Inventory and Modelling of Different Objective Functions and their Impact on Optimal Design of an Offshore Wind Farm
Inventory and Modelling of Different Objective Functions and their Impact on Optimal Design of an Offshore Wind Farm

By

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Without your involvement you can’t succeed. With your involvement you can’t fail.

A.P.J Abdul Kalam
Abstract

The design of an offshore wind farm (OWF) is multidisciplinary in nature as it involves the design of many disciplines such as the wake effects, support structure, electrical cables etc. For the optimal design of an OWF, an optimization procedure is required where all the disciplines are optimized simultaneously. The objective function plays a significant role in optimization as it expresses the main aim of the model which is to be either minimized or maximized. So far, cost of energy (COE) and annual energy production (AEP) are one of the commonly used objective functions for OWF optimization as far as the author is aware. However, there might be other objective functions that may influence the optimal design of an OWF as well. This may include maximizing the profit, minimizing the environmental impact, reducing their carbon emissions etc. Hence, this thesis investigates the overview of different objective functions and understand its impact on the optimal design of an OWF.

An inventory of different objective functions is prepared, and relevant ones are selected for further study. It is observed that even though some objectives are dissimilar, they still depend on the same wind farm parameters and are therefore expected to give similar design results. From the list of objective functions, net present value (NPV) and risk management objectives are chosen for further research.

The selected objective functions are then formulated in a metric for optimization. The price of electricity plays a significant role in determining the NPV. It is learnt that electricity price varies with the power supply depending on the site conditions. The electricity price is low if the supply of power is high in a region where there are many OWF’s and vice versa. Moreover, OWF investors value constant power output without any fluctuations. Hence, taking all these aspects into consideration, the electricity price in the NPV function is modelled for a constant value, wind variability and wind power predictability.

The risk management function, on the other hand, aims at minimizing the uncertainty associated with an OWF project. The risk here refers to the uncertainty associated with the profit obtained from the OWF. A set of annual average wind speeds is computed using monte carlo simulations and the AEP and NPV are estimated. The mean( NPV\text{mean} ) and standard deviation ( NPV\text{std} ) of NPV are then calculated. NPV\text{std} represents the uncertainty in this scenario and is minimized to reduce the risk.

A suitable method is then identified to deal with multiple objectives. The NPV function is maximized for maximum profit and this objective is evaluated using a single objective optimization technique. The risk management objective involves the calculation of NPV\text{mean} and NPV\text{std}. Both objectives are contrasting in nature as a significant reduction in NPV\text{std} corresponds to an undesirable reduction in NPV\text{mean}. A tradeoff between both these objectives is the best possible solution. Therefore, a multi-objective optimization technique is used, and a list of solutions is obtained by generating a pareto front.

The new approach is then evaluated by implementing different case studies. It is observed that optimum rotor diameter and number of turbines for the single objective optimization technique are influenced by economic indicators such as the real interest rate and lifetime. However, they are not influenced by variation in the electricity price. Nevertheless, the NPV function is sensitive to the economic indicators and variation in the electricity price.

For the multi-objective optimization technique, multi criteria analysis was used to determine the weight to the objective functions while moving along the pareto curve. It was observed that the improvement of one objective led to the deterioration of the other objective. Hence, the pareto front
provides opportunities to investors to negotiate and decide on the weight they want to specify for their objectives.
Acknowledgement

The thesis was a result of encouragement from many people, who helped me in shaping the work by providing feedback, support and direction throughout. It is with sincere gratefulness that I acknowledge their contributions.

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Vishal Murali
Delft, October 2018
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### Abbreviations

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<thead>
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<th>Description</th>
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<tbody>
<tr>
<td>AEP</td>
<td>Annual energy production</td>
</tr>
<tr>
<td>CES</td>
<td>Carbon emissions signature</td>
</tr>
<tr>
<td>COE</td>
<td>Cost of energy</td>
</tr>
<tr>
<td>CPP</td>
<td>Carbon payback period</td>
</tr>
<tr>
<td>IRR</td>
<td>Internal rate of return</td>
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<tr>
<td>ISO</td>
<td>Independent service operator</td>
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<tr>
<td>FB</td>
<td>Financial balance</td>
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<tr>
<td>LCOE</td>
<td>Levelized cost of energy</td>
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<tr>
<td>LPC</td>
<td>Levelized production cost</td>
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<tr>
<td>MC</td>
<td>Monte carlo</td>
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<tr>
<td>MCA</td>
<td>Multi criteria analysis</td>
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<tr>
<td>MDAO</td>
<td>Multi - disciplinary design and optimization</td>
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<tr>
<td>NPV</td>
<td>Net present value</td>
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<td>NSGAII</td>
<td>Non - sorting genetic algorithm</td>
</tr>
<tr>
<td>N5RT</td>
<td>NREL 5MW offshore reference turbine</td>
</tr>
<tr>
<td>OWF</td>
<td>offshore wind farm</td>
</tr>
<tr>
<td>OWP</td>
<td>Offshore wind power</td>
</tr>
<tr>
<td>PSO</td>
<td>Particle swarm optimization</td>
</tr>
<tr>
<td>RNA</td>
<td>Rotor nacelle assembly</td>
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<tr>
<td>WINDOW</td>
<td>Wind farm integrated design and optimization workflow</td>
</tr>
<tr>
<td>XDSM</td>
<td>Extended design structure matrix</td>
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### Subscripts

<table>
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<th>Subscript</th>
<th>Description</th>
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<td>avg</td>
<td>average</td>
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<tr>
<td>D</td>
<td>degradation</td>
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<tr>
<td>dec</td>
<td>decommissioning</td>
</tr>
<tr>
<td>ew</td>
<td>east west</td>
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<tr>
<td>F</td>
<td>fatigue</td>
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<td>G</td>
<td>electrical infrastructure</td>
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<tr>
<td>in</td>
<td>investment</td>
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<tr>
<td>k</td>
<td>current iteration</td>
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<td>l</td>
<td>loan</td>
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<tr>
<td>m</td>
<td>mean</td>
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<tr>
<td>max</td>
<td>maximum</td>
</tr>
<tr>
<td>ns</td>
<td>north south</td>
</tr>
<tr>
<td>std</td>
<td>standard deviation</td>
</tr>
<tr>
<td>t</td>
<td>turbines</td>
</tr>
<tr>
<td>u</td>
<td>wind speed</td>
</tr>
<tr>
<td>( f )</td>
<td>farm</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>50</td>
<td>50% probability of exceedance</td>
</tr>
<tr>
<td>90</td>
<td>90% probability of exceedance</td>
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**Superscripts**

<table>
<thead>
<tr>
<th>( g )</th>
<th>best</th>
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<tbody>
<tr>
<td>( i )</td>
<td>particle index</td>
</tr>
<tr>
<td>( ref )</td>
<td>reference</td>
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</table>

**Symbols**

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<thead>
<tr>
<th>( a )</th>
<th>annuity factor</th>
<th>[-]</th>
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<tbody>
<tr>
<td>( C )</td>
<td>coal</td>
<td>[-]</td>
</tr>
<tr>
<td>( c )</td>
<td>scale factor</td>
<td>[m/s]</td>
</tr>
<tr>
<td>( D )</td>
<td>rotor diameter</td>
<td>[m]</td>
</tr>
<tr>
<td>( ep )</td>
<td>electricity price</td>
<td>[c€/kWh]</td>
</tr>
<tr>
<td>( f(U) )</td>
<td>probability of occurrence of the wind speeds</td>
<td>[-]</td>
</tr>
<tr>
<td>( i )</td>
<td>interest rate</td>
<td>[%]</td>
</tr>
<tr>
<td>( k )</td>
<td>shape factor</td>
<td>[-]</td>
</tr>
<tr>
<td>( N )</td>
<td>number of wind farms</td>
<td>[-]</td>
</tr>
<tr>
<td>( n )</td>
<td>energy conversion efficiency</td>
<td>[%]</td>
</tr>
<tr>
<td>( NG )</td>
<td>natural gas</td>
<td>[-]</td>
</tr>
<tr>
<td>( P )</td>
<td>other sources</td>
<td>[-]</td>
</tr>
<tr>
<td>( p )</td>
<td>penalty</td>
<td>[c€/kWh]</td>
</tr>
<tr>
<td>( P(U) )</td>
<td>power of the wind farm for every wind speed</td>
<td>[W]</td>
</tr>
<tr>
<td>( R )</td>
<td>rotor radius</td>
<td>[m]</td>
</tr>
<tr>
<td>( r )</td>
<td>real rate of interest</td>
<td>[%]</td>
</tr>
<tr>
<td>( T )</td>
<td>lifetime</td>
<td>[years]</td>
</tr>
<tr>
<td>( u )</td>
<td>uncertainty</td>
<td>[%]</td>
</tr>
<tr>
<td>( v )</td>
<td>inflation rate</td>
<td>[%]</td>
</tr>
<tr>
<td>( V1 )</td>
<td>cut in wind speed</td>
<td>[m/s]</td>
</tr>
<tr>
<td>( V2 )</td>
<td>cut out wind speed</td>
<td>[m/s]</td>
</tr>
<tr>
<td>( z )</td>
<td>value based on probability table</td>
<td>[-]</td>
</tr>
<tr>
<td>( \Delta_{tip} )</td>
<td>deflection</td>
<td>[m]</td>
</tr>
</tbody>
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1. Introduction

This chapter gives a background which forms the foundation of this thesis. Firstly, in Section 1.1 information is provided on growing interest in the offshore wind energy sector and its emerging challenges. Secondly, section 1.2 gives a brief description of wind farm optimization. Section 1.3 discusses the commonly used objective functions for optimization of an offshore wind farm (OWF). These three sections provide the necessary context for the research motivation in Section 1.4. This chapter ends with sections 1.5 and 1.6 where a description of the research approach and thesis outline is deliberated.

1.1. Offshore wind energy

The world is constantly looking for innovations in the field of energy to endure sustainability. Energy generating companies are always exploring new ways and means of providing clean energy in their quest to replace non-renewable sources [1]. Among renewable energy sources, wind power plays a major role around the globe. With an additional capacity of 52GW in 2017, the global installed capacity has risen to 539GW. [2].

In the field of wind power, onshore wind is an established technology which is proven to be cost effective. While it is cost effective, the visual impact and noise levels in the environment compels researchers to look at other viable options within wind power.

One such option is offshore wind power (OWP), which promises higher electricity generation with less noise and visual impact versus onshore wind power and hence has shown lots of promise as alternate technology, thus generating a significant interest in this area ever since the last decade. With the cost of generating OWP being relatively expensive compared to well entrenched existing technologies, extensive research is on at a rapid pace to make OWP generation a competitive energy source. [3].

A total of 4.3 GW of global OWP was added cumulatively in 2017. The overall installed capacity of OWP now is around 18.8 GW across 17 countries. It is predicted that global OWP will have around 34GW of installed capacity by 2020 [4].
On the European scale, UK has the highest contribution in the offshore wind energy market with around 36% of global installed capacity. UK is closely followed by Germany, which contributes to 28% of the market share [5]. It is anticipated that OWP will be instrumental in achieving the renewable energy target set by the European Union for the year 2020 [6].

Offshore wind has grown consistently and is moving from niche to conventional market stage since the last decade. The last few years has seen offshore wind energy steadily grow in Europe and expand in new markets around the world in Asia and USA [13]. However, offshore wind energy still faces a lot of challenges. Costs are very high for the installation of wind turbines, foundations, electricity cables and connecting wind farm to the grid [9]. For offshore wind energy to be competitive with other energy sources, major research is required in almost every discipline associated with it [10]. The current trend followed by the researchers is to carry out an optimization approach wherein all required disciplines are included.

1.2. Wind farm optimization – An introduction

Even though research was underway on planning and modelling of wind farms by 1970’s, the study of wake models based on momentum conservation and linearized wake assumptions in 1980 started the main groundwork for the study on wind farm design and optimization [14-20]. The main purpose of the research was to maximize power by minimizing wake effects in an array. This resulted in an increase of power percentage by 1.29%. The work by Mosetti is still considered to be a breakthrough in the field of wind farm optimization [21]. Significant improvements were made in both wind farm performance and numerical approach to solve the problem. The objective function used by Mosetti was to minimize cost of energy (COE) and a genetic algorithm was used to solve the optimization problem. The breakthrough by Mosetti led to researchers using similar objective functions for optimizing a wind farm [11]. The commonly identified objective functions were COE and annual energy production (AEP), which are further elaborated in the next sections.
1.3. Commonly used objective functions for optimization

1.3.1. AEP

AEP is one of the frequently used objectives for wind farm optimization [28,29]. Four metrics from the literature were found to be equivalent in definition to AEP: [22 – 26].

