THE INFLUENCE OF TRIM ON THE DIRECTIONAL STABILITY OF A RO-RO SHIP IN SHALLOW WATER

by

W. Beukelman

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CONTENTS

Abstract

On account of the disaster with the 'Herald of Free Enterprise' the Shiphydromechanics Laboratory of the Delft University of Technology carried out an investigation into the directional stability of a Ro - Ro ship as function of waterdepth, trimcondition and forward speed. This study was commissioned by the 'Directoraat - Generaal Scheepvaart en Maritieme zaken (DGSM)' of the Ministry of Public Works in The Netherlands.
Forced oscillations were carried out by means of a Planar Motion Mechanism (PMM) with the model of a Ro-Ro ship at four different waterdepth's.
Except the evenkeel position, four trimconditions, including one for trim by stern were also taken into consideration.
The model was equipped with one rudder and three propellers.
After measurement of the static drift coefficients the hydrodynamic derivatives were determined for both the sway- and yaw motion. The different coefficients together with the stability roots are presented in tabular form for three low frequencies of oscillation related to each condition considered. For all these conditions it was concluded that the vessel possessed negative directional stability characteristics.

1. Motive

On 6 March 1987 the roll on/roll off passenger and freight ferry 'Herald of Free Enterprise' capsized 23 minutes after departure from the inner harbour of Zeebrugge. During the last moments before capsize the ship turned rapidly to starboard against the rudder. She capsized 90° to port and came to rest in shallow water some 930 metres from the channel centreline.
After passing the "Zand 1" buoy the quartermaster reported that he was having difficulty with the handling of the ship and that she was not answering the helm. Very soon afterwards capsizing started. Also in the turn to starboard which followed, the ship did not answer the helm to port.

1.1 Simulation carried out by BMT

The hydrodynamic aspects of the disaster have been investigated by 'Britsh Maritime Technology Ltd' for which use has been made of computer modelling, model tests and a full scale trial on a sister ship, the 'Pride of Free Enterprise' to check a number of suppositions [1]. The research had a twofold purpose: at first purpose was to investigate the water ingress through the opened bow doors and secondly the dynamic aspects of the capsize which followed. Because there was some doubt about the exact draught and the ship's trim at the time of the accident, model tests were performed to determine the water ingress on the main vehicle deck for a series of draughts and trim conditions.
Computer simulations suggested that the likely speed of the ship as it approached the accident site was somewhere between 15 and 18 knots. Estimates of squat and trim based on calculations and model tests, see Figure 1 [1], confirm that at high speed the ship shows significant sinkage at the bow more than at the stern which is augmented by shallow water effects.
For a speed of more than 16 knots the bow sinkage is predicted to be of the order of one metre which for a static freeboard at the bow of 2 metre means a very substantial reduction of the freeboard.
The 'Herald of Free Enterprise' possessed adequate power to achieve speeds between 16 and 18 knots in the approach channel at a waterdepth of 16.5 metre. The Froude-number related to the waterdepth is for a waterdepth of 16.5 metre and a ship speed of 18 knots: $\frac{U}{\sqrt{gh}} = 0.73$, so that significant shallow water influences may be expected for such a situation.
Moreover the height of the bow wave is also important in reducing the effective freeboard substantially. See Figure 2 from [1].
Combination of the static bow trim, which at departure from the harbour amounts about 0.80 m, the sinkage and trim by hydrodynamic influences and the bow wave caused the ingress of water through the opened bow doors.
This effect took place as soon as a certain critical speed rela-

ted to waterdepth and ship speed was achieved as shown in Figure 3 from [1].
Modeltests showed an ingress of substantial quantities of water on the vehicle deck at that critical speed, after which the model maintained position at a heeling angle of about 30° to 40°. At the end of the turn the starboard propeller was above the waterlevel and the equivalent of 1600 ton water on the vehicle deck was present.

Very important is the following conclusion of BMT [1]:
No problems in steering were encountered with the self propelled model before water entered the vehicle deck. As speed increased substantial quantities of water entered the vehicle deck once a 'critical speed' had been reached. The ship heeled to port after water ingress which heeling had been enlarged by centrifugal forces during the uncontrollable turn to starboard. The combined effect of heeling by water ingress on deck and the dynamic effects during the sheer to starboard caused a rapid capsizing: 51 seconds at full scale.

The certification that no steering problems could be established with the modeltests, at least not before water ingress on the vehicle deck, does not seem to fit with the warning of the helmsman of the 'Herald of Free Enterprise'.
It is not known whether the test with the self propelled model was carried out at the 'self propulsion point of ship'. If no correction was applied for the relatively larger friction coefficient of the model, for instance by means of an air-propeller or a small added propeller under the model, then a more significant influence of the model-rudder upon the directional stability may be expected on account of the relatively higher load of the model-propellers.
In other words: the model of the 'Herald of Free Enterprise' might have had a substantially higher directional stability than the ship itself. So one might also suppose that as a result of initial directional instability the ship made an uncontrollable sheer to starboard in such a way that due to centrifugal forces a heeling angle to port side arose, allowing asymmetrically water ingress in large quantities.

