Acknowledgments

I would like to thank NedMag Industries B.V. for their tremendous help with this research. I especially would like to mention Joren Bullen, as he has provided assistance in numerous ways.

Furthermore I would like to express my gratitude to Guy Drijkoningen, Jan Thorbecke and Rob Arts from the Delft University of Technology for helping me out with the many problems I came across during this research.

Guus van Noort
August 2009
Abstract

In the northern part of the Netherlands the (magnesium containing) salt minerals carnallite and bischofite are extracted using solution mining methods, such as squeeze mining. As a consequence subsidence up to the surface is taking place, potentially causing harm to the environment. Given this fact the local government allows extraction of this salt under the condition that subsidence at the surface does not exceed a pre-defined limit. This makes the importance of salt mining induced subsidence evident. For an accurate prediction the 3-D characteristics of the caverns are important. Currently it is impossible to predict what effect squeeze mining has on the characteristics of the caverns, and thus on the subsidence.

This thesis addresses the question whether time-lapse seismic reflection techniques can be used to image and quantify the effects of mining magnesium salt in the north of The Netherlands. The use of seismic time-lapse techniques to indentify produced salt zones has not been investigated before and this study must be considered as a feasibility study, using synthetic seismic data. The questions addressed in this thesis are:

- Can the effects of solution salt mining be detected in seismic time-lapse mode?
- If so, can these effects be quantified?

In our approach we studied the time-lapse effects of different scenarios; representing a vertical and a lateral extension of the mine due to salt production have been evaluated. These scenarios have been transformed into different subsurface models that were an input to an acoustic and elastic finite difference scheme in order to create synthetic data. The geometric and material properties in the scenarios are based on the interpretation of real seismic data. A combination of well data and empirical relations has been used to derive the necessary seismic parameters.

The main findings can be summarized as:

1) A seismic reflection of the salt mine is visible in seismic shot records, CMP-gathers and migrated sections. The exact geometry of the mine cannot be distinguished in the data, because of interference effects.

2) To derive time-shifts and amplitude changes caused by geometry and property changes in seismic time-lapse mode, 2-D cross correlation on migrated data was used. This technique allows deriving a horizontal shift as well as a time-shift. The amplitude changes were calculated by comparing the amplitude maximum from a 2-D cross correlation window with the maximum amplitude from a 2-D auto correlation window. The difference is expressed as a percentage.

3) A vertical extension of 5 m causes a potentially detectable time shift of 1.5 ms for the acoustic case and 2.0 ms for the elastic case. The amplitude changes are respectively 5.3% and 7.1%. For a purely lateral extension of 100 m of the caverns no time shift is found for the acoustic and elastic case and the amplitude change is 0.2% and 2.0% respectively. These results show that the amplitude change caused by a vertical extension is significantly higher than the one caused by a lateral extension of the mine. In order to make lateral changes of the salt mine visible one could opt for 1-D cross correlation. The time shifts and amplitudes found are comparable as those found in literature for the oil and gas industry.

The final conclusion yields that the effects of solution salt mining can be detected and quantified in seismic time-lapse mode. Some effects in this seismic study are large enough to be seen in real seismic data. It is therefore feasible to use time-lapse seismic to monitor geometric and material changes of an underground solution salt mine.
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CHAPTER 10 CONCLUSIONS AND RECOMMENDATIONS

10.1 Conclusions
10.2 Recommendations

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List of symbols

\( t \)  
Time

\( M \)  
Number of traces in a CMP gather

\( x_m \)  
Offset

\( v_i = v_i(x,t) \)  
Particle velocity

\( f_i = f_i(x,t) \)  
Volume density of external force

\( q = q(x,t) \)  
Volume injection source

\( \rho = \rho(x,t) \)  
Volume density of mass

\( p = p(x,t) \)  
Pressure

\( \lambda \)  
Lamé's first parameter, or

\( \lambda \)  
Wave length

\( \mu \)  
Shear modulus, also called Lamé's second parameter

\( \kappa = \kappa(x,t) \)  
Compressibility

\( \tau_{ij} = \tau_{ij}(x,t) \)  
Stress tensor

\( C_v \)  
Specific heat at constant volume

\( C_p \)  
Specific heat at constant pressure

\( c_p \)  
Compressional wave velocity

\( c_s \)  
Shear wave velocity

\( \delta_{ij} \)  
Kronecker delta: \( \delta_{ij} = 1 \) for \( i = j \) and \( \delta_{ij} = 0 \) for \( i \neq j \)

\( u_i = u_i(x,t) \)  
Particle displacement

\( Z = Z(x) \)  
Acoustic impedance

\( R = R(x) \)  
Reflection coefficient

\( R = R(t) \)  
Ricker wavelet

\( A_i \)  
Amplitude of wave in medium \( i \)

\( s_h \)  
Horizontal slowness
Chapter 1  Introduction

In this thesis the issue whether time-lapse seismic reflection techniques can be used to image and quantify the effects of mining magnesium salt in the north of The Netherlands is addressed. Reflection seismic techniques are widely used in the hydrocarbon industry for imaging the subsurface until a few kilometers depth. Time-lapse seismic is essentially used in the hydrocarbon industry for inversion to saturation changes in producing oil and gas fields. For solution salt mining a similar approach can be thought of to identify produced salt zones (or cavity growth) in time. However, no time-lapse studies have been carried out on solution salt mines up to now. In order to check whether this is possible this feasibility study has been performed.

1.1  NedMag

Because of the discovery of the large Groningen gas field in 1959, all of the subsurface of the north of The Netherlands was imaged using the seismic reflection method. During the exploration drilling for the new found gas and oil in the Groningen province, they had to drill through the thick Zechstein formation, which serves as seal. Within this formation layers that contain a high content of carnallite and bischofite were found. These two minerals are magnesium salt minerals. Especially bischofite, which contains the highest amount of magnesium and therefore delivers very high quality magnesium products, is very rare in the world.

The mining part Shell, which existed until 1972, was interested in mining the magnesium salts to make magnesium metals. Because there was no experience yet with mining of magnesium salts a test was needed to prove whether it was possible and/or profitable. The first test location was near Barradeel, province of Friesland in 1969. Only carnallite was present here. The mining proved to be successful, however non profitable. (Renier, 2006)

A later study by P.J. Coeleweij was set up to find more magnesium salts. The total volume of magnesium salts in the north of The Netherlands was estimated during this study. Based on this study Tripscompagnie, near Veendam in the province of Groningen, was chosen to mine magnesium salts. In 1972 prices of magnesium steel dropped dramatically and therefore it was not profitable anymore to mine magnesium salts. Making magnesium oxide for fireproof material proved to be a good alternative though but needed a higher quality production. A study by J. F. Holtrop in 1974 showed this was possible, and therefore production continued. (Renier, 2006)

Figure 1-1: The provinces of The Netherlands. Groningen is marked in red.
The mining company started of as a part of Shell, Shell Delftstoffen B.V. This name changed to Billiton International Metals after reorganization in 1972. In 1980 a factory was built for the actual production of magnesium oxide. In order to split costs and risks Billiton and the Noordelijke Ontwikkelings Maatschappij (NOM) merged interests in Magnesia International. In 1984 NOM withdrew from Magnesia International, after which the company was renamed Billiton Refractories. In 1994 Shell sold the whole Billiton group, which then sold Billiton Refractories. Billiton Refractories was bought by Lhoist, a couple of former board members and again NOM. The name was then changed to NedMag. (Renier, 2006)

Nowadays NedMag produces different types of magnesium and calcium products that are used for a wide variety of applications. In the table below the produced products are summed up and some examples of their use are given.

<table>
<thead>
<tr>
<th>Name</th>
<th>Formula</th>
<th>Examples of use</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnesium oxide</td>
<td>MgO</td>
<td>Cement and steel industry</td>
</tr>
<tr>
<td>Magnesium chloride</td>
<td>MgCl₂</td>
<td>Textile finishing, catalyst production</td>
</tr>
<tr>
<td>Magnesium hydroxide</td>
<td>Mg(OH)₂</td>
<td>Neutralizing acid waters</td>
</tr>
<tr>
<td>Calcium chloride</td>
<td>CaCl₂</td>
<td>Dust control, road stabilization</td>
</tr>
</tbody>
</table>

### 1.2 Mining history and methodology

The production of carnallite started in 1972. The production in 1972 started with one well (VE-1) straight into the top of the salt body. This well was drilled from the village of Borgercompagnie, around 7 kilometers north-west of Veendam. From this site three more wells (VE-2 to VE-4) were drilled in 1976 to increase production. From another site between Tripscompagnie and Veendam another six were drilled in 1982 (TR-1 to TR-6), and two more (TR-7 and TR-8) in 1992. The wells VE-1 and VE-2 from the site at Borgercompagnie were abandoned in 1982 and 1998 respectively, while the other six wells are all still producing. The location of the wells is shown in Figure 1-3.
Figure 1-3: Location of the wells.

Until 1996 conventional solution mining (See Figure 1-4) was used to mine mainly carnallite. Conventional solution mining is done by pumping water down the well and pumping up the brine through a second, but smaller pipe within the same well. At that time NedMag did not have enough knowledge to mine the bischofite as well. They were afraid that because of the expansion of the bischotitic brine, which will be further discussed in Chapter 3, the salt surrounding the caverns would crack. This would lead to losing brine to surrounding layers.

Based on production data and a simple geometry the Dutch ‘Staatstoezicht op de mijnen’ (Ministry of Economic Affairs) imposed a maximum diameter of 100 m per cavern in order to prevent too much subsidence. Therefore NedMag started mining in the bottom layer, where an oil roof prevented halite to dissolve. When production data showed that the maximum diameter had been reached, one would shoot off the production and injection pipe at the height of the top magnesium layer and raise the oil roof. When the cavern in that layer had also reached 100 m production in from well had to be stopped completely.

Figure 1-4: Conventional solution mining, as practiced by NedMag until 1996.
In 1996 the method was changed to squeeze mining (See Figure 1-5). This method is based on the fact that magnesium salts become mobile under large pressures. The salts are pressurized by the overburden and creep towards zones of lower pressure. Bischofite is most sensitive to creep, followed by carnallite and then halite. The mobility ratios for halite, carnallite and bischofite are assumed by NedMag to be 1: 10 : 100 respectively. Caverns created by the original solution mining are kept at a pressure of 65 bar lower than the lithostatic pressure. The bischofite will therefore creep towards the caverns, resulting in a gradual thinning of the salt layer resulting in elastic and inelastic deformation of the overburden.

Figure 1-5: Squeeze mining, as practiced by NedMag at this moment.

Squeeze mining leads to a more rapid surface subsidence than conventional lithostatic mining in which there is no net volume change in the salt layers. Squeeze mining causes a different effect, because a part of the created brine is replaced by salt. This enables a production of more salt from the same cavern. So far theories to predict subsidence caused by squeeze mining have shown not to be accurate enough (See Section 3.4). This is mostly due to the fact that very little is known about the mechanism in the bedded bischofite layers as an effect of squeeze mining.

The local governments and ‘Staatstoezicht op de mijnen’ therefore imposed new regulations regarding subsidence and the damages associated with them. The most important one is that the subsidence at any point should not exceed 65 centimeters. NedMag needs to propose a new production plan to produce the remaining salt when the subsidence has reached 65 cm. Because the maximum subsidence allowed has not been reached at any point yet, no new wells have had to be drilled from 1995 onwards.

1.3 Seismic monitoring

During the last decade, seismic monitoring of the subsurface has become more popular in the hydrocarbon industry although it is still not a standard production technique. The technique allows observing changes in the subsurface, which are caused by the production of hydrocarbons. Nowadays such monitoring is often established via so-called time-lapse seismics, i.e., seismic is done again after some lapsed time of, say, one year. Via comparison and analysis of differences, conclusions can be drawn on the changes of the reservoir itself. A very striking
feature of such analyses is that it may happen that while the image itself does not reveal so much structure or too much structure, the difference may be significant. This has been shown for carbonate reservoirs (see Calvert, 2005).

In the north of the Netherlands, recently an initiative has been taken to permanently monitor the subsurface: the LOFAR program. Although initially started up as an astronomical program, it has become a very fast multi-sensor network to which other applications than astronomy can be accommodated. The seismic technique has been applied here. A dense array has been installed near Annerveen, just south of Veendam where the mining of magnesium salts is taking place. It could be a possibility to link up this monitoring network to the salt mine, to observe changes in the subsurface due to mining.

1.4 Problem definition and research goals

Since the local governments demand that subsidence should not exceed 65 centimeters at any point at the surface it is important to predict subsidence. For an accurate prediction the geometry of the caverns is important. Currently it is impossible to predict what effect squeeze mining has on the geometry of the caverns in the subsurface, and thus on the subsidence of the surface. The main goal of this research is to find out the feasibility of detecting how the geometry of the cavern changes in time using time-lapse seismic data, so that we might be able to relate these geometry changes to subsidence at later stages.

4D-seismic potentially is a very useful method to image the effect of squeeze mining on salt layers in the subsurface and thus help in predicting subsidence. This research focuses on the feasibility of 4D-seismic on the magnesium salt layers in the north of The Netherlands through a study on synthetic data. The questions to be are answered in this report are:

- Can the effects of solution salt mining be detected in seismic time-lapse mode?
- If so, can these effects be quantified?

In order to address these questions it is important to understand what the salt body we are interested in looks like (Chapter 2), what the implications of the chemistry involved with salt mining are (Chapter 3). In Chapter 4 to Chapter 7 is explained how the geometry changes of the mine are modeled in seismic sense. Different tools to analyze the data qualitatively and quantitatively are discussed in Chapter 8. The results can be found in Chapter 9. In Chapter 10 conclusions are drawn and the research goals are answered.

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1 LOFAR stands for LOw Frequency ARray. For more information the program’s website offers a great deal of information: www.lofar.org.
Chapter 2  Geology of the Zechstein formation

This chapter gives a brief description of the geological history that is relevant for the forming of the salt body near Veendam. Many details are skipped, as they are not relevant for this seismic study, the goal is only to provide necessary background information on the forming of the Zechstein formation.

The magnesium salts in the north of The Netherlands were formed in the Permian. The Permian lasted from 299 to 251 million years ago (See Figure 2-1). The Permian is subdivided in the early Permian (Rotliegendes) and the late Perm, which is also called Zechstein. The magnesium salts we are interested in were formed during the Zechstein era. The formation in which they are found is therefore called the Zechstein formation.