1. Maximize wind speed reaching to each turbine
2. Minimize wind speed deficit at each wind turbine
3. Optimize capacity factor of the wind farm
4. Maximize the efficiency of the wind farm

AEP is maximized for the wind farm and the following constraints are commonly adopted:

1. Minimum spacing between the turbines
2. The boundary of the wind farm

Wake effects are taken into consideration and Jensen wake model is commonly used to determine the wind speed deficit in a wind farm. The commonly used design variables are as follows:

1. Number of turbines
2. Cartesian coordinates of the turbines

The AEP can be calculated using equation 1.1:

$$AEP = T \int_{V_1}^{V_2} P(U) \cdot f(U) \, dU$$  \hspace{1cm} \text{Equation 1.1}

In which, $T$ represents the number of hours in a year, $P(U)$ is the power of the wind farm for every wind speed and $f(U)$ is the probability of occurrence of the wind speeds. $V_1$ is the cut in wind speed and $V_2$ is the cut-out wind speed.

The AEP calculated in the above equation is for a single turbine. For turbines in a wind farm, wind direction also plays a significant role and cannot be neglected. This is because the wind turbine wakes will reduce the total AEP of the wind farm due to a reduction of power of the wake affected wind turbines. Hence AEP for a wind farm is calculated using the equation 1.2 as proposed by [28,32]:

$$AEP_{Farm} = T \int_{0}^{360} \int_{0}^{cut \, out} P(U) \cdot f(U, \Theta) \, dU \, d\Theta$$  \hspace{1cm} \text{Equation 1.2}

In the above equation, $f(U, \Theta)$ refers to a bivariate probability density function. It represents the probability of occurrence of a wind condition characterized by wind speed from the range $0$ to $cut \, out$ wind speed in m/s and wind direction from $0 \leq \Theta \leq 360$ in degrees at the position of a specific turbine.

$AEP_{Farm}$ of a wind farm considers the conversion of kinetic energy in the wind into mechanical energy at the rotor of the wind turbine. The estimation of $AEP_{Farm}$ also includes net mechanical to electrical efficiency and availability factors. To make calculations easier, manufacturers usually provide electrical power curves which include all the possible electrical and mechanical losses [11].
The availability of the wind turbine refers to the fraction of time that the wind turbine is in operation mode and supplies electricity. The total availability of the wind turbines and wind farm are often assumed to be constant factors. This factor is usually assumed to be in the range of 95 to 98 % [32]. For accurate measurements, IEC 61400 – 26 availability standards for wind turbines can be referred [51].

1.3.2. COE

COE is defined as the cost of kWh of energy converted from the wind and denotes the price of energy at which wind farms will neither have profit nor loss over its lifetime i.e. it represents the breakeven price of energy. The general expression for COE is given as:

\[ COE = \frac{\text{Cost}_{\text{Farm}}}{\text{AEP}_{\text{Farm}}} \quad \text{Equation 1.3} \]

Lot of definitions were identified in literature differing in the way the total cost of the wind farm was defined. More than 33% of the works reviewed for the current study used Mosetti’s cost function which assumed the total cost of the wind farm to be a function of the number of wind turbines present in the farm [21]. The equation below represents Mosetti’s cost function:

\[ \text{Cost}_{\text{Mosetti}} = N\left(\frac{2}{3} + \frac{1}{3}e^{-0.00174N^2}\right) \quad \text{Equation 1.4} \]

Equation 1.4 is dimensionless and represents only a minor portion of the total costs of the wind farm. A dimension of \(\text{cost/ year}\) is assumed which is unreliable because, for farms with more than 35 turbines, the function value converges to 2/3N.

Similarly, two equivalent definitions related to the COE were identified from literature: Levelized cost of energy (LCOE) and Levelized Production Cost (LPC) which will be explained in detail in chapter 2.

1.4. Research motivation

It was stated that AEP and COE are the most common objective functions used in a wind farm optimization problem. However, while designing a wind farm (OWF in this research), AEP is not a good objective function as it principally considers energy obtained from the wind with the consideration of wake effects. Nevertheless, costs also play a major role in the design of an OWF and therefore, there needs to be a tradeoff between costs and energy produced. In general terms, energy needs to be maximized and cost needs to be minimized. Hence, considering only AEP as an objective function is not a reasonable choice. A more rational choice would be if AEP is considered as one of the modules for the optimization problem instead of an objective function. Thus, COE is a better objective function as it takes both energy and costs into consideration. COE gives an indication of the breakeven price of the OWF. Moreover, there might be other objective functions that might be of interest to the OWF designer apart from AEP and COE. For example, COE indicates the price at which costs are equal to the revenues, but it does not provide any detail on how much profit is obtained from the OWF which might be of interest to the OWF investors. Similarly, it has been observed that carbon is emitted during the installation, operation and maintenance phase of the OWF which needs to be controlled [8]. OWFs are also a threat to some marine animals and birds for which guidelines are provided by environmental regulations to minimize environmental impacts.
for the betterment of marine species [7]. Thus, while designing an OWF, multiple objectives need to be taken into consideration to arrive at a commercially and socially viable model. On the other hand, there might also be a possibility where designers might be interested in analyzing more than one objective for the same OWF at the same time. For example, one designer would want a tradeoff between COE and carbon emissions. Another designer might be attracted towards a tradeoff between AEP and noise levels as shown in figure 1.2. 

![Figure 1.2: Optimized tradeoff between AEP and noise generated [34,35]](image)

These tradeoffs can be attained by using a multi-objective approach. The advantage of a multi-objective approach is that investors can have an insight into optimized tradeoffs between different objectives [12].

Even though optimization of wind farms is a common area of research, examination of different tradeoffs has been very few and far between [11]. There has been a study on the tradeoff between the capacity factor of the wind farm and the power density within the wind farm area [31]. Researchers have also analyzed the conflict between minimizing the spacing of the wind turbines and maximizing the energy production via reducing the wake effects. The work done by [32] optimized the AEP where the constraints were considered as the second objective function. In another study, AEP was maximized, and the sum of the number of turbines and wind farm area was treated as a second objective function [33]. The noise effects and AEP were optimized in [34,35]. Some researchers used three objective functions for optimization [36].

In short, there are only a few frameworks available as far as the author is aware which give a good tradeoff between different objectives and provide descriptive insight to wind farm designers. Therefore, the current research effort will develop a method to model different objective functions for optimization of an OWF. Tradeoffs between different objectives and their influence on the optimal design of an OWF will be investigated as well. In the following sections, goals and the report structure of this thesis are briefly explained.
1.5. **Objective and approach**

The objective of the thesis study is outlined as:

*“To have an overview of different objective functions and understand their impact on the optimal design of an OWF”*

Eventually, the result of the project is to have a technique that provides flexibility to OWF designers in using different objective functions and estimating tradeoffs between different objectives for optimal design of an OWF. The points below encapsulate the different drafts required to achieve the goal mentioned above:

1. A list of different objective functions for the OWF optimization and selection of relevant ones.
2. Formalize the relevant objective function in metrics suitable for optimization.
3. A suitable method to deal with multiple objectives.
4. Devise case studies to reflect the new approach towards solving the OWF optimization problem.

Figure 1.3 shows the aspects which will be worked within this thesis. The boxes which are colored are the ones which will be worked with and the boxes with no color are adopted from literature. This research work focuses mainly on the objective functions, design variables and constraints. A desirable optimization algorithm is chosen based on the literature study. OPENMDAO framework will be adopted in this thesis. This framework will be explained more conclusively in chapter 3.

1.6. **Thesis outline**

The outline of the thesis is shown in figure 1.4. Chapter 1 provides an introduction with some background information on the OWF optimization problem and the setup of the research is also conferred. Chapter 2 introduces to the reader the inventory of different objective functions. Applicable ones are selected and are formulated in a metric and a description is provided on how these selected objective functions are modelled. Chapter 3 discusses the OpenMDAO framework and converses the
optimization approach used in this research where an explanation will be provided on the use of design variables, constraints and objective function. Chapter 4 and 5 present the different case studies implemented in the project. Finally, chapter 6 includes the conclusion of the thesis work and appends the recommendations for future research.

Figure 1.4: Overview of thesis outline and the numbers indicate respective chapters
2. **Inventory of different objective functions, their selection and modelling**

2.1. **Introduction**

The list of different objective functions in a metric form will be discussed in this section. Relevant ones are then selected for the optimization problem and arguments are provided for the choices made. Modelling approach will also be discussed for the selected objective functions.

2.2. **Objective functions**

2.2.1. **Levelized cost of energy/Levelized production cost**

It was mentioned in section 1.3 that COE was one of the commonly used objective function for wind farm optimization. It was also stated that two equivalent definitions for COE were found in literature:

- Levelized cost of energy (LCOE)
- Levelized Production Cost (LPC)

Both LCOE and LPC are very similar in definitions with subtle variations. However, as far as wind farm optimization is concerned both the definitions have been used interchangeably as per the author’s knowledge. Both provide a good estimate as both costs and AEP are taken into consideration.

\[ LCOE = \frac{C_{in}}{aAEP} + \frac{C_{om}}{AEP} + \frac{C_{dec}(1+r)^{-T}}{aAEP} \]  \hspace{1cm} \text{Equation 2.1}

In the above equation, \( AEP \) is the annual energy production, \( C_{in} \) represents the investment costs of the wind farm, \( C_{om} \) operation and maintenance cost, \( C_{dec} \) refers to the decommissioning cost of the wind farm and \( T \) denotes lifetime of the wind farm during operation. The annuity factor is defined as a financial value, when multiplied by a periodic amount calculates the present or future value of that amount. Annuity factor is denoted by \( a \) in the above expression:

\[ a = \frac{1}{r} \left( 1 - \left( \frac{1}{1+r} \right)^T \right) \]  \hspace{1cm} \text{Equation 2.2}

Where \( r \) is the real rate of interest and is calculated by using equation 2.3:
\[ 1 + r = \frac{1+i}{1+v} \quad \text{Equation 2.3} \]

In the above equation, \( i \) is the interest rate and \( v \) is the inflation rate.

There are other LCOE models defined in literature as well [39,41]. A detailed review of economics of wind energy and concepts of COE and LCOE can be found in [37,38,40].

As stated in chapter 1, OpenMDAO framework will be adopted in this research. LCOE is used in WINDOW, which is built within OpenMDAO. The framework and WINDOW will be explained in detail in chapter 3. This research uses LCOE as one of the references for the selection of the objective functions.

2.2.2. Net present value

Net present value (NPV) represents the value and worth of a stream of payments in a single number taking into consideration that the same nominal payment made at different times will have different worth [42]. NPV of a project examines the cost (cash outflows) and revenues (cash inflows) together and it consists of many different cost and revenue streams. In general terms, the form of different streams should be known for the correct use of discount rate for NPV analysis. The internal rate of return (IRR) of a payment stream is defined as a discount rate for which NPV of the payment stream is zero.

A positive NPV represents a good investment and a negative NPV indicates that the income is lower than the costs. The formulation of NPV is given by equation 2.4:

\[ NPV = (R - OM) * a - C_{in} - \frac{Dec}{(1+r)^t} \quad \text{Equation 2.4} \]

Revenue is defined using equation 2.5:

\[ R = ep * AEP \quad \text{Equation 2.5} \]

In equation 2.4 \( OM \) is the operation and maintenance cost, \( a \) denotes the annuity factor, \( C_{in} \) represents the total investment cost, \( Dec \) signifies the decommissioning cost, \( T \) is the number of years the wind farm is in operation and \( r \) denotes the real rate of interest. Comparing equations 2.1 and 2.4, the formulation of LCOE is very similar to NPV in the sense that both require almost the same economic parameters. The only addition is the electricity price in the NPV calculation as shown in equation 2.4. If LCOE is substituted instead of the electricity price in equation 2.4, then the value of NPV will be zero.

2.2.3. Risk management

The estimation of AEP over the entire project life cycle is one of the most important factors to determine the profitability of wind power project. AEP is determined using equation 1.2.

