Uncertainty Analysis applied to Numerical Models of River Bed Morphology

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Uncertainty Analysis
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Proefschrift

ter verkrijging van de graad van doctor
aan de Technische Universiteit Delft,
op gezag van de Rector Magnificus prof.dr.ir. J.T. Fokkema,
voorzitter van het College voor Promoties,
in het openbaar te verdedigen

op maandag 17 november 2003 om 13:00 uur

door

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wiskundig ingenieur
geboren te Zaandam.
Published and distributed by: DUP Science

DUP Science is an imprint of
Delft University Press
P.O. Box 98
2600 MG Delft
The Netherlands
Telephone: +31 15 27 85 678
Telefax: +31 15 27 85 706
E-mail: info@library.tudelft.nl

ISBN 90-407-2440-7

Keywords: river bed morphology, uncertainty analysis, numerical model

Front cover: Free impression of Figure 6.1e. Designed by Serge van der Schaft.

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Printed in The Netherlands
Summary

Knowledge of the behaviour of a river is important in decision making on river management and in river engineering. One of the important river processes is the transport of sediment and the resulting changes of the river bed topography. These processes are studied in the field of river morphology.

Much effort has been put into the development of sophisticated numerical models which simulate morphological changes of the river bed. These numerical models are based on a deterministic approach, that is, the results are presented in terms of the bed level at a certain time and location. However, the modelling of river morphology involves numerous uncertainties, for example the future river discharge, or the relation between the sediment transport and the flow conditions. Knowledge of the effect of these uncertainties on the model results is essential for a meaningful interpretation of the computed river bed. To gain this knowledge, it is necessary to perform an uncertainty analysis.

An uncertainty analysis includes the following actions:

1. making an inventory of the uncertainties involved,
2. identifying the uncertainties that have a relatively large effect on the model results and quantifying them,
3. quantifying the effect of those uncertainties on the model results,
4. interpreting the uncertainty in the model results.

In practice, little attention is paid to uncertainties in the modelling of river morphology. We expect that explicitly taking into account uncertainties will improve the understanding of river morphological processes and the completeness of morphological modelling studies. This conviction forms the background of the study described in this thesis.

The objective of this study is to inventory the difficulties concerning uncertainty analysis of numerical models of river bed morphology. Furthermore, we search a suitable method to estimate the effects of uncertainties in the model parameters and input variables on the morphological model output. With this objective the study has a methodological character. That is, we investigate how uncertainty analyses can be performed, not the magnitude of the uncertainty for specific cases. This methodological character is also reflected by the choice
to focus mainly on the effect of one particular source of uncertainty, namely the future river discharge.

For investigation and illustration purposes, three idealised case studies are used. These cases consist of one-dimensional models of a constricted main channel, a lowered floodplain and a widened floodplain, each in an otherwise uniform straight river.

Uncertainty analysis starts with making an inventory of the uncertainties in a model. For this purpose, we discuss in this thesis the method of Van Asselt (2000) to classify uncertainties. This method arranges uncertainties both by their source and by the way they manifest themselves in numerical models.

Two main sources of uncertainty are distinguished: variability and limited knowledge. Uncertainty due to variability is inherent to the process, and therefore cannot be eliminated. An example of uncertainty due to natural variability is the uncertainty concerning the future river discharge. Uncertainty due to limited knowledge may be reduced by further research or more data. An example of such an uncertainty is the uncertainty concerning the bed roughness in a specific river reach.

Four levels are distinguished for the manifestation of uncertainties in numerical models: technical uncertainties, methodological uncertainties, epistemological uncertainties, and model operation uncertainties. In this study we focus on the assessment of technical uncertainties.

The next part of an uncertainty analysis is the identification of the uncertainties that have a large influence on the model results and the quantification of them. In this study we perform a rough sensitivity analysis to get an impression which of the technical uncertainties are important. We conclude that the uncertainty about the future discharge is one of the important technical uncertainties. Other important technical uncertainties are the roughness of the river bed and the grain size of the bed material.

We examine the uncertainty about the future discharge in further detail. The variability of the river discharge often needs to be taken into account in order not to lose important information about the morphological effect of an intervention in a river reach. For the analysis of long-term morphological process in our case studies, it appears that the discharge series can be schematised by averaging the periods of low discharge, and schematising the floods as rectangles or triangles. Based on this conclusion we construct a statistical model to synthesise random discharge series.

The third part of the uncertainty analysis is the quantification of the effect of uncertainties on the model results. Many methods for uncertainty analysis are not suitable, because of the complexity of river morphological models. These models are time- and space-dependent, non-linear and the uncertainties are large. Furthermore, numerical simulations with these models are time consuming.

We examine the applicability of two Monte Carlo methods, one based on crude sampling (MCS) and another based on Latin Hypercube Sampling (LHS). Both methods give accurate estimates of the uncertainty about the computed river bed levels, as long as the sample
size is large enough and the description of the input uncertainties adequate. An advantage of MCS over LHS is the possibility to estimate the required sample size beforehand and verifying the sample size conveniently afterwards. Therefore, we conclude that especially MCS is a suitable method to quantify the uncertainty in the results of numerical models of river bed morphology. To reduce the computation time required by this method, it is important to require the least acceptable accuracy of both the MCS results and the underlying deterministic model.

Besides these two methods, we examine the applicability of the First Order Reliability Method (FORM), mainly because of its good reputation in civil engineering. We show, however, that FORM is not suitable to estimate the uncertainty in river morphological model results. The combination of non-linearity and large uncertainties leads to unreliable results.

The interpretation of the uncertainty in the model results is the final step of an uncertainty analysis. Knowledge of the magnitude and the sources of uncertainty in the model results gives more insight into the behaviour of the river bed. In this thesis we show that spatial and time-dependent characteristics of the confidence intervals are found in all case studies. We also illustrate specific relations between the river geometry and the uncertainty in the river bed level.

Knowledge of the uncertainty in model results can be used for risk analysis studies, such as for the probabilistic design of river structures, or the estimation of the risks and costs involved in the maintenance of the river. It appears that in our case studies the uncertainty about the bed level has a limited influence on the estimates of high water levels, but an important influence on the estimates of small water depths (e.g. the minimum navigable depth).

We conclude that experience with uncertainty analysis in one case study simplifies the analysis of other cases and of more complex river schematisations. Knowledge of the effect of uncertainties on the model output can help to reduce the required computation time for uncertainty analysis considerably. Investments into gaining experience will pay off in the long run.

To increase the possibilities of uncertainty analysis in river morphological models, further research is required. An important issue is the quantification of the uncertainties with a relatively large influence on the model results, such as the roughness of the river bed. Because of the time- and space-dependency and the restricted availability of data, the quantification of many uncertainties is not easy. Furthermore, the effect of methodological uncertainties on the model results should be investigated. Their effects on the reliability of the model results are expected to be large.

The analysis of uncertainties should become a standard issue in river morphological case studies. A time consuming Monte Carlo analysis is not always possible in practice. Identification of the most important uncertainties and the performance of a sensitivity analysis, however, already give much information about the reliability of the morphological results.
Samenvatting

Kennis over het gedrag van rivieren is belangrijk in, bijvoorbeeld, de besluitvorming op het gebied van rivierbeheer, of voor het ontwerpen van waterbouwkundige constructies. Het transport van sediment en de resulterende veranderingen in de rivierbodem zijn belangrijke rivierkundige processen, welke bestudeerd worden binnen het onderzoeksgebied Riviermorphologie.

Veel riviermorphologisch onderzoek is gericht op de ontwikkeling van geavanceerde numerieke modellen die bodemveranderingen simuleren. Deze numerieke modellen zijn deterministisch enigszins van aard: de resultaten worden uitgedrukt in de ligging van de rivierbodem op een bepaalde locatie en op een bepaald tijdstip. Het modelleren van riviermorphologische processen is echter onderhevig aan tal van onzekerheden, zoals bijvoorbeeld het verloop van de toekomstige rivierafvoer, of de relatie tussen het sedimenttransport en de stromingscondities. Kennis over het effect van deze onzekerheden op de modelresultaten is essentieel voor het geven van een betekenisvolle interpretatie van de berekende bodemligging. Om deze kennis te verkrijgen is het noodzakelijk om een onzekerheidsanalyse uit te voeren. Een onzekerheidsanalyse bestaat uit de volgende onderdelen:

1. het inventariseren van de onzekerheden,
2. het identificeren van de onzekerheden die relatief veel invloed hebben op de modelresultaten en het kwantificeren ervan,
3. het kwantificeren van het effect van deze onzekerheden op de modelresultaten,
4. de interpretatie van de onzekerheid in de modelresultaten.

In de praktijk wordt aan onzekerheden weinig aandacht besteed bij het modelleren van riviermorphologische processen. Wij verwachten dat het expliciet meenemen van onzekerheden in numerieke modellen het begrip van riviermorphologische processen zal verbeteren en dat het morfologische modelstudies completer zal maken. Deze overtuiging vormt de achtergrond van het onderzoek dat in dit proefschrift wordt beschreven.

De doelstelling van dit onderzoek is het inventariseren van de problematiek rond het uitvoeren van een onzekerheidsanalyse op numerieke riviermorphologische modellen. Daarnaast zoeken we een geschikte methode om het effect op de modelresultaten te schatten van de
onzekerheden die zich manifesteren in modelvariabelen en -parameters. Met deze doelstelling heeft dit onderzoek een methodologisch karakter: we onderzoeken hoe een onzekerheidsanalyse kan worden uitgevoerd. Het gaat in ons onderzoek niet om de grootte van de onzekerheid in specifieke gevallen. Dit methodologische karakter wordt ook weerspiegeld in de keuze om het onderzoek hoofdzakelijk te richten op het effect van één specifieke bron van onzekerheid, namelijk de toekomstige rivierafvoer.

Ter onderbouwing en illustratie van het onderzoek worden drie geïdealiseerde case studies gebruikt. Deze cases bestaan uit cendimensionale modellen van achtereenvolgens een versmalde hoofdgeul, verlaagde uiterwaarden en verbrede uiterwaarden, elk in een verder uniforme en rechte rivier.


Er worden twee hoofdbronnen van onzekerheid onderscheiden: variabiliteit en beperkte kennis. Onzekerheid door variabiliteit is inherent aan het proces en kan daarom niet worden geëlimineerd. Een voorbeeld van een onzekerheid door natuurlijke variabiliteit is de onzekerheid over het toekomstig afvoeroverloop in een rivier. Onzekerheid door beperkte kennis kan soms worden gereduceerd door verder onderzoek of door het verzamelen van meer meetgegevens. Een voorbeeld hiervan is de onzekerheid omtrent de bodemruwheid van een specifiek riviertraject.

Voor de manifestatie van onzekerheden in numerieke modellen worden vier niveaus onderscheiden: technische onzekerheid, methodologische onzekerheid, epistemologische onzekerheid en operationele onzekerheid. In dit onderzoek richten we ons op de analyse van technische onzekerheden.

Het volgende onderdeel van een onzekerheidsanalyse is het identificeren van de onzekerheden die relatief veel invloed hebben op de modelresultaten en het kwantificeren ervan. In dit onderzoek krijgen we op basis van een ruwe gevoeligheidsanalyse een indruk welke van de technische onzekerheden hieronder vallen. We concluderen dat de onzekerheid over de toekomstige rivierafvoer een van de belangrijkste technische onzekerheden is. Andere belangrijke technische onzekerheden zijn de ruwheid van de rivierbodem en de korrelgrootte van het bodemmateriaal.

We analyseren de onzekerheid over het toekomstig afvoeroverloop in meer detail. De variaties in het afvoeroverloop kunnen vaak niet worden verwaarloosd zonder belangrijke informatie te verliezen over het morfologische effect van een ingreep in een riviertraject. Voor de analyse van langetermijn morfologische processen in onze case studies blijken de afvoerreeksen te kunnen worden geschematiseerd door periodes van lage afvoer te middelen en periodes van hoge afvoer te schematiseren als rechthoeken of driehoeken. Op basis van deze conclusie presenteren we een statistisch model voor het genereren van random afvoerreksen.
Als derde onderdeel van een onzekerheidsanalyse noemden we het kwantificeren van het effect van onzekerheden op de modelresultaten. Vanwege de complexiteit van riviermorfológische modellen zijn veel bestaande rekenmethoden voor onzekerheidsanalyse niet geschikt. De modellen zijn tijd- en plaatsafhankelijk, niet-lineair en de onzekerheden zijn groot. Tevens kosten de numerieke simulaties met deze modellen veel rekentijd.

We onderzoeken de toepasbaarheid van twee Monte Carlo methoden, de een op basis van eenvoudige random trekkingen (MCS) en de ander op basis van Latin Hypercube Sampling (LHS). Beide methoden geven een nauwkeurige schatting van de onzekerheid rond de berekende ligging van de rivierbodem, zolang het aantal trekkingen groot genoeg is en de beschrijving van de onzekerheden in de modellinvoer accuraat. Een voordeel van MCS boven LHS is, dat vooraf het benodigde aantal trekkingen ge schat kan worden terwijl dit aantal achteraf op een acceptabele manier geverifieerd kan worden. We concluderen daarom dat met name MCS een geschikte methode is voor het kwantificeren van de onzekerheid in de resultaten van riviermorfológische modellen. Om de rekentijd die deze methode vraagt te beperken is het belangrijk om de nauwkeurigheidsseisen aan zowel de Monte Carlo resultaten als aan het onderliggende deterministische model zo laag mogelijk te stellen.

Naast deze twee methodes onderzoeken we de toepasbaarheid van de First Order Reliability Method (FORM), vooral vanwege zijn goede reputatie in de civiele techniek. We laten echter zien dat FORM niet geschikt is om de onzekerheid in riviermorfológische modelresultaten te schatten. De combinatie van niet-lineariteit en grote onzekerheden leidt tot onbetrouwbare resultaten.

De interpretatie van de onzekerheid in de modelresultaten vormt het laatste onderdeel van een onzekerheidsanalyse. Kennis over de grootte en de bronnen van de onzekerheid in de modelresultaten geeft meer inzicht in het gedrag van een rivierbodem. In dit proefschrift laten we zien dat karakteristieke ruimtelijke en tijdsafhankelijke kenmerken van de betrouwbaarheidsintervallen rond gemedellieerde bodemveranderingen herkenbaar zijn in elk van de case studies. Ook hebben we aan de hand van de case studies specifieke verbanden tussen de riviergeometrie en de onzekerheid in de bodemligging geïllustreerd.

Kennis over de onzekerheid in de modelresultaten kan onder andere worden gebruikt in risicoanalyses, bijvoorbeeld voor het probabilistisch ontwerp van constructies in een rivier, of het schatten van de risico's en kosten voor het rivieronderhoud. Het blijkt bijvoorbeeld dat voor de case studies in dit onderzoek de onzekerheid in de bodemligging een beperkte invloed heeft op schattingen van extreem hoge waterstanden, maar een belangrijke invloed op schattingen van geringe waterdieptes (bijvoorbeeld de minimale vaardiepte).

We concluderen dat ervaring met onzekerheidsanalyse in één case studie de analyse van andere case studies en van complexere modellen vereenvoudigt. Kennis over de invloed van onzekerheden op de resultaten van riviermorfológische modellen kan helpen de benodigde rekentijd voor onzekerheidsanalyses aanzienlijk te reduceren. Investeringen in het opdoen van dergelijke kennis zullen zichzelf daarom op termijn terugbetalen.

Om de mogelijkheden voor het toepassen van onzekerheidsanalyse op riviermorfológische modellen te vergroten is verder onderzoek nodig. Een belangrijk onderwerp hierbij is het
kwantificeren van de onzekerheden die een relatief grote invloed hebben op de modelresultaten, zoals bijvoorbeeld de bodemruwheid. Door de ruimte- en tijdsafhankelijkheid en de beperkte beschikbaarheid van meetgegevens is het kwantificeren van veel onzekerheden niet eenvoudig. Daarnaast moet het effect van methodologische onzekerheid op de modelresultaten worden onderzocht. De invloed van deze onzekerheden op de betrouwbaarheid van de resultaten is naar verwachting groot.

De analyse van onzekerheden moet een standaard onderdeel gaan vormen van riviermorfolo-gische studies. Een tijdrovende Monte Carlo analyse is praktisch gezien niet altijd mogelijk. Door de belangrijkste onzekerheden te identifieren en een gevoeligheidsanalyse uit te voeren wordt echter al veel informatie verkregen over de betrouwbaarheid van morfologische modelresultaten.
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1 Introduction

This thesis combines two research fields, namely river morphology and uncertainty analysis. With this combination we aim to improve the modelling of the former.

1.1 Context of the study

1.1.1 Modelling of river bed morphology

Knowledge of the behaviour of a river is important in river engineering and for decision making on river management. Examples are questions concerning safety against flooding, discussions about nature rehabilitation, planning of navigation channel maintenance and technical problems like the foundation of bridge piers. An example of a project currently in preparation is the Dutch project Room for the Rivers, which aims at the restoration of the Rhine and Meuse rivers, in such a way that future design-floods can be accommodated without loss of other functions (Silva and Kok, 1996). To control the transport of water and sediment through a river, river managers intervene with the river system, for example by constructing groynes, lowering the floodplains, or dredging the main channel. These interventions can have a large influence on the flow and the sediment transport through the river. To satisfy planning and design specifications in engineering and to avoid the occurrence of undesirable effects, understanding river processes is essential. One of the important processes is the transport of sediment and the resulting changes of the river bed topography. These processes are studied in the field of river morphology.

To improve our understanding and prediction capabilities concerning river morphological processes, a lot of research has been carried out. Much effort has been put into the development of sophisticated numerical model systems, which couple modules that estimate the flow, the sediment transport and morphological changes of the river bed. Examples are the one-dimensional model SOBEK and the two- and three-dimensional model system Delft3D of WL | Delft Hydraulics and the MIKE model series of Danish Hydraulics Institute. The use of this kind of models is common practice in present river engineering and river management.
1.1.2 The need for uncertainty analysis

The numerical models of river morphology are all based on a deterministic approach, that is, the results are presented in terms of the bed level at a certain time and location. However, as for other natural processes, the modelling of river morphology involves numerous uncertainties. For example, the river geometry contains irregular spatial variations and the composition of the bed material is not exactly known. This results in uncertain morphological estimates. Some uncertainties are inherent to the morphological process, for example the uncertainty about the future discharge, which cannot be eliminated. As a consequence, the model output of a deterministic model represents only one of many potential bed level realisations. In that respect one can claim that ignoring uncertainties leads to an incorrect, or at least an incomplete answer.

From this reasoning we conclude that uncertainty analysis is necessary in order to gain insight into the temporal and spatial variability of morphological processes and to judge the adequacy of the model results. By uncertainty analysis we understand the whole procedure of making an inventory of the uncertainties involved, quantifying them as far as possible, estimating their (relative) effect on the model output and interpreting the resulting uncertainty in the model results (cf. Morgan and Henrion, 1990, p. 39, and Bedford and Cooke, 2001, pp. 326–327). The result of an uncertainty analysis is no longer a deterministic prediction. Instead, we get a probability distribution over the output values, or the probability of occurrence of a bed level at a certain time and location (Bedford and Cooke, 2001, p. 317).

Besides the fact that it is a significant addition to the model results and that it is an advised part of each model based study following the rules of ‘good modelling practices’ (cf. Van der Molen, 1999), uncertainty analysis has other advantages, such as,

- it gives the possibility to judge whether the accuracy of the model results is acceptable for a specific purpose (cf. Van der Molen, 1999), and whether it is possible to improve the accuracy when necessary (Booij, 2002),

- it gives objectives of further research to improve the numerical model efficiently (De Vries, 1977),

- it can guide data collection so that model accuracy is enhanced at a reasonable data-collection cost (Melching, 1995),

- it enables the estimation of the risk of undesired morphological effects.

The assessment of uncertainties in the model results should not be interpreted as weakening the image of the model. For the application of a river morphological estimate in a wider context (e.g. river management) a quantified uncertainty is of more use than a statement without the knowledge of its reliability.
1.2 Present state of knowledge

Despite the fact that uncertainty analysis is necessary for a thorough river morphological study, little attention is paid to it in practice. The deterministic character of the model systems mentioned in Section 1.1.1 illustrates this. None of them contains built-in facilities to estimate the effect of uncertainties in the model parameters and input variables on the morphological output. In practice, uncertainty is sometimes addressed qualitatively, or by adopting conservative assumptions and applying safety factors. A major drawback of this approach is the lack of insight into the likelihood of the predictions (Ragas, 2000), that is, into the uncertainty about the calculated bed level changes.

In the literature a limited number of studies have been presented on the assessment of uncertainties in river morphological models. Here, we give a short literature review in order to illustrate the present state of knowledge. Most references are discussed more extensively in other chapters of this thesis.

Literature review

Some authors focus on analytical morphological models, which are simpler than numerical model systems, but also less generally applicable. Yeh and Tung (1993) analyse the uncertainty in an analytical model of pit migration through a river, resulting in the uncertainty about the maximum pit depth after a certain migration distance. A number of papers analyse uncertainties in bridge pier scouring due to various uncertain model parameters (Johnson and Ayyub, 1996; Johnson and Simon, 1997; Johnson and Dock, 1998). De Vries (1985) illustrates the uncertainty in morphological predictions based on analytical solutions to one-dimensional differential equations describing the river morphology. In earlier work De Vries illustrates that the effect of uncertainties on the model output depends on the problem studied and on the river involved (De Vries, 1979).

Other authors combine a deterministic approach and uncertainty analysis to investigate effects due to uncertain boundary conditions. Klaassen et al. (1993), for example, describe such a combination in a conceptual prediction method for planform changes of the braided sand-bed Jamuna River in Bangladesh. This method has been implemented in a numerical model by Jagers (2001), who compared it with other methods to predict braiding river processes. Zavala Zunino (1996) uses a combined approach to perform a risk analysis of the flooding of mountain rivers where landslides occur regularly. Jagers (2003) applied a neural network to estimate the probability that bank erosion occurs at a certain location within a braiding river.

We found some studies in which uncertainty analysis has been applied to a particular module of a numerical river morphological model system. Yeh and Deng (1998) describe an uncertainty analysis with respect to the sediment transport formulae of Einstein and Yang. Al-Zahrani (1995) and Gates and Al-Zahrani (1996a and 1996b) focus on the flow mo-
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dule of a one-dimensional model system. They describe the statistical characteristics of the model input and examine the effect on the model output, applied to a reach of the Columbia river (USA) concerning the backwater effect of a barrage under conditions of a peak annual discharge.

Little research has been published on uncertainty analysis of numerical models of river morphology as a whole, that is, on the combination of flow, sediment transport and bed changing modules the numerical model consists of. Chang et al. (1993) analyse the sensitivity of the model output (i.e. the water level, the sediment transport and the bed level) for various uncertain model parameters at several time points during a design-hydrograph. They study a case concerning the installation of pipe and bank protections within a section of the Santa Cruz River. Maurer et al. (1997) apply uncertainty analysis to a one-dimensional hypothetical case study of deposition of suspended sediment upstream of a river dam. They estimate the probability that the deposition of suspended sediment exceeds a threshold level.

1.3 Research definition

The objective of this study is to inventory the difficulties concerning uncertainty analysis of numerical models of river bed morphology, and to find a suitable method to estimate the effect of uncertainties in the model parameters and input variables on the morphological model output.

With this objective the study has a methodological character. That is, we are interested in how uncertainty analyses can be performed to river morphological models, not in the actual magnitude of the uncertainty in model results for specific cases. This methodological character is also reflected by the choice to focus mainly on the effect of one particular source of uncertainty, namely the uncertainty about the future river discharge.

To meet the objective, we focus on the following research questions:

1. Which types of uncertainty can be distinguished concerning river morphological modelling?

2. Which uncertainties in the parameters and the input variables of a river morphological model are relatively important with respect to the uncertainty in the model results?

3. How can the uncertainty about the future river discharge be described?

4. Which calculation methods are suitable to estimate the effect of uncertainties in model parameters and input variables on the morphological model results?

5. What opportunities do results of an uncertainty analysis offer for practice?
To specify the limitations of this study, we focus on some central concepts from the research objective and questions:

**River**: Despite the fact that in the end uncertainty analysis is desired for arbitrary morphodynamic processes in all kinds of rivers, we restrict this study to rivers with characteristics similar to those of the main Dutch rivers. This implies lowland rivers without tidal influences, continuously carrying discharge. Furthermore, we assume a stable planform.

**River morphology**: In the Netherlands, the increasing pressure on space and the wish to enhance nature restoration lead to river improvement measures, often at a local scale (i.e. a few kilometres along the river). Therefore, we focus on the morphological effects of local interventions on a river. These effects are not limited to the immediate vicinity of the river works. Both upstream and downstream of the works morphological effects can occur over larger space- and time-scales (cf. Mosselman and Sloff, 2002). In this study, we focus on morphological effects over river reaches of tens of kilometres. For the Dutch rivers, this implies processes that occur within a few decades.

**Numerical model**: This thesis is dedicated to uncertainty analysis of numerical models, suitable to describe river morphological processes in a wide variety of cases. For practical reasons this study focuses on one-dimensional numerical models, because these require a relatively small calculation time. At the end of this thesis we discuss the consequences for two-dimensional river morphological models.

**Uncertainty**: Modelling river morphology gives rise to different types of uncertainty. These uncertainties will be classified and, to a certain extent, discussed. The methodological question of how to quantify the effect of the uncertainties on the model results is restricted to the quantifiable uncertainties in the parameters and the input variables of the model.

**River discharge**: Various uncertainties concerning river morphology are complex and therefore difficult to describe. The time scale of this study is too short to investigate all uncertainties. The river discharge is an important initiator of bed disturbances, and its uncertainty appears to be one of the important sources of uncertainty in the model results. Therefore, this thesis mainly focuses on the uncertainty about the future river discharge.

**Use of terminology**

To prevent confusion, we clarify some of the terminology as it is applied in this thesis:

- *stochastic, random*: In literature, these words have different meanings. In this thesis, we use them as synonyms.

- *uncertainty analysis, stochastic approach*: Our understanding of uncertainty analysis is given in Section 1.1.2. The term stochastic approach is used for the same concept.
1.4 Outline of the thesis

As schematised in Figure 1.1, the heart of this thesis is formed by Chapters 3, 5 and 6:

- In Chapter 3 the uncertainties involved in river morphological modelling are classified and described (research question 1). A sensitivity analysis is applied to technical uncertainties, i.e. uncertainties in the model parameters and input variables (research question 2). It follows that the uncertainty due to the variability of the discharge is one of the major technical uncertainties.

- In Chapter 5 methods for uncertainty analysis are discussed in view of their applicability to numerical models of river bed morphology (research question 4). We will argue that the Monte Carlo method is the best available method to perform uncertainty analysis of numerical morphodynamic models. Also the applicability of Latin Hypercube Sampling and the First Order Reliability Method is discussed.

- Chapter 6 discusses the opportunities offered by uncertainty analysis for morphological practice (research question 5). We illustrate the presentation and interpretation of computed river bed changes in terms of confidence intervals. Furthermore, we show how to estimate the probability of occurrence of undesired river bed states.

A detailed discussion of the uncertainty in the river discharge, as formulated in research question 3, is subject of Chapter 4. Here, a statistical model is derived for the synthesis of discharge hydrographs. This model is applied to simulations performed in Chapters 5 and 6.

Throughout the thesis, three case studies are used for investigation and illustration purposes. These cases concern the morphological effects of a constricted main channel, of lowered floodplains and of widened floodplains. Chapter 2 defines these case studies, followed by a discussion of some basic theory about river morphology.

