Full Wavefield Migration of Vertical Seismic Profiling data
Full Wavefield Migration of Vertical Seismic Profiling data

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Alok Kumar SONI

Master of Technology in Applied Geophysics,
Indian Institute of Technology Roorkee, India
geboren te Gomia, India.
Dit proefschrift is goedgekeurd door de promotor:
Prof. dr. ir. L.J. van Vliet

en de copromotor:
Dr. ir. D.J. Verschuur

Samenstelling promotiecommissie:
Rector Magnificus, voorzitter
Prof. dr. ir. L.J. van Vliet, Technische Universiteit Delft, promotor
Dr. ir. D.J. Verschuur, Technische Universiteit Delft, copromotor
Prof. dr. ir. A. Gisolf, Technische Universiteit Delft
Prof. dr. ir. C.P.A. Wapenaar, Technische Universiteit Delft
Prof. Y. Wang, Imperial College London
Dr. R. Fletcher, Schlumberger Gatwick, advisor
Prof. dr. ir. P.M. van den Berg, Technische Universiteit Delft, reservelid

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In loving memory of my closest friend and brother Akash Kumar (1981-2007)
Dedicated to my parents and wife
Karmanye Vadhikaraste, Ma phaleshou kada chana,
Ma Karma Phala Hetur Bhurmatey Sangostva Akarmani

(Bhagavad Gita, Chapter II, Verse 47, Language: Sanskrit)

Meaning:
You have the right to perform your actions,
but you are not entitled to the fruits of the actions.
Do not let the fruit be the purpose of your actions,
and therefore you won’t be attached to not doing your duty.
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Introduction to borehole seismic

1.1 Introduction

In exploration seismology, active man-made sources are used as an input to the earth system and we measure the resulting responses. The response of the earth comprises of reflections, refractions, diffractions and possible noises (both random and coherent noises). For an overview on exploration seismic, see Dobrin and Savit [1988]; Telford et al. [1990]. The objective of the oil and gas exploration industry is to extract reliable, high-resolution structural and stratigraphic details of the prospect area using seismic measurements. The structural and stratigraphic details aid the interpretation geologist, petrophysicist as well as reservoir engineers in understanding the prospect and eventually helps in decision making towards the drilling and production processes.

Surface seismic has been the major exploration tool in the oil and gas industry over the last six decades. Therefore, seismic prospecting has become one of the most dynamic field in terms of innovative research towards a better understanding, of both the acquisition as well as the processing of the data. Vertical Seismic Profiling (VSP) has been one of the important innovations in the data acquisition design. For a detailed introduction to this technology, see Gal’perin [1974]; DiSiena et al. [1981]; Balch et al. [1982]; Hardage [1985]; Hinds et al. [1996]; Pereira and Jones [2010]. Basically, vertical seismic profiling is a borehole seismic technique. As the name suggests, the detectors are placed in a borehole (usually) near the reservoir, while the seismic sources are located near the surface of the earth.

In fact, in seismic measurements, we see all possible geometries such as surface
seismic (Figure 1.1a) with both sources and receivers being located at the surface; vertical seismic profiling (Figure 1.1b) with sources located on the surface and the receivers being located in a borehole; crosshole seismic profiling (Figure 1.1c) with both sources and receivers, being located in different boreholes; and horizontal or deviated well (Figure 1.1d) profiling with the sources located on the surface and the receivers located in a near-horizontal well closer to the reservoir. An example common-shot gather is simulated using a 2D acoustic finite difference method, shown in Figures 1.1e, 1.1f, 1.1g and 1.1h for their corresponding geometries given in Figures 1.1a, 1.1b, 1.1c and 1.1d, respectively.

These different geometries to acquire seismic data meet different exploration objectives. Further, in the past, we have also seen different VSP geometries, such as zero-offset VSP, offset VSP, walkaway VSP, walkabove VSP, salt-proximity VSP, drill-bit VSP and multi-offset VSP. For an overview of these different geometries, see for example Hope et al. [1998]. The VSP geometries vary according to the geological objectives and challenges in the prospecting area [Oristaglio, 1985]. A discussion on various kinds of VSP field geometries is beyond the scope of this thesis.

In this research, our objective is to obtain the best possible image from walkaway VSP data. A walkaway VSP geometry comprises of geophones being located in a vertical or deviated borehole and an array of surface sources in a line moving away from the well. In fact, among different VSP acquisition geometries, walkaway VSP provides a reasonable laterally extended high-resolution image of the reservoir around the well trajectory, see e.g. Payne et al. [1990]. Figure 1.2 shows a schematic diagram of a typical 2D and 3D walkaway VSP geometry in a marine environment.

In the following sections, we will discuss the advantages and the limitations of VSP imaging" in current industrial practice, followed by the motivation and proposals of our research and a brief outline of this thesis.

1.2 VSP imaging: advantages and limitations

Over the past two decades, vertical seismic profiling has proven to provide high resolution and reliable images of oil and gas reservoirs around the globe [Arroyo et al., 2003; Müller et al., 2010a,b]. In VSP acquisition, since the receivers are placed...
1.2 VSP imaging: advantages and limitations

Figure 1.1: Schematic diagram showing a) surface seismic geometry, b) vertical seismic profiling, c) crosshole seismic geometry and d) horizontal well seismic geometry. An example common-shot gather from all the corresponding geometries are shown in e), f), g) and h), respectively.

in a borehole, it provides a unique relationship between the measured wavefield in time and the true depth of the receivers. This property has been exploited to obtain a reliable time-to-depth conversion for surface seismic data around the well. Further, the recorded wavefields in the borehole geophones travel less distance through the complex overburden and thereby, suffer less from wavefield distortions and contain higher frequency signals [Arroyo et al., 2003]. In other words, the bandwidth of VSP data is generally wider compared to surface seismic data. VSP has proven to obtain images in very difficult geological situations such as gas-
clouds or complex salt structure, where the surface seismic data often struggles to provide reliable images [Hornby et al., 2006].

Another important advantage of the receivers being located in a borehole is that it can record both the upgoing and downgoing wavefield (including the direct arrivals from source to receivers), which are easy to distinguish in the measured data and has helped in various ways to interpret and closely understand the subsurface [Hartse and Knapp, 1990]. Figure 1.3 shows a comparison of surface seismic and VSP data for a zero-offset source, simulated using a 2D acoustic finite-difference method. In the past, we also saw proposals to generate pseudo-VSP data from surface seismic for better understanding of the subsurface [Alá’i and Wapenaar, 1994].

However, despite of the high-resolution images and reliable time-depth relationship, images from VSP often suffer from migration artefacts and limited illumination extent around the well trajectory. This is because the current migration algorithms in the industry mostly rely on the primary-only assumption. The illumination due to the primary upgoing wavefields cover only a narrow triangular or trapezoidal area with increasing lateral extent with depth along the well trajectory, as shown in Figure 1.4a. The poor illumination and limited aperture in addition to other limiting factors such as inaccurate wavefield separation and contamination by multiples often account for poor-quality images when using conventional imaging techniques. Therefore, the economic-value(s) of VSP surveys

---

**Seismic Migration:** Seismic migration can be defined as an inverse operation involving rearrangement of seismic information elements so that reflections and diffractions are plotted at their true locations. The need for this arises since variable velocities and dipping horizons cause elements to be recorded at surface positions different from the subsurface positions. [Sherrif, 2002]
are debatable in terms of the trade-off between high acquisition costs involved and poor illumination or limited-extent images obtained from the primary wavefields. To overcome these limitations by incorporation of the entire wavefield (primaries and multiples), is the main driver for our research.

1.3 VSP imaging: from removing to using multiples

Intuitively, we can see the advantages of using the multiples in imaging based on VSP geometries. Figure 1.4a schematically shows the illumination due to the upgoing primary-only wavefield that is confined around the well trajectory (as mentioned earlier in the previous section), while Figure 1.4b shows an extension of the illumination area if the information is retrieved from the full wavefield, i.e., including the surface multiples and the internal multiples.
In this research, our motivation is to develop a novel imaging technology that can handle some of the above limitations. The main directions of the algorithm can be enumerated as:

a) an imaging algorithm that can handle all the multiples (both surface and internal multiples) in the data to enhance the overall illumination in the imaging;

b) a feedback loop-based or inversion-based \(^3\) imaging algorithm, that uses observed data and data misfit to steer the imaging scheme in the correct direction;

c) an algorithm that requires a minimum amount of data pre-processing, e.g. can handle un-separated upgoing and downgoing wavefields together, and can handle different wavefield modes (P and S waves) together;

d) an algorithm that can handle any complexities in the source wavefield (blended or simultaneous sources) or complexities in subsurface geology;

e) an algorithm that yields a high-resolution, reliable, true-amplitude reflectivity image that can be either angle-independent (for structural studies) or angle-dependent (for detailed AVO-type analysis, velocity analysis, etc) depending on the survey objective(s);

---

\(^3\)Inversion: Inversion can be defined as deriving from field data a model to describe the subsurface that is consistent with the data; determining the cause from observation of effects. In other words, inversion means solving for a spatial distribution of parameters which could have produced an observed set of measurements. [Sherrif, 2002]
f) an algorithm whose results comply with the laws of physics.

In recent years, we have seen that the seismic industry is slowly shifting its emphasis from removing the multiples to using the multiples. Multiple removal or elimination has been an extensive topic of research. For a comprehensive discussion on various methods of multiple elimination or removal see Verschuur [2006]. Some of the interesting examples include water-bottom multiple attenuation based on wave-extrapolation based redatuming [see for example Berryhill and Kim, 1986], multiple suppression based on F-K dip filtering [see for example Zhou and Greenhalgh, 1994], multiple suppression in the tau-p domain [see for example Zhou and Greenhalgh, 1994], multiple attenuation in post-migration angle-domain image-space [for example Sava and Guitton, 2005], multiple attenuation based on up/down deconvolution [see for example Majdański et al., 2011], inversion-based approaches such as SRME (Surface-Related Multiple Elimination)|Verschuur, 1991; Verschuur et al., 1992; Berkhout and Verschuur, 1997; Weglein et al., 1997; Wang, 2004; Dragoset et al., 2010], EPSI (Estimation of Primaries and multiples by Sparse Inversion) [van Groenestijn and Verschuur, 2008, 2009a,b; Lopez and Verschuur, 2013], etc.

We also see discussions on imaging using multiples through various approaches for surface as well as borehole seismic data. For some interesting examples of using surface multiples in surface seismic case, see Berkhout and Verschuur [1994]; Youn and Zhou [2001]; Berkhout and Verschuur [2004]; Brown and Guitton [2005]; Dash et al. [2009]; Verschuur and Berkhout [2011]; Liu et al. [2011b]; Lu et al. [2011]; Zheng and Schuster [2014]. Similarly, for examples of including surface multiples in imaging VSP data, see Jiang et al. [2005]; He et al. [2006]; Jiang et al. [2007]; Soni et al. [2012c,a]; O’Brien et al. [2013a]. We will discuss some of these VSP imaging methods in the later chapters of this thesis.

To illustrate the importance of surface multiples, we use a dipping layer velocity model as shown in Figure 1.5a and compute the beam illumination due to sparsely distributed sources at the surface. Figures 1.5b, 1.5c and 1.5d illustrate the single (primary-only) and Figures 1.5e, 1.5f and 1.5g illustrate the double illumination (primary plus surface multiples) for 1, 4 and 11 equally spaced surface sources [Verschuur and Berkhout, 2011]. The normalized illumination energy values at 1000m and 2000m depth are plotted for comparison and shown in Figures 1.5h, 1.5i and 1.5j. Note the extra energy in illumination added from the surface multiples, that can help to improve the resolution of the images.

Surface multiples in marine environments are relatively easy to include into an imaging process because we know the location of the air-water reflecting boundary as well as a decent approximation of the surface reflectivity. However, including the internal multiples or full-wavefield into imaging is now a hot subject of re-
Introduction to borehole seismic

Figure 1.5: a) Velocity model used to compute illumination plots for sparsely distributed sources at the surface. b), c), d) Normalized illumination beam for single (using only primary energy) for 1, 4, and 11 equally spaced sources, respectively, and e), f), g) double illumination (using primary and free surface multiples) for 1, 4, and 11 equally spaced sources, respectively, computed on this velocity model. h), i) and j) Normalized illumination energy extracted at 1000m for single illumination (black dotted curve) and double illumination (black bold curve) and extracted at 2000m for single illumination (red dotted curve) and double illumination (red bold curve).

search, where the prediction of internal multiples is a non-trivial problem. In the past decades, the prediction, elimination or use of internal multiples was a vast area of research. Some interesting examples on prediction and removal of internal multiples include data-driven approaches discussed in Jakubowicz [1998a,b]; Berkhout and Verschuur [2005]; Verschuur and Berkhout [2005]; Ikelle [2006]; Ikelle et al. [2009], model-driven approaches on pre-stack data discussed
in Berkhout and Verschuur [2005]; Verschuur and Berkhout [2005], model-driven approach on post-stack image domain discussed in Reshef et al. [2006], inverse-scattering-based prediction of internal multiples in the image domain discussed in Malcolm et al. [2007] and inverse-scattering-based schemes discussed in Weglein et al. [1997]; ten Kroode [2002].

On the other hand, some recent publications showed interesting examples towards using the internal multiples as an extra source of illumination. Some methods include imaging different order of internal multiples independently via an inverse-scattering approach as discussed in Malcolm et al. [2008]. Vasconcelos et al. [2008] considered internal multiples for imaging using interferometric principles in very specific scenarios, using VSP geometry. Fleury [2013] illustrates the potential of using internal multiples using reverse time migration. More recently, we see the concept of data-driven Marchenko imaging to obtain target images free from overburden multiples, discussed in Behura et al. [2012]; Wapenaar et al. [2013]; Slob et al. [2014]; Wapenaar et al. [2014]. Although, these algorithms include internal multiples in imaging, they are not posed as an inversion scheme.

Berkhout [2012] introduced the concept of Full Wavefield Migration (FWM) and Joint Migration Inversion (JMI). Full wavefield migration is defined as a least-squares inversion-based imaging scheme that utilizes primaries and all the multiples (both surface and internal multiples) in the observed data to estimate the subsurface reflectivity\(^4\). The forward modelling engine termed as full wavefield modelling in the FWM inversion scheme takes the non-linearity of the wavefield due to subsurface reflectivity into account and iteratively models the full wavefield by adding one higher order of scattering in each iteration.

Joint migration inversion is also a least-squares inversion-based scheme that utilizes primaries and all the multiples (both surface and internal multiples) in the observed data to simultaneously estimate the subsurface reflectivity image and the migration velocity. A detailed discussion on JMI is beyond the scope of this thesis, however, the readers are referred to discussions in Staal and Verschuur [2012, 2013]; Staal et al. [2014].

Imaging both surface seismic data [Davydenko and Verschuur, 2012; Davydenko et al., 2012] and borehole seismic or vertical seismic profile data [Soni et al., 2012b; Soni and Verschuur, 2013a,c] using full wavefield migration is a topic of ongoing research. In this thesis, we propose full wavefield migration to image VSP data. Now, if we go back to the beginning of this section and note the criteria we listed in order to aim for a novel imaging approach, we propose that FWM satisfies all of those points. Using multiples for imaging in VSP data is important because of the associated limitations of the poor illumination and small aperture from the primary-only wavefields.

\(^4\)For the detailed concepts of least-squares inversion, the readers are referred to the comprehensive books by Menke [1984]; Tarantola [1987]; Strang [2003]
In the next section, we present an overview of VSP acquisition, processing, imaging and inversion practices currently employed in the oil and gas industry. We also discuss the data requirements and / or assumptions made in this thesis for VSP imaging.

1.4 VSP acquisition, processing, imaging and inversion: an overview of historical and recent developments

VSP Acquisition
The chosen VSP acquisition geometry depends on the survey objectives of the oil and gas industry, as mentioned earlier. Walkaway VSP data are typically acquired to obtain a high-resolution structural image of the reservoir close to the borehole as well as information on the anisotropy in the subsurface layers. The other important steps include knowledge of the borehole site conditions and estimating the acquisition parameters such as adequate source-strength, desired bandwidth in the data, source lateral sampling, receiver depth sampling, etc. Once the laboratory or computer simulation aided analysis is done to get an acquisition design, a decision on the acquisition equipment such as the specifications of airguns (for marine cases) or vibrators (for land cases), geophones, recording systems, etc are taken. For a comparative guide to these aspects see Gilpatrick and Fouquet [1989]. Furthermore, we have seen several innovative advances both in VSP acquisition design [van Gestel et al., 2002; Gulati et al., 2011; Coles et al., 2012] and acquisition source and receiver equipment that are deployed in the field [Poster, 1983; Haldorsen et al., 1995; Moldoveanu et al., 2000; Dirks et al., 2002; Fuller et al., 2006; House et al., 2008; Greenwood et al., 2012; Dean et al., 2013; Mateeva et al., 2013].

In this thesis, we assume to have walkaway marine VSP data with an objective to obtain a high-resolution image of the reservoir.

VSP Processing
Due to the VSP geometry design, signal processing of VSP data is different in several aspects from processing surface seismic data. For details on the basic processing steps of VSP data, see Hardage [1985]; Hinds et al. [1996]; MacBeth [2002]; Pereira and Jones [2010]. The processing specific to multicomponent VSP data include tube-wave noise attenuation, tool orientation correction, up-down wavefield separation and $P-SV$ wavemode separation, downgoing wavefield deconvolution for multiple attenuation. We will not go into details of these methods in this thesis, but for an acoustic imaging of 2D walkaway VSP data, we assume that the wavefields are mode-separated, i.e. for the 2D-2C case, the horizontal component ($v_x$) and vertical component ($v_z$) are decomposed into their equivalent $P-P$ components. In the literature and practice, several methods regarding
multicomponent data decomposition are discussed in Dankbaar [1987], Hermann and Wapenaar [1992], Leaney [1990], Blias [2008], Sun et al. [2009], Lou et al. [2013] and Palacios et al. [2013]. Further, we assume to have walkaway VSP data with tube-wave noise removed. However, in full wavefield migration, we are not required to separate the upgoing and downgoing wavefields. Also, multiple attenuation is not required, because we aim to use the full wavefield for estimating the reservoir image. The transmitted direct arrival does not contribute to imaging directly, but is used to estimate the correct source-wavelet to be used in FWM.

**VSP Imaging**

Several VSP imaging techniques are discussed in the literature. Some of the many methods include single-channel mapping [Dillon and Thomson, 1984], imaging by single shot record inversion [Harwijanto et al., 1987], imaging using generalized Radon transforms [Beylkin et al., 1987; Miller et al., 1987; Krasovec, 2001], vector-image isochron imaging [Krasovec, 2001], Kirchhoff migration [Keho, 1986; Gherasim, 2005], reverse time migration [Hokstad et al., 1988; Neklyudov and Borodin, 2009], interferometric migration using extrapolated VSP Green’s functions [Xiao and Schuster, 2009], wavefield extrapolation and depth imaging [Amundsen, 1993], VSP-CDP mapping techniques [Gulati et al., 1997; Chen et al., 2000], using common-reflection stack technology (CRS) [von Steht, 2008] and the use of image point transformation [Cosma et al., 2010]. Almost all of these mentioned imaging methods can only migrate the upgoing primary wavefields. Hence, they ideally require wavefield separated data with no upgoing or downgoing multiples being present. As introduced earlier, we propose FWM for imaging VSP data in this thesis, which uses the full wavefield of the data to estimate the subsurface reflectivity image.

**VSP Inversion**

VSP inversion is a very broad term and includes any algorithm that uses VSP data to estimate any set of subsurface model parameters from it. Of course, the choice of model parameter is based on the objective of the inversion scheme and, hence, the underlying forward model equation that connects the observed data to these model parameters. In the past, we saw mainly two categories of inversion on VSP data, namely - (1) traveltime inversion (using predominantly the transmission data, i.e. the direct arrivals) to estimate an interval-velocity profile near the well and (2) waveform inversion (using predominantly the reflection data). For details on traveltime inversion, readers are referred to Stewart [1983]; Lines et al. [1984]; Lee [1990]; Mao and Stuart [1997]; Lizarralde and Swift [1999]; Moret et al. [2004]. VSP inversion of multicomponent data was used to estimate the anisotropy and stiffness tensor [Dewangan and Grechka, 2003; Grechka and Mateeva, 2007]. For a discussion on acoustic or visco-acoustic full waveform inversion, see Barnes and Charara [2009]; Liang et al. [2013]; Silvestrov et al. [2013]. Recently, Podgornova et al. [2013b,a] also discussed anisotropic elastic full waveform inversion on VSP
data. In this thesis, we present an inversion-based imaging - FWM, where our model parameters are reflectivity images, and the underlying non-linear forward model connects the reflectivity with the full wavefield. In chapter 9 on future recommendations, we also discuss the concept of Joint Migration Inversion (JMI), where the inversion model parameters are reflectivity and migration velocity simultaneously [Staal and Verschuur, 2012, 2013]. Another future recommendation on VSP inversion include high-resolution estimation of the elastic properties of a reservoir using VSP data using the full waveform inversion algorithm as discussed in Haffinger [2013]; Haffinger et al. [2013]; Rizzuti and Gisolf [2014a].

The next section provides an overview of the rest of this thesis.

1.5 Thesis outline

Chapter 2

This chapter discusses the full wavefield modelling for walkaway VSP geometries. We first give an overview of various seismic modelling algorithms and compare full wavefield modelling with the existing concepts. As mentioned earlier, full wavefield modelling uses subsurface reflectivity instead of subsurface elastic parameters, so it is different from finite-difference or finite-element type schemes. It is recursive in depth and iterative, where each iteration adds the next higher order of multiples. Further, it accounts for both reflection and transmission of the wavefield, and, hence accounts for the non-linearity of the wavefield with respect to reflectivities. We will illustrate the modelling scheme using numerical examples.

Chapter 3

This chapter discusses the full wavefield migration scheme for walkaway VSP data. We first give an overview of various VSP imaging algorithms. Then we discuss the unconstrained and constrained inversion-based imaging scheme to obtain structural images from VSP data. Using a numerical example, we compare the effect of primaries, surface and internal multiples on the estimation of reflectivity via the constrained FWM inversion scheme. Finally, we discuss angle-dependent imaging of laterally invariant media. For angle-dependent imaging, the reflectivity matrix is parametrized in the space-ray parameter domain. We illustrate the importance of including multiples in imaging to extend the subsurface illumination and obtain high-resolution images, using numerical examples.

Chapter 4

This chapter illustrates the potential of FWM through synthetic examples for various case scenarios such as a complex reservoir geology; imaging vertical and
deviated well geometry data; imaging multi-well scenarios. Also, we discuss the sensitivity of the FWM algorithm using an erroneous velocity field in migration. As expected, the error in the migration velocity model not only gives an error in image depth, but also have a higher data misfit or data residual. JMI (joint migration inversion) can make use of the residual to estimate the velocity model (see also chapter 9 with conclusions and recommendations).

Chapter 5

This chapter applies full wavefield migration (FWM) to blended VSP data. We illustrate that FWM can handle VSP data from blended sources to estimate the subsurface reflectivity such that it explains the total blended data. Hence, active deblending is not required as an extra pre-processing step in the proposed method. We discuss the forward modelling and imaging of blended VSP data for different blending factors. Further, we see that a constrained least-squares inversion scheme helps to reduce the blending interference noise in the image space.

Chapter 6

This chapter introduces the extension of FWM scheme of VSP data to the elastic case. The acoustic full wavefield modelling equations are extended to incorporate the converted wavefield at each depth level using angle-dependent reflection coefficients for a laterally invariant medium. The elastic modelling is illustrated using synthetic examples for 1.5D models, where the angle of the incidence is restricted well below its critical value. We have used the linearized Aki and Richards approximation of the Zoeppritz equation to obtain reflectivities and to illustrate the full wavefield elastic modelling scheme. Further, we discuss possible full wavefield elastic imaging schemes, that could be used to get high-resolution P-P, P-S, S-P as well as S-S images through an inversion scheme.

Chapter 7

This chapter discusses an extension of the FWM scheme to image steeply-dipping or overturned salt-flanks using the turning waves in VSP data. The extended FWM uses both vertical and horizontal wavefield extrapolation in the imaging scheme and, therefore, is able to image complex and steep-dip structures. We discuss the forward modelling and inversion scheme, followed by some numerical examples to illustrate the imaging of steep structures. These examples involve both unblended and blended VSP experiments, and also illustrate the effect of near-surface scatterers or complex overburden in the imaging of salt-flanks.
Chapter 8
This chapter demonstrates an application of the FWM scheme on field data acquired in deep-water of the Gulf of Mexico. As expected, using the multiples in imaging VSP data enhances the illumination beyond the region illuminated by the upgoing primary wavefield only.

Chapter 9
This final chapter discusses the conclusions drawn from this research as well as highlights some future recommendations towards joint migration-inversion of VSP data.
Full Wavefield Modelling for VSP geometries

2.1 Introduction

Seismic forward modelling is an essential part of any seismic inversion technology. Over the years, several modelling algorithms have been presented in the literature. For example, the primary-only modelling using ray-based approaches, such as the analytical Cagniard de-Hoop Pekeris method [Chapman, 2004] for a homogeneous medium and the WKBJ approximation method [Aki and Richards, 1980; Chapman, 2004] extended to smooth, inhomogeneous media. Another example includes the plane wave superposition-based approach for a layered system like the well-known reflectivity method. The Reflectivity method generates a full elastic response, and can model primaries as well as multiples [Fuchs and Muller, 1971; Kennett, 1979]. The most popular modelling algorithms used in current migration technology - Reverse Time Migration (RTM) [Baysal et al., 1983] - as well as most of the full waveform inversion algorithms are based on numerical solution to the differential wave equation, i.e. the finite-difference methods [Kelly et al., 1976; Virieux, 1986] or its variants such as finite-element or pseudo-spectral approaches [Komatitsch and Tromp, 1999; Galis et al., 2008; Liu et al., 2011a].

Further, specifically for synthetic VSP, there are several methods discussed in the literature. For example Wyatt [1981] introduced the synthesis method for normal incidence VSP; Aminzadeh and Mendel [1985]; Ferber [1988]; Xu [1990] extended the concept of state-space algorithms [Frasier, 1970; Aminzadeh and Mendel, 1982] to model elastic VSP data for non-normal incidence computed
for a given angle and layered media. McMechan [1985] discussed a time-domain
finite-difference scheme to model VSP data for laterally varying media. Suprajitno
and Greenhalgh [1986] and Mallick and Frazer [1988] used reflectivity methods to
model VSP data.

We introduce the full wavefield modelling algorithm [Berkhout and Verschuur,
2011; Berkhout, 2012], that is based on a numerical solution of the integral wave
equation. Full wavefield modelling uses the subsurface reflectivity as a modelling
parameter instead of the elastic medium parameters, which are required in finite-
difference methods. In full wavefield modelling, the continuity boundary condition
and reflectivity-wavefield relationship at a discontinuity is similar to the above-
mentioned reflectivity method [Claerbout, 1976; Kennett, 1979; Wyatt, 1981].

The full wavefield modelling algorithm is recursive in depth from top to bottom,
and formulated in the space-frequency domain using one-way extrapolation op-
erators and two-way scattered wavefields at each depth level. Furthermore, the
algorithm is iterative, in which every iteration adds the next order of scattering
to the wavefields. This means that in the first iteration, primaries are modelled,
followed by first-order surface and internal multiples in the second iteration, and
so on. Therefore, it can be exploited very flexibly and efficiently in an inversion-
based full wavefield migration scheme. In the following sections the full wavefield
modelling method is further explained and exemplified.

2.2 Wavefield conventions and boundary condition

To begin, we will define the full wavefield forward modelling for the VSP geome-
try in the common receiver domain, i.e. in the reciprocal domain. The reciprocal
domain is equivalent to sources being located in the borehole and the receivers
being located at the surface. For more details on reciprocity principles and appli-
cation to VSP data, please see Knopoff and Gangi [1959] and Mittet and Hokstad
[1995]. Figure 2.1 schematically illustrates this reciprocity relation that we will
use to formulate the full wavefield modelling scheme.

For the mathematical formulation of the iterative full wavefield modelling scheme,
we will use a similar matrix notation as introduced by Berkhout [1982], where a
bold symbol represents a full prestack dataset for one frequency component i.e.
the wavefields for all source and receiver locations. One column vector then repre-
sents the wavefield from one seismic source experiment i.e. a common shot gather
and one row vector represents one common receiver gather. Figures 2.2 and 2.3
show example common-shot gathers and common-receiver gathers, respectively.
Specifically, Figure 2.2a shows a velocity model, annotated with three surface
sources and receivers being located in the borehole. The three shot gathers corre-
spond to three column vectors in the data matrix $P$ as shown by the dotted box
2.2 Wavefield conventions and boundary condition

Figure 2.1: Schematic diagram illustrating a) general walkaway VSP geometry, where the sources are located at the surface and receivers are located in the borehole. b) Transformed VSP to reverse-VSP geometry, based on the reciprocity theorem, where the sources are located in the borehole and the receivers are at the surface.

Let us define the wavefield convention for the upgoing and downgoing wavefield just above and just below a depth level \( z_n \) [see also Berkhout, 2012]. The upgoing and downgoing wavefield just above depth level \( z_n \) can be represented by \( \vec{Q}^- (z_n) \) and \( \vec{P}^+ (z_n) \), respectively. The upgoing and downgoing wavefield just below depth level \( z_n \) can be represented by \( \vec{P}^- (z_n) \) and \( \vec{Q}^+ (z_n) \) respectively. Please note that the + and − superscript signs represent the downgoing and upgoing directions, respectively. Figure 2.4 illustrates this wavefield convention. The matrices \( \mathbf{R}^\cup (z_n) \) and \( \mathbf{R}^\cap (z_n) \) represent reflectivity matrices related to the discontinuities at depth level \( z_n \) for the wavefield coming from above and from below the layer, respectively. Note that wavefields \( \vec{P}^+ (z_n) \) and \( \vec{P}^- (z_n) \) can be designated as incoming wavefields and the wavefields \( \vec{Q}^+ (z_n) \) and \( \vec{Q}^- (z_n) \) as the outgoing ones.

Now, we will define the boundary conditions using the wavefield continuity equations as described in Claerbout [1976] [see also Berkhout, 2012]. In Figure 2.4b, the downgoing and the upgoing wavefields leaving depth level \( z_n \) can be defined in terms of the downgoing and upgoing wavefields incident at depth level \( z_n \), and the two-way scattered wavefield at that depth level. Hence the full wavefield equations from the continuity relationship becomes:

\[
\vec{Q}^+ (z_n) = \vec{P}^+ (z_n) + [\mathbf{R}^\cup (z_n) \vec{P}^+ (z_n) + \mathbf{R}^\cap (z_n) \vec{P}^- (z_n)]
= \vec{P}^+ (z_n) + \delta \vec{P} (z_n),
\]  

(2.2.1)
which states that the downgoing wavefield $\vec{Q}^+(z_n)$ departing from just below the depth level is the sum of the downgoing wavefield $\vec{P}^+(z_n)$ incident just above the depth level and the two-way scattered wavefield represented by $\vec{\delta P}(z_n)$. Similarly, the upgoing wavefield $\vec{Q}^-(z_n)$ leaving from just above the depth level is the sum of the upgoing wavefield $\vec{P}^-(z_n)$ incident on the depth level, i.e. just below the level and the two-way scattered wavefield represented by $\vec{\delta P}(z_n)$. This can be written as:

$$
\vec{Q}^-(z_n) = \vec{P}^-(z_n) + \left[ R^\cup(z_n) \vec{P}^+(z_n) + R^\cap(z_n) \vec{P}^- (z_n) \right]
$$

$$
= \vec{P}^-(z_n) + \vec{\delta P}(z_n).
$$

The two-way scattered wavefield $\vec{\delta P}(z_n)$ can be written as:

$$
\vec{\delta P}(z_n) = \left[ \begin{array}{c}
R^\cup(z_n) \\
R^\cap(z_n)
\end{array} \right] \left[ \begin{array}{c}
\vec{P}^+(z_n) \\
\vec{P}^-(z_n)
\end{array} \right] = R(z_n) \vec{P}(z_n),
$$

(2.2.3)

where $R$ is the total reflectivity matrix and $\vec{P}(z_n)$ is the total incident wavefield, for a depth level $z_n$, both from above and from below. Please note that in the acoustic approximation, $R^\cap(z_n) = -R^\cup(z_n)$. Also note that in Equation 5.2.6, the terms $\vec{P}^+(z_n) + R^\cup(z_n)\vec{P}^+(z_n) = T^\cup(z_n)\vec{P}^+(z_n)$, can be recognized as the
2.2 Wavefield conventions and boundary condition

Figure 2.3: a) Velocity model, annotated with a walkaway VSP geometry, schematically showing three receivers located in a borehole and dense sources located at the surface. b) Data matrix $\mathbf{P}$, highlighted with three row vectors related to the shown common-receiver gathers. c), d) and e) Three common-receiver gathers corresponding to the three receivers in (a) from top to bottom, respectively.

Figure 2.4: a) Schematic diagram annotated with a walkaway VSP geometry in the reciprocal domain, i.e. sources located in the borehole and receivers are at the surface. $z_n$ is an arbitrary depth level used to illustrate the wavefield conventions. b) Schematic diagram showing the upgoing and downgoing wavefields at depth level $z_n$. The upgoing and the downgoing wavefields just above depth level $z_n$ are represented by $\vec{Q}^-(z_n)$ and $\vec{P}^+(z_n)$, respectively, and the ones just below depth level $z_n$ are represented by $\vec{P}^-(z_n)$ and $\vec{Q}^+(z_n)$, respectively. The matrices $\mathbf{R}^+(z_n)$ and $\mathbf{R}^-(z_n)$ represent reflectivity matrices related to the discontinuities at depth level $z_n$ for the wavefield from above and below the depth level, respectively.

Transmitted downgoing wavefield through level $z_n$. Similarly, in Equation 2.2.2, the terms $\vec{P}^-(z_n) + \mathbf{R}^-(z_n)\vec{P}^-(z_n) = \mathbf{T}^-(z_n)\vec{P}^-(z_n)$, can be recognized as the
transmitted upgoing wavefield through level \( z_n \). \( \mathbf{T}^+(z_n) \) and \( \mathbf{T}^-(z_n) \) represent the transmittivity matrices for depth level \( z_n \).

