0,2 mm. The transition length describes the length that is needed for a crack to develop to a full crack. This value depends on many parameters. From practice it is known that in reinforced elements cracks will occur between 2 (average) and 3,7 (maximum) times the concrete cover thickness. The crack distance will therefore be 2 to 3,7 times the cover plus the length which is needed to create a full crack (Eq. 43).

\[ w_{mv} = 2U_{r,cr} = 2 \left\{ \frac{1 + b}{4} \frac{D^*_{cr}}{a \cdot f_{cm,cube} E_s} \frac{1}{1 + \alpha_c \rho} \right\}^{\frac{1}{1 + b}} \quad \text{Eq. 41a} \]

\[ D^*_{cr} = D_{s,cr} \frac{A_{bar}}{A_{nbar}} \quad \text{Eq. 41b} \]
\[ w_{mo} = 2 \left( \frac{0.4 D_{s}}{f_{cm,cube} E_s} \cdot \left( \frac{\sigma_{cr}}{\rho} \right)^2 \cdot (1 + \alpha, \rho) \right)^{0.85} \]  
Eq. 41c  
\[ w_{mo} = 2 \left( \frac{0.38 D^*}{f_{cm,cube} E_s} \cdot \left( \frac{\sigma_{cr}}{\rho} \right)^2 \cdot (1 + \alpha, \rho) \right)^{0.77} \]  
Eq. 41d  
\[ l_m = 1.5 l_{st} = 1.5 \cdot \frac{w_{mo} E_s}{1 - b \sigma_{s,cr}} \]  
Eq. 42a  
\[ l_m = 1.8 \frac{w_{mo} E_s}{\sigma_{s,cr}} \]  
Eq. 42b  
\[ l_m = 2.1 \frac{w_{mo} E_s}{\sigma_{s,cr}} \]  
Eq. 42c  
\[ l_{cr,ave} = 2 \epsilon_g + l_m \]  
Eq. 43a  
\[ l_{cr,max} = 3.7 \epsilon_g + l_m \]  
Eq. 43b  
\[ w_{w,ave} = \frac{l_{cr,ave} \left( \sigma_s - 0.5 \sigma_{s,cr} \right)}{E_s} \]  
Eq. 44a  
\[ w_{w,max} = \frac{l_{cr,max} \left( \sigma_s - 0.5 \sigma_{s,cr} \right)}{E_s} \]  
Eq. 44b  
\[ \sigma_{cr} = 0.60 f_{clm} \]  
Eq. 45  

**This calculation is done using a micropile with a hollow reinforcement: TITAN 103/78. The diameter of the grout is 300 mm and properties are given in Table 3.1. The results of the calculations for a load of 1500 (about 60% of Rt) are given in Table 3.2.**

**Table 3.1 – Dimensions of the TITAN pile and soil**

| D grout | 0.3 m | Esteel | 200*10^6 kN/m²^2 | f₁ | 1 |
| D steel in | 0.103 m | Egrout | 20*10^6 kN/m²^2 | f₂ | 1 |
| D steel out | 0.078 m | Esteel | 710785 kN | f₃ | 1 |
| α₁ | 0.012 | EAgroun | 1247070 kN | μₘₙ | 1 |
| a₀ | 15 MPa | Lₙ | 15 m |
| μ₀ | 0.005 m | Lₙ | 0 m | Rₜ | 2545 kN |

\[ D_{s} \quad \text{Diameter of the steel reinforcement} \]  
\[ c_{g} \quad \text{Grout cover} \]
Table 3.2 - The results are given for the TITAN pile under a loading of 1500 kN.

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>D_s out</td>
<td>103 mm</td>
</tr>
<tr>
<td>D_s in</td>
<td>78 mm</td>
</tr>
<tr>
<td>D_g</td>
<td>300 mm</td>
</tr>
<tr>
<td>E_s</td>
<td>20000 N/mm²</td>
</tr>
<tr>
<td>E_g</td>
<td>20000 N/mm²</td>
</tr>
<tr>
<td>ε_cr</td>
<td>0,1 %</td>
</tr>
<tr>
<td>ε_s,cr</td>
<td>0,35 %</td>
</tr>
<tr>
<td>ε_s,cr</td>
<td>0,44 %</td>
</tr>
<tr>
<td>f_cm,cube</td>
<td>23 N/mm²</td>
</tr>
<tr>
<td>σ_s,cr</td>
<td>88,4 N/mm²</td>
</tr>
<tr>
<td>σ_cr</td>
<td>9,66 N/mm²</td>
</tr>
<tr>
<td>σ_s</td>
<td>371 N/mm²</td>
</tr>
<tr>
<td>σ_s,cr</td>
<td>88,4 N/mm²</td>
</tr>
<tr>
<td>E_s</td>
<td>200000 N/mm²</td>
</tr>
<tr>
<td>E_g</td>
<td>20000 N/mm²</td>
</tr>
<tr>
<td>ε_s</td>
<td>0,35 %</td>
</tr>
<tr>
<td>ε_s,cr</td>
<td>0,44 %</td>
</tr>
<tr>
<td>ε_cr</td>
<td>0,1 %</td>
</tr>
<tr>
<td>f_cm,cube</td>
<td>23 N/mm²</td>
</tr>
<tr>
<td>σ_s,cr</td>
<td>88,4 N/mm²</td>
</tr>
<tr>
<td>σ_cr</td>
<td>9,66 N/mm²</td>
</tr>
<tr>
<td>σ_s</td>
<td>371 N/mm²</td>
</tr>
<tr>
<td>σ_s,cr</td>
<td>88,4 N/mm²</td>
</tr>
</tbody>
</table>

Output:

| l_cm max       | 438,4 mm        |
| α_e            | 10 [-]          |
| ρ              | 0,053 [-]       |
| w_mv ave       | 0,44 mm         |
| w_mv max       | 0,72 mm         |

The stress in the steel is 371, the maximum expected stress in serviceability state. In ultimate limit state the cracks are wider: at 454 N/mm² the crack width is about 0,54 mm. However, the permanent situation is valid for the calculation of the crack widths. The crack width exceeds the (assumed for micropiles) limit of 0,2 mm. When looking to this norm, the stress might not be more than 90 N/mm². This is a very low value and with normal loading corrosion might be expected.

Table 3.3 - The crack width for the TITAN pile under certain loadings. *Due to the material, the stresses above 371 N/mm² are not expected to happen under normal loading conditions.

