S.A. Nagy
GUSTO ENGINEERING C.V.
Schiedam, The Netherlands
February, 1986
Dynamics of jack-up platforms in elevated condition

CONTENTS

1 Combined leg stiffness 2
   1.1 Introduction 2
   1.2 Bending stiffness of the leg 3
   1.3 Shear stiffness of one bay 5
   1.4 Reduced bending stiffness 10
   1.5 Rotational stiffness of the leg-soil connection 12
   1.6 Rotational stiffness of the leg-hull connection 13
   1.7 Deflection of the leg 22
   1.8 Result 26
   1.9 Verification 27

2 Mass distribution 29
   2.1 Legs 29
   2.2 Platform 32

3 Linearization of the drag force 34
   3.1 Introduction 34
   3.2 Deterministic approach 34
   3.3 Stochastic approach 37

4 Three-degrees-of-freedom-system 40

5 Tables 45
1 COMBINED LEG STIFFNESS

1.1 INTRODUCTION

The combined leg stiffness, $k_c$, is defined as the stiffness of the horizontal spring at lower guide level, replacing one leg (see figure 1.1). It is a combination of the following parameters:

- $EI$ = bending stiffness of the leg
- $k_{sb}$ = shear stiffness of one bay of the leg
- $k_b$ = rotational stiffness of the leg-hull connection
- $k_A$ = rotational stiffness of the leg-soil connection
- $F_2$ = axial leg load, causing second order bending of the leg. This axial leg load is exerted by the total deck mass

The parameters $EI$ and $k_{sb}$ are combined to one parameter, called the reduced bending stiffness $EI'$. After this, $EI'$, $k_A$ and $k_b$ are used to find an expression for the displacement of the leg, including second order bending. The combined leg stiffness is derived from this expression.
1.2 BENDING STIFFNESS OF THE LEG

For the computation of the legs it is assumed that the bending forces in the leg are mainly carried by the chords, thus allowing the use of Steiner’s rule and neglecting of the bending stiffness of the chords. The moment of inertia of the leg about the x-axis shown in figure 1.2 is equal to:

\[ I_x = (y_1^2 + y_2^2 + y_3^2)A_c \]  \hspace{1cm} (1.01)

where:

- \( I_x \) = moment of inertia about the x-axis
- \( y_i \) = y-coordinate of chord \( i \)
- \( A_c \) = cross-sectional area of one chord

The y-coordinates of the chords can be written as:

\[ y_1 = r \sin(\phi) \]  \hspace{1cm} (1.02)
\[ y_2 = r \sin(\phi + 120^\circ) \]  \hspace{1cm} (1.03)
\[ y_3 = r [\sin(\phi) \cos(120^\circ) + \cos(\phi) \sin(120^\circ)] \]
Dynamics of jack-up platforms in elevated condition

\[ y_3 = r \sin(\phi + 240^\circ) \]
\[ y_3 = r(\sin(\phi)\cos(240^\circ) + \cos(\phi)\sin(240^\circ)) \]  
\( (1.04) \)

where:

- \( r \) = distance between centre chord and centre leg

Substitution of (1.02) to (1.04) into (1.01) yields:

\[ I_x = -\frac{r^2 A_c}{2} \]  
\( (1.05) \)

The relationship between \( r \) and \( d \), the distance between two chords, is:

\[ r = d \tan(30^\circ) \]  
\( (1.06) \)

Combination of (1.05) and (1.06) leads to the expression for the moment of inertia of the leg, which is valid for all directions:

\[ I = \frac{A_c d^2}{2} \]  
\( (1.07) \)

where:

- \( I \) = moment of inertia of one leg
- \( A_c \) = cross-sectional area of one chord
- \( d \) = distance between the chords

The cross-sectional area of one chord, \( A_c \), is not constant, but increases from the spud can to the top of the legs. The cross-sectional area of these three chord sections is listed in table 1.1, together with the respective section lengths.
Dynamics of jack-up platforms in elevated condition

<table>
<thead>
<tr>
<th>cross-sectional area of one chord [mm²]</th>
<th>section length [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>146986</td>
<td>14.50</td>
</tr>
<tr>
<td>162590</td>
<td>34.50</td>
</tr>
<tr>
<td>184521</td>
<td>89.50</td>
</tr>
<tr>
<td>total leg length, without spudcan</td>
<td>138.50</td>
</tr>
</tbody>
</table>

Table 1.1: Cross-sectional area of one chord

For the computation of the bending stiffness of the leg the weighed average of the cross-sectional area of the chords is taken:

\[
A = \frac{14.50 \times 146986 + 34.50 \times 162590 + 89.50 \times 184521}{138.50}
\]

\[
A = 175128 \text{ mm}
\]

The moment of inertia of one leg, \( I \), is found by substituting this into equation (1.07), together with:

\[
d = 10.50 \text{ m}
\]

This yields:

\[
I = \frac{0.175128 \times 10.50^2}{2} = 9.65 \text{ m}
\]

The bending stiffness of one leg, \( EI \), is:

\[
EI = 210 \times 10^6 \times 9.65 = 2.03 \times 10^9 \text{ kNm}
\]

1.3 SHEAR STIFFNESS OF ONE BAY

A bay is the element of the leg shown in figure 1.3. For the computation of the shear stiffness of one bay the assumption is made that the shear forces of the leg are mainly carried by the braces. This is correct, since the chords are slender and therefore cannot carry any shear forces. This assumption allows the modelling of the bay as described in this paragraph.
First, one of the three bay sides is considered (see figure 1.4). A horizontal load, $P$, at the top of the bay side causes a displacement, $\delta$, of the top.

The axial forces in the members, $N_1$ to $N_6$, and the extension of the members, $\Delta l_1$ to $\Delta l_6$, resulting from this are listed in table 1.2.
Table 1.2: Axial forces and extensions in one bay side due to a horizontal load $P$ at the top

<table>
<thead>
<tr>
<th>member</th>
<th>length 1</th>
<th>axial force N</th>
<th>extension $\Delta l$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$h$</td>
<td>$+Ph/2l$</td>
<td>$+Ph^2/2EA_c l$</td>
</tr>
<tr>
<td>2</td>
<td>$1/2\cos\alpha$</td>
<td>$-P/2\cos\alpha$</td>
<td>$-Pl/4EA_c \cos^2\alpha$</td>
</tr>
<tr>
<td>3</td>
<td>$1/2\cos\alpha$</td>
<td>$+P/2\cos\alpha$</td>
<td>$+Pl/4EA_c \cos^2\alpha$</td>
</tr>
<tr>
<td>4</td>
<td>$1/2\cos\alpha$</td>
<td>$+P/2\cos\alpha$</td>
<td>$+Pl/4EA_c \cos^2\alpha$</td>
</tr>
<tr>
<td>5</td>
<td>$1/2\cos\alpha$</td>
<td>$-P/2\cos\alpha$</td>
<td>$-Pl/4EA_c \cos^2\alpha$</td>
</tr>
<tr>
<td>6</td>
<td>$h$</td>
<td>$-Ph/2l$</td>
<td>$-Ph^2/2EA_c l$</td>
</tr>
</tbody>
</table>

In this table:

$E$ = Young's modulus

$A_c$ = cross-sectional area of one chord
Dynamics of jack-up platforms in elevated condition

\[ A_b = \text{cross-sectional area of one bracing} \]

