DELFT UNIVERSITY OF TECHNOLOGY

DEPARTMENT OF AERONAUTICAL ENGINEERING

Report VTH-193

PREDICTION OF FATIGUE CRACK PROPAGATION
IN AIRCRAFT MATERIALS UNDER
VARIABLE-AMPLITUDE LOADING

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AIRCRAFT MATERIALS UNDER VARIABLE-AMPLITUDE LOADING

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Delft University of Technology, Aeronautical Department

ABSTRACT: The paper starts with a discussion on loads in service, after which a
survey is given of various types of variable-amplitude loading as applied in
test programs. The various phenomenological aspects of fatigue damage associated
with fatigue cracks are indicated. Interaction effects between cycles of different
magnitudes are defined. Methods for measuring interaction effects, examples of
interaction effects and possible explanations are reviewed. This includes both
tests with simple types of variable-amplitude loading (overloads and step loading)
and more complex load-time histories (program loading, random load and flight-
simulation loading). New evidence on crack closure is presented. Various types
of prediction methods are discussed. The paper is primarily a survey of the present
knowledge with an analysis of the consequences for prediction techniques.
KEY WORDS: fatigue properties, crack propagation, variable-amplitude, sequence
effects, crack closure, prediction techniques, service load-time histories.

NOMENCLATURE

overload: a high positive load, preceded and followed by lower constant-
amplitude loading
overload cycle: a high positive and a high negative load
step loading: a step wise change of the magnitude of constant-amplitude
loading (change of $S_a$ or $S_m$ or both)
lo - hi: low - high referring to amplitude sequence of step loading or
hi - lo: high - low program loading
lo - hi - lo: amplitude sequence in program loading
INTRODUCTION

For an operator of a machine or a structure cracks are of little interest as long as they cannot be detected by available inspection techniques. However, as soon as detection is possible the situation is different. If crack growth is sufficiently slow routine inspections can be adopted to prevent failures in service. In aircraft structures this has led to the well known fail-safe philosophy. The crack propagation curve in Fig. 1 is illustrating this point. Obviously the time available for crack detection is depending on both \( a_0 \) and \( a_f \). Since the growth of a crack is usually an accelerating process the time is less dependent on \( a_f \) and more dependent on \( a_0 \). This emphasizes the significance of inspection techniques. For a long time an arbitrary choice was \( a_0 = \frac{1}{2} \) inch (12.6 mm), but more optimistic values, say a few millimeters, appear to be justified, provided the crack location is accurately known.

The time available for crack detection is clearly related to the crack propagation curve (Fig. 1). The problem of predicting this curve is the leading question of the present paper. Crack growth is dependent on numerous variables, see Table 1, listing the main groups.

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Variables affecting crack growth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variables outside the structure . load-time history . chemical environment</td>
<td></td>
</tr>
<tr>
<td>Variables inside the structure . type of structure . material</td>
<td></td>
</tr>
</tbody>
</table>

The present paper attempts to survey the various aspects of predicting crack growth curves. Actually this should include a discussion of many variables. However, the main topic will be the effect of the load-time history with a few comments on environmental effects. It is well known, especially for aircraft structures, that crack growth can be retarded and temporarily stopped by adopting specific design features. Fail-safe design procedures, however, will not be discussed. Similarly material selection for slow crack growth will not be covered.

The discussion in this paper includes the following steps

(i) Loads in service
(ii) Present knowledge about crack growth under variable-amplitude loading
(iii) Present prediction methods
The problems are discussed as they occur in aeronautics, but the situation will be partly similar for other disciplines.

LOADS IN SERVICE

The major aspects of loads in service are listed below.

<table>
<thead>
<tr>
<th>Table 2</th>
<th>Aspects of service load-time histories</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>load occurrences</td>
</tr>
<tr>
<td></td>
<td>load sequences</td>
</tr>
<tr>
<td></td>
<td>speed of load variations</td>
</tr>
<tr>
<td></td>
<td>environment</td>
</tr>
</tbody>
</table>

The definition of the varying load is a problem of its own. Two samples are shown in Fig. 2 to illustrate this point. The first sample has the character of an amplitude-modulated signal with a constant mean and frequency. The random feature is in the modulation. The second sample shows a random nature, which is less easily defined as for the first sample. Such samples can be statistically analysed with respect to peak loads, load ranges, etc. (counting methods, \[2, 3\]). However, information on the sequence of load occurrences is lost in such a statistical description.

Investigations on sequence effects have clearly shown its significance. Results of a comparative study on crack growth with random and programmed sequences are summarized in Fig. 3 \[4\]. Although all sequences were statistically equivalent with respect to load occurrences the crack growth rates were significantly different. Moreover, the fracture surfaces were also different, even macroscopically. This will be discussed later.