From equation 1.2, it is observed that there is a certain uncertainty involved while specifying the AEP for the wind farm. The first uncertainty lies in the fact that the wind speed is uncertain as it is unpredictable in nature. Secondly, the availability of wind turbines plays a crucial role. For instance, there might be a situation where the wind turbine has a certain issue and it is being worked upon. In this case, the wind turbine does not operate, and AEP generated for the wind farm will decrease. To finance a wind power project, it would be favorable if the investors have an accurate knowledge on
these uncertainties to mitigate the errors and increase the reliability of the project [43] Hence, there is a need to calculate this uncertainty. Usually, project investors are interested in the estimation of the P90 value of the AEP. This value refers to a 90% probability of the AEP being attained or exceeded. P90 (AEP) can be calculated using the equation 2.6:

\[ P_{90} = P_{50} (1 - z \times u) \]  

Equation 2.6

P50 refers to the \( AEP_{\text{Farm}} \) calculated in equation 1.2. It is also referred to as central energy production estimate in normal Gaussian distribution. This represents an energy value with 50% probability of being exceeded as shown in figure 2.1. The \( z \) value is dependent on the desired probability and appendix A1 shows the \( z \) values for various probability levels. It is seen that for higher values of uncertainty (represented by \( u \) in the equation), higher will be the difference between P50 and the other levels of probability of exceedance.

\[ P_{90} = P_{50} (1 - z \times u) \]

P50 refers to the \( AEP_{\text{Farm}} \) calculated in equation 1.2. It is also referred to as central energy production estimate in normal Gaussian distribution. This represents an energy value with 50% probability of being exceeded as shown in figure 2.1. The \( z \) value is dependent on the desired probability and appendix A1 shows the \( z \) values for various probability levels. It is seen that for higher values of uncertainty (represented by \( u \) in the equation), higher will be the difference between P50 and the other levels of probability of exceedance.

\[ P_{90} = P_{50} (1 - z \times u) \]

2.2.4. Reducing carbon emissions

Wind turbines do not emit any carbon during their operation. However, they can release greenhouse gases at the rate of 72% and 90% of cumulative emissions during their lifetime especially in the manufacturing, installation and operation and maintenance stage. [44,45] It was estimated that a 1.65 MW turbine emitted 394 t of CO\(_2\) during its lifetime [46]. Hence, this number will be even higher for a wind farm. For this reason, there should be a system to facilitate wind farm design to reduce these emissions. The main goal of this objective is to reduce the carbon emissions by decreasing the carbon payback period. The equation for carbon payback period is shown in 2.7:

\[ CPP = \frac{CO_2 \text{emissions} \times \text{lifespans}}{CO_2 \text{emissions avoided}} \]  

Equation 2.7

\[ CPP = \frac{CO_2 \text{emissions} \times \text{lifespans}}{CO_2 \text{emissions avoided}} \]

\[ CO_2 \text{emissions avoided} \] can be calculated by:

\[ CO_2 \text{emissions avoided} = AEP \times CES \times \text{lifespans} \]  

Equation 2.8

\[ CO_2 \text{emissions avoided} = AEP \times CES \times \text{lifespans} \]}
CES refers to carbon emissions signature. Equation 2.9 is used to calculate CES [47]

\[
CES = \left( n \times \frac{112 \times %C + 49 \times %NG + 66 \times %P}{100} \right)
\]

Equation 2.9

- \(n\) = energy conversion efficiency
- \(%C\) = percentage of coal contribution to the electrical grid
- \(%NG\) = percentage of natural gas contribution to the electrical grid
- \(%P\) = percentage of (other) sources contribution to the electrical grid

CES is specific to an electrical energy grid and can be used by manufacturers or anyone using energy which has been generated by fossil fuels [46]. An electrical grid has many primary sources of energy such as coal, petroleum, biofuel, hydro, solar, wind, geothermal, natural gas, wave and tidal. Each of these sources can be expressed as a fraction and a grid which provides electrical energy is made up of the sum of fractions of the primary energy sources and multiplied by the conversion efficiency. The coefficients 112, 66 and 49 used in equation 2.9 are the kilograms of carbon emitted per gigajoule of heat released in each case. Therefore, for any system that uses electrical energy from the grid, carbon emissions can be calculated by multiplying energy consumed by CES.

### 2.2.5. Financial balance

Financial balance (FB) and profit are equivalent in definition. This model includes time variation in electricity prices, taxes, payments O&M costs of wind farm etc. [48]. Equation 2.10 is generally used to calculate FB:

\[
FB = WP - C_D - C_{O&M} - (C_F + C_G) \left( 1 + \frac{i-v}{nT} \right)^{T-nT}
\]

Equation 2.10

In the above equation, \(WP\) represents the revenue got from the wind farm. \(C_D\) is the accumulated cost of degradation. \(C_{O&M}\) is the operation and maintenance cost. \(C_F\) and \(C_G\) represent the cost of foundations and electrical infrastructure. The interest and inflation rate is represented by \(i\) and \(v\) respectively. The number of times the loan must be paid is represented by \(nT\). \(T\) is the number of years the wind farm will be in operation. For simplicity, the operating costs can be interpreted as referring to year zero with an assumption that the development of these costs over time follows the inflation rate and that the inflation rate can be represented as the discounting factor for transforming these running costs into NPV.

### 2.3. Selection of objective functions

This section will discuss the objective functions that are selected for further study in the research. The objective functions in consideration are as follows:

- NPV
- Risk Management
- Carbon Emissions
- Financial Balance
NPV

NPV is chosen as one of the objective functions in this thesis as it estimates the total profit of an OWF. It is also suitable to evaluate exclusive projects because it can distinguish the size of different OWFs [51]. The LCOE is used as a reference for selecting NPV as one of the objectives. It was mentioned earlier that COE(LCOE) is one of the commonly used objectives used for wind farm optimization. From equation 2.4, revenue plays an important role in the estimation of NPV. To calculate the revenue, the electricity price plays a significant role. The electricity price is in some ways determined by estimating the LCOE. The LCOE represents the breakeven price of energy and to attain any profit, the electricity price should be higher than the estimated LCOE. WINDOW uses LCOE as the objective function as mentioned before and the electricity price in the NPV function will be assigned based on the estimation of the LCOE.

Risk management

Risk Management is chosen as one of the objective functions in this thesis as well. As stated earlier, it will be motivating to see the AEP values based on different values of probability of exceedance. AEP is used as a reference for selecting risk management as an objective function. For determining uncertainty, AEP needs to be calculated. The application of this objective function will benefit the investor in estimating the profit obtained from OWF for different levels of probability of exceedance. The application of this objective function will be explained more in detail in section 2.4.2.

Carbon emissions

The modelling of this objective involves calculating two important factors: CO₂ emissions emitted by the wind farm and the estimation of AEP. It can be clearly seen that the AEP needs to be maximized to reduce the carbon payback period. However, AEP is already being maximized in the NPV objective function and is also used in the risk management objective. For this research, it is a redundant objective as it would give an extra weight to AEP. Secondly, modelling the CO₂ emissions generated during the installation, O&M and decommissioning is a complex task. It is out of scope in this research. Hence, this objective function is not chosen for further study.

Financial balance

The definition and representation of FB are identical to that of NPV. FB also helps in determining the profit of OWF, like NPV. Degradation cost is the additional term in FB equation. This cost is estimated from the fatigue of the wind turbines and support structure. The calculation of fatigue for the degradation cost requires complex modelling and simulations. With the limited resources available for the project and the limited difference with respect to NPV, FB is not chosen as one of the objectives for further study.

To summarize, the table below shows the inventory of objective functions. A tick means that the objective function is chosen for further study and cross means vice versa.
### Table 2.1: Selection of objective functions for further research

<table>
<thead>
<tr>
<th>Objective function</th>
<th>Selection for further study</th>
</tr>
</thead>
<tbody>
<tr>
<td>NPV</td>
<td>✔</td>
</tr>
<tr>
<td>Risk Management</td>
<td>✔</td>
</tr>
<tr>
<td>Carbon Emissions</td>
<td>✗</td>
</tr>
<tr>
<td>Financial Balance</td>
<td>✗</td>
</tr>
</tbody>
</table>

### 2.4. Modelling of objective functions

#### 2.4.1. NPV

To estimate the NPV, electricity price plays a significant role. In industrial standards, electricity prices reflect the cost to build, finance, maintain and operate OWFs and electricity grid. Independent Service Operator (ISO) manages the auctions that determine the day ahead and real-time electricity prices in the market. The prices fluctuate mainly due to the demand and bidding strategies. For OWP, predicting electricity price is not straightforward as the power output is variable and hence there is a certain level of uncertainty for wind generators and power market. For example, if the wind generator cannot supply the bidding amount of power, it must pay a penalty for the unsupplied amount of electricity. The ISO must balance the system from the reserves which charge more, so the electricity price goes up. On the other hand, if the wind generator produces more than the bidding amount, the excess power may not be sold to the grid as there might be voltage fluctuations in the grid. In this case, the electricity price goes down [52]. Hence, it is important to assess the value of electricity price based on the power supply. In this thesis, the price of electricity is modelled in the following ways:

- Constant electricity price
- Electricity price based on wind variability
- Electricity price based on wind power predictability

#### Constant electricity price

In this scenario, the electricity price is assumed to be a constant for the OWF. The revenue is calculated based on this constant electricity price and NPV is estimated. As mentioned earlier, LCOE determines the breakeven price of the OWF. Therefore, in general, LCOE is first estimated for the OWF and it is used as the base (lower limit) for the determination of the electricity price. For example, if the LCOE is $9 \text{ €/kWh}$ for an OWF, this will be the minimum value for the electricity price for determination of NPV. This value is then substituted in equation 2.5 and revenue for the OWF is estimated. NPV is calculated using equation 2.4.

#### Electricity price based on wind variability

As mentioned earlier, the electricity price varies based on the power supply. Power produced by an OWF is directly dependent on the wind speed of that site. Therefore, if there are many OWFs in a
region where wind speed is high, the electricity price will be reduced. Similarly, in a region where there are many OWFs and the wind speed is low, there will be an increase in the electricity price. [49].

To make electricity price a function of the varying wind speed, following technique is adopted. It is known that at low wind speeds the electricity price is high and vice versa. Therefore, in this modelling technique, a range of realistic electricity prices is assumed and the revenue and NPV is calculated for this range of prices. Let’s consider an OWF’s power curve as shown in the figure 2.2:

![Figure 2.2: Power curve of a virtual OWF](image)

As shown in figure 2.2, the cut in wind speed is 3.5 m/s and the rated wind speed is 14m/s for the OWF. The region between these two wind speeds is referred to as partial load region where the power of the wind farm is proportional to the cube of the wind speed. The cut-out wind speed is 25 m/s and the region between the rated wind speed and cut out wind speed is known as the full load region. In this region, the power output from the OWF is a constant i.e. it delivers rated power. As mentioned earlier, LCOE is used as a basis to determine the electricity price. Let’s say the LCOE for this OWF is 9 \( \frac{\text{euro}}{\text{kWh}} \). Therefore, the assumed electricity price for the respective wind speeds is tabulated in table 2.2.

Using the data from table 2.2, the revenue and NPV can be calculated for the list of electricity prices. It should be noted that the electricity price in the full load region is the same as the OWF delivers rated power i.e. constant power in this region. The revenue is calculated with the sum over the wind speeds and wind directions. To represent the above words in an equation, the total revenue can be estimated using equation 2.11:

\[
R = \sum_{3.5}^{25} ep(U) \cdot P(U, \Theta) \cdot f(U, \Theta) \, dU \, d\Theta \quad \text{Equation 2.11}
\]

The NPV is calculated again using equation 2.4.
It should be noted that the values of wind speed and electricity price used here are just an example for the reader to understand the concepts. Different values will be used in chapter 4 while implementing the case study.

Table 2.2: List of wind speeds and the corresponding electricity price

<table>
<thead>
<tr>
<th>Wind speed (m/s)</th>
<th>Electricity price (€/kWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.5</td>
<td>14.75</td>
</tr>
<tr>
<td>4</td>
<td>14.5</td>
</tr>
<tr>
<td>5</td>
<td>14.5</td>
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<td>6</td>
<td>13.5</td>
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<td>7</td>
<td>13.5</td>
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<td>8</td>
<td>12.5</td>
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<td>9</td>
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<td>10</td>
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<td>24</td>
<td>9.5</td>
</tr>
<tr>
<td>25</td>
<td>9.5</td>
</tr>
</tbody>
</table>

Electricity price based on wind power predictability

Predicting the power output from an OWF is very essential. Developers/investors value OWFs that deliver constant power output. Delivery of constant power output leads to fewer penalties being provided to OWF. It is observed that a slight change in wind direction leads to fluctuations in the wind farm power output [50]. Figure 2.3 shows the fluctuation of OWF power output for a range of wind speeds and all wind directions. The solid lines represent the fluctuations in power output and the dashed lines represent constant power output which the investors/developers are looking for.
To tackle this case, a penalty is provided for the fluctuation in power output for different wind directions. The power output is obtained for all possible wind directions and the mean and standard deviation of the power output is determined. The concept of percentage difference is then used to estimate the new electricity price. The percentage difference is defined as the difference between two values divided by the average of two values and is shown as a percentage. Hence, the two values, in this case are the total power output and its standard deviation. The percentage difference is estimated for these two values and a penalty is given for this estimated difference and finally, the remodeled electricity price is calculated using equation 2.12:

$$ep_{new} = ep - p \cdot \frac{|mp - std|}{\sqrt{mp \cdot std}}$$  \hspace{1cm} \text{Equation 2.12}$$

Where $ep_{new}$ is the remodeled electricity price, $p$ is the penalty provided, $mp$ is the total power and $std$ is the standard deviation of the power output in all directions. NPV is then calculated using equation 2.4 using this remodeled electricity price.

### 2.4.2. Risk management

The uncertainty factor plays a significant role in estimating P90(AEP). It is seen that higher the value of uncertainty, lower the value of P90. In industrial standards, the uncertainty is assumed to be around 10 to 15% [43]. In this thesis, the uncertainty value is modelled. To estimate this value, the following approach is adopted. 100 values of annual average wind speeds are taken into consideration using a normal distribution. The Weibull parameters are then estimated for these annual average wind speeds and AEP is computed. The NPV is then calculated and the mean and standard deviation of NPV is estimated. The focus is on minimizing the standard deviation of NPV and the uncertainty is minimized.
2.5. Conclusion

This chapter helped to answer the first two sub-goals of the thesis:

- A list of different objective functions for the research and selection of relevant ones.
- Formalize the relevant objective function in metrics suitable for optimization.