The displacement of \( A \) can be written as (see figure 1.5):

\[ \delta_A = \frac{\Delta l_3}{\cos(\alpha)} \quad (1.08) \]

![Diagram showing displacement due to load P](image)

\[ \Delta \text{d} = \Delta_3 \cos \alpha \]

\[ \Delta_4 \cos \alpha \]

\[ \Delta_6 \tan \alpha \]

Figure 1.5: Displacement due to load \( P \)

From figure 1.5 it can be seen that the displacement of the top of the bay side, \( \delta \), is equal to:

\[ \delta = \delta_A - \frac{\Delta l_4}{\cos(\alpha)} - \Delta l_6 \tan(\alpha) \quad (1.09) \]

Substituting (1.08) into (1.09) and using table 1.2 yields:

\[ \delta = \frac{P l}{2E A_b \cos^3(\alpha)} + \frac{P h^2 \tan(\alpha)}{2E A_c l} \quad (1.10) \]

In order to find the shear stiffness of one complete bay, its topview is considered. For the loading case as shown in figure 1.6, the relationship between the load at the top of one bay, \( P \), and the load at the top of the complete bay, \( F \), is described by:

\[ F = 2P \cos(30^\circ) \]

\[ F = \sqrt{3} P \quad (1.11) \]

In a similar way, the displacement of the bay top, \( \delta' \), can be expressed in terms of bay side displacement, \( \delta \):
Dynamics of jack-up platforms in elevated condition

\[ \delta^* = \cos(30^\circ) \]

\[ \delta^* = \frac{1}{2\sqrt{3}} \delta \]  \hspace{1cm} (1.12)

Due to this displacement, the rotation of the chords about a horizontal axis is:

\[ \phi = \frac{\delta^*}{h} \]  \hspace{1cm} (1.13)

The shear stiffness of one bay, \( k_{sb} \), is defined as:

\[ k_{sb} = -\frac{F}{\phi} \]  \hspace{1cm} (1.14)

Substitution of (1.13), (1.12), (1.11) and (1.10) into (1.14) yields:

\[ k_{sb} = 4hE \frac{1}{A_c \cos^3(\alpha) + A_c l} \]  \hspace{1cm} (1.15)

where:

\[ k_{sb} \] = shear stiffness of one leg

\[ h \] = bay height

Figure 1.6: Shear deformation of one bay
Dynamics of jack-up platforms
in elevated condition

1 = distance between two chords

\[ A_c = \text{cross-sectional area} \]

\[ A_b = \text{cross-sectional area of one bracing} \]

\[ \theta = \text{inclination of the braces} \]

\[ E = \text{Young's modulus} \]

The following figures apply to the Russian jack-up:

\[ h = 5.76 \text{ m} \]

\[ l = 10.50 \text{ m} \]

\[ A_b = \frac{n(419^2 - 379^2)}{4} = 25070 \text{ mm}^2 \]

\[ A_c = 175128 \text{ mm}^2 \]

\[ \theta = 26^\circ \]

\[ E = 210 \times 10^6 \text{ kN/m}^2 \]

The shear stiffness of one bay becomes:

\[ k_{sb} = \frac{4 \times 5.76 \times 210 \times 10^6}{10.50 \times 5.76^2 \times \tan(26^\circ)} \]

\[ k_{sb} = 8.26 \times 10^6 \text{ kN/rad} \]

1.4 REDUCED BENDING STIFFNESS

The reduced bending stiffness, \( E I_s \), combines the influence of the bending stiffness, \( E I \), and the shear stiffness, \( k_{sb} \), in order to simplify the differential equation for the leg displacements, which will be solved in paragraph 1.7 of the appendix. The error, caused by this approximate inclusion of the shear stiffness, will be small if the bending stiffness governs the reduced bending stiffness.

For the purpose of computing the reduced bending stiffness, the leg is assumed to be a cantilever, as shown in figure 1.7.

Due to a horizontal load at the top, \( F \), the deflection of this
Dynamics of jack-up platforms in elevated condition

Figure 1.7: Definition of the reduced leg bending stiffness

The reduced bending stiffness, $EI''$, is now given by:

$$EI'' = f\times EI$$

where:

$$\delta = \frac{Fl^3}{3EI} + \frac{F}{k_{sb}}$$

$$\delta = \frac{F l^3}{3} \left( \frac{1}{EI} + \frac{3}{k_{sb} l^2} \right)$$

Rewriting this expression yields:

$$F = \frac{3\delta}{l^3} EI'$$

$$EI'' = f\times EI$$
Dynamics of jack-up platforms in elevated condition

\[ f = \frac{1}{1 + \frac{3EI}{k_s b l^2}} \]

\( f \) = reduction factor

\( EI \) = bending stiffness of the leg

\( k_s b \) = shear stiffness of one bay

\( l \) = free leg length

The numerical values of the Russian jack-up are:

\( EI = 2.03 \times 10^9 \text{ kNm}^2 \)

\( k_s b = 8.26 \times 10^6 \text{ kN/rad} \)

\( l = 120 \text{ m} \)

With these data, the reduction factor, \( f \), and the reduced bending stiffness, \( EI' \), become:

\[ f = \frac{1}{1 + \frac{3 \times 2.03 \times 10^9}{8.26 \times 10^6 \times 120^2}} = 0.951 \]

\[ EI' = 2.03 \times 10^9 \times 0.951 = 1.93 \times 10^9 \text{ kNm} \]

As can be seen, the bending stiffness governs the reduced bending stiffness and shear deformation of the leg is small. This means that the substitution of \( EI' \) into the differential equation for leg displacements, which is presented in paragraph 1.7 of the appendix, will give a good approximation of reality.

1.5 ROTATIONAL STIFFNESS OF THE LEG-SOIL CONNECTION

The rotational stiffness of the leg-soil connection depends on:

a. Soil conditions

b. Penetration depth

c. Spudcan dimensions
Soil conditions as well as penetration depth are highly site-dependent and therefore the parameters involved are hard to estimate.

There are no reliable expressions describing the dynamic behaviour of the rotational restraint, but in order to get a rough estimate, the following D.N.V. formula is used for the leg-soil stiffness:

\[ k_A = \frac{8GR^3}{3(1-\nu)} \]  \hspace{1cm} (1.19)

where:

- \( k_A \) = rotational stiffness of leg-soil connection
- \( G \) = shear modulus of soil
- \( R \) = spud can radius
- \( \nu \) = Poisson's ratio

Values of the soil parameters applying to certain areas of the North Sea are:

- \( G = 15000 \text{ kN/m}^2 \)
- \( \nu = 0.4 \)

The equivalent spud can radius is:

- \( R = 7.57 \text{ m} \)

Substitution of these values into (1.19) yields the following rotational leg-soil stiffness:

- \( k_A = 3 \times 10^7 \text{ kNm/rad} \)

1.6 ROTATIONAL STIFFNESS OF THE LEG-HULL CONNECTION

1.6.1 Introduction

For the computation of the leg-hull connection, the hull itself is assumed to be rigid, so displacements of the lower guides are neglected. As outlined in the main report, all horizontal forces in the jacking system are carried by the guides. The position of these guides is shown in figures 1.8 and 1.9.

The rotational stiffness of the leg-hull connection will de-
Dynamics of jack-up platforms in elevated condition

Figure 1.8: Position of the guides, sideview

Figure 1.9: Position of the guides, topview

a. Horizontal displacements of the upper guides.

