Second Order Roll of Semi-Submersible Crane Vessel
Second order roll of semi-submersible crane vessel

By

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An electronic version of this thesis is available at http://repository.tudelft.nl/.
This thesis presents the study on “Second order roll of the semi-submersible crane vessel” in order to fulfil the requirement for the Master degree in Offshore and Dredging Engineering. The topic is proposed and supported by Heerema Marine Contractors to gain a better understanding of the nonlinear roll motion of the new semi-submersible under construction.

It is really a great experience trying to explain a physical phenomenon as a complete story in this thesis, I learnt a lot during the whole process and met many nice and smart people. I would like to thank my company supervisor Bob Hoogendoorn a lot, for him not only shares a lot of hydrodynamics knowledge with me, but also gives me a lot of advice with regard to expressing my ideas and communicating with people in a better way. I also learn a lot from my committee members within Heerema Marine Contractors, Geert Meskers, Radboud van Dijk, Job Bokhorst and Eelco Harmsen, they are very intelligent and knowledgeable people, and always provide a lot new inspirations for my thesis. Also special thanks for Shen Qing, she is very smart and always willing to help, and many thanks for Yannis, for his time and interest discussing second order problems with me. Of course, I would like to thank all colleagues in Heerema Marine Contractors who support me during my internship.

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Last but not least, I would like to thank all my friends and family members, for your support during my 2 years study in Netherlands.

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Heerema Marine Contractors is building the world’s largest semi-submersible crane vessel ‘Sleipnir’. During the design phase of this vessel, model tests were performed to evaluate the hydrodynamic properties. In the model test, a significant roll motion at a period of 40s was found. Because this response is not within the range of first order wave excitation, it received great attention. Later, the 40s is proved to be the natural roll period of the vessel which can be excited by the difference frequency of the two first order wave components, thus it is called a 2nd order roll in this thesis. The aim of this thesis is to explain this phenomenon in a better way and to investigate whether this 2nd order roll identified in the model test will be a true problem in reality.

In the first part of the thesis, the physical reasons for the existence of 2nd order roll as well as the methodology to quantify 2nd order roll are discussed. Besides the resonance effect, the nonlinear excitation is proved to be quite considerable and the damping ratio is low within the natural frequency range because of the ship’s shape features. For the nonlinear excitations, Quadratic Transfer Functions (QTF’s) are firstly calculated based on the Pinkster’s theory. The second order forces and moments are further simulated by three methods in both frequency domain and time domain, where the advantages and limitations of each method are discussed. For the damping, special attention is paid to the linearization method in the frequency domain and the application of Cummins’ equation in the time domain. The use of a cubic damping term is proposed to fix the inaccuracies of the current methods. Furthermore, the second order roll motion is simulated both in the frequency domain and time domain. All simulations are validated by comparing the results with the measured data from the model test. Multi-directional wave conditions are also simulated to make the methodology applicable in the real world.

The second part of the thesis discusses an important phenomenon in the second order roll, i.e. the variance of the standard deviation of each simulation is quite large. The reasons for this phenomenon are the non-uniqueness in the excitation and the low damping ratio. Thus 2nd order roll should be predicted as a probability distribution instead of a specific value because it has a much larger variance compared to the first order motions. The sensitivity of the factors makes the model test and the real situation different, and the factors causing a difference in the real situation are discussed by an explanation of the physical reasons supported by simulations.

In the third part, the discussion is moved on to the reality. The second order roll is identified from offshore measurements of the existing semi-submersible ‘Thialf’, in both the free floating and free hanging stage. It is further used to validate the simulation methodology and the discussed characteristics for 2nd order roll.

In the end the operability of the Sleipnir is evaluated, and it is concluded that the 2nd order roll actually has a dominant contribution to the overall roll motion. However, with the 2nd order roll included, Sleipnir’s roll motion is still acceptable.
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- **QTF for reflection model**

## QTF for SLEIPNIR

- **Introduction**
- **Roll**
- **Sway**
- **Heave**
- **Surge**
- **Pitch**
- **Yaw**

## Roll QTF for THIALF

ERROR! BOOKMARK NOT DEFINED.

## Power Spectrum Density

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1 INTRODUCTION

1.1 Problem statement

The stability of the vessel is one of the most critical issues to be considered when designing a ship. According to the hydrostatic calculation, the initial metacentric height (GM) should be high enough in order to provide sufficient restoring moment when the ship is inclined. The GM here can be regarded as the recovering ability, the higher the GM, the larger the restoring moment.

On the other hand, if the metacentric height (GM) is too large, it will cause a low natural roll period (high frequency). If the roll natural period is close to the first order wave period, it would lead to a severe resonant roll motion in the waves. The GM here can be seen as the spring for the roll spring-mass-damper system, a high stiffness of the spring results in a low natural roll period. Severe roll motions not only bring discomfort to the crew, but also harms the offshore heavy lifting operation. It would add to the swinging motion of the load carried by the hoists, and further cause problems during the lifting operation, reducing the workability of the vessel.

![Figure 1-1: Natural frequency of ship type vessels and semi-submersibles, comparing with the first order wave excitation frequency range.](image-url)
In order to minimize the roll motion of the vessel, the natural roll period of the vessel should be designed to avoid the period range of the first order wave spectrum (normally from 5 to 15 seconds). The traditional ship type vessel normally has a natural roll period slightly above 10s which is regarded as not long enough to make the ship as motionless as to meet the requirement of heavy lifting, and due to the stability as well as the ship-type restriction, there’s little room to change the natural period or the GM. Thus a semi-submersible type of vessel is chosen instead, Heerema has ‘Thialf’, ‘Hermod’, ‘Balder’ and ‘Sleipnir’, this type of vessel has a much longer natural roll period than normal ships, so they completely avoid the natural period of first order waves (see Figure 1-1). Also the shape with the large pontoons deep under the water surface and only “slender” columns sticking through the water surface makes them less influenced by the waves.

However, during the model test of Heerema’s new semi-submersible vessel ‘Sleipnir’, a significant roll motion was still identified (See Figure 1-2). The frequency of this roll motion (0.16rad/s) is obviously not within the range of the first order wave excitation (See Figure 1-3), which indicates that the roll motion is not a response from the first order wave excitation. (Figure 1-2 is the measured roll motion time series and the corresponding frequency band along the time series in a model test, while Figure 1-3 is the incoming wave time series and frequency band corresponding to the roll motion.)

According to the inertia-stiffness calculation, it is further found that the 0.16rad/s is exactly the natural roll frequency of the model semi, which indicates it is a resonance phenomenon.

Because the response is not at the frequency of 1st order wave range, but is around the difference frequency, we call it a second order problem.

Figure 1-2: Top: Time series of the measured roll motion
Bottom: Frequency band of the measured roll motion extracted by short time Fourier Transform.
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1.2 Research objectives

Although the second order problem is widely noticed for the horizontal motions under the mooring situation, the second order roll motion has never been reported as a problem so far. It is of great interest to investigate whether the severe second order roll motion in Sleipnir’s model test would be a true problem in real operation or not. To investigate in whether the second order roll motion would happen in reality, the research is divided into the following steps:

First step is to investigate in the external excitation force and damping values of this resonance problem, by studying the questions:

- Where does the external excitation come from?
- What kind of factors have an impact on the excitation and the damping ratio?
- How to quantify the excitation and the damping ratio at the natural roll frequency?

Next step is to simulate the 2nd order roll and validate the results by comparing with the model test results:

- How to calculate 2nd order roll in frequency domain?
- How to calculate 2nd order roll in time domain?

A simulation tool is developed and the 2nd order roll is simulated in varied cases, in order to verify whether the differences between the model test and real environment cause a different behaviour of the 2nd order roll. Focuses are on the questions:

- What is the influence of reflection?
- What is the influence of wave spreading?
- What kind of sea state can cause severe roll motion?
- In order to mitigate 2nd roll motion, how much viscous damping ratio should be achieved?

Figure 1-3: Top: Time series of the measured wave elevation
Bottom: Frequency band of the measured wave elevation extracted by short time Fourier Transform.
• What's the influence of different load cases?
  Especially a discussion on the statistics of the second order motions would be performed:
  • What is the influence of the wave seed on the statistics?
  • Does the damping have an influence on these statistics?

Then offshore data is checked to see how severe 2nd order roll is in reality:
  • How severe is 2nd order roll in reality?
  • Are the 2nd order roll simulation results able to match the measurement data for real offshore heavy lifting?

The operability for the new vessel ‘Sleipnir’ is evaluated based on overall roll motion considering both 1st, 2nd order and the influence of the load’s swing motion.

For the study of how to mitigate the second order roll, following methods are investigated
  • Change the dynamic properties of the suspended load - ship coupled system
  • Bilge keels
  • Use of the DP system to provide a roll damping moment

1.3 Thesis outline

The thesis has 13 chapters, divide into 3 parts.

Part 1 Raise the 2nd order roll problem from model test, analysis and quantify the two main reasons: the nonlinear excitation and the low damping ratio. After that, the complete simulation procedure for the 2nd order roll motion is built both in time domain and frequency domain.

Chapter 2 firstly introduces the model test in which 2nd order roll is identified. The measured data is further processed to reveal some phenomenon such as reflection, random group effect in the response time series and etc.

Chapter 3 explains the second order wave and the second order force, which is the core theoretical base of the nonlinear excitation part, mathematical derivation and related physical meaning is explained according to the Pinkster’s theory.

Chapter 4 explains the QTF(Quadratic Transfer Function) calculation procedure. A reflection model and a spreading model are built as the input. Some phenomenon, why QTF values for certain frequency and direction combination are higher is explained by various theories.

Chapter 5 explains another reason which causes severe 2nd order roll: the low damping ratio. Different methods are then used to evaluate damping ratio both in the model test and in reality, the possible reasons for inaccuracies are explained. And finally cubic damping is proposed to solve the problems.

Chapter 6 introduces the frequency domain method of calculating the second order excitation and roll motion. Uni-directional and multi-directional wave cases are explained respectively. The existing method which only allows single degree of
INTRODUCTION

freedom is firstly applied, and then a new frequency domain method is proposed to deal with the coupled motion. 

**Chapter 7** introduces the time domain method of calculating the second order excitation and roll motion, with both uni-directional and multi-directional wave cases. Comparison between frequency domain and time domain method is performed by matching the model test results.

**Part 2** Base on the calculation method, 2nd order roll under different environment is simulated to investigate in how much the difference issues between the model test and reality influence the 2nd order roll. Some characteristics of the 2nd order roll are concluded and revealed.

**Chapter 8** propose the large variance of second order roll, and explain the two main reasons: phasing and low damping. Possible statistics is investigated.

**Chapter 9** some influential factors of second order roll is discussed, such as sea state, wave spreading, water depth, loading condition, ship heading and etc. The sensitivity of some factors is compared with first order roll.

**Part 3** Investigate the 2nd order roll for Thialf in real offshore heavy lifting, give the prediction and propose the mitigation for Sleipnir’s future application.

**Chapter 10** first explains the real offshore measured data in which 2nd order roll is identified. The dynamic model of ship-load coupling system is introduced. After that an overall roll motion of the ship is predicted taking 1st, 2nd motion and all coupling effect into account. The calculated result are compared with the real data, a discussion is performed.

**Chapter 11** base on all the discussion, predictions for Sleipnir’s overall roll motion in reality under some typical loading conditions are given.

**Chapter 12** investigates in the possible ways to mitigate the second order roll, using tugger damper, bilge keels and DP systems.

**Chapter 13** is the final conclusion.
2

MODEL TEST DISCUSSION

2.1 Introduction

A model test has been performed in MARIN for Heerema’s new semi-submersible crane vessel in order to evaluate all kind of hydrodynamic properties.[2]

Figure 2-1: Model test of Sleipnir in MARIN, Left: A picture of the model, scale 40:1, Right: The seakeeping and manoeuvring basin, Dimension: 170x40x5 m.

Figure 2-2: Basin layout during the model test.

There are three tests which are relevant to this thesis, as described below:

Decay test
Decay tests helps to determine the natural frequency of ship motion and the damping ratios. A force is exerted on the edge of the ship in order to give an initial inclined angle.
The ship is then set free, it will roll at its natural frequency while the amplitude gradually decreases. The roll angle is recorded and the linear damping coefficient \( p \) and quadratic damping coefficient \( q \) can be derived from the measured roll time series, which is discussed in Chapter 5.

**Calibration of irregular wave**

Altogether 3 sea states are tested and measured in the model test:

1. \( H_s = 1.5 \text{ m}, T_p = 7 \text{ s}, \gamma = 1 \)
2. \( H_s = 1.5 \text{ m}, T_p = 12 \text{ s}, \gamma = 1 \)
3. \( H_s = 3 \text{ m}, T_p = 7 \text{ s}, \gamma = 3 \)

Where,

- \( H_s \) Significant wave height
- \( T_p \) Peak period
- \( \gamma \) Wave steepness

The wave is made by wave maker as a unidirectional wave according to the theoretical JONSWAP spectrum. These waves are firstly calibrated when there's no model in the basin to make sure the measured wave spectrum matches the theoretical one. The buoys are set at the spots: Wave_270, Wave_cl, Wave_180 which are marked by the red dots on the right hand. After putting the model into the basin, only the wave elevations at Wave_270, and Wave_180 spots are measured.

**Soft mooring test in irregular wave**

The motion responses of the vessel are then measured under the three sea states respectively, as mentioned above. In order to prevent the ship from drifting away in the waves, soft mooring lines are applied, which should not add any additional roll moment to the vessel.

### 2.2 Relevant results and discussion

By the model test observation and further data process, several kinds of phenomena are found, which are discussed and explained as follows:
Severe 2nd order roll
As the problem is raised in the introduction chapter, an obvious second order roll is identified in the model test. The measured roll response data is further post-processed to obtain below the statistics:

<table>
<thead>
<tr>
<th>Max. roll angle (degree)</th>
<th>Std. total (degree)</th>
<th>Std. 2nd order roll (degree)</th>
<th>Std. 1st order roll (degree)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>1.5</td>
<td>0.5315</td>
<td>0.5287</td>
</tr>
<tr>
<td>1.5</td>
<td>12</td>
<td>0.6</td>
<td>0.1475</td>
</tr>
<tr>
<td>0.5</td>
<td>7</td>
<td>0.1868</td>
<td>0.1841</td>
</tr>
</tbody>
</table>

Table 2-1: Roll response parameters under three test sea states

Where std. is the abbreviation of standard deviation.

The roll response spectrum is plotted as follows (A discussion about how to create wave spectrum from measured time series is discussed in Appendix E), zoom in 2nd order motion and 1st order motion separately:

![Roll response spectrum](image)

Figure 2-5: Measured roll response spectrum under 3 sea states, **Left**: 2nd order roll response spectrum, **Right**: 1st order roll response spectrum.

The following conclusions can be made from the above table and plots:

- In general, the 2nd order response is much more severe than the 1st order response.
- The 2nd order roll is influenced by both Hs and Tp. Peak response occurs under the sea state Hs = 3m, Tp = 7s. When comparing the three test cases, it is dominated by the influence of Hs.
- The 1st order roll is also influenced by both Hs and Tp. Peak response occurs under the sea state Hs = 1.5m, Tp = 12s. When comparing the three test cases, it is dominated by the influence of Tp.
- The 2nd order roll is nonlinear. If comparing the two sea state Hs = 3m, Tp = 7s and Hs = 6m, Tp = 7s, the latter is with Hs twice higher, theoretically the 2nd order excitation should be 4 times higher (see Chapter 3), but the maximum angle is only 3 times higher while the standard deviation is only 2.87 times higher.
MODEL TEST DISCUSSION

- The 1st order roll is also kind of nonlinear, that with the excitation twice higher, the response is only 1.7 times higher. However, its nonlinearity is much smaller than the 2nd order roll.

Inaccurate damping values for low motions

Decay tests for the roll motion have been performed 4 times under different situation. The decay tests for other motions are also listed as a comparison:

<table>
<thead>
<tr>
<th>MARIN test No.</th>
<th>Period</th>
<th>Proportional damping</th>
<th>Quadratic damping</th>
<th>Logarithmic decrement</th>
<th>Percentage of critical damping</th>
</tr>
</thead>
<tbody>
<tr>
<td>802005 ROLL (5deg)</td>
<td>27.2 s</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>100</td>
</tr>
<tr>
<td>802006 ROLL (2deg)</td>
<td>27.2 s</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>100</td>
</tr>
<tr>
<td>802007 ROLL (5deg)</td>
<td>27.2 s</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>100</td>
</tr>
<tr>
<td>802008 PITCH (3 deg)</td>
<td>27.2 s</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>100</td>
</tr>
<tr>
<td>802009 PITCH (2deg)</td>
<td>27.2 s</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>100</td>
</tr>
<tr>
<td>802010 PITCH (3deg)</td>
<td>27.2 s</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>100</td>
</tr>
<tr>
<td>802012 HEAVE</td>
<td>27.2 s</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>100</td>
</tr>
<tr>
<td>803008 SURGE</td>
<td>27.2 s</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>100</td>
</tr>
<tr>
<td>803009 SWAY</td>
<td>27.2 s</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>100</td>
</tr>
<tr>
<td>803007 HEAVE</td>
<td>27.2 s</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>100</td>
</tr>
<tr>
<td>803005 ROLL (5deg)</td>
<td>27.2 s</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>100</td>
</tr>
<tr>
<td>803006 PITCH (3deg)</td>
<td>27.2 s</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>100</td>
</tr>
<tr>
<td>803010 YAW</td>
<td>27.2 s</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>100</td>
</tr>
<tr>
<td>803012 SURGE</td>
<td>27.2 s</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>100</td>
</tr>
<tr>
<td>803011 SWAY</td>
<td>27.2 s</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>100</td>
</tr>
<tr>
<td>402033 SURGE</td>
<td>27.2 s</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>100</td>
</tr>
</tbody>
</table>

*In the above table between brackets are added in additional information*

Table 2-2: Decay test results, the values in the red rectangular marks are the results of roll decay tests.

From the above table, following phenomenon can be seen:

- The damping ratio for roll motion at natural frequency is very low compared to other motions. Actually the damping can be evaluated quite well by current engineering methods for all the other motions, while the roll damping is still an unsolved problem.
- The initial roll angle has a huge influence on the damping coefficient resulted from decay test. Test No.802005, 802007, 803005 which start with the same initial angle of 5 degree has similar p and q ratios, while test No. 802006 with an initial angle of 2 degree has obviously different p and q values. As can be seen in Figure 2-6 the linearization data of the roll decay test with low initial angle is very scattered, that the damping coefficient cannot be determined properly by decay test with low initial angle.
MODEL TEST DISCUSSION

Figure 2-6: Linearization of roll decay tests. **Left:** test No. 802005 free floating with 5 degree initial angle; **Right:** test No. 802006 free floating with 2 degree initial angle.

- The soft mooring line does introduce small amount of extra roll moment, which can be seen from the difference of damping ratios between test No. 802005, 802007 and test No. 803005.

And according to the literature\[^{[17]}\], the damping in the model test can deviate from the reality by another two mechanism:

- Reynolds number: according to the scaling effect, not all parameters can achieve similarities at the same time, the Reynolds number in the model test is much smaller, thus can lead to all the damping coefficients larger than the reality.
- Relative velocity: the model test decay is performed in a situation without waves, thus the relative velocity might be larger than the reality and cause the damping coefficient to be smaller.

**Reflection**
An obvious wave reflection can be seen by comparing the measured wave spectrum at point Wave_270 (Figure 2-7) both with and without model in the basin model in the basin. The reflection causes the spectrum density at certain frequency much higher (An similar explanation for this please refer to Chapter 4, the standing wave), which means the measured roll motion is not a pure response from theoretical spectrum, but also interfered by wave reflection between the model and the basin.
MODEL TEST DISCUSSION

Figure 2-7: Wave spectrum based on measured wave elevation in the model test. Red line: without reflection; Blue line: with reflection.

Large variance in response
The roll response has obvious group phenomenon. This group phenomenon would cause the variance of roll response during different time period quite large, as an example, the red part and the magenta part would obviously result in a different spectrum.

Figure 2-8: Obvious group effects in the measured roll response

After ticking out the starting period and shut down period in the measurement data above, the roll response spectrum is plotted separately for each hour, we can conclude for the 2nd order roll dominated motion, the variance of roll spectrum in each hour measurement is very large.

Figure 2-9: Roll response spectrums for different measured time periods under the same sea state excitation
MODEL TEST DISCUSSION

Another reason model test could fail to match reality is that the model test simulate a uni-directional wave while in reality the waves come from all directions.

All the phenomenon mentioned above may have an influence on 2\textsuperscript{nd} order roll and they will be further simulated and discussed in the Part 2 of the thesis.
3

SECOND ORDER THEORY

3.1 Wave group

Second order forces and moments are strongly related with wave groups, thus in this chapter, we first give an introduction to the wave group.

A wave group is the envelop of the wave elevation. This can be visualized in Figure 3-1:

![Figure 3-1: Wave group time series, blue line is a measured wave elevation time series from the model test and the red line is the wave group extracted by mathematical method.](image)

We can extract the wave group from the original time series by taking the square of the wave elevation, multiply by 2, then subsequently add a low frequency pass filter:

\[
A(t) = 2\zeta(t)^2
\]  

(3 - 1)

Where,

\[
\begin{align*}
A(t) & \quad \text{Wave group elevation} \\
\zeta(t) & \quad \text{Original wave elevation}
\end{align*}
\]

The factor 2 in Equation 3-1 can be explained as, if we consider two wave components of unit wave amplitude, there's naturally a 2 in the mathematics for the envelop part (red):

\[
\zeta_a \cdot \sin(\omega_i t + \xi_i) + \zeta_a \cdot \sin(\omega_j t + \xi_j) \\
= 2\zeta_a \cos \left( \frac{(\omega_i - \omega_j)t + (\xi_i - \xi_j)}{2} \right) \sin \left( \frac{(\omega_i + \omega_j)t + (\xi_i + \xi_j)}{2} \right)
\]

(3 - 2)
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Where,
\[ \zeta \] Unit amplitude wave
\[ \omega_i, \omega_j \] The frequency of the first and the second wave component
\[ \vec{\varepsilon}_i, \vec{\varepsilon}_j \] The phasing of the first and the second wave component

As an alternative to extract the wave group, one can first use a Fourier transform to transfer the original time series into frequency domain, multiply by 2 and then take the inverse Fourier transform, which is called a Hilbert transform\[28\]. Both of the methods retrieve a good envelop of the original time series.

An analytical expression for wave group caused by two wave components can be written as:
\[
\zeta(t)^2 = (\zeta_i \sin(\omega_i t + \vec{\varepsilon}_i) + \zeta_j \sin(\omega_j t + \vec{\varepsilon}_j))^2
= \zeta_i \zeta_j \cos[(\omega_i - \omega_j) t + (\vec{\varepsilon}_i - \vec{\varepsilon}_j)] + \zeta_i \zeta_j \sin[(\omega_i + \omega_j) t + (\vec{\varepsilon}_i + \vec{\varepsilon}_j)] + \frac{\zeta_i^2}{2} + \frac{\zeta_j^2}{2} \\
+ \frac{\zeta_i^2}{2} \cos(2\omega_i t + 2\vec{\varepsilon}_i) + \frac{\zeta_j^2}{2} \cos(2\omega_j t + 2\vec{\varepsilon}_j)
\]

(3 - 3)

Where,
\[ \zeta_i, \zeta_j \] The wave elevation of the first and the second wave component

Taking the difference frequency part of Equation 3-3 only, and substituting it into Equation 3-1 the wave group takes the form:
\[
A(t) = 2\zeta_i \zeta_j \cos[(\omega_i - \omega_j) t + (\vec{\varepsilon}_i - \vec{\varepsilon}_j)]
\]

(3 - 4)

Thus in the following, we call the physical quantities proportional to the term \[\cos[(\omega_i - \omega_j) t + (\vec{\varepsilon}_i - \vec{\varepsilon}_j)]\] as ‘in phase’ with the wave group, while the physical quantities proportional to the term \[\sin[(\omega_i - \omega_j) t + (\vec{\varepsilon}_i - \vec{\varepsilon}_j)]\] as ‘out of phase’ with the wave group.

By further deriving Equation 3-4, Wichers\[18\] gives an equation to represent the second order wave spectrum by using first order wave spectrum:
\[
S_g(\mu) \Delta \omega = \frac{1}{2} (2\zeta_i \zeta_j)^2 = 8S_\xi(\omega_i) \Delta \omega S_\xi(\omega_j) \Delta \omega
\]

(3 - 5)

Where
\[ \mu \] Difference frequency of \( \omega_i \) and \( \omega_j \)

Then consider all the components cause the same difference frequency, the spectrum density of a specific difference frequency can be written as:
\[
S_g(\mu) = 8 \int_0^\infty S_\xi(\omega + \mu) S_\xi(\omega) d\omega
\]

(3 - 6)

Where,
\[ S_\xi(\omega) \] First order wave spectrum
\[ S_g(\mu) \] Second order wave spectrum
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Figure 3-2 presents the wave spectrum and the wave group spectrum. Here the spectrum in the middle frequency band is the normal first order wave spectrum, while the spectrum in the lower frequency band is the wave group spectrum. Actually, ships and offshore structures can get wave force excitation from both of the frequency range.

![Figure 3-2: Wave spectrum and wave group spectrum](image)

The wave group spectrum is in the lower frequency band where locates the natural frequencies of surge and sway motion (under mooring situation, usually around 0.03 rad/s) for ships and offshore structures, as well as the roll, pitch motion (usually around 0.2 rad/s) for some type of vessels. Resonance problems should be paid attention to.

3.2 2nd order force and moment

Pinkster presents in 1980s the famous mathematical model to calculate the second order force and moment which can be used as the basic theory for the QTF calculation in this thesis. In the following, his mathematics model is introduced and the second order force components are briefly described.

We start off from calculating hydrodynamic force exerting on ship body by integrating the pressure normal to the ship surface. $\varepsilon$ is a symbol means the value of higher order is much smaller compare to lower order terms, usually they can be neglected, but because of their frequency band, resonance problem might be caused, that’s why we pay attention to higher order terms in this thesis.

$$F = -\iint_{S_0} \left( P^{(0)} + \varepsilon P^{(1)} + \varepsilon^2 P^{(2)} \right) \cdot \left( \vec{n} + \varepsilon \overline{N^{(1)}} \right) dS$$

$$-\iint_{S} \left( P^{(0)} + \varepsilon P^{(1)} + \varepsilon^2 P^{(2)} \right) \cdot \left( \vec{n} + \varepsilon \overline{N^{(1)}} \right) dS \quad (3-7)$$
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Where:

- \( p^{(0)} \) Hydrostatic pressure
- \( p^{(1)} \) First order pressure
- \( p^{(2)} \) Second order pressure
- \( S_0 \) Constant part of wet surface
- \( S \) Oscillating part of wet surface
- \( \vec{n} \) Normal vector
- \( \vec{N} \) Normal vector oscillating with rotational motion

The core assumption and derivation basis of this theory is to expand all the physical quantities into three parts, distinguished by the number in the superscript. And their physical meaning are as follows:

(0): Static value
(1): First order quantity oscillating with wave frequency
(2): Second order quantity oscillating with low(difference) frequency

The static parts are caused by the weight and buoyancy, this part will be gradually neglected in the following derivation, since the focus is on wave caused force.

From Bernoulli Equation, we can decompose \( p^{(0)} \), \( p^{(1)} \) and \( p^{(2)} \) in Equation 3-7 as follows:

\[ p = -\rho g X_3 - \rho \frac{\partial \phi}{\partial t} - \frac{1}{2} \rho (\bar{\psi} \phi)^2 \]  

(3 - 8)

\[ p^{(0)} \) - hydrostatic pressure
\[ p^{(0)} = -\rho g X_3^{(0)} \]  

(3 - 9)

\[ p^{(1)} \) - first order pressure
\[ p^{(1)} = -\rho g X_3^{(1)} - \rho \frac{\partial \phi^{(1)}}{\partial t} \]  

(3 - 10)

\[ p^{(2)} \) - second order pressure
\[ p^{(2)} = -\rho g X_3^{(2)} - \rho \frac{\partial \phi^{(2)}}{\partial t} - \frac{1}{2} \rho (\bar{\psi} \phi^{(1)})^2 - \rho \left( X^{(1)} \cdot \bar{\psi} \frac{\partial \phi^{(1)}}{\partial t} \right) \]  

(3 - 11)

Where:

- \( \rho \) Fluid density
- \( X_3^{(0)} \) Vertical position of the vessel
- \( X_3^{(1)} \) Heave velocity of the vessel
- \( X_3^{(2)} \) Heave acceleration of the vessel
- \( \phi^{(1)} \) First order wave potential
- \( \phi^{(2)} \) Second order wave potential

It is noticed that the last term of \( p^{(2)} \) does not originally exist in the Bernoulli Equation, it is a Taylor expansion term of \(-\rho \frac{\partial \phi^{(1)}}{\partial t}\). The purpose of this Taylor expansion is that we usually simplify the first order problem by assuming the ship is not in motion when calculating incoming wave and diffraction wave potentials. But for the second order problems, it is no longer suitable to make this assumptions, so all the wave potential related terms for the second order calculation should be based on the ship is performing a first order motion. This is visualized in Figure 3-3.
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Figure 3-3: Assumptions of ship motion when calculate first and second order hydrodynamic problems

The physical meaning of the normal vector terms in Equation 3-7 can be explained as:

\[ \vec{N} \] The main normal vector determined by horizontal and vertical motions

\[ \vec{n} \] The part of normal vector which is oscillating with rotational motion

The \( \vec{N} \) is caused by coupled motion, which means mode 4,5,6 (Roll, Pitch, Yaw) can have an influence on mode 1,2,3 (surge, sway heave) which determines the normal vectors \( \vec{n} \).