The selected objective functions are:

1. Net Present Value (NPV)
2. Risk Management

Electricity price is the main parameter in focus while estimating NPV and is modelled for three different cases:

1. Constant Electricity price
2. Electricity price based on wind variability
3. Electricity price based on wind power predictability

Similarly, to calculate the uncertainty the standard deviation of NPV is estimated and needs to be minimized to minimize uncertainty. For this purpose, a certain optimization procedure needs to be carried out. Chapter 3 will discuss the optimization framework and approach in detail. A general description will also be provided on the objective function, design variables and constraints.
3. **Background on MDAO and optimization approach**

3.1. Introduction

This chapter will provide a brief description of the framework that is used in this thesis. An introduction and development of the framework will be provided followed by its application in OWF design. Details will also be listed on the different analysis blocks used in the framework which will be further adopted in this research. Finally, this section will also present the readers with the optimization approach that will be implemented while carrying out the different case studies.

3.2. Multi-disciplinary analysis and optimization

Multi-disciplinary analysis and optimization (MDAO) is a methodology that allows the analysis and optimization of a system by explicit and implicit consideration of important interactions between all the disciplines [54]. It is used to couple multiple computational tools to a driver for solving a problem that requires the estimation of the overall performance of the system. It is a system’s engineering approach in the sense that MDAO helps in the design and management of a complex engineering system. MDAO exhibits the following characteristics [54]:

- Create workflows encircling analysis tools and processes from different disciplines.
- Execute these workflows to explore the performance, cost and risk of many different design alternatives.
- Perform sensitivity analyses to find the most important variables and discover the key relationships.
- Run optimization algorithms to find the best optimum design and evaluate the robustness and reliability of their designs.
Figure 3.1 shows a simple MDAO framework. The coupled tools are referred to as analysis blocks as shown in the figure. These blocks are repeatedly called by the driver. The driver regulates the flow of inputs and outputs for the optimization purpose through the analysis blocks. The driver in this diagram refers to any algorithm that calls the analysis block for a specific purpose. This specific purpose is referred to as the use case. Examples of drivers include optimization algorithms, sensitivity analysis, design certification etc. with multiple use cases. The workflow helps in the coupling of the disciplines in the form of input and output interactions between the models. This is depicted through the arrows in the figure.

However, there are challenges while implementing MDAO. One of the main challenges is the need for reconfigurability and reusability of analysis techniques. This means that a possibility is missed out on improving the performance of MDAO workflow by not exploring the coupling of other tools with different levels of fidelity. To counter this challenge, OpenMDAO framework was developed. The illustration of this framework will be given in the next section.

3.2.1. Open – MDAO framework

OpenMDAO is developed by NASA Glenn Research Centre and is an open source framework. It provides users with a combination of tools and interface that helps in the setup for complex engineering design, analysis and optimization problems as shown in figure 3.2. MDAO involves optimization around complex system models where they are made up of many analysis blocks and are linked with each other. OpenMDAO is an open source high - performance computing platform for analysis of a system and multidisciplinary optimization. It simplifies the implementation of tools and methods for multidisciplinary design, analysis and optimization. It has been designed to handle variable problem formulations and promote the reuse of the model. OpenMDAO provides tools and interface that helps in solving complex problems [56]. Special emphasis has been placed to ensure robustness, flexibility and reconfigurability. This framework is written in Python programming language and leverages the object-oriented format to decompose MDAO algorithms. The OpenMDAO library provides access to
many gradient and non-gradient optimizers and the performance of various optimizers can be readily compared in this framework. OpenMDAO was initially famous for solving problems related to aircraft design, however, it has gained popularity in other disciplines as well [55].

![Figure 3.2: Different domains that OpenMDAO works on [56]](image)

The OpenMDAO framework is managed through a website, http://OpenMDAO.org. This website hosts discussion forums and provides downloads for all official OpenMDAO release versions. It is also home to all the documentation concerning the OpenMDAO. The OpenMDAO team is currently working on problems involving cubesat design, mission planning, wind turbine design, wind farm layout, boundary layer ingestion for aircraft etc [56].

### 3.2.2. Application of OpenMDAO in OWF

OWF design is a multidisciplinary problem as it has a lot of disciplines like the support structure, cables, aerodynamics etc. associated with it. Considering the requirements of an OWF optimization problem and framework required, the wind energy research group at the faculty of aerospace engineering, TU Delft has been designing software, Windfarm Integrated Design and Optimization Workflow (WINDOW) for multi-disciplinary design analysis and optimization of OWF [30]. The feature of this software is the ability of the workflow to suit the desired case study which makes this software distinctive.
Figure 3.3: XDSM diagram of the WINDOW workflow for an OWF optimization problem

The interactions between the different disciplines can be investigated using Extended Design Structure Matrix (XDSM) [57]. The XDSM diagram represents a particular workflow for a given use case. In figure 3.3, the spacing of the turbines is optimized for a minimum LCOE. A more descriptive information of WINDOW and the respective workflows can be found in [30].

The first row in the XDSM diagram includes all the fixed parameters for the respective disciplines while the given MDAO is performed. The green rectangle boxes represent the different disciplines comprising of the OWF design in WINDOW.

The second row comprises of the design variables and the spacing of the turbines. The spacing of the turbines here refers to the crosswind spacing and downwind spacing and is controlled by the MDAO driver. The driver here is represented by the blue rounded rectangle. The vertical lines connected to each discipline represent the flow of inputs and the horizontal lines represent the flow of output.

The coordinates of the turbine and substation are set by the layout discipline. This is based on the spacing provided by the driver. The water depth at each turbine and substation is computed using the bathymetry data of the site.

The wake effect is calculated by the wake aerodynamics discipline. The wind is sampled into discrete speed and directions. For every sample, the power and the thrust coefficient is interpolated and computed and is used to calculate the wind speed deficit in each turbine. The wake merge model then calculates the overall wind speed deficit and the overall power output from each turbine for each wind sampling is found out. The power output is then integrated with a Weibull distribution and wind rose of the site condition and the AEP of the OWF is estimated.

The support structure is designed by using the wake - induced turbulence, water depth at each turbine, the mass of rotor and nacelle assembly (RNA), yaw radius, rated wind speed and maximum rotor thrust.
The cabling discipline determines the layout of the cables, length and cost based on Esau – Williams heuristic algorithm [58]. This algorithm uses the coordinates of the turbines and substations as inputs. A cable type is also accepted from the database for a given capacity of the turbine.

The cost model sums the cost of RNA, support structure and cabling for all turbines. The operation and maintenance cost is scaled linearly with the AEP. The capital expenditure, operating expenditure and decommissioning cost is then calculated. Lastly, LCOE is calculated for the OWF which is sent back to the optimizer driver which generates a new set of design variables.

3.2.3. Application of WINDOW in this research

The focus of this research is on objective functions, constraints, design variables and optimization approach as mentioned in chapter 1. The analysis blocks are all adopted from WINDOW and are not changed in this study.

3.3. Optimization approach

The formulation of an optimization problem involves the selection of design variables, constraints and objective function, wherein the consideration of one parameter may be influenced by the other. This section will brief on the above-mentioned parameters.

3.3.1. Design variables

The design variables are varied during the optimization process. A design problem involves the use of many parameters, of which some are highly sensitive to the proper working of the design. These parameters are referred to as design variables. Other parameters which are not so important usually remain fixed or vary in accordance to the design variables and these parameters are referred to as fixed variables. There is no rigid guideline to choose a list of parameters which may affect the problem, as one parameter may affect the cost of the design while maybe totally insensitive with maximizing the life of the product [59]. Hence, the choice of design variables in the optimization problem largely depends on the user. However, it should be noted that the efficiency and speed of the optimization algorithms to some extent depend on the number of chosen design variables. Thus, by effectively choosing the required design variables, the accuracy and efficacy of the optimization problem can be increased. Hence, it is generally recommended to choose as few design variables as possible.

3.3.2. Constraints

Constraint is a condition of an optimization problem that the solution must satisfy. Constraints are generally there to limit the design space, hence to forbid the search to step through an area of the design space that is unfeasible. They determine the feasible region in the design space. They are the functions that describe the relationships among the design variables and define the allowable values for the design variables.
The constraints might represent some functional relationships between the design variables and other fixed variables satisfying a physical phenomenon and resource limitations. The nature and number of constraints depend on the user again and there is no unique way to formulate constraints in an optimization problem. It is not necessary to have a mathematical formulation of a constraint, however an algorithm or a mechanism to calculate the constraint is mandatory. There are two types of constraints that are commonly used in optimization problems:

- Equality constraints
- Inequality constraints

Inequality constraints state that the functional relationship between the design variables is generally lesser or greater than a resource value \([59]\]. On the other hand, equality constraints state that the functional relationships should exactly equal the resource value defined in the optimization problem. Generally, equality constraints are difficult to handle and thus need to be avoided wherever possible. There are certain optimization algorithms which are specifically designed to handle this type of constraints. However, in most cases, equality constraints are converted to inequality constraints as it allows a smoother operation of the optimization algorithm. Thus, the thumb rule is to keep the number of equality constraints as low as possible.

### 3.3.3. Objective functions

The previous chapter selected the objective functions for the optimization problem. This section will illustrate the use of the objective functions in general for the optimization problem. Firstly, the objective function can be of two types. Either the objective function must be maximized like the NPV objective function or minimized like the COE objective function. However, most of the optimization algorithms comply only with minimization problems. Nevertheless, tinkering with the objective function, a maximizing problem can be converted to a minimization problem. This is made possible by multiplying the objective function with \(-1\).

The most common optimization techniques involve the use of a single objective function as mentioned in chapter 1. However, a conflict between two or more objectives enhances the need for a multi-objective approach. For example, from the selected objective functions, an investor would want to maximize the NPV and at the same time minimize the uncertainty. To satisfy both demands, a multi-objective optimization can be used. The objective functions can be represented as a vector function and its optimization will lead to a non-unique solution to the problem. Pareto technique is most commonly used technique for multi-objective optimization nowadays \([60]\). The pareto technique states the following:

- An objective function \(x_1\) is said to dominate another objective function \(x_2\) if the solution of \(x_1\) is no worse than solution of \(x_2\) in all cases.
- The design variable \(y_1\) dominates \(y_2\) if \(f(y_1)\) dominates \(f(y_2)\) where \(f(y_1)\) and \(f(y_2)\) are the objective functions respectively.
- All the non-dominated solutions are the optimal solutions of the given problem. This means that these solutions are not dominated by any other solutions. The set of these solutions is called as the Pareto set and its image in objective space is named pareto front depicted in figure 3.4.
The optimization solver then searches for all non-dominated solutions that agree to the tradeoffs between the concerned objectives. Typically, an ideal point is one which corresponds to the minimum point of all the objectives. This point is called as a utopia point and is generally not a real or a feasible point.

**Type of pareto fronts:**

The computation of the pareto front is not an easy task. Non-continuous design space, high dimensionality and clustered solutions are some of the issues that can make the problem very complex. Local pareto frontiers can sometimes cause bad convergence of the multi-objective optimization approaches. There are two main types of pareto fronts that arise while solving a multi-objective problem:

- **Convex pareto front**

  This type of front is of most interest to the decision makers and is displayed in figure 3.5. The decision makers can negotiate, fighting for their own objective and they can easily agree for a tradeoff point which is much better than the linear combination of the original objectives. For example, if a decision maker gives up a percentage of the target, say 40%, another decision maker may have an improvement of more than 40% on his/her personal target.
Figure 3.5: Convex pareto front [61]

- Non-convex pareto front

Figure 3.6 is an example of a non–convex front. This front is exactly the opposite to the previous case. The negotiation between the decision makers is hard. The decision maker should give up more than 40% of his/her goal to give a minimum 40% advantage to the other decision maker. The final solution is not based on a democratic negotiation but rather influenced by the decision maker.

Figure 3.6: Non–Convex pareto front [61]
3.4. Conclusion

This chapter briefed on the OpenMDAO technique, its implementation and helped in answering the third sub goal of the thesis:

- **A suitable method to deal with multiple objectives.**

Further research on the objective functions is carried out in the following ways:

- Single – objective Optimization
- Multi – objective Optimization

Maximizing NPV is the objective function used in the single – objective optimization case study which is presented in chapter 4. The NPV is maximized for three different models of electricity price for two different set of design variables. Chapter 5 discusses the results of multi – objective optimization. The two objective functions that are used are maximizing the mean of NPV \((NPV_{\text{mean}})\) and minimizing the standard deviation of NPV \((NPV_{\text{std}})\). Both these objectives are conflicting with each other and a tradeoff is the best possible result for the user, based on the requirement.