Copyright of GUSTO ENGINEERING CV whose property this document remains. No part thereof may be disclosed, copied, duplicated, or in any other way made use of, except with approval of GUSTO ENGINEERING CV SCHIEDAM.
b. Bending of the chord sections facing the guides. Both items will be considered in terms of rotational stiffness, after which they are combined to the rotational leg-hull stiffness.

1.6.2 Horizontal displacements of the upper guides

For an exact computation of the upper guide displacements a detailed analysis framework supporting the guides is needed, which will not be done here. Instead of this, a rough estimate is provided by the assumption that the displacement of the upper guides causes a platform displacement is of the same order of magnitude as the displacement due to bending of the leg. For this purpose, the guiding system is replaced by a rotational spring, $k_i$. This is shown in figure 1.10.

\[ \delta_i = \frac{F l^3}{3EI'} \]  

(1.20)

where:

- $\delta_i$ = deck displacement due to leg bending
- $F$ = horizontal load at lower guide level

Figure 1.10: Model of the leg-hull connection
1 = free leg length

\[ EI' = \text{reduced leg bending stiffness} \]

The rotational spring with stiffness \( k_1 \) causes a displacement \( \delta_l \):

\[ \delta_l = \frac{F l^2}{k_1} \quad (1.21) \]

where:

\( \delta_l \) = deck displacement due to upper guide displacement

\( k_1 \) = rotational stiffness replacing the upper guide displacement

Equation of (1.20) and (1.21) yields an estimate of \( k_1 \):

\[ k_1 = \frac{3EI'}{l} \quad (1.22) \]

Substitution of the following values:

\[ EI' = 1.93 \times 10^9 \text{ kNm}^2 \]

\( l = 120 \text{ m} \)

yields:

\[ k_1 = \frac{3 \times 1.93 \times 10^9}{120} = 4.8 \times 10^7 \text{ kNm/rad} \]

Comparison with a more detailed computation by D.N.V. for a similar platform results in the following range of values to be used for further computations:

\[ k_1 = 4.8 \times 10^7 \text{ kNm/rad to } 9.6 \times 10^7 \text{ kNm/rad} \]

1.6.3 Bending of the chords

This effect will also be represented by a rotational spring. Before this is done, the moment of inertia of one chord is computed.

In elevated condition of the platform, the guides will face the section of the chords with the largest cross-section,
which is shown in figure 1.11. The contribution from the toothing to the cross-section of the toothplate is averaged.

![Diagram](attachment:diagram.png)

**Figure 1.11: Cross-section of one chord**

It can be seen in figure 1.9 that the guide forces act in the x-direction of the coordinate system shown in figure 1.11, so the moment of inertia about the y-axis, $I_y$, is calculated:

$$I_y = \frac{1}{12} \left( 90 \times 1000^3 + 318 \times 127^3 + 601.5 \times 45^3 \right) + 2 \times 601.5 \times 45 \times 270^2$$

$$I_y = 1.2 \times 10^{10} \text{ mm}^4 = 1.2 \times 10^{-7} \text{ m}^4$$

Now the effect of the guide forces is considered. It is assumed that the bracings act as rigid supports of the bracings.

The amount of chord bending due to guide forces depends on the position of the guides facing the chord. Two extreme cases are shown in figure 1.12.

If all guides face a node, no chord bending occurs. This is called case A.
In case B, all guides face the centres of the chord sections, yielding maximum chord bending. The bending of this section is reduced by stiffness of the adjacent sections, but this reduction is neglected by assuming the loaded chord section to be a simply supported beam. In this way, an upper boundary is found for the amount of chord bending. The guide force is assumed to act as a distributed load, $f$, over the length of the guide:

$$f = -\frac{F}{d}$$

(1.23)

where:

$F =$ guide force

$d =$ length of one guide

The resulting model is shown in figure 1.13, together with the curve of bending moments.

First, the deflection at the centre of the chord section, $\delta$, is computed:

$$\delta = \frac{1}{2} \cdot 0.48 \cdot \frac{F}{0.96}$$

(1.24)
Dynamics of jack-up platforms in elevated condition

\[ \phi_2 = \frac{1}{2} \frac{-0.48F + 1.92}{EI} \quad (1.25) \]

Figure 1.13: Case B of chord bending

\[ \phi_3 = \frac{1}{2} \frac{-0.48F + 1.92}{EI} \quad (1.26) \]

\[ \phi_A = \phi_1 + \phi_2 + \phi_3 \quad (1.27) \]

\[ \delta = 2.88 \phi_A - (2.88 - \frac{2}{3} -0.96) \phi_1 - 0.96 \phi_2 \]

\[ -0.96 \phi_3 \quad (1.28) \]

Substituting (1.27), (1.26), (1.25) and (1.24) into (1.28) yields:

\[ \delta = 3.10 \frac{F}{EI} \quad (1.29) \]
This is the deflection of the center of one chord section due to the guide force, in SI-units.

The rotation in the leg-hull connection is computed as follows. For a rotation of the leg about the y-axis shown in figure 1.14, the counteracting moment is delivered by chords 1 and 2, while the guide forces at chord 3 are zero.

\[
\phi = \frac{\sqrt{3} \delta}{l}
\]

where:

\( \phi \) = rotation of the leg about an horizontal axis

Copyright of GUSTO ENGINEERING CV whose property this document remains. No part thereof may be disclosed, copied, duplicated, or in any other way made use of, except with approval of GUSTO ENGINEERING CV SCHIEDAM.
6 = deflection of the chord
1 = vertical guide distance

The counteracting moment is given by:

\[ M = 2 \times \sqrt{3} \times \frac{1}{2} = \sqrt{3} \times 1 \]  \hspace{1cm} (1.31)

The chord bending effect is replaced by a rotational stiffness of the leg-hull connection, \( k_z \), defined as:

\[ k_z = \frac{M}{\phi} \]  \hspace{1cm} (1.32)

Substitution of (1.30), (1.31) and (1.29) into (1.32) yields, in SI-units:

\[ k_z = \frac{\varepsilon I I^2}{3.10} \]  \hspace{1cm} (1.33)

The moment of inertia of the chord, \( I \), has already been computed:

\[ I = 1.2 \times 10^{-2} \text{ m}^4 \]

The other values needed are:

\[ E = 210 \times 10^6 \text{ kN/m}^2 \]

\[ l = 16.50 \text{ m} \]

Now the rotational stiffness replacing the chord bending becomes:

\[ k_z = \frac{16.50 \times 210 \times 10^6 \times 1.2 \times 10^{-2}}{3.10} = 2.2 \times 10^8 \text{ kNm/rad} \]

This is the minimum stiffness, found if all guides face the centre of a chord section. If they face nodes of the leg, the chords "are" rigid:

\[ k_z = \infty \]

As a result of this, the leg-hull stiffness replacing the effect of chord bending due to guide forces has the following range of values:
1.6.4 Conclusion

The rotational stiffness of the leg-hull connection, \( k_\theta \), is found as follows:

\[
\frac{1}{k_\theta} = \frac{1}{k_1} + \frac{1}{k_2}
\]  

(1.34)

where:

- \( k_a \) = rotational leg-hull stiffness
- \( k_1 \) = replacing the displacement of the upper guides
- \( k_2 \) = replacing the bending of the chords due to guide forces

Substitution of the values found for \( k_1 \) and \( k_2 \) leads to:

\[ k_\theta = 3.9 \times 10^7 \text{ kNm/rad to } 9.6 \times 10^7 \text{ kNm/rad} \]

1.7 DEFLECTION OF THE LEG

1.7.1 Differential equation

The model of one leg is shown in figure 1.16. It is loaded by a horizontal load at lower guide level, \( F_1 \), and a vertical load, \( F_2 \).