Its mathematical expression reads:

\[ \vec{N} = \vec{R}^{(1)} \cdot \vec{n} \]  \hspace{1cm} (3-12)

Where

\[
\vec{R}^{(1)} = \begin{bmatrix}
-X_6^{(1)} & +X_5^{(1)} \\
+X_6^{(1)} & -X_4^{(1)} \\
-X_5^{(1)} & +X_6^{(1)}
\end{bmatrix}
\hspace{1cm} (3-13)
\]

\[ X_4^{(1)} \] Roll velocity

\[ X_5^{(1)} \] Pitch velocity

\[ X_6^{(1)} \] Yaw velocity

From Equation 3-7, the second-order terms (the summation of superscripts equals 2) are isolated, i.e. the static terms and first order terms are not incorporated. Some terms after integration on the wet surface, their values are zero, thus, not listed here, the following three terms are the remaining non-zero terms.

\[
\varepsilon^2 F^{(2)} = -\iint_{S_0} \varepsilon P^{(1)} \cdot \varepsilon \vec{N}^{(1)} dS - \iint_{S_0} \varepsilon^2 P^{(2)} \cdot \vec{n} dS - \iint_{S} \varepsilon P^{(1)} \cdot \vec{n} dS \]  \hspace{1cm} (3-14)

By substituting the pressure (Equation 3-10 and Equation 3-11) and normal vector expressions (Equation 3-12 and Equation 3-13) above into Equation 3-14, and
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making some extra assumptions in mathematics (see Appendix A). The second order force and moment then can be rewritten as a composition of five terms:

\[
\begin{align*}
\vec{F}^{(2)} &= m \ddot{R}^{(1)} X_G^{(1)} + \int \int_{S_0} \left( \rho \frac{\partial \phi^{(2)}}{\partial t} + \frac{1}{2} \rho \left( \vec{v} \phi^{(1)} \right)^2 + \rho \left( X^{(1)} \cdot \vec{v} \frac{\partial \phi^{(1)}}{\partial t} \right) \right) \cdot \vec{n} dS \\
&\quad - \oint_{\text{wl}} \frac{1}{2} \rho g \left( \zeta_r^{(1)} \right)^2 \vec{n} dl \\
\end{align*}
\]

\[
\begin{align*}
\vec{M}^{(2)} &= I \ddot{\alpha}^{(1)} + \int \int_{S_0} \left( \rho \frac{\partial \phi^{(2)}}{\partial t} + \frac{1}{2} \rho \left( \vec{v} \phi^{(1)} \right)^2 + \rho \left( X^{(1)} \cdot \vec{v} \frac{\partial \phi^{(1)}}{\partial t} \right) \right) \cdot (\vec{x} \times \vec{n}) dS \\
&\quad - \oint_{\text{wl}} \frac{1}{2} \rho g \left( \zeta_r^{(1)} \right)^2 (\vec{x} \times \vec{n}) dl \\
\end{align*}
\]

Where,

\[
\begin{align*}
X_G^{(1)}, \alpha^{(1)} & \quad \text{The acceleration of first order horizontal motion and rotational motion} \\
\zeta_r^{(1)} & \quad \text{The relative height between wave elevation and ship heave motion}
\end{align*}
\]

The physical meaning of each terms are explained by Pinkster as follows:

I. First order relative wave elevation

\[
\begin{align*}
&\quad - \oint_{\text{wl}} \frac{1}{2} \rho g \left( \zeta_r^{(1)} \right)^2 \vec{n} dl \\
\end{align*}
\]

II. Pressure drop due to first order velocity

\[
\begin{align*}
&\quad \int \int_{S_0} \frac{1}{2} \rho \left( \vec{v} \phi^{(1)} \right)^2 \cdot \vec{n} dS \\
\end{align*}
\]

III. Pressure due to product of gradient of first order pressure and first order motion

\[
\begin{align*}
&\quad \int \int_{S_0} \rho \left( X^{(1)} \cdot \vec{v} \frac{\partial \phi^{(1)}}{\partial t} \right) \cdot \vec{n} dS \\
\end{align*}
\]

IV. Contribution due to products of first order angular motions and inertial forces

\[
\begin{align*}
&\quad m \ddot{R}^{(1)} X_G^{(1)} \\
\end{align*}
\]

V. Contribution due to second order potential

\[
\begin{align*}
&\quad \int \int_{S_0} \rho \frac{\partial \phi^{(2)}}{\partial t} \cdot \vec{n} dS \\
\end{align*}
\]

\[
\begin{align*}
\phi^{(2)} &= \phi_w^{(2)} + \phi_d^{(2)} + \phi_b^{(2)} \\
\phi_w^{(2)} &= \phi_{ww}^{(2)} + \phi_{wd}^{(2)} + \phi_{wb}^{(2)} + \phi_{dd}^{(2)} + \phi_{dw}^{(2)} + \phi_{db}^{(2)} + \phi_{bb}^{(2)} + \phi_{bw}^{(2)}
\end{align*}
\]

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If calculating Equation 3-17 to Equation 3-21 for one pair of unit amplitude waves, the results are called Quadratic transfer function (QTF). Equation 3-16 to Equation 3-20 are further defined as the first to the fifth term of QTF. A QTF is only valid for a pair of wave components with certain frequency and direction combination, if all the nonlinearities in an irregular wave spectrum should be considered, extra methods should be taken to sum up all QTFs for each pair of wave components (see Chapter 6 and Chapter 7).

The first four components are the products of the first order terms, and the two first order terms can either originate from one wave component of an irregular wave or from two wave components of different wave frequencies.

Since in first order hydrodynamic theories, one can assume all the first order terms are proportional to the first order wave elevation, here we explain how the second order terms appears by representing the first order terms as first order wave components. The irregular wave is composed of a large number of harmonic wave components:

\[
\zeta(t) = \sum_{i=1}^{N} \zeta_i \cdot \sin(\omega_i t + \bar{\varepsilon}_i) \quad (3-24)
\]

Considering term one and two, we sum up all the harmonic terms and then take the square of them, by doing this, we can get a difference term (red) as follows:

\[
(\zeta_i \cdot \sin(\omega_i t + \bar{\varepsilon}_i) + \zeta_j \cdot \sin(\omega_j t + \bar{\varepsilon}_j))^2
= \zeta_i \zeta_j \cos[\omega_i - \omega_j]t + (\bar{\varepsilon}_i - \bar{\varepsilon}_j)] + \zeta_i \zeta_j \sin[\omega_i + \omega_j]t + (\bar{\varepsilon}_i + \bar{\varepsilon}_j)] + \frac{\zeta_i^2}{2}
+ \frac{\zeta_j^2}{2} + \zeta_i^2 \cos(2\omega_i t + 2\bar{\varepsilon}_i) + \zeta_j^2 \cos(2\omega_j t + 2\bar{\varepsilon}_j) \quad (3-25)
\]

Term three and four is a direct product of two harmonic wave components. This could also lead to difference frequency term (red) as follows:

\[
\zeta_i \cdot \sin(\omega_i t + \bar{\varepsilon}_i) \cdot \zeta_j \cdot \sin(\omega_j t + \bar{\varepsilon}_j)
= 0.5 \zeta_i \zeta_j \cos[(\omega_i - \omega_j) t + (\bar{\varepsilon}_i - \bar{\varepsilon}_j)]
- 0.5 \zeta_i \zeta_j \cos[(\omega_i + \omega_j) t + (\bar{\varepsilon}_i + \bar{\varepsilon}_j)] \quad (3-26)
\]

The reason to only focus on the difference term is that the difference frequencies are within the range of the vessel’s natural roll period. Which could cause resonance problem. And finally all the first four terms can be written by the following expression:

\[
\zeta_i \zeta_j \cos/sin[(\omega_i - \omega_j) t + (\bar{\varepsilon}_i - \bar{\varepsilon}_j)]
\]

Thus, they are proportional to the wave group, and have in-phase terms and out-of-phase terms as we mentioned previous subchapter.

And for the fifth term, the physical meaning of second order wave potential is indicated in the following picture, we take \( \Phi_{wb}^{(2)} \) as an example, this potential is caused by the interaction between the incoming wave and body motion caused wave. There is no
relation with the ship body, but only the nonlinear effect caused by two wave components of different frequency:

![Diagram](image)

**Figure 3-4**: Physical meaning of the second order potential

The terms $\Phi_d^{(2)} \Phi_b^{(2)}$ are connected to $\Phi_w^{(2)}$ by the second order boundary condition (see Appendix A), on the ship body $\Phi_b^{(2)}$ is especially used to compensate second order velocity of the ship in the normal direction. It is assumed the $\Phi_{ww}^{(2)}$ term should be larger than other components (see Pinkster's approximation), so we can also use $\Phi_{ww}^{(2)}$ to represent all other terms.

This assumption can make the calculation much easier, because the ship body cannot be represented by an analytical equation, thus, complicated and time-consuming numerical calculation should be involved. Only for component $\Phi_{ww}^{(2)}$, it has an analytical solution, and obviously it is proportional to the wave group:

$$
\Phi_{ww}^{(2)} = - \sum_{i=1}^{N} \sum_{j=1}^{N} \zeta_i^{(1)} \zeta_j^{(2)} A_{ij} \frac{\cosh\left((k_i - k_j)(X_3 - h)\right)}{\cosh(k_i - k_j) h} \sin\left((k_i - k_j)X_1 - (\omega_i - \omega_j)t + (\epsilon_i - \epsilon_j)\right)
$$

In this thesis, the second order potentials are calculated according to their full mathematical expression. In the reflection model, however, the Pinkster's assumption is employed.

The computation of the first and second order potentials is called diffraction analysis and is based on potential theory (See Appendix A), the software WAMIT is chosen to perform such kind of calculation.
4.1 Introduction

In this Chapter, all the QTFs used in the thesis are calculated based on Pinkster’s theory. A reflection model is used to simulate the model test environment, see Chapter 2, Chapter 6 and Chapter 7. And a spreading model is used to simulate the real offshore situation for both ‘Thialf’ and ‘Sleipnir’, the former will be used to simulate the second order motion of ‘Thialf’ and compare with the real measured data in Chapter 10. And the latter will be used to give a prediction for ‘Sleipnir’’s roll motion in the future in Chapter 11.

For each case, the input models and parameters for WAMIT are first introduced, and the output values of QTF are plotted and discussed afterwards. Note in this Chapter, only the QTFs are calculated, it is not yet the excitation, in Chapter 6 and Chapter 7, the QTFs will be further processed into 2nd order forces and moments both in frequency domain and time domain.

4.2 QTFs for reflection model

The reflection model is simulated by setting a wall (represent the wave maker and the boundary of the basin) 2160 m to the centre of the ship (25), this size is corresponding to the model test size with a scale factor of 40. And the water depth is 200m full scale. More information about the model test, please refer to Chapter 2.

Figure 4-1: Concept of the reflection model for WAMIT simulation, An unidirectional beam wave is made to excite the ship motion, in the red point, a buoy is set to measure the incident wave.
4.2.1 Panel model

Low order method is chosen to be used in WAMIT to avoid the possible singularity point problem in high order method\(^\text{[25]}\)(see Appendix A, Figure A-2 and related explanations introduce the low order method). Thus, panel models have to be built for the vessel ‘Sleipnir’ and the free surface by the software MultiSurf as the input. The panel model for free surface is a special input for second order problems, used to solve second order potentials according to nonlinear free surface boundary condition (also see Appendix A).

![Figure 4-2: Panel model for Sleipnir, LEFT: panel model for the ship, RIGHT: panel model for the free surface](image)

Mesh

A mesh refinement study is performed\(^\text{[3]}\), the final mesh is chosen based on the balance of computational time (about 9 days to calculate the QTFs for both with and without reflection) and the accuracy of output.

For the following calculation, the number of panels (represents the number of sources calculated in Green's Function, see Appendix A) for the vessel is 9982, for the free surface, the number of panels is 6368.

4.2.2 Input parameters

The QTFs calculated in the reflection model are two dimensional identified by the frequency of the first wave component and the frequency of second wave component. The frequency range calculated for the first wave component is 0.16rad/s to 2.40rad/s and for the second wave component is from 0.02rad/s to 2.24rad/s.

The combination of two wave frequencies is chosen to satisfy their difference frequency equals 0.14rad/s~0.26rad/s, which corresponding to the natural roll frequency of Sleipnir from full loading to no loading in the crane. With a frequency step of 0.02 rad/s, there are altogether 770 frequency combinations of 2 wave components.

4.2.3 Calculation result
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QTF
The first four terms of QTF are calculated using the model with reflection, while the fifth term is calculated under the circumstances without reflection, because the calculation for second order potentials (included in the fifth term of QTF) could be very complicated and time-consuming after just adding a wall, and WAMIT doesn't support this calculation. This assumption is reasonable according to Pinkster's approximation, which indicates in the fifth term, $\Phi_{ww}(2)$ dominates over all other second order potentials, so the contribution of all others can be ignored. And $\Phi_{ww}(2)$ only take the incident wave into consideration, there's no reflection issues in this physical quantity, so it's reasonable to calculate the fifth term under non-reflection case. The result of QTFs simulated by WAMIT are plotted by the following concept, each diagonal line represents one difference frequency:

![Figure 4-3: Concept of 2D QTF, each diagonal line represents one single difference frequency.](image)

To make the comparison between reflection and non-reflection more clearly, the diagonal lines with different difference frequency are plotted separately. Here the diagonal line of 0.16 rad/s (the natural roll frequency of the vessel in the model test) is taken as an example, for all the others, please refer to Appendix B:

![Figure 4-4: QTFs for difference frequency equals 0.16 rad/s. Blue line: with reflection, Red line: without reflection.](image)

QTF Discussion
The QTF data curve for the case with reflection is serrated, with peaks at certain frequencies, while the QTF data curve for the case without reflection is more smooth along the whole frequency band. This can be explained by standing wave theory:
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Considering the distance between model and wave maker, some wave frequencies reaches its crest at the model while other wave frequencies doesn’t. The standing wave would add up to the wave crest/trough, so for frequencies reaching its crest/trough at the model, the QTF value in the case with reflection would be larger, while for the frequencies reach its node point at the model, the QTF value in the case with reflection would even be smaller compared to cases without reflection.

Figure 4-5: Concept of stand wave

4.3 QTFs for spreading model

When doing model test, we simulate a long crested wave (the upper part of the following figure). A long crested wave propagating in one direction, so theoretically for all the points with a certain distance to the wave maker, the wave elevation should always be the same, at certain time, they all arrive at wave crest, which forms a long crest, this is where its name comes from. While in reality, waves propagate in all directions, the “long crest” cannot be formed, the wave in the real world is often called short crested wave (the lower part of the following figure).

Figure 4-6: Upper: Long crest wave, Lower: Short crest wave [4]
So in this subchapter, all the calculation are extended by taking into account the wave directionality in order to study the influence of wave spreading on the second order moment and roll response spectrum.

In the following, the panel model for ship body and free surface built in Multi-Surf as well as the input parameters of WAMIT is explained.

4.3.1 Sleipnir

Panel model
The panel models and the mesh used for ‘Sleipnir’ and the free surface are the same as the reflection calculation in Chapter 4.2.

Input parameters
A full scale simulation is performed, with all directions wave spreading and infinite water depth. The frequency range calculated for the first wave component is from 0.16rad/s to 2.40rad/s and the frequency of the second wave component is from 0.02rad/s to 2.24rad/s. The combination of two wave frequencies is chosen to satisfy their difference frequency equals 0.14rad/s~0.26rad/s, which corresponding to the natural roll frequency of Sleipnir from full loading to no loading on the crane. So there are altogether 770 frequency combinations of the two wave components. And for each combination, consider both wave components can come from 0 degree to 360 degree, we choose the direction step to be 10 degree for this case, so there are altogether 36x36=1296 direction combinations. Finally 6 modes of motion are considered. So altogether 770x1296x6=5987520 sets of QTFs are calculated for this model, which takes 5 weeks computational time.

4.3.2 Thialf

Panel model
The panel models for ‘Thialf’ and the free surface are showed as follows:

![Panel model for Thialf](image)

**Figure 4-7:** Panel model for Thialf, **LEFT:** panel model for the ship, **RIGHT:** panel model for the free surface

Mesh
After investigating the balance of computational time, the number of mesh panels for Thialf vessel is chosen to be 4572, and for the free surface, it is chosen to be 4384.
Input parameters
The input parameters are set to be slightly different with Sleipnir calculation. The difference frequency band is chosen from 0 rad/s to 0.25 rad/s, we calculate the lower frequency from zero is aimed at catching the natural frequency of sway which is much lower than the natural frequency of roll motion, and this 2 motion are coupled with each other.

The frequency range calculated for both the first and the second wave component is from 0.00 rad/s to 2.00 rad/s. The combination of two wave frequencies is chosen to satisfy their difference frequency equals 0.00 rad/s~0.25 rad/s, the frequency step is chosen to be 0.025 rad/s this time, so there are altogether 825 frequency combinations of two wave components. And for each frequency combination, consider both wave can come from 0 degree to 360 degree, the direction step is chosen to be 15 degree for this case, so there are altogether 24x24=576 direction combinations. Finally 6 modes of motion are considered. So altogether 825x576x6=2851200 sets of QTFs are calculated for this model.

In this Chapter, only the QTF results for Sleipnir are discussed, however, since both vessel are semi-submersibles, the discussion below is also applicable to Thialf which can be approved by taking a look at the Thialf QTF results in Appendix D.

4.3.3 QTF plotting
The QTFs are expanded to 4-dimension domain for the wave spreading model. The four dimensions include:
1. the frequency of the first wave component
2. the frequency of the second wave component
3. the direction of the first wave component
4. the direction of the second wave component

Two methods are used to plot the 4D QTFs:
1. With the direction of two wave components chosen, plot all the frequency combinations of two wave components.
2. With the frequency of two wave components chosen, plot all the direction combinations of two wave components.

In the following, the first method is used to plot the QTFs of two wave components coming from the same direction, but with different frequency combinations. Figure 4-8 and Figure 4-9 show a concept of this plotting.
QUADRATIC TRANSFER FUNCTION

Figure 4-8: Concept of QTFs by 2 wave component coming from the same direction

Figure 4-9: Concept of diagonal lines representing difference frequencies
All frequency combinations are further divided into 7 groups, identified by the difference frequency of the two wave components from 0.14 rad/s to 0.26 rad/s with the step of 0.02 rad/s. See Appendix C, Figure 4-10 takes the difference frequency of 0.16 rad/s as an example:

![Figure 4-10](image)

**Figure 4-10:** QTFs by 2 wave component coming from the same direction with difference frequency 0.16 rad/s

And next we plot the direction combinations of QTFs for pairs of wave components with the difference frequency of 0.16 rad/s, because the number of this kind of plots are large, they are put in the Appendix C. The following explains the concept of this plotting.

![Figure 4-11](image)

**Figure 4-11:** Concept of direction combinations of QTFs with difference frequency of 0.16 rad/s
4.3.4 QTF Discussion

The following phenomena can be concluded from the plots above and Appendix C,D:

**Phenomenon 1:**
In general, the QTF for roll motion reaches its peak when both of the two wave components are coming from the direction of 270 degree, and the more the angle deviate from 270 degree, the smaller the QTF value is. This is because roll is the motion around longitudinal axis of the ship length, only the wave proportion perpendicular to this axis can cause a roll moment.

![Diagram](image1)

**Figure 4-12:** Direction of roll moment

**Phenomenon 2:**
For wave pairs around 270 degree, an obvious peak is observed at about 0.6~0.8 rad/s, the peaks shift to the lower frequency side when difference frequency becomes larger. And also when the wave direction deviate from 270 degree, the frequency corresponding to the peak QTF becomes lower and lower (see Figure 4-10 and Appendix C,D).

To explain these phenomenon, first the QTF is decomposed to its original five terms.

![Diagram](image2)

**Figure 4-13:** Five components of QTF
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As Figure 4-13 shows, the peak is mainly due to the second terms of QTF (Equation 3-18), it is a product of first order potentials. This means the main peaks are the result of first order wave. Thus, the reason for peaks exist at 0.6~0.8rad/s can be explained as follows.

If the wave crest occurs at the centre line of one pontoon, and wave trough exists at another (see Figure 4-14), then we can expect the difference of hydrostatic force act on the bottom of the pontoon to be largest, which would further cause the largest roll moment. Considering the geometry of the vessel, the corresponding wave length which cause the maximum roll moment should be around 120m.

According to the wave length equation:

\[ \lambda = \frac{2\pi}{k} = \frac{2\pi g}{\omega^2} \]  \hspace{1cm} (4 – 1)

The wave frequency then should be 0.7165 rad/s, which explains the peak in QTF plots quite well for 270 degree wave components. The dimension of Thialf is smaller than the Sleipnir, thus in Appendix D, the peak values for Thialf roll QTFs exist at higher frequencies.

As for the reason why the more the direction deviated from 270 degree, the more left the peak moves is because the wave length needed to cause the largest roll motion becomes larger, thus the corresponding peak frequencies become smaller.
QUADRATIC TRANSFER FUNCTION

And for the same analogy, the peak value is becoming lower and lower, because less pontoon area is perpendicular to wave length cause the maximum roll moment when wave direction deviated from 270 degree.

Phenomenon 3:
The QTFs for two wave components from the same direction are obviously higher than the QTFs for two wave components from different directions (see Appendix C, obvious peaks can be seen in the main diagonal line for the direction combination plots). This is mainly due to the fifth term, the direction of second order physical quantity is different for the fifth terms compare to the first four terms. The first four terms still follows the first order wave direction, while the direction of the fifth term (difference frequency) can be explained as follows [5][6]:

\[ k_{ij} = |k_i - k_j| = \sqrt{k_i^2 + k_j^2 - 2k_ik_j\cos\Delta\theta} \] (4-2)

And according to Sharma and Dean (1979), the second order wave interaction (showed in the fifth term of QTF) is extremely sensitive to the difference angle of the two wave components, the mathematics model is as follows:

\[ \eta^{(2)} = \frac{1}{4} \sum_{i=1}^{N} \sum_{j=1}^{N} a_ia_j \{K^- \cos(\psi_i - \psi_j) + K^+ \cos(\psi_i + \psi_j)\} \]

\[ K^- = [D_{ij} - (k_i \cdot k_j + R_i \cdot R_j)](R_i \cdot R_j)^{-\frac{1}{2}} + (R_i + R_j) \]

\[ K^+ = [D_{ij}^* - (k_i \cdot k_j - R_i \cdot R_j)](R_i \cdot R_j)^{-\frac{1}{2}} + (R_i + R_j) \]

\[ D_{ij} = \frac{(\sqrt{R_i} - \sqrt{R_j})(\sqrt{R_i}(k_i^2 - R_i^2) - \sqrt{R_i}(k_j^2 - R_j^2)) + 2(\sqrt{R_i} - \sqrt{R_j})^2(k_i \cdot k_j + R_i \cdot R_j)}{(\sqrt{R_i} - \sqrt{R_j})^2 - k_{ij}^{-1}\text{tanh}k_{ij}^{-1}d} \]

\[ D_{ij}^* = \frac{(\sqrt{R_i} - \sqrt{R_j})(\sqrt{R_i}(k_i^2 - R_i^2) + \sqrt{R_i}(k_j^2 - R_j^2)) + 2(\sqrt{R_i} + \sqrt{R_j})^2(k_i \cdot k_j - R_i \cdot R_j)}{(\sqrt{R_i} + \sqrt{R_j})^2 - k_{ij}^+\text{tanh}k_{ij}^+d} \] (4-3)

Where;

\[ k_{ij} = |k_i - k_j| \]
\[ k_{ij}^* = |k_i + k_j| \]
QUADRATIC TRANSFER FUNCTION

\[ R_i = |k_i| \tanh(|k_i|d) \]
\[ \psi_i = k_i x - \omega_i t + \epsilon_i \]

If only considering the difference frequency term, its sensitivity can be showed in the following picture:

![Figure 4-17: The influence of difference direction to second order wave interaction](image)

So in Appendix C,D, we can see in the high frequency range, the roll moment QTFs which are dominate by the fifth term (see Figure 4-17) have an obvious peak in the diagonal line where two wave components have the same direction.

**QTF for other motions**
The QTFs for other modes of motion: surge, sway, heave, pitch, yaw please refer to Appendix C, the values are all used in the coupled motion calculation in later chapters. In this Chapter, we don't discuss them further.

### 4.4 Conclusion

In this Chapter, full QTFs for the Semi-Submersible are calculated and discussed. Following conclusions and suggestions are made:
- The QTF curves for semi-submersibles are very steep due to the body shape, the traditional Newman's approximation method for ship type vessels is not suitable any more, and currently no approximation method is proven to be suitable for semi-submersibles.
- It is strongly advised to calculate the QTF only for the natural frequency range, so as to save time. However, the natural frequencies for coupled motion is also suggested to be included.
- For the roll QTF at natural frequency range, it is dominated by the second term of QTF, note this conclusion cannot be extended to all frequencies and other motions.
QUADRATIC TRANSFER FUNCTION

- The fifth term of QTF can be replaced by $\Phi^{(2)}_{ww}$ which can be calculated analytically, if no 2nd order diffraction software is available (very few software has this function, if WAMIT is not the only one), and this can save great amount of calculation time.

- For two wave components from different directions, the fifth term can be even neglected.

- The peaks in the QTF curves are not only frequency dependent but also direction dependent, which means if ship heading can be changed, we can change it to mitigate the second order roll.
5.1 Introduction

In this Chapter, we discuss how the low damping ratio at the natural roll frequency adds to the severity of second order roll. Since we are studying a problem at the resonance peak, the response is very sensitive to the damping ratio, thus a good prediction of the damping is required.

Each subchapter below follows a small topic on roll damping, they all aim at contributing to a better evaluation of damping in our frequency and time domain simulation of second order roll later, the logic is built according to the following flow chart:

Figure 5-1: Flow chart of Chapter 5
First, the low damping problem for the 2nd order roll motion of Sleipnir is introduced and the possible reasons are explained in Chapter 5.2. Then, as we all know the hydrodynamic damping mainly includes two parts: the potential damping and the viscous damping. The potential damping is evaluated by diffraction analysis[11] and the viscous damping coefficients are predicted by model test and CFD simulated decay test. In this thesis, we neglect the discussion of the former because the calculation method is very mature and the value is negligible at the roll natural frequency, but put focus on the latter in Chapter 5.3.

Afterwards, we investigate the methods of involving damping terms in the equation of motion both in frequency and time domain: In frequency domain, the linearization method for viscous damping would be paid attention to in Chapter 5.4. In time domain, the application of potential damping using Cummins equation along the full frequency range is discussed in Chapter 5.5.

After that, we identified the damping ratio from the response spectrum measured in both model test and reality(Chapter 2 and Chapter 10) by half band method in Chapter 5.6. It is then compared with all the predicted damping values by the calculation methods introduced above.

The limitation of the current engineering methods in predicting damping would be found after the comparison. A cubic damping term is proposed to fix the problem in Chapter 5.7 and the accuracy of the linearization factor after introducing the cubic term is further discussed in Chapter 5.8 by a new calculation method.

### 5.2 Low roll damping for Sleipnir

The damping value in low frequency range of Sleipnir is very low compare to other vessels, part is due to the characters of semi-submersible, part is due to some special features of Sleipnir only.

**Low potential damping**

Figure 5-2 is the potential damping(radiation damping) calculated by diffraction analysis, the potential damping for roll motion is negligible in low frequency range(<0.3 rad/s). Thus the total damping is mostly dependent on viscous damping instead of the potential damping.
DAMPING

One reason is because the low frequency is corresponding to a very long wave length, and the ship dimension become relatively very small. The motion of the tiny ship can only follow the long wave instead of creating long waves to dissipate the energy. Another reason explained by Hong et al. is that due to the shape of semi-submersibles, huge energy is trapped between the two pontoons and has difficulties to be dissipated away.

Hong et al. changed the geometry of the pontoons, making the energy easier to escape from the trap, and they conclude after the energy escapes, the roll motion of semi-submersible can be decreased which means the potential damping becomes higher. This phenomenon can be calculated by diffraction analysis.