Rewriting the boundary conditions and using an initial acoustic approximation for the boundary at depth level \( z_n \), \( \mathbf{R}^-(z_n) = -\mathbf{R}^+(z_n) \), we can also describe the upgoing and downgoing wavefields just below the depth level \( z_n \) in terms of the upgoing and downgoing wavefields just above the depth level \( z_n \) as [Claerbout, 1976]:

\[
\begin{bmatrix}
\vec{P}^-(z_n) \\
\vec{Q}^+(z_n)
\end{bmatrix} = \frac{1}{1 - \mathbf{R}^+(z_n)} \begin{bmatrix}
1 & -\mathbf{R}^+(z_n) \\
-\mathbf{R}^+(z_n) & 1
\end{bmatrix} \begin{bmatrix}
\vec{Q}^-(z_n) \\
\vec{P}^+(z_n)
\end{bmatrix}.
\]
(2.2.4)

### 2.3 Full wavefield modelling: mathematical formulation

To describe the recursive full wavefield modelling algorithm, we also need to define the convention for one-way wavefield propagation between consecutive depth levels. Figure 2.5a shows the downgoing wavefield \( \vec{Q}^+(z_{n-1}) \) from just below depth level \( z_{n-1} \) propagated down to depth level \( z_n \) using the propagation operator \( \mathbf{W}^+(z_n, z_{n-1}) \) and represented by the downgoing wavefield \( \vec{P}^+(z_n) \) just above the depth level \( z_n \). Similarly, Figure 2.5b shows the upgoing wavefield \( \vec{Q}^-(z_{n+1}) \) from just above the depth level \( z_{n+1} \) propagated up to depth level \( z_n \) using the propagation operator \( \mathbf{W}^-(z_n, z_{n+1}) \) and represented by the upgoing wavefield \( \vec{P}^-(z_n) \) just below the depth level \( z_n \). The propagation operators are the phase-shift operators, implemented as space-frequency domain convolution operators that can be calculated using an inhomogeneous background velocity model [Thorbecke et al., 2004]. Mathematically, we can write the one-way wavefield propagation for downward propagation as:

\[
\vec{P}^+(z_n) = \mathbf{W}^+(z_n, z_{n-1})\vec{Q}^+(z_{n-1}),
\]
(2.3.5)

and for upward propagation as:

\[
\vec{P}^-(z_n) = \mathbf{W}^-(z_n, z_{n+1})\vec{Q}^-(z_{n+1}).
\]
(2.3.6)

The iterative full wavefield modelling can be formulated in terms of iterative modelling of total incident wavefields \( \vec{P}(z_n) \) recursively along all depth levels. In the modelling scheme, the first step involves the modelling of the direct wavefields at all depth levels from the source. Thus, \( \vec{P}^+(z_n) \) and \( \vec{P}^-(z_n) \) in the first iteration are given by:

\[
[\vec{P}^+(z_n)]^{(1)} = \sum_{m=0}^{n-1} \mathbf{W}^+(z_n, z_m)\vec{S}^+(z_m),
\]
(2.3.7)

\[
[\vec{P}^-(z_n)]^{(1)} = \sum_{m=n+1}^{N} \mathbf{W}^-(z_n, z_m)\vec{S}^-(z_m),
\]
(2.3.8)
where the superscript (1) indicates the iteration number; $\vec{S}^+$ and $\vec{S}^-$ are the downgoing and upgoing sources wavefields, respectively, which are non-zero when there is an active source at that depth level. Now, mathematically, the incident wavefield from above, i.e. $\vec{P}^+(z_n)$, and the one from below, i.e. $\vec{P}^-(z_n)$, for a given iteration $i$ can be written as:

\[
\vec{P}^+(z_n)^{(i)} = \sum_{m=0}^{n-1} \vec{W}^+(z_n, z_m)[\delta \vec{P}^+(z_m)^{(i-1)} + \vec{S}^+(z_m)], \tag{2.3.9}
\]

\[
\vec{P}^-(z_n)^{(i)} = \sum_{m=n+1}^{N} \vec{W}^-(z_n, z_m)[\delta \vec{P}^-(z_m)^{(i-1)} + \vec{S}^-(z_m)]. \tag{2.3.10}
\]

Figures 2.6a and 2.6b schematically show the recursive modelling steps for $\vec{P}^+(z_n)$ and $\vec{P}^-(z_n)$, respectively. We can see that the incident wavefield from above, i.e. $\vec{P}^+(z_n)$ is a recursive summation of all the scattered wavefields above the depth level $z_n$, propagated to the level $z_n$ using the downward propagation operator $\vec{W}^+$ plus the direct wavefield contribution from all the sources in the subsurface above the level $z_n$ (via $\vec{W}^+\vec{S}^+$ term). Similarly, the incident wavefield from below, i.e. $\vec{P}^-(z_n)$, is a recursive summation of all the scattered wavefields below the depth level $z_n$, propagated to the level $z_n$ using the upward propagation operator $\vec{W}^-$ plus the direct wavefield contribution from all the sources (via $\vec{W}^-\vec{S}^-$ term) in the subsurface below the level $z_n$.

In subsequent iterations, $\delta \vec{P}(z_n)$ includes the $\vec{P}^+(z_n)$ and $\vec{P}^-(z_n)$ from the previous iteration. Thus, each iteration leads to one full round-trip of the wavefield, i.e. adds one higher order of scattering. The iterative full wavefield modelling scheme can be represented as a block diagram, see Figure 2.7 [Berkhout, 2012]. In other words, for a given iteration $i$, $\delta \vec{P}(z_n) = \mathbf{R}^\cup(z_n)\vec{P}^+(z_n) + \mathbf{R}^\cap(z_n)\vec{P}^-(z_n)$ in which the terms $\vec{P}^+(z_n)$ and $\vec{P}^-(z_n)$ were computed in the previous iteration $(i - 1)$.
2.4 Including angle-dependent effects in full wavefield modelling

In the previous section, we presented the mathematical formulation of the iterative full wavefield modelling. It is important to note that the parameter for the modelling engine, i.e. the $R(\hat{z}_n)$ matrix, incorporates the complete angle-dependent properties of the interface at any depth $z_n$ [de Bruin et al., 1990]. The diagonal elements of $R(\hat{z}_n)$ defines the structural image. On the other hand, the off-diagonal elements define the angle-dependent changes in amplitude as well as phase in the reflectivity.

We know that in the true subsurface, the reflectivity behaves as a complex function of the angle of incidence at an interface [Castagna and Backus, 1993]. So it is obvious that in such a case, if we only estimate the structural image in FWM, i.e. the diagonal elements of the reflectivity matrix, we will not be able to explain the amplitude and phase variation in the data with respect to the angle of incidence. In this section, we will describe how to parameterize the angle-dependent reflectivity in the ray parameter - tau domain. We will present the inversion or estimation of angle-dependent reflectivity in the next chapter.

In order to illustrate the angle-dependent behavior of reflectivity, we modelled surface data for a one-reflector model with a high-velocity contrast as shown in Figure 2.8a. Figure 2.8b shows a reflectivity operator of a grid point in the space-
2.4 Including angle-dependent effects in full wavefield modelling

Figure 2.7: Block diagram showing the iterative upgoing and downgoing wavefields being modelled at each depth level, using a given reflectivity model as well as a background velocity field. The scattering due to reflectivities are represented by the boxes 1a and 1b, [adapted from Berkhout, 2012]

time domain for the location indicated by the red dot on the flat reflector interface at lateral location 1500m shown in Figure 2.8a. The output of the reflectivity operator or the grid-point gather is the sum of the dual focused-wavefields on both the source and the receiver side of the reflector at a depth level (500m) for data from regular surface acquisition [see also Berkhout, 1997]. Such a reflectivity operator can be interpreted as a response measured at receivers just above the reflector from a source at the red dot in Figure 2.8a. We can clearly see the large amplitude reflectivity value at time zero. The lateral crossing is due to lateral bandwidth limitations, i.e. due to the propagation velocity of 1500m/s. Also note the two additional linear events that represent the head waves or refraction beyond post-critical angles. Figure 2.9 shows the reflection coefficient becoming complex beyond the critical angle for an interface with a positive velocity contrast (note that in this example the S velocity is zero).

Now, to include the angle-dependent behavior in the forward modelling, we will use a similar concept as described in de Bruin et al. [1990]. Hence, the two-way scattered wavefield (as described in previous section) $\delta \tilde{P}(z_n)$ is now written in terms of the angle-dependent reflectivity matrix $A$ in the linear Radon domain
as:

$$\delta \tilde{P}(z_n) = \begin{bmatrix} LA \cup (z_n) & LA \cap (z_n) \end{bmatrix} \begin{bmatrix} \tilde{P}^+(z_n) \\ \tilde{P}^-(z_n) \end{bmatrix} = LA(z_n)\tilde{P}(z_n).$$  \hspace{1cm} (2.4.11)

$A$ is the total angle-dependent reflectivity matrix, where each column describes reflection amplitudes as a function of horizontal slowness, $L$ is the inverse linear Radon transform operator matrix which transforms the frequency-ray parameter domain reflectivity to frequency-space domain reflectivity i.e. $LA \cup (z_n) = R \cup (z_n)$, and $\tilde{P}(z_n)$ contains the total incident wavefield both from above and below, for a depth level $z_n$. Again, in the acoustic approximation, we can write $A \cap (z_n) = -A \cup (z_n)$.

To define the inverse Radon transform operator $L$, we can write the relationship between model space (Radon domain) and data space in terms of a matrix-vector operator for a single frequency as:

$$\tilde{R} = L\tilde{A},$$  \hspace{1cm} (2.4.12)

where operator $L$ is the inverse linear Radon transform operator, and where $\tilde{R}$ and $\tilde{A}$ are the space domain and the Radon domain reflectivity, respectively. Note that
2.4 Including angle-dependent effects in full wavefield modelling

\[ v_{p_1} = 1500 \text{ m/sec}; \]
\[ \rho_1 = 1000 \text{ kg/m}^3; \]
\[ v_{p_2} = 3000 \text{ m/sec}; \]
\[ \rho_2 = 1000 \text{ kg/m}^3; \]

\[ v \]
\[ \rho \]

\[ \text{Real } R_{pp} \]
\[ \text{Im } R_{pp} \]
\[ \text{Real } T_{pp} \]
\[ \text{Im } T_{pp} \]

**Figure 2.9:** Plot of the acoustic P-P only reflection and transmission coefficient across two half-spaces separated by an interface as shown on the left hand side. The velocity of the upper half-space and the lower half-space are 1500 m/s and 3000 m/s respectively. The red and blue continuous curves show the real reflection and transmission coefficient, respectively; and the red and blue dotted curves show the imaginary reflection and transmission coefficient, respectively. (courtesy: CREWES Zoeppritz explorer website)

the adjoint operator \( L^H \) is approximately the forward Radon transform operator. The operator \( L \) can be expanded as:

\[
L = \begin{bmatrix}
    e^{-j\omega p_1 x_1} & \ldots & e^{-j\omega p_1 x_n} \\
    \ldots & \ldots & \ldots \\
    e^{-j\omega p_m x_1} & \ldots & e^{-j\omega p_m x_n}
\end{bmatrix}.
\]

Note that we will restrict ourselves to laterally invariant medium for angle-dependent reflectivity estimation. This indeed can be extended to geological interfaces with dips, where the transformation of space-frequency domain reflectivity matrix to ray-parameter domain AVP matrix take the dip term into account. For details, please see de Bruin [1992]. Also, we know that for using AVP in linearized inversion and AVO-type analysis, to obtain elastic properties of the subsurface uses various approximations of Zoeppritz equations. Zoeppritz equation [Aki and Richards, 1980] inherently defined for locally horizontal reflectors.

In chapter 3, we will present examples of the imaging of angle-dependent reflectivity for VSP data using full wavefield migration.
2.5 Numerical examples

To illustrate the iterative full wavefield modelling scheme, we use a 2D density model and a corresponding reflectivity model as shown in Figure 2.10a and 2.10b, respectively. The modelling examples are shown for both the common-receiver domain as well as for the common-shot domain. Figures 2.10a and 2.10b are annotated with the reverse-VSP geometry showing two sources in the borehole, one at 250\,m depth and the other at 1100\,m depth, for illustration purposes. Also, the models in Figures 2.10 show a black dotted line at a depth of 700\,m. It is the depth used to illustrate the iterative modelling of the downgoing wavefield $\vec{P}^+(z_{700})$ and the upgoing wavefield $\vec{P}^-(z_{700})$. Further, Figures 2.10c and 2.10d are annotated with the true walkaway VSP geometry showing two sources at the surface, one for illustrating a near-offset VSP at 500\,m and other for illustrating a far-offset VSP at 1450\,m.

Figures 2.11 and 2.12 illustrate the iteratively modelled upgoing and downgoing wavefields at depth level 700\,m for sources at a depth of 250\,m and 1100\,m, respectively (as shown in Figure 2.10a).

Figures 2.13 and 2.14 show common-receiver gathers for reverse VSP geometry, for equivalent sources at a depth of 250\,m and 1100\,m respectively. Specifically, 2.13a and 2.14a show the data modelled after the 1st iteration, 2.13b and 2.14b after the 2nd iteration, 2.13c and 2.14c after the 3rd iteration and 2.13d and 2.14d after the 4th iteration. Note, how each iteration adds a higher order of scattering to the modelled data.

Figures 2.15, 2.16 and 2.17 show common-shot gathers for the true VSP geometry, for a near-offset source, laterally located at 1450\,m. Similarly, Figures 2.18, 2.19 and 2.20 show example common-shot gathers for true VSP geometry, for a far-offset source, laterally located at 500\,m. Specifically, Figures 2.15a, 2.16a, 2.17a, 2.18a, 2.19a and 2.20a show the upgoing wavefield, Figures 2.15b, 2.16b, 2.17b, 2.18b, 2.19b and 2.20b show the downgoing wavefield and Figures 2.15c, 2.16c, 2.17c, 2.18c, 2.19c and 2.20c show the total wavefield (sum of the upgoing and downgoing wavefields) after the 1st, 2nd and 3rd iteration, respectively. Note that these results were extracted from modelling in the reciprocal domain. Again, we see how each iteration adds a higher order of scattering to the modelled data.

Finally, we compare the modelled data using full wavefield modelling with that simulated using an acoustic finite-difference scheme. The examples of modelled data in the reciprocal domain for a shot at depth 700\,m are shown in Figures 2.21a, 2.21b, 2.21c and 2.21d, for the first, second, third and fourth iteration, respectively. For comparison, Figure 2.21e shows an example simulated data for same source-receiver geometry, using an acoustic finite difference modelling scheme.
2.6 Discussion

In this chapter, we have presented the concept of full wavefield forward modelling illustrated with some simple numerical examples. This shows how the wavefields are built iteratively and every iteration adds a higher order of scattering. We also described how the parametrization of reflectivity can be formulated in the ray pa-
rameter - tau domain to include angle-dependent behavior of layer discontinuities. We have restricted our discussion of angle-dependent reflectivities to horizontal reflector cases only. We can extend this analysis for dipping interface, by taking the dip of the structure into account [de Bruin, 1992, see for example]. Although, as a parameter, we can estimate the angle-dependent reflectivity for a complex subsurface model, however, the interpretation of the estimates in terms of true incident angles and further AVO-type (AVO - Amplitude Variation with respect to Offset, see for example Castagna and Backus [1993]) linearized inversions are non-trivial.

Figure 2.11: Upgoing and downgoing wavefields at depth level 700m (shown by dotted line in Figure 2.10) for a source at a depth of 250m as shown in Figure 2.10a. a), c), e) and g) show the modelled downgoing wavefield using full wavefield modelling after the 1\textsuperscript{st}, 2\textsuperscript{nd}, 3\textsuperscript{rd} and 4\textsuperscript{th} iteration, respectively. Similarly, b), d), f) and h) show the modelled upgoing wavefield using full wavefield modelling after the 1\textsuperscript{st}, 2\textsuperscript{nd}, 3\textsuperscript{rd} and 4\textsuperscript{th} iteration, respectively.
2.6 Discussion

Figure 2.12: Upgoing and downgoing wavefields at depth level 700m (shown by dotted line in Figure 2.10) for a source at a depth of 1100m as shown in Figure 2.10a. a), c), e) and g) show the modelled downgoing wavefield using full wavefield modelling after the 1st, 2nd, 3rd and 4th iteration, respectively. Similarly, b), d), f) and h) show the modelled upgoing wavefield using full wavefield modelling after the 1st, 2nd, 3rd and 4th iteration, respectively.

Also, note that although the matrix $R$ in this chapter is defined under acoustic approximation, later in chapter six, we will extend the concept of full wavefield modelling and migration to include the converted waves. In the next chapter, we will discuss the least-squares inversion scheme in full wavefield migration.

Further, for more details on the propagation matrix $W$ as well as reflectivity matrix $R$, see Appendix B.
Figure 2.13: Modelled data in the common-receiver domain, for an equivalent source at 250m (shown in Figure 2.10a) after the a) 1st, b) 2nd, c) 3rd and d) 4th iteration. Note that every iteration adds a higher order of scattering.
Figure 2.14: Modelled data in the common-receiver domain, for an equivalent source at 1100m (shown in Figure 2.10a) after the a) 1st, b) 2nd, c) 3rd and d) 4th iteration. Note that every iteration adds a higher order of scattering.
Figure 2.15: Near-offset VSP, common-shot gather for a source located laterally at 1450 m (shown in Figure 2.10c), after the 1st iteration showing a) the downgoing wavefield, b) the upgoing wavefield and c) the total wavefield.

Figure 2.16: Near-offset VSP, common-shot gather for a source located laterally at 1450 m (shown in Figure 2.10c), after the 2nd iteration showing a) the downgoing wavefield, b) the upgoing wavefield and c) the total wavefield.
2.6 Discussion

Figure 2.17: Near-offset VSP, common-shot gather for a source located laterally at 1450m (shown in Figure 2.10c), after the 3rd iteration showing a) the downgoing wavefield, b) the upgoing wavefield and c) the total wavefield.

Figure 2.18: Far-offset VSP, common-shot gather for a source located laterally at 500m (shown in Figure 2.10c), after the 1st iteration showing a) the downgoing wavefield, b) the upgoing wavefield and c) the total wavefield.
Figure 2.19: Far-offset VSP, common-shot gather for a source located laterally at 500m (shown in Figure 2.10c), after the 2\textsuperscript{nd} iteration showing a) the downgoing wavefield, b) the upgoing wavefield and c) the total wavefield.

Figure 2.20: Far-offset VSP, common-shot gather for a source located laterally at 500m (shown in Figure 2.10c), after the 3\textsuperscript{rd} iteration showing a) the downgoing wavefield, b) the upgoing wavefield and c) the total wavefield.
2.6 Discussion

Figure 2.21: Modelled data for the density model shown in Figure 2.10, using full wavefield modelling for source at depth 700m (in reciprocal domain) after a) iteration 1, b) iteration 2, c) iteration 3 and d) iteration 4. Modelled data using full wavefield modelling for an offset source at lateral location 1000m after f) iteration 1, g) iteration 2, h) iteration 3 and i) iteration 4. For comparison, e) and j) show modelled data using an acoustic finite-difference scheme for the corresponding source-receiver geometry.
Full Wavefield Modelling for VSP geometries
3 Full Wavefield Migration of VSP data

3.1 Introduction

Seismic migration has been an extensive topic of research and development in the seismic industry over the last six decades. Some of the many algorithms that have been developed and used in the industry, especially for surface seismic data, include Kirchhoff migration [Schneider, 1978], wave-extrapolation migration [Gazdag, 1978; Berkhout, 1981, 1982; Claerbout, 1985], reverse time migration [Baysal et al., 1983], Gaussian Beam migration [Hill, 1990] and the inversion-based least-squares migration [Nemeth et al., 1999]. All of these algorithms have associated advantages and some drawbacks. For the details of various aspects, please refer to their specific articles. However, it is important to note that all of these existing imaging technologies only use the primary reflections and treat multiples as noise (i.e. it assumes that multiples are removed from the data before migration).

In general, VSP data have an additional limitation compared to surface seismic data due to its acquisition geometry. Even the advanced imaging techniques used for VSP data mostly rely on migration of primaries-only upgoing wavefields. Hence, the images are often hampered by poor illumination, limited aperture as well as incomplete wavefield separation. That causes migration artefacts, especially away from the well trajectory. Several VSP imaging techniques are discussed in the literature addressing these limitations. Some of the many examples specifically for VSP includes imaging by single shot record inversion [Harwijanto et
al., 1987], reverse time migration [Hokstad et al., 1988; Neklyudov and Borodin, 2009], interferometric migration using extrapolated VSP Green’s functions [Xiao and Schuster, 2009], wavefield extrapolation and depth imaging [Amundsen, 1993], VSP-CDP mapping techniques [Gulati et al., 1997; Chen et al., 2000] and the use of image point transformations [Cosma et al., 2010].

In order to suppress imaging artefacts in VSP imaging, some modified processing or imaging techniques have been proposed. For example, Yu and Hornby [2008] introduced a stereographic local beam imaging scheme to reduce migration noise; Kiyashchenko et al. [2009] used an extended wave-equation vector migration to image the noise separately; Lou et al. [2009] showed that adjusting the migration aperture angle appropriately helps in reducing migration artefacts and Leaney et al. [2009] showed that approaching the migration as a constrained least-squares inversion can also help in reducing the migration artefacts. They have all used the image dip field as a regularization term.

Further, including surface multiples in imaging VSP data proved to enhance the illumination region away from the well trajectory. There are various methods proposed in the literature to handle primaries and multiples together in VSP imaging. Jiang et al. [2005, 2007] discussed an interferometric approach to migrate the first-order multiples. Vasconcelos et al. [2008] also illustrated some examples of interferometric imaging of internal multiples in specific geological scenarios. He et al. [2006] showed the result of 3D wave-equation interferometric migration of VSP multiples. In most cases, the interferometric scheme suffers from crosstalk due to high-order multiples. Soni et al. [2012c,a] discusses a constrained least-squares inversion approach to include surface multiples in imaging to enhance illumination. In a slightly different example, Nasyrov et al. [2009] demonstrated that the image from primaries and from first-order surface multiples can be used to update the velocity model based on a cross-correlation objective functional of the two images and hence improve illumination in the image. Recently, O’Brien et al. [2013a] discussed a detailed case-study from the Gulf of Mexico, on using free-surface multiples in VSP imaging.

In most situations, the internal multiples are usually considered as noise. This is because internal multiples are difficult to predict for complex geological settings or when weak internal multiple generating boundaries are present. For some examples in literature from the recent past for internal multiple prediction, removal and imaging, please see Jakubowicz [1998a,b], ten Kroode [2002], Berkhout and Verschuur [2005], Verschuur and Berkhout [2005], Ikelle [2006], Jiang [2006b,a], Jiang et al. [2007], Vasconcelos et al. [2008], Ikelle et al. [2009] and Malcolm et al. [2008]. In our approach the internal multiples are considered signal.

In this chapter, we will discuss the full wavefield migration (FWM) approach to image VSP data. The details for imaging surface data can be found in Berkhout and Verschuur [2011], Berkhout [2012], Davydenko and Verschuur [2012] and
Davydenko et al. [2012]; to image the VSP data see also Soni et al. [2012b]; Soni and Verschuur [2013a,c]. FWM aims at estimating the true angle-dependent reflectivity of the subsurface utilizing primaries, surface multiples and internal multiples.

In the following sections, we discuss the basics of the FWM inversion scheme - posed as an unconstrained as well as a constrained least-squares inversion problem. Finally, the extension to include estimation of angle-dependent reflectivity is discussed using laterally invariant medium.

### 3.2 Full wavefield migration as a least-squares problem

As mentioned, FWM utilizes the full wavefield of the seismic data to estimate the subsurface reflectivity. However, FWM does not depend on the prediction of surface multiples or internal multiples explicitly, but it explains the angle-dependent reflectivity of the subsurface using the total upgoing and downgoing wavefield at each depth level. In this way, multiples are considered as part of the illuminating wavefield and can provide extra information during the imaging process. Also, no up/down wavefield separation is required, unlike the methods such as discussed in Ross and Shah [1987]. Ross and Shah [1987] discussed estimation of angle independent reflectivity for 1D or normal-incident VSP data, by dividing the upgoing wavefield by the downgoing wavefield. Their method requires up and down wavefield separation, followed by multiple attenuation to get a multiple-free 1D reflectivity profile. Furthermore, in FWM, the transmission effects are incorporated recursively in depth in order to determine the upgoing and downgoing wavefield from one depth level to the next in an accurate manner.

In the previous chapter, we discussed the iterative recursive full wavefield modelling. The aim of full wavefield migration is to estimate the true-amplitude reflectivity image. Hence, using the forward modelling method, we can pose the imaging as an inversion problem. Figure 3.1 shows a generalized block diagram for the inversion scheme in FWM. As we can see from the block diagram, the migration is performed as a feedback process, where the first iteration is similar to conventional imaging of the primary wavefield. However in FWM, there is no multiple suppression or attenuation processing applied to the measured data prior to imaging. The reflectivity image obtained in the first iteration is used to simulate the response using full wavefield modelling, and the simulated data is compared with the measured data. The residual of measured and simulated data after least-squares subtraction or adaptive subtraction is fed back in the loop to update the reflectivity in such a way that the data residual is minimized. Please note that each iteration of FWM involves an additional round-trip of full wavefield modelling, and hence, adds or uses a higher order of multiples to estimate the reflectivity. Also, it is interesting to note that if we replace the non-linear full-
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Wavefield modelling with a linear Born-modelling operator, which models only the upgoing primary wavefield, the block diagram is equivalent to what we call least-squares migration [see for example Nemeth et al., 1999; Duijndam et al., 2000].

**Figure 3.1:** Block diagram showing the general feedback loop for the inversion in FWM. The measured data is imaged, yielding subsurface reflectivity. The estimated reflectivity is used to simulate the response using full wavefield modelling, which is compared with the measured data. The residual of the measured and simulated data after adaptive subtraction is fed back in the loop to iteratively update the reflectivity. Each iteration adds or uses a higher order of multiples.

Mathematically, we formulate the estimation of the reflectivity as an optimization problem, where the simulation of the data is compared to the measured or observed data. The simulation involves one-way propagation of the two-way scattered wavefields, i.e. \( \delta \vec{P}(z_j) = R(z_n)\vec{P}(z_n) \), from all depth levels in the subsurface to the surface, plus the direct wavefield from the sources propagated to the surface. All propagation takes place in a background velocity model. Like all other migration algorithms, we assume that a migration velocity model is known before FWM. Based on this, we can formulate the estimation of the reflectivity as minimization of the following objective function \( J \) in a least-squares sense:

\[
J = \sum_k \sum_\omega ||P_{\text{obs},k}(z_0) - \sum_{m=0}^N W^-(z_0, z_m)[R(z_m)\vec{P}_k(z_m) + \vec{S}^-_k(z_m)]||_2^2,
\]

(3.2.1)

where \( P_{\text{obs},k}(z_0) \) is the observed data at the surface due to the \( k^{th} \) source, \( W^- \)
3.2 Full wavefield migration as a least-squares problem

is the one-way propagation matrix in the background velocity model, $\mathbf{R} \vec{P}_k$ is the two-way scattered wavefield at all depth levels due to the $k^{th}$ source (see Equation 2.2.3), and $\vec{S}_k$ is the upgoing source wavefield for the $k^{th}$ source. Further, note that in the acoustic approximation, we can write $\mathbf{R}^\cap(z_n) = -\mathbf{R}^\cup(z_n)$. This reduces the number of parameters to estimate in the inversion from two reflectivity matrices per depth level ($\mathbf{R}^\cap$, $\mathbf{R}^\cup$) to one ($\mathbf{R}^\cup$).

The above optimization problem can be solved by an iterative optimization scheme. Here, we have used the conjugate gradient (CG) scheme [Hestenes and Stiefel, 1952]. In the first iteration, the steepest descent is the negative of the gradient, and can be written for depth level $z_n$ as:

$$\Delta \mathbf{R}^\cup(z_n) = -\sum_k [\mathbf{W}^-(z_0, z_n)]^H \vec{E}_k(z_0) [\vec{P}_k(z_n)]^H, \quad (3.2.2)$$

where $\vec{E}_k(z_0)$ is the data misfit for the $k^{th}$ source, given by:

$$\vec{E}_k(z_0) = \vec{P}_{obs,k}(z_0) - \sum_{m=0}^N \mathbf{W}^{-}(z_0, z_m)[\mathbf{R}(z_m)\vec{P}_k(z_m) + \vec{S}_k(z_m)]. \quad (3.2.3)$$

The superscript $H$ represents the Hermitian of a matrix or vector. Also, note that we only use $\vec{P}^+(z_n)$ wavefield instead of the total $\vec{P}(z_n)$ wavefield in the gradient computation, which helps in avoiding the low-frequency artefacts due to transmission effects. This artefacts are similar in nature what is observed in reverse time migration when diving waves, head-waves and backscattered waves crosscorrelate [Guitton et al., 2007; Zhang and Sun, 2009]. Hence, the reduced steepest-descent direction at depth level $z_n$ becomes:

$$\Delta \mathbf{R}^\cup(z_n) = \sum_k [\mathbf{W}^-(z_0, z_n)]^H \vec{E}_k(z_0) [\vec{P}_k^+(z_n)]^H. \quad (3.2.4)$$

Further, in the CG scheme the new search direction is made orthogonal to the previous gradient using the gradients of two consecutive iterations $i$ and $i-1$. We have used the Polak-Ribiere algorithm [Polak and Ribiére, 1969] to update the search direction. At any iteration $i$, the update can be computed by finding the appropriate step-length $\alpha$ in the gradient scheme, as:

$$\mathbf{R}^\cup(i) = \mathbf{R}^\cup(i-1) + \alpha(i) \Delta \mathbf{R}^\cup_{cg}(i), \quad (3.2.5)$$

such that objective function $J$ is minimized. $\Delta \mathbf{R}^\cup_{cg}(i)$ is the computed conjugate direction. Again, each modelling step in the iterative scheme adds one order of scattering, and we perform the iteration until the estimated reflectivity image explains the total measured data, i.e. the residual goes below a pre-defined level of error tolerance or until a predefined maximum number of iteration is exceeded. In Table 5.1, the pseudo-code of the FWM inversion algorithm using the conjugate-gradient scheme is given.
**3.3 Numerical examples for structural imaging**

To illustrate the above mentioned least-squares inversion concept, we used a synthetic density model (as shown in Figure 3.2a) to simulate VSP data using a 2D acoustic finite difference algorithm. Note that for homogeneous velocity and varying density model, the reflectivities are scalar values, i.e. angle-independent [for a derivation see de Bruin et al., 1990]. The least-squares inversion was performed on the simulated VSP data to estimate the scalar reflectivity image. Figure 3.2b shows the empty image at the initialization of the conjugate gradient scheme; 3.2c shows the estimated reflectivity image after the 1\textsuperscript{st} iteration, which is equivalent to the image obtained from conventional imaging of upgoing primaries-only wavefields and Figure 3.2d shows the estimated reflectivity image after the 10\textsuperscript{th} iteration of FWM. Clearly, we note the extension of the illumination as well as resolution in the image using the full wavefield.
3.3 Numerical examples for structural imaging

Figure 3.2: a) Density model annotated with the VSP acquisition geometry, used to model VSP data for illustrating the FWM least-squares inversion concept. b) Zero reflectivity image to initialize the iterative gradient scheme. c) Reflectivity image estimated after the 1\textsuperscript{st} iteration, i.e. equivalent to the image computed using conventional imaging techniques using only the upgoing primary wavefield. d) Reflectivity image estimated after the 10\textsuperscript{th} iteration of FWM.

Further, Figure 3.3 shows the comparison of the true and the estimated data and the data misfit after these iterations. Specifically, Figure 3.3a, 3.3d and 3.3g shows an example common shot gather from the true data. Figure 3.3b, 3.3e and 3.3h shows the estimated data at initialization, after the 1\textsuperscript{st} iteration and after the 10\textsuperscript{th} iteration of FWM, respectively. Note that the estimated data in Figures 3.3b, 3.3e and 3.3h are modelled using the reflectivity shown in Figures 3.3b, 3.3c and 3.3d, respectively. Also, note that at initialization, we assume that the source wavefield is known and hence, the direct wavefield is estimated in the background medium. In practice, the source wavefield can be estimated in the VSP data with large accuracy, because we measure the direct wavefield transmitted in the true medium. Again, in FWM, each iteration utilizes a higher order of scattering to estimate the subsurface reflectivity and the data misfit is gradually reduced. Figure 3.3c, 3.3f and 3.3i shows the data misfit i.e. the difference between the true and estimated data at initialization, after the 1\textsuperscript{st} iteration and after the 10\textsuperscript{th} iteration, respectively.
Figure 3.3: a), d) and g) show an example true common shot gather; b), e) and h) show the estimated common shot gather at initialization (i.e. modelled using the reflectivity image shown in Figure 3.2b), after the 1\textsuperscript{st} iteration (i.e. modelled using the reflectivity image shown in Figure 3.2c) and after the 10\textsuperscript{th} iteration (i.e. modelled using the reflectivity image shown in Figure 3.2d), respectively; c), f) and i) show the corresponding data misfit. Note that at initialization of the iteration, the direct arrival is known, which means we assume that the source wavefield can be properly estimated.
3.4 Constrained least-squares inversion

In general, least-squares inversion gives a high-resolution image in FWM. However, in order to suppress some remaining imaging or extrapolation artefacts or crosstalk in the image, we could use a constraint in the inversion scheme. One example is using a sparsity promoting norm on the estimated reflectivity. Hence, the objective function for the constrained least-squares inversion can be written as:

\[
J = \sum_k \sum_\omega \| \vec{P}_{obs,k}(z_0) - \sum_{m=1}^{N} \vec{W}^{-}(z_0, z_m)[\vec{R}(z_m)\vec{P}_k(z_m) + \vec{S}_k^{-}(z_m)] \|^2_2 + \epsilon^2 F(\vec{R}),
\]

where, \(F(\vec{R})\) can be any sparsity-promoting norm imposed on the reflectivity e.g. the L1 norm [see Claerbout and Muir, 1973] or the Cauchy norm [see Amundsen, 1991; Sacchi et al., 1998] and \(\epsilon^2\) is a weighting parameter that governs the trade-off between the data misfit and the model priority usually depending on the noise content in the data. Note again, the subscript \(k\) is for the \(k^{th}\) source. For the numerical examples ahead, we have used the Cauchy norm that is defined as:

\[
F(\vec{R}) = \sum_n \sum_j \log(1 + \frac{R_{jj,n}^2}{\sigma_r^2}),
\]

where \(R_{jj,n}\) is a sample of the reflectivity image (i.e. a diagonal element from matrix \(\vec{R}(z_n)\)) and \(\sigma_r\) is the weighting parameter in the Cauchy norm. Similar to the unconstrained gradient, the gradient of the constrained objective function can be written as:

\[
\Delta \vec{R}^U(z_n) = \sum_k [\vec{W}^{-}(z_0, z_n)]^H \vec{E}_k(z_0)[\vec{P}_k^+(z_n)]^H + \epsilon^2 \sum_j \left( \frac{R_{jj,n}}{\sigma_r^2 + R_{jj,n}^2} \right).
\]

Again, the estimation of the reflectivity is solved using the iterative conjugate gradient scheme as discussed earlier.