<table>
<thead>
<tr>
<th>ERA</th>
<th>PERIOD</th>
<th>EPOCH</th>
<th>AGE*</th>
<th>MAJOR EVENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>CENOZOIC</td>
<td>Quaternary</td>
<td>Holocene</td>
<td>0.01</td>
<td>Earliest Homo sapiens</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Pleistocene</td>
<td>1.8</td>
<td>Earliest hominids</td>
</tr>
<tr>
<td></td>
<td>Tertiary</td>
<td>Pliocene</td>
<td>5.3</td>
<td>Dominance of mammals</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Miocene</td>
<td>23.8</td>
<td>Widespread extinctions</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Oligocene</td>
<td>23.7</td>
<td>First flowering plants</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Eocene</td>
<td>55</td>
<td>Dinosaurs dominant</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Cretaceous</td>
<td>65</td>
<td>Widespread extinctions</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Jurassic</td>
<td>145</td>
<td>First reptiles</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Triassic</td>
<td>200</td>
<td>First fish</td>
</tr>
<tr>
<td></td>
<td>Permain</td>
<td></td>
<td>251</td>
<td>Fishes dominant</td>
</tr>
<tr>
<td></td>
<td>Carboniferous</td>
<td></td>
<td>299</td>
<td>Soft-bodied animals</td>
</tr>
<tr>
<td></td>
<td>Devenian</td>
<td></td>
<td>350</td>
<td>Appearance of fossils</td>
</tr>
<tr>
<td></td>
<td>Silurian</td>
<td></td>
<td>417</td>
<td>First one-celled organisms</td>
</tr>
<tr>
<td></td>
<td>Ordovician</td>
<td></td>
<td>443</td>
<td>Origin of the earth</td>
</tr>
<tr>
<td></td>
<td>Cambrian</td>
<td></td>
<td>490</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Precambrian</td>
<td></td>
<td>542</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3000</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>4600</td>
<td></td>
</tr>
</tbody>
</table>

*Age in millions of years (Ma)

Figure 2-1: The geological time scale. (www.teara.govt.nz)

2.1 Pangea

As a result of plate tectonics climate conditions and the locations of the continents have always changed in the geological time. During the Permian, all the continents collided under influence of plate tectonics into a super continent called Pangea (See Figure 2-2) was formed. The collision of the different tectonic plates caused mountains to form. This can be imagined as two continental crusts folding over each other. The total of continental crust therefore decreased, so that the oceanic crust by definition had to increase. The volume of mid-oceanic ridges decreased so that the oceans could hold more water, which caused a sea level fall (Berendsen, 2004.) A large ocean, the Tethys Ocean, existed east of Pangea.

2 Pangea is sometimes written as Pangaea or Pan\textgreek{a} in literature.
The massive area of the continent caused extreme climate conditions, different from place to place. The mountains formed a large rain shadow so that wind containing rain could not reach the center of the continent. Arid conditions were therefore abundant. Eastern Pangea was very warm, because of warm sea streams. The south of Pangaea was still covered with ice. (Berendsen, 2004.)

### 2.2 The Netherlands during the Permian

The location of The Netherlands changed tremendously over the past geological eras (See Figure 2-3). During the Permian, The Netherlands lay at the current position of the Sahara, just north of the equator. The greenhouse effect was very high, around a factor 2 higher than nowadays. These conditions led to a very humid and warm desert climate, with high evaporation.
2.3 Evaporation in the Zechstein Sea

During the Zechstein the climate stayed the same as during the Rotliegendes, but a relative sea level rise created an enormous epeiric sea (i.e. an enormous sea that overlies a large part of a continent). It covered what now includes the North Sea, lowland areas of Britain and the north European plain through Germany and Poland. The north of The Netherlands lay on the border of this sea. The epeiric sea, called the Zechstein Sea, only had a very shallow connection to the Tethys Ocean. Water could only pour into this sea in case of relative sea level rise, whereas at low stand conditions the sea would retain from inflow of fresh sea water. During periods of relative low sea level, the water in the sea evaporated and evaporites were formed. The evaporation cycle belonging to this sea will be discussed in Section 3.2. Obviously the sea must have been situated in a subsiding area, since otherwise there would not be enough space for new evaporites to form.

![Figure 2-4: (Left) The location of the Zechstein Sea. In the background the different countries can be recognized. (www.zechsteinmagnesium.com) (Right) The current extension of the Zechstein formation. (magtopical.com)](image)

![Figure 2-5: All the different steps involved with the forming of the Zechstein evaporites. (www.chemie-master.de)](image)

It has been proven that many cycles of inflow of fresh sea water and evaporation of salts have taken place. In the Zechstein formation in the north of The Netherlands 5 main cycles are recognized, all divided up in many sub cycles. Only the third main cycle contains magnesium salts.

The magnesium minerals are not found anywhere else in the neighborhood of Veendam, so it is most likely that this area was situated in a large depression, where all the not yet evaporated sea water was collected.
The Zechstein formation was conserved during the Triassic, the period that directly followed the Permian, when continental sediments were deposited. These continental deposits have covered up the Zechstein formation and have thus prevented it from dissolving again at a period of relative sea level rise.
Chapter 3  Precipitation and solution of salts

In this chapter it is explained how the evaporites found within the Zechstein formation are formed in terms of chemistry, and how this affects solution mining.

Solution mining is a usually more economical alternative for the mechanical excavation of salt ores. It is based on the idea that the desired ores dissolve while the undesired waste does not and accumulates at the bottom of the cavern.

In the case of the NedMag-mine near Veendam the situation is more complex, since bischofite is produced from beds that are a mixture of halite, sylvite, carnallite and bischofite that all dissolve. With techniques based on the solubility of the different salts brine is produced that consists almost only of bischofite. How this is achieved will be explained in this chapter. The work of Renier (2006) will be used as a lead for this chapter.

3.1 Types of salt

As mentioned previously, many types of salt are found within the Zechstein formation. All of them are summed up in this section, including their respective chemical formula, molecular weight and a picture of the crystals.

**Calcite**
Chemical formula: CaCO₃
Molecular weight: 100.09 g

*Figure 3-1: Calcite crystals. (Dave Barthelmy)*

**Gypsum**
Chemical formula: CaSO₄ · 2 H₂O
Molecular weight: 172.17 g

*Figure 3-2: Gypsum crystals. (Dave Barthelmy)*
**Halite**
Chemical formula: NaCl
Molecular weight: 58.44 g

*Figure 3-3: Halite crystals. (John Veevaert)*

**Kieserite**
Chemical formula: MgSO₄·H₂O
Molecular weight: 138.38 g

*Figure 3-4: Kieserite crystals. (Jeff Weissman)*

**Sylvite**
Chemical formula: KCl
Molecular weight: 74.55 g

*Figure 3-5: Sylvite crystals. (Jeff Weissman)*

**Carnallite**
Chemical formula: KCl·MgCl₂·6H₂O
Molecular weight: 277.85 g

*Figure 3-6: Carnallite crystals. (Jeff Weissman)*

**Bischofite**
Chemical formula: MgCl₂·6H₂O
Molecular weight: 203.30 g

*Figure 3-7: Bischofite crystals. (Thomas Witzke)*
3.2 Precipitation of salts

In this section it is explained what determines the precipitation of salt minerals and how this has affected the Zechstein Sea. It is beyond the scope of this project to provide a detailed description, thus only the most important factors are shortly highlighted. For more information Braitsch (1971) and Renier (2006) are suggested.

The order in which salts precipitate in an evaporating sea depend on a number of factors. First of all it depends on the quantity of the different ions available in the sea water. Obviously some salts cannot precipitate, as their ions are not available.

The order of precipitation also depends on the thermodynamic area of stability for the different salts. The area of stability depends mainly on the solubility of the salts, but also concentrations in the brine and temperature.

Combining these factors leads to a precipitation sequence for sea water, as a function of the amount of water that has been evaporated. This is also shown in Figure 3-8.

<table>
<thead>
<tr>
<th>Type of salt</th>
<th>Sea water evaporated</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calcite (CaCO₃)</td>
<td>50%</td>
</tr>
<tr>
<td>Gypsum (CaSO₄ · 2 H₂O)</td>
<td>70%</td>
</tr>
<tr>
<td>Halite (NaCl)</td>
<td>90%</td>
</tr>
<tr>
<td>Kieserite (MgSO₄ · H₂O)</td>
<td>&gt; 90%</td>
</tr>
<tr>
<td>Sylvite (KCl)</td>
<td>&gt; 90%</td>
</tr>
<tr>
<td>Carnallite (KCl · MgCl₂ · 6 H₂O)</td>
<td>&gt; 90%</td>
</tr>
<tr>
<td>Bischofite (MgCl₂ · 6 H₂O)</td>
<td>&gt; 90%</td>
</tr>
</tbody>
</table>

In this order it has been taken into account that some minerals are converted into other minerals during precipitation. The order of solution is the exact opposite of the order of precipitation.

Figure 3-8: Evaporites precipitation order of sea water. (www.zechsteininside.com)
Halite precipitates when 90% of the sea water already has evaporated. Kieserite, sylvite, carnallite and bischofite precipitate from the last little bit of sea water. According to Renier (2006) the precipitation of these four salts does not depend on solubility, and they thus solute at the same time. This means that it is hard to distinguish different layers of these separate minerals, but usually a combination of them is found within one layer.

In the salt body near Veendam there is almost no kieserite available, so kieserite is not taken into account in the remainder of this chapter.

### 3.3 Solution of salts

Calcite and gypsum are absent in the Zechstein formation, since they had already been precipitated. Halite sits lower in the solution sequence than the other available salts. Generally injection water mixes with the cavern MgCl₂ brine. Due to the still relative high MgCl₂ concentration the water has very little halite dissolution potential. If the water first contacts carnallite salt a carnallitic brine results and sylvite remains as a solid residue. When the resulting carnallitic brine contacts bischofite salt the bischofite goes into solution and a bischofitic brine results at the expense of the precipitation of carnallite salt. When the injection water first contacts bischofite salt a bischofitic brine is formed immediately.

When carnallite dissolves K+ and I- ions are released and sylvite will precipitate since it is higher in the precipitation sequence. This lowers the salinity of the brine, so that more carnallite can dissolve and the brine gets richer in magnesium. If this brine comes in touch with bischofite, carnallite will precipitate and the brine becomes even richer in magnesium content. Now, it is clear that in the caverns at Veendam two types of brine can exist, either a more bischofitic brine or a more carnallitic brine, depending on with which salt(s) the brine has been in contact with. The composition of the brine in the caverns can be determined from phase diagrams in combination with stability diagrams.

When hydrated salts – like carnallite and bischofite – dissolve, water is released. Per kilogram bischofite, for instance, this is more than half a kilogram of water. In this new water more bischofite can dissolve. The same holds for carnallite, but this effect also causes sylvite to precipitate. The precipitation of sylvite however causes more carnallite to dissolve. This whole effect can be determined using mass balances. The whole derivation of these mass balances is not relevant for this research, but is important to realize that the volume expansion of dissolving carnallite and bischofite can be derived from these mass balances. This can be seen in Table 3-1. For the mass balances mentioned above the following assumptions have been made (Renier, 2006):

- The theoretically determined composition of the different dissolutions is the final dissolution that is obtained in an equilibrium state, after a reaction time that is long enough to obtain such a composition when enough necessary salts are available all the time;
- The solution reactions are always complete;
- Halite, sylvite and kieserite do not dissolve in the presence of carnallite and/or bischofite;
- The amount of Na⁺ en SO₄²⁻ ions in a solution of bischofite are negligible;
- When a carnallitic brine comes in contact with bischofite the conversion to bischofitic brine is complete and the reaction times are fast.
### Table 3-1: Volumes that are created when creating different types of brine. (Renier, 2006)

<table>
<thead>
<tr>
<th></th>
<th>Solution</th>
<th>Bischofite brine (direct)</th>
<th>Carnallite brine (direct)</th>
<th>B. brine (from C. brine)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(-)</td>
<td>(m³)</td>
<td>(m³)</td>
<td>(m³)</td>
</tr>
<tr>
<td><strong>Input</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Water</td>
<td>1.00</td>
<td>1.00</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>C. brine</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1.00</td>
</tr>
<tr>
<td><strong>Salt in solution</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bischofite</td>
<td>2.29</td>
<td>-</td>
<td>0.67</td>
<td></td>
</tr>
<tr>
<td>Carnallite</td>
<td>-</td>
<td>1.46</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Halite</td>
<td>-</td>
<td>0.01</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Kieserite</td>
<td>-</td>
<td>0.01</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td><strong>Precipitation of salt</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Carnallite</td>
<td>-</td>
<td>-</td>
<td>0.15</td>
<td></td>
</tr>
<tr>
<td>Sylvite</td>
<td>-</td>
<td>0.24</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Halite</td>
<td>-</td>
<td>-</td>
<td>0.01</td>
<td></td>
</tr>
<tr>
<td>Kieserite</td>
<td>-</td>
<td>-</td>
<td>0.01</td>
<td></td>
</tr>
<tr>
<td><strong>Created brine</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B. brine</td>
<td>3.38</td>
<td>-</td>
<td>1.52</td>
<td></td>
</tr>
<tr>
<td>C. brine</td>
<td>-</td>
<td>2.22</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td><strong>Expansion</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-</td>
<td>0.09</td>
<td>-0.02</td>
<td>0.03</td>
<td></td>
</tr>
<tr>
<td><strong>Cavern size</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-</td>
<td>2.29</td>
<td>1.24</td>
<td>1.73</td>
<td></td>
</tr>
<tr>
<td>Production</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B. brine</td>
<td>1.09</td>
<td>-</td>
<td>1.03</td>
<td></td>
</tr>
<tr>
<td>C. brine</td>
<td>-</td>
<td>0.98</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>

### 3.4 Creep of salts

The convergence of the caverns, caused by the volume effect of the forming of brines, is mainly determined by the creep of the different types of salt. As mentioned in Section 1.2, bischofite is most sensitive to creep. The degree of creep is not yet well understood. From literature we know that creep depends on the type of salt, temperature, applied pressure and stadium of creep.

Different models have been proposed to predict creep of salts. All of them, however, describe experiments at laboratory scale. The conditions for these laboratory experiments are fundamentally different from the actual conditions within the caverns. The most fundamental differences that the models do not describe are (Renier, 2006):

- An increase of pressure of the brine and salt with depth;
- An increase of temperature with depth;
- Heterogeneity of the salt body in which the caverns are situated;
- Within the Veendam salt body individual salts are not recognizable.

For these reasons it is very hard to describe creep. The only conclusion that can be drawn is that the creep of halite is much slower than the creep of carnallite. The creep of carnallite again, is much slower than the creep of bischofite.
3.5 Implications for the forming of caverns

Because of the dissolution of the salts, caverns are formed. The exact shape of these caverns is not known. The expansion of the bischofitic brine (See Section 3.3) in particular was the reason bischofite was not mined until 1996. Fear existed that the expansion of the brine would lead to a dramatic increase in pressure and therefore cracks in salt around the cavern, which then would lead to leakage of the brine. When bischofite was accidentally produced, it was found out that the increase of pressure was not so dramatic after all, and bischofite could therefore be mined without trouble.