1.2 Previous research

In the past research has been performed by Gerritsma [2] into the directional stability of a Todd 60 series model. For this case hydrodynamic derivatives were determined as a function of draught and forward speed by means of PMM tests.

These experiments were carried out in deep water for two conditions namely without rudder and propeller and with rudder and rotating propeller. In the last case a positive directional stability could generally be established for the ship type considered.
It also appears that the influence of forward speed on the direc-

tional stability is small compared to that of the draught variation in terms of the load condition and the trim.
Without rudder and propeller directional instability was always established except for the condition of trim by stern.
The influence of trim on the hydrodynamic derivatives has also been investigated by Inoue et al as reported in [3] and [4].
From these studies it appears that just as from Gerritsma's study [2] the lateral drift force increases for trim by the stern.
Also research carried out by Beukelman [5] and [6] shows that for trim by the stern the drift force indeed increases but the distribution over the ships's length is more symmetrical fore and aft resulting in a reduction of the drift force moment.
The influence of waterdepth on the hydrodynamic derivatives for manoeuvring has been investigated by Hirano et al [7] and by Beukelman and Gerritsma [8].
These investigations found an increase of the damping coefficient (drift force coefficient) Y_v' and the added mass coefficient $Y_{\dot{v}}'$ with decrease of the waterdepth. Measurements and strip theory calculations taking into account the influence of restricted waterdepth agree reasonably except for the damping coefficient Y_v' in the aft part of the ship.

Bishop and Price published a theoretical study entitled 'On the dangers of trim by the bow' [9]. They put forward there that 'the vessels loss by hydrostatic instability was preceded by loss of control. She (Herald of Free Enterprise) became directionally unstable at high speed in shallow water while trimmed by the bow'.
With the aid of linear equations of motion for the ship motion in the horizontal plane the authors put forward as condition for positive stability of the yawing, swaying and rolling system:

$$\rho g \nabla GM\ [Y_v N_r + (mU - Y_r) N_v] > 0 \qquad (1)$$

The first factor should be positive because the metacentric height GM > 0.
The term between the brackets is the well known criterium of stability for the coupled yaw and sway motion for which the separate stability derivatives Y_v, N_r, Y_r, N_v only depend on the underwater hull form and consequently also on trim and sinkage; m is the mass and U the forward speed of the ship.

The stability criterion:

$$[Y_v N_r + (mU - Y_r) N_v] > 0 \qquad (2)$$

may be written as function of the trim

$$\gamma = \frac{T_{APP} - T_{FPP}}{T_m} \qquad (3)$$

as follows:

$$mU_c(\gamma) = \left[\frac{Y_v(\gamma)N_r(\gamma)}{-N_v(\gamma)} \right] + Y_r(\gamma) \tag{4}$$

in which U_c = critical speed with respect to stability.
After some suppositions about the stability derivatives and their dependence on γ analysis of condition (4) leads Bishop and Price to the conclusion that trim by stern increases the critical speed above which instability appears.

1.3 Proposed research

The supposition mentioned under 1.1 and discussed in the 'Commissie Zeegaande Ro-Ro schepen' was the motive for research into the influence of waterdepth, trim and speed on the directional stability of the 'Herald of Free Enterprise' on modelscale.
Generally it seemed desirable to carry out qualitative research into the influence of trim, speed and waterdepth on the directional stability of a Ro-Ro passengership.
In this case forced oscillation tests by means of which the linear stability derivatives may be determined were selected.
Harmonic motion tests with the shipmodel were carried out for both sway and yaw motions.
These tests were performed with rotating propellers in such a way that the thrust agreed with the self propulsion point of ship in order to avoid scale influence in this case. The stability of the system is easy to quantify based on the measured stability derivatives. For instance by means of the criterion of stability or by requirement that the real part of the roots σ should be negative. See (8).
A complication at these modeltests is caused by the restricted width of the towing tank while at shallow water the critical speed is also determined by the breadth/waterdepth ratio of the waterway.
The speeds to be investigated have been chosen in such a way that the critical speed as result of bottom- and wallinfluence was not achieved so that also the waterlevel fall beside the model could be neglected. See pt. 4.1 and the tables 10a and 10b.
For the largest waterdepth only the even keel and one trim condition and 3 high speeds have been taken into consideration with the maximum revolution number to obtain some impression about the tendency of the directional stability at these high speeds.

2. Modeldata and testconditions

The model is almost equal to the 'Herald of Free Enterprise', but not an exact copy of this ship.
Data of ship and model are presented in table 1. The model was manufactured to a scale 1:40. The longitudinal radius of inertia was known for each trim condition and amounts about $0.25L_{pp}$.