To illustrate the inversion results using this constraint, we compare reflectivity images estimated using conventional imaging, constrained and unconstrained least-squares migration of primaries only wavefield, unconstrained least-squares FWM and the constrained least-squares FWM, for the same model shown in Figure 3.2a. Figure 3.4a shows the estimated reflectivity image after the 1\(st\) iteration, which is equivalent to the image obtained from conventional imaging of upgoing primary-only wavefields. Figure 3.4b shows the image obtained after the 10\(th\) iteration of least-squares migration of the primary-only wavefield. This is equivalent to what we expect in least-squares RTM. Figure 3.4d shows the estimated reflectivity image after the 10\(th\) iteration of FWM. Certainly, the least-squares migration
of primaries only wavefield (Figure 3.4b) does show improvement in illumination and amplitude compared to conventional primary-only migration (Figure 3.4a). However, note the extension of illumination area using multiples in migration as shown in Figures 3.4d. Also, note the effect of the sparsity constraint in 3.4d, yielding a sharper image and suppression of side lobes and artefacts. However, due to unoptimized inversion parameters for sparsity constraint, we also notice some energy at the edge of the image are suppressed in image obtained from constrained FWM, i.e. illumination extent get slightly worsen.

The next section illustrates the image analysis to compare the contribution of surface and internal multiples in the imaging.

### 3.5 Contribution of surface and internal multiples

In this section, we show an analysis using controlled FWM inversion. Controlled inversion in this case means that we control the use of multiple type, i.e. we either use surface multiples or internal multiples or both. In order to illustrate this analysis, we simulated walkaway VSP data on a reservoir-oriented density model using a 2D finite-difference algorithm shown in Figure 3.5. Figure 3.5a also shows the walkaway acquisition geometry schematically annotated on the model.

Now, for illustrating the contribution of different types of multiples, Figure 3.6a shows the image using the primaries-only wavefield, Figure 3.6b shows the image after the 10\textsuperscript{th} iteration of FWM using only the primaries and surface multiples (i.e. internal multiples are not used), Figure 3.6c shows the image after the 10\textsuperscript{th} iteration using only the primaries and internal multiples (i.e. surface multiples are not used) and Figure 3.6d shows the image after the 10\textsuperscript{th} iteration using the full wavefield, i.e. including primaries, surface multiples as well as internal multiples. The inversion parameters are kept the same for all results in Figure 3.6. It is interesting to note that neither the surface multiples alone nor the internal multiples alone can lead to a satisfactory solution.

Although, we observe an extension of the illumination, both in Figure 3.6b (using primaries + surface multiples) and 3.6c (using primaries + internal multiples), we see that the amplitudes of the reflectivity images are not accurate and the images suffer from crosstalk noise. In the aforementioned two cases, the inversion did not converge to a reasonable solution and also yield a reasonable high residual. However, when the full wavefield is used (Figure 3.6d), the inversion is more stable, and does converges towards an accurate and noise-free image. In addition, the effects of the sparsity constraint becomes clearly visible.

We have seen the principles of unconstrained and constrained least-squares inversion in full wavefield migration. In the next section, we discuss the angle-dependent reflectivity estimation for VSP data using FWM.
3.6 Angle-dependent full wavefield migration

In the previous sections, we have assumed that reflectivity can be described by a single scalar, being the diagonal elements of the $R$ matrix at each depth level. However, as discussed in chapter 2, in reality reflectivity has an angle-dependency, yielding also off-diagonal elements in $R$ to be non-zero [see also de Bruin et al., 1990]. Mathematically, we can formulate the estimation of angle-dependent...
Figure 3.5: Density model, annotated schematically with the VSP acquisition geometry. The dotted box shows the target area to image.

Figure 3.6: a) Target reflectivity image related to Figure 3.5 after the first FWM iteration, i.e. primaries-only image, comparable to the conventional imaging technique. b) Reflectivity image after the 10th iteration of constrained least-squares inversion using primaries + surface multiples. c) Reflectivity image after the 10th iteration of constrained least-squares inversion using primaries + internal multiples. d) Reflectivity image the after the 10th iteration of constrained least-squares inversion using primaries + all multiples.

reflectivity also as an optimization problem. If we bear in mind that the columns of the $R$ matrix transformed to the linear Radon domain provide the reflectivity as
3.6 Angle-dependent full wavefield migration

A function of ray-parameter (see also chapter 2), we can formulate the estimation of the AVP (amplitude variations as a function of ray parameter) image $A$ at all depth levels as minimization of the following objective function $J$ in a least-squares sense:

$$J = \sum_k \sum_\omega \| \vec{P}_{obs,k}(z_0) - \sum_{n=1}^{N} \mathbf{W}^{-}(z_0, z_n)[\mathbf{L} \mathbf{A}(z_n)\vec{P}_k(z_n) + \vec{S}_k^{-}(z_n)] \|_2^2,$$  (3.6.9)

where $\vec{P}_{obs,k}(z_0)$ is the observed data at the surface due to the $k^{th}$ source, $\mathbf{W}^-$ is the one-way propagation matrix in a background velocity model, where $\mathbf{L}$ is the inverse Radon transform given by Equation (B.2.13), $\mathbf{L} \mathbf{A} \vec{P}_k$ is the two-way scattered wavefield at all depth levels due to the $k^{th}$ source, and $\vec{S}_k^{-}$ is the upgoing source wavefield for the $k^{th}$ source. We solve this by an iterative optimization scheme, where the gradient of the objective function can be written as:

$$\Delta A^U(z_n) = -\sum_k [\mathbf{L}]^H [\mathbf{W}^{-}(z_0, z_n)]^H \vec{E}_k(z_0)[\vec{P}_k(z_n)]^H,$$  (3.6.10)

where $\vec{E}_k(z_0)$ is the data misfit for the $k^{th}$ source, given by:

$$\vec{E}_k = \vec{P}_{obs,k}(z_0) - \sum_{n=0}^{N} \mathbf{W}^{-}(z_0, z_n)[\mathbf{L} \mathbf{A}(z_n)\vec{P}_k(z_n) + \vec{S}_k^{-}(z_n)].$$  (3.6.11)

Note, in Equation 3.6.10, to apply de Bruin’s imaging condition [de Bruin et al., 1990], we have to use the summation over all frequencies of this frequency-ray parameter domain reflectivity matrix, which is equivalent of choosing time zero in the time domain. Thus, we can write the frequency-independent image gradient $\Delta A^U_0(z_n)$ as:

$$\Delta A^U_0(z_n) = -\sum_k [\mathbf{L}]^H [\mathbf{W}^{-}(z_0, z_n)]^H \vec{E}_k(z_0)[\vec{P}_k(z_n)]^H.$$  (3.6.12)

In addition to this, we have modified the gradient by adding the phase term for the post-critical phase variation (see Figure 2.9), which makes the reflectivity complex-valued. In this case, we have used a simple Hilbert transform operator $\mathcal{H}$ to obtain the real and imaginary part of the angle-dependent reflectivity, similar to one of the methods described in Zhu and McMechan [2011]. So the new complex gradient $\Delta A^U_c(z_n)$ can be defined as:

$$\Delta A^U_c(z_n) = [\Delta A^U(z_n)] + i\mathcal{H}[\Delta A^U(z_n)],$$  (3.6.13)

where the real part of the AVP function is the same as the amplitude at zero time and the imaginary part is the Hilbert transform of the real part (which is equivalent to a 90° phase shift). Note that the Hilbert transform is applied before
time zero is selected for the imaging condition. So, we can write the frequency independent complex-valued image gradient $\Delta A^{\cup,0}(z_n)$ as:

$$\Delta A^{\cup,0}(z_n) = -\sum_{\omega} [\Delta A^{\cup}(z_n)] + i \mathcal{H}[\Delta A^{\cup}(z_n)].$$  \hspace{1cm} (3.6.14)

Figure 3.7: a) Reflectivity in the tau-p (time-ray parameter) domain of a single grid point at the reflector as shown in Figure 2.8b. This is obtained by applying the linear Radon transform operator to convert the space-frequency data to the ray parameter-frequency data and inverse Fourier transform this to the time domain. b) Real part of the AVP function $\Delta A^{\cup}$, same as (a). c) Imaginary part of the AVP function, which is the Hilbert transform of the Real part $\mathcal{H}[\Delta A^{\cup}]$. d) and e) show the temporal trace selected from (b) and (c) at ray parameter=0 or for 0° angle. Note that the imaginary part or the Hilbert transform part has a 90° phase shift compared to the real part. f) and g) show the time zero amplitude variation with respect to the ray parameter. These are the real and imaginary part of the modified gradient after applying the imaging condition ($\Delta A^{\cup,0}$).

Figure 3.7 and 3.8 show the modified de Bruin’s imaging condition, as discussed above. We have used the same example from Figure 2.8 to illustrate the concept. Figure 3.7a shows the tau-p domain reflectivity that is obtained by applying a
3.7 Numerical examples for angle-dependent imaging

To illustrate the concept of the proposed least-squares AVP or angle-dependent reflectivity estimation using FWM, we have used two velocity models: case 1, as shown in Figure 3.9a,b with a positive high-impedance contrast at the top of the reservoir level and case 2, as shown in Figure 3.10c,d with a negative high-impedance contrast at the top of the reservoir level. It is well understood that the problem of post-critical reflection will be more pronounced in case 1, i.e. with a positive high-contrast impedance model. Note that for the two cases, we have used a constant density model of 1000 kg/m$^3$. The VSP data is simulated using
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Figure 3.8: a) Original reflectivity in the space-time domain for a grid point on the reflector after double-focusing of the source and receiver side; b) The reconstructed reflectivity in the space-time domain, after applying the inverse Radon transform to the selected time zero amplitude of the AVP function in the tau-p domain and c) the reconstructed reflectivity in space-time, after applying the inverse Radon transform to the selected time zero of the complex AVP function, i.e. sum of the real amplitude plus the imaginary Hilbert transformed AVP function in the tau-p domain.

a 2D acoustic finite difference method using the acquisition geometry as shown by the annotations in Figure 3.9a and 3.9c.

We performed iterative least-squares inversion using the FWM approach that includes the estimation of the AVP image for the two aforementioned cases. For case 1, where the model has a positive high-contrast impedance at the top of the reservoir, Figure 3.10 compares the estimated data and data misfit or residual after 25 iterations, when the inversion is parametrized to compute only the scalar reflectivity (i.e. angle-independent reflectivity), when the inversion is parametrized to compute the angle-dependent real AVP matrix and when the inversion is parameterized to compute the complex angle-dependent (real part of AVP plus the imaginary part being the Hilbert transform of the real part) AVP matrix, represented by $A_c$. A similar comparison is shown in Figure 3.11 for case 2, where the model has a negative high-contrast impedance at the top of the reservoir.

Next, Figures 3.12 and 3.13 show the AVP image of the three reflectors estimated using FWM for case 1 and case 2, respectively. Figures 3.12 and 3.13 show the real-valued AVP image after the 1st iteration, the real-valued AVP image after the 25th iteration when only the real AVP was estimated in the FWM inversion and the real and the imaginary part of the complex AVP image after 25 iterations, when the complex-valued AVP was estimated in the FWM inversion scheme. Also, Figure 3.14a and 3.14b show the normalized objective function with respect to iteration number for case 1 and case 2. The red curve represents the least-squares inversion when only the scalar $R$ was estimated, the blue curve represents the least-squares inversion when the real-valued AVP image was estimated and the black curve represents the least-squares inversion when the complex-valued AVP image was estimated.
3.7 Numerical examples for angle-dependent imaging

Figure 3.9: a) Case 1: 1.5D velocity model with positive high-impedance contrast at the top of the reservoir level, schematically annotated with the VSP acquisition geometry. b) Case 1: 1D velocity function profile. c) Case 2: 1.5D velocity model with negative high-impedance contrast at the top of the reservoir level, schematically annotated with the VSP acquisition geometry. d) Case 2: 1D velocity function profile.

We observe in Figure 3.10 and 3.14a that for case 1, the data misfit is significantly reduced when the complex-valued AVP image is estimated compared to when only the real-valued AVP image is estimated. However, in the data-misfit Figure 3.10h we still observe some remaining unexplained data at the very far offsets. This is due to limitation in aperture during wavefield extrapolation as well as due to stopping the inversion after 25 iterations for this test. On the other hand, as expected due to the negative velocity contrast, the difference in the data misfit is not so significant for case 2 as shown in Figure 3.11 and 3.14b for the two approaches in AVP imaging i.e. estimating the real-valued AVP only and estimating the complex-valued AVP. Also, when we compare the data misfit of the angle-dependent imaging with angle-independent imaging we do see a significant
difference, as expected in both cases, because the estimated angle-independent parameter is not good enough to explain all the data in the case of strong velocity or impedance contrasts. Note that the residual event around 1.5s in all data misfit plots is probably a modelling artefact from the lower edge of the model (due to sponge-boundary condition used in finite-difference scheme, see for example Cerjan et al. [1985]; Shin [1995] for details). This event should not be explained by FWM.

Also, from the AVP images in Figures 3.12 and 3.13, we see the influence of multiples in the illumination. If we just compare the illumination extent between AVP images after the 1\textsuperscript{st} and 25\textsuperscript{th} iteration, we clearly see the illumination is enlarged due to multiples. However, we also see some extrapolation noise at the
3.8 Discussion

In this chapter, we discussed the concept of full wavefield migration for application to VSP data. We observe how including all the multiples in the migration enhances the illumination as well as the resolution of VSP images. We illustrate the concept for both angle-dependent and angle-independent imaging cases. A sparseness constraint was introduced as a priori knowledge about the reflectivity edges after 25 iterations of FWM. This noise is due to the highly-underdetermined inversion problem, where the AVP image suffers from poor illumination. Please see Appendix B (Figure B.7), which illustrates angle gathers in AVP imaging using surface and VSP data. We can clearly see that even for correct velocity model used in imaging, the angle gathers shows artefacts at far angles especially away from the well. Further, if we compare the images from the real-valued AVP and complex-valued AVP estimation, the noise level is slightly lower in the latter case, which also shows the advantage of the complex-valued AVP parametrization.

**Figure 3.11:** Same as Figure 3.10, for case 2.
of the subsurface, which helps in reducing the extrapolation / migration artifacts as well as in improving convergence of the inversion. In the past, there are some other types of constraints also discussed in the literature to impose sparseness to the subsurface reflectivity, such as in Oldenburg et al. [1983] and Zhang and Castagna [2011].

Note that in this research, for 2D VSP data, we found that doing one iteration of modelling followed by one iteration of inversion in the feedback loop lead to
3.8 Discussion

a) b) c) d) e) f) g) h) i) j) k) l)

Figure 3.13: Same as Figure 3.12, for case 2.

converge the algorithm smoothly and yield a reasonably good result. However, for 3D surface seismic or OBC (Ocean-bottom cable) data, this strategy may not always work. Hence, for practical cases, one must test strategies like multiple iteration of modelling followed by one iteration of inversion or vice-versa.

Further, we have used one-way propagation operators which are tapered at very high angles. Therefore, using such operator have a dip-limitations in our current migration scheme. However, later in chapter 7, we have extended the scheme to incorporate turning waves using horizontal extrapolation operators. This extension help us to image steeply-dipping salt-flanks. In future research, we can also try to use full-wave equation extrapolation operator as described in Sandberg and Beylkin [2009]. Additionally, for land data with varying topography, propagation operators discussed in Al-Saleh et al. [2009] can also be incorporated.
For angle-dependent imaging, we have modified de Bruin’s imaging principle to address the phase variation in the inversion. Hence, the revised inversion parameters describe a complex-valued AVP image, where the imaginary part is defined by the Hilbert transform of the real part. This modified imaging condition does show potential to explain most of the data including the far-offset phase variations. In addition, we noticed that introducing multiples in FWM also leads to crosstalk and extrapolation noise, especially at the edges of the AVP images. Note that in the examples for angle-dependent imaging illustrated in this chapter, the imaging is performed only at the reflector levels. If we allow any AVP function to fit the data at any depth level, it may give too much freedom for an unconstrained least-squares inversion. In order to solve such issues, Wang and Sacchi [2007] discussed a constrained AVP inversion for primary-only wavefield, using a sparsity constraint along the depth axis and a smoothness constraint along the ray-parameter axis.

Again, we mention that we have restricted our discussion of angle-dependent reflectivities to horizontal reflector cases only. We can extend this analysis for dipping interface, by taking the dip of the structure in to account [de Bruin, 1992, see for example]. Although, as a parameter, we can estimate the angle-dependent reflectivity for a complex subsurface model, however, the interpretation of the estimates in terms of true incident angles and further AVO-type analysis(AVO - Amplitude Variation with respect to Offset, Castagna and Backus [1993]) and linearized inversions are non-trivial. Also, AVO/AVP inversion to obtain elastic properties of the subsurface uses various approximations of Zoeppritz equations. Zoeppritz equation [Aki and Richards, 1980] inherently defined for locally hori-
zontal reflectors.

In the next chapter, we will illustrate some more FWM for VSP data with more numerical examples and demonstrate its capabilities in obtaining structural images for different complex geological scenarios.
Synthetic examples: different case scenarios

4.1 Introduction

In this chapter, we will illustrate the potential of FWM through synthetic examples for various case scenarios. The latter sections include - a) examples of a complex reservoir geology with simple overburden and deviated well geometries, b) an example of imaging VSP data from multiple wells simultaneously, c) an example of a well passing through a complex overburden and d) finally, an example illustrating imaging of overburden while the receivers are located below the overburden. In addition, we will study the sensitivity of the FWM algorithm for an erroneous velocity field in migration.

4.2 Complex reservoir case

In this section we will illustrate the potential of FWM to image a complex reservoir scenario. We have used the well-known Marmousi density model to represent the reservoir structure, and assumed a constant velocity model, such that the reflectivities are scalars in the acoustic approximation. In this example, the overburden is assumed to be relatively simple, containing a vertical gradient only. We also demonstrate the flexibility of the FWM algorithm to handle any well geometry. Our aim is to image the complex Marmousi reservoir using either a vertical well geometry or deviated wells as shown in Figure 4.1a. In the latter
we distinguish deviated from left to right as shown in Figure 4.1b or deviated from right to left as shown in Figure 4.1c. The deviated wells extend laterally from 2000m to 4000m for the left to right situation and from 4000m to 2000m for the right to left situation. We assumed that the source wavelet is estimated from the direct wavefield recorded in the VSP data. The data is modelled using a 2D acoustic finite difference method for the three different well geometries. The FWM inversion scheme is applied to all three cases.

Figure 4.1: Marmousi reservoir model (density) annotated with different well geometries (in the reciprocal domain) i.e. a) vertical well, b) deviated well from left to right and c) deviated well from right to left.

Figure 4.2 shows images of the primaries-only image, i.e. the image obtained after the first iteration of the FWM scheme, and the image using all the multiples after 15 iterations of the FWM scheme. More specifically, Figure 4.2a, 4.2b and 4.2c show the images after the 1st iteration of FWM for the well geometries shown in Figure 4.1a, 4.1b and 4.1c, respectively. Note the poor illumination in the vertical well geometry. The image is improved slightly in terms of illumination extent in the case of the deviated wells. However, the images suffer from significant amounts of crosstalk and poor amplitude estimation after the 1st iteration. Figure 4.2d, 4.2e and 4.2f show the images after the 15th iteration of FWM for the well geometries shown in Figure 4.1a, 4.1b and 4.1c, respectively. We can clearly observe a drastic improvement in image quality both in terms of amplitude as
4.3 Multi-well case

In this section, we demonstrate the potential of FWM to perform simultaneous imaging of multiple well data, if available. Figure 4.3 shows a synthetic $P$-wave...
velocity model, a reservoir-oriented $S$-wave velocity model and a density model, annotated with two walkaway VSP acquisition geometries. The wells are 5000\textit{m} apart from each other. Walkaway VSP data were simulated using both an acoustic 2D finite difference scheme (i.e. keeping the $S$-wave velocity zero) and an elastic finite difference scheme.

Figure 4.3: a) Homogeneous $P$-wave velocity model; b) reservoir-oriented $S$-wave velocity model and c) density model, schematically annotated with two well geometries, laterally separated by 5000\textit{m}.

Figure 4.4 shows an example of a common-receiver gather simulated using an acoustic and an elastic finite difference method. More specifically, Figure 4.4a shows an example of a $P$-wave common-receiver gather simulated using an acoustic finite difference method, Figure 4.4b shows an example of a common-receiver gather simulated using an elastic finite difference method, and Figure 4.4c shows the difference between the two gathers. Note, the events annotated in circles 1 and 2. Circle 1 depicts the difference due to converted wavefields and circle 2...
shows the angle-dependent effects due to a space-variant $S$-wave velocity model, in the data simulated using an elastic finite difference method. For the elastic data, the $P$-wave gather is the true vertical-component ($z$ component) gather, after rotation correction.

![Graphs showing angle-dependent effects](image)

**Figure 4.4:** a) An example of a common-receiver gather simulated using an acoustic finite difference method, b) An example common-receiver gather simulated using an elastic finite difference method and c) their difference. The two annotations in Figure c) show 1) a converted wave and 2) differences due to angle-dependent effects caused by the $S$-wave velocity model.

The well data from the two wells are simultaneously fed into the FWM algorithm to perform acoustic imaging (to estimate $PP$-reflectivity). Figure 4.5 shows the reflectivity image using the primaries-only wavefield migration and the result after 10 iterations of FWM for both data simulated using an acoustic finite difference method and an elastic finite difference method. Again, as expected, we can observe that the information in-between the two wells is nicely retrieved using multiples in the imaging process. Note that in Figure 4.5d, although the information in-
between the two wells is retrieved using multiples, the image suffers from cross-talk noise due to converted waves present in the $P$-wave data. Also, note that the image shows a weaker amplitude as we go further away from the well due to the angle-dependent effects in the input data due to the $S$-wave velocities.

**Figure 4.5:** Images from the data simulated using an acoustic finite difference method where a) is the image obtained using primaries-only wavefields and b) is the image obtained using all multiples, after the $10^{th}$ iteration of FWM. Note that the information in between the wells is retrieved nicely using all the multiples. Images from the data simulated using an elastic finite difference method where c) is the image obtained using primaries-only wavefields and d) is the image obtained using all multiples, after the $10^{th}$ iteration of FWM. Note again that the information in between the wells is retrieved nicely using all the multiples, however, the image suffers from cross-talk noise due to converted waves present in the $P$-wave data. Also, note the image shows weaker amplitude as we go further away from the well for the case of elastic data. This is due to the angle-dependent effects in the input data due to the $S$-wave velocities.

### 4.4 Complex overburden case

The examples in the previous sections had a relatively simple or even homogeneous overburden. In this section, we have used a part of the Marmousi model as
the complex overburden model to simulate walkaway VSP data (shown in 4.6a). Figure 4.6b shows the true reflectivity model. In this scenario with a complex overburden, the reflectivity image estimated using the primaries-only upgoing wavefields is poor as expected, see Figure 4.6a. On the other hand, the image estimated using the iterative FWM including all multiples does improve the image quality and retrieved the information significantly better.

Figure 4.6: a) Marmousi density model used to simulate walkaway VSP data, annotated schematically with the well geometry and b) true reflectivity image. c) Reflectivity image estimated using the primaries-only wavefields and d) reflectivity image estimated after the 10\textsuperscript{th} iteration of FWM.

4.5 Imaging overburden using receivers below the overburden

The examples in the previous sections had receiver(s) extending all the way to the surface. In this section, we have illustrated the potential of FWM to image the overburden even if the entire receiver array lies below the overburden. Figure 4.7a shows the synthetic anticline density model, schematically annotated with the walkaway VSP geometry, showing that the receivers are located between 450m to 650m in depth. Sources are located at the surface.
Figure 4.7b shows the image obtained using primaries and all multiples (surface and internal multiples) after the $10^{th}$ iteration of FWM. Again, we can clearly observe that, even using a small receiver array, the multiples provide a high-resolution image of the whole subsurface, with an extended illumination. Despite, the receivers lying below the overburden, the downgoing multiples illuminate the overburden and hence, help in retrieving the overburden information.

![Figure 4.7a: Density model, annotated schematically with a walkaway VSP geometry showing the receiver array below the overburden, located between 450m to 650m in depth.](image)

![Figure 4.7b: Image obtained after the $10^{th}$ iteration of FWM. Note how using the multiples (surface and internal) help in imaging the overburden and yield a high-resolution image with extended illumination.](image)

**Figure 4.7:** a) Density model, annotated schematically with a walkaway VSP geometry showing the receiver array below the overburden, located between 450m to 650m in depth. b) Image obtained after the $10^{th}$ iteration of FWM. Note how using the multiples (surface and internal) help in imaging the overburden and yield a high-resolution image with extended illumination.

### 4.6 Velocity sensitivity test

This section illustrates the velocity sensitivity of images for VSP data imaging using FWM. For this test, a simple reservoir-oriented density model (shown in Figure 4.9a) is used to simulate data using acoustic finite-difference modelling. FWM was performed with a correct and a wrong velocity model. A velocity error is introduced in either the overburden or the reservoir or both. Figure 4.8 shows the 1D velocity profile for the correct velocity (represented by the dotted blue graph) and the migration velocity (represented by the red graph) used for migration of the VSP data using FWM. Specifically, Figure 4.8a shows the same correct and migration velocities. Figures 4.8b and 4.8c show respectively a $+5\%$ and a $-5\%$ error in the reservoir velocity used in migration, and Figures 4.8d and 4.8e show respectively a $+5\%$ and a $-5\%$ error in the total velocity (i.e. both in overburden and reservoir) used in migration.

Furthermore, Figure 4.9 shows images obtained using the different migration ve-
4.6 Velocity sensitivity test

Figure 4.8: 1D velocity profiles comparing the true velocity and the migration velocity. An error is introduced in the migration velocity to perform the velocity sensitivity test. a) Correct migration velocity, b) a +5% error introduced in the reservoir velocity for migration, c) a −5% error introduced in the reservoir velocity for migration, d) a +5% error introduced in the total velocity (in both overburden and reservoir) for migration and d) a −5% error introduced in the total velocity (in both overburden and reservoir) for migration.

Locities. Figure 4.9 shows the image after the 1st and the 10th iteration of FWM. Specifically, Figures 4.9b, 4.9d, 4.9f, 4.9h and 4.9j show images obtained using the correct migration velocity (shown in Figure 4.8a), using a +5% error in the reservoir velocity (shown in Figure 4.8b), using a −5% error in the reservoir velocity (shown in Figure 4.8c), using a +5% error in the total velocity (shown in Figure 4.8d) and using a −5% error in the total velocity (shown in Figure 4.8e), after the 1st iteration of FWM, respectively. Similarly, Figures 4.9c, 4.9e, 4.9g, 4.9i and 4.9k shows images obtained using true migration velocity (shown in Figure 4.8a), using a +5% error in the reservoir velocity (shown in Figure 4.8b), using a −5% error in the reservoir velocity (shown in Figure 4.8c), using a +5% error in the total velocity (shown in Figure 4.8d) and using a −5% error in the total velocity (shown in Figure 4.8e), after the 10th iteration of FWM, respectively.

Note how an erroneous velocity model in migration causes the errors in the struc-
Figure 4.9: Velocity sensitivity test: a) reservoir-oriented density model, b), d), f), h) and j) image obtained using the correct migration velocity (shown in Figure 4.8a), using a +5% error in the reservoir velocity (shown in Figure 4.8b), using a −5% error in the reservoir velocity (shown in Figure 4.8c), using a +5% error in the total velocity (shown in Figure 4.8d) and using a −5% error in the total velocity (shown in Figure 4.8e), after the 1st iteration of FWM, respectively. c), e), g), i) and k) same as b), d), f), h) and j), show respectively the results for the same migration velocity as b)-j), but now after the 10th iteration of FWM.

tures after imaging. As expected, the image suffers from larger depth errors in the areas far from the well trajectory. This is due to the VSP acquisition geometry, where the wavefields that explain the subsurface image far away from the
well travels further compared to wavefields closer to the well. Also, note how the overburden velocity is important in obtaining a reasonable image. Errors in overburden velocity causes pronounced depth-errors in imaging.

Figure 4.10 shows the data residual or data misfit for the various velocity models. Although, the imaging of VSP data is sensitive to velocity error, the inversion based imaging scheme in FWM helps in assessing the possible error in the model by inspection of the data misfit or data residual.

![Figure 4.10](image)

**Figure 4.10:** a) An example of a common-receiver gather from observed data. Data misfit after the 10th iteration of FWM when migration is performed using b) the correct velocity, c) a +5% error in the total velocity field and c) a −5% error in the total velocity field.

This is a key aspects towards using 'Joint-Migration Inversion' (JMI), also in VSP data. JMI is a multi-parameter inversion scheme that aims to estimate both the reflectivity as well as the macro-velocity model for migration simultaneously. For theoretical details as well as application of JMI in surface seismic data, the readers are referred to Berkhout [2012], Staal and Verschuur [2012] and Staal and Verschuur [2013].
4.7 Discussion

In this chapter, we presented several numerical examples of imaging for walkaway VSP data using FWM. As discussed in chapter three, we clearly note the extension of illumination using all multiples in imaging. Note that in all examples, we assume to have a dense source sampling at the surface. In today’s industrial practice, 3D VSP data are acquired using such a dense source sampling at the surface. Note also that the sensitivity to a velocity error in the migration algorithm is higher as we move further away from the well trajectory. We propose to incorporate the simultaneous imaging and velocity model update using JMI for VSP data.

In the next chapter, we will discuss the application of FWM for blended VSP data.
5

Application of full wavefield migration to blended VSP data

5.1 Introduction

In the previous chapters, we introduced full wavefield modelling and full wavefield migration (FWM) for walkaway VSP data. In this chapter, we will present the application of FWM in the case of blended source VSP acquisition. In blended source (also called simultaneous source) experiments, more than one shot are fired simultaneously or with a delayed time. The sources may vary in spatial location as well as source strength. Blended source acquisition [Beasley et al., 1998; Ikelle, 2007; Berkhout, 2008; Neelamani et al., 2010] in the surface seismic case is slowly becoming a routine practice in the oil and gas industry. Blending in surface seismic has made huge 3D surveys possible in an economical survey time. It has proven to improve both the quality as well as economic aspects. It reduces the costs of data acquisition and survey time, while still acquiring a dense survey [see for example Berkhout, 2008; Howe et al., 2008; Bouska, 2010; Berkhout et al., 2012; Doulgeris, 2013]. The word 'quality' here indicate the illumination capability of blended source arrays compared to unblended source. For a theoretical discussion, please see Berkhout et al. [2012], where it is described that even for a very simple blending code, e.g., time delays only, the incident wavefield at a particular subsurface grid point is represented by a dispersed time series, corresponding to a complex code. This time series is grid point dependent and contains multi-offset, multi-azimuth information and therefore enhances grid point illumination.
Recently, Gulati et al. [2011] proposed acquiring 3D VSP data using simultaneous sources to reduce the borehole acquisition costs significantly. Note that VSP acquisition is relatively costly because of the fact that all activities need to be stopped (strictly production). Thus, reducing downtime via blended acquisition is of great importance. Nawaz and Borland [2013] discussed the processing sequence for simultaneous source 3D VSP data. In a similar way, Morley [2013] discussed the application of compressed sensing\(^1\) in 3D VSP acquisition and processing.

For the processing, imaging and (full waveform) inversion of the blended data, we have seen methods of deblending sources from the acquired seismic data [for example see Spitz et al., 2008; Moore et al., 2008; Kim et al., 2009; Mahdad et al., 2011, 2012; Beasley et al., 2012; van Borselen et al., 2012; Wapenaar et al., 2012], the result of which can be fed into conventional processing and imaging methods. On the other hand, there have been investigations in performing processing [see Hou et al., 2012; Bagaini et al., 2012], imaging [see Verschuur and Berkhout, 2009; Tang and Biondi, 2009; Jiang and Abma, 2010; Verschuur and Berkhout, 2011; Berkhout et al., 2012; Dai et al., 2012; Huang and Schuster, 2012] and inversion [see Guitton and Diaz, 2012; Choi and Alkhalifah, 2012; Plessix et al., 2012] of the blended seismic data directly without separating the sources or deblending them.

Along this latter approach, we propose full wavefield migration (FWM) to image blended VSP data [Soni and Verschuur, 2013b]. Hence, the least-squares inversion process in FWM can help in estimating the subsurface reflectivity such that it explains the total blended data. In this chapter we describe the extension of the forward modelling algorithm to include blended VSP data. Furthermore, we discuss the potential of FWM in imaging blended VSP data by illustrating some examples using a density-only, synthetic dipping-layer model as well as reservoir-oriented modified Marmousi model. Indeed, we will notice some crosstalk noise in the image due to wavefield interference for blending with high blending factors. However, the blending leakage noise can be reduced by using a constrained least-squares inversion scheme.

