Effect of hysteresis on the stability of an embankment under transient seepage

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Abstract. Hysteresis is a well-known phenomenon that exists in the soil water retention behaviour of unsaturated soils. However, there is little research on the effects of hysteresis on slope stability. If included in slope stability analyses, commonly the suction in the unsaturated zone is taken as non-hysteretic. In this paper, the authors investigate the effect of hysteresis on the stability of an embankment under transient seepage. A scenario of water level fluctuation has been assessed, in which a cyclic external water level fluctuates between a low and high level. It was found that the factor of safety (FOS), the volumetric water content and the suction in the unsaturated zone are significantly affected by hysteresis. It was also found that, when the period of water level fluctuation in one cycle is relatively small, there is little difference in the FOS between the hysteretic case and non-hysteretic case. However, when the period exceeds a certain threshold value, significant differences between these two cases can be observed. Compared to the case in which hysteresis is considered, the FOS is higher in the case which does not consider hysteresis. This suggests that the non-hysteretic case may overestimate slope stability, leading to a potentially dangerous situation. Moreover, the period under which there emerge large differences between the hysteretic and non-hysteretic case is strongly related to the magnitude of hydraulic conductivity and the period of the cyclic water level fluctuation.

1. Introduction

Wetting-drying hysteresis in the soil water retention curve (SWRC) of unsaturated soils has been recognised for a long time. The relationships between water content and suction for the wetting path and the drying path are not identical. The reason for hysteresis is, in the main, attributed to the “ink-bottle” effect, i.e. entrapped air in the voids and the contact angle of the meniscus [1, 2]. Normally, the hysteresis in water retention behaviour is ignored in seepage analysis [3, 4]. Yang et al. [5, 6] investigated the influence of hysteresis on the seepage in a soil column by comparing the numerical results to three infiltration tests. The authors showed that the difference in the variation of suction profiles using hysteretic and non-hysteretic formulations was significant. They found that, when hysteresis was considered, the computed suction and volumetric water content in the soil column were closer to the experimental results. Tsai and Chen [7] and Ma et al. [8] studied the influence of hysteresis on the stability of infinite slopes subjected to rainfall and pointed out that hysteresis behaviour affects the distribution of water content and suction, and thus the slope stability.

This paper investigates the effect of wetting-drying hysteresis on the stability of an embankment slope under transient seepage conditions. The authors first introduce the
formulations and numerical implementation for analysing transient seepage in an embankment considering the hysteresis in water retention behaviour, and the computation of factor of safety (FOS). Then, the authors present a comparison between hysteretic and non-hysteretic cases in terms of matric suction profiles and the variation of FOS. They also present a brief analysis of the parameters which can affect the emergence of the differences between non-hysteretic and hysteretic cases.

This paper follows on from recent work undertaken in this research group. Arnold and Hicks [9, 10] investigated the effect of rainfall on the stability of heterogeneous soil slopes under steady state and transient conditions, respectively. Li et al. [11], Li and Hicks [12] and Hicks et al. [13] focused on slope stability computations in 3D, investigating the impact of material property variability, and trying to improve the realism of slope stability computation in comparison to 2D. Wang et al. [14] extended the research to the propagation of slope failure using the material point method and de Gast et al. [15] has investigated how analyses are used to determine the FOS in the Netherlands.

2. Formulation of transient seepage considering hysteresis and slope stability

2.1. Governing flow equation

The governing equation of 2D transient flow is based on mass conservation. In this paper, the soil skeleton is considered rigid for the flow analysis, which implies that any volume change potentially affecting the flow is not accounted for. Therefore, the hydraulic governing equation is given as follows,

$$\frac{\partial}{\partial x} \left( k_x \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left( k_y \frac{\partial h}{\partial y} + k_y \right) = C(h) \frac{\partial h}{\partial t}$$

where $h$ is the pore water pressure head, and $k_x$ and $k_y$ are the hydraulic conductivities in the $x$ and $y$ directions, respectively, $C(h) = \frac{\partial \theta}{\partial h}$ is the specific moisture capacity function, $\theta$ is the volumetric water content and $t$ is time.