The conclusion to be drawn here is that complete knowledge about loads in service should include information on load sequences. If the loading is a stationary random process the sequence appears to be well defined in statistical terms. However, in many practical situations the loading conditions are non-stationary. Moreover mixtures of random loads and deterministic loads frequently occur. A well-known example is the (non-stationary) random gust
loading on a wing combined with the deterministic ground-to-air cycles.

An inherent problem of predicting loads in service is the scanty information on the rarely occurring very high loads. The disturbing feature is the large effect, which these high loads can have on crack growth. High tensile loads can drastically reduce crack growth. Unfortunately, accurate predictions on the occurrence of such loads in service can hardly be made. This implies a severe limitation to the practical significance of predicted crack rates.

Service conditions also include the environment in which the structure is operating. If the environment is affecting crack growth the phenomenon has to be associated with corrosion fatigue. The speed of load variations can be significant (effects depending on time and loading rates). Three groups of environments should be mentioned here.

<table>
<thead>
<tr>
<th>Table 3</th>
<th>Environments</th>
</tr>
</thead>
<tbody>
<tr>
<td>. non-agressive dry environments</td>
<td></td>
</tr>
<tr>
<td>. water vapor as the most detrimental element</td>
<td></td>
</tr>
<tr>
<td>. more agressive electrolytical conditions</td>
<td></td>
</tr>
</tbody>
</table>

The non-agressive environments do not frequently occur in aerospace, although they are relevant to structures in space and aircraft at very high altitudes. However, the second group is applicable in many cases. The third group encompasses wet environments.

The detrimental effect of water vapor on crack growth in Al-alloys is well known. The present knowledge appears to indicated as a first approximation a constant effect of water vapor for many service conditions as well as for testing in the laboratory. Sufficient water vapor is available for the same maximum detrimental effect. Comparative flight-simulation tests at 10 Hz, 1 Hz and 0.1 Hz have shown the same crack rates in 2024-T3 Alclad and 7075-T6 clad material [5]. This simplified picture is a most desirable result in view of the relevance of laboratory results for service conditions. It should be clearly recognized, however, that the picture is near to wishful thinking if it cannot be backed up by physical understanding. Moreover, for more agressive wet environments the time scale will certainly be significant. An outstanding example is the growth of cracks in marine environments, which evidently applies to off-shore structures. It then becomes extremely difficult to obtain relevant information from high-speed laboratory tests.
Differences between crack growth studies

The first impression of the literature is the overwhelming variety of different types of variable-amplitude loading. For an analysis of the present knowledge the various types of loading have been classified in a number of groups in the table below.

<table>
<thead>
<tr>
<th>Table 4: Types of variable-amplitude loading with main variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. Overloads</td>
</tr>
<tr>
<td>. single overload</td>
</tr>
<tr>
<td>. repeated overloads</td>
</tr>
<tr>
<td>. blocks of overloads</td>
</tr>
<tr>
<td>. magnitude of overloads (including $R$)</td>
</tr>
<tr>
<td>. sequence in overload cycles</td>
</tr>
<tr>
<td>II Step loading</td>
</tr>
<tr>
<td>. sequence of steps (hi-lo or lo-hi)</td>
</tr>
<tr>
<td>. magnitude of steps (including $R$)</td>
</tr>
<tr>
<td>III Programmed block loading</td>
</tr>
<tr>
<td>. sequence of amplitudes</td>
</tr>
<tr>
<td>. size of blocks</td>
</tr>
<tr>
<td>. distribution function of amplitudes</td>
</tr>
<tr>
<td>IV Random loading</td>
</tr>
<tr>
<td>. spectral density function (narrow band, broad band)</td>
</tr>
<tr>
<td>. crest factor (clipping ratio)</td>
</tr>
<tr>
<td>. irregularity factor</td>
</tr>
<tr>
<td>V Flight-simulation loading</td>
</tr>
<tr>
<td>. distribution function of load cycles</td>
</tr>
<tr>
<td>. sequence of flights</td>
</tr>
<tr>
<td>. sequence of loads in flight</td>
</tr>
<tr>
<td>. maximum load in the test</td>
</tr>
</tbody>
</table>

The variety of fatigue loads is partly but not fully illustrated by the second column of the table. The variety of investigations on crack growth is significantly larger because in addition to the loading history there are several additional variables. It is sufficient to mention:

- type of specimen
- material
- loading system (tension, bending, etc.)
- loading rate (frequency)
- environmental conditions

The effect of the type of specimen is generally supposed to be covered by relevant K-values. However, differences between thick and thin specimens are significant (plane strain/plane stress). The other variables further contribute to a large variety of investigations reported in the literature. It then becomes possible that the relative amount of duplication is rather limited, despite an impressive number of publications.