Since creep – and thus the actual geometry of the mine – is so hard to predict, it is hard to predict subsidence. Prediction of subsidence is however necessary in order to predict when production has to be stopped, according to local regulations (See Section 1.2), or where to drill new wells.
Chapter 4  Seismic modeling

The aim of this work is to investigate the effect of salt production on the seismic response over time. Crucial here is the process of the propagation of waves in the subsurface. In this chapter it will be discussed how waves propagate through different media, and how this affects the measured data. Furthermore the numerical implementation of seismic wave propagation will be discussed.

4.1 Seismic wave propagation

In this thesis two approaches are used for the seismic modeling, namely an acoustic approach and an elastic approach. The main difference between these two approaches is that in an acoustic approach the stress field is considered as a scalar, whereas in the elastic approach the stress field is considered a tensor. An acoustic medium, like fluid, therefore only supports longitudinal (P) waves that propagate by compressional and dilatational unaxial strains in the direction of wave propagation. An elastic medium, on the other hand, can support P-waves as well as Shear (S) waves that propagate by a pure shear strain in a direction perpendicular to the direction of wave propagation. These two wave types are shown in Figure 4-1.

![Compressional and shear waves](geology.csupomona.edu)

Figure 4-1: Compressional and shear waves. (geology.csupomona.edu)

4.1.1 Acoustic wave propagation

Let us first focus on the acoustic case. The wave propagation is described by three equations, namely the conservation of mass (continuity equation), conservation of momentum (equation of motion) and the constitutive relation. All these equations are derived from the well-known Navier-Stokes equations. If we assume a non-flowing fluid and linearize the equation of motion, conservation of mass and the equation of continuity we get according to Wapenaar (2009):

Eq. 4-1: $$ \frac{1}{\rho_0} \frac{\partial \rho}{\partial t} + \frac{\partial \vec{v}_i}{\partial x_i} = q $$
where:

\( t \) 
Time

\( v_i = v_i(x, t) \) 
Particle velocity

\( q = q(x, t) \) 
Volume injection source

\( \rho = \rho(x, t) \) 
Volume density of mass

Please note that \( \rho^0 \) denotes the static mass density that is not affected by the propagating wave. It relates to the total density as: \( \rho^{\text{tot}} = \rho^0 + \rho \), where \( \rho \) denotes the acoustic field density, which is induced by the propagating wave. The linearization process means that we assume that the acoustic field parameters are much smaller than the static properties.

In words Eq. 4-1 states that the change of mass density within an infinitely small block plus the mass density that flows out of that block equals the volume density of volume injection source that is injected into that block.

**Eq. 4-2:**

\[
\rho^0 \frac{\partial v_i}{\partial t} + \frac{\partial \rho}{\partial x_i} = f_i
\]

where:

\( f_i = f_i(x, t) \) 
Volume density of external force

\( p = p(x, t) \) 
Pressure

The same reasoning as for Eq. 4-1 holds for Eq. 4-2, but in stead of describing a mass density balance it now describes a force balance.

**Eq. 4-3:**

\[
\kappa^0 dp = \frac{d\rho}{\rho^0},
\]

where:

**Eq. 4-4:**

\[
\kappa^0 = \frac{C_v}{C_p} \frac{1}{\rho^0}
\]

where:

\( \kappa = \kappa(x, t) \) 
Compressibility

\( C_v \) 
Specific heat at constant volume

\( C_p \) 
Specific heat at constant pressure

Eq. 4-3 and Eq. 4-4 describe the effect that pressure has on mass density, or in other words how much the mass density will increase as a result of an increase in pressure.

Substitution and rewriting all the above equations leads to the two equations that describe linear acoustic wave motion in an inhomogeneous non-flowing fluid:

**Eq. 4-5:**

\[
\rho^0 \frac{\partial v_i}{\partial t} + \frac{\partial \rho}{\partial x_i} = f_i,
\]

which is the linearized equation of motion, and:
Eq. 4-6: \[ \kappa^0 \frac{\partial p}{\partial t} + \frac{\partial v_i}{\partial x_i} = q, \]

which is the linearized stress-strain relation.

From now on the superscript 0 for the mass density and compressibility is skipped for notational convenience:

Eq. 4-7: \[ \rho^0 \rightarrow \rho, \text{ and:} \]

Eq. 4-8: \[ \kappa^0 \rightarrow \kappa \]

Such a medium only supports P-waves, involving an unaxial compressional strain, which has a wave speed \( c_p \) of:

Eq. 4-9: \[ c_p = (\kappa \rho)^{\frac{1}{2}} \]

In these equations, many assumptions are made. The most important are:

- Linear acoustic material behavior, i.e. Young’s modulus does not depend on strain and so the material obeys Hooke’s Law;
- Lossless media, i.e. no acoustic energy is lost within the medium;
- Isotropic media, i.e. its physical properties are independent of the direction in which they are measured;
- The fluid is non-flowing.

### 4.1.2 Elastic wave propagation

Let us now focus on the elastic case. Here the wave propagation can be described by two equations, but now they are more elaborate since we consider an elastic medium instead of an acoustic medium.

According to Wapenaar and Berkhout (1989) the linearized equation of motion for the elastic case reads:

Eq. 4-10: \[ \rho \frac{\partial v_i}{\partial t} - \frac{\partial \tau_{ij}}{\partial x_j} = f_i, \]

where the stress tensor reads:

Eq. 4-11: \[ \tau_{ij} = \lambda \delta_{ij} \frac{\partial u_k}{\partial x_i} + \mu \frac{\partial u_j}{\partial x_i} + \mu \frac{\partial u_i}{\partial x_j} \]

Where:

\[ \tau_{ij} = \tau_{ij}(x,t) \] Stress tensor, where \( i \) denotes the direction normal to the plane the tensor is working on, and \( j \) denotes the direction in which the stress tensor works

\[ f_i = f_i(x,t) \] Volume density of external force
\[ u_i = u_i(\mathbf{x}, t) \]

Particle displacement

\[ \lambda \]

Lamé’s first parameter, or

\[ \mu \]

Shear modulus, also called Lamé’s second parameter

\[ \delta_{ij} \]

Kronecker delta, \( \delta_{ij} = 1 \) for \( i = j \), and \( \delta_{ij} = 0 \) for \( i \neq j \)

The time-differentiated stress tensor gives the equivalent of the equation of continuity in the acoustic case:

**Eq. 4-12:**

\[
\frac{\partial}{\partial t} \sigma_{ij} - \lambda \delta_{ij} \frac{\partial}{\partial x_k} v_k + \mu \frac{\partial}{\partial x_i} v_j + \mu \frac{\partial}{\partial x_j} v_i = 0
\]

Based on these equations, both P and S-waves can exist. Their respective wave speeds are given by:

**Eq. 4-13:**

\[
c_p = \sqrt{\frac{\lambda + 2\mu}{\rho}}, \text{ and:}
\]

**Eq. 4-14:**

\[
c_s = \sqrt{\frac{\mu}{\rho}}
\]

In these equations, many assumptions are made. The most important are again:

- Linear elastic material behavior;
- Lossless media;
- Isotropic media;
- Non-flowing fluid.
4.1.3 Reflection of incident waves

Now that we know how waves propagate through elastic and acoustic media it is time to look at what happens at boundaries between different interfaces. We are especially interested in the reflection on such boundaries, because in reflection seismology we derive information from the energy of seismic reflections at interfaces in the subsurface i.e., at changes in layer properties of the subsurface. At such an interface different things can happen, and they are all shown in Figure 4-2. Table 4-1 explains what the different symbols in Figure 4-2 mean, and discusses all the different types of waves shortly.

![Figure 4-2: The different manner a boundary between two interfaces influences the incident wave. (www.gns.cri.nz)](image)

**Table 4-1:** An explanation of the symbols shown in Figure 4-2. (www.gns.cri.nz)

<table>
<thead>
<tr>
<th>Wave path</th>
<th>How the wave travels in the upper medium</th>
<th>What happens at the boundary</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Direct wave - along the surface</td>
<td>(doesn’t get to the boundary)</td>
</tr>
<tr>
<td>B</td>
<td>Super-critical wave - down &amp; along at an angle greater than the critical angle</td>
<td>B’ The wave is totally reflected back up to the surface (at an equal angle)</td>
</tr>
<tr>
<td>C</td>
<td>Critical wave - down &amp; along at the critical angle</td>
<td>C’ refracted so much by the lower medium that it travels along the boundary ...</td>
</tr>
<tr>
<td>D</td>
<td>sub-critical wave - down &amp; along at an angle less than the critical angle</td>
<td>C’’ and refracts back up towards the surface at the critical angle.</td>
</tr>
<tr>
<td>D’</td>
<td>some of the energy is reflected back to the surface at an equal angle.</td>
<td>D’’ while the rest of the waves energy is transmitted into the lower medium but at a new or refracted angle.</td>
</tr>
<tr>
<td>E</td>
<td>Vertical wave - straight down perpendicular to the boundary</td>
<td>E’ a small part of the waves energy is reflected straight back up</td>
</tr>
<tr>
<td>E’</td>
<td>while most is transmitted. The direction does not change but the speed does.</td>
<td>E’’ while most is transmitted. The direction does not change but the speed does.</td>
</tr>
</tbody>
</table>
In reflection seismic we are only interested in the upgoing reflections, i.e. B' and D' in Figure 4-2. We do however measure all the other types of waves in seismic experiments, but we neglect those events. Therefore only the derivation of reflections is discussed in this report.

The full derivation of reflections at interfaces between media with impedance differences is beyond the scope of this report. A summary for elastic media is useful however, because it shows why we can look at amplitude effects. For this research only a P-wave source has been used (See Section 7.1.3), therefore only incident P-waves are considered in this section.

We are only interested in the reflections from the mine that we measure at the surface, so we need to find a way to relate the properties of this measured reflection to the media properties. This can be obtained by defining a reflection coefficient, which describes nothing more than the complex angle-dependent amplitude of a reflected wave relative to an incident wave.

Theoretically, we assume there is always a welded contact between the interfaces, and we assume that the pressure, particle-velocities and stresses are continuous. This gives one more assumption of the wave-propagation model:

- Welded contacts between media

The seismic impedance ($Z$) is defined as the product of the mass density of the medium and the wave speed of the medium, i.e.:

Eq. 4-15: \[ Z = \rho c \]

On an interface between two media with different acoustic impedances an incident P-wave reflects on and transmits through the interface. Part of the transmitted energy is converted from P-waves to S-waves (See Figure 4-3).

![Figure 4-3: The reflected and transmitted waves, caused by an incident P-wave.](image)

From the law of conservation of energy, we know that the total energy of the 5 waves depicted in Figure 4-4 must stay constant. Furthermore, along the interface the following boundary conditions hold:

- Continuity of horizontal displacement along the interface;
- Continuity of vertical traction along the interface.

The continuity equations can only be fulfilled if, with the angles defined as in Figure 4-4:
Eq. 4-16: \[ s_h = \frac{\sin \theta_{p,1}}{c_{p,1}} = \frac{\sin \theta_{p,2}}{c_{p,2}} = \frac{\sin \theta_{s,1}}{c_{s,1}} = \frac{\sin \theta_{s,2}}{c_{s,2}}, \]

where \( s_h \) represents the horizontal slowness.

\[ \sin \theta_{p,1} \cos \theta_{s,1} \quad -\cos \theta_{s,1} \quad -\sin \theta_{p,1} \quad \cos \theta_{p,1} \\
-\cos \theta_{p,2} \quad \sin \theta_{s,2} \quad -\cos \theta_{s,2} \quad -\sin \theta_{p,2} \\
\sin(2\theta_{p,1}) \quad \frac{c_{p,1}}{c_{s,1}} \cos(2\theta_{s,1}) \quad \frac{\rho_{p} c_{s,1}^2 c_{p,1}}{\rho_{p} c_{s,1}^2 c_{p,2}} \cos(2\theta_{s,2}) \quad \frac{\rho_{p} c_{s,2}^2 c_{p,2}}{\rho_{p} c_{s,1}^2 c_{p,2}} \cos(2\theta_{s,2}) \\
\cos(2\theta_{p,2}) \quad \frac{c_{s,2}}{c_{s,1}} \sin(2\theta_{s,2}) \quad -\frac{\rho_{p} c_{s,2}^2 c_{p,2}}{\rho_{p} c_{s,1}^2 c_{p,2}} \cos(2\theta_{s,2}) \quad -\frac{\rho_{p} c_{s,1}^2 c_{p,1}}{\rho_{p} c_{s,1}^2 c_{p,2}} \sin(2\theta_{s,2}) \]

Eq. 4-17 shows that the amplitudes of the reflected and transmitted waves depend on the angle of incidence, as well as on the medium parameters. The fact that the amplitudes depend on the angle of incidence is the main principle for Amplitude-versus-Offset analysis, where one tries to invert the different amplitudes to changes in the subsurface.

For practical use Eq. 4-17 is too complicated. One might therefore rather opt for an approximation\(^3\).

### 4.2 Numerical implementation

In the above, the continuous equations are written, but they need to be implemented on the computer in order to simulate responses over different models.

A standard approach for laterally heterogeneous models is the finite-difference scheme (Virieux, 1986). In a finite-difference scheme the derivates are approximated by finite difference

\(^3\)For a relatively easy to use approximation Shuey (1985) is advised
equations, obtained by a Taylor-series expansion, in order to approximate the solutions of the differential equations.

In this research the numerical scheme has been defined as follows:

- 4th -order scheme in space
- 2nd -order scheme in time
- 2D implementation, e.g. only the x and z-directions are considered
- Staggered and explicit grid

In this scheme the second order wave equation is not used directly, but defined through the first order system of Newton’s second law and Hooke’s law. The first order derivates in the spatial coordinates are approximated by a 4th order Crank-Nicolson approximation and the first order time derivate is approximated by a 2nd order system (Virieux, 1986):

\[
\frac{\partial p_{i,j,k}}{\partial x} \approx \frac{-p_{i+\frac{1}{2},j,k} + 27p_{i+\frac{1}{2},j,k} - 27p_{i-\frac{1}{2},j,k} + p_{i-\frac{1}{2},j,k}}{24\Delta x}
\]

\[
\frac{\partial p_{i,j,k}}{\partial z} \approx \frac{-p_{i,j+\frac{1}{2},k} + 27p_{i,j+\frac{1}{2},k} - 27p_{i,j-\frac{1}{2},k} + p_{i,j-\frac{1}{2},k}}{24\Delta z}
\]

\[
\frac{\partial p_{i,j,k}}{\partial t} \approx \frac{p_{i,j,k+\frac{1}{2}} - p_{i,j,k-\frac{1}{2}}}{\Delta t}
\]

In these equations the subscripts i and j denote the x- and z-coordinate of the grid blocks respectively, and the k denotes the time sample.