After the investigation of the research questions, in Chapter 7 we focus on some topics concerning the complete study. We discuss the relative importance of different types of uncertainty concerning the estimated river bed levels. Since uncertainty analysis usually requires large computation times, we pay attention to appropriate modelling in this respect. We emphasise the importance of gaining expert knowledge of uncertainty in model results. Finally, we discuss the consequences of this study for two-dimensional river morphological models.
1.4. Outline of the thesis

Figure 1.1: Outline of this thesis.

In Chapter 8 we give the conclusions of this study, that is, the answers to the research questions. We also give recommendations for further research and for the application of uncertainty analysis in practice.

We assume that the reader of this thesis is familiar with basic concepts of probability theory. The subject is thoroughly described in literature (e.g. Benjamin and Cornell, 1970; Ang and Tang, 1975; Morgan and Henrion, 1990).
1. Introduction
2 Case studies

2.1 Introduction

An important component of this thesis is the investigation of idealised cases. They are used to investigate both the impact of uncertainties on the model results (Chapter 3 and 4) and the complexity of a morphodynamic model (Chapter 5). Furthermore, we use case studies to illustrate the applicability of stochastic methods (Chapter 5) and the opportunities offered by uncertainty analysis in practice (Chapter 6).

As will become clear from this chapter, the schematised river reaches we used for the case studies are highly simplified compared to reality. The main reason for this choice is our conviction that a new modelling feature can best be understood by first examining simplified cases and gradually increasing complexity thereafter. By using case studies in which the morphological processes are fully understood, the uncertainty about the model results and its time- and space-dependence become clear. In a later stage, this enables the interpretation of uncertainty estimates of complex river schematisations.

The following sections describe the choice and schematisation of the cases. Furthermore, we discuss some river morphological theory to explain the bed level changes in these particular cases. Further details about the theoretical background of river morphology can be found in the literature (e.g. Jansen et al., 1979). The final section of this chapter discusses a series of discharge measurements of the river Rhine, used for the selection of discharge time series in the case studies.

2.2 Choice and schematisation

In the Netherlands a major project is in preparation, called Room for the Rivers. It aims at the improvement of the Rhine and Meuse rivers, in such a way that future design floods can be accommodated without loss of other functions, and, as far as possible, without further strengthening of the dikes (Silva and Kok, 1996). The hydraulic and morphological effects of a wide variety of river works are investigated, for example lowering of the groynes, floodplain lowering, lowering of the main channel bed, removal of hydraulic obstacles, and large-scale repositioning of river dykes.
Inspired by this project, we choose the following three cases:

- Case 1: constriction of the main channel, in an otherwise uniform straight river with floodplains,
- Case 2: lowering of the floodplains in an otherwise uniform straight river,
- Case 3: widening of the floodplains in an otherwise uniform straight river.

These cases meet our demands since they concern local-scale river works (cf. Section 1.3) of which the morphological effects are fully understood (cf. Section 2.1). Furthermore, the cases complement each other, since each of the river works reacts differently on discharge variations (which is the source of uncertainty this thesis focuses on).

The one-dimensional morphodynamic model SOBEK (WL | Delft Hydraulics, 2000) is used to simulate the morphological processes in each of the cases (see Appendix A for technical information about SOBEK). The next sections describe the schematisation of the cases and the boundary conditions and control parameters as required for the numerical model.

**Schematisation of the planforms**

The dimensions and parameter settings of the case studies have been based on the Dutch river Waal. The river Waal is one of the main branches of the river Rhine in the Netherlands. It is a non-tidal lowland river with subcritical flow. Training works have resulted in a fixed planform of the river.

Figure 2.1 gives an overview of the planform of each of the cases. The corresponding dimensions are presented in Table 2.1:

- Case 1: The main channel is constricted instantaneously over a distance of 10 km, reducing the channel width by about 20%. The transition from the wide to the small channel, and vice versa, occurs within one grid cell, that is, within 500 m.
- Case 2: The floodplains are lowered instantaneously with 1 m, over a distance of 10 km. The transition from the high to the lowered floodplains, and vice versa, occurs within a river reach of 1 km.
- Case 3: The floodplains are widened instantaneously by a factor 2.5 over a distance of 5 km. The transition from the reference to the widened floodplains, and vice versa, occurs within a reach of 1.5 km.

For each of the cases, the morphological changes are investigated relative to the corresponding reference state, that is, the state of the river before the construction of the intervention. Each reference state is in equilibrium, implying that morphological changes only result from the influence of the river work.
2.2. Choice and schematisation

(a) Case 1: constricted main channel

(b) Case 2: lowered floodplains

(c) Case 3: widened floodplains

Figure 2.1: Overview and cross-sections of the case studies, where the index 0 refers to the unadapted river reach, c to the constricted reach and w to the widened reach (note: dimensions not equally scaled). The dimensions are listed in Table 2.1.
Table 2.1: Dimensions used in the case studies, with changed geometry between brackets: constricted main channel (Case 1), lowered floodplains (Case 2), widened floodplains (Case 3)

<table>
<thead>
<tr>
<th>description</th>
<th>symbol</th>
<th>value</th>
<th>unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>bed slope</td>
<td>$i$</td>
<td>0.11</td>
<td>0.11</td>
</tr>
<tr>
<td>width main channel</td>
<td>$B_m$</td>
<td>260 (210)</td>
<td>260</td>
</tr>
<tr>
<td>width floodplains</td>
<td>$B_f$</td>
<td>1100</td>
<td>1100</td>
</tr>
<tr>
<td>length river work</td>
<td>$L$</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>height main channel bank</td>
<td>$h_b$</td>
<td>6</td>
<td>6 (5)</td>
</tr>
<tr>
<td>Chézy coeff. main channel</td>
<td>$C_m$</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>Chézy coeff. floodplains</td>
<td>$C_f$</td>
<td>35</td>
<td>35</td>
</tr>
<tr>
<td>Strickler coeff. main channel</td>
<td>$k_{s,m}$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Strickler coeff. floodplains</td>
<td>$k_{s,f}$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>grain size bed material</td>
<td>$D$</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>numerical grid distance</td>
<td>$\Delta x$</td>
<td>500</td>
<td>500</td>
</tr>
<tr>
<td>numerical time step</td>
<td>$\Delta t$</td>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>

Note that the reference state of Case 3 differs from that of Cases 1 and 2. Since this difference does not essentially affect the aim of the case study for this thesis, we have left this case as it is. Cases 1 and 2 have also been used in earlier publications by Van der Klis (2000 and 2001) and Van Vuren et al. (2002). Case 3 is described in further depth by Rath (2001).

Boundary conditions and control parameters

For each of the case studies the boundary conditions have been defined as follows:

- the hydraulic condition at the upstream boundary is a discharge hydrograph;
- the morphological condition at the upstream boundary is that the incoming sediment transport equals the local transport capacity at the boundary;
- the hydraulic condition at the downstream boundary is a rating curve, which specifies the water level as a function of the discharge. The location of the boundary is such that it does not significantly influence the morphology in and near the reach of intervention;
- both the main channel and the floodplains have fixed vertical banks.
Besides the river geometry, the schematisations of the hydraulic roughness and the grain size of the bed material have been simplified. Both parameters are considered constant in time and place, and the grain size is supposed to be uniform.

In Cases 1 and 2 the hydraulic roughness is expressed in terms of Chézy coefficients. In Case 3 the hydraulic roughness is expressed in terms of Strickler coefficients ($k_s$). The relation between the Strickler and Chézy coefficients is as follows:

$$C = 25 \left( \frac{R}{k_s} \right)^{1/6},$$

(2.1)

where $R$ is the hydraulic radius.

### 2.3 Morphological impact of the river works

To describe and explain the morphological consequences of the river works in the three cases, we first discuss the one-dimensional mathematical model involved, and subsequently its application to the cases.

#### 2.3.1 Mathematical model

**The dynamic model**

In the description of the equations describing the dynamic river morphological process we closely follow the technical reference manual of SOBEK (WL | Delft Hydraulics, 2000), the one-dimensional numerical modelling system used within this study. More information about these dynamic equations can be found in river morphological textbooks, such as Jansen et al. (1979).

The morphodynamic model consists of equations describing water flow, sediment transport and bed level changes. The water flow is computed by solving the Saint Venant-equations (also called the one-dimensional shallow water equations), which are a continuity equation and a momentum equation. The continuity equation reads

$$\frac{\partial A_f}{\partial t} + \frac{\partial Q}{\partial x} = q_{lat},$$

(2.2)

in which $A_f$ is the cross-sectional flow area, $Q$ the discharge and $q_{lat}$ the discharge added to or withdrawn from the river per unit length. The time and space parameters are represented by $t$ and $x$ respectively. The momentum equation reads

$$\frac{\partial Q}{\partial t} + \frac{1}{\alpha_B} \left( \frac{Q^2}{A_f} \right) + gA_f \frac{\partial h}{\partial x} + \frac{gQ|Q|}{C^2 RA_f} = 0,$$

(2.3)

in which $\alpha_B$ is the Boussinesq constant, $g$ the gravity acceleration, $h$ the water level relative to a chosen reference level, $C$ the hydraulic roughness Chézy coefficient and $R$ the hydraulic
radius. The flow velocity $u$ is defined as the average flow velocity in the flow section of the
cross-section (i.e. $u = \frac{Q}{A_f}$). This set of equations can also be used in steady mode, by
setting the time derivatives in the equations equal to zero.

The transport of sand in rivers is often estimated by making use of empirical sediment trans-
port formulae, of which a large number is available from literature. We use the formula of
Engelund and Hansen (1967), which computes the total sediment load (i.e. the bed load and
the suspended load). This transport formula is usually applied for rivers with relatively fine
sediment. The formula does not account for a threshold of motion, implying that water flow
always results in sediment transport. In case of an uniform grain size $D$, the sediment trans-
port per unit width (including the pore volume) according to Engelund-Hansen is computed by

$$s = \frac{0.05u^5}{(1-\epsilon)\sqrt{gC^3\Delta_d^2 D}},$$  \hspace{1cm} (2.4)

in which $u$ is the average flow velocity in the main channel, $\epsilon$ the packing factor of the
bed material and $\Delta_d$ the relative density of the sediment. The total sediment transport in a
cross-section of the river, in terms of deposited bed volume, is calculated by

$$S = W_s s,$$  \hspace{1cm} (2.5)

in which $W_s$ is the sediment transporting width. In the case studies, sediment transport only
takes place in the main channel.

Sediment transport gradients will lead to changes in the bed level of a cross-section. If
only bed load transport is taken into account, the bed level changes are described by the
continuity equation of bed material,

$$\frac{\partial z_b}{\partial t} + \frac{\partial s}{\partial x} = 0,$$ \hspace{1cm} (2.6)

in which $z_b$ is the bed level relative to the chosen reference level.

The most important assumptions made for the dynamic model are:

- two- and three-dimensional influences (e.g. asymmetric floodplains on left and
right banks) and morphological effects (e.g. inner versus outer bend) are negligible;
- the river is shallow, that is, the flow width is large compared to the flow depth;
- the pressure distribution is hydrostatic;
- the banks are non-erodible;
- the sediment is uniform, so effects of variations in the bed material size are negli-
gible;
- the sediment transport in a point is determined by local hydrodynamic conditions
without lag effects.
The equilibrium model

For a first analysis of the morphological effect of an intervention on a river, the dynamic model described above is complicated. Here, we describe a highly simplified model describing the equilibrium state under constant discharge.

In case of a constant discharge through a straight river reach with a rectangular cross-section without floodplains, the river bed will approach an equilibrium state that is described by the following equations (Jansen et al., 1979, p. 287),

\begin{align*}
Q &= Bh \quad \text{(2.7)} \\
\frac{u}{i} &= C \sqrt{hi} \quad \text{(2.8)} \\
S &= Bs(u) \quad \text{(2.9)}
\end{align*}

in which \( B \) the river width and \( i \) the bed slope. As mentioned above, we used the Engelund-Hansen formula to describe the sediment transport \( s \). The notation of this formula can be simplified to \( s = au^b \), where \( b = 5 \) in case of the Engelund-Hansen formula (cf. Equation (2.4)).

These equations hold for each river reach where the model parameters are constant. In the equilibrium state, the flow is uniform and the water surface is parallel to the river bed. As a result, the equilibrium state of a prismatic straight alluvial channel with fixed banks, supplied with a constant discharge \( Q \) and a constant sediment influx \( S \), is described by

\begin{align*}
h_e &= \left( \frac{S}{aB} \right)^{-\frac{1}{6}} \frac{Q}{B}, \quad \text{and} \quad \text{(2.10)} \\
i_e &= \left( \frac{S}{aB} \right)^{\frac{3}{5}} \frac{B}{C^2 Q}. \quad \text{(2.11)}
\end{align*}

For river reaches that include floodplains, the water depth increases more slowly with discharges beyond bankfull than suggested by Equation (2.10), because part of the water flows over the much wider floodplains (Jansen et al., 1979, p. 58). For each of the cases, a bankfull discharge of about 1600 m³/s holds.

2.3.2 Application to the case studies

By applying the mathematical models just described, information is obtained about the morphological processes in the case studies. We present the equilibrium states and the initial morphological effects. The latter gives an indication of the dynamic behaviour of the river bed under a variable discharge.

The equilibrium state

The equilibrium state of the river bed gives a first impression of the morphological effect of a river work. It reveals the river bed topography that would occur under a constant discharge,
Figure 2.2: Relation between the total river discharge and the discharge conveyed through the main channel, for Case 1, with and without floodplains.

Giving insight into which river reaches will undergo net sedimentation or erosion of the bed. We determine the equilibrium states in each of the cases in two ways:

1. by applying the equilibrium model (Equations (2.10) – (2.11)) to a discharge of 2000 m³/s, which requires a simplified treatment of the floodplains;

2. by performing a numerical simulation under a constant discharge of 2000 m³/s, in which the influence of the floodplains is automatically considered.

Case 1 (constricted main channel): Since the constriction in Case 1 only concerns the river main channel, the floodplains can simply be neglected when applying the equilibrium model. From the expressions for $h_e$ and $i_e$ (Equations (2.10) – (2.11)), it follows that in the equilibrium state the water depth in the constriction is larger and the bed slope is gentler than before (Figure 2.3(a)). Hence, upstream of the constriction the river bed is slightly lowered.

The equilibrium state resulting from the numerical simulation under constant discharge reveals the influence of the floodplains (Figure 2.3(a)). For discharges higher than bankfull a substantial part of the water is conveyed through the floodplains (Figure 2.2), implying that an increasing portion of the discharge is not effective in transporting sediment. This results in a reduction of the morphological effect within the constricted reach. Furthermore, the redistribution of the high discharge over the river width upstream and downstream of the constriction does not occur abruptly, but gradually over a certain distance. The effect is a further increase of the bed slope within the constriction, and a sedimentation reach downstream.

For low discharges, in which the flow is carried only through the main channel, the equilibrium state resulting from the numerical simulation under constant discharge coincides with the state following from the equilibrium equations.
2.3. Morphological impact of the river works

Figure 2.3: Equilibrium states for each case study under a constant discharge of 2000 m³/s, by applying both the equilibrium model and a numerical simulation under a constant discharge.

Cases 2 and 3 (lowered and widened floodplains): Since in Cases 2 and 3 the river works concern the geometry of the floodplains, they have to be schematised alternatively in order to investigate the morphological consequences in the equilibrium state through the equilibrium model. Both floodplain adaptations enlarge the flood conveyance capacity, resulting in a decreased part of the discharge conveyed through the main channel. When we assume that the sediment transport only takes place within the main channel, the changes within the floodplains can be schematised by extracting discharge from the main channel at one location and supplying the same amount at a location further downstream. The expressions for $h_e$ and $i_e$ (Equations (2.10) – (2.11)) imply an increasing bed slope, thus a steeper bed, and a decreasing water depth, thus an increased bed level (Figures 2.3(b) and 2.3(c)). The bed level upstream of the adapted reach rises. The results of the numerical simulation under constant discharge (Figures 2.3(b) and 2.3(c))
Figure 2.4: The three possible equilibrium states of Case 2, for three distinct discharge intervals, according to numerical simulations under constant discharge.

shows an essentially different effect. Firstly, systematic erosion in the reach downstream of the lowered floodplains is shown. Secondly, the equilibrium profile is curved, due to the gradual spatial variation of the discharge distribution between the main channel and the floodplains.

Both for Case 2 and 3 no morphological effect occurs under a low constant discharge, i.e. lower than the bankfull discharge. For Case 2 even three different equilibrium states exist, for three distinct discharge intervals (Figure 2.4). In case of the intermediate interval, the discharge is under bankfull upstream and downstream of the lowered reach, but above bankfull within this reach (where a bankfull discharge of 1250 m$^3$/s holds).

**Initial morphological effects**

Based on the dynamic equations, the initial effect of the river works on the bed level can be constructed. This information helps to understand the dynamic behaviour of the river bed. Figures 2.5 and 2.6 show schematically how the initial effect can be constructed, by successively determining the initial effect on the water depth ($h$), the flow velocity ($u$), the sediment transport ($s$), and finally the time derivative of the bed level ($\frac{\partial z}{\partial t}$). Figure 2.5 shows the results for Case 1, and Figure 2.6 for the Cases 2 and 3. For the changed floodplains we assume a discharge above bankfull, since only then the bed level of the main channel is affected.

From this construction of the initial morphological effects we can conclude that the most prominent effects occur at either end of the adapted river reaches. In Case 1, initially an erosion wave develops at the upstream edge of the constriction, and a sedimentation wave at the downstream edge. In Cases 2 and 3 the opposite effect occurs, namely a sedimentation wave at the upstream end of the reach with the changed floodplains, and an erosion wave downstream of this reach.
Figure 2.5: Construction of the initial morphological effect of a constricted main channel (Case 1).
Figure 2.6: Construction of the initial morphological effect of lowered or widened floodplains(Cases 2 and 3).
2.4 Analysis of bed disturbances

Under the variable discharge that occurs in reality, an equilibrium state as described in the foregoing will never be reached. With each flood bed waves are formed, following the initial effect just described. At low discharge the bed waves decrease in height and propagate downstream (see Section 2.4). Since the constriction in Case 1 is located within the main channel, the bed is affected under all discharge conditions. This means that the bed within the constricted reach never reaches a steady state. Contrastingly, the changes in the floodplain geometry of Cases 2 and 3 only affect the main channel bed under relatively high discharge conditions. This implies that the bed in the main channel tends to its initial level during low discharges. Whether this initial level is reached or not depends on the period of time over which the low discharge holds and on the time scale of the morphological changes (see Section 2.4).

2.4 Analysis of bed disturbances

As discussed in the previous section, changes in the river geometry induce bed waves which propagate downstream through the river system (Figures 2.5–2.6). By analysing the development of these bed waves, estimates can be obtained about the time scale of certain morphological processes, and about the distance over which the river bed is affected by an intervention. This kind of information also helps to interpret the results of uncertainty analysis on river morphological models. Therefore, we now focus on the development of bed disturbances.

About advection and diffusion

The analysis of the initial morphological effects of a river work showed that bed waves are initiated at locations of a relatively abrupt change in the river geometry. The largest bed waves are initiated during periods of high discharge. At low discharges, they propagate downstream, an advection process, and reduce in height, partly a diffusion process (that is, the eight can also reduce without diffusion, when a shock wave has been developed). The advection and diffusion processes are characterised by the propagation velocity of the bed waves, $c_b$, and the diffusion coefficient, $K$. These parameters are derived from simplified versions of the full one-dimensional model of river morphology (Appendix B) and read

$$c_b = \frac{b}{h} s$$

(2.12)

$$K = \frac{b}{3} \frac{s}{t}$$

(2.13)

where the sediment transport $s$ includes the pore volume. For small bed disturbances, $c_b$ represents the propagation speed of the bed waves, whereas $K$ defines the diffusion process. For the large bed waves induced by river works, these expressions are no longer valid. Ribberink and Van der Sande (1985) show that $c_b$ and $K$ increase with increasing magnitude of the bed disturbance.
The simplified models from which \( c_b \) and \( K \) have been derived are valid in different parameter domains, so that the combination of the advection and diffusion processes is not just the superposition of the two (Appendix B). This also follows from the fact that the effective propagation speed and diffusion coefficient differ from \( c_b \) and \( K \). Vreugdenhil (1982) deduces the following expressions for the effective propagation speed \( (c_e) \) and diffusion coefficient \( (K_e) \):

\[
\frac{c_e}{c_b} = \frac{1}{1 + P^2}
\]

\[
\frac{K_e}{K} = \frac{P^2}{1 + P^2},
\]

where \( P = \frac{l_{\text{cb}}}{l} = \frac{l}{\frac{\delta}{h}} \), the Péclet number. Note that \( P \) is fully determined by the ratio of the length scale of the backwater curve (\( \sim \frac{\delta}{h} \)) to the length scale of the bed disturbance (\( l \)). Given the validity domains of the advection and the diffusion model (Appendix B), \( P \) serves as a discriminator between the two approaches. The relation between \( P \) and these dimensionless advection and diffusion terms is illustrated in Figure 2.7, showing that for small values of \( P \) the advection process prevails, whereas for large values the bed waves are mainly influenced by diffusion.

To find out what the theory about the advection and diffusion of bed waves implies for our case studies, we estimated \( c_b \) and \( K \) for the river reaches downstream of the river works (Table 2.2). Since the dimensions of the downstream river reaches are identical for Cases 1 and 2, the estimates for the parameters are also equal. From the estimates we notice, that both \( c_b \) and \( K \) increase with the discharge. The values for Case 3 are relatively small, implying that bed disturbances propagate and decrease more slowly. The values for \( P \) for a
Table 2.2: Estimates of the propagation speed ($c_b$) and diffusion parameter ($K$) of reaches downstream of the river works for all cases, based on numerical simulations, and the concerning Péclet number ($P$) for a wave length of 5 km.

<table>
<thead>
<tr>
<th>$Q$ [m$^3$/s]</th>
<th>Cases 1 and 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$c_b$ [mm/s]</td>
<td>$K$ [m$^2$/s]</td>
</tr>
<tr>
<td>500</td>
<td>0.05</td>
<td>0.2</td>
</tr>
<tr>
<td>1000</td>
<td>0.09</td>
<td>0.7</td>
</tr>
<tr>
<td>1500</td>
<td>0.14</td>
<td>1.4</td>
</tr>
<tr>
<td>2000</td>
<td>0.17</td>
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<tr>
<td>2500</td>
<td>0.19</td>
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<tr>
<td>3000</td>
<td>0.20</td>
<td>6.7</td>
</tr>
<tr>
<td>4000</td>
<td>0.23</td>
<td>13.0</td>
</tr>
</tbody>
</table>

typical bed wave length of 5 km (Table 2.2), combined with Figure 2.7, imply that during floods the propagation of the bed waves prevails, whereas under decreasing discharge the relative influence of diffusion increases.

Within the adapted reaches $c_b$ and $K$ are slightly different from the values presented in Table 2.2. In the first period after the construction of a river work, the flow and transport conditions change rapidly, making a theoretical estimation of the parameters difficult. When a new dynamic equilibrium has been reached, the average sediment transport is the same as outside the constriction, while the water depth and the bed slope have changed. In case of the constricted main channel the water depth has increased, causing a decreased $c_b$, whereas the slope has decreased, causing an increased $K$. For the other cases the opposite holds, so an increased $c_b$ and a decreased $K$ within the reach of the river work.

**Use of this information**

The time it takes for a specific morphological process to evolve depends on various parameters, such as the length scale of the process, the discharge and the characteristics of the bed material. Knowledge of the time scale of morphological processes simplifies the design of numerical experiments, including uncertainty analysis, and the interpretation of results. For example, a good estimate of the time scale covered by the process of interest enables an efficient choice of the duration of a simulation.

To estimate the time scale of a morphological process in a one-dimensional model, the propagation speed $c_b$ can be used. It will take in the order of $L/c_b$ seconds for the front of a bed wave to propagate through a river reach of length $L$. For example, considering Case 1, the period in which the bed throughout the constricted river reach has been affected under a constant discharge of 1500 m$^3$/s (i.e. about the mean of the variable discharge, Section 2.5)
can be estimated by

$$T = \frac{L}{c_{b,1}} \geq \frac{10,000}{0.14 \cdot 10^{-3}} \text{ m/s} \approx 2.3 \text{ years.}$$

(2.16)

As discussed before, the propagation speed within the constricted reach, $c_{b,1}$, is smaller than in the undisturbed river reaches. Since under a variable discharge a couple of bed waves must have travelled the distance $L$ before the river bed has been adapted to the constriction, we estimate that it takes in the order of 5–10 years to reach a new dynamic equilibrium. A similar estimation for the widened reach in Case 3 gives a larger period:

$$T = \frac{L}{c_{b,1}} \leq \frac{5000}{0.03 \cdot 10^{-3}} \text{ m/s} \approx 5.4 \text{ years.}$$

(2.17)

Another example is the product of the time scale of floods and the propagation speed of a bed wave. This estimates the length of the river reach which is affected during one high-discharge season. As an example we analyse Case 2. If the product of the flood time scale and the propagation speed is large compared to the adapted reach, the whole lowered reach is affected during the high discharge period. During low discharge this sedimentation wave would propagate immediately out of the lowered reach. In the present case study (and in case of the river Waal), however, the product is small compared to the adapted reach. Only the first kilometres within the lowered reach are directly affected during one high discharge season. Further downstream within the lowered reach sedimentation occurs due to the propagation of this relatively short bed wave. Therefore, it takes a few floods before the lowered reach has adjusted completely.

### 2.5 Discharge measurements and schematisation

The hydraulic upstream boundary conditions for all three case studies are based on discharge measurements in the river Rhine near Lobith. To be more precise, we used the daily discharge data from 1946 to 2000. Since the dimensions of the case studies are based on the river Waal, which is a Dutch branch of the river Rhine, these measured discharges are too high. Therefore, we used a scaled version of the measured discharge series, that is, scaled with the factor $\frac{2}{3}$. This scaling factor roughly corresponds with the fraction of the Rhine discharge that flows into the river Waal.

To fit the discharge series with the numerical time step applied in the simulations ($\Delta t = 10$ days), we used the 10-days averaged discharges based on the measurements. To that end, we defined one year to consist of 360 days, subdivided in 36 periods of about 10 days (that is, the remaining 5 or 6 days per year are divided between these 36 periods). The 10 discharge measurements per 10-days period were averaged with such a weight that the sediment transport capacity was maintained. In case of the Engelund-Hansen sediment
transport formula the weighted averaged discharge $\bar{Q}$ can be calculated by

$$\bar{Q} = \left( \frac{\sum_{i=1}^{N} Q_i^{5/3}}{N} \right)^{3/5},$$

(2.18)

where $Q_i$ is the discharge measured at day $i$, and $N$ the number of daily discharges averaged. Figure 2.8 illustrates the resulting 10-days discharge series. Clearly visible are the floods and the seasonal periodicity, with both large and small discharges during the year. Floods generally occur in winter or spring, whereas in summer and autumn the discharge is low. More information about the characteristics of this discharge series can be obtained from its statistics:

- Table 2.3 contains some general statistics. For comparison, also the statistics of the daily (scaled Rhine) discharge series are given.
- Figure 2.9 shows the frequency and cumulative frequency distributions of the discharge levels. The high discharges show a wider spread than the low discharges.
- Figure 2.9(c) shows the mean, the 90%-confidence interval and the extreme values per 10-days averaged discharge. The seasonal periodicity is clear, with a larger confidence interval in the high discharge season. The discharge generally is lowest during July and August.
Figure 2.9: Statistics of the 10-days discharge series applied to the cases.