Low viscous damping
The viscous damping ratio for Sleipnir is also very low compare to other semi-submersibles of Heerema. According to Himeno, viscous roll damping can separate into several parts: friction damping, eddy damping, lift damping, wave damping and etc. Specially bilge keels can add a lot to the roll damping both linearly and nonlinearly (see Chapter 12). And for our case, the viscous damping is dominated by the nonlinear eddy damping, the mechanism is by a vortex introduced pressure difference, a damping roll moment is introduced. It is influenced a lot by the shape of bottom pontoon corner.
Among Sleipnir, Thialf and Balder (three different semi-submersibles of Heerema Marine Contractors), Sleipnir has a very large radius for the bottom corner, while Thialf has a much smaller radius, Balder has bilge keel which adds a lot to the viscous damping ratio. The least vortices could be created by Sleipnir, which cause the least viscous damping. (see Figure 5-4).

### 5.3 Calculation of viscous damping

In Chapter 2, model decay test is used to get the linear and quadratic damping coefficients. However model test is very expensive, sometimes CFD software is used instead. Both of the methods can solve the viscosity and rotation of the fluid by Naiver-Stokes equation instead of the Laplace equation (calculating non-viscous and irrotational flow, which is normally used as the fluid model for ship motion calculation, all hydrodynamic parameters below are solved by Laplace equation based on the potential theory except for the viscous damping). Within Heerema ‘STAR-CCM+’ software is used for CFD method, the decay test is simulated using a transient multiphase solver with a RANS(Reynolds Averaged Naiver-Stokes) turbulent model and a rigid body motion model of the vessel.[7] The RANS method is chosen because it requires the least computational resource and time, it only solves the time averaged Naiver-Stokes equation, all the fluctuations are modeled instead of calculated, but it is sufficient enough to solve the viscous damping problem in our case.

For both model test and CFD method, roll angle time series is recorded, and linear & quadratic damping coefficients can be derived accordingly by logarithmic method.[2][7][8][9][10]

Figure 5-3: Recorded roll decay motion time series
Starting from the equation of motion, the damping will only be extended to quadratic term at maximum according to the literatures in most hydrodynamic problems:

\[ M\phi\ddot{\phi} + b_1^{(1)} \phi + b_2^{(2)} \phi \dot{\phi} + k\phi = 0 \]  

(5 - 1)

Where:
- \( M\phi \): Roll moment of inertia + added mass for roll motion
- \( b_1^{(1)} \): Linear damping coefficient, [kNms/rad]
- \( b_2^{(2)} \): Quadratic damping coefficient, [kNms²/rad²]
- \( k\phi \): Restoring coefficient of roll motion, [kNm/rad]

Consider linear damping value, a general solution to Equation 5-1 which represents the motion time series in Figure 5-3 can be written as:

\[ \phi(t) = e^{-\frac{b_1^{(1)} t}{2M\phi}} (A \cos(\omega_1 t) + B \sin(\omega_1 t)) \]  

(5 - 2)

Where:
- \( \omega_1 = \omega_n \sqrt{1 - \zeta^2} \): \( \omega_n \) natural frequency
- \( A, B \): Constants

Consider \( \zeta \) always quite small at the natural frequencies, \( \omega_1 \approx \omega_n \). For logarithmic decay in one period:

\[ \frac{\phi_{n+1} - \phi_n}{\phi_n} = e^{-\frac{b_1^{(1)} T_n}{2M\phi}} \]  

(5 - 3)

Expand Equation 5-3 to \( N \) periods, take logarithm of both side:

\[ \frac{\ln \phi_n - \ln \phi_{n+N}}{N} = \frac{b_1^{(1)} T_n}{2M\phi} \]  

(5 - 4)

Linearize Equation 5-4 into \( y = qx + p \), p part is contributed by \( b_1^{(1)} \), while q part is contributed by \( b_2^{(2)} \) after linearization:

\[ p = \frac{b_1^{(1)} T_n}{2M\phi} \]  

(5 - 5)

\[ q = \frac{b_2^{(2)} \text{linear} T_n}{2M\phi} \]  

(5 - 6)

Where:

\[ b_2^{(2)} \text{linear} = \frac{8}{3\pi} b_2^{(2)} \Phi_n \omega \]  

(5 - 7)

Equation 5-7 is from Equations 5-11.

Figure 5-5: The determination of linear damping coefficient p and quadratic damping coefficient q
On the other hand, by linearizing the measured roll decay curve, we can get the \( p \) and \( q \) coefficients, and calculate \( b_\phi^{(1)} \) and \( b_\phi^{(2)} \) by inverting Equation 5-5 and Equation 5-6:

\[
b_\phi^{(1)} = \frac{2M_\phi}{T_n} p \tag{5 - 8}
\]

\[
b_\phi^{(2)} = \frac{3}{8} M_\phi q \tag{5 - 9}
\]

Note this method actually is used to predict the total damping ratio at the natural frequency where the ship is performing a harmonic decay motion, however since the potential damping is almost zero at the natural frequency of the roll motion, here we consider \( b_\phi^{(1)} \) and \( b_\phi^{(2)} \) are all contributed by viscous damping.

Also note in this thesis, the viscous damping term is originally calculated by model test and CFD at only the natural roll frequency, it is then added to all the frequencies as a frequency independent value. This treatment is performed because it is too expensive (model test) or too time-consuming (CFD) to solve (Reynolds-averaged) Naiver-Stokes Equations for all frequencies. Due to the fact only the natural frequency is sensitive to damping, others is not and the normal first order frequency range is much dominated by potential damping which can be solved very quickly by Laplace equation at all frequencies, it is reasonable to simply the problem by solving (Reynolds-averaged) Naiver-Stokes Equation only at the natural frequency. Thus in this thesis, the reader can find only the potential damping term is frequency dependent and the viscous damping is treated as a constant number.

### 5.4 Linearization of viscous damping (Frequency domain implementation)

In frequency domain, the potential damping is easy to apply, while for the nonlinear part of viscous damping, linearization methods should be taken. By equalizing the energy dissipation within one period, an equivalent damping can be used to represent the influence of both linear damping and quadratic damping, neglecting the cubic and higher terms:

\[
\frac{1}{T} \int_0^T \{b^{(eq)} \phi \} \cdot \{\phi dt\} = \frac{1}{T} \int_0^T \{b^{(1)} \phi + b^{(2)} \phi |\phi|\} \cdot \{\phi dt\} \tag{5 - 10}
\]

Substitute \( \dot{\phi} = -\phi_0 \omega \sin(\omega t + \epsilon) \) in Equation 5-10, and solve the integral on both side, the equivalent damping can be written as:

\[
b^{(eq)} = b^{(1)}_\phi + \frac{8}{3\pi} \Phi_\phi \omega b^{(2)}_\phi \tag{5 - 11}
\]

However Equation 5-11 is based on the assumption the vessel is performing harmonic roll motion, while in reality, an irregular motion should be considered instead, numerous researches have been done in linearizing equivalent damping for irregular motion. The most widely used one is given by Borgman\(^{[13]}\), based on extra assumption
that the velocity follows a narrow banded Gaussian random distribution around the zero mean:
\[ Q = \int_{-\infty}^{\infty} |v| |v| - c v|^2 e^{-v^2/2\sigma^2} \frac{1}{\sqrt{2\pi}\sigma} dv \]  
(5 - 12)

Where,
\[ c \quad \text{Linearization factor} \]

By solving \( \frac{\partial Q}{\partial c} = 0 \), we can get \( c = \sqrt{\frac{8}{\pi}} \sigma \dot{\phi} \), thus the total equivalent damping term consider both linear damping and quadratic damping can be written as:
\[ b^{(eq)} = b^{(1)} + \sqrt{\frac{8}{\pi}} \sigma \dot{\phi} b^{(2)} \]  
(5 - 13)

Many researchers report the under-prediction of his method afterwards and propose their own linearization factors. Although none of them replace the usage of Equation 5-13 in modern engineering, the fact of under-prediction by using Borgman’s factor seems to be true.

Wolfram\(^{[14]}\) explains this under-prediction is due to the narrow band Gaussian distribution assumption. And further proposes his linearization method based on the peak value of each period and that has no restriction to the distribution of ship motion. Wolfram presents his result as a function of KC number, in another word, the linearization factor is determined by the proportion between drag term and inertial term, here we choose the drag dominate case for 1000 cycles simulation given in Wolfram’s paper, note this number of cycles would be longer than 3 hours, thus might cause over-prediction in the final damping ratio.

<table>
<thead>
<tr>
<th>Borgman(^{[13]})</th>
<th>1.596 (( \sqrt{\frac{8}{\pi}} ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wolfram(^{[14]})</td>
<td>3.902 (1000 cycles)</td>
</tr>
</tbody>
</table>

Table 5-1: Linearization factors for damping term

Although there are other linearization factors proposed by other researchers, Borgman and Wolfram's linearization factor are almost the lower and upper limit we can find in the literatures, thus this range is chosen as a reference for the discussion in the next subchapters.

Another point worth attention is that in Equation 5-13, there is a term 'the standard deviation of the velocity \( \sigma \dot{\phi} \)', which means the damping ratio depends on the response. On the other hand, we know the response is dependent on the damping as well, thus the determination of the damping ratio has to be an iteration process as follows:
5.5 Retardation function (Time domain implementation)

In time domain, nonlinear viscous damping is applied, however, the potential (radiation) damping coefficient is frequency dependent, in order to apply it in time domain when the vessel is performing an irregular motion (for example, considering a combined first and second order roll motion), Cummins Equation is introduced here. The Cummins Equation with retardation function in time domain is:

\[(M + A)x(\dot{t}) + \int_{0}^{t} B(\tau) \dot{x}(t - \tau) d\tau + Cx(t) = X(t)\]  \hspace{1cm} (5 - 14)

By assuming harmonic response, \(x = \cos \omega t\) is substituted into the equation, we can get:

\[B(\tau) = \frac{2}{\pi} \int_{0}^{\infty} b(\omega) \cos(\omega t) d\omega\] \hspace{1cm} (5 - 15)

And inversely:

\[b(\omega) = \int_{0}^{t} B(\tau) \cos(\omega t) d\tau\] \hspace{1cm} (5 - 16)

On the left of Figure 5-16, it is the frequency dependent potential damping coefficient \(b(\omega)\), while on the right hand, it is the retardation function \(B(\tau)\). With \(d\omega = 0.025 \text{rad/s}\) (the frequency step chosen in WAMIT to calculate the potential damping), \(d\tau\) should be as small as 0.004s, it is important to choose the time step right, otherwise the damping ratio at the low frequency will deviate from its real value, for the natural frequency point, small deviation could cause large problem.

By transfer the retardation function back to frequency domain, we can always check whether the potential damping ratio we add in time domain is in accordance with its original value.
5.6 Identified damping ratio from response spectrum

Besides directly calculating the damping ratio, we can also identify the damping ratio from the measured spectrum. Here the corrected classical damping ratio proposed by [12] is used:

$$\frac{\omega_2 - \omega_1}{\omega_r} = \frac{2\xi}{\sqrt{1 - 2\xi^2}}$$  \hspace{1cm} (5 - 17)

The half-power band method is calculated based on model test measured data and offshore real measured data([Chapter 2](#) and [Chapter 10](#)). Due to the limitation of coarse sampling frequency, the identified damping ratio might lose some accuracy, and in general it should be higher than the real value, we can regard it as the higher limit.

In the following, we mean the same thing by using the term ‘identifying damping from the measured spectrum’ and ‘half-band method’, despite of the inaccuracy, we still
assume this method can represent the real value best, it is now compared with the calculated damping by different linearization factors:

For the linearization methods, the p and q values from both the model test and CFD simulation are separately applied. In order to calculate the standard deviation of the roll velocity when performing linearization, both the 1st order and the 2nd order roll are taken into consideration, assuming their combined velocity follows a Gaussian distribution.

The 'Harmonic' term in the following tables, we applied Equation 5-11 assuming the vessel is performing a harmonic roll at natural frequency. This method always much underestimate the damping ratio, and it is reasonable to explain it as the first order motion (high frequency range) also contribute to the damping ratio at the resonance peak (low frequency range).

Model test (Sleipnir)
In the following, the damping is identified separately from the measured spectrum corresponding to the three sea states we have tested in the model test.

<table>
<thead>
<tr>
<th>Linearization factor</th>
<th>$b_{eq}$</th>
<th>$\xi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Half power band</td>
<td></td>
<td></td>
</tr>
<tr>
<td>From p and q values</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(model test)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>P = 0.0136</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Q = 0.7047 /rad</td>
<td></td>
<td></td>
</tr>
<tr>
<td>From p and q values</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(CFD)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>P = 0.0134</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Q = 0.6704 /rad</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5-2: Damping ratio comparison for model test sea
DAMPING

Figure 5-10: The half-power band method for model test sea state \( H_s = 1.5 \, m \, T_p = 12 \, s \) \( \gamma = 1 \)

<table>
<thead>
<tr>
<th>Half power band</th>
<th>( b_{eq} )</th>
<th>( \xi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-</td>
<td>1.4088e+6</td>
<td>0.60 %</td>
</tr>
</tbody>
</table>

From \( p \) and \( q \) values (model test)

- Wolfram: 1.8526e+6, 0.79 %
- Borgman: 1.0575e+6, 0.45 %
- Harmonic: 5.7365e+5, 0.24 %

Table 5-3: Damping ratio comparison for model test sea state \( H_s = 1.5 \, m \, T_p = 12 \, s \) \( \gamma = 1 \)

3. Sea state

Figure 5-11: The half-power band method for model test sea state \( H_s = 1.5 \, m \, T_p = 7 \, s \) \( \gamma = 1 \)

<table>
<thead>
<tr>
<th>Half power band</th>
<th>( b_{eq} )</th>
<th>( \xi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-</td>
<td>1.1271e+6</td>
<td>0.48 %</td>
</tr>
</tbody>
</table>

From \( p \) and \( q \) values

- Wolfram: 1.6737e+6, 0.71 %
- Borgman: 1.0163e+5, 0.43 %
- Harmonic: 6.3581e+5, 0.27 %
From Chapter 2 and above, we can conclude there might be inaccuracies in p, q and linearization factors at the same time. In the three model test cases, compare the identified damping value from measured spectrum and the equivalent damping linearized by the famous Borgman’s factor, there is a good match especially in $H_s = 3$ m $T_p = 7$ s $\gamma = 3$ sea state, where the vessel has the largest roll motion amplitude (closer to the roll amplitude when performing a decay test). The motion amplitude of the third sea state is smaller and matches the identified damping less, the second sea state matches worst. On the other hand, the performance of Wolfram’s factor get closer result in the second sea state where has a larger response in the first order frequency range than in the other two sea state. In general, the Borgman’s linearization has a small under-prediction as most of the literature reports, the Wolfram’s linearization method results in an over-prediction of damping, while the Harmonic method underestimate the damping to a large extent.

**Offshore measured data (Thialf)**

![Figure 5-12: The half-power band method for real offshore data – free floating](image1)

![Figure 5-13: The half-power band method for real offshore data – free hanging](image2)
Seeing from the real measured cases, by all linearization factors the damping ratio calculated by p and q values cannot match the identified damping from the measurement. The free hanging case even results in negative damping, which cannot explained by any physical meaning in our case.

Conclude from above, the possible reasons for the error are as following:

1. **The half-band method largely depend on the sampling frequency.** The time step for the real measurement is much larger than the model test, and the measured time series is limited, in order to get a spectrum with smooth curve, windows and overlaps are applied artificially, these all introduce inaccuracies in the half-band method, thus, it is not always accurate and sometimes can only represent the upper boundary of the real value.

2. **The single slope line identified by p and q is not sufficient to represent the damping ratio for small amplitude ship motion.** We can see from the Figure 5-15 to 5-18 when the roll motion amplitude is small, the measured data is very scattered, thus in the model test and CFD decay test, it is common practice to start with a large initial angle and neglect the data measured in smaller motion part to get a good fit with single slope line as we showed in Chapter 2, however if we extend this single slope line to the small motion part, it always happen the damping ratio at the small motion part is very low and even negative as Table 5-5 shows. And according to our simulation in the later Chapters, with single slope fitted damping, the big motions can be simulated well, while the small motion under a mild sea state is always simulated much larger than the measured value because of the low damping ratio. As a result, we would expect the damping ratio at low motion amplitude part to be larger than the result calculated by single slope fitting.

In the current theory, the quadratic damping is a function of ship motion while the viscosity is a mutual effect between the vessel and fluid, it should be more accurately determined by the relative motion.

For large ship motions, the decay curve can be fit by single slope line identified by p and q coefficients very well, it is because the relative motion is mainly introduced by the motion of the ship. While in reality, the flow of the fluid itself (not caused by ship motion) should be taken into account to increase the viscous effect when the ship motion is low.
DAMPING

Judging from the real offshore case we discussed above, with p and q only, it is always much underestimated the damping ratios for lower motion amplitude.

3. **It might also be the problem of linearization factor.**

As we discussed above, a lot of linearization factors are proposed by different literatures and none of them is proved to be a very accurate one. For the cases discussed in this thesis, the Borgman’s factor has the best performance. The velocities distribution is further checked in Figure 5-14, for a 3h simulation, both the pure second order roll case and the first order motion dominated case is well Gaussian distributed. Thus, they are in accordance with the Gaussian distribution assumption.

However, as we discussed above, the Borgman’s factor still has a small underprediction for damping. The hypothesis we would like to give here is that the higher velocity part weight more when contribute to the total damping ratio.

![Figure 5-14: Roll velocity distribution, Left: First order dominate case. Right: Second order dominate case.](image)

But since we can use time domain calculation method to avoid this linearization problem, and as in the real offshore situation discussion, with all range of linearization factors, the theoretical methods just cannot work. So the second issue seems to be more important.

5.7 **Cubic damping**

The use of cubic damping is proposed to fix the issue with linear and quadratic damping, it is difficult to match the identified damping ratio.

The derivation in the first subchapters can be extended into the following:

\[
 b^{(eq)} = b^{(1)}_\phi + \frac{8}{3\pi} \Phi_\alpha \omega b^{(2)}_\phi + \frac{3}{4} \omega^2 \Phi_\alpha^2 b^{(3)}_\phi
\]  

(5 - 18)
DAMPING

\[ b_{\phi}^{(3)} = \frac{2}{3\pi^2} M_{\phi} T_r n r \]  (5 – 19)

Where
\[ b_{\phi}^{(3)} \]  Cubic damping coefficient, [kNms^3/rad^3]

The decay curve is now fit by quadratic polynomial: \( y = rx^2 + qx + p \), where \( r \) is the quadratic constant in Equation 5-19.

As for the linearization factor, we still derived it based on Gaussian distribution Equation 5-12, thus the equivalent damping include the cubic term can be extended as:

\[ b^{(e)} = b_{\phi}^{(1)} + \frac{8}{\pi} \sigma_{\phi} b_{\phi}^{(2)} + 3 \sigma_{\phi}^2 b_{\phi}^{(3)} \]  (5 – 20)

Equation 5-20 will be used in frequency domain calculation later.

We first validate the use of the cubic damping by re-fitting the model test decay result. The whole decay curve is divided into four parts, for the first three parts, each one include 10 periods, the fourth part is from the 31th period to the very end. In general, we find for the first part with the highest roll amplitude (blue line), the \( p \) value is quite small and \( q \) value is quite large compare to other parts, and with roll amplitude smaller and smaller, the \( p \) value gradually becomes larger while the \( q \) value becomes smaller. This phenomenon verify the necessity to apply quadratic polynomial fitting. Thus the roll decay curves from the model tests are refit as follows:

Figure 5-15: Quadratic polynomial fitting for model test ‘802005’, Free floating, initial angle around 5 degree
**Figure 5-16:** Quadratic polynomial fitting for model test ‘802006’, Free floating, initial angle around 2 degree

**Figure 5-17:** Quadratic polynomial fitting for model test ‘802007’, Free floating, initial angle around 5 degree
The quadratic polynomial fitting results are compared to the linear fitting result given by the Marin report [2] in Table 5-6.
We can conclude that the quadratic polynomial fitting can reach consistency for all the decay tests, especially for test number ‘802006’, where the initial angle is much smaller than the other three and the linear fitting in MARIN report fail to reach consistency with others.

<table>
<thead>
<tr>
<th>Quadratic polynomial fitting</th>
<th>Linear fitting</th>
</tr>
</thead>
</table>

Table 5-6: Comparison between quadratic polynomial fitting and linear fitting for Sleipnir model test results.

By the same method, the linear, quadratic and cubic damping ratio of Thialf can be determined based on CFD simulation and a very old model test record [16], the results are presented as follows, a comparison with the original linear curve fitting is also performed:
Figure 5-19: Quadratic polynomial fitting for Thialf based on CFD simulation and old model test

Table 5-7: Comparison between quadratic polynomial fitting and linear fitting for Thialf roll damping.

The data samplings for Thialf is not that sufficient, especially for the full load case, no model test is available and CFD simulation is only given for the first 10 periods due to the simulation time limitation. The so caused inaccuracy would be discussed in Chapter 10, but it is worth to mention in advance that by introducing cubic damping term, the simulation results can fit the real measured results much better than using linear and quadratic terms only.

5.8 New method determining linearization factor and damping coefficients

Finally, after solving the p q r coefficients, we perform a short discussion on linearization factor and propose a new method determine both linearization factors and damping coefficients.

By substituting the identified damping ratio using half-band method on the left of Equation 5-21, and the calculated damping on the right hand, for each two measured cases, we can solve the linearization factors for quadratic damping and cubic damping:
DAMPING

\[ b^{(eq)} = b^{(1)}_{\phi} + X_1 \sigma_{\phi} b^{(2)}_{\phi} + X_2 \sigma_{\phi}^2 b^{(3)}_{\phi} \]  

(5 – 21)

Where:
- \( b^{(1)}_{\phi} \), \( b^{(2)}_{\phi} \), \( b^{(3)}_{\phi} \): Linear, quadratic, cubic damping based on p, q, r values linearized in Chapter 5.7.
- \( X_1, X_2 \): Unknown linearization factors for quadratic damping and cubic damping.

<table>
<thead>
<tr>
<th>Sleipnir</th>
<th>Thialf</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5-8: Linear, quadratic and cubic damping term for Sleipnir and Thialf.

In Table 5-8 we again find the problem that Sleipnir has a much lower damping ratio comparing to Thialf.

The linearization factors are solved and compared in the following:

<table>
<thead>
<tr>
<th>Based on Gaussian distribution</th>
<th>X₁</th>
<th>X₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wolfram's method</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Solved based on measurement (Model test sea state 1 and 3)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5-9: Linearization factors

By solving Equation 5-21 based on the measured data of Model test sea state 1 and 3, where the roll velocities follow narrow banded Gaussian distribution in a fair way (see Figure 5-14), the calculated linearization factors are very close to the analytical solution.

Thus for the cases discussed in this thesis, it can be concluded that the use of linear, quadratic and cubic damping can solve the damping problem both in frequency domain and time domain well.

On the other hand, Equation 5-21 can be used as an alternative method of logarithmic fitting (see Chapter 5.3) to determine damping coefficients, with the linearization factors proven to be accurate, it is possible to determine \( b^{(1)}_{\phi}, b^{(2)}_{\phi}, b^{(3)}_{\phi} \) by three model tests. Putting the identified damping ratio at the left hand of the equations, we have three equations and three unknowns, thus \( b^{(1)}_{\phi}, b^{(2)}_{\phi}, b^{(3)}_{\phi} \) can be solved.

### 5.9 Conclusion

Many damping topics are discussed in this Chapter, a general conclusion of the important points are listed in the following:

- The general damping ratio for Sleipnir at the natural frequency is very low due to the low frequency and the large-radius pontoon corner.
DAMPING

- A linearized equivalent damping should be used in the frequency domain calculation, the most common used Borgman factor and its extension to the cubic term is proven to match the real damping ratio best.
- Since the linearized equivalent damping is dependent on the response, an iteration loop should be applied when doing frequency domain calculation.
- Special attention should be paid when apply Cummins equation along the full frequency range, very small time step should be applied in order to avoid the over-prediction of damping in the low frequency range.
- Only by linear and quadratic damping terms derived from the model test and CFD simulated decay test cannot predict the damping ratio at low roll motion amplitude, a cubic damping term should be introduced.
6.1 Introduction

In this Chapter, the second order roll moment (by post-processing the QTFs) and second order roll motion are calculated in frequency domain. A series of methods are combined and the complete frequency domain calculation procedure is showed as Figure 6-1:

The QTFs are first computed using WAMIT (Chapter 4) according to the Pinkster's theory (Chapter 3), the second order roll moment can be calculated by applying the QTFs according to certain wave spectrum. And after that roll motion is solved based on the equation of motion.

There are two most critical treatments among the whole procedure: one is the linearization of the damping as already discussed (see Chapter 5), another is the calculation of 2nd order forces/moments.
In this Chapter, we will calculate 2nd order force and moment first based on the equation of Wichers, this is a widely used method for second order effect of mooring systems, a reflection model and a spreading model will be calculated as examples corresponding to the QTFs in Chapter 4. Especially, we will also explain the physical reasons for the high nonlinear excitation by Wichers’ equation.

After that we also explore a new way to post process QTFs in frequency domain in this thesis aimed for coupled motion. Finally the 2nd order roll motion of the vessel will be calculated based on the equation of motion, in which the damping is linearized.

### 6.2 Method based on Wichers’ 2nd force equation

Wichers\(^{[18]}\) proposed an equation which enables us to calculate 2nd order force and moment in frequency domain, it is based on the integration of all QTFs for pairs of wave components which have the same difference frequency (it is related to the wave group expression **Equation 3-6**):

\[
S_F(\mu) = 8 \int_0^\infty S_\zeta(\omega + \mu)T(\omega + \mu, \omega)\|^2\ d\omega 
\]  

(6 – 1)

This equation can be expanded into directional domain as well\(^{[19]}\):

\[
S_F(\mu) = 8 \int_0^\infty \int_0^{2\pi} \int_0^{2\pi} S_\zeta(\omega + \mu, \alpha)S_\zeta(\omega, \beta)\|T(\omega + \mu, \omega, \alpha, \beta)\|^2\ d\alpha d\beta d\omega 
\]  

(6 – 2)

Where,
\[T(\omega + \mu, \omega), T(\omega + \mu, \omega, \alpha, \beta)\] – The total QTF for certain frequency and direction combination.

\[T^2 = P^2 + Q^2\]  

(6 – 3)

\(P\) – the in phase part of the QTF

\(Q\) – the out of phase part of the QTF

The roll response spectrum is further calculated by the moment transfer function as below:

\[
H_4(\omega) = \frac{1}{[M_{44} + A_{44}(\omega)]\omega^2 + (D_{vis} + D_{pot}(\omega))i\omega + K_{res} + K_{moor}] 
\]  

(6 – 4)

Where in the equation:

\(M_{44}\) – Roll moments of inertia.

\(A_{44}(\omega)\) – Added mass for roll motion.

\(D_{vis}\) – Viscous damping (linearized by the methods discussed in Chapter 5).

\(D_{pot}(\omega)\) – Potential damping.

\(k_{xx}\) – The radius of inertia.

\(M_{44} = \rho \nabla k_{xx}^2\)  

(6 – 5)
FREQUENCY DOMAIN SIMULATION

\[ K_{res} \quad - \quad \text{Hydrodynamic stiffness.} \]
\[ K_{moor} \quad - \quad \text{Mooring line caused stiffness.} \]

Finally the roll response spectrum is resulted:

\[ S_\varphi(\omega) = H_4(\omega)^2 S_F(\omega) \] \hspace{1cm} (6-6)

Where,
\[ S_\varphi(\omega) \quad - \quad \text{Second order roll spectrum.} \]

### 6.3 High second order excitation of Sleipnir

In **Chapter 5.2**, we give the physical reasons of the low damping problem, here we explain another important issue which cause significant second order roll for Sleipnir, and it is the high second order excitation:

The equation of Wichers can be visually explained as **Figure 6-2**. As we explain in **Chapter 4**, the peak of QTFs in the red circle is mainly due to the certain distance between the two pontoons, and according to the dimension of Sleipnir, the peak located at 0.7165rad/s which corresponding to a period of 8.8s, this is very close to the model test sea state which has a 7s peak period. The second order roll moment excitation would be high if the 2 peaks in the wave spectrum are located in the same frequency range as the peak in the QTF, because the three peaks will be multiplied together according to Wichers’ equation (see **Figure 6-2**). Thus in the model test, the second order excitation is quite high, and it is another very important reason cause obvious second order roll motion.

Further we can conclude for Sleipnir, because of its large size, it is more sensitive to the wave with larger period, this could be a problem. Usually larger period means the wave has more time to build up, thus the significant wave height is tend to be higher for
longer wave period. The second order roll will be higher because of the double influence from both the higher $H_s$ and more sensitive $T_p$.

### 6.4 Uni-direction 2nd order roll moment and motion prediction (Frequency domain)

In the following, uni-directional second order roll moment and roll motion is calculated based on the procedure in Chapter 6.2. In the following, we simulate the model test environment (See Chapter 2) as an example:

**Moment spectrum**

The 2nd order roll moment spectrums calculated based on Equation 6-1 are plotted as follows (see Figure 6-3).

Three cases are calculated here, for the no reflection case, we calculate it based on measured first order wave spectrum without reflection (Figure 2-7 red line) and the QTF values without reflection (See Chapter 4).

For the reflection case, two results are calculated, both use the QTF values with reflection (See Chapter 4), for the first order spectrum, one uses the spectrum without reflection (Figure 2-7 red line) and another uses the first order spectrum with reflection (Figure 2-7 blue line) as measured in model test.

