The nonlinear effect of combining uncertainties on the energy yield of an offshore wind farm

A case study for array efficiency and availability

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by

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Delft, 16 November 2015

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Thank you,

Lotte Engelen
Summary

Offshore wind energy is expected to grow in the coming years: future plans for offshore wind farms total more than 98 GW. Financing is needed to realise these plans. Investment decisions partly depend on the uncertainty in energy yield predictions. It is therefore important that these energy yield predictions and their corresponding uncertainty are determined as accurately as possible. Current methods for determining annual energy production assume that there is a linear relation between input uncertainties and output uncertainty, allowing the use of simple methods for determining annual energy production and its uncertainty. It is however known that this assumption is incorrect: nonlinear relations do exist. This means that it is unclear whether the use of these simple methods can be justified.

This thesis has developed a methodology that can be used to determine if, and how, the nonlinear effect of combining two uncertainty sources should be incorporated in the energy yield prediction. This has been done by investigating the case study of the nonlinear effect of combining array efficiency and availability. The investigation was split up in five main steps. First, the physical relations between array efficiency and availability have been explored, revealing that downtime of a turbine affects the array efficiency of a wind farm. Figuring out the state-of-the-art methods pointed out that this interdependency is currently not taken into account. By adjusting the current models, an adapted model was developed that is able to consider this effect. Simulations have been performed on both the current and the adapted model. The results show that for a typical mean availability value of 96.2%, the differences between the current and the adapted model are smaller than 1%. This means that the current methods used by industry can be justified. However, if availability values drop, the difference between both models becomes significant. Due to the high development and computational time of the adapted model, an approximation of the mean annual energy production was developed that uses existing tools. This approximation yields accurate results: the difference between the approximation and the results of the adapted model is lower than 0.5%.

Since the approach that was used in this research has proven to be successful, it can be translated to a generic methodology. This methodology can be followed to determine the nonlinear effect of two other uncertainty sources.
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<th>Description</th>
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<tr>
<td>AEP</td>
<td>Annual Energy Production</td>
</tr>
<tr>
<td>$AEP_{gross}$</td>
<td>Gross Annual Energy Production</td>
</tr>
<tr>
<td>CBM</td>
<td>Condition Based Maintenance</td>
</tr>
<tr>
<td>EUROS</td>
<td>Excellence in Uncertainty Reduction of Offshore-wind Systems</td>
</tr>
<tr>
<td>FBM</td>
<td>Failure Based Maintenance</td>
</tr>
<tr>
<td>MTBF</td>
<td>Mean Time Between Failure</td>
</tr>
<tr>
<td>MTTR</td>
<td>Mean Time To Repair</td>
</tr>
<tr>
<td>P50</td>
<td>Value indicating the AEP for which there is a 50% probability that this value is exceeded</td>
</tr>
<tr>
<td>P90</td>
<td>Value indicating the AEP for which there is a 90% probability that this value is exceeded</td>
</tr>
<tr>
<td>RSS</td>
<td>Root-Sum-Square</td>
</tr>
<tr>
<td>SR</td>
<td>Sensitivity Ratio</td>
</tr>
<tr>
<td>TBM</td>
<td>Time Based Maintenance</td>
</tr>
<tr>
<td>WMEP</td>
<td>Scientific Measurement and Evaluation Programme (Wissenschaftlichen Mess- und Evaluierungsprogramm)</td>
</tr>
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## List of Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>Scale parameter of Weibull distribution</td>
</tr>
<tr>
<td>$A_{\text{lens}}$</td>
<td>Area of asymmetric lens where the wake of one turbine partially overlaps a downwind rotor</td>
</tr>
<tr>
<td>$A_{\text{rotor}}$</td>
<td>Area of rotor</td>
</tr>
<tr>
<td>$c$</td>
<td>Autocovariance</td>
</tr>
<tr>
<td>$c_0$</td>
<td>Variance of a wind speed series</td>
</tr>
<tr>
<td>$C_T$</td>
<td>Thrust coefficient</td>
</tr>
<tr>
<td>$D$</td>
<td>Rotor diameter</td>
</tr>
<tr>
<td>$d_{2h}$</td>
<td>Distance between two turbines</td>
</tr>
<tr>
<td>$D_W$</td>
<td>Diameter of the wake</td>
</tr>
<tr>
<td>$E_{\text{array}}$</td>
<td>Annual energy produced by a wind farm</td>
</tr>
<tr>
<td>$E_T$</td>
<td>Annual energy produced by a single, isolated turbine</td>
</tr>
<tr>
<td>$f_i$</td>
<td>Frequency of occurrence of the wind speed bin $i$</td>
</tr>
<tr>
<td>$g$</td>
<td>Gravitational constant</td>
</tr>
<tr>
<td>$h$</td>
<td>Height</td>
</tr>
<tr>
<td>$H_s$</td>
<td>Significant wave height</td>
</tr>
<tr>
<td>$j$</td>
<td>Lag</td>
</tr>
<tr>
<td>$k$</td>
<td>Wake decay constant</td>
</tr>
<tr>
<td>$k$</td>
<td>Shape parameter of Weibull distribution</td>
</tr>
<tr>
<td>$L_i$</td>
<td>Production losses of loss category $i$</td>
</tr>
<tr>
<td>$M$</td>
<td>Markov transition matrix</td>
</tr>
<tr>
<td>$m_{ij}$</td>
<td>Element $i,j$ of Markov transition matrix</td>
</tr>
<tr>
<td>$n$</td>
<td>Number of observations</td>
</tr>
<tr>
<td>$N$</td>
<td>Number of samples</td>
</tr>
<tr>
<td>$N_{\text{bins}}$</td>
<td>Number of wind speed bins</td>
</tr>
<tr>
<td>$N_{\text{turbines}}$</td>
<td>Number of turbines in a wind farm</td>
</tr>
<tr>
<td>$P$</td>
<td>Probability matrix</td>
</tr>
<tr>
<td>$P_{\text{actual}}$</td>
<td>Actual power output of a wind farm</td>
</tr>
<tr>
<td>$P_i$</td>
<td>Power produced by a wind speed bin $i$</td>
</tr>
<tr>
<td>$P_{ij}$</td>
<td>Element $i,j$ of probability matrix</td>
</tr>
<tr>
<td>$P_{\text{ref}}$</td>
<td>Reference power output</td>
</tr>
<tr>
<td>$r$</td>
<td>Autocorrelation coefficient</td>
</tr>
<tr>
<td>$R$</td>
<td>Radius of circle</td>
</tr>
<tr>
<td>$s$</td>
<td>Number of states</td>
</tr>
</tbody>
</table>
t


Hour

X

Position downwind of a turbine that creates a wake

\( x_i \)

Horizontal coordinate of turbine \( i \)

\( U \)

Wind speed

\( \bar{u} \)

Mean wind speed of wind speed series

\( U_0 \)

Undisturbed wind speed

\( U_i \)

Wind speed in wake \( i \)

\( U_m \)

Wind speed in a mixed wake

\( U_{\text{max}} \)

Upper boundary of wind speed state

\( U_{\text{min}} \)

Lower boundary of wind speed state

\( U_{\text{pot}} \)

Potential wind speed

\( U_R \)

Local wind speed just behind the rotor

\( u_t \)

Wind speed at hour \( t \)

\( U_W \)

Wind speed in the wake

\( y_i \)

Vertical coordinate of turbine \( i \)

\( z_0 \)

Roughness length

**Greek symbols**

\( \alpha \)

Angle between two turbines

\( \gamma \)

Intermediate angle used to calculate \( \delta \alpha_{\text{partial}} \)

\( \delta \alpha_{\text{full}} \)

Full wake angle

\( \delta \alpha_{\text{partial}} \)

Partial wake angle

\( \epsilon \)

Uniform random number between 0 and 1

\( \eta_{\text{array}} \)

Array efficiency of a wind farm

\( \eta_{\text{availability}} \)

Availability of a wind farm

\( \eta_{\text{combined}} \)

Combined efficiency of a wind farm when considering only array efficiency and availability

\( \eta_i \)

Efficiency of production loss category \( i \)

\( \kappa \)

Inverse tangent of the wake decay constant \( k \)

\( \lambda \)

Failure rate

\( \lambda_i \)

Failure rate of failure mode \( i \)

\( \lambda_s \)

Failure rate of entire system

\( \mu \)

Mean of normal distribution

\( \sigma \)

Standard deviation of normal distribution

\( \sigma_{\text{AEP}} \)

Uncertainty in the annual energy production

\( \sigma_{\text{array efficiency}} \)

Uncertainty in the array efficiency

\( \sigma_{\text{availability}} \)

Uncertainty in the availability

\( \sigma_{\text{other}} \)

Other uncertainties

\( \sigma_{\text{turbine performance}} \)

Uncertainty in the turbine performance

\( \sigma_{\text{wind speed}} \)

Uncertainty in the wind speed

\( \sigma^*_{\text{wind speed}} \)

Uncertainty in the energy yield due to the wind speed
Chapter 1

Introduction

In this thesis a methodology will be developed to determine the nonlinear effect of combining uncertainties on the energy yield of offshore wind farms. This introduction will briefly outline the importance of uncertainty quantification for the progress of offshore wind energy, the motivation for this specific research topic, the objectives of the project and the structure of the report.

1.1 The implications of uncertainty on the development of offshore wind energy

Offshore wind energy has known a strong increase over the past decade. Figure 1.1 shows the increase in annual and cumulative installed capacity in Europe from 1993 to 2014. This capacity is expected to grow in the future; the European Wind Energy Association has identified future plans for offshore wind farms totalling more than 98 GW [16].

Figure 1.1: Annual and cumulative installed offshore wind energy capacity from 1993 to 2014 [16].
To be able to realise these plans, financing is needed. According to the European Wind Energy Association, the industry needs to attract between €90 billion and €123 billion by 2020. Financing decisions by potential investors and banks are usually based on the return on investment; a measure depending on the costs of the wind farm and its predicted energy yield. These energy yield predictions are uncertain, since they are based on uncertain input: the wind, turbine performance and losses can never be known exactly. The higher the uncertainty in the energy yield prediction, the greater the risk that an investor or bank takes. This results in unfavourable financial conditions for the wind farm project developer [34], or might even lead to the decision to not build the wind farm at all. Accurate energy yield predictions, with uncertainties that are quantified to the best possible extent, are therefore important.

1.2 Motivation for the research topic

Section 1.1 described the importance of accurate uncertainty quantification for the wind energy industry. Industry currently incorporates a form of uncertainty quantification in the energy yield prediction. A typical energy yield assessment determines the gross energy yield by using a wind speed frequency distribution in combination with the power curve of a wind turbine. Technical loss factors are then subtracted from this gross yield to come up with the expected net energy generation [13]. In each of these steps uncertainty occurs. These various uncertainty sources are determined separately and combined using the root-sum-square approach, implying that the assumption is made that the uncertainty sources are independent and that the relation between the input uncertainties and the output uncertainty is linear [18]. It is however known that correlations between several uncertainty sources exist and that the relation between some of the input uncertainties and the output uncertainty is nonlinear [15]. This means that it is unclear yet how valid this assumption is. There is therefore a need for scientific knowledge on how a combination of uncertainties can best be dealt with. This is one of the reasons for the setup of the EUROS (Excellence in Uncertainty Reduction of Offshore-wind Systems) programme. The research presented in this report connects seamlessly to the research goal of the EUROS programme. It aims to develop a methodology to determine the nonlinear effect of combining uncertainties on the energy yield of offshore wind farms. This methodology will be developed by means of a case study. In this case study, the nonlinear effect of combining array efficiency and availability will be investigated.

1.3 Objectives

Section 1.2 has shown that the nonlinear effect of combining uncertainties has up to now not been taken into account in the energy yield predictions of wind farms. The EUROS programme indicates that better knowledge of the effect of the combination of uncertainties is needed. This leads to the main research question of this project:

How can the nonlinear effect of combining uncertainties on the energy yield of an offshore wind farm be determined?

This project aims to answer this question by looking at the specific case study of combining uncertainties in array efficiency and availability on the energy yield of offshore wind farms. The main question can be divided into the following subquestions:

1. What are the physical relations between array efficiency and availability?
   *Mapping the physics of the process helps to identify how the combination of uncertainties should be modelled.*

2. Should the nonlinear effect of combining uncertainties on the energy yield of an offshore wind farm be incorporated in the current energy yield predictions?
   *Up to now, the nonlinear effect of uncertainties has not been taken into account in industry practice. Answering this subquestion can prove if this assumption can be justified or not.*
   
   (a) How is the mean of the annual energy production influenced by treating uncertainty combinations?
(b) How is the standard deviation of the annual energy production influenced by treating uncertainty combinations?

3. If the nonlinear effect of combining uncertainties is not negligible, how should it be modelled?
   (a) Can the nonlinear effect of combining uncertainties be approached using an approximation?
   (b) How do the results of an approximation and a full sampling technique compare?

4. How can the case study of combining uncertainties in array efficiency and availability of an offshore wind farm be translated to the general case of combining uncertainties?

These questions lead to the main objective of the MSc thesis: to develop a methodology that can be used to determine if, and how, the nonlinear effect of combining two uncertainty sources should be incorporated in the energy yield prediction, by investigating the case study of the nonlinear effect between array efficiency and availability. Again, this objective can be divided into subgoals:

- Construct a mutually exclusive, collectively exhaustive overview of the physical relations between array efficiency and availability that affect the energy yield of an offshore wind farm.
- Develop a model that determines the nonlinear effect of combining uncertainties in array efficiency and availability of an offshore wind farm.
- Translate this model and its outcomes to the general case of combining uncertainties.
- Determine whether this should be incorporated in the energy yield predictions.

1.4 Structure of the report

The structure of this report is as follows. Chapter 2 describes the physical relations that exist between wake effects and availability. Chapter 3 discusses the methods that are currently used to predict the array efficiency and the availability, and how these two are combined to determine annual energy production and the corresponding uncertainty. This chapter is concluded by identifying mismatches between the physical relations and the current procedure. These shortcomings are addressed in Chapter 4, in which an adapted method is developed. Chapter 5 describes the verification of the model. Its results and the comparison with the results of the current procedure are given in Chapter 6. Chapter 7 proposes an approximation that is able to indicate whether such an adapted procedure is needed. The main objective of this research – the development of a methodology that can be used to determine the correlated effect of combining two uncertainty sources – is documented in Chapter 8.
Overview of the physical relations between array efficiency and availability

Section 1.2 described that the current procedure to determine the uncertainty in the annual energy production neglects possible nonlinear effects between the various uncertainty sources. It is important that these effects are identified in detail, to be able to get an idea what the consequences of this assumption are. Therefore, this chapter will examine the physics of both the array efficiency and availability, in order to create an overview of the physical relations between the two. The results of this chapter are used in Chapter 4 to adapt the current procedure for determining the annual energy production. Section 2.1 maps the physics concerned with the wake effects of an offshore wind farm. The same will be done for availability in Section 2.2. Relations between both effects are investigated in Section 2.3.

2.1 Physics of wake effects

Before the physical relations between both effects are identified, it is important to have an overview of the physics of each effect separately. This section describes the physics involved in the wake effects. To map the physics accurately, it is important not to be guided by equations since these can contain assumptions regarding the physics. Therefore, the choice was made to look at how wake effects arise. The wind plays an important role herein. More specifically, the nature and the size of the wake effects are caused by a particular wind speed and wind direction. The position of the turbines — that is, the wind farm layout — is crucial. A wind farm in which all turbines are placed in one line has a different array efficiency than a wind farm with a rectangular layout. The turbine itself matters as well. Its performance determines the wind speed deficit behind each turbine. One aspect of the turbine performance that is worth mentioning explicitly is the power curve, since it determines the energy production of the farm eventually. Last but not least, the properties of the wake affect the wake effect, especially its expansion. In short, six inputs can be defined: wind speed, wind direction, wind farm layout, turbine performance, power curve and wake expansion. There are however still some intermediate “steps”. Not all turbines in a wind farm experience the undisturbed wind speed. Since some of them are potentially in the wake of an upwind turbine, they experience a local wind speed. This local wind speed, combined with the turbine performance, defines the thrust coefficient of the turbine — the metric that eventually ensures a certain wind speed deficit. The thrust coefficient, local wind speed, wind direction, farm layout and the wake expansion eventually add up to the wake effects of a farm.
The efficiency factor in which a project developer is interested – the array efficiency\(^1\) – can be calculated using the wake effects and the power curve of the turbine. The process explained above is visualized schematically in Figure 2.1.