Table 2.3: Statistics of the discharge series applied to the cases (i.e. the scaled Rhine discharge), both for the 10-days averaged discharges and the daily discharges.

<table>
<thead>
<tr>
<th>quantity</th>
<th>10-days $Q$</th>
<th>daily $Q$</th>
<th>unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td>1485</td>
<td>1481</td>
<td>m$^3$/s</td>
</tr>
<tr>
<td>standard deviation</td>
<td>744</td>
<td>787</td>
<td>-</td>
</tr>
<tr>
<td>skewness</td>
<td>1.7</td>
<td>2.0</td>
<td>-</td>
</tr>
<tr>
<td>kurtosis</td>
<td>6.8</td>
<td>9.5</td>
<td>-</td>
</tr>
<tr>
<td>maximum</td>
<td>5578</td>
<td>7923</td>
<td>m$^3$/s</td>
</tr>
<tr>
<td>minimum</td>
<td>422</td>
<td>413</td>
<td>m$^3$/s</td>
</tr>
</tbody>
</table>
3 Uncertainties

3.1 Introduction

In this chapter we focus on the central theme of this thesis: uncertainty. Presenting a definition of uncertainty is less trivial than it might seem. Baecker and Christian (2003) give a review of definitions used by different authors. Bedford and Cooke (2001) discuss that in practical scientific and engineering contexts, certainty is achieved through observations, implying that uncertainty concerns the results of possible observations. Walker et al. (2003) adopt a general definition of uncertainty as being “any deviation from the unachievable ideal of completely deterministic knowledge of the relevant system”.

With the analysis of uncertainties questions arise like 'Which uncertainties play a role?', 'How can their influence on the model results be analysed?', 'How should the uncertainty about the results be interpreted?' and 'Can the uncertainty be reduced?'. To answer this kind of questions, an inventory must be made of all uncertainties of potential importance. For that purpose we discuss a classification method to order uncertainties. We apply the method to some uncertainties that are potentially important for river morphological problems (Section 3.2).

Different types of uncertainty require different methods to handle them in an uncertainty analysis. We discuss this briefly in Section 3.3.

In Section 3.4 we focus on the type of uncertainty that is the subject of Chapters 4 to 6: uncertainties in model parameters and input variables (i.e. 'technical uncertainties in empirical quantities'). We discuss specific uncertainties of this type and argue, based on a sensitivity analysis, which are relatively of importance.

3.2 A classification of uncertainties

In this section we describe a method to classify uncertainties. Matching specific uncertainties to this classification has a number of advantages: it simplifies the communication about uncertainties, it indicates how to assess the uncertainties, and it helps to interpret the variance in the output of the model under consideration (cf. Walker et al., 2003).
Several authors have tried to order the different types of uncertainties (e.g. Morgan and Henrion, 1990; Van Gelder, 2000; Bedford and Cooke, 2001; Baecher and Christian, 2003). We follow the classification of Van Asselt (Van Asselt, 2000; Van Asselt and Rotmans, 2002), because this classification distinguishes between a taxonomy of sources of uncertainty and the way uncertainties manifest themselves. In our opinion, this distinction clarifies the definition of the uncertainties in specific applications, and it corresponds to the different analyses of the uncertainties required for an uncertainty analysis. Furthermore, this classification is meant to be universally applicable, which simplifies the comparison of different problems on the basis of the uncertainties involved. In this section we first discuss the taxonomy of sources of uncertainty, followed by the ways uncertainty manifests itself in numerical models.

### 3.2.1 Taxonomy of sources of uncertainty

In her classification of uncertainties, Van Asselt distinguishes between two main sources of uncertainty: *variability* and *limited knowledge*:

- **Variability**: A process or system behaves in different ways or has a random character. A number of sources of variability can be distinguished (Figure 3.1). Uncertainty due to variability is inherent to the particular process, elimination is not possible.

- **Limited knowledge**: This is a property of the modeller or of the general state of knowledge. Limited knowledge partly results from variability, but knowledge of deterministic processes can also be incomplete and uncertain.

Van Asselt describes the sources of limited knowledge as a continuum, ranging from unreliability to more fundamental, structural uncertainty (Figure 3.1). Uncertainty due to unreliability is caused by inexactness or lack of measurements, and can in principle be reduced with more research and more accurate measurements. Structural uncertainty is due to practically immeasurable data, processes that we do not yet observe and indeterminacy of some natural processes. This type of uncertainty can at best be roughly estimated, and reduction is usually difficult or even impossible. Whether uncertainties can be reduced or not is mainly interesting for the interpretation of the uncertainty in the model results, for example, in the light of recommendations for future research (De Vries, 1977). This difference normally does not lead to practical difficulties for uncertainty analyses (Morgan and Henrion, 1990).

To make this taxonomy of sources of uncertainty concrete, we applied it to a variety of uncertainties concerning the description or prediction of river morphology (Table 3.1). In gathering these uncertainties, we focussed on long-term morphological processes modelled by a numerical model (compare the case studies described in Chapter 2). This list of uncertainties is not complete for a particular river morphological problem.
Bedford and Cooke (2001, p. 33) remark that the categorisation of uncertainties is for the purpose of a particular model, since the ‘same’ uncertainty in a different model with a different goal might be classified differently (also compare Baecher and Christian, 2003). We agree with this remark, in the sense that some uncertainties can be categorised at more than one place within the taxonomy of sources of uncertainty. The final choice of the categorisation can help to make the modelling choices clear. Take, for example, the uncertainty about the river discharge. In a river morphological model the discharge is often considered as a boundary condition that forces the river processes, implying an uncertainty due to variability. In a hydrological model, however, that aims to model the discharge as a function of weather conditions and catchment characteristics, the uncertainty about the discharge will be classified as due to limited knowledge of the underlying processes. In the latter model, the uncertainty due to variability is recognised in, for example, the weather conditions.

### 3.2.2 Manifestation of uncertainties in numerical models

The ordering of uncertainties of a particular problem according to the taxonomy of their sources reveals to what extent the uncertainties can be quantified or eliminated. When, however, a method has to be chosen to assess the effect of uncertainties on model results, a classification in terms of the manifestation of uncertainties in the particular model is desired. Building upon Funtowicz and Ravetz (1990) and Van der Sluijs (1997), Van Asselt (2000) and Van Asselt and Rotmans (2002) distinguish between four levels at which uncertainties...
<table>
<thead>
<tr>
<th>Sources of uncertainty due to limited knowledge</th>
<th>Sources of uncertainty due to variability</th>
</tr>
</thead>
<tbody>
<tr>
<td>morphological process under extreme discharges</td>
<td>new measurement techniques</td>
</tr>
<tr>
<td>- containing results from model calibration</td>
<td>- technological surprise</td>
</tr>
<tr>
<td>- sediment distribution at bifurcation (1-D model)</td>
<td>- Flint measurement</td>
</tr>
<tr>
<td>- sediment transport model</td>
<td>- Flint over management</td>
</tr>
<tr>
<td>- application of numerical model to new river work</td>
<td>- Flint river management</td>
</tr>
<tr>
<td>- interaction morphology/navigation</td>
<td>- Fragile river behaviour</td>
</tr>
<tr>
<td>- grain size bed material</td>
<td>- nutrient, climate change</td>
</tr>
<tr>
<td>- initial bed level</td>
<td>- nutrient, discharge</td>
</tr>
<tr>
<td>- river geometry</td>
<td>- nutrient, water discharge</td>
</tr>
<tr>
<td>- probability distribution parameters nutrient discharge</td>
<td>- nutrient, temperature</td>
</tr>
<tr>
<td>- riverine curve</td>
<td>- nutrient, discharge</td>
</tr>
<tr>
<td>- water depth</td>
<td>- nutrient, discharge</td>
</tr>
<tr>
<td>- riverine flow</td>
<td>- nutrient, discharge</td>
</tr>
</tbody>
</table>

Table 3.1: Examples of sources of uncertainty for river morphological models, ordered by the lexonomy of sources of uncertainties.
manifest themselves in computer simulation models:

- **Technical uncertainties**, which are uncertainties in model quantities (input data and model parameters). An individual uncertainty at this level can be caused both by variability and limited knowledge (Ragas, 2000).

- **Methodological uncertainties**, which are uncertainties due to the assumptions underlying the model structure and model equations. Uncertainties at this level primarily result from limited knowledge. A nuance of the term ‘limited knowledge’ is appropriate here, since available knowledge is sometimes intentionally omitted from a numerical model. In such a case the modeller assumes that the omitted knowledge is of minor influence to the model results. To verify this assumption, the omitted knowledge can be considered to be the source of an uncertainty due to limited knowledge and as such be included in an uncertainty analysis.

- **Epistemological uncertainties**, which are uncertainties about model completeness and validity. This type of uncertainty refers to whether the model is an adequate or at least relevant representation of the system under concern. These uncertainties result from limited knowledge (due to structural uncertainty) and variability.

- **Model operation uncertainties**, which are uncertainties due to numerical and implementation errors. These uncertainties are due to limited knowledge.

With the quantification of uncertainties based on a limited amount of observations, *statistical uncertainty* is introduced (Van Gelder, 2000). In particular, when an uncertainty is described in terms of a probability distribution function, the choice of the distribution type and the value of its parameters are uncertain. When enough data are available, the distribution function can be chosen in an objective way through statistical tests, which also give an estimate of the statistical uncertainty. Based on extensive literature review, Van Gelder (2000, pp. 73–79) discusses the performance of various methods to estimate distribution parameters for six commonly used distribution types. Van Noortwijk et al. (2001) discuss a method, based on Bayesian statistics, to estimate extreme percentiles of loads on a structure while taking statistical uncertainty into account. They apply the method to estimate the design discharge of the river Rhine. In cases of few or no data, the probability distribution function can be based on subjective expert judgement (see Bedford and Cooke (2001) for theory about the use of expert opinions).

Statistical uncertainty manifests itself at the level of methodological uncertainties, in the uncertainty about the choice of the distribution type, and at the level of technical uncertainties, in the uncertain value of the distribution parameters. It is, however, not always possible to draw the line between these two levels. In case of unknown parameters (through lack of observations), the distribution type will be uncertain as well (Van Gelder, 2000). In the taxonomy of sources the statistical uncertainty fits mainly under limited knowledge.

Since in this study we focus on the assessment of technical uncertainties (see Sections 1.3 and 3.3 for explanation), we have a closer look at this category. The description, and therefore also the assessment, of a specific technical uncertainty depends on the type of model
<table>
<thead>
<tr>
<th>Model Operation Uncertainties</th>
<th>Methodological Uncertainties</th>
<th>Residual Uncertainties</th>
</tr>
</thead>
<tbody>
<tr>
<td>New measurement techniques</td>
<td>- Clustering realistic model calibration</td>
<td></td>
</tr>
<tr>
<td>Uncertainty due to function error</td>
<td>- Hydraulic parameters</td>
<td></td>
</tr>
<tr>
<td>- Estimating fluvial depositional processes</td>
<td>- Initial sea level</td>
<td></td>
</tr>
<tr>
<td>- Application of numerical models</td>
<td>- Seismicity</td>
<td></td>
</tr>
<tr>
<td>- New flow discharge</td>
<td>- Tectonic stress</td>
<td></td>
</tr>
<tr>
<td>- Future climate changes</td>
<td>- Water depth</td>
<td></td>
</tr>
<tr>
<td>- Future geology</td>
<td>- Water quality</td>
<td></td>
</tr>
<tr>
<td>- Future management</td>
<td>- Groundwater discharge</td>
<td></td>
</tr>
<tr>
<td>- River salt discharge</td>
<td>- Erodible sediments</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.2: Examples of uncertainties in river morphological models, classified by their manifestation levels.
quantity it is related to. Building upon Morgan and Henrion (1990, Section 4.3), we distinguish between the following types of model quantities:

- **Empirical quantities**, which are, in principle, measurable properties of the real-world system being modelled. Morgan and Henrion suggest that this is the only type of quantity that can appropriately be represented in terms of a probability distribution function, since it is the only type that is both uncertain and can be said to have a **true**, as opposed to an **appropriate** or **good**, value. In a numerical model, empirical quantities are sometimes used for calibration purposes (see Section 3.3 for a further discussion).

We also define probability distribution parameters as empirical quantities. These may be indirectly measurable through data describing the empirical quantities.

- **Model domain parameters**, which specify the domain of the system being modelled, generally by specifying the range and increments for the spatial and temporal grid. It is common to be uncertain about what values are **appropriate** for them, but they have no **true** values. In order to examine how they affect the results of the analysis, they can be varied parametrically.

This describes a classification of uncertainties through their manifestation levels in numerical models. Table 3.2 shows an application of this classification to a variety of concrete uncertainties with respect to the description or prediction of river morphology. Furthermore, since Tables 3.1 and 3.2 contain the same set of uncertainties, together they illustrate the connection between the taxonomy of sources of uncertainty and the manifestation levels of uncertainties in numerical models.

### 3.3 Uncertainty analysis for different manifestation levels

The various types of uncertainty, as distinguished in the previous section, differ in the extent to which they can be quantified and in the way the order of magnitude can be expressed (e.g. in terms of a probability distribution function, or in a countable number of alternatives). As a consequence, different approaches are required to assess the influence of the various types of uncertainty on the model results. In this section we shortly describe possible approaches at each of the manifestation levels of the uncertainties. We end with an explanation of the limitation of this study to technical uncertainties.

#### Technical uncertainties

Various methods have been developed to estimate the influence of technical uncertainties in empirical quantities on the model output. Many of these methods start from a deterministic model. In Chapter 5 we discuss the applicability of these methods to numerical models of
river bed morphology. Here we will discuss the assessment of some special types of technical uncertainty.

**Calibration quantities** – Traditional calibration techniques assume that one optimal set of values exists for the calibration parameters. Not believing in the existence of such an optimal parameter set, some authors developed alternative strategies for model calibration. Beven and Binley (1994), for example, developed the Generalized Likelihood Uncertainty Estimation (GLUE). This method is based upon Monte Carlo Simulation, where random samples are drawn from specified probability distributions of the calibration parameters. On a basis of comparing modelled and observed responses, each set of parameter values is assigned a likelihood of being a simulator of the system. GLUE results in a confidence interval of the model output, thus quantifying the uncertainty resulting from the calibration parameters (cf. Werner, 2002).

**Probability distribution parameters** (Morgan and Henrion, 1990, p. 55) – The approximation error caused by the chosen model domain parameters, and thus the uncertainty manifesting itself at this level, can be regulated by adapting the range and increments of the spatial and temporal grid. Since these parameters have a considerable potential impact, the influence of how parametrical variations on the model results should be examined. In good modelling practice this is part of the model design process. Model domain parameters should be chosen such that the model deals adequately within the full range of the system of interest, while avoiding undue approximation resulting in excessive computation costs. If the model domain parameters have been chosen carefully, the uncertainty due to the approximation error will generally be small compared to other uncertainties involved.

The uncertainty in an empirical quantity is often described by a probability distribution function. Because of the lack of data often encountered, the choice of a distribution function is to some extent subjective, which is often used as an argument in favour of the uniform distribution. Slob (1994) argues that the more informative lognormal distribution is often a better choice. Especially data relating to nonnegative empirical quantities usually appear to be quite well described by the lognormal distribution. Kallen and Lewandowski (2002) show that the choice of a probability distribution function can make a significant difference. They show that a less informative distribution, like the uniform distribution, leads to larger variations in the model output than, for example, normal, lognormal or other distributions (see also Morgan and Henrion, 1990).
Methodological uncertainties

To analyse methodological uncertainties, validation research is generally the best option (Ragas, 2000). However, this is not always practically possible, for instance by lack of data. Another way to get an impression of the extent of these uncertainties is by applying models with a different structure to identical problems and comparing the results (Ragas, 2000; Van Asselt, 2000). A third way is to develop a model containing choice parameters controlling the selection of competitive theories or equations concerning the problem and processes involved. A sensitivity analysis would then point out the choices that are important in the overall uncertainty analysis (Ragas, 2000; Van Asselt, 2000). For the general case, practical methods for analysis of methodological uncertainties are not available (McKay, 1995).

Epistemological uncertainties

Uncertainties about model completeness is the most fundamental and crucial for the quality of the model. This uncertainty is addressed in the validation phase of the model design process. However, complete validation is impossible in case of complex systems due to inherent uncertainties, especially due to ignorance and indeterminacy (Van Asselt and Rotmans, 2002). Epistemological uncertainties are difficult to describe, if it is possible at all. Similar to methodological uncertainties, these cannot be adequately addressed with existing methods.

Model operation uncertainties

Following Van Asselt and Rotmans (2002) model operation uncertainties occur partly due to numerical errors and bugs in hard- and software (Van der Sluijs, 1997), but above all due to accumulation of uncertainties propagated through the model (Beck, 1987; Van Asselt and Rotmans, 1996).

Research limitation

As mentioned in Section 1.3, we focus in this study on the assessment of technical uncertainties in empirical quantities. A reason for this limitation is the fact that only for this type of uncertainty generally applicable methods are available, enabling a methodological study. We leave the specific problems concerning the calibration quantities and pay only little attention to uncertainty in probability distribution parameters. With our focus on technical uncertainties we do not want to suggest that other uncertainties are not important (cf. Chapter 7).

3.4 Relative importance of technical uncertainties

In the previous sections, we identified and classified uncertainties that possibly affect the reliability or the variability of the model results. We mentioned already that this thesis
focuses on the technical uncertainties in the empirical quantities in the one-dimensional morphological model used for the case studies. In this section we examine which of these uncertainties are relatively important and should therefore be taken into account in an uncertainty analysis. It is a common approach to analyse the relative sensitivities prior to an uncertainty analysis (compare, for example, Janssen et al., 1990; Van Asselt and Rotmans, 2002). This is a classical chicken and egg problem, since a correct description of the uncertainties would be required to determine the definite order of influence on the model results, whereas the research effort required to correctly describe the uncertainties is so large that one would only want to describe the relatively important uncertainties in detail. We address this problem by describing the uncertainties and their order of magnitude, combined with a rough sensitivity analysis resulting in a first distinction between important and less important parameters.

The order of magnitude of the uncertainties is expressed in terms of the coefficient of variation ($\text{CV}$), which is the ratio between the standard deviation and the mean of the uncertain quantity. This coefficient only gives restricted information about the probability distribution of the uncertainties (for example, it says nothing about the skewness of the distribution). However, combined with the sensitivity analysis, the $\text{CV}$ should suffice to identify the major technical uncertainties in the empirical quantities.

In the next subsections we first investigate the sensitivity of the model results to variations in the river discharge. Thereafter the uncertainties in the sediment transport parameters and in the hydraulic roughness are examined. Based on literature and experience we expect these uncertainties to be important. Finally, the uncertainties in the boundary and initial conditions and in the river geometry are discussed.

### 3.4.1 River discharge

This separate discussion of the influence of the variability in the river discharge on the modelled bed level serves two purposes. Firstly, the central position of this uncertainty supports the research definition (Section 1.3), stating that this study focuses on the uncertainty in the river discharge. We aim to show that this is one of the important technical uncertainties. Secondly, the conclusion of this section is used for the sensitivity analysis of the uncertainties discussed in Section 3.4.2.

**Description**

Due to the variability of nature, the future discharge hydrograph is uncertain. Or, in other words, the future variations of the discharge are uncertain. This uncertainty is inherent to nature, it cannot be removed. With available discharge measurements this uncertainty can be quantified: a statistical description of the future discharge hydrograph can be derived, enabling the estimation of the probability of a particular discharge curve. In Chapter 4 we will discuss the statistical description of the discharge through the river Waal in detail.
Order of magnitude

The variability of the discharge can be considerable. For example, the CV of the averaged daily discharge through the river Waal varies between 0.3 in the low discharge season and 0.7 in the season with high discharges. Figure 2.9(c) shows that in the high discharge season the 10-days discharge roughly varies between 500 and 5500 m³/s.

Sensitivity

Firstly, we examine the importance of the fact that the discharge varies at all. To that end, we compare the morphological effect of a variable discharge with that of its dominant discharge (that is, the constant discharge with the same sediment load capacity within a certain period). Figure 3.2 shows a considerable difference (i.e. tens of centimetres) between the bed level changes under one of the applied discharge series and the representative constant discharge. Also the mean effects under the variable discharges differ from the effect under the constant discharge. These differences are caused by the non-linear relation between the flow velocity and the sediment discharge (Equation (2.4)) and by the bed waves initiated continuously under a variable discharge. Some processes occur just because the discharge fluctuates, such as the inundation of floodplains. This is illustrated by the results of Case 3, where the river bed is not affected under representative discharge conditions, since this discharge is below bankfull (Figure 3.2(b)).

This effect of variations in the discharge confirms the findings in Van der Klis (2001), namely that the effect on the bed level changes of an uncertain, but constant representative discharge is considerably smaller than that of a variable discharge.

Secondly, we roughly examine the sensitivity of the bed level changes to the uncertainty about the discharge variations. To that end, we performed three 10-year simulations forced by variable discharges based on the measured discharge through the river Waal in the years 1946-1956. From this measured hydrograph we interchanged the hydrographs of the individual years, based on random permutations. In this way, we obtained three comparable hydrographs regarding their identical statistical properties and total sediment load capacity. Figure 3.2 shows that the difference between the bed levels resulting from different comparable hydrographs is also in the order of tens of centimetres. In particular downstream of the adapted river reach, this can make a difference between erosion and sedimentation at corresponding moments in time.

Wiersma (1997) also compares bed level effects under various hydrographs versus a constant representative discharge. He studies the two-dimensional morphological effects of submerged vanes in the main channel of a reach of the river Waal. He concludes, for example, that the transition time (i.e. the period in which the river bed adapts to the installed vanes) is less under a variable discharge, even compared to a constant discharge with the same average sediment transport. This transition time appears to depend strongly on the occurring discharges.
3. Uncertainties

Figure 3.2: Effect of variable discharge versus representative discharge (1445 m³/s) for two cases, after 10 simulated years.

(a) Case 2: lowered floodplains  
(b) Case 3: widened floodplains

Conclusion

Since time-dependent variations of the river bed are practically relevant and a constant discharge cannot simulate all morphological processes, a constant discharge is often an oversimplification of reality. This also holds for morphological effects after tens of years, since the differences in bed level change will not decrease through time. Chapter 4 further discusses the influence of discharge variations on the river bed. For now, the illustrated sensitivity to the discharge variations, combined with the large uncertainty about these variations, shows that the uncertainty about the discharge variations is of significant importance to the model results.

3.4.2 Sediment transport parameters and hydraulic roughness

The sediment transport model is an important part of a river morphological model, but surrounded by large uncertainty (Jansen et al., 1979). This uncertainty is illustrated by the large deviation in the sediment transport capacities resulting from different models, as shown by Van der Scheer et al. (2002). In a numerical model, the uncertainty concerning the sediment transport manifests itself through methodological uncertainty (in the choice for a particular transport model) and through technical uncertainty (in the transport model parameters). The latter concerns, for example, uncertainty about the grain size of the bed material and the multiplier used for calibration purposes. The roughness of the river bed occurs as a parameter both in sediment transport models and in the water movement equations. It is often used as a hydraulic calibration parameter, which indicates the sensitivity of the river flow to variations in the hydraulic roughness.
Chang et al. (1993) apply a sensitivity analysis to a case study concerning the installation of pipe and bank protection within a section of the Santa Cruz River. Based on a numerical model, the bed elevation is examined at three time points during a design-hydrograph. They find that the roughness coefficient is one of the main contributors to the uncertainty in the model output. Furthermore, they find the bed porosity, the surface layer thickness and grain sizes as important contributors. In their research, uncertainty in the future river discharge is not taken into account.

Because of their potential importance, we will examine the relative sensitivity of the model results in our case studies to the uncertainties in the hydraulic roughness, the grain size, the relative density and the packing factor of the bed material and in the multiplier of the transport model.

**Description**

*Hydraulic roughness:* A distinction must be made between the roughness within the main channel and the roughness on the floodplains. Within the main channel, the hydraulic roughness of sand-bed rivers is determined by the bed forms, varying in space and time due to the river geometry, the characteristics of the bed material and the river discharge. Within the floodplain the roughness is determined by the vegetation, also varying in space and time. The variability in the hydraulic roughness is inherent to the river processes and the development of vegetation in the floodplain. Apart from this variability, uncertainty is introduced by the translation of the measured roughness heights into the roughness coefficients used in the numerical model (e.g., Chézy or Strickler). This can be interpreted as a methodological uncertainty.

*Grain size:* The uncertainty in the grain size distribution is due to natural variability, both in space and time. Local spatial variations are due to, for example, bedforms, local armouring, or bend effects. These variations increase with the complexity of the river system. Variations in time are caused by graded sediment processes, such as the inflow of coarse sediment, or vertical sorting processes (cf. Blom, 2003).

Uncertainty about the grain size also manifests itself as a methodological uncertainty, for example, by summarising two-dimensional data into a one-dimensional model, or by neglecting the gradation of the grain size (as in our case studies).

*Relative density:* The relative density of the sediment is defined as $\Delta \rho = \frac{\rho_s - \rho_w}{\rho_w}$, in which $\rho_s$ is the density of the sediment and $\rho_w$ the density of water. The uncertainty in the relative density is due to natural variability and to limited knowledge of modelling the information into a single constant parameter.

*Packing factor:* The packing factor ($\epsilon$) indicates the relative pore volume of the river bed. The uncertainty about the packing factor is due to natural variability and to limited knowledge of modelling the information into a single constant parameter.
Multiplier: Most sediment transport models are based on sand flume experiments. For application of these models to realistic cases the transport must often be multiplied by a constant factor ($c$). This factor is mostly larger than 1 due to the non-linear relation between the sediment transport and the Shields parameter (Paola and Seal, 1995). This can be interpreted as a methodological uncertainty, but here, we interpret it as a technical uncertainty.

Order of magnitude

Hydraulic roughness: Johnson (1996) gathered information about the CV of the Manning roughness coefficient from literature and her own findings, resulting in values ranging from 0.05 to 0.35. Duits and Van Noortwijk (1999) suggest a CV of about 0.1 for the spatial variability of the Chézy coefficient in the main channel of the river Waal.

Grain size: Little research has been done into the order of magnitude of the uncertainty in the grain size. Based on data analyses in project reports (Sloff and Mosselman, 1998; Sloff and Stolker, 2000) we estimate the CV of the spatial variability to be about 0.3, and the temporal variability about 0.5.

Relative density and packing factor: No literature has been found about the magnitude of these uncertainties. Based on the judgement of one expert we estimate the CV of the relative density to be 0.05, and the CV of the bed packing factor about 0.1.

Multiplier: We assume a moderate uncertainty in the multiplier ($0.1 < CV < 0.3$).

Relative sensitivities

The previous section showed that the variability of the river discharge cannot be neglected without losing important information about the morphological effects of an intervention on the river. Therefore, we expect that also the relative sensitivities of other uncertainties should be examined under a variable discharge. This is also suggested by the results of Chang et al. (1993) and by Van der Klis (2001), namely that the sensitivity of the model results to technical uncertainties depends on the magnitude of the discharge.

The most basic method to investigate the relative influence of uncertainties on model response is the Individual Parameter Variation (Janssen et al., 1990). In this method one parameter is varied at a time, keeping all others fixed. The information obtained is qualitative, and shows the relative sensitivity of the model output to each parameter. We varied the reference value of each of the uncertain empirical quantities by 15%, resulting in the values presented in Table 3.3. Note that this variation of the reference values does not correspond with the actual deviation of the uncertainties as roughly discussed above.