The goals of investigations can also be grouped into different categories. Three general goals are frequently recognized from the literature.

<table>
<thead>
<tr>
<th>Table 5: Goals of investigations</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. To increase the fundamental understanding of the crack growth mechanism under variable-amplitude loading</td>
</tr>
<tr>
<td>2. To check crack growth prediction models</td>
</tr>
<tr>
<td>3. To generate data from which useful empirical trends might be derived</td>
</tr>
</tbody>
</table>

Investigations on overload effects and step loading are usually aiming at the first goal. Conclusions of papers suggest some degree of understanding of a mechanism. Investigations on program loading, random loading or flight-simulation are more directed to the second and the third goal. Relevant papers suggest that a technical problem is treated in a practical way.

DEFINITION AND MEASUREMENT OF INTERACTION EFFECTS

It would be very convenient for the prediction of crack growth if the growth was a simple addition process of crack length increments (\( \Delta a \)) in each load cycle:

\[
a = a_0 + \Sigma \Delta a_i
\]

(1)

This process should be understood to be simple if \( \Delta a \) was dependent on the momentary size of the crack (a) only and independent of the history of the preceding crack growth. Unfortunately this is not true, which is attributed to so-called interaction effects. The crack growth increment (\( \Delta a \)) in a certain load cycle will be a function of \([6]\):
(i) the crack geometry being present before the cycle started
(ii) the condition of the crack tip material
(iii) the magnitude of the load cycle
These arguments are further illustrated in table 6 below.

<table>
<thead>
<tr>
<th>Table 6: Δa in a certain load cycle will depend on:</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Amount of cracking</td>
</tr>
<tr>
<td>. crack length</td>
</tr>
<tr>
<td>. shape of crack front</td>
</tr>
<tr>
<td>b. Crack front orientation</td>
</tr>
<tr>
<td>. tensile mode</td>
</tr>
<tr>
<td>. shear mode</td>
</tr>
<tr>
<td>. mixed modes</td>
</tr>
<tr>
<td>. crack geometry</td>
</tr>
<tr>
<td>c. Crack tip blunting</td>
</tr>
<tr>
<td>. shape of crack tip</td>
</tr>
<tr>
<td>. blunted/sharpened</td>
</tr>
<tr>
<td>d. Crack closure</td>
</tr>
<tr>
<td>. plastic deformation</td>
</tr>
<tr>
<td>. in wake of crack</td>
</tr>
<tr>
<td>e. (Cyclic) strain hardening</td>
</tr>
<tr>
<td>. distribution in crack</td>
</tr>
<tr>
<td>. condition</td>
</tr>
<tr>
<td>f. Residual stress and strain</td>
</tr>
<tr>
<td>. tip zone</td>
</tr>
<tr>
<td>. of material at tip of crack</td>
</tr>
<tr>
<td>g. Magnitude of load cycle</td>
</tr>
<tr>
<td>. Δ S, R</td>
</tr>
<tr>
<td>h. Environment and frequency</td>
</tr>
<tr>
<td>. present load cycle</td>
</tr>
</tbody>
</table>

The arguments compiled in table 6 imply that Δa will be a function of the preceding cyclic load history. Similarly a stress cycle will affect crack growth increments in subsequent cycles. These effects are referred to as "interaction effects". Several examples will be shown later on.

Initially interaction effects were measured from crack propagation curves as obtained by visual observations of the tip of a crack at the surface of the specimen. Examples for a thin and a thick specimen are given in Fig. 4 [7]. It should be pointed out that the tip of a crack cannot always be located accurately. Especially after an overload the more extensive plastic deformation
at the tip of a crack will give some extra blurring of the surface image. Moreover, it is generally recognized now, that the two-dimensional picture of the surface does not give sufficient information about the three-dimensional phenomenon occurring in the material. This was borne out by fractographic observations which showed that the curvature of the crack front could change by varying the fatigue load (e.g. forming of "tongues" by peak loads). The topography could also vary (amount of shear lips).

Another fractographic refinement was due to measurements of striation spacings. It essentially added to a more detailed picture of interaction effects. Local accelerations or retardations of crack growth can hardly be observed from crack growth curves, but striations can reveal such effects [8].

A different approach is associated with crack closure measurements [9,10]. Such measurements give indirect indications about plastic deformations left in the wake of the crack (see Fig. 5). Unfortunately more information on the changes of the fatigue damage in the material can hardly be obtained. It is difficult, if not impossible, to measure crack tip blunting and sharpening. The same applies to the cyclic strain-hardening in the crack tip zone and the related residual stress field.