<table>
<thead>
<tr>
<th>P₀ [kN]</th>
<th>σ₀ * [N/mm²]</th>
<th>w_mv ave [mm]</th>
<th>w_mv max [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>150</td>
<td>42</td>
<td>0,00</td>
<td>0,00</td>
</tr>
<tr>
<td>300</td>
<td>84</td>
<td>0,05</td>
<td>0,09</td>
</tr>
<tr>
<td>500</td>
<td>141</td>
<td>0,13</td>
<td>0,21</td>
</tr>
<tr>
<td>700</td>
<td>197</td>
<td>0,21</td>
<td>0,33</td>
</tr>
<tr>
<td>1000</td>
<td>281</td>
<td>0,32</td>
<td>0,52</td>
</tr>
<tr>
<td>1200</td>
<td>338</td>
<td>0,40</td>
<td>0,64</td>
</tr>
<tr>
<td>1400</td>
<td>394</td>
<td>0,47</td>
<td>0,77</td>
</tr>
<tr>
<td>1600</td>
<td>450</td>
<td>0,55</td>
<td>0,89</td>
</tr>
<tr>
<td>1800</td>
<td>506</td>
<td>0,63</td>
<td>1,01</td>
</tr>
</tbody>
</table>

Figure 3.3 – Crack widths for the TITAN pile, under various loads.

**Contribution of the grout due to tension stiffening**

After the cracks have fully developed the stiffness (EA) of the reinforced concrete element is equal to that of steel but the less strain has developed due to the tension stiffening (Figure 3.2). In this third branch of the graph the lengthening of the element is fully depended of the steel stiffness, but the grout between the ribs does not lengthen. Therefore it can not be concluded that the EA of the pile increases due to tension stiffening, but it is the strain that is lowered. The contribution of the grout to tension stiffening can be expressed by factor α_s, or in strain Δε_s, and calculated by:

\[
\alpha_s = \frac{\varepsilon_{s,cr}}{\varepsilon_{fdc}}
\]

\[
\varepsilon_{s,cr} = \sigma_{s,cr} / E_s
\]

Eq. 46

Eq. 47
\[ \Delta \varepsilon_{ts} = \varepsilon_{s,cr} - \varepsilon_{fdc} \quad \text{Eq. 48} \]

\[ \Delta \varepsilon_{ts} \quad \text{Tension stiffening} \quad [\text{m/m}] \]
\[ \alpha_{ts} \quad \text{Tension stiffening factor} \quad [-] \]

Not the tension stiffening factor but the decrease in strain due to tension stiffening will be used in calculations. The strain in the cracked area would therefore be the strain assuming only the \( EA \) of steel minus the tension stiffening \( \Delta \varepsilon_{ts} \).

For the TITAN pile the strain in the steel corresponding to the crack force is \( \varepsilon_{s,cr} = 0.44 \% \), while the strain at the fully developed crack force \( \varepsilon_{fdc} = 0.35 \% \). The decrease in strain due to the tension stiffening in the cracked area is therefore \( \Delta \varepsilon_{ts} = 0.09 \% \), and the tension stiffening factor \( \alpha_{ts} \) is 1.22. Due this tension stiffening the pile head displacement of the TITAN pile corresponding to Table 3.1 is reduced from 11.6 mm to 9.9 mm. This 1.7 mm decrease comes fully due to the decrease in lengthening.

From the Basic pile example it can be concluded that the contribution of the grout after cracking to the stiffness of the pile is small but still present. Also for other piles, hollow bar 103/73 mm with a grout diameter of 0.2 and 0.3 m but other properties equal to the Basic pile, the calculated tension stiffening factor is 1.2. This leads to a decrease of the displacements of 1.1 mm. A comment has to be made to these calculated increases of the stiffness of the micropile. In these calculations a good bond and perfect grout is assumed. Due to local situations and the execution method the bond and grout quality might be different. Furthermore, this calculated tension stiffening is based on a pull-out test of a TITAN tube. The bond measured at a test using GEWI steel might be a better input for realistic calculation of cracks at GEWI steel.

Table 3.4 – Calculated pile head displacement of the TITAN pile, 103/78 and Dg=0.3m, with different contribution of the grout to the pile’s stiffness. *Steel+Grout fully is not realist to occur, so only added as a fictive situation.

<table>
<thead>
<tr>
<th>TITAN</th>
<th>Steel</th>
<th>Steel+GroutNcr</th>
<th>TensStif</th>
<th>Steel+Grout fully*</th>
</tr>
</thead>
<tbody>
<tr>
<td>P0=</td>
<td>1500</td>
<td>1500</td>
<td>1500</td>
<td>1500</td>
</tr>
<tr>
<td>u_length</td>
<td>10.91</td>
<td>10.17</td>
<td>9.03</td>
<td>3.43</td>
</tr>
<tr>
<td>u_tip</td>
<td>0.67</td>
<td>0.85</td>
<td>0.85</td>
<td>1.35</td>
</tr>
<tr>
<td>U_head</td>
<td>11.58</td>
<td>11.01</td>
<td>9.87</td>
<td>4.77</td>
</tr>
<tr>
<td>reduction</td>
<td>5%</td>
<td>15%</td>
<td>59%</td>
<td></td>
</tr>
</tbody>
</table>

3.3. Combined stiffness

When looking to the influence of grout to the micropile performance 4 situations are assumed (Figure 3.4):

- Steel: only the contribution of the EA of steel along the full length (as in chapter 4 and CUR236)
- Steel+Grout Ncr: at the lower part of the pile where no cracks have developed both steel and grout contributes to the EA of the pile. Where the grout is cracked, only the contribution of the steel is assumed
- TensStif: at the lower part of the pile where no cracks have developed both steel and grout contributes to the EA of the pile. Where the grout is cracked, a tension stiffening factor of 1.2 is assumed.*
- Steel+Grout fully: along the total length both grout and steel contribute to the stiffness EA. (Due to the low

Figure 3.4 – Schematisation of the four
tension capacity of grout this is not realistic, and calculation is just for imagination)

Due to the decrease in displacements or local strain, the developed shear stress will be a bit less, resulting in less displacement. This iteration process is not taken into account.

The results for the micropile with hollow reinforcement TITAN 103/78 are given in Table 3.4, Figure 3.5 and Figure 3.6. The Steel+grout fully-situation is not realistic but gives an idea of the possible influence of grout. Within the 3 more realistic situations it can be concluded that the differences in micropile performance are small.

Figure 3.5 - Development of the shear stresses along the TITAN pile, with different contribution of the grout to the pile's stiffness.

Figure 3.6 – Load-displacement diagram of the Basic pile, with different contribution of the grout to the pile’s stiffness.
Two topics will be discussed: the influence of the loading direction to the possible maximum shear stress and the occurrence of softening.