### 4.2.1 Acoustic scheme

If we assume no external forces Eq. 4-1 and Eq. 4-2 can be rewritten to:

\[
\frac{\partial v_x}{\partial t} = \frac{1}{\rho} \frac{\partial p}{\partial x}
\]

\[
\frac{\partial v_z}{\partial t} = \frac{1}{\rho} \frac{\partial p}{\partial z}
\]

\[
\frac{\partial p}{\partial t} = (\lambda + 2\mu) \left( \frac{\partial v_x}{\partial x} + \frac{\partial v_z}{\partial z} \right)
\]

With help Eq. 4-18 to Eq. 4-20 we discretize equations Eq. 4-21 and Eq. 4-23.

\[
\frac{\partial v_{x,i,j,k}}{\partial t} = \frac{1}{\rho \Delta x} \left( -p_{i+\frac{1}{2},j,k} + 27p_{i+\frac{1}{2},j,k} - 27p_{i-\frac{1}{2},j,k} + p_{i-\frac{1}{2},j,k} \right)
\]
Eq. 4-25: \[
\frac{\partial v_{i,j,k}}{\partial t} = \frac{1}{\rho \Delta t} \left( -p_{i,j,k} + 27 p_{i,j,k-\Delta x} - 27 p_{i,j,k+\Delta x} + p_{i,j,k+2\Delta x} \right)
\]

Eq. 4-26: \[
p_{i,j,k} = (\lambda + 2\mu) \left( \frac{1}{\Delta t} \right) \left( v_{i+1,j,k} + v_{i-1,j,k} + v_{i,j+1,k} + v_{i,j-1,k} + v_{i,j,k+1} + v_{i,j,k-1} \right)
\]

The different time steps can then be calculated via:

\[
p_{i,j,k+\Delta t/2} = p_{i,j,k-\Delta t/2} + p_{i,j,k} \Delta t
\]

The model uses a model grid (represented by \(\rho\) and \(c_p\)). This grid is placed at an offset of 1 or 2 grid points in the calculation grid for efficient handling of two of the boundaries. This requires that extra points are added at the other two boundaries. At the boundaries all parameters are set to 0, so the boundaries describe a free surface. This type of boundary condition is usually called a Dirichlet boundary condition.

Figure 4-5 shows the model grid and calculation grid that is used to calculate the discretized equations Eq. 4-24 to Eq. 4-26.
4.2.2 Elastic scheme

To derive the numerical scheme for the elastic case Eq. 4-10 and Eq. 4-12 are transformed into a first-order hyperbolic system:

**Eq. 4-27:**
\[
\frac{\partial v_x}{\partial t} = \frac{1}{\rho} \left( \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xz}}{\partial z} \right)
\]

**Eq. 4-28:**
\[
\frac{\partial v_z}{\partial t} = \frac{1}{\rho} \left( \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{zz}}{\partial z} \right)
\]

**Eq. 4-29:**
\[
\frac{\partial \tau_{xx}}{\partial t} = (\lambda + 2\mu) \frac{\partial v_x}{\partial x} + \lambda \frac{\partial v_z}{\partial z}
\]

**Eq. 4-30:**
\[
\frac{\partial \tau_{zz}}{\partial t} = (\lambda + 2\mu) \frac{\partial v_z}{\partial z} + \lambda \frac{\partial v_x}{\partial x}
\]

**Eq. 4-31:**
\[
\frac{\partial \tau_{xz}}{\partial t} = \mu \left( \frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x} \right)
\]

Like in the acoustic case we can use Eq. 4-18 to Eq. 4-20 to discretize Eq. 4-27 to Eq. 4-31, such that they can be used for finite difference modeling.

**Eq. 4-32:**
\[
\frac{v_{x,i,j,k}}{\Delta t} = \frac{1}{\rho_{x,i,j}} \frac{27}{24} \left[ \left( \tau_{xx,i,j+1,k} - \tau_{xx,i,j-1,k} \right) + \left( \tau_{xx,i+1,j,k} - \tau_{xx,i-1,j,k} \right) + \left( \tau_{x,i,j+1,k} - \tau_{x,i,j-1,k} \right) + \left( \tau_{x,i+1,j,k} - \tau_{x,i-1,j,k} \right) + \left( \tau_{xx,i,j+1,k} - \tau_{xx,i,j-1,k} \right) \right]
\]

**Eq. 4-33:**
\[
\frac{v_{z,i,j,k}}{\Delta t} = \frac{1}{\rho_{z,i,j}} \frac{27}{24} \left[ \left( \tau_{zz,i,j+1,k} - \tau_{zz,i,j-1,k} \right) + \left( \tau_{zz,i+1,j,k} - \tau_{zz,i-1,j,k} \right) + \left( \tau_{z,i,j+1,k} - \tau_{z,i,j-1,k} \right) + \left( \tau_{z,i+1,j,k} - \tau_{z,i-1,j,k} \right) + \left( \tau_{zz,i,j+1,k} - \tau_{zz,i,j-1,k} \right) \right]
\]

**Eq. 4-34:**
\[
\frac{\partial \tau_{x,i,j,k}}{\partial t} = \left( \lambda_{x,i,j} + 2\mu_{x,i,j} \right) \frac{1}{\Delta x} \frac{27}{24} \left[ v_{x,i+1,j,k} + v_{x,i-1,j,k} - 2v_{x,i,j,k} \right] + \lambda_{x,i,j} \frac{1}{\Delta z} \frac{27}{24} \left[ v_{z,i,j+1,k} + v_{z,i,j-1,k} - 2v_{z,i,j,k} \right]
\]

**Eq. 4-35:**
\[
\frac{\partial \tau_{z,i,j,k}}{\partial t} = \left( \lambda_{z,i,j} + 2\mu_{z,i,j} \right) \frac{1}{\Delta z} \frac{27}{24} \left[ v_{z,i+1,j,k} + v_{z,i-1,j,k} - 2v_{z,i,j,k} \right] + \lambda_{z,i,j} \frac{1}{\Delta x} \frac{27}{24} \left[ v_{x,i,j+1,k} + v_{x,i,j-1,k} - 2v_{x,i,j,k} \right]
\]
Figure 4-6 shows the staggered grid that has been used to calculate equations Eq. 4-32 to Eq. 4-36. The same Dirichlet boundary conditions have been used, as for the acoustic scheme.

\[ \frac{\partial \tau_{x,z,i,j,k}}{\partial t} = \mu_{i,j} \left( \frac{27v_{x,i,j+\frac{1}{2},k} - 27v_{x,i,j-\frac{1}{2},k} - v_{x,i,j+\frac{1}{2},k} + v_{x,i,j-\frac{1}{2},k}}{24} \right) + \mu_{i,j} \frac{1}{\Delta x} \left( 27v_{z,i,j+\frac{1}{2},k} - 27v_{z,i,j-\frac{1}{2},k} - v_{z,i,j+\frac{1}{2},k} + v_{z,i,j-\frac{1}{2},k} \right) \]

4.2.3 Grid

In the numerical system all parameters first need to be discretized. This is done by defining values for all parameters for all grid blocks. Theoretically the 2-D grid blocks can have different dimensions in the x- and z-directions, in practice however they are almost always chosen the same.

Choosing a correct grid size is important for a couple of reasons:

- In order for the numerical system to be stable the size grid blocks have to follow the numerical stability condition. In the finite difference system I have used the stability conditions (for grid blocks with \( \Delta x = \Delta z \)) are:
(1) **Eq. 4-37:**  \[ f_{\text{max}} < \frac{c_{\text{min}}}{5\Delta x} \]

(2) **Eq. 4-38:**  \[ c_{p,\text{max}} < \frac{0.606\Delta x}{\Delta t} \]

- The smaller the grid blocks are the more grid blocks you need, which leads to a higher computational time

### 4.2.4 Limitations

A pitfall of describing wave propagation through a numerical scheme is the numerical dispersion. Numerical dispersion prevents waves from propagating over large distances. Therefore relatively low frequent waves need to be used. Another known problem with many numerical schemes is that interpretation of the synthetic seismograms might be troublesome for complex geometries (Virieux, 1986). This has to be kept in mind when looking at the results of this research.

In the ideal case the acoustic scheme and the elastic scheme should give same the exact same times but amplitudes will be different. This stems from fact that the amplitudes that come from acoustically and elastically modeled data are different by nature. The acoustic scheme uses wave velocities as input and considers the pressure as a scalar, while the elastic scheme works with Lamé's parameters as input and considers the stress tensor. This also causes the wave signature to be different in the acoustic and elastic case.

Another pitfall of the specific numeric schemes that are used for this research is that the absolute amplitudes are not modeled correctly for the elastic case. For the acoustic scheme a factor that accounts for the fact that the source is implemented as a volume source, to ensure that the amplitudes found are of the same order of magnitude as real data. For the elastic scheme this factor has not yet been implemented. The relative amplitudes are however modeled correctly. Therefore the amplitudes found by acoustic and elastic numeric modeling cannot be compared with each other in an absolute sense, but the relative amplitudes between different acoustic data sets or different elastic data sets can be compared.
Chapter 5  Scenario definition

In Section 1.4, the research goals were defined, namely:
• Can the effects of solution salt mining be detected in seismic time-lapse mode?
• If so, can these effects be quantified?

In order to answer these questions, as part of a 4-D seismic feasibility study, one has to define how it is believed that the salt-production process is taking place. No exact information of the geometry of the caverns or the connecting layers is available. The only exact information that is available is the production data. Therefore different scenarios have been defined, all with a different geometry, and each of these scenarios have been evaluated on their impact on the time-lapse seismic.

At this moment magnesium salt is only mined from the lower of the two magnesium salt layers, using squeeze mining. Squeeze mining leads to bischofite being ‘pushed’ towards the caverns at a lower pressure. The mobility ratios of halite, carnallite and bischofite are 1 : 10 : 100. It is therefore assumed that bischofite creeps towards the cavern, while carnallite and halite will not move at all.

From pressure measurements in the wells it is known that a brine connection exists between the wells. At this moment it is not exactly clear what this connection looks like, but NedMag’s engineers assume it exists in the lower of the two magnesium salt layers. This brine connection has been modeled as a layer of constant thickness at the top of the magnesium layers. Different thicknesses are applied to different scenarios. These different thicknesses thus only relate to a vertical extension of the mine. This is schematically shown in See Figure 5-1.

For the purpose of scenario definition it is assumed that a lateral extension of the mine is purely caused by magnesium salt squeezed towards the caverns from the magnesium salt layer outside both sides of caverns (See Figure 5-1). This idea means that both the vertical effect and the lateral effect of squeeze mining need to be researched for time lapse seismic.

![Figure 5-1: The assumed effects of squeeze mining.](image-url)
Based on real production data an average production of 27,500 m$^3$ of salt per year per well is assumed. Although not completely correct (See Table 3-1), it is assumed in this research that this production leads to an increase in underground brine volume of exactly 27,500 m$^3$ per year per well. This assumption suits the purpose of this scenario definition well.

In order to study the effect mentioned above four scenarios are suggested. All four of them are discussed in this chapter.

The pictures below all have the legend shown in Figure 5-2.

<table>
<thead>
<tr>
<th>Legend</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Non-magnesium Salt</td>
</tr>
<tr>
<td></td>
<td>Brine</td>
</tr>
<tr>
<td></td>
<td>Magnesium Salt</td>
</tr>
<tr>
<td></td>
<td>Production and Injection Pipes</td>
</tr>
</tbody>
</table>

**Figure 5-2:** Legend.

### 5.1 Scenario A: No mining

This is the so-called base model. This model represents the original state of the subsurface without salt mining.

**Figure 5-3:** A schematic view of scenario A.

### 5.2 Scenario B: The current state

Scenario B represents the current state of the mine. There are two caverns at every well. In order to make diffraction effects as easy to interpret as possible the caverns are modeled as rectangles.

The dimensions of the caverns are based on NedMag’s ideas. Pressure measurements in both wells show that there must be a connection between the caverns. It is assumed that the caverns are connected through the bottom magnesium layer only.

**Figure 5-4:** A schematic view of scenario B.
5.3 **Scenario C: One year of production - A vertical effect**

For this scenario it is assumed that during a year of production the magnesium salt layer has been replaced by brine for 5 more meters in the vertical direction between the two main caverns. This scenario is meant specifically to see the effect of a vertical change in salt mine.

![Figure 5-5: A schematic view of scenario C.](image)

5.4 **Scenario D: Four years of production - A lateral effect**

Since it is unknown how the vertical and horizontal effects of squeeze mining relate, a scenario in which only a purely lateral effect is considered is also proposed. Based on the real salt production of 27,500 m³ per year per well and a height of 10 m over which this takes place this is a production of three years after scenario C, or four years after scenario B.

![Figure 5-6: A schematic view of scenario D.](image)
Chapter 6  Seismic model parameters

In this chapter it is explained how the seismic model parameters required for numerical modeling of the different scenarios are derived. The models basically contain three parameters, namely the compressional wave velocity, the shear wave velocity and the mass density. Other parameters used in the numerical schemes are derived from these three parameters.

A 2-D section through the salt mine of a 3-D seismic cube from 1992 is shown in Figure 6-1. The mine is visible as a bright spot. In the figure the mine is indicated by a black arrow.

![Figure 6-1: Seismic section through the salt mine. The data is taken from a 3D data set, shot by the Nederlandse Aardolie Maatschappij (NAM) in 1992. The arrow indicates the location of the mine.](image)

An effort has been made to reconstruct the structure shown in the real seismic data (Figure 6-1). When the densities and the acoustic wave velocities are known it is possible to numerically compute the seismic response of the geometry. This is necessary to understand certain events in your measured seismic responses or to even perform a feasibility study.

6.1 Compressional wave velocity model

For the P-wave velocity the Velmod-1 velocity model, suggested by Van Dalfsen et.al. (2006) is used. The model is developed by the Dutch geological survey TNO in cooperation with exploration and production companies. It is based on information from 720 wells and a isochore map of the Zechstein Group. Via least-squares analysis parameters for a linear velocity function were found:

\[ c(z) = c_0 + Kz \]

The Velmod-1 model provides values for \( c_0 \) and \( K \) for every formation. In the table below the values for the layers relevant for this research are given.
Table 6-1: $c_0$ and $K$ values for the different layers.

<table>
<thead>
<tr>
<th>Era</th>
<th>$c_0$ [m/s]</th>
<th>$K$ [1/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cenozoicum</td>
<td>1750</td>
<td>0.32</td>
</tr>
<tr>
<td>Chalk Group</td>
<td>2500</td>
<td>0.86</td>
</tr>
<tr>
<td>Trias Group</td>
<td>2900</td>
<td>0.37</td>
</tr>
</tbody>
</table>

The model does not provide information on the velocities within the Zechstein formation. The Zechstein formation is obviously the most important for this research, since it contains the relevant magnesium salt layers. For the Zechstein formation velocities derived from sonic logs from wells of the mine are used. These sonic logs have been obtained by Shell in the 1970s.

Table 6-2: Compressional wave velocities for the salt layers, derived from sonic logs.