For deflections which are small compared to the free leg length, \( l \), the bending moment in the leg, \( M \), is given by

\[
M = F_1 z + F_2 x + k_a \phi_A
\]  

(1.35)

Since the shear stiffness of the leg was included in the reduced bending stiffness, \( EI' \), the bending moment is approximately equal to:

\[
M = EI' \frac{d^2 x}{dz^2}
\]  

(1.36)

Equation of (1.35) and (1.36) and rewriting of the result leads to the following differential equation:
Dynamics of jack-up platforms in elevated condition

\[ \frac{d^2 x}{dz^2} + \alpha^2_x = -\frac{F_i z}{EI'} - \frac{k_A \phi_A}{EI'} \]  \hspace{1cm} (1.37)

\[ \alpha^2 = \frac{F}{EI'} \]  \hspace{1cm} (1.38)

\[ \phi_A = -\frac{dx}{dx} \bigg|_{z=0} \]  \hspace{1cm} (1.39)

The general solution of equation (1.37) is given by:

\[ x(z) = c_4 \sin(\alpha z) + c_5 \cos(\alpha z) - \frac{F_i}{\alpha^2 EI'} z - \frac{k_A \phi_A}{\alpha^2 EI'} \]  \hspace{1cm} (1.40)

\[ \frac{dx}{dz} = c_4 \alpha \cos(\alpha z) - c_5 \alpha \sin(\alpha z) - \frac{F_i}{\alpha^2 EI'} \]  \hspace{1cm} (1.41)

Substitution of (1.41) into (1.39) yields:

\[ \phi_A = \frac{F_i}{\alpha^2 EI'} - c_4 \alpha \]  \hspace{1cm} (1.42)
With this, equation (1.40) can be written as:

\[ x(z) = c_4 \sin(\Delta z) + \frac{k_A}{dEI^*} + c_5 \cos(\Delta z) \]

\[ - \frac{F_1}{d^2EI^*} (z + \frac{k_A}{d^2EI^*}) \]

(1.43)

The constraints are:

\[ z = 0 : \quad x = 0 \]  

(1.44)

\[ z = 1 : \quad M = -EI^* \frac{d^2x}{dz^2} = k_b \frac{dx}{dz} \]  

(1.45)

Substitution of (1.40) and (1.41) into these constraints leads to the following formula for the deflection of the leg:

\[ x(z) = c_2 F_1 \sin(\Delta z) + c_3 F_1 (\cos(\Delta z) - 1) - \frac{F_1 z}{d^2EI^*} \]

(1.46)

where:

\[ c_2 = -\frac{k_b}{d^2EI^*} c \]  

(1.47)

\[ c_3 = \frac{k_A k_b}{d^3EI^*} c + \frac{k_A}{d^4EI^*^2} \]

(1.48)

\[ c = \frac{1 + k_A \cos(\Delta l)/k_b + k_A \sin(\Delta l)/dEI^*}{(EI^* d^2 - k_A k_b / EI^*) \sin(\Delta l) - (k_A + k_b) d \cos(\Delta l)} \]

(1.49)

\[ \Delta l = \sqrt{\frac{F_2}{EI^*}} \]

\[ x(z) = \text{leg displacement} \]

\[ F_2 = \text{axial leg load} \]

\[ EI^* = \text{reduced leg bending stiffness} \]

\[ k_A = \text{rotational stiffness of the leg-soil connection} \]

\[ k_b = \text{rotational stiffness of the leg-hull connection} \]
\[ l = \text{free leg length} \]
\[ F_1 = \text{horizontal load at lower guide level} \]

It should be noted that this solution is valid only for values of \( F_1 \) less than the buckling load of the leg.

1.7.2 Axial leg load

Due to deck displacements, the individual axial leg loads will vary. For the situation shown in figure 1.17, the equilibrium of horizontal and vertical forces yields:

\[ z_A + z_B - 2F_2 = 0 \]  \hspace{1cm} (1.50)
\[ x_A + x_B - F_1 = 0 \]  \hspace{1cm} (1.51)

Figure 1.17: Variation of the axial leg loads due to deck displacement.

The result of the equilibrium of moments about the leg-soil connection of leg \( B \) is:
Dynamics of jack-up platforms
in elevated condition

\[-z_A \times b - \frac{F_1}{1} + 2z_F \times (-b - \delta) = 0\]  \hspace{1cm} (1.52)

Combination of the last three equations leads to the vertical soil reactions:

\[z_A = \frac{F_2}{b} (1 - \frac{26}{b})\]  \hspace{1cm} (1.53)

\[z_F = \frac{F_2}{b} (1 + \frac{26}{b})\]  \hspace{1cm} (1.54)

For relatively small deck displacements, these expressions are good approximations of the axial leg loads, \(F_{zA}\) and \(F_{zB}\). The range of values of the axial leg load can be found by substituting the following data from the Russian jack-up:

\[F_2 = 3.63 \times 10^4\] kN

\[b = 54\] m

Assuming the maximum deck displacement is:

\[\delta = 2\] m

the axial load range becomes:

\[F_{zA} = 3.36 \times 10^4\] kN to \[3.90 \times 10^4\] kN

The maximum relative fluctuation of the axial leg load is in the order of:

\[\frac{26}{b} \approx \frac{2 \times 2}{54} \approx 8\%\]

1.8 RESULT

The combined leg stiffness, \(k_c\), is defined as the stiffness of a spring at lower guide level, replacing the lateral stiffness of the leg. The reciprocal of the combined leg stiffness is given by:
Dynamics of jack-up platforms in elevated condition

\[
\frac{1}{k_c} = \frac{x(l)}{F}
\]

\[
\frac{1}{k_c} = c_2 \sin(\alpha l) + c_3 (\cos(\alpha l) - 1) - \frac{1}{\alpha^2 E I}
\]

(1.55)

The following values were computed in the previous sections:

- \( E I^* = 1.93 \times 10^9 \) kNm²
- \( k_b = 3.9 \times 10^2 \) kNm/rad to \( 9.6 \times 10^2 \) kNm/rad
- \( F_z = 3.63 \times 10^4 \) kN
- \( k_\alpha = 0 \) kNm/rad to \( 4 \times 10^3 \) kNm/rad

The combined leg stiffness is found by substitution of these values into expression (1.55):

\[ k_c = 1.2 \times 10^3 \) kN/m to \( 4.8 \times 10^3 \) kN/m

1.9 VERIFICATION

For the purpose of verifying the order of magnitude of this result, the combined leg stiffness is computed for the cantilever shown in figure 1.18.