INTERACTION EFFECTS IN TESTS WITH OVERLOADS OR STEP LOADING

Originally most information was obtained on aluminium alloys. Later on similar tests were conducted on titanium alloys [11-20] and high-strength steels [11,12,15,21-29]. More or less similar trends were observed. Several test series were also performed on mild steel and other non-aircraft steels [30-39]. The behaviour of these materials is not fully similar, but crack growth delays were found as well. Apparently similar results may be expected for materials with a similar (cyclic) strain behaviour. For most aeronautical materials this implies a fairly high $\sigma_{0.2}/\sigma_u$ ratio, a limited ductility and a rapid cyclic strain hardening. Mild steel does not conform to this picture, which should be related to its characteristic yielding and strain ageing behaviour.

The more prominent observations on interaction effects in tests with overloads or step loading are summarized below.

a. Positive overloads introduce significant crack growth delays 6,12,14-20, 22,27,29,34,40-50. In general larger delays are obtained by:
- increasing the magnitude of the overload
- repeating the overload during the crack propagation life
- application of blocks of overloads instead of single overloads

The retarded growth need not immediately follow the overloads. Some further growth may be required before the crack rate decreases \([29, 34, 43, 45-47, 50]\). Sometimes a small initial acceleration was even observed. This delayed retardation was clearly verified by observations of striation spacings.

b. The crack extension caused by the overloads themselves is larger than expected from constant-amplitude tests \([49, 51]\). This acceleration usually requires fractography also.

c. Negative overloads have a relatively small detrimental effect on crack growth \([20, 40, 52]\). However, a negative overload added immediately after positive overloads can significantly reduce the crack growth delay of the latter ones. If the negative overload precedes the positive overload the reduction of the delay is much smaller. There is an apparent sequence effect of the overload cycles \([12, 20, 40, 41, 43, 49]\).

d. In step loading a hi-lo sequence produces qualitatively similar results as overload cycles \([9, 10, 40, 41, 45, 46, 49, 50, 53-55]\). Once again delayed retardation was observed. Interaction effects after a lo-hi sequence are hardly detected from macroscopical crack growth observations. However, more accurate measurements and striations do reveal locally accelerated crack growth \([21, 25, 45, 46, 50]\).

e. Delays are clearly depending on the ductility of the material. If the ductility of an alloy is controlled by heat treatment a lower yield strength will produce larger delays \([27, 29, 54]\).

Originally the explanation of crack growth delays was based on residual stresses in the crack tip zone \([40, 54]\). Later it turned out that this picture was too simple. Crack closure, discovered by Elber \([9, 10]\) entered the literature and he explained both retardations and accelerations by the mechanism. Elber pointed out that crack growth retardation did not occur immediately after an overload. The crack had to penetrate into the plastic zone created by the overload before crack closure could become effective (delayed retardation). This was amply confirmed by the work of others.

Some exploratory tests were recently carried out \([7]\) to study crack closure and delays in relation to specimen thickness. The crack-opening stress, \(S_{op}\) (i.e. the stress at which crack closure is removed) was measured in constant-amplitude tests for three different thicknesses. A small COD meter
was used for this purpose. Results in Fig. 6 show that $S_{op}$ is lower for a larger thickness. In the thicker specimen the plastic zone size will be smaller (appr. plane strain) than in the thinner specimen (appr. plane stress). Similarly the elongation in the plastic zone is smaller and the residual deformation in the wake of the crack will be smaller. This explains the lower crack opening stress. Adding one overload cycle gives a significant reduction of $S_{op}$ immediately after the overload cycle, see Fig. 6. This has to be expected, because the plasticity ahead of the crack opens the wake of the crack. The results of these overloads on subsequent crack growth are shown in Fig. 4. The crack growth delay is twice as large in the thinner specimen. In view of more crack closure occurring in the thinner specimen this result had to be expected. The investigation is still in progress, but some data on measurements of $S_{op}$ during the delay period for a thin specimen can already be given here. As shown by Fig. 7 $S_{op} < S_{min}$ before the overload is applied. Immediately after the overload $S_{op}$ is reduced still further. This would allow an accelerated growth for a small number of cycles, which was reported to occur in the literature [43]. However, shortly afterwards $S_{op}$ is significantly raised beyond $S_{min}$. Later on $S_{op}$ again decreases and $S_{eff}$ increases. After $S_{op} = S_{min}$ the retardation of the growth has vanished. In Fig. 4 the crack length increment affected by the overload is approximately equal to its estimated plastic zone size. In Fig. 7 this increment is larger than the plastic zone. As pointed out by Van Lipzig and Nowack [50] the retardation due to crack closure can very well be effective beyond the overload plastic zone.