### 2.2 Physics of availability

Section 2.1 described the physics in the wake effects of an offshore wind farm. This section looks at the causes of unavailability and maps the physics involved. Availability of a wind turbine is defined as the amount of time that a turbine is available to operate when scheduled service and repair time is taken into account [7]. Repair time is needed when a turbine experiences some kind of failure. Occurrences of failures are therefore an important physical factor. Combined, they determine the total downtime of a farm due to turbine failures. Extra downtime of a farm results due to the need for scheduled service. A turbine can only be repaired or serviced when the weather allows it. These so-called weather windows are dependent on wave heights and wind speeds. Eventually the theoretical downtime due to failures and service and weather windows determine the actual downtime, and thus the availability, of a farm. This process is once again visualized schematically. It can be found in Figure 2.2.

### 2.3 Physical relations between both effects

Section 2.1 and Section 2.2 mapped the physics involved in the wake effects and availability, respectively. Truly of interest are however the physical relations between both effects. This section tries to find these

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\(^1\)The array efficiency is defined as $\eta_{\text{array}} = \frac{E_{\text{array}}}{E_T \cdot N_{\text{turbines}}}$. In this case, $E_{\text{array}}$ is the annual energy produced by the farm. $E_T$ is the annual energy produced by a single, isolated turbine. $N_{\text{turbines}}$ is the number of turbines in the farm. [24]
Chapter 2: Overview of the physical relations between array efficiency and availability

relations, based on the knowledge of the previous two sections.
The most apparent similarity between Figures 2.1 and 2.2 is that the undisturbed wind speed exists in both diagrams. Nevertheless, the nonlinear effect of the combination appears somewhere else in the process: downtime of a turbine directly affects the wake effects of a wind farm. When a turbine is shut down, it does not generate a wake, meaning that downwind turbines do not experience a wind speed deficit.
This leaves two relations that are a little less incontrovertible. First of all, a high wind speed might lead to higher failure rates due to higher forces on the turbine. The same holds for wake effects: if their size changes, this might also affect the forces on the turbine and thus also the failure rates. These two relations are however considered as second-order effects, since they are both dependent on the forces on the turbines.
The four relations between both effects are added to the diagrams in Figures 2.1 and 2.2 for the sake of clarity. The combined diagram can be found in Figure 2.3.

Figure 2.3: Overview of the physics involved in the wake effects and availability of an offshore wind farm.
The four physical relations are indicated in red.
Current procedure to predict the combined effect of array efficiency and availability

A brief description of the current method for predicting the annual energy production was given in Section 1.2. This chapter discusses this procedure in more detail, with a focus on the determination of the wake effects and the availability. Section 3.1 describes the current steps taken to predict the array efficiency of an offshore wind farm. The same is done for availability in Section 3.2. The combination of both loss factors to come up with a combined efficiency is presented in Section 3.3. The chapter is concluded by comparing the current procedure to the physical relations derived in Chapter 2 and identifying the shortcomings of the current procedure.

3.1 Predicting the array efficiency of an offshore wind farm

Power losses that occur due to the situation that a wind turbine is in the wake of another wind turbine are the most substantial losses in an offshore wind farm. The method used for predicting these wake losses – or in other words, for forecasting the array efficiency – is explained in this section. Section 3.1.1 describes the theory of the wake model developed by Katic et al. Its application to determine the array efficiency of offshore wind farms and corresponding difficulties is addressed in Section 3.1.2.

3.1.1 Theory of the wake model developed by Katić et al.

One of the most popular wake models to determine wind farm power outputs was originally developed by N.O. Jensen [23] and extended by I. Katić, J. Højstrup and N.O. Jensen [25]. From now on, this model will be called the *Jensen wake model*. It is a semi-empirical model to describe the wake of a wind turbine, and was originally developed to enable industry to decide on the ideal wind turbine configuration at a given site. Because it is one of the most used wake models by industry and relatively simple to implement in a numerical model it is used in this research. This section will briefly discuss the theory behind the model. The Jensen wake model is based on the description of a single wake. This wake is assumed to expand linearly with respect to the downwind distance and its start diameter is equal to the rotor diameter. A schematic view of this description can be found in Figure 3.1.
In this figure can be seen that this wake profile has a “top hat” distribution. The gradient of the width of the top hat corresponds to the wake decay constant $k$, which defines the linear expansion. At each downwind position $X$ the wind speed in the wake $U_W$ is assumed to be constant. This yields a highly idealized velocity profile and leads to high errors in the near-wake region\(^1\), but yields satisfactory results from four diameters \([25]\) onwards. This is the significant region for project developers, since turbines in offshore farms are seldomly placed at a distance smaller than this \([31]\).

Katić et al. use conservation of mass to calculate $U_W$ at a particular downwind distance $X$. This yields Equation 3.1.

$$1 - \frac{U_W}{U_0} = \frac{1 - \sqrt{1 - \frac{C_T}{(1 + \frac{2kX}{D})^2}}}{(1 + \frac{2kX}{D})^2}$$

Using this equation yields a velocity profile for a specific turbine under specific wind conditions. In Figure 3.2 this velocity profile is shown for an undisturbed wind speed $U_0$ of 9 m/s.

\(^1\)Several definitions for the near-wake region exist. The one considered in this research is the following: the near-wake region holds up to a certain downstream distance, usually between two and five turbine diameters \([12]\). The region behind the near-wake region is called the far-wake region.
3.1.2 Applying the wake model to an offshore wind farm

Section 3.1.1 described the theory of a single wake according to Katić, Højstrup and Jensen. To determine the array efficiency of an offshore wind farm, this theory needs to be applied to an offshore wind farm. Expanding the single wake to handle an entire farm poses a few challenges. Situations to consider include for example how to treat partial wakes, multiple wakes and mixing wakes. The treatment of these situations depends on the choices made during the application of the theory in the model: several possibilities for each of the situations exist. Since this chapter aims to give a more general overview of the current procedures, the specific choices made in this research are discussed in Section 4.2.1.

3.2 Predicting the availability of an offshore wind farm

The previous section elaborated on the model used for predicting the array efficiency of an offshore wind farm. This section will do the same for availability. Before the actual procedure for predicting availability is explained, definitions of frequently used terms are given. This is done in Section 3.2.1. Section 3.2.2 describes the state-of-the-art strategy to predict the availability of an offshore farm.

3.2.1 Definition of frequently used terms in predicting availability

Terms like availability, reliability and accessibility are regularly mistaken for each other. Therefore, this section will briefly give a definition of the terms that are frequently used throughout this research.

- **Availability**
  The definition of the term “availability” was mentioned in Section 2.2: it corresponds to the amount of time that a turbine is available to operate when maintenance and repair time is taken into account [7].

- **Reliability**
  Another term that is often used – and mistaken for availability – is reliability. Reliability of a wind turbine is the probability that the turbine will perform its tasks [11]. Reliability can also be expressed as the number of failures per time frame. It thus has a strong relation with the term availability. Having a high reliability not necessarily implies that a system has a high availability: when a system rarely fails but no maintenance/repair action is taken, the reliability is high but the availability is low.

- **Accessibility**
  An important part of the availability prediction process is the accessibility. This term can be defined as the percentage of time that the offshore wind turbine can be approached and maintained. This depends upon the wind speed and wave height – in a storm, no maintenance or repair activities will take place.

- **Maintainability**
  Maintainability is a qualitative term that expresses the ease with which a certain system is repaired. This term can be made quantitative by expressing it as repair time.

- **Serviceability**
  Serviceability is a qualitative term as well, used to express the ease of regular (scheduled) service.

3.2.2 State-of-the-art strategy for predicting availability

Section 3.2.1 gave a short explanation of commonly used terms in the procedure of predicting availability. This section shows the connection between all terms and gives an overview of the state-of-the-art strategy for predicting availability. This strategy is schematically visualized in Figure 3.3. The theoretical availability of a wind farm is a function of the reliability, maintainability and serviceability [9]. The reliability can be determined using failure statistics from historical data [10]. The same holds for the maintainability (time needed to repair the failure – also called corrective maintenance) and the serviceability (time needed for preventive maintenance). These three data sources determine the theoretical amount of
time that a turbine is available to operate. However, exploiting a wind farm offshore has a major impact on the accessibility. Due to high waves or high winds it may well be that a repair or maintenance crew cannot access the site for a few days, weeks or even months. These circumstances might decrease the actual availability dramatically with respect to the theoretical availability. The last factor that determines the actual availability of an offshore wind farm is the maintenance strategy. There are several types of maintenance strategies, including ‘Time Based Maintenance’ (TBM), ‘Failure Based Maintenance’ (FBM) and ‘Condition Based Maintenance’ (CBM) [2]. As the term implies, TBM means maintenance is performed on regular, predetermined time intervals. FBM means that maintenance is performed once a component has failed. CBM means that maintenance is carried out after a particular deterioration of a component, preventing failures. Most maintenance policies include all three strategies [40].

To put it shortly, data is used to estimate the reliability, maintainability, serviceability and accessibility. Combined with the maintenance strategy foreseen the actual availability can be predicted. A sophisticated way to calculate this is by using a Monte Carlo approach. Wind and wave series, failures, crew deployment and availability of equipment are randomly simulated to determine realistic maintenance actions [8]. An example of such an approach is the CONTOFAX code, developed by Delft University of Technology [10].

![Figure 3.3: Visualization of the relations needed to determine the actual availability of an offshore wind farm. Adopted from [11].](image)

### 3.3 Combining array efficiency and availability and their effect on energy yield predictions and wind farm financing

The previous sections described how the array efficiency and availability of an offshore wind farm are predicted. To be able to answer subquestion 2, as stated in Section 1.3, it is however important to know how both factors are combined. This section describes the procedure that is followed to combine uncertainties (Section 3.3.1) and how the results of this procedure affect wind farm financing (Section 3.3.2).

#### 3.3.1 Current method for combining array efficiency, availability and other factors

Sections 3.1 and 3.2 explained the methods that are used to determine the array efficiency and availability of an offshore wind farm. The quantity of interest however is the combination of both factors and their effect
on the annual energy production (AEP). Since the array efficiency and availability are both uncertain, this section is split up in two paragraphs. The first paragraph discusses the role of array efficiency and availability in the determination of the mean AEP. The second paragraph describes how uncertainties are combined to calculate the standard deviation of the AEP.

Combining array efficiency, availability and other factors to determine mean AEP

Section 1.2 already explained that a typical energy yield assessment consists of a determination of the gross energy yield, from which loss factors are subtracted. A more detailed description of this assessment is needed to be able to explain the combination of the efficiency factors. First of all, the term “gross annual energy yield” ($AEP_{\text{gross}}$) needs to be defined more accurately. The inputs to determine this gross annual energy yield are a wind speed frequency distribution and the power curve of a wind turbine. A wind speed frequency distribution is a histogram representing wind speeds categorized in bins of a specific width, and the frequency of hours per year that are expected for each category. Multiplying each wind speed bin with the corresponding power production for that wind speed (derived from the power curve of a specific turbine) and summing all the bins yields the gross energy produced by one turbine during one year. This is shown in Equation 3.2.

$$AEP_{\text{gross, turbine}} = \sum_{i} f_i P_i$$  \hspace{1cm} (3.2)

In this equation, $f_i$ is the frequency of occurrence of the wind speed bin $i$ and $P_i$ is the corresponding power production.

The gross annual energy yield of the entire farm is then calculated by multiplying $AEP_{\text{gross, turbine}}$ with the number of turbines in the farm, $N_{\text{turbines}}$. This is shown in Equation 3.3.

$$AEP_{\text{gross}} = N_{\text{turbines}} \cdot \sum_{i} f_i P_i$$  \hspace{1cm} (3.3)

It was already mentioned that the net annual energy production is determined by subtracting losses from $AEP_{\text{gross}}$. Six categories of production losses can be observed: wake losses, availability losses, turbine performance losses, electrical losses, environmental losses and losses due to curtailment [13]. These losses can happen simultaneously, meaning that adding them up in an absolute sense would double count the overlaps [45]. Therefore, the losses are converted to efficiencies ($\eta_i = 1 - L_i$, where $i$ represents the category and $L_i$ is expressed as a percentage) and multiplied, yielding an overall efficiency of the farm. This is shown in Equation 3.4.

$$\eta_{\text{overall}} = \prod_{i} \eta_i$$  \hspace{1cm} (3.4)

If the overall efficiency is found, the gross annual energy production can be multiplied with this efficiency, yielding the net annual energy production. This is shown in Equation 3.5.

$$AEP = \eta_{\text{overall}} \cdot AEP_{\text{gross}}$$  \hspace{1cm} (3.5)

Since this research only considers array efficiency, $\eta_{\text{array}}$, and availability, $\eta_{\text{availability}}$ (once again expressed as a percentage), the equations given above are given once more for the sake of clarity, using $\eta_{\text{array}}$ and $\eta_{\text{availability}}$ explicitly. This is shown in Equation 3.6.

$$\eta_{\text{overall}} = \eta_{\text{array}} \cdot \eta_{\text{availability}} \cdot \eta_{\text{other}}$$  \hspace{1cm} (3.6)

$AEP = \eta_{\text{array}} \cdot \eta_{\text{availability}} \cdot \eta_{\text{other}} \cdot AEP_{\text{gross}}$

Equation 3.6 presents a linear relation between the net annual energy production and the efficiency factors and gross annual energy production. A derivation to show that this relation is linear is given in Appendix A. In current industry methods, the net annual energy production resulting from the equations above forms the mean annual energy production, $AEP_{\text{mean}}$. The determination of the standard deviation is treated in the next paragraph.
Combining array efficiency, availability and other factors to determine standard deviation of AEP

The previous paragraph described how the gross AEP, array efficiency and availability of an offshore wind farm are combined to determine a net value for the AEP. It was however already mentioned that array efficiency and availability are not deterministic: they are uncertain values. The gross AEP is uncertain as well, due to uncertainty in the wind. This means that the AEP calculated by Equation 3.6 is also uncertain. This paragraph describes how the uncertainty in this AEP is calculated by combining input uncertainties. Currently, industry assumes that all (uncertain) inputs needed to determine the AEP are normally distributed [27] [45]. The net AEP calculated by Equation 3.6 forms the mean of this normal distribution. The uncertainty is defined to be the standard deviation of the normal distribution [20], and is usually expressed as a percentage [14]. Inputs that are considered to affect the uncertainty in the AEP the most are uncertainties related to the wind speed (for example wind speed measurement uncertainty and future wind variability [13]). Lackner provides a detailed description on how to determine the total uncertainty related to the wind speed, from now on called $\sigma_{\text{wind speed}}$ [27]. It is however important to note that this uncertainty is the standard deviation of the wind speed: in order to see which effect it has on the AEP, it needs to be converted [32]. This is done by means of the sensitivity ratio. The sensitivity ratio is the ratio of the relative increase in energy yield based on a relative increase in wind speed. A detailed explanation of the sensitivity ratio can be found in the literature study prior to this research [15]. For clarity, the expression for the uncertainty of the wind in terms of energy, $\sigma_{\text{wind speed}}^*$ is given by Equation 3.7, in which $SR$ represents the sensitivity ratio.

$$\sigma_{\text{wind speed}}^* = SR \cdot \sigma_{\text{wind speed}}$$

(3.7)

Other uncertainties are related to plant performance and losses [13]: for example $\sigma_{\text{array efficiency}}$, $\sigma_{\text{availability}}$ and $\sigma_{\text{turbine performance}}$. Many of these uncertainties are estimated by the wind farm developers and therefore highly subjective. For example, a typical value of the uncertainty in wake modelling is 3% [45]. This subjectiveness is also apparent from the ‘Loss & Uncertainty’ module of the wind farm developer tool WindPRO, where the loss uncertainties can be set manually by the user [14]. Uncertainties related to plant performance and losses are directly related to energy and thus do not have to be converted, as opposed to $\sigma_{\text{wind speed}}$.