![Figure 6-3: Second order roll moment spectrums for reflection and non-reflection case, Unit: kNm²/s](image)

The reasons for calculating 2 wave reflection cases are based on:

1. The measured reflected wave is larger than the wave that causes the vessel’s roll motion. Because it measures not only the wave propagating to the model but also the wave traveling away from the model. However, the latter doesn’t cause ship motions.
2. On the other hand, the non-reflected wave is smaller than the real wave which cause vessel’s roll motion. So neither the reflection wave nor the non-reflection wave in Figure 2-7 can represent the exact incoming wave to the vessel, and that is the reason why we calculate both in order to investigate how the reflection in the model test can be modelled based on the fact no accurate incoming wave spectrum is available.

The moment spectrum density of 0.16rad/s difference frequency (natural roll frequency of Sleipnir in the model test) with and without reflection corresponding to Figure 6-3 is listed in the Table 6-1, and the percentage on the right of the spectrum density value indicates how much the reflection added to the non-reflection case.

<table>
<thead>
<tr>
<th>Reflections</th>
<th>Spectrum Density</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>No reflection</td>
<td>1.53E+09</td>
<td>100.00%</td>
</tr>
<tr>
<td>Reflection QTF only</td>
<td>2.71E+09</td>
<td>177.14%</td>
</tr>
<tr>
<td>Reflection QTF &amp; wave spectrum</td>
<td>3.81E+09</td>
<td>248.91%</td>
</tr>
</tbody>
</table>

Table 6-1: Second order roll moment spectrum density at roll natural frequency 0.16 rad/s. Unit: kNm²/s

**Moment transfer function**

The roll moment transfer function is further calculated based on Equation 6-4, where the added mass and added damping are calculated by diffraction analysis, the result is showed as follows:

![Added Mass for Roll motion](image1)

![Added Damping for Roll motion](image2)

Figure 6-5: Added mass and added damping (Reflection case)

And the equivalent damping is calculated by the linearization method explained in Chapter 5 and it is a result of iteration as Figure 5-6:
After the iteration, with reflection, the ship roll motion is expected to be larger, thus they have larger damping ratio.

**Figure 6-6** is the transfer function from the second order roll moment to the second order roll motion, the peak value appears at 0.155 rad/s which indicated the natural roll frequency. The transfer function at the peak value is extremely sensitive to the damping value, for the three cases calculated, it is obviously different because of their slightly varied damping ratios. And compare **Table 6-2** and **Table 6-3**, the quadratic polynomial fitting is closer to the identified damping we discussed in **Chapter 5**, thus all the calculation below is based on quadratic polynomial fitted damping.

**Roll response spectrum**
The roll response spectrum can be further achieved by multiplying the force spectrum with the moment transfer function, see **Equation 6-6**.
The total spectrum density which represents the variation is calculated by integrating the spectrum density over the whole frequency band, the standard deviation is calculated by taking the square root (see Equation 6-7).

\[
\sigma_\varphi = \sqrt{ \int S_M(\mu) \cdot \left| \frac{\varphi_A(\mu)}{F_a} \right|^2 d\mu }
\]  
\[
\varphi_{\text{max}} = \varphi_{\text{mean}} + \sqrt{2\ln N \sigma_\varphi}
\]  

The mean value for the roll angle of semi-submersible equals zero, thus we can achieve the maximum roll angle for each simulation case as follows:

\[
\begin{array}{|c|c|c|c|}
\hline
\text{Measured} & \text{No reflection} & \text{Reflection QTF only} & \text{Reflection QTF & wave spectrum} \\
\hline
\end{array}
\]

Table 6-4: The standard deviations of the second order roll response spectrums (unit: degree)

The most probable maximum roll angle is further calculated by:\(^{(11)}\):

\[
\sigma_\varphi = \sqrt{ \int S_M(\mu) \cdot \left| \frac{\varphi_A(\mu)}{F_a} \right|^2 d\mu }
\]  
\[
\varphi_{\text{max}} = \varphi_{\text{mean}} + \sqrt{2\ln N \sigma_\varphi}
\]  

From Figure 2-7, an obvious higher spectrum density can be seen in the reflected wave spectrum, however, Table 6-4 and Table 6-5 shows the non-reflected case we have simulated actually match the real measured data best. The possible reasons are explained as follows:

- The second order roll moment is overestimated by applying Wichers’ equation, we will evaluate the possibility of overestimation by comparing the time domain calculation in Chapter 7 and a new frequency domain method we use later in this Chapter.
After introducing cubic damping, the damping ratio is still slightly underestimated compared to the real value identified in Chapter 5 and Table 6-3.

The measured reflected wave is the wave some distance away from the vessel, while the true excitation is the wave exactly on the vessel, there is a difference in between: On one hand the measured spectrum is close to the wave maker which results in a higher energy level, on the other hand, the peak frequency modes of stand wave could be totally different, which cause the peak frequencies in QTF also totally different.

The reflected QTF, we propose to calculate it by adding the first four terms of a reflected model and the fifth term of a non-reflected model in Chapter 4. This treatment may have a shortage, which is by reflection, the phasing of the wave could be changed a lot, the simple summation of the two parts could cause problem.

6.5 Multi-direction 2nd order roll moment and motion prediction (Frequency domain)

In this subchapter, the QTF results are further processed for multidirectional wave which is more close to the real open sea situation.

Wave spreading spectrum
The wave spectrum used for the following calculation is a directional wave spectrum according to DNV-205[20] considering spreading.

\[ S(\omega, \theta) = S(\omega)D(\theta) \]  \hspace{1cm} (6 - 9)

\( S(\omega) \) is chosen to be JONSWAP spectrum while the expression for \( D(\theta) \) is as follows:

\[ D(\theta) = \frac{\Gamma(s+1)}{2\sqrt{\pi}\Gamma(s+1/2)} \cos^{2s}\left(\frac{1}{2}(\theta - \theta_p)\right) \]  \hspace{1cm} (6 - 10)

Where \( |\theta - \theta_p| \leq \pi \)

The six sea state we used to perform the calculation is plotted as follows from the least spreading to almost 360 degree spreading. The spreading angle is chosen based on whether there is non-zero spectrum energy in the corresponding spreading angle domain.
FREQUENCY DOMAIN SIMULATION

$s = 80$ spreading 60 degree
Wave spreading spectrum $S(\omega)$ [m$^2$ s$^{-1}$]

$s = 20$ spreading 120 degree
Wave spreading spectrum $S(\omega)$ [m$^2$ s$^{-1}$]

$s = 7$ spreading 180 degree
Wave spreading spectrum $S(\omega)$ [m$^2$ s$^{-1}$]

$s = 4$ spreading 240 degree
Wave spreading spectrum $S(\omega)$ [m$^2$ s$^{-1}$]

$s = 2$ spreading 300 degree
Wave spreading spectrum $S(\omega)$ [m$^2$ s$^{-1}$]

$s = 1$ spreading 360 degree
Wave spreading spectrum $S(\omega)$ [m$^2$ s$^{-1}$]

Figure 6-8: Wave spreading spectrums, unit: m$^2$s$^{-1}$10degree
FREQUENCY DOMAIN SIMULATION

Roll moment spectrum
The roll moment spectrum can be further calculated by Equation 6-2, the result of all the six spreading models are plotted and compared as follows:

Figure 6-9: Second order roll moment spectrums for spreading models

As we can see, the roll moment is decreased a lot by wave spreading, and the larger the spreading angle is, the smaller the roll moment density is. The values for each wave spreading case at the natural frequency(0.16 rad/s) are listed as follows, the percentage on the right indicates how much energy is still left after considering certain degree of wave spreading.

<table>
<thead>
<tr>
<th>Unidirectional</th>
<th>s=80 +30 degree</th>
<th>s=20 +60 degree</th>
<th>s=7 +90 degree</th>
<th>s=4 +120 degree</th>
<th>s=2 +150 degree</th>
<th>s=1 +180 degree</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.53E+09</td>
<td>9.89E+08</td>
<td>5.86E+08</td>
<td>3.52E+08</td>
<td>2.63E+08</td>
<td>1.82E+08</td>
</tr>
<tr>
<td>Percentage</td>
<td>100.00%</td>
<td>64.84%</td>
<td>38.44%</td>
<td>23.11%</td>
<td>17.23%</td>
<td>11.93%</td>
</tr>
</tbody>
</table>

Table 6-6: Second order roll moment spectrum density for spreading models at the natural roll frequency

Equivalent Damping
For each spreading sea state, different roll response is expected, so accordingly their damping ratio is different due to nonlinearity. The final used equivalent dampings for each sea state after iteration are listed as follows:

<table>
<thead>
<tr>
<th>Unidirectional</th>
<th>s=80 +30 deg</th>
<th>s=20 +60 deg</th>
<th>s=7 +90 deg</th>
<th>s=4 +120 deg</th>
<th>s=2 +150 deg</th>
<th>s=1 +180 deg</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 6-7: Equivalent damping and damping ratio for each spreading case
FREQUENCY DOMAIN SIMULATION

**Moment transfer function**
Added mass and added damping are still calculated by diffraction analysis, and the moment transfer function is calculated by **Equation 6-4**:

![Added Mass and Damping](image)

**Figure 6-10**: Added mass and added damping (spreading model)

The roll moment transfer function is further calculated based on **Equation 6-4**, the peak value still appears at 0.155 rad/s which indicates the natural roll frequency. The peak of transfer function is different for each sea state because of their different damping ratios.

![Transfer Function](image)

**Figure 6-11**: Moment transfer function at the natural frequency band

**Roll response spectrum**
The roll response spectrums for all kind of spreading angles are plotted and compared as follows, the roll response spectrum could be mitigated significantly by wave spreading.
Figure 6.12: Second order roll response spectrum for different wave spreading

The standard deviation of each wave spectrum are listed as follows:

<table>
<thead>
<tr>
<th>Uni-directional</th>
<th>s=80 +30 deg</th>
<th>s=20 +60 deg</th>
<th>s=7 +90 deg</th>
<th>s=4 +120 deg</th>
<th>s=2 +150 deg</th>
<th>s=1 +180 deg</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td></td>
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</tr>
</tbody>
</table>

Still assume the second order roll motion follows a normal distribution, then the maximum roll angle can be calculated as follows by Equation 6.8. According to DNV rules, the normal spreading factor for wind sea is \( s = 4 \sim 9 \). And for swell \( s > 13 \). So for a wave spreading spectrum with the same total spectrum density as uni-direction JONSWAP spectrum \( H_S = 3m \) \( T_P = 7s \) \( \gamma = 3 \), in wind sea, the maximum angle according to the simulation is expected to be about 1 degree, and for swell, it is larger than 1 degree.

6.6 Summation of QTFs with phasing information

We notice the treatment based on the equation of Wichers has the following limitation:

Equation 6.1 and Equation 6.2 don’t include any phasing issues, for single degree of freedom, we can take the absolute value of both side in the equation of motion as Equation 6.4 to calculate the transfer function, however, to solve a multiple degrees of freedom problem, we need a complex number of force in the right hand of the equation of motion instead of an absolute value of force amplitude(see Equation 6.12).
This limitation (the phasing information for the second order roll moment is missing) makes the Wichers’ equation not able to calculate multi degrees of freedom system. For those problems where 2nd roll motion not only caused by 2nd order roll moment but also influenced a lot by 2nd order sway force or the swing motion of the load on the deck as we will calculate in Chapter 10, a new method has to be proposed to solve the problem.

Since the output from WAMIT has an in-phase part and an out-of-phase part for each QTF value corresponding to two wave components, so for each wave pair we do have the phasing information. However, since the phasing of P and Q is only valid for single pair, we have to establish a phasing connection between all the QTFs values which have the same difference frequency, so as to get a combined phasing information for each difference frequency, the following method is proposed:

According to the definition, the P part is in-phase with the wave group while the Q part is out-of-phase with the wave group, so we feed a reference phasing for this wave group which equals the difference phasing of the two first order wave components \( \varepsilon_1 - \varepsilon_2 \), and then rotate the QTF by an angle equals \(- (\varepsilon_1 - \varepsilon_2)\), thus all the QTFs of the same difference frequency now have the same reference system, we can add them up by Equation 6-11 then:

\[
F(\mu)^{(2)} = \sum_{n=1}^{N} \zeta_{n,1} \zeta_{n,2} (P_n + Q_n i) e^{-i(\varepsilon_{n,1} - \varepsilon_{n,2})} 
\]  

(6 – 11)
Please note depend on what kind of phasing we feed for first order wave, the second order force calculated by Equation 6-11 is not unique.

We use this method to calculate the second order force or moment for sway, roll and yaw motion as well as the first order force and moment. And feed the results in the right hand of Equation 6-12:

\[
\begin{align*}
-A^2 & \begin{pmatrix}
\rho \psi & \rho \psi & l_x & l_y \\
\rho \psi & \rho \psi & l_x & l_y \\
\end{pmatrix}
\begin{pmatrix}
P \lambda_1 \\
D_{22} \\
D_{33} \\
D_{55} \\
\end{pmatrix}
\begin{pmatrix}
\omega \\
\omega \\
\end{pmatrix}
+ \begin{pmatrix}
[A_{11}(\omega)] & [A_{12}(\omega)] & [A_{13}(\omega)] & [A_{14}(\omega)] & [A_{15}(\omega)] & [A_{16}(\omega)] \\
0 & [A_{22}(\omega)] & [A_{23}(\omega)] & [A_{24}(\omega)] & [A_{25}(\omega)] & [A_{26}(\omega)] \\
0 & 0 & [A_{33}(\omega)] & [A_{34}(\omega)] & [A_{35}(\omega)] & [A_{36}(\omega)] \\
0 & 0 & 0 & [A_{44}(\omega)] & [A_{45}(\omega)] & [A_{46}(\omega)] \\
0 & 0 & 0 & 0 & [A_{55}(\omega)] & [A_{56}(\omega)] \\
0 & 0 & 0 & 0 & 0 & [A_{66}(\omega)] \\
\end{pmatrix}
\begin{pmatrix}
X \\
Y \\
Z \\
\Phi \\
\Theta \\
\Psi \\
\end{pmatrix}
+ \begin{pmatrix}
[B_{11}(\omega)] & [B_{12}(\omega)] & [B_{13}(\omega)] & [B_{14}(\omega)] & [B_{15}(\omega)] & [B_{16}(\omega)] \\
0 & [B_{22}(\omega)] & [B_{23}(\omega)] & [B_{24}(\omega)] & [B_{25}(\omega)] & [B_{26}(\omega)] \\
0 & 0 & [B_{33}(\omega)] & [B_{34}(\omega)] & [B_{35}(\omega)] & [B_{36}(\omega)] \\
0 & 0 & 0 & [B_{44}(\omega)] & [B_{45}(\omega)] & [B_{46}(\omega)] \\
0 & 0 & 0 & 0 & [B_{55}(\omega)] & [B_{56}(\omega)] \\
0 & 0 & 0 & 0 & 0 & [B_{66}(\omega)] \\
\end{pmatrix}
\begin{pmatrix}
X \\
Y \\
Z \\
\Phi \\
\Theta \\
\Psi \\
\end{pmatrix}
+ \begin{pmatrix}
[K_{11}] & [K_{12}] & [K_{13}] & [K_{14}] & [K_{15}] & [K_{16}] \\
K_{22} & K_{33} + \rho g A_w & K_{44} + \rho \psi \cdot G M^T & K_{55} + \rho \psi \cdot G M^L & K_{66} \\
\end{pmatrix}
\begin{pmatrix}
P \lambda_1 \\
D_{22} \\
D_{33} \\
D_{55} \\
\end{pmatrix}
\begin{pmatrix}
\omega \\
\omega \\
\end{pmatrix}
\begin{pmatrix}
[X] \\
[Y] \\
[Z] \\
[\Phi] \\
[\Theta] \\
[\Psi] \\
\end{pmatrix}
\end{align*}
\]

Where,
\( \nabla \) - The displacement of the hull.
\( A_{ij}(\omega) \) - Frequency dependent added mass, if \( j \neq i \), the added mass is caused by coupled motion, \( j \) motion influence on \( i \) motion.
\( B_{ij}(\omega) \) - Frequency dependent potential damping, \( j \) motion influence on \( i \) motion.
\( D_{ii} \) - The equivalent viscous damping.
\( K_{ii} \) - Stiffness caused by mooring line.
\( G M^T \) - Transverse initial metacentric height.
\( G M^L \) - Longitudinal initial metacentric height.
\( A_w \) - Water line area.
\( F_i^{(2)} M_i^{(2)} \) - Second order force and moment.
\( F_i^{(1)} M_i^{(1)} \) - First order force and moment.

Here we consider the coupled motion, the motion is coupled because the moment of inertia, the standard viscous damping and the stiffness caused by both mooring and restoring moment are defined at the centre of gravity, while the added mass, added damping, the excitations and the final roll motion are calculated at the hydrodynamic centre which is located at the water line. So the physical quantities defined at the centre of gravity have to be transferred to the hydrodynamic centre to make the coordinate consistent. And this treatment will lead to non-zero diagonal terms.

The \( P \) matrix is used as a transfer function for this purpose. It is defined as:
FREQUENCY DOMAIN SIMULATION

\[
P = \begin{bmatrix}
1 & 1 & 1 & 1 \\
OG_z & 1 & 1 & 1 \\
OG_y & -OG_z & OG_x & 1 \\
OG_x & -OG_y & OG_z & 1
\end{bmatrix}
\] (6 – 13)

Where O is the hydrodynamic centre and G is the centre of gravity. And \( P^T \) is the transport matrix.

The damping ratio in Equation 6-12 has to be iterated for each seed, which make this frequency domain method very time consuming. By solving Equation 6-12, an amplitude value of \([X Y Z Φ θ Ψ]\) can be resulted, and by Equation 6-7 and Equation 6-8, standard deviation and possible maximum roll angle can be calculated.

Because of the non-uniqueness caused by phasing, a random seed experiment should be performed to see the statistics of results. Please note the same random seeds experiment would be performed in Chapter 7 in time domain as well, the results will be compared, and finally in Chapter 8, a discussion about the statistics would be further performed.

The following are the statistics of the results for 230 random seeds, all the three cases mentioned above are investigated:

![Figure 6-14](image1.jpg)

**Figure 6-14:** The statistics of the variance of roll response spectrum for the non-reflection case, 230 random seeds.

![Figure 6-15](image2.jpg)

**Figure 6-15:** The statistics of the variance of roll response spectrum for the reflection case (Considering QTF only), 230 random seeds.
The distribution of the variance doesn’t seem to follow any distribution, it obviously skews to the lower side of the mean value. We pick the mean value of variance here and calculate the corresponding standard deviation, the mean variance is chosen instead of the mean standard deviation is because it directly represent the mean of energy level. The results of the new frequency domain method are compared to the result with Wichers’ equation as follows:

<table>
<thead>
<tr>
<th></th>
<th>New method</th>
<th>Wichers’ equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>No reflection</td>
<td>0.2382</td>
<td>0.4881</td>
</tr>
<tr>
<td>Reflection QTF only</td>
<td>0.3801</td>
<td>0.6165</td>
</tr>
<tr>
<td>Reflection QTF &amp; wave</td>
<td>0.5100</td>
<td>0.7141</td>
</tr>
</tbody>
</table>

Table 6-10: The comparison of the variance/standard deviation between the new method and Wichers’ equation. Unit: degree$^2$/degree

Table 6-10 shows the simulation results of the new method are smaller than the results of Wichers’ equation. Although here we compare the roll response results, the procedure from the second order forces/moments to the roll motions are exactly the same for both methods, thus the difference in the final results is a direct reflection of the difference in the calculation of second order excitations.

The measured standard deviation in the model test for a 3 hour simulation is 0.56 degree. For the results of Wichers’ equation, all three cases are larger than the measured results. For the new method, the non-reflection case is smaller than the measurement while the two reflection cases are larger.

As a conclusion based on our case only, the new method fits the measurement better. However, the Wichers’ equation is still a good method, because it can give a conservative result and the simulation time is much less than the new method. Thus in the following, when performing a rough estimation, the Wichers’ equation would still be applied by simplifying the problems as single degree of freedom.

6.7 Conclusion

In this Chapter, the frequency domain methods of calculating 2nd order force or moment, and 2nd order roll are investigated.
FREQUENCY DOMAIN SIMULATION

- First, the equation of Wichers is used and then based on its limitation (it is a Force spectrum without phasing information), we propose a method by applying the original definition of QTF. Both methods are capable of giving a rough estimation of 2\textsuperscript{nd} order roll, while the result of the Wichers’ equation is more conservative.
- Three uni-directional wave cases, with one non-reflection case and two reflection cases are simulated according to the model test environment. The reflection phenomenon in the model test do add to the roll response theoretically.
- Several spreading wave cases are simulated, and with the same total spectrum density, the larger the spreading angle, the smaller the final roll amplitude is expected.

Since our problem is a nonlinear problem, after linearization, the frequency domain methods are more or less inaccurate. However, on the other hand, the nonlinearities also make time domain methods very time consuming, not to say the large variance characteristics of 2\textsuperscript{nd} order roll (discussed in Chapter 8), thus, the use of frequency domain method as a fast estimation do have its value in the discussions in our following chapters.
7

TIME DOMAIN SIMULATION

7.1 Introduction

The second order roll is a nonlinear problem, considering both 2nd order roll moment and damping. In the previous frequency domain calculation, we introduced linearization methods for the nonlinearities, this will definitely lead to inaccuracies. Thus although the time domain calculation is very time consuming, it is still regarded a better way to solve our problem. The complete time domain procedure of calculating second order roll is showed in Figure 7-1:

![Diagram showing the time domain procedure of calculating the 2nd force/moment spectrum and the 2nd order motion.]

There are only two big difference between frequency domain and time domain, one is the calculation of second order force, the other is damping (see Chapter 5). The QTFs we apply in time domain is exactly the same as in frequency domain calculated in Chapter 4.

In this Chapter, we introduce the time domain method to calculate second order forces and moments, and further give the equation of motion to calculate the second order roll. The results will be compared with frequency domain methods.
7.2 Concept of time domain method

In the time domain, the second order force time series can be calculated by\[^{[11]}\]:

$$F^{(2)} = \sum_{i=1}^{N} \sum_{j=1}^{N} \zeta_i \zeta_j P_{ij}(\omega_i, \omega_j, \alpha, \beta) \cos\left[\left(\omega_i - \omega_j \right)t + \left(\bar{\varepsilon}_i - \bar{\varepsilon}_j \right)\right]$$

$$+ \sum_{i=1}^{N} \sum_{j=1}^{N} \zeta_i \zeta_j Q_{ij}(\omega_i, \omega_j, \alpha, \beta) \sin\left[\left(\omega_i - \omega_j \right)t + \left(\bar{\varepsilon}_i - \bar{\varepsilon}_j \right)\right]$$  \hspace{1cm} (7 - 1)

Where

- $\omega_i, \omega_j$ The frequency of the first and the second wave component
- $\bar{\varepsilon}_i, \bar{\varepsilon}_j$ The phasing of the first and the second wave component
- $\alpha, \beta$ The incoming direction of the first and the second wave component
- $\zeta_i, \zeta_j$ The wave amplitude of the first and the second wave component
- $P_{ij}(\omega_i, \omega_j, \alpha, \beta)$ The in phase part of QTF
- $Q_{ij}(\omega_i, \omega_j, \alpha, \beta)$ The out of phase part of QTF

The N in Equation 7-1 stands for the number of wave components in a certain wave spectrum. It should be chosen on one hand fully represents the wave spectrum, on the other hand, not too large which would make the calculation time too long. In this thesis, the frequency step of wave components are chosen to be non-equivalent\[^{[30]}\]. This trick is made because we want to simulate a non-repetitive time series as long as three hours, if we use equivalent frequency step, the wave components would be too large for the calculation of Equation 7-1.

For the directional wave spectrum, not all directions and all frequencies have the same spectrum density, thus another trick made to limit the number of wave components is that we choose more components in the direction and frequency range with higher spectrum density and less components in the direction and frequency range with less spectrum density.

After calculating the second order forces and moments, they can be substituted into the right hand of the equation of motion. The equation of motion can be written as follows, we substitute the Cummins equation and its application (Equation 5-14, Equation 5-15, Equation 5-16) and the treatment of nonlinear damping as explained in Chapter 5 into a normal 6 degree of freedom equation, hydrodynamic stiffness are especially added:
The equation is explained as follows:

P is the transfer function from centre of gravity to the hydrodynamic centre (see Equation 6-13 and the related explanation above that equation).

For the inertial terms, we have the original mass and the added mass, all the notations for the original mass are the same as the frequency domain expression (see Equation 6-5 and Equation 6-12). For the added mass term, it is originally frequency dependent, we use the same treatment as the potential damping terms to make it applicable in time domain, see Chapter 5. The derivation is neglected, the final results are directly written in Equation 7-2.
TIME DOMAIN SIMULATION

For the damping terms, the first three terms are corresponding to the linear, quadratic and cubic viscous damping terms as introduced in Chapter 5, Equation 5-8, Equation 5-9 and Equation 5-19. The last damping term is the application of frequency dependent potential damping in time domain (Equation 5-14, Equation 5-15, Equation 5-16).

For the stiffness term, still we have mooring stiffness and hydrodynamic stiffness, the notations please refer to Equation 6-5 and Equation 6-12.

On the right hand, the external forces are consisted of 1st and 2nd order wave excitation, for second order, it is calculated by Equation 7-1. First order can refer to the following equation, only one wave component is involved each time, the notations are similar as Equation 7-1, $P^{(1)}$ and $Q^{(1)}$ are the in phase and out of phase first order force for unit amplitude wave.

\[
F^{(1)}(t) = \sum_{\omega_0}^{\omega_{\text{max}}} \sum_{\alpha}^{2\pi} \zeta_{\alpha,i} P^{(1)}(\omega, \alpha) \cos(\omega_0 t + \epsilon_i) + \zeta_{\alpha,i} Q^{(1)}(\omega, \alpha) \sin(\omega_0 t + \epsilon_i) \quad (7-3)
\]

7.3 Uni-direction 2nd order roll moment and motion prediction (Time domain)

We first use Equation 7-1 to calculate the 2nd order force and moment time series in the model test environment where the vessel encounters a uni-directional beam wave. The three cases corresponding to whether there’s reflection or not as explained in Chapter 6 are also used here.

Altogether 256 wave components with non-equivalent spaced wave frequencies and random phasing are made according to the wave spectrum in Figure 2-7, they are further used in calculating the 2nd force and moment time series.

A random seed experiment of 230 different phasing is performed, one example is given in Figure 7-3, the time series of other 229 simulations are not plotted in this thesis, we only focus on the main parameters of each simulation and how they are distributed.

In the following, the 230 second order roll moment time series simulated by different phasing are transferred into frequency domain to see how much deviation could be caused by phasing. The frequency domain results are also plotted as a comparison.
TIME DOMAIN SIMULATION

Compare with the frequency domain results, the time domain results are relatively lower, the frequency domain results are at about the upper limit of 230 random seeds calculation of time domain results. Since the time domain method is believed to be more accurate, it is calculated without any linearization and assumptions, it can conclude that the Wichers’ equation in general over-estimate the 2nd order roll moment in our case.

And afterwards the roll motion time series is calculated by MATLAB ODE solver, where the input is the equation of motion (see Equation 7-2). In which, the frequency dependent added mass and potential damping coefficients are calculated additionally from diffraction analysis. Roll, yaw second order moments and sway second order forces are input as time-dependent external force on the right hand of the equation. One simulation results for non-reflection case is plotted as follows:

Figure 7-2: 2nd order roll moment spectrum simulated both in frequency domain and time domain
TIME DOMAIN SIMULATION

Figure 7-3: One seed simulation of wave elevation, 2nd order roll moment and roll angle time series for non-reflection case

Figure 8-6 shows more time series. In the following, the general distributions of the simulation results are discussed.

Starting from the non-reflection case:
Seeing from the plot below, the values of maximum roll angles and the standard deviations of the 230 simulations (3 hour per simulation) are fit by NORMAL, RAYLEIGH and WEIBULL distribution separately by MATLAB function ‘fitdist’, the fitting of RAYLEIGH distribution need to adjust the minimum value to zero at the beginning and shift the horizontal axis back after fitting. The bar plot for original data is normalized to have a summation value of 1, so that it can be compared to the fitting distributions. All the fitting parameters are plotted in the Figure 7-4.
The maximum roll angles tends to follow a NORMAL distribution, with a mean value of 1.49 degree, thus we can define the most probable maximum angle during 3 hours for the non-reflection case is 1.49 degree, and if we consider 95% confidence band, the most probable maximum extreme roll angle during a 3h time series is about 1.86 degree.

The standard deviation is also calculated per simulation. They seem to follow a normal distribution. The mean value is 0.52 degree while the maximum extreme with 95% confidence band is about 0.63 degree.

Then comes the reflection cases, first we calculate the case with reflected QTF but non-reflected wave. We also fit this set of data by various of distributions as the procedure explained above in Figure 7-5.