4.1. **Shear stress and vertical loading direction**

Based on the basics of theory and instrumented pile tests, there might be a difference in shaft capacity under tension or compression loading in non-cohesive soils. After all, the shaft capacity depends on the shear stress on the shaft, which can be created by the lateral stress. When loaded in compression the grains are pulled downwards, creating denser sand around the pile shaft. Tension loading would be expected to do the opposite: all grains around the shaft are pulled upwards. This will create looser sand which results in lower horizontal stresses. Previous theory might be a simple explanation for the difference in shaft capacity measured at pile tests since the 1960’s-70’s. Beringen et al [23] reviewed piles driven into sand and found ratios of tensile and compressive shaft capacities between 0,65 and 0,75. Randolph [9] also investigated this difference by tests and uses a factor of 0,7 to 0,85 in his design criteria. Lehane [23], executed instrumented pile tests and found a significant difference of 20 %. The CUR 236 (and 2001-4) also implements this difference in loading behaviour (only for non-cohesive soil) by the factor $f_t$.

However, a debate is going on about the existence of this difference in shaft capacity. Measurements would be interpreted wrong, therefore indicating the difference. Fellenius [24] indicates that the bearing capacity of the tip is underestimated, leading to over estimating the shaft capacity in compression. Moreover, residual stresses are present in driven piles. When instrumentation is re-zeroed, this is not taken into account following in lower shaft capacity in tension. Finally, difference based on theory would be too small to have a significance influence, according to Fellenius. The API is following this idea and advises no difference in shaft capacity.

While it would be safe to implement a lower shaft capacity for tension piles, based on empirical data, for economic prospective it would be investigate if this is a hidden safety or reality. Theory and practice are therefore discussed.
4.1.1. Theory

Going back to the theory first, the shaft friction is related to the stresses in the soil. Coulomb’s equation is given in Eq. 49. The (theoretical) radial effective stress can be divided into two parts: the constant equilibrium stress ($\sigma'_{rc}$) and a change during loading ($\Delta\sigma'_{r}$) (Figure 4.5).

$$\tau_f = \sigma'_{sf} \tan \delta_f$$  \hspace{1cm} Eq. 49

$$\sigma'_{sf} = \sigma'_{rc} + \Delta\sigma'_{r}$$  \hspace{1cm} Eq. 50

- $\tau_f$: Ultimate shaft shear stress [N/mm$^2$]
- $\sigma'_{sf}$: Radial effective stresses on the shaft at failure [N/mm$^2$]
- $\delta_f$: Interface friction angle at failure between pile and soil [degrees]
- $\sigma'_{rc}$: Local equilibrium effective stress [N/mm$^2$]
- $\Delta\sigma'_{r}$: Change in the effective stress during loading [N/mm$^2$]

The local equilibrium effective stress depends on the stress in and density of the soil. The value can be calculated by the relative density, or using the cone resistance. A relation based on field tests is described by Lehane and Jardine, which also implements the radius and distance from pile tip in the calculation. For more information is referred to [25].

**Example, using relative density:**

$$\sigma'_{rc} = \sigma'_{bc} = K \sigma'_{v}$$  \hspace{1cm} Eq. 51

$$\gamma_{sand} = 20kN / m^3$$  \hspace{1cm} Eq. 52

$$\sigma'_{v,15m} = 15m \cdot 10kN / m^3 = 150kN / m^2$$

$$K = 0.9$$

$$\sigma'_{rc} = 0.9 \cdot 150kN / m^2 = 135kN / m^2$$  \hspace{1cm} Eq. 53

The change in effective radial stress during pile loading can be divided in the effect of the principal stress rotation in the sand $\Delta\sigma'_{rp}$, dilation due to slip at the interface $\Delta\sigma'_{rd}$ and the Poisson’s effect $\Delta\sigma'_{rv}$.

The parameters that may affect the ratio of tensile to compressive shaft capacity are Poisson’s ratio of the pile and soil, pile length, diameter of the pile, Young’s modulus of the pile, the average and local soil shear modulus and the maximum (ideal) shaft friction at any depth. They are grouped in dimensionless parts in Eq. 54.

$$\frac{Q_{tens}}{Q_{comp}} = f \left( v_p, \frac{L}{D}, \frac{E_p}{G_{ave}}, \frac{G}{\tau_s}, v_s \right)$$  \hspace{1cm} Eq. 54

- $v_p$: Poissons ratio for the pile=0.2 [-]
- $v_s$: Poissons ratio for the soil=0.33 [-]
- $L$: Pile length [m]
- $D$: Pile diameter [m]
- $E_p$: Young’s modulus of the pile [kN/m$^2$]
- $G_{ave}$: Average shear modulus of the soil [kN/m$^2$]
- $G$: Soil shear modulus on the location [kN/m$^2$]
- $\tau_s$: Ideal shaft friction at any depth [kN/m$^2$]
**Principle stress rotation $\Delta \sigma'_{rp}$**

At the pile tests from Lehane in Labenne, reductions in $\sigma'$ are observed over the early loading stages of the pile tests. These reductions were small in compression loading and more significant for tension piles. This can be explained by the principle stress rotation, using the circles of Mohr. When there are no shear stresses, the stresses on both axis in the graph are assumed to be equal. Loading will cause shear stresses, resulting in a shift of the circle and corresponding stresses. Laboratory studies suggest that sensitivity to principal stress rotation decreases with increasing relative density. For loose sands and silts the difference in shaft friction at tension and compression loading will be the largest. The difference in lateral stress must be determined from laboratory tests. (Figure 4.5)

**Dilatation due to slip $\Delta \sigma'_{rd}$**

In loading condition, friction is developed between micropile shaft and soil. The grain skeleton of the soil around the pile will deform. Starting with a loose soil, compaction (grains closer together) and therefore a denser soil is created. Starting with a dense soil, the grains will deform to a less dense soil. This process is called dilatation and illustrated in Figure 4.3. Piles executed by a soil replacing method will normally create a dense sand around the pile, so compaction is assumed. For piles made with the removal of soil the dilatation effect is not expected.

The shear zone, where dilatation and particle reorientation takes place, is assumed to be 10 to 15 $D_{50}$, about 7 mm. However, the effect of dilatation on the soil stresses will extend much further into the surrounding soil was, and this effect will decrease radially.

The shear zone as well as boundary displacement $\delta h$ due to the dilation are influenced by the natural dilatancy of the soil, grain diameter and the surface roughness of the pile. Using the cavity expansion theory, the radial stress change resulting from a boundary displacement applied to an elastic soil mass is given by:

$$\sigma'_{rd} = 2\delta hG / R$$

Eq. 55

<table>
<thead>
<tr>
<th>R</th>
<th>Pile radius [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta h$</td>
<td>Boundary displacement [mm]</td>
</tr>
<tr>
<td>G</td>
<td>Soil shear stiffness [kN/m²]</td>
</tr>
</tbody>
</table>

The change in stress is a function of the shear stress of the soil and pile radius. For same soil conditions, the influence of dilatation will be more on a pile with a small radius compared with a big radius.