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\(^1\)Compressed sampling is an alternative subsampling method, different from blending. In compressed sensing, randomized sub-Nyquist sampling is used to capture the structure of the data with the assumption that it is sparse or compressible in some transform domain, such as curvelet domain. For more details on this subject, the readers are referred to Candès and Tao [2006]; Donoho [2006]; Hermann et al. [2012]; Mansour et al. [2012]; Herrmann and Li [2012]. For a discussion on recovery conditions from compressed sensing measurements, see for example Friedlander et al. [2012]. Further discussion on compressed sensing is beyond the scope of this thesis.
5.2 Full wavefield modelling of blended VSP data

To formulate the iterative forward modelling of blended VSP data, we will use similar expressions as in chapter two. In the case of a blended source experiment, we can define the modelling either in its true source-receiver domain or in the reciprocal domain. Let us first define the modelling in the true source-receiver domain. The iterative full wavefield modelling can be formulated in terms of iterative modelling of the total incident wavefields $\vec{P}_{bl}(z_n)$ recursively for all depth levels. The subscript 'bl' represents a blended experiment. The total incident wavefields $\vec{P}_{bl}(z_n)$ comprise a downgoing incident wavefield $\vec{P}^+_{bl}(z_n)$ and an upgoing incident wavefield $\vec{P}^-_{bl}(z_n)$ at depth level $z_n$. In the modelling scheme, the first step involves the modelling of the direct downgoing wavefields at all depth levels due to blended source $\vec{S}_{bl}(z_0)$ located at the surface. Thus, the downgoing incident wavefield $\vec{P}^+_{bl}(z_n)$, in the first iteration is given by:

$$[\vec{P}^+_{bl}(z_n)]^{(1)} = \sum_{m=0}^{n-1} W^+(z_n, z_m) \vec{S}_{bl}(z_0), \quad (5.2.1)$$

where the superscript $(1)$ indicates the iteration number. Similar as discussed in chapter two, mathematically, the incident wavefield from above, i.e. $\vec{P}^+_{bl}(z_n)$, and the one from below, i.e. $\vec{P}^-_{bl}(z_n)$, for a given iteration $i$ can be written as:

$$\vec{P}^+_{bl}(z_n)^{(i)} = \sum_{m=0}^{n-1} W^+(z_n, z_m)[\delta \vec{P}_{bl}(z_m)^{(i-1)} + \vec{S}_{bl}(z_0)], \quad (5.2.2)$$

$$\vec{P}^-_{bl}(z_n)^{(i)} = \sum_{m=n+1}^N W^-(z_n, z_m)\delta \vec{P}_{bl}(z_m)^{(i-1)}, \quad (5.2.3)$$

where the two-way scattered wavefield $\delta \vec{P}_{bl}(z_n)$ can be written as:

$$\delta \vec{P}_{bl}(z_n) = \begin{bmatrix} R^+(z_n) & R^-(z_n) \end{bmatrix} \begin{bmatrix} \vec{P}^+_{bl}(z_n) \\ \vec{P}^-_{bl}(z_n) \end{bmatrix} = R(z_n) \vec{P}_{bl}(z_n). \quad (5.2.4)$$

Here, the blended source vector $\vec{S}^+_{bl}(z_0)$ can be defined using the complete or full source matrix at the surface $S(z_0)$ and a blending operator $\vec{\Gamma}_{bl}(z_0)$ [see Berkhout, 2008] as:

$$\vec{S}^+_{bl}(z_0) = S^+(z_0) \vec{\Gamma}_{bl}(z_0), \quad (5.2.5)$$

where the blending operator $\vec{\Gamma}_{bl}(z_0)$ can be written as $\vec{\Gamma}_{bl}(z_0) = [\gamma_1, \gamma_2, \gamma_3, \ldots, \gamma_N]$, with $\gamma_n = a_n e^{-j\omega T_n}$. In this case, $T_n$ is a random time-shift applied to blend the sources and $a_n$ is a scale factor that can be $a_n = 0$ for those sources not included in the blended experiment. We will use the term 'blending factor' to define the number of shots blended together i.e. number of $a_n \neq 0$. We will use the term
'blending factor ($N_{bl}$)' to define the number of shots added together to make one blended source experiment. In other words, the blending factor in this paper is defined as the ratio between the number of sources in the unblended survey and the number of blended records in the blended survey. In the numerical blending, we have designed the operator such that it applies a random time-shift to all the conventional sources at the surface and then add the regularly sampled sources in space at a distance separated by $\Delta x_{bls}$ to yield the blended survey, where $\Delta x_{bls}$ is given by:

$$\Delta x_{bls} = \frac{n_s}{N_{bl}} \times \Delta x$$

(5.2.6)

The maximum randomized time-shift allowed in the blending process do not exceed ±0.1s. Figure 5.1 shows an example of the blending operator for blending factors two, three and four.

![Figure 5.1](image)

**Figure 5.1:** An example of the blending operator depicting a random time-shift which is applied to a regularly separated sources in space (separated by a distance of $\Delta x_{bls}$) and then summed to obtain blended source arrays, for blending factors a) two, b) three and c) four. In these figures, the colour of the elements of the matrices represent the time-shift applied to different sources in the blending process, which is between −0.1s to 0.1s. Also note that $\Delta x_{bls}$ only shows the lateral distance between the sources in a schematic way and does not comply with the axis of these figures, which indicate the source numbers.

We have defined the incident wavefields at all depth levels due to a blended source experiment located at the surface. Once the blended source experiment is defined, the full wavefield modelling scheme is similar to the unblended one [Berkhout and Verschuur, 2011; Davydenko and Verschuur, 2012]. In the iterative modelling
5.3 Numerical examples of the forward modelling

scheme, the upgoing wavefield at the surface $\vec{P}^{-}_{bl}(z_0)$ is equivalent to the modelled surface seismic data. Now, for walkaway VSP data modelling, we can select the upgoing and downgoing wavefields for all the spatial locations i.e. both lateral and vertical location where the receivers are present. The modelled VSP data is obtained by taking the sum of the upgoing and downgoing wavefields measured in the borehole receivers.

Above, we discussed the modelling for a blended source experiment in the true source-receiver domain. As mentioned earlier, we can also define the modelling in the reciprocal domain, which gives an equivalent blended data. In the inversion scheme, we perform modelling in the reciprocal domain (discussed in detail in later sections). The important point to note here is that in the reciprocal domain, we assume unblended sources in the borehole (all corresponding to true receiver locations) and estimate the unblended modelled data for receivers located on the surface (same as described in chapter two). After each iteration, we perform the receiver-side blending to obtain the equivalent blended data. Note that the receiver-side blending can be done using the transpose of the same operator used in data acquisition.

In data matrix notations (for one frequency component), $\mathbf{P}_{vsp}$ and $\mathbf{P}_{vsp,bl}$ represents unblended and blended VSP data, respectively. A column-vector and a row-vector of these matrices represents a common-source gather and a common-receiver gather, respectively. In the reciprocal domain, the data matrices are represented by $\mathbf{P}^{T}_{vsp}$ and $\mathbf{P}^{T}_{vsp,bl}$ (transpose of the original matrices), where a column-vector and a row-vector of the matrices now represents a common-receiver gather and a common-source gather, respectively. Furthermore, in terms of matrix multiplication, blending operator $\Gamma_{bl}$ when act on the right side of the data matrix $\mathbf{P}_{vsp}$, it is equivalent to source-side blending. While, in the reciprocal domain, when the transposed blending operator $\Gamma_{bl}^{T}$ act on the left side of the reciprocal domain data matrix $\mathbf{P}^{T}_{vsp}$, it indicates the receiver-side blending [Soni and Verschuur, 2014b]. Figure 5.2 schematically illustrates this relationship. Please see the caption for specifications.

5.3 Numerical examples of the forward modelling

In order to illustrate the modelling scheme, we used a synthetic density model (with constant velocity, same as used in chapter two, shown in Figure 2.10). Figure 5.3 shows the density model and the corresponding scalar reflectivity model used to illustrate full wavefield modelling for blended source experiments. To illustrate, the models are annotated schematically with a blended source experiment located at the surface and the receivers being located in a borehole. For the conventional (unblended) acquisition geometry, the sources are located at the surface between 0m to 3000m, laterally, at a spacing of 20m (i.e. we have 151
Figure 5.2: Schematic diagram showing the equivalence of a) source-side blending $P_{vsp,bl} = P_{vsp} \Gamma_{bl}$ for original surface-source walkaway VSP data with b) receiver-side blending $\Gamma_{bl}^{T} P_{vsp}^{T}$, for walkaway VSP data in the reciprocal domain. The matrix-multiplication are depicted schematically, where $n_s$ denotes the number of unblended surface-sources, $n_r$ denotes the number of borehole receivers, $n_{bls}$ denotes the number of blended surface-sources for the original experiment. The circled shots in a) and receivers in b) represents schematically, blending factor of two, in this diagram.

shots for an unblended geometry. Using the blending operator (as shown in Figure 5.1) the data is blended for blending factors two, three and four. Hence, the resulting number of shots for blending factor two, three and four are 76, 51 and 38, respectively. The receivers are located in the borehole, between a depth of 100m to 1000m, at a depth spacing of 10m. We will show the modelled direct source wavefields after the 1st iteration for a horizontal depth of 700m and as recorded at the receivers located in the vertical borehole for different blending factors.

Figures 5.4, 5.5, 5.6 and 5.7 show the modelled direct source wavefields due to a blended source located at the surface for blending factors one, two, three and four, respectively. Specifically, Figures 5.4a, 5.5a, 5.6a and 5.7a show the reflectivity model, schematically annotated with a blended source at the surface and receivers in the borehole, for blending factors one, two, three and four, respectively.
5.3 Numerical examples of the forward modelling

Figure 5.3: a) Density model and b) corresponding scalar reflectivity model (same as Figure 2.10) used to illustrate the full wavefield modelling scheme for a blended source VSP. In a) and b) the models are annotated schematically with a VSP geometry, showing an example blended source at the surface with random time-delays and receivers being located in the borehole. In the later figures, the direct wavefield due to this blended source experiment is illustrated for a depth of 700m, shown by the black dotted line. Note that the sources being located at the surface and the receivers being located at the borehole. The varying depth of the sources as shown schematically in these figures are only to indicate that they are fired at different time.

5.4b, 5.5b, 5.6b and 5.7b show the direct wavefield at a depth of 700m (shown by dotted black line on reflectivity model), and Figures 5.4c, 5.5c, 5.6c and 5.7c show the direct wavefield recorded at the receivers located in the borehole at a lateral location of 1500m and a depth of 100m to 1000m, for blending factors one, two, three and four, respectively. Note that a blending factor of one means unblended data. Also, note again that the direct wavefield at a horizontal depth level is computed by equation 5.2.1, while the the direct wavefield at the vertical receiver depths are evaluated by selecting the direct wavefields at all depth levels for the lateral location of 1500m for depths of 100m to 1000m. The borehole receiver depth interval is assumed to be 10m, which is the same as the wavefield extrapolation depth interval for this illustration.

Furthermore, Figures 5.8, 5.9 and 5.10 illustrate the full wavefield modelling for the blended data with blending factor three after the 1st, 2nd and 3rd iteration, respectively. Specifically, Figures 5.8a, 5.9a and 5.10a show the downgoing wavefields, Figures 5.8b, 5.9b and 5.10b show the upgoing wavefields and Figures 5.8c, 5.9c and 5.10c shows the total (sum of downgoing and upgoing wavefields) wavefields, recorded by the receivers located in the vertical borehole after the the 1st, 2nd and 3rd iteration, respectively. Also, Figures 5.8d, 5.9d and 5.10d show the upgoing wavefield at the surface i.e. surface seismic profile data for blending fac-
tor three, after the $1^{st}$, $2^{nd}$ and $3^{rd}$ iteration, respectively. Again, note that each iteration in the full wavefield modelling adds one higher order of scattering. Also, note that due to the blended sources, the wavefield becomes very complex and incoherent due to the interfering events.

In this section, we discussed incorporating blended source for the VSP geometry in the full wavefield forward modelling engine. Previously, in chapter two, we have defined the full wavefield modelling algorithm for a VSP geometry as analogous to a special case of surface seismic profiling, where the sources are buried in the subsurface. The buried sources were equivalent to the receivers located in the borehole in the reciprocal domain. In chapter three, we discussed the inversion scheme in full wavefield migration in the common-receiver domain.

In this section, we have defined the full wavefield modelling for a VSP geometry using blended sources in its true domain, which is again similar to the modelling of surface seismic profiling. Considering the special case for VSP, here we select the upgoing and downgoing wavefields at the known receiver’s spatial locations i.e. both laterally and in depth. In the next section, we will discuss the full wavefield migration for blended VSP data. We propose to perform the migration in the common-receiver domain to make the algorithm work efficiently as well as to achieve a significant improvement in the illumination of the image. This is because when we perform the migration in common-shot domain, the propagation operators are defined for wavefield from a horizontal depth level to the vertical
5.3 Numerical examples of the forward modelling

Figure 5.5: a) Reflectivity model, schematically showing a blended source, with blending factor=2, located at the surface and receivers being located in the borehole. Modelled direct source wavefield b) at a horizontal level at a depth of 700m (shown by the dotted line in (a)) and c) at the receivers located in the vertical borehole, due to a blended source of blending factor=2, located at the surface.

Figure 5.6: a) Reflectivity model; direct source wavefield at b) depth 700m and c) in the vertical borehole. Similar as Figure 5.5, but with blending factor=3.

plane at the well, and it often suffer from dip-limitations. Therefore, the common-shot migration is not an effective way to image blended VSP data. Further, note again that in the actual inversion scheme, we perform modelling in the reciprocal
Figure 5.7: a) Reflectivity model; direct source wavefield at b) depth 700m and c) in the vertical borehole. Similar as Figure 5.5, but with blending factor=4.

Figure 5.8: An example of a common-shot gather for a blended source experiment with blending factor=3, after the 1st iteration showing a) the downgoing wavefield, b) the upgoing wavefield and c) the total wavefield for the VSP geometry and d) the upgoing wavefield recorded at the surface for the surface seismic geometry.
5.3 Numerical examples of the forward modelling

**Figure 5.9:** Same as Figure 5.8 after the 2\textsuperscript{nd} iteration.

**Figure 5.10:** Same as Figure 5.8 after the 3\textsuperscript{rd} iteration.

domain and blend the results afterwards, which gives equivalent blended data.
5.4 Full wavefield migration of blended VSP data

In the previous section we discussed how to incorporate the blended source experiments to model VSP data in the true source domain, followed by selection of the total wavefields at the known receiver’s spatial locations. However, we propose to perform the imaging in the common-receiver domain, similar as for the unblended VSP data, that was discussed extensively in chapter three. For the blended VSP data, it is impossible to perform the imaging in the common-receiver domain, because the common-receiver gathers show random events [see also Mahdad, 2012; Doulgeris, 2013]. Therefore, to circumvent this problem, pseudo-blending is included as part of the imaging scheme. We believe that doing the full deblending (active deblending) followed by FWM will give a similar result in quality, compared to simultaneous deblending (passive deblending) and migration in FWM. For conventional migration, this has been discussed in Gulati et al. [2011]. But these strategies are still a subject of further research. The active deblending becomes more and more difficult with increasing blending factors due to large null-space or leakage subspace. For detailed discussion on this, please see Doulgeris et al. [2012]; Doulgeris [2013]. On the other hand, because imaging the blended data is also highly-underdetermined problem and suffer from a large null-space, therefore a constrained least-squares inversion based imaging algorithm can help in reducing the blending crosstalk, and hence reducing the null-space of the problem. Therefore, we can say that simultaneous deblending and imaging makes the algorithm more efficient and robust. In other words, the FWM scheme actually acts as a deblending algorithm.

Figure 5.11 shows a generalized block diagram for the inversion scheme in FWM for blended VSP data. This is similar to Figure 3.1 with two intermediate steps added, being pseudo-deblending of the data residual and blending of the estimated data.

Again, the migration is performed as a feedback process. In order to perform the migration in the common-receiver domain, we need to apply an intermediate step of pseudo-deblending to the data residual in the feedback loop. Note that the intermediate pseudo-deblended is imaged, yielding a subsurface reflectivity that is used to simulate the response using full wavefield modeling (in the reciprocal domain), which is subsequently blended by the same blending operator used in data acquisition. The estimated blended data is compared with the measured blended data. The residual of the measured and simulated data, after

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2 Passive deblending: Passive deblending is same as pseudo-deblending. Pseudo-deblending is basically finding the generalized inverse of the blending matrix.

3 Null-space or leakage subspace: In leakage subspace, the vectors corresponds to energy that is coherent for more than one source contributing to the same blended experiment and therefore, this energy cannot be assigned uniquely to one particular source. For a geometrical interpretation of data and model null-space in linear algebra, see Strang [2003]
5.4 Full wavefield migration of blended VSP data

Figure 5.11: Block diagram (similar to Figure 3.1), FWM for blended VSP data, showing the general feedback loop for inversion in the common-receiver domain. The intermediate pseudo-deblended data is imaged, yielding subsurface reflectivity. The estimated reflectivity is used to simulate the response using full wavefield modelling (in the reciprocal domain) and then blended by the same blending operator used in data acquisition. The blended estimated data is compared with the measured blended data. The residual of the measured and simulated data after adaptive subtraction is fed back in the loop to update the reflectivity iteratively. Each iteration adds or uses a higher order of multiples.

adaptive subtraction, is fed back in the loop to update the reflectivity iteratively. Note that in FWM, each iteration adds and uses a higher order of multiples. Since, the migration is performed as a feedback process, the first iteration is similar to conventional imaging of the primary wavefields. Next, each iteration of FWM involves an iteration of full wavefield modeling, and hence, adds or uses a higher order of multiples to estimate the reflectivity. With subsequent iterations of FWM, the image becomes more accurate and sharper, i.e. the vertical resolution increases and the full-wavefield is better explained. This is the same as in other least-squares imaging schemes: with subsequent iterations, the estimated reflectivity converges to a reasonable solution. However, in addition, in FWM, with each additional iteration also the higher-order scattering effects in the data are explained. Hence, the first iteration is equivalent to conventional imaging of the primary wavefields of pseudo-deblended data containing blending interference noise.

Pseudo-deblending is basically finding the generalized inverse of the blending matrix [see also Mahdad, 2012; Doulgeris, 2013]. Mathematically, if the blended VSP data matrix, for one frequency component is written as:

$$P_{vsp,bl} = P_{vsp} \Gamma_{bl},$$

(5.4.7)
then we can estimate the pseudo-deblended VSP data $P_{vsp,pdbl}$ by solving the minimum norm or least-squares solution for equation above as:

$$P_{vsp,pdbl} = P_{vsp,bl} \left[\Gamma_{bl}^H \Gamma_{bl}\right]^{-1} \Gamma_{bl}^H.$$  \hspace{1cm} (5.4.8)

The subscripts ‘bl’ and ‘pdbl’ refer to the blended and pseudo-deblended VSP data, respectively. For our examples, the matrix $[\Gamma_{bl}^H \Gamma_{bl}] = \frac{1}{b} I$, where $b$ is the number of blended sources. Hence the pseudo-deblending in these examples reduces to:

$$P_{vsp,pdbl} = \frac{1}{b} P_{vsp,bl} \Gamma_{bl}^H.$$  \hspace{1cm} (5.4.9)

Figure 5.12 shows an example of a common shot gather and a common-receiver gather from pseudo-deblended VSP data, with blending factor of 3. Note that in the common-shot domain, the signal and the interference noise due to blending are not well separated. However, in the common-receiver domain, the interference noise spreads out as random noise, with the random time-shifts associated to each blended source experiment.

**Figure 5.12:** An example a) a common-shot gather and b) a common-receiver gather from pseudo-deblended VSP data, with blending factor=3.

In the inversion scheme for blended VSP data, we need to evaluate the source wavefields. As we discussed in chapter three, the source wavefield can be easily and accurately estimated from the direct arrivals in the unblended VSP data case. However, in the case of blended VSP data, estimating the source wavelet directly from the blended gathers is not straightforward. To illustrate this fact,
5.4 Full wavefield migration of blended VSP data

Figures 5.13 and 5.14 show the direct wavefield (they are modelled separately) from the data for an example common-shot and a common-receiver gather, respectively. From Figure 5.13, we can observe that picking the direct arrivals for the blended source experiment might not be trivial in the common-shot domain due to interference of different events. Further, from Figure 5.14, we observe that it is not possible to extract the direct arrival in the common-receiver domain because the events are randomized after blending. On the other hand, Figure 5.15 shows the direct wavefield picked from the pseudo-deblended data in an example of a common-receiver gather. The direct arrival is comparatively easier to pick in this domain as the blending noise gets spread out as random noise and the direct-arrival stands out as a coherent event.

![Figure 5.13: An example of blended VSP data in the common-shot domain, with blending factor 3, showing a) the full wavefield, b) the direct arrival and c) the difference wavefield, i.e. wavefield without the direct arrivals. Note that although the shown direct arrival is accurate here, picking the blended direct arrival in common-shot domain may not always be accurate step due to interfering events.](image)

Hence, the source wavefields are estimated from the pseudo-deblended data by picking direct arrival in the common-receiver domain. This gives the equivalent source wavefield due to buried sources in the reciprocal domain. Therefore, the full wavefield modelling is also performed in the reciprocal domain in order to address the amplitude difference of the estimated source wavefield due to pseudo-deblending. Note that this amplitude changes are taken care of in the inversion scheme, by iteratively fitting the estimated blended data to the measured blended data.
Figure 5.14: An example of blended VSP data in the common-receiver domain, with blending factor 3, showing a) the full wavefield, b) the direct arrival and c) the difference wavefield. Note that, although the shown direct arrival is accurate here (picked in common-shot domain), picking the blended direct arrival in the common-receiver domain is impossible due to the randomized events.

Figure 5.15: An example of pseudo-deblended VSP data in the common-receiver domain, with blending factor 3, showing the a) the full wavefield, b) the direct arrival and c) the difference wavefield. Note that picking the direct arrival in the pseudo-deblended common-receiver domain is easier and more accurate than estimating the source wavefield in the reciprocal domain.
Furthermore, similar as discussed in chapter three, the reflectivity image obtained in the first iteration is used to simulate the response using full wavefield modelling in the reciprocal domain. The simulated data gives the wavefields for the equivalent receivers at the surface in the reciprocal domain, where the locations resemble the location of unblended sources in the observed data. In order to compare the simulated data to the blended data, we apply blending to the simulated data using the same operator as used in data acquisition. Please note here that the modelling scheme discussed in the previous sections for the blended sources at the surface for true domain VSP data is equivalent to the two steps in this inversion block diagram. They are the full wavefield modelling in the reciprocal domain to simulate unblended data at the surface (which is geometrically equivalent to the true unblended data in the common-receiver domain) plus equivalent receiver-blending in the reciprocal domain. Figure 5.2 illustrated this relationship using the simple matrix identity, where Figure 5.2a schematically shows source-side blending to get a blended VSP data matrix and Figure 5.2b shows that the transpose of equation (5.4.7) is also true, i.e. it is equivalent to receiver-side blending for common-receiver data.

As Figure 5.11 shows, the residual after least-squares subtraction or adaptive subtraction of the simulated blended data and the measured blended data are subjected to pseudo-deblending, after which it is used to update the reflectivity, iteratively, in such a way that the data residual is minimized. Again, note that each iteration of FWM involves a round-trip iteration of full wavefield modelling, and hence, uses a higher order of multiples to estimate the reflectivity.

Similar to the optimization scheme discussed in chapter three, imaging blended VSP data using FWM is also defined as an optimization problem, where the simulation of the data is compared to the measured or observed data. We can write the objective function $J$ to minimize in a least-squares sense as:

$$J = \min \sum_k \sum_\omega \| \vec{P}_{bl,obs,k} - \vec{P}_{bl,est,k} \|^2_2, \quad (5.4.10)$$

where $\vec{P}_{bl,obs,k}(z_0)$ and $\vec{P}_{bl,est,k}(z_0)$ are the observed and estimated blended VSP data due to the $k^{th}$ blended source term. Note that the estimated blended VSP data is a function of the reflectivity matrix $\mathbf{R}$. Basically, to simulate data using full wavefield modelling at the surface due to the $l^{th}$ source $\tilde{S}_l$ in the borehole in the reciprocal domain can be written as:

$$\vec{P}_l^{-}(z_0) = \sum_{m=0}^{\infty} \mathbf{W}^{-}(z_0, z_m) [\mathbf{R}(z_j) \vec{P}_l(z_m) + \tilde{S}_l^{-}(z_m)]. \quad (5.4.11)$$

After modelling of the data in the reciprocal domain, the receiver-side blending is performed in the common-receiver domain as:

$$\vec{P}^T_{vsp,bl} = \Gamma_{bl}^T \vec{P}^T_{vsp}. \quad (5.4.12)$$
The $k^{th}$ row of the data matrix $P_{vsp,bl}^T$ is equivalent to the estimated blended common-source gather $\vec{P}_{bl,est,k}(z_0)$. Note again that in the acoustic approximation, we can write $R \cap (z_n) = -R \cup (z_n)$. This reduces the number of parameters to estimate in the inversion from two reflectivity matrices per depth level ($R \cap$, $R \cup$) to one ($R \cup$).

The least-squares inversion for blended VSP data suffers from blending noise leaking into the image space especially for higher blending factors. So, in order to suppress the blending noise leaking into the image, we could use a sparsity-promoting constraint in the inversion scheme, similar to what was done in chapter three. Hence, the new objective function for the constrained least-squares inversion can be written as:

$$J = \sum_k \sum_\omega \| \vec{P}_{bl,obs,k} - \vec{P}_{bl,est,k} \|^2_2 + \epsilon^2 F(R),$$

(5.4.13)

where, $F(R)$ is a sparsity-promoting norm imposed on the reflectivity and $\epsilon^2$ is a weighting parameter that governs the trade-off between the data misfit and the model prior usually depending on the noise content of the data. Note again, the subscript $k$ is for the $k^{th}$ blended source. For the numerical examples ahead, we have used the Cauchy norm (same as in chapter three), that is defined as:

$$F(R) = \sum_n \sum_j \log(1 + \frac{R_{jj,n}^2}{\sigma_r^2}),$$

(5.4.14)

where $R_{jj,n}$ is a sample of the reflectivity image at lateral location $j$ (i.e. a diagonal element from matrix $R(z_n)$) and $\sigma_r$ is the weighting parameter in the Cauchy norm. The above optimization problem can be solved by an iterative optimization scheme in the same way as discussed in chapter three, to estimate the reflectivity of the subsurface. In Table 5.1, the pseudo-code of the FWM inversion algorithm for blended VSP data using conjugate gradient scheme is given.

### 5.5 Numerical examples of FWM for blended VSP data

To illustrate the FWM inversion scheme, we have modelled VSP data using an acoustic 2D finite-difference method, with the full source geometry (conventional geometry) and using the density model as shown in Figure 5.3a. The unblended VSP data are simulated for uniformly distributed sources, laterally located between 0m and 3000m, with a source spacing of 20m, at the surface. The receivers are located between a depth of 100m to 1000m, at a depth spacing of 10m. The simulated unblended VSP data were then numerically blended by adding shots with random time shifts. In order to test the imaging process for the blended
PSEUDO-CODE

- initialization: \( \mathbf{R}^{(0)} = 0, \vec{E}_{bl,k} = \vec{P}_{bl,obs,k}^T, i = 1 \)

- while \( i \leq i_{max} \) or \( |\vec{E}_{bl,k}| < |\vec{E}_{tolerance}| \)
  - compute \( \vec{P}_k^{+(i)}, \vec{P}_k^{-(i)} \) for all sources \( k \), at all depth levels
  - estimate unblended \( \vec{P}_{est,k}^T \) at the surface
  - blend the estimated data at the surface \( \vec{P}_{est,k}^T = \Gamma_{bl}^T \vec{E}_{est,k} \)
  - blended data misfit \( \vec{E}_{bl,k} = \vec{P}_{bl,obs,k}^T - \vec{P}_{bl,est,k}^T \)
  - pseudo-deblended data misfit \( \vec{E}_k = \frac{1}{b} \vec{E}_{k} \Gamma_{bl}^H \)
  - compute gradient \( \Delta \mathbf{R}^{\cup(i)} \) for all depth levels
    if \( i = 1 \)
    \[ \beta^{(i)} = 0 \]
    else
    \[ \beta^{(i)} = \frac{\Delta \mathbf{R}^{(i-1)}H[\Delta \mathbf{R}^{(i)}] - \Delta \mathbf{R}^{(i-1)}H[\Delta \mathbf{R}^{(i-1)}]}{\Delta \mathbf{R}^{(i-1)}H[\Delta \mathbf{R}^{(i-1)}]} \]
    estimate conjugate direction, \( \Delta \mathbf{R}^{\cup(i)}_{cg} = \Delta \mathbf{R}^{\cup(i)} + \beta^{(i)} \Delta \mathbf{R}^{\cup(i-1)}_{cg} \)
    search for \( \alpha^{(i)} \), \( \alpha^{(i)} = \text{argmin}_{\alpha} [J(\mathbf{R}^{\cup(i-1)} + \alpha^{(i)} \Delta \mathbf{R}^{\cup(i)}_{cg})] \)
    update reflectivity matrix, \( \mathbf{R}^{(i)} = \mathbf{R}^{(i-1)} + \alpha^{(i)} \Delta \mathbf{R}^{\cup(i)}_{cg} \)
    \( i = i + 1 \)

\[ \text{Table 5.1: Pseudo-code for the full wavefield migration algorithm using an iterative conjugate-gradient scheme to image blended VSP data in common-receiver domain.} \]

data, we did the numerical blending by adding shots with random time shifts, the number of shots added to make one blended shot is defined by the ‘blending factor’. We have tested the scheme for blending factor one, two, three and four. Figure 5.16 shows snapshots of the wavefield propagation in the finite-difference modeling for numerically blended shot experiments with blending factors one, two and three.

Figures 5.17, 5.18, 5.19 and 5.20 illustrate schematically the numerical blending process, showing a frequency slice of the blending operator \( \Gamma_{bl} \), an example of the common-shot and common-receiver gathers after numerical blending for blending factor one, two, three and four, respectively. Specifically, Figures 5.17a, 5.18a, 5.19a and 5.20a show an example of an unblended common-shot gather and Figures 5.17b, 5.18b, 5.19b and 5.20b show a frequency slice of the corresponding blending operator, with blending factor one, two, three and four respectively. Figures 5.17c, 5.18c, 5.19c and 5.20c show an example of a blended common-source
Figure 5.16: Snapshots of the wavefield propagating in the medium while performing the numerical blending of the sources using a finite-difference scheme for blending factors one, two and three. a), d), g) shows the wavefields at 0.3s, 0.6s and 0.9s for blending factor = 1; b), e), h) shows the wavefields at 0.3s, 0.6s and 0.9s for blending factor = 2; and c), f), i) shows the wavefields at 0.3s, 0.6s and 0.9s for blending factor = 3. The reflectivity curve are overlaid on all of these figures.

gather and Figures 5.17d, 5.18d, 5.19d and 5.20d show an example of a blended common-receiver gather, with blending factor one (equivalent to unblended data), two, three and four, respectively. Note that that the blended data in the common receiver domain appears to be completely random, as expected, and the number of traces (i.e. number of blended source experiments) reduces with increasing blending factor. Also, note that numerical blending leads to interference of the events in the common-shot domain.

To illustrate the potential of the imaging blended VSP data, we perform FWM
5.5 Numerical examples of FWM for blended VSP data

![Diagram](image)

**Figure 5.17:** a) An example of an unblended common-shot gather; b) a blending matrix with blending factor=1 (no blending); c) an example of a blended common shot gather and d) an example of a blended common-receiver gather.

![Diagram](image)

**Figure 5.18:** Same as Figure 5.17, now for blending factor=2.

Inversion on the blended data. Figures 5.21, 5.22, 5.23 and 5.24 show the images obtained after (a) the 1st and (b) the 10th iteration using the blended data with blending factor one, two, three and four, respectively. Note the illumination extent of the image increases if we compare the images after the 1st and 10th
Figure 5.19: Same as Figure 5.17, now for blending factor=3.

Figure 5.20: Same as Figure 5.17, now for blending factor=4.

iteration. This is expected in FWM, because we use the full wavefield (including all multiples) to estimate the subsurface reflectivity. Again, each iteration uses one higher order of multiples in the data. However, we also note that with in-
creasing blending factor, the image increasingly suffers from crosstalk, although the sparsity constraint helps in reducing the blending noise in the image space. Note that these crosstalk appears as noise-bursts in images shown in Figures 5.22, 5.23 and 5.24. We also believe that a proper design of the acquisition geometry, i.e. the parameters of the blending operator, can help in reducing the crosstalk noise. Note that for these examples, the Cauchy norm is used as a regularization term in the inversion scheme to promote sparseness in the solution and hence, reduce the random blending noise in the image space.

![Figure 5.21](image1.png)

**Figure 5.21:** a) Image after the 1st iteration of FWM and b) image after the 10th iteration. Both for blending factor = 1.

![Figure 5.22](image2.png)

**Figure 5.22:** a) Image after the 1st iteration of FWM and b) image after the 10th iteration. Both for blending factor = 2.

Furthermore, Figures 5.25, 5.26, 5.27 and 5.28 shows the observed (true) blended data and the estimated blended data, and the data misfit after the 10th iteration of FWM for blending factors one, two, three and four, respectively. Note that, as the blending factor increases, the convergence is slower (i.e. the residual is larger)
Figure 5.23: a) Image after the 1$^{st}$ iteration of FWM and b) image after the 10$^{th}$ iteration. Both for blending factor = 3.

Figure 5.24: a) Image after the 1$^{st}$ iteration of FWM and b) image after the 10$^{th}$ iteration. Both for blending factor = 4.

due to more blending interference noise in the data.

Figure 5.29 shows the convergence curves for the normalized objective function with respect to number of iterations of FWM for blended VSP data. Note again that the convergence rate is slower for data with higher blending factor. Also, in these examples, the convergence of the algorithm becomes slow beyond the 10$^{th}$ iteration. So, we set this maximum number of iterations as a stopping criteria for these tests.

As another example to illustrate the FWM inversion scheme, we have used a complex reservoir model (modified marmousi density model) as shown in Figure 5.30. Blended VSP data are simulated using an acoustic 2D finite-difference method.

The unblended VSP data are simulated for uniformly distributed sources, laterally located between 0m and 6000m, with a source spacing of 25m, at the surface. The
5.5 Numerical examples of FWM for blended VSP data

![Diagram showing true data, estimated data, and data misfit for blending factor=1, after the 10th iteration.]

**Figure 5.25:** An example of a common-shot gather showing a) the true data, b) the estimated data and c) the data misfit, for blending factor=1, after the 10th iteration.

![Diagram showing true data, estimated data, and data misfit for blending factor=2, after the 10th iteration.]

**Figure 5.26:** An example of a common-shot gather showing a) the true data, b) the estimated data and c) the data misfit, for blending factor=2, after the 10th iteration.
Figure 5.27: An example of a common-shot gather showing a) the true data, b) the estimated data and c) the data misfit, for blending factor=3, after the 10th iteration.