The maximum roll angle still tends to follow a NORMAL distribution, with a mean value (most probable maximum roll angle) of 1.70 degree, and if 95% confidence band is taken, the maximum extreme roll angle during a 3h simulation is expected to be 2.06 degree. The standard deviation of each 3 hour simulation for this reflection case is also tend to follow a normal distribution. The mean value is 0.58 degree while the maximum is about 0.71 degree.
Figure 7-5: The max. roll angle and the standard deviation statistics for 230 random seeds experiment (Reflection case: Reflected QTF with non-reflected wave)

For the reflection case considering both reflected QTF and reflected waves:

Figure 7-6: The max. roll angle and the standard deviation statistics for 230 random seeds experiment (Reflection case: Reflected QTF with reflected wave)

We treat the data as the procedure above, both the max. roll angle and spectrum variance tends to follow a normal distribution.
The mean of the maximum roll angles in the 230 times 3 hour simulations, which represents the most probable maximum roll angle within 3 hours simulation for this reflection case is expected be 1.9 degree, while its maximum extreme is about 2.3 degree considering a 95% confidence band. For the standard deviation, the mean value is 0.66 degree and the maximum value is expected to be 0.81 degree after taking 95% confidence band.

A comparison between frequency and time domain results considering the standard deviation as well as the MPM (most probable maximum) roll angle within 3 hours simulation for the three cases are listed as below:

<table>
<thead>
<tr>
<th></th>
<th>Measured (Max./MPM)</th>
<th>Frequency domain (MPM)</th>
<th>Time domain (MPM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No reflection</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reflection QTF only</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reflection QTF &amp; wave spectrum</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 7-1: Comparison of frequency and time domain methods on the std. of the spectrum density. Unit: degree

<table>
<thead>
<tr>
<th></th>
<th>Measured (Max./MPM)</th>
<th>Frequency domain (MPM)</th>
<th>Time domain (MPM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No reflection</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reflection QTF only</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reflection QTF &amp; wave spectrum</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 7-2: Comparison of frequency and time domain methods on the maximum and MPM of the roll angle. Unit: degree

The first value in the frequency domain column is calculated by Wichers’ equation and the second is calculated by the new method we proposed last Chapter.

As we can see, both of the frequency domain method result in a higher value than the time domain method, while the latter is believe to be more accurate, because no linearization is applied.

For the time domain prediction, the measured MPM (most probable maximum) roll angle is located between the two reflection case, which means the reflection in the model test do add to the roll motion.

7.4 Multi-direction 2nd order roll moment and motion prediction (Time domain)

There are only two big difference between simulating roll motion in uni-directional wave and multi-directional wave in time domain.

1. More wave components are required to represent a directional wave spectrum, which means longer simulation time and we have to choose wave components more smartly.
2. The relative ship heading in the wave spectrum should be paid attention to, this will be discussed in the following chapters.
In the following, we give a single seed simulation for multi-directional wave. The target spectrum used to make wave components and time series is as follows. The parameter for this wave spectrum is:

This spectrum model is chosen because it is close to real wind sea situation according to DNV as described before. Altogether 720 wave components are made following the two tricks mentioned before.

Figure 7-7: Target wave spreading spectrum for random seed experiment

The results of wave elevation, second order roll moment and roll response time series for a single seed one hour simulation are plotted as follows:

Figure 7-8: One seed simulation of wave elevation, 2nd order roll moment and roll angle time series for multidirectional wave

We can see from this example, the roll motion is obviously lower than the simulation for the non-reflected uni-direction wave, although difference is expected for different
phasing, it can still be concluded that the spreading of wave energy plays an important role here. Because we are going to simulate the real cases in Chapter 10 for multi-directional wave, here we just introduce the concept by calculating one seed, for the possible difference caused by different phase, it will be discussed in the later chapters.

### 7.5 Conclusion

In this Chapter, the time domain methods of calculating 2\textsuperscript{nd} order forces or moments, and 2\textsuperscript{nd} order roll are investigated.

- To mitigate the time-consuming problem of time domain simulation, we propose two tricks in making wave components based on the programme practice.
- The results for both frequency and time domain method are compared, and the time domain method fits the measurement better, thus, it is believed the time domain method is more accurate for this nonlinear problem.
- The phenomenon that the reflection add to the 2\textsuperscript{nd} order roll and the wave spreading decrease the 2\textsuperscript{nd} order roll can be still found in time domain simulation.

Base on the characteristics of being accurate and the time-consuming, the time domain method will be applied in the situation where high accuracy is required in the following chapters.
LARGE VARIANCE IN RESPONSE

8.1 Introduction

In Chapter 7, random seed experiments are performed for the three uni-directional wave cases and probability distributions are fit for the results (see Figure 7-4, Figure 7-5 and Figure 7-6). For each case, the variance of the maximum angles (the extreme value of each simulation) and the standard deviations are quite large, the former is expected to be true even for 1st order motion and it can be explained well by most of the literatures [11]:

![Figure 8-1: Probability Distributions of the zero-crossing maxima (red dash line) and the maxima during each simulation (blue dash line)](image)

The zero-crossing maxima in one simulation (distribution in red dash line Figure 8-1) and the maxima of each simulation (distribution in blue dash line Figure 8-1) naturally follow a distribution (Both are Rayleigh distribution for 1st order motion).

In Figure 7-4, Figure 7-5 and Figure 7-6, the distribution in cyan is actually the distribution in blue dash line (the distribution of extreme value of each simulation see Figure 8-1), in the discussion comparing frequency and time domain methods at the end of last Chapter, we pick the most probable maximum value (blue dot as the results are Gaussian distributed) in time domain and compare it with the calculated most probable maximum (red dot) by Equation 6-7 and Equation 6-8 in frequency domain.
LARGE VARIANCE IN RESPONSE

However, the large variance of the standard deviations for each simulation is quite different from what we are used to in first order motions (see Figure 7-4, Figure 7-5 and Figure 7-6, the distribution in green). In normal first order motion calculation, the response spectrum is expected to be quite unique under the situation where the excitations are coming from the same first order wave spectrum, and for different simulations, the only difference is the phasing of the chosen wave components.

In this Chapter, we will explain the two main reasons: Non-uniqueness of wave group excitation and Low damping caused group effect which result in the large variance in the standard deviations of different simulations.

8.2 Non-uniqueness of wave group excitation

In this subchapter, we first explain why the wave group excitation is not unique. In the model test, only one set of random phase is given to the chosen wave components. However, as we all know, with the same wave spectrum but different seeding, the wave time series could be different. (See Figure 8-2)

The deviation in wave time series will lead to difference in wave group time series. As the theory in Chapter 3.1, there are in general two ways to calculate wave group spectrum, one is using Equation 3-6 to represent wave group by first order spectrum, and the other is derived from the low frequency part of the squared real wave time series as described in Chapter 3.1.
We first try two seeds of phasing for the same sea bi-directional wave to investigate how big the difference introduced by phasing could be. Three lines are plotted, they are:

- **Theoretical wave group spectrum**: represent wave group by theoretical first order spectrum.
- **Measured wave group spectrum**: represent wave group by measured first order spectrum (the wave spectrum transfer back from the measured time series).
- **Envelop (Hilbert)**: taking the low frequency part of squared wave elevation record.

The name ‘Theoretical wave group spectrum’ and ‘Measured wave group spectrum’ are the expression from the MARIN report. Thus we use the same name in this thesis.

![Figure 8-3: Wave group spectrums of different phasing](image)

We can see from the plot above, for **theoretical wave group spectrum**, according to its definition, there’s no difference at all.

For **measured wave group spectrum**, there is a slight difference and the difference is caused by the variance of first order wave spectrum in each simulation.

While for the method **Envelop (Hilbert)**, the variance is quite significant. This method is considered as matching the real wave group time series best because the methods use the first order wave spectrum to represent the second order wave spectrum lack their reasoning in phasing issues (as explained in **Chapter 6**), and mathematically because the wave group squares the original time series, so its uniqueness has been lost.

Fernandes in LabOceano and Hennig et al. in MARIN also conclude in 2008 according to their model test experience, that starting from the same first order spectrum, when different seed for phasing are given, the resulted wave group spectrum derived from measured sea state can be very different. [21] [22] However, their discussion is based on
measured wave group spectrum, and in the author’s view, they should use Envelop (Hilbert) instead.

A 1000 seeds random phasing experiment is further performed as follows:
First, we try to fit distributions for the ‘measured wave group spectrum’ calculated using 1000 different phasing. This group spectrum is a product of the first order spectrum, the latter is not a theoretical spectrum, it is calculated by inverse Fourier transform from 1000 simulations of 3 hour wave elevation time series, each time with same sampling frequencies but different phasing. So actually the variance reflects the difference in first order spectrum caused by different phasing.

![Figure 8-4: Measured wave group spectrum and the spectrum density at the natural frequency with 1000 random phasing](image)

Then, we investigate the influence of phasing on Hilbert wave envelop. This spectrum is corresponding to the wave groups which can be seen visually in the time series plot, it reflects better the variance for wave group spectrums.

![Figure 8-5: Hilbert wave group spectrum and the spectrum density at the natural frequency with 1000 random phasing](image)

The frequency domain spectrum for each simulation are plotted on the left hand of Figure 8-4 and Figure 8-5, the spectrum density at 0.16rad/s(natural roll frequency of...
the ship model in the model test) are fit with normal distribution and the distribution parameters are listed in Table 8-1.

<table>
<thead>
<tr>
<th></th>
<th>$\mu$ [m$^4$]</th>
<th>$\sigma$ [m$^4$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sample 2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 8-1: The statistics of wave group spectrum

As can be seen from the table above, all the three methods have very similar mean value for the spectrum density at the natural frequency 0.16 rad/s, which means the result of 'The theoretical wave group spectrum' can be regarded as the mean value if random seed experiments are not performed.

As for the standard deviation, the first method, it has no variance, while the second and the third method has an obvious variance. The Hilbert method has much larger variance than the 'Measured wave group spectrum', which can be explained as the difference of phasing cause much larger difference in wave groups than in first order waves. For Hilbert method, within the 1000 simulations, the maximum is almost 230% of the minimum value.

Since the wave group represents the excitation, if the excitation is not unique, then no matter the problem is linear or nonlinear, we would expect the response is also not unique.

8.3 Large variance caused by low damping

In the following, another reason cause large variance of second order roll is explained. As we mentioned in Chapter 2, the measured roll response has obvious group phenomenon, so as for the simulated roll motion time series. In the following, simulations of 5 different seeds are plotted to give a general feeling:
The variance or the standard deviation is dominated by the big groups in the time series, and Figure 8-6 shows the appearance of the groups are quite random along the time series, thus, the standard deviations of roll response are also expected to vary a lot.

The reason of the group phenomenon as explained by Naciri et al.[23] is due to the low damping ratio, the Dynamic Amplification Factor (DAF) becomes very narrow banded, thus the system becomes very sensitive to even small changes in the external excitation.

8.4 Statistics of 2nd order roll

Because of the large variance of second order roll exist in the response. It is concluded no certain value can be given when predicting the second order roll in reality, it can only be predicted by giving a statistics distribution for the possible values.
For first order motions, the zero-crossing maxima in one simulation (distribution in red dash line Figure 8-1) and the maxima of each simulation (distribution in blue dash line Figure 8-1) usually follows a Rayleigh distribution because the wave elevation which is the excitation naturally follows a Rayleigh distribution. However, for second order motions, because of its nonlinearity, it is not proportional to wave elevation any more, thus it doesn’t necessarily follow a Rayleigh distribution. Actually according to the simulation cases performed in this thesis, the wave groups spectral densities at the natural frequency is more likely to follow a Gaussian distribution (Figure 8-5). Further because the linear part is dominate in the viscous damping after we introduce cubic damping, thus it is slightly nonlinear, we can see that the roll response (the maxima of each simulation) also tends to follow a Gaussian distribution. Figure 7-4, Figure 7-5 and Figure 7-6 is Gaussian distributed, Figure 10-17 is more close to a Rayleigh distribution because it is dominated by first order roll, Figure 10-25 tends to be more Gaussian, because second order is more dominant. It should be noted, because a pure 2nd order roll is Gaussian distributed, the MPM value equals the mean of the maxima in each simulations. However, for 1st order motions, the MPM doesn’t equal the mean of the maxima any more, the mean would be slightly larger than the MPM.

Another question arises: how many simulations should we do to get a fair distribution?
Figure 8-8: Maximum angle and the standard deviation distributions with different sampling numbers

Figure 8-8 is a discussion on how many samples can result in a fair distribution based on the unidirectional wave case, we can see 150 samples can give a good fit for the maximum roll angles and the standard deviations of each simulation, while 50 samples enable us to build a preliminary distribution shape. Based on the complexity of 2nd order roll calculation and the limitation of the MATLAB efficiency, the time domain simulation is very time consuming, and because the 1st order roll is also considered in the cases presented in the following chapters, where less variation is expected. Thus, for all the cases in the following Chapters, it is decided to perform 50 times 3 hour simulations (seeds) in order to reach a balance between simulation time and accuracy.
9

INFLUENTIAL FACTORS

9.1 Introduction

In this Chapter, the influential factors based on the possible difference between model test and reality as well as their influences on the 2nd order roll are discussed. The wave spreading and reflection phenomenon make the model test and real situation quite different, and they do have a great influence on the 2nd order roll, which have already been proved in Chapter 6 and Chapter 7. In the following, the effects of water depth, sea state, loading condition and ship heading are further discussed. In order to reduce the computational time, most of the discussion in this Chapter will be supported only by frequency domain calculation, and according to the discussion in the above chapters, the frequency domain method is a conservative method, so it is safe to apply the results of this Chapter to the reality.

9.2 Water depth influence

Wave components in shallow water have stronger second order wave interaction\[5\][6], also see Figure 4-17, thus, at least the fifth term of QTF will be a little larger in shallow water. We first investigate the influence of water depth on the QTF, 2 sets of QTFs have been calculated in the Chapter 4, one is with 200m water depth as we calculated for reflection model, one is with infinite water depth for the spreading model. The diagonal line of 0.16rad/s (difference) frequency is plotted as follows, and the difference between the two only located in the very low frequency range. We can expect the influence would expand to higher frequency range as the water depth becomes lower and lower as the red arrow indicates in Figure 9-1. However, the type of semi-submersible vessel is usually used for deep water operation, thus the influence of water depth wouldn't go far into the high frequency range. According to Wichers’ equation (see Figure 6-2), the difference in lower frequency band has no influence on second order roll moment because the first order wave spectrum has no spectrum density there. For our 200 meter water depth and infinite water depth cases, the difference in second order roll moment is negligible (see Figure 9-1).
INFLUENTIAL FACTORS

Figure 9-1: Difference in QTF and roll moment spectrum corresponding to different water depth. Red: 200m water depth. Blue: Infinite water depth.

Thus as long as the water depth can be regarded as a deep water situation, we can always use the same set of QTFs.

9.3 Loading condition influence

The model test is performed in a situation where the GM is very low, it is tested according to minimum GM criteria for stability, in reality, the low GM is corresponding to the situation where the maximum load is put on the crane, all the above calculations are based on such a load case.

In the following a zero load case at the same operation draft is calculated as a comparison. The load on the crane is compensated by ballast water to maintain the draft, and this causes the VCG much lower, which further makes GM value much higher in the zero load case. The radius of inertia also changes, and the roll moment caused stiffness and the roll moment of inertia would varied accordingly, finally it would cause a difference in the natural roll frequency.

<table>
<thead>
<tr>
<th></th>
<th>GM (m)</th>
<th>Total weight (mT)</th>
<th>Radius of inertia Rxx (m)</th>
<th>Natural frequency (rad/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No load case</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Full load case</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 9-1: Loading condition

The diagonal line of QTFs at 0.16rad/s difference frequency are plotted as follows, as we can see, the main difference of the two load cases is located in their natural frequencies. This is because in the QTF analytical expressions, there are motion related terms, which makes an obvious peak in QTFs at the natural frequency. For other frequency range, there’s no obvious difference, thus we can conclude the influence of loading condition on QTF and 2nd order roll moment is also limited.

For different loading cases with the same draft, we can save computational time using the same set of QTF.
Figure 9-2: Difference in QTF and roll moment spectrum corresponding to different load cases. **Blue:** No load case, **Red:** Full load case.

However, although the second order moment is very similar, different natural frequency would also cause difference in the final roll motion. The transfer function and the final roll moment spectrum are plotted as follows:

Figure 9-3: Difference in Transfer function and final roll response spectrum corresponding to different load cases. **Blue:** No load case, **Red:** Full load case.

In our cases, the no load situation has much less 2nd order roll. It is in general, the higher the natural roll frequency, the lower the 2nd order roll, which can still be explained by visualizing Wichers' equation (see Figure 9-4), with the difference frequency higher, the overlap between the three peaks in Wichers' equation become smaller and smaller, thus there's less wave pairs result in wave groups of higher difference frequency, the 2nd order force and moment usually would become less.
INFLUENTIAL FACTORS

Figure 9-4: Concept of difference in second order excitations corresponding to different natural frequency.

9.4 Sea state influence

In the following, the sensitivity of $H_S, T_P$ is further discussed based on frequency domain calculation. All the calculations below are performed in beam waves for a uni-directional wave case and a multi-directional wave case with spreading factor $s=4$. First, keep $T_P = 7s, \gamma = 3$ constant and see how much the significant wave height influences the second order roll motion.

<table>
<thead>
<tr>
<th>$H_S$</th>
<th>MPM (degree)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.5 m</td>
<td></td>
</tr>
<tr>
<td>3 m</td>
<td></td>
</tr>
<tr>
<td>2.5 m</td>
<td></td>
</tr>
<tr>
<td>2 m</td>
<td></td>
</tr>
<tr>
<td>1.5 m</td>
<td></td>
</tr>
</tbody>
</table>

Table 9-2: The sensitivity of significant wave height (uni-directional wave)

<table>
<thead>
<tr>
<th>$H_S$</th>
<th>MPM (degree)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.5 m</td>
<td></td>
</tr>
<tr>
<td>3 m</td>
<td></td>
</tr>
<tr>
<td>2.5 m</td>
<td></td>
</tr>
<tr>
<td>2 m</td>
<td></td>
</tr>
<tr>
<td>1.5 m</td>
<td></td>
</tr>
</tbody>
</table>

Table 9-3: The sensitivity of significant wave height (wave with spreading factor $s=4$)

![Frequency band](image)

Figure 9-5: The sensitivity of significant wave height (uni-directional wave)

Figure 9-6: The sensitivity of significant wave height (wave with spreading factor $s=4$)
As we can see, the influence of significant wave height on the second order roll is monotonic, the higher the significant wave height, the larger the second order roll. The changing of the MPM is much larger than just proportional to $H_S$, but a little smaller than proportional to $H_S^2$.

Then, $H_S = 3m$, $\gamma = 3$ are kept as constant, the influence of peak period is checked.

We see the influence of peak period is not monotonic, with peak at the period around 7s~11s, the second order roll is much larger than other peak wave periods. This is because in this period/frequency range, the peaks of two first order spectrum are located within the same range of the peak of QTF, and according to the frequency domain calculation, this three peaks will be multiplied together when calculating the second order force and moment, which cause the latter to be much higher (see Figure 6-2). And according to the previous calculation for uni-directional wave, the peak of total QTF located in 0.6~0.9 rad/s for low frequency component, 0.75~1.07 rad/s for high frequency component, which is caused by the geometry of the semi-submersible. While the peak of first order wave spectrum (JONSWAP spectrum) If the peak period changes, for example, $T_P = 5s$ corresponds to peak frequency of 1.26rad/s could cause three peaks deviate from each other, thus 2nd order moment decreases. And the reason is the same for much higher peak period.
9.5 Wave spreading influence

Although the influence of wave spreading has already been discussed in Chapter 6 when we explain the frequency domain method and reach the conclusion that the spreading of wave energy is able to decrease 2nd order roll. However, seeing the sensitivity discussion for peak period above, the maximum value for uni-directional wave appears at 9s close to 7s, while for spreading wave, it appears at 9s close to 11s. It means for different peak periods, the wave spreading doesn’t always decrease second order roll to the same extent, for a peak period of 13s, the spectrum with full spreading even increases 2nd order roll as follows:

![Figure 9-9: Wave spreading influence for different peak period](image)

The reason can be explained by Figure 4-10, if we focus on the 0.4~0.6 rad/s (from 10.5~15.7s), the QTFs for beam wave direction(270 degree) is even smaller than the QTFs for near head direction(340~350 degree).

Thus whether wave spreading decrease 2nd order roll depend on the peak period a lot.

9.6 Vessel heading influence

Another interesting phenomenon can be explained by the same reasoning is the sensitivity of ship heading.

The ship heading is defined as Figure 9-10, with beam wave, the ship heading is defined as 270 degree, with head wave, the ship heading is defined as 360 degree. For the others, they are interpolated in between.
INFLUENTIAL FACTORS

Figure 9-10: Ship heading definition, Left: 270 degree beam wave, Right: 360 degree head wave.

Table 9-6 indicates for a certain peak period, which ship heading is expected to have the maximum second order roll, they are marked in red. It is interesting to find not always the beam wave result in the largest roll motion, sometimes even head position would cause the largest 2nd order roll. Thus, as a possible mitigation of 2nd order roll, the ship heading can be changed artificially according to certain wave peak period.

Table 9-6: The sensitivity of ship heading
10

SECOND ORDER ROLL IN REALITY

10.1 Introduction

In this Chapter, the typical four stages of offshore heavy lifting are introduced, and their corresponding data for the roll motion of ‘Thialf’ measured in one offshore project ‘Montrose’ of HMC\(^{[24]}\) is investigated so as to check whether there's any 2\(^{nd}\) order roll in reality.

We use theoretical methods introduced in the above chapters to set up models and perform a combined 1\(^{st}\), 2\(^{nd}\) and hoisting load influenced overall roll motion simulation both in frequency domain and time domain. A comparison with the measured data will be performed and the statistics will be investigated for the variance of calculation data.

10.2 Offshore heavy lifting stages and measured data

The typical offshore heavy lifting process is defined as four stages: Free floating, Pretension, Free hanging and Set down. There are transition periods between each two stages as well, during the transition, the vessel and the crane are moved artificially which is beyond the discussion of this thesis.

Figure 10-1: Overall roll time series and the chosen time period for each stage\(^{[24]}\)
SECOND ORDER ROLL IN REALITY

We only choose the parts of time series related to a stable stage, in other words, there’s no artificial motion for ships and cranes. The four stages and their corresponding measured data are discussed in detail in the following:

**Free Floating:**
Free floating is a stage before the heavy lifting, in this stage, the vessel is waiting near the operation spot. There’s no hook load and the vessel itself doesn’t have velocity, thus don’t have to consider the mutual influence between the load and vessel motion.

*Figure 10-3* shows in this stage, there's visible second order roll motion, however it is not as severe as the 1st order motion. The total roll amplitude consider both 1st and 2nd order motion is about 0.03~0.04 degree, it is very mild.

**Pretention:**
In this stage, the crane is lifting up the load on the barge, part of the weight is carried by the hoisting wires, while the other part is still carried by the barge. The vessel motion is coupled with the load carrying barge. In fact, the roll motion of the barge is expected to be more severe, because its ship shape would make it have a natural roll frequency within the range of first order wave frequencies. Thus in order to mitigate the more severe barge roll motion, in offshore heavy lifting, the barge is usually chosen to be put head on in the waves and the semi-submersible will beam on (which would cause large roll in general).
**SECOND ORDER ROLL IN REALITY**

**Figure 10-5** shows in this stage, there's no second order roll motion.

![Figure 10-5: Roll time series and spectrum for Pretension stage](image)

**Free hanging:**
In this stage, the load is fully carried by the crane, thus the swing, pitch motion of the load and the roll, sway motion of the vessel are strongly coupled with each other. The coupled motion would not only influence the roll amplitude of the ship but change its natural roll frequency as well.

The incoming wave direction is still a beam wave because once carrying the load after the pretension stage, it should be avoided to change the heading of the vessel any more.

**Figure 10-7** shows in this stage, there's very obvious second order roll motion.

![Figure 10-7: Roll time series and spectrum for Free Hanging stage](image)
Set down:
In this stage, the load which is a topside deck is put off onto a Jacket. The swing motion of the load no longer influence the vessel, actually when the load is partly set down, the hoist wire tends to help restricting the ship motion, the Eigen frequency of the roll motion the ship becomes larger and larger in this stage.

Figure 10-9 shows in this stage, there’s no visible second order roll motion. Compare to the obvious 2nd order roll in free hanging stage, this indicates when the load is set down, the stiffness/damping of the wire would actually play a role mitigating the 2nd order roll.

For the free floating stage, there is a quite long measured record and a very stable period is chosen here for study, it can be confirmed that the low frequency peak exist in the roll response spectrum is a second order roll.

For the free hanging stage, a very obvious low frequency peak exists in the roll response spectrum, however, it is hard to determine whether all the low frequency motion is caused by 2nd order wave excitation based on the short time of measurement. However, it can be concluded second order roll do exist, because for the whole half hour, low frequency motion continuously exist, no other continuous excitation has ever been proved at the low frequency and the load motion influence is an in-phase one which means the roll of the ship is not caused by the swing of the load.

Since free floating and free hanging are the two stages which have the 2nd order roll problem, thus we calculate the ship roll motion in this two stages as follows.

10.3 Free Floating stage simulation model

In this subchapter, we give the model and the input parameters simulating the overall roll motion(consider both first and second order) of the vessel in the free floating stage.
SECOND ORDER ROLL IN REALITY

First we do a coarse frequency domain approximation to check the model and then a more precise time domain calculation is performed. The calculation procedure is explained below while the results are discussed in Chapter 10.5.

In this stage, the vessel is encountering a sea state as the right hand. The wave spectrum is measured from a wave buoy setting near the vessel when the vessel is in operation. The significant wave height for this sea state is and the peak period . This spectrum is chosen for its measured time matches the free floating period we defined before.

The vessel has the heading of degree, which means the beam wave is coming from degree, we have to rotate the QTF values accordingly, make the beam wave direction originally at degree turn to degree.

Frequency domain:
The first order force and the second order force are calculated by multiplying the wave spectrum with the transfer function from unit wave height to force. For second order force and moment, this transfer function is the QTF which has been calculated in Chapter 4, and the second order forces and moments calculation have been discussed in Chapter 6 (Frequency domain) and Chapter 7 (Time domain). The 1st order force spectrum is quite similar in calculation, we can use Equation 10-1. Note the WAMIT output value should be normalized before use by multiplying . (The sea water density and the gravity acceleration.)

\[
S_F^{(1)}(\mu) = \int_0^\infty \int_0^{2\pi} S_c(\omega, \alpha)|T^{(1)}(\omega, \alpha)|^2 \, d\alpha \, d\omega \quad (10-1)
\]

Where,
- \(S_F^{(1)}(\mu)\) First order force spectrum
- \(S_c(\omega, \alpha)\) First order wave spectrum (\(\omega\) frequency domain; \(\alpha\) directional domain)
- \(T^{(1)}(\omega, \alpha)\) Transfer Function from first order unit amplitude wave to first order force (\(\omega\) frequency domain; \(\alpha\) directional domain)

The resulted 1st and 2nd order force and moment are plotted as follows considering roll motion is coupled with sway (If hydrodynamic centre is not in line with CoG in vertical direction), and heave (If hydrodynamic centre is not in line with CoG in horizontal direction), the 1st order forces and moments are much larger than the second order ones, thus we only calculate 2nd forces and moment for the low frequency range (0~0.25 rad/s) where there’s no 1st order force. And for the higher frequencies, it is assumed the influence of 2nd order force is negligible compared to the first order force.
SECOND ORDER ROLL IN REALITY

Figure 10-11: 1\textsuperscript{st} and 2\textsuperscript{nd} order sway force, heave force and roll moment (free floating)

Then we calculate the transfer function from force and moment (both 1\textsuperscript{st} order and 2\textsuperscript{nd} order) to roll motion by solving the equation of motion as follows, the viscous damping terms \( D_{ii} \) are linearized by the method discussed in Chapter 5:

\[
\begin{pmatrix}
M_{22} & M_{23} & M_{24} \\
M_{32} & M_{33} & M_{34} \\
M_{42} & M_{43} & M_{44}
\end{pmatrix}
\begin{pmatrix}
P \\
P
\end{pmatrix}
+ 
\begin{pmatrix}
A_{22}(\omega) & A_{24}(\omega) \\
A_{32}(\omega) & A_{34}(\omega) \\
A_{42}(\omega) & A_{44}(\omega)
\end{pmatrix}
\begin{pmatrix}
-\omega^2A_{y}(\omega) \\
-\omega^2A_{z}(\omega) \\
-\omega^2A_{\phi}(\omega)
\end{pmatrix}
+ 
\begin{pmatrix}
D_{22} & D_{23} & D_{24} \\
D_{32} & D_{33} & D_{34} \\
D_{42} & D_{43} & D_{44}
\end{pmatrix}
\begin{pmatrix}
P \\
P
\end{pmatrix}
+ 
\begin{pmatrix}
F_{a,sway}^{(1)}(\omega) + F_{a,sway}^{(2)}(\omega) \\
F_{a,heave}^{(1)}(\omega) + F_{a,heave}^{(2)}(\omega) \\
M_{a,roll}^{(1)}(\omega) + M_{a,roll}^{(2)}(\omega)
\end{pmatrix}

(10 - 2)

Where,
- \( M_{ii} \) - The mass for motion i.
- \( \vec{V} \) - The displacement of the hull.
- \( A_{ij}(\omega) \) - Frequency dependent added mass, if \( j \neq i \), the added mass is caused by coupled motion, j motion influence on i motion.
- \( B_{ij}(\omega) \) - Frequency dependent potential damping, j motion influence on i motion.
- \( D_{ii} \) - The equivalent viscous damping.
- \( K_{ii} \) - Stiffness caused by mooring line.
- \( GM^T \) - Transverse initial metacentric height.
- \( A_w \) - Water line area.
SECOND ORDER ROLL IN REALITY

\[ F_{a,i}^{(2)} M_{a,i}^{(2)} \] – Second order force and moment amplitude.
\[ F_{a,i}^{(1)} M_{a,i}^{(1)} \] – First order force and moment amplitude.
\[ A_y, A_z, A_\varphi \] – Sway, heave, roll amplitude.