The deflection, \( \delta \), due to a load at lower guide level, \( F \), is given by:

\[
\delta = \frac{F l^3}{3 E I^*}
\]

(1.56)

The combined leg stiffness of the cantilever becomes:

\[
k_{c,\text{cant}} = \frac{F}{3 E I^*}
\]

\[
k_{c,\text{cant}} = \frac{3 E I^*}{l^3}
\]

(1.57)

Using the data from the jack-up leg:

\[ E I^* = 1.93 \times 10^9 \) kNm²
\]

\[ l = 120 \) m
Dynamics of jack-up platforms in elevated condition

Figure 1.18: Cantilever with bending stiffness equal to the reduced leg bending stiffness

\[ k_{c,cant} = 3.4 \times 10^3 \text{ kN/m} \]
Dynamics of jack-up platforms in elevated condition

2 MASS DISTRIBUTION

2.1 LEGS

The legs have the following parameters:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length overall</td>
<td>141.50 m</td>
</tr>
<tr>
<td>Nominal mass, incl. spud can</td>
<td>1160 tons</td>
</tr>
<tr>
<td>Spud can mass</td>
<td>60 tons</td>
</tr>
<tr>
<td>Spud can height</td>
<td>3.50 m</td>
</tr>
</tbody>
</table>

Since the spud can is very near the horizontal rotation axis of the leg-soil connection, it is assumed that the spud can mass does not influence the dynamic behaviour, and therefore it is neglected for the computation of the leg mass per unit length, $m_l^o$:

$$m_l^o = \frac{(1160 - 60)}{(141.50 - 3.50)} = 8 \text{ tons/m}$$

During motions, the leg will contribute to the dynamic mass of the platform. This contribution consists of two parts:

a. The leg part above the lower guide, subdue to the same motion amplitude as the deck. Its mass is called $M_1^*$. 

b. A part of the mass of free leg length. This mass contribution is denoted as $M_2$.

Since the length of the leg part is about 20 m, its mass is equal to:

$$M_1 = 20 \times 8 = 160 \text{ tons}$$

The mass contribution from the free leg length, $M_2$, is computed assuming quasi static leg motions. This is verified in section 2.4 of the main report, together with the introduction of the dimensionless static leg deflection, $h(z)$:

$$h(z) = \frac{x(z)}{x(l)}$$

(2.01)

$h(z)$ = dimensionless static leg deflection

$x(z)$ = static deflection of the leg

$x(l)$ = static deflection at lower guide level

With this dimensionless leg deflection, the quasi-static leg
motions can be written in terms of the deck motions. For the acceleration this yields:

\[ x(t,z) = h(z)x(t,1) \]  \hspace{1cm} (2.02)

where:

\[ x(t,z) = \text{structure acceleration} \]
\[ x(t,1) = \text{deck acceleration} \]

For platform motions, the inertia force per unit length of the leg, denoted as \( f_z(t,z) \), is given by:

\[ f_z(t,z) = m_l x(t,z) \] \hspace{1cm} (2.03)

where:

\[ f_z(t,z) = \text{inertia force per unit length} \]
\[ m_l = \text{leg mass per unit length} \]

Substitution of (2.02) into (2.03) and integration of expression (2.03) with respect to \( z \) yields the overall inertia force of the leg, \( F_z(t) \):

\[ F_z(t) = \int_0^l m_l h(z)x(t,1)dz \]
\[ F_z(t) = x(t,1) \int_0^l m_l h(z)dz \] \hspace{1cm} (2.04)

The integral on the right of expression (2.04) is the mass contribution of the leg to the dynamic mass of the platform, \( M_z \), and can also be written as:

\[ M_z = m_l l_{CUTR} \] \hspace{1cm} (2.05)

where:

\[ l_{CUTR} = \int_0^l h(z)dz \] \hspace{1cm} (2.06)

This expression represents the part of the free leg length contributing to the dynamic deck mass and is equal to the shaded area in figure 2.1.
As can be seen the contributing leg length will be minimal for a maximum of \( k_A \) and a minimum of \( k_b \) and vice versa.

Introduction of the deflection formula found in the previous chapter and integration of (2.06) leads to:

\[
\begin{align*}
1_{\text{CONTR}} &= k_c \left[ \frac{c_2}{d} \left(1 - \cos(dl)\right) + \frac{\sin(dl)}{d} \right] - 1 \\
1^2 &= \frac{1}{2d^2 EI} \tag{2.07}
\end{align*}
\]

where:

\[
\begin{align*}
c_2 &= -\frac{k_b}{d^2 EI'} \tag{2.08} \\
c_3 &= \frac{k_A k_b}{d^3 EI'} c + \frac{k_4}{d^4 EI'^2} \tag{2.09} \\
c &= \frac{1 + k_A \cos(dl)/k_b + k_A \sin(dl)/dEI'}{(EI' d^2 - k_A k_b / EI') \sin(dl) - (k_A + k_b) d \cos(dl)} \tag{2.10}
\end{align*}
\]
Dynamics of jack-up platforms in elevated condition

\[ \alpha_c^2 = \sqrt{\frac{F_2}{EI'}} \]

- \( k_c \) = combined leg stiffness
- \( F_2 \) = axial leg load
- \( EI' \) = reduced leg bending stiffness
- \( k_h \) = rotational stiffness of the leg-soil connection
- \( k_b \) = rotational stiffness of the leg-hull connection
- \( l \) = free leg length

Substitution of the following values:

- \( k_c = 1.2 \times 10^3 \text{ kN/m} \) to \( 4.8 \times 10^3 \text{ kN/m} \)
- \( EI' = 1.93 \times 10^7 \text{ kNm}^2 \)
- \( k_h = 3.9 \times 10^7 \text{ kN/m/rad} \) to \( 9.6 \times 10^7 \text{ kN/m/rad} \)
- \( F_2 = 3.63 \times 10^4 \text{ kN} \)
- \( k_b = 0 \text{ kN/m/rad} \) to \( 4 \times 10^7 \text{ kN/m/rad} \)
- \( l = 120 \text{ m} \)
- \( m_l = 8 \text{ tons/m} \)

yields:

- \( l_{\text{CONTR}} = 59.9 \text{ m} \) to \( 70.1 \text{ m} \)

and:

- \( M_\perp = 479 \text{ tons} \) to \( 561 \text{ tons} \)

2.2 PLATFORM

The dynamic mass for translation of the three-legged platform is given by:

\[ M_{\text{DY}} = M_0 + 3(M_1 + M_\perp) \]  \hspace{1cm} (2.11)

where:

- \( M_{\text{DY}} \) = dynamic deck mass
M₀ = total deck mass
M₁ = mass of the leg part above lower guide level
M₂ = mass contribution from the free leg length

With all previous results, the dynamic deck mass becomes:

M_DYN = 12527 tons to 12773 tons

The legs contribute to the mass moment of inertia of the three-legged platform as follows:

\[ J = J₀ + 3(M₁ + M₂)r^2 \] (2.12)

where:

J = overall mass moment of inertia
J₀ = mass moment of inertia of the deck
r = distance between the gravity centre of the platform and the legs

With the following numerical value computed by D.N.V. for the deck of a similar platform:

\[ J₀ = 4.3 \times 10^6 \text{ tonm}^2 \]

and:

r = 31 m

the following mass moment of inertia is found for the platform:

\[ J = 6.1 \times 10^6 \text{ tonm}^2 \text{ to } 6.4 \times 10^6 \text{ tonm}^2 \]
Dynamics of jack-up platforms in elevated condition

3 LINEARIZATION OF THE DRAGFORCE

3.1 GENERAL

The drag force per unit length on the equivalent tubular leg is given by:

\[ f_d(t,z) = C_r |\dot{r}(t,z)| |r(t,z)| \]  \hspace{1cm} (3.01)

where:

- \( f_d(t,z) \) = drag force per unit length
- \( t \) = time
- \( z \) = vertical coordinate
- \( C_r \) = modified drag coefficient
- \( \dot{r}(t,z) \) = relative velocity

The relative velocity is split into two components:

\[ \dot{r}(t,z) = v(z) + \dot{r}^\nu(t,z) \]  \hspace{1cm} (3.02)

where:

- \( v(z) \) = current
- \( \dot{r}^\nu(t,z) \) = relative velocity due to waves

As the deterministic wave model is only used as a basis for the complete frequency domain analysis method, current is not accounted for in the computation of the deterministic linearization coefficient. The stochastic coefficients, however, include the influence of current.