The boundary displacement can be measured by direct shear tests. The surface roughness is very important in this. Axelsson [26] found from full scale tests $\delta h=0,04$mm for a concrete pile and $\delta h=0,02$mm in laboratory tests on steel piles. Except for the difference in
roughness, the way of testing may influence the displacement: in full scale tests the pile can deform and the lateral movement of the pile may influence the measurements.

De Nicola and Randolph describe this dilatation behaviour as ‘differences in total stress field, with compressive loading tending to increase and tensile loading tending to decrease the mean stress level in the soil’. They investigated the dilatancy effect during loading with numerical analysis using the finite-difference program FLAC. A simple elastic-perfectly plastic soil model with a Mohr-Coulomb failure criterion in the soil mass and at the pile-soil interface has been adopted. By using a Poisson’s ratio of 0 for the pile, a rigid pile is assumed and Poisson’s effect is excluded. It was found that the length/diameter ratio influences the difference in radial shear stress during loading. De Nicola and Randolph found the following relation:

\[
\frac{Q_{\text{tens}}}{Q_{\text{comp}}} = 1 - 0.2 \log_{10} \left( \frac{100}{L/D} \right)
\]

Eq. 56

This formulation is valid for length/diameter ratios up to 100, above these ratios there is no difference in capacity.

Micropiles have higher slenderness ratios, indicating that this formulation cannot be used. However, this is about the load-transfer and therefore only the bond length in which load transfer occurs must be used. From bond lengths with a higher slenderness ratio of 100 there is no (significance) difference in loading direction-capacity.

While Lehane and De Nicola both assume this dilatation as lowering factor for tensile capacity, NEN 9997-1 discusses this factor as valid for both loading conditions. So, this dilatancy doesn’t explain a difference between shaft tensile and compressive loading. To be safe, it is assumed that is increases the horizontal stress at compressive loading, and doesn’t exist in tensile loading.

Example:
For the basic pile, \( q_c = 15 \) MPa, \( E_{150} \) would be \( 45 \) MPa, Poisson’s ratio of the soil 0,33 and \( D_p = 0,2 \)m, \( G_s = 17 \)MPa, \( \delta h = 0,02 \)mm. The increase in effective stress due to dilation would be 6,8 kPa following Lehane. Compared with \( \sigma'_{hc} \) this is small and concludes in a tensile/compression ratio of 0,95. Randolph gives a factor of 0,98.
Figure 4.4 – Displacement due to the Poisson’s effect.

**Poisson’s effect** $\Delta \sigma'_{\text{poisson}}$

Under loading the pile will deform: an expansion at compression loading and contraction at tensile loading. This might influence the radial stresses in the soil. While De Nicola and Randolph [27] assume this Poisson’s effect as an important reason for the difference, Fellenius [24] refutes this argument. The analytical method of Alawneh, [28] will be used to determine if the Poisson’s effect has a significant influence on the soil stresses.

First, the free lateral expansion of the pile material is called $u$ and can be calculated by:

$$|u| = V_p \frac{P_r r_p}{A_p E_p}$$  \hspace{1cm} \text{Eq. 57}

This lateral displacement is negative in compression and positive in tension, because of the negative respectively positive direction of the load.

At tensile loading the lateral displacement can happen free: the displacement will be negative, so the soil has no influence on it. The difference in lateral stress can then be given by:

$$\Delta \sigma_{r, \text{poisson,} t} = K_s u = V_p \frac{2 P_s G_s}{A_p E_p}$$  \hspace{1cm} \text{Eq. 58}

$$K_s = \frac{2G_s}{E_s} = \frac{E_s}{(1 + v_s) r_p}$$  \hspace{1cm} \text{Eq. 59}

In case of compression, the free lateral expansion $u$ would be stopped by the surrounding soil acting in displacement $u_2$. The final displacement due to the lateral expansion of the pile is then $u_1$ (Eq. 60).

$$u_1 = u - u_2$$  \hspace{1cm} \text{Eq. 60}

At the pile/soil interface the equilibrium given in Eq. 61 is valid. The increase in effective stress can then be given by Eq. 62.

$$K_s u_1 = K_p u_2$$  \hspace{1cm} \text{Eq. 61}

$$\Delta \sigma_{r, \text{poisson,} c} = K_s u_1 = K_p u_2$$  \hspace{1cm} \text{Eq. 62}

This can be calculated using the (horizontal) stiffness of the pile, but is now assumed as half of the displacement at free behaviour. (De Nicola and Randolph use for both tension and compression the difference in effective stress calculation of Eq. 58. (free behavior))
Example:
For the basic pile, the free lateral displacement due to contraction of the micropile, assuming a load of 1200kN (about 70% of $R_t$) is 0.03mm. Poisson’s ratio of the pile is 0.2. This corresponds to a decrease in horizontal stress of 6.8kN/m². At compression loading, the expansion movement would be about half of it, resulting in an increase of 3.4kN/m² of the horizontal stress. Comparison with the static horizontal stress of 135kN/m², this might be a significant difference in shaft capacity. The tension/compression ratio would be 0.93.

Fellenius also discusses the stress change due to the Poisson’s effect. Assuming a lateral movement of 0.02-0.1mm is in accordance with the assumptions before. However, Fellenius assumes that the zone of influence is about 0.2-0.5m and that changing this height with 0.1mm would give an insignificant change in stresses. Due to re-organisation of the grain skeleton this might be true for the average change of stress in the area, but it is expected that the change of stress at the pile/soil interface will be more.

Difference in tensile/compression shaft ratio
The final difference between the capacity in tension loading and compression loading can be taken as a ratio. The ratio of tensile to compressive shaft resistance can be written as

$$\frac{Q_t}{Q_c} = \frac{Q_o - \Delta Q_t}{Q_o + \Delta Q_c}$$  \hspace{1cm} \text{Eq. 63}

$$Q_o = \pi D \int_0^L \sigma'_{nc} \tan \delta$$  \hspace{1cm} \text{Eq. 64}

$$\Delta Q_t = \pi D \int_0^L (\Delta \sigma'_{nc}) \tan \delta$$  \hspace{1cm} \text{Eq. 65}

$$\Delta Q_c = \pi D \int_0^L (\Delta \sigma'_{nc}) \csc \delta$$  \hspace{1cm} \text{Eq. 66}