Note theoretically the right hand of Equation 10-2 is a complex number, it should be treated by the method we proposed in Chapter 6 (considering phasing). However, this treatment would be very complicated for multi-directional waves and a lot of samplings should be calculated because of the large variance in second order response (see Chapter 8). Here as a quick approximation and also a check whether Wichers’ equation is applicable in this case, we put the absolute value of force amplitude calculated from the force spectrum in the right hand, for single degree of freedom system, this treatment is the same as the application of Wichers’ equation. Later, a precise calculation will be done in time domain, and because there’re no linearization simplification, it is much more accurate and worth the time doing random phasing test.

The value of \( M_{ii}, D_{ii}, K_{ii} \) are chosen from the standard database of the vessel ‘Thialf’, which could be a little different from the real situation.

<table>
<thead>
<tr>
<th>Table 10-1: Input M, D, K matrix (free floating)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moment transfer function</td>
</tr>
<tr>
<td>Sway force to roll (Real)</td>
</tr>
<tr>
<td>Heave force to roll (Real)</td>
</tr>
<tr>
<td>Roll moment to roll (Real)</td>
</tr>
<tr>
<td>Sway force to roll (Imaginary)</td>
</tr>
<tr>
<td>Heave force to roll (Imaginary)</td>
</tr>
<tr>
<td>Roll moment to roll (Imaginary)</td>
</tr>
</tbody>
</table>

All the above values are defined at CoG, they should be transferred to the hydrodynamic centre, because the added mass, added damping as well as 1st and 2nd order force and moment are all defined at hydrodynamic centre.

The CoG of this case is defined at [72.44, 0, 24.03], while the hydrodynamic centre is defined at [72.44, 0, 27.6], their y axis is the same, so the heave motion is not coupled in this case, the transfer functions are plotted as below:

Figure 10-12: Transfer function from forces, moments to roll motion (free floating)
Finally, by multiplying 1\textsuperscript{st} and 2\textsuperscript{nd} force or moment spectrum with the transfer function, we can calculate the final roll response spectrum and it is compared with the measured one as follows:

![Roll response spectrum](image)

\textbf{Figure 10-13: Roll response spectrum (free floating simulated in frequency domain)}

The two spectrum can be compared as follows:

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
 & Measured & Calculated \\
\hline
Max. roll angle/MPM 1h (degree) & 0.0518 & 0.0501 \\
STD (degree) & 0.0146 & 0.0167 \\
\hline
\end{tabular}
\caption{Roll response comparison (free floating frequency domain)}
\end{table}

\textbf{Time domain:}
In time domain, we first make some wave components according to the measured wave spectrum, still non-equivalent frequency step is used, and by integraling the wave spectrum density of each direction, we detect the first three directions with the highest wave energy and the first five directions with lowest wave spectrum density (basically almost zero), make 64 wave components for the highest three and 0 components for the lowest five, for the rest 16 directions, 32 wave components are made for each. Thus altogether 704 wave components are made.

A new set of random seeds are fed to this 704 wave components each time for random seeds experiment, altogether 50 seeds are tried.

Wave elevation time series are made, code is designed to check the whether the significant wave height and peak period of the simulated wave matches the measured one to ensure the quality of wave elevation simulation.

For the next step, the first and the second order force and moment time series are made, the second order force and moment are made according to the method explained in \textbf{Chapter 7} (2\textsuperscript{nd} order in \textbf{Equation 7-1} and 1\textsuperscript{st} order in \textbf{Equation 7-3}), frequencies are interpolated and directions are rotated so that the wave spectrum can match the frequency and direction step calculating QTFs.

The first seed is taken as an example, the wave elevation, the first order roll moment and second order roll moment time series are plotted as follows:
Sway and heave forces are also calculated for coupled motion, a time domain version equation of motion is used to solve this problem:

\[
\begin{align*}
&P \begin{bmatrix}
M_{22} & M_{33} & M_{44} \\
M_{33} & M_{44} & M_{55} \\
M_{44} & M_{55} & M_{66}
\end{bmatrix} y(t) + \begin{bmatrix}
A_{22}(t) & A_{24}(t) \\
A_{33}(t) & A_{34}(t) \\
A_{44}(t) & A_{44}(t)
\end{bmatrix} \begin{bmatrix}
\dot{y}(t) \\
\dot{z}(t) \\
\dot{\phi}(t)
\end{bmatrix} + \\
2p_{22,cog} \frac{(M_{\text{sway}} + A_{22}(t))}{T_{n,\text{sway}}} + 2p_{33,cog} \frac{(M_{\text{heave}} + A_{33}(t))}{T_{n,\text{heave}}} + 2p_{44,cog} \frac{(M_{\text{roll}} + A_{44}(t))}{T_{n,\text{roll}}}
\end{align*}
\]

\[
\begin{align*}
&\frac{3}{8} P \begin{bmatrix}
q_{22,cog}(M_{\text{sway}} + A_{22}(t)) \\
q_{33,cog}(M_{\text{heave}} + A_{33}(t)) \\
q_{44,cog}(M_{\text{roll}} + A_{44}(t))
\end{bmatrix} \begin{bmatrix}
\dot{y}(t) \\
\dot{z}(t) \\
\dot{\phi}(t)
\end{bmatrix} + \\
\frac{2}{3\pi^2} P \begin{bmatrix}
r_{22,cog}(M_{\text{sway}} + A_{22}(t))T_{n,\text{sway}} \\
r_{33,cog}(M_{\text{heave}} + A_{33}(t))T_{n,\text{heave}} \\
r_{44,cog}(M_{\text{roll}} + A_{44}(t))T_{n,\text{roll}}
\end{bmatrix} \begin{bmatrix}
\dot{y}(t) \\
\dot{z}(t) \\
\dot{\phi}(t)
\end{bmatrix} + \\
\int_0^\infty \left( \begin{bmatrix}
B_{22}(\omega) & B_{24}(\omega) \\
B_{33}(\omega) & B_{34}(\omega) \\
B_{42}(\omega) & B_{44}(\omega)
\end{bmatrix} \cos(\omega t) d\omega \cdot \begin{bmatrix}
\dot{y}(t) \\
\dot{z}(t) \\
\dot{\phi}(t)
\end{bmatrix} \right) \begin{bmatrix}
\dot{y}(t) \\
\dot{z}(t) \\
\dot{\phi}(t)
\end{bmatrix} = \\
\begin{bmatrix}
F_{a,\text{sway}}^{(1)}(t) + F_{a,\text{sway}}^{(2)}(t) \\
F_{a,\text{heave}}^{(1)}(t) + F_{a,\text{heave}}^{(2)}(t) \\
M_{a,\text{roll}}^{(1)}(t) + M_{a,\text{roll}}^{(2)}(t)
\end{bmatrix}
\end{align*}
\]

The notations are the same as before. Where the added mass should be further written as:

\[
\begin{bmatrix}
K_{22} & K_{33} + \rho g A_w \\
K_{33} + \rho g A_w & K_{44} + \rho g \nabla \cdot GM
\end{bmatrix} P \begin{bmatrix}
\dot{y}(t) \\
\dot{z}(t) \\
\dot{\phi}(t)
\end{bmatrix} = \\
\begin{bmatrix}
F_{a,\text{sway}}^{(1)}(t) + F_{a,\text{sway}}^{(2)}(t) \\
F_{a,\text{heave}}^{(1)}(t) + F_{a,\text{heave}}^{(2)}(t) \\
M_{a,\text{roll}}^{(1)}(t) + M_{a,\text{roll}}^{(2)}(t)
\end{bmatrix}
\]
\[ A_{ij}(t) = A_{ij}(\mu) + \frac{1}{\mu} \int_0^t B_{ij}(\tau) \sin(\omega \tau) d\tau \]  

(10 - 4)

One realization of the roll motion time series is plotted as follows:

Figure 10-15: Roll response realization for seed 1 (free floating)

All the 50 seeds simulation are plotted as follows and distributions are fit correspondingly (see Figure 10-16), we first give the overall roll response spectrums for every 3 hour simulation then the spectrums for every 1 hour simulation in order to match the time length of the measurement:

Figure 10-16: Roll response spectrum (free floating simulated in time domain). Left: 3h per spectrum. Right: 1h per spectrum.

The statistics and distribution of the results are given as follows:
SECOND ORDER ROLL IN REALITY

![Image](image_url)

**Figure 10-17:** The statistics of Roll response spectrum. **Left:** 3h per spectrum. **Right:** 1h per spectrum.

<table>
<thead>
<tr>
<th>Mean of the extreme roll angle in 1h (degree)</th>
<th>Measured</th>
<th>Calculated</th>
</tr>
</thead>
<tbody>
<tr>
<td>STD (degree)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 10-3:** Roll response comparison (free floating time domain)

10.4 Free Hanging stage simulation model

In this subchapter, the overall roll motion for the free hanging stage is simulated both in frequency domain and time domain.

In this stage, the vessel is encountering a sea state as the right hand.
The significant wave height for this sea state is 0.89m and the peak period is 4.87s.
This spectrum is chosen for its measured time matches the free hanging period we defined before.
As we already explained in Chapter 10.2, in free hanging stage the vessel is more likely to encounter a beam wave.
The ship heading is 215 degree, which means the beam wave is coming from 125 degree, we have to rotate the QTF values carefully, make the beam wave direction originally at 90 degree turn to 125 degree.

![Image](image_url)

**Figure 10-18:** Wave spectrum (free hanging)
SECOND ORDER ROLL IN REALITY

Frequency domain
The 1st and 2nd order force and moment are calculated the same way as above, their spectrum are plotted as follows:

Figure 10-19: 1st and 2nd order sway force, heave force and roll moment (free hanging)

The transfer function from force and moment to roll motion should be calculated in a different way in this case:

Figure 10-20: Dynamic model for free hanging

Figure 10-21: Left: In phase mode (0.16rad/s); Right: Out of phase mode (0.45rad/s);
SECOND ORDER ROLL IN REALITY

Assume small vibrations, for the load, its displacement in x and y direction are as follows:

\[ x_{\text{load}}(t) = x_{\text{ship}}(t) + L \sin \theta(t) - l \sin \theta(t) \]  \hspace{1cm} (10 – 5)

\[ y_{\text{load}}(t) = y_{\text{ship}}(t) - L + L \cos \varphi(t) + l - l \cos \theta(t) \]  \hspace{1cm} (10 – 6)

The dimensions of l and L please refer to Figure 10-20. The kinetic energy of the whole system can be written as:

\[ T = \frac{1}{2} m x_{\text{load}}(t)^2 + \frac{1}{2} m y_{\text{load}}(t)^2 + \frac{1}{2} M x_{\text{ship}}(t)^2 + \frac{1}{2} M y_{\text{ship}}(t)^2 + \frac{1}{2} M \varphi(t)^2 \]  \hspace{1cm} (10 – 7)

Where,

\[ M \] stands for the general mass of the ship

\[ m \] stands for the mass of the load

The potential energy of the whole system can be written as:

\[ V = m g y_{\text{load}}(t) + M g y_{\text{ship}}(t) + \frac{1}{2} k_{\text{sway}} x_{\text{ship}}(t)^2 + \frac{1}{2} k_{\text{heave}} y_{\text{ship}}(t)^2 \]  \hspace{1cm} (10 – 8)

The energy dissipated by damping of the whole system can be written as:

\[ D = \frac{1}{2} D_{\text{sway}} x_{\text{ship}}(t)^2 + \frac{1}{2} D_{\text{heave}} y_{\text{ship}}(t)^2 + \frac{1}{2} D_{\text{sway}} \theta(t)^2 \]  \hspace{1cm} (10 – 9)

Using the Euler-Lagrange equations:

\[
\begin{align*}
\frac{d}{dt} \left( \frac{\partial (T - V)}{\partial x_{\text{ship}}(t)} \right) + \frac{\partial (T - V)}{\partial x_{\text{ship}}(t)} - \frac{\partial (T - V)}{\partial x_{\text{ship}}(t)} &= F_{\text{sway}}(t) \\
\frac{d}{dt} \left( \frac{\partial (T - V)}{\partial y_{\text{ship}}(t)} \right) + \frac{\partial (T - V)}{\partial y_{\text{ship}}(t)} - \frac{\partial (T - V)}{\partial y_{\text{ship}}(t)} &= F_{\text{heave}}(t) \\
\frac{d}{dt} \left( \frac{\partial (T - V)}{\partial \varphi(t)} \right) + \frac{\partial (T - V)}{\partial \varphi(t)} - \frac{\partial (T - V)}{\partial \varphi(t)} &= M_{\varphi}(t) \\
\frac{d}{dt} \left( \frac{\partial (T - V)}{\partial \theta(t)} \right) + \frac{\partial (T - V)}{\partial \theta(t)} - \frac{\partial (T - V)}{\partial \theta(t)} &= M_{\theta}(t)
\end{align*}
\]  \hspace{1cm} (10 – 10 – 13)

After a series of substitution and simplification, the above equations can be written as:

\[
\begin{align*}
& m x_{\text{ship}}''(t) + m L \varphi_{\text{ship}}''(t) \cos \varphi_{\text{ship}}(t) - m L \varphi_{\text{ship}}''(t) \sin \varphi_{\text{ship}}(t) - m L \varphi(t) \cos \theta(t) + \\
& m \theta''(t)^2 \sin \theta(t) + M x_{\text{ship}}''(t) + D_{\text{sway}} x_{\text{ship}}(t) + k_{\text{sway}} x_{\text{ship}}(t) = F_{\text{sway}}(t) \\
& -m L \varphi_{\text{ship}}''(t) \sin \varphi_{\text{ship}}(t) - m L \varphi_{\text{ship}}''(t) \cos \varphi_{\text{ship}}(t) + m \varphi(t) \sin \theta(t) + \\
& m \varphi''(t) \cos \theta(t) + M y_{\text{ship}}''(t) + M y_{\text{ship}}''(t) + D_{\text{heave}} y_{\text{ship}}(t) + k_{\text{heave}} y_{\text{ship}}(t) + M g + \\
& m g = F_{\text{heave}}(t)
\end{align*}
\]  \hspace{1cm} (10 – 14 – 15)

\[
\begin{align*}
& m L x_{\text{ship}}''(t) \cos \varphi_{\text{ship}}(t) + m L^2 \varphi_{\text{ship}}''(t) - m L \theta''(t) \cos \theta(t) \cos \varphi_{\text{ship}}(t) + \\
& m L \theta''(t)^2 \sin \theta(t) \cos \varphi_{\text{ship}}(t) - m L \theta(t) \sin \theta(t) \sin \varphi_{\text{ship}}(t) -
\end{align*}
\]
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\[
ml\theta(t)^2\cos\theta(t)\sin\varphi_{\text{ship}}(t) - m\gamma_{\text{ship}}(t)L\sin\varphi_{\text{ship}}(t) + f_m\dot{\varphi}_{\text{ship}}(t) - mgL\sin\varphi_{\text{ship}}(t) + D\varphi_{\text{ship}}(t) + k\varphi_{\text{ship}}(t) = M\varphi(t) \quad (10-16)
\]

\[
-mlx_{\text{ship}}(t)\cos\theta(t) - ml\dot{\varphi}_{\text{ship}}(t)\cos\varphi_{\text{ship}}(t)(\cos\theta(t) + ml\varphi_{\text{ship}}(t)^2\sin\varphi_{\text{ship}}(t)\cos\theta(t) + ml\varphi_{\text{ship}}(t)^2\cos\varphi_{\text{ship}}(t)\sin\theta(t) - ml\varphi_{\text{ship}}(t)^2\cos\varphi_{\text{ship}}(t)\sin\theta(t) + mly_{\text{ship}}(t)\sin\theta(t) + mgL\sin\theta(t) + D_{\text{sway}}\dot{\theta}(t) = 0
\quad (10-17)
\]

Put all the mass and acceleration terms on left hand and others on the right hand:

\[
\begin{bmatrix}
M + m & ml\cos\varphi_{\text{ship}}(t) & -ml\cos\theta(t) \\
ml\cos\varphi_{\text{ship}}(t) & -ml\sin\varphi_{\text{ship}}(t) & I_\text{m} + ml^2 \\
-ml\cos\theta(t) & ml\sin\theta(t) & -ml\cos(\varphi_{\text{ship}}(t) - \theta(t))
\end{bmatrix}
\begin{bmatrix}
x_{\text{ship}}(t) \\
y_{\text{ship}}(t) \\
\varphi_{\text{ship}}(t)
\end{bmatrix}

= \begin{bmatrix}
ml\varphi_{\text{ship}}(t)^2\sin\varphi_{\text{ship}}(t) - m\theta(t)^2\sin\theta(t) - D_{\text{sway}}x_{\text{ship}}(t) - k_{\text{sway}}x_{\text{ship}}(t) + F_{\text{sway}}(t) \\
ml\varphi_{\text{ship}}(t)^2\cos\varphi_{\text{ship}}(t) - m\theta(t)^2\cos\theta(t) - D_{\text{heave}}y_{\text{ship}}(t) - k_{\text{heave}}y_{\text{ship}}(t) - Mg - mg + F_{\text{heave}}(t) \\
ml\theta(t)^2\sin(\theta(t) - \varphi_{\text{ship}}(t)) + mgL\sin\varphi_{\text{ship}}(t) - D\varphi_{\text{ship}}(t) - k\varphi_{\text{ship}}(t) + M\varphi(t)
\end{bmatrix}
\]

(10-18)

First we try to solve the problem in frequency domain: the equation has to be further linearized. We rewrite the motion by substituting hydrodynamic expressions, transfer the coordinate and turn the equation into frequency domain:

\[
\begin{align*}
(P) & \begin{bmatrix}
M_{22} & M_{33} \\
M_{33} & M_{44}
\end{bmatrix} P^T + \begin{bmatrix}
A_{22}(\omega) + m & A_{23}(\omega) + m & A_{24}(\omega) + mL & -ml \\
A_{23}(\omega) + mL & A_{33}(\omega) + m & A_{44}(\omega) + mL^2 & -mlL \\
-ml & -mlL & A_{44}(\omega) + mL^2 & -mlL \\
-ml & -mlL & -mlL & A_{44}(\omega) + mL^2
\end{bmatrix} \begin{bmatrix}
x_{\text{ship}}(\omega) \\
y_{\text{ship}}(\omega) \\
\varphi_{\text{ship}}(\omega) \\
\theta(\omega)
\end{bmatrix} \\
&+ (P) \begin{bmatrix}
D_{22} & D_{33} \\
D_{33} & D_{44}
\end{bmatrix} P^T + \begin{bmatrix}
B_{22}(\omega) & B_{23}(\omega) & B_{24}(\omega) \\
B_{32}(\omega) & B_{33}(\omega) & B_{34}(\omega) \\
B_{42}(\omega) & B_{43}(\omega) & B_{44}(\omega)
\end{bmatrix} \begin{bmatrix}
[A_y(\omega)] \\
[iA_x(\omega)] \\
[iA_z(\omega)] \\
[iA_y(\omega)]
\end{bmatrix} \\
&+ (P) \begin{bmatrix}
K_{22} & K_{33} + \rho g A_w \\
K_{33} + \rho g A_w & K_{44} + \rho g \nabla \cdot GM^T
\end{bmatrix} P^T + \begin{bmatrix}
-A_y(\omega)^2 \\
-A_x(\omega)^2 \\
-A_z(\omega)^2 \\
-A_y(\omega)^2
\end{bmatrix} \begin{bmatrix}
A_y(\omega) \\
A_x(\omega) \\
A_z(\omega) \\
A_y(\omega)
\end{bmatrix}
\end{align*}
\]

\[
= \begin{bmatrix}
F_{a,\text{sway}}(\omega)^{(1)} + F_{a,\text{sway}}(\omega)^{(2)} \\
F_{a,\text{heave}}(\omega)^{(1)} + F_{a,\text{heave}}(\omega)^{(2)} \\
M_{a,\text{roll}}(\omega)^{(1)} + M_{a,\text{roll}}(\omega)^{(2)} \\
0
\end{bmatrix}
\quad (10-19)
\]

Where additionally we have,

\[A_\theta \quad \text{- Swing amplitude}\]
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The value of $M_{ii}$, $D_{ii}$, $K_{ii}$ are chosen from the standard database of the vessel ‘Thialf’, which could be a little different from the real situation. The related input parameters are given as follows.

Table 10-4: Input M,D,K matrix (free hanging)

L and l value are calculated by simple geometry relation, other input parameters are list as follows:

Table 10-5: Parameters related to the coupled load motion

Still $P$ and $P^T$ are used to transfer coordinate from CoG to hydrodynamic centre in order to make all physical parameters defined in the same reference point.

By solving this equation, we can get the transfer function from unit forces or moments to motion, which are plotted and further discussed as follows:

Figure 10-22: Transfer function from forces, moments to roll motion (free hanging)
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The two spectrum can be compared as follows:

<table>
<thead>
<tr>
<th></th>
<th>Measured</th>
<th>Calculated</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max. roll angle/MPM 0.5h (degree)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>STD (degree)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 10-6: Roll response comparison (free hanging)

**Time domain**

In time domain, we follow the same procedure as the free floating case, first simulate wave and excitations, the first seed results are still plotted as an example:

*Figure 10-24: Wave elevation, first order and second order roll moment for seed 1 (free hanging)*
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As for the equation of motion, we use the its nonlinear term Equation 10-18 directly. However the mass, damping and stiffness terms should be written in a more complicated way as discussed in the above chapters. The mass term should be written as:

\[
\begin{bmatrix}
M_{22} + A_{22}(t) + m & M_{32} + A_{32}(t) \\
M_{32} + A_{32}(t) & M_{33} + A_{33}(t) + m
\end{bmatrix}
\]

Where the added mass term please refer to Equation 10-4. The damping term should be written as:

\[
D_{\text{sway}} x_{\text{ship}}(t) = \\
(D_{22,\text{hyd.visp}} + D_{22,\text{hyd.visq}}) \dot{x}_{\text{ship}}(t) + D_{22,\text{hyd.visr}} x_{\text{ship}}(t)^2 + D_{22,\text{hyd.pot}} \dot{x}_{\text{ship}}(t) + \\
(D_{32,\text{hyd.visp}} + D_{32,\text{hyd.visq}}) \dot{y}_{\text{ship}}(t) + D_{32,\text{hyd.visr}} y_{\text{ship}}(t)^2 + D_{32,\text{hyd.pot}} \dot{y}_{\text{ship}}(t) + \\
(D_{24,\text{hyd.visp}} + D_{24,\text{hyd.visq}}) \dot{\varphi}_{\text{ship}}(t) + D_{24,\text{hyd.visr}} \varphi_{\text{ship}}(t)^2 + D_{24,\text{hyd.pot}} \dot{\varphi}_{\text{ship}}(t)
\]

\[
D_{\text{heave}} y_{\text{ship}}(t) = \\
(D_{33,\text{hyd.visp}} + D_{33,\text{hyd.visq}}) \dot{x}_{\text{ship}}(t) + D_{33,\text{hyd.visr}} x_{\text{ship}}(t)^2 + D_{33,\text{hyd.pot}} \dot{x}_{\text{ship}}(t) + \\
(D_{43,\text{hyd.visp}} + D_{43,\text{hyd.visq}}) \dot{y}_{\text{ship}}(t) + D_{43,\text{hyd.visr}} y_{\text{ship}}(t)^2 + D_{43,\text{hyd.pot}} \dot{y}_{\text{ship}}(t) + \\
(D_{44,\text{hyd.visp}} + D_{44,\text{hyd.visq}}) \dot{\varphi}_{\text{ship}}(t) + D_{44,\text{hyd.visr}} \varphi_{\text{ship}}(t)^2 + D_{44,\text{hyd.pot}} \dot{\varphi}_{\text{ship}}(t)
\]

Where for viscous damping:

\[
\begin{bmatrix}
2p_{22,cog} \\
2p_{32,cog} \\
2p_{42,cog}
\end{bmatrix}
\]

\[
\begin{bmatrix}
D_{22,\text{hyd.visp}} \\
D_{32,\text{hyd.visp}} \\
D_{42,\text{hyd.visp}}
\end{bmatrix}
\begin{bmatrix}
D_{23,\text{hyd.visp}} \\
D_{33,\text{hyd.visp}} \\
D_{43,\text{hyd.visp}}
\end{bmatrix}
\begin{bmatrix}
D_{24,\text{hyd.visp}} \\
D_{34,\text{hyd.visp}} \\
D_{44,\text{hyd.visp}}
\end{bmatrix}
\]

\[
\frac{(M_{\text{sway}} + A_{22}(t))}{T_{n,\text{sway}}}
\]

\[
\frac{(M_{\text{heave}} + A_{33}(t))}{T_{n,\text{heave}}}
\]

\[
\frac{(M_{\text{roll}} + A_{44}(t))}{T_{n,\text{roll}}}
\]

\[
\begin{bmatrix}
p^T
\end{bmatrix}
\]
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\[
\begin{bmatrix}
D_{22, \text{hyd.vis,q}} & D_{23, \text{hyd.vis,q}} & D_{24, \text{hyd.vis,q}} \\
D_{32, \text{hyd.vis,q}} & D_{33, \text{hyd.vis,q}} & D_{34, \text{hyd.vis,q}} \\
D_{42, \text{hyd.vis,q}} & D_{43, \text{hyd.vis,q}} & D_{44, \text{hyd.vis,q}}
\end{bmatrix} - \frac{3}{8} q_{22, \text{cog}} (M_{\text{sway}} + A_{22}(t)) = P \begin{bmatrix}
\frac{3}{8} q_{33, \text{cog}} (M_{\text{heave}} + A_{33}(t)) \\
\frac{3}{8} q_{44, \text{cog}} (M_{\text{roll}} + A_{44}(t))
\end{bmatrix} p^T
\]

(10 - 25)

\[
\begin{bmatrix}
D_{22, \text{hyd.vis,r}} & D_{23, \text{hyd.vis.r}} & D_{24, \text{hyd.vis.r}} \\
D_{32, \text{hyd.vis,r}} & D_{33, \text{hyd.vis.r}} & D_{34, \text{hyd.vis.r}} \\
D_{42, \text{hyd.vis.r}} & D_{43, \text{hyd.vis.r}} & D_{44, \text{hyd.vis.r}}
\end{bmatrix} - \frac{2}{3\pi^2} \frac{\tau_{22, \text{cog}} (M_{\text{sway}} + A_{22}(t)) T_{n, \text{sway}}}{\tau_{33, \text{cog}} (M_{\text{heave}} + A_{33}(t)) T_{n, \text{heave}}} - \frac{\tau_{44, \text{cog}} (M_{\text{roll}} + A_{44}(t)) T_{n, \text{roll}}}{p^T}
\]

(10 - 26)

And for the potential damping terms:

\[
\begin{bmatrix}
D_{22, \text{hyd.pot}} & D_{23, \text{hyd.pot}} & D_{24, \text{hyd.pot}} \\
D_{32, \text{hyd.pot}} & D_{33, \text{hyd.pot}} & D_{34, \text{hyd.pot}} \\
D_{42, \text{hyd.pot}} & D_{43, \text{hyd.pot}} & D_{44, \text{hyd.pot}}
\end{bmatrix} = \int_0^{\infty} \int_0^{\infty} \begin{bmatrix}
B_{22}(\omega) & B_{24}(\omega) \\
B_{33}(\omega) & B_{44}(\omega)
\end{bmatrix} \cos(\omega t) d\omega \cdot \begin{bmatrix}
y(t - \tau) \\
z(t - \tau) \\
\phi(t - \tau)
\end{bmatrix} d\tau
\]

(10 - 27)

For the stiffness part:

\[
\begin{bmatrix}
K_{22, \text{hyd.pot}} & K_{23, \text{hyd.pot}} & K_{24, \text{hyd.pot}} \\
K_{32, \text{hyd.pot}} & K_{33, \text{hyd.pot}} & K_{34, \text{hyd.pot}} \\
K_{42, \text{hyd.pot}} & K_{43, \text{hyd.pot}} & K_{44, \text{hyd.pot}}
\end{bmatrix} = P \begin{bmatrix}
K_{22, \text{cog}} + \rho g A_w \\
K_{33, \text{cog}} + \rho g V \cdot GM^T \\
K_{44, \text{cog}} + \rho g V \cdot GM^T
\end{bmatrix} p^T
\]

(10 - 28)

\[
k_{\text{sway}} x_{\text{ship}}(t) = K_{22, \text{hyd.pot}} x_{\text{ship}}(t) + K_{23, \text{hyd.pot}} y_{\text{ship}}(t) + K_{24, \text{hyd.pot}} \varphi_{\text{ship}}(t)
\]

(10 - 29)

\[
k_{\text{heave}} y_{\text{ship}}(t) = K_{32, \text{hyd.pot}} x_{\text{ship}}(t) + K_{33, \text{hyd.pot}} y_{\text{ship}}(t) + K_{34, \text{hyd.pot}} \varphi_{\text{ship}}(t)
\]

(10 - 30)

\[
k_{\varphi} \varphi_{\text{ship}}(t) = K_{42, \text{hyd.pot}} x_{\text{ship}}(t) + K_{43, \text{hyd.pot}} y_{\text{ship}}(t) + K_{44, \text{hyd.pot}} \varphi_{\text{ship}}(t)
\]

(10 - 31)
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One realization of the roll motion time series is plotted as follows:

![Roll response realization for seed 1 (free hanging)](image)

**Figure 10-25**: Roll response realization for seed 1 (free hanging)

All the 50 seeds simulations are plotted as follows and distributions are fit correspondingly, we first give the spectrums for every 3 hour simulation then the spectrums for every 0.5 hour simulation in order to match the time length of the measurement:

![Roll response spectrum freehanging stage](image)

**Figure 10-26**: Roll response spectrum (free hanging simulated in time domain). **Left**: 3h per spectrum. **Right**: 0.5h per spectrum.