The optimization algorithm for both the single – objective and the multi – objective case study will be discussed briefly in chapter 4 and 5 respectively. The design variables and constraints will also be mentioned for each case in the respective chapters.
4. Single – objective optimization

4.1. Introduction

This chapter will focus on maximizing the NPV for three models of electricity price with two different set of design variables. The structure of this chapter is as follows:

First, a brief description will be provided on the choice of design variables for the optimization problem. The scope and the choices made for evaluating the case studies are shown next. Further, the reader will be presented with different sub - models of the electricity price that will be treated for sensitivity studies. The constraints and the optimization algorithm to evaluate the respective cases will be described as well. Finally, the last section of this chapter will evaluate the results obtained for both the studies.

4.2. Choice of design variables

The rated power, rotor diameter and hub height of offshore wind turbines are increasing at a rapid rate since the initial installations. The average rated power of turbines installed in 2017 is 5MW offshore in UK and Germany [62] and is on a constant rise. Hence, the current trend indicates that the offshore wind industry prefers larger wind turbines.

For an OWF, the number, location and model of wind turbines must be optimized to obtain the best possible design for an OWF. The location of turbines has a robust impact on the overall efficiency of the wind farm. The turbine’s energy production is directly proportional to its power curve and the wind resource specifically from the wind farm area. However, installing turbines close to each other causes shadowing effects which leads to reduced power production and hence leads to lower wind farm efficiencies.

Moreover, different turbine models have different market prices. Therefore, it is a necessity to assess the influence of the number of turbines and the model of turbines in the capital costs. Support structures also need to be evaluated since the turbine model, water depth and soil properties play a significant role. The cost of the support structure is determined by the water depth and is dominated by the steel price and the structure design [12]. Hence, all these factors need to be taken into consideration for designing an optimal layout for an OWF.

In this thesis, only the wind turbines and its parameters are taken into consideration for the role of design variables. As specified earlier, the turbine models play a crucial role in the energy production and costs. Similarly, the number of turbines and location coordinates of the turbines play a crucial role in determining the optimal layout.

Therefore, in this chapter, NPV is optimized for two independent sets of design variables to understand the individual effect of these parameters on the objective function. These are:

1. Rotor diameter
2. Number of turbines

In the first case, the number of turbines and their position are fixed, and the rotor diameter is varied. In the second case, the rotor diameter is fixed, and the number and position of turbines are design variables. For convenience, the first case will be referred to as rotor diameter optimization and the second case will be referred to as layout optimization from now. The fixed variables will be mentioned in section 4.3.
4.3. Scope and choices made

This section will discuss the scope and choices made for the respective case studies.

4.3.1. Rotor diameter optimization

The scope of this case study lies in the juxtaposition of the RNA model and OWF optimization. A complete optimization of both these domains together is a cumbersome procedure and is computationally very expensive. Therefore, a case study with realistic assumptions is carried out. At the RNA level, the blade design from NREL 5MW Reference Turbine (N5RT) is scaled to make the rotor $C_{p,\text{max}}$ and $C_{t,\text{max}}$ constant. The values for the design tip speed ratio, shaft tilt angle, gearbox ratio and drive – train efficiency are adopted from design iteration 2 performed in [63]. The constraints and the design vectors for the optimization problem is also adopted from [63]. However, the focus of this study is only the rotor diameter and hence, only this parameter will be treated.

Table 4.1 shows the choices made for carrying out this case study.

Table 4.1: Choices made for rotor diameter optimization

<table>
<thead>
<tr>
<th>Design Parameter</th>
<th>Choices Made</th>
</tr>
</thead>
<tbody>
<tr>
<td>No of turbines</td>
<td>49 turbines</td>
</tr>
<tr>
<td>Power Rating</td>
<td>5MW</td>
</tr>
<tr>
<td>Cut in and rated wind speed</td>
<td>4 m/s and 12 m/s</td>
</tr>
<tr>
<td>Wind speed distribution</td>
<td>Shape factor = 2.11, scale factors = 9 m/s</td>
</tr>
<tr>
<td>Bathymetry</td>
<td>Fixed water depth of 20 m</td>
</tr>
<tr>
<td>Farm layout</td>
<td>Rectangular layout with 7 X 7 turbines and 1 substation</td>
</tr>
<tr>
<td>Wake model</td>
<td>Jensen wake model with the directional sampling of 10 and speed sampling of 1 m/s</td>
</tr>
<tr>
<td>Electrical Infrastructure</td>
<td>7 turbines per cable, grid 60 km, harbor 40km, onshore transport distance 100km, collection voltage 66kV, transmission voltage 220kV</td>
</tr>
<tr>
<td>Availability of the OWF</td>
<td>98%</td>
</tr>
</tbody>
</table>

The layout of the OWF is given in figure 4.1:
4.3.2. Layout optimization

Similarly, certain choices were made for evaluating this case study as well. They are represented in table 4.2.

<table>
<thead>
<tr>
<th>Design Parameter</th>
<th>Choices Made</th>
</tr>
</thead>
<tbody>
<tr>
<td>Turbine rating</td>
<td>5MW, 126D</td>
</tr>
<tr>
<td>Bathymetry</td>
<td>Varying water depth ranging from 30 to 70 m</td>
</tr>
<tr>
<td>Farm layout</td>
<td>Layout with a fixed boundary</td>
</tr>
<tr>
<td>Wake model</td>
<td>Jensen wake model with the directional sampling of 10 and speed sampling of 1 m/s</td>
</tr>
<tr>
<td>Availability of the OWF</td>
<td>98%</td>
</tr>
</tbody>
</table>

4.4. Sensitivity study

This section will help the reader understand the different sub models that will be evaluated in this study. Figure 4.2 shows the different electricity models that are evaluated in separate optimization runs. The electricity models have already been presented in brief in chapter 2.
From Figure 4.2, the three models of electricity price that are considered are as follows:

4.4.1. **Constant electricity price - Model 1**

The electricity price in this model is assumed to be a constant. The work done in [63] optimized the LCOE and for rotor diameter optimization, the optimized LCOE was in the range of 9 to 10 €/kWh. Hence, this value of LCOE is used as a reference for assuming a realistic electricity price. The electricity price is presumed to be 11 €/kWh. For layout optimization as well, this value of electricity price is used. In this model, the economic indicators are varied. The NPV function is dependent on the following economic indicators:

- Annuity factor
- Real interest rate
- Lifetime
- Electricity price

The annuity factor is a function of both the real rate of interest and lifetime of the OWF as represented in equation 2.2. Hence, the interest rate and lifetime are varied for a fixed electricity price and rotor diameter optimization and layout optimization is carried out for four different scenarios. The four scenarios are listed in table 4.3:
Table 4.3: Four scenarios for model 1

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Lifetime [years]</th>
<th>Real Interest rate [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scenario 1</td>
<td>25</td>
<td>7.5</td>
</tr>
<tr>
<td>Scenario 2</td>
<td>20</td>
<td>7.5</td>
</tr>
<tr>
<td>Scenario 3</td>
<td>25</td>
<td>10</td>
</tr>
<tr>
<td>Scenario 4</td>
<td>25</td>
<td>10</td>
</tr>
</tbody>
</table>

4.4.2. Electricity price based on wind variability – Model 2

The operational wind speed range for an OWF is assumed to be between 4 m/s and 25 m/s. As stated in chapter 2, the lowest wind speed will have the highest electricity price and the highest wind speed will have the lowest electricity price. It should be noted that the electricity price between 12 m/s and 25 m/s is assumed to be a constant as the power generated for wind speeds from 12 m/s to 25 m/s by a single wind turbine is assumed to be a constant. Again, in this model, there are two sub models:

4.4.2.1. Large variation in electricity price – Model 2a

In this case, for every 1 m/s wind speed increase, the electricity price is decreased by 0.5\( c\€/Kwh \). This is shown in table 4.4.

4.4.2.2. Small variation in electricity price – Model 2b

In this model, the electricity price is reduced by 0.2 for every 1 m/s wind speed increase. This is represented in table 4.5.

4.4.3. Electricity price based on wind power predictability – Model 3

The electricity price for this model is estimated using equation 2.12. From equation 2.12, \( ep \) is assumed to be 11 \( c\€/kWh \). The total power output for all directions is obtained and the standard deviation of the power output is estimated. The percentage difference is calculated for the total power output and the standard deviation of the power output. A penalty \( p \) of 1 \( c\€/kWh \) is provided, and the remodeled electricity price is calculated.
Table 4.4: Large variation in electricity price based on wind variability

<table>
<thead>
<tr>
<th>Wind Speed (m/s)</th>
<th>Electricity Price ((\epsilon p \ (U)) \ (\text{c€/kWh})) (large variation)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>15</td>
</tr>
<tr>
<td>5</td>
<td>14.5</td>
</tr>
<tr>
<td>6</td>
<td>14</td>
</tr>
<tr>
<td>7</td>
<td>13.5</td>
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<td>8</td>
<td>13</td>
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<td>9</td>
<td>12.5</td>
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<td>11</td>
</tr>
<tr>
<td>25</td>
<td>11</td>
</tr>
</tbody>
</table>
Table 4.5: Small variation in electricity price based on wind variability

<table>
<thead>
<tr>
<th>Wind Speed (m/s)</th>
<th>Electricity Price (( \text{ep}(U) )) ((\text{c€/kWh} )) (small variation)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>12.6</td>
</tr>
<tr>
<td>5</td>
<td>12.4</td>
</tr>
<tr>
<td>6</td>
<td>12.2</td>
</tr>
<tr>
<td>7</td>
<td>12.0</td>
</tr>
<tr>
<td>8</td>
<td>11.8</td>
</tr>
<tr>
<td>9</td>
<td>11.6</td>
</tr>
<tr>
<td>10</td>
<td>11.4</td>
</tr>
<tr>
<td>11</td>
<td>11.2</td>
</tr>
<tr>
<td>12</td>
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<tr>
<td>24</td>
<td>11</td>
</tr>
<tr>
<td>25</td>
<td>11</td>
</tr>
</tbody>
</table>

4.5. Optimization study

This section will describe the optimization procedure of the respective case studies. The optimization problem will be first defined followed by a brief description of the optimization algorithm.

4.5.1. Rotor diameter optimization

4.5.1.1. Optimization problem definition

The formulation of the optimization problem is given below.
\[ f(x) = \begin{cases} 
\text{Min } F(-NPV) & \text{Objective function} \\
D_{ew}, D_{ns} \geq 12 \times R_{rotor} & \text{Spacing Constraint} \\
\Delta_{margin} = \Delta_{tip} * \frac{R_{rotor}^{ref}}{(\Delta_{tip}^{ref} \times R_{rotor})} \leq 1 & \text{Tip deflection constraint} \\
\vec{x} = [R_{rotor}, D_{ew}, D_{ns}]^T & \text{Design Vector}
\end{cases} \]

Where, \( \Delta_{margin} \) refers to maximum tip deflection margin, \( \Delta_{tip} \) is the tip deflection, and \( D_{ew}, D_{ns} \) refers to the east west and north south spacing. For more description on these parameters, the reader is recommended to refer [63].

Table 4.6 displays the minimum and maximum range of diameter used in this study. The initial value is also specified.

**Table 4.6: Minimum and maximum scaling of rotor diameter for the Optimization of NPV**

<table>
<thead>
<tr>
<th>Design Variable</th>
<th>Initial Value</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotor Diameter  [m]</td>
<td>126.0</td>
<td>101.0</td>
<td>200.0</td>
</tr>
</tbody>
</table>

4.5.1.2. Optimization algorithm

This section will brief on the optimization algorithm used in this case study. The algorithm used is COBYLA and it stands for constrained optimization by linear approximation. The COBYLA algorithm works based on linear approximations to the objective functions and constraints. If a function is minimized over x variables, at the i\textsuperscript{th} iteration, the algorithm has x + 1 points, an approximate solution \( k_i \) and a radius \( RHO_i \) approximations to the objective function and constraints such that their function values satisfy with the linear approximation on the x+1 points [64]. This gives a linear problem to solve where the linear approximations of the constraint functions are constrained to be non-negative. In other words, a candidate is obtained for an optimal solution during an iteration. The candidate solution is evaluated using the objective function and the constraints which thereby yields a new point in the solution space. This information is then used to improve the approximating linear programming problem used for the next iteration. When the solution cannot be improved anymore, the step size is reduced and ultimately when the search space becomes infinitely small, the algorithm terminates.
4.5.2. Layout optimization

4.5.2.1. Optimization problem definition

The optimization problem is given by $f(x) =$

\[
\begin{align*}
    \text{Min } F(-NPV) \quad \text{Objective function} \\
    X_{\text{min}} \geq 0, X_{\text{max}} \leq 5000 \quad \text{Area constraint} \\
    Y_{\text{min}} \geq 0, Y_{\text{max}} \leq 5000 \quad \text{Area constraint} \\
    D \geq 2 * R_{\text{rotor}} \quad \text{Spacing constraint} \\
    \bar{x} = [n_t, X, Y]^T \quad \text{Design vector}
\end{align*}
\]

Where $n_t$ represents the number of turbines. The constraints are explained more specifically in the next section.