When all individual uncertainty components are determined (or estimated), they have to be combined to determine the uncertainty in the AEP. The method for this depends on the calculation of the net AEP, shown in Equation 3.5, and on the assumption that the uncertainty components are independent [22]. Taylor states that if the output is calculated by taking the product of input parameters (with independent uncertainties), the fractional uncertainty of the output can be determined by summing the fractional uncertainties of the inputs in quadrature [39]. This method of combining uncertainties is also called the root-sum-square (RSS) method and is shown in Equation 3.8. Please note that the terms relevant for this research are explicitly mentioned in this equation. All other uncertainties are clustered in the term $\sigma_{\text{other}}$.

$$\sigma_{\text{AEP}} = \sqrt{\sigma_{\text{wind speed}}^*^2 + \sigma_{\text{array efficiency}}^2 + \sigma_{\text{availability}}^2 + \sigma_{\text{other}}^2}$$

(3.8)

### 3.3.2 Effect of combination on energy yield prediction and its implications on wind farm financing

The previous subsection has shown that array efficiency and availability affect the energy yield prediction. This has implications for the financial conditions of a wind farm project. The introduction of this report mentioned that high uncertainties in the energy yield prediction result in unfavourable financial conditions for the wind farm project. This section gives extra information on this statement and links it to the combination of array efficiency and availability. Section 3.3.1 explained that the AEP is not a deterministic value, but an uncertain one. This means that it can be modelled as a probability density function. The mean of this function, $AEP_{\text{mean}}$, is called the P50. This means that there is a probability of 50% that the actual AEP of the wind farm is below this mean value, and there is a probability of 50% that the actual AEP of the wind farm is above this mean value. However, banks usually do not use the P50 since this poses a risk that is too high (a 50% chance that the AEP is lower than expected). Instead, they use the P90, a value that is lower than $AEP_{\text{mean}}$. P90 means that there
Chapter 3: Current procedure to predict the combined effect of array efficiency and availability

is a chance of 10% that this value will not be reached, and a chance of 90% that it is exceeded. Figure 3.4 shows the P90 points for two normal distributions with a different \( \sigma \). From this figure can be seen that both distributions have the same \( AEP_{\text{mean}} \) and thus the same P50, but due to the different standard deviations the P90 values are different. This shows that an inaccurate uncertainty quantification might have negative consequences for the financing of an offshore wind farm.

Figure 3.4: P90 points for two normal distributions with the same mean, but a different standard deviation.

3.4 Shortcoming of current procedure

Section 3.3 has shown how array efficiency, availability and other factors are currently combined in the energy yield prediction. This section compares this current method to the physics identified in Chapter 2. An important shortcoming can be observed: the effect of availability on wake losses is neglected. The previous chapter has shown that when a turbine is shut down, it does not generate a wake, meaning that downwind turbines do not experience a wind speed deficit. This effect is neglected in the current methods, since both the array efficiency and availability are determined separately. This combination might actually affect the mean AEP in a less negative sense: it seems logical that having lower wake losses at certain times leads to a higher AEP overall.

Neglecting the effect of availability on wake losses has another effect on AEP predictions. Section 3.3.1 explained that currently, Equations 3.6 and 3.8 are used to determine the mean and standard deviation of the AEP of an offshore wind farm. This method implies that the assumption is made that all uncertainties are independent and that their relation is linear. The previous chapter, however, has shown that this is not the case. Using these equations therefore seems invalid. It might as well be that using the linear relation results in a lower P50, and using the RSS leads to a standard deviation in the AEP that is too high, resulting in lower P90 values and therefore unfavourable financial conditions.

Another important notice is that the availability is partly dependent of the wind speed, since the wind speed affects the weather windows and thus the accessibility of a wind farm. At the same time, this same wind speed affects the size of the wake effects. From the descriptions of the methods that are used to determine array efficiency and availability, it is unclear whether these methods use the same wind series as an input.

Neglecting the effect of availability on wake losses implies that the nonlinear effect of combining array efficiency and availability is not taken into account in the state-of-the-art method for determining the mean AEP and the uncertainty of the AEP. This is addressed in the next chapter, where an adaptation of the current method is proposed.
Development of an adapted model

One of the goals of this report is to develop a model that determines the nonlinear effect of combining uncertainties in array efficiency and availability of an offshore wind farm. The previous two chapters provided all the ingredients to be able to develop this model. This chapter proposes an adaptation to the current procedure, based on the shortcoming that was identified in Section 3.4. Section 4.1 describes the high-level development of this model, whereas Section 4.2 enters into detail by describing the choices made when applying the theory for array efficiency and availability in the adapted model.

4.1 Development of a model that addresses the shortcoming of the current procedure

Chapter 3 described the current methods that are used to determine array efficiency and availability, and how both factors are combined. An important shortcoming was identified: the effect of availability on wake incidence is currently neglected, meaning that the nonlinear effect between both uncertainty components is not taken into account. This results in an incorrect energy yield prediction, in terms of the mean AEP as well as the uncertainty in the AEP. This section explains how the shortcoming mentioned above is resolved by using an adapted model. Section 4.1.1 addresses the steps that need to be taken to adapt the current procedure. Section 4.1.2 explains how this adapted model affects the determination of the mean AEP and uncertainty in the AEP.

4.1.1 Including the effect of availability on the array efficiency

Chapter 2 described an important relation between availability and array efficiency. If a wind turbine in a farm experiences downtime, it does not create a wake and thus has no effect on downwind turbines. These downwind turbines are therefore likely to produce more energy than they would if they did experience a wind speed deficit resulting from a wake. The current method for energy yield predictions does not include this relation, since both array efficiency and availability are determined separately. It does however make sense to include it, since it is plausible that this results in a higher predicted annual energy yield, simply because the farm experiences less wake losses in a year due to unavailability. To see how the current method can best be adapted, a basic flowchart is made of the current model using the information from Chapter 3. This flowchart can be found in Figure 4.1.
Figure 4.1: Basic flowchart of the current model used to determine the combined efficiency.

From this flowchart follows a relatively easy adjustment: instead of using the output of the wake and availability model to determine the combined efficiency, the availability should be used as an input for the wake model. This adjustment should lead to the desired effect; possible unavailability will result in lower annual wake losses due to this measure. Eventually, this means a higher combined efficiency. The adjustment is visualized in Figure 4.2.

Figure 4.2: Basic flowchart showing the adjustment of the current model. Using availability as an input for the wake model leads to lower annual wake losses and thus to a higher combined efficiency.

However, the flowchart shown in Figure 4.2 omits one important input parameter that was already mentioned in Section 3.4, being the wind speed. It is important that the same wind speed is used as an input for both the availability and wake model. After all, a high wind speed can cause low accessibility and thus a longer period of unavailability, whilst at the same time lowering the wake losses. The negative effect of unavailability might counterbalance the less negative effect of wake losses. Omitting the wind speed from this flowchart might thus misrepresent reality. Therefore, this wind speed is explicitly added to the flowchart in Figure 4.3. A detailed flowchart can be found in Appendix B.

4.1.2 Consequences of adapted model on the determination of mean AEP and uncertainty in AEP

The previous section briefly explained how the state-of-the-art method is adapted in order to take into account the effect of availability on the wake incidence – the important shortcoming identified in Chapter 3. This has consequences for the determination of the mean AEP and the uncertainty in the AEP: Equations 3.5 and 3.8 can no longer be used. This section discusses an approach to stay clear of these equations.

In a literature study conducted prior to this research, several methods for propagating uncertainties were investigated [15]. Table 4.1 gives an overview of the approaches identified in this literature study, including their advantages and drawbacks. Based on the information in this table, the literature study concluded that a sampling technique is the most suitable method for the propagation of uncertainties.

The most famous sampling technique is the Monte Carlo approach. In a Monte Carlo simulation, a process is simulated a great amount of times, each time using different inputs. These inputs are generated randomly based on their probabilistic characteristics. The result of this collection of simulations is a high number of output samples. These samples can be plotted in a histogram, from which a particular probability distribution follows. Statistical parameters are then determined from this probability distribution, depending on the type of distribution. The results of this distribution can be compared to the mean $\mu_{AEP}$ and $\sigma_{AEP}$ that result from the current procedure.
Chapter 4: Development of an adapted model

Figure 4.3: Basic flowchart showing the adjustment of the current model. Using availability as an input for the wake model leads to lower annual wake losses and thus a higher combined efficiency. In this flowchart, the wind speed is stated explicitly as an input parameter for both the wake model and the availability model.

Table 4.1: Advantages and drawbacks of the most common uncertainty propagation techniques.

<table>
<thead>
<tr>
<th>Category</th>
<th>Advantages</th>
<th>Disadvantages</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Sampling techniques</strong></td>
<td>+ Relatively simple</td>
<td>– Slow convergence [5]</td>
</tr>
<tr>
<td></td>
<td>+ Universally applicable</td>
<td></td>
</tr>
<tr>
<td></td>
<td>+ Non-intrusive</td>
<td></td>
</tr>
<tr>
<td><strong>Quadrature methods</strong></td>
<td>+ Much more efficient than sampling techniques at low dimensions [44]</td>
<td>– Not suitable for high dimensions [44]</td>
</tr>
<tr>
<td></td>
<td>+ Non-intrusive</td>
<td>– Fairly complicated</td>
</tr>
<tr>
<td><strong>Spectral methods</strong></td>
<td>+ High accuracy [28]</td>
<td>– Intrusive [29]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>– Computationally expensive for high dimensions [21]</td>
</tr>
</tbody>
</table>

Although there are more sampling techniques, the Monte Carlo method is a widely used one that is easy to implement. Therefore, this research will use the Monte Carlo method to determine the nonlinear effect of combining array efficiency and availability. The approach is applied to the model shown in Figure 4.3. This means that the inputs of this model will be varied. How this is done is explained in Section 6.1.

4.2 Choices made in applying the theory in the models

In the previous section, the high-level layout of the adapted model was presented. This layout was developed by rearranging several process blocks in order to take into account the nonlinear effect of availability and array efficiency. The models shown in Figures 4.1 and 4.3 are both based on theory explained in the previous chapter. The choices that were made when applying this theory in the models are briefly discussed in this section. Section 4.2.1 describes the choices made in applying the Jensen wake model, Section 4.2.2 the
determination of the availability and Section 4.2.3 the requirements of the input parameters.

### 4.2.1 Choices made in applying the theory of the Jensen wake model

Section 3.1.1 described the theory of a single wake according to Katić, Højstrup and Jensen. To determine the array efficiency of an offshore wind farm, this theory needs to be applied to an entire farm. Section 3.1.2 explained that in expanding the single wake to handle an entire farm, several choices have to be made. Since Section 3.1 described the Jensen wake model in a more general fashion, the choices during the application of the theory are explained here. A detailed flowchart of the entire model with a brief explanation about each step can be found in Appendix B, but three choices are addressed in this section: the angle conventions used, the types of wake incidence and the mixing of wakes.

#### Angle conventions

An important choice that needs to be made when applying the theory of the Jensen wake model to an offshore wind farm is how to define the angles that are used. Section 2.1 described that the wake effects of an offshore farm strongly depend on the wind direction. Usually, wind direction is reported by the direction from which it originates and a northerly wind is defined as $0^\circ$. All other angles are defined with respect to the $0^\circ$ point in a clockwise fashion.

It makes sense to also know the angle between two turbines, since this angle can then be compared to the wind direction to see whether the downwind turbine is in the wake of the upwind turbine. Therefore, it is important to define the angle between two turbines in a similar sense as the wind direction. The angle conventions used are shown in Figure 4.4. In this figure, the upwind turbine is denoted by $(x_1, y_1)$ and the downwind turbine by $(x_2, y_2)$.

![Figure 4.4: Overview of the angle conventions used in this report. The upwind turbine is denoted by $(x_1, y_1)$ and the downwind turbine by $(x_2, y_2)$.

#### Three types of wake incidence

The previous paragraph described the importance of a consistent use of angles, because the angles between turbines have to be compared to the wind direction. This comparison is needed to check whether any downwind turbine is in the wake of an upwind turbine. There are three types of wake incidence. A turbine can experience **no** wake (1), a **partial** wake (2) and a **full** wake (3). To determine which of the three types
a turbine experiences, two critical angles can be defined: $\delta\alpha_{\text{full}}$ and $\delta\alpha_{\text{partial}}$. Figure 4.5 shows how $\delta\alpha_{\text{full}}$ is determined.

From Figure 4.5 can be seen that $\delta\alpha_{\text{full}}$ is the final angle for which turbine 2 is in the full wake of turbine 1. This ultimate angle corresponds to the size of the wake decay constant $k$. Their relation is given by Equation 4.1. The eventual wind speed that the downwind turbine experiences is dependent on the distance between the two turbines, $d_{h2h}$.

$$\delta\alpha_{\text{full}} = \tan^{-1}(k) = \kappa \quad (4.1)$$

The partial wake angle $\delta\alpha_{\text{partial}}$ is a little less straightforward. Figure 4.6 shows the intermediate angles used to determine $\delta\alpha_{\text{partial}}$.

Once again, Figure 4.6 shows the ultimate angle for which turbine 2 experiences a partial wake from turbine 1. To calculate this angle, two intermediate angles are necessary: $\kappa$ and $\gamma$. The definitions of both angles are given by Equations 4.2 and 4.3. $\delta\alpha_{\text{partial}}$ is eventually determined by adding up $\kappa$ and $\gamma$, shown in Equation 4.4.

$$\kappa = \tan^{-1}(k) \quad (4.2)$$
\[
\gamma = \sin^{-1}\left(\frac{D \cos(\kappa)}{d_{k2h}}\right)
\]

(4.3)

\[
\delta \alpha_{\text{partial}} = \kappa + \gamma
\]

(4.4)

The rotor of a downwind turbine subject to a partial wake experiences two wind speeds: one area experiences the undisturbed wind speed \(U_0\), the other experiences a particular wind speed in the wake of the upwind turbine \(U_W\). Figure 4.7 visualizes these areas.

![Figure 4.7: Visualization of the rotor area subject to a partial wake.](image)

From Figure 4.7 can be seen that the shape of the area where the rotor experiences the wake is an asymmetric lens. The area of this asymmetric lens can be determined analytically. The procedure for this can be found in Appendix C. To determine the rotor averaged wind speed, a weighted average approach is used based on the work of Attias et al. [3]. This approach is shown in Equation 4.5.

\[
U_r = U_W \cdot \frac{A_{\text{lens}}}{A_{\text{rotor}}} + U_0 \cdot \frac{A_{\text{rotor}} - A_{\text{lens}}}{A_{\text{rotor}}}
\]

(4.5)

**Mixing wakes**

The previous information only treated the appearance of a single wake. It however also happens that one downwind turbine experiences wakes of multiple upwind turbines. To determine the power output of this downwind turbine, the resultant wind speed on this turbine – once again called the rotor averaged wind speed – needs to be known. A commonly used assumption is developed by Mosetti [33] and is given in Equation 4.6 to calculate the resulting velocity downstream of \(n\) turbines.

\[
\left(1 - \frac{U_m}{U_0}\right)^2 = \sum_{i=1}^{n} \left(1 - \frac{U_i}{U_0}\right)^2
\]

(4.6)

Figure 4.8 shows a scenario where a turbine experiences multiple wakes, which partly mix. In the figure can be seen that the purple area is the area where wake 1 and wake 2 mix. The red and blue areas are the areas where the rotor experiences only wake 1 or wake 2, respectively.
Chapter 4: Development of an adapted model

Figure 4.8: Schematic overview of a downwind turbine experiencing multiple partial wakes.