Results on crack closure in flight-simulation tests, discussed later on, also confirm that crack closure is a real phenomenon. Surprisingly enough its existence was overlooked for a long time. Recently the first attempts to incorporate crack closure into calculations were published [17,56-58]. The more rigorous calculations made by Newman and Armen [58] produced promising results. Such calculations are still rather expensive as yet. More progress, however, may be expected. The question to be raised here is whether interaction effects might be due to crack closure alone. Having in mind the fatigue damage picture outlined in the previous chapter (table 6) this assumption appears to be somewhat too optimistic. It would be a surprise if the cyclic straining of the material in the crack tip zone, the crack tip geometry and the crack front orientation would be fully irrelevant. Interaction effects
as described in [49] cannot all be explained by crack closure alone. The crack
extension during an overload (for instance tongues [29,42,49,55]) is too large
to be due to a low $S_{op}$. It is thought that the conditioning of the material in
the crack tip zone during the preceding low-amplitude cycles is another
contributing factor. Incompatible crack front orientation [49] may further
add to the observed behaviour. More research is still needed to unravel the
complex phenomena occurring during variable-amplitude loading. For further
studies on this issue a mandatory requirement is to include measurements on
crack closure. Tests with load sequences avoiding the occurrence of crack
closure should also be elucidating, as shown by Shih and Wei [17]. It should
be recognized, however, that highly localized crack tip closure might not be
detected by "macro crack closure measurements".

INTERACTION EFFECTS IN TESTS WITH PROGRAM LOADING, RANDOM LOADING OR FLIGHT-
SIMULATION LOADING

During a complex load sequence it is more difficult to observe the local
interaction effects separately. The overall effect, however, can easily be
deduced from macroscopical measurements of the crack growth. The predicted
crack rate without interaction effects is obtained from the predicted crack
growth curve:

$$a = a_0 + \sum \Delta a_i$$

(1)

with $\Delta a_i = \frac{da}{dn} = f_p(\Delta K)$

(2)

It is generally observed that the actual crack rates are considerably different
from the predicted values. More detailed observations were obtained by
fractographic techniques and crack closure measurements. A summary of the main
results is given below.

a Crack rates derived from crack growth curve were usually found to be
significantly lower than the values predicted by Eqs (1) and (2) [5,15,24,26,28,
[41,59-62]. Values 2-8 times lower were frequently found. Apparently retardation
effects are predominating the possibilities for acceleration effects.

b In program tests similar sequence effects were found as observed in step
loading [4,8,16,19,25,26]. Especially a retarded crack growth after a drop
of the stress amplitude was clearly observed. In tests with a lo-hi-lo sequence
the crack rate in the descending part was lower than in the ascending part of the program.

C. Nowack [63a] studied crack growth under random loading with a constant \( S_{\text{rms}} \) but a stepwise change of \( S_m \). A decrease of \( S_m \) caused retarded crack growth.

d. The effect of high loads is similar to the effect of overloads in constant-amplitude loading. In flight-simulation tests it is well established that the application of rarely occurring very high loads can decrease the crack rate significantly. Truncation of these high loads to lower levels gives faster growth [6,13,23,26,61,64,65]. Overloads applied to a structure (for instance fail-safe loads) can drastically reduce subsequent crack growth [65].

e. Comparison of program loading and random loading has revealed significant sequence effects [4]. This effect was not restricted to crack rates (see Fig. 3), but it also applied to the topography of the fracture surface. Macrocopically the roughness of the fracture surface and the transition from the tensile mode to the shear mode were completely different for the two types of loading, in spite of the same load spectrum applying to both.

f. Crack closure was recently shown to occur during flight simulation tests [66,67]. A sample of the loads in a flight with severe gusts is shown in Fig. 8. Crack closure measurements made by a COD meter during a severe flight are shown in Fig. 9. The measurements were made only for larger stress amplitudes of the flight. When the load passed the mean stress in flight small horizontal shifts were given to the recorder to separate the loops of the various loads. Two interesting observations can be made. First, five maximum peak loads occurring in the same flight do not have the same effect. The non-linearity at the top of the first one is mainly due to crack extension and crack tip plasticity. Apparently the other four peak loads produce smaller contributions. This is in agreement with results from a more elementary study [49] (see also [47]). The second observation is related to crack closure which is easily observed from the non-linearities in the bottom part of the recording. Arrows indicate the stress levels below which the crack is partly closed \( S_{\text{op}} \). The first high load widens the crack and thus causes a decrease of \( S_{\text{op}} \) similarly as reported in the previous chapter. After further crack growth during subsequent flights with milder gust spectra the crack opening stress was again restored to a higher level.
It appears that the observations of the tests with more complex load histories are qualitatively in good agreement with the results of the more simple types of loading, discussed in the previous chapter. The agreement concerns the effect of high loads, delays, sequence effects and crack closure. In view of all this evidence it is opportune to reconsider the occurrence of both crack growth delaying and acceleration effects during complex load-time histories. A useful comment was recently made by Katcher [15]. He started from the observation that crack growth retardation after a high load requires a number of cycles before it becomes effective (delayed retardation). He then pointed out, that a re-application of a high load, before the retardation could become effective, may considerably reduce the delay effect. A similar suggestion can be made for acceleration effects. Accelerations were observed after lo-hi sequences, occurring immediately after the lo-hi transition. This is in agreement with the crack closure concept, although it need not be the only explanation. Anyhow the acceleration requires some prehistory of lower-amplitude cycles. Consequently, high-load cycles which have not seen a prehistory of lower-amplitude cycles, will not be associated with the same acceleration effect. This was illustrated before (see e and Fig. 9).