Since in the Individual Parameter Variation (IPV) the parameters are varied one by one, the effect of mutual correlations on the relative sensitivities is not examined. This effect, however can be significant. Chang et al. (1994) show that neglecting correlations can lead to overestimation of the uncertainty in the model output.
3.4. Relative importance of technical uncertainties

Table 3.3: Input for the Individual Parameter Variation.

<table>
<thead>
<tr>
<th></th>
<th>Cases 1 and 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>reference</td>
<td>variation</td>
</tr>
<tr>
<td>$C_m$</td>
<td>40</td>
<td>46</td>
</tr>
<tr>
<td>$C_f$</td>
<td>35</td>
<td>29.75</td>
</tr>
<tr>
<td>$k_{s,m}$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$k_{s,f}$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$D$</td>
<td>1</td>
<td>1.15</td>
</tr>
<tr>
<td>$e$</td>
<td>0.4</td>
<td>0.46</td>
</tr>
<tr>
<td>$\Delta_d$</td>
<td>1.65</td>
<td>1.90</td>
</tr>
</tbody>
</table>

Table 3.4: Results of the IPV: the relative effect on the bed level of the varied quantities (Table 3.3), averaged over the last 5 years of the 10-years simulations, for two locations per case study.

<table>
<thead>
<tr>
<th></th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>km 101</td>
<td>km 109</td>
<td>km 109</td>
</tr>
<tr>
<td>$C_m$</td>
<td>26%</td>
<td>35%</td>
<td>33%</td>
</tr>
<tr>
<td>$C_f$</td>
<td>5%</td>
<td>7%</td>
<td>8%</td>
</tr>
<tr>
<td>$k_{s,m}$</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$k_{s,f}$</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$D$</td>
<td>14%</td>
<td>12%</td>
<td>13%</td>
</tr>
<tr>
<td>$e$</td>
<td>15%</td>
<td>16%</td>
<td>13%</td>
</tr>
<tr>
<td>$\Delta_d$</td>
<td>12%</td>
<td>12%</td>
<td>10%</td>
</tr>
<tr>
<td></td>
<td>28%</td>
<td>18%</td>
<td>24%</td>
</tr>
</tbody>
</table>
A summary of the results is presented in Table 3.4. The percentages represent the relative effects of the varied quantities on the bed level, averaged over the last 5 years of the 10-years simulations. Higher percentages indicate relatively higher sensitivity of the model results. These values show that, of the quantities examined, the model output is relatively sensitive to variations in the hydraulic roughness of the main channel and to variations in the relative density of the bed material. The sensitivities to the grain size, the multiplier and the packing factor are moderate. The effect of variations in the hydraulic roughness of the floodplains is only relevant in Case 2 at the downstream end of the lowered reach.

The visualisation of the results of Case 1 in Figure 3.3 illustrates how the varied parameters affect the estimate of the bed level changes:

- The initial response of the river bed to discharge variations, that is, at locations of changes in the river geometry (Section 2.3.2), is affected by some parameter variations. For example, the variations in $C_m$ and $D$ at km 101 of Case 1 have an increasing and decreasing effect respectively, on the initial bed level changes (Figure 3.3(b)).

- The effect on the propagation speed of the bed waves is illustrated downstream of the constriction. For example, an increase of $C_m$ has an increasing effect on the propagation speed, as shown by the location of the edge of the large bed wave downstream of the constriction, around km 150 (Figure 3.3(a)).

- The effect on the damping rate of the bed waves, that is, the speed at which the magnitude of the bed waves decreases, is illustrated in Figure 3.3(a). An increased $D$, for example, results in a decreased damping rate, as shown by the higher level of the large bed wave downstream of the constriction.
3.4. Relative importance of technical uncertainties

The effects on the propagation speed and damping rate of the bed waves are confirmed by the theory discussed in Section 2.4. From the expressions for $c_b$ and $K$ (Equations (2.12) and (2.13)), combined with Equations (2.4) and (2.7) – (2.9), it follows that $c_b \sim \frac{C}{D}$ and $K \sim \frac{C^3}{D}$. 

**Conclusion**

We examined the relative sensitivity of the model results to some sediment transport parameters and the hydraulic roughness, together with their order of magnitude. To obtain a rough impression of the relative importance of the uncertainties in these quantities for the uncertainty in the bed level effects, we combine this information. The broad outline of this reasoning is that large uncertainties to which the model results are highly sensitive are relatively important for the uncertainty in the results, in contrast with small uncertainties to which the results are not sensitive. Figure 3.4 arranges the uncertainties examined according to their magnitude and the associated relative sensitivity. We conclude that from these quantities the uncertainty in the grain size of the bed material and the hydraulic roughness of the main channel are relatively the most important for the uncertainty in the bed level effects, followed by the uncertainties in the multiplier and the packing factor of the sediment.
3.4.3 Boundary and initial conditions and river geometry

Description

Sediment entrance: The morphological upstream boundary condition of a numerical model concerns either the sediment flux entering the modelled river reach, or the bed level (i.e. the sediment transport that equals the local transport capacity). This sediment influx is uncertain due to natural variability in time.

Rating curve: The downstream boundary condition is often a rating curve, i.e. the relation between the river discharge and the water level at a cross-section. The rating curve at a certain cross-section in a river is uncertain due to limited knowledge, caused by unreliability of the measurements and the question whether the measured rating curve is still valid after interventions on the modelled river reach. Collecting more data can reduce this uncertainty only to a certain extent: if there are many interventions on the river within a short time, it is unlikely that a sufficiently wide range of hydraulic conditions can be covered by the measurements.

Initial conditions: Both the initial bed level and the initial flow conditions within the modelled river reach are uncertain due to natural variability.

River geometry: The geometry of a river reach covers aspects like the cross-sectional geometry, the layout of groin fields, the width and height of the floodplain and all kinds of obstacles within the floodplain. Uncertainties about these features are due to variability, limited availability of data and measurement errors.

Order of magnitude

Sediment entrance: The magnitude of the sediment entrance is often highly uncertain (Jansen et al., 1979, p. 293).

Rating curve: Duits and Van Noortwijk (1999) estimated the CV of the rating curve in the river Waal to be about 0.01, which is small compared to other uncertainties.

Initial conditions: The uncertainty in the initial flow conditions is expected to be small compared to other uncertainties. The uncertainty in the initial morphological conditions depends on the availability of measurements and the spatial variability of the river reach.

River geometry: Gates and Al-Zahrani (1996a) analysed the spatial variability of the bed slope and the cross-sectional geometry in 26 distinct river reaches. They conclude that this variability can be considerable (i.e. CV of the bed slope up to 260 and CV of the cross-sectional characteristics up to 3). The uncertainty about the geometry of a specific river reach depends on the available data in relation to its variability, that is, for a highly variable river reach many measurements are required to model its geometry accurately.
3.4. Relative importance of technical uncertainties

Figure 3.5: Propagation of uncertainties at the upstream boundary through the \((x, t)\)-domain (following Jansen et al., 1979, p. 293).

**Relative sensitivities**

*Sediment entrance*: Bed disturbances, and thus uncertainty at the upstream boundary, propagate through the model with the propagation speed of bed waves \(c_b\) (Section 2.4). This implies that a location at a distance \(L\) from the boundary is only affected by this uncertainty after a period \(\frac{L}{c_b}\) (Figure 3.5). The sensitivity of the estimates of the bed levels to this uncertainty will decrease in space in downstream direction, because of the reduction of the bed disturbances. As a consequence, the influence of the uncertainty about the sediment entrance can be limited by locating the upstream boundary far enough from the river reach of interest, depending on the length of the simulation period and the characteristics of the river reach (Jansen et al., 1979, p. 293).

*Rating curve*: The discussion of the sensitivity of the model results to uncertainty in the rating curve is similar to that of the sediment entrance. The influence of this uncertainty reaches a limited distance from the downstream boundary, diminishing in space in upstream direction. Therefore, the influence can be reduced or abandoned by locating the downstream boundary far enough from the river reach of interest (Jansen et al., 1979, p. 293).

*Initial conditions*: Since the flow conditions adapt much faster to changing circumstances than the river bed, their initial conditions do not influence the morphological model outcome significantly.

As opposed to the uncertainties at the upstream and downstream boundaries, the effect of uncertainties in the initial bed level decreases while the uncertainties propagate out of the model domain. In that respect, the opposite of Figure 3.5 holds, namely for this uncertainty
the region in the \((x, t)\)-domain \textit{above} the line marked \(c_b\) is not influenced. As a consequence, the sensitivity of the model results to uncertainty in the initial bed level can be reduced by defining a simulation period before the period of interest.

\textit{River geometry:} Concerning the uncertainties in the river geometry no simulation period exists in which the effect on the model results is absent. As discussed in Chapter 2, variable discharge continuously induces bed disturbances at locations where in the river geometry varies. Uncertainty in the size or form of these geometry variations results in uncertainty about the characteristics of the bed level effects.

\subsection*{Conclusions and discussion}

We conclude that the relative importance of uncertainties in the boundary and initial conditions for the model results for a specific river reach depends on the distance of the reach of interest to the model boundaries and the simulation period of interest relative to the total simulation period. A balance must be found between the reduction of the influence of each of these uncertainties, and the increase in computation time due to a larger model reach and a longer simulation period. Because of the potential importance of these uncertainties, they cannot be neglected beforehand in an uncertainty analysis.

For a specific case study, the relative importance of uncertainties in the river geometry depends on their magnitude. For these uncertainties, as for those in the boundary and initial conditions, we expect a different influence on the estimates of bed levels as compared with bed level changes. When the bed level effects of a river work are estimated by comparing the results of two model simulations, i.e. one with and one without the presence of the river work, the uncertainties discussed in this section will compensate each other for the greater part. Their influence will not be reduced to zero, since the river work changes the behaviour of the bed and thus the influence of the uncertainties on the bed disturbances.

\section{Conclusion and discussion}

We started this chapter with the description of a classification method in which uncertainties are ordered by their sources and by the way they manifest themselves in numerical models. On the basis of the application of this classification method to a variety of uncertainties in river morphological problems, we conclude that the classification is applicable to this kind of models. The use of this classification method in river morphological practice would help effectively to communicate about uncertainties and to interpret the results of an uncertainty analysis. Furthermore, it helps to decide which approach is required to perform an uncertainty analysis.

Different approaches are required to assess the influence of uncertainties at the various manifestation levels in a numerical model. Only for the assessment of technical uncertainties
in empirical quantities generally applicable methods are available. Which types of uncertainty are most important for a particular morphological case depends on the problem and its context. The fact that this study focuses on technical uncertainties does not imply that these are the most important for each morphological problem. In Section 7.1 we will discuss the relative importance of the different types of uncertainty.

With simulations of our case studies, we illustrated that the variability of the river discharge often cannot be neglected without losing important information about the morphological effect of an intervention on a river reach. Moreover, the uncertainty about the future variations of the discharge is large. Based on this information, we conclude that the uncertainty about the future discharge hydrograph is one of the relatively important technical uncertainties in river morphology.

Based on the discussion of some other technical uncertainties and the results of a rough sensitivity analysis, we expect that the uncertainties about the hydraulic roughness and the grain size of the bed material are also relatively important. A sensitivity analysis based on more adequate descriptions of the spatial and time-dependent uncertainties in these parameters will further clarify their relative importance, also compared to the uncertainty about the future discharge.

The influence of the uncertainty in the boundary conditions, other than the water discharge, and in the initial bed level is potentially important. Their relative importance depends on the choices made in the model design process, about the location of the model boundaries and the duration of the simulation period before the period of interest. The relative importance of the uncertainty in the river geometry depends on the magnitude of this uncertainty in a particular case. Especially if absolute bed levels are estimated (as opposed to bed level changes) this uncertainty can be relatively important.

Note that disturbances in the bed level are initiated by several causes and their mutual interaction. In the non-realistic case of a constant discharge, disturbances of the river bed are induced by, for example, variations in the river geometry, and spatial variations in the hydraulic roughness or in the characteristics of the bed material. However, the 'breathing' of the river bed near and downstream of these variations is typically induced by variations in the discharge. On the other hand, in another non-realistic scenario with a variable discharge in a uniform channel, with uniform roughness and bed material, no disturbances of the bed level will occur. These examples illustrate that the influence of the discharge variations on the bed level is relatively large, but owing to variations in other features of the river.
3. Uncertainties
4 The synthesis of discharge hydrographs

4.1 Introduction

As we showed in Section 3.4, the variability of the river discharge is an important source of the uncertainty in morphological predictions. This implies that the uncertainty about the river discharge should be included in an uncertainty analysis of a river morphological model, often requiring random samples of the discharge series. To enable the synthesis of random discharge series a statistical model is required. Ideally, the random discharge series force the morphological changes in a realistic way, on the one hand, and enable an efficient uncertainty analysis, on the other. We define this efficiency as the calculation time required for an uncertainty analysis. In the case of river morphological models, the calculation time is dominated by both the number of simulations and the calculation time per simulation. As we will discuss in the next chapter, for some methods of uncertainty analysis the number of simulations required depends on the number of random parameters involved. This motivates a statistical model with few parameters. Furthermore, the calculation time of one single simulation is reduced considerably if one can use discharge series containing long periods of constant discharge (i.e. periods of several numerical time steps).

Summarizing, the aim of this chapter is to construct a statistical model that enables the synthesis of discharge series that force realistic morphological changes, contain few random parameters, and restrict the calculation time per simulation.

In theory, two extreme statistical descriptions can be applied:

- a detailed description containing probability distribution functions of the averaged discharges over short periods of time (e.g. a few days) and their mutual correlations (Duits, 1997; Van Vuren et al., 2002), or

- an extremely simplified description, consisting of the probability distribution function of a constant discharge that somehow represents the variable discharge (for definitions of these dominant discharges, see, e.g., Jansen, 1979).
The advantage of the detailed description is that it automatically includes all features of a discharge series forcing the morphological changes. It leads, however, to a large number of stochastic parameters and time consuming simulations. A constant discharge, on the other hand, enables relatively small computation times, but does not lead to accurate predictions of time dependent morphological changes (as shown in Section 3.4). Thus, we are looking for a compromise.

In practice, measured hydrographs are often schematised by hand, such that low discharges are averaged over relatively large time-intervals and floods over small intervals. For example, Wiersma (1997) shows that a relatively coarse schematisation still gives acceptable results concerning the bed level effects of submerged vanes in the main channel. In this chapter we look for a similar schematisation of the hydrograph, suitable to be synthesised automatically.

The river discharge has a variety of random aspects, such as the height and duration of the floods, the time of occurrence of those floods, the magnitude of the low and moderate discharges, or the chronology of the floods. So far, little information has been published about the relative importance of such aspects to the statistics of morphological predictions. Insight into the relevant aspects of a hydrograph would enable the construction of a representative statistical model. Therefore, in the first section of this chapter we examine the sensitivity of the morphological processes to several aspects of a hydrograph. In the second section we use this knowledge to construct a statistical model for the river discharge. All tests in this chapter are based on the case studies described in Chapter 2 and the scaled discharge measurements of the river Rhine (Section 2.5).

### 4.2 Features of a hydrograph forcing the morphology

The selection we examine in this section of potentially dominant features of a hydrograph is based on physics of river morphology. We focus again on lowland rivers with a stable planform, and on the morphological effects of local-scale river works (cf. Section 1.3). When the river bed has reached a new dynamic equilibrium after the construction of the river work, the periods of high discharge and the periods of low discharge each have their own type of influence on the bed topography. The high discharges, which usually occur during relatively short periods (floods), change the bed topography relatively much. The morphological impact will therefore depend strongly on the height and the duration of the flood. The low discharges, which occur during a longer period, slowly reduce the effects of the floods. This suggests that the periods of low discharges are responsible for the overall, average bed topography, whereas the floods give rise to small-scale bed disturbances. Furthermore, the non-linear response of river morphology to the water discharge makes the chronology of discharge variations potentially important.
4.2. Features of a hydrograph forcing the morphology

Based on these considerations, we will examine the following random features of a hydrograph:

1. the variability of low discharges,
2. the shape of the flood hydrograph,
3. the chronology of floods.

We choose a threshold discharge of 2000 m³/s, so as to separate the low discharges from the high discharges. For this rather subjective choice we took into account the distribution function of the 10-days averaged discharges (400 – 5500 m³/s, Section 2.5) and the magnitude of the bankfull discharge (1600 m³/s). A balance must be sought between a range of low discharges as large as possible (expecting that especially the schematisation of the low discharges reduces the computation time), and a range of high discharges that includes all discharge levels with relatively large influence on the river morphology.

To test the effect of schematisations of the discharge series on the computed mean bed level changes and their 90%-confidence interval, we perform Monte Carlo simulations with crude sampling (MCS – for an explanation of this method, see Chapter 5), with a sample size of 300. In all simulations we assume that only the discharge time-series contains uncertainties. Other uncertainties are neglected. This implies that random samples are only required of the discharge series.

For the Monte Carlo simulation that will serve as a reference, we base the random samples on the series of 10-days averaged discharges presented in Section 2.5. To that end, we split this discharge series into 54 single years from August to July. By defining the year boundaries within this period of low discharges (Figure 2.9(c)), we prevent floods to be split into two parts. Next, 20-years hydrographs are constructed by sampling 20 of these 54 single years, in which repetition of a year is possible. To find the dominant features of a hydrograph, we applied suitable schematisations to the 10-days averaged discharges series and constructed 20-years hydrographs by sampling similarly from these adapted series.

Where in the schematisations periods of discharges are replaced by their average, it always concerns a weighted average, such that the total sediment load over the entire period remains unchanged (Equation 2.18).

To compare the reference results with the effect of the schematisations we present graphs of the 5-, 50- and 95-percentiles of the reference results in terms of their 90%-confidence intervals (cf. Appendix C.1). This enables the analysis of whether the results of the schematisation deviate significantly from the reference results. In addition, Appendix E contains quantitative results of some of the simulations discussed in the following paragraphs.

For our purpose, the (statistically) significance of the difference between the results is too strict to serve as a condition for the acceptability of the differences. In the light of other uncertainties influencing the model results and depending on the purpose of a specific river morphological study, a difference of at least several centimetres will be acceptable. In this study, we assume a difference of about 5 cm to be acceptable.
4. The synthesis of discharge hydrographs

(a) discharge schematisation

(b) Case 1: constricted main channel

(c) Case 2: lowered floodplain

(d) Case 3: widened floodplain

(e) Case 2 with $Q_{th} = 1600$ m$^3$/s

Figure 4.1: Results of simulations constant low and variable low compared to the reference simulations.
4.2.1 The variability of low discharges

To test whether the variability in low discharges may be neglected, we consider two simulations with the following schematisations of the hydrograph (Figure 4.1(a)):

- **constant low**: the low discharges replaced by one constant discharge, equal to the weighted averaged low discharge over the 54 years of measurements (1237 m$^3$/s); high discharges unchanged.

- **variable low**: the low discharges per period of low discharge replaced by their weighted average discharge over that period (resulting in an average low discharge of 1370 m$^3$/s and a standard deviation of 310 m$^3$/s); high discharges unchanged.

The results of these simulations are presented in Figure 4.1 and compared with the reference results. The figures show the estimated morphological effect in the 20th year after the construction of the particular river work, at the end of the flood season. The results at other time steps show similar effects and the comparison of other percentiles also lead to the same conclusions (cf. Appendix E, Table E.1).

From the results it appears that the variation in the low discharges cannot always be neglected. In Case 1 the results of the constant low schematisation differ considerably from the reference results, compared to the statistical uncertainty of the latter. Maintaining some variation, like in the variable low schematisation, gives acceptable results. In Case 3 the variations in the low discharges appear to be of minor importance, that is, the results of both schematisations lie within the confidence intervals of the reference results.

In Case 2 both schematisations underestimate the morphological effect considerably. This is due to the fact that the threshold discharge of 2000 m$^3$/s is relatively far above bankfull discharge in the lowered reach (which is 1250 m$^3$/s). This leads to the replacement of too much relevant discharge by low discharge, of which the latter does not lead to sedimentation within the lowered reach. A better correspondence with the reference result is obtained with a threshold discharge of 1600 m$^3$/s (Figure 4.1(e)).

4.2.2 The shape of a flood hydrograph

We consider two schematisations of the floods. Firstly, we model the floods as triangles, with the peak equal to the maximum discharge of the particular period of high discharge, and the basis equal to the duration of that period. This description does not respect the volume of the flood. Secondly, we model the floods as rectangles, with the peak equal to the weighted average discharge and the duration equal to the length of the period. This description does not reflect the maximum discharge.

The results of the corresponding simulations:

- **triangular flood**: high discharges schematised as triangles; low discharges unchanged.

- **rectangular flood**: high discharges schematised as rectangles; low discharges unchanged.
are shown in Figure 4.2. Both schematisations turn out to yield reasonable estimates of the morphological impact of the modelled river works (cf. Appendix E, Table E.2).

### 4.2.3 The chronology of floods

The non-linear response of river morphology to the water discharge makes the chronology of discharge variations potentially important. On the long term, that is a number of years, this concerns the order in which floods of a particular magnitude and duration occur. On the short term, that is within one year, the chronology concerns the variability in periods of low and high discharge.
4.2. Features of a hydrograph forcing the morphology

Figure 4.3: Results of long-term chronology simulations, showing the envelope of the 20 simulations results and some examples of realisations.
To examine the importance of the long-term chronology, we follow the approach described by Southgate (1995). Two sets of 20 simulations for each case study are performed, based on the 9 years of 10-days averaged discharges between 1946–1955, namely

- *direct effect*: the first four years as measured and the last five years in random order;
- *historical effect*: the first four years in random order and the last five years as measured.

The envelope of the bed level effects at the end of the simulations (Figures 4.3) indicate the influence of the order in which the one-year hydrographs occur on the deviation in the model outcome. Since only the order has been changed, the total sediment transport capacity in the nine modelled years was constant over the simulations. The results show a *direct effect* that is large (> 50%) compared to the total effect of uncertainties in the discharge (see previous results in this chapter). The *historical effect* differs per case study. As expected, in all cases the affected river reach is located more downstream than the direct effect, while both reaches overlap in the middle. In Cases 1 and 2 the historical effect is smaller than the direct effect. In Case 3 both effects are of the same order of magnitude. This difference from the other cases can be explained by the difference in propagation speed of the bed waves, that is, in Case 3 this speed is less than half the speed in the other cases (Chapter 2). As far as the relevance of the long-term chronology is concerned, it can be concluded that this source of uncertainty cannot be neglected.

On the short term, that is within one year, floods above a certain threshold level occur at different moments in time, and they may occur more than once a year, or not at all. We would like to know whether this short-term chronology may be simplified to a schematisation of not more than one flood per year. This is tested for the rectangular flood schematisation by performing the following simulation:

- *one rectangular flood*: All floods in one year (i.e. from August to July) are combined into one single flood, occurring at the same time as the flood with the maximum peak. The duration of this flood equals the total duration of the single floods, whereas the height is the weighted average of the single flood heights (Equation 2.18). The low discharges are replaced by their weighted average over the year.

Applying this schematisation to the case studies shows good correspondence with the reference results (Figure 4.4). The only exception is km 100 of Case 2, where the schematisation results in an overestimated morphological effect. This, however, is very much a local effect for which a two-dimensional model would be more appropriate. Apart from this location, the deviation from the reference results is larger than its statistical uncertainty, but remains restricted to approximately 5 cm. For many applications this will be acceptable, as we also assume for our case studies.
4.2. Features of a hydrograph forcing the morphology

(a) discharge schematisation

(b) Case 1: constricted main channel

(c) Case 2: lowered floodplain

(d) Case 3: widened floodplain

Figure 4.4: Results of simulations one rectangular flood compared to the reference simulations.
4.3 Statistical model to synthesise hydrographs

In the previous section, we constructed a simplified schematisation of the discharge hydrograph, still representing the features that determine the morphological effects of river works on the main channel. In this section we describe a statistical model, based on this schematisation, to enable the synthesis of random discharge series. These series can, for example, be applied to explore possible future morphological changes. Since in our case studies the triangular and rectangular schematised floods led to similar results, we only focus on a statistical model for the latter. The derivation of a statistical model for triangular floods proceeds analogously.

4.3.1 The model

The schematisation is defined by four parameters per year (Figure 4.5), namely the flood height ($Q_H$), the flood duration ($Q_D$), the time of occurrence of the middle of the flood ($Q_T$) and the magnitude of the low discharge ($Q_L$). We have shown that the long-term chronology cannot be neglected, implying that these parameters must be sampled separately for each year. Since no significant correlation exists between the discharge hydrographs of consecutive years (Van Gelder et al., 2000), the parameters can be drawn independently for each year. The statistical model we construct consists of probability distribution functions for each of the four parameters and mutual correlations, if and when necessary. The statistical model and its sensitivity to the choices for distribution functions and correlation factors are tested via MCS based on randomly synthesised discharge hydrographs. The results are compared to the reference results of the previous section, which are directly based on the 10-days averaged discharge series. For a correct statistical model, these results are expected to be similar to the reference results, since the statistical model is based on
4.3. Statistical model to synthesise hydrographs

Table 4.1: Gaussian distribution parameters of the discharge parameters and their coefficients of variation (CV) following a bootstrap procedure of 1000 samples.

<table>
<thead>
<tr>
<th></th>
<th>best fit $\mu$</th>
<th>best fit $\sigma$</th>
<th>$CV_{\mu}$</th>
<th>$CV_{\sigma}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_H$</td>
<td>2688 m$^3$/s</td>
<td>318 m$^3$/s</td>
<td>2%</td>
<td>11%</td>
</tr>
<tr>
<td>$Q_D$</td>
<td>7.3 *10 days</td>
<td>4.6 *10 days</td>
<td>9%</td>
<td>9%</td>
</tr>
<tr>
<td>$Q_T$</td>
<td>14.9 *10 days</td>
<td>4.4 *10 days</td>
<td>4%</td>
<td>9%</td>
</tr>
<tr>
<td>$Q_L$</td>
<td>1233 m$^3$/s</td>
<td>199 m$^3$/s</td>
<td>2%</td>
<td>7%</td>
</tr>
</tbody>
</table>

the same data. Samples may contain higher discharge peaks than the data, because of the extrapolation of the probability distribution to higher discharges than measured. This may result in a larger variation in the morphological impact, in particular concerning the more extreme fractiles.

Discharge measurements are used to support the choice of the probability distribution functions. Once again, we use the measurements described in Section 2.5. Applying the schematisation to 10-days averaged discharges yields 54 data points for each parameter. This number is further reduced by the years in which no discharge peak above the threshold level occurred. In case of a threshold discharge of 2000 m$^3$/s, six of the 54 years do not contain a discharge event that exceeded this threshold.

Figure 4.6 shows the 90%-confidence interval of the empirical distribution function of the discharge parameters (Appendix C.1), together with some Least Square-fits of well known distribution functions. This figure shows that, on sight, several distribution functions fit well to the data. Since the Gaussian distribution function fits all four discharge parameters as good as other distribution functions and is easy to work with, we use a statistical model consisting of four Gaussian distribution functions. Since the data available is limited, the parameters of these distribution functions are uncertain, by definition. This is an example of statistical uncertainty (Section 3.2). To describe the uncertainty in the distribution function parameters, the bootstrap method can be used (Appendix C.2). Table 4.1 contains the best fits of the Gaussian distribution parameters and their coefficients of variation following a bootstrap procedure of 1000 samples. It appears, that the statistical uncertainty in the distribution parameters can be approximated by Gaussian distributions. A sensitivity analysis in the next subsection will show the importance of this uncertainty.