De Nicola and Randolph found a direct method for this ratio, implementing the dilation and Poisson effect:

$$\frac{Q_{t,\text{DI}}}{Q_{c,\text{DI}}} = \left[ 1 - 0.2 \log_{10} \left( \frac{100}{L/d} \right) \right] \left( 1 - 8\eta + 25\eta^2 \right)$$  \hspace{1cm} \text{Eq. 67}

$$\eta = \nu_p \tan \delta \left( \frac{L G_{av}}{d E_p} \right)$$  \hspace{1cm} \text{Eq. 68}

De Nicola and Randolph, using eq 55, found a ratio of 0.97, while with Lehane/Alwaneh a ratio of 0.89 is calculated. Based on previous theory, there might be influence of the loading direction on the shaft capacity. However, the reduction might be smaller than calculated in the examples. While in this calculation dilation is taken as increasing factor for the horizontal stresses in compression, it should also work positive in tension. Dilatation therefore causes no difference in tension/compression ratio. Moreover, the difference in stress due to lateral movement of the pile is taken as acting in a small shear zone, while its influence zone might be larger. This can lower the change in stress.
4.1.2. Practise

The difference in shaft capacity is concluded often from instrumented pile tests. Both Lehane and De Nicola and Randolph found their theoretical calculation compatible with results from pile tests. The pile test used by Lehane is discussed below.

Lehane [23] executed field tests on two displacement piles in sand; one loaded under compression and the other under tensile loading. The piles were instrumented on pile head and three locations with a surface stress transducer (which recorded radial stress, shear stress and temperature). The steel piles were installed by fast-jacking, and had a bond length of 6 meter. Residual stresses were measured and taken into account. It was found that the shaft capacity at the tension pile was 20% less than at the compression pile, and the pile head displacement at maximum load was much larger (13.5 vs 3.6 mm) at the pile loaded on tension. The local radial effective stress \(\sigma'_r\) had been plotted against the shear stress \(\tau_{rz}\). (Figure 4.5) Lehane relates the depth of the measurement point to the \(h/R\) ratio, the height measured from pile tip. A difference in radial stress at failure can be seen. Based on previous described (and other) pile test, the shaft capacity depends on the direction of the vertical loading.

\[\text{Figure 4.5 - Results from a tension and compression test of a jacked 6 m instrumented pile in sand [23]}\]

\[\text{Figure 4.6 – The instrumented pile [23]}\]

On the other hand, authors as Fellenius state that there is no difference in shaft capacity with the loading direction, as a conclusion from (other) pile tests. The differences found in instrumented pile tests as well as laboratory tests can be explained due to different things. At first, underestimating (wrong interpretation) of the bearing capacity of the base would overestimate the shaft capacity at compression loading. Ignoring residual stresses in the pile after driving also influences the calculated shaft capacity ratio. Often instrumentation in piles is re-zeroed before starting a loading test, and therefore assumed the pile as free of stresses. For bored piles this is valid, but the residual stresses in driven piles are significantly present. In small scale model tests boundary conditions and scaling may have its influence. An example of scaling – problems can be showed by the dilatancy effect: while stresses can be scaled, the grain diameter and therefore interlocking is not scaled.

Fellenius showed in different examples that taking into account residual stresses in the pile, the shaft capacity is equal at tension and compression loading [29].
4.1.3. **Conclusion**

Theoretically there are a couple of reasons why the shaft capacity in tension is different from that in compression: the lateral movement of the pile due to the Poisson's effect, dilatation and the principal shear stress rotation. The calculated difference in shaft capacity of the Basic pile is small.

Field tests show a higher shaft capacity in compression than for piles loaded in tension. However, this is not directly indicating the influence of the loading direction. Different factors could explain this measured difference: underestimating of the end bearing capacity or residual stresses. There are many factors that can be measured with an error or estimated wrong.

Taking all possible mistakes into account, measurements from pile tests might indicate a small difference in shaft capacity. Since there are so many ‘guessed’ factors, it could be discussed if these differences are due to a significant difference depending on the loading direction or due to measurement/calculation-errors.

4.2. **Softening and the lengthening effect**

Softening of the soil may have a big influence on the shaft capacity of the micropile: the total capacity might be 70% of the maximum capacity. However, not all design norms take the softening effect into account. The CUR 236 and Randolph both have a reduction due to the lengthening effect implemented. A closer look is given to both methods. After that, a closer look into the BBRI field tests is given.

**CUR 236**

The CUR 236 uses the factor $f_3$ to implement a reduction due to tensile stress. This reduction depends on the stress in the steel and the fixed length.

$$f_3 = \frac{f_{3, \text{workingpile}}}{f_{3, \text{testpile}}} \leq 1.0$$  
\text{Eq. 69}

- $f_3$ (corrected) reduction factor due to the length effect [-]
- $f_{3, \text{workingpile}}$ (uncorrected) reduction factor of the final pile, following Figure 4.7
- $f_{3, \text{testpile}}$ (uncorrected) reduction factor of the testpile, following Figure 4.7 (if there is no test done $f_{3, \text{testpile}}=100\%$)
Randolph discusses two effects that reduce the bearing capacity: the length effect and friction degradation. Assuming softening of the soil, the residual shear stress might low in loose compressible sands (particularly calcareous sediments) and in soft slightly overconsolidated clays. Tests in clay have indicated that already after a 30 mm movement of the pile, the shear stress may be lowered to 50% [9]. While the pile compressibility indicates that the relative state along load-transfer curve is different along the pile length, this might be of big influence on the bearing capacity. Friction degradation is discussed before.

Randolph introduced a reduction factor to implement this lengthening effect. This pile compressibility depends on the pile capacity and local displacement required for degradation from peak to residual shaft friction. The reduction factor will be affected to some degree by the soil stiffness and the shape of the load-transfer curve. The reduction therefore should be evaluated by means of a numerical analysis. However, a first approximation is expressed in Eq. 73.

\[ R_f = \frac{Q_{\text{actual}}}{Q_{\text{rigid}}} \]  
\[ \xi = \frac{\tau_{\text{residual}}}{\tau_{\text{peak}}} \]  
\[ K = \frac{\pi d L^2 \tau_{\text{ult}} / EA}{\Delta w_{\text{res}}} \]  
\[ R_f \approx 1 - (1 - \xi) \left(1 - \frac{1}{2\sqrt{K}}\right)^2 \text{ for } K > 0.25 \]
Ring shear tests suggest a \( \xi \) -factor of 0.5-0.8, in which the lower range is possible for high-plasticity clays at moderate to large effective stress levels. Strain softening is found to start from relatively small displacements. \( \Delta w_{\text{res}} \) can be determined with a direct shear box test, and will be around 10-30 mm. This value will be independent of the shaft diameter, since degradation is concentrated on a thin band close to the pile shaft. The re-orientation of soil particles is responsible for most of the decrease in angle of friction. For offshore piles with a \( L/d \) ratio about 60, typical \( K \)-values are not more than 5-10. The reduction factor will be in the range between 0.65 and 0.9.