Figure 5.28: An example of a common-shot gather showing a) the true data, b) the estimated data and c) the data misfit, for blending factor=4, after the 10th iteration.
5.5 Numerical examples of FWM for blended VSP data

**Figure 5.29:** Convergence curves depicting the minimization of the normalized objective function with respect to number of iterations. Note that the convergence rate is slower for data with higher blending factor.

**Figure 5.30:** a) Reservoir-oriented density model (modified Marmousi model) used to illustrate imaging results for blended VSP data.
receivers are located between a depth of 50m to 600m, at a depth spacing of 10m. The simulated unblended VSP data were then numerically blended by adding shots with random time shifts. We have tested the scheme for blending factor one, two and three. Figure 5.31 shows images obtained after the 1st and 15th iterations of FWM for blending factors of one, two and three.

![Images](image_url)

Figure 5.31: a), b), and c) Images after the 1st iteration of FWM for blending factor = 1, 2, and 3, respectively; d), e), and f) images after the 15th iteration of FWM for blending factor = 1, 2, and 3, respectively.

### 5.6 Discussion

Blended VSP acquisition can help in reducing the expensive rig-time or borehole acquisition time. Therefore, we expect to see more blended VSP acquisition in the future. In this chapter, we have demonstrated that FWM has a potential to be effectively used in imaging blended VSP data without a need for an active deblending as a pre-processing step. However, the option to perform an active deblending before FWM or performing deblending and imaging simultaneously
for blended VSP data as a strategy require further research and will depend on the measured data. In this chapter, we propose the latter, to perform FWM on blended VSP data in the common-receiver domain. Hence the algorithm works for simultaneous deblending and imaging, and estimating the subsurface reflectivity such that the modelled data fits the true blended data in a least-square sense.

The estimation of the source wavefield is highly dependent on how well we can pick the direct arrivals in the measured VSP data. However, picking direct arrivals on blended data directly is challenging. The direct arrivals for blended data can be picked effectively after applying pseudo-deblending. Furthermore, in this paper, we have demonstrated examples from an offshore environment. A similar technique can be extended to use VSP data acquired onshore. However, in land data, there will be other processing aspects to be taken care of such as near-surface issues and statics. A detailed discussion on these issues are beyond the scope of this chapter. We have shown simple synthetic examples to illustrate the imaging of blended VSP data, with different blending factors. Again, using the full wavefield helps to get a significant improvement in the illumination in the images compared to conventional approaches. Note also that, using a constrained inversion scheme does help in reducing the blending interference noise in the image space. Furthermore, we expect that designing the acquisition geometry with optimum blending parameters will also help in reducing the blending noise in the image space. In this chapter, we have not included the concept of compressed sensing in acquiring VSP data and estimating images, however, it is certainly an interesting subject for future research.

Finally, in this research, for 2D VSP data, we found that doing one iteration of modelling followed by one iteration of inversion in the feedback loop lead to converge the algorithm smoothly and yield a reasonably good result. However, for 3D blended surface seismic or OBC (Ocean-bottom cable) data, this strategy may not always work. Hence, for practical cases, one must test strategies like multiple iteration of modelling followed by one iteration of inversion or vice-versa.
6.1 Introduction to multi-component VSP data

In practice, VSP data are mostly multi-component records, i.e. the geophones in the borehole can register the x-, the y- and the z- component of the wavefields. Hence, shear-waves or converted waves play a major role in VSP data processing, imaging and interpretation. The \( P \)-waves and \( S \)-waves have different wave properties, such as the particle velocity and particle direction, which in fact aids in providing additional stratigraphical as well as structural information of the subsurface. Compared to surface seismic data, it is easier to identify the reflected and transmitted primaries and multiples of the converted wavefield in VSP data and, thus, it helps in deriving reliable information about the subsurface lithological properties. Assuming a horizontally layered earth, the location of P-P and P-S reflection points in the VSP geometry are curved and P-S reflection points are displaced toward the receivers, see Figure 6.1, after Stewart et al. [2002]. The P-P and P-S reflection point trajectories are computed for an offset source located 3000\( m \) away from a vertical well, for a given horizontally layered elastic medium obeying Snell’s law. Furthermore, in practice, P-S images have higher spatial resolution than the associated P-P sections from VSP data [see, for example, in Stewart et al., 2002].

In the past, several example applications of multi-component VSP data are dis-
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cussed in the literature. For example, Omnes [1980] discussed an application to derive log-data and corresponding lithological parameters using P- and S-wavefields in VSP data, Daley et al. [1988] and Geis et al. [1990] discussed a detailed analysis and applications of the shear-waves in VSP data, Aminzadeh [1989] showed how synthetic modelling of elastic VSP data aids in reservoir interpretation, Gulati et al. [2004]; Xu and Stewart [2004] discussed an integrated interpretation of multicomponent surface and vertical seismic data and Yan et al. [2012] illustrated an application of 3D multi-component VSP data to image tight gas sands. Furthermore, in the literature, we also see some examples of 9-C VSP data, i.e. using a three component source wavefield and recording a three component wavefield at the borehole geophones. Some examples where the importance and application of 9C VSPs are discussed are in Lefeuvre et al. [1992], MacBeth et al. [1995], MacBeth et al. [1997], MacBeth [2002] and Hardage et al. [2003].

We will illustrate an example for this multi-component source VSP in a 2D case, where two orthogonal source wavefields at the surface are used to simulate the vertical and the transverse components of the receiver wavefields. Hence, the simulated example is 4C. Figure 6.2 illustrates an example of a simulated common-shot gather for the four components of the VSP data, calculated using an elastic finite difference algorithm. Specifically, Figure 6.2a shows a schematic diagram of the 1.5D model used to simulate the VSP data, annotated with two far-offset dipole sources, one oriented in the z-direction and the other in the x-direction.
Figure 6.2b shows the four different components of data measured in matrix form. Figures 6.2c, 6.2d, 6.2e and 6.2f show an example of a simulated common-shot record for the xx, xz, zx and zz components, respectively.

Figure 6.2: 2D-4C VSP data modelling in 2D case. a) Schematic diagram showing a 1.5D elastic earth model, with three reflectors at a depth of 300m, 550m and 780m. The layers are annotated with their corresponding P-wave velocities, S-wave velocities and densities. VSP data is modelled based on this 1.5D layered model using an elastic finite difference method. The two dipole P-sources, schematically shown, are situated at a lateral distance of 1000m away from the well. The sources have their orientation in the x- and in the z-direction and the borehole receivers record the multi-component wavefield i.e. the x- (transverse) and z- (vertical) component of the wavefield. b) Matrix showing four possible data simulated in the 2D scenario, i.e. xx, xz, zx and zz wavefields, where the first letter represents the receiver orientation and the second letter represents the source orientation. c), d), e) and f) show an example of a common-shot record from the simulated VSP data for the xx, xz, zx and zz wavefields, respectively. Note the strong S-waves in xx and xz gathers (i.e. for the transverse receiver component) and strong P-waves in the zx and zz gathers (i.e. for the vertical receiver component).
With VSP data acquisition, the orientation of the geophones in the borehole are generally not aligned to the referenced x-, y- and z- direction. Hence, in terms of the processing of multi-component VSP data, one of the major steps is a rotation correction applied to the transverse and vertical component wavefields. After the correction, the recorded three component wavefield are oriented to the referenced x-, y- and z- directions. A subsequent $P$-$S$ wavefield mode-separation is applied to separate $P$ waves and $S$ waves. We will not discuss the details of $P$ and $S$ wavefield mode-separation in this thesis. For examples in literature, please refer to methods discussed in Dankbaar [1987], Hermann and Wapenaar [1992], Leaney [1990], Blias [2008], Sun et al. [2009], Lou et al. [2013] and Palacios et al. [2013].

We assume to have a $P$-$S$ mode-separated wavefield from VSP data. So towards elastic FWM, the next step is to incorporate the simulation of converted waves in the full wavefield modelling. In the previous chapters, we have discussed full wavefield modelling and full wavefield migration (FWM) for VSP data in an acoustic scenario, where we assumed the $S$-wave velocity to be zero, yielding no effect of converted wavefields and angle-dependent effects due to $S$-wave velocities. An example in section 4.3 does show the effect of non-zero $S$-wave velocity on $R_{pp}$ imaging (acoustic imaging), where the observed $P$-wave data contains converted waves and also shows angle-dependent reflection effects. In the next section, we will extend the concept of acoustic full wavefield modelling to elastic multicomponent full wavefield modelling for VSP data. In the subsequent sections, we will discuss an approach to incorporate the converted waves in imaging.

### 6.2 Elastic full wavefield modelling: mathematical formulation

In the past, we have seen some examples of elastic multi-component VSP modelling in the literature. Some of the examples are mentioned in chapter two of this thesis, such as the reflectivity method discussed in Mallick and Frazer [1988] and the state-space algorithm discussed in Aminzadeh and Mendel [1985]; Berger [1988]; Xu [1990]. Also, Young et al. [1984] presented a comparative study of modelling algorithms based on geometric ray theory, asymptotic ray theory, generalized ray theory, Kirchhoff wave theory, Fourier synthesis, finite differences, and finite elements to simulate elastic VSP data. Dietrich and Bouchon [1985] discussed modelling of VSP in elastic media using a discrete wavenumber method [Bouchon and Aki, 1977].

In chapter two of this thesis, we introduced the concept of iterative full wavefield modelling and illustrated examples in the 2D acoustic case. In this section, we will extend the concept of the full wavefield modelling scheme to include the converted wavefields. In order to discuss the mathematical formulation, we will use a similar matrix-vector notation as introduced earlier in chapter two of this thesis. To
account for converted waves, we have to define the continuity boundary condition of both $P$- and $S$-wavefields at an interface. Note that $P$ and $S$-wavefields are the $P$-wave potential and $S$-wave potential (Lame potentials) represented by $\phi$ and $\vec{\psi}$, respectively [for details, please refer to Wapenaar and Berkhout, 1989]. For an isotropic homogeneous elastic medium, the $P$-wave and $S$-wave potentials are related to the particle velocities $\vec{v}_p$ and $\vec{v}_s$ and can be written as:

$$
\vec{v}_p = \nabla \phi,
\vec{v}_s = \nabla \times \vec{\psi}.
$$

(6.2.1)

The total particle velocity is given by $\vec{v} = \vec{v}_p + \vec{v}_s$, and related to the $P$- and $S$-wave potentials by (Newton’s law):

$$
\frac{\partial \vec{v}}{\partial t} = -\frac{1}{\rho} [\nabla \phi + \nabla \times \vec{\psi}].
$$

(6.2.2)

Figure 6.3 schematically shows the upgoing and downgoing $P$- and $S$- wavefields across a discontinuity at depth level $z_n$. The upgoing wavefields have a superscript '-' and the the downgoing wavefields have a superscript '+'. Also, similar to chapter two, the wavefields that are incident to a depth level are represented by a vector $\vec{P}$ and the wavefield that are leaving a depth level are represented by a vector $\vec{Q}$. Further, to specify the mode of the wavefield, subscript $P$ and $S$ are used to represent $P$-waves and $S$-waves.

Hence, the upgoing and the downgoing $P$-wavefields just above the discontinuity are represented by $\vec{Q}_p^-$ and $\vec{P}_p^+$, respectively, and the ones just below the discontinuity are represented by $\vec{P}_p^-$ and $\vec{Q}_p^+$, respectively. Similarly, the upgoing and the downgoing $S$-wavefields just above the discontinuity are represented by $\vec{Q}_s^-$ and $\vec{P}_s^+$, respectively, and the ones just below the discontinuity are represented by $\vec{P}_s^-$ and $\vec{Q}_s^+$, respectively. Now, in order to incorporate the converted wavefields, we will define the reflectivity and transmissivity matrices with subscripts representing the mode of the incident and the corresponding reflected or transmitted wavefields. In such notation, $\mathbf{R}_{pp}^\cup$ and $\mathbf{R}_{pp}^\cap$ represent reflectivity matrices related to the discontinuity for an incident and reflected $P$-wavefield, incident from above and below the depth level, respectively (as shown in Figure 6.3b). $\mathbf{R}_{ss}^\cup$ and $\mathbf{R}_{ss}^\cap$ represent reflectivity matrices related to the discontinuity for an incident and reflected $S$-wavefield, incident from above and below the depth level, respectively (as shown in Figure 6.3c). $\mathbf{R}_{ps}^\cup$ and $\mathbf{R}_{ps}^\cap$ represent reflectivity matrices related to the discontinuity for an incident $S$-wavefield but reflected $P$-wavefield, incident from above and below the depth level, respectively (as shown in Figure 6.3d). Similarly, $\mathbf{R}_{sp}^\cup$ and $\mathbf{R}_{sp}^\cap$ represent reflectivity matrices related to the discontinuity for an incident $P$-wavefield but reflected $S$-wavefield, incident from above and below the depth level, respectively (as shown in Figure 6.3e). $\mathbf{T}_{pp}^+$, $\mathbf{T}_{ss}^+$, $\mathbf{T}_{ps}^+$, $\mathbf{T}_{sp}^+$,
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\( T_{pp}, T_{ss}, T_{ps}, T_{sp} \) represents the corresponding transmissivity matrices as shown in Figures 6.3b, 6.3c, 6.3d and 6.3e. Note that ‘+’ and ‘−’ signs represents the transmission for downgoing and upgoing wavefields, respectively.

We will define the boundary conditions using the wavefield continuity equations as introduced in Claerbout [1976] [see also Frasier, 1970; Aminzadeh and Mendel, 1982; Berkhout, 2012].

Using the wavefields as well as the reflectivity and the transmissivity matrices notation as discussed above, and as shown in Figure 6.3, the full wavefield relations from the continuity relationship become:

\[
\begin{align*}
\vec{Q}_p^+ &= T_{pp}^p \vec{P}_p^+ + T_{ps}^s \vec{P}_s^+ + R_{pp}^p \vec{P}_p^- + R_{ps}^s \vec{P}_s^- , \\
\vec{Q}_s^+ &= T_{sp}^s \vec{P}_p^+ + T_{ss}^s \vec{P}_s^+ + R_{sp}^s \vec{P}_p^- + R_{ss}^s \vec{P}_s^- , \\
\vec{Q}_p^- &= R_{pp}^p \vec{P}_p^- + R_{ps}^s \vec{P}_s^- + T_{pp}^p \vec{P}_p^+ + T_{ps}^s \vec{P}_s^+ , \\
\vec{Q}_s^- &= R_{sp}^s \vec{P}_p^- + R_{ss}^s \vec{P}_s^- + T_{sp}^s \vec{P}_p^+ + T_{ss}^s \vec{P}_s^+ .
\end{align*}
\]

The above equations can be written as a matrix equation given by:

\[
\begin{bmatrix}
\vec{Q}_p^+ \\
\vec{Q}_s^+ \\
\vec{Q}_p^- \\
\vec{Q}_s^-
\end{bmatrix} =
\begin{bmatrix}
T_{pp} & T_{ps} & R_{pp} & R_{ps} \\
T_{ps}^+ & T_{ss}^+ & R_{ps}^+ & R_{ss}^+ \\
R_{pp}^+ & R_{ps}^+ & T_{pp} & T_{ps} \\
R_{sp}^+ & R_{ss}^+ & T_{sp} & T_{ss}
\end{bmatrix}
\begin{bmatrix}
\vec{P}_p^+ \\
\vec{P}_p^- \\
\vec{P}_s^- \\
\vec{P}_s^-
\end{bmatrix},
\]

(6.2.7)

Rearranging the above equation in order to formulate the outgoing wavefields at the interface - \( \vec{Q}_p^+, \vec{Q}_s^+, \vec{Q}_p^-, \vec{Q}_s^- \) as a sum of their corresponding incident wavefields \( \vec{P}_p^+, \vec{P}_s^+, \vec{P}_p^-, \vec{P}_s^- \), respectively and the corresponding two-way scattered terms, we get:

\[
\begin{bmatrix}
\vec{Q}_p^+ \\
\vec{Q}_s^+ \\
\vec{Q}_p^- \\
\vec{Q}_s^-
\end{bmatrix} =
\begin{bmatrix}
\vec{P}_p^+ + \delta \vec{P}_p^+ \\
\vec{P}_p^- + \delta \vec{P}_p^- \\
\vec{P}_s^+ + \delta \vec{P}_s^+ \\
\vec{P}_s^- + \delta \vec{P}_s^-
\end{bmatrix},
\]

(6.2.8)

where \( \delta \vec{P}_p^+, \delta \vec{P}_s^+, \delta \vec{P}_p^- \) and \( \delta \vec{P}_s^- \) are the scattered downgoing \( P \) wavefield, scattered downgoing \( S \) wavefield, scattered upgoing \( P \) wavefield and scattered upgoing \( S \) wavefield, respectively. These scattered wavefields are given by:
6.2 Elastic full wavefield modelling: mathematical formulation

Figure 6.3: a) Schematic diagram showing the upgoing and downgoing \textit{P}- and \textit{S}- wave-fields at a depth level $z_n$. The upgoing and the downgoing \textit{P}-wavefields just above the discontinuity are represented by $\vec{Q}_p^-$ and $\vec{P}_p^+$, respectively, and the ones just below the discontinuity are represented by $\vec{P}_p^-$ and $\vec{Q}_p^+$, respectively. Similarly, the upgoing and the downgoing \textit{S}-wavefields just above the discontinuity are represented by $\vec{Q}_s^-$ and $\vec{P}_s^+$, respectively, and the ones just below the discontinuity are represented by $\vec{P}_s^-$ and $\vec{Q}_s^+$, respectively. b), c), d) and e) show schematic diagrams annotated with the reflectivity and the transmissivity matrices for different wavefield modes. $R_{pp}^{\cup}$, $R_{ss}^{\cup}$, $R_{ps}^{\cup}$, $R_{sp}^{\cup}$, $R_{pp}^{\cap}$, $R_{ps}^{\cap}$, $R_{sp}^{\cap}$ represents the reflectivity matrices for different incident and reflected wavefield modes as shown in b), c), d) and e). Similarly, $T_{pp}^{+}$, $T_{ss}^{+}$, $T_{ps}^{+}$, $T_{sp}^{+}$, $T_{pp}^{-}$, $T_{ss}^{-}$, $T_{ps}^{-}$, $T_{sp}^{-}$ represents the corresponding transmissivity matrices for different incident and transmitted wavefield modes as shown in b), c) d) and e).
First, let us define the convention for one-way wavefield extrapolation between consecutive depth levels. Figure 6.4a and 6.4c show the downgoing $P$- and $S$-wavefield $\vec{Q}_p^+ (z_{n-1})$ and $\vec{Q}_s^+ (z_{n-1})$ from just below depth level $z_{n-1}$ propagated down to depth level $z_n$ using the propagation operator $W_p^+ (z_n, z_{n-1})$ and $W_s^+ (z_n, z_{n-1})$, and represented by the downgoing $P$- and $S$-wavefield $\vec{P}_p^+ (z_n)$ and $\vec{P}_s^+ (z_n)$ just above depth level $z_n$, respectively.

Similarly, Figure 6.4b and 6.4d show the upgoing $P$- and $S$-wavefield $\vec{Q}_p^- (z_{n+1})$ and $\vec{Q}_s^- (z_{n+1})$ from just above depth level $z_{n+1}$ propagated up to depth level $z_n$ using the propagation operator $W_p^- (z_n, z_{n+1})$ and $W_s^- (z_n, z_{n+1})$, and represented by the upgoing $P$- and $S$-wavefield $\vec{P}_p^- (z_n)$ and $\vec{P}_s^- (z_n)$ just below depth level $z_n$, respectively. The propagation operators $W_p$ and $W_s$ are the phase-shift operators, implemented as space-frequency domain convolution operators that can be calculated using inhomogeneous $P$- and $S$-wave background velocity models [Thorbecke et al., 2004]. Mathematically, we can write the one-way $P$- and $S$-wavefield propagation for downward propagation as:

$$\vec{P}_p^+ (z_n) = W_p^+ (z_n, z_{n-1}) \vec{Q}_p^+ (z_{n-1}), \quad (6.2.10)$$

$$\vec{P}_s^+ (z_n) = W_s^+ (z_n, z_{n-1}) \vec{Q}_s^+ (z_{n-1}), \quad (6.2.11)$$

and for upward propagation as:

$$\vec{P}_p^- (z_n) = W_p^- (z_n, z_{n+1}) \vec{Q}_p^- (z_{n+1}), \quad (6.2.12)$$

$$\vec{P}_s^- (z_n) = W_s^- (z_n, z_{n+1}) \vec{Q}_s^- (z_{n+1}). \quad (6.2.13)$$

For a detailed derivation of the extrapolation of the elastic $P$- and $S$-wavefields, please refer to Wapenaar and Berkhout [1989] and Wapenaar and Haime [1990].

The iterative full wavefield modelling of the $P$- and $S$-wavefields is done in a similar way as described in chapter two of this thesis. However, the scattered $P$-wavefield now comprise of the two-way full-elastic scattered $P$-waves i.e. $\delta \vec{P}_p^+$ and $\delta \vec{P}_p^-$, and the scattered $S$-wavefield now comprise of two-way full-elastic scattered $S$-waves i.e. $\delta \vec{P}_s^+$ and $\delta \vec{P}_s^-$. 

$$\begin{bmatrix}
\delta \vec{P}_p^+ \\
\delta \vec{P}_s^+
\end{bmatrix} =
\begin{bmatrix}
T_{pp}^+ - 1 & T_{ps}^+ & R_{pp}^+ & R_{ps}^+
\end{bmatrix}
\begin{bmatrix}
\vec{P}_p^+ \\
\vec{P}_s^+
\end{bmatrix}.
$$

(6.2.9)
As described in chapter two of this thesis, the iterative full wavefield modelling includes iterative modelling of the total incident wavefield from above and from below. For elastic VSP data, we will define the modelling in the true source-receiver geometry, which is similar to modelling surface seismic wavefields, followed by selecting the upgoing and the downgoing wavefields at known receiver depths. In order to incorporate the converted wavefield in the modelling scheme, we have to model the incident downgoing P- and S- wavefields, i.e. $\vec{P}_p^+(z_n)$ and $\vec{P}_s^+(z_n)$, from above, and the incident P- and S- upgoing wavefields, i.e. $\vec{P}_p^-(z_n)$ and $\vec{P}_s^-(z_n)$, from below, respectively. We assume that for the time being that only P-sources are present at the surface $z_0$. For a given iteration $i$, the modelling equations can be written as:
\[ \vec{P}^+(z_n)^{(i)} = \mathbf{W}_p^+(z_n, z_0) \vec{S}^+(z_0) + \sum_{m=0}^{n-1} \mathbf{W}_p^+(z_n, z_m)[\delta \vec{P}_p^+(z_m)]^{(i-1)}, \quad (6.2.14) \]

\[ \vec{P}^-(z_n)^{(i)} = \sum_{m=n+1}^{N} \mathbf{W}_p^-(z_n, z_m)[\delta \vec{P}_p^-(z_m)]^{(i-1)}, \quad (6.2.15) \]

\[ \vec{P}^+(z_n)^{(i)} = \sum_{m=0}^{n-1} \mathbf{W}_s^+(z_n, z_m)[\delta \vec{P}_s^+(z_m)]^{(i-1)}, \quad (6.2.16) \]

\[ \vec{P}^-(z_n)^{(i)} = \sum_{m=n+1}^{N} \mathbf{W}_s^-(z_n, z_m)[\delta \vec{P}_s^-(z_m)]^{(i-1)}, \quad (6.2.17) \]

where \( \vec{S}_p^+(z_0) \) represents the P-source wavefield at the surface. Note again that in the first step, \( \vec{P}^+(z_n)^{(1)} = \mathbf{W}^+(z_n, z_0) \vec{S}_p^+(z_0) \), i.e. the direct source wavefield at all depth levels and other wavefields i.e. \( \vec{P}^-(z_n)^{(1)} \), \( \vec{P}^+(z_n)^{(1)} \) and \( \vec{P}^-\)\( (z_n)^{(1)} \) are zero. Furthermore, as mentioned earlier, for walkaway VSP data modelling, we can select the upgoing and downgoing wavefields for all the spatial locations, i.e. both lateral and vertical location, where the receivers are present. The modelled VSP data is obtained by taking the sum of the upgoing and downgoing wavefields measured by the borehole receivers.

### 6.3 Elastic full wavefield modelling for a layered medium: numerical examples

In the previous section, we have discussed the mathematical formulation of the elastic full wavefield modelling. The iterative modelling of the incident P- and S-wavefields are a function of the two-way scattered P wavefield (\( \delta \vec{P}_p^+ \) and \( \delta \vec{P}_p^- \)) and the two-way scattered S wavefield (\( \delta \vec{P}_s^+ \) and \( \delta \vec{P}_s^- \)), which in turn are a function of the reflectivity and transmissivity matrices i.e. \( \mathbf{R}_{pp}, \mathbf{R}_{ps}, \mathbf{R}_{sp}, \mathbf{R}_{ss}, \mathbf{T}_{pp}, \mathbf{T}_{ps}, \mathbf{T}_{sp}, \mathbf{T}_{ss}, \mathbf{T}_{sp}^+, \mathbf{T}_{ss}^+, \mathbf{T}_{sp}^- \) and \( \mathbf{T}_{ss}^- \), given by Equation (6.2.9). Note that all these reflectivity matrices are angle-dependent, which shows complex behavior at critical and post-critical angles. Hence, the modelling as well as inversion of all these parameters together is not trivial. In order to simplify
the situation, we will restrict ourself to a layered medium. Furthermore, we will use an approximation to the reflectivity matrices introduced by Aki and Richards [1980] [see also Gisolf and Verschuur, 2010; Dey and Gisolf, 2007] for a relatively low contrast medium and small incident angles (pre-critical angles, approximately up to 30 degrees).

The approximation to the angle-dependent reflectivities can be written as a function of the ray-parameters \( s_x \), background elastic medium parameters (velocities and density) as well as the contrast of the elastic parameters of the medium, i.e. \( \Delta \ln \rho \) (density contrast), \( \Delta \ln \alpha \) (P-wave velocity contrast) and \( \Delta \ln \beta \) (S-wave velocity contrast).

The contrast of the medium parameters are given by:

\[
\begin{align*}
\Delta \ln \rho &= \ln \rho_2 - \ln \rho_1, \\
\Delta \ln \alpha &= \ln \alpha_2 - \ln \alpha_1, \\
\Delta \ln \beta &= \ln \beta_2 - \ln \beta_1,
\end{align*}
\]  

(6.3.18)

where \( \ln \rho_1, \ln \alpha_1 \) and \( \ln \beta_1 \) are the logarithmic elastic parameters just above a discontinuity, and \( \ln \rho_2, \ln \alpha_2 \) and \( \ln \beta_2 \) are the logarithmic elastic parameters just below a discontinuity (schematically shown in Figure 6.5).

![Figure 6.5: Schematic diagram showing the elastic P-wave velocities \( \alpha_1 \) and \( \alpha_2 \), S-wave velocities \( \beta_1 \) and \( \beta_2 \), and densities \( \rho_1 \) and \( \rho_2 \) in the medium above and below a discontinuity, respectively.](image)

The angle-dependent reflectivity vectors of a grid point at an interface or discontinuity are given by (Aki and Richards [1980]):
\[
\tilde{R}_{pp}(s_x) = \frac{1}{2}(1 - 4\beta_0^2 s_x^2)\Delta \ln \rho + \frac{1}{2(1 - \alpha_0^2 s_x^2)}\Delta \ln \alpha - 4\beta_0^2 s_x^2 \Delta \ln \beta, \quad (6.3.19)
\]

\[
\tilde{R}_{sp}(s_x) = -\frac{s_x \alpha_0}{2\sqrt{1 - \beta_0^2 s_x^2}} \left[ \left( 1 - \beta_0^2 s_x^2 + 2\beta_0^2 \sqrt{1 - \alpha_0^2 s_x^2} \sqrt{1 - \beta_0^2 s_x^2} \right) \Delta \ln \rho \right] - \frac{s_x \alpha_0}{2\sqrt{1 - \beta_0^2 s_x^2}} \left[ 4\beta_0^2 \left( s_x^2 - \frac{\sqrt{1 - \alpha_0^2 s_x^2} \sqrt{1 - \beta_0^2 s_x^2}}{\alpha_0 \beta_0} \right) \Delta \ln \beta \right] \quad (6.3.20)
\]

\[
\tilde{R}_{ss}(s_x) = -\frac{1}{2}(1 - 4\beta_0^2 s_x^2)\Delta \ln \rho - \left[ \frac{1}{2(1 - \beta_0^2 s_x^2)} - 4\beta_0^2 s_x^2 \right] \Delta \ln \beta, \quad (6.3.21)
\]

\[
\tilde{R}_{ps}(s_x) = \frac{\beta_0 \sqrt{1 - s_x^2 \beta_0^2}}{\alpha_0 \sqrt{1 - s_x^2 \alpha_0}} \tilde{R}_{sp}(s_x). \quad (6.3.22)
\]

Similarly, the angle-dependent transmissivity vectors of a grid point at an interface or discontinuity are given by (Aki and Richards [1980]):

\[
\tilde{T}_{pp}(s_x) = 1 - \frac{1}{2} \Delta \ln \rho + \left[ \frac{1}{2(1 - \alpha_0^2 s_x^2)} - 1 \right] \Delta \ln \alpha, \quad (6.3.23)
\]

\[
\tilde{T}_{sp}(s_x) = -\frac{s_x \alpha_0}{2\sqrt{1 - \beta_0^2 s_x^2}} \left[ \left( 1 - \beta_0^2 s_x^2 - 2\beta_0^2 \sqrt{1 - \alpha_0^2 s_x^2} \sqrt{1 - \beta_0^2 s_x^2} \right) \Delta \ln \rho \right] - \frac{s_x \alpha_0}{2\sqrt{1 - \beta_0^2 s_x^2}} \left[ 4\beta_0^2 \left( s_x^2 + \frac{\sqrt{1 - \alpha_0^2 s_x^2} \sqrt{1 - \beta_0^2 s_x^2}}{\alpha_0 \beta_0} \right) \Delta \ln \beta \right] \quad (6.3.24)
\]

\[
\tilde{T}_{ss}(s_x) = 1 - \frac{1}{2} \Delta \ln \rho + \left[ \frac{1}{2(1 - \beta_0^2 s_x^2)} - 1 \right] \Delta \ln \beta, \quad (6.3.25)
\]

\[
\tilde{T}_{ps}(s_x) = -\frac{\beta_0 \sqrt{1 - s_x^2 \beta_0^2}}{\alpha_0 \sqrt{1 - s_x^2 \alpha_0}} \tilde{T}_{sp}(s_x). \quad (6.3.26)
\]

Here, \( \alpha_0 \) and \( \beta_0 \) are the \( P \)-wave and \( S \)-wave background velocities. In the forward modelling, we assume that both the background \( P \)- and \( S \)-wave velocity models are known. Also in the forward modelling, we know the true logarithmic elastic contrast parameters. We can re-write the above four equations for \( \tilde{R}_{pp}, \tilde{R}_{sp}, \tilde{R}_{ss} \) and \( \tilde{R}_{sp} \) and the four equations for \( \tilde{T}_{pp}, \tilde{T}_{sp}, \tilde{T}_{ss} \) and \( \tilde{T}_{sp} \) in a simple way as a
function of elastic contrast parameters i.e. $\Delta \ln \rho$, $\Delta \ln \alpha$ and $\Delta \ln \beta$ as:

$$
\begin{align*}
\vec{R}_{pp}(s_x) &= a_{11}(s_x) \Delta \ln \rho + a_{12}(s_x) \ln \alpha + a_{13}(s_x) \ln \beta, \\
\vec{R}_{sp}(s_x) &= a_{21}(s_x) \Delta \ln \rho + a_{22}(s_x) \ln \alpha + a_{23}(s_x) \ln \beta, \\
\vec{R}_{ps}(s_x) &= a_{31}(s_x) \Delta \ln \rho + a_{32}(s_x) \ln \alpha + a_{33}(s_x) \ln \beta, \\
\vec{R}_{ss}(s_x) &= a_{41}(s_x) \Delta \ln \rho + a_{42}(s_x) \ln \alpha + a_{43}(s_x) \ln \beta,
\end{align*}
$$

(6.3.27) (6.3.28) (6.3.29) (6.3.30)

$$
\begin{align*}
\vec{T}_{pp}(s_x) &= 1 + b_{11}(s_x) \Delta \ln \rho + b_{12}(s_x) \ln \alpha + b_{13}(s_x) \ln \beta, \\
\vec{T}_{sp}(s_x) &= b_{21}(s_x) \Delta \ln \rho + b_{22}(s_x) \ln \alpha + b_{23}(s_x) \ln \beta, \\
\vec{T}_{ps}(s_x) &= b_{31}(s_x) \Delta \ln \rho + b_{32}(s_x) \ln \alpha + b_{33}(s_x) \ln \beta, \\
\vec{T}_{ss}(s_x) &= 1 + b_{41}(s_x) \Delta \ln \rho + b_{42}(s_x) \ln \alpha + b_{43}(s_x) \ln \beta,
\end{align*}
$$

(6.3.31) (6.3.32) (6.3.33) (6.3.34)

which can be written as a matrix equations as:

$$
\begin{bmatrix}
\vec{R}_{pp} \\
\vec{R}_{sp} \\
\vec{R}_{ps} \\
\vec{R}_{ss}
\end{bmatrix}
= 
\begin{bmatrix}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23} \\
a_{31} & a_{32} & a_{33} \\
a_{41} & a_{42} & a_{43}
\end{bmatrix}
\begin{bmatrix}
\Delta \ln \rho \\
\Delta \ln \alpha \\
\Delta \ln \alpha \\
\Delta \ln \beta
\end{bmatrix}
= 
A
\begin{bmatrix}
\Delta \ln \rho \\
\Delta \ln \alpha \\
\Delta \ln \beta
\end{bmatrix}
$$

(6.3.35)

and

$$
\begin{bmatrix}
\vec{T}_{pp} \\
\vec{T}_{sp} \\
\vec{T}_{ps} \\
\vec{T}_{ss}
\end{bmatrix}
= 
\begin{bmatrix}
1 & b_{11} & b_{12} & b_{13} \\
0 & b_{21} & b_{22} & b_{23} \\
0 & b_{31} & b_{32} & b_{33} \\
1 & b_{41} & b_{42} & b_{43}
\end{bmatrix}
\begin{bmatrix}
1 \\
\Delta \ln \rho \\
\Delta \ln \alpha \\
\Delta \ln \beta
\end{bmatrix}
$$

(6.3.36)

where the coefficients of matrices $A$ and $B$ are the functions of $P$-wave and $S$-wave background velocities, and ray-parameter $s_x$. Note that the superscripts ’∪’ and ’+’ indicates that wavefield is traveling downwards before reflection and transmission, respectively. The computation of reflectivity and transmissivity for the wavefields traveling upwards are performed by flipping the model upside down, i.e. making a change in sign in the medium contrasts.