The procedure used in such a scenario to come up with the rotor averaged wind speed is a combination of Equations 4.5 and 4.6. The speed in the purple area in Figure 4.8 is calculated using Equation 4.6. The wind speeds in the blue, purple and red areas are averaged using Equation 4.5. In the implementation of the above equations it was chosen to neglect the small areas in which the turbine experiences undisturbed wind speed.

4.2.2 Choices made in determining the availability

Section 3.2 explained that reliability, maintainability and accessibility are combined to come up with the availability. It was however not yet made clear how this is done. This section will briefly elaborate on this determination.

First of all, it was mentioned in Section 3.2.2 that reliability (or failures per year) and maintainability and serviceability are combined to come up with a theoretical availability. This means that the number of failures per year – the failure rate – of the turbine(s) used in the offshore wind farm needs to be known. The maintainability is expressed as the repair time, that is the amount of time needed to repair a certain failure. Multiplying the failure rate of the turbine with the repair time yields the theoretical time that the turbine is unavailable due to failures. This can be expressed as a percentage. For this research, it was chosen not to take the serviceability and maintenance strategies into account.

This leaves the accessibility, which is determined by wind and wave data. Whenever the wind speed and/or the wave height reach a certain threshold, the wind farm becomes inaccessible. The amount of time that this happens can also be expressed as a percentage.

However, when theoretical availability and accessibility need to be combined in order to come up with the actual availability, using percentages does not work. To be able to make an accurate estimate of the actual availability, the accessibility needs to be checked each time a turbine experiences a failure and needs to be repaired. In other words: time series are needed. In the availability model presented in Figures 4.1 and 4.3, this means that at each time step, the program checks if any of the turbines experiences downtime due to a failure. If this is the case, the program checks whether the wind farm is accessible for maintenance. If the weather allows it, the standard repair time for the failure can be used. However, if the wave heights are too high, the maintenance needs to be postponed until the site becomes accessible again. This thus results in a longer period of unavailability. This is schematically visualized in Figure 4.9.

From this figure can be seen that at the moment that failure 1 occurs, the wind turbine is inaccessible. Therefore, the shaded failure area (representing the time that it takes before the failure is fixed), needs to be shifted slightly. This results in a slightly longer period of unavailability. At the moment that failure 2 occurs, the wind turbine is accessible. However, it is not accessible for the entire maintenance period. This means that the entire shaded failure area needs to be shifted until the weather window allows it. This results in a significantly longer period of unavailability. When failure 3 occurs, the wind turbine is accessible for the entire maintenance period. This means that the turbine can be repaired straight away, avoiding a longer period of unavailability.
4.2.3 Requirements of the input parameters: time series

Section 4.2.2 already mentioned briefly that time series are needed in order to determine the actual availability. Since the usage of time series is an essential part of the adapted model, it is explicitly treated in this section. Section 4.2.2 described that the availability model checks for each time step if any turbine experiences downtime and if the wind turbine is accessible. This implies that the failure rate and the corresponding repair time of the turbine need to be converted to a time series for a specific period. Since the interest in this research mainly lies with the determination of the AEP, this period is taken to be a year. This time series, consisting of only ones (the turbine is operative) and zeros (the turbine experiences downtime) should represent the failure rate of the data.

To compare the theoretical availability to the accessibility of the farm, the accessibility also needs to be a time series (with the same time step as the theoretical availability). Once again, this series consists only of
ones (the site is accessible) and zeros (the site is inaccessible). To be able to generate the series, the wind and wave data also need to be time series, since for each time step, the program checks whether the tresholds are reached. The wind time series are also used in the determination of the wind farm wake losses. The use of time series is an essential part of the adapted model. Therefore, a flowchart that is more detailed than the flowcharts given in Figures 4.1 – 4.3 is given in Figure 4.10.

Figure 4.10: Detailed flowchart of the adapted model.
Chapter 5

Verification of the models

The previous chapter described the development of a model that is able to take the nonlinear effect of combining array efficiency and availability into account. This adapted model, but also the model of the current procedure that is used for comparison, need to be verified to ensure that the results are usable. Section 5.1 explains the choices that were made in the selection of the data sources for this verification. Section 5.2 describes the steps that are taken to verify the models.

5.1 Data used for the verification

The models that were shown in Figures 4.1 and 4.3 need several inputs. This requires data sources. The data needs to be selected in such a way that the verification is convincing. This section describes the main requirement for these data sources and the choices that were made for the wind farm, turbine, wind, wave and failure data.

5.1.1 Main requirement for the data used

In order to obtain plausible results, the verification of the models has to be convincing. Using plausible data contributes to a good verification. Therefore, it is prudent to state a main requirement that the data sources have to meet. Since the main objective of this research is to develop a methodology rather than produce results that comply with reality, data sources that are acquired at the same farm in the same year (that are difficult to find) are not necessary. It is however important that the results on which the methodology is based, are plausible. Therefore, the main requirement for the data used can be formulated as follows: the data should represent typical, but no exceptional circumstances. All data sources were put to the test of this requirement.

5.1.2 Wind farm data

Chapter 2 explained that the layout of a wind farm affects its array efficiency. To satisfy the requirement stated in Section 5.1.1, a typical offshore wind farm layout needs to be selected. In this research, a typical layout is considered to be a layout in which all types of wake incidence can occur – that is, single full and partial wakes and multiple full and partial wakes. This leads to the selection of a wind farm in which the turbines are placed in a rectangular fashion. The placement of the turbines is based on the Barrow offshore
wind farm, located in the East Irish Sea, since it is convenient that the \(X\)- and \(Y\)-coordinates of this farm are known. The layout of this offshore wind farm is visualized in Figure 5.1.

![Figure 5.1: Layout of the offshore wind farm used.](image)

**5.1.3 Turbine data**

The turbine data is important for correct execution of both the current and the adapted model. Section 3.1 explained that an important input parameter of the Jensen wake model is the thrust coefficient \(C_T\). This thrust coefficient depends on the performance of the turbine used. Furthermore, a turbine power curve is needed to predict the energy production. Therefore, it was necessary to choose a turbine from which these data were available. Based on this requirement, and the main requirement from Section 5.1.1, the Vestas V80-2.0MW turbine was chosen. This turbine has been used extensively in offshore wind farms over the past years [42]. It has a rotor diameter \(D\) of 80 m. The thrust curve and power curve of this turbine are shown in Figure 5.2.

![Figure 5.2: Thrust coefficient and power curve of the Vestas V80-2.0MW [42].](image)
5.1.4 Wind data

Section 4.2.3 already explained that the wind input for the adapted model needs to be a time series. Furthermore, it is important that the wind data contains information on the wind speed as well as the wind direction and that this data is measured at a typical offshore location. The KNMI Hydra project provides such measurements. This project comprises hourly wind speed and direction records of 65 locations, with a length of multiple years [26]. The wind data selected for this research is the data from Station 254, also called “Meetpost Noordwijk”. Since the focus of this research is the AEP, the wind data of one year (specifically: 2005) was used. The histogram and corresponding Weibull distribution of the wind speed data is shown in Figure 5.3a, the wind direction distribution in Figure 5.3b.

![Histogram and Weibull distribution of wind speed data.](image)

![Number of occurrences of wind directions [in degrees].](image)

Figure 5.3: Overview of wind data used for verification.

All wind speeds measured in the KNMI Hydra project are converted from their original measuring height to potential wind speeds, $U_{pot}$, at the standard height, $h_{standard} = 10\text{m}$, corresponding to the standard roughness length, $z_{0,standard} = 0.03\text{m}$. To be able to determine the power output of the turbines, these wind speeds need to be converted to hub height. This is done using a sequence of equations. First of all, the potential wind speed at standard height needs to be converted to wind speed at blending height, $h_{blending} = 60\text{m}$, using the standard roughness length. This is done using Equation 5.1.

$$U_{h_{blending}} = U_{pot} \cdot \frac{\log 60}{\log 0.03} \quad (5.1)$$

When the wind speed at blending height is obtained, the original measured wind speed can be calculated. This is done using the roughness length specified in the dataset, $z_0 = 0.002\text{m}$, and the height of the measuring equipment, $h_{meas} = 27.6\text{m}$, by applying Equation 5.2.

$$U_{h_{meas}} = U_{h_{blending}} \cdot \frac{\log 27.6}{\log 0.002} \quad (5.2)$$

The original measured wind speeds can be scaled to the wind speeds at hub height, $h_{hub} = 90.0\text{m}$. The roughness length used in this case has the classification ‘sea’ according to Stull; $z_0 = 0.0002\text{m}$ [38]. The scaling is shown in Equation 5.3.

$$U_{h_{hub}} = U_{h_{meas}} \cdot \frac{\log 90.0}{\log 0.0002} \quad (5.3)$$
5.1.5 Wave data

Section 3.2.1 explained that the accessibility of a wind farm depends on both the wind speed and the wave height. Accordingly, wave data is also needed. However, plausible hourly wave height data was not found. The choice was therefore made to estimate the wave height by using a relation between wind speed and wave height selected by Germanisher Lloyd [19]. This relation can be found in Equation 5.4.

\[ H_s = 0.246 \cdot \frac{U_{10}^2}{g} \]  \hspace{1cm} (5.4)

In this equation, \( H_s \) represents the significant wave height, \( U_{10} \) the wind speed at a height of 10m, and \( g \) the gravitational constant. Significant wave height is usually calculated for three-hour periods. Therefore, the wind speed is converted to three-hour means. These three-hour means are then used in Equation 5.4 to compute the three-hour significant wave height. Because the hourly accessibility is needed, these three-hour significant wave heights are converted to an hourly time series. Since wave height is now dependent on the wind speed, only one criterion is used to determine whether the wind farm site is accessible. This criterion corresponds to a maximum significant wave height of 1.5m.

5.1.6 Failure data

The final data needed to execute the models shown in Figures 4.1 and 4.3 is failure data: failure rates and repair times. Faulstich et al. provide both [17]. An overview of the annual failure rate and corresponding downtime (mean time to repair, MTTR) can be found in Table 5.1.

<table>
<thead>
<tr>
<th>Subassembly</th>
<th>Annual failure rate</th>
<th>Downtime per failure [days]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electrical system</td>
<td>0.57</td>
<td>1.53</td>
</tr>
<tr>
<td>Electronic control</td>
<td>0.43</td>
<td>1.59</td>
</tr>
<tr>
<td>Sensors</td>
<td>0.25</td>
<td>1.41</td>
</tr>
<tr>
<td>Hydraulic system</td>
<td>0.23</td>
<td>1.36</td>
</tr>
<tr>
<td>Yaw system</td>
<td>0.18</td>
<td>2.70</td>
</tr>
<tr>
<td>Rotor hub</td>
<td>0.17</td>
<td>3.71</td>
</tr>
<tr>
<td>Mechanical brake</td>
<td>0.13</td>
<td>2.89</td>
</tr>
<tr>
<td>Rotor blades</td>
<td>0.11</td>
<td>2.60</td>
</tr>
<tr>
<td>Gearbox</td>
<td>0.10</td>
<td>6.21</td>
</tr>
<tr>
<td>Generator</td>
<td>0.11</td>
<td>5.39</td>
</tr>
<tr>
<td>Support &amp; housing</td>
<td>0.10</td>
<td>4.90</td>
</tr>
<tr>
<td>Drive train</td>
<td>0.05</td>
<td>5.71</td>
</tr>
</tbody>
</table>

The characteristics shown in the table above were collected during the Scientific Measurement and Evaluation Programme (WMEP), a large monitoring survey for onshore wind turbines, whereas this research is interested in offshore wind farms. Faulstich et al. however reason that offshore wind turbines are directly derived from onshore technology, meaning that similar faults can be expected [17]. Due to this argument and due to the lack of offshore wind turbine reliability data, it was decided that the data is suitable for this research.
5.2 Verification of the adapted model

The previous section gave an overview of the data sources used to verify both the current and adapted model. This section discusses the steps taken to verify the models and their outcomes in detail. Ideally, the model would also be validated. Validation however requires real-life data. Due to the lack of this data, validation of the model was impossible. Therefore, a thorough verification has been carried out, by checking parts of the code analytically and by seeing if the model produces results that fulfil reasoned expectations.

First of all, it is important to check if the wake model, that is used in both the current and the adapted model, functions as expected. This is described in Section 5.2.1. Then, a quick check is carried out to see if the correct actual availability is determined. This check is presented in Section 5.2.2. Finally, steps are taken to verify the adapted model. This verification is explained in Section 5.2.3.

5.2.1 Verification of the wake model

From the flowcharts presented in Figures 4.1 and 4.3 can be seen that the same wake model is used in both the current and adapted model. Therefore, this process block needs to be verified before the verification of the entire model can be executed. Five tests are conducted to verify the wake model:

1. Determine types of wake incidence for a set of turbines by hand calculations. Check if the model finds the same types of incidence.
2. Determine the wind speed deficit of a wind turbine wake and determine the power output of a turbine situated in this wake by hand calculations. Compare these values to the results from the wake model.
3. Run the model for one specific wind speed and wind direction, in which it is known that some of the turbines experience a wake. Check the power output of each of the turbines against reasoned expectations.
4. Perform a simulation using the data presented in Section 5.1 and see whether this gives a plausible result for the array efficiency.
5. Perform a simulation on wind farms with another layout, as an extra check to see whether plausible array efficiencies result from the model.

The execution of these tests and their corresponding results are described in this section.

Section 4.2.1 explained that three types of wake incidence exist: a turbine can experience no wake, a partial wake or a full wake. The formulas to determine the wake angles are given by Equations 4.1 – 4.4. To see whether the model computes these incidences correctly, several test layouts were made to which a wind with a certain direction was applied. As an example, a test layout consisting of four turbines experiencing an Eastern wind is presented. This test layout is shown in Figure 5.4.

Figure 5.4: Fictional layout used to verify the determination of the wake incidence.

The coordinates of the turbines are used to determine the distance between the turbines, $d_{h2h}$, and the angle between the turbines, $\alpha_{h2h}$, using the angle conventions explained in Section 4.2.1. Equations 4.1 – 4.4 are then used to determine the full and partial wake angles. All these parameters are summarized in Table 5.2.
Please note that the values in this table have been rounded. The last column indicates which wake type follows from comparing the parameters with the wind direction. Section 4.2.1 explained that wind direction is reported by the direction from which it originates. An Eastern wind thus corresponds to a wind direction of $90^\circ$. However, in order to obtain correct results for the wake types, the wind direction should be expressed by the direction opposite from where it originates – in this case $270^\circ$. This can also be logically derived from Figures 4.4 and 5.4.

It can be seen that turbine 4 experiences a partial wake resulting from turbine 1, whereas the rest of the turbines experience no wake. These wake types seem plausible when compared to Figure 5.4. All values were checked with the numerical values calculated by the wake model. These yielded the exact same results, for all the test layouts considered. This part of the code is therefore considered to be verified.

Table 5.2: Parameters determined by hand calculations for the test layout shown in Figure 5.4. The wind direction to which the parameters are compared is $270^\circ$: the direction opposite from where the wind originates.