**Field tests of BBRI**

In the research to ‘Ground Anchors’ BBRI investigated the development of the unit shaft friction and therefore the existence of softening. The average shaft friction of inclined and vertical, hollow bars and strand anchors in sand and a heterogeneous clayey sand layer is plotted against the anchor head displacement in Figure 4.8. The fixed length of the anchors was only 5 to 6 meter. It can be seen that there is a non-linear relation in shaft friction, but at large displacements no peak value and therefore also no residual value has been observed. Only for one (inclined) anchor (E18-19, Figure 4.9) that did not fail under the maximum load softening was observed when the anchor was submitted to cyclic loadings, and with a pile in loam (cohesive layer). The BBRI also investigated the difference in shaft capacity for inclined and vertical anchors, the differences seemed small.

It can be concluded that for a static loaded pile in sand and a clayey sand layer no softening/lengthening effect is seen for piles with a fixed length of 5 meter. When cyclic loading is assumed, or piles are installed in loam, the softening has to be taken into account.

The design methods in the UK and Germany are calibrated with cyclic test methods, and might therefore use the lengthening effect.

![Figure 4.8 – Self boring anchors and IGU anchors, vertical and inclined](image-url)
Conclusion

While design methods sometimes advise a reduction of shaft capacity due to softening of the soil (and therefore the lengthening effect), it seems only important for cyclic loadings or very long piles. With static loading on piles of only 5 m length softening is not found in sand or clay. When piles are tested on cyclic loading in sand or static loading in loam, softening is observed. From direct shear tests between sand and prestressed concrete interfaces also softening is observed.
In this appendix the full calculation method to obtain the performance of micropiles is given. At first the maximum bearing capacity is important when designing micropiles. This will be calculated using material factors because it definitely has to be safe. The actual capacity and corresponding displacements during loading are leading to the micropile performance. Calculation is done by dividing the pile and soil in elements. These can be corresponding to soil layering. For each pile element the soil and pile condition can be different.

5.1. Bearing capacity (ULS)

Micropiles are foundation elements and therefore they are designed on their maximum bearing capacity. This is calculated in the ultimate limit state using material factors. When unpredicted events occur the bearing capacity must still be enough.

The calculation of the maximum bearing capacity \( R_{t,d} \) will follow the method given in CUR 236 and is given in Eq. 74. For details on calculation of factors \( f_1, f_2 \) and \( f_3 \), and factors \( \xi_{m,n}, \gamma_{s,t} \) and \( \gamma_{m,var,qc} \) a reference is made to the CUR 236 chapter 6, CUR 2001-4 chapter 8 and NEN 9997-1 section 7.6.3.3 + appendix A. The value of the cone resistance measured with a CPT has to be corrected for over unrealistic strong peaks in the CPT, consolidation and excavation. Furthermore a limit is given for the cone resistance. The rules for consolidation, excavation and limits are given in the norms.

The (excel) results of a CPT can directly be implemented in the model. Correcting the cone resistance can be done by taking the geometric average instead of the arithmetic average. At the last one, all the measured cone resistances have the same influence so when measuring a very thin but strong layer; the average cone resistance is relative high. When using the geometric averages these peaks have less influence [30]. This formulation is given in Eq. 75. The shaft friction coefficient \( \alpha_t \) can be taken from the regulations or can be obtained from pile tests. When taken from field tests the calculation has to be done using the limited cone resistance.

\[
R_{t,d} = \int_0^t \frac{\pi \cdot D \cdot f_1 \cdot f_2 \cdot f_3 \cdot \alpha_t \cdot q_{c,z,d} \cdot \xi_{m,n} \cdot \gamma_{s,t} \cdot \gamma_{m,var,qc}}{} dz
\]  
Eq. 74

\[
q_c = (q_{c1} \cdot q_{c2} \cdot q_{c3} \cdot ..., q_{cn})^{1/n}
\]  
Eq. 75

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_{t,d} )</td>
<td>Design value of bearing capacity</td>
<td>[MN]</td>
</tr>
<tr>
<td>( L_b )</td>
<td>Length grout body for calculating capacity (bond length)</td>
<td>[m]</td>
</tr>
<tr>
<td>( D_g )</td>
<td>Diameter of the grout</td>
<td>[m]</td>
</tr>
<tr>
<td>( f_1 )</td>
<td>Factor for the effect of compressing, is with sand and soil</td>
<td>[-]</td>
</tr>
<tr>
<td>( f_2 )</td>
<td>Factor for lowering the effective stress by the tension force,</td>
<td>[-]</td>
</tr>
<tr>
<td>( f_3 )</td>
<td>Length factor (CUR 236)</td>
<td>[-]</td>
</tr>
<tr>
<td>( \alpha_t )</td>
<td>Shaft friction coefficient</td>
<td>[-]</td>
</tr>
<tr>
<td>( q_{c,z,exe} )</td>
<td>Cone resistance at depth ( z ), taking into account the possibility of an over-consolidation and an excavation</td>
<td>[MPa]</td>
</tr>
<tr>
<td>( \xi_{m,n}, \gamma_{s,t}, \gamma_{m,var,qc} )</td>
<td>Factors in material and load</td>
<td>[-]</td>
</tr>
</tbody>
</table>
Important in the design of tension piles is that the micropiles are not pulled out of the soil: the weight of the soil in the influence zone must be more than the tension load. The calculation method is given in NEN 9997 and it has to be checked for each layer.

The maximum bearing capacity does not only depend on the soil properties but also on the material. This may not fail and therefore a limit is set depending on the yield and failure strength. Following CUR 236, the maximum load may not be more than the minimum of:

$$R_d = \frac{0.9 f_t \cdot A_s}{\gamma_{m,2}}$$  \hspace{1cm} \text{Eq. 76a}$$

$$R_d = \frac{f_{yk} \cdot A_g}{\gamma_s}$$  \hspace{1cm} \text{Eq. 76b}$$

- $f_t$ Failure stress [N/mm$^2$]
- $f_{yk}$ Yield stress [N/mm$^2$]
- $R_d$ Design value of the steel strength [kN]
- $A_s$ Area of the steel [m$^2$]
- $A_g$ Area of the steel including screw treat [m$^2$]
- $\gamma_{m,2}$ Material factor of screw treat connection (=1,25) [-]
- $\gamma_s$ Material factor of the reinforcement (=1,15) [-]