For the testconditions see table 2.
Including even keel four trim conditions were considered of which three were with bow trim and one with trim by the stern.
For all trimconditions, waterdepths, oscillation frequencies and static drift angles three forward speeds have been considered namely Fn = 0.08, 0.10 and 0.12. Three high speeds corresponding to ship speeds of 14, 16 and 18 knots have been tested only for the highest waterdepth, the even keel condition (nr. 0) and the smallest bow trim (condition nr. I).
At these high speeds there was a substantial fall of the water level beside the constrained model. The resistance for this condition was so high that insufficient thrust at the maximum number of revolutions could be delivered. Nevertheless this condition has been taken into consideration as a special addition to determine a possible tendency in directional stability at speeds for this ship type when leaving the harbour.
For oscillation use has been made of the Planar Motion Mechanism (PMM) which was connected to a lengthened bridge construction under the towing carriage in such a way that the required oscillations at shallow water could be realized. See figure 4.
The model was connected to the legs of the PMM by means of dynamometers at two points situated 0.5 m fore and aft the centre of gravity for the trim-condition considered.
These dynamometers, situated in the ship longitudinal symmetrical plane were sensitive for forces perpendicular to this plane.
Before carrying out tests for a new trim condition and waterdepth these dynamometers were turned 90 degrees to measure the resistance in order to adjust the required number of propeller revolutions with respect to the correction for the frictional resistance. Two motions were carried out by the PMM,viz. sway and yaw, for 3 frequencies: ω = 0.25, 0.50 and 0.75 rad/s.

The experiments were executed on 4 waterdepths viz. 1.4, 1.6, 2.0 and 2.2 times the models draught in even keel position.
Static drift tests were performed for all trim conditions considered and waterdepths with the rudder in zero position and rotating propellers. The adjusted drift angles for these tests were β = ± 2, 4 and 6 degrees. Except for the largest waterdepth and the three highest model speeds the number of revolutions of the propellers had been adjusted to a value corresponding to the ship self propulsion point. For this purpose the frictional resistance correction was determined according to the procedure as denoted by the ITTC 1973 friction extrapolator:

$$C_V = \frac{0.075}{(\log R_e - 2)^2} \qquad (5)$$

For the adjusted number of propeller revolutions and the frictional resistance correction, see table 3.

3. Results of experiments

The transverse drift force Y and the moment N as measured for the oscillatory motions and the static drift experiments are presented in the tables 5-9 in a non-dimensional way as Y_v', N_v', Y_r', etc. This non-dimensional values for the derivatives correspond to those for non-dimensional forces and moments:

$$Y' = \frac{Y}{\frac{1}{2}\rho U^2 \; L_{pp}^2} \qquad \text{en} \qquad N' = \frac{Y}{\frac{1}{2}\rho U^2 \; L_{pp}^3} \qquad (6)$$

For a review of the way in which each of the measured derivatives had been made non-dimensional see table 4.
The mass moment of inertia I_{zz} with respect to the vertical z-axis as used in the derivative $(I_{zz} - N_{\dot{r}})$ has been determined for each condition. The structural mass radius of inertia had an average value $k_{zz} = 0.25L$.

The static drift test showed a strong non-linear behaviour for all conditions.
As an example the driftforce-coefficient Y_β' has been presented in figure 5 for h/T = 2.2 and trimcondition IV (trim by the stern).
For calculation of the directional stability roots in table 5 - 9 the value of the static drift coefficients Y_β' and N_β' is based on the lowest drift angle, that means $\beta = \pm\ 2$ degrees.
There are some exceptions, but generally these values do not differ much from the results obtained from forced oscillation tests, $-Y_v'$ and $-N_v'$. The static and hydrodynamic derivatives determine the roots σ_1' and σ_2' for the straight line stability.

The equation of motion for the sway- and yaw motion of a model with fixed rudder are as follows:

$$\left.\begin{aligned} (m - Y_{\dot{v}})'\dot{\beta}' + Y_\beta'\beta + Y_{\dot{r}}'\dot{r} + (Y_r - m)'r' &= 0 \\ N_{\dot{v}}'\beta' - N_\beta'\beta + (I_{zz} - N_{\dot{r}})'\dot{r}' - N_r'r' &= 0 \end{aligned}\right\} \qquad (7)$$