4.5.2.2. Constraints

To obtain the desired wind farm layouts, the following constraints were implemented in this study:

1. A minimum distance between the turbines is always necessary to guarantee the good function and integrity of the turbines. Hence, a minimum separation is considered equal to 1 time the rotor diameter for all the models.
2. The turbines need to be placed in the farm area. As seen in figure 4.3 a specific area constraint is employed to make sure that the turbines do not fall out of the boundary.

![Figure 4.3: Example of an area constraint [12]](image-url)
4.5.2.3. Optimization algorithm

PSO was developed by Eberhart and Kennedy [65] in 1995 and is based on an idea of mimicking the flocking of birds or fish schooling. PSO does not guarantee a global optimum but it works well in challenging, non–convex and non–continuous environments. The two equations shown below make up the bare bones of the PSO algorithm.

\[
x_{k+1}^i = x_k^i + v_{k+1}^i \tag{Equation 4.1}
\]

\[
v_{k+1}^i = w_k v_k^i + c_1 r_1 (p_k^i - x_k^i) + c_2 r_2 (p_g^i - x_k^i) \tag{Equation 4.2}
\]

Equation 4.1 is associated with particle position and equation 4.2 is linked to particle velocity. In PSO, each candidate solution is called a particle and represents a point in the design space, representing the values of the design variables. The table below defines the different variables used in equations 4.1 and 4.2.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x_k^i)</td>
<td>Particle position</td>
</tr>
<tr>
<td>(v_k^i)</td>
<td>Particle position</td>
</tr>
<tr>
<td>(p_k^i)</td>
<td>Best individual particle position</td>
</tr>
<tr>
<td>(p_g^i)</td>
<td>Best swarm position</td>
</tr>
<tr>
<td>(w_k)</td>
<td>Constant inertia weight</td>
</tr>
<tr>
<td>(c_1, c_2)</td>
<td>Cognitive and social parameters</td>
</tr>
<tr>
<td>(r_1, r_2)</td>
<td>Random numbers between 0 and 1</td>
</tr>
<tr>
<td>(k, k+1)</td>
<td>Current iteration and next iteration</td>
</tr>
<tr>
<td>(i)</td>
<td>Particle index</td>
</tr>
</tbody>
</table>

Two sets of equations emerge from the particle velocity term:

- Social term: \(c_2 r_2 (p_g^i - x_k^i)\)
- Cognitive term: \(c_1 r_1 (p_k^i - x_k^i)\)

Using the two equations (4.1 and 4.2), the flow structure of the PSO algorithm is as follows:

**Initialize**

In this step, \(k_{\text{max}}, w_k, c_1 \text{ and } c_2\) values are fixed. The particle positions and particle velocities are initiated in a random manner.
**Optimize**

For a minimization problem, the following procedure is followed. The objective function is evaluated at each particle position and if the value of the objective function for the current iteration is lesser than the best value of particle position, then this current value is taken as the best value. Similarly, if the current iteration value of swarm position is lesser than the nest swarm position, the best swarm position value is replaced by the current iteration value. After this step, all the particle velocities and particle positions are updated, and the value of k is incremented and the same procedure repeats.

Let’s say an objective function is represented by $f_k^i$:

At each particle position $x_k^i$:

- If $f_k^i \leq f_{\text{best}}^i$ then $f_{\text{best}}^i = f_k^i$ and $p_k^i = x_k^i$
- If $f_k^i \leq f_{\text{best}}^g$ then $f_{\text{best}}^g = f_k^i$ and $p_k^g = x_k^i$

**Terminate**

The main concept behind PSO is that there is a continuous balance between three distinct forces pulling on each particle:

- The particles previous velocity (inertia)
- Distance from the individual particles best - known position (cognitive force)
- Distance from the swarms best - known position (social force)

Specific weights $w_k, c_1, c_2$ are assorted to the three forces and are randomly perturbed by $r_1, r_2$. Hence, depending on the weight specified, either the particles best position or swarm’s best position will pull harder on the particle and dictate the overall direction. It is also assumed that inertia won’t cause the particle to wander around. The three forces can be represented in vector form as shown in the figure below. Here, the magnitude of the vector represents the value of weight specified for that force.

![Figure 4.4: Example of how PSO works](image)

In the above example, inertia and individual best overpowers swarms influence as more weight is given to the former two forces. In this case, the particle will keep exploring the search space instead of converging on the swarm. However, if more weight is specified to the swarm, it will result in faster
convergence and the search space, in this case, will not be fully explored. This will result in not obtaining a better solution.

4.6. Results and discussion

4.6.1. Rotor diameter optimization

The optimization was run 10 times and the time taken for one run was roughly around 45 minutes. The results for all the models are shown below in table 4.8 and 4.9. The optimization properties are tabulated in appendix A3. A3 indicates the minimum value, maximum value and average value obtained for each model.

Table 4.8: Results for Model 1

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Lifetime [years]</th>
<th>Real Interest Rate [%]</th>
<th>NPV [Euros]</th>
<th>Optimized Rotor Diameter [meter]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scenario 1</td>
<td>25</td>
<td>7.5</td>
<td>1.7E +10</td>
<td>140.00</td>
</tr>
<tr>
<td>Scenario 2</td>
<td>20</td>
<td>7.5</td>
<td>1.0E +10</td>
<td>136.70</td>
</tr>
<tr>
<td>Scenario 3</td>
<td>25</td>
<td>10</td>
<td>0.3E +10</td>
<td>128.00</td>
</tr>
<tr>
<td>Scenario 4</td>
<td>20</td>
<td>10</td>
<td>0.1E + 10</td>
<td>127.75</td>
</tr>
</tbody>
</table>

Table 4.9: Results for Model 2 and Model 3

<table>
<thead>
<tr>
<th>Parameter Varied</th>
<th>Rotor Diameter [m]</th>
<th>NPV [Euros]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small Variation in Electricity price</td>
<td>139.0</td>
<td>2.25E +10</td>
</tr>
<tr>
<td>Large variation in electricity price</td>
<td>141.0</td>
<td>3.6E + 10</td>
</tr>
<tr>
<td>Wind power predictability</td>
<td>141.0</td>
<td>1.37E + 10</td>
</tr>
</tbody>
</table>
The results obtained indicate the following trend. It is observed that the economic indicators i.e. the lifetime and real interest rate influence the optimum rotor diameter and the NPV function. This is indicated in table 4.8 and the highest value for the NPV function and rotor diameter is obtained for maximum lifetime and minimum interest rate. To understand this sensitivity, let’s consider a simpler version of the NPV formulation:

\[
NPV = (R) \times a - E \quad \text{Equation 4.3}
\]

Where,

\[
E = (OM) \times a - C_{in} - \frac{Dec}{(1+r)^t} \quad \text{Equation 4.4}
\]

As mentioned earlier in chapter 2, a represents the annuity factor and it has a fixed value for a fixed interest rate and a fixed lifetime. Hence equation 4.4 can be reformulated as:

\[
NPV = (R) \times C - E \quad \text{Equation 4.5}
\]

Since rotor diameter is the design variable that is being varied, both R and E can be generalized as functions of rotor diameter, D.

\[
NPV = C \times R(D) - E(D) \quad \text{Equation 4.6}
\]

The maximum NPV is found by differentiating the above equation with respect to the rotor diameter. And setting it equal to zero as shown below.

\[
\frac{dNPV}{dD} = C \frac{dR(D)}{dD} - \frac{dE(D)}{dD} = 0 \quad \text{Equation 4.7}
\]

Equation 4.7 shows that constant C influences the rotor diameter that maximizes the NPV. Hence, the financial assumptions that go into determining the real rate of interest and lifetime will change the optimum design.

Therefore, for model 2, the best lifetime and real interest rate are chosen from model 1, and the results are tabulated in table 4.9. For both cases, the optimum rotor diameter obtained is very similar but there is a large variation in the NPV function. For model 3 as well, the electricity price model does not influence the rotor diameter but influences the NPV function. Hence, it can be concluded that the variation in electricity price influences the NPV function but does not influence the rotor diameter optimum.

4.6.2. Layout optimization

The optimization was run 10 ten times to avoid the effect of the initial population. The parameters used for implementing the PSO algorithm is shown in table 4.10. The results obtained are displayed in table 4.11 and table 4.12.
Table 4.10: Basic parameters used in PSO

<table>
<thead>
<tr>
<th>Parameters Varied</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Particle size</td>
<td>20</td>
</tr>
<tr>
<td>Generations</td>
<td>200</td>
</tr>
<tr>
<td>Total runs</td>
<td>10</td>
</tr>
<tr>
<td>Stopping Criteria</td>
<td>Maximum iterations (200)</td>
</tr>
<tr>
<td>Constant inertia weight</td>
<td>0.8</td>
</tr>
</tbody>
</table>

Table 4.11: Results for Model 1

<table>
<thead>
<tr>
<th>Lifetime [years]</th>
<th>Real Interest Rate [%]</th>
<th>NPV [Euros]</th>
<th>Optimized Number of turbines</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>7.5</td>
<td>4.71E+10</td>
<td>69</td>
</tr>
<tr>
<td>20</td>
<td>7.5</td>
<td>4.10E+10</td>
<td>59</td>
</tr>
<tr>
<td>25</td>
<td>10</td>
<td>4.00E+10</td>
<td>64</td>
</tr>
<tr>
<td>20</td>
<td>10</td>
<td>3.80E+10</td>
<td>60</td>
</tr>
</tbody>
</table>

Table 4.12: Results for Model 2 and Model 3

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Varied</th>
<th>Turbines</th>
<th>NPV [Euros]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small Variation in Electricity price</td>
<td>65</td>
<td></td>
<td>4.82E+10</td>
</tr>
<tr>
<td>Large variation in electricity price</td>
<td>66</td>
<td></td>
<td>6.64E+10</td>
</tr>
<tr>
<td>Wind power predictability</td>
<td>66</td>
<td></td>
<td>4.50E+10</td>
</tr>
</tbody>
</table>
The results obtained indicate a similar trend as observed in rotor diameter optimization. From table 4.12, it is seen that lifetime and the real rate of interest play an influential role in determining the optimal number of turbines and NPV function. Again, Scenario 1 has the highest NPV function value and the highest number of turbines. Referring to equation 4.3, both \( R \) and \( E \) are a function of the number of turbines as well. Hence, the financial assumptions that go in determining the real rate of interest and lifetime will change the optimum number of turbines as well. The optimized layout for model 1 is shown in figure 4.5 and 4.6.

The second observation is that the variation of electricity price has minimum effect on the optimum number of turbines. Again, the best real rate of interest and lifetime is chosen from table 4.11. It is observed that the optimal number of turbines for all three models of electricity price is very similar as shown in table 4.12. However, the variation in electricity price has a drastic influence on the outcome of the NPV function. Appendix A4 shows the optimized layout of model 2 and 3 respectively.
5. Multi – objective optimization

5.1. Introduction

This chapter will discuss the results of the multi-objective optimization technique. The aim of this study is to provide OWF investors with tradeoffs by implementing a multi-objective approach. The objective functions used are $NPV_{mean}$ and $NPV_{std}$ where $NPV_{mean}$ is maximized and $NPV_{std}$ is minimized. Like the previous chapter, the choice of design variables will be discussed, and the scope and assumptions of this case study will be listed. A short description will be provided on the optimization study as well which includes a brief explanation of the optimization problem and the optimization algorithm. The pareto front is obtained and multi – criteria analysis (MCA) is used to rank the solutions in the pareto front for three different strategies.

5.2. Choice of design variables

As mentioned in the previous chapter, two sets of design variables were taken into contemplation for evaluating the case studies. In this chapter, only rotor diameter optimization is carried out due to time constraints. However, a brief description is provided below on the choice of the rotor diameter as a design variable.

In this study, 100 annual average wind speeds are computed using Monte Carlo Simulations (MC). A MC simulation performs risk analysis by building models of possible results by substituting a range of values, say a probability distribution, for any factor that has inherent uncertainty. It then calculates the results, each time using a different set of random values from the probability functions. Depending on the uncertainties and the ranges specified for them, a MC simulation could involve many recalculations before it is complete.

The probability distribution used in this study is the normal distribution. In a normal distribution, the user defines the mean and a standard deviation to describe the variation about the mean where values in the middle i.e. close to the mean are most likely to occur.

The mean of the average wind speed is assumed to be 8 m/s and the standard deviation is assumed to be 0.1 m/s in this study. The average wind speeds from the MC simulation are then used to estimate the scale and shape factors respectively using the following relation:

$$c = \left( \frac{U_{avg}}{\Gamma(1+\frac{1}{k})} \right)$$  

Equation 5.1

$$k = \left( \frac{\sigma_u}{U_{avg}} \right)^{-1.086}$$  

Equation 5.2

Where $U_{avg}$ is the annual average wind speed, $c$ is the shape factor, $k$ is the scale factor and $\sigma_u$ is the standard deviation of the average wind speeds. The AEP is then calculated for the list of 100 shape and scale factors. The NPV is then estimated for the list of 100 AEP’s. In this way, MC simulation is used to evaluate the probability distribution of NPV and $NPV_{mean}$ and $NPV_{std}$ is then depicted.