Given the true and background $P$-wave and $S$-wave velocities and the true density model in a forward problem, we can evaluate the angle-dependent reflectivity and transmissivity vectors as a function of ray-parameter $s_x$ (from the approximations
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mentioned earlier), restricted to pre-critical angles. Furthermore, using the modelling equations (6.2.14), (6.2.15), (6.2.16) and (6.2.17) described earlier, we can model the upgoing and downgoing incident $P$ and $S$ wavefields at all depth levels iteratively. In order to illustrate this modelling scheme, we use a 1.5D model (layered earth model) as shown in Figure 6.6.

![Figure 6.6: 1D profiles for a) true and background $P$-wave velocity, b) true and background $S$-wave velocity and c) true density, used to illustrate elastic full wavefield modelling.](image)

Now, in the elastic full wavefield modelling, the first step is to compute the coefficients of matrix $A$ and $B$, using the known $P$- and $S$-wave background velocity models. The elements of this matrix are computed at three reflector depths. Figures 6.7 and 6.8 shows the elements of the matrices $A$ and $B$, respectively, for all three reflectors. Note that for larger $p$-values, i.e., for higher angle of incidence, the value of the coefficient matrix is zero. This is to ensure that the modelling will only take the pre-critical angles into account.

Furthermore, using the matrix $A$ and the known logarithmic contrast for the elastic parameters i.e. $\Delta \ln \rho$, $\Delta \ln \alpha$ and $\Delta \ln \beta$, the reflectivities $\tilde{R}_{pp}^{(s)}(s_x)$, $\tilde{R}_{ps}^{(s)}(s_x)$,
Ray parameter [s/m]

Figure 6.7: Elements of the matrix $\mathbf{A}$, computed using the $P$- and $S$- waves background velocity models at three reflector depths. Note that all three curves for three reflectors are plotted together in different colors.

\[\frac{\tilde{R}_{sp}(s_x)}{\tilde{R}_{ss}(s_x)}\text{ and } \frac{\tilde{R}_{ps}(s_x)}{\tilde{R}_{ss}(s_x)}\text{ as well as the transmissivities } \tilde{T}_{pp}(s_x), \tilde{T}_{ps}(s_x), \tilde{T}_{sp}(s_x)\text{ and } \tilde{T}_{ss}(s_x) \text{ are computed. Note that under the Aki and Richards approximation, the reflectivities from below are the same as reflectivities from above a discontinuity in magnitude and opposite in sign. Therefore, } \tilde{R}^{\cap}_{pp}(s_x) = -\tilde{R}^{\cup}_{pp}(s_x), \tilde{R}^{\cap}_{ps}(s_x) = -\tilde{R}^{\cup}_{ps}(s_x), \tilde{R}^{\cap}_{sp}(s_x) = -\tilde{R}^{\cup}_{sp}(s_x)\text{ and } \tilde{R}^{\cap}_{ss}(s_x) = -\tilde{R}^{\cup}_{ss}(s_x).\text{ The same approximation holds for transmissivities. Figure 6.9 and 6.10 show the computed}\]
Figure 6.8: Elements of the matrix $B$, computed using the $P$- and $S$-waves background velocity models at three reflector depths. Note that all three curves for three reflectors are plotted together in different colors.

reflectivities and transmissivities for the three reflectors in the 1.5D model. Note again, the reflectivities and transmissivities are vectors for one grid point location and for all the grid points at a depth level, these become matrices.
Figure 6.9: Computed reflectivities for the three reflectors as a function of ray-parameter $s_x$. $\bar{R}_{pp}(s_x)$ for a) $1^{st}$, e) $2^{nd}$ and i) $3^{rd}$ reflector from top to bottom. $\bar{R}_{ps}(s_x)$ for b) $1^{st}$, f) $2^{nd}$ and j) $3^{rd}$ reflector from top to bottom. $\bar{R}_{sp}(s_x)$ for c) $1^{st}$, g) $2^{nd}$ and k) $3^{rd}$ reflector from top to bottom. $\bar{R}_{ss}(s_x)$ for d) $1^{st}$, h) $2^{nd}$ and l) $3^{rd}$ reflector from top to bottom.
Figure 6.10: Computed transmissivities for the three reflectors as a function of ray-parameter $s_x$. $\overrightarrow{T}_{pp}(s_x)$ for a) 1st, e) 2nd and i) 3rd reflector from top to bottom. $\overrightarrow{T}_{ps}(s_x)$ for b) 1st, f) 2nd and j) 3rd reflector from top to bottom. $\overrightarrow{T}_{sp}(s_x)$ for c) 1st, g) 2nd and k) 3rd reflector from top to bottom. $\overrightarrow{T}_{ss}(s_x)$ for d) 1st, h) 2nd and l) 3rd reflector from top to bottom.
6.3 Elastic full wavefield modelling for a layered medium: numerical examples

Figure 6.11: Exact solution of the Zoeppritz equation (courtesy of CREWES Zoeppritz explorer 2.2) at one grid-point location for a) $R_{pp}$, b) $R_{sp}$, c) $R_{ps}$ and d) $R_{ss}$ for an interface separating two half spaces. The density, P-wave velocity and S-wave velocity for the medium above the interface are 1000 kg/m$^3$, 2700 m/s and 1500 m/s, respectively, and for the medium below the interface are 2000 kg/m$^3$, 3000 m/s and 1700 m/s, respectively. Note that in all figures, the bold curve shows the real part and the dotted curve shows the imaginary part of the complex reflectivity at and above the critical angle. Also, note the reflectivity becomes very complex at post-critical angles for incident S-waves i.e. for $R_{ps}$ and $R_{ss}$. The dotted blue curve at +/- 30 degrees shows the cut-off point up to where the reflectivity value is used in the elastic forward modelling scheme. Note that the reflector in this example corresponds to the first reflector shown in Figure 6.6, and the reflectivity curves within the cut-off value are equivalent to the reflectivity curves shown in Figures 6.9a, 6.9b, 6.9c and 6.9d.

Note again that values of all reflectivities are cut-off at pre-critical angle to avoid complex reflectivity behavior at and above the post-critical angles of incidence.
Figure 6.11 illustrates an exact solution of the Zoeppritz equation, courtesy of CREWES Zoeppritz explorer 2.2, for the first reflector (as shown in Figure 6.6).

Specifically, Figure 6.11a, 6.11b, 6.11c and 6.11d show $\vec{R}_{pp}$, $\vec{R}_{sp}$, $\vec{R}_{ps}$ and $\vec{R}_{ss}$ respectively at one grid-point location for an interface separating two half spaces. The density, $P$-wave velocity and $S$-wave velocity for medium above the interface are $1000\text{kg/m}^3$, $2700\text{m/s}$ and $1500\text{m/s}$, respectively, and for the medium below the interface are $2000\text{kg/m}^3$, $3000\text{m/s}$ and $1700\text{m/s}$, respectively. Note that in all figures, the bold curve shows the real part and the dotted curve shows the imaginary part of the complex reflectivity at and above critical angle. Also, note the reflectivity becomes very complex at post-critical angles for incident $S$-waves i.e. for $R_{ps}$ and $R_{ss}$. The dotted blue curve at +/- 30 degrees shows the cut-off point up to where the reflectivity value is used in the elastic forward modelling scheme. This corresponds to the reflectivity curves shown in Figures 6.9a, 6.9b, 6.9c and 6.9d.

Figures 6.12 to 6.19 show an example of a common-shot gather modelled iteratively using the elastic full wavefield modelling scheme. Specifically, Figures 6.12, 6.14, 6.16 and 6.18 show an example common-shot gather for the $P$-wavefield modelled after the 1$^{st}$, 2$^{nd}$, 3$^{rd}$ and 4$^{th}$ iteration, respectively. Similarly, Figures 6.13, 6.15, 6.17 and 6.19 show an example common-shot gather for the $S$-wavefield modelled after the 1$^{st}$, 2$^{nd}$, 3$^{rd}$ and 4$^{th}$ iteration, respectively. Note that the dotted axis at depth 300m, 550m and 780m annotated in all figures indicate the depth of the three reflectors used in the model. Also note that the surface multiples are not included in these modelling examples. As expected, in the elastic full wavefield modelling scheme, each iteration adds a higher order of scattered wavefields, i.e. a higher order of multiples for both $P$- and $S$-wavefields.

Furthermore, in the modelling scheme, the total upgoing wavefields at the surface represent the data equivalent to surface seismic data. Figures 6.20 shows an example of a common-shot gather for receivers located at the surface (i.e. surface seismic data) modelled after the 1$^{st}$ and the 4$^{th}$ iteration, showing the upgoing $P$-wavefield and the upgoing $S$ wavefield.
6.3 Elastic full wavefield modelling for a layered medium: numerical examples

**Figure 6.12:** Modelled P-wave after the 1\textsuperscript{st} iteration of elastic full wavefield modelling, showing an example of a common-shot gather for a) the downgoing wavefield, b) the upgoing wavefield and c) the total wavefield.

**Figure 6.13:** Same as Figure 6.12, but showing the modelled S-wavefield.
Figure 6.14: Modelled P-wave after the 2nd iteration of elastic full wavefield modelling, showing an example of a common-shot gather for a) the downgoing wavefield, b) the upgoing wavefield and c) the total wavefield.

Figure 6.15: Same as Figure 6.14, but showing the modelled S-wavefield.
Figure 6.16: Modelled P-wave after the 3rd iteration of elastic full wavefield modelling, showing an example of a common-shot gather for a) the downgoing wavefield, b) the upgoing wavefield and c) the total wavefield.

Figure 6.17: Same as Figure 6.16, but showing the modelled S-wavefield.
Figure 6.18: modelled P-wave after the 4th iteration of elastic full wavefield modeling, showing an example of a common-shot gather for a) downgoing wavefield, b) upgoing wavefield and c) total wavefield.

Figure 6.19: Same as Figure 6.18, but showing the modelled S-wavefield.
Figure 6.20: An example of a common-shot gather for receivers located at the surface (i.e. surface seismic data) modelled after the 1st iteration of elastic full wavefield modelling showing a) the upgoing $P$-waves and b) the upgoing $S$ waves; and after the 4th iteration of elastic full wavefield modelling showing c) the upgoing $P$-waves and d) the upgoing $S$ waves.

6.4 Elastic full wavefield migration: future recommendations

In the previous sections, we discussed the extension of full wavefield modelling to incorporate the converted wavefields for an elastic medium. In the numerical examples, we illustrated the concept using a layered medium and a linearized approximation of reflectivity matrices introduced in Aki and Richards [1980]. Note that the Aki and Richards approximation used for reflectivities in the modelling scheme are just to illustrate modelling examples. In this section, we will discuss a possible scheme to migrate the multicomponent VSP data, to be able to obtain angle-dependent PP and PS reflectivities. In the past, several methods have been discussed in the literature to image multicomponent VSP data to obtain P-P as well as P-S reflectivity images. Sun and McMechan [1986], Hokstad et al. [1988] and Neklyudov and Borodin [2009] discussed imaging using elastic reverse time migration. Dillon et al. [1988] and Agnihotri and McMechan [2007] discussed imaging using Kirchhoff migration for mode separated data. Jackson et al. [1991] discussed ‘scalar-vector migration’ which does not require pre-migration mode separation. However, it involves two scalar migration steps, along with a

As mentioned in the modelling section, we noticed that Aki and Richards linearized approximations to the reflectivities are valid for small angles. One such simplification was also introduced by Shuey [1985], stating \( R_{pp} \) as a function of angle of incidence \( \theta \) as:

\[
R_{pp} = R_0 + R_1 \sin^2 \theta,
\]

(6.4.37)

where \( R_0 \) is the normal incident reflectivity and \( R_1 \) is a coefficient term governing the angle-dependence of \( R_{pp} \). The equation above can be re-written in the wavenumber-frequency domain as:

\[
R_{pp} = R_0 + R_1 \frac{k_x^2}{k^2},
\]

(6.4.38)

where \( k_x \) is the horizontal wavenumber for an acoustic inversion scheme with an angle-dependent \( R_{pp} \) using such the linearized approximations. A similar approach can be used to incorporate the P-S imaging as well. However, the approximation to \( R_{sp} \) can be slightly more complex as it includes the angle of incidence for \( P \)-wave \( \theta^p \) and the angle of reflection for \( S \)-wave \( \theta^s \). One such example for \( R_{sp} \) can be written as [Jin et al., 2000]:

\[
R_{sp} = 0.5 \frac{\sin \theta^p}{\sin \theta^s} \left[ 4(\sin^2 \theta^s - \gamma \cos \theta^p \cos \theta^s) \frac{\Delta V_s}{V_s} \right] - 0.5 \frac{\sin \theta^p}{\sin \theta^s} \left[ (\cos 2 \theta^s + 2 \gamma \cos \theta^p \cos \theta^s) \frac{\Delta \rho}{\rho} \right],
\]

(6.4.39)

where \( \theta^p \) is the angle of incidence of the \( P \)-wave, and \( \theta^s \) is the reflection angle of the \( S \)-wave; \( \theta^s \) can be calculated from \( \theta^p \) using Snell’s law; \( V_p, V_s, \) and \( \rho \) represent, respectively, the average \( P \)-wave velocity, \( S \)-wave velocity, and density; \( \Delta V_s \) and \( \Delta \rho \) are the changes in \( S \)-wave velocity and density at the boundary of two layers; and \( \gamma \) is the velocity ratio \( V_s/V_p \). Unlike the \( P \)-wave reflection coefficient, the \( P-S \) reflection coefficient does not depend on the \( P \)-wave velocity contrast.

As future work towards estimating angle-dependent elastic reflectivities from multicomponent VSP data, the simplified equations can be used to reduce the parameters to be inverted in the inversion scheme. It is important to note that even if the medium is not layered, FWM will find the best matching parameters to explain the data.
6.5 Discussion

In this chapter, we have introduced an extension to the applicability of full wavefield migration to incorporate the converted wavefields for multicomponent VSP data. Elastic full wavefield modelling has been briefly discussed. We have illustrated the modelling using a 1.5D elastic model, for reflectivities restricted to pre-critical angles only, using Aki and Richards approximation to the Zoeppritz equations. We further discussed a possible approach towards estimation of angle dependent $R_{pp}$, $R_{sp}$, etc using simplified equations in order to reduce the inversion parameters. This will be a part of future research.
Elastic FWM: incorporating converted waves in multicomponent VSP data
Imaging steeply dipping salt-flanks using VSP data

7.1 Introduction

Imaging of steeply dipping or overturned salt-flanks is still a challenge for conventional migration techniques applied to surface seismic data [Buur and Kühnel, 2008]. On the other hand, walkaway Vertical Seismic Profiling (VSP) data has proven advantages. It performs better in mapping or imaging the steep salt-boundaries compared to the surface seismic data [Whitmore and Lines, 1986; McMechan et al., 1988; O’Brien et al., 2002; O’Brien, 2005], where the receivers are placed closer to the target. In the past, we have seen several approaches to use VSP data to image steeply-dipping salt-flanks. Some of the examples include imaging the turning waves through horizontal wavefield extrapolation [Brandsberg-Dahl et al., 2003], imaging P-S / P-P reflected and transmitted wavefields in VSP data using Reverse Time Migration (RTM) [McMechan et al., 1988] or using interferometric imaging principles [Sheley and Schuster, 2003; Xiao et al., 2003; Zhao et al., 2006; Luo et al., 2006; Hornby and Yu, 2007; Lu et al., 2009; Karrenbach et al., 2010; Hendrickson et al., 2011; Brand et al., 2013], employing the concept of reverse-time acoustics [Willis et al., 2006; Lu et al., 2008] and vector migration [Li et al., 2005].

In the previous chapters of this thesis, the described FWM algorithm is based on one-way propagation using a given migration velocity and two-way scattering across all depth levels due to the estimated reflectivity image. Therefore, using the current implementation, it has a limitation to image steep dips caused by sharp
lateral contrast in the medium impedance. Recently, FWM of surface seismic data has been extended to incorporate duplex and turning waves [Davydenko and Verschuur, 2013, 2014], using both vertical and horizontal wavefield extrapolation in the imaging scheme [Zhang and McMechan, 1997; Sava and Guitton, 2005; Brandsberg-Dahl et al., 2003; Jia and Wu, 2009] and therefore, is able to image complex and steeply-dipping structures. Using the same concept, we will extend the FWM method for VSP data to incorporate turning waves to image steep salt-flanks. In the next section, we will discuss the forward modelling and inversion scheme.

### 7.2 Incorporating turning-waves in full wavefield migration

Full wavefield forward modelling and imaging of turning waves for walkaway VSP data can be formulated in a similar way as described in chapter two and three of this thesis [Berkhout, 2012; Davydenko et al., 2012; Soni and Verschuur, 2013c]. Now the wavefield extrapolation is performed in a rotated coordinate system in order to image a target domain which encloses the salt-structure. See Figure 7.1 for a schematic illustration of this scheme.

**Figure 7.1:** a) Schematic diagram showing a borehole at $x_0$, placed close to the near-vertical target and an offset source $S(z_0)$ located at the surface. The dotted box defines our target domain for imaging. The wavefields arriving at the borehole receivers from the source-side comprise both direct arrivals (A) and scattered coda (B) due to possible scatterers in the near-surface. $X_s$ represents the impulse response of the subsurface outside the target domain. b) The target domain is shown in the rotated coordinate system $(x'-z')$, where the incident wavefield $\tilde{P}_{inc}$ appears as a downgoing wavefield, illuminating the target. We assume that in the rotated coordinate system, the medium below $z_N'$ is homogeneous, i.e. the upgoing wavefield from below the target domain is zero. $R^\cup$ and $R^\cap$ represent the reflectivity matrices at any depth level from above and from below, respectively, in the rotated coordinate system.
7.2 Incorporating turning-waves in full wavefield migration

We assume that the turning-waves incident on the borehole act as a nearly-horizontal wavefield that illuminates the salt-body beyond the borehole after which the receivers can record the reflections from the salt boundaries, as shown in Figure 7.1. Hence, we could formulate the forward modelling of the downgoing wavefield \( \vec{P}^+(z'_n) \) and the upgoing wavefield \( \vec{P}^-(z'_n) \) at any depth level \( z'_n \) in the orthogonal rotated coordinate system as:

\[
\vec{P}^+(z'_n) = \sum_{m=0}^{n-1} W^+(z'_n, z'_m)[\delta\vec{P}(z'_m) + P_{inc}(z'_0)],
\]

\[
\vec{P}^-(z_n) = \sum_{m=n+1}^{N} W^-(z'_n, z'_m)[\delta\vec{P}(z'_m)],
\]

where \( W^+ \) and \( W^- \) represent the downward and upward wavefield propagation operators, respectively, in the rotated coordinate system and \( \vec{P}_{inc}(z'_0) \) represents the total wavefield coming from the source-side towards the borehole. Note that the vectors and matrices in the above formulation represent one frequency component of their respective wavefields or operators. The two-way scattered wavefield is given in terms of the reflectivity matrices as:

\[
\delta\vec{P}(z'_n) = R^\cup(z'_n) \vec{P}^+(z'_n) - R^\cap(z'_n) \vec{P}^-(z'_n).
\]

For small S-wave velocity contrasts and for pre-critical angles, \( R^\cup(z'_n) = -R^\cap(z'_n) \), therefore, we can re-write the above equation for a two-way scattered wavefield as:

\[
\delta\vec{P}(z'_n) = R^\cup(z'_n)[\vec{P}^+(z'_n) - \vec{P}^-(z'_n)].
\]

Note that the incident wavefield \( \vec{P}_{inc}(z'_0) \) on the receivers in the borehole is equivalent to the convolution of the source wavefield at the surface \( \vec{S}(z_0) \) with the impulse response, \( \vec{X}_s \) of the subsurface outside the target domain, at the source-side. Hence, the impulse response \( \vec{X}_s \) accounts for all the scattering outside the target domain. If we have a blended source experiment (as discussed in chapter five), where the blended source is represented by \( \vec{S}_{bl}(z_0) \), then the incident wavefield \( \vec{P}_{inc,bl} \) can be written as:

\[
\vec{P}_{inc,bl}(z'_0) = \vec{X}_s \vec{S}_{bl}(z_0) = \vec{X}_s \vec{S}(z_0) \vec{I}_{bl}(z_0),
\]

where the blending operator \( \vec{I}_{bl}(z_0) \) can be written as \( \vec{I}_{bl}(z_0) = [\gamma_1, \gamma_2, \gamma_3, \ldots, \gamma_N] \), with \( \gamma_n = a_n e^{-j\omega T_n} \) [see Berkhout, 2008]. In this case, \( T_n \) is a random time-shift applied to blend the sources and \( a_n \) is a scale factor. Note that \( a_n = 0 \) eliminate these sources in the blended experiment. We will use the term ‘blending factor’ to define the number of shots blended together in one experiment for which \( a_n \neq 0 \) [Soni and Verschuur, 2013b, 2014b].
To illustrate the illumination from the turning waves and reflections, we used a
1.5D sediment-flooded velocity model and a 2D density model with a vertical salt
structure. The waves are simulated using a 2D acoustic finite-difference scheme
for an offset source laterally located at 2700m. Figure 7.2 shows the velocity
and density models, and the simulated wavefield snapshots at different times with
reference to the source excitation time at \( t = 0 \)s. Note that the turning wavefront
illuminating the vertical salt-boundary generates primaries and multiples upon
reflection. In this example, we have assume a rigid sea-air boundary at \( z_0 \) (i.e.
without surface-multiples). A vertical borehole laterally located at 1000m, close
to the salt structure will record the primary and multiple-reflected turning waves.
Hence, the measurements can be used to image the salt boundary.

We have discussed the forward model for the turning waves which laterally illu-
minate the steeply-dipping salt structures in the target domain. We have also de-
defined the modelling equations for both unblended and blended VSP experiments.
Hence, in the rotated coordinate system, we can also write the full wavefield mi-
gration as an optimization scheme to minimize the following objective function:

\[
J = \sum_s \sum_\omega \| \tilde{P}^-(z'_0) - \tilde{P}^-_{est}(z'_0) \|^2_2,
\]

where \( \tilde{P}^-(z'_0) \) and \( \tilde{P}^-_{est}(z'_0) \) are the observed and estimated data at the borehole
in the rotated coordinate system, for unblended or blended VSP experiments,
summed over all frequencies and for all source experiments. The objective func-
tion is minimized using an iterative conjugate gradient scheme to estimate the
subsurface reflectivity (as described in chapter three).

Note that the estimated data at the receivers are computed using Equation (7.2.2)
for an upgoing wavefield at depth level \( z'_0 \), in the rotated coordinate system. In
the migration, we have a smooth migration velocity model for the target domain.
However, we do not require the velocity model for the domain outside the target.
For the incident source-wavefield, we can use the direct arrival recorded at the
borehole receivers. In case of scatters in the domain outside the target, we expect
to get some crosstalk in the image, however, the inversion-based imaging process
iteratively suppresses the noise, providing a reasonable image. For real situations,
we do require some knowledge of the subsurface sediment-flood velocity, in order
to design the VSP acquisition geometry. We do not discuss the acquisition design
methodology to acquire turning waves in this paper.

For illustration, we perform FWM to image the data measured using the config-
uration shown in Figure 7.2. Figure 7.3 illustrates the image obtained after the
10\( ^{th} \) iteration of FWM for the target domain.
7.3 Numerical examples of full wavefield modelling and migration: BP benchmark model

To illustrate the imaging of steeply-dipping or overturned salt-flanks, we have modified the BP benchmark model [Billette and Brandsberg-Dahl, 2005], and used the salt structure in the density model, embedded in a homogeneous density...
Imaging steeply dipping salt-flanks using VSP data

Figure 7.3: a) True effective horizontal reflectivity for the salt-structure inside the target domain and b) the estimated image after the 10th iteration of FWM for the target domain.

Figure 7.4a and 7.4b show two density models used to generate VSP data, one with and one without a near-surface scatterer outside the target domain (shown by the yellow-dotted box). Figures 7.4c and 7.4d show the illumination map (unnormalized) due to a far-offset source for the given two models. Note the scattering due to near-surface complexity in Figure 7.4d. In FWM, we will be able to use all the scattered coda\(^1\) in addition to the direct incident wavefield to image the target salt-structure. Figure 7.5 show the effective horizontal reflectivity of the target salt-structure, which we aim to estimate in FWM.

Furthermore, the P-wave velocity profile used in this example is similar to the velocity model shown in Figure 7.2a, increasing with depth as \(v_z = v_0 + kz\), where \(v_0 = 1500 \text{ m/s}\) just below the water bottom at 250 m and \(k = 0.5(\text{m/s})/\text{m}\). We assumed a negligible S-wave velocity field. The well is located at a lateral location of 2500 m, and a pressure source array is located at the surface from 250 m to 1250 m. The receiver arrays are located in a vertical borehole between 500 m and 1500 m, at a depth interval of 10 m. Figures 7.4c and 7.4d show the illumination maps of a source located laterally at 250 m at the surface. These maps are computed using the two density models, shown in Figures 7.4a and 7.4b, respectively, and using the same sediment-flood velocity model. Note that for this particular example, the near-surface scatterer in the density model leads to an enhancement of the total scattered wavefield illuminating the salt-flank in

\(^1\)Coda: In seismology, coda is defined as the concluding portion of a seismic record after identifiable waves have passed, presumably caused by scattering, etc. [Sherrif, 2002]
7.3 Numerical examples of full wavefield modelling and migration: BP benchmark model

Figure 7.4: a) and b) show the density models used to generate VSP data, annotated schematically with the VSP acquisition geometry without (a) and with (b) a near surface scatterer, located outside the target domain (depicted by the yellow dotted box). c) and d) show the illumination maps (unnormalized) for the two density models (a and b, respectively) using the same sediment-flood velocity model and a source laterally located at 250 m at the surface.

We have used a 2D acoustic finite-difference scheme to generate the VSP data for both unblended and blended source geometry. In this chapter, we will discuss only the P-P imaging.
The density model in Figure 7.4b has a near-surface scatterer, that generates a scattered coda. Figures 7.6, 7.7 and 7.8 illustrate the simulation of the observed data using finite-difference scheme for unblended and blended source experiment, showing wavefield propagation for incident field in two different density models (shown in Figures 7.4a and 7.4b as well as an example common-shot gather for incident and observed wavefield for both the models. Please see the caption for details. Note the complexity in the incident wavefield for the model with near-surface scatterer. Also, with higher blending factor, the observed data becomes more and more complex due to wavefield interference.

Further, using the known incident wavefield $\vec{P}_{inc}$ for unblended and blended source experiments, a known effective horizontal reflectivity (as shown in Figure 7.5), known velocity model and using horizontal extrapolation operators, we illustrate examples for full-wavefield modelling for both models shown in Figure 7.4a and 7.4b. Figure 7.9 show an example common-shot gather and common-receiver gather for unblended source experiment after the 0th (i.e. only the incident wavefield), 1st, 2nd and 3rd iteration. Similarly, Figure 7.10 show an example common-shot gather and common-receiver gather for blended source experiment, with blending factor=4 after the 0th (i.e. only the incident wavefield), 1st, 2nd and 3rd iteration. For specific description, please see the caption.
7.3 Numerical examples of full wavefield modelling and migration: BP benchmark model

**Figure 7.6:** Simulated wavefield for unblended far-offset source: a) and b) show the wavefield propagation in finite-difference scheme at 0.2s and 1.2s for model without the near-surface or overburden complexity (shown by Figure 7.4a). Similarly, c) and d) show the wavefield propagation in finite-difference scheme at 0.2s and 1.2s for model with the near-surface or overburden complexity (shown by Figure 7.4b). e) and f) show an example common-shot gather for the incident wavefield $\vec{P}_{\text{inc}}$ from the left side of the borehole for the two models without and with near-surface scatterer, respectively. Similarly, g) and h) show an example common-shot gather for total observed wavefield $\vec{P}_{\text{obs}}$ at the borehole for the two models without and with near-surface scatterer, respectively. Note that the total observed wavefield is the sum of $\vec{P}_{\text{inc}}$ and reflected wavefield $\vec{P}_{\text{refl}}$ from the target domain (right side of the borehole), i.e. $\vec{P}_{\text{obs}} = \vec{P}_{\text{bl,inc}} + \vec{P}_{\text{refl}}$. 
Figure 7.7: Same as Figure 7.6, but now the simulation example is for blended source experiment with blending factor=2. Note again that the total observed wavefield is the sum of the blended incident wavefield $\vec{P}_{bl,inc}$ from the left side of the borehole and reflected wavefield $\vec{P}_{refl}$ from the target domain (right side of the borehole), i.e. $\vec{P}_{bl,obs} = \vec{P}_{bl,inc} + \vec{P}_{refl}$. 
7.3 Numerical examples of full wavefield modelling and migration: BP benchmark model

Figure 7.8: Same as Figure 7.7, but now the simulation example is for blended source experiment with blending factor=3. Note again that the total observed wavefield is the sum of the blended incident wavefield $\vec{P}_{bl,inc}$ from the left side of the borehole and reflected wavefield $\vec{P}_{refl}$ from the target domain (right side of the borehole), i.e. $\vec{P}_{bl,obs} = \vec{P}_{inc} + \vec{P}_{refl}$. 
Imaging steeply dipping salt-flanks using VSP data

Figure 7.9: a), c), e) and g) show an example common-shot gather computed using full wavefield modelling using the model without the near-surface scatterer (Figure 7.4a) after the 0th (i.e. the incident field), 1st, 2nd and 3rd iteration, respectively, for an unblended VSP data. b), d), f) and h) show the corresponding common-receiver gathers. Similarly, i), k), m) and o) same as a), b), c) and d), respectively, and j), l), n) and p) same as b), d), f) and h), respectively, but computed using the model with the near-surface scatterer (Figure 7.4b).
Figure 7.10: a), c), e) and g) show an example common-shot gather computed using full wavefield modelling using the model without the near-surface scatterer (Figure 7.4a) after the 0th (i.e. the incident field), 1st, 2nd and 3rd iteration, respectively, for a blended VSP data, with blending factor=4. b), d), f) and h) show the corresponding common-receiver gathers. Similarly, i), k), m) and o) same as a), b), c) and d), respectively, and j), l), n) and p) same as b), d), f) and h), respectively, but computed using the model with the near-surface scatterer (Figure 7.4b).
We performed the full wavefield migration on the VSP data simulated using both the density models (shown in Figure 7.4 a and 7.4b) to estimate the P-P image of the target domain in the rotated coordinate system (as explained earlier). The imaging was performed for unblended and blended data. Note that we have numerically blended the measured data to illustrate imaging of the blended VSP data [for more details on blended VSP imaging, see Soni and Verschuur, 2013b, 2014a]. We illustrate the imaging for blending factors one (unblended) and four in this chapter.

Figure 7.11 show the images obtained after the 1st and the 10th iteration of FWM. Specifically, Figures 7.11a and 7.11e show the images for VSP data simulated with the density model without a near-surface scatterer (Figure 7.4a) for blending factors one and four, respectively, after the 1st iteration. The corresponding images after the 10th iteration are shown in Figures 7.11b and 7.11f. Similarly, Figures 7.11c and 7.11g show the images for VSP data simulated with the density model with a near-surface scatterer (Figure 7.4b) for blending factors one and four, respectively, after the 1st iteration. Again, the corresponding images after the 10th iteration are shown by Figures 7.11d and 7.11h.

Note that the images after the 1st iteration are equivalent to results from conventional imaging techniques using the primary-reflections only. The inversion-based FWM helps to improve the amplitude of the steeply-dipping or overturned salt flank, using the full-wavefield. Furthermore, we illustrate imaging of the blended VSP experiment, where FWM also helps to suppress the crosstalk noise in the image due to wavefield interference as expected and yields a reasonable image [Verschuur and Berkhout, 2011; Soni and Verschuur, 2014b]. Finally, we illustrate the effect of a near-surface high-contrast scatterer in the imaging, where the total illuminating wavefield enhances due to the coda. The FWM also effectively handles the crosstalk due the scattering in the illumination wavefield that occurs outside the target domain.

### 7.4 Discussion

Walkaway VSP data has proven to be useful in imaging steeply dipping or overturned salt-flanks. We have discussed an extended FWM approach using horizontal wavefield extrapolation to incorporate turning waves in VSP data to image steep structures. In this chapter, we assume that we know the total incident wavefield at the borehole that illuminates the target domain. In practice, since VSP measurements are multicomponent, we can perform directional decomposition of the wavefield to estimate incident wavefield and reflected wavefield from the total observed data. Furthermore, we have illustrated that FWM can handle both unblended and blended VSP data effectively for imaging salt-flanks. The inversion-based imaging helps to suppress the crosstalk due to a blended experi-
ment and complex incident fields caused by scatterers present outside the target domain.

**Figure 7.11**: FWM images of the target domain for the model shown in Figure 7.4a: after the 1rst iteration for a) unblended and e) blended VSP data (blending factor=4) and after the 10th iteration for b) unblended and f) blended VSP data (blending factor=4). Similarly, for the model shown in Figure 7.4b, c) (unblended) and g)(blended, blending factor=4) are images after the 1rst iteration and d) (unblended) and h)(blended, blending factor=4) are images after the 10th iteration. Note the portion of image shown by orange arrows. Comparing this part of images between b) and d) or between f) and h) show an improvement in image due to use of scattered wavefield in the incident wavefield in presence of near-surface scatterer.
Imaging steeply dipping salt-flanks using VSP data
Field data application: deep-water Gulf of Mexico data

8.1 Introduction

In this chapter, we will illustrate the application of full wavefield migration on a field data set from deep-water in the Gulf of Mexico. The dataset comprises of two nearly orthogonal walkaway VSP lines. The following sections will discuss the VSP acquisition geometry, preprocessing used before FWM and finally the imaging results.