The stability roots derived from equations (7) are:

$$\left.\begin{aligned} \sigma_1' &= \frac{-B + \sqrt{B^2 - 4AC}}{2A} \\ \sigma_2' &= \frac{-B - \sqrt{B^2 - 4AC}}{2A} \end{aligned}\right\} \qquad (8)$$

in which:

$$A = (m - Y_{\dot{v}})' \quad (I_{zz} - N_{\dot{r}})' - Y_{\dot{r}}'N_{\dot{v}}'$$

$$B = Y_{\beta}'(I_{zz} - N_{\dot{r}})' - N_{r}'(m - Y_{\dot{v}})' + N_{\beta}'Y_{\dot{r}}' - N_{\dot{v}}'(Y_{r} - m)'$$

$$C = N_{\beta}'(Y_{r} - m)' - Y_{\beta}'N_{r}'$$

The calculated stability roots are presented in the tables 5 - 9, one set derived from the static coefficients Y_{β}' and N_{β}' together with the remaining hydrodynamic derivatives and one set determined with hydrodynamic derivatives only, which means $-Y_{v}'$ and $-N_{v}'$ instead of Y_{β}' and N_{β}'.
The differences between these 'static' and 'dynamic' stability roots, σ'(stat) and σ'(dyn) are generally small with some exceptions. This could be expected due to the small differences between $-Y_{v}'$ and Y_{β}' and $-N_{v}'$ and N_{β}'.
As an example σ_{1}'(dyn) with positive value has been plotted in the figures 6 and 7 as function of speed(Fn) for each waterdepth, trim-condition and $\omega = 0.25$.
Figure 8 shows the stability root σ_{1}'(dyn) in the same way for $h/T = 2.2$ in case of the two trim-conditions considered and three higher speeds.

4. Discussion of results

4.1 Restricted waterconditions

At first it was the intention to consider higher speeds viz. Fn = 0.15, 0.20 and 0.25 but on account of the strong bottom and wall influence of the towing tank these speeds were not tested.
At these speeds a strong water level fall could be established leading to a substantial trim and sinkage if the model had not been constrained. In such conditions the model power and number of propeller revolutions were too much restricted to apply the required friction correction. It was therefore decided to lower the speeds so that no significant wall and bottom influence should arise. See table 10a and 10b.

Previous investigation on deep water carried out by Gerritsma [2] showed that the speed influence on the hydrostatic and dynamic derivatives and the stability roots were small.
From the present research, however it appeared that the influence of restricted water conditions is rather important especially related to trim.
Further investigations into the influence of the bottom with or without channel walls partly under water on the trim, sinkage, hydrodynamic derivatives and stability roots as function of high forward speeds are required for this ship type.
It should be remarked that at high speeds (Fn = 0.205, 0.233 and 0.262 corresponding to ship speeds of V = 14, 16 and 18 knots) which for h/T = 2.2 only have been taken into account the above

mentioned phenomena as a result of bottom and wall influence occurred. The adjusted maximum number of revolutions of the propeller was insufficient related to the speeds considered.
The reason for still taking these speeds into consideration is presented under pt.2.

To give an impression of the critical boundary speeds, sinkage and return speeds calculations have been carried out for all the speeds considered in accordance with the method presented by Bouwmeester in [10] and drafted by Schijf. The critical boundary speed for the condition considered has been determined with the graphic curves (α = 1.1) of Schijf as presented in figure 9 (figure 36 from [10]) which curves have been determined according to:

$$F_r = V_{cr}/\sqrt{g\bar{h}} = 0.78[1 - A_s/A_c]^{2.25} \qquad (9)$$

in which:

$A_c = B_o h_o$ = undisturbed channel profile surface

A_s = surface of submerged part of the midship cross section

V_{cr} = critical boundary speed in a channel

F_r = ratio of critical boundary speed in a channel to critical speed on shallow water $c = \sqrt{g\bar{h}}$

α = correction coefficient

$\bar{h}$ = average waterdepth (= A_c/B_o)

Three conditions are distinguished:

1. $U_m < V_{cr}$

The water level fall is iteratively determined with:

$$Z/\bar{h} = \frac{U_m^2}{gh}\{\alpha(1 - A_s/A_c - Z/\bar{h})^{-2} - 1\}/2 \qquad (10)$$

and the return flow u with:

$$u/\sqrt{g\bar{h}} = U_m/\sqrt{g\bar{h}}\ \{1/(1 - A_s/A_c - Z/\bar{h}) - 1\} \qquad (11)$$

2. $U_m = V_{cr}$

Then will be:

$$Z_{cr}/\bar{h} = \frac{1}{3}\{1 - A_s/A_c - V_{cr}^2/g\bar{h}\} \qquad (12)$$

and

$$U_{cr}/\sqrt{gh} = \{\frac{2}{3}(1 - A_s/A_c + \tfrac{1}{2}V_{cr}^2/gh)\}^{\frac{1}{2}} - V_{cr}/\sqrt{gh} \qquad (13)$$

3. $U_m > V_{cr}$

The maximum transported quantity of water per time unit is now first determined:

$$Q_{max} = V_{cr}A_c \qquad (14)$$

and the waterlevel fall than amounts:

$$Z = \frac{-Q_{max}/(U_m + u_{cr}) + A_c - A_s}{B_o} \qquad (15)$$

The calculated values are presented in the tables 10a and 10b for the conditions considered. From these it appears that for the three lowest speeds, Fn = 0.08, 0.10 and 0.12, the model speed remains far below the critical boundary speed, so $U_m < V_{cr}$. The average water level fall amounts about 0.005 m.