The scale factor varies with the $U_{avg}$ in the same order of magnitude. The variation in $U_{avg}$ represents uncertainty for the site and hence there is an uncertainty as well in estimating the rotor diameter and...
the rated wind speed for that site. As $U_{avg}$ increases, the rotor diameter decreases and rated wind speed increases. Similarly, as $U_{avg}$ decreases, the rotor diameter increases and the rated wind speed decreases. Therefore, the rotor diameter is varied for a specific range in this study to find the list of solutions for the $NPV_{mean}$ and $NPV_{std}$ in a pareto front.

5.3. Scope and choices made

Like the previous chapter, the framework described in chapter 3 is adopted. The assumptions implemented in this case study are synonymous to those considered in case study 1. The major difference is the estimation of the Weibull parameters which are calculated using the average wind speed. The average wind speeds are determined using a normal distribution as mentioned in section 5.2. The development of this case study required a slight change in the framework. The $NPV_{mean}$ and $NPV_{std}$ were added to the finance module as objective functions.

For computational purposes, 25 turbines are considered in a rectangular layout in this case study as shown in figure 5.1. The choices made for this case study are tabulated in table 5.1:

Table 5.1: Design parameters adopted for this case study

<table>
<thead>
<tr>
<th>Design Parameter</th>
<th>Choices Made</th>
</tr>
</thead>
<tbody>
<tr>
<td>No of turbines</td>
<td>25</td>
</tr>
<tr>
<td>Power Rating</td>
<td>5MW</td>
</tr>
<tr>
<td>Cut in and rated wind speed</td>
<td>4 m/s and 12 m/s</td>
</tr>
<tr>
<td>Bathymetry</td>
<td>Fixed water depth of 20 m</td>
</tr>
<tr>
<td>Farm layout</td>
<td>Rectangular layout with 5 X 5 turbines</td>
</tr>
<tr>
<td>Wake model</td>
<td>Jensen wake model with the directional sampling of 10 and speed sampling of 1 m/s</td>
</tr>
<tr>
<td>Electrical Infrastructure</td>
<td>5 turbines per cable, grid 60 km, harbor 40km, onshore transport distance 100km, collection voltage 66kV, transmission voltage 220kV</td>
</tr>
<tr>
<td>Availability of the OWF</td>
<td>98%</td>
</tr>
</tbody>
</table>

![Figure 5.1: Layout of the OWF for this case study](image)
5.4. Optimization study

This section will discuss the process of carrying out the multi-objective optimization and obtaining the pareto front. The optimization problem will be first described followed by a brief description of the optimization algorithm.

5.4.1. Optimization problem

The optimization problem is given by:

\[
f(x) = \begin{cases} 
\text{Max}(NPV_{\text{mean}}), \text{Min}(NPV_{\text{std}}) & \text{Objective function} \\
D_{ew}, D_{nz} \geq 12 \times R_{rotor} & \text{Spacing Constraint} \\
\Delta_{\text{margin}} = \Delta_{\text{tip}} \times \frac{R_{\text{rot}}^{\text{ref}}}{\Delta_{\text{tip}}^{\text{ref}} \times R_{\text{rotor}}} \leq 1 & \text{Tip deflection constraint} \\
\hat{x} = [R_{\text{rotor}}, D_{ew}, D_{nz}] & \text{Design Vector}
\end{cases}
\]

The optimization problem is adopted from [63]. Again, rotor diameter is the main design variable in focus in this study as well.

Table 5.2 displays the lower and upper range of the design variable used in the optimization technique. It should be noted that the \( NPV_{\text{mean}} \) values are multiplied with -1 to convert it into a minimization problem.

<table>
<thead>
<tr>
<th>Design Variable</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotor Diameter</td>
<td>100.0</td>
<td>200.0</td>
</tr>
</tbody>
</table>

5.4.2. Optimization algorithm

A suitable optimization technique must be selected to carry out the multi-objective optimization. It is necessary to understand the two phases of a multi-objective problem. Firstly, the optimization of objective functions involved and secondly, it is important to decide what kind of tradeoffs are suitable from the decision maker perspective. Within the operation research community, [66] proposed a popular technique that focuses on the way in which every method handles the two problems of making decisions and searching:
• **A priori preference articulation**: Decision is taken before searching. It is assumed that the decision maker chooses a certain set of desirable goals and pre-ordering of objectives prior to the search.

• **A posteriori preference articulation**: In this method, searching is done before decision making. These techniques do not require prior information of preference from the decision maker.

• **Progressive Preference Articulation**: This method integrates search and decision making. Operation of this technique takes place in three stages:
  
  ➢ Find a non-dominated solution
  ➢ The decision maker then modifies the preferences of the objectives according to the requirement.
  ➢ The two previous steps are repeated until the decision maker is totally satisfied, or no further improvement is possible.

Evolutionary algorithms can be used to solve multi-objective problems. These algorithms are based on Darwin’s theory of survival of the fittest. The main idea is that the population evolves in a genetic algorithm and the solutions that are non-dominated are chosen to remain in the population. The basic idea behind the evolutionary algorithms is represented in figure 5.2.

![Flowchart representing Evolutionary Algorithms](image)

**Figure 5.2**: Flowchart representing Evolutionary Algorithms [61]

In figure 5.2, the memory module contains the current solutions. In the selection module, the solutions that should be kept in the memory are determined. This is done in two ways, by mating and environmental selection.

• The mating selection consists of a fitness selection phase where promising solutions are picked for variation.
• Environmental selection determines which of the stored solutions are transferred to the memory module.

A variation module modifies the set of solutions systematically or randomly to generate better solutions by using specific operators:
- Crossover operator which produces new individuals by combining the information of two or more parents
- Mutation operator which alters individuals with a low probability of survival.

In an evolutionary algorithm, by analogy to natural evolution, the solutions obtained are called as candidates and a set of candidates is called as a population. The objective function is also known as the fitness function in this case. It characterizes the problem measuring how close a given solution is to achieve the target, considering all the problem constraints as well in the process.

The drawback of evolutionary algorithms is that they require many iterations. However, they always lead to an accurate identification of the entire pareto front. Hence, based on correctness and accuracy, evolutionary algorithms are chosen for the multi-objective optimization problem. Non-sorting genetic algorithm (NSGA II) is one of the most effective algorithms for implementing multi-objective optimization problems [67]. This algorithm was developed by [68]. The working of NSGAIi is briefed below:

The population is divided into multiple fronts and the algorithm uses the front to determine the fitness. This algorithm sorts the solutions into levels of non-dominance. For example, the pareto front is solved for non-dominated solutions of the population. The pareto front of solutions are removed from the population and pareto front is calculated again. These two pareto fronts are called as level 1 and level 2 respectively. This is represented in figure 5.3.

![Figure 5.3: Non-Dominated Sorting in NSGA-II](image)

In figure 5.3, Level 1 is non-dominated by all points in the solution space. Level 2 is dominated by level 1 but non-dominated by level 3 and so on. This process of finding different levels of pareto front is continued up to the $n^{th}$ front. All the points are removed from fronts 1 to n-1 and the pareto front is calculated for the remaining points.

The NSGAIi retains all the members in the population for the higher-level fronts so that later, crossovers might generate offspring that are even fitter and closer to the pareto front than the current members.
5.5. Results and discussion

5.5.1. Basic parameters used in NSGAII

The basic parameters used in the implementation of NSGAII is given in table 5.3:

Table 5.3: Parameters used in NSGAII

<table>
<thead>
<tr>
<th>Parameters Varied</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population size</td>
<td>20</td>
</tr>
<tr>
<td>Generations</td>
<td>200</td>
</tr>
<tr>
<td>Total runs</td>
<td>10</td>
</tr>
<tr>
<td>Crossover probability</td>
<td>0.9</td>
</tr>
<tr>
<td>Mutation Probability</td>
<td>0.5</td>
</tr>
<tr>
<td>Stopping Criteria</td>
<td>Maximum iterations (200)</td>
</tr>
</tbody>
</table>

The optimization was run, and the entire population of the last generation is obtained as shown in figure 5.4. The black dots in figure 5.4 represent the solutions in the pareto front. These solutions are analyzed in the next section.

![Entire Population of Last Generation](image)

Figure 5.4: Entire population of last generation

5.5.2. Analysis of the front

In this section, the solutions in the pareto front are sorted based on the rank. The ranking is given based on the weight provided to the two objective functions. In a general sense, the $NPV_{std}$ must be minimized and the $NPV_{mean}$ must be maximized. As these objectives are conflicting, the investors need to compromise on one of the objectives to achieve the other objective. For example, a certain investor might be interested in obtaining maximum NPV. However, with maximum NPV comes maximum risk, hence the $NPV_{std}$ also increases. Similarly, another investor might opt for a
conservative approach and aim at minimizing the risk associated with the OWF. Therefore, in this section, different weighting is provided for the objective functions and the best solution is obtained for that weight.

To achieve this result, the technique of multi-criteria analysis (MCA) is adopted. MCA technically compares different solutions according to a variety of criteria or policies [69]. This method is based on the evaluation of actions by means of a weighted average. The main advantage of MCA is its ability to find the best scenario that suits the decision makers expectations. However, the decision maker must reach a consensus on a weighted set of criteria with which to judge the performance of the project. Hence, the solutions from the pareto front are taken into consideration and appropriate scores are given. For example, the highest value of $NPV_{mean}$ in the pareto front is given a score of 1. Based on this value, the weighted scores of the other values of $NPV_{mean}$ are calculated proportional to the highest score. Therefore, if $m$ is the best value for $NPV_{mean}$, it is given a score of one. The weighted scores of the other values are calculated using equation 5.4.

Similarly, if $m$ is the best value for $NPV_{std}$, it is given a score of 1. The weighted scores for the other values of $NPV_{std}$ are calculated using equation 5.3.

\[
\frac{m}{p} \cdot 1 \quad \text{Equation 5.3}
\]

\[
\frac{c}{m} \cdot 1 \quad \text{Equation 5.4}
\]

After calculating the weighted score, the MCA analysis is carried out for three cases:

- Ambitious investor strategy
- Conservative investor strategy
- Balanced investor strategy

In the ambitious investor strategy, $NPV_{mean}$ is given more preference with 80% weight and the remaining 20% to $NPV_{std}$. In the conservative investor strategy, $NPV_{std}$ is given more preference with 80% weight and the remaining 20% to $NPV_{mean}$. In the balanced investor strategy, both the objective functions are given equal preference with a weight of 50% each. Table 5.4 represents the weighted sum for all the points for three cases:

**Table 5.4: The three scenarios and their respective results**

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Solution with highest sum</th>
<th>Weighted sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ambitious investor strategy</td>
<td>(0.523, -0.826)</td>
<td>0.888</td>
</tr>
<tr>
<td>Conservative investor strategy</td>
<td>(0.163, -0.365)</td>
<td>0.862</td>
</tr>
<tr>
<td>Balanced investor strategy</td>
<td>(0.337, -0.635)</td>
<td>0.756</td>
</tr>
</tbody>
</table>

The best solution for all the three scenarios is shown in figure 5.5.
The first scenario provides which provides 80% weightage to $NPV_{\text{mean}}$ and 20% weightage to the $NPV_{\text{std}}$. The solution (0.523, -0.826) has the highest score of 0.88. This solution is represented inside a square in figure 5.5. This seems logical, as more weightage is provided to the $NPV_{\text{mean}}$. As the value of $NPV_{\text{mean}}$ decreases along the curve, the overall score also decreases. This is because a significant weightage is provided to the $NPV_{\text{mean}}$ which dominates the weightage given to the $NPV_{\text{std}}$.

The second scenario allocates 80% weightage to the $NPV_{\text{std}}$ and 20% to $NPV_{\text{mean}}$. In this scenario, the solution (0.163, -0.365) has the highest score of 0.862. This point is shown inside a triangle in figure 5.5. This is true as the weightage provided to the $NPV_{\text{std}}$ dominates the $NPV_{\text{mean}}$. Hence, the solution with the lowest standard deviation will be given the preference.

In the balanced investor and conservative strategy, a weightage of 50% is each provided to both the $NPV_{\text{mean}}$ and $NPV_{\text{std}}$. In this case, solution (0.337, -0.635) is ranked first by the MCA which has an overall score of 0.756. This solution is represented inside a circle in figure 5.5. This solution lies in the middle of the pareto curve which is reasonable as equal weights are provided to both the objectives.

5.6. Concluding remarks

The main observation of the pareto curve is discussed here. The property of the pareto front states that one criterion cannot be improved without degrading the other criterion. Therefore, this means that the improvement of $NPV_{\text{mean}}$ solutions will lead to debasing the solutions of $NPV_{\text{std}}$ in the pareto front. For example, the best possible solution obtained for the conservative investor strategy is the worst possible solution for the ambitious investor strategy. Similarly, the best possible solution for the ambitious investor strategy is the worst solution for the conservative investor strategy.