5.2. Displacement (SLS)

The displacement of the pile head can be subdivided into displacement due to lengthening and displacement of the pile tip. The displacement is calculated for the normal situation (serviceability limit state), this means that the material factors $\gamma_{s,t}$ and $\gamma_{m,\varphi,\varphi}$ are not included, but the factor $\xi_{m,n}$ is included and the limitation of the cone resistance is applied. The lengthening of the pile is a summation of the deformations of all elements. This lengthening following from the local axial force and stiffness $EA$ of the pile. The stiffness of the pile will decrease due to grout-cracking, this is discussed in A5.2.2. The displacement at the pile tip follows from the shear stress that is acting on the pile tip (last element). Using the Hyperbolic soil function described in the CUR 236 the displacement can be determined.

$$u_{\text{head}} = u_{\text{length}} + u_{\text{tip}}$$  \hspace{1cm} \text{Eq. 77}$$

$$u_{\text{length}} = \int_0^L \varepsilon \cdot \frac{N(z)}{EA_{\text{pile}}} \hspace{1cm} \text{Eq. 78}$$

$$u_{\text{tip}} = -\frac{2 - r_f}{2 \cdot k_f} \cdot \frac{\tau_{\text{tip}}}{1 - \tau_{\text{tip}}/\tau_u}$$  \hspace{1cm} \text{Eq. 79}$$

$$k = k_f \cdot \xi_{\text{tip}} \cdot \tau_{\text{max}}$$  \hspace{1cm} \text{Eq. 80}$$

- $u_{\text{head}}$ Total displacement of the pile head [m]
- $u_{\text{length}}$ Displacement due to lengthening of the pile [m]
- $u_{\text{tip}}$ Displacement of the pile tip [m]
- $N(z)$ Normal force in the pile [kN]
- $EA_{\text{pile}}$ Stiffness of the pile [kN]
- $\varepsilon$ (Local) deformation [m/m]
- $u_{\text{tip}}$ Displacement pile tip [m]
- $r_f$ Failure ratio $(R_c/F_a) = 0.8$ (this value is taken from the original formula in [CUR 236]) [-]
- $k_c$ Soil spring stiffness [-]
5.2.1. Axial force in the micropile: \( N(z) \)
The axial force in the pile determines the strain on that location and is therefore important in the displacement. The calculation is made from head to tip: a load \( P_0 \) is placed on top. The axial force in the pile on depth \( z \) is the load on top minus the already developed shaft friction in the length above. In the plastic part, above transition depth \( z_t \), this is given by Eq. 81 and the shear stress is the maximum shear stress. In the part with elastic micropile-soil interaction the shear stress is less. Assuming a linear elastic pile-soil interaction this axial force decreases hyperbolic and can be calculated by Eq. 82. The effective reference length \( \lambda \) is used for this relation to implement the soil properties. Because the shear stress cannot be higher than the ultimate shear stress the maximum of the normal force in elastic and plastic phase is the governing force Eq. 83.

When a non-homogeneous soil is assumed the calculation of the axial force in the pile considering plastic and elastic pile soil interaction has to be done for each element. The axial force calculated at an element is input for the element below. The transition depth \( z_t \) is then taken at the top of the element.

The shear stress can be calculated analytically but due to limitations it is easier to use the shear stress acting on element \( i \) determined by the normal force on the top and bottom of that element (60).

\[
N_{pl}(z) = P_0 - \pi \cdot D_g \cdot \tau_{max} \cdot z \quad \text{for } z < z_t
\]

\[
N_{el}(z) = \int_0^z \tau_{el}(z) \cdot \pi D_g = \frac{N_s \sinh \left( \frac{L_p - z}{\lambda} \right)}{\sinh \left( \frac{L_p - z_t}{\lambda} \right)} \quad \text{for } z > z_t
\]

\[
N(z) = \max \{ N_{pl}(z), N_{el}(z) \}
\]

\[
\lambda = \sqrt{\frac{EA}{k_c \pi D_g}}
\]

\[
\tau_i(z) = \frac{N(z_i) - N(z_{i-1})}{(z_i - z_{i-1}) \cdot \pi D_g} \leq \tau_{max}
\]

\[
N(z) \quad \text{Normal force in the pile} \quad [\text{kN}]
\]

\[
\tau_i(z) \quad \text{Shear stress acting on element } i \quad [\text{kN/m}^2]
\]

\[
\lambda \quad \text{Effective reference length} \quad [\text{m}]
\]

5.2.2. Stiffness of the micropile: \( EA_{pile} \)
Grout and steel are materials with different properties. While steel can bear large tensile strains, grout cannot and will crack when the tensile strain is too much. This means that during loading the stiffness of the pile can change from the combined value to only that of only the steel.
The axial force at which the grout starts to crack can be determined using Eq. 86-Eq. 88. In this calculation the influence of the reinforcement is taken into account: the cracking-force is higher than only would be expected in grout. To avoid an iteration process in the modelling, the axial force at element i will be governing for the stiffness at element i+1 (the element below).

\[
N_{g,cr} = \varepsilon_{g,cr} E_c A_c (1 + \alpha \rho) \quad \text{Eq. 86}
\]

\[
\alpha_s = E_s / E_g \quad \text{Eq. 87}
\]

\[
\rho = A_s / A_g \quad \text{Eq. 88}
\]

\[
\alpha_e \quad \text{Ratio of E-moduli} \quad [-]\n\]

\[
\rho \quad \text{Reinforcement ratio} \quad [-]\n\]

\[
E_c A_s \quad \text{Combined stiffness grout+steel} \quad [\text{kN}]\n\]

\[
\varepsilon_{g,cr} \quad \text{Strain at which the grout starts to crack (=0,1\%)}\n\]

The contribution of the grout to the pile's stiffness due to tension stiffening is implemented as well. Starting with the crack force the strain at which the crack pattern has fully developed is given by (empirical) Eq. 89. The long term factor is assumed to have its maximum of 1,3. The contribution of the grout to tension stiffening is expressed in strain \(\Delta \varepsilon_{ts}\) and calculated by:

\[
\varepsilon_{fde,longterm} = \gamma_{oo} \cdot (60 + 2,4\sigma_{s,cr}) \cdot 10^{-6} \quad \text{Eq. 89}
\]

\[
\sigma_{s,cr} = N_{cr} / A_s \quad \text{Eq. 90}
\]

\[
\varepsilon_{s,cr} = \sigma_{s,cr} / E_s \quad \text{Eq. 91}
\]

\[
\Delta \varepsilon_{ts} = \varepsilon_{s,cr} - \varepsilon_{fde} \quad \text{Eq. 92}
\]

\[
\alpha_{ts} = \frac{\varepsilon_{s,cr}}{\varepsilon_{fde}} \quad \text{Eq. 93}
\]

\[
\varepsilon_{dc} \quad \text{Strain in grout at fully development crack pattern} \quad [\text{m/m}]
\]

\[
\sigma_{s,cr} \quad \text{Stress in the steel directly after cracking} \quad [\text{N/mm}^2]
\]

\[
\gamma_{oo} \quad \text{Long term factor, will be taken as 1,3 (the maximum)} \quad [\text{mm}^2/\text{N}]
\]

\[
\Delta \varepsilon_{ts} \quad \text{Difference in strain due to tension stiffening} \quad [\text{m/m}]
\]

\[
\alpha_{ts} \quad \text{Tension stiffening factor} \quad [-]
\]