<table>
<thead>
<tr>
<th>Type of rock</th>
<th>$C$ [m/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gypsum and halite</td>
<td>4600</td>
</tr>
<tr>
<td>Carnallite and bischofite</td>
<td>4300</td>
</tr>
</tbody>
</table>

The compressional wave velocities for the brine are assumed to be the same as those of water. This means a compressional wave velocity of 1500 m/s. The actual compressional wave velocity is probably slightly higher, because of the dissolved salts and the higher pressure; however no measurements are available the velocity of 1500 m/s has been adopted.

The compressional wave velocity models for scenarios A and B are plotted Figure 6-2 and Figure 6-3 respectively.

Figure 6-2: P-wave velocity model for scenario A.
6.2 Shear wave velocity model

No measurements have been taken in the NedMag’s boreholes to determine the shear wave velocity. In order to make a S-wave velocity model Castagna’s empirical relation (Castagna, 1985) that relates the S-wave velocity to the P-wave velocity has been used. Castagna’s relation was found using a least-squares fit on laboratory data. The equation only holds for rocks that mainly consist of sand.

Eq. 6-1:  
\[ c_s = 0.804c_p - 0.856 \]

To derive the shear wave velocities of the Zechstein formation the following well know relations have been used. The input in this formula was the compressional wave velocity of the sonic logs and a Poisson’s ratio of 0.35 as suggested by Jeremic (1994).

Eq. 6-2:  
\[ c_s = \sqrt{\frac{\mu}{\rho}}, \text{ and:} \]

Eq. 6-3:  
\[ \left( \frac{c_p}{c_s} \right)^2 = \frac{2(1-\nu)}{1-2\nu} \]

Since water does not support shear waves the caverns filled up with brine are modeled with a shear-wave velocity of 0 m/s.

This leads to a shear-wave velocity model for scenario B, which is shown in Figure 6-4. In order to clearly see the difference between the compressional and shear wave velocities they are both plotted as a function of depth, and constant x-position (13.3 km) for scenario B (See Figure 6-5).
**Figure 6-4:** S-wave velocity model for scenario B.

**Figure 6-5:** Compressional (red line) and shear-wave (blue line) velocity profile at \( x=13.3 \) for scenario B.
6.3 Density model

The densities used in the numerical model are derived from density logs. The original logs and their interpretation from the NedMag archives were used for this research. In order to make the model manageable only one density value is assigned to each layer. The density of the brine has been measured by NedMag and is 1.335 kg/m³.

The full density log is shown in Appendix A. The result of this method is a density model. To give a quick insight the density model for scenario A is shown in Figure 6-6.

Figure 6-6: The density model for scenario A.
Chapter 7  Baseline survey: modeling migrated image from field data

In this chapter the seismic images that are obtained with the finite different schemes are discussed. Before doing that it is first explained how the models were defined in terms of a numeric scheme, and what processing steps were performed in order to obtain the images. This requires processing of the modeled shots which includes the essential process of migration.

7.1 Modeling

In this section it is discussed how the finite difference scheme was implemented in order to gain the necessary data for time lapse seismic analysis.

7.1.1 Model definition

The first model made is a 2-D model referred to as the base model (Scenario A), which does not contain a mine. The full model is 17000 meters (x-direction) by 4500 meters (z-direction), divided up in grid blocks of 5 x 5 m in the acoustic case, and 2 x 2 m in the elastic case.

For the acoustic models extra grid points at the bottom of the model were added in order to prevent reflections from the bottom of the model to appear in the migrated sections. Reflections from the sides and top of the model are prevented by including a taper zone of 300 grid blocks at both side edges of the model. The acoustic finite difference algorithm adds extra points to the model, over which is tapered. The elastic finite difference algorithm however tapers of points that are already in the model; therefore 900 extra meters at both sides and the top of the model in the elastic case were added, and tapered only 50 grid points (100 m) at the edges. This approach also ensures that no unwanted reflections from the sides of the model appear in the migrated sections. The extra added meters of the models are not shown in pictures in this report, as they are not actually part of the model.

7.1.2 Shot and receiver spacing and time sampling

At the surface 341 shots with an offset of 50 meters were modeled for both the acoustic and elastic case. In the acoustic case the receivers were placed at a spacing of 25 m. In the elastic case the receivers were placed at a spacing of 26 m, starting at x=0. It was the intention to place the receivers at a spacing of 25 m - which is very common in seismic field studies - but since the grid blocks are 2 x 2 meters the finite difference program placed them at a spacing of 26 meters. This results in the fact that the receiver and shot positions are not coinciding. Although this has no implications for the imaging, it causes sparsely filled CMP-gathers, i.e. the fold of the CMP-gathers is too small for practical use. This problem was resolved by using super-gathers consisting of a range of sparsely filled CMP’s. The resulting super-gather then has a fold which is good enough for practical use. This solution was chosen in stead of rerunning the finite difference models, since that would take a very long time (See Section 7.1.4).

The time sampling of the receiver data was 0.004 s. The choice for this sampling rate is based on two factors, namely:

1. The total calculation time, as the sampling rate is linearly related to the total calculation time;
2. The stability conditions for the numeric scheme, as discussed in Section 4.2.3. A higher sampling would not allow high enough compressional wave speeds, such as those of the salts (>4300 m/s).

### 7.1.3 Wavelet

The study has been performed with a Ricker wavelet. This type of wavelet is very common to make synthetic seismic data. The Ricker wavelet is the second derivative of the Gaussian function. According to Ryan (2004) the Ricker wavelet can be mathematically expressed as a force source, according to:

\[
R(t) = (1 - 2\pi^2 f^2 t^2) e^{(-x^2 f^2 t^2)}
\]

Where:
- \( f \) Peak frequency [Hz]

The Ricker wavelet is a zero phase wavelet (i.e. symmetrical around zero time) with a very clean signal. The wavelet can be uniquely specified with only one parameter, namely the peak frequency. The spectrum of the Ricker wavelet is shown in Figure 7-1 (Left).

**Figure 7-1: (Left) A zero phase Ricker wavelet. (Right) The shifted Ricker wavelet, as is used in this research.**

In order to be able to use the Ricker wavelet it should be shifted. Otherwise half of the signal would occur before \( t=0 \), and not be contributing to the outcome (See Figure 7-1 (Right)). In order to obtain zero phase data the wavelet should be shifted back after modeling. This has not been done in this research however.

The Ricker-wavelet used for this research has a peak frequency 30 Hz, and a maximum frequency of 46 Hz. The choice for this frequency is mainly determined by the Nyquist criterion, which allows us to have a maximum frequency of 46.3 Hz in the model described in this chapter. The frequency spectrum of the chosen Ricker wavelet is shown in Figure 7-2. The peak frequency is relatively low, compared to other seismic modeling studies, but it represents frequencies used in real seismic surveys well.
7.1.4 Calculation time

The acoustic and elastic finite difference schemes are very time intensive to calculate on modern computers. The calculation time for both schemes is mainly determined by the number of grid blocks. Since the model is 2-D, a decrease of the grid block size for a square grid blocks increases the calculation time with a factor that is the square of the factor with which the grid block size was decreased.

Acoustic modeling of a typical model used for this research takes about 1 hour per shot, while there are 341 shots per model. In the elastic case the calculation time is much higher, since the pressures are defined as tensors in stead of scalars. For the elastic case the calculation time is more than 15 hours per shot. This means the calculation of the full elastic model takes more than 5100 hours. The calculation time can be party reduced by using parallel computing. This means that different processors are used that all calculate a different shot simultaneously. For this research a cluster of 26 CPU’s was used.

As it has become clear now, for this research it was important to define the model such that the total calculation time would be manageable.

Figure 7-2: The amplitude spectrum used of the Ricker wavelet that was used for this research.
7.2 Processing

Several steps need to be taken in order to obtain an image of the subsurface. In this section all the steps are briefly discussed.

First of all, the direct wave needs to be removed. This was done by simply muting out the direct wave from the migrated section. This means that the direct wave is still present in all the pre-stack data.

One of the main reverse processes of seismic forward modeling is seismic migration: it is the inverse of the one-way propagation operator. A pre-stack time migration of the shots, using a Kirchhoff migration algorithm, was used for this research. A Kirchhoff migration calculates the travel times of the waves from the source to the receiver for all different sources and receivers and all different imaging points. This allows us to create coinciding source and receiver pairs at the imaging points in the subsurface. The needed travel times are obtained through a ray tracing algorithm that uses Normal Move Out (NMO) velocities of the model. Because of the modeling, it is possible to directly obtain the correct NMO-velocities, but using these in the migration scheme did not give satisfactory results. Therefore a more processing oriented approach was chosen. The velocity model for the pre-stack time migration was thus obtained via so-called semblance gathers of Common-Mid-Point gathers, a standard way in seismic analysis.

Only scenario A was used to obtain the velocity model. This implicates that all the other scenarios have a slight migration error, because of the velocity differences. However, time-lapse seismic requires the processing steps to be as equal as possible.

A semblance gather is a measure of coherency between the data itself. The semblance \( S(t, c_p) \) at a time \( t \) for a (compressional wave) velocity \( c_p \) can be measured as follows:

\[
S(t, c_p) = \frac{1}{M} \left( \sum_{m=1}^{M} A(x_m, t, c_p) \right)^2
\]

Eq. 7-2:

Using the compressional wave velocity only implicates by definition that shear wave events that appear in the elastic modeling will not be migrated correctly. Since this research does not focus on shear waves, this introduces “noise” in the final image.

The velocities have been obtained for 25 points. Interpolation between those 25 points leads to the velocity model shown in Figure 7-3.

![Figure 7-3: The compressional wave velocity model obtained by a semblance gather.](image)
7.3 **Numeric modeling results**

Now that it is clear how the four models are defined and modeled it is time to look at the migrated images and the differences between them. In this section migrated sections are quantitatively described. The seismic events are interpreted.

First the acoustic models (Section 7.3.1) are discussed and then from page 44 onwards, the elastic models are discussed (Section 7.3.2).

### 7.3.1 Acoustic models

In this subsection the final pre stack time migrated and stacked pictures of all the four models that are used for this research are briefly discussed.

The migrated sections are built from shot records. It is therefore a good idea to first have a look at such a shot record. A shot record for scenario B is shown in Figure 7-4. The shot is positioned at \( x=13.3 \) km. All the different events that are relevant to this research are indicated. A reflection of the mine is visible. It does not look very clean, because of interference of reflections of all the different sides of the caverns and the layer in between.

**Figure 7-4:** Acoustic shot record for a shot record at \( x=13.3 \) km of scenario B.
7.3.1.1 Scenario A

In Figure 7-5 the migrated response of scenario A, the scenario without mine, is shown. This section compares very well with the real data as shown in Figure 6-1. It can also be seen that the main reflectors match up nicely.

Figure 7-5: Pre stack time migrated and stacked image of the acoustically modeled scenario A.

7.3.1.2 Scenario B

In Figure 7-6 the response of scenario B, the scenario that represents the current state is given. The reflections of the salt mine are obvious when Figure 7-5 is compared with Figure 7-6. It is not possible to see the full geometry of the different caverns, because of the interference of all the different reflections that come from the salt mine.

The reflections of the mine are not migrated quite properly. Diffractions and multiples are still visible. The main reason is that the velocity model used for the migration is not completely correct near the mine, as discussed in Section 7.2. This effect is also visible for all other models that contain a mine, and will not be discussed in the remaining sections.
Figure 7-6: Pre stack time migrated and stacked image of the acoustically modeled scenario B.

7.3.1.3 Scenario C

In Figure 7-7 the response of scenario C, the scenario that compared to scenario B represents a vertical extension of the mine is given. Comparing Figure 7-6 with Figure 7-7, a difference is not visible at first sight. However, a closer look at this difference is taken in Section 9.2.2, where the differences are also discussed quantitatively.

Figure 7-7: Pre stack time migrated and stacked image of the acoustically modeled scenario C.
7.3.1.4 Scenario D

In Figure 7-8 the response of scenario D, the effect of three or four years of production, compared to scenarios C and B respectively, is shown. If we compare scenario C with scenario D we are looking at a lateral change of the salt mine, due to three more years of salt production. Comparing Figure 7-7 with Figure 7-8, we do not see a difference at first sight. The effect is barely visible, but if you look very closely you see an amplitude change at both side lobes of the reflections. These changes are indicated with an arrow. We will have a closer look in Section 9.2.3.

Figure 7-8: Pre stack time migrated and stacked image of the acoustically imaged scenario D. The arrows indicate the places where the amplitude has changes in comparison with the migrated section of scenario C.
7.3.2 Elastic models

The stacked and pre-stack time migrated sections are discussed in this section. Because of time limitations (bear in mind that modeling one elastically modeled shot takes more than 15 hours) only three scenarios have been modeled elastically, namely scenarios B, C and D. These three scenarios give insight in the effects of actual production, rather than then a comparison with a hypothetic state without a mine. In order to further reduce calculation time only the shots right around the salt mine have been modeled. This does not influence the fold of the CMP’s and therefore it does not influence the migrated and stacked sections.

Let us, like in the acoustic case, first have a look at a shot record again (Figure 7-9). The shot record is from scenario B. The shot was placed at x=13.3 km. The same events can be indentified as in the acoustic case. A very pronounced difference is that the reflection of the mine shows up much brighter in the elastic case than in the acoustic case.

![Figure 7-9: Elastic shot record for a shot record at x=13.3 km of scenario B.](image)

7.3.2.1 Scenario B

The time migrated and stacked section through the salt mine for scenario B (See Section 5.2) is shown in Figure 7-10. The reflection of the mine comes forward as a very clear bright spot, caused by the large impedance differences. The mine is much better imaged than in the acoustic case (Figure 7-6).

The reflection also gives some idea about the geometry of the salt mine, but we have to be aware that the maximum vertical resolution is not good enough to see all the details in the mine.
Figure 7-10: Pre stack time migrated and stacked image of the elastically modeled scenario B. Zoomed-in section at the reflection of the salt mine.

7.3.2.2 Scenario C

From Figure 7-11 is very obvious that the extension of the brine layer between the two bottom caverns leads to an increase in amplitude. The figure gives an good idea on where the changes of the geometry of the mine have taken place.

Figure 7-11: Pre stack time migrated and stacked image of the elastically modeled scenario C. Zoomed-in section at the reflection of the salt mine.
7.3.2.3 Scenario D

It is striking to see that the migrated sections of scenario D (Figure 7-12) and scenario C (Figure 7-11) look so similar. The reflection in scenario D is a little bit longer, which is caused by the extension of the mine in the lateral direction.

Figure 7-12: Pre stack time migrated and stacked image of the elastically modeled scenario D. Zoomed-in section at the reflection of the salt mine.
Chapter 8  
Time-lapse analysis

In this feasibility study it is researched if and how the time lapse seismic effects caused by geometry changes of the solution salt mine can be determined. In the oil and gas industry time lapse seismic is mainly used to look at reservoir changes such as saturation and pressure changes. Even though time lapse seismic is a fairly new subject in the oil and gas industry, it is already discussed in many papers and books, such as Calvert (2005). However, how to use the time-lapse seismic attributes to detect changes in salt caverns has not been described in literature yet. Therefore this chapter addresses the topic on how different analyzing tools of time lapse seismic data can be used in order to get an insight in the effects of salt production from an underground solution salt mine.