2.2. Water retention model

The soil water retention curve is a function relating the suction head, $h$, with the volumetric water content, $\theta$. In this paper, the suction head is,

$$h = -s/\gamma_w = -(u_a - u_w)/\gamma_w$$

where $s$ is the matric suction, $u_w$ is the pore water pressure, $u_a$ is the pore air pressure, which in this paper is assumed to be atmospheric, and $\gamma_w$ is the unit weight of water.

The van Genuchten model [16], in combination with the pore size distribution relationship proposed by Mualem [17], is used:

$$S = \frac{\theta - \theta_d}{\theta_s - \theta_r} = \frac{1}{\left[1 + (\alpha|h|)^n\right]^{1-\frac{1}{n}}} \quad h < 0$$

$$S = 1 \quad h \geq 0$$

where $S$ is effective degree of saturation, $\alpha$ is the inverse of the air-entry suction head and $\theta_s$ and $\theta_r$ are the saturated and residual volumetric water contents, respectively (see Figure 1). Due to the hysteretic behaviour of the water retention curve, the main drying and wetting curves have different values of $\alpha$, i.e. $\alpha_w \neq \alpha_d$. The model parameter $n$ defines the slope of the water retention curve and is assumed to be identical for the main wetting and drying curves.

The function of hydraulic conductivity is derived from the SWRC, which is proposed by van Genuchten [16]:

$$k(h) = k_s \left[1 - \left(\frac{\theta - \theta_d}{\theta_s - \theta_d}\right)^n\right]^{\frac{1}{n}}$$

where $k_s$ is the saturated hydraulic conductivity.

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where $k_s$ is the saturated hydraulic conductivity.
Figure 1. Soil water retention curve.

\[ k = k_{sat} \sqrt{S} \left[ 1 - \left( 1 - S \left( \frac{n}{n-1} \right) \right)^2 \right] \tag{4} \]

In which \( k_{sat} \) is the saturated hydraulic conductivity of the soil.

In this paper, the authors use linear scanning curves to model the transition from drying to wetting and vice versa (Figure 1) [5, 6]. The slope of the scanning curve, \( \kappa \), is constant and defined as:

\[ -\kappa = \frac{d\theta}{dh} \tag{5} \]

2.3. Slope stability assessment

In this work, Bishop’s effective stress combined with the extended Mohr-Coulomb failure criterion has been used to calculate the shear strength [9, 10]:

\[ \tau = c' + \sigma \tan \phi' + \chi s \tan \phi' \tag{6} \]

where \( \tau \) is the shear strength, \( c' \) and \( \phi' \) are effective cohesion and friction angle, \( \chi \) is a scalar parameter defining the suction, \( s \), induced effective stress and \( \sigma \) is the normal stress to the shear strength plane due to gravitational loading. In this paper, the suction stress, \( \chi s \), is defined as,

\[ \chi s = \begin{cases} \frac{\theta - \theta_r}{\theta - \theta_r} s & s > 0 \\ 0 & s \leq 0 \end{cases} \tag{7} \]

The unit weight, \( \gamma \), of the unsaturated soil is influenced by the volumetric water content, \( \theta \), [7] and can be expressed as,

\[ \gamma = [(1 - \theta_s) \rho_w G_s + \rho_w \theta] g \tag{8} \]

where \( \rho_w \) is the density of water, \( G_s \) is the specific gravity of the soil solid and \( g \) is the gravitational acceleration.

3. Numerical implementation

In the flow analysis, 4-node quadrilateral elements are used in the finite element method (FEM) program based on Smith and Griffiths [19]. The governing flow equation has been solved by use of the modified Picard iteration method [20, 21]. In the flow analysis code, the volumetric water
content for the main drying and wetting curves is computed via Equation 3. The water retention behaviour along the linear scanning curve is computed via an algorithm which determines the current volumetric water content as a function of the variation in the suction head, $\Delta h$, and the previous volumetric water content $\theta_{t+1} = f(\Delta h, \theta_t)$ [5].