From the data sets, the linear correlation between the four parameters for each year can be estimated. The data sets suggest a correlation between the parameters $Q_H$ and $Q_D$ (of $\rho \approx 0.495$), and between $Q_D$ and $Q_L$ (of $\rho \approx 0.6$). An important advantage of the choice to model the discharge parameters with Gaussian distributions, is the possibility to apply the multivariate Gaussian distribution function, thus introducing correlations between the parameters. In case of other distribution functions the introduction of correlation factors is more complicated. We will test the sensitivity of the results to these correlations in the next section.
Figure 4.6: Best fit of several distribution functions to the discharge parameters.
4.3. Statistical model to synthesise hydrographs

Figure 4.7: Results of randomly synthesised discharge hydrographs compared with the reference results; sensitivity for choice distribution function and correlation between \( Q_H \) and \( Q_D \).

4.3.2 Sensitivities

The results (Figure 4.7) show good correspondence between the results based on the statistical model and the reference results for Cases 1 and 3. Furthermore, the influence of the correlations between \( Q_D \) and \( Q_L \) and between \( Q_H \) and \( Q_D \) appears to be small: the maximum differences between the uncorrelated and correlated samples are in the order of 5 cm. Whether this is reason enough to take the correlations into account depends on the aim of a specific uncertainty analysis. In practice, however, an effect of a couple of centimetres in the bed level will be acceptable.

The coefficients of variation of the Gaussian distribution parameters (Table 4.1) show a relatively large uncertainty in the standard deviations (\( \sigma_x \)) of the discharge parameters and in the mean of the flood duration (\( \mu_Q \)). In order to give an impression of the effect of this statistical uncertainty on the reliability of the estimated confidence intervals we perform a sensitivity analysis. To that end, we compare MCS-results based on the following Gaussian distribution parameters:

- **small mean**: the 5%-percentile of \( \mu_Q \) (= 6.2); best fit values for the other parameters,
- **large mean**: the 95%-percentile of \( \mu_Q \) (= 8.4); best fit values for the other parameters,
- **small deviation**: the 5%-percentiles of the standard deviations (\( \sigma_Q = 252 \), \( \sigma_{Q_D} = 3.8 \), \( \sigma_{Q_T} = 3.7 \), \( \sigma_{Q_L} = 173 \)); best fit values for the means,
- **large deviation**: the 95%-percentiles of the standard deviations (\( \sigma_Q = 375 \), \( \sigma_{Q_D} = 5.2 \), \( \sigma_{Q_T} = 5.0 \), \( \sigma_{Q_L} = 219 \)); best fit values for the means.
Figure 4.8 shows the results of the simulations. Case 3 appeared to be more sensitive to these uncertain distribution parameters than Case 1. The largest effects, however, are still only a couple of centimetres, which is little for river morphological problems. Moreover, when more discharge measurements are used to determine the best fit values for the distribution parameters, the statistical uncertainty will be further reduced.
4.4 Discussion

In Section 4.2 we constructed a schematisation of the (scaled) discharge hydrograph of the river Rhine (see Section 2.5), tested on the case studies of this thesis. About the general applicability of this schematisation we make the following remarks:

- The schematisation is suitable for long-term morphological modelling, that is, in the order of ten years or more. If short-term results are required on a river reach of a few kilometres, a more detailed description of the discharge will be necessary (e.g. a prediction concerning dredging work needed after a flood season).

- In natural rivers, the geometry of the main channel is not as smooth as in the cases examined. That makes the low discharge potentially more important, since bed disturbances will be initiated continuously (natural variability of the river bed). It would be interesting to investigate whether this makes an important difference when the morphological impact of a particular human intervention on the river (e.g. an engineering work) is examined. If the natural variability of the river bed is of the same order of magnitude in the situations with and without the intervention, it does not affect the estimate of the bed level difference between both situations (Section 3.4.3). If that is the case, our schematisation of the low discharge would still be suitable for more realistic models of natural rivers.

- For the cases examined, the triangular and rectangular floods gave comparable results. In more realistic river reaches, where floodplains contain levees, vegetation and other obstacles, the actual peak of the floods becomes more important. In cases where these obstacles have to be taken into account, the triangular model is more advisable.

In the introduction of this chapter we already mentioned that long periods of constant discharge are advantageous for the computation time required per simulation. The computation times required for the simulations in this chapter illustrate this effect: based on the same MCS-sample size and the same computer, the rectangular schematisation of the floods reduced the computation time of the reference simulation with 10% and the schematisation of the low discharge resulted in a reduction of even 50%.

We concluded that the statistical model derived in Section 4.3 is suitable to synthesise discharge series for morphological modelling in our case studies. This conclusion holds as long as the statistical properties of the discharge do not change during the simulated period. The statistical properties may change through, for example, changed land use in the upstream river catchment, or through climatic changes. These are examples of epistemological uncertainties due to variability (Section 3.2).
4.5 Conclusion

The aim of this chapter was to construct a statistical model to enable the synthesis of representative discharge hydrographs. These random hydrographs should force the morphological process in a realistic way. As extra criteria we imposed that the number of random parameters in the model should be small and that the discharge schematisation had to contain periods of constant discharge as long as possible, to limit the computation time required for a numerical simulation.

To find the features of a discharge hydrograph that force the morphological process, we performed a sensitivity analysis on the bed level changes resulting from our case studies. From this we conclude that discharge series can be schematised by averaging the discharge within periods of low and high discharge separately. We recommend to choose the threshold, distinguishing between low and high discharges, slightly higher than the bankfull discharge along the lowest floodplains within the river reach of interest. All events of discharges exceeding the threshold within one year may be combined into one event. Finally, the random order in which floods of a particular magnitude and duration occur over the years cannot be neglected.

The computation times required for the MCS-simulations of this chapter were up to 50% less than that for the reference simulation in which a detailed description of the discharge series was applied.

The statistical model proposed is based on Gaussian probability distributions for four random parameters per year, namely the flood height ($Q_H$), the flood duration ($Q_D$), the time of occurrence of the flood ($Q_T$), and the magnitude of the low discharge ($Q_L$). Based on a sensitivity analysis of the results of the case studies with 54 years of data, we conclude that the mutual correlations between these parameters have a negligible effect on the estimate of the bed level changes. Furthermore, the effect of the statistical uncertainty about their description on the results is acceptable.

To facilitate the construction of a similar model based on other discharge measurements we describe the steps to be performed (schematised in Figure 4.9):

1. Start with a detailed time-series of the discharge to be modelled, schematised to fit the numerical time-step of the model (e.g. a 10-days averaged discharge series).

2. Split this series into separate years (or other periods), by defining the year boundaries within a period of relatively low discharges.

3. Schematise these series following the model to be built. For our model this means: all discharge events in a year exceeding the threshold are combined to one rectangular flood, whereas the low discharges per year are substituted by their weighted average.

4. Derive the data points for each model parameter from the schematised series (e.g. $Q_H$, $Q_D$, $Q_T$ and $Q_L$ for each year).
Figure 4.9: Steps to construct the statistical model and synthesise discharge time-series.
5. Choose appropriate probability distribution functions to represent the parameters.
   a. Quantify the statistical uncertainty about the description of the parameters with, for example, the bootstrap procedure, and perform a sensitivity analysis to estimate the effect on the morphological model results.

6. Estimate the mutual correlations of the parameters and analyse whether they should be included in the model.

7. The statistical model to synthesise discharge hydrographs is now complete, namely the combination of distribution functions of the parameters and their mutual correlations, if and when necessary.

To synthesise discharge hydrographs with this statistical model, the following steps must be performed:

1. Sample one value per model parameter per simulated year from the appropriate distribution functions, including their mutual correlations if necessary.

2. Combine the sampled values to a discharge time-series (compare Figure 4.5).
5 Methods of stochastic river morphology

5.1 Introduction

In Chapters 3 and 4 we discussed the uncertainties in river morphological models, in particular the technical uncertainties. These uncertainties imply that the dynamic morphological process in a river can be described mathematically by a set of stochastic partial differential equations: Equations (2.2) – (2.6) with random initial conditions (the initial bed level and hydraulic conditions), random boundary conditions (e.g. the water discharge) and random parameters (e.g. the hydraulic roughness and the grain characteristics). In this chapter we investigate how to solve this stochastic problem. For that purpose we analyse existing calculation methods on their applicability to river morphodynamic models.

Many methods exist to handle uncertainties in different types of models and problem descriptions. For a calculation method to be applicable to numerical morphodynamic models, it must be suitable to deal with the complexity of these models. As discussed in Section 5.2 such models contain non-linear relations, are time- and space-dependent and contain large uncertainties. Furthermore, numerical simulations of morphodynamic processes are time consuming, limiting the number of simulations that can be performed in practice. Besides this model complexity, a calculation method must fit the type of river morphological problems we want to solve. We aim to estimate the overall uncertainty of the model output in terms of confidence intervals, or the probability of occurrence of a particular state of the river bed. In the first instance we are not interested in the estimation of extreme percentiles (say < 5%), since this requires detailed information about the uncertainties involved that is not available (Section 3.4).

Based on these criteria many calculation methods can be eliminated from the list of potentially suitable methods for river morphology. In Table 5.1 some regularly used methods are listed, with a short description, the reason of elimination, and a suggestion for literature about the particular method.
Table 5.1: Examples of calculation methods that are not analysed any further for applicability to river morphology.

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
<th>Reference(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fourier Amplitude Sensitivity Test</td>
<td>The multi-dimensional integral over the failure domain is reduced to a 1-dimensional integral, followed by numerical integration. It is mainly powerful for linear models with independent stochastic inputs. (Tung and Yen, 1993; Saltelli et al., 2000)</td>
<td></td>
</tr>
<tr>
<td>Itô Integration</td>
<td>Technique to deal with irregular stochastic processes with nondifferentiable sample paths, regularly applied in diffusion problems and particle models. Technical uncertainties in this thesis are regular with differentiable samples. (Arnold, 1974; Kloeden and Platen, 1992)</td>
<td></td>
</tr>
<tr>
<td>Importance Sampling</td>
<td>Monte Carlo technique, sampling concentrated in the area of high likelihood of failure, possibly combined with an increased variance of the input variables. Especially efficient for estimation of small probabilities of failure. (Clark, 1961; Saltelli et al., 2000)</td>
<td></td>
</tr>
</tbody>
</table>

The calculation methods we do investigate in this chapter are:

1. Monte Carlo Simulation: crude sampling (Section 5.3) – This seems the most appropriate method in our case. The method is regularly used in a broad range of applications.

2. Monte Carlo Simulation: Latin Hypercube Sampling (Section 5.4) – A variation to Monte Carlo Simulation with crude sampling, possibly leading to more efficient simulations.

3. First Order Reliability Method (Section 5.5) – Based on local linearisation of the model, yet examined because of its good reputation gained in civil engineering.

This chapter ends with conclusions about the applicability of these methods to river morphological models.

5.2 The complexity of river morphological models

The complexity of river morphodynamic models is one of the main reasons for many stochastic methods to be unsuitable for this kind of problems. In this section, we explain the model’s complexity by discussing four of its properties:
5.2. The complexity of river morphological models

(a) Bed level change versus grain size in Case 1 at one moment in time.

(b) Bed level change at km 57 of Case 3 versus combined height and duration of the floods (according to Chapter 4).

Figure 5.1: Illustration of the non-linear and non-monotonic relation between model output and parameters based on numerical simulations.

- non-linearity,
- time- and space-dependence,
- large input uncertainties,
- large computation times.

5.2.1 Non-linearity

The equations describing the river morphology (Chapter 2) show that the morphological changes of the river bed are non-linearly related to the model parameters and inputs. Especially the sediment transport formula is often highly non-linear. For example, in case of the Engelund-Hansen formula the sediment transport depends on the flow velocity to the power 5 ($s = au^5$). From these equations, however, it is difficult to extract to which extent the relations are non-linear in practice since that also depends on the range in which the parameters and inputs vary. This is important information, since methods based on linearisation of the model, such as the First Order Reliability Method (Section 5.5), often lead to satisfactory results, even for moderately non-linear processes. In this respect, the large uncertainties about the parameters and inputs of a morphological model are an extra complication (Subsection 5.2.3).

To show the extent of non-linearity, Figure 5.1 visualises the relations between the bed level effect and some model parameters. Both figures show non-linear relations, considering the curved lines where straight lines would have been expected in case of linearity. Or even stronger, the relations can be non-monotonic, in which case linearisation might only
serve locally as a relevant approximation. An important cause of this non-linear or non-monotonic behaviour are the bed disturbances propagating through the main channel. The propagation speed and rate of deformation of these disturbances depend (non-linearly) on model input like the discharge and the grain size of the bed material. This dependence leads, at a particular moment in time, to different forms and locations of the bed disturbances for different values of the model input (Figure 5.2), resulting in relations like those shown in Figure 5.1. Because of these non-linear relations we would expect accuracy problems for methods based on linearisation of the model, such as the First Order Reliability Method (e.g. Cooke and Van Noortwijk, 1999). Also for a statistical reason the non-linearity complicates an uncertainty analysis, since Gaussian distributed input parameters do not lead to Gaussian distributed model output. This property of a linear system is often used to simplify uncertainty analyses, since the Gaussian distribution is easy to work with and often suitable to describe uncertainties. Even to predict only the expected value of the model output non-linearity requires special attention, since the expected values of the model input do not lead to the expected value of the model output (Gardner and O’Neill, 1983).

5.2.2 Time- and space-dependence

River morphological processes are both time- and space-dependent, since bed level changes are induced by both spatial variations (e.g. in river geometry and grain size of the bed material) and temporal variations (e.g. in the river discharge). This multidimensional character leads to difficulties concerning uncertainty analysis. One difficulty is the propagation of bed disturbances, as we already mentioned in the light of the non-linear character of the process. A transformation of the model output in the \((x, t)\)-space seems attractive to circumvent this non-linearity. This, however, is practically not possible, since the propagation speed of the
bed disturbances is not constant in time, but depends on the variable river discharge. Another difficulty is the time- and space-dependence of, for example, local shoals in the river, that are decisive for the navigability of a river reach. If an estimate of the probability of occurrence of certain local shoals is needed, both time and space have to be searched for such situations. This can lead to large numerical models, the storage of much data and a large number of simulations, depending on the method applied.

5.2.3 Large uncertainties

Some of the uncertainties in the model input are large, such as the uncertainties about the river discharge and the grain size of the bed material (Section 3.4). This may cause a problem for methods based on linearisation or other model simplifications. The idea is illustrated in Figure 5.3, where the relation between the maximum erosion depth and the grain size $D$ in Case 1 is used as an illustration. Let us assume for this example that linearisation is considered in the mean grain size of 1 mm. The linear relation shown is only relevant for the uncertainty analysis within a small interval of $D$. Then, the deviation between the actual relation and the linearised model is small. In reality, however, the uncertainty about $D$ is large, implying that a large interval of the shown relation has to be considered in an uncertainty analysis. As shown in the figure, this leads to a deviation of the linearisation of up to 30% from the non-linear model. Whether this deviation is unacceptable depends on the required precision, but caution is needed.
5.2.4 Large computation times

Unless an analytical solution is available, stochastic modelling always requires more simulations than a deterministic approach. In general, high complexity of a model leads to a larger number of simulations required. Herein, the aforementioned properties of river morphological models are to their disadvantage. In many applications, methods for uncertainty analysis are applied to analytical functions or relatively small models, the evaluation of which takes little computation time. Under such circumstances, a couple of hundreds or even thousands of simulations do not pose major problems. In case of large models, however, the relatively large computation time required per model evaluation limits the number of simulations that can be performed in practice.

5.3 Monte Carlo Simulation: crude sampling

5.3.1 Theory

Basically, a Monte Carlo Simulation (MCS) consists of a large number of deterministic simulations, where the uncertain model input is randomly generated according to prescribed probability distributions (Figure 5.4). Thus, it is assumed that the input uncertainties can be characterised in statistical terms, that is, in probability distribution functions and mutual correlations. Because of the randomness of the input, the output values constitute random samples from the probability distribution of the output. Therefore, standard statistical techniques can be used to estimate the statistical properties of the model output and the precision of the output distribution. The basic and more advanced theory of MCS is explained by many authors (e.g. Hammersley and Handscomb, 1964; Rubinstein, 1982; Morgan and Henrion, 1990).

Because of the following advantages, MCS is often applied to a variety of numerical models:

- the method can easily be implemented;
- the method maintains the non-linear character of the model;
- there are no limitations to the nature or magnitude of the input uncertainties, as long as they can be described in statistical terms;
- information can be obtained about the complete probability distribution of the model output, and therefore about all its statistical properties;
- the precision of the estimate of the output distribution can be estimated from the output samples with standard statistical techniques.

As Melching (1995, p. 88) notes, "MCS is an extremely flexible and robust method capable of solving a great variety of problems. In fact, it may be the only method that can estimate the cumulative distribution function of the model output for cases with highly non-linear
and/or complex system relationships”. Further, he cites Beck (1985) who notes that “when computing power is available, there can, in general, be no strong argument against the use of MCS”. Here, however, lies exactly the main disadvantage of MCS. For the time consuming numerical simulations concerning river morphology the number of model evaluations required may constitute a practical problem.

An important question is whether the required number of samples to obtain a desired accuracy of the results can be estimated beforehand. The MCS results consist of independent random samples from the probability distribution of the output, irrespective of the number of uncertain inputs. This means that the required number of samples depends on the probability distribution function of the output and of the desired accuracy of its estimate. The required number of samples does not depend directly on the number of uncertain inputs, nor on their deviation. In general, when uncertainties are added to an MCS the required number of samples does not change, except when the variance in the output changes considerably. However, usually an uncertainty analysis is preceded by a sensitivity analysis to find those uncertain inputs that influence the variation in the output most. These dominant uncertain inputs largely determine the number of samples needed.

Morgan and Henrion (1990, p. 202) describe how to estimate the sample size required for a desired degree of accuracy (Appendix C.1). In their approach the accuracy is expressed as a fractile that must be estimated and the allowed deviation in terms of an interval of fractiles. We extend this approach, such that the desired accuracy can be expressed as a deviation from an estimated fractile in metres of bed level change. This corresponds better with the present river morphological practice. We assume that the distribution type of the model output is known, otherwise the Gaussian distribution is suitable as a first assumption.
To estimate the required sample size the following steps have to be made:

1. Translate the desired degree of accuracy into a fractile \( p \) that should be estimated and the width of the confidence interval (corresponding to a probability \( \alpha \)) that should contain the actual value of \( p \).

2. Give a rough estimate of the distribution parameters (the mean and standard deviation for Gaussian distributions) based on a small sample of outputs, that is, in the order of 20.

3. Find those fractiles \((p_1, p_2)\) of the assumed output distribution that correspond to the confidence interval defined in step 1, by applying the inverse probability distribution function with the estimated parameters of step 2. This results in the fractile interval of width \(2\Delta p\) (\(\Delta p = (p_2 - p_1)\)).

4. The required sample size \( m \) can now be estimated following Equation (C.6):

\[
m = p(1 - p) \left( \frac{c_\alpha}{\Delta p} \right)^2,
\]

where \( c_\alpha \) is the interval of the standard Gaussian distribution \( \Phi \) that reflects the desired confidence level \((P(-c_\alpha < \Phi < c_\alpha) = \alpha)\).

We will explain these steps by an example. Suppose we are interested in the maximum erosion depth that occurs in Case 1. We assume only the river discharge to be uncertain. It is modelled according to Chapter 4. For simplicity in this example we consider only the bed level effect at one moment in time, just after the high discharge period. We have no knowledge, yet, about the probability distribution of the maximum erosion depth, so we assume a Gaussian distribution. To estimate the required sample size for an MCS, we follow the steps above:

1. We specify our aim, by claiming an estimate of the erosion depth at time \( t_{out} \) that has a probability of exceedance of 10\%, with a precision of plus or minus 5 cm. We allow a probability of only 5\% that this degree of precision is not reached. In other words, we want to be 95\% confident \((\alpha = 0.95)\) that the actual 90th percentile of the maximum erosion depth \((p = 0.90)\) lies within an estimated interval of maximum 10 cm.

2. To estimate the mean and the standard deviation of the assumed Gaussian distribution we perform a first set of 20 random simulations. From the 20 output (i.e. the maximum erosion depth at \( t_{out} \) for each of the 20 simulations) results we obtain: \( \mu \approx 1.20 \text{ m}, \sigma \approx 0.13 \text{ m} \).

3. Assuming that the output distribution is Gaussian with \( \mu \) and \( \sigma \) as estimated, the 90th percentile of the erosion depth is about 1.37 m (Figure 5.5). The maximum confidence interval we desire would be \([1.32 \text{ m}, 1.42 \text{ m}]\), which corresponds with the 82th and 95th percentiles respectively \((\Delta p = (0.95 - 0.82) \approx 0.07)\).
4. Now we have enough information to apply Equation (C.6):

\[ m = p(1 - p) \left( \frac{c_\alpha}{\Delta p} \right)^2 = 0.9(1 - 0.9) \left( \frac{2}{0.07} \right)^2 \approx 73. \]

Thus we estimate that we need about 53 additional simulations, apart from the 20 we performed in step 2, to reach the desired precision. For the sake of a round value, we performed an MCS of 75 model evaluations.

Since this method gives only an estimate of the required sample size, based on an assumed distribution function with roughly estimated parameters, it is necessary to check the accuracy afterwards. When the desired accuracy is not yet reached, more samples must be added to the MCS.

Morgan and Henrion (1990, pp. 81–83) describe a method to estimate a confidence interval for the \( p^{th} \) fractile. In case of our example, the 95%-confidence interval is determined by the two values \( x_i \) and \( x_k \) of the ordered series of the 75 model outputs, with

\[
i = \left[ m_p - c \sqrt{m_p (1 - p)} \right] = \left[ 75 \times 0.90 - 2 \sqrt{75 \times 0.90 (1 - 0.90)} \right] = 62
\]

\[
k = \left[ m_p + c \sqrt{m_p (1 - p)} \right] = \left[ 75 \times 0.90 + 2 \sqrt{75 \times 0.90 (1 - 0.90)} \right] = 73.
\]

For the MCS we performed, this results in the 95%-confidence interval [1.32, 1.38]. Since the width of this interval (1.38 – 1.32 = 6 cm) is smaller than the desired precision (10 cm), the sample size of 75 appears to be sufficient.

The accuracy can also be checked by plotting the 90th percentile against the number of simulations performed (Figure 5.6). This plot also shows that the desired precision has been reached with the sample size of 75. For this particular example, both of these checks reveal an overestimation of the required sample size.
A point of concern with MCS is the generation of correlated random numbers (Tung and Yen, 1993). The generation of correlated normal or lognormal random variables poses no difficulty. However, the difficulty arises when the uncertain variables consist of a mixture of correlated non-normal and normal random variables. Chang et al. (1994) propose a practical method to generate random samples of correlated random variables, while preserving the marginal distributions of the random variables.

5.3.2 Applicability

As stated before, MCS is a robust method that can handle the characteristics of a complex model. In the light of the complexities of river morphodynamic models (Section 5.2) this implies that

- both the non-linearity and the time- and space-dependence are automatically dealt with, since the deterministic numerical models are used in their original form, and

- the large uncertainties involved do not lead to difficulties, at most to a larger sample size when they cause a large variation in the morphological effects.

Because of its important advantages, MCS has been applied to many research fields, like watershed hydrology (for a review, see Melching, 1995), water quality modelling (e.g. Gardner and O’Neill, 1983), and groundwater modelling (e.g. Kunstmann, 1999). As far as river morphology is concerned, MCS has been applied to analytical, relatively simple models. Johnson and Dock (1998) and Johnson and Simon (1997) apply MCS to models of scour around bridge piers. The authors examine the propagation of uncertainties in the effective pier width, the flow velocity, the flow depth and model correction factors, and the effect of changes in the river geometry due to channel adjustment processes. Al-Zahrani (1995) and
Gates and Al-Zahrani (1996a and 1996b) apply MCS to the Saint-Venant equations. They examine the effect of uncertainties in the model parameters and initial and boundary conditions on the model output. The method is applied to a reach of the Columbia river in the USA, in order to determine the backwater effect of a barrage during the peak annual discharge.

Since, theoretically, any accuracy of MCS results can be obtained by performing enough simulations (as long as the description of the input uncertainties permits), MCS results are often used as reference results in comparative studies (e.g. Iman and Helton, 1988; Kunsmann, 1999).

A negative point about MCS is the large computation time required when it is applied to complex models like those in river morphology. This computation time is dominated by the numerical simulations. The time required to sample input data and analyse the results is minor. This implies that the practical applicability of MCS to river morphodynamic modelling will stand or fall with the number of simulations that has to be performed, in other words, with the sample size required. In the light of keeping the sample size as small as possible, we have the following remarks:

1. From Equation (C.6) it follows that the required sample size \( m \) is inversely proportional to the square of the desired degree of accuracy \( (m \sim \frac{1}{(\Delta P)^2}) \). This unfavourable relation emphasizes the importance of being critical of the demanded precision. In this light, it is useful to bear in mind that when the uncertainty in the model input is known only roughly, it is pointless to demand a very high precision of the output statistics (Morgan and Henrion, 1990, p. 203).

2. For a given precision, the required sample size increases when aiming at more extreme percentiles. Therefore, it is advisable to focus on precise estimates of extreme percentiles only if this is inevitable and meaningful with respect to the accuracy of the description of the input uncertainties. If little information is available to quantify the input uncertainty, the tails of the probability distribution (that is, the low probability values) are only vaguely known, which restricts the feasible precision of extreme percentile estimates.

It is interesting to investigate whether output samples could be gathered over time instead of over separate simulations, when the effect of random time series on the model results is examined. If that is the case, one long-term simulation would suffice, instead of many shorter simulations.

In order to estimate the statistics of a set of output results, these results must be mutually independent and statistically homogeneous (i.e. the statistical properties may not change over the samples). In case of a random discharge series, bed level states at a particular location at different time steps are mutually independent if the period in between is large enough. The length of this period depends on the activity of the bed at a particular location.
For the statistical homogeneity of the results the following holds:

- Homogeneity does not hold during the transition period of a river reach, in which the bed adapts to a new river work.

- Because of the periodicity of the discharge hydrograph, the statistical properties of the bed level depend on the relative moment within a period.

- For realistic rivers, long-term simulations might reveal long-term changes of the bed topography. In case of the Dutch river Waal, for example, statistical homogeneity does not hold for long-term simulations, because of the large-scale erosion taking place.

Thus, gathering suitable samples over time instead of over separate simulations, in order to estimate the statistics, is only possible for special cases. In those cases it might save computation time, if only the effect of random time series is investigated. If random time series are combined with other uncertainties (e.g. the grain size, or the hydraulic roughness), separate simulations are required anyway.

5.4 Monte Carlo Simulation: Latin Hypercube Sampling

In order to reduce the computational effort required for MCS, techniques have been developed to reduce the variance of the statistical error in the MCS results (Hammersley and Handscomb, 1969; Rubinstein, 1982). Although many of these 'variance reduction techniques' exist, they are by no means always readily available, since each technique has been developed for specific types of problems. Many variance reduction techniques focus on more efficient sampling of the input scenarios. A method used regularly is the Latin Hypercube Sampling.