The comparable speed on unrestricted water is presented by $\frac{u + U_m}{\sqrt{gL}}$

For the three highest speeds at h/T = 2.2 it yields, that $U_m > V_{cr}$. while water level fall varies from 0.065 to 0.103 m. The lowest one of these three speeds, Fn = 0.205, corresponds on unrestricted water to:

$$Fn = \frac{u + Um}{\sqrt{gL}} = 0.276$$

which means a ship speed of 18.9 knots.

4.2 Hydrostatic- and dynamic derivatives.

From the tables 5 to 9 it appears that the values for the hydrostatic derivatives Y_β'and N_β' generally agree well with respectively the measured hydrodynamic derivatives $-Y_v$' and $-N_v$' for the lowest and considered frequency of oscillation $\omega = 0.25$.
An exception should be made for the even keel condition (0) for which the hydrodynamic derivatives are a good deal lower than the hydrostatic derivatives.
The influence of waterdepth on the hydrostatic- and dynamic force coefficients may be characterized as follows:

Y_β' and $-Y_v$' increase with decreasing h/T

while the moment-coefficients N_β' and $-N_v$' experience little influence of variation in waterdepth. This agrees with what has been found in [5] and [6].

The influence of forward speed on the hydrostatic- and dynamic derivatives is small for both the drift force and moment.
This only concerns the three lowest speeds up to Fn = 0.12. For the three highest speeds at h/T = 2.2 the moment-coefficients N_β' and $-N_v$' appear to decrease with increasing speed.

The influence of trim on the hydrostatic- and dynamic coefficients may be described as follows:

- the drift force coefficients Y_β' and $-Y_v$' are independent of trim by the bow, but they increase for trim by stern.
- the drift force moment coefficients N_β' and $-N_v$', however, mostly increase with trim by the bow and reduce considerably for trim by stern.

Similar behaviour of the hydrostatic- and dynamic coefficients related to trim in deep water was also ascertained in [2] and [3].

4.3 Stability roots

The static- and dynamic stability roots are presented in the tables 5 to 9 for all conditions considered. From these tables it appears that the lowest oscillation frequency generally shows the lowest absolute value for the stability roots with exception of the three highest speeds at the deepest water. For this condition that relation is not clear and sometimes even reversed.

It also appears from the tables 5 - 9 that generally good agreement is shown between the 'static' and 'dynamic' stability roots. Exceptional is the even keel condition nr.0 which phenomenon was also valid for the hydrodynamic derivatives as denoted under pt. 4.2. As most principal characterization it appears from the tables 5 - 9 that for all conditions a negative directional stability should established because in no case did both roots show a negative value. Henceforth for discussion of the influence of speed, waterdepth and trim the degree of negative directional stability or in other words the degree of directional instability will be used as a basis for comparison. For this reason the positive value of the dynamic stability root σ_1'(dyn) is taken into consideration.

It should be remarked that directional instability had not been observed with experiments related to the 'Herald of Free Enterprise' as presented by Dand in [1].

The dynamic stability root σ_1'(dyn) as function of forward speed is shown in figures 6 and 7 for each waterdepth related to the lowest frequency of oscillation $\omega = 0.25$.
In figure 8 the basis for h/T = 2.2 has been extended up to the highest speed considered (Fn = 0.262, V = 18 knots).
From the figures it is clear that directional instability reduces with increasing speed up to a certain value (Fn ≈ 0.18) after which the directional instability increases with forward speed. One should, however, keep in mind that the conditions at the higher speeds were not completely realistic as discussed under pt. 4.1.
The same tendency may also be perceived for both lowest waterdepths in case of some trim conditions related to lower speeds (Fn = 0.08 - 0.12).
The influence of waterdepth on the directional instability in the considered area h/T is in general small (see the figures 6 and 7). In deep water some improvement may be observed, especially for speed Fn = 0.12.
Trim by the bow appears to provide little improvement in the directional stability for both low speeds in shallow water (h/T = 1.4 and 1.6) and for Fn = 0.12 in h/T = 2.0 and 2.2.
In the remaining cases trim by the bow causes a deterioration of the directional stability. Trim by the stern, condition IV, generally provides a pronounced improvement in directional stability, especially as result of the strong reduction of the hydrostatic- and dynamic moments.
For manoeuvrability preference is given to this condition because the rudder will react more effectively even if the ship is still lightly directionally instable.
It should be remarked that a directionally instable ship may be made stable in a cybernetic way by a correct choice of the constants of the automatic pilot. The question, however, remains up to which conditions such a control still may be valid.
It is questionable if for $\sigma > 1$ to 2 the ship considered remains manoeuvrable. Definite answer about this can only be obtained by full scale simulation manoeuvring experiments.