In the FEM slope stability code, 8-node quadrilateral elements are used. After the flow analysis, the suction stress and volumetric water content are imported into the stability analysis and mapped to the Gaussian points of the 8-node elements. If the pore water pressure at the Gaussian point is negative, the unit weight is calculated by Equation 8; otherwise, the unit weight is the saturated unit weight. The initial stresses are based on the suction stress and gravitational loading. The strength reduction method [22] is applied here to compute the FOS of the slope. The FOS is defined as the factor by which the original shear strength is reduced in order to cause the slope to fail:

$$c'_{f} = \frac{c'_{f}}{FOS}$$
$$\phi'_{f} = \arctan\left(\frac{\tan \phi'_{f}}{FOS}\right)$$

where $c'_{f}$ and $\phi'_{f}$ are the factored shear strength parameters.

4. Example set-up

In this paper, an embankment under water level fluctuation is analysed to show the influence of hysteresis on slope stability. The geometry of the embankment is shown in Figure 2. The height of the earth embankment is 12m. The width of the embankment crest and foundation are 4m and 52m, respectively. The upstream and downstream side slopes are both 1:2. The upstream water level is changing with time and the downstream water level remains at foundation level ($z=0$m). The upstream water level fluctuation is simulated by the superposition of two sinusoidal curves (Figure 3).

![Figure 2. Geometry of the embankment under water level fluctuation.](image)

In Figures 2 and 3, $WL_1$ and $WL_2$ are the highest and lowest water levels in the process of water level fluctuation. The two sinusoidal curves have different periods, i.e. $T = 10$d and $T_1 = 0.2T = 2$d, but the same amplitude. The saturated hydraulic conductivity, $k_{sat}$, of the soil has been set to be 0.0864m/d. The saturated and residual volumetric water contents, $\theta_s$ and $\theta_r$, are 0.38 and 0.0038, and the retention curve parameter, $n$, is 1.226. The inverse of the air-entry suction head of the main drying and wetting curves, $\alpha_d$ and $\alpha_w$, are 0.1m$^{-1}$ and 0.2m$^{-1}$, respectively, with $\alpha = \alpha_d$ being used for the non-hysteretic case. The slope of the scanning curves is $\kappa = 0.00006$. The high and low water levels, $WL_1$ and $WL_2$, are 10m and 4m, respectively.

The transient seepage starts from the result of a steady state analysis using the drying curve when the water level is 10m. The bottom boundary of the embankment is impermeable and fixed.
Figure 3. Two cyclic fluctuations of the simulated water level.

5. Results

5.1. Influence of hysteresis on seepage behaviour

The hysteresis in water retention behaviour implies that the variation of volumetric water content in soil becomes more complex. In Figure 4, the suction and water content have been tracked at position A shown in Figure 2. It can be seen from Figure 4 (a) that, when the soil is considered as non-hysteretic, the variation of volumetric water content are always on the main drying curve. However, if the hysteretic behaviour is taken into account (Figure 4 (b)), the variation of volumetric water content with suction can follow the scanning curve and can even reach the main wetting curve (although not in this case at this point).

Figure 4. Volumetric water content vs. suction at point A for: (a) non-hysteretic case, and (b) hysteretic case, with $t_1$=0d, $t_2$=5d, $t_3$=10d, $t_4$=15d and $t_5$=20d.

Figure 5 shows the comparison of pore water pressure and volumetric water content, between non-hysteretic and hysteretic cases, at the end of the first cycle, i.e. $t = T = 10$d. The internal gradients of pore pressure and volumetric water content for the hysteretic case less steep than those for the non-hysteretic case.
5.2. Influence of hysteresis on slope stability

The influence of hysteresis on the pore water pressure and the volumetric water content results in an impact on the shear strength (Equation 6), and thus on the slope stability. The variation of the FOS during the transient process of water level fluctuation is shown in Figure 6.