5.4.1 Theory

The aim of Latin Hypercube Sampling (LHS) is to divide the samples more equally over the probability distribution compared to random sampling (McKay et al., 1979). To reach this, the range of each independent variable is divided into \( n \) non-overlapping intervals of equal probability. Values of these variables are chosen at random, but such that each range is sampled only once. These values are combined in a random manner to \( n \) sets of Latin Hypercube samples, in which each sample contains one value for each independent variable (Figure 5.7). The sets of samples obtained in this way serve as input scenarios for numerical simulations, resulting in \( n \) outputs. Since these outputs are equi-distributionally divided over the (yet unknown) output distribution, standard statistical techniques can be used to estimate the empirical distribution function and the distribution parameters (like the mean and standard deviation).
Through the use of a technique introduced by Iman and Conover (1982), which restricts the pairing of the values of the individual variables, unwanted large pair-wise correlations between the uncertain variables can be avoided. Furthermore, this restricted pairing technique can be used to induce correlation among the variables if desired (Iman and Helton, 1988).

Several authors have compared the efficiency of LHS and MCS and came to the following conclusions:

- In case of linear and of monotonic problems, LHS results are more precise than MCS results based on the same number of model runs (McKay et al., 1979; Helton and Davis, 2002), but

- in case of non-monotonic problems, LHS may or may not be more efficient than MCS (Iman and Helton, 1988; Helton and Davis, 2002).

In case of non-monotonic problems, like river morphological problems, it cannot be reasoned beforehand whether LHS is more efficient or not. Despite the fact that LHS is often better and never worse than MCS, there can be a reason to choose the latter. Since the LHS outputs are not completely independent, the method to estimate the accuracy of the results as described in Section 5.3 is inaccurate for LHS.
(Morgan and Henrion, 1990, p. 205). It typically underestimates the accuracy. Although a more precise result than estimated is not bad, no methods are available to estimate the required sample size beforehand.

5.4.2 Applicability

Since the development of LHS in the 1970’s (McKay et al., 1979), the method has been applied many times. We mention three examples:

- Chang et al. (1993) apply LHS, in combination with a regression analysis, in a sensitivity analysis on a numerical model describing river morphology. They consider the sensitivity of the model output (that is, the water level, the bed level and the sediment transport) to uncertainty in the Manning’s roughness coefficients, the contraction and expansion coefficients, the porosity, eight grain sizes and the surface layer thickness. All uncertainties are described by uniform distributions. They use 30 samples of these uncertain inputs, without discussing the accuracy obtained with this sample size. Since the study concerns a sensitivity analysis based on rough descriptions of the uncertain inputs, this sample size is probably large enough.

- Yeh and Tung (1993) apply LHS in comparison with other methods, among which the First Order Reliability Method (Section 5.5), to an analytical model of pit migration through a river. The model output of their interest is the maximum pit depth after a specific travelled distance. They apply three methods to investigate the influence of uncertainties in 28 parameters. Most of these parameters are regression coefficients, besides Manning’s roughness coefficient, the friction slope and bed material characteristics. The analysis is performed under one particular constant discharge. An LHS sample size of 60 is used. The authors base this sample size on McKay (1988), who suggests that a sample size of about twice the number of random variables is sufficient for LHS. McKay’s suggestion, however, is not generally applicable, and Yeh and Tung do not quantify the accuracy of their LHS results further. Their qualitative conclusion that the LHS results “yield an approximation to the true statistical moments of the model output” seems reasonable, since all methods applied give results of the same order of magnitude.

- Christiaens and Feyen (2001) apply LHS to a spatially distributed, physically based hydrological model. This model consists of one- and two-dimensional non-linear partial differential equations, among which the Saint-Venant equations, which makes it comparable to the model considered in this thesis. The authors examine the propagation of uncertainties in soil hydraulic parameters to the model outputs, which are the river discharge, the ground water level and the soil water content. In this investigation the results are based on 25 samples of the uncertain inputs. Unfortunately, the precision reached with this sample size is not discussed.
Because of the non-linear character of the process, it is not possible to reason beforehand whether this sampling method is more efficient than MCS in case of river morphodynamic modelling. Furthermore, no method exists to estimate the accuracy of LHS results better than presuming it has been an MCS result and applying the method described in Section 5.3. However, since we are interested in the question whether LHS is more efficient than MCS, this rough estimate is of no use to us. The only way to investigate the performance of LHS for particular cases, is to repeat the LHS method several times (cf. McKay, 1995; Helton and Davis, 2002). This is what we have done for the same example as we used in Section 5.3. Again, we focus on the maximum erosion depth at a moment in time, due to the constricted main channel of Case 1. We only assumed the discharge to be uncertain. The LHS sampling of a discharge series, however, is less straightforward than in case of MCS. In Chapter 4 we showed that the discharge time series can be modelled with four parameters per year, that is, the height $Q_H$, duration $Q_D$ and time of occurrence $Q_T$ of a flood and the magnitude of the low discharges $Q_L$. Furthermore, the sensitivity analysis showed that the correlations among these parameters may be neglected. It is now possible to consider the discharge series as a group of stochastic parameters, where the number of parameters depends on the simulation time.

For two moments in time, Figure 5.8 compares LHS results, based on a sample size of 75, with reference MCS results, based on a large sample size of 1000. These results show that LHS estimates the correct CDF. To compare the accuracy of LHS and MCS estimates, we repeated each method 15 times, both based on 75 samples. Figure 5.9 shows the estimates for the maximum erosion depth after high and low discharge. Figure 5.10 shows the estimates of the bed level effects at one
location at the upstream part of the constricted reach. For all estimates we see that LHS is more efficient than MCS, based on the spread of the estimates. The extent to which LHS is more efficient varies per location and moment in time, and per type of model output. As mentioned before, it is not possible to estimate this efficiency without repeating the LHS procedure several times, which, however, would annihilate the potential advantage of LHS. Another difficulty concerning LHS is that the sample size cannot easily be extended if the first choice appears to be too small. Namely, for a larger sample size, the ranges of the uncertain inputs must be divided into more intervals of equal probability (Figure 5.7).
5.5 First Order Reliability Method

5.5.1 Theory

The First Order Reliability Method (FORM), based on the linearisation of a model, is commonly and successfully used in risk assessment studies of hydraulic structures (e.g. Vrijling et al., 1999). The attractive property of this method for those applications is the small number of computations required. Other names for the same method are First Order Second Moment method, First Order Variance Estimation method and First Order Analysis.

The theory about FORM is thoroughly explained in the literature (e.g. Yen et al., 1986; Morgan and Henrion, 1990). Here, only a summary is given.

Consider the model output \( Y \) as a function of the stochastic input parameters \( X \):

\[
Y = f(X) = f(X_1, X_2, \ldots, X_n),
\]

in which \( X_1, X_2, \ldots, X_n \) are assumed statistically independent and Gaussian distributed with mean \( \mu_X \), and standard deviation \( \sigma_X \). The Taylor series expansion of this equation is

\[
f(X) = f(X_0) + \sum_{i=1}^{n} (X_i - X_{0i}) \left( \frac{\partial f}{\partial X_i} \right)_{X_0} + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{m} (X_i - X_{0i}) (X_j - X_{0j}) \left( \frac{\partial^2 f}{\partial X_i \partial X_j} \right)_{X_0} + \text{higher order terms}
\]

(5.2)

where the subscript \( X_0 \) indicates that the derivative is evaluated at this point. From this expression the expected value and the variance of \( Y \) can be obtained by using the linearity of the expected value \( \mu_Y \) in its argument and the relation \( \sigma_Y^2 = \mu_Y^2 + \mu_X^2 \). In FORM, the second and higher order terms in Equation (5.2) are neglected, leading to the following mean and variance,

\[
\mu_Y = f(X_0) + \sum_{i=1}^{n} (\mu_{X_i} - X_{0i}) \left( \frac{\partial f}{\partial X_i} \right)_{X_0}
\]

(5.3)

\[
\sigma_Y^2 = \sum_{i=1}^{n} \left( \frac{\partial f}{\partial X_i} \right)_{X_0}^2 \sigma_{X_i}^2.
\]

(5.4)

The partial derivative terms can be approximated numerically, for example with the central difference scheme:

\[
\frac{\partial f}{\partial X_i} \approx \frac{f(X_1, \ldots, X_i + \frac{1}{2} \Delta X_i, \ldots, X_n) - f(X_1, \ldots, X_i - \frac{1}{2} \Delta X_i, \ldots, X_n)}{\Delta X_i},
\]

(5.5)
where $\Delta X_i$ is small compared to $\sigma_{Xi}$. The more non-linear the function $f()$ is, the more difference it makes which value is chosen for $\Delta X_i$.

FORM has been developed for problems in which the probability of a particular state of a system has to be estimated, especially if this probability is small. Examples of such problems in river morphology are the exceedance of a minimum water depth, or of a maximum erosion depth near a structure foundation. For this kind of problems a refinement of FORM has been developed (e.g. Yen et al., 1986). Consider the function $g(X)$ describing the state of the system, such that $g(X) > 0$ describes the desired state and $g(X) < 0$ the undesired state. We call $g(X)$ the State Function, and $g(X) = 0$ the Limit State Function. The FORM results can now be improved by evaluating the Equations (5.3) – (5.4) in the so-called 'design point', rather than in the mean values of $X$. The design point $X^*$ is the point on the Limit State Function with the highest probability of occurrence. Figure 5.11 illustrates this for a case with two random variables, both transformed into standard Gaussian distributions. An iterative minimisation method is required to find the design point on the Limit State Function. Waarts (2000) discusses methods to find the design point for different types of Limit State Functions. Most iteration methods use the partial derivative terms of the Limit State Function to each of the random parameters, thus requiring at least $n + 1$ simulations per iteration step. Considering the importance of the uncertainties about at least the last part of the discharge series and about the hydraulic roughness and grain size, 5–10 random parameters seems a reasonable assumption. Herein, we assume that the statistical model of Chapter 4 is used to include the discharge series in the FORM procedure. Let it take 10 iteration steps to find the design point (cf. Waarts, 2000). Then an iteration procedure requires 50–100 simulations.

![Figure 5.11: Illustration of linearisation in the design-point, for two random variables, transformed into standard Gaussian distributions](image-url)
The theory of FORM is based on the following assumptions about the model (Morgan and Henrion, 1990; Gates and Al-Zahrani, 1996a):

- The relation between model output and input is linear or close to linear within the range of variation considered.

- The uncertainty about the model input is small (CV < 0.2-0.25 – Gates and Al-Zahrani, 1996a). This assumption becomes more important for stronger non-linearity.

- The first two moments of the probability distributions of the uncertainties exist (i.e. the mean and standard deviation).

Since in general these assumptions do not hold for river morphological problems, the accuracy of FORM for this application is questionable (e.g. Melching, 1995, p. 93). In the following, we will investigate this.

### 5.5.2 Applicability

We first discuss a few studies from literature on the applicability of FORM in case of non-linear models in the field of river morphology:

- Yeh and Tung (1993) apply FORM to an uncertainty analysis on their analytical pit migration model, under the condition of a constant discharge. The model output of interest is the maximum pit depth after a specific travelled distance. The model is non-linear in some of the random parameters. A couple of parameters have large coefficients of variation (CV > 0.3), but they appeared to be relatively unimportant to the uncertainty in the model output. The results show a slightly (≈ 6%) underestimated standard deviation of the model output compared with corresponding LHS results (of which, however, the accuracy is unknown – cf. Section 5.4.2). The number of simulations is comparable for both methods. The authors extended this work to the evaluation of the probability that the migrating pit poses a safety threat to a hydraulic structure downstream of an initial pit (Tung and Yeh, 1993). They compare the results of FORM, based on linearisation in the mean values of the random parameters, with those evaluated in the design point. The design point approach results in a slightly more conservative estimate. Unfortunately, the authors do not compare the FORM results to MCS or LHS estimates. Therefore, it is difficult to evaluate the accuracy of the FORM results in this work.

- Yeh and Deng (1998) apply FORM in a sensitivity analysis of two sediment transport formulae, namely the Einstein bedload function and Yang’s formula. For these analyses constant discharge conditions are assumed. The formulae are non-linear in the uncertain model parameters, and the uncertainty in these parameters is assumed to be small (CV < 0.1). The results of FORM are compared to results of
a corresponding LHS analysis. The correspondence between those results suggest reasonable FORM results.

Maurer et al. (1997) apply the method to a one-dimensional hypothetical case study of deposition of suspended sediment upstream of a river dam. The non-linear model consists of the Saint-Venant equations and an advection-diffusion equation describing the suspended sediment transport. Four parameters are considered to be random, which are Manning’s hydraulic roughness coefficient ($\text{CV} = 0.1$), the critical deposition shear stress ($\text{CV} = 0.2$), the settling velocity ($\text{CV} = 0.2$) and the longitudinal dispersion coefficient ($\text{CV} = 0.3$). The latter parameter appeared to be of minor importance. The discharge varies in time, but not in a random way. The probability of the deposition of suspended sediment to exceed a threshold level is estimated. The FORM results closely match the corresponding MCS results.

Despite the non-linear models in these studies, the FORM results are accepted as reasonable. We expect that the main reasons for these good results are the limited extent of the non-linearity of the models and the limited magnitude of the uncertainties examined (i.e. $\text{CV} \leq 0.2$ for important uncertainties).

From a preliminary study (Van der Klis, 2001) we conclude that FORM gives acceptable results (i.e. the results agree with MCS results) in the estimation of the equilibrium state of the river bed under an unrealistic constant discharge. This study suggests that FORM estimates of a non-equilibrium state are inaccurate, which we now analyse further. We make a distinction between the estimation of confidence intervals of the model output, on the one hand, and the estimation of the probability of occurrence of specific river bed states, on the other.

Figure 5.12(a) shows two FORM estimates of the 90%-confidence interval of the bed level effect in Case 1 at one moment in time (20 years after the construction of the constricted). Only an uncertainty about the grain size $D$ has been taken into account (with $\mu_D = 1$ mm and $\sigma_D = 0.5$ mm), under a variable, but certain discharge hydrograph. The two estimates result from different choices of $\Delta D$ in the approximation of the derivative terms (Equation (5.5)). The derivatives have been evaluated in $\mu_D$.

These results show that the FORM results are inaccurate for the estimation of a confidence interval of the bed level effect. From the theory discussed in Chapter 2 and the simulation results presented in the previous chapters, we know that a variable discharge induces bed disturbances at either end of the constricted river reach. These disturbances propagate downstream, where uncertainty about $D$ causes uncertainty about their propagation speed and damping rate. Consequently, the relatively large confidence interval kilometres from the constriction ends (e.g. km 120) are physically not realistic, nor is the frequently changing width of the confidence interval.

These inaccurate results are caused by the combination of the non-linear relation between the bed level and the grain size, and the large uncertainty (cf. Gates and Al-Zahrani, 1996a). This is confirmed by the difference it makes which value of $\Delta D$ is chosen, as explained in
the previous subsection. To clarify the results further, Figure 5.12(b) shows the numerically obtained relation between the bed level change and the grain size at the same moment in time as Figure 5.12(a). The non-monotonicity of this relation has been explained previously (Figure 5.2). As illustrated for km 120, the linear approximation of the non-monotonic relation depends significantly on $\Delta D$. Furthermore, it is clear that linearisation in $\mu_D = 1$ mm, or in any other point, does not yield a satisfactory approximation of the non-linear relation.

For the estimation of the probability of occurrence of a specific river bed state we expect similar inaccuracies. But even if the results would be acceptable, FORM is not an ideal method for uncertainty analysis of the models we examine. Many practical problems in river morphology are time or space dependent. An example is the question where local shoals of the fairway must be expected and hence a need for dredging. To answer such questions the probability of a particular state of the river bed has to be estimated for several locations or moments in time. For each location or each point in time FORM requires an iteration procedure to locate a design point. As we estimated previously, this will take 50–100 simulations per iteration procedure, thus undoing the potential advantage of FORM concerning the computation time.

The only type of problem for which FORM might be acceptable is the probability of occurrence of an extreme state of the river bed that is time- and space-independent. One example is the maximum erosion depth to be expected during one year within the constricted reach of Case 1. Since the accuracy of FORM results cannot be estimated without an extensive analysis of the process, such results must be used with care. Even if FORM appears to be suitable for this kind of problems, experience must be gained in order to interpret the results in a reliable way.
Portielje et al. (2000) investigate the efficiency (i.e. the required number of simulations) of the combinations of FORM with LHS and with Importance Sampling respectively, for the computation of extreme event statistics in water quality models. For the non-linear models they examined, the combined methods are more efficient than MCS or LHS only for small probabilities of exceedance (below 5%).

5.6 Conclusion and discussion

In this chapter we asked the methodological question how to perform an uncertainty analysis of technical uncertainties in numerical models of river bed morphology. We focused on the estimation of moderate percentiles (say between 5 and 95%). First of all, potentially applicable methods must be suitable to deal with the complexity of river morphological models. In this context, we discussed the non-linearity, the large uncertainties, the time- and space-dependence, and the large computation time of the models. Because of these model properties, many existing methods of uncertainty analysis are not suitable.

We examined the applicability of two potentially suitable methods, namely Monte Carlo Simulation with crude sampling (MCS) and with Latin Hypercube Sampling (LHS). Both MCS and LHS give accurate results for river morphological problems, as long as the sample size is large enough and the description of the input uncertainties adequate. However, where the accuracy of the MCS results can be estimated conveniently, the accuracy of the LHS results can only be estimated by repeating the procedure several times. This would undo the benefits from the potential reduction of the sample size. Another advantage of MCS above LHS is the possibility to estimate the required sample size beforehand, for which we described a procedure. If the sample size appears to be too small afterwards, the set of samples can easily be enlarged in case of MCS, but not in case of an LHS simulation.

Besides these two methods, we examined the applicability of the First Order Reliability Method (FORM). Our main reason for this has been its good reputation in civil engineering. We showed, however, that FORM is not suitable to estimate the uncertainty in river morphological model results. The combination of non-linearity and large uncertainties leads to inaccurate results. Furthermore, if the model results are time- and space-dependent, a large number of simulations would be required, thus annihilating the potential advantage of a small sample size.

We conclude that MCS is the best available method to model the uncertainties in a river morphological model. To restrict the required sample size, it is important to formulate the stochastic problem accurately. The sample size depends on the desired accuracy of the results and on the magnitude of the output uncertainty. Therefore, the sample size can be restricted by desiring an accuracy as low as possible, and by a critical choice of the location and moment in time where this accuracy should be reached. Furthermore, it is not always necessary to estimate the complete probability distribution of the model output. When, for
example, the standard deviation gives enough information, the sample size can be strongly reduced. In river morphological practice, we expect a required sample size between some tens and a couple of hundreds of simulations. We expect that, in general, gathering output samples over time from a long-term simulation does not improve the computation time significantly. If only the uncertainty in time series (like the discharge) is examined, it might be interesting to investigate this further. However, it is only applicable to special cases, because of the required statistical homogeneity of the samples.

As mentioned before, we focused on the non-extreme percentiles of morphological model results. Our main reason for this has been the limited available knowledge of the probability distribution of the technical uncertainties. This limits the knowledge of the tails of the distributions in particular, leading to large statistical uncertainty in estimates of extreme percentiles (Melching, 1995, p. 87). If, despite this, the estimation of extreme morphological events is desired, we recommend further research on

- the quantification of technical uncertainties involved in river morphological modelling,

- the applicability of sampling techniques developed for this type of problems (e.g. Importance Sampling), and

- the applicability of combinations of FORM with LHS and other sampling techniques.
6 Opportunities offered by uncertainty analysis

6.1 Introduction

In the previous chapters we investigated the type and magnitude of uncertainties and methods to model the effect of technical uncertainties on the model output. In this chapter, we discuss possible applications of knowledge of output uncertainties. What is the added value of the stochastic results compared with deterministic model results? The extra information obtained from a stochastic model is the uncertainty about the results which we have to take into account, whether this is due to variability or to limited knowledge. This extra knowledge can be applied in several ways. On the one hand, it gives more insight into the behaviour of the river bed, both in time and space. For example, the effect of a river work can be considered in a wider context, namely not only on the so-called mean bed level, but also on the variability of the bed and the uncertainty about it. Furthermore, the extra knowledge can be used for risk analysis studies, such as the probabilistic design of river structures, or the estimation of the risks and costs in the maintenance of the river. Of both these applications we give some examples based on the case studies of this thesis. Section 6.2 discusses the spatial and time-dependent uncertainty about the river bed behaviour, and explains the interpretation of these stochastic results. Section 6.3 gives some examples of risk analysis. In Section 6.4 we discuss the relevance of the presented results for complex rivers, and the opportunity given by general knowledge of the effect of uncertainties on river bed changes. We end this chapter with conclusions about the opportunities offered by uncertainty analyses of river morphological models.

6.2 Variability of the river bed

Knowledge of the magnitude and source of the uncertainty in the model output is essential for a meaningful interpretation of the computed river bed. We mention the following advantages:
it reveals whether the model results are accurate enough for the problem to be solved,

- it shows the natural variability of the results, thus the uncertainty inherent to the process,

- it indicates efficient ways to improve the reliability of the results by giving the dominant uncertainties,

- it enables the comparison of different model results in a statistical way.

In this section we discuss some results of each of the cases discussed in Chapter 2. To this end, we apply MCS with a sample size big enough to estimate the 5% and 95% percentiles of the bed level changes with 95% accuracy within an interval smaller than 10 cm (see Section 5.3). In most simulations a sample size of 300 appeared to be enough. Most examples only take the uncertainty due to variability in the discharge into account (described according to Chapter 4). As a consequence, the confidence intervals and probabilities presented should not be interpreted as the uncertainty in the modelled bed level effects. Their relative importance with respect to other uncertainties is further discussed in Section 7.1.

### 6.2.1 Interpretation of the model output

**Spatial uncertainty in the model output**

To discuss the interpretation of the stochastic model results of the three cases we first performed 20-year Monte Carlo simulations with random discharge series. Figure 6.1 shows the 90%-confidence intervals and the mean bed level changes for each case, at two moments in time. It is important to realise that these percentiles do not represent output samples themselves, but rather an envelope of 90% of the bed level changes resulting from the MCS. To illustrate this, the figures show some samples resulting from arbitrary simulations in the MCS. The 90% confidence interval, formed by the 5% and 95% percentiles, should be interpreted as the interval containing 90% of the possible bed level effects. Thus, with 90% probability the bed level effect lies within this interval, that is, if only the discharge would have been uncertain.

In the spatial variations of the confidence interval the analysis of Chapter 2 is recognised:

- Bed disturbances are initiated at the upstream and downstream ends of the adapted river reaches, recognised by the relatively large magnitude of the confidence intervals at these locations in the high discharge season.

- The bed disturbances decrease in magnitude while propagating downstream, as recognised by the decreasing width of the confidence interval.
6.2. Variability of the river bed

(a) high discharge season, Case 1 (constricted main channel)

(b) low discharge season, Case 1

(c) high discharge season, Case 2 (lowered floodplains)

(d) low discharge season, Case 2

(e) high discharge season, Case 3 (widenened floodplains)

(f) low discharge season, Case 3

Figure 6.1: Uncertainty about the bed level changes due to the uncertainty in the discharge.
In Cases 2 and 3 the low discharge season is too short to undo the morphological effect of the preceding flood throughout the adapted river reach, as indicated by the positive values of the lower bound of the confidence intervals. In Case 1 the river bed is eroded under all discharge conditions, resulting in the negative values of the complete interval within the constricted reach.

The mean bed level changes and the confidence intervals show a wavy pattern, especially in Figures 6.1(a) – 6.1(c). This pattern results from the periodicity of the discharge hydrograph, in combination with the ratio of the flood time scale and the morphological time scale. In Figure 6.1(a), for example, the relatively large width of the confidence interval at km 100 results from bed disturbances induced during the last flood season. The next widening of this confidence interval (around km 104) results from the bed disturbances induced by the flood season of the year before, which have propagated downstream during the last low discharges season.

More information about the characteristics of the uncertainty in the bed level changes can be obtained from distribution functions. As an example, Figure 6.2 shows the density distribution and the cumulative distribution for two locations of Case 1, at the end of the high discharge season. These distributions show, for example, the shifted mean bed level change and the decreased variation in downstream direction. Furthermore, the uncertainty about the morphological effect in the upstream part of the constriction (km 100), where the bed level is directly affected during high discharge, is more skewed than further downstream, where changes occur due to propagating bed disturbances.

![Cumulative distribution functions](a) (b) density distribution functions

Figure 6.2: Distribution functions of the bed level changes at two locations in Case 1, due to the uncertainty in the discharge.
Temporal uncertainty in model output

Besides analysing the spatial aspects of the output uncertainty, it is interesting to focus on its development in time. Figure 6.3 shows the mean and the 5% and 95% percentiles of the bed level change for each case, at two locations. From these results we observe the following:

- In Cases 1 and 2 the width of the confidence interval at km 100 varies relatively strongly in a periodic rhythm. This is a direct effect of the seasonal discharge variations. The non-linear relation between the discharge and the bed level, with relatively large effects during high discharges, results in relatively large uncertainty in the high discharge season. The width of the confidence interval at km 56.5 of Case 3 also varies periodically, but less pronounced due to the smaller morphological effect.

- The periodic variations in the percentiles at km 105 (Cases 1 and 2) and km 58 (Case 3) are relatively weak. The morphological effects at these locations are caused by propagating bed disturbances and therefore only indirectly by discharge variations.

- The bed levels within the adapted river reaches are in a dynamical equilibrium within 5–10 years. This corresponds with the estimate of Section 2.4.

Relative sensitivities

In Section 3.4 we concluded that the uncertainty in the discharge is one of the important technical uncertainties, along with the uncertainties about the hydraulic roughness ($C$) and the grain size of the bed material ($D$). As a comparison, Figure 6.4 shows both the results of an MCS with only a random discharge and of an MCS with random discharge and random $C$ and $D$, for Cases 1 and 2. The results show a dominant influence of the uncertain discharge, since the major part of the confidence intervals is due to this uncertainty. The fact that a large part of the output uncertainty is due to the uncertain variability of the discharge is important for the interpretation of the results. The source of this uncertainty is natural variability, implying that its effect on the results is inherent to the river processes.

Figure 6.4 also shows a reduction of the wavy pattern in the main bed level change and the confidence interval, if the uncertainty in $C$ and $D$ is taken into account. As explained before, this wavy pattern results, among other factors, from the ratio between the flood time scale and the morphological time scale. The parameters $C$ and $D$ affect the morphological time scale. Therefore, the bed disturbances, induced during a flood, propagate at a bigger variety of speeds than in case of fixed values of $C$ and $D$. This larger spread in the propagation speed results in the reduced waviness of the pattern.
Figure 6.3: Time dependence of the uncertainty in the bed level change for each of the cases at two locations along the river.
Figure 6.4: The relative sensitivities of the model output to uncertainties in the discharge ($Q$), the hydraulic roughness Chézy coefficient ($C$), and the grain size of the bed material ($D$).
6.2.2 Variants on the case studies

The presence of floodplains

From the analytical considerations in Section 2.3 it already appeared that the presence of floodplains affects the equilibrium state of a river reach under a constant discharge. Also in the light of uncertainties it is interesting to investigate the influence of floodplains. Figure 6.5 shows the effect of uncertainty about the discharge on the model output of Case 1, both for the situations with and without floodplains. From this comparison it is clear that not only the mean bed level effect is reduced by the floodplains, but in particular the output uncertainty. The width of the 90%-confidence interval is more than halved by the present floodplains. This is explained by the reduced conveyance of discharge through the main channel (Figure 2.2), causing a reduction of the discharge variations in the main channel.

The effect of floodplains on the uncertainty in the model output for other river works within the main channel will be similar to the effect illustrated for Case 1: the presence of floodplains reduces the effect of the uncertainty. Moreover, since the conveyance of discharge through the main channel is further reduced by wide floodplains than by small floodplains, the uncertainty in the estimated bed level effects is larger for the latter.