The crack width and crack distance can also be determined, however they are not used in the calculation of the pile head displacement.

\[
w_{mo} = 2U_{r,c} = 2 \left\{ \frac{1 + b \, D^*_{s}}{2} \cdot \frac{1}{4 \cdot a \cdot f_{cm,cube} E_s} \cdot \frac{\sigma_{s,cr}^2}{1 + \alpha_s \rho} \right\}^{\frac{1}{1+b}} \quad \text{Eq. 93a}
\]

\[
D^*_{s} = D_{s,outr} \frac{A_{bar}}{A_{nbar}} \quad \text{Eq. 93b}
\]

\[
w_{mo} = 2 \left\{ \frac{0,4D^*_{s}}{f_{cm,cube} E_s} \cdot \left( \frac{\sigma_{s,cr}}{\rho} \right)^2 \right\}^{0.85} \quad \text{Eq. 93c} \quad \text{Mas}
\]

\[
w_{mo} = 2 \left\{ \frac{0,38D^*_{s}}{f_{cm,cube} E_s} \cdot \left( \frac{\sigma_{s,cr}}{\rho} \right)^2 \right\}^{0.77} \quad \text{Eq. 93d} \quad \text{Hol}
\]
\[ l_m = 1.5l_{st} = 1.5 \cdot \frac{w_{mo} E_s}{1 - b \sigma_{s,cr}} \]  
\[ l_m = 1.8 \frac{w_{mo} E_s}{\sigma_{s,cr}} \]  
\[ l_m = 2.1 \frac{w_{mo} E_s}{\sigma_{s,cr}} \]  
\[ l_{cr,ave} = 2c_g + l_m \]  
\[ l_{cr,max} = 3.7c_g + l_m \]  
\[ w_{mv} = \frac{l_{cr}}{E_s} \left( \sigma_s - 0.5\sigma_{s,cr} \right) \]  
\[ \sigma_{cr} = 0.60f_{c,cm} \]  

**5.3. Micropile’s performance**

Finally the behaviour of the micropile in the soil can be calculated. This is expressed in the stiffness of the pile and can be determined for all loads following Eq. 98.

\[ K_{pile} = \frac{P_0}{u_{head}} = \frac{P_0}{u_{length} + u_{tip}} \]  

- \( w_{mv} \): Mean crack width at fully developed crack phase [mm]
- \( w_{mo} \): Mean crack width at not fully developed crack phase [mm]
- \( \sigma_{cr} \): Concrete tensile strength at which cracking occurs [N/mm²]
- \( \sigma_s \): Stress in the steel at which cracking occurs [N/mm²]
- \( l_m \): Transition length at fully developed crack phase [mm]
- \( l_{st} \): Transition length at not fully developed crack phase [mm]
- \( f_{ck} \): Cylindrical compressive strength [N/mm²]
- \( D_s \): Diameter of the steel reinforcement [mm]
- \( c_g \): Grout cover [mm]
The Excel sheet consists of 10 pages. First the three pages will be discussed with input and output.

1. Invoer en uitkomsten

All output and almost all input is on this sheet. A picture of the sheet is given in Figure 6.1. The upper part in red is the input, in green below the output is given.

The input is divided in different categories (from left to right): pile, materials, soil, material factors and the load. The parameters that have to be filled in are the ‘normal ones’ as needed for the calculation of the bearing capacity of a pile following NEN 9997-1 and some material-parameters as Young’s moduli, tension strain of grout, strength of grout and the soil spring stiffness coefficient. It is also possible to include or exclude the different parts of the contribution due to grout. In the right box ‘invoer belasting’ the expected load on the micropile can be filled in. To fill in the soil profile another page will be used (‘invoer GEF sondering’).

Figure 6.1 - Picture of the page ‘Invoer en uitkomsten’. 
The output is given in the green boxes. In the left upper the capacities, displacements, grout cracking properties and the axial spring stiffness are given. On the right and below graphs are plotted which give insight to the pile performance and behaviour.

2. Invoer GEF sondering
The results of the CPT can be imported using GEF into Excell format. These results can be paste in this sheet.

3. Figuren FOR
This page shows the development of the shear stress, displacements and axial force in the pile. The input of the load \( P_0 \) has to be linked to the input on this page.

The following sheets are only for calculation or to get insight in the model:

4. Draagkracht UGT
Page following the NEN 997-1 (and CUR 2001-4). The bearing capacity in ULS is calculated. Including the calculation against pull out. Also explained in A5.1.

5. Draagkracht BGT
Page following the NEN 997-1 (and CUR 2001-4). The bearing capacity in SLS is calculated.

6. FOR verpl
The lengthening and tip displacement are calculated following A5.2 and A5.2.1.

7. Grout cracking
The cracking force, width and distance are calculated using A5.2.2.

8. CUR
Calculation of the pile head displacement and pile spring stiffness following the CUR 236.

9. HS-model
Just a help for implementing the non-linear hyperbolic soil model.

10. Nette sondering
Just a help to make a nice overview on the CPT.
MODEL VALIDATION

The model is validated using a case and by comparison with other models. The results of this comparison are given in the report section 7.2.1. In this appendix of the results of the comparison with other models is given.

The two other models that are compared with the Revised FOREVER model are the numerical models INTER 3 of Van Dalen [19] and the Axial stiffness method of Ad Vriend [31]. A description of the models is given in the report section 7.2.2. The results of the study are given in the following tables. Table 7.1 represents the calculation for micropile types A and B and Table 7.2 represents the pile behavior of micropile types C and D. It can be seen that the results are similar.

Table 7.1 - Comparison between the RFM and the Axial spring stiffness method of A. Vriend and INTER. Micropile type A/B, \( q_c=20\text{MPa} \), \( \alpha_t=1\% \), \( D_g=0,16\text{m} \) and \( k_{50}=200 \) (=curve 1 of NEN9997-1)

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Table 7.2 - Comparison between the RFM and the Axial spring stiffness method of A. Vriend, and INTER. Micropile type C/D, \( q_c=20\text{MPa} \), \( \alpha_t=1\% \), \( D_g=0,16\text{m} \) and \( k_{50}=120 \) (=curve 2,3 of NEN9997-1)

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