The conclusion now is that accelerations and delays in a complex load history are very sensitive to the sequence of the various loads. The assumption that random sequences and equivalent programmed sequences could give the same overall crack rate has to be considered as wishfull thinking.

There is another conclusion to be drawn from the present knowledge. During variable-amplitude loading accelerations occur during the high loads. Retardations occur during the low-amplitude cycles. The result may well be that the major part of the crack extension will occur during the high-amplitude cycles. Crack growth observations during flight-simulation tests and fractography appear to confirm this view [59,64]. This appears to be important for the mutually related effects of environment and frequency. Environmental effects during crack growth (corrosion fatigue) were amply shown in the literature to be large for low $\Delta K$ values and relatively small during high $\Delta K$ values. If it is true that the major contribution in $\Sigma \Delta a$ is coming from the higher load-amplitude cycles a relatively small environmental effect should be expected. This is now investigated at the NLR. Flight-simulation tests are carried out on 2 mm and 10 mm sheet specimens (width 160 mm) of 2024 - T3 and 7075 - T6 material at
15 cps. Some preliminary results for 2 mm specimens of 2024 - T3 are shown below.

<table>
<thead>
<tr>
<th>Environment</th>
<th>Crack growth life (a = 10 mm to failure)</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dry air ($\text{H}_2\text{O} &lt; 20$ ppm)</td>
<td>23120 flights</td>
<td>1.3</td>
</tr>
<tr>
<td>Lab air (R.H. 50%, 20-25°C)</td>
<td>18210 flights</td>
<td>1</td>
</tr>
<tr>
<td>Salt water</td>
<td>7845 flights</td>
<td>0.43</td>
</tr>
</tbody>
</table>

The life in dry air is only 30 percent larger than in wet air, whereas constant-amplitude data [68] would suggest it to be about three times larger. The effect of salt water is probably not much different from the expected ratio derived from constant-amplitude data. The problem of environmental effects under service loading requires further clarification [19]. The present considerations, however, already emphasize the need for realistic testing if realistic answers are looked for.

PREDICTION METHODS FOR VARIABLE-AMPLITUDE LOADING

After the conclusive proof of the usefulness of $\Delta K$ for correlating constant-amplitude crack rate data it was all but natural that extensions to variable amplitude loading have been proposed. Available propositions can be classified in five groups.

<table>
<thead>
<tr>
<th>Table 7</th>
<th>Base of prediction methods</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Non-interaction</td>
<td></td>
</tr>
<tr>
<td>2. Interaction based on $K_{\text{eff.}}$</td>
<td></td>
</tr>
<tr>
<td>3. Equivalent $K$-concept</td>
<td></td>
</tr>
<tr>
<td>4. Characteristic $K$-concept</td>
<td></td>
</tr>
<tr>
<td>5. Empirical trends</td>
<td></td>
</tr>
</tbody>
</table>

The non-interaction method simply assumes that $\Delta a$ in any load cycle is dependent on the applicable $K$-value pertaining to that cycle. As discussed before it is physically incorrect and it leads to overconservative crack rate estimates.

Interaction methods based on $K_{\text{eff.}}$ also start from the idea that the crack
rate is uniquely related to $\Delta K$ and $R$ values. However, in these methods attempts are made to account for the effect of crack tip plasticity in preceding load cycles on the real values of $K$ ($K_{\text{eff.}}$) and $R$. A notable example is the Willenburg model \cite{62}. More comments on this model and the Wheeler model \cite{11} are given later on.

The equivalent $K$ method was proposed for random loading \cite{60,62,71}. The basic assumption is that an equivalent $\Delta K$ can be indicated, which under constant-amplitude (CA) loading will give the same crack rate as the random loading.

\[
\left[ \frac{da}{dn} = f(\Delta K_{\text{eq.}}) \right]_{\text{random loading}} = \left[ \frac{da}{dn} = f(\Delta K) \right]_{\text{CA loading}}
\]

The rms value of the random loading was proposed for this purpose: $\Delta K_{\text{eq.}} = \Delta K_{\text{rms}}$. There are no theoretical reasons to see that the basic assumption is plausible. Checks in the literature have shown systematic deviations \cite{60,62,71}.