Since the variation in the model output is considerably larger without the presence of floodplains, more simulations are required for an MCS to meet the same accuracy demands. The estimation method described in Section 5.3 results in a sample size of 475 for the MCS of Case 1 without floodplains, which is verified by the MCS results.

![Figure 6.5: Effect of the presence of floodplains on the uncertainty in the model output (Case 1) at one moment in time.](image)
6.2. Variability of the river bed

Figure 6.6: Mean and 5% and 95% percentiles for different grain sizes, representing different morphological time scales (Case 1); situation at one moment in time after 20 simulated years.

**Time scale of the morphological processes**

In the case studies of this thesis the parameter values characterising the time scale of the morphological processes correspond to the situation in the Dutch river Waal. In other rivers both the morphological time scale and the discharge characteristics differ, leading to other patterns in the model output uncertainty. To illustrate possible differences we varied the grain size of the bed material, thus altering the morphological time scale: a larger grain size causes a slower morphological process and therefore a larger morphological time scale. Figure 6.6 shows results of Case 1 in the low discharge season. We observe the following:

- The confidence interval is relatively large for the fast process and small for the slow process. This is explained by the different ratios between the time scale of the floods and the time scale of the morphological process, as discussed in Section 2.4. When the duration of the high discharges is relatively long, the river bed can adapt to the new conditions, resulting in a large confidence interval.

- Figure 6.6(a) shows that the uncertainty is not necessarily largest just downstream of the ends of the constriction. When the low discharge season lasts long enough, the bed disturbances propagate further downstream while the bed at the ends recovers from the flood.

- The remarkable difference in the downstream reach of Figure 6.6(b) is explained by the fact that the bulk of sediment released during the first years of the presence of the constriction has not propagated further than about 20 kilometres in case of the larger grain size. In case of a grain size of 1 mm, the simulated period has been long enough for the bulk of sediment to propagate out of the model domain.
Figure 6.7: Uncertainty about the bed level changes in Case 2 with roughened floodplains \((C_f = 30\sqrt{m/s})\) due to the uncertain discharge.

**Floodplain roughness**

Within the Dutch project Room for the Rivers (Section 2.2), attention is being paid to nature development within the floodplains. A changing vegetation will affect the hydraulic roughness of the floodplains. Figure 6.7 shows the effect of rougher floodplains within the lowered reach \((C_f = 30\sqrt{m/s})\) on the bed level of Case 2. It is clear that not only the mean bed level effect is smaller, but also the uncertainty about the effect. This is explained by the fact that less water flows through the lowered floodplains, resulting in a smaller discharge reduction in the main channel within the lowered river reach. This smaller reduction of the discharge results in a smaller effect on the bed level.

**Combined effects**

In a real river, various changes in geometry are present and combinations of river works are planned or carried out. To illustrate possible effects in terms of uncertainty, we estimated the effect of an uncertain discharge for two variants of Case 3:

- The floodplain widening is combined with a floodplain lowering of 0.5 m. Figure 6.8 shows that both the mean effect and the corresponding uncertainty are enlarged by the lowering.

- The floodplain widening is repeated twice, with 1.5 km in between. Figure 6.9 shows the results after a simulation of 30 years. The morphological effects of the two downstream widenings are slightly affected by bed disturbances induced by the one(s) upstream.
6.2. Variability of the river bed

(a) high discharge season

(b) low discharge season

Figure 6.8: The bed level effect of Case 3 combined with a floodplain lowering of 0.5 m, due to uncertainty in the discharge at one moment in time.

Figure 6.9: Uncertainty about the bed level changes for three successive floodplain widenings according to Case 3 when only the uncertainty in the discharge time series has been taken into account, after 30 simulated years at the end of the high discharge season.
6.3 Risk analysis

A type of question that cannot be answered by the traditional deterministic approach is the probability of occurrence of unwanted morphological effects. An example is the probability of the erosion depth exceeding the foundation depth of a structure. This kind of question is asked in risk analysis studies, where the probability of occurrence of extreme situations is combined with their negative effects on safety or usage of the river. In this section we discuss some examples of questions that stem from risk analysis.

Effect of morphological uncertainty on high water levels

The aim of modelling the morphological effects of a river work can be the investigation of changes in the probability of flooding. In that sense, it is interesting to examine the effect of uncertainty in the bed level on the water level under an extremely high discharge. To test this we perform an MCS with random discharge, where each discharge series ends in an extreme discharge level of 10,000 m³/s. The results show a 90%-confidence interval of the water level, with a maximum width of about 1 cm for a maximum water level effect of about 6 cm (Figure 6.10). Van Vuren and Kok (2003) investigate the impact of uncertainty about the river morphology on extreme flood levels in the river Waal. They also conclude that this impact is restricted to a couple of centimetres in the extreme water levels. Whether this effect on the water level is large depends on the problem to be solved. We expect, however, that other sources of uncertainty (e.g. in the hydraulic roughness) are more important than the uncertainty in the river bed level.

Note that this conclusion can be different for other type of rivers, or, for example, for river reaches near a bifurcation.

Figure 6.10: Estimation of 90%-confidence intervals of the bed level changes and the water level in Case 1 during an extreme discharge after a random discharge series.
6.3. Risk analysis

(a) bed level changes and water level relative to the initial bed

(b) water depth in detail

Figure 6.11: Uncertainty about the bed level changes and the water level and depth under OLR in Case 2 after a random discharge series.

Figure 6.12: Cumulative frequency distribution of minimum water depth under a low discharge of 680 m$^3$/s within the lowered reach of Case 2 for an MCS with sample size 300.

**Effect of morphological uncertainty on small water depths**

Under low discharges the water depth is of interest for the river’s navigability. As an example, Figure 6.11 shows the effect of uncertainty about the bed level on the water level and water depth under a low discharge for Case 2. This figure is based on an MCS with random discharge where each discharge series ends in a low discharge of 680 m$^3$/s. The results show a considerable uncertainty in the water depth, namely, within and around the lowered floodplain reach the width of the confidence interval is larger than the mean effect on the water depth.
From the MCS results it is possible to estimate the probability of a water depth smaller than a limit value. Figure 6.12 shows the estimated cumulative frequency distribution of the minimum water depth within the lowered reach, together with its confidence interval based on 300 simulations. For example, it follows that the probability of a water depth less than 2.5 m under a low discharge of 680 m$^3$/s is about 10%. That is, according to our model schematisation and if only the uncertainty about the discharge is taken into account.

**Maximum erosion depth**

Assume that a constriction has to be built within the main channel of a river. One of the design problems will be the foundation depth in order to build a stable construction. In case of a probabilistic design this question will be formulated as: what should be the foundation depth in order to realise a probability of failure less than $p\%$ per year?

We illustrate a way to answer this question by performing an MCS to Case 1. From the results in Section 6.2.1 we already know that the maximum erosion occurs at the upstream edge of the constriction, at km 100 (Figure 6.1(a)). Therefore, the model output of interest is the maximum bed level change within one year at this location (compare Figure 6.3(a)). In order to estimate the probability distribution of this maximum depth we can use the MCS performed in Section 6.2.1: a set of 300 20-years simulations with random discharge series. Note that 5-years simulations would have been sufficient since a dynamic equilibrium has been reached at km 100 in less than 5 years (Figure 6.3(a)). From each simulation we obtained the maximum depth within the 20th year, thus finding the cumulative distribution function in Figure 6.13. From this figure we can read the answer to our question. For example, the erosion depth that has a probability of exceedance of less than 2% per year is estimated to be 1.40 m, within a confidence interval of [1.39 m, 1.43 m].

![Figure 6.13: Cumulative frequency of maximum erosion depth per year within the constricted reach of Case 1.](image)
6.4 Discussion about the results presented

6.4.1 Relevance of the results for complex rivers

The results presented in this chapter, and elsewhere in this thesis, are the effects of isolated river works in a uniform river. In practice, a river reach contains many river works and variations in geometry. To illustrate the meaning of the simplified case studies for more complex models, we discuss a result of Van Vuren and Van Breen (2002). Figure 6.14 shows their result of an MCS with a one-dimensional model of the river Waal, with only the time-dependent discharge as random input. The figure shows the result after about 100 simulated years, relative to the initial bed level. The large-scale erosion in this river reach is also observed in reality, possibly caused by earlier human interventions on this river.

From this result we recognise the following patterns from the results of our simple case studies:

- km 873–876 (Erlecom): The mean bed level has barely been eroded and the confidence interval is relatively small. At this location submerged groins prevent the river bed to erode, hence the bed maintains the level imposed by these groins and morphological activity is reduced.

- km 882–885 (Nijmegen): A similar situation holds as near Erlecom, due to a stabilised bed.

- km 876 and 885: Directly downstream of the submerged groins and the stabilised bed the river bed erodes freely. The bed disturbances generated here cause relatively large confidence intervals.
- km 889–897: This reach has a relatively constant geometry. Here, we recognise a relatively small confidence interval which reduces gradually in magnitude, similar to the river reaches downstream of the river works in each of our three cases.

- km 897–913 (around Ochten): This reach contains significant variations in the floodplain width. We recognise the pattern of Figure 6.9 in the alternate sedimentation and erosion, and in the variations in the magnitude of the confidence interval.

From this comparison between the results of the complex and the simplified models, we conclude that the simple case studies help to interpret the complex results. Figure 6.14 shows an estimate of the variability of the river bed in the present state of the river. For new river works, it is interesting to investigate whether this variability changes. Van Vuren and Van Breen (2002) investigated the morphological response and the corresponding uncertainty to large scale floodplain lowering, combined with removal of levees within the floodplains, along the river Waal. They found an increased uncertainty, especially within the reach km 900–913. This corresponds with our results in Figure 6.8.

6.4.2 Importance of expert knowledge

In Chapter 5 we concluded that MCS is the best available method to analyse technical uncertainties in river morphological numerical models. In practice, however, the required sample size may lead to unacceptable computation times, even when the desired accuracy of the results is minimal. In this light, the results of the current chapter show that it is profitable to invest in gaining experience with uncertainty analysis and the characteristics of its results. First, the examples in Section 6.2 show spatial and time-dependent characteristics in the confidence intervals that are recognised in all case studies. For example, relatively large uncertainty occurs where disturbances are generated, and periodicity is caused by the discharge series. These characteristics are explained by the theoretical analysis of Chapter 2, suggesting that the theory can also be used to reason out the uncertainty in the effects of other river works. Furthermore, we showed some river characteristics that influence the magnitude of the uncertainty about the results. For example, the decreasing effect of roughened floodplains in Case 2, or the increasing effect of reduced floodplains in Case 1. Such knowledge helps to anticipate the effects of a changed river geometry on the uncertainty in the bed level.

Second, in the previous section we showed the correspondence between the results of the simplified cases and the more complex model of the river Waal. It appeared that knowledge of isolated river works helps to interpret the more complex results. This also implies that knowledge of the simple cases enables a first evaluation of the output uncertainty in the complex models.

Expert knowledge gained can be applied in various ways. In practice, an underpinned judgement about the uncertainty in morphological results will be sufficient in many problems. In
those cases, it may be sufficient to fully rely on expert judgement. In other problems a combination of expert judgement and MCS saves a lot of computation time. For example, a new river work can be isolated from a complex model to analyse its morphological effect with MCS. The results, combined with gained experience may lead to a convenient judgement about the complex model. Or another example, when several versions of a morphological model have to be analysed, an MCS on one base version may be enough to comment on the spread in other versions. When a complete MCS is necessary, which will often be the case for risk analysis studies, gained experience can help to define the MCS as efficiently as possible.

Concluding, investing into gaining experience with uncertainty analysis will pay off in the end, when the uncertainty in model results can be interpreted without too much extra effort.

6.5 Conclusion

In this chapter we showed which opportunities are offered by the results of an uncertainty analysis. For that, we discussed the interpretation of stochastic model output and gave the results of some illustrative variants on the case studies. Furthermore, three examples of applications to risk analysis have been given.

An uncertainty analysis results in knowledge of the magnitude and type of the uncertainty in the model output. This gives the opportunity to judge whether the morphological effects have been estimated with the desired accuracy. The uncertainty about the bed level effect can be of the same order of magnitude as the mean effect. Lack of this knowledge may lead to wrong conclusions about the morphological consequences of an intervention on a river reach. Furthermore, from the characteristics of the most important sources of the uncertainty in the model output it follows whether or not the accuracy can be improved, that is, whether this uncertainty is inherent to the process or due to a reducible lack of knowledge.

Stochastic results also give the opportunity to estimate the probability of occurrence of unwanted morphological effects, required for risk analysis. Where in some practical situations a simple sensitivity analysis might give enough quantitative information about the model accuracy, risk analysis problems require information about the probability distribution of the output. In the latter case, an uncertainty analysis is always necessary.

From results presented in this chapter, it follows that spatial and time-dependent characteristics of the confidence intervals are recognised in all case studies. Examples are the large width of the confidence interval at locations where bed disturbances are generated, and the spatial and time-dependent periodicity in the output uncertainty that is caused by the uncertain discharge. These characteristics can be explained by the common theory about river morphology (Chapter 2). We also illustrated that these characteristics depend on the ratio between the time scale of floods and the morphological time scale. We illustrated some specific relations between the river geometry and the output uncertainty:
- the presence of floodplains reduces the effect of the variable discharge on the bed level in reaches with geometry variations or river works within the main channel,
- increased roughness of the floodplains decreases the uncertainty about bed level changes that occur from causes within those floodplains,
- lowering of an existing wide floodplain increases the uncertainty about the bed level,
- through propagating bed disturbances, successive river works affect each other's uncertainty bounds,
- uncertainty about the bed level has a limited influence on high water levels, but an important influence on small water depths.

The conclusion is that experience with uncertainty analysis in one case study simplifies the analysis of other cases and of more complex river schematisations. Expert knowledge of the effect of uncertainties on the model output can help considerably to reduce the required computation time for a stochastic approach. Investments into gaining experience will pay off in the end.
7 Discussion

In the previous chapters, we have investigated the research questions of this study. In this chapter, we will discuss some topics concerning the complete study. First we discuss the relative importance of the different types of uncertainty for the overall uncertainty in the results of river morphological models (Section 7.1). In Section 7.2 we discuss the appropriate modelling for uncertainty analysis, given the large computation time required for a stochastic approach. Finally, in Section 7.3 we discuss the consequences of this study for uncertainty analysis in two-dimensional models of river morphology.

7.1 Relative importance of the types of uncertainty

In several of the previous chapters we have emphasized that most results presented in this thesis only show the uncertainty in the bed level changes due to the uncertain variability in the river discharge. In most simulations no other technical uncertainties have been included, nor other types of uncertainty (Chapter 4). This implies that the uncertainties presented should not be interpreted as the uncertainties in the model output, but as underestimates. In this section we will discuss them in the context of other uncertainties.

The sensitivity analysis in Section 3.4 showed that the uncertainty due to the variability in the river discharge is one of the important technical uncertainties in river morphological models. In our cases it even holds that the uncertainty about the bed level changes is dominated by this uncertainty (Figure 6.4). In case of a more detailed river schematisation, however, uncertainties in other empirical quantities will gain importance. Spatial or time-dependent variations in the hydraulic roughness or the sediment grain size, for example, result in additional bed disturbances under a variable discharge. Uncertainty about the characteristics of the variations in these quantities result in uncertainty about the bed disturbances and therefore in the morphological model results.

The relative importance of our results with respect to other types of uncertainty has not been examined in this thesis. Yet, there are a few remarks to be made about this relative importance, based on experience with river morphological models and the interpretation of the type of uncertainty in the light of possible contexts of morphological studies.
Apart from technical uncertainties we expect methodological uncertainties to contribute considerably to the uncertainty in the estimated bed levels. Take for example the uncertainty concerning the sediment transport model. In Section 3.4.2 we already mentioned how much difference the choice of the transport model can make. In complex river reaches, the influence of this uncertainty may override that of technical uncertainties.

The relative importance of epistemological uncertainties is difficult to estimate, not only for the general case, but also for specific case studies. The reason is that epistemological uncertainties are often difficult to quantify, if it is possible at all. Nevertheless, we expect the relative importance of these uncertainties to be limited when modelling reaches of well-known rivers under non-extreme circumstances (e.g. river morphological studies to Dutch rivers under a moderate river regime). For such studies, numerical models have often been used successfully, which gives confidence in the validity and completeness of the models. Epistemological uncertainties will gain importance when river morphology is studied under extreme conditions, for completely new types of river interventions, or, for example, when future climate changes have to be considered. Also for models of rivers in which, for example, complex meandering behaviour or braiding are important, the validity and completeness of the model will be less certain.

### 7.2 Appropriate modelling of uncertainty analysis

From the previous section it follows that an uncertainty analysis, in particular on river morphological model results, should not automatically result in an MCS. The uncertainties involved must first be classified and the problem to be solved needs to be described accurately. Based on this information a judgement on the relative importance of the types of uncertainty must reveal whether technical uncertainties need to be included in the uncertainty analysis. If this is the case, the modeller may conclude that an MCS is an appropriate method to perform (part of) the analysis.

If an MCS is used to perform an uncertainty analysis, the computation time is a point of concern. To restrict the computation time as far as possible, attention must be paid both to the degree of model detail and the sample size of the MCS (Section 5.3).

We first discuss the choice of the degree of model detail. In the so-called *appropriate modelling* of a physical process a balance is sought between the uncertainty in the model results (partly determined by the complexity of the model) and the costs necessary for the modelling, including the computation time (Booij, 2002). It is important to realise that including more knowledge of the internal processes of a system into the model only leads to better predictions if it is matched by extended information concerning the model parameters and input variables (cf. Jagers, 2003). By lack of data of sufficient quality, this is not always possible. In defining a numerical model, appropriate modelling should always be an issue, but an intended uncertainty analysis by MCS enlarges this necessity. The comparison between a simple model with knowledge of the uncertainty in the results and a detailed model without this knowledge may lead to preference for the former. Because of the cross-
dependencies between model detail, accuracy of the results and required calculation time, no simple recipe exists for appropriate modelling. It is an iterative process and requires expert knowledge of the modeller.

The need for appropriate modelling also applies to the definition of the MCS. Here it concerns the balance between the accuracy of the statistical estimates and the sample size required (Section 5.3). Again a larger sample size will only lead to a higher precision of the output statistics if it is matched by a better quantification of the technical uncertainties (Johnson, 1996). Often, the latter will not be available (Chapter 3).

Other ways to save computation time could be found in faster computer systems or programming techniques. For example, MCS is excellently suitable for parallel programming, since the separate deterministic simulations are mutually independent.

All time-saving techniques cannot prevent MCS to be too time consuming in some situations. In this light, we make the following comments:

- A sensitivity analysis can give a rough impression of the effect of uncertainties on the model output based on a few simulations. This impression, however, is restricted to a qualitative statement. If an estimate in terms of probabilities is required, e.g. for risk analysis problems, a sensitivity analysis does not suffice.

- Expert knowledge of uncertainties in model results can save a lot of computation time. Therefore, we recommend to invest in gaining this knowledge by performing MCS for different practical problems and analysing and comparing the results (Section 6.4.2).

7.3 Stochastic modelling of two-dimensional models

In this thesis we focus on one-dimensional models. In river morphological practice, however, two-dimensional depth-averaged models (2-DH models) are also being used. Therefore the question arises how to interpret our results in the light of 2-DH models. Compared to one-dimensional models, an important extra complication in 2-DH models is the modelling of the direction of the bedload transport, instead of only its magnitude (e.g. Struiksma et al., 1985). Therefore, 2-DH models exhibit a much richer morphological behaviour. The results of a 2-DH model can be analysed at a more detailed level than 1-D results, like bank patterns, or channel development. Other examples of morphological processes that can be analysed with 2-DH models are river meandering (e.g. Seminara and Tubino, 1989; Mosselman, 1998), braiding of rivers (e.g. Jagers, 2003), or the river bed morphology at bifurcations (Mosselman et al., 1999; Mosselman et al., 2003; Sloff et al., 2003). The computation time required for 2-DH models is usually larger than for 1-D models, that is, a day per simulation is not exceptional.

Without going into an extensive investigation, we will discuss the modelling of uncertainties in 2-DH models.
First, we consider the uncertainties involved. The classification method in Chapter 3 is directly applicable to 2-DH models of river morphology. This holds both for the taxonomy of sources of uncertainties and for the manifestation of the uncertainties in numerical models. The examples of sources of uncertainty concerning river morphology presented in Tables 3.1 and 3.2 also apply to 2-DH models. Sources of uncertainty typical of a 2-DH model are, for example, the parameters describing the direction of the sediment transport, or the limited knowledge underlying local scour models (Mosselman and Stoff, 2002).

Concerning the relative sensitivity of the model results to technical uncertainties, we expect similar results as found in Section 3.4. That is, we expect the uncertainty due to the variability in the river discharge to be one of the important technical uncertainties. Furthermore, through linear analyses Mosselman et al. (1999) show that spatial variations in grain size and hydraulic roughness can have large effects on the bed topography in rivers (also Mosselman et al., 2003; Stoff et al., 2003). Stoff et al. (2003) show that horizontal and vertical sorting of graded sediment is a relevant parameter in the morphology at bifurcations (cf. Blom, 2003).

To synthesise discharge hydrographs for 2-DH model applications we expect that a more detailed description is required than derived in Chapter 4. Namely, 2-DH models are often used for short-term simulations (i.e. a few years) of river reaches of a few kilometres, because of the large computation time required. As we discussed in Chapter 4, more details in the discharge schematisation are required for short-term simulations compared to our statistical model. Especially the description of the floods will require more detail, because of their relatively large effect on the sediment transport. Wiersma (1997) shows that a relatively rough schematisation of the low discharges still gives acceptable two-dimensional morphological results in the case of a river bend.

Our conclusions about the methodology to model technical uncertainties in 1-D models of river morphology also hold for 2-DH models. For the quantification of the uncertainty in the results of a dynamic model, Monte Carlo Simulation is robust and the best available method. Concerning Latin Hypercube Sampling, the conclusion remains that its estimates are reliable, but do not give the opportunity to calculate their accuracy in a suitable way. Since in practice the computation time for 2-DH simulations is much larger than for 1-D simulations, the total computational burden of an MCS will often be prohibitive. Therefore, our discussion about appropriate modelling in the previous section and the recommendation to gain expert knowledge of the influence of particular uncertainties on the model results are of even more importance.

To reduce the computational burden of uncertainty analyses of 2-DH models, it might seem attractive to replace this complex model with a simplified model, a so-called reduced model. The derivation of such reduced models for specific cases is a research subject itself (e.g. Hooimeijer, 2001). The derivation of a verified reduced model requires many simulations with the original model, to provide enough data describing the input-output relation of the model. Further research on this subject is recommended, especially for studies in which many variants of a reference case have to be analysed.
In general, the quantitative stochastic results presented in Chapter 6 cannot be translated directly to the uncertainty in the results of corresponding 2-DH models. Compared to 2-DH models, one-dimensional models give a width averaged representation of the morphodynamics of a river bed. This 1-D results are representative of reality as long as the two-dimensional processes do not dominate the morphological processes being modelled. An analogy holds with the estimated uncertainty in the model results. As long as a 1-D model is applicable, its output uncertainty inherent to the modelled process (e.g. due to the uncertain discharge) is expected to give a sensible representation of the width averaged uncertainty in a 2-DH model. We expect the uncertainty in the width-averaged profiles to be smaller than the uncertainty in individual longitudinal profiles of the river bed, because of the averaging out of the relatively large and small uncertainties.

Often, a 2-DH model is applied to situations in which two-dimensional processes are particularly important, such as in strongly meandering river reaches, or in a detailed model of a river bifurcation (cf. Wang et al., 1995; Sloff et al., 2003). In such cases, the output uncertainty in lateral direction is also interesting. In both directions, longitudinal and lateral, the uncertainty will be largest at locations of relatively high morphological activity.
8 Conclusions and recommendations

The objective of this study was to inventory the difficulties concerning uncertainty analysis of numerical models of river bed morphology, and to find a suitable method to estimate the effect of uncertainties in the model parameters and input variables on the morphological model output. Herein, we mainly focussed on the effect of one particular uncertainty, namely the uncertainty due to the variability in the future river discharge.

To meet the objective, we sought answers to the following research questions:

1. Which types of uncertainty can be distinguished concerning river morphological modelling?

2. Which uncertainties in the parameters and the input variables of a river morphological model are relatively important with respect to the uncertainty in the model results?

3. How can the uncertainty about the future river discharge be described?

4. Which calculation methods are suitable to estimate the effect of uncertainties in model parameters and input variables on the morphological model results?

5. What opportunities do results of an uncertainty analysis offer for practice?

These questions have been addressed and answered in the previous chapters (see Figure 1.1). Three case studies have been used for investigation and illustration purposes. These cases concerned the one-dimensional morphological effects of a constricted main channel, of lowered floodplains and of widened floodplains. In this chapter we present the conclusions of this study by successively summarising the answers to the research questions.

To stimulate a follow-up of this study, we end this thesis with recommendations for future research and for the performance of uncertainty analysis in practice.
8. Conclusions

8.1 Classification of uncertainties

An uncertainty analysis starts with making an inventory of the uncertainties that might influence the uncertainty about the model results. Various types of uncertainty exist, ranging from quantifiable to non-quantifiable and from reducible to inherent to the process. To simplify the interpretation of the uncertainties we presented a method (found in the literature) to classify uncertainties. This classification method has been based on a taxonomy of sources of uncertainties, on the one hand, and on the manifestation of uncertainties in numerical models, on the other. The two main sources of uncertainty distinguished are variability and limited knowledge. Uncertainties of both sources manifest themselves in numerical models at four levels, namely technical, methodological, epistemological and model operation uncertainties.

We applied the classification method to a variety of uncertainties in river morphological models. Based on this application and on the fact that the classification method has been developed as a generally applicable method, we conclude that it is suitable for river morphological modelling. The use of this classification method in river morphological practice would help effectively to communicate about uncertainties and to interpret the results of an uncertainty analysis. Furthermore, it helps to decide which approach is required to perform an uncertainty analysis.

8.1.2 Relative importance of technical uncertainties

We base our conclusion about the relative importance of technical uncertainties (i.e. uncertainties in the model parameters and the input variables) on a rough sensitivity analysis of our case studies, combined with an estimate of their order of magnitude. We illustrated that the variability of the river discharge can often not be neglected without losing important information about the morphological processes in a river. Since the uncertainty about the future discharge variations is large, we conclude that this uncertainty is one of the important technical uncertainties in river morphological models. In our cases it even holds that the uncertainty about the bed level changes is dominated by this uncertainty. Besides the uncertain variations in the discharge, we expect the hydraulic roughness and the grain size of the bed material to be relatively important uncertainties. For further investigation of their relative importance, research on more adequate descriptions of these uncertainties is required.

The importance of uncertainties in the boundary conditions, other than the discharge, and in the initial bed level depends on the location of the model boundaries with respect to the river reach of interest, and to the length of the simulation period before the period of interest.

The focus of this study on technical uncertainties does not imply that uncertainties at other manifestation levels are not important. The uncertainties presented should not be interpreted as the uncertainties in the model output. We expect that methodological uncertainty contributes considerably to the uncertainty in the model output. This expectation is based on
the sensitivity of the results to modelling choices (e.g. the sediment transport formula), as described in the literature. The relative importance of epistemological uncertainty depends on the river reach modelled and the conditions under which the morphological process is examined.