3.2 DETERMINISTIC APPROACH

For the deterministic case without current, the steady state relative velocity consists of a harmonic platform velocity and a harmonic fluid velocity. Since both components have the same period, the relative velocity can be written as:

\[ \dot{r}(t,z) = \hat{r}(z) \sin(\omega t^\nu) \]  \hspace{1cm} (3.03)

where:

- \( \hat{r}(z) \) = amplitude of the relative velocity
- \( \omega t^\nu = \omega t + \psi \)
\[ \omega = \text{angular wave frequency} \]
\[ \psi = \text{constant phase difference between structure and fluid motions} \]

Now the drag force becomes:
\[ f_\text{D}(t,z) = C_1 r(z)^2 f_\text{p}(t) \]  \hspace{1cm} (3.04)

where:
\[ f_\text{p}(t) = \sin(\omega t') \sin(\omega t') \]  \hspace{1cm} (3.05)

This expression is linearized to:
\[ f_\text{D}(t,z) = C_1 a_\text{D}(z) r(z) \sin(\omega t') \]  \hspace{1cm} (3.06)

where:
\[ a_\text{D}(z) = \text{deterministic linearization coefficient} \]

This linearization coefficient is found by developing the harmonic part of the drag force, represented by equation (3.05), in a Fourier progression. All cosine terms of this progression are zero, so only the sine components are left:

\[ f_\text{p}(t) = \sum_{n=1}^{\infty} a_n \sin(n \omega t') \]  \hspace{1cm} (3.07)

\[ a_n = \frac{1}{\pi} \int_{0}^{\pi} f_\text{p}(t) \sin(n \omega t') d(\omega t') \]  \hspace{1cm} (3.08)

Substitution of (3.05) into (3.08) yields the following Fourier coefficients:
Dynamics of jack-up platforms in elevated condition

\[ a_1 = \frac{1}{\pi} \int_0^{2\pi} \sin(\omega t') \sin(\omega t') \sin(\omega t') d(\omega t') \]
\[ = -\frac{2}{\pi} \int_0^{2\pi} \sin^3(\omega t') d(\omega t') \]
\[ = -\frac{2}{\pi} \int_0^{2\pi} \sin(\omega t') d(\omega t') - \frac{2}{\pi} \int_0^{2\pi} \sin(\omega t') \cos(\omega t') d(\omega t') \]
\[ a_1 = -\frac{4}{3\pi} = \frac{8}{3\pi} \]  \hspace{1cm} (3.09)

\[ a_2 = \frac{1}{\pi} \int_0^{2\pi} \sin(\omega t') \sin(\omega t') \sin(2\omega t') d(\omega t') \]
\[ = -\frac{2}{\pi} \int_0^{2\pi} \sin^2(\omega t') \sin(2\omega t') d(\omega t') \]
\[ = -\frac{4}{\pi} \int_0^{2\pi} \sin^3(\omega t') \cos(\omega t') d(\omega t') \]
\[ a_2 = 0 \]  \hspace{1cm} (3.10)

\[ a_3 = \frac{1}{\pi} \int_0^{2\pi} \sin(\omega t') \sin(\omega t') \sin(3\omega t') d(\omega t') \]
\[ a_3 = \frac{8}{15\pi} \]  \hspace{1cm} (3.11)

\[ a_4 = 0 \]  \hspace{1cm} (3.12)

\[ a_5 = \frac{-8}{105\pi} \]  \hspace{1cm} (3.13)

The deterministic linearization coefficient is found by only using the first Fourier coefficient. Substitution of expressions (3.05), (3.07), (3.08) and (3.09) into (3.04) and equation of (3.04) and (3.06) leads to:

\[ C_1 \hat{f}(z) \frac{8}{3\pi} \sin(\omega t') = C_1 \hat{f}(z) a_0(z) \sin(\omega t') \]
The linearization coefficient resulting from this is given by:

\[ a_0(z) = \frac{3}{3n} r(z) \quad (3.14) \]

### 3.3 STOCHASTIC APPROACH

In the case of the stochastic wave model, the excursion of the sea surface is assumed to be a Gaussian process in time, and thus the fluid velocity has a Gaussian distribution as well. The drag force is given by:

\[ f_0(t,z) = C \hat{\dot{r}}(t,z) \quad (3.15) \]

where:

\[ \hat{\dot{r}}(t,z) = \text{relative velocity} \]

This expression is linearized to:

\[ f_0(t,z) = C (a_\xi(z) \dot{r}(t,z) + b_\xi(z)) \quad (3.16) \]

where:

\[ \dot{r}(t,z) = \text{relative velocity due to waves} \]

\[ a_\xi(z), b_\xi(z) = \text{stochastic linearization coefficients} \]

The linearization coefficients are found by minimizing the expectation value of the squared error due to this linearization of the drag force:

\[ \text{min} \quad E \left[ (\dot{r}(t,z) | \dot{r}(t,z) \right] - a_\xi(z) \dot{r}(t,z) - b_\xi(z) \right] \]

This is achieved in the following way:

\[ \frac{\partial}{\partial a_\xi} E \left[ \dot{r}(t,z) | \dot{r}(t,z) \right] - a_\xi(z) \dot{r}(t,z) + b_\xi(z)(-\dot{r}(z)) = 0 \quad (3.17) \]

\[ \frac{\partial}{\partial b_\xi} E \left[ \dot{r}(t,z) | \dot{r}(t,z) \right] - a_\xi(z) \dot{r}(t,z) + b_\xi(z)(-1)) = 0 \quad (3.18) \]

This yields:
Dynamics of jack-up platforms in elevated condition

\[
\begin{align*}
& \mathbb{E}[\dot{r}^2(t,z) | \dot{r}(t,z)] + \alpha_z(z)\dot{r}^2(t,z) + \\
& b_z(z)\dot{r}^2(t,z) = 0 \\
& \mathbb{E}[\dot{r}(t,z) | \dot{r}(t,z)] + \alpha_z(z)\dot{r}^2(t,z) + b_z(z) = 0
\end{align*}
\]

or:

\[
\begin{align*}
& \alpha_z(z)\mathbb{E}[\dot{r}^2(t,z)] + b_z(z)\mathbb{E}[\dot{r}^2(t,z)] = \\
& \mathbb{E}[\dot{r}^2(t,z) | \dot{r}(t,z)] \\
& \alpha_z(z)\mathbb{E}[\dot{r}^2(t,z)] + b_z(z) = \\
& \mathbb{E}[\dot{r}(t,z) | \dot{r}(t,z)] \quad (3.19) \\
& \mathbb{E}[\dot{r}(t,z) | \dot{r}(t,z)] \quad (3.20)
\end{align*}
\]