In general, the following trend is observed in the pareto curve. As one moves from the first solution in the curve, the weightage for the conservative investor reduces and at the same time, the weightage for the ambitious investor starts increasing. This is the movement which is followed till the end of the curve where the weightage of the ambitious investor totally dominates the conservative one.
6. Concluding observations

6.1. Introduction

This section will enumerate the findings of this thesis and provide directions for further research. It is divided into two parts:

1. Conclusion
2. Recommendation

6.2. Conclusion

The design of an OWF is interdisciplinary in nature. The common approach to design an OWF is to optimize all the disciplines simultaneously. Objective functions play a significant role in optimization as they express the main aim of the model which is to be either minimized or maximized. In common approaches, COE and AEP are the commonly used objective functions for OWF optimization. However, there might be other objective functions that might influence the optimal design of an OWF. Therefore, this report presented an overview of different objective functions and their impact on the optimal design of an OWF. To achieve this main objective, the following sub goals were achieved.

➢ A list of different objective functions for the OWF optimization and selection of relevant ones
➢ Formalize the relevant objective function in metrics suitable for optimization
➢ A suitable method to deal with multiple objectives
➢ Devise case studies to reflect the new approach towards solving the OWF optimization problem.

6.2.1. Sub goal 1

A list of different objective functions for the OWF optimization and selection of relevant ones

The objective functions that were included in the list are:

I. NPV
II. Risk Management
III. Carbon emissions
IV. Financial Balance

The main conclusion that can be drawn from this sub goal is that some seemingly different objectives relate to the same basic properties of the OWF. For example, for reducing carbon emissions, AEP must be maximized. However, AEP is also maximized for maximizing NPV. Similarly, FB is very identical to NPV and depends on the same factors as NPV. Therefore, similar design results are expected when these objectives are taken into consideration.

NPV and risk management were the chosen objective functions for further study.
6.2.2. Sub goal 2

**Formalize the relevant objective function in metrics suitable for optimization**

The conclusions drawn from this sub goal is as follows:
The formulation of the NPV objective function requires the same economic parameters as the LCOE function, the only exception being the electricity price. It is observed that the electricity price is dependent on the power supply in the sense that if the power supply is low in a region where there are many OWF’s, the electricity price is high and vice versa. Moreover, OWF investors value constant power output from the wind farm. Hence, taking these aspects into consideration, the electricity price in this research is modelled in three ways:

- Constant electricity price
- Price based on wind variability
- Price based on wind power predictability

Similarly, the formulation of the risk management objective is to minimize the risks associated with an OWF project. There are many uncertain elements associated with an OWF and the risk can be minimized by reducing these uncertainties. Average wind speed is one such uncertainty. Accurate prediction of average wind speed is very difficult as they are stochastic in nature. In this research, 100 average annual wind speeds are estimated using MC simulations and the AEP and NPV are depicted. The $NPV_{mean}$ and $NPV_{std}$ are calculated and the $NPV_{std}$ is minimized. Minimizing $NPV_{std}$ minimizes the uncertainty.

6.2.3. Sub goal 3

**A suitable method to deal with multiple objectives**

In this research, the selected objective functions are dealt with in two ways for optimization. First, the NPV function is maximized using a single objective optimization technique. In this case, NPV will be optimized for a specific set of design variables and constraints. In the second case, the two objectives are $NPV_{mean}$ and $NPV_{std}$. Both these objectives are conflicting in nature as $NPV_{mean}$ must be maximized and $NPV_{std}$ must be minimized. Hence, a tradeoff between both the objectives is the best possible result. A multi-optimization technique is used, and a list of solutions are obtained by generating a pareto front.

6.2.4. Sub goal 4

**Devise case studies to reflect the new approach towards solving the OWF optimization problem.**

**Single objective approach**

The objective of the first case study was to study the effect of rotor diameter on the NPV of an OWF and the aim of the second case study was to study the effect of the number of turbines on the NPV of...
an OWF. Similar trends were obtained from the studies and the key conclusions drawn from the studies are as follows:

- The economic indicators i.e. the real rate of interest and lifetime influence the optimum value of the design variables and the NPV function as seen in chapter 4.
- The variation in electricity price does not play any significant role in the estimation of the optimum value of the design variables.
- The NPV function however, is very sensitive to the variation in the electricity price.

The multi-objective approach

This study helped in providing the ambitious and conservative investors with tradeoffs for the different objective functions. The concept of MCA was used to determine the weight to the objective functions while moving along the pareto curve and rank the solutions on the front. It was observed that the improvement of one objective led to the deterioration of the other objective. Hence, a pareto front provides both these investors an opportunity to negotiate and decide on the weight they want to specify for their objectives.

6.3. Future recommendations

This section will provide recommendations for further research.

1. Additional objective functions can be taken into consideration. It will be interesting to observe if many other objective functions like Benefit to Cost ratio, Utilization Factor, Payback period do depend on similar parameters as observed with the list of objectives chosen in this study.

2. In this research, the electricity price was predicted for wind variability and wind power predictability. It would be more accurate if the electricity price is modelled based on the mentioned parameters using a GARCH model to accurately forecast the electricity prices.

3. In the OWF layout optimization, the average spacing between the turbines can be deduced and the power density of the OWF can be calculated for different models of the electricity price. This will give a better picture on whether there is any effect of the electricity price models on the spacing of the wind turbines in the wind farm.

4. MCA analysis was used to rank the solutions obtained from the pareto front. However, a cluster analysis algorithm can be used to form clusters on the Pareto set. The implementation of the cluster analysis algorithm will provide the investor with a promising solution for system application.


[51] Asma Arshad, Net Present Value is better than Internal Rate of return, 2012.


[67] Arvind Seshadri, A FAST ELITIST MULTIOBJECTIVE GENETIC ALGORITHM: NSGA-II.


Appendix

A1 – chapter 2 – Calculation of P90

Table A1.1: Z probability table

<table>
<thead>
<tr>
<th>Probability of exceedance %</th>
<th>z</th>
</tr>
</thead>
<tbody>
<tr>
<td>99</td>
<td>2.326</td>
</tr>
<tr>
<td>95</td>
<td>1.645</td>
</tr>
<tr>
<td>90</td>
<td>1.282</td>
</tr>
<tr>
<td>85</td>
<td>1.036</td>
</tr>
<tr>
<td>84</td>
<td>1.000</td>
</tr>
<tr>
<td>80</td>
<td>0.842</td>
</tr>
<tr>
<td>75</td>
<td>0.674</td>
</tr>
<tr>
<td>50</td>
<td>0</td>
</tr>
<tr>
<td>25</td>
<td>0.674</td>
</tr>
<tr>
<td>10</td>
<td>1.282</td>
</tr>
<tr>
<td>1</td>
<td>2.326</td>
</tr>
</tbody>
</table>

In equation 2.6, the uncertainty parameter is calculated by

\[ u = \frac{|NPV_m - NPV_{std}|}{NPV_m + NPV_{std}} \]  
Equation A1.1

where, \( NPV_m \) is the average value of 100 NPV’s and \( NPV_{std} \) is the standard deviation.

Using the corresponding z value for p90 from table A1.1, P90 is calculated using equation 2.6.

A2 – chapter 4 – Sensitivity of rotor diameter w.r.t NPV

NPV

This section analyses the sensitivity of rotor diameter with respect to the NPV function. The spacing is fixed as 6D.

NPV in a simplified form can be thought of as a difference between revenues and total cost of the OWF. Revenues can be further simplified as the product between electricity price and farm AEP. Thus, the main response variables are AEP and costs and these two parameters will be validated in this section.

Taking costs into consideration, the main drivers with respect to the change in rotor diameter are the RNA cost, support structure cost, cabling cost and other investment costs in the form of procurement, installation and management costs. The operation and maintenance cost is scaled empirically with AEP in WINDOW.

As mentioned in section 1.3.1, AEP for an OWF depends on the individual AEP of the wind turbines which in turn depends on the power curve of the individual turbines and Weibull characteristics. Wake
losses also play an unprecedented role which depends on the rotor diameter. The power curve of the turbine can also be altered based on the rated wind speed and controlling the rotor diameter. The rotor diameter is normalized using a specific minimum and maximum scaling to visualize the sensitivity of rotor diameter on NPV. The minimum value is assumed to be zero and maximum value to be 1. The rotor diameter is varied from 100m to 200m. A constant electricity price of 11 \( \text{€/kWh} \) is used and a lifetime of 25 years and real interest rate of 7.5% is assumed.

The above graph indicates the following trend. As the rotor diameter increases, the NPV function first increases. It obtains an optimum and then starts to decrease with the increase in rotor diameter. To understand this phenomenon, a sensitivity study was carried out for the AEP and cost constituents of the NPV with respect to the rotor diameter.

**AEP**

![Figure A2.2: Sensitivity analysis of AEP](image)
The sensitivity analysis for the AEP is shown in figure A2.2. An increase in rotor diameter leads to the wake effect being increased over a large area. Hence, wake losses increase with the increase in rotor diameter. The AEP curve increases with the increase in rotor diameter. However, the slope of the AEP curve is steep at the start but gradually becomes smoother as the rotor diameter increases. This can be attributed to the following reason: The increase in rotor diameter leads to a decrease in rated wind speed. This leads to the wind turbine spending more time in the full load region. At the full load region, the power is constant and is independent of the rotor diameter. In the partial load region, the power is proportional to the cube of wind speed. Hence, as the rotor diameter increases, the effect of rotor diameter on the AEP is less pronounced. Therefore, the slope gradually decreases, and the curve becomes less steep.

Cost

The sensitivity analyses of the support structure cost, RNA cost and other investment costs are shown in figure A2.3 and A2.4 respectively. It should be noted that RNA and the support structure costs consider all 49 turbines in the OWF. The support structure and RNA cost have a relation with the rotor diameter. It is seen that both the costs increase with the increase in rotor diameter. However, the slope of the graph in both these cases is low at the start but gradually becomes steep as the rotor diameter increases. The increase in rotor diameter leads to a decrease in rated wind speed. However, the torque of the rotor is proportional to the square of the rotor diameter, hence the support structure costs increase with the increase in rotor diameter. The mass of the RNA also contributes to the increase in the cost. The blade is part of the RNA and is one of the most expensive components of the RNA in terms of costs. Hence, as the diameter increases, the RNA cost also increases.

![Figure A2.3: Sensitivity Analysis of RNA cost](image-url)
Appendix

A3 – chapter 4 – Optimization properties

The optimization algorithm was run 10 times for each model of the electricity price for both the case studies. The minimum, maximum and the average values of the optimum design variable for all models of the electricity price is depicted below.

Rotor diameter optimization

The optimization algorithm used is COBYLA.

Table A3.1: Optimization properties for model 1

<table>
<thead>
<tr>
<th>Rotor diameter [m]</th>
<th>Scenario 1</th>
<th>Scenario 2</th>
<th>Scenario 3</th>
<th>Scenario 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum value</td>
<td>144</td>
<td>139</td>
<td>129</td>
<td>128.5</td>
</tr>
<tr>
<td>Minimum value</td>
<td>137</td>
<td>136.70</td>
<td>126</td>
<td>126</td>
</tr>
<tr>
<td>Average value</td>
<td>140</td>
<td>138</td>
<td>128</td>
<td>127.75</td>
</tr>
</tbody>
</table>

Table A3.2: Optimization properties for model 2 and model 3

<table>
<thead>
<tr>
<th>Rotor diameter [m]</th>
<th>Large variation</th>
<th>Small variation</th>
<th>Wind predictability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum value</td>
<td>143</td>
<td>141</td>
<td>144</td>
</tr>
<tr>
<td>Minimum value</td>
<td>137</td>
<td>136</td>
<td>138</td>
</tr>
<tr>
<td>Average value</td>
<td>141</td>
<td>139</td>
<td>141</td>
</tr>
</tbody>
</table>
Layout optimization

In this case, PSO algorithm was implemented and the results are shown in the tables below:

Table A3.3: Optimization properties for model 1

<table>
<thead>
<tr>
<th>Number of turbines</th>
<th>Scenario 1</th>
<th>Scenario 2</th>
<th>Scenario 3</th>
<th>Scenario 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum value</td>
<td>72</td>
<td>64</td>
<td>68</td>
<td>63</td>
</tr>
<tr>
<td>Minimum value</td>
<td>65</td>
<td>56</td>
<td>62</td>
<td>58</td>
</tr>
<tr>
<td>Average value</td>
<td>69</td>
<td>59</td>
<td>64</td>
<td>60</td>
</tr>
</tbody>
</table>

Table A3.4: Optimization properties for model 2 and model 3

<table>
<thead>
<tr>
<th>Number of turbines</th>
<th>Large variation</th>
<th>Small variation</th>
<th>Wind power predictability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum value</td>
<td>70</td>
<td>70</td>
<td>72</td>
</tr>
<tr>
<td>Minimum value</td>
<td>61</td>
<td>62</td>
<td>62</td>
</tr>
<tr>
<td>Average value</td>
<td>66</td>
<td>65</td>
<td>66</td>
</tr>
</tbody>
</table>

A4 – chapter 4 – Layout optimization

Figure A4.1: OWF layout for model 2
Figure A4.2: OWF layout for model 3