<table>
<thead>
<tr>
<th>Turbine combination</th>
<th>$d_{h2h}$ [m]</th>
<th>$\alpha_{h2h}$ [$^\circ$]</th>
<th>$\delta_{\alpha_{full}}$ [min$^2$-max$^2$]</th>
<th>$\delta_{\alpha_{partial}}$ [min$^2$-max$^2$]</th>
<th>Wake type</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-2</td>
<td>602.1</td>
<td>318</td>
<td>316 – 321</td>
<td>308 – 328</td>
<td>no</td>
</tr>
<tr>
<td>1-3</td>
<td>838.2</td>
<td>253</td>
<td>250 – 255</td>
<td>245 – 260</td>
<td>no</td>
</tr>
<tr>
<td>1-4</td>
<td>1201</td>
<td>272</td>
<td>270 – 275</td>
<td>266 – 278</td>
<td>partial</td>
</tr>
<tr>
<td>2-3</td>
<td>806.2</td>
<td>210</td>
<td>207 – 212</td>
<td>202 – 218</td>
<td>no</td>
</tr>
<tr>
<td>2-4</td>
<td>894.4</td>
<td>243</td>
<td>241 – 246</td>
<td>236 – 251</td>
<td>no</td>
</tr>
<tr>
<td>3-4</td>
<td>500.0</td>
<td>307</td>
<td>305 – 309</td>
<td>295 – 318</td>
<td>no</td>
</tr>
</tbody>
</table>

The next verification step is to check whether the wind speed deficits and power outputs of downwind turbines are correctly calculated by the model. The case that is verified here is a simple case in which one turbine experiences a full wake from an upwind turbine. The spacing between the turbines is 400 m. The undisturbed wind speed that the upwind turbine experiences is 12 m/s. The corresponding thrust coefficient follows from the turbine data shown in Figure 5.2: $C_T = 0.709$. Substituting these values in Equation 3.1 yields $U_W = 9.18$ m/s.

The downwind turbine thus experiences a wind speed of 9.18 m/s. From the Vestas V80-2.0MW it follows that the turbine generates a power of 1058 kW at this wind speed. This simple setup was also tested in the model and once again gave exactly the same results. This test was carried out for multiple simple setups. For each of the tests, the numerical results were exactly the same as the analytical values. Therefore, this part of the model is also considered verified.

Now that the simple setups have been tested and verified, test cases can be performed on farm level. The first test that is performed is to run the model for the wind farm presented in Section 5.1.2 with one wind speed and wind direction. The wind direction is chosen such that some turbines in the farm experience a wake from upwind turbines (in this case $270^\circ$). The wind speed is chosen to be 12 m/s. By looking at the power generated by each of the turbines, one can see whether the results seem likely. The result of this test is shown in Figure 5.5. In this test, the turbines that are indicated in dark red experience no wake. Six turbines in the second row (when looked at the turbines from the perspective of the lower left corner) experience partial wakes from the upstream turbines. It can be seen that the power between these rows drops significantly, from over 1850 kW to around 1700 kW. Five of the turbines in the third row experience a partial mixed wake (from the turbines from the first and second row), a partial wake only from the turbines in the second row and the undisturbed wind speed. The power of these turbines is slightly lower than 1700 kW. The difference between the powers of the third and second row is barely noticeable. There are two turbines in the last row that produce the lowest amount of power: around 1650 kW. These two turbines experience both a full wake (from the two turbines that are most upstream) and partial wakes from turbines in the second and third row. The result visualized in this figure seems plausible. It is known that the wake of the first upstream turbine
Chapter 5: Verification of the models

is most important and has the most significant impact on the power output of the farm. This can also be observed in the figure.

The same test was done for several wind directions and wind speeds; each time yielding plausible and satisfactory results.

Figure 5.5: Power production of a rectangular wind farm for a wind speed of 12 m/s and a wind direction of $270^\circ$. It can be seen that the turbines that experience a wake produce less power than the upwind turbines.

Test case number 4 is checking the annual array efficiency of the farm. In literature can be found that typical wake losses are in the order of 10-15% [4], [32], corresponding to an array efficiency of 85-90%. Seeing whether this wind farm experiences similar annual wake losses is a good check on whether the wake model was coded correctly. To perform this test, the wind data from Section 5.1.4 was used. This data yields an array efficiency of 89.2%, well in the range of the typical array efficiency found in literature.

The last verification test case is to test the model for other wind farm layouts (but with the same wind and turbine data). This was done for wind farms with the layouts of Sheringham Shoal (88 turbines) and Walney 1 (51 turbines). The layouts of these wind farms can be found in Appendix D. The array efficiency of Sheringham Shoal was found to be 88.7% and the array efficiency of Walney 1 90.7%. These results also fall well in the range of typical array efficiencies. This means that the model has passed the five verification tests mentioned above and is therefore considered verified.

5.2.2 Verification of the availability model

Now that the wake model has been tested and verified to a good extent, similar plausibility studies should be conducted on the availability. Section 5.1.6 presented the failure data that was used for this research. However, Section 4.2.2 explained that time series are needed to be able to compare the theoretical availability to the accessibility and by doing so, determining the actual availability. The failure rates and corresponding downtimes shown in Table 5.1 are used to create such a time series. As a start, a matrix of $8760 \times 30$ (total hours in a year $\times$ number of turbines in the farm) is created. All entries are set to 1: this means that the
turbine is theoretically available. By multiplying each of the 12 failure rates with the number of turbines in the farm (in this case 30), the total number of failures in the entire farm in a year can be determined. The corresponding downtimes are converted to hours. Each failure can now be represented by an array of zeros: each 0 represents one hour of unavailability. These arrays of zeros are spread over the turbines and over the year randomly. This results in a matrix of $8760 \times 30$, consisting of only 1’s and 0’s, and forms the time series used in the model. The relative number of occurrences of 1’s represents the theoretical availability of the wind farm. By using the failure data of Faulstich et al., a theoretical availability of 98.3% is realized.

The accessibility of the wind farm site is determined by using the wind speed data presented in Section 5.1.4 in combination with Equation 5.4 and the wave threshold of 1.5m mentioned in Section 5.1.5. The accessibility is an array consisting of 8760 entries. If the wind turbines are accessible, this is indicated with a 1 in the accessibility array. Inaccessibility is indicated with a 0.

Figure 4.9 has shown the procedure that is taken to determine the actual availability of a wind farm: at each time step, the availability model checks if any of the turbines experiences downtime due to a failure. If a failure occurs but the site is inaccessible during the necessary repair time, the maintenance needs to be postponed until the site becomes accessible again. This results in a longer period of unavailability, meaning that actual availability is always lower than theoretical availability. For the time series described in this section, the actual availability turned out to be 93.4%. This seems a realistic value compared with typical values stated in literature [11].

The procedure that the availability model follows can easily be checked manually. To do this, the theoretical availability matrix and the accessibility array, which are inputs to the model, are put side by side with the actual availability matrix, which is computed by the model. If there is a 0 in the theoretical availability matrix, this should automatically result in a 0 in the actual availability matrix. If there is a 0 in the theoretical availability matrix as well as in the accessibility array, this should lead to a longer array of zeros in the actual availability matrix. This was the case for the time series described in this section.

Another check that has been done on the availability model is to see whether the availability (both theoretical and actual) goes down when the failure rate is increased. To perform this check, one overall failure rate is used, which is varied from 0.1 to 700 in 250 steps. The result is shown in Figure 5.6. From this figure can be seen that at a failure rate of 0 (that is, no failures occur), the theoretical availability is 100%. The actual availability is also 100%, which makes sense since none of the turbines experiences a failure. As the failure rate increases, the theoretical availability decreases. The actual availability decreases with a steeper slope, since the weather windows now play a role in the determination of the actual availability. As expected, the actual availability always lies below the theoretical availability. Both the theoretical availability and the actual availability approach 0 at high failure rates. This means that this part of the code functions properly.

![Figure 5.6: Theoretical and actual availability for a varying failure rate. It can be seen that both values go to zero.](image)
5.2.3 Verification of the adapted model

Now that both the wake and the availability model have been verified, verification of the adapted model can be performed. The best way to see whether this adapted model yields plausible results, is to compare them to the results of the current model shown in Figure 4.1. Four cases have been tested:

1. Run both models using the theoretical availability of 98.3%, omitting the step that determines the actual availability. This means that, in the adapted model, the availability input for the wake model is equal to 98.3%.
2. Run both models using the theoretical availability of 98.3%, including the step that determines the actual availability. In the adapted model, this results in an availability input for the wake model of 93.4%.
3. Run both models for an artificial theoretical availability of 0.01%, omitting the step that determines the actual availability. This means that, in the adapted model, the availability input for the wake model is equal to 0.01%.
4. Run both models for an artificial theoretical availability of 100%, including the step that determines the actual availability. In the adapted model, this results in an availability input for the wake model of 100%.

These cases were selected based on expectations of the behaviour of the models. Case 1 has a high availability, which is used in the adapted model as an input for the wake model. It is expected that the combined efficiency resulting from the adapted model is higher than the one resulting from the current model, but that this difference is small. Case 2 has an availability which is slightly lower. Because of the nonlinearity, it is expected that the difference between the adapted and the current model is slightly higher than the difference in case 1. Case 3 and case 4 both represent extreme availability values. It is expected that both models give the same results at an availability of 0% and 100%. This means that the results of case 3 should be almost the same (after all, the availability is nearly 0%). The results of case 4 should be exactly the same. The results are shown in Table 5.3.

<table>
<thead>
<tr>
<th>Model</th>
<th>Array efficiency</th>
<th>Availability</th>
<th>Combined efficiency</th>
<th>Annual energy production</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model from Figure 4.1, representing state-of-the-art method</td>
<td>89.2%</td>
<td>98.3%</td>
<td>87.7%</td>
<td>237.6 GWh</td>
</tr>
<tr>
<td></td>
<td>89.2%</td>
<td>93.4%</td>
<td>83.3%</td>
<td>225.8 GWh</td>
</tr>
<tr>
<td></td>
<td>89.2%</td>
<td>0.01%</td>
<td>0.00892%</td>
<td>0.024 GWh</td>
</tr>
<tr>
<td></td>
<td>89.2%</td>
<td>100%</td>
<td>89.2%</td>
<td>241.8 GWh</td>
</tr>
<tr>
<td>Model from Figure 4.3, representing adapted method</td>
<td>n.a.</td>
<td>98.3%</td>
<td>88.0%</td>
<td>238.5 GWh</td>
</tr>
<tr>
<td></td>
<td>n.a.</td>
<td>93.4%</td>
<td>84.0%</td>
<td>227.6 GWh</td>
</tr>
<tr>
<td></td>
<td>n.a.</td>
<td>0.01%</td>
<td>0.014%</td>
<td>0.038 GWh</td>
</tr>
<tr>
<td></td>
<td>n.a.</td>
<td>100%</td>
<td>89.2%</td>
<td>241.8 GWh</td>
</tr>
</tbody>
</table>

From Table 5.3 can be seen that the adapted model functions qualitatively as expected. The difference between the current and adapted model in case 2 is higher than the difference is case 1. Furthermore, the results of case 4 are exactly the same in both models. For case 3, these are almost the same. It is therefore considered as a suitable model to answer the main research question of this thesis.
The previous chapter described the verification of the current and adapted model. Now that both models are verified, results can be obtained for the mean AEP, uncertainty in the AEP and the P90. Section 4.1.2 already described that the Monte Carlo approach is used to achieve these results. This chapter presents the pre-processing and the results of this approach. Section 6.1 describes the stochastic input generation required for the Monte Carlo method. The results of the current model are discussed in Section 6.2. The same is done for the adapted model in Section 6.3. Section 6.4 compares both results.

6.1 Stochastic input generation

Before the Monte Carlo simulation can be performed, inputs are needed. Section 4.1.2 explained that in a Monte Carlo simulation, a process is simulated a great amount of times, each time using different inputs. This means that not all the data presented in Section 5.1 is suitable. The wind speed, wind direction and downtime time series can only be used to generate one sample, since these are stochastic inputs. Ideally, thousands of samples are generated. Thousands of wind and failure time series do however not exist. Therefore, representative time series have to be generated. This section describes the procedures for generating these inputs. Section 6.1.1 explains how wind series are generated. The same is done for availability in Section 6.1.2. The wind farm and turbine data presented in Sections 5.1.2 and 5.1.3 are not treated stochastically and therefore still suitable for use in this chapter.

6.1.1 Wind series

The introduction of this section already described that thousands of time series are needed in order to obtain good results using the Monte Carlo approach. This section will discuss the method used to generate the wind time series. Use is made of the Markov chain approach, which will be described in the first paragraph of this section. Subsequently, an overview of the generated inputs is given and their suitability is discussed.
Markov chain approach

To obtain realistic results, the generated wind series have to be realistic: their autocorrelation needs to be credible. For example, it is unrealistic that the wind speed is 1 m/s in the first hour, 24 m/s in the second hour and 3 m/s in the third hour. Furthermore, it is important that the generated wind series approximate the wind speed and direction distribution. A way to do this is by using the Markov chain method. This approach can be used to generate wind speed as well as wind direction time series. This paragraph explains the Markov chain method on the basis of wind speed series generation. The approach below is based on research from Aksoy et al., Sahin and Sen, Shamshad et al. and Veldkamp [1] [35] [36] [41].

First of all, a measured dataset containing hourly wind speeds is needed. The data presented in Section 5.1.4 is used for this. The wind speeds in this dataset are then divided into a number of states, \( s \). The choice of \( s \) is subjective [35]; in this research \( s = 10 \) is used based on research conducted by Aksoy et al. [1]. An overview of the wind speeds per state is shown in Table 6.1.

Table 6.1: Overview of wind speeds per state used in this research.

<table>
<thead>
<tr>
<th>State</th>
<th>Wind speeds [m/s]</th>
<th>State</th>
<th>Wind speeds [m/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0 – 2</td>
<td>6</td>
<td>10 – 12</td>
</tr>
<tr>
<td>2</td>
<td>2 – 4</td>
<td>7</td>
<td>12 – 14</td>
</tr>
<tr>
<td>3</td>
<td>4 – 6</td>
<td>8</td>
<td>14 – 16</td>
</tr>
<tr>
<td>4</td>
<td>6 – 8</td>
<td>9</td>
<td>16 – 18</td>
</tr>
<tr>
<td>5</td>
<td>8 – 10</td>
<td>10</td>
<td>18 – 22</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>( = max)</td>
</tr>
</tbody>
</table>

When this is done, a Markov transition matrix, \( M \), is made of size \( s \times s \). The element \( m_{ij} \) of matrix \( M \) represents how many times a transition from state \( i \) to state \( j \) occurs in the data wind series. This matrix is converted to a probability matrix \( P \) by using Equation 6.1.

\[
P_{ij} = \frac{m_{ij}}{\sum_{i,j=1...s} m_{ij}} \quad (6.1)
\]

When \( P \) is obtained, the cumulative transition probability matrix is calculated, meaning each row has 1 as the final value. This matrix is shown in Table 6.2.

Table 6.2: Cumulative transition probability matrix resulting from the wind data described in Section 5.1.4.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.596</td>
<td>0.967</td>
<td>0.993</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>0.075</td>
<td>0.780</td>
<td>0.941</td>
<td>0.992</td>
<td>0.996</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>0.008</td>
<td>0.249</td>
<td>0.683</td>
<td>0.980</td>
<td>0.995</td>
<td>0.999</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>0.001</td>
<td>0.032</td>
<td>0.152</td>
<td>0.835</td>
<td>0.967</td>
<td>0.998</td>
<td>0.999</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>0.0005</td>
<td>0.017</td>
<td>0.273</td>
<td>0.738</td>
<td>0.987</td>
<td>0.997</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>0.003</td>
<td>0.005</td>
<td>0.044</td>
<td>0.200</td>
<td>0.870</td>
<td>0.977</td>
<td>0.999</td>
<td>0.999</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>0.0005</td>
<td>0.005</td>
<td>0.020</td>
<td>0.272</td>
<td>0.792</td>
<td>0.991</td>
<td>0.998</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>0.002</td>
<td>0.006</td>
<td>0.048</td>
<td>0.214</td>
<td>0.882</td>
<td>0.980</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>9</td>
<td>0.0002</td>
<td>0.020</td>
<td>0.321</td>
<td>0.746</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>0.002</td>
<td>0.005</td>
<td>0.037</td>
<td>0.165</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Using the matrix in Table 6.2, the wind series can be generated. This process is summarized by the following steps:

1. An initial state \( i \) is chosen randomly.
2. A uniform random number between 0 and 1 is chosen and is compared with row \( i \) of the cumulative transition probability matrix. This determines the next state, \( j \).
3. Another uniform random number \( \epsilon \) between 0 and 1 is chosen to convert the wind speed state to an actual wind speed, using \( U = U_{\text{min}} + \epsilon (U_{\text{max}} - U_{\text{min}}) \), where \( U_{\text{min}} \) and \( U_{\text{max}} \) are the lower and upper boundaries of the wind speed state [36].
Chapter 6: Results: comparing the adapted model to the current model

Overview of the generated inputs for wind speed and wind direction

The previous paragraph explained the Markov chain method by means of wind speed series generation. This approach is used to generate synthetic wind speed as well as wind direction series. This paragraph shows the results of this synthetic generation. In total, 5000 time series are generated. Figure 6.1 shows the Weibull fits of the generated wind speed time series in blue and the Weibull fit of the data wind speed in red.