The definition of time lapse seismic data is as follows: “Seismic data from the surface or a borehole acquired at different times over the same area to assess changes in the subsurface with time.” (Schlumberger Oilfield Glossary, http://www.glossary.oilfield.slb.com)

Of course we then have to determine how the seismic data changes, and what causes the changes. If we keep the principles of reflection seismology in mind (Chapter 4), we know that seismic data can change because of:

- A change in amplitude of a particular reflection, which can only be caused by a change of the reflection coefficient of the interface, i.e. a change of the acoustic impedances.
- A change in arrival time of a particular reflection. This can only be caused by geometry changes or changes in acoustic wave velocities along the ray path.

![Figure 8-1: The principle of time lapse seismics. (Calvert, 2005)](image)

Visually, a difference dataset allows a good visualization whether differences exist; especially time shifts (see Calvert, 2005). Based on the subtraction of the two datasets, the so-called NRMS attribute can be determined. The subtraction of two data sets is discussed in Section 8.1.

In this research cross correlation techniques are used for the quantitative determination of the time shifts and amplitude changes. Based on the correlation results, the so-called predictability attribute can be determined. In Section 8.2 these attributes are described in detail.

In the last section of this chapter a plane wave example is discussed. This example gives an idea of the effects we are interested in. Based on these simple examples, we show the results of the cross-correlation and the results of subtraction of datasets before discussing the results of the actual data in Chapter 9.
8.1 Subtraction of two data sets

Another good approach to look at the differences is to subtract the two datasets from each other. As said before, this is very sensitive to time shifts between the two datasets. It very much highlights where changes are taking place in the subsurface. However, it should be realized that a difference section does not give you the time shift and/or amplitude change itself.

Based on the difference section, the so-called Normalized Root-Mean-Square (NRMS) attribute can easily be determined (Kragh et. al., 2002). The NMRS is expressed as a percentage:

Eq. 8-1: \[ NRMS(a, b) = \frac{200 \text{RMS} (a(t) - b(t))}{\text{RMS}(a(t)) + \text{RMS}(b(t))}, \]

where \( \text{RMS}(x) \) is given by:

Eq. 8-2: \[ \text{RMS} = \sqrt{\frac{\sum_{t_1}^{t_2} (x(t))^2}{N}}, \]

where \( N \) denotes the number of samples in the interval \( t_1 \) to \( t_2 \).

8.2 Cross correlation of two data sets

A fairly easy way to study time shifts is through a cross correlation of the two data sets. A cross correlation is a measure of the similarity of two data sets. Cross correlation can for instance be used to figure out how much a signal has to be shifted along any axis to make it identical to another signal. It can therefore be used to see the difference of a seismic event that changes in time. Since the correlation value should be largest when the traces are aligned, the lag value at maximum correlation is considered the time shift that we are interested in.

Mathematically a 1-D cross correlation is a convolution of a signal with the complex conjugate time-reversed version of the other. The equations are given in the time domain, as well as in the frequency domain, as the cross correlation used in this research was performed in the frequency domain.

Eq. 8-3: \[ \varphi_{ab}(\tau) = \int_{-\infty}^{+\infty} a(t) b^*(t-\tau) dt \]

A temporal Fourier transformation of a cross correlation leads to a simple multiplication:

Eq. 8-4: \[ \Phi_{ab}(\omega) = \mathcal{F} \left( \varphi_{ab}(\tau) \right) = A(\omega) B^*(\omega) \]

In 2D the cross correlation formulas read:
Eq. 8-5:  \[ \varphi_{ab}^{2D}(\chi, \tau) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} a(x, t) b^*(x - \chi, t - \tau) dx dt \]

and

Eq. 8-6:  \[ \Phi_{ab}^{2D}(\chi, \omega) = F \left( \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} a(x, t) b^*(x - \chi, t - \tau) dx dt \right) = A(\omega, \chi)B^*(\omega, \chi) \]

As becomes clear now a 2-D correlation does not only give a measure of similarity along the time axis, but also along the x-axis, as is represented by different traces in seismic data. It therefore gives averaged values over the window over which the cross correlation was performed. It will however show where the maximum change has taken place. One can now determine the time and lateral shifts between two data sets by simply picking the maximum amplitude of the 2-D cross correlation of the two data sets and the 2-D auto correlation of the reference data set. The different time and spatial coordinates of these two maximum values give the time shift and lateral shift respectively. The amplitude change is expressed as a percentage compared to the reference data set. A pitfall of this approach is that it does not take Amplitude- Versus-Offset (AVO-) effects into account. In order to do have a look at AVO-effects we need to study the pre-stack CMP-gathers.

The time shift is determined by comparing the shift in time of the maximum amplitude between the cross correlation with the auto correlation of the reference data set. As time shifts between two seismic data sets are usually in the order of less than a millisecond and the time sampling of those data sets is usually 2 or 4 milliseconds cross correlation of the originally sampled dataset cannot be used to determine time shifts.

There is a way to determine time shifts smaller than the temporal sampling density of the data. This can be obtained by adding extra time samples to the data by interpolating between the original time samples. In order to interpolate we use the fact that adding zeroes in the frequency domain means interpolation in the time domain. This method is preferred over indirect methods where the effect needs to be modeled.

When the cross correlations have been calculated, it is relatively easy to obtain some standard attributes as used in time-lapse analysis, like the predictability (Kragh and Christie, 2002):

Eq. 8-7:  \[ PRED = \frac{\sum_{t_{h}} \varphi_{ab}(t)\varphi_{ab}(t)}{\sum_{t_{h}} \varphi_{aa}(t)\varphi_{bb}(t)} \]

As can be seen from Eq. 8-7, the prediction function gives the degree of repeatability, expressed the summed and squared cross correlation within a time window divided by the summed product of the auto correlations. The result is expressed as a fraction.

### 8.3 A plane wave example

To a feeling for the data that is obtained by 2-D cross correlation and the subtraction of two data sets an example with some plane waves is discussed in this section.

A plane wave is considered as a band-limited pulse which has a plane wave front, i.e., constant wave slowness. In Figure 8-2 three plane waves are depicted. The first one is a reference plane.
wave. The second plane wave is basically the same, but is has a time shift of 0.0005 seconds. The third plane wave has an amplitude that is 10% higher than the reference wave.

Figure 8-2: A reference plane wave (Left), a plane wave with a time shift of 0.0005 s compared to the reference plane wave (Middle) and a plane wave with a 10% higher amplitude than the reference plane wave (Right).

It is clear that the different plane waves depicted in Figure 8-2 do not show a visible difference. To show the difference, the reference plane wave has been subtracted from all three plane waves (See Figure 8-3). Obviously the result of subtracting the reference plane wave from itself is 0 everywhere. This approach does show the difference between the three waves very clearly though.

Figure 8-3: The reference section subtracted from (Left) itself, (Middle) A reference plane wave, a plane wave with a time shift of 0.0005 s compared to the reference plane wave and (Right) a plane wave with a 10% higher amplitude than the reference plane wave.

The differences are in the order of the actual data used for this research however. This proves that it is almost impossible to quantify these differences without help of techniques, such as subtraction.

In order to see what to expect from a 2-D correlation, the auto correlation of the reference plane has been plotted as well as the cross correlation of the reference plane wave with the time shifted plane wave and the plane wave with an increased amplitude in Figure 8-5. In order to make the differences visible they are plotted in the same image. A legend is included in Figure 8-4 to interpret the images. In these pictures we see that a time shift and amplitude
change both show up in a 2-D cross correlation plot, but they are again hard to see, therefore
the differences between the reference section and the different scenarios are depicted in Figure
8-6. Of course, since two plane waves are used for the correlation there are no changes along
the different traces.

In Figure 8-6 (Middle) a positive time shift is clearly visible; the figure on the right shows an
increase in amplitude.

\[
\text{Relative Amplitude}
\]

\[
\text{Figure 8-4: Legend for 2D cross correlation pictures.}
\]

\[
\begin{align*}
\text{Figure 8-5:} & \quad \text{2-D auto correlation of the reference plane wave (Left),} \\
& \quad \text{2-D cross correlation of the reference plane wave with a 0.0005 s time shifted plane wave} \\
& \quad \text{(Middle), and a 2-D cross correlation of the reference plane wave with a plane wave} \\
& \quad \text{with a 10\% higher amplitude (Right).}
\end{align*}
\]

\[
\begin{align*}
\text{Figure 8-6:} & \quad \text{The 2-D auto correlation of the reference section subtracted from (Left)} \\
& \quad \text{itself, (Middle) the 2-D cross correlation of the reference plane wave with a 0.0005 s} \\
& \quad \text{time shifted plane wave, and (Right) the 2-D cross correlation of the reference plane} \\
& \quad \text{wave with a plane wave with a 10\% higher amplitude.}
\end{align*}
\]
Chapter 9  Results for different scenarios

In this chapter the time shifts, lateral shifts and amplitude changes found by correlating all the different scenarios with each other are discussed. This has been done for the acoustic data, as well as the elastic data.

With time-lapse analysis we are interested in the effect of time shifted seismic events and their amplitude effects between two data sets acquired at different times. This means that one has to pick an event and see how much it has shifted between the two data sets. Obviously this time shift also depends on the changes in the overburden and on the repeatability of the acquisition surface. In the case of this research it is assumed that this effect is very small. This assumption is correct for the purposes of this research as the overburden only changes as a result of subsidence, which occurs at a very low rate (in the order of centimeters per year, compared to an extension of the underground caverns in the order of several meters to tens of meters per year.)

As explained in Section 4.2.4, it is expected that the elastic finite difference scheme and the acoustic finite difference scheme both give the same results for the times P-wave reflections arrive at the receivers, so one can choose which data is used to extract the time shifts from. This does not hold for amplitudes however, so a difference in the amplitude change found with acoustic and elastic modeling is expected.

![Figure 9-1: Migrated seismic section of scenario B. The red box indicates the area over which the 2-D cross correlation is performed. The window is between x=13286 m and 14066 m, and t=1.22 s to t=1.32 s.](image)
9.1 **Expected amplitude behavior**

In this section it is shown how vertical and lateral changes of the brine affects the amplitude. These changes are implemented in the four scenarios that have been discussed in Chapter 5.

One has to realize that if one uses 2-D cross correlation results of migrated and stacked sections to extract amplitudes one does not take useful Amplitude-Versus-Offset (AVO) information into account. This information might be very useful to derive the geometry changes from the seismic data.

The need for AVO-analysis becomes evident if we look at an elastic CMP-gather of scenario D (Figure 9-2). In this figure amplitude decay with offset is clearly visible. Because of time limitations it was not possible to perform a full AVO-analysis as part of this thesis.

Since stacking changes the amplitudes found according to the fold of the different CMP’s it is necessary to have a look at the pre-stack amplitude behavior in order to judge the amplitude changes derived from 2-D cross correlation.

Therefore only the vertical and lateral changes of the brine layer that are expected to occur were modeled; i.e. only the changing layer was modeled. These changes are based on the assumptions made in Chapter 5, and that are plotted in Figure 5-1. The modeling has been done using the elastic scheme, as we then get the full set of P-wave and S-wave reflections of the brine layer and their complete interference pattern.

In order to predict the expected amplitude behavior, only the vertical and lateral changes of the brine layer that are expected to occur were modeled; i.e. only the changing layer was modeled. This has been done by building models that represent the different thicknesses of the brine layer in between the bottom cavern and models that represent the different lengths of the side lobes. The reference state for the vertical extension has a thickness of 5 m (Figure 9-3 (Left)). The reference state for the lateral extension has two side lobes of 100 m length (Figure 9-3 (Right)).

---

*Figure 9-2: Supergather of all the CMPs between 13300 m and 13400 m. The black arrow indicates the reflection of the mine.*

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4 See Section 4.1.3 for more information on the main principle of AVO-behavior.
One shot at \( x=0 \) was elastically modeled for all the states depicted in Figure 9-4 and Figure 9-6. From the zero-offset trace the amplitude of the reflection was derived. The amplitude change for all the states was compared to the reference states and expressed as a percentage.

![Figure 9-3: The reference models used to model the expected amplitude behavior.](image)

*Left* The reference state for the vertical extension. *Right* The reference state for the lateral extension of the side lobes.

Let us first have a look at the vertical changes (Figure 9-4). All the amplitudes have been compared with the amplitude of the reference section of 5 m thickness. The points are shown in Figure 9-5 and a logarithmic trend line has been fitted through the found data points. As can be seen in Figure 9-5 the amplitude change increases with an increasing vertical extent of the brine layer. This change seems to become constant from on a thickness of 20 m. The difference between 10 m thickness and 5 m thickness, which is exactly the difference between scenarios B and C, leads to an amplitude change of 12.0%. In absolute terms we could say that the amplitude change is 1.43% per meter of vertical change.

The amplitude change caused by a lateral extension of the side lobes (Figure 9-6) of the mine shows a linear behavior (See Figure 9-7). The increase of amplitude is around 0.11% per meter extension. Scenario D has a side lobe of 100 m, while scenario C has not. We could thus expect an amplitude change of around \( 100 \times 0.11 = 11.0\% \).

The absolute amplitude effect of a lateral extension (0.11%/m) is significantly less than that of a vertical extension (1.43%/m). Amplitude changes are thus much more pronounced for vertical changes than for lateral changes of the salt mine.
Figure 9-4: A schematic view of the vertical extension of the brine layer in between the two bottom caverns.

Figure 9-5: Amplitude change versus brine layer thickness. All compared to the reference thickness of 5 m.
Figure 9-6: A schematic view of the lateral extension of one of the two side lobes to the bottom caverns.

Figure 9-7: Amplitude change versus lateral extension of the side lobes. All compared to the reference side lobe length of 100 m.
9.2 Acoustic results

First a qualitative discussion of the comparison of all the acoustic data sets is given. In the last sub-section the results are discussed quantitatively.

In this research, modeling was performed with a temporal sampling rate of 4 milliseconds. The data was sub-sampled with a factor 8 to 0.50 ms by adding zeros in the frequency domain. The same was done to the sampling in the spatial direction to obtain a spatial sampling rate of 2.75 m.

9.2.1 Scenario A versus scenario B

Let us first have a look at the difference between both sections (Figure 9-8). As scenario B represents the current state, and scenario A represents a state without a mine, we are only left over with the reflections of the mine when the migrated section of scenario A is subtracted from the migrated section of scenario B. In this figure the mine appears, exactly at the spot where it has been implemented in the model. Furthermore the diffractions are obvious. Because of these diffractions it is handier to zoom-in at the main reflection of the mine (Figure 9-9). This figure indeed gives an idea of the geometry of the mine as it was implemented in the model, although due to multiples and interference it is not possible to distinguish the effect of the caverns that are positioned at the same horizontal coordinate.