8.1.3 The synthesis of discharge series

Based on a sensitivity analysis to our cases, we determined the characteristics of the river discharge forcing the morphological process: the magnitude, duration and time of occurrence of floods and the average magnitude of lower discharges in between. Since our case studies have been based on the Dutch river Waal, we expect that the conclusions also hold for the Dutch Rhine branches. By deriving statistical descriptions of these characteristics, we constructed a statistical model of the discharge. This model can be used to synthesise random discharge series. These random hydrographs, first of all, force the morphological process in a realistic way. Furthermore, the number of random parameters in the model is relatively small. The discharge schematisation contains relatively long periods of constant discharge, thus limiting the computation time required for a numerical simulation.

In the statistical model a series of mutually independent one-year discharge hydrographs is sampled. Each one-year hydrograph consists of one rectangular or triangular flood, enclosed by a constant low discharge. This schematisation results in four random parameters per year: the flood height and duration, the time of occurrence of the flood and the magnitude of the low discharge. For the case studies of this thesis a Gaussian distribution appeared to be suitable for each of these parameters. The effect of the mutual correlations on the estimates of the bed level changes could be neglected.

With this statistical model representative discharge series can be synthesised for long-term morphological processes (say ten years or longer). For short-term simulations more details will be required, especially in the description of the floods. The statistical model does not take account of possible changes in the statistical properties of the discharge characteristics through, for example, climatic changes.

8.1.4 Methods to estimate uncertainty in the model results

Many existing methods to estimate the uncertainty in the model results are not suitable to deal with the complexity of river morphological models. Concerning this complexity, we discussed the non-linearity, the large uncertainties, the time- and space-dependence and the large computation times of the models. We investigated the applicability of two potentially suitable methods. It concerns the Monte Carlo Sampling with crude sampling (MCS) and with Latin Hypercube Sampling (LHS).

Both MCS and LHS give accurate results for river morphological problems, as long as the sample size is large enough and the description of the input uncertainties adequate. However, where the accuracy of the MCS results can be estimated conveniently, the accuracy of the LHS results can only be estimated by repeating the procedure several times. This would undo the benefits from the potential reduction of the sample size. We described a procedure
to estimate the required sample size of an MCS beforehand. Such a procedure is not available for LHS.

We also examined the applicability of the First Order Reliability Method (FORM), because of its good reputation in civil engineering. We showed, however, that FORM is not suitable to estimate the uncertainty in river morphological model results. The combination of non-linearity and large uncertainties leads to inaccurate results. Furthermore, if the model results are time- and space-dependent, a large number of simulations would be required, thus annihilating the potential advantage of a small sample size.

We conclude that MCS is the best available method to model the uncertainties in a river morphological model. To restrict the computation time, appropriate modelling is required of both the numerical model and the MCS. On both levels it concerns the balance between the accuracy of the results and the required simulation time or sample size respectively. Note that more accurate results can only be reached by more modelling effort if it is matched by appropriate model input.

8.1.5 Opportunities offered by the results of uncertainty analysis

An uncertainty analysis results in knowledge of the magnitude and type of the uncertainty in the model output, e.g. the estimated bed level effects. First of all, this gives the opportunity to judge whether the model results have the desired accuracy. From the characteristics of the sources of the uncertainty it follows whether the accuracy can be improved or not, that is, whether this uncertainty is inherent to the process or due to a reducible lack of knowledge. Furthermore, we illustrated that knowledge of the uncertainties also give the opportunity to estimate the probability of occurrence of unwanted morphological effects, which is required for risk analysis.

Based on examples of uncertainty analysis on our case studies in this thesis, we conclude that typical time- and space-dependent characteristics of the confidence intervals are recognised in all results. Examples are the large width of the confidence interval at locations where bed disturbances are generated, and the spatial and time-dependent periodicity in the output uncertainty that is caused by the uncertain discharge. We also illustrated that these characteristics depend on the ratio between the time scale of floods and the morphological time scale. Furthermore, comparison of the results of our simplified cases with the results of a corresponding uncertainty analysis on a more detailed model of the river Waal showed useful correspondence.

We conclude that experience with the results of uncertainty analysis for a number of relatively simple case studies simplifies the analysis of other cases and of more complex river schematisations. This makes that expert knowledge of the effect of uncertainties on estimated morphological effects can considerably reduce the computation time required for a stochastic approach.
8.2 Recommendations

To increase and improve the facilities to perform uncertainty analysis of numerical river morphological models, we recommend further research on the following topics:

- The quantification of important technical uncertainties is a point of concern. Both the often limited availability of data, and the complex structure of the quantities concerning their space- and time-dependence (e.g. hydraulic roughness or grain size) are a handicap. Further research on this topic should have a high priority, since the results of uncertainty analyses may not be meaningful when the assumed distributions of the input uncertainties are incorrect.

- We advise to perform more advanced sensitivity analyses concerning technical uncertainties on a variety of practical river morphological case studies. This would result in a better understanding of the relative importance of the various uncertainties to the uncertainty in the model results.

- With a better understanding of the characteristics of the technical uncertainties and with ongoing research in other fields, sampling techniques may be found to reduce the required sample size of Monte Carlo simulations.

- Depending on the specific morphological problem, methodological uncertainties may have a significant influence on the accuracy of the model results. We recommend research on the quantification of these uncertainties.

Concerning the performance of uncertainty analysis in practice we have the following recommendations:

- The analysis of uncertainties should become a standard issue in river morphological case studies. This does not necessarily imply a full Monte Carlo simulation. The awareness of the influence of uncertainties is already enlarged by making an inventory of the uncertainties involved and discussing, on the basis of expert knowledge, their effect on the model results.

- The presentation of uncertainty in model results to, for example, decision makers asks for attention. Firstly, the combination of different types of uncertainty into one confidence interval (or other measures of uncertainty) hinders a clear interpretation. Then, for example, it is unclear which part of the uncertainty is inherent to the problem and which part may be reduced. Secondly, making a decision on the
basis of a range of possibilities instead of one deterministic result is difficult. To enlarge the acceptance of uncertainties, a modeller should take the objective of the decision maker into account when presenting uncertain model results.
Bibliography


A Numerical schematisation of the case studies

SOBEK is, in concise technical terms, a one-dimensional open-channel dynamic numerical modelling system. It numerically solves the dynamic model described in Section 2.3.1. An extensive description of the numerical method can be found in the Technical Reference Manual of SOBEK (WL | Delft Hydraulics, 2000). We only mention some characteristics. In SOBEK the equations are solved in a quasi-uncoupled way. The flow equations are solved using a fixed known river bed, being the latest bed computed. After this computation, a new bed level is computed by applying the continuity equation of bed material, using the latest flow field computed. This approach is schematised in Figure A.1. This method is allowed if the propagation speeds of flow and bed level perturbations are of different order of magnitude. For morphological changes in a river with Froude numbers lower than 0.6 to 0.8 this condition is usually fulfilled.

The two equations describing the water flow are solved using a finite difference method based on the Preissmann box scheme (which is a numerical discretisation). The sediment continuity equation is solved by an explicit numerical method based on an adapted Lax-Wendroff scheme.

For stability of the numerical solution of the sediment continuity equation the Courant number $\sigma$ must satisfy the following condition:

$$\sigma = c_b \frac{\Delta t}{\Delta x} \leq 1,$$

(A.1)

with $c_b$ is the propagation speed of bed disturbances, $\Delta t$ the morphodynamic time step and $\Delta x$ the grid space. SOBEK checks for this condition. When the condition is not met, the morphodynamic time step is reduced. Based on the information in Table 2.1 and the estimates of $c_b$ in Table 2.2 we find that $\sigma \approx 0.4$ for Cases 1 and 2 and $\sigma \approx 0.2$ for Case 3, under high discharges. This implies that the stability condition is met.
Figure A.1: Quasi-steady solution method applied in SOBEK.
B  Simplified morphological models

River morphological processes are often described by the mathematical model discussed in Section 2.3.1. Solutions of this model can only be obtained through numerical approximation. Analytical solutions, however, are sometimes more attractive because they are suitable for rough guesses and getting insight in the importance of various parameters.

Three simplifications of the full one-dimensional river morphological model are regularly used in order to find analytical solutions: the simple-wave model, the parabolic model and the hyperbolic model. Here we only discuss the mathematical description of the models and the assumptions made. Extensive discussions can be found in, for example, Jansen et al. (1979, p. 122) and Ribberink and Van der Sande (1985).

B.1  Simple-wave model

The assumptions made for this model are the following:

- A steady flow holds without sources or sinks in a channel of constant width. With this assumption the continuity equation reduces to the statement that the discharge per unit width is constant:

  \[
  \frac{\partial uh}{\partial x} = 0.
  \]

- The water surface is approximated by a frictionless rigid lid, i.e. the water depth variations stem from the bed level variations only:

  \[
  \frac{\partial h}{\partial x} = -\frac{\partial z_b}{\partial x}.
  \]

This assumption implies that the model is valid for distances much smaller than the length scale of the backwater curve (\(|x| << \frac{k}{3\gamma}\)).
These two equations combined with \( s = au^b \) and the continuity equation of bed material (Equation 2.6) lead to the simple-wave approximation:

\[
\frac{\partial z_b}{\partial t} + c_b \frac{\partial z_b}{\partial x} = 0,
\]

with \( c_b = b \frac{s}{h} \). From this simple-wave model the propagation speed of small bed disturbances (\( c_b \)) is deduced.

When the propagation speed \( c_b \) is considered as a constant, the simple-wave approximation is linear and holds for small bed disturbances. When the non-linear variant is considered (i.e. \( c_b \) increases with increasing bed level), the solution includes the development of a shock-wave.

### B.2 Parabolic model

The only difference from the simple-wave approximation is that now the bed slope equals the friction slope, meaning that friction-dominated flow is assumed and the water surface approximately follows the bed. With this, the hydrodynamic equations are reduced to

\[
-\frac{g}{c^2} \frac{\partial z_b}{\partial x} - \frac{g}{C^2} \frac{u^2}{h} = 0.
\]

In general, this approximation is valid at spatial scales much larger than the scale of the backwater effect (\( |x| >> \frac{h}{s_1} \)). With these equations we can rewrite the velocity gradient in terms of bed level variations:

\[
\frac{\partial u}{\partial x} = -\frac{C^2 q}{3u^2} \frac{\partial^2 z_b}{\partial x^2}.
\]

Combined with the sediment transport formula (\( s = au^b \)) and the continuity equation of bed material, this leads to the parabolic approximation:

\[
\frac{\partial z_b}{\partial t} - K \frac{\partial^2 z_b}{\partial x^2} = 0,
\]

with \( K = \frac{b}{3} \frac{s}{s_1} \). From this model the diffusion coefficient of small bed disturbances (\( D \)) is deduced.

### B.3 Hyperbolic model

Again, the only difference from the other approximation lies in the equations of water motion. Now steady friction-dominated flow is assumed, i.e. backwater effects are allowed, but
the Froude number remains small:

\[
\frac{\partial uh}{\partial x} = 0
\]
\[
-g \frac{\partial h}{\partial x} - g \frac{\partial z_b}{\partial x} - \frac{g}{C^2} \frac{u^2}{h} = 0.
\]

Since in the derivation of this model the propagation speed \( c_b \) is assumed to be constant, the model holds for small bed disturbances.

From the reduced hydrodynamic equations it follows that

\[
\frac{\partial u}{\partial x} = \frac{1}{h} \left( \frac{\partial z_b}{\partial x} + \frac{1}{C^2} \frac{u^3}{q} \right).
\]

Substituting this into the continuity equation of bed material and eliminating the velocity \( u \) from the resulting equation leads to the hyperbolic approximation:

\[
\frac{\partial z_b}{\partial t} - K \frac{\partial^2 z_b}{\partial x^2} - \frac{K}{c_b} \frac{\partial^2 z_b}{\partial x \partial t} = 0,
\]

which is basically a combination of the simple-wave model and the parabolic model.
B. Simplified morphological models
C Statistics

C.1 Empirical distribution function

The probability distribution for a stochastic variable $X$ is often represented by its cumulative distribution function (CDF). This function gives the probability that $X$ will be less than or equal each possible value $x$:

$$F(x) = P(X \leq x), \quad \text{for } x \in \mathbb{R}.$$  

The CDF of a stochastic process can be estimated from a set of observations $x_1, x_2, \ldots, x_n$ by the empirical distribution function $F_n$. This distribution function is defined as

$$F_n(x) = \frac{\# \{i \in \{1, 2, \ldots, n\} : x_i \leq x\} }{n}, \quad \text{for } x \in \mathbb{R},$$  

(C.1)

or in words, the number of observations smaller than or equal to $x$, divided by the total number of observation $n$.

Suppose we wish to estimate the $p^{th}$ fractile $X_p$ from a sample of $m$ values $x_i$. Without loss of generality, we have the sample in increasing order and relabel the values:

$$x_1 \leq x_2 \leq \ldots \leq x_m$$

A simple estimate of the $p^{th}$ fractile $X_p$ is the corresponding sample value $x_j$, where $j = mp$. If $mp$ is not a whole number, it is round to the nearest sample value.

Morgan and Henrion (1990, pp. 81–83) describe a method to estimate a confidence interval for the $p^{th}$ fractile, again from a sample of $m$ values $x_i$. The interval is determined by the two values $x_i$ and $x_k$ that enclose the fractile with a specified probability $\alpha$, with

$$i = \left\lfloor mp - c \sqrt{mp(1-p)} \right\rfloor,$$  

(C.2)

$$k = \left\lceil mp + c \sqrt{mp(1-p)} \right\rceil,$$  

(C.3)
where $mp$ is the estimated $p$th fractile, $\sqrt{mp(1 - p)}$ an estimate for its standard deviation, and $c$ the corresponding interval in a unit Gaussian distribution:

$$P[-c < \Phi < c] = \alpha.$$  

The notation $\lfloor \cdot \rfloor$ and $\lceil \cdot \rceil$ indicates that we round these two quantities down and up respectively to the nearest whole numbers. This estimation is based on the assumption that the probability that $mp$ represents the actual value of the $p$th fractile is normally distributed. This approximation generally holds for the non-extreme percentiles.

As an example, suppose we have a sample size $m = 100$, and wish to obtain a 95% confidence interval for the median, so $c \approx 2$, $p = 0.5$. From C.2-C.3 it follows:

$$i = \left\lfloor 100 \times 0.5 - 2\sqrt{100 \times 0.5 \times 0.5} \right\rfloor = 40 \tag{C.4}$$

$$k = \left\lceil 100 \times 0.5 + 2\sqrt{100 \times 0.5 \times 0.5} \right\rceil = 60 \tag{C.5}$$

So $(x_{40}, x_{60})$ gives an approximate 95% confidence interval for the median.

Morgan and Henrion (1990, p. 202) also show how the estimation of the $\alpha$ confidence interval (Eqs. C.2–C.3) can be used to estimate the sample size required for a specific degree of accuracy. For $\alpha$ confidence of the $p$th fractile being between the sampled values used as estimates of the $p - \Delta p$th and $p + \Delta p$th fractiles, the required sample size $m$ is estimated by

$$m = p(1 - p) \left( \frac{c}{\Delta p} \right)^2. \tag{C.6}$$

For example, suppose a 95% confidence is desired of the actual 90th percentile being between the estimates of the 85th and 95th percentiles. So, with $\Delta p = 0.05$ and $c \approx 2$, equation C.6 gives

$$m = 0.90 \times (1 - 0.90) \times \left( \frac{2}{0.05} \right)^2 = 144.$$ 

Equation C.6 is identical to the estimation formula known in structural engineering (CUR–190, 1997, p. 90).

### C.2 The bootstrap method

Bootstrapping methods are described by, for example, Efron and Tibshirani (1993). The methods estimate the uncertainty in the parameters $\phi$ of a model that has been fitted to a set of data $x = (x_1, x_2, ..., x_n)$. The following algorithm is used in Chapter 4:

1. Resample data sets from $x = (x_1, x_2, ..., x_n)$ by randomly drawing with replacement $n$ elements. This results in $B$ resampled data sets $x_1^*, x_2^*, ..., x_B^*$.
2. Estimate the model parameters $\phi$ for each of the bootstrap samples, for example by determining the Least Square Fit of the model to the samples. This results in $B$ parameter estimates ($\phi_1^*, \phi_2^*, ..., \phi_B^*$) = $\phi^*$.

3. Determine the parameter uncertainty by the empirical distribution function (Equation C.1) of $\phi^*$.

In Chapter 4 we apply this method to estimate the statistical uncertainty in the parameters of the Gaussian distribution functions, which on their turn describe the variability in the discharge parameters (Figure 4.5). Herein, $x = (x_1, x_2, ..., x_n)$ are the data from the discharge measurements, $n = 48$ (i.e. the number of measured years that contain a flood), $\phi = (\mu, \sigma)$ (i.e. the mean and standard deviation) and $B = 1000$. 
D Explanation of figures

To clarify the reading of some characteristic figures presented in this thesis, two of them will be explained further in this Appendix.

Figure D.1 shows a typical figure of Chapter 6. In this type of figures, the two outer lines form the 90%-confidence interval of the estimated bed level changes. The line in the middle represents the mean bed level change. In some figures the results of two or more simulations are compared. In those figures both the confidence interval and the mean of one simulation are presented with the same line type. The legend within the figure shows which line type corresponds to which simulation.

Figure D.2 shows a typical figure of Chapter 4. In this type of figures, results of various simulations are compared to one reference simulation. The results of this reference simulation are presented in terms of grey areas. The width of each of these grey areas represent the 90%-confidence interval of the corresponding percentile, thus indicating the statistical uncertainty concerning the estimate of this percentile.

Figure D.1: Explanation of a characteristic figure of Chapter 6.
Figure D.2: Explanation of a characteristic figure of Chapter 4.
Tables E.1 and E.2 show quantitative results of the simulations discussed in Sections 4.2.1 and 4.2.2, respectively. For each case study the following information is presented for seven different percentiles ($P$):

- the width of the 90%-confidence interval of the reference results (representing the statistical uncertainty of the results), averaged over the river reach containing the river work,

- the difference between the best estimates of the reference results and each of the other simulations, averaged over the river reach containing the river work.

Table E.1: Quantitative comparison between the simulations constant low (CL) and variable low (VL) and the reference results (REF). Presented are the bed level effects (in centimetres) averaged over the river reach containing the river work.

<table>
<thead>
<tr>
<th>P</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>width CI</td>
<td>deviation</td>
<td>width CI</td>
</tr>
<tr>
<td></td>
<td>REF CL VL</td>
<td>width CL</td>
<td>deviation</td>
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<tr>
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<td>CI CL VL</td>
<td>CI CL VL</td>
</tr>
<tr>
<td>1%</td>
<td>3.0 4.7 0.1</td>
<td>&gt; 8.5</td>
<td>16.5</td>
</tr>
<tr>
<td>5%</td>
<td>2.1 4.5 0.2</td>
<td>8.1</td>
<td>23.3</td>
</tr>
<tr>
<td>20%</td>
<td>1.8 5.4 0.2</td>
<td>8.5</td>
<td>29.7</td>
</tr>
<tr>
<td>50%</td>
<td>2.3 4.1 0.8</td>
<td>7.2</td>
<td>33.2</td>
</tr>
<tr>
<td>70%</td>
<td>2.8 1.0 1.5</td>
<td>7.3</td>
<td>33.3</td>
</tr>
<tr>
<td>95%</td>
<td>3.0 6.3 1.1</td>
<td>10.0</td>
<td>31.9</td>
</tr>
<tr>
<td>99%</td>
<td>6.5 10.7 0.2</td>
<td>&gt; 10.7</td>
<td>34.1</td>
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</tbody>
</table>
Table E.2: Quantitative comparison between the simulations *triangular flood* (TF) and *rectangular flood* (RF) and the reference results (REF). Presented are the bed level effects (in centimetres) averaged over the river reach containing the river work.

<table>
<thead>
<tr>
<th>P</th>
<th>Case 1</th>
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<th>Case 2</th>
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<th>Case 3</th>
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<td>8.5</td>
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<td>70%</td>
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<td>95%</td>
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<tr>
<td></td>
<td>&gt; 6.6</td>
<td>0.2</td>
<td>2.4</td>
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</tr>
</tbody>
</table>
List of symbols and abbreviations

This list explains the symbols and abbreviations used throughout the thesis. Symbols only used locally have been omitted.

\[A_f\] cross-sectional flow area \[m^2\]
\[a\] parameter of the sediment transport formula \[m\]
\[B\] width of a river (equilibrium model) \[m\]
\[B_f\] total width of both floodplains \[m\]
\[B_m\] width of the main channel \[m\]
\[b\] parameter of the sediment transport formula \[-\]
\[C\] Chézy roughness coefficient \[\sqrt{m/s}\]
\[C_f\] Chézy roughness coefficient of the floodplains \[\sqrt{m/s}\]
\[C_m\] Chézy roughness coefficient of the main channel \[\sqrt{m/s}\]
\[c\] correction factor (multiplier) of sediment transport formula \[-\]
\[c_b\] propagation velocity of bed waves \[m/s\]
\[c_e\] effective propagation velocity of bed waves \[m/s\]
\[D\] grain size of the bed material \[m\]
\[g\] gravity acceleration \[m/s^2\]
\[h\] water level \[m\]
\[h_b\] height of the bank of the main channel \[m\]
\[i\] bed slope \[m/m\]
\[K\] diffusion coefficient \[m^2/s\]
\[K_e\] effective diffusion coefficient \[m^2/s\]
\[k_s\] Strickler roughness coefficient \[m\]
\[k_{s,f}\] Strickler roughness coefficient of the floodplains \[m\]
\[k_{s,m}\] Strickler roughness coefficient of the main channel \[m\]
\[L\] length of the modelled river work in flow direction \[m\]
\[P\] Péclet number \[-\]
\[Q\] river discharge \[m^3/s\]
\[Q_H\] flood height \[m^3/s\]
\[Q_D\] flood duration \[m^3/s\]
$$Q_T$$  time of occurrence of the middle of the flood  [m$^3$/s]  
$$Q_L$$  magnitude of the low discharge  [m$^3$/s]  
$$S$$  total sediment transport in a cross-section of the river  [m$^3$/s]  
$$s$$  sediment transport per unit width, including the pore volume  [m$^2$/s]  
$$t$$  time parameter  [s]  
$$\Delta t$$  numerical time step  [s]  
$$u$$  average flow velocity in the main channel  [m/s]  
$$x$$  space parameter  [m]  
$$\Delta x$$  numerical grid distance  [m]  
$$z_b$$  bed level  [m]  
$$\Delta_d$$  relative density of the bed material  [-]  
$$\varepsilon$$  packing factor of the bed material  [-]  
$$\mu$$  statistical mean  
$$\mu_y$$  statistical mean of random variable $y$  
$$\rho$$  statistical linear correlation  
$$\rho_s$$  density of the sediment  [kg/m$^3$]  
$$\rho_w$$  density of water  [kg/m$^3$]  
$$\sigma$$  standard deviation  
$$\sigma_y$$  standard deviation of random variable $y$  

1-D  one-dimensional  
2-DH  two-dimensional depth averaged  
CV  coefficient of variation  
FORM  First Order Reliability Method  
MCS  Monte Carlo Simulation with crude sampling  
LHS  Monte Carlo Simulation with Latin Hypercube Sampling
Acknowledgements

Writing these pages, I am at the end of a period of more than four years of research, which I, most of the time, greatly enjoyed. One of the best parts of a Ph.D. research, however, is that it is finished at a certain moment. I happily use these pages to thank a number of people and institutes who contributed to me finishing my dissertation.

The study presented here was carried out as a Ph.D. study at the Civil Engineering department of the Delft University of Technology (TUD). The project was supported by WL | Delft Hydraulics (WL), through guidance and a working place. The first two years of my research have been funded by LWI. The last two years have been supported by the Delft Cluster (project 03.03.02).

I want to thank my supervisors and other people who gave me advise. Huib de Vriend, I experienced your enthusiasm about the subject and the freedom you gave me as very pleasant. Erik Mosselman, thank you for your involvement during the full period of my Ph.D. period and for your pleasant way of giving constructive criticism. Matthijs Kok, I am glad that you got involved in my research. I enjoyed the hours of talking and discussing very much.

I thank Ron Agtersloot and Gerrit Klaassen for their contribution during the first part of my Ph.D. period. Ron, thanks for your thinking along with the setting up of my research. Gerrit, I owe a lot of my basic knowledge of river morphology to you. During the writing process of my dissertation the comments of Marjolein van Asselt and Pieter van Gelder were valuable. Marjolein, your enthusiasm as a researcher and your energy to sharpen ideas more and more are contagious. Pieter, thanks for your fresh view on my writings.

There are a couple of students I would like to thank for there contributions to my research. Sebastian Rath, your thorough and hard work led to an excellent report and a considerable contribution to this dissertation. Maarten-Jan Kallen and Daniel Lewandowski, your mathematical approach sharpened my ideas and understanding of the matter.
Colleagues and former colleagues at WL and at the section Hydraulic Engineering of the TUD, thanks for your support and the pleasant atmosphere. You determined for a great part the pleasure in my work, whether it was as a roommate (Simone, Chris, Lucie, and in the final stage Ron and Ferdinand), in a Japanese bathhouse (Saskia and Anneke :)), or by pep talks at the corridor (Astrid, Bert).

The fact that a Ph.D. study does not only keep you busy during working hours is one thing I no longer need to explain to my family and friends. Thanks for your support and patience, and for the relaxation during nice weekends (Klissen, Enschede friends!), girls talk (Juuls, Anne-Marie!) and weekly squash games (Jeanette, Sander!).

Special thanks to Simone van Schijndel and Saskia van Vuren. I am very glad that you are my paranimfs, as friends and colleagues. Simone, after six years of being colleagues, of which a large part as roommates, you know more than anybody else how my research passed off. Thank you for your flexible response to my changing moods. Saskia, you have been my one and only colleague in the struggle with uncertainties in the last years. The possibility to tackle a problem together appeared to be much more fun. Thanks girls!

And than Serge... Traditionally, the most important word of thanks comes at the end. You have been a great support to me on all fronts. You handled my most intense ups and downs, as well as the very first versions of my writings. Thanks a lot! I look forward to the time ahead of us, in which, for the first time, neither of us have a study or research in hand!

Hanneke van der Klis

Gouda, September 2003
About the author

Hanneke van der Klis was born in Zaandam, The Netherlands, on 12 January 1972. From 1984 to 1990 she attended the Farel college in Amersfoort and received her VWO diploma in 1990.

She continued her education with the studies of Applied Mathematics at the University of Twente in Enschede. As a traineeship, she worked at the research department of electricity producer National Power in Swindon, U.K., where she worked on a computer model of the chemistry within a dispersing plume. Her masters thesis dealt with a numerical model describing a linear shear flow of a periodic suspension of viscous drops. She received her masters degree in 1997.

From 1997 to 1998, Hanneke worked as an engineer at the river engineering department of WL | Delft Hydraulics. In 1998 she started her Ph.D. research at the department of Civil Engineering at the Delft University of Technology. In 2002 she returned to WL at the Hydrology department as a researcher/advisor on uncertainty in hydraulic and hydrological models.
The morphology of the river bed is an important process to understand. The knowledge is used for purposes like the planning of navigation channel maintenance or safety measurements against flooding. Modelling river bed morphology involves numerous uncertainties. Examples are the uncertain future river discharge and the roughness of the river bed.

Hanneke van der Klis describes in her dissertation how to deal with uncertainties in numerical models of river bed morphology. Based on idealised case studies, she examines which uncertainties are important and how to quantify their effect on the model results.

By combining the fields of river morphology and uncertainty analysis Hanneke aims to improve the modelling of the river bed.