Figure 6.1: Overview of the generated wind speed series. The Weibull fits of the generated series are shown in blue, the Weibull fit of the data series in red.

From this figure can be seen that the wind speed data series is not exactly in the middle of the generated wind speed series. Ideally, this would be the case. To see whether better fits could be realised, the procedure explained in the previous paragraph was repeated with different values for $s$. It was found that $s = 10$ yielded the best results. Furthermore, in energy yield predictions in practice wind series with other $A$ and $k$ values are also used: Weibull distributions are usually generated for annual wind speed data, but annual variability in the wind speed exists. This thus yields different $A$ and $k$ values. Therefore, it is decided that these 5000 series are usable. Figures 6.2b and 6.2a show the distributions of the scale parameter $A$ and shape parameter $k$, through which a normal probability density function (PDF) is fitted.

(a) Distribution of the scale parameter $A$, through which a normal PDF is fitted with $\mu_A = 8.124$, $\sigma_A = 0.240$.

(b) Distribution of the shape parameter $k$, through which a normal PDF is fitted with $\mu_k = 2.162$, $\sigma_k = 0.061$.

Figure 6.2: Weibull parameters $A$ and $k$ of the generated wind speed series.
Another check that should be done on the generated wind speed series is whether they possess a plausible autocorrelation. The autocorrelation coefficient $r$ can be calculated using the autocovariance $c$ at a certain lag $j$. This is shown in Equations 6.2 and 6.3 [6].

$$c_j = \frac{1}{n-j-1} \sum_{t=j+1}^{n} (u_t - \bar{u})(u_{t-j} - \bar{u})$$ \hspace{1cm} (6.2)

$$r_j = \frac{c_j}{c_0}$$ \hspace{1cm} (6.3)

In these equations, $n$ represents the number of observations, $j$ the lag chosen (in this case, the lags checked are one hour, $j = 1$ and two hours, $j = 2$), $u_t$ the wind speed at hour $t$, $\bar{u}$ the mean of the series and $c_0$ the variance of the series. For the wind speed data series, $r_1 = 0.961$ and $r_2 = 0.919$. For the generated wind series, the mean values are lower: $r_1 = 0.907$ and $r_2 = 0.850$. However, these autocorrelation coefficients are still well in the range of real autocorrelation coefficients of measured wind speeds, based on research done by Brett and Tuller [6]. They determined the autocorrelation coefficients of hourly wind speeds for seven stations in Canada. The values for $r_1$ that they found are in the range from 0.773 to 0.909, and the values for $r_2$ from 0.657 to 0.855. Therefore, it is concluded that the wind speed series are good enough to use in this research.

The wind direction time series are also generated using the Markov chain method, only in this case, $s$ was chosen to be 36 (one state for each $10^\circ$ bin). Figure 6.3 shows the wind rose for the generated time series compared to the data time series. It can be seen that the mean of the generated time series fits the data time series. It was therefore concluded that these direction series are usable for this research.

![Figure 6.3: Overview of the generated wind direction series. The generated series are shown in blue, the data series in red.](image)

### 6.1.2 Theoretical availability series

Now that the necessary wind series are all generated, the same needs to be done for the theoretical availability time series. This section explains the procedure followed in order to create these time series.

Section 5.1.6 presented the failure rates and corresponding downtimes of different subassemblies of wind turbines in the WMEP programme. In total, 12 failure modes were considered. Using these 12 separate failure modes to generate 5000 theoretical availability time series is rather complex. Therefore, the choice was made to combine these 12 failure modes into 1 failure mode. According to the Institute of Electrical and Electronics Engineers, this can be done by summing up the separate failure rates. Mathematically this is shown in Equation 6.4.
Chapter 6: Results: comparing the adapted model to the current model

\[ \lambda_s = \sum_{i=1}^{n} \lambda_i \]  

(6.4)

In this equation, \( \lambda_i \) represents the failure rate of failure mode \( i \) and \( \lambda_s \) represents the failure rate of the entire system. Summing the failure rates shown in Table 5.1 gives a system failure rate of 2.43.

Besides the system failure rate, a mean time to repair (MTTR) of the entire system is also needed. A possibility for calculating the MTTR is to compute the weighted average of the downtimes shown in Table 5.1. This results in a system MTTR of 59 hours. This MTTR causes a low mean actual availability of 90.57%, significantly below the actual availability of 93.4% presented in Section 5.2.2. This implausible low availability is due to the width of the weather windows: a large weather window occurs less often than a small one. For a high system MTTR, the maintenance needs to be postponed until such a large weather window occurs, resulting in a low availability. Therefore, it was chosen to use the mode of the MTTR: the value that appears most often. This corresponds to the MTTR with the highest probability and is therefore in this case 1.53 days = 37 hours. Using this MTTR results in a mean actual availability of 96.21%. This value significantly lies above the actual availability of 93.4%, but is more realistic than an availability of 90.52% when compared to standard industry values.

Now that these values are known, it is possible to generate 5000 time series containing the theoretical availability. A common way to predict the probability of a failure is by using an exponential failure distribution, shown in Equation 6.5.

\[ F(t) = 1 - e^{-\lambda t} \]  

(6.5)

Using this distribution implies that a constant failure rate \( \lambda \) is assumed with respect to time, which is done in this research. To generate the time series, a random number from this exponential distribution with \( \lambda = 2.43 \) is chosen. This random number represents the hour at which a turbine fails after the repair of a failure. The corresponding downtime (for 100% accessibility) is 37 hours, after which the turbine switches on again. This is done until the entire year for each of the turbines in the farm is finished. When 5000 time series are combined, the annual failure rate of a turbine should be 2.43 and the MTTR should be 37 hours (for 100% accessibility). These conditions were checked and are satisfied: the mean failure rate of the 5000 time series is 2.43, with a minimum of 1.50 and a maximum of 3.67. The resulting theoretical availability is shown in Figure 6.4.

Figure 6.4: Histogram of theoretical availability. It can be seen that the result is a normal distribution with \( \mu = 0.9895 \) and \( \sigma = 0.0012 \).
6.2 Result of the current model

Now that the inputs for the models have been generated, Monte Carlo simulations can be performed and the results can be presented. First of all, the result of the current model is discussed. This result can be obtained in two ways. The first method closely resembles the processes explained in literature and was addressed in Chapter 3. This method uses normal distribution fits of the samples to obtain values for the mean AEP and the uncertainty in the AEP. The second method makes effective use of the available time series by using the samples directly, instead of the fits. This procedure is therefore slightly more advanced. Section 6.2.1 presents the result from the first method, Section 6.2.2 the result from the second.

6.2.1 Result of the current model using the procedure presented in literature

Section 3.3 presented the procedure that is followed to combine efficiencies and uncertainties in order to come up with a net annual energy production. This section follows this procedure to obtain a result for the AEP and uncertainty in the AEP, yielding a P90 value. For clarity, a brief step-by-step plan of the process as explained in Section 3.3 is given below:

1. Calculate the gross annual energy production, $AEP_{\text{gross}}$
2. Determine the array efficiency, $\eta_{\text{array}}$ and availability, $\eta_{\text{availability}}$
3. Calculate the combined efficiency, $\eta_{\text{combined}} = \eta_{\text{array}} \cdot \eta_{\text{availability}}$
4. Compute the net annual energy production → this forms the mean AEP
5. Determine the standard deviation of the wind on the energy yield, $\sigma_{\text{wind}}$
6. Determine the standard deviation of the array efficiency, $\sigma_{\text{array efficiency}}$, and the standard deviation of the availability, $\sigma_{\text{availability}}$
7. Use the root-sum-square method to determine the standard deviation of the AEP, $\sigma_{AEP}$

An advantage of the use of time series is that step 1 & 5 and step 2 & 6 can be combined. This works in the following way: the 5000 generated time series for wind speed, wind direction and theoretical availability, as explained in Section 6.1, are all different. These time series are used as an input for the model shown in Figure 4.1. This results in 5000 realisations of $AEP_{\text{gross}}$, 5000 realisations of the array efficiency and 5000 realisations of the availability. This means that $AEP_{\text{gross}}$, the array efficiency and the availability can be visualized in a histogram and represented by a distribution. Section 3.3 explained that industry currently assumes that these distributions are normal distributions. Therefore, normal distributions are fitted through the histograms. These are shown in Figures 6.5, 6.6 and 6.7. It can be seen that the normal distribution fits $AEP_{\text{gross}}$ and the array efficiency well, but this does not hold for the availability.

![Histogram of $AEP_{\text{gross}}$ resulting from 5000 samples. It can be seen that it can be approximated with a normal distribution with $\mu = 276.2$ GWh and $\sigma = 11.23$ GWh.](image)

Figure 6.5: Histogram of $AEP_{\text{gross}}$ resulting from 5000 samples. It can be seen that it can be approximated with a normal distribution with $\mu = 276.2$ GWh and $\sigma = 11.23$ GWh.
Chapter 6: Results: comparing the adapted model to the current model

Figure 6.6: Histogram of the array efficiency resulting from 5000 samples. It can be seen that it can be approximated with a normal distribution with $\mu = 89.59\%$ and $\sigma = 0.406\%$.

Figure 6.7: Histogram of the availability resulting from 5000 samples. It can be seen that it can be approximated with a normal distribution with $\mu = 96.21\%$ and $\sigma = 0.762\%$.

In the captions of Figures 6.5 – 6.7, the means and standard deviations are given. These are the means and standard deviations of the normal distribution fits. The means are used in Equation 3.6 to compute the combined efficiency and subsequently the net AEP (step 3 and 4 of the step-by-step plan). Please note that $\eta_{\text{other}}$ is not taken into account in this research. The combined efficiency resulting from this is $86.20\%$, the net AEP $238.1$ GWh.

Now that the mean AEP – or the P50 for a normal distribution – has been determined, the standard deviation can be calculated. Section 3.3 explained that this is currently done by using the RSS method. To do this, the standard deviations (in percent) of the wind on the energy yield, the array efficiency and the availability are needed. Once again, these conveniently follow directly from the fits in Figures 6.5 – 6.7. An extra advantage is that the steps to determine the SR, described in Section 3.3.1, are not necessary now; the standard deviation of the normal distribution shown in Figure 6.5 already represents the uncertainty in the wind on the energy yield. When this is converted to a fractional uncertainty, this yields $\sigma_{\text{wind}} = 4.066\%$. The uncertainty in AEP, $\sigma_{\text{AEP}}$, is calculated using Equation 3.8 and is $4.157\% = 9.895$ GWh.

Now that both the mean AEP and $\sigma_{\text{AEP}}$ have been determined, the normal distribution of the AEP can be constructed. This normal distribution is shown in Figure 6.8. From this normal distribution, the P90-value is derived. This corresponds to a value of $225.4$ GWh.

Figure 6.8: Normal distribution of AEP. The P90-value is indicated with the vertical line.
6.2.2 Result of the current model using the samples instead of the fits

The previous section presented the result of the current model, using a procedure that closely resembles the processes explained in literature. This involved using normal distribution fits to determine the mean and standard deviation of $AEP_{\text{gross}}$, the array efficiency and the availability. However, from Figure 6.7 is apparent that a normal distribution does not fit the 5000 availability realisations well. This might influence the result presented in Section 6.2.1.

There is however another way to obtain a result using the current model. This approach uses the time series in a more effective manner. Once again, the intermediate results for $AEP_{\text{gross}}$, the array efficiency and the availability from Figures 6.5 – 6.7 are used. The main difference of this slightly advanced approach with the method explained in Section 6.2.1 is that not the normal distribution fit is used, but the samples themselves. This works in the following way: the 5000 realisations of $AEP_{\text{gross}}$, the array efficiency and the availability are obtained in the same way as was explained in Section 6.2.1. Then, a large number $N$ is chosen. Randomly, $N$ samples are taken from the 5000 realisations. This results in $N$ times a single $AEP_{\text{gross}}$, a single array efficiency and a single availability. These are combined using Equation 3.6, resulting in $N$ times a value for AEP. The result is a histogram of the AEP, which can once again be represented by a normal distribution. The result for $N = 100000$ is shown in Figure 6.9.

![Figure 6.9: Histogram and normal distribution of the AEP using a slightly more advanced approach.](image)

The normal distribution that is fitted through the histogram has a mean of $238.1 \text{ GWh}$ and a standard deviation of $9.894 \text{ GWh}$. This yields almost the same result as the method presented in Section 6.2.1.

The P90 value following from the normal distribution fit is $225.4 \text{ GWh}$ as well. The P90 value that follows directly from the 100000 samples is $225.5 \text{ GWh}$, only marginally higher than that of the fit.

6.3 Result of the adapted model

The previous section presented the results of the current method. These results were obtained using two different approaches. The main interest however, is how these results compare to the result of the adapted model. This result is presented in this section.

To obtain the results for the current model, some post-processing was needed. This is however unnecessary for the adapted model; the actual power output that the model calculates is immediately the net AEP. Once
again, the model is run using the 5000 time series for wind speed, wind direction and theoretical availability. The 5000 samples that result from this simulation are shown in the histogram in Figure 6.10. The shape can once again be approximated by a normal distribution; this is also visible in the figure. From Figure 6.10 can be seen that the normal distribution fits the distribution of the samples well. The normal distribution has a mean of $238.9 \text{ GWh}$ and a standard deviation of $9.988 \text{ GWh}$. The P90-value following from this normal distribution is $226.1 \text{ GWh}$. The P90-value that follows directly from the 5000 samples is $226.3 \text{ GWh}$.

![Figure 6.10: Histogram and normal distribution of the AEP using the adapted model.](image)

### 6.4 Comparison of the results

Sections 6.2 and 6.3 presented the results of the current and the adapted model. This section compares these results and by doing that, answers subquestion 2 from Section 1.3. The results that were found in the previous two sections are gathered in Table 6.3. In the last column of this table, the difference between the current and adapted model is shown as a percentage. Furthermore, the normal distributions are plotted in one graph, to make the differences between the two models graphically visible. This can be observed in Figure 6.11.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Current model: approach 1</th>
<th>Current model: approach 2</th>
<th>Adapted model</th>
<th>Difference w.r.t. approach 1</th>
<th>Difference w.r.t. approach 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>AEP = P50 [GWh]</td>
<td>238.1</td>
<td>238.1</td>
<td>238.9</td>
<td>0.34%</td>
<td>0.34%</td>
</tr>
<tr>
<td>$\sigma_{AEP}$ [GWh]</td>
<td>9.895</td>
<td>9.894</td>
<td>9.988</td>
<td>0.93%</td>
<td>0.95%</td>
</tr>
<tr>
<td>P90$_{fit}$ [GWh]</td>
<td>225.4</td>
<td>225.4</td>
<td>226.1</td>
<td>0.31%</td>
<td>0.31%</td>
</tr>
<tr>
<td>P90$_{samples}$ [GWh]</td>
<td>n.a.</td>
<td>225.5</td>
<td>226.3</td>
<td>n.a.</td>
<td>0.35%</td>
</tr>
</tbody>
</table>
First of all, it can be noticed that there is hardly any difference between the two approaches that were used to obtain a result for the current model. There is a slight difference of 0.02% in the uncertainty of the two methods, but this is considered to be negligible. Furthermore, a minor difference can be observed between the P90 resulting from the normal fit and the P90 resulting directly from the samples. This, however, is only 0.04% and therefore also considered insignificant.