In Figure 6, there is a significant difference in FOS between the non-hysteretic and hysteretic cases. The FOS results of the non-hysteretic case overestimate the slope stability compared to the hysteretic case. This means that the FOS computed without considering the hysteretic water retention behaviour is non-conservative and could lead to a higher risk in design. The higher FOS for the non-hysteretic case and the lower FOS for the hysteretic case are due to the combined effects of gravitational loading, overturning force and suction. For example, in the drying process of the first cycle, the response of the pore water pressure and volumetric water content in the non-hysteretic case is slower than that of the hysteretic case. The combined effect of those three factors results in a slightly higher FOS. In addition, during the wetting process in the hysteretic case, the variation of volumetric water content with suction follows the scanning curve; there is a rapid decrease in suction, while the increase of volumetric water content is insignificant, which means that the gravitational loading and overturning force do not change much. The rapidly reduced suction results in a significant decrease in the effective stress, which is the reason for the decrease in FOS for the hysteretic case at the end of the first cycle.
cycle. Moreover, the highest FOS does not occur at the same time as the water level reaches the lowest point; it occurs at a slightly later time.

5.3. Parameters which affect the difference between non-hysteretic and hysteretic cases

In this section, the authors discuss the influence of the saturated hydraulic conductivity and the period of water level fluctuation on the final FOS. Figure 7 shows the evolution of FOS corresponding to four different periods using the same soil property values. To simplify the variation of water level, only a single sinusoidal curve is used here, which is component 1 in Figure 3. \( WL_1 \) and \( WL_2 \) are now 12m and 6m, respectively.

![Figure 7](image_url)

**Figure 7.** Differences between non-hysteretic and hysteretic cases corresponding to four different values of \( L_k \).

The difference in the results is due to the interplay between the period of the water level fluctuation and the saturated hydraulic conductivity. The authors proposed a non-dimensional variable to explore the result of this interplay, \( L_k = \frac{TK_{sat}}{H} \) (where \( H \) is the height of the embankment). Although the amplitude of the water level change and the width of the embankment are also important, for this specific embankment these two features are ignored in the expression of \( L_k \). It can be seen from Figure 7 that there are only negligible differences between the non-hysteretic and hysteretic cases when \( L_k \) is relatively small. This is because, when the period is small compared to the saturated hydraulic conductivity, there is little time for the pore water pressure and volumetric water content changes to propagate and their variations are limited into a small range for both cases. Therefore, the variation of FOS for each case during the transient process is small, meanwhile the difference between these two cases is not apparent. The authors found that when the variable \( L_k \) increases, there is a greater difference between the non-hysteretic and hysteretic cases. In Figure 7, the approximate minimum period for which there is a noticeable difference is 1d (\( L_k \) =0.072), although it is still small, but it increases further with a 10d period (\( L_k \) =0.072).

In Figure 8, the variation of the maximum difference between the non-hysteretic and hysteretic cases with the non-dimensional variable \( L_k \) is presented. It can be seen from Figure 8 that the maximum difference in FOS between the two cases in the transient seepage process increases with increasing \( L_k \).
This non-dimensional variable could be a good indication for design purposes. As to a specific type of embankment or slope, the engineer could find a critical value of $L_k$ and quickly decide whether it is necessary to compute the slope stability by considering hysteresis or not.

6. Conclusions
This paper investigates the effects of hysteresis in the water retention behaviour of unsaturated soil on the slope stability of an embankment under transient seepage. Due to the existence of hysteresis, the distribution of pore water pressure and volumetric water content are different from the non-hysteretic case. The differences in pore water pressure and volumetric water content result in differences in the slope stability between the non-hysteretic and hysteretic cases. In all cases, the hysteretic soil has a lower FOS.

The authors found that the differences can be related to a non-dimensional variable which shows the interplay between period, saturated hydraulic conductivity and slope dimensions. When the saturated hydraulic conductivity is large, the period for which there is a difference between non-hysteretic and hysteretic cases is small, and vice versa.

References


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