It becomes very obvious that the mine does indeed cause a very distinctive reflection which is clearly observable. We can therefore already conclude that the mine itself is visible in seismic sections. From this observation alone we cannot yet conclude whether the effects of salt mining are visible though.

![Figure 9-8](image.png)

**Figure 9-8:** Migrated section of scenario A subtracted from the migrated section of scenario B.
In order to quantify the effect of the difference between the two migrated sections the 2-D cross correlation between the two migrated sections (Figure 9-10) has been compared with the auto correlation of scenario A. The time shift is 0.5 ms and the amplitude change is 49.6%.

**Figure 9-10:** 2-D cross correlation result between scenarios A and B.

### 9.2.2 Scenario B versus scenario C

Scenario B represents the current state, and scenario C represents one year of production. The only difference between the two scenarios is that the brine connection between the two bottom caverns has become 5 m thicker. We need time-lapse analysis tools to see the difference between these migrated sections, as the migrated sections itself do not show a clear difference (See Sections 7.3.1.2 and 7.3.1.3).
Figure 9-11 shows the section where the two data sets are subtracted from each other. This figure shows a difference right between the four caverns of the mine, at the exact spot of the vertical difference of the brine connection between scenario B and C. It can therefore be concluded that a vertical extension of the mine is visible in seismic time-lapse mode.

Figure 9-12 shows a very clean 2-D cross correlation result, since there is only a very small difference between the two migrated sections. The time-shift and amplitude change are respectively 1.5 ms and 5.3%.

Figure 9-11: Section of scenario B subtracted from the section of scenario C, zoomed-in around the main reflection of the salt mine.

Figure 9-12: 2-D cross correlation result between scenarios B and C.
9.2.3 Scenario C versus scenario D

In Section 9.2.2 it was shown what effect a vertical extension of the brine layer between the two bottom caverns has on the seismic signal. Now it is shown what the effect of a purely lateral change of the mine has. Therefore the migrated section of scenario C has been subtracted from the migrated section of scenario D (Figure 9-13). The two extra side lobes to the bottom caverns show up nicely, while the rest of the mine does not show a difference. Lateral changes of the salt mine can thus be made visible.

A comparison between the 2-D auto correlation of scenario C and the 2-D cross correlation of the migrated sections of scenario C and scenario D (Figure 9-14) gives a time shift of 0.0 ms and an amplitude change of 0.2%. These seem very small, especially compared to the time shift and amplitude change caused by going a vertical extension of the mine (Section 9.2.2). In order to be able to answer the question why the time shift and amplitude change are so small we first have to compare scenario B with scenario D, which includes a vertical as well as a lateral extension of the mine (See Section 9.2.4).

Figure 9-13: Section of scenario C subtracted from the section of scenario D, zoomed-in around the main reflection of the salt mine.
9.2.4 Scenario B versus scenario D

When looking at four years of production after the current state (scenario B), a vertical change of the brine layer between the bottom cavern, and also a lateral extension of the mine at both sides of the bottom caverns is assumed. So in that case we are looking at a change of the seismic signal caused by a vertical and lateral extension of the mine.

The subtracted section (Figure 9-15) clearly shows where the differences between scenario D and B are. The subtracted section is basically a combination between Figure 9-11 and Figure 9-13. It can therefore be concluded that a vertical extension of the mine, a lateral extension of the mine as well as a combination of the two can be made visible using subtraction of the two data sets.

A comparison between the 2-D auto correlation of scenario B and the 2-D cross correlation of the migrated sections of scenario B and scenario D (Figure 9-16) gives a time shift of 1.5 ms and an amplitude change of 4.9%. These numbers are almost the same as when comparing scenario B with scenario C (only a vertical change), but are very different from the time shift and amplitude change obtained by comparing scenario C with scenario D (a purely lateral change). This observation suggests that time shifts and amplitude changes are purely caused by vertical changes of the geometry of the mine. This fact will be discussed in more detail in section 9.2.5.

![Figure 9-14: 2-D cross correlation result between scenarios C and D.](image)
9.2.5 Quantitative results of acoustic numerical modeling study

It is now possible to combine all the time shifts and amplitude changes found in one table. The differences are explained in this section.

In order to quantify time shifts the exact time sample of the maximum amplitude for the 2-D auto correlation of all scenarios and all 2-D cross correlations between the different scenarios has been determined. The difference between the maximum time of the auto correlations and
cross correlations gives the time shift. Table 9-1 shows an overview of all the results of the time shift analysis. On the diagonal the auto correlation results are shown.

**Table 9-1: Time shifts, derived from 2-D cross correlations on the acoustic data.**

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Effect</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Time shift [ms]</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>Amplitude change</td>
<td>49.6%</td>
<td>50.4%</td>
<td>54.1%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Lateral shift [m]</td>
<td>3.3</td>
<td>0.0</td>
<td>3.3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Time shift [ms]</td>
<td>0.5</td>
<td>1.5</td>
<td>1.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Amplitude change</td>
<td>49.6%</td>
<td>5.3%</td>
<td>4.9%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Lateral shift [m]</td>
<td>3.3</td>
<td>0.0</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Time shift [ms]</td>
<td>0.5</td>
<td>1.5</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>Amplitude change</td>
<td>50.4%</td>
<td>5.3%</td>
<td>0.2%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Lateral shift [m]</td>
<td>0</td>
<td>0</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Time shift [ms]</td>
<td>0.5</td>
<td>1.5</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Amplitude change</td>
<td>54.1%</td>
<td>4.9%</td>
<td>0.2%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Lateral shift [m]</td>
<td>3.3</td>
<td>0.0</td>
<td>0.0</td>
<td></td>
</tr>
</tbody>
</table>

Table 9-1 shows that the time shifts for scenario A and all the other scenarios that do have a mine is 0.5 ms. These are considerably smaller than the time shifts of around 2 ms for oil production mentioned in literature (Zadeh, 2007). This seems awkward, since there is a gigantic change in seismic signal going from a scenario without a mine to a scenario with a mine. These small time shifts can only be explained by the fact the 2-D cross correlation algorithm can not handle large differences in signal, i.e. the algorithm tries to correlate the reflection of the mine with a signal that does not contain this reflection. The time shifts found for scenario A are therefore not reliable. The found amplitudes are still reliable though.

The amplitude changes between scenario A and all other scenarios B, C and D are 49.6%, 50.4% and 54.1% respectively. They are obviously very high, since a reflection appears where it was not present before. The amplitude also increases with a larger mine, i.e. an increase in brine volume has a direct link with an increase in amplitude.

A lateral change cannot be observed. The lateral change of 3.3 meters is namely within the error margin of one sample.

If we compare scenario B, the current state with scenarios C and D, which represent 1 and 4 years of production we see a time shift of 1.5 milliseconds for both cases, and an amplitude change of 5.3% and 4.9% respectively. The time shift of 1.5 ms is what we expect if we keep literature in mind. This time shift means that changes in seismic signal due to salt production are observable with the use of 2-D cross correlation.

The purely lateral change of the mine, which distinguishes scenario C from scenario D does not cause a time shift or a pronounced amplitude change. This is obviously not what we expected, since the volume of brine, has increased tremendously between the scenarios. This effect can be explained if we take a closer look at the migrated sections that have been used as input for the 2-D cross correlation (See Figure 9-17).
Figure 9-17: Migrated sections of scenarios C (left) and D (right), zoomed in on the reflection of the salt mine. The arrows indicate the zones that contribute most to the maximum amplitude found by a 2-D cross correlation analysis.

Figure 9-17 clearly shows that the maximum amplitudes of the reflection of the mine are right in the middle of the full reflection. The side lobes of the mine do give such a pronounced effect. 2-D cross correlations therefore tends to give a higher correlation amplitude to the zones in the middle. Therefore the final amplitude changes and time shifts are mainly determined by the zones in the middle of the reflection. The side lobes will therefore not give a distinguished difference in the cross correlation between scenarios C and D. This effect can even be better seen if we look at a 1-D cross correlation between scenarios C and D (Figure 9-18). A 1-D cross correlation gives a correlation result per trace, rather than an averaged value over all traces. Therefore a 1-D cross correlation plot will show which horizontal positions of the mine give the largest time shifts and amplitude changes.

Figure 9-18: 1-D cross correlation result between the migrated sections of scenarios C and D.

Figure 9-18 indeed shows that the amplitude change and time shifts as a result of the vertical extension of the brine layer between the bottom caverns is higher than the amplitude changes and time shifts as a result of the extension of the brine layer in the lateral sense. This means that 2-D cross correlation is not a useful tool to determine a lateral extension of the salt mine, but in the case of lateral extensions we can use subtraction of the data sets, or use 1-D cross correlation in order to quantify the time shifts and amplitude changes for every lateral coordinate.
9.3 Elastic results

The elastic data has also been analyzed using the subtraction and cross correlation techniques. Not all the results are discussed as extensively as the acoustic case, as many results are quite similar.

9.3.1 Scenario B versus scenario C

Subtraction of the migrated section of scenario B from the migrated section of scenario C (Figure 9-19) gives already a lot of insight in the time-lapse behavior of the salt mine caused by one year of production. The figure clearly shows an amplitude change caused by the vertical extension of the brine layer between the two bottom caverns.

With 2-D cross correlation (Figure 9-20) a time shift of 2.0 ms and an amplitude change of 7.1% is found. There is no horizontal shift.

Figure 9-19: Section of scenario B subtracted from the section of scenario C, zoomed-in around the main reflection of the salt mine.
9.3.2 Scenario C versus scenario D

Subtraction of the migrated section of scenario C from the migrated section of scenario D (Figure 9-21) gives the effect of three years of production in the lateral direction. From this figure and Figure 9-22, which shows the 2-D cross correlation result it becomes clear, that just like in the acoustic case, the extension of the mine in the lateral direction barely affects the cross correlation result.

The time shift found between these two scenarios is 0.0 ms, the horizontal shift is 0.0 m and the amplitude change is 2.0%.

Figure 9-20: 2-D cross correlation result between scenarios B and C.

Figure 9-21: Migrated section of scenario C subtracted from the migrated section of scenario D, zoomed-in around the reflection of the mine.
9.3.3 Scenario B versus scenario D

In the subtracted section it can be seen where the differences between scenario B and C are, namely outside the caverns and right in between the caverns. From the knowledge of the acoustic modeling it is to be suspected that the time-shift and amplitude change between scenarios B and D will be in the order of magnitude of the time-shift and amplitude change between scenario B and C, rather than scenarios C and D. In order to check this we compare again the 2-D cross correlation result between the data of scenario B and D with the auto correlation result of the data of scenario B. The results are indeed as expected: the time-shift is 2.0 ms, the amplitude change 13.1% and there is no horizontal shift.

Figure 9-22: 2-D cross correlation result between scenarios C and D.

Figure 9-23: Migrated section of scenario B subtracted from the migrated section of scenario D, zoomed-in around the reflection of the mine.
9.3.4 Quantitative results of elastic numerical modeling study

All the time-shifts, horizontal shifts and amplitude changes are combined in Table 9-2. The observed time-shifts are almost the same as in the acoustic case. The amplitude changes are slightly higher.

The amplitude change between scenarios C and D is 7.1%. The relating amplitude change for pre-stack data was 12.0% (See Section 9.1). We can therefore conclude that stacking leads to a pronounced decrease in amplitude change. 2-D cross correlation of scenarios C and D (a purely lateral effect) causes a much smaller amplitude change than a vertical change. This effect was already predicted by the pre-stack study. This observation confirms that the amplitude behavior found by 2-D cross correlation is indeed correct.

Table 9-2: Time-shifts, derived from 2-D cross correlations on the elastic data.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Effect</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>Time shift [ms]</td>
<td>2.0</td>
<td>2.0</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>Amplitude change</td>
<td>7.1%</td>
<td>13.1%</td>
<td>2.0%</td>
</tr>
<tr>
<td></td>
<td>Lateral shift [m]</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>C</td>
<td>Time shift [ms]</td>
<td>2.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td></td>
<td>Amplitude change</td>
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Chapter 10  Conclusions and recommendations

In this chapter all the results are summed up, and the research goals will be answered. Afterwards some recommendations for future work are given.

10.1  Conclusions

The change from conventional solution mining to solution squeeze mining has led to a higher subsidence rate. In order to be able to predict the subsidence more information of the changing geometry of the mine has to be known.

In this research it has been proven that the salt mine is visible is in shot records, CMP-gathers and migrated sections. The exact geometry of the mine cannot be distinguished in the data, because of interference effects.

A preliminary study on pre-stack data shows that a vertical extension of the mine causes an increase in amplitude. A lateral extension of the mine shows a linear relationship with amplitude. The amplitude effect of a vertical extension of the mine is much higher than the amplitude effect of a lateral extension.

Amplitude decay with offset is visible in the elastic CMP-gathers. This observation suggests that an AVO-analysis could be successfully performed. Because of time limitations this has not been done in this thesis.

In order to quantify the time-shift, lateral shift and amplitude change between two data sets 2-D cross correlation can be used. The 2-D cross correlation is performed on the migrated and stacked data. This means that the amplitude changes found depend on the fold of the CMP's. The data resampled through adding zeroes in the frequency domain to obtain a sampling rate that allows determining smaller time shifts.

A 5 m vertical extension of the brine layer between the two bottom caverns causes a time shift of 1.5 ms for the acoustic case and 2.0 ms for the elastic case. The amplitude changes are respectively 5.3% and 7.1%. For a purely lateral extension of the two side lobes to the cavern no time shift is found for the acoustic and elastic case and the amplitude change is 0.2% and 2.0% respectively. The time-shifts have the same order of magnitude as the ones that are worked with in the oil and gas industry. The amplitude changes correlate well to the amplitude changes found by the preliminary study on pre-stack data. In order to make lateral changes of the salt mine visible one could opt for 1-D cross correlation. The conclusion therefore yields that the effects of solution salt mining on time-lapse seismic can be quantified.

A time shift between the acoustically and elastically modeled results exists. This is probably due to a difference in the source signal.

The final conclusion yields that the effects of solution salt mining can be detected and quantified in seismic time-lapse mode. Some of the effects in this seismic study are large enough to be seen in real seismic data. It is therefore feasible to use time-lapse seismic to monitor geometry changes of NedMag's underground solution salt mine.
**10.2 Recommendations**

It has been proven in this research that geometry changes in the vertical and lateral directions give different time-shifts and amplitude changes. However, it is not yet clear how to relate the found differences into exact geometry changes. A follow-up study should therefore focus on this inversion process.

In this study only a 2-D line has been considered. In order to get a better idea of the changing geometry a 3-D study needs to be carried out. A 3-D study also allows more complex geometry changes to be implemented.