The statistics and distributions of the results are given as follows:
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Figure 10-27: The statistics of Roll response spectrum. **Left:** 3h per spectrum. **Right:** 0.5h per spectrum.

The most probable results from the distribution are compared to the real measured data as follows:

<table>
<thead>
<tr>
<th>Mean of the extreme roll angle in 0.5h (degree)</th>
<th>Measured</th>
<th>Calculated</th>
</tr>
</thead>
<tbody>
<tr>
<td>STD (degree)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 10-7: Roll response comparison (free hanging)

10.5 Discussion and conclusion

From the results comparison above, the following issues can be concluded:

In general, the roll response and its range found in different simulations by time domain method can match the measured value in a fair way. The frequency domain method has a good performance in free floating case, however, fail to predict the 2\textsuperscript{nd} order roll in free hanging case. The reason is the force put on the right hand is a force amplitude --- a real number, as we mentioned in Chapter 6, this treatment is theoretically not right for multi degrees of freedom system. It is applicable to predict the result only when the target motion is not (largely) coupled with other motions. The poor result showed above does prove the frequency domain method by using amplitude force from Wichers’ equation just doesn’t work for complicated situations. The only way to simulate free hanging case is using time domain method (Chapter 7).

As for the damping, the effect of introducing cubic damping term in this thesis (see discussion in Chapter 5) can be seen in the following comparison.
For the free floating case, where the motion is very mild, with damping up to quadratic terms only, the response is much over estimated, while for the free hanging case, the performance with quadratic damping can on the other hand fit the reality quite well. It proves that the quadratic damping underestimates the damping ratios when the motion is very mild, but with high motions (close to the motion in the decay test), its performance of estimating damping is good. For the free hanging case, the fact that the cubic damping calculated motion seems lower than the measurement doesn’t necessarily mean cubic damping is not good, it can be explained as follows:

- The measurement is only half an hour length, if we plot the simulation in half hour length as well, see Figure 10-26, the measurement is still within the simulation range.
- A guess from the author is that due to the external issues in previous stage, if we further model the low frequency motion as a summation of a free roll decay (cause by an initial roll angle resulted from the period ahead of
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measurement) and a second order roll from nonlinear wave excitation, it will also cause the second order roll much higher (see Figure 10-30).

Figure 10-30: Roll response spectrum for free hanging stage with initial roll angle 0.1 degree.

For different simulations, much larger variance can be seen in the second order motion frequency range than the first order motion frequency range. This again proves the large variance of 2nd order roll as proposed in Chapter 8. What’s more, the two cases, especially the free floating case is dominated by first order motion, thus not much variance can be seen in the standard deviation as the model test environment simulation, and the statistics distribution of the extreme values during different simulations is more tend to be a Rayleigh distribution.

In general, the simulation of Thialf’s roll motion in reality matches all the characteristics discussed in the previous chapters, and the time domain simulation method gives a reasonable result compare to the measurement, thus in the next chapter, we use the same simulation method to evaluate the operability of Sleipnir.
11

INFLUENCE OF SECOND ORDER ROLL MOTIONS ON THE OPERABILITY OF SLEIPNIR

11.1 Introduction

In this Chapter, we give a prediction for Sleipnir’s overall roll motion, in order to see whether 2nd order roll cause safety problem in reality. A criteria is first set to judge whether the roll motion is severe or not, and then based on this criteria, three typical load cases are checked in time domain.

11.2 Operability criteria

Based on Heerema Marine Contractors’ previous offshore heavy lifting experience, offshore heavy lifting operation should be safe when the sea state meets the following criteria:

\[ H_s T_p^2 < 75 \]

(11 - 1)

The first part of the curve doesn’t exist in the reality according to the wave steepness curve by DNV[20]:

\[ S_p = \frac{2\pi H_s}{g T_p^2} < \frac{1}{15} \quad T_p \leq 8s \]

(11 - 2)

The criterion for the operability check in this thesis is: the extreme roll angle in a 3 hour simulation should not exceed 0.5 degree. The 0.5 degree is set for a combined consideration of many factors, for example,

1. Strength of the cranes
2. Relative motions of the hooks while performing the hook-up
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3. Uncontrolled swinging of the load leading to large relative motions and difficulties during set down and controlling the load.

In all, because Sleipnir is going to be used for offshore heavy lifting, the requirement for stable operation is very strict. Thus, although 0.5 degree seems to be very low for normal ship roll motion, it is fair to set it as the high limit in the case of heavy lifting.

As for the extreme roll angle during a 3h simulation, the mean value is taken from 50 simulations. The 10% non-exceedance value is also reported as a more conservative prediction.

![Operability curve](image)

**Figure 11-1:** The operability curve and the checked sea states.

Altogether 6 sea states are picked to be checked along the expected operability curve. The wave spectrum used for the check is still a theoretical JONSWAP spectrum with the spreading factor of 4 (see **Figure 7-7**), because it can represent the real sea state best. The ship is always encountering a beam wave because in general it is the wave direction in offshore heavy lifting and has the highest roll motion, although in **Chapter 9.6**, we proved it is not true for wave spectrum with higher \( T_p \).

After the wave spectrum and the ship heading are chosen, the excitations can already be determined without considering load condition. Because of the discussion in **Chapter 9.3**, it is concluded the loading condition doesn’t change the QTFs a lot, the changing of loading condition doesn’t have a lot of influence on the excitation, only have a large impact on the response. Thus, the 1st and 2nd order wave excitations can be regarded as the same for all the load cases. The excitations simulated by the first seed according to different wave spectrums are plotted as an example.
In general, the first order excitation is much higher than the second order excitation, if considering the force amplitude only, the latter is negligible. It is a problem in this thesis, is because its frequency cause resonance.

The first order wave excitation reaches its highest value under the sea state $H_S = 0.8 \, m \, T_p = 9.68 \, s$ while the second order excitation reaches highest under the sea state $H_S = 2.8 \, m \, T_p = 5.18 \, s$. This indicates the first order force is more sensitive to $T_p$ while the second order force is more sensitive to $H_S$. The sensitivity of $T_p$ is related to the vessel dimension (see Chapter 9.2), both 1st and 2nd order roll is influenced a lot by the vessel dimension issues. However, the $H_S$ becomes a more domanate issue for 2nd order...
INFLUENCE OF SECOND ORDER ROLL MOTIONS ON THE OPERABILITY OF SLEIPNIR

is as we explained in Chapter 9, for the sensitivity of 2nd order force, it is almost proportional to \( H_S^2 \), while for the first order, it is proportional to \( H_S \) only.

A certain amount of variance (especially for 2nd order force) is expected for different seeds (see Chapter 8), however, it wouldn't influence the relative amplitude that much. For the chosen 6 sea states, we see the sea has both smaller first and second order excitation than the sea, thus the check of this sea state is kind of meaningless, we will remove this sea state in the following.

11.3 Checked loading conditions

In the following, we simulate the overall roll responses for three typical loading condition cases, each one we give a 50 seeds 3 hour simulation for every checked sea state. 50 seeds is chosen as the number for simulation according to the discussion in Chapter 8, the loading conditions are calculated based on the design parameters of the 'Sleipnir'[26];

<table>
<thead>
<tr>
<th>Draft [m]</th>
<th>GM [m]</th>
<th>Hook load [mT]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Free Floating (no loading condition)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Free Hanging (no loading condition, simulated by adjusting CoG)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Free Hanging (load simulated by load coupling dynamic model)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 11-1: Loading condition parameters

All the three cases are defined with the same draft, although Sleipnir's operational draft is designed from 27 – 32 m, larger draft usually means lower motions. Since draft definitely influence QTFs a lot and the full QTF needs a calculation time of more than a month, here we only check the draft at 27m where the roll motion is expected to be more severe.

The GM value is calculated based on traditional hydrostatic theories, three cases are different in GM values due to weight distribution of ballast water and hook load. The GM is higher means the hydrodynamic stiffness is higher.

For the third case, two GM values are presented in the table, the first one is calculated based on the weight of the load counted on the top of the crane. The second GM we consider the vessel only, the load is connected to the vessel by a dynamic model.

**Loading condition 1 --- Free Floating no loading condition / High GM case**

There's no load on the crane in this case, thus the ship motion is not interfered by load motion. The GM is large, with the centre of gravity quite low (cause the operation draft is maintained by large amount of ballast water in the bottom tanks).
INFLUENCE OF SECOND ORDER ROLL MOTIONS ON THE OPERABILITY OF SLEIPNIR

This case is chosen because it represents a typical free floating situation for Sleipnir’s future operation.

The roll response time series simulated by the first seed under all sea states and the corresponding roll spectrums are plotted as follows:

Figure 11-4: Loading condition 1 first seed simulation

Variance is expected for different time period of simulation, distributions are fit for the zero crossing maximums during the whole length of simulations (150 hours) for each sea state.

Figure 11-5: Loading condition 1 zero crossing maximum roll angle distribution (150 hours length)

The final check based on the mean of the extreme roll angle in all 3 hour simulations and the probability of exceedance are listed as follows:
The 'Mean of Maxima' and the 'Highest 10% of Maxima' criteria are checked based on the extreme values in all 3 hour simulations. The 'Probability of exceedance' criteria is calculated based on the zero crossing maximum roll angles along the whole 150 hours simulations.

**Loading condition 2 --- Free Hanging simulated by adjusting CoG / Low GM case**

This case is defined as a free hanging case, it is chosen because it is the model test way of simulating free hanging: with no load motion influence, however the centre of the gravity is adjusted to a point calculated by the weight average of ship weight and load weight, where the latter is counted on the top point of the crane. The adjusted CoG becomes much higher and corresponding to lower GM. In this case, the GM is chosen based on the lowest GM allowed by HMC heavy lift criteria, thus due to the stability consideration, this case is also worth checking.

The roll response time series simulated by the first seed and the corresponding roll spectrums are plotted as follows:

**Figure 11-6: Loading condition 2 first seed simulation**
INFLUENCE OF SECOND ORDER ROLL MOTIONS ON THE OPERABILITY OF SLEIPNIR

Variance is expected for different time period of simulation, distributions are fit for the zero crossing maximums during the whole length of simulations (150 hours) for each sea state.

![Graphs showing roll angle distributions](image)

**Figure 11-7:** Loading condition 2 zero crossing maximum roll angle distribution (150 hours length)

The final check based on the mean of the extreme roll angle in all 3 hour simulations and the probability of exceedance are listed as follows:

<table>
<thead>
<tr>
<th>Mean of Maxima (degree)</th>
<th>Highest 10% of Maxima (degree)</th>
<th>Probability of exceedance (&gt;0.5 degree)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.65</td>
<td>3.87</td>
<td>0.0050</td>
</tr>
<tr>
<td>3.25</td>
<td>4.56</td>
<td>0.0074</td>
</tr>
<tr>
<td>3.85</td>
<td>5.25</td>
<td>0.0098</td>
</tr>
<tr>
<td>4.45</td>
<td>6.05</td>
<td>0.0124</td>
</tr>
</tbody>
</table>

**Table 11-3:** Operability check for loading condition 2

**Loading condition 3 —— Free Hanging simulated by load coupling dynamic model**

This case is also defined as a free hanging case, it is simulated by the vessel and load coupling model described in Chapter 10, with the mutual influence the original natural frequencies of the vessel’s roll motion and the load’s swing motion further split away. The model is showed in Figure 11-8. This case is chosen because it represents a typical free hanging situation for Sleipnir’s future operation.
INFLUENCE OF SECOND ORDER ROLL MOTIONS ON THE OPERABILITY OF SLEIPNIR

Figure 11-8: Dimension of loading condition 3

The roll response time series simulated by the first seed and the corresponding roll spectra are plotted as follows:

Figure 11-9: Loading condition 3 first seed simulation
INFLUENCE OF SECOND ORDER ROLL MOTIONS ON THE OPERABILITY OF SLEIPNIR

Variance is expected for different time period of simulation, distributions are fit for the zero crossing maximums during the whole length of simulations for each sea state.

![Graph showing distribution of roll angles](image)

Figure 11-10: Loading condition 2 zero crossing maximum roll angle distribution (150 hours length)

The final check based on the mean of the extreme roll angle in all 3 hour simulations and the probability of exceedance are listed as follows:

<table>
<thead>
<tr>
<th>Mean of Maxima (degree)</th>
<th>Highest 10% of Maxima (degree)</th>
<th>Probability of exceedance (&gt;0.5 degree)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.85</td>
<td>0.85</td>
<td>0.2387</td>
</tr>
<tr>
<td>0.85</td>
<td>0.85</td>
<td>0.2682</td>
</tr>
<tr>
<td>0.85</td>
<td>0.85</td>
<td>0.2682</td>
</tr>
<tr>
<td>0.85</td>
<td>0.85</td>
<td>0.2682</td>
</tr>
</tbody>
</table>

Table 11-4: Operability check for loading condition 3

11.4 Conclusions

As a conclusion from Figure 11-4, Figure 11-6 and Figure 11-9, the 2nd order roll do contribute a lot to the overall roll motion of Sleipnir especially under the sea state where $H_S$ is high. Furthermore, the more 2nd order roll is dominate, the more group phenomenon can be seen in the simulation time series.

For all the three loading condition cases and under all the checked sea states, the mean of the extreme roll angles within 3 hours never exceed 0.5 degree, thus in general, Sleipnir is expected to perform safe operations with regard to the overall roll motion. The check in this thesis is only based on wave excitation. Wind, current and some sudden external issues should also be added into consideration according to certain situations, however, in general, 0.5 degree leaves enough margin for all these issues.
12 MITIGATION

12.1 Introduction

After predicting the overall roll motion for Sleipnir, we find the second order roll do have a large contribution (although not severe under the checked sea state), the mitigations of 2nd order roll are discussed for interest in this chapter. We will focus more on the mechanism of how the mitigation works instead of simulating them very precisely. Three methods are investigated, which are adding bilge keels (originally used to mitigate 1st order roll), applying tugger dampers (originally used to mitigate swing of the load for crane vessels), and using DP systems (originally used to mitigate horizontal motions).

12.2 Bilge keels

The resonance phenomenon is very sensitive to the damping ratio. Thus by adding a small amount of damping ratio, the 2nd order roll can be mitigated a lot. According to Himeno\(^9\), bilge keels can add to the damping in three ways, the first two contribute to the viscous damping, which is the 'normal-force damping of the bilge keel' and the 'hull pressure damping due to the bilge keel', the former can be quantified by Morison equation while the latter is caused by a pressure increase before the bilge keel and a pressure drop after the bilge keel, which finally leads to a damping moment for the ship. Bilge keels can also leads to an increase of radiation (potential) damping, but according to Himeno\(^9\), this effect is negligible compared to its contribution to viscous damping. Based on the CFD simulation of a decay test with bilge keels by Ottens\(^{27}\), we fit damping coefficients with the influence of bilge keels as follows:

![Vortices created by adding bilge keel to pontoon corners\(^{10}\)](image)

**Figure 12-1:** Vortices created by adding bilge keel to pontoon corners\(^{10}\)
As can be seen, the bilge keels add a lot to the quadratic damping coefficient $q$. This can be explained, the normal-force damping of the bilge keel is analytically calculated by Morison equation, and the drag term is a velocity squared term. The hull pressure damping due to bilge keels is actually a vortices phenomenon, which is also calculated by a velocity squared term. Thus, the effect of adding bilge keels is mainly increasing the quadratic viscous damping.

The effect of bilge keel is simulated for loading condition 2 under the sea state $H_S = 2.8m$ $T_P = 5.18s$ and $H_S = 0.8m$ $T_P = 9.68s$ described in the last Chapter. The results are plotted as follows.

For this mitigation, seeing from Figure 12-3, the bilge keel doesn't mitigate first order response a lot, and it can be explained by first order response (not within natural frequency range) is not sensitive to damping ratio and it's originally much dominated by the potential damping which is not contributed by bilge keels. Also note the viscous damping coefficients are derived based on the decay tests at the natural frequency, thus the damping value added by bilge keel might not be accurate for first order frequency range. We cannot give a solid conclusion for first order here, but for the 2nd order roll, it can be largely mitigated by adding bilge keel.
**MITIGATION**

**Figure 12-3:** Mitigation effect with bilge keel for loading condition \( H_s = 0.8 \text{m} \) \( T_p = 9.68 \text{s} \). **Left:** Roll response spectrum with and without bilge keel, **Right** The statistics of zero crossing maximum roll angle, **Top:** Without bilge keel, **Bottom:** With bilge keel.

**Figure 12-4:** Mitigation effect with bilge keel for loading condition \( H_s = 0.8 \text{m} \) \( T_p = 9.68 \text{s} \). **Left:** Roll response spectrum with and without bilge keel, **Right** The statistics of zero crossing maximum roll angle, **Top:** Without bilge keel, **Bottom:** With bilge keel.

**Figure 12-3** is a combined 1st and 2nd order motion case, the mean of the extreme roll angle is reduced from [value]. **Figure 12-4** is a 2nd order roll dominated case, the mean of the extreme roll angle is reduced from [value].

As we can see, for the high 2nd order roll case, the mitigation of adding bilge keel is very obvious.
12.3 Tugger damper

A tugger damper can be added between the load and the vessel to mitigate both the roll motion of the vessel and the swing motion of the load.

By connecting the load and the crane with horizontal wires, and controlling the length of the wire by tugger winches, damping can be achieved by pay-in and pay-out of the wire, which artificially introduces a wire length changing speed dependent tension force.\(^{[29]}\) We assume this can result in a constant damping coefficient, which means the pay-in and the pay-out velocity of the wire is inversely proportional to the relative velocity between the load and the vessel.

Although because the crane arm is stretched out, the tugger damper has to be set in a 3D dimension, however since we only consider roll motion in this thesis, and it isn’t influenced a lot by issues along the ship length, thus the 2D model in Chapter 10 is still used here, only the component along the ship width is taken into consideration.

For the tugger damper, Equation 10-8 should be modified into:

\[
D = \frac{1}{2} D_\varphi \dot{\varphi}(t)^2 + \frac{1}{2} D_{\text{sway}} x_{\text{ship}}(t)^2 + \frac{1}{2} D_{\text{heave}} y_{\text{ship}}(t)^2 + \frac{1}{2} D_{\text{swing}} \theta(t)^2 + \frac{1}{2} D_{\text{tugger}} ((L - l) \dot{\varphi}(t) - l \dot{\theta}(t))^2
\]

(12 - 1)

The final equation of motion is further changed into:

\[
\begin{bmatrix}
M + m & ml\cos\varphi_{\text{ship}}(t) & -ml\cos\theta(t) \\
ml\cos\varphi_{\text{ship}}(t) & M + m & -ml\sin\varphi_{\text{ship}}(t) \\
-m\cos\theta(t) & -ml\sin\varphi_{\text{ship}}(t) & J_M + ml^2
\end{bmatrix}
\begin{bmatrix}
x_{\text{ship}}(t) \\
y_{\text{ship}}(t) \\
\varphi_{\text{ship}}(t) \\
\theta(t)
\end{bmatrix}
= \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix}
\]

(12 - 2)

Where,

\[
A = ml\dot{\varphi}_{\text{ship}}(t)^2 \sin\varphi_{\text{ship}}(t) - m\dot{\theta}(t)^2 \sin\theta(t) - D_{\text{sway}} x_{\text{ship}}(t) - k_{\text{sway}} x_{\text{ship}}(t) + F_{\text{sway}}(t)
\]

(12 - 3)
\[ B = mL\dot{\varphi}_{\text{ship}}(t)^2 \cos \varphi_{\text{ship}}(t) - ml\dot{\theta}(t)^2 \cos \theta(t) - D_{\text{heave}} y_{\text{ship}}(t) - k_{\text{heave}} y_{\text{ship}}(t) - Mg - mg + F_{\text{heave}}(t) \]  
\[ C = ml\dot{\theta}(t)^2 \sin \left( \theta(t) - \varphi_{\text{ship}}(t) \right) + mg L \sin \varphi_{\text{ship}}(t) - D_{\varphi} \dot{\varphi}_{\text{ship}}(t) - k_{\varphi} \varphi_{\text{ship}}(t) + M_{\varphi}(t) - (L - l)D_{\text{tugger}}((L - l)\varphi(t) - l\dot{\theta}(t)) \]  
\[ D = -ml\dot{\varphi}_{\text{ship}}(t)^2 \sin \varphi_{\text{ship}}(t) \cos \theta(t) + mlL\dot{\varphi}_{\text{ship}}(t)^2 \cos \varphi_{\text{ship}}(t) \sin \theta(t) - D_{\text{swing}} \dot{\theta}(t) - mgL \sin \theta(t) - lD_{\text{tugger}}(l\dot{\theta}(t) - (L - l)\varphi(t)) \]  

The effect of tugger damper is simulated by loading condition \[ A \]  with a 12000 mT load swing motion coupled with the roll motion of the ship under the sea state \[ H_S = 0.8 \text{ m} \]  and \[ T_P = 9.68 \text{ s} \]  in Chapter 3. With \[ D_{\text{tugger}} = 10 \text{ e}^{5} \text{ kN} \text{m} \]  the results are plotted in Figure 12-6 and Figure 12-7. Figure 12-6 is a combined 1st and 2nd order case, the mean of the extreme roll angle is reduced from 0.2387 degree to 0.1830 degree. Figure 12-7 is a 2nd order roll dominated case, the mean of the extreme roll angle is reduced from 0.2786 degree to 0.2139 degree. As we can see, tugger damper is able to mitigate both 1st and 2nd order roll motion to a great extent, both in phase and out of phase motion in the vessel-load coupled system are able to be mitigated because the damping is acted by the relative velocity between the CoG of the load and the point on the crane which has the same horizontal level, thus they have relative velocity both at the 'in-phase' and 'out of phase' frequency.

Figure 12-6: Mitigation effect with tugger damper for loading condition \[ A \]  Roll response spectrum with and without tugger damper. Roll angles of load crossing zero, Top: Without tugger damper, Bottom: With tugger damper.
12.4 DP system

A DP system is originally designed to compensate horizontal motions, such as surge, sway and yaw. Conventional DP systems cannot compensate roll motion, because the roll motion signal would be filtered away because of its high frequency (too high for the control system to react to the input signals). However, 2nd order roll has a very low frequency, thus it does not necessarily have to have the problem of being filtered away, although a DP system normally disregards anything under ~ 60 to 100 seconds.

Here, the simplest solution of a DP system is applied by combining Kalman filter and PID controller to see whether it can be used to compensate the low frequency roll motion. Trust allocations and the so caused influence on other motions are all neglected. To avoid the possible instability problem of DP systems during heavy lifting reported by a lot of literature, only the situation without load coupling is considered here.

The Kalman filter works as an observer to predict the motion of next time step based on measurement. \[^{[33]}\]

\[
\begin{align*}
\hat{x}_k &= A_k \hat{x}_{k-1} + H_k (y_k - C_k A_k \hat{x}_{k-1}) \\
H_k &= P_k' C_k^T (C_k P_k' C_k^T + R_k)^{-1} \\
P_k' &= A_k P_{k-1}' A_k^T + Q_{k-1} \\
P_k &= (1 - H_k C_k) P_k'
\end{align*}
\]

Where,
- \(A_k\) System matrix
- \(C_k\) Output matrix
- \(Q_k\) Covariance matrix of process
- \(R_k\) Covariance matrix of measurement
- \(H_k\) Kalman gain
MITIGATION

\[ \begin{align*}
\hat{x}_k & \quad \text{A posteriori state estimate at time step } k \\
\hat{x}_{k-1} & \quad \text{A priori state estimate at time step } k \\
P'_k & \quad \text{Error covariances } P'_k = E\{e'_k e'_k^T\} \\
P_k & \quad \text{Error covariances } P_k = E\{e_k e_k^T\} \\
e'_k & \quad \text{A priori estimate error} \\
e_k & \quad \text{A posteriori estimate error}
\end{align*} \]

After predicting the motion of the next time step, it is fed into a PID controller to calculate a thruster force which will be added to the ship in the next time step to compensate the wave excitation force.

\[ T = f \left( X_4, \int X_4 \, dt, \dot{X}_4 \right) \]  \hspace{1cm} (12-11)
\[ (M + A)\dddot{X}_4(t) + BX_4(t) + KX_4(t) = T(t) + F^{(2)}(t) \]  \hspace{1cm} (12-12)

**Figure 12-8:** Motion prediction by Kalman filter. **Upper:** Prediction of the roll motion. **Bottom:** Prediction of the roll velocity.

**Figure 12-9:** Roll motion mitigations by DP system.
As a result, with the DP system the roll motion in Figure 12-9 is much mitigated. Thus, it is also possible for DP system to be applied to compensate 2nd order roll motion in the future. Of course, more complicated issues should be taken into consideration to validate this method further.
13.1 The reasoning of the 2nd order roll problem

Issues of highly importance:
The second order roll problem found in the model test of Sleipnir is a resonance phenomenon, caused by relatively high nonlinear excitations and very low roll damping ratio at the natural frequency band.

13.2 Nonlinear roll moment excitation

Issues of highly importance:
There's a nonlinear wave excitation at the natural frequency band of Sleipnir’s roll motion, this wave excitation is introduced by the difference frequency of the two wave components, the shape features of Sleipnir cause the 2\textsuperscript{nd} order roll moment excitation quite considerable.

Some conclusions:
- With an obvious peak in roll QTF caused by the certain distance between the two pontoons of semi-submersible vessel, the 2\textsuperscript{nd} order roll moment would reach a high value if the first order wave spectrum frequency range matches this peak in the QTF.
CONCLUSION

- For Sleipnir, because of its large size, it is more sensitive to waves with larger period compared to small semi-submersibles HMC owns.
- The nonlinear effect (the fifth term of QTF, consider wave only) for two wave components from different direction will be much smaller than the situation two wave regular components coming from the same direction.
- In general, the QTF and the excitation reaches the highest value in the beam wave direction, however, exceptions exist for some certain wave period due to the shape features of the vessel.
- Standing wave phenomenon can be seen in the serrated QTF curve when reflection environment is modelled.

13.3 Roll damping

Issues of highly importance:
The total damping ratio at the natural frequency band of Sleipnir’s roll motion is very low which add to the severity of 2nd order roll, it is also caused by the shape features and especially the large-radius pontoon of Sleipnir.

Some conclusions:
- The potential damping is almost zero at the low frequency band, it is because the ship motion can hardly create waves with extremely large wave length (corresponding to low frequency), and the semi-submersible type vessel traps a lot energy within the area between the columns, which also causes less energy dissipation.
- The viscous damping is also low because of the large radius of the pontoon corners.
- Linearization methods should be applied to the viscous damping when apply a frequency domain simulation, the most commonly used Borgman’s linearization factor have a good performance and in general underestimate damping ratios a little.
- When applying the Cummins equation, the time step should be given small enough in order to match the frequency step, otherwise obvious error would be introduced in low frequency band.
- With linear and quadratic damping only, it is difficult to predict the viscous damping accurately when the motion response is small, cubic damping term should be introduced.

13.4 Simulation method

Frequency domain method:

Method 1: Wichers’ equation + Damping linearization by Borgman’s factor (extend to cubic term)
Advantage:
1. Quickest method.
2. Both Wichers’ equation and damping linearization by Borgman’s factor are widely used in hydrodynamic field. The two method is supported by a lot of literatures separately.

**Disadvantage:**

1. Unable to calculate 2\textsuperscript{nd} order motion when the target motion is strongly coupled with other motions. Because the force spectrum calculated by Wichers’ equation doesn’t contain phasing information.