8.2 VSP geometry and preprocessing

The walkaway VSP data used to illustrate FWM were acquired in deep-water of the Gulf of Mexico in a block under Anadarko Petroleum Corporation. The water-bottom depth in this area is approximately 1200m. We have received the PP reflection data after basic preprocessing. The data comprises of two nearly orthogonal walkaway VSP lines and a set of nine receivers in a deep borehole. The source separation on the surface is approximately 60m, with the maximum source offset from the wellhead around 3000m. The receivers were located from 3230m to 3350m, with an inter-geophone spacing of 15m. For a detailed description of this data, please see O’Brien et al. [2013a,b]. These papers discuss imaging of first-order free-surface multiples using the mirror-model concept. Figure 8.1 illustrates the map view of the two VSP lines. Furthermore, Figures 8.2 and 8.3 show all nine received pre-processed common-receiver gathers for line A and line
As a preprocessing step to get the data suited for FWM, we performed regularization and interpolation of the received VSP data in the common-receiver domain. The source lateral sampling is irregular and has a gap near the well and, therefore, show gaps for the near-offset in the common-receiver domain. We have used a sparse hybrid-Radon transform approach to do regularization and interpolation, and estimating traces every $30\, m$. Figure 8.4 shows an example of a common-receiver gather from line A and line B, before and after the regularization and interpolation, using sparse hybrid-Radon transforms. For more details on the algorithm used, see Verschuur et al. [2012]. In the past, the Radon transform was also used to interpolate 3D VSP data [see for example Hindriks and Duijndam, 1998]. In the next section we discuss the estimation of the buried source wavefield in the reciprocal domain, followed by imaging using full-wavefield migration.

![Figure 8.1: Map view of the two orthogonal walkaway VSP lines - line A (dip line) and Line B (strike line) and the schematic location of the well. The colors annotate the varying number of sources for each set of three receivers available.](image)

### 8.3 FWM: source estimation and imaging

As mentioned in chapter two and three, the direct arrivals of the VSP data are picked and inversely extrapolated to estimate the equivalent buried sources in the common-receiver domain. Figure 8.5 shows an example of a common-receiver and
common-shot gather from the two lines and the picked direct arrivals. See the figure caption for specifications. For the source estimation and imaging, we have got a 3D PSDM (pre-stack depth migration) velocity cube from the same area.
Figure 8.4: An example of a common-receiver gather showing the irregular traces and near-offset gap from a) line A and c) line B. The corresponding common-receiver gather after regularization and interpolation from b) line A and d) line B, using sparse hybrid Radon transforms.

Figure 8.6 shows the aerial view map of the 3D velocity field annotated with the two walkaway VSP lines - line A and line B and the extracted 2D velocity fields along the VSP acquisition geometry. The 2D extracted velocity fields for the two VSP lines are shown in Figure 8.7.

Furthermore, Figure 8.8 shows an example estimated source vector in the space-time domain using the method discussed above. Note that the overburden image obtained using the free surface multiples help to calibrate the amplitude of this estimated source wavefield by incorporating the overburden transmission effects. Using the estimated source wavefield, the given velocity model and the pre-processed VSP data, we perform full wavefield migration (as described in chapter three). The images are compared after the 1st and the 5th iteration of FWM.

Note again, the image obtained after the 1st iteration is equivalent to a conventional image obtained using the primary-only upgoing wavefields. Figures 8.9 and 8.10 show the FWM results and compares the images for line A and line B obtained using only the upgoing primaries wavefield, using only the multiples in
8.3 FWM: source estimation and imaging

Figure 8.5: An example of a common-receiver gather from a) line A and e) line B. An example of a common-shot gather from c) line A and g) line B. Picked direct arrival shown in the corresponding common-receiver domain for b) line A and f) line B, and in the corresponding common-shot domain for d) line A and h) line B.

Figure 8.6: Aerial map of the 3D survey (extracted using the 3D PSDM velocity model), annotated with the two walkaway VSP lines A and B (in black) and the spatial location of extracted 2D velocity fields along the two VSP lines, using the 3D PSDM velocity model for migration.
least-squares imaging and using the full wavefield, respectively. Figure 8.11 shows the same image comparison for two VSP lines, plotted in wiggle-traces overlaid on their corresponding migration velocities. Clearly, we see a significant extension in illumination from including the full wavefield in the inversion, away from the well trajectory as well as above the borehole receiver locations. Furthermore, Figure 8.12 shows an example of a common-receiver gather comparing the observed and estimated data and the difference i.e. data misfit after the 5th iteration of FWM.

8.4 Discussion

In this chapter, we have discussed an application of full wavefield migration on walkaway VSP field data acquired in deep-water in the Gulf of Mexico. We have received a pre-processed PP reflection data, along with a 3D velocity field derived from a 3D seismic survey in that region. We performed a sparse hybrid Radon-transform based interpolation and regularization in the common-receiver domain to fill the near-offset gaps and increase the source sampling. The interpolated data is used in full wavefield migration. The source is estimated using the picked direct arrivals. From the images obtained by our FWM, we clearly see a significant improvement in illumination and resolution in areas away from the well trajectory and also in the overburden, where the primaries-only upgoing wavefields failed to illuminate.

In the field-data experiment, we not only have a limited number of receivers, but they are also all located at a similar depth, very deep near the reservoir. This
limits the effectiveness of the FWM scheme, in suppressing remnant crosstalk as well as limits the increase of illumination in the deeper section. This could have been resolved, and thereby yielding a much better illumination at the reservoir level if we would have measured data from three of four receiver arrays at different depth levels. In the next chapter, we discuss the advantage of such type of acquisition design for VSP to effectively image the reservoir and overburden using full wavefield migration.

Furthermore, since we have estimated the source wavefield in FWM process, the image obtained through the inversion scheme is a true-amplitude image. However, if we look at the data misfit carefully, we see higher residual for the primary reflections. This is due to imperfect transmission corrections applied to the estimated source wavefield. This effect can also be solved with more receivers at different depths in VSP acquisition, which can help to estimate the effective transmission accurately through the overburden more accurately.

We also noticed that after a few iterations of FWM, the convergence slowed and the algorithm struggle to explain all of the observed data. This is due to the fact that in this illustration, we only estimate the angle-independent scalar reflectivity images, however in the real world, the reflection properties have an angle-dependent behavior. In addition, the estimated source is not perfect and that could also lead to slow convergence of the algorithm.
Figure 8.9: Image of line A obtained using a) only the upgoing primaries i.e. after the 1st iteration of FWM, b) only the multiples, after the 5th iteration of FWM, and c) the full wavefield, after the 5th iteration of FWM.
Figure 8.10: Image of line B obtained using a) only the upgoing primaries i.e. after the 1\textsuperscript{st} iteration of FWM, b) only the multiples, after the 5\textsuperscript{th} iteration of FWM and c) the full wavefield, after the 5\textsuperscript{th} iteration of FWM.
Figure 8.11: FWM Images plotted in wiggle-traces overlaid on the migration velocity after the 1st iteration for a) line A and c) line B, and after the 5th iteration for line b) line A and d) line B.
Figure 8.12: a), b) and c) show an example of a common-receiver gather of the observed data, the estimated data and the data misfit or residual for line A, respectively, after the 5th iteration of FWM. Similarly, d), e) and f) show an example of a common-receiver gather comparing the observed data, the estimated data and the data misfit or residual for line B, respectively, after the 5th iteration of FWM.
Conclusions, recommendations and future research

In this chapter, we will discuss the conclusions and recommendations based on this research and future research plans. Finally, we will discuss an overview of various seismic inversion schemes.

9.1 Conclusions

In seismic imaging, multiples in the measured data are generally considered as noise. If they are not removed prior to a conventional primaries-only migration schemes, they create artifacts i.e. false structures in the image that can make the interpretation deceptive. However, the multiples provide an extra source of illumination in the area where the primary wavefield may have failed to illuminate the field. In this research, we have introduced an imaging technology termed full wavefield migration, which aims to utilize the multiples (both surface multiples and internal multiples) in the observed data in imaging. This technology can handle both surface and borehole seismic data. In this research we discuss its application on borehole seismic or vertical seismic profiling (VSP) data. The primary-only images of VSP data suffer from imaging artifacts and low illumination as we move further away from the well. Hence, using multiples in the imaging has proven to improve illumination and resolution significantly in VSP images.

In chapter two, three and four, we have discussed the forward modelling, the inversion scheme and illustrated various synthetic examples, respectively. The
Conclusions, recommendations and future research

Numerical examples illustrate the advantages of using the full wavefield in the imaging of VSP data in terms of better illumination and improved resolution of VSP images. The multiples help to image in the overburden area above the receivers where the primaries-only wavefields fail to illuminate. The closed-loop or inversion-based imaging scheme helps us to get a true-amplitude image that can explain the full wavefield in the measured data. Furthermore, the inversion scheme depends on an accurate estimation of the source wavefield. In VSP data, it is relatively easier compared to surface seismic data to estimate the source wavefield from the direct arrivals in the measured data.

In chapter five, we have illustrated the potential of FWM to image blended source VSP data, without the need of a separate active deblending step. Again, the inversion-based imaging process can handle complex source wavefields more effectively than a cross-correlation based imaging scheme. The estimated reflectivity image explains the full wavefield of the measured blended VSP data.

In chapter six, we introduced the extension of acoustic full wavefield migration to incorporate converted waves, i.e. elastic full wavefield migration. We have shown elastic full wavefield modelling using a horizontally layered model and the angle-dependent elastic reflectivity parameters derived from Aki and Richards approximation of the Zoeppritz equations. Understanding the elastic full wavefield modelling is critical to begin the inversion scheme. The inversion of elastic or multi-component VSP data is discussed as one of the future research plans.

In chapter seven, we have extended FWM by incorporating the turning waves using horizontal or orthogonal wavefield extrapolation schemes in a rotated coordinate system. We demonstrate its use in imaging near-vertical or steeply-dipping salt flanks.

In chapter eight, we have illustrated the application of the proposed imaging scheme on two sets of walkaway VSP data from a deep-water region in the Gulf of Mexico. Only one set of receiver array was placed very deep and close to the reservoir to record the data. We have shown that the image obtained from full-wavefield migration has a wider illumination especially in the shallow region due to strong surface multiples present in the data. However, the image could have been better in terms of illumination at the reservoir level if the acquired data had two or three sets of receivers recorded at different depths both in the overburden and close to the reservoir. The proposed ideal geometry of the VSP data to make better use of full-wavefield migration must have receiver data at different depth levels. This aspect is discussed in the next section.

In terms of an overall conclusion from this research work - we have shown the feasibility and applicability of full wavefield migration in imaging VSP data. We have demonstrated that multiples, when handled properly, can add extra illumination, especially away from the well vicinity. However, in the current scheme, we have mostly restricted ourselves to estimate an angle-independent $P-P$ reflec-
tivity of the subsurface. VSP data are often multicomponent records. Hence, ideally we need to estimate the elastic angle-dependent reflectivity to explain the multicomponent measurements. Also, the current imaging scheme assumes isotropic medium properties, whereas there are various geological scenarios where the anisotropic behavior of $P$ and $S$-wave velocities are pronounced. Furthermore, from the field data experiment, we found that the acquisition geometry is crucial to make FWM more effective in terms of illumination in the deep as well as suppressing remnant crosstalk. A walkaway VSP data with multi-level receiver arrays placed at different depth levels can help to improve illumination especially at the deep reservoir level. Finally, like any migration technology, we require a smooth background velocity model.

In the next section, based on this research, we would like to discuss some recommendations and future research plans to image VSP data using FWM.

### 9.2 Recommendations and future research plans

#### Angle-dependent elastic imaging of VSP data

In this thesis, we pre-dominantly focused on structural imaging of VSP data in an acoustical sense (considering only $PP$ reflections). However, in reality, the earth layers have angle-dependent reflection properties that cause wave mode conversion. A simplified elastic full wavefield modelling scheme is discussed in chapter six. Our future research plan includes incorporating the converted waves in the measured multi-component VSP data to estimate the full elastic reflectivity parameters i.e. $R_{pp}$, $R_{ps}$, $R_{sp}$ and $R_{ss}$. Figure 9.1 shows a block diagram for such a scheme.

One of the major challenges towards elastic imaging would be proper parametrization of the reflection coefficients, especially for high-contrast media where the angle-dependent reflection coefficients become complex-valued at and above post-critical angles.

#### Extension to anisotropic velocities

In this thesis, we have assumed an isotropic medium for imaging. As a future extension to FWM on VSP and surface data, anisotropic full wavefield migration is an ongoing research topic [for some introductory examples, see Alshuhail et al., 2014].

#### Extension to 3D VSP data and simultaneous VSP-surface data imaging

One of the future research plans is to perform 3D full wavefield migration on 3D VSP data. The concepts discussed in this thesis are illustrated for 2D walkaway VSP data. However, the concepts of FWM are equally applicable in 3D and can be extended to handle 3D VSP geometries. Furthermore, handling surface
Conclusions, recommendations and future research

Figure 9.1: Block diagram showing the feedback loop for future elastic FWM. The aim will be to estimate the full angle-dependent elastic reflection coefficients from the measured data, through an inversion-based elastic full wavefield migration scheme. Note that the migration velocity in such case would include both P-wave and S-wave velocity fields.

Seismic data and VSP data simultaneously in JMI is an important topic for future research [see an introductory discussion in Marhfof and Verschuur, 2014]. The simultaneous inversion of VSP and surface seismic data will yield more reliable depth images and velocity models, taking advantages of both surface seismic and VSP data.

Suggestions on multi-level receiver measurements

From experience on field-data, we realize that in many cases, walkaway VSP data is acquired at one depth level, close to the reservoir. The limited data measured at one depth level (using one set of receiver arrays in the borehole) can in fact make it difficult for the least-squares inversion based FWM to suppress all crosstalk due to strong multiples, and hence lead to slow convergence of the algorithm. However, measuring data at different depth levels in two or more locations can help to suppress any remaining crosstalk and improve convergence. To illustrate this suggestion, we have mimicked a seismic section with a fault velocity model (shown in Figure 9.2a), from one of the VSP lines shown in chapter eight. The corresponding effective angle-independent reflectivity model is shown in Figure 9.2b.
9.2 Recommendations and future research plans

Figure 9.2: a) Fault velocity model, inspired by the Gulf of Mexico image (for reference, see O’Brien et al. [2013a,b]), and b) corresponding effective angle-independent reflectivity model. c), e), g) and i) show primary-wavefield images using receiver set one, two, three and all, respectively. Similarly, d), f), h) and j) show full-wavefield images using receiver set one, two, three and all, respectively. Note that the crosstalk is suppressed when multi-level receivers are used.
Now we have used three sets of 11 receivers, with depth spacing of 10m each, at depth levels starting from 1250m, 1550m, and 1950m. Figure 9.2 shows the images obtained using the three receiver sets separately as well as all together, for both primaries-only wavefields and the full-wavefield. Note that the crosstalk that appeared at different depth levels due to the limited number of receivers at one depth level eventually got suppressed when the simultaneous imaging is done for all three sets. Hence, using multi-level receivers helps better convergence of the algorithm.

Joint migration-inversion of VSP data

In this thesis, we have assumed that for the imaging of VSP data using full wavefield migration, we have a proper migration velocity model derived from surface seismic data or well logs. However, this might not be true in all cases. Furthermore, in chapter four, we illustrated that if we use a wrong velocity model in full wavefield migration, the data-misfit in the inversion scheme is considerably higher. This residual could also be used to drive the update of the velocity model.

The scheme to update or estimate both the reflectivity image and the migration velocity model is termed joint-migration inversion (JMI) [for an introduction to JMI and applications in surface seismic data, see Berkhout, 2012; Staal and Verschuur, 2012, 2013; Staal et al., 2014]. We propose that joint-migration inversion can be equally important for borehole seismic or VSP data. Figure 9.3 shows a block diagram for the inversion scheme in JMI.

As a preliminary test, we will demonstrate an example of JMI for VSP data. Figure 9.4 shows the true velocity model, the initial velocity model and the update velocity model using JMI. The imaging results are depicted in Figure 9.5. Figure 9.6 shows the same images overlaid with the true reflectivity curves. Note that the FWM image using the updated velocity model, as shown in Figure 9.5d, is improved in terms of imaging quality as well as the depth of the reflectors. However, in this preliminary test, we found that the strong transmitted direct arrivals are too dominant in the inversion schemes. We recommend that in future research, we need to test different weighting for the reflection and transmitted data to make better use of both components.

Incorporating multiples in 3D VSP acquisition-design

In today’s industrial practice, 3D VSP acquisition design involves generating synthetic data using a finite-difference scheme for various possible geometry designs and performing conventional imaging using the simulated data. A prior 3D velocity model derived using surface seismic or well-logs are used in the finite-difference schemes. Illumination by primary wavefields are also studied using 3D ray-tracing tools to understand the impact of different geometries. The acquisition geometry that yields the best image is selected.

However, these huge 3D VSP acquisitions can be optimized by including strong
9.2 Recommendations and future research plans

![Figure 9.3: Block diagram showing the general feedback loops for JMI. The measured data is imaged, yielding a subsurface reflectivity. The estimated reflectivity is used to simulate the response using full wavefield modelling, which is compared to the measured data. The residual of the measured and simulated data after adaptive subtraction is fed back in the two loops to iteratively update both the reflectivity and the migration velocity model. Similar to FWM, in JMI each iteration adds or uses a higher order of multiples to estimate the image and velocity. R denotes the estimated reflectivity map.](image)

multiples in the illumination studies and subsequent imaging. In future research, we can perform illumination studies of primaries and strong surface and internal multiples to aid in optimization of acquisition design as well as obtaining high-resolution VSP images using all the multiples. Such a scheme is already an ongoing research topic for optimization of surface seismic geometry design. The concept is discussed in Kumar et al. [2014b,a].
Figure 9.4: a) True velocity model, b) initial velocity model (1.5D model built using 90% of the original velocity model and smoothed, at the well location), c) update velocity model using JMI and e) the difference between the initial and the updated velocity models. e) 1D velocity profiles at the well location from the true velocity model (blue curve), from the initial velocity model (green curve) and the updated velocity model (red curve).
9.2 Recommendations and future research plans

Figure 9.5: a) True velocity model, b) FWM image using the true velocity model, c) FWM image using the initial velocity model and d) FWM using the updated velocity model through JMI.

Figure 9.6: Same as Figure 9.6, overlaid with the true reflectivity curve (black curve) on the estimated images. The dotted lines in d) show the extent of image corrected using the updated velocity model.
Incorporating seismic absorption or quality factor 'Q' in imaging and inversion

Seismic inelastic absorption\(^1\) or attenuation leads to loss of seismic energy into heat and mathematically, often described by seismic-quality factor \(Q\) or the absorption coefficient \(\alpha\). Quality factor \(Q\) is defined as the ratio of \(2\pi\) times the peak energy to the energy dissipated in a cycle i.e. the ratio of \(2\pi\) times the power stored to the power dissipated. The seismic \(Q\) of rocks is of the order of 50 to 300. \(Q\) is related to other measures of absorption (see below) [Sherrif, 2002]:

\[
\frac{1}{Q} = \frac{\alpha V}{\pi f} = \frac{\alpha \lambda}{\pi} = \frac{hT}{\pi} = \frac{\delta}{\pi} = \frac{2\Delta f}{f_r},
\]

where \(V\), \(f\), \(\lambda\) and \(T\) are, respectively, velocity, frequency, wavelength, and period. The absorption coefficient \(\alpha\) is the term for the exponential decrease of amplitude with distance because of absorption; the amplitude of plane harmonic waves is often written as \(Ae^{-\alpha x}\sin2\pi f(t - \frac{x}{V})\), where \(x\) is the distance traveled. The logarithmic decrement \(\delta\) is the natural log of the ratio of the amplitudes of two successive cycles. The last equation above relates \(Q\) to the sharpness of a resonance condition; \(f_r\) is the resonance frequency and \(\Delta f\) is the change in frequency that reduces the amplitude by \(1/\sqrt{2}\). The damping factor \(h\) relates to the decrease in amplitude with time, \(A(t) = A_0e^{-ht}\cos\omega t\). Figure 9.7 shows a summary of absorption terminology.

<table>
<thead>
<tr>
<th>Term</th>
<th>Symbol</th>
<th>Equation</th>
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<th>(Q)</th>
<th>(\eta)</th>
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<td>Quality factor</td>
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<td>(\pi/\nu)</td>
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<tr>
<td>Absorption coefficient</td>
<td>(\eta)</td>
<td>(=(-1/\nu) \ln (A/A_0))</td>
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<td>Damping factor</td>
<td>(h)</td>
<td>(=(-1/\nu) \ln (A/A_0))</td>
<td>(\pi/f Q)</td>
<td>(\pi/\nu f)</td>
<td>(f/\delta)</td>
<td>(h)</td>
<td>(f)</td>
</tr>
<tr>
<td>Logarithmic decrement</td>
<td>(\delta)</td>
<td>(=\ln (A_1/A_2))</td>
<td>(\pi/\delta)</td>
<td>(\pi Q)</td>
<td>(\nu)</td>
<td>(\nu/\delta)</td>
<td></td>
</tr>
<tr>
<td>Loss angle</td>
<td>(\phi)</td>
<td>(=\cot^{-1} Q)</td>
<td>(\cot^{-1} Q)</td>
<td>(\cot^{-1} Q)</td>
<td>(\cot^{-1} Q)</td>
<td>(\cot^{-1} Q)</td>
<td></td>
</tr>
</tbody>
</table>

Figure 9.7: Absorption terminology: sometimes this terminology is used for attenuation because of factors other than absorption. \(E\) = energy, \(\Delta E\) = energy lost in one cycle, \(\lambda\) = wavelength, \(f\) = frequency, \(x\) = distance, \(t\) = time, \(A/A_0\) = amplitude/(initial amplitude), \(A_1/A_2\) = amplitude/(amplitude one cycle later) [Sherrif, 2002].

Seismic absorption is a vast subject of research by its own. In the past, we have seen several work on seismic modelling and wave propagation concepts in

\(^1\)A process whereby energy is converted into heat while passing through a medium. Absorption for seismic waves is typically about 0.25 dB/cycle and may be as large as 0.5 dB/cycle, [Sherrif, 2002].
attenuating media [for example see Kjartansson, 1979; Johnston and Toksoz, 1980; Krebes and Hron, 1980; Bourbié and Serrano, 1983; Varela et al., 1993; Liao and McMechan, 1996]; and on the subject of inverse Q-filtering and estimation of Q-factor using both surface seismic and VSP data [for example see Hargreaves and Calvert, 1991; Wang, 2002; Guerra and Leaney, 2006; Wang, 2006, 2008; Blias, 2012; Wang, 2014]. The details of various algorithm on Q estimation is beyond the scope of this thesis, however, we will discuss briefly a possible way to incorporate absorption in FWM and JMI by including the effects in the wavefield extrapolation.

Basically, if we see in 1D case using the phase shift operator concept [Gazdag, 1978], we can define the wavefield extrapolation between two depth levels separated by $\Delta z$ as [Varela et al., 1993; Wang, 2008]:

$$P(z + \Delta z, \omega) = P(z, \omega)e^{jk_z\Delta z},$$

(9.2.2)

where $P$ is the Fourier transform of the wavefield, $z$ is the depth location where wavefield is measured, $\omega$ is the angular frequency and $k_z$ is the wavenumber in $z$-direction. Note again that forward or inverse extrapolation implies taking $\Delta z$ as negative or positive, respectively. Now, to include absorption in the above equation, $k_z$ is modified and become a complex function of frequency, given by:

$$k_z = -\left(\frac{\omega}{V} + j\alpha\right),$$

(9.2.3)

where $V$ is the frequency-dependent phase velocity\(^2\) and $\alpha$ is the absorption coefficient. Generally, $\alpha$ is a function of frequency which represents the dissipative attenuation and given by $\alpha = \omega/2QV$. Quality factor $Q$ is assumed to be constant within the seismic bandwidth. Further, the phase velocity is also a function of frequency that accounts for dissipative dispersion in the model. Note that there are several discussions on the form of frequency dependence on the phase velocity, see for example Kolsky [1956]; Futterman [1962]; Robinson [1979]; Kjartansson [1979]. Also, Toverud and Ursin [2005] discuss various attenuation models using VSP data comprehensively. The resulting expression, which is valid for relatively large and constant $Q$ is given by:

$$\frac{V}{V_r} \approx 1 - \frac{1}{\pi Q} \ln\left(\frac{\omega}{\omega_r}\right),$$

(9.2.4)

where $\omega_r$ is the reference angular frequency, $V_r$ is the propagation velocity in the reference frequency, which is given by the ratio of the distance traveled and its time equivalent, $\Delta \tau$, i.e. $V_r = \Delta z/\Delta \tau$. In other words, $\Delta \tau$ is the incremental traveltime at the reference frequency $\omega_r$. Substituting Equations (9.2.3) and

\(^2\)Phase velocity: The velocity of any given phase (such as a trough) or a wave of single frequency; it may differ from group velocity because of dispersion. Sometimes called ŚtroughŚ velocity or ŚpeakŚ velocity [Sherrif, 2002].
(9.2.4) in (9.2.2), and changing depth to its equivalent in time, we get:

\[ P(\tau + \Delta \tau, \omega) = P(\tau, \omega)U(\Delta \tau, \omega)e^{-j\omega \Delta \tau}, \]

(9.2.5)

where \( U(\Delta \tau, \omega) \) is the absorption operator given by:

\[ U(\Delta \tau, \omega) = \exp\left(\frac{\omega \Delta \tau}{Q}\left[0.5 + \frac{j}{\pi} \ln\left(\frac{\omega}{\omega_r}\right)\right]\right). \]

(9.2.6)

From Equation (9.2.5), we can clearly see that the propagation of the wavefield from one depth level to the next include two separate terms, one accounting for the phase shift (or time shift) and other being the absorption operator \( U \).

A similar approach in the propagation operator can be used for incorporating FWM in attenuating media, if \( Q \)-factor is known. For unknown \( Q \)-factor, future research can include simultaneous estimation of reflectivity and a complex velocity model in JMI, where the imaginary term in the velocity model accounts for the absorption in the observed data. In slightly different approaches, we have also seen estimation of \( Q \)-factor in recent research where they are estimated in ray-based tomography [for example, see Cavalca et al., 2011] and full waveform inversion [for example, see Bai and Yingst, 2013].

9.3 Seismic inversion: an overview of various schemes

In this section, we discuss the similarities and differences in the concept of full wavefield migration (FWM) and joint migration-inversion (JMI), full waveform inversion (FWI) for velocity estimation, simultaneous impedance-background model inversion in FWI, least-squares reverse time migration (LSQRTM), velocity perturbation and impedance perturbation inversion using modified RTM and contrast-source inversion (CSI). Note that all of these inversion-based processes aimed to estimate a set of parameters from the measured seismic data. The main difference in all these schemes lies in the model parametrization, the forward modelling operator used in the inversion as well as the underlying assumption on the type of data (primary-only, full wavefield, diving waves, etc). In this discussion, we will present an overview of various schemes, and exclude the comparison of their computational costs involved. Figure 9.8 list these inversion schemes in a chart diagram, highlighting the output parameters in them.
Figure 9.8: Flavors of seismic inversion, starting from top to down in the figure above: full waveform inversion (FWI) using mostly diving waves and primary wavefield to yield $V_p$ (P-wave velocity), $V_s$ (S-wave velocity) and $\rho$ (density) [Lailly, 1983; Tarantola, 1986, 1987; Virieux and Operto, 2009; Warner et al., 2013]; full waveform inversion (FWI) using mostly diving waves and primary wavefield to yield A.I. (acoustic impedance), E.I. (elastic impedance) and $\rho$ [Snieder et al., 1989; Cao et al., 1990]; Reverse time migration (RTM) using primary-only wavefield to yield structural image or reflectivity ($R$). [Baysal et al., 1983; McMechan, 1983] and Least-squares RTM, again using primary-only wavefield but in a closed-loop inversion scheme [Dai et al., 2012; Yao and Jakubowicz, 2012a,b; Zheng and Schuster, 2014; Zhang et al., 2014a; Tan and Huang, 2014]; A modified Reverse-time migration using primary-only wavefield to yield acoustic impedance perturbations ($\delta A.I.$)and velocity perturbations ($\delta V_p$); Full wavefield migration (FWM) and Joint Migration-Inversion (JMI) using the full wavefield i.e. primaries and multiples (both surface and internal) to yield subsurface angle-dependent image $R$ and background velocity model $V_p$ and $V_s$ [Berkhout, 2012; Staal and Verschuur, 2012; Davydenko et al., 2012; Berkhout, 2014a,b,c; Soni and Verschuur, 2014a]; Contrast Source inversion (CSI) using the full wavefield to yield high-resolution elastic contrast parameters ($\chi$) [van den Berg and Kleinman, 1997; van den Berg et al., 1999; Abubakar et al., 2008; Haffinger et al., 2013; Rizzuti and Gisolf, 2014a,b]. Note that in the figure above, the order in which the schemes are arranged does not have a specific meaning.
In JMI, the parameters to estimate are reflectivity and background (migration) velocity. We assume a scale separation between these two parameters, i.e. reflectivity accounts for the amplitude behavior and background velocity accounts for phase changes (wavefield propagation) in the seismic measurements. In FWM, we assume that a background velocity is known and, therefore, we aim to estimate only the subsurface reflectivity. In JMI and FWM, the forward modelling operator iteratively builds all the scattering in the wavefield and corrects for transmission effects. Hence, the inversion process uses all the orders of scattering to estimate the subsurface parameters. Note that the scale separation also helps to make the JMI process robust in terms of initial velocity model and it does not suffer from cycle skipping\textsuperscript{3} issues [for a discussion, please see Staal et al., 2014]. For an extensive discussion on FWM and JMI, see also Berkhout [2012, 2014a,b,c]. Further, the estimated reflectivity could include the angle-dependent behavior of the main interfaces that in turn can be used for AVO-type inversion or detailed reservoir studies. Also note that the forward iterative equation used in FWM complies with the Generalized Bremmer series (See Appendix A for a discussion on this).

In FWI, the parameters to estimate are the high-resolution elastic properties of the subsurface i.e. $P$-wave velocity, $S$-wave velocity and density. In most cases, the forward modelling operator used in FWI is a finite-difference operator, therefore there is no scale-separation between the amplitude and propagation behavior of the wavefield. This make the inversion scheme highly non-linear with respect to model parameters and often suffers from cycle skipping. Therefore, the success of the scheme is highly dependent on the availability of very low frequencies in the data as well as the starting model. For an extensive discussion on FWI, see Lailly [1983]; Tarantola [1986, 1987]; Virieux and Operto [2009]; Warner et al. [2013]. Recent advancement in broadband acquisition is proving to play a major role in improving the FWI results [ten Kroode et al., 2013]. Also note that most of the recent practical applications are restricted to use only diving waves in the measured data. Also, the final product - a high-resolution velocity model is not yet appreciated in the current practice, because the obtained model are in turn smoothed and used in the conventional migration algorithms to obtain subsurface images. The latter usually do not account for multiples in the data.

In the same family of FWI, we also saw proposals to estimate simultaneously the background model and impedances (both $P$-wave impedance and $S$-wave impedance). This is another way to do scale separation, where the forward modelling operator is based on the impedance and background model. For introductory discussions on this type of inversion, see Snieder et al. [1989]; Cao et al. [1990]. Let us compare this inversion scheme with JMI in terms of parameterization. We know that reflectivities are the boundary properties of the subsurface.

\textsuperscript{3}Cycle skipping: jumping a leg in correlating events, as may occur in matching non corresponding peaks in automatic statics programs. [Sherrif, 2002]
They are physically equivalent to the impedance contrasts or directional gradient of impedance, but not impedance itself. Unlike impedance (acoustic or elastic), it includes the angle-dependent reflection behavior at an interface. On the other hand, impedance are the layer properties without any angle-dependent information. Further, the earlier development of this type of inversion scheme assumes Born-approximation i.e. accounting only for primary wavefields in the data, to obtain a smooth background model. This inherently assumes that higher order of scattering is removed from the measured data.

Reverse time migration (RTM) [Baysal et al., 1983; McMechan, 1983] aim to estimate subsurface reflectivity with an underlying linear single-scattering assumption. The least-squares RTM makes the scheme closed-loop, where the reflectivity or the structural image can be updated iteratively using only primary wavefields [Yao and Jakubowicz, 2012a,b; Dai et al., 2012; Zheng and Schuster, 2014; Zhang et al., 2014a; Tan and Huang, 2014]. A modified version of RTM has been proposed recently Zhang et al. [2014b], which aim to estimate subsurface impedance perturbation and velocity perturbation using the primary-only wavefield. In this scheme, the output angle-gathers using a modified imaging condition allows to estimate the impedance perturbation from the near-angle stacked image and the velocity perturbation from the far-angle image. This scheme makes a good intermediate link between imaging using RTM to FWI. In this modified RTM scheme, the velocity perturbation can provide high-resolution details, although it is limited, of course by the small-angle approximation.

Further, we also saw proposals to extend the concept of RTM (generally termed as nonlinear RTM) to incorporate multiples and also converted wavefields through modifying the imaging condition. Non-linear or multiple least-squares reverse time migration (NLSQRTM) extends the parameter estimation by including the interaction of multiples through modifying the imaging condition, modifying wavefield extrapolation and aim to estimate a scattering contrast model [Fleury, 2013] in an acoustic scenario. However, this inversion scheme does not give an integrated subsurface reflectivity as an output. Instead of one output, it gives four sub-images that represent same subsurface from different aspects. The non-linear imaging condition is further extended in elastic case and discussed comprehensively in Ravasi and Curtis [2013]. The modified nonlinear imaging condition accounts for multiply scattered and multiply converted waves.

Finally, Contrast Source inversion (CSI) [van den Berg and Kleinman, 1997; van den Berg et al., 1999; Abubakar et al., 2008; Haffinger et al., 2013; Rizzuti and Gisolf, 2014a,b] aim to estimate the high-resolution elastic contrast parameters from seismic measurements. The method iteratively solves the forward scattering equation through simultaneous update of contrast-sources (i.e. the perturbed wavefields) and the actual medium contrast for a known reference medium. It uses the full wavefield by iteratively adding higher order of scattering with each iteration. The contrast parameters ($\chi$) for an elastic case include $\chi_\kappa = (\kappa - \kappa_b)/\kappa_b$, 

$$
\chi_\kappa = \frac{\kappa - \kappa_b}{\kappa_b}
$$
\( \chi_M = (M - M_b)/M_b \) and \( \chi_\rho = (\rho - \rho_b)/\rho_b \), where \( \kappa \) is the compressibility, \( M \) is the shear compliance and \( \rho \) is the density [Rizzuti and Gisolf, 2014b].