Some factors that play a role with real field data have not been taken into consideration, such as noise conditions, dispersion and damping. Before starting tests in the field it is advised that these effects are studied on synthetic models.

The receiver sampling used in this research was 26 m, while the source spacing was 50 m. This means that there are not many coinciding source and receiver pairs. The fold of the different CMP’s is therefore very small. This makes AVO-analysis difficult. It is therefore suggested that the modeling is redone with a receiver spacing that is exactly half of the source spacing.
References

Articles


J. Thorbecke, 2009. 2D Finite Difference Wavefield Modelling, In progress.


**Books**


Appendices

Appendix A: Density log of well VE-02

Appendix B: Abstract 71st EAGE Conference & Exhibition, Amsterdam

Appendix C: SU-script to make model for scenario B
Appendix A: Density log of well VE-02

This gamma ray/density log was made on February 2nd, 1976.
Appendix B: Abstract 71st EAGE Conference & Exhibition, Amsterdam

Title: Quantifying Time-Lapse effects of Solution Squeeze Mining
Authors: G. van Noort, G.G. Drijkoningen, R.J. Arts, J.W. Thorbecke (Delft University of Technology), J.G. Bullen & J. Visser (Nedmag Industries Mining & Manufacturing BV)
Presentation: Oral presentation on 11 June 2009
Presenter: G. van Noort
T017
Quantifying Time-lapse Effects of Solution Squeeze Mining

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SUMMARY

In the northern part of The Netherlands the magnesium salts carnalite and bischofite are mined in a deep solution mine. For optimal production and also for environmental impact such as subsidence, it is vital to understand the process of salt production. To this aim, it is studied whether seismic in time-lapse mode may help to visualize and quantify the changes due to the salt production. The main questions addressed are: Can changes due to salt production be detected in seismic time-lapse mode? And if so, how big are these changes? To address these issues, a modelling study was performed. Seismic data were synthesized using acoustic and elastic finite-difference schemes, and further processed to obtain migrated seismic images. For the models themselves, different production scenarios were studied. Analysis of these data clearly shows that salt mining causes detectable changes in time-lapse seismic signals and can be quantified. The amplitude effect seems relatively larger than the induced time shift.
Introduction

Nowadays, time-lapse seismic is a well established technique for monitoring hydrocarbon production (Calvert, 2005) and, e.g., CO₂ storage (Arts et al., 2006). However, for many other subsurface processes such monitoring is often too expensive or its gains are less obvious, like in the case we discuss in this paper: Seismic monitoring of salt production through solution mining at depths larger than 1.5 km. In this paper we try to demonstrate the feasibility and the added value of seismic monitoring of deep salt mining.

In the northern part of the Netherlands the Zechstein formation, deposited in the late Perm, consists mainly of salts and is found at a depth range between 1400 and 1800 meters. The main components for commercial exploration are magnesium salts, more precisely carnalite and bischofite. Mining started in 1972 in a conventional way, where the salt was dissolved by flushing fresh water through different wells. This produced brine-filled caverns of some tens of meters height and around 100 meters in diameter. In 1996, the method was adapted to squeeze mining where the salts are made more mobile using a pressure difference, i.e. a underpressure in the caverns, between the magnesium salt layers and the caverns. This makes the salt creep towards the caverns.

Apart from the commercial reasons for monitoring salt mining, there are also environmental and safety reasons: The salt mining leads to subsidence of the surface in habituated areas. To prevent the caverns from collapsing, a safety margin was set by the government of a maximum cavern size of 100 m in diameter. Furthermore, the Dutch government prescribes that the maximum subsidence of the surface may not exceed 65 cm at any point to limit damage to constructions. At the currently observed rate of subsidence this limit will be reached in a few years’ time.

Due to these restrictions and the commercial stakes, it has become paramount to better understand and monitor the full process of this salt production. Although the mining engineers control the mining process, but they have little idea of the 3D geometry of the caverns in the subsurface since very no geophysical imaging and very little logging was done in the past. Seismic and its time-lapse mode, if proven successful for this application, would solve those questions and help to better understand and control the process of mining, including the overburden effect. The study presented in this paper is a seismic-modelling study where the following questions are answered:

- Can changes due to salt production be detected in seismic time-lapse mode?
- If so, can these changes be quantified?

Creating a model including synthetic seismics

Based on the interpretation of 3D time migrated data, a subsurface model of the salt mining area was constructed. To limit the computation time for the synthetic seismics, a 2D cross section of the model was selected. The properties of this model, i.e. the P-wave velocity, S-wave velocity and mass density, were derived as follows:

- The P-wave velocities were obtained from a regional velocity model, determined by Van Dalosen et al (2006). This velocity model gives the velocities for the main formations. For the Zechstein group a sonic exists from the 1970’s giving a more detailed velocity of 4600 m/s for the gypsum and halite, and 4300 m/s for the carnalite and bischofite. For the brine/liquid that replaced the rock salts, a velocity of 1500 m/s has been assumed;
- The S-wave velocities were obtained by using rock physics models. For the rocks other than the salt, Castagna’s (1985) empirical relation for sandstones was used: \( V_s = 0.804 V_p - 0.856 \). For the rocks themselves, the shear-wave velocity has been obtained by assuming a Poisson’s ratio of 0.35, as determined by Jeremic (1994). No shear-wave velocity has been assumed in the brine;
- The mass densities were extrapolated directly from a density log in a well.
Based on the model information above, pre-stack synthetic seismic data were created, using both acoustic and elastic 2D finite-difference schemes. These data were then processed up to a migrated seismic P-wave image. For the imaging, a pre-stack time-migration approach was taken. For optimal imaging results, the velocities used have been obtained through a velocity semblance gather. The resulting seismic image can be seen in Figure 1 (left), next to the corresponding 2D section extracted from the 3D data (right). Note the similarity between the main horizons of the synthetic model and the real seismic.

![Figure 1: Seismic images obtained from: (left:) synthetic modelling, pre-stack time migrated. (right:) real 3D seismic.](image)

As a next step, the model was adapted to mimic one year of salt production. Based on real production data, 27,500 m$^3$ per year per well was assumed. Since the formation of caverns due to production is uncertain, several scenarios were defined. Here we will show the effects of three scenarios (Figure 2), with the production taking place in the vertical direction (changing layer/slab thicknesses) and in the horizontal direction (extending finite slab, one year of production compared after scenario 2).

![Figure 2: (Top:) Scenario 1: The reference model. Salt production changing layer thicknesses. (Middle:) Scenario 2: Salt production changing the vertical extent of slab. (Bottom:) Scenario 3: Salt production changing lateral extent of slab. This scenario represents one year of production after scenario 2.](image)
Results for time-lapse seismic

Based on the two scenarios, we have produced seismic time-migrated images. The initial synthetic data were obtained with purely acoustic modelling. For the processing, we used the same velocity model as for the reference model. The results for Scenario 1 (see Figure 2) and the differences with the reference section are shown in Figure 3. One can clearly see time shifts and amplitude changes occur due to the salt production.

![Image 1](image1.png)

**Figure 3:** (Left:) Migrated seismic image for scenario 1. The red box indicates the area of interest. (Right:) Differences with reference image.

In order to quantify the time-lapse changes, we performed 2D cross-correlations between the sections and normalised them with the autocorrelation of the reference section. For this correlation, we only used the area of interest, around the main reflection from the salt mine, to avoid the effect of unwanted diffractions and multiples, that are still present in the time migrated images. The main conclusion here is that no time-shift of about can be observed between scenario 1 and 2, and a time-shift of 0.25 ms between reference model and scenario 3. For amplitudes comparing the time-lapse seismic data to the reference model, a change of 3.4% can be observed for scenario 2 and 6.0% for scenario 3.

![Image 2](image2.png)

**Figure 4:** Zoomed-in normalised auto-correlation of reference section (left), cross-correlation of the reference model with scenario 2 (middle) and cross-correlation of the reference model with scenario 3 (right), using acoustic modelling.

So far, we showed the results for acoustic modelling, but because of the displacement of rock by a fluid/brine we would expect also a large effect in the elastic response. In the migrated post-stack domain the results are higher, namely a amplitude change of 4.8% and 18.3% between the reference model and scenario 2. The time-shifts is 1.0 ms.
Discussions and conclusions

This seismic modelling study clearly shows that salt mining can cause detectable changes in time-lapse seismic signals. Elastic modelling gives much more pronounced results than purely acoustic modelling. The amplitude effect seems relatively larger than the induced time-shift. The inversion of the time-lapse seismic data to the geometry of the cavern is less obvious. A rather complex interference pattern arises, similar as observed for example in CO₂ storage at Sleipner, where the CO₂ spreads in thin layers below thin shales (Arts et al., 2006). The lateral extent of the anomaly is easier to detect. In a follow-up study we intend to focus more on the inversion process.

Of course success for field surveys would still depend on several other factors such as noise conditions encountered in the field, with a particular challenge on land, repeatability of the time-lapse seismic data and the successfullness of migration/imaging due to structural complexity in and around the salt body. However, especially by using fixed arrays, we are optimistic about detecting caverns with time-lapse seismic data.

References


**Appendix C: SU-script to make model for scenario B**

This appendix shows the Seismic Unix script that defines the compressional wave velocity and density model for scenario B. These models are implemented in the acoustic modeling, and— together with a shear wave velocity model—in the elastic modeling.

```plaintext
#Initializing the background model
makemod sizex=17000 sizez=4500 dx=2 dz=2 cp0=1750 ro0=2000
file_base=temp.su \
  intt=def poly=0 x=0,17000 z=10,10 cp=1750 gradcp=0.32 ro=2000 \
  intt=def poly=0 x=0,17000 z=300,300 cp=1750 gradcp=0.32 ro=1950 \
  intt=def poly=2 x=0,3759,13078,17000 z=989,1066,422,700 cp=2500
gradcp=0.86 ro=2600 \
  intt=def poly=2 x=0,4706,12795,17000 z=1375,1570,814,1130 cp=2900
gradcp=0.37 ro=2700 \
  intt=def poly=2 x=0,4506,13106,17000 z=1999,2114,1258,1506 cp=4600
gradcp=0.0 ro=2170 \
  intt=def poly=0 x=0,4506,13106,17000 z=2049,2134,1480,1606 cp=4300
gradcp=0.0 ro=1600 \
  intt=def poly=2 x=0,4506,13106,17000 z=2069,2140,1500,1626
gradcp=0.0 ro=2170 \
  intt=def poly=2 x=0,6352,13976,17000 z=2140,2147,1840,1995 cp=3000
gradcp=0.2 ro=2800
```

```plaintext
# Making a cavern with incorrect velocity and density
makemod sizex=17000 sizez=4500 dx=2 dz=2 cp0=0 ro0=0 file_base=tempcav.su \
  intt=def poly=0 x=13200,13400 z=1517,1517 cp=10000 ro=10000 \
  intt=def poly=0 x=13200,13400 z=1543,1543 ro=0 cp=0
```

```plaintext
# Making a cavern incorrect velocity and density
makemod sizex=17000 sizez=4500 dx=2 dz=2 cp0=0 ro0=0
file_base=tempcav2.su \
  intt=def poly=0 x=13700,13900 z=1517,1517 cp=10000 ro=10000 \
  intt=def poly=0 x=13700,13900 z=1543,1543 ro=0 cp=0
```

```plaintext
# Making a cavern incorrect velocity and density
makemod sizex=17000 sizez=4500 dx=2 dz=2 cp0=0 ro0=0
file_base=tempcav3.su \
  intt=def poly=0 x=13200,13400 z=1472,1472 cp=10000 ro=10000 \
  intt=def poly=0 x=13200,13400 z=1498,1498 ro=0 cp=0
```

```plaintext
# Making a cavern incorrect velocity and density
makemod sizex=17000 sizez=4500 dx=2 dz=2 cp0=0 ro0=0
file_base=tempcav4.su \
  intt=def poly=0 x=13700,13900 z=1472,1472 cp=10000 ro=10000 \
  intt=def poly=0 x=13700,13900 z=1498,1498 ro=0 cp=0
```

Echo 'make model'

```plaintext
makemod sizex=17000 sizez=4500 dx=2 dz=2 cp0=0 ro0=0 file_base=ztemp.su \
  intt=def poly=2 x=0,4506,13106,17000 z=2089,2145,1520,1646 cp=4300
gradcp=0.0 ro=2170 \
  intt=def poly=2 x=0,4506,13106,17000 z=2094,2150,1525,1651 cp=0
gradcp=0.0 ro=0
```

Echo 'making lhs'

# Making a cavern

makemod sizex=17000 sizez=4500 dx=2 dz=2 cp0=0 ro0=0
file_base=ztempcav.su
    intt=def poly=0 x=0,13200 z=0,0 cp=100000 ro=100000 \n    intt=def poly=0 x=0,13200 z=4500,4500 ro=0 cp=0
echo 'making rhs'
# Making a cavern
makemod sizex=17000 sizez=4500 dx=2 dz=2 cp0=0 ro0=0
file_base=ztempcav2.su
    intt=def poly=0 x=13900,17000 z=0,0 cp=100000 ro=100000 \n    intt=def poly=0 x=13900,17000 z=4500,4500 ro=0 cp=0
echo 'combining lhs with model'
# Combining the original model with the cavern
suop2 ztemp_cp.su ztempcav_cp.su op=diff | clip minclip=0 > ztemp2_cp.su
suop2 ztemp_ro.su ztempcav_ro.su op=diff | clip minclip=0 > ztemp2_ro.su
echo 'combining rhs with model'
# Combining the original model with the cavern
suop2 ztemp2_cp.su ztempcav2_cp.su op=diff | clip minclip=0 > ztemp3_cp.su
suop2 ztemp2_ro.su ztempcav2_ro.su op=diff | clip minclip=0 > ztemp3_ro.su
suop2 temp_cp.su ztemp3_cp.su op=diff | clip minclip=1500 > temp6_cp.su
suop2 temp_ro.su ztemp3_ro.su op=diff | clip minclip=1335 > temp6_ro.su
# Filling in the correct velocities and densities
suop2 temp6_cp.su tempcav_cp.su op=diff | clip minclip=1500 > temp7_cp.su
suop2 temp6_ro.su tempcav_ro.su op=diff | clip minclip=1335 > temp7_ro.su
suop2 temp7_cp.su tempcav3_cp.su op=diff | clip minclip=1500 > temp8_cp.su
suop2 temp7_ro.su tempcav3_ro.su op=diff | clip minclip=1335 > temp8_ro.su
suop2 temp8_cp.su tempcav4_cp.su op=diff | clip minclip=1500 > temp9_cp.su
suop2 temp8_ro.su tempcav4_ro.su op=diff | clip minclip=1335 > temp9_ro.su
suop2 temp9_cp.su tempcav2_cp.su op=diff | clip minclip=1500 > complete-model_cp.su
suop2 temp9_ro.su tempcav2_ro.su op=diff | clip minclip=1335 > complete-model_ro.su