5. Conclusions and recommendations

From the preceding investigation the following conclusions and recommendations may be derived:

1. It appears that for the considered Ro-Ro ship type directional instability could be established for all investigated waterdepths, forward speeds and trim conditions.

2. This directional instability decreases if speed increases up to a certain value (Fn ≈ 0.18) after which probably a strong growth occurs in directional instability.

3. The influence of variation of waterdepth in the directional instability is generally small for the considered area.

4. Trim by the bow shows light improvement of the directional instability in shallow water, but some decrease in deeper water.

5. Trim by the stern generally provides a clear rectification of the directional instability mainly as result of the decrease of the hydrostatic- and/or dynamic moment.
   The rudder will be more effective for this condition.

6. Drift forces increase with decreasing waterdepth and trim by the stern.
   Trim by the bow and speed show little influence on the drift force coefficients.

7. Drift moment coefficients are rather independent of variation of waterdepth and forward speed.
   The drift moment increases with bow trim and reduces with trim by the stern.

8. It is recommended to carry out further investigations into the influence of restricted waterdepth with and without channel walls under water on the trim, sinkage, hydrodynamic derivatives and stability roots as function of high forward speeds.

9. Restriction of forward speed dependent on the waterdepth is also strongly recommended.

6. Nomenclature

A	propeller blade area
A_c	area of undisturbed cross channel profile
A_s	area of submerged part of the ship's cross section
B	breadth, beam
B_o	width of waterlevel of undisturbed channel
D	propeller diameter

$Fn = \frac{U}{\sqrt{gL_{pp}}}$	Froude number
F_r	ratio of critical boundary speed in a channel to critical shallow water speed
G	centre of gravity
g	acceleration due to gravity
h	waterdepth (maximum)
$\bar{h}$	average waterdepth (= A_C/B_O)
I	mass moment of inertia
k	radius of mass moment of inertia
L_{PP}, L	length between perpendiculars
L_{OA}	overall length
m	mass of ship model
N	moment of drift force
n	number of revolutions
P	pitch
R_a	friction correction
$R_e = \frac{UL}{\nu}$	Reynolds number
r	yaw velocity
T	draught
t	trim (=T_{VLL} - T_{ALL})
U	forward speed
u	return speed or flow
V_{cr}	critical boundary speed
v	transverse forward speed component (positive to starboard)
x, y, z	coordinate-system fixed to model

Y	drift force
Z	waterlevel fall
z	number of propeller blades
α	correlation coefficients
β	drift angle
γ	trim coefficient
ν	kinematic viscosity coefficient
ω	circular frequency of oscillation
ρ	density of water
σ	stability root

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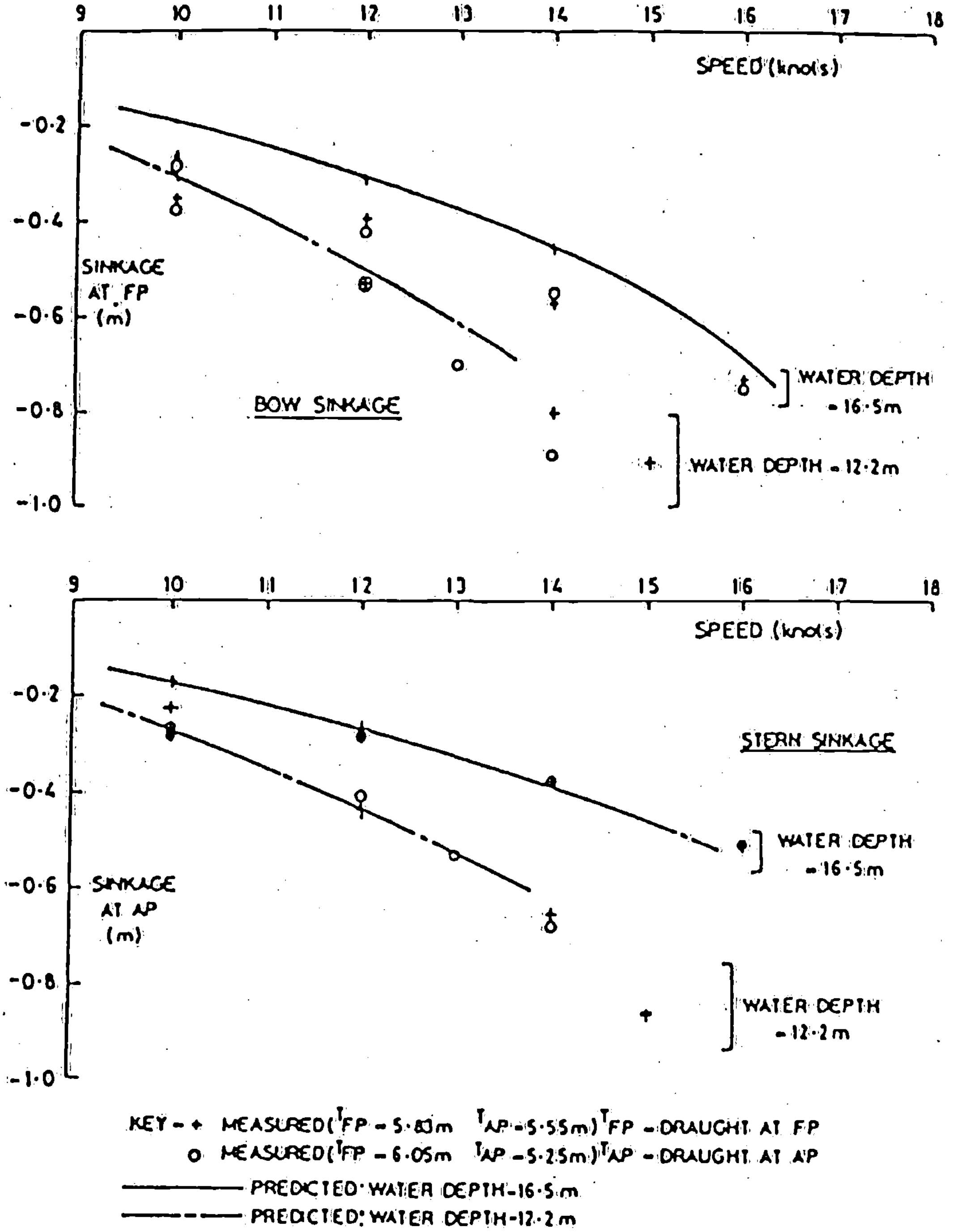
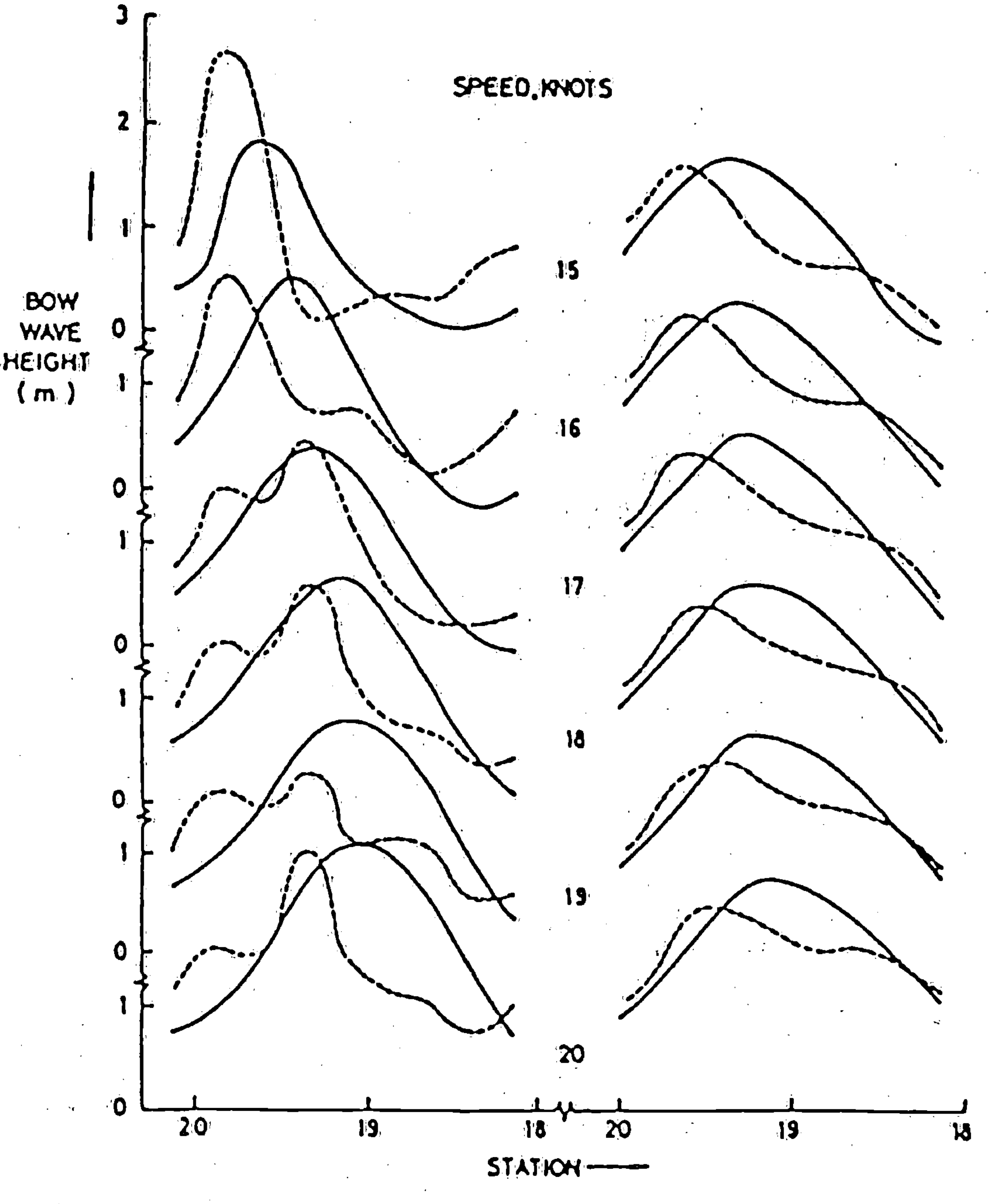


Figure 1. Predicted and measured squat from model tests [1].



Calculated rise of water level along hull side near bow for 'Herald of Free Enterprise'. L H S, non linear theory, R H S, linear theory. Dotted curves with approximate flare correction.

Figure 2. Calculated bow wave profiles [1].

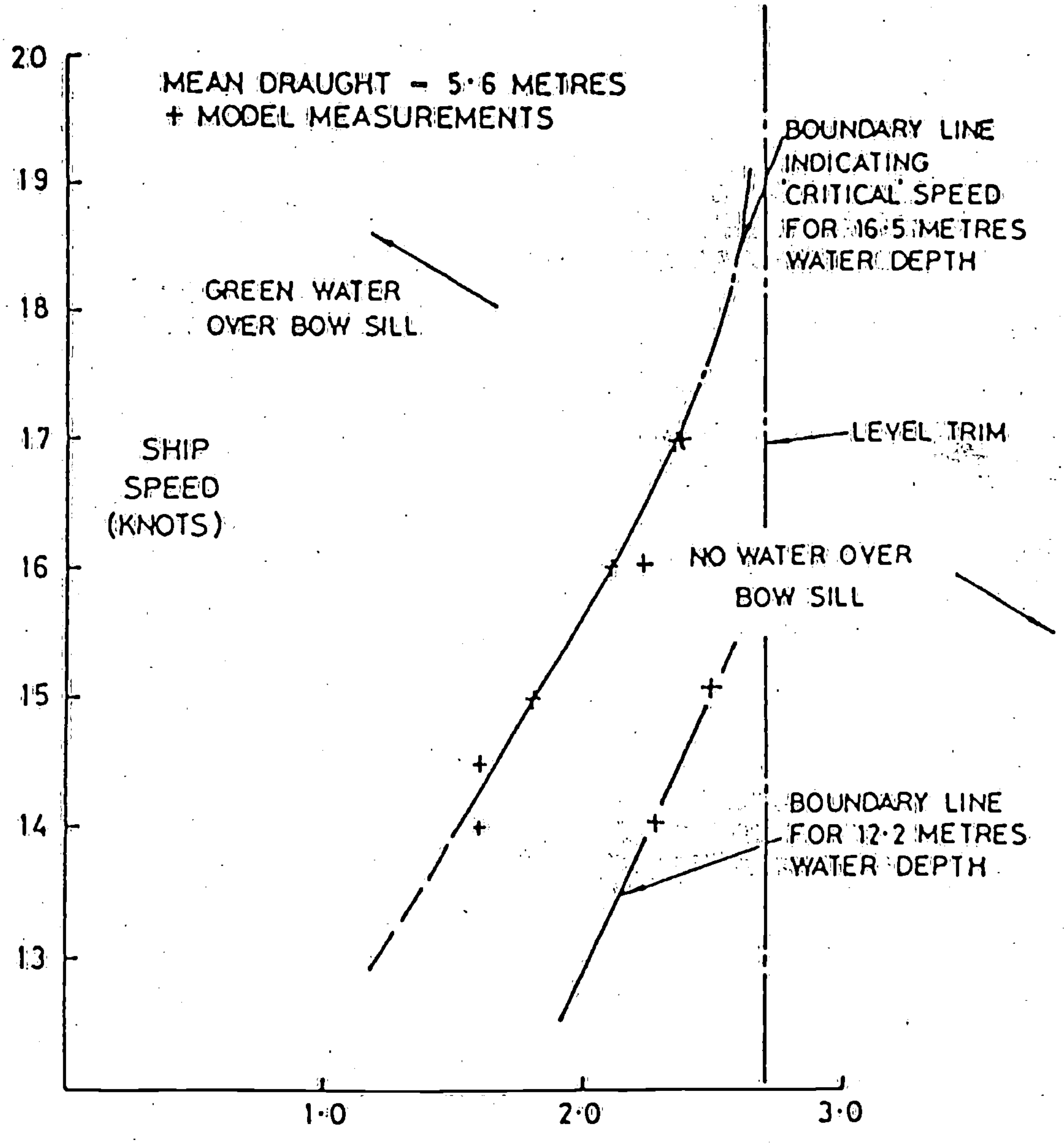


Figure 3. Critical speed plots [1].