The differences between the results from the current model and the adapted model are slightly larger, as would be expected. There is a difference of 0.8 GWh between the P50-values, and a difference 0.7/0.8 GWh between the P90-values between the two models. The percentages stated in the table – 0.31%-0.35% – indicate that this difference is not striking. This answers subquestion 2 from the introduction; it seems justified that industry currently neglects the nonlinear effect of combining uncertainties, at least for conditions similar to the ones of this study.

This small difference does however make sense. A logical hypothesis is that the current model and the adapted model should give the exact same results at an availability of 0% and at an availability of 100%. After all, at an availability of 100%, all turbines are operative all year, meaning that the net annual energy production is simply the gross annual energy production multiplied with the array efficiency for both models. At an availability of 0%, none of the turbines is operative, leading to a net annual energy production of 0 GWh. Furthermore, it is known that the current model applies a linear relationship between the net AEP and the efficiency factors (as shown in Equation 3.6), while the adapted model tries to exhibit the nonlinear effect of combining uncertainties. Combining the hypothesis that both models yield the same results at 0% and 100%, and the difference between linearity and nonlinearity, leads to the supposition that the difference between the current and the adapted model should be low near the extremes in availability.

To prove and quantify this assumption, the adapted model was run for 200 different theoretical availability series, whilst using only one wind speed and wind direction time series (see Appendix E for the wind speed frequency distribution and wind rose). The 200 theoretical availability time series were generated by varying the system failure rate from 2 to 250 in 200 steps. Eventually, the net annual energy production was plotted against the availability. The line of the current model was plotted in the same graph. The result is shown in Figure 6.12.
Chapter 6: Results: comparing the adapted model to the current model

Figure 6.12: Graph showing the nonlinear effect of availability on the annual energy production.

From Figure 6.12 can be seen that the hypothesis is correct. The models perform exactly the same at 0% and 100% availability. The nonlinearity of the adapted model can clearly be seen from this figure as well. As expected, the difference between the current and the adapted model is negligible near the extremes. However, in the middle, this difference becomes higher. This is visualized in Figures 6.13 and 6.14, in which the absolute difference (in GWh) and relative difference (in %, with respect to the current model) is shown.

![Figure 6.13: Absolute difference between the current and adapted model.](image)

From these figures can be seen that the difference near the extremes is indeed quite small. However, already at an availability of 90%, this difference becomes almost 3 GWh. This means that – especially in the later life of a wind farm, when the number of failures might increase – this nonlinear effect might become relevant for the wind farm developer.
Approximation of the nonlinear effect using existing tools

The previous chapter presented the results of the current and adapted model and explained why these results make sense. However, it took a while to run both models, since 5000 years—over 40 million hours—were simulated. Depending on the hardware of the computer that is used for these simulations, this can take days. Next to that, it takes weeks up to months to develop an adapted model. This might be expensive, especially if the result of the adapted model turns out to be insignificant with respect to the current model. It is therefore desirable to use some sort of approximation for the nonlinear effect, in order to get a sense of its size before developing and implementing an adapted model. This chapter presents such an approximation, using tools that already exist. The previous chapter has shown that the main difference between the current and the adapted model lies in the mean AEP. This approximation therefore only considers the nonlinear effect on the mean and not on the standard deviation.

7.1 Proposal for approximation

Due to the high development and computational time of the adapted model, it would be advantageous to have a simple approximation that enables the wind farm developer to make a guesstimate of the nonlinear effect. This approximation should provide a reasonable estimate of the size of the nonlinear effect. The wind farm developer can choose, on the basis of the outcome of the approximation, whether he would like to perform a full sampling simulation. One of the main requirements of this approximation is that it makes use of existing tools. In that way, a wind farm developer does not need to acquire or build new tools—which also cost money. The proposal done in this section is therefore rather basic.

It is assumed that most wind farm developers make use of some sort of software to predict the energy yield of a wind farm, for example WindPRO, WindFarmer or WindSim. These software packages are able to make an energy yield prediction using the current method, presented in Chapter 3. This means that these software packages incorporate models that calculate the gross annual energy production and array efficiency for a chosen farm layout. The proposed approximation uses such packages. Furthermore, it is assumed that wind farm developers have a good idea of the yearly availability of a wind farm, due to their earlier experience. This availability can be translated to a number of wind turbines that is shut down. For example: if a wind farm consisting of 30 wind turbines has an availability of 97%, this means...
that on average, 1 turbine is switched off the entire year. For an availability of 93%, 2 turbines are switched off the entire year. These numbers are used to create test-layouts. These test-layouts are exactly the same as the proposed wind farm layout, except that a number of turbines corresponding to the estimated availability is deleted from this layout. The next section elaborates on which turbines should be deleted. These test-layouts are then used as an input in the wind farm software package. The software calculates a specific array efficiency. This array efficiency is likely higher than the array efficiency of the original wind farm layout. The array efficiency of the test-layout is then multiplied with $AEP_{\text{gross}}$ (of the entire farm) and the estimate of the availability. This yields a net annual energy production that is supposed to approximate the annual energy production calculated by the adapted model.

In short, the following steps should be taken by the wind farm developer:

1. Convert the first estimate of wind farm availability to a number of turbines that is switched off.
2. Delete this number of turbines from the proposed wind farm: this creates a test layout. If the number resulting from step 1 is not a round figure, create two test layouts: one in which the rounded-down number is deleted from the proposed farm, and one in which the rounded-up number is deleted.
3. Use this test-layout as an input for the wind farm software package. This yields a specific array efficiency. If the number of turbines resulting from step 1 was not round and two test layouts have been used, use a weighted average on the results from the two test layouts to determine the specific array efficiency.
4. Multiply $AEP_{\text{gross}}$ of the complete farm with the array efficiency from the previous step and the availability: this gives an indication of the combined effect of array efficiency and availability on the annual energy production.

### 7.2 Examples of approximation for various availability values

The previous section proposed a method to approximate the nonlinear effect of combining array efficiency and availability on the mean of the annual energy production. But how accurate is this approximation? This section gives examples of the approach explained above and compares them to the nonlinear curve presented in Section 6.4.

First of all, it is sensible to make a decision about which cases are tested. It is important that realistic examples are carried out (for these examples, the availability percentages should be higher than 90%). However, the previous chapter has shown that near the extremes, the nonlinear effect is negligible. For the purpose of showing the effectiveness of the approximation, it makes sense to also include availabilities that are lower, but better indicate the accuracy of this simple approximation. Therefore, it was chosen to carry out the approach for the availabilities stated in Table 7.1. The number of turbines that are switched off, corresponding to these availabilities, are stated in the second column.

#### Table 7.1: Overview of cases tested using the simple approach.

<table>
<thead>
<tr>
<th>Availability [%]</th>
<th>Number of affected turbines</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>15</td>
</tr>
<tr>
<td>70</td>
<td>9</td>
</tr>
<tr>
<td>90</td>
<td>3</td>
</tr>
<tr>
<td>93.3</td>
<td>2</td>
</tr>
<tr>
<td>96.7</td>
<td>1</td>
</tr>
</tbody>
</table>

An important question that already arose in the previous section, is which of the turbines in the farm should be deleted. To find out if there is a “best practice” for this, different turbines are deleted for the several cases. The corresponding values for the array efficiency are gathered and checked. To do this in a systematic way, the turbines have been numbered as shown in Figure 7.1.
Chapter 7: Approximation of the nonlinear effect using existing tools

Figure 7.1: Overview of the numbered turbines.

For the case of an availability of 96.7%, each turbine was deleted once to see how this influences the array efficiency. This layout was run in the current model, using the wind speed frequency distribution and wind rose from Appendix E in order to be able to compare the result with Figures 6.12 and 6.13. Figure 7.2 shows the result. It can be seen that the maximum difference between the highest and lowest array efficiency is only 0.62 percentage point. A comparison of the AEP is shown in Figure 7.3.

Figure 7.2: Array efficiency of the test-layout, depending on which turbine was deleted from the original layout. The yellow line indicates the array efficiency of the original layout.
Figure 7.3: AEP of the test-layout, depending on which turbine was deleted from the original layout. The yellow line indicates the AEP of the current model at an availability of 96.7%. The purple line indicates the AEP of the adapted model at an availability of 96.7%.

From Figure 7.3 can be seen that the mean of the approximations closely resembles the result from the adapted model; their difference is only 0.025%. Furthermore, it can be seen that the result of the approximation is always higher than the result of the current model. The effect of the deletion of specific turbines on the result seems rather arbitrary. The deletion of turbines 30 or 19 yield the lowest results. For turbine 30 this makes sense: from Figure 7.1 can be seen that this turbine is located at the absolute edge of the farm. Turbine 19 however is located in the middle of the farm. It is therefore difficult to make a particular recommendation on which turbine to delete.

The same approach was carried out for the 93.3% case. The difference in array efficiencies for various turbine combinations is shown in Appendix F. For the cases where 3, 9 or 15 turbines are affected, only one approximation was made, because the number of possible combinations is too high to test them all (4060 for the 3 turbine case, over 14 billion for the 9 turbine case and even more for the 15 turbine case). For the 90% case, turbines 5, 20 and 28 were deleted. For the 70% case, turbines 1, 3, 12, 14, 16, 20, 23, 28 and 30 were deleted. For the 50% case, turbines 2, 3, 4, 5, 6, 9, 14, 15, 17, 19, 21, 25, 28, 29 and 30 were deleted. The results are gathered in Table 7.2, and graphically visualized in Figure 7.4.

Table 7.2: Result of the approximation compared to the results of the current and adapted model.

<table>
<thead>
<tr>
<th>Availability [%]</th>
<th>Result current model [GWh]</th>
<th>Result adapted model [GWh]</th>
<th>Result approximation [GWh]</th>
<th>Difference between approximation and adapted model [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>123.69</td>
<td>131.65</td>
<td>131.78</td>
<td>0.099</td>
</tr>
<tr>
<td>70</td>
<td>173.13</td>
<td>179.85</td>
<td>179.33</td>
<td>0.290</td>
</tr>
<tr>
<td>90</td>
<td>222.64</td>
<td>225.51</td>
<td>225.57</td>
<td>0.027</td>
</tr>
<tr>
<td>93.3</td>
<td>230.89</td>
<td>232.87</td>
<td>233.04 (mean)</td>
<td>0.073</td>
</tr>
<tr>
<td>96.7</td>
<td>239.13</td>
<td>240.16</td>
<td>240.22 (mean)</td>
<td>0.025</td>
</tr>
</tbody>
</table>
Figure 7.4: Graphical representation of the results of the approximation. The test cases are plotted on the lines that were also shown in Figure 6.12.

From this table and figure can be seen that the approximation presented in Section 7.1 gives a very good indication of the size of the nonlinear effect. This method might therefore help wind farm developers in deciding whether they want to spend time and money on the development of a similar model as presented in Chapter 4. They might even decide to use the result of this approximation as the final answer.
Generic methodology for determining nonlinear effects of combining uncertainties

The previous chapters have shown how the nonlinear effect between array efficiency and availability can be determined and approximated. Although the results from Chapter 6 did point out that this nonlinear effect is quite small for high availabilities, it does affect the energy yield predictions significantly for lower availability values. The combination of array efficiency and availability is not the only situation in which nonlinearity occurs. It is therefore recommended to investigate what effect other nonlinear combinations might have on the energy yield predictions.

Next to the nonlinear effect of combining uncertainties on the energy yield, the combination of uncertainties might also have a nonlinear effect on other parameters of an offshore wind farm. Loading conditions are for example also subject to uncertainty. Currently, safety factors in design are used as a compensation for this uncertainty, leading to high construction and maintenance costs [37]. If the uncertainty in the loading conditions are quantified more accurately, these costs might be lowered.

Since the methodology that was developed to determine the nonlinear effect for this case study has proven to work, the steps described by the previous chapters have been converted into a generic methodology. This chapter presents this methodology.

Section 1.2 explained that it is already known that possible correlations and nonlinear relations between several uncertainty sources that affect the energy yield exist. This was investigated during a literature study prior to this research. Figure 8.1 shows eight interdependencies affecting the energy yield that were identified during this study. A short description of each interdependency can be found in Appendix G.

It is interesting to see if, and how, these interdependencies influence the energy yield predictions. Next to that, it is important that a similar overview as shown in Figure 8.1 is created for uncertainties affecting other parameters, like the loading conditions. When these overviews are created, the nonlinear effect of each of the interdependencies can be determined. Based on the research presented in the previous chapters, a step-by-step procedure has been formulated.

1. Map the physical relations of the remaining interdependencies
   Chapter 2 zoomed in on interdependency 8 of Figure 8.1 and identified the physical relations between array efficiency and availability. By doing this, the nonlinear effect became clear: the effect of downtime on the array efficiency is something that is not yet taken into account in industry. Besides investigating
2. Investigate how the separate uncertainty sources are treated currently
Chapter 3 was devoted to the description of the current methodology entirely. This is an important step for multiple reasons. First of all, knowing the state-of-the-art methods is required in order to make comparisons between current models and possible adapted models. It however also gives an indication of the complexity of the adjustments that should be made to the current model. In this case study, the adjustments proved to be relatively straightforward – instead of determining array efficiency and availability separately, availability had to be used as an input for the array efficiency calculations. This might not be the case for other interdependencies.
An important subtask of this step that should be emphasized is the investigation of wind farm developer software. It was already mentioned that wind farm developers often make use of software packages. It might be the case that some of the interdependencies from Figure 8.1 are well hidden in this software. It is important to find out how the software packages treat the combinations, since it might even be that possible nonlinearities are already incorporated in their models. On the other hand, the negligence of the nonlinear effect can become clear immediately, when separate software packages are used to generate intermediate results.

3. Try to develop an approximation
This step deviates from the procedure followed for this case study. In this research, first an adapted model was developed, verified and tested. Afterwards, an approximation of the nonlinear effect was proposed. For future research it is however recommended to swap these two steps. If a rough but credible approximation can already be made, this might affect the decision to develop an adapted model: for negligible differences between including and not including nonlinearity, the choice can be made to skip the remaining steps, saving resources. It is important to add to this that it can be difficult or even
impossible to develop such an approximation.

The approximation proposed in this research uses average values to determine the effect of the interaction: an availability of 90% was translated to an average of 3 turbines that are switched off during one year. The adapted model applies this interaction within a time series, which is stronger and more accurate but at the same time costs more time. Furthermore, the existing tools might not support time dependent interactions. A suggestion for developing the approximation is therefore to try to convert the time dependent interaction to an average, similar to the approximation in this research. This is more time-efficient and better suitable for existing tools.

4. Develop and verify (and validate) an adapted model

If the decision has been made to proceed with the identification of the nonlinear effect, an adapted model needs to be developed. To do this, the results of step 1 and 2 are particularly important. The investigation of the current model should have revealed where the state-of-the-art procedures are inadequate. At the same time, the mapping of the physical relations has indicated where the nonlinear effect arises. The combination of both determines what the adapted model should look like. When the adapted model has been developed, it needs to be verified and ideally also validated. For this case study, only verification could be performed. This was done by comparing numerical (intermediate) results to analytically obtained results, but also by checking if the code produced results that matched reasoned expectations.