**Method 2: Add phasing information by QTF definition**

**Advantage:**

1. Able to calculate second order motion when the target motion is coupled with other motion.

**Disadvantage:**

1. Introduce non-uniqueness, thus much longer simulation time(a number of seeds needs to be calculated to give the distribution of probable results).
2. Not tested by other literature.

**Time domain method:**

**Advantage:**

1. Applicable for any case.
2. No linearization caused inaccuracies.

**Disadvantage:**

1. Very large computational time due to:
   - For each time step, multiply a 4D QTF with two 2 dimensional wave spectrum.
   - Retardation function needs a very small time step to ensure accuracy.
   - Large variance exists in different simulations of 2\textsuperscript{nd} order roll, thus a need for a large number of simulations (seeds) to fit distributions.

**Solutions:**

- Using non-uniqueness frequency step to limit the number of wave components.
- Only retardation function is calculated according to small time step, other physical quantities are all interpolated.
- According to the discussion, the number of random simulations is limited to 50.

### 13.5 Characteristics of 2\textsuperscript{nd} order roll motion

**Issues of highly importance:**

- Large variance exists when performing different simulations for 2\textsuperscript{nd} order roll under the same first order wave spectrum(with different phasing).
- With the same vessel and the same draft, the QTFs can be used repeatedly, regardless of water depth, loading condition and other issues. One do not need to recalculate the QTF for different damping levels or when the natural frequency shifts.
- 2nd order roll is almost proportional to Hs\textsuperscript{2}, while Tp is also a sensitive factor because of ship shape features.
Some conclusions:

- Because of the large variance of 2nd order roll motion, it is not sufficient to give a prediction with only one simulation, at least 50 simulations, ideally 150 simulations should be performed to give a fair prediction for the probability distribution.
- The large variance for different simulations is caused by the non-unique wave group excitation and the low damping ratio, the later would cause an obvious group effect.
- The extreme roll angles from each simulation tend to follow a Gaussian distribution instead of Rayleigh distribution (Which is the case for pure first order roll).
- For the QTF, different water depth (when the water depth is not too low) only causes differences in the very low frequency range, where there’s no wave spectrum density.
- For the QTF, different loading conditions only cause obvious difference at the natural frequency where also no wave spectrum density exists. However, because the natural frequency changes for different load cases, the final roll response would change accordingly.
- The reflection phenomenon of the model test adds to the second order roll.
- The spreading of wave in reality will decrease the 2\textsuperscript{nd} order roll a lot compared to the unidirectional wave with the same spectrum density. However, exceptions do exist for certain Tp and direction, wave spreading would add to second order roll.
- The ship’s heading influence is also dependent on Tp, in most of the time, the beam wave situation (which is likely to happen for semi-submersible during the heavy lifting) cause the largest 2\textsuperscript{nd} order roll.

13.6 2\textsuperscript{nd} order roll in reality

Issues of highly importance:
In reality, second order roll could happen in the free floating and free hanging stage of offshore heavy lifting operation.

Some conclusions:

- For the free floating stage, it can be confirmed the low frequency peak exists in the roll response spectrum is a second order roll.
- For the free hanging stage, it is hard to determine whether all the low frequency motion is cause by 2\textsuperscript{nd} order wave excitation based on the short time of measurement. However, it can be concluded second order roll do exist because for the whole half hour, low frequency motion continuously exists, there is no other low frequency excitation and the load motion influence is an in-phase one.
- As for the simulation, both frequency domain and time domain method can give a quite fair result for the free floating stage. Because in the free floating stage 2\textsuperscript{nd} order roll motion is not strongly coupled with other motions.
Due to the complexity of free hanging stage, it is suggested to use time domain method to understand the phenomenon better. And the frequency domain method is not applicable for this multiple degrees of freedom system.

13.7 Mitigation

2nd order roll could be quite severe under the sea state where the significant wave is high, thus mitigations are proposed in this thesis.

- Bilge keels can be used to mitigate 2nd order roll by the mechanism of increasing viscous damping.
- Tugger damper also works well by the mechanism of adding damping between the load and the vessel.
- DP systems works by giving a predicted trust force to compensate environmental forces, so as to decrease the wave excited roll motion.

Thus, a lot of mitigation methods can be used to decrease the 2nd order roll, which makes it even possible for the new semi-submersible to operate in more severe sea state in the future.

13.8 Answer to the research question

Will second order roll happen in reality?

Yes, and it has a dominant contribution to the overall roll motion.

Good news is even 2nd order roll is added to the overall roll motion, Sleipnir is still able to perform a stable operation according to operability evaluation. The overall roll motion stays below a maximum amplitude of 0.5 degree in normal operational sea states.
13.9 Recommendation for future study

- It is suggested to use very detailed CFD simulation and if possible model test to investigate how loading condition and waves influence the roll damping ratio.
- It is suggested to study the possible frequency dependence of the viscous damping by performing forced oscillation tests. And study the so caused influence on the added mass part based on Kramers-Koning method.
- It is suggested to try more decay tests for different cases to figure out whether, why and where cubic damping term is necessary. If necessary, what issues are related to the cubic terms.
- It is suggested to measure a long time record (preferably over 1 hour) offshore for the free hanging stage in order to understand the 2nd order roll better in this stage.
- It would be interesting to investigate whether the 2nd order roll can be mitigated by changing the shape feature of the semi-submersible and by what mechanism.
- It is suggested to rewrite all the calculation programme in more efficient language instead of MATLAB, thus more simulation can be done to conclude on the statistics.
- It is suggested to find an open source diffraction software, and programme it to calculate the 2nd term of QTF only. To see whether it is a good approximation for the full QTF.
- It is suggested to validate the methods proposed in Chapter 5.8 to determine damping coefficients and compare the results with the logarithmic fitting method in Chapter 5.3.
- For the author's interest, another new damping model is proposed as follows:
  - **Phase 1:** by adding constant force (irrelevant with ship motions)
  - **Phase 2:** \( b_2 \dot{\varphi}^{(1)} + b_2 \varphi^{(2)} \dot{\varphi} \)
  - **Phase 3:** \( b_3 \varphi^{(1)}(\omega) \dot{\varphi} \)

Phase 1 is dependent on micro fluid, i.e. the viscosity or the micro force between the water particles (cannot be solved by potential theory). This phase is irrelevant with ship motion.

Phase 2, the ship motion would add to the movement of water particles.

Phase 3, the ship motion would even create macro water motions, i.e. waves to dissipate more energy. It is corresponding to the potential damping as we considered for normal ship motions.

Phase 2 and phase 3 is paralleled introduced by ship motion, the limit value of the phase 1 should always be added actually as a constant value to the right side of the equation, despite of the fact under most circumstances it is negligible.

It is interested to validate this model and investigate how to determine the 'constant value' which should be added in phase 1.


SECOND ORDER BOUNDARY CONDITION AND DIFFRACTION ANALYSIS

Introduction

In the main text, Pinkster’s theory is introduced in a simple way. Some intermediate derivations and assumptions are neglected, they are further explained in this appendix. What’s more, the second order potential mentioned as an important term in the QTF actually should be solved by second order boundary conditions, they will also be derived based on the author’s understanding in this appendix chapter. The theory in this Appendix chapter combined with the theory in Chapter 3 is the complete theory for the calculation of QTF.

Additional derivations for QTFs[1][11]

The derivations from Equation 3-14 to Equation 3-15, 3-16 are neglected in the main text, they are added as follows:
The first term of Equation 3-14.

\[- \iint_{S_0} p^{(1)} \cdot N^{(1)} dS = - \vec{R}^{(1)} \iint_{S_0} p^{(1)} \cdot \vec{n} dS = \vec{R}^{(1)} \{ \vec{F}^{(1)} - \vec{R}^{(1)}(0,0,mg) \} = m\vec{R}^{(1)} X_g^{(1)} - \vec{R}^{(1)} \cdot \vec{R}^{(1)}(0,0,mg)\]  

(A-1)

This term can be further explained as the rotation part of first order force, only the term \(m\vec{R}^{(1)} X_g^{(1)}\) remains in the final QTF expression because the \(- \vec{R}^{(1)} \cdot \vec{R}^{(1)}(0,0,mg)\) is a constant term and has nothing to do with wave excitations.
The second term of Equation 3-14.

\[ - \iint_{S_0} P^{(2)} \cdot \mathbf{n} dS = - \iint_{S_0} \left( - \rho g X_3^{(2)} - \rho \frac{\partial \Phi^{(2)}}{\partial t} - \frac{1}{2} \rho \left( \nabla \Phi^{(1)} \right)^2 - \rho \left( X^{(1)} \cdot \nabla \frac{\partial \Phi^{(1)}}{\partial t} \right) \right) \cdot \mathbf{n} dS = \iint_{S_0} \left( \frac{\partial \Phi^{(2)}}{\partial t} + \frac{1}{2} \rho \left( \nabla \Phi^{(1)} \right)^2 + \rho \left( X^{(1)} \cdot \nabla \frac{\partial \Phi^{(1)}}{\partial t} \right) \right) \cdot \mathbf{n} dS \]  

(A−2)

The third term of Equation 3-14.

\[ - \iint_S P^{(1)} \cdot \mathbf{n} dS = \iint_S \left( - \rho g X_3^{(1)} - \rho \frac{\partial \Phi^{(1)}}{\partial t} \right) \mathbf{n} dX_3 dl = \iint_S \rho g \left( -X_3^{(1)} + \zeta^{(1)} \right) \mathbf{n} dX_3 dl = - \oint_{\omega} \frac{1}{2} \rho g \left( \zeta_r^{(1)} \right)^2 \mathbf{n} dl \]  

(A−3)

The oscillation wet surface is further decomposed into \( dX_3 \) and \( dl \), and by integrating the influence of \( dX_3 \), another term of QTF can be resulted.

**Diffraction analysis**[31][32]

Except \( \Phi_{ww}^{(1)} \) and \( \Phi_{ww}^{(2)} \), there’s no analytical solution for complex 3-D floating structure. So other potentials have to be solved numerically by performing diffraction analysis according to the following boundary conditions.

Separate potentials into space dependent part and time dependent part. The space dependent part need to be further solved.

\[ \Phi_j(x, y, z, t) = \phi_j(x, y, z)v_j(t) \]  

(A−4)

Incident wave potential \( \phi_w^{(1)} \) can be solved analytically. \( \phi_a^{(1)} \) and \( \phi_b^{(1)} \) can be represented by Green’s theorem.
$$\Phi_j(x,y,z) = \frac{1}{4\pi } \iint_{S_0} \sigma_j(\hat{x},\hat{y},\hat{z}) \cdot G(x,y,z,\hat{x},\hat{y},\hat{z}) dS_0$$

Where,

- $j = 1,2,3,4,5,6$ stands for the potential due to the six body motion
- $j = 7$ stands for the diffraction potential
- $\Phi(x,y,z)$ stands for the potential at a certain point $(x,y,z)$, this point represent the mean of the panel it located on. Figure A-2
- $\sigma(\hat{x},\hat{y},\hat{z})$ stands for the source strength at a certain point $(\hat{x},\hat{y},\hat{z})$, assume source strength is constant on the panel it locates.

The Green’s function $G(x,y,z,\hat{x},\hat{y},\hat{z})$ represents how a source at $(\hat{x},\hat{y},\hat{z})$ contributes to the potentials at $(x,y,z)$, by integrating the influences of all sources on the ship, the potential at the point $(x,y,z)$ can be determined.

The Green's function $G(x,y,z,\hat{x},\hat{y},\hat{z})$ is a very magic function, it automatically satisfy the Laplace equation, the boundary condition on the sea bed, the free surface and the infinity, with the Greens' function, the only thing needed to be considered is the boundary condition on the ship surface. Because of the complexity of this function, we don’t discuss its mathematical expression in this thesis.

The diffraction potential can be solved by the boundary condition on the ship:

$$\frac{\partial \phi_0}{\partial n} + \frac{\partial \phi_7}{\partial n} = 0$$ (A - 5)

Substitute the wave potential by its analytical expression:

$$\frac{\partial (\frac{\partial}{\partial z} \cdot e^{kz} \cdot e^{ik(x \cos \mu + y \sin \mu)})}{\partial n} + \frac{\partial (\frac{1}{4\pi } \iint_{S_0} \sigma_7(\hat{x},\hat{y},\hat{z}) \cdot G(x,y,z,\hat{x},\hat{y},\hat{z}) \cdot dS_0)}{\partial n} = 0$$ (A - 6)

Consider $(x, y, z)$ could be the same point as $(\hat{x}, \hat{y}, \hat{z})$, the equation has to be rewritten in case of infinity result.
\[
\frac{\partial}{\partial n} \left( \frac{g}{\omega^2} e^{ikz} e^{i(kx\cos \mu + y\sin \mu)} \right) - \frac{1}{2} \sigma_7(x, y, z) + \frac{1}{4\pi} \int_{S_0} \sigma_7(\hat{x}, \hat{y}, \hat{z}) \frac{\partial (G(x, y, z, \hat{x}, \hat{y}, \hat{z}) \cdot dS)}{\partial n} = 0 \tag{A - 7}
\]

Discretize the equation, and solve for each panel:

\[
\begin{pmatrix}
-\frac{1}{2} & \cdots & \frac{1}{4\pi} \frac{\partial G_{1N}}{\partial n} \Delta S_N \\
\vdots & \ddots & \vdots \\
\frac{1}{4\pi} \frac{\partial G_{N1}}{\partial n} \Delta S_1 & \cdots & -\frac{1}{2}
\end{pmatrix}
\begin{pmatrix}
\sigma_{1,7} \\
\vdots \\
\sigma_{N,7}
\end{pmatrix} =
\begin{pmatrix}
-\frac{\partial \phi_0}{\partial n}_1 \\
\vdots \\
-\frac{\partial \phi_0}{\partial n}_N
\end{pmatrix} \tag{A - 8}
\]

The first term on the left hand of Equation A-8 has \(-\frac{1}{2}\) as the diagonal line while for the non-diagonal line \(\frac{1}{4\pi} \frac{\partial m_n}{\partial n} \Delta S_n\) represents the influence caused by the source of panel on the potential on panel \(m\), altogether \(N\) potential needs to be solved by \(N\) equations, thus the space dependent part of diffraction potential can be solved.

Same for solving potentials caused by ship motion. Still start from the ship body boundary condition, Fluid velocity equals body velocity in the normal direction of ship surface

\[
\frac{\partial \Phi}{\partial n} = \frac{\partial \phi_1}{\partial n}_j v_j(t) = f_j v_j(t) \tag{A - 9}
\]

\(f_j\) stands for the normal vector of each panel. Where,

Surge: \(f_1 = n_1\) \tag{A - 11}

Sway: \(f_2 = n_2\) \tag{A - 12}

Heave: \(f_3 = n_3\) \tag{A - 13}

Roll: \(f_4 = (r \times n)_1\) \tag{A - 14}

Pitch: \(f_5 = (r \times n)_2\) \tag{A - 15}

Yaw: \(f_6 = (r \times n)_3\) \tag{A - 16}

Thus just change the right hand of Equation A-8, we have:

\[
\begin{pmatrix}
-\frac{1}{2} & \cdots & \frac{1}{4\pi} \frac{\partial G_{1N}}{\partial n} \Delta S_N \\
\vdots & \ddots & \vdots \\
\frac{1}{4\pi} \frac{\partial G_{N1}}{\partial n} \Delta S_1 & \cdots & -\frac{1}{2}
\end{pmatrix}
\begin{pmatrix}
\sigma_{1,j} \\
\vdots \\
\sigma_{N,j}
\end{pmatrix} =
\begin{pmatrix}
-(f_j)_1 \\
\vdots \\
-(f_j)_N
\end{pmatrix} \tag{A - 17}
\]

Thus the space dependent part of motion introduced potential can be solved.

**Second order boundary conditions**

To solve second order potential, in addition to adding source and perform green function on the ship body, we also add sources on the free surface.
\( \Phi_d^{(2)} \Phi_b^{(2)} \) is calculated by sources defined on ship body. They only have to satisfy homogeneous (1st order) boundary condition.

\[
g \frac{\partial \Phi_d^{(2)}}{\partial X_3} + \frac{\partial^2 \Phi_d^{(2)}}{\partial t^2} = 0 \quad (A - 18)
\]

\( \Phi_w^{(2)} \) is calculated by sources defined on free surface. They have to satisfy the non-homogeneous boundary condition. That's why we input a free surface model in Chapter 4.

\[
g \frac{\partial \Phi_w^{(2)}}{\partial X_3} + \frac{\partial^2 \Phi_w^{(2)}}{\partial t^2} = -2 \left( \nabla \Phi^{(1)} \cdot \nabla \frac{\partial \Phi^{(1)}}{\partial t} \right) + \frac{\partial \Phi^{(1)}}{\partial t} \left( \frac{\partial^2 \Phi^{(1)}}{\partial X_3^2} + \frac{1}{g} \frac{\partial^2}{\partial t^2} \frac{\partial \Phi^{(1)}}{\partial X_3^2} \right) \quad (A - 19)
\]

Source type of free surface should be chosen not to qualify the homogenous surface condition.

The whole calculation procedure follows:

1. Solve second order wave potentials, right handed part of first order potentials has already be solved, so they can be regarded as known:

\[
g \frac{\partial \Phi_w^{(2)}}{\partial X_3} + \frac{\partial^2 \Phi_w^{(2)}}{\partial t^2} = -2 \left( \nabla \Phi^{(1)} \cdot \nabla \frac{\partial \Phi^{(1)}}{\partial t} \right) + \frac{\partial \Phi^{(1)}}{\partial t} \left( \frac{\partial^2 \Phi^{(1)}}{\partial X_3^2} + \frac{1}{g} \frac{\partial^2}{\partial t^2} \frac{\partial \Phi^{(1)}}{\partial X_3^2} \right) \quad (A - 20)
\]

2. Solve second order diffraction potentials, with second order wave potential and all first order potentials known:

\[
\nabla \Phi_d^{(2)} \vec{n} = -\nabla \Phi_w^{(2)} \vec{n} - (\vec{X}^{(1)} \vec{v}) \nabla \Phi^{(1)} \vec{n} + (\vec{v}^{(1)} - \nabla \Phi^{(1)}) \vec{N}^{(1)} \quad (A - 21)
\]

3. Solve second order potentials caused by body motions, the second order velocity can be regarded as a low frequency velocity which is not caused by first order wave, then it’s procedure is the same as solving the first order potentials of ship motion, just take the low frequency part:

\[
\nabla \Phi_b^{(2)} \vec{n} = \vec{V}^{(2)} \vec{n} \quad (A - 22)
\]

The second wave potential \( \Phi_w^{(2)} \) should be calculated for each two wave components causing the same difference frequency.

While \( \Phi_d^{(2)} \Phi_b^{(2)} \) only needs to be calculated once for one difference frequency, they don’t care how the difference frequency is composed.

Boundary conditions:

**Continuity**

1st order:

\[
\nabla^2 \Phi^{(1)} = 0 \quad (A - 23)
\]

2nd order:

\[
\nabla^2 \Phi^{(2)} = 0 \quad (A - 24)
\]
On the body
1st order:

\[ \nabla \Phi^{(1)} \vec{n} = \vec{v}^{(1)} \vec{n} \]  \hspace{1cm} (A - 25)

2nd order:

\[ \nabla \Phi^{(2)} \vec{n} = - (\vec{X}^{(1)} \nabla \Phi^{(1)} \vec{n}) + \left( \vec{v}^{(1)} - \nabla \Phi^{(1)} \right) \vec{n}^{(1)} + \vec{v}^{(2)} \vec{n} \]  \hspace{1cm} (A - 26)

Derivation of 2nd order boundary on the body

\[ \nabla \Phi^{(1)} \vec{n} = \vec{v}^{(1)} \vec{n} \]  \hspace{1cm} (A - 27)

\[ \nabla (\varepsilon \Phi^{(1)} + \varepsilon \vec{X}^{(1)} \vec{v}^{(1)} + \varepsilon^2 \phi^{(2)}) (\vec{n} + \varepsilon \vec{N}^{(1)}) = (\varepsilon \vec{v}^{(1)} + \varepsilon^2 \vec{v}^{(2)}) (\vec{n} + \varepsilon \vec{N}^{(1)}) \]  \hspace{1cm} (A - 28)

Only take the second order terms:

\[ \nabla \Phi^{(1)} \vec{n} + \nabla (\varepsilon \vec{X}^{(1)} \vec{v}^{(1)} + \varepsilon \phi^{(2)}) = (\varepsilon \vec{v}^{(1)} + \varepsilon^2 \vec{v}^{(2)}) (\vec{n} + \varepsilon \vec{N}^{(1)}) \]  \hspace{1cm} (A - 30)

\[ \nabla \Phi^{(2)} \vec{n} = - (\vec{X}^{(1)} \nabla \Phi^{(1)} \vec{n}) + \left( \vec{v}^{(1)} - \nabla \Phi^{(1)} \right) \vec{n}^{(1)} + \vec{v}^{(2)} \vec{n} \]  \hspace{1cm} (A - 31)

Free surface
1st order:

\[ g \left( \frac{\partial \phi^{(1)}}{\partial X_3} + \frac{\partial^2 \phi^{(1)}}{\partial t^2} \right) = 0 \]  \hspace{1cm} (A - 32)

2nd order:

\[ g \left( \frac{\partial \phi^{(2)}}{\partial X_3} + \frac{\partial^2 \phi^{(2)}}{\partial t^2} \right) = -2 \left( \nabla \Phi^{(1)} \cdot \nabla \frac{\partial \phi^{(1)}}{\partial t} \right) + \frac{\partial \phi^{(1)}}{\partial t} \left( \frac{\partial^2 \phi^{(1)}}{\partial X_3^2} + \frac{1}{g} \frac{\partial^2}{\partial t^2} \left( \frac{\partial \phi^{(1)}}{\partial X_3} \right) \right) \]  \hspace{1cm} (A - 33)

Derivation of nonlinear free-surface condition
Starting from Bernoulli’s equation

\[ \frac{\partial \Phi}{\partial t} + \frac{1}{2} \nabla \Phi \cdot \nabla \Phi + \frac{P}{\rho} + gz = C \]  \hspace{1cm} (A - 34)

Then the dynamic boundary condition can be written as:

\[ \frac{\partial \Phi}{\partial t} + \frac{1}{2} \nabla \Phi \cdot \nabla \Phi + g\zeta = \frac{P}{\rho} - C \quad \text{at } z = \zeta \]  \hspace{1cm} (A - 35)

\[ \frac{P}{\rho} - C \] is constant, it can be include in \[ \frac{\partial \Phi}{\partial t} \], thus

\[ \frac{\partial \Phi}{\partial t} + \frac{1}{2} \nabla \Phi \cdot \nabla \Phi + g\zeta = 0 \quad \text{at } z = \zeta \]  \hspace{1cm} (A - 36)
\[
\zeta(x, y, t) = -\frac{1}{g} \left( \frac{\partial \Phi}{\partial t} + \frac{1}{2} \nabla \Phi \cdot \nabla \Phi \right) \bigg|_{z=\zeta}
\]  

(A – 37)

Perform a Taylor expansion at \( z = 0 \)

\[
\zeta(x, y, t) = -\frac{1}{g} \left( \frac{\partial \Phi}{\partial t} + \frac{1}{2} \nabla \Phi \cdot \nabla \Phi \right) \bigg|_{z=0} - \frac{1}{g} \zeta \left( \frac{\partial \Phi}{\partial t} + \frac{1}{2} \nabla \Phi \cdot \nabla \Phi \right) \bigg|_{z=0} + \ldots
\]  

(A – 38)

Where

\[
\Phi = \Phi^{(1)} + \Phi^{(2)}
\]  

(A – 39)

\[
\zeta = \zeta^{(1)} + \zeta^{(2)}
\]  

(A – 40)

Only take the first order terms from both side:

\[
\zeta^{(1)} = -\frac{1}{g} \left( \frac{\partial \Phi^{(1)}}{\partial t} \right) \bigg|_{z=0}
\]  

(A – 41)

Only take the second order terms from both side

\[
\zeta^{(2)} = -\frac{1}{g} \left( \frac{\partial \Phi^{(2)}}{\partial t} + \frac{1}{2} \nabla \Phi^{(1)} \cdot \nabla \Phi^{(1)} + \zeta^{(1)} \frac{\partial^2 \Phi^{(1)}}{\partial z \partial t} \right) \bigg|_{z=0}
\]  

(A – 42)

Then derive the static boundary condition

\[
\tilde{f}(x, y, z, t) \equiv z - \zeta(x, y, t)
\]  

(A – 43)

Always zero at free surface, Perform a total derivation

\[
\frac{D \tilde{f}}{Dt} = \frac{\partial \tilde{f}}{\partial t} + \tilde{v} \nabla f = \frac{\partial (z - \zeta(x, y, t))}{\partial t} + \tilde{v} \nabla (z - \zeta(x, y, t)) = 0
\]  

(A – 44)

\[
\frac{\partial \zeta}{\partial t} + \frac{\partial \Phi}{\partial x} \frac{\partial \zeta}{\partial x} + \frac{\partial \Phi}{\partial y} \frac{\partial \zeta}{\partial y} = \frac{\partial \Phi}{\partial z} \quad \text{at } z = \zeta
\]  

(A – 45)

\[
\frac{\partial \zeta}{\partial t} + \nabla \Phi \nabla \zeta = \frac{\partial \Phi}{\partial z} \quad \text{at } z = \zeta
\]  

(A – 46)

Still where

\[
\Phi = \Phi^{(1)} + \Phi^{(2)}
\]  

(A – 39)

\[
\zeta = \zeta^{(1)} + \zeta^{(2)}
\]  

(A – 40)

Only take the first order terms from both side:

\[
\frac{\partial \zeta^{(1)}}{\partial t} = \frac{\partial \Phi^{(1)}}{\partial z} \quad \text{at } z = \zeta
\]  

(A – 47)
Only take the second order terms from both side

\[
\frac{\partial \zeta^{(2)}}{\partial t} + \nabla \phi^{(1)} \nabla \zeta^{(1)} = \frac{\partial \phi^{(2)}}{\partial z} \quad \text{at } z = \zeta
\]  

(A - 48)

**At seabed**  \(X_3 = -h \) *(water depth)*

1st order:

\[
\frac{\partial \phi^{(1)}}{\partial X_3} = 0
\]  

(A - 49)

2nd order:

\[
\frac{\partial \phi^{(2)}}{\partial X_3} = 0
\]  

(A - 50)

**At infinity**

\( \Phi_{d}^{(1)}, \Phi_{b}^{(1)}, \Phi_{d}^{(2)}, \Phi_{b}^{(2)} \) all equals zero at infinity.
Introduction

In this Appendix Chapter, QTF results for the uni-directional reflection model are given, as a comparison, results without reflection are also plotted. Diagonal line from 0.14rad/s to 0.26rad/s difference frequency are plotted.

QTF for reflection model

Figure B-1: QTFs for difference frequency equals 0.14 rad/s

Figure B-2: QTFs for difference frequency equals 0.16 rad/s
Figure B-3: QTFs for difference frequency equals 0.18 rad/s

Figure B-4: QTFs for difference frequency equals 0.20 rad/s

Figure B-5: QTFs for difference frequency equals 0.22 rad/s

Figure B-6: QTFs for difference frequency equals 0.24 rad/s

Figure B-7: QTFs for difference frequency equals 0.26 rad/s
POWER SPECTRUM DENSITY

The power spectrum density can be calculated based on Wiener–Khinchin theorem for a stationary process.\[^{[34]}\]

\[
S(\mu) = \int X(t) \cdot X(t - \tau)e^{-i\mu\tau} d\tau \quad (E - 1)
\]

Within Heerema Marine Contractors, a standard code 'specdens.m' is developed by van Dijk to create frequency domain spectrum according to the time series. And the base of this script is:

\[
S(\mu) = |F(X(t))|^2 \quad (E - 2)
\]

F() stands for the Fourier transform.

**Equation E-2** actually is equal to **Equation E-1** in its amplitude by convolution theorem:

\[
F(X(t) \cdot X(t - \tau)) = F(X(t)) \cdot F(X(t)) \quad (E - 3)
\]

Right hand is \(F(X(t)) \cdot F(X(t))\), take the absolute value, it is \(|F(X(t)) \cdot F(X(t))|\)

\(F(X(t))\) is a complex number, for each frequency, it can be written as \(a+bi\)

For the theoretical method to determine frequency domain spectrum:

\[
|F(X(t)) \cdot F(X(t))| \rightarrow |(a+bi) \cdot (a+bi)| = |a^2 - b^2 + 2abi| = \sqrt{(a^2 - b^2)^2 + 4a^2b^2} = \sqrt{a^4 + b^4 - 2a^2b^2 + 4a^2b^2} = \sqrt{(a^2 + b^2)^2} = a^2 + b^2 \quad (E - 4)
\]

For Heerema Marine Contractors’ standard code:

\[
|F(X(t))|^2 \rightarrow |a + bi|^2 = a^2 + b^2 \quad (E - 5)
\]

So to the author's point of view, Heerema Marine Contractors' standard code is valid to be used to determine frequency domain spectrum.