The characteristic $K$-concept was proposed by Paris \cite{70} for stationary random loading. Contrary to the equivalent $K$-method a relation to constant-amplitude data was abandoned. The basic idea is that the random variations of the stress in the crack tip zone are fully described by $\Delta K_{\text{rms}}$

\[
\frac{da}{dn} = g_R (\Delta K_{\text{rms}})
\]

with $\Delta K_{\text{rms}} = C S_{\text{rms}} \sqrt{\pi a}$

Available evidence in the literature \cite{60,71,72} appear to confirm the validity of this approach. It should not be overlooked that the relationship in Eq. (4) will be a function of a stress ratio, which was defined by S.H. Smith \cite{60} as $\gamma = S_m / S_{\text{rms}}$. Two more important variables of random loading are the spectral density function and the crest factor (clipping ratio). The first one appears to have a small effect on the crack rate \cite{60,71}, although the evidence is still limited. It is difficult to indicate theoretical expectations on this topic. The effect of the crest factor is practically unknown, but a significant effect should be expected in view of results obtained in flight-simulation tests \cite{6,61}.

The promising results obtained for random loading have promoted a similar approach to flight-simulation loading \cite{5,73}. A characteristic $K$-value can also be defined easily for this type of loading. Unfortunately the test results
indicated that a relation similar to Eq. (4) did not hold. In a theoretical analysis of this observation it was shown that Eq. (4) should be replaced by

$$\frac{da}{dn} = f_R(\Delta K, \frac{dK}{da})$$  \hspace{1cm} (6)

In constant-amplitude tests and random load tests the effect of $dK/da$ turned out to be negligible, but for flight-simulation loading this did no longer apply. A more extensive discussion is given in [5,73].


Both models were proposed to explain crack growth delays induced by high loads. Both employ plastic zone sizes as indicated in Fig. 10. Further the concepts are different, however. Wheeler related the retardation to $r_{pl}/\lambda$. Willenborg made a rather strange assumption about the effective stress as affected by the plastic deformation of the overload. As a result $\Delta K_{eff}$ and $R_{eff}$ are calculated and these values are used to obtain the applicable $da/dn$ values.

Both models were checked by various authors [14,15,26,62,63,74], but a systematic agreement with test results was rarely found. The advantage of the Willenborg model was said to be that no empirical material constants were required. However, in order to improve the reliability of the Willenborg model a material constant was again introduced [29,62]. Actually the number of variables of complex load-time histories is rather large and it would be surprising if a single empirical constant would be sufficient to account for all variables. This can only be expected if the model itself is physically correct. Unfortunately this does not appear to be true. The Willenborg model and the Wheeler model consider plasticity in the crack tip zone only with a simple assumption about the plastic zone size. A reversed plastic zone as discussed by Rice [75] is not included. Both models predict maximum retardation immediately after an overload, while the retardation is assumed to pass off as soon as $r_{pl} = \lambda$ (Fig. 10). These features were shown to be incorrect, which can easily be explained by crack closure. The models exclude the possibility of accelerated growth, which is also a real phenomenon.

In [67] requirements for a more realistic model were indicated. The complexity of the problem became clearly evident. The first approach should be to incorporate crack closure. Nevertheless, it cannot be ruled out that this will not be enough to make a model sufficiently reliable for prediction
problems. Extensive research is still required to reach this goal.

**Empirical approach**

If fundamentally correct laws are not available it is a practical solution to look for a systematic empirical rule. Attempts were made to start from delays as observed in tests with overloads. Delay factors were derived from such tests, which then were translated to more complex loading programs [12,14,18, 19,48]. The more complex programs were still rather simple as compared to service load-time histories. Since these propositions start from delays only, ignoring accelerations it is doubted whether it will ever lead to a useful rule. Checking empirical rules by non-practical types of loading is not a logical approach. Similarly it is also illogical to disprove an empirical rule by tests with non-practical types of loading.

The prediction method proposed by Habibie [76-78] for application to service load-time histories is starting from crack growth retardations as observed in flight-simulation tests. Apparently this is a more practical approach. Habibie employs the K faktor to arrive at crack growth retardation formulae. Originally he adopted 8 material constants, but this number was reduced later on. His predictions for flight-simulation test results are quite good, but the method was less successful in predicting the trends for program tests. For an empirical method the latter result need not be a disadvantage if the prediction for realistic load sequences is envisaged.