5. Determine what the nonlinear effect is on the annual energy production

Now that the adapted model has been developed and verified, results can be generated. It is important that these results are compared with the result from the current model, such that the effect of the nonlinearity becomes clear. It is important that not only the mean AEP is taken into account, but also the uncertainty in the AEP. For this case study, it turned out that the difference in uncertainty between the current and the adapted model was insignificantly small. This might however be different for other interdependencies.

When the steps above have been carried out for all interdependencies, the overall nonlinear effect of combining uncertainties on the energy yield of an offshore wind farm can be determined. This is an important goal of the EUROS programme mentioned in the introduction of this report. By more accurately combining uncertainties, offshore wind farms can be developed more efficiently. This might ensure that costs of offshore wind energy are lowered.
Chapter 9

Conclusions and recommendations

The main purpose of this research was to develop a methodology that can be used to determine if, and how, the nonlinear effect of combining two uncertainty sources should be incorporated in the energy yield prediction, by investigating the case study of the nonlinear effect of combining array efficiency and availability. This objective was divided into four subgoals. This chapter states the most important conclusions of this research in Section 9.1. Recommendations for future work are given in Section 9.2.

9.1 Conclusions of the work presented in this report

This research has developed a methodology on how to determine the nonlinear effect of combining uncertainties on the energy yield of an offshore wind farm, by investigating the case of combining array efficiency and availability. By conducting a structured approach, it was demonstrated that the size of the nonlinear effect can be determined and approximated successfully. This approach can therefore be translated to a generic methodology, meaning that the main objective of this research has been achieved.

The methodology consists of five steps. First, the physical relations between two uncertainty sources need to be mapped. For this case study, the physical relations between array efficiency and availability revealed that downtime of a wind farm affects its array efficiency.

The second step is to investigate the state-of-the-art methods and compare these to the physical relations of the first step, such that possible shortcomings can be identified. In this case, this step showed that industry assumes a linear relationship between efficiency factors, the gross annual energy production and the net annual energy production, and that the uncertainties of these components are independent. These assumptions allow a simplified method for determining the combined uncertainty, but neglect the nonlinear effect that was revealed in the first step.

The third step suggests to develop an approximation that is able to estimate the size of this nonlinear effect. If this approximation indicates a significant nonlinear effect, the development of an adapted model is worthwhile. In this case study, the availability of a wind farm was converted to an average number of turbines that is switched off during the year and used to create temporary test layouts. These test layouts were used to calculate the array efficiency, such that the effect revealed in the first step was taken into account whilst at the same time using the existing tools. This proved to be an effective method; the differences between the
approximation and the results of the developed adapted model were less than 0.5%. However, it is difficult to generalise the approach for this approximation for other interactions.

The fourth step is to develop an adapted model. The layout of this model is determined by the results of the first and second step. The shortcoming that was identified in the second step should be incorporated in the adapted model. For this case study, this meant that the output of the availability model was used as an input for the wake model. Furthermore, the adapted model had to be able to handle time series. Once again, it is difficult to generalise this step for other interdependencies, since the adapted model might deviate substantially from the current, state-of-the-art models.

The fifth and final step is to execute this model to obtain results for the P50, P90 and uncertainty of the annual energy production. An effective approach to obtain these results is to use the Monte Carlo method. This method is able to generate multiple samples. These samples follow a particular distribution, from which the mean and standard deviation can directly be derived and compared to the results of the state-of-the-art methods. This approach is therefore suggested to determine the effect of other interdependencies as well. For this case study, it was found that for a high mean availability of 96.2%, the difference between the current and the adapted model is less than 1% for both the mean annual energy production and the uncertainty in annual energy production. The state-of-the-art methods can therefore be justified, since this availability falls in the range of typical industry values. However, for lower availabilities, the nonlinear effect does become significant.

9.2 Recommendations for future work

Although the research questions have been answered and the research goals have been achieved, several recommendations for future work can be done. These recommendations can be split up in two classes. The first considers the combination of array efficiency and availability specifically, whereas the second considers the general case of combining uncertainties.

Recommendations for the combination of array efficiency and availability

- The model that was developed in this research has been programmed in the programming environment MATLAB®. However, most wind farm developers use dedicated software packages, like WindPRO, WindFarmer or WindSim. It would be interesting to see if the models used in these packages can be adapted in a straightforward way such that they include the nonlinear effect.

- The adapted model that was developed in this research has not been validated. If, in the future, data sources can be obtained that belong to one particular farm, it would be desirable to validate the model.

- Due to the large computational time, simulations have only been performed on one, relatively small, wind farm. In the future, having more results, for various types of wind farms, would increase the knowledge of the nonlinear effect of combining array efficiency and availability. Particularly, it would be interesting to see if the relative difference between the current and adapted model for a specific availability value stays the same or varies for different types of farms.

Recommendations for the general case of combining uncertainties

- The methodology presented above works for the case of combining array efficiency and availability. It is on the other hand by no means a best practice. It is recommended to evaluate the methodology for multiple nonlinear effects and adjust the procedure where necessary.

- In this research, only the nonlinear effect of combining array efficiency and availability was investigated. It would be interesting to know how other nonlinear effects contribute to the annual energy production, and what effect all these nonlinear effects on the total annual energy production.
• In addition to the previous recommendation: next to the uncertainty in energy yield of an offshore wind farm, there are also uncertainties in other parameters, such as the loading conditions. In order to cut costs of offshore wind energy in general, the knowledge about all these uncertainties should be increased. It is therefore recommended that a similar approach as the one developed in this research is carried out for uncertainties in these other parameters.

• Of the methodology presented above, the approximation of the (nonlinear) effect of an interdependency is most difficult to generalise. Subsequent research into these approximations could perhaps provide additional handles to develop such an approximation.

• One of the conclusions of this research was that there is little difference between the uncertainty in the AEP calculated using the current and the adapted model, even though it is known that the current model makes incorrect assumptions. The uncertainty was determined by the adapted model by using the Monte Carlo approach. It would be interesting to see if using other approaches yield uncertainties of the same magnitude as those that were found in this research.


Appendix A

Proof of linear relationship between efficiencies and net annual energy production

Section 3.3.1 described that the mean annual energy production is calculated by multiplying efficiency factors with the gross annual energy production. At the same time, literature often states that the net AEP is equal to the gross AEP minus the losses. To find out what the actual relation is, a derivation was carried out. This derivation is presented in this appendix. Please note that only array efficiency and availability are used – other efficiency factors are omitted.

The relation for determining the net annual energy production is given by Equation A.1.

$$AEP_{\text{net}} = AEP_{\text{gross}} \cdot \eta_{\text{availability}} \cdot \eta_{\text{array efficiency}}$$  \hspace{1cm} (A.1)

The efficiency factors can be written as functions of the corresponding losses. This is shown in Equation A.2.

$$\eta_{\text{array efficiency}} = \frac{AEP_{\text{gross}} - L_{\text{array}}}{AEP_{\text{gross}}}$$
$$\eta_{\text{availability}} = \frac{\text{hours operational}}{\text{total hours}}$$
$$L_{\text{availability}} = \frac{\text{hours off}}{\text{total hours}} \cdot AEP_{\text{gross}}$$
$$= \frac{\text{total hours} - \text{hours operational}}{\text{total hours}} \cdot AEP_{\text{gross}}$$
$$= (1 - \eta_{\text{availability}}) AEP_{\text{gross}}$$

These expressions can be substituted in Equation A.2. This is shown in Equation A.3.

$$AEP_{\text{net}} = AEP_{\text{gross}} \cdot \left(1 - \frac{L_{\text{availability}}}{AEP_{\text{gross}}} \right) \cdot \left( \frac{AEP_{\text{gross}} - L_{\text{array}}}{AEP_{\text{gross}}} \right)$$
$$= (AEP_{\text{gross}} - L_{\text{array}}) \left(1 - \frac{L_{\text{availability}}}{AEP_{\text{gross}}} \right)$$  \hspace{1cm} (A.3)
This can be rewritten into the final expression for $AEP_{\text{gross}}$, shown in Equation A.4.

$$AEP_{\text{net}} = AEP_{\text{gross}} - L_{\text{availability}} - L_{\text{array}} + \frac{L_{\text{array}} \cdot L_{\text{availability}}}{AEP_{\text{gross}}} \quad (A.4)$$

It can be seen that the positive term at the end of this equation differs from simply subtracting the losses from the gross annual energy production. This is also visualized in Figure A.1.
Appendix B

Detailed flowchart and explanation of the adapted model

This appendix gives an overview of the steps taken in the adapted model. For clarity, the flowchart is given in Figure B.1.

The first steps are relatively straightforward. The input data is loaded into the program. The wave height can be approximated using Equation 5.4. This (hourly) wave height is then converted to a three-hour time series. The three-hour time series is used to determine the accessibility of the farm. Then, the wind data can be converted to wind speeds at hub height, using Equations 5.1–5.3.

Before the loop is entered, the fixed parameters can be calculated. This is more efficient. Based on the X− and Y−coordinates of the turbines in the wind farm, all turbine distances, turbine angles and ultimate wake angles can be calculated. This is done using the procedure explained in Section 4.2.1.

Then, the time loop is started. At first, the wind speed and wind direction at the specific hour $t_t$ are identified. This wind direction is then used to sort the turbines from most upwind to most downwind. This is convenient for later use: to determine the power output of the downwind turbines, the wind speed deficits of the upwind turbines are needed.

Subsequently, the wind direction is compared to the wake angles defined before the loop. By doing this, the model knows which turbines experience a (full or partial) wake. The lens areas (explained in Appendix C) are then calculated only for the turbines that experience a partial wake.

If the lens areas and wake types are known, the model checks whether there are turbines that experience multiple wakes. If this is the case, the overlapping areas are determined. These areas are used to determine the local wind speed that each turbine experiences.

When this is done, the power module of the model is entered. First of all, the model checks how much energy a single turbine can generate if it experiences the undisturbed wind speed. This is multiplied by the number of turbines in the farm, yielding the reference power output. The local wind speeds determined in the previous module function as the input for the actual power output. For each turbine, it is checked how much power it actually generates.

These steps are repeated for each hour of the year. If the entire year is done, the powers can be added to determine the annual energy production. The actual annual energy production divided by the reference annual energy production determines the combined efficiency of the farm.
**Input**: atmospheric conditions, turbine data, farm layout, wind data, failure data

Approximate wave height using wind data (at $h = 10m$)

Use wave height, failure data and number of turbines to determine actual availability

Convert wind speed data at $h = 10m$ to wind speed at hub height

Determine $d_{w,h}$, full and partial wake angles for each turbine combination

For $tt = 1/8760$ (hrs/year)

Sort turbines from most upwind turbine to most downwind turbine

Determine wake type for each operative turbine (full, partial or null)

Sum $P_{act,t}$ and $P_{ref}$ of farm over entire year: this yields the combined efficiency

Entire year done?

Determine $P_{act,t}$ and $P_{ref}$ of entire farm during this specific hour

Determine actual power output of each turbine that is operative: $P_{act,turbine}$

Determine power output of a single turbine under these wind conditions: $P_{ref,turbine}$

Determine local wind speeds on each operative rotor

Check if rotor is subject to multiple wakes and determine overlapping areas

Determine lens areas of partial wakes (if any)

Figure B.1: Detailed flowchart of the adapted model.
Appendix C

Determination of asymmetric lens areas

Section 4.2.1 described that partial wakes cause an area with the shape of an asymmetric lens on the downwind rotor. This appendix describes how to calculate the area of an asymmetric lens [43].

Consider two circles with radii $R_1$ and $R_2$ that intersect each other. Circle 1 is centered at $(0, 0)$ whereas circle 2 is centered at $(d, 0)$. This is visualized schematically in Figure C.1.

![Figure C.1: Two circles intersecting.](image)

The equations for the two circles are:

\[
\begin{align*}
    x^2 + y^2 &= R_1^2 \\
    (x - d)^2 + y^2 &= R_2^2
\end{align*}
\]
Combining yields:

\[(x - d)^2 + (R_1^2 - x^2) = R_2^2\]
\[x^2 - 2dx + d^2 - x^2 = R_2^2 - R_1^2\]
\[x = \frac{d^2 - R_2^2 + R_1^2}{2d} = d_1\]

d_2 can also be defined:

\[d_2 = d - x = \frac{d^2 + R_2^2 - R_1^2}{2d}\]

The area can now be calculated using the following equation:

\[
A = R_2^2 \cos^{-1} \left( \frac{d^2 + R_2^2 - R_1^2}{2d} \right) + R_1^2 \cos^{-1} \left( \frac{d^2 - R_2^2 + R_1^2}{2d} \right) - \frac{1}{2} \sqrt{(-d + R_2 + R_1)(d + R_2 - R_1)(d - R_2 + R_1)(d + R_2 + R_1)}
\]
Appendix D

Wind farm layout of Sheringham Shoal and Walney 1

Figure D.1: Layout of the offshore wind farm Sheringham Shoal.
Figure D.2: Layout of the offshore wind farm Walney 1.
Appendix E

Wind speed and wind direction distribution used for visualizing the nonlinear effect and the approximation.

Figure E.1: Histogram and Weibull distribution of wind speeds used for the nonlinear effect and approximation.
Figure E.2: Number of occurrences of wind directions used for the nonlinear effect and approximation.
Appendix F

Difference in approximation array efficiency for various turbine combinations

Figure F.1: Variation in approximation of array efficiency of various deleted turbine combinations.
Appendix G

Description of interdependencies between uncertainty sources

1. Relation between air conditions and wind
   Wind is caused by differences in the atmospheric pressure. Pressure is related to the air temperature and density through the equation of state, \( P = R \cdot \rho \cdot T \). This means that each uncertainty in the atmospheric conditions has an effect on the (uncertainty in the) wind speed.

2. Relation between air conditions and turbine performance
   The interdependence between the air conditions and turbine performance is relatively straightforward. The power curve of the turbine is established by the manufacturer and holds under certain atmospheric conditions; if these conditions are different, the turbine might also perform differently. Next to that, the equation to calculate the power contains the air density. The air density thus influences the generated power directly.

3. Relation between wind and turbine performance
   The relationship between wind and turbine performance is also easy to see. The wind speed is the most important parameter in the power equation and directly influences the turbine performance.

4. Relation between wind and wake loss
   Wake effects are dependent on a number of parameters. Important wind parameters are the incoming wind speed and the wind direction. This means that slight changes in these parameters influence the wake effects. Since wake losses range between 10% and 20%, this influence might have a significant effect on the total energy loss.

5. Relation between wind and availability
   The relation between wind and availability is a little less trivial. Availability is defined as the percentage of time that a wind turbine is available for power production, and thus depends on maintenance, faults and component failures. When the wind behaves differently than expected, this might have an effect on component failures. For example; higher wind speeds than predicted might lead to a higher component failure rate or reduced site access for maintenance. Uncertainty in the wind therefore might lead to additional uncertainty in the availability.
6. **Relation between turbine performance and wake loss**

Wake losses are not only affected by the wind, but also by the turbine performance. Wake models use the thrust coefficient as an input to determine the wake effects. The thrust coefficient is a component of the turbine performance and strongly dependent of the blade performance. Any uncertainty in the blade performance leads to an uncertainty in the thrust coefficient and thus to an uncertainty in the wake loss.

7. **Relation between turbine performance and electrical losses**

Electrical losses occur mostly in the cables and wiring of the wind farm electrical system. These losses are influenced by the power that is transferred through the cables and the operating temperature inside the nacelle and tower [30]. The generated power and the temperature produced are both effects from the turbine performance.

8. **Relation between availability and wake loss**

The relation between availability and wake loss can have a significant effect on the energy yield of an offshore wind farm. If a wind turbine is not producing power due to for example scheduled maintenance, the turbine will not create a wake. This means that turbines that would normally be affected by the wake, can now produce energy with the unperturbed wind speed. This yields a lower energy loss and might thus be beneficial for the AEP.