Based on the overview of various inversion schemes as discussed above, we can say that for the future research, we will see efforts to develop an inversion-scheme that will help to obtain the best possible structural image as well as true-amplitude angle-dependent image gathers, using the full-wavefield. Additionally, it should also aim to estimate the redatumed wavefields (full-wavefield redatuming) above the reservoir level (or depth of interest), with all the overburden effects removed, for a high-resolution reservoir-oriented elastic inversion.
A

Derivation of iterative full wavefield modelling using the seismic representation theorem

A.1 Introduction: two-way and one-way wave equations

This appendix is inspired by the lectures on ‘Advanced wave theory for geoscientists’, taught by professor Kees Wapenaar and professor Jacob Fokkema during February-March 2014 at CiTG, Delft University of Technology, The Netherlands. Here, we are going to derive the iterative full wavefield modelling scheme using the well-known Seismic Representation theorem [Fokkema and van den Berg, 1993]. In this appendix, we will use the lecture notes from the course, the details of which can be found in Wapenaar and Berkhout [1989], Wapenaar and Grimbergen [1996]; Wapenaar [1996a] and Wapenaar [1996b].

We aim to relate the one-way representation of seismic data to arrive at the Generalized Bremmer series which is an iterative scheme for computing upgoing and downgoing wavefield and then relate it to the concept of full wavefield modelling (discussed in chapter two). Note that in this appendix, the discussion is restricted to the acoustic case, however, using the same principles, the derivation can be extended to elastic wavefields case. Further, we will derive the equations in wavenumber-frequency domain (for an laterally invariant medium), however the derivation can be easily generalized in space-frequency domain (incorporating laterally heterogeneous medium). Let us start by defining the two-way wave equation.
We assume a laterally invariant medium as shown in Figure A.1, with $z$ the direction of wavefield propagation. Begin figure End figure For a laterally homogeneous medium, the linearized equation of continuity and motion in the wavenumber-frequency domain can be written as:

$$\rho \left( \frac{\partial P}{\partial z} + k^2 P \right) = j\omega I_v,$$  

(A.1.1)

where $V_x, V_y$ and $V_z$ represent velocity vector components in $x, y,$ and $z$ direction, respectively; $I_v$ represents the volume vector components and $P$ represents the acoustic pressure. Note that $V_x, V_y, V_z, I_v$ and $P$ are all functions of $k_x, k_y, z$ and $\omega$. They are not used in the equation above for brevity. Similarly, the linearized equation of motion can be written as:

$$\begin{bmatrix} -jk_x V_x - jk_y V_y + \frac{\partial V_z}{\partial z} \frac{j\omega}{K} P = j\omega I_v \end{bmatrix} + \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix},$$

(A.1.2)

where $F_x, F_y$ and $F_z$ are the vector components in $x, y$ and $z$ for volume density of the external force, respectively. Note again that $F_x, F_y$ and $F_z$ are all functions of $k_x, k_y, z$ and $\omega$. Also, $\rho = \rho(z)$ and $K = K(z)$ represents the density and the compression modulus, respectively. Now eliminating $V_x, V_y$ and $V_z$ from Equations (A.1.1) and (A.1.2) (for detail derivation, see [Wapenaar and Berkhout, 1989]), we get:

$$\rho \frac{\partial}{\partial z} \left( \frac{1}{\rho} \frac{\partial P}{\partial z} \right) + (k^2 - k_x^2 - k_y^2) P = -D,$$  

(A.1.3)
where \( D = D(k_x, k_y, z, \omega) \) represents the source term given by:

\[
D = -\omega^2 \rho I_v + jk_x F_x + jk_y F_y - \rho \frac{\partial}{\partial z} \left( \frac{1}{\rho} F_z \right). \tag{A.1.4}
\]

Note that \( k^2 = \frac{\omega^2}{c^2(z)} \) and \( c(z) = \sqrt{\frac{K(z)}{\rho(z)}} \).

Now, eliminating the \( V_x \) and \( V_y \) related terms for laterally invariant medium, we can represent the two-way wave equation in a compact operator notation as [Wapenaar and Berkhout, 1989]:

\[
\partial_z Q - AQ = D. \tag{A.1.5}
\]

Note again that in above equation, \( Q = Q(k_x, k_y, z, \omega), A = A(k_x, k_y, z, \omega) \) and \( D = D(k_x, k_y, z, \omega) \). For brevity, the terms between brackets is not explicitly written. Furthermore, in the aforementioned equation, \( Q \) is the two-way wavefield matrix defined as:

\[
Q = \begin{bmatrix} P \\ V_z \end{bmatrix}, \tag{A.1.6}
\]

\( D \) is the two-way source matrix defined as:

\[
D = \begin{bmatrix} F_z \\ \frac{1}{j\omega\rho} (-\omega^2 \rho I_v + jk_x F_x + jk_y F_y) \end{bmatrix}, \tag{A.1.7}
\]

and the two-way operator \( A \) connecting the wavefields and sources is defined as:

\[
A = \begin{bmatrix} 0 & -j\omega \rho(z) \\ \frac{k_z^2(z)}{j\omega \rho(z)} & 0 \end{bmatrix}. \tag{A.1.8}
\]

Note that \( k_z^2(z) = k^2(z) - k_x^2 - k_y^2 \). Further, the two-way operator \( A \) can be decomposed using eigenvalue decomposition, and can be written as:

\[
A = L \Lambda L^{-1} = \begin{bmatrix} 1 & 1 \\ \frac{k_z}{\omega \rho} & -\frac{k_z}{\omega \rho} \end{bmatrix} \begin{bmatrix} -jk_z & 0 \\ 0 & jk_z \end{bmatrix} \frac{1}{2} \begin{bmatrix} 1 & \frac{\omega \rho}{k_z} \\ \frac{1}{k_z} & -\frac{\omega \rho}{k_z} \end{bmatrix}. \tag{A.1.9}
\]

Note that \( k_z = \sqrt{(k_2 - k_x^2 - k_y^2)} \) when \( k_x^2 + k_y^2 \leq k_2 \) and \( k_z = -j \sqrt{(k_x^2k_y^2 - k_2)} \) when \( k_x^2 + k_y^2 > k_2 \).

Now, let us discuss the one-way wave equation. We define the one-way wave vector \( P \) and the one-way source vector \( S \), respectively, as:

\[
P = \begin{bmatrix} P^+ \\ P^- \end{bmatrix}, S = \begin{bmatrix} S^+ \\ S^- \end{bmatrix}. \tag{A.1.10}
\]
where $P^+$ and $P^-$ represent the downgoing and the upgoing pressure waves and $S^+$ and $S^-$ represent the downgoing and the upgoing source wavefield.

Further, the one-way and two-way wave vectors are related as [Ursin, 1983]:

$$Q = LP$$
$$D = LS$$

(A.1.11)

Substituting Equations (A.1) and (A.1) in (B.1.7) and solving systematically, we get the one-way wave equation from the two-way wave equation:

$$\partial_z Q - AQ = D$$
$$\Leftrightarrow \partial_z (LP) - L\Lambda P = LS$$
$$\Leftrightarrow L^{-1} \partial_z (LP) - \Lambda P = S$$
$$\Leftrightarrow \partial_z P - \{\Lambda - L^{-1} \partial_z L\} P = S$$
$$\Leftrightarrow \partial_z P - BP = S$$

(A.1.12)

Here, operator $B = \Lambda - L^{-1} \partial_z L$ connects the one-way wavefield matrix $P$ to the one-way source matrix $S$. Note that $\Lambda$ governs the one-way wavefield propagation and matrix $\Theta = -L^{-1} \partial_z L$ governs the scattering due to vertical variation in the medium parameters. The matrix $\Lambda$ is a function of medium slowness (as given in Equation (A.1)) and $\Theta$ is a function of reflection and transmission coefficients written as:

$$\Theta = \begin{bmatrix} T^+ & R^\cap \\ -R^\cup & -T^- \end{bmatrix} = \begin{bmatrix} 1 + R^\cup & -R^\cup \\ -R^\cup & -(1 - R^\cup) \end{bmatrix}$$

(A.1.13)

where $R^\cup$ and $R^\cap$ represent the downgoing and upgoing reflection coefficients, and as mentioned in chapter two, $R^\cup = -R^\cap$ in the acoustic approximation. $T^+$ and $T^-$ represent the downgoing and upgoing transmission coefficients and from the continuity relationship, we have $T^+ = 1 + R^\cup$. The relationship between the propagation term $\Lambda$ and propagation operator matrices $W$ (as introduced in chapter two) can be written as:

$$W(z + \Delta z, z) = \exp(\Lambda \Delta z) = \begin{bmatrix} e^{-jk^z \Delta z} & 0 \\ 0 & e^{jk^z \Delta z} \end{bmatrix} = \begin{bmatrix} W^+ & 0 \\ 0 & W^- \end{bmatrix}$$

(A.1.14)
A.2 Seismic Representation theorem and Generalized Bremmer series

Seismic representation can be described as a special form of the reciprocity theorem [Fokkema and van den Berg, 1993], obtained if one of the acoustic wavefields represents the impulse response of a reference medium, i.e. Green’s function whereas the other wavefield represents the physical wavefield in the true medium [Wapenaar and Berkhout, 1989]. Using the representation theorem, from the one-way wave equation derived in the previous section (in wavenumber-frequency domain), we have:

\[
\partial_z P = BP + S \tag{A.2.15}
\]

Further, similar to the above equation, we can also write the one-way wave equation for Green’s function as:

\[
\partial_z G = \overline{B}G + I\delta(z-z'), \tag{A.2.16}
\]

where the source is located at \(z'\). Using \(\overline{B} = \Lambda\) and \(B = \Lambda + \Theta\), we have \(B - \overline{B} = -L^{-1}\partial_z L = \Theta\), as mentioned in the previous section. Again, note that \(\overline{B}\) account for the primary-only wavefield propagation in the correct medium. Therefore, \(G = G_p\), i.e. the Green’s matrix for primaries. Now substituting this relationship is Equations (A.2.15) and (A.2.16), we get:

\[
\partial_z P - \Lambda P = \Theta P + S \tag{A.2.17}
\]

and

\[
\partial_z G_p - \Lambda G_p = I\delta(z-z'). \tag{A.2.18}
\]

Note that \(G_p(z, z')\) is the Green’s matrix for primaries and can be written as:

\[
G_p(z, z') = \begin{bmatrix}
H(z-z')W^+(z, z') & 0 \\
0 & -H(z-z')W^-(z, z')
\end{bmatrix} \tag{A.2.19}
\]

where \(H(z)\) is the heaviside step-function, \(W^+\) and \(W^-\) are the upgoing and the downgoing propagation operators, as mentioned earlier (shown schematically in Figure A.2).

From Equations (A.2.17) and (A.2.18), we can write the wavefield at any depth \(z\) as a function of scattering matrix \(\Theta\) and source term \(S\), as:

\[
P(z) = \int G_p(z, z') [\Theta(z') P(z') + S(z')] dz' \tag{A.2.20}
\]
Derivation of iterative full wavefield modelling using the seismic representation theorem

Homogeneous half-space

Figure A.2: Schematic diagram showing a Green’s matrix for primaries $G_p$ and a downgoing propagation operator $W^+$ between locations $z'$ and $z$. Figure adapted from Wapenaar [1996a].

Since the above equation is nonlinear in $P$, we can represent it as an iterative formulation, which in fact is termed as the the Generalized Bremmer series representation, given by:

$$P^{(k)}(z) = P^{(0)}(z) + \int G_p(z, z') \Theta(z') P^{(k-1)}(z') dz',$$

where

$$P^{(0)}(z) = \int G_p(z, z') S(z') dz',$$

and with $k$ the iteration number. Note Equations (A.2.21) and (A.2.22) carefully. Substituting the value for $G_p$ (Equation (A.2.19)), $\Theta$ (Equation (A.1)) and $P$ (Equation (A.1)) in equations (A.2.21) and (A.2.22), and re-writing the integral equation as a matrix-vector operator equation, we get:

$$
\begin{bmatrix}
P^+ \\
P^-
\end{bmatrix}^{(k)} = 
\begin{bmatrix}
P^+ \\
P^-
\end{bmatrix}^{(0)}
+ 
\begin{bmatrix}
W^+(z, z') & 0 \\
0 & -W^-(z, z')
\end{bmatrix}
\begin{bmatrix}
1 + R^U & -R^U \\
-R^U & -(1 - R^U)
\end{bmatrix}
\begin{bmatrix}
P^+ \\
P^-
\end{bmatrix}^{(k-1)},
$$

(A.2.23)
Splitting the equations for $P^+$ and $P^-$, we get:

\[
P^{\pm(0)} = W^{\pm}S^{\pm},\quad P^{-(0)} = W^-S^-
\]

\[
P^{\pm(k)} = P^{+(0)} + W^+[P^+ + R^\cup(P^+ - P^-)]^{(k-1)}
\]

\[
P^{-(k)} = P^{-(0)} + W^-[P^- + R^\cup(P^+ - P^-)]^{(k-1)}.
\]

Equation (A.2.24) is the same as the equation of the full wavefield model. They iteratively model the downgoing and the upgoing wavefield between two layers for a laterally invariant medium, as described in chapter two. Note again that for simplicity, the derivation in this appendix is shown in wavenumber-frequency domain, however, they can be generalized in space-frequency domain for 3D inhomogeneous medium.

Hence, in this appendix, we used to concepts of Seismic Representation theorem to describe an iterative modelling scheme which is termed as the Generalized Bremmer series\(^1\). In fact, it is equivalent to the iterative full-wavefield modelling scheme.

---

\(^1\)Generalized Bremmer series reduces a complex scattering problem to a sequence of single scattering problems. In the Generalized Bremmer series, the wave equation is first decomposed into a coupled system of one-way wave equations. For more details on Bremmer series, the readers are referred to [Bremmer, 1951; Corones, 1975; Li, 1994; Gustafsson, 2000].
Derivation of iterative full wavefield modelling using the seismic representation theorem
Matrix description used in this thesis

B.1 Extrapolation operators : W

The wavefield extrapolation operators in this thesis are based on the discrete Rayleigh integral. Using the acoustic wave equation and Green’s theorem, we can derive the Kirchhoff integral, given by [Berkhout, 1981; Wapenaar and Berkhout, 1989; Gisolf and Verschuur, 2010]:

\[
P(r_A, \omega) = \frac{-1}{4\pi} \int_{s} \frac{1}{\rho(r)} \left[ P(r, \omega) \frac{\partial G(r, r_A, \omega)}{\partial n} - \frac{\partial P(r, \omega)}{\partial n} G(r, r_A, \omega) \right] dS, \quad (B.1.1)
\]

where \( P \) is the acoustic pressure in frequency domain, \( \rho \) is the mass density distribution, \( G \) is the Green’s function and represents the pressure associated with the wavefield resulting from a point source in \( A \), where \( A \) is an arbitrary point in a volume, enclosed by a surface \( S \) (as shown in Figure B.1).

Further, considering the surface seismic acquisition from one side and following the Sommerfield radiation principle, the closed Kirchhoff integral is simplified to an integral over a plane. The Kirchhoff integral can be used further to derive the Rayleigh integrals I and II using the Neumann boundary condition\(^1\) and the Dirichlet boundary condition\(^2\) for Green’s function, respectively. We can split the total wavefield and the Green’s function into the upgoing and the downgoing

\(^1\)Neumann boundary condition: \( \frac{\partial G}{\partial n} = 0 \) at \( z = z_0 \)
\(^2\)Dirichlet boundary condition: \( G = 0 \) at \( z = z_0 \)
components, which satisfies the one-way wave equations as below:

\[
\begin{align*}
\frac{\partial P^+}{\partial z} + jH_1 * P^+ &= 0 \\
\frac{\partial P^-}{\partial z} - jH_1 * P^- &= 0 \\
\frac{\partial G^+}{\partial z} + jH_1 * G^+ &= 0 \\
\frac{\partial G^-}{\partial z} - jH_1 * G^- &= 0,
\end{align*}
\] (B.1.2)

where \( P = P^+ + P^- \), \( G = G^+ + G^- \) and for a laterally invariant velocity and density medium, \( H_1(z_0) \) at the surface can be written as:

\[
\tilde{H}_1(k_x, k_y, z_0, \omega) = (x, y, x, \omega) e^{j(xz + ky)} dxdy = \sqrt{k^2(z_0) - k_x^2 - k_y^2}. \] (B.1.3)

Further, by choosing the medium for the Green’s function homogeneous above the surface, we get \( G^+ = 0 \). Using the one-way wavefield components in the Kirchhoff integral, we arrive to Rayleigh integral I:

\[
P(r_A, \omega) = -\frac{1}{2\pi} \int_\Sigma \frac{1}{\rho(r)} \frac{\partial P^+(r, \omega)}{\partial z} G^-(r, r_A, \omega) dS \] (B.1.4)

and Rayleigh integral II:

\[
P(r_A, \omega) = -\frac{1}{2\pi} \int_\Sigma \frac{1}{\rho(r)} P^+(r, \omega) \frac{\partial G^-(r, r_A, \omega)}{\partial z} dS. \] (B.1.5)

The above Rayleigh II integral in discrete form can be written as propagation of the downgoing wavefield from depth level \( z_n \) to \( z_{n+1} \) as:

\[
P(z_{n+1}) = W(z_{n+1}, z_n) P^+(z_n). \] (B.1.6)

Further, one-way wavefield extrapolation operators are computed using the algorithm described in Thorbecke et al. [2004]. Wavefield extrapolation operator (as
shown in Figure B.2) in the wavenumber-frequency domain for a 2D medium is given by:

\[ \tilde{W}(k_x, \omega, \Delta z) = e^{-jk_x \Delta z}, \]  

(B.1.7)

where \( k_z = \sqrt{k^2 - k_x^2} \) for \( k_x^2 \leq k^2 \) and \( k_z = -j \sqrt{k^2 - k_x^2} \) for \( k_x^2 > k^2 \); \( k \) is defined as \( \omega/c \), \( \delta z \) is a small extrapolation step, \( c \) is the propagation velocity of the layer, \( j \) is the imaginary unit, and \( \omega \) is the angular frequency. This operator is same as the phase shift operator [Gazdag, 1978].

![Figure B.2: Schematic diagram showing the downward extrapolation operator.](image)

Note that for \( k_x^2 > k^2 \), the wavefield becomes evanescent i.e. exponentially decaying. For laterally varying media, the operator is applied as space-frequency convolution with the data to extrapolate it from depth level \( z_n \) to \( z_{n+1} \). Further, the analytical inverse Fourier transform of equation (B.1.7) is a scaled Hankel function [Berkhout, 1984] given by:

\[ W(x, \omega, \Delta z) = -jk \frac{\Delta z}{2r} H_1^{(2)}(kr), \]  

(B.1.8)

where the distance \( r = \sqrt{x^2 + \Delta z^2} \), \( H_1^{(2)}(kr) = J_1(kr) - jY_1(kr) \) is the first-order Hankel function of the second kind, \( J_1 \) and \( Y_1 \) are the first-order Bessel functions of the first and second kind, respectively. Note that the inverse wavefield extrapolation operator \( F \) should invert for the wave propagation effects as \( \tilde{W}F = I \).

An inverse wavefield operator \( F \) for the phase shift operator is given by:

\[ \tilde{F}(x, \omega, \Delta z) = \frac{1}{W(x, \omega, \Delta z)}. \]  

(B.1.9)

The accuracy and stability should always be investigated for an inverse wavefield propagation operator. This inverse is accurate if \( \tilde{W}F = I \). However, for the evanescent part of the wavefield this inverse operator increases exponentially with \( \Delta z \). Therefore it is not a stable operator. In order to stabilize the inverse operator, the following approximation is made \( \tilde{F}(x, \omega, \Delta z) = W^H(x, \omega, \Delta z) \), where \( H \) represents the hermitian.
B.2 Reflectivity matrices \( R^\cup \) and \( R^\cap \)

\( R^\cup(z_n) \) and \( R^\cap(z_n) \) represent reflectivity matrices related to a discontinuities at depth level \( z_n \) for the wavefield coming from above and from below the layer, respectively. The diagonal of the reflectivity matrices indicate the zero-offset reflection coefficients and the angle-dependent reflectivity information is contained in the full reflectivity matrices [see also de Bruin et al., 1990]. Consider two homogeneous acoustic half-spaces separated by an interface at \( z_n \), then angle dependent reflection coefficient at a grid point, i.e. one element of the matrix \( R^\cup \) can be written as a function of incident angle \( \alpha \) (shown in Figure B.3):

\[
r^\cup(\alpha, z_n) = \frac{\rho_2 c_2 \cos \alpha - \rho_1 \sqrt{c_1^2 - c_2^2 \sin^2 \alpha}}{\rho_2 c_2 \cos \alpha + \rho_1 \sqrt{c_1^2 - c_2^2 \sin^2 \alpha}}.
\]  

(B.2.10)

\[ \text{Figure B.3: Reflection of an incident downgoing plane wave at an interface between two acoustic homogeneous half-spaces.} \]

Clearly we see that if \( c_2 > c_1 \), then the reflection function becomes complex for \( \sin|\alpha| > c_2/c_1 \), i.e. \( r^\cup = 1 \) and total reflection occurs. We can define the critical angle as \( \alpha_c = \sin^{-1}(c_1/c_2) \). An example \( r^\cup(\alpha) \) is plotted (courtesy of CREWES Zoeppritz explorer) for \( c_1 = 1500m/s \), \( c_1 = 3000m/s \) and \( \rho_1 = \rho_2 = 1000m/s \).

Figure B.5 schematically illustrate the structure of the \( R \) matrix and illustrates an example of the angle-dependent and the angle-independent reflectivity vector in the space-time domain at a grid point located on a reflector.

Further, note also that for \( \rho_1 \neq \rho_2 \) and \( c_1 = c_2 \), the reflection coefficient becomes scalar i.e. angle-independent quantity. We can also write the expression for reflection coefficient for monochromatic plane waves as a function of horizontal wavenumber \( k_x \), using \( k_x = k_1 \sin \alpha \) and \( k_1 = \omega/c \), where \( \omega \) is the angular frequency, given by \( \omega = 2\pi f \), with \( f \) the frequency. Therefore the reflection
B.2 Reflectivity matrices $\mathbf{R}^\cup$ and $\mathbf{R}^\cap$

Coefficient can be written as:

$$r^\cup(k_x, z_n, \omega) = \frac{\rho_2 \sqrt{\frac{1}{c_1^2} - k_x^2} - \rho_1 \sqrt{\frac{1}{c_1^2} - k_x^2}}{\rho_2 \sqrt{\frac{1}{c_2^2} - k_x^2} + \rho_1 \sqrt{\frac{1}{c_2^2} - k_x^2}}.$$  \hspace{1cm} (B.2.11)

For angle of incidence range $-90^\circ \leq \alpha \leq +90^\circ$, the equivalent range for wavenumber is $-\infty < k_x < \infty$. However, the discretized wavenumber is bounded by $k_{Nyq}$, where the spatial Nyquist frequency is given by $\pi/\Delta x$. Therefore, if $c_2 > c_1$, the wavenumber-domain reflection coefficient becomes complex for $k_2 < |k_x| < k_1$ and if $c_2 < c_1$, the wavenumber-domain reflection coefficient becomes complex for $k_1 < |k_x| < k_2$. Beyond these limits in two cases, the wavenumber domain reflection coefficients represents evanescent wavefields.

Finally, let us define the reflection coefficient as a function of ray-parameter $p$, where $p = k_x/\omega = \sin \alpha/c_1$:

$$a^\cup = r^\cup(p, z_n) = \frac{\rho_2 \sqrt{\frac{1}{c_1^2} - p^2} - \rho_1 \sqrt{\frac{1}{c_1^2} - p^2}}{\rho_2 \sqrt{\frac{1}{c_2^2} - p^2} + \rho_1 \sqrt{\frac{1}{c_2^2} - p^2}}.$$  \hspace{1cm} (B.2.12)

Note that in our definition of angle-dependent reflectivity (AVP image) as introduced in chapter 2, we have defined the reflectivity as a function of space and ray-parameter. Therefore, in the above equation, the element $a^\cup$ is an element in the matrix $\mathbf{A}^\cup$, as introduced in chapter 2. In terms of operator relationship,
Matrix description used in this thesis

Figure B.5: Description of Reflectivity matrix $\mathbf{R}$. Matrix $\mathbf{R}^\cup(z_n)$ represents reflectivity for a) an arbitrary depth level $z_n$ for all lateral locations $x_1$ to $x_M$ in a gridded model. b) show the reflectivity matrix $\mathbf{R}$ for one frequency component, where one column-vector at grid point location $x_i$ represents the corresponding angle-dependent reflectivity. The reflectivity vector of the grid point $x_i$ is shown in space-time domain for both c) angle-dependent and d) angle-independent reflection. Note the complex angle-dependent reflectivity behavior computed for a high-velocity contrast horizontally layered medium in (c), showing both the pre-critical and post-critical reflections. On the other hand (d) shows the angle-independent reflectivity which is a scalar value for one grid-point location, i.e. when the reflectivity matrix $\mathbf{R}$ is a diagonal matrix.

we have expressed $\mathbf{LA}^\cap = \mathbf{R}^\cup$, where $\mathbf{L}$ is the inverse linear radon transform operator given by:

$$
\mathbf{L} = \begin{bmatrix}
e^{-j\omega p_1 x_1} & \ldots & e^{-j\omega p_1 x_n} \\
\ldots & \ldots & \ldots \\
e^{-j\omega p_m x_1} & \ldots & e^{-j\omega p_m x_n}
\end{bmatrix}.
$$

(B.2.13)

Note again, in ray-parameter domain, the two cases to consider are - when $c_2 > c_1$, 

\[ \mathbf{LA}^\cap = \mathbf{R}^\cup \]
the ray-parameter domain reflection coefficient becomes complex for \( 1/c_2 < |p| < 1/c_1 \) and when \( c_2 < c_1 \), the ray-parameter domain reflection coefficient becomes complex for \( 1/c_1 < |p| < 1/c_2 \).

To illustrate the AVP images, we used a synthetic velocity model shown in Figure B.6, for a surface seismic case (Figure B.6a) and for a VSP case (Figure B.6b). Figure B.7 shows example angle gathers obtained from the primary-only imaging, for surface seismic profile (Figures B.7a, b, c) and for vertical seismic profile (Figure B.7d, e, f). Angle gathers is the AVP response representing one lateral location and all depth. We observe imaging artifacts (non-flat reflection events even when a correct velocity model is used in imaging) in the VSP case, due to poor illumination away from the well trajectory. Also, Figure B.8 compares the AVP images at depth level 700m and 1400m obtained from the primary-only wavefield for surface seismic and VSP cases.

![Figure B.6](image_url)

*Figure B.6: Velocity models used to illustrate the primary-only AVP imaging for a) surface seismic profile and for b) VSP. Two black dotted lines are the depth levels (700m and 1400m) used to illustrate the AVP map at the 2 depth levels for different geometries in Figures B.7 and B.8.*
Figure B.7: Example angle gathers for the surface seismic profile at lateral locations a) 1250m, b) 1500, and c) 1750m; and for the VSP at lateral locations d) 1250m, e) 1500m and f) 1750m.

Figure B.8: AVP image comparison, for depth levels shown by black dotted lines in Figure1. AVP images for surface seismic profile at depth a) 700m and b) 1400m; and AVP images for VSP at depth c) 700m and d) 1400m.


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Summary

Until now, in most seismic imaging technologies, both surface and internal multiples are considered as noise. In today’s industrial practice, we see various methods for suppressing multiples before migration. This means that only a fraction of the recorded wavefield is used in imaging. In this thesis, we present a method termed full wavefield migration (FWM) that uses the multiple-reflections in the data to improve the illumination of the field in areas that cannot be reached by the primaries, to yield a better vertical resolution as well as to suppress migration artefacts caused by crosstalk of multiple-reflections.

This thesis demonstrates the feasibility of full wavefield migration on a kind of borehole seismic known as vertical seismic profiling (VSP). We know that in today’s practice, images obtained using VSP data always suffer from poor illumination and small aperture effects. Therefore, we expect in VSP acquisition geometry, multiples can lead to significant improvement in illumination, both at the reservoir level as well as away from the well region. In this thesis, the advantage of using multiples in full wavefield migration has been demonstrated. We validated our algorithm on 2D synthetic and field VSP data.

Full wavefield migration is posed as an inverse problem, where the parameters to be estimated are the subsurface reflectivities. We discuss an iterative forward modelling engine termed full wavefield modelling which is used in the inversion scheme. Full wavefield modelling allows us to compute the full wavefield (primaries and all multiples) in terms of estimated reflectivities. In the full wavefield modelling engine, we assume a scale-separation between the background migration velocity that governs only the one-way wavefield propagation and the reflectivity model that governs the two-way scattering. The modelling engine accounts for the non-linearity of the wavefield due to reflectivity, incorporating the transmission effects and multiple scattering at all depth levels. To solve the inverse problem, we have used iterative conjugate-gradient scheme, which is a local optimization method.
We also present a solution for imaging of blended source VSP data using FWM. The inversion-based imaging algorithm allows us to use any complex source wavefield without the need for a separate deblending (pre-processing) step. This thesis introduces the concepts of elastic full wavefield modelling and inversion. The elastic modelling of P and S waves is illustrated for a horizontally layered medium using a VSP geometry. The elastic imaging to estimate angle-dependent reflectivity parameters that incorporates mode-conversions in subsurface layers is an important area of future research. Nearly vertical structures such as salt-flanks pose a migration challenge for conventional FWM. We have also extended the FWM algorithm to incorporate turning-waves using horizontal one-way wavefield extrapolation. Using this extension, we illustrate that FWM can be used to image steep dips or near-vertical structures using the turning wavefield in VSP data.

Alok Kumar Soni.
Samenvatting

Tot nu toe worden zowel surface en interne multiples door de meeste seismische afbeeldingstechnieken als ruis behandeld. In de huidige industriële praktijk zien we diverse methodes om multiples te onderdrukken voor migratie. Dit betekent dat slechts een fractie van het gemeten golfveld wordt gebruikt voor het afbeelden. In dit proefschrift presenteren we een methode, genaamd full wavefield migration (FWM), waarbij multiples in de data worden gebruikt om de belichting van de ondergrond te verbeteren op die plekken waar de primaries niet komen, om de resolutie te verbeteren dankzij een grotere apertuur en om artefacten in de migratie wegens interferentie van multiples te onderdrukken.

Dit proefschrift toont de haalbaarheid van full wavefield migration aan voor een type boorgat seismiek die vertical seismic profiling (VSP) wordt genoemd. We weten dat bij de huidige praktijk, de afbeeldingen op basis van VSP data altijd lijden onder slechte illuminatie en kleine apertuur effecten. Om deze reden verwachten we dat bij een VSP acquisitie, multiples kunnen leiden tot een significante verbetering van de illuminatie, zowel in het reservoir als buiten de omgeving van het boorgat. In dit proefschrift is het voordeel van het gebruik van multiples in full wavefield migration aangetoond. We hebben de werking van ons algoritme bevestigd met 2D synthetische data en field VSP data.

Full wavefield migration is gesteld als een invers probleem, waarbij de reflectiviteit van de ondergrond de te schatten parameters zijn. We beschrijven een iteratief voorwaarts model, full wavefield modeling, die gebruikt wordt in het inversieschema. Full wavefield modelling stelt ons in staat om het volledige golfveld (primaries en alle multiples) te berekenen in termen van geschatte reflectiviteiten. In het voorwaartse model gebruiken we de aanname dat er een scheiding van schalen is tussen de achtergrond migratiesnelheden die de een-wegs golfpropagatie bepaalt en het reflectiviteitsmodel die de twee-wegs verstrooiing bepaalt. Het voorwaartse model omvat de niet-lineariteiten van het golfveld die veroorzaakt worden door reflectiviteiten, inclusief de transmissie effecten en meervoudige reflecties op alle diepteniveaus. Om het inverse probleem op te lossen hebben we
een iteratief conjugate-gradient schema gebruikt, wat een lokale optimisatiemethode is.

We laten ook een oplossing zien voor het afbeelden van blended source VSP data met FWM. Dankzij het afbeelden op basis van een inversie algoritme kunnen we elk willekeurig complexe bronveld gebruiken zonder de noodzaak voor een apart deblending (pre-processing) stap. Dit proefschrift introduceert de concepten van elastische full wavefield modeling en inversie. Het elastisch modelleren van P en S golven wordt geïllustreerd voor een horizontaal gelaagd medium met een VSP geometrie. Het elastisch afbeelden van hoekafhankelijke reflectiviteitsparameters die mode-conversies in aardlagen omschrijven is een belangrijk onderwerp voor verder onderzoek.

Bijna-verticale structuren zoals zoutflanken vormen een uitdaging voor conventionele FWM. We hebben het FWM algoritme ook uitgebreid om turning-waves te omvatten met behulp van horizontale een-wegs golfveld extrapolatie. Met deze uitbreiding laten we zien dat FWM kan worden gebruikt om steile of bijna-verticale structuren af te beelden met gebruik van het de turning-waves in de VSP data.

Alok Kumar Soni.
Brief CV

Alok Kumar Soni was born in year 1982, in a town called Gomia (in Jharkhand state), India. He finished his early schooling between year 1987 to 2000 at Pitts Modern School, Gomia. His major in high-school was Physics, Chemistry, Mathematics and Biology. After high-school, he finished his B.Sc. Honors (Bachelors in Science, Honors) in Physics at Hansraj College, University of Delhi between year 2000 to 2003. After B.Sc, he completed M.Tech (Master of Technology) in Applied Geophysics at Indian Institute of Technology, Roorkee, India between year 2003 to 2006. After masters, he joined WesternGeco, Schlumberger UK, where he worked as depth imaging geophysicist. He worked in WesternGeco UK at Gatwick centre between September 2006 to January 2011. His main job role was to manage full projects for real 3D land and marine surface seismic data processing, velocity model building and depth imaging. In his four and half years of industrial exposure, he has worked with data from many parts of the world. On February, 2011, he joined Delphi Consortium, Delft University of Technology, The Netherlands to pursue his PhD research under the supervision of Dr. Eric Verschuur. His research interest broadly includes seismic imaging, seismic inversion, numerical simulation and reservoir characterization. His PhD research project (this thesis) was to develop full wavefield migration algorithm and study its feasibility on borehole seismic data (VSP data). He completed his PhD on 23rd December, 2014.
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