**Multi-variable regression analysis**

Habibie was still employing the K-faktor to support his method by physical arguments. A more rigorous empirical approach is to start directly from empirical data and to look for mathematical relations, which represent the trends best. This leads to multi-variable regression analysis of test data. An illustrative example has recently been suggested for flight-simulation loading by Simpkins, Neulieb and Golden [79]. The regression function proposed is:

\[ N = D \cdot (\sigma^2)^a \cdot (\frac{S}{\sigma})^b \cdot (n)^c \cdot (1-\frac{Z}{2}) \]  

(7)
where $D$ is a constant, $\sigma^2$ is the variance of the load spectrum in flight, $\bar{S}_{1g}$ is the average meanstress in flight, $\bar{n}$ is the average number of cycles per flight and $\bar{Z}$ is the average of $S_{min}$ of the ground-to-air cycle divided by $S_{1g}$. The equation is accounting for four independent variables, which are supposed to characterize the severity of the load history. The five constants $D$, $a$, $b$, $c$ and $d$ are determined by a regression analysis applied to empirical data. Equation (7) applies to the fatigue life $N$ under flight loading. A similar approach was proposed for random loading [80]. Applications to crack propagation although not yet made so far, could occur in the same way. The problem is to indicate the independent variables, which sufficiently characterize the load-time history.

As a matter of fact a rigorous analysis is still hampered by insufficient available test data. For this purpose a systematic test program was proposed earlier [61]. The purpose should be to obtain systematic data on the effect of the more important variables of realistic load-time histories.

CONCLUSIONS

1. The problem of predicting crack growth rates in service cannot be solved without a thorough knowledge about the load-time histories occurring in service. A statistical distribution function of peak loads is insufficient. Knowledge about load sequences is essential.

2. During crack growth under variable-amplitude loading significant interaction effects will occur. Both crack growth retardations and accelerations have amply been demonstrated. Relevant evidence is available from tests with simple types of loading (overloads, step loading), but similar trends were observed in tests with more complex load sequences (random loading, program loading and flight-simulation loading).

3. Several mechanisms can contribute to interaction effects and it is difficult to separate the contributions of each of them. Actually the picture of damage accumulation during crack growth under variable-amplitude loading is very complex. However, it has well been shown that crack closure gives a significant contribution. New evidence is reported in the present paper. It has to be strongly recommended that empirical studies on crack growth under variable-amplitude loading should anyhow include crack closure measurements.
4. During complex load-time histories high-amplitude cycles contribute more crack extension than the non-interaction conception will predict, whereas low-amplitude cycles contribute less. This observation could be significant for environmental effects.

5. Since interaction effects are very sensitive to the sequence of the loads the "equivalence" of complex load-time histories and simplified histories (e.g. program loading) is illusory. Actually realistic answers from tests can only be expected if realistic load-time histories are simulated.

6. There is an increasing activity reported in the literature to develop prediction techniques. Unfortunately available crack growth models are too simplistic and reliable predictions cannot be expected with any certainty. Observations on crack closure as affected by overloads in relation to material thickness and yield strength strongly emphasize the controlling influence of plastic deformation on fatigue crack growth. It should therefore be attempted to include crack closure in a fatigue model, although this alone may not be sufficient.

7. For random loading the applicability of $\Delta K_{\text{rms}}$ is promising. Information on the effect of the crest factor (clipping ratio) is insufficient.

8. For the time being a prediction method based on multi-variable regression analysis could provide useful information for practical problems. However, insufficient empirical data is available as yet. Empirical investigations to fill this gap should be recommended.

REFERENCES

[7] Recent test results of W.J. Arkema, Aerospace Department, Delft University of Technology.


FIG. 1 - Limitations of fail-safety

FIG. 2 - Two samples of service load-time histories
### Fig. 3
Comparison between crack propagation lives under random loading and "statistically equivalent" programmed loading [4]

<table>
<thead>
<tr>
<th>2024-T3 Alclad</th>
<th>random</th>
<th>programmed, short period</th>
<th>programmed, long period</th>
</tr>
</thead>
<tbody>
<tr>
<td>cycles for crack growth from 24 mm to 100 mm</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Fig. 4
The effect of sheet thickness on crack growth delays [7]

- Material: 2024 - T3
- $B = \text{thickness (mm)}$
- $S_{\text{peak}} = 196$ (28.4)
- $S_{\text{max}} = 147$ (21.3)
- $S_{\text{min}} = 98$ (14.2)
- $\text{MN/m}^2$ (ksi)

### Fig. 5
Plastic deformation in the wake of a crack (real size).
Crack viewed through a window. The upper light part is the reflection of a FL tube.
Fig. 6  Effect of sheet thickness and overload on crack closure [7]

Fig. 7  Crack-opening stress before and during delay period (material and stress levels, see Fig. 6) [7]
FIG. 8 - Load history during a flight-simulation test [66, 67]. Flight with severe gust loads

FIG. 9 - COD measurements during a severe flight of a flight-simulation test [66, 67]

Fig. 10 Plastic zone size concepts in the models of Wheeler and Willenborg