From the assumption that the excursion of the sea surface is a Gaussian process it follows that the fluid velocity is Gaussian as well. In the frequency domain, it is assumed that the platform motions, and thus the relative velocity, also have a Gaussian distribution. The distribution of the relative velocity due to waves is now given by:

\[
p(\dot{r}^2) = \frac{1}{\sigma_\dot{r}^2 \sqrt{2\pi}} \exp\left(-\frac{\dot{r}^2}{2\sigma_\dot{r}^2}\right) \quad (3.21)
\]

where:

\[
p(\dot{r}^2) = \text{probability of occurrence of } \dot{r}^2
\]

\[
\sigma_\dot{r}^2 = \text{standard deviation}
\]

\[
\mu_\dot{r}^2 = \text{mean value}
\]

With this distribution, the following expressions are found:

\[
\begin{align*}
& \mathbb{E}[\dot{r}^2(t,z)] = \int_{-\infty}^{+\infty} \dot{r}^2(t,z) \exp\left(-\frac{\dot{r}^2}{2\sigma_\dot{r}^2}\right) \, d\dot{r} = 0 \\
& \mathbb{E}[\dot{r}^2(t,z)^2] = \int_{-\infty}^{+\infty} \dot{r}^2(t,z)^2 \exp\left(-\frac{\dot{r}^2}{2\sigma_\dot{r}^2}\right) \, d\dot{r} = \\
& \mathbb{E}[\dot{r}(t,z) | \dot{r}(t,z)] = \int_{-\infty}^{+\infty} \dot{r}(t,z) | \dot{r}(t,z) | \exp\left(-\frac{\dot{r}^2}{2\sigma_\dot{r}^2}\right) \, d\dot{r} = \\
& \frac{2}{\sqrt{2\pi}^2 \sigma_\dot{r}^2 v(z)} \exp\left(-\frac{v^2}{2\sigma_\dot{r}^2}\right) + 2(v(z)^2 + \sigma_\dot{r}^2) \exp\left(-\frac{v^2}{2\sigma_\dot{r}^2}\right) \text{erf}\left(-\frac{v}{\sigma_\dot{r}^2}\right) \quad (3.24)
\end{align*}
\]
Dynamics of jack-up platforms in elevated condition

\[
\mathbb{E}[\hat{r}(t,z)\hat{r}(t,z)|\hat{r}(t,z)l] = \int_{-\infty}^{\infty} \frac{\hat{r}(t,z)\hat{r}(t,z)|\hat{r}(t,z)l}{\sigma_{\alpha}^2 \sqrt{2\pi}} \exp\left(-\frac{\hat{r}^2}{2\sigma_{\alpha}^2}\right) d\hat{r} = \frac{4\sigma_{\alpha}^3}{v^2} \exp\left(-\frac{v^2}{2}\right) + 4v(z) \text{erf}\left(\frac{v}{\sigma_{\alpha}^2}\right) \frac{v(z)}{\sigma_{\alpha}^2} \tag{3.25}
\]

Substitution of (3.22) to (3.25) into (3.19) and (3.20) yields the stochastic linearization coefficients:

\[
a_{\xi}(z) = \frac{4\sigma_{\alpha}^3}{\sqrt{2\pi} v^2} \exp\left(-\frac{v^2}{2}\right) + 4v(z) \text{erf}\left(\frac{v}{\sigma_{\alpha}^2}\right) \frac{v(z)}{\sigma_{\alpha}^2} \tag{3.26}
\]

\[
b_{\xi}(z) = \frac{2\sigma_{\alpha}^3 v(z)}{\sqrt{2\pi} v^2} \exp\left(-\frac{v^2}{2}\right) + 2(v(z)^2 + \sigma_{\alpha}^2) \text{erf}\left(\frac{v}{\sigma_{\alpha}^2}\right) \frac{v(z)}{\sigma_{\alpha}^2} \tag{3.27}
\]

where:

\[
\text{erf}(x) = \frac{1}{\sqrt{2\pi}} \int_{0}^{x} \exp\left(-\frac{y^2}{2}\right) dy
\]

\[
\sigma_{\alpha}^2 = \text{standard deviation of the relative velocity due to waves}
\]

\[
v(z) = \text{current}
\]

If there is no current, expressions (3.26) and (3.27) reduce to:

\[
a(z) = \frac{4\sigma_{\alpha}^3}{\sqrt{2\pi}} \tag{3.29}
\]

\[
b(z) = 0 \tag{3.29}
\]
The equation of motion of the three-degrees of freedom system is found by examining the three motion modes separately and for the dry condition. Both translational modes are considered for the x- and y-direction of the platform-fixed coordinate system, but the resulting description of the dynamic system is valid for all directions.

For a translation in the x-direction, denoted as $x(t)$, the following internal forces and moments are generated:

$$ F'(t) = -M\ddot{x}(t) - \sum_{i=1}^{n_l} c_{x,i} \dot{x}(t) - \sum_{i=1}^{n_l} k_{c,i} x(t) \quad (4.01) $$

$$ T'(t) = + \sum_{i=1}^{n_l} c_{x,i} y_i \dot{x}(t) + \sum_{i=1}^{n_l} k_{c,i} y_i x(t) \quad (4.02) $$

where:

- $i$ = leg number
- $n_l$ = total number of legs
- $F'(t)$ = internal force due to x-motions
- $T'(t)$ = internal moment due to x-motions
- $x(t)$ = deck displacement
- $\dot{x}(t)$ = deck velocity
- $\ddot{x}(t)$ = deck acceleration
- $y_i$ = y-coordinate of leg $i$
- $M$ = dynamic mass
- $c_{x,i}$ = damping of leg $i$ in the x-direction
- $k_{c,i}$ = combined leg stiffness of leg $i$

The internal forces and moments due to a translation in the y-direction, $y(t)$, are identical:

$$ F'(t) = -M\ddot{y}(t) - \sum_{i=1}^{n_l} c_{y,i} \dot{y}(t) - \sum_{i=1}^{n_l} k_{c,i} y(t) \quad (4.03) $$

Copyright of GUSTO ENGINEERING CV whose property this document remains. No part thereof may be disclosed, copied, duplicated, or in any other way made use of, except with approval of GUSTO ENGINEERING CV SCHIEDAM.
Dynamics of jack-up platforms in elevated condition

\[ T'(t) = - \sum_{i=1}^{n_L} c_{x,i} x_i \dot{y}(t) + \sum_{i=1}^{n_L} k_{y,i} y(t) \]  

where:

- \( F'(t) \) = internal force due to \( y \)-motions
- \( T'(t) \) = internal moment due to \( y \)-motions
- \( y(t) \) = deck displacement
- \( \dot{y}(t) \) = deck velocity
- \( \ddot{y}(t) \) = deck acceleration
- \( x_i \) = \( x \)-coordinate of leg \( i \)
- \( c_{y,i} \) = damping of leg \( i \) in the \( y \)-direction

The internal forces and moments caused by a rotation \( \phi(t) \) are given by:

\[ F'(t) = - \sum_{i=1}^{n_L} c_{x,i} x_i \dot{\phi}(t) + \sum_{i=1}^{n_L} c_{y,i} y_i \dot{\phi}(t) - \sum_{i=1}^{n_L} k_{x,i} x_i \ddot{\phi}(t) - \sum_{i=1}^{n_L} k_{y,i} y_i \ddot{\phi}(t) \]  

\[ T'(t) = - J \ddot{\phi}(t) - \sum_{i=1}^{n_L} c_{x,i} x_i \ddot{\phi}(t) - \sum_{i=1}^{n_L} c_{y,i} y_i \ddot{\phi}(t) - \sum_{i=1}^{n_L} k_{x,i} x_i \dddot{\phi}(t) - \sum_{i=1}^{n_L} k_{y,i} y_i \dddot{\phi}(t) \]  

where:

- \( F'(t) \) = internal force due to rotation
- \( T'(t) \) = internal moment due to rotation
- \( \phi(t) \) = deck rotation
- \( \dot{\phi}(t) \) = angular deck velocity
- \( \ddot{\phi}(t) \) = angular deck acceleration
- \( J \) = mass moment of inertia of the platform
Dynamics of jack-up platforms in elevated condition

If there are no external forces and moments, the summation of internal forces is equal to zero. Since this is the case for each of the three motion modes, this condition yields the following three equations:

\[ M \ddot{X}(t) + \sum_{i=1}^{n} c_{x_i} \dot{X}_i(t) + \sum_{i=1}^{n} k_{x_i} X_i(t) = 0 \]  
\[ M \ddot{Y}(t) + \sum_{i=1}^{n} c_{y_i} \dot{Y}_i(t) + \sum_{i=1}^{n} k_{y_i} Y_i(t) = 0 \]  
\[ J \ddot{\phi}(t) - \sum_{i=1}^{n} c_{\phi_i} \phi_i(t) + \sum_{i=1}^{n} k_{\phi_i} \phi_i(t) = 0 \]

At the introduction of the influence of the surrounding sea, the expression for the damping need not be changed. The dynamic mass now becomes:

\[ M_{dy} = M_0 + \sum_{i=1}^{n} M_{A_i} \]

where:

\[ M_{dy} = \text{dynamic deck mass, including contributions from the legs} \]

\[ M_{A_i} = \text{added water mass of leg } i \]

The leg stiffness is an omnidirectional parameter and the same for all legs, so:

Copyright of GUSTO ENGINEERING CV whose property this document remains. No part thereof may be disclosed, copied, duplicated, or in any other way made use of, except with approval of GUSTO ENGINEERING CV SCHIEDAM.
\[ k_{c_i, l} = k_c \]  

(4.11)

where:

\[ k_c = \text{combined leg stiffness} \]

Together with these two adjustments, equations (4.07) to (4.09) yield the following equation of motion of the complete platform, in matrix notation:

\[ M \ddot{x} + C \dot{x} + Kx = 0 \]  

(4.12)

where:

\[ x(t) = [x(t), y(t), \phi(t)] \]

\[ \dot{x} = [\dot{x}(t), \dot{y}(t), \dot{\phi}(t)] \]

\[ \ddot{x} = [\ddot{x}(t), \ddot{y}(t), \ddot{\phi}(t)] \]

\[ M = \begin{bmatrix} M_{DC} + \sum_{i=1}^{n} M_{A_i, l} & 0 & 0 \\ 0 & M_{DC} + \sum_{i=1}^{n} M_{A_i, l} & 0 \\ 0 & 0 & J \end{bmatrix} \]

\[ C = \begin{bmatrix} \sum_{i=1}^{n} c_{x, i, l} & 0 & -\sum_{i=1}^{n} c_{y, i, l} \\ 0 & \sum_{i=1}^{n} c_{y, i, l} & 0 \\ -\sum_{i=1}^{n} c_{x, i, l} & \sum_{i=1}^{n} c_{y, i, l} & \sum_{i=1}^{n} [c_{x, i, l} y_{i, l} + c_{y, i, l} x_{i, l}] \end{bmatrix} \]
Dynamics of jack-up platforms in elevated condition

If the platform is subjected to loads, the general equation of motion becomes:

\[ M \ddot{x} + C \dot{x} + Kx = F \]  

(4.13)

where:

\[ F = \begin{bmatrix} F_x(t) \\ F_y(t) \\ T_z(t) \end{bmatrix} \]

- \( F_x(t) = \) load in \( x \)-direction
- \( F_y(t) = \) load in \( y \)-direction
- \( T_z(t) = \) moment
<table>
<thead>
<tr>
<th>wave direction [°]</th>
<th>equivalent drag coefficient</th>
<th>equivalent mass coefficient</th>
<th>added mass conversion factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.5660</td>
<td>0.0234</td>
<td>0.0234</td>
</tr>
<tr>
<td>15</td>
<td>0.5660</td>
<td>0.0234</td>
<td>0.0234</td>
</tr>
<tr>
<td>30</td>
<td>0.5194</td>
<td>0.0234</td>
<td>0.0234</td>
</tr>
<tr>
<td>45</td>
<td>0.4964</td>
<td>0.0234</td>
<td>0.0234</td>
</tr>
<tr>
<td>60</td>
<td>0.4860</td>
<td>0.0234</td>
<td>0.0234</td>
</tr>
<tr>
<td>75</td>
<td>0.4964</td>
<td>0.0234</td>
<td>0.0234</td>
</tr>
<tr>
<td>90</td>
<td>0.5194</td>
<td>0.0234</td>
<td>0.0234</td>
</tr>
<tr>
<td>105</td>
<td>0.5660</td>
<td>0.0234</td>
<td>0.0234</td>
</tr>
<tr>
<td>120</td>
<td>0.5660</td>
<td>0.0234</td>
<td>0.0234</td>
</tr>
<tr>
<td>135</td>
<td>0.5660</td>
<td>0.0234</td>
<td>0.0234</td>
</tr>
<tr>
<td>150</td>
<td>0.5194</td>
<td>0.0234</td>
<td>0.0234</td>
</tr>
<tr>
<td>165</td>
<td>0.4964</td>
<td>0.0234</td>
<td>0.0234</td>
</tr>
<tr>
<td>180</td>
<td>0.4860</td>
<td>0.0234</td>
<td>0.0234</td>
</tr>
<tr>
<td>195</td>
<td>0.4964</td>
<td>0.0234</td>
<td>0.0234</td>
</tr>
<tr>
<td>210</td>
<td>0.5194</td>
<td>0.0234</td>
<td>0.0234</td>
</tr>
<tr>
<td>225</td>
<td>0.5660</td>
<td>0.0234</td>
<td>0.0234</td>
</tr>
<tr>
<td>240</td>
<td>0.5660</td>
<td>0.0234</td>
<td>0.0234</td>
</tr>
<tr>
<td>255</td>
<td>0.5660</td>
<td>0.0234</td>
<td>0.0234</td>
</tr>
<tr>
<td>270</td>
<td>0.5194</td>
<td>0.0234</td>
<td>0.0234</td>
</tr>
<tr>
<td>285</td>
<td>0.4964</td>
<td>0.0234</td>
<td>0.0234</td>
</tr>
<tr>
<td>300</td>
<td>0.4860</td>
<td>0.0234</td>
<td>0.0234</td>
</tr>
<tr>
<td>315</td>
<td>0.4964</td>
<td>0.0234</td>
<td>0.0234</td>
</tr>
<tr>
<td>330</td>
<td>0.5194</td>
<td>0.0234</td>
<td>0.0234</td>
</tr>
<tr>
<td>345</td>
<td>0.5660</td>
<td>0.0234</td>
<td>0.0234</td>
</tr>
</tbody>
</table>

Table 5.1: Drag and inertia parameters of the equivalent leg with $D_{eq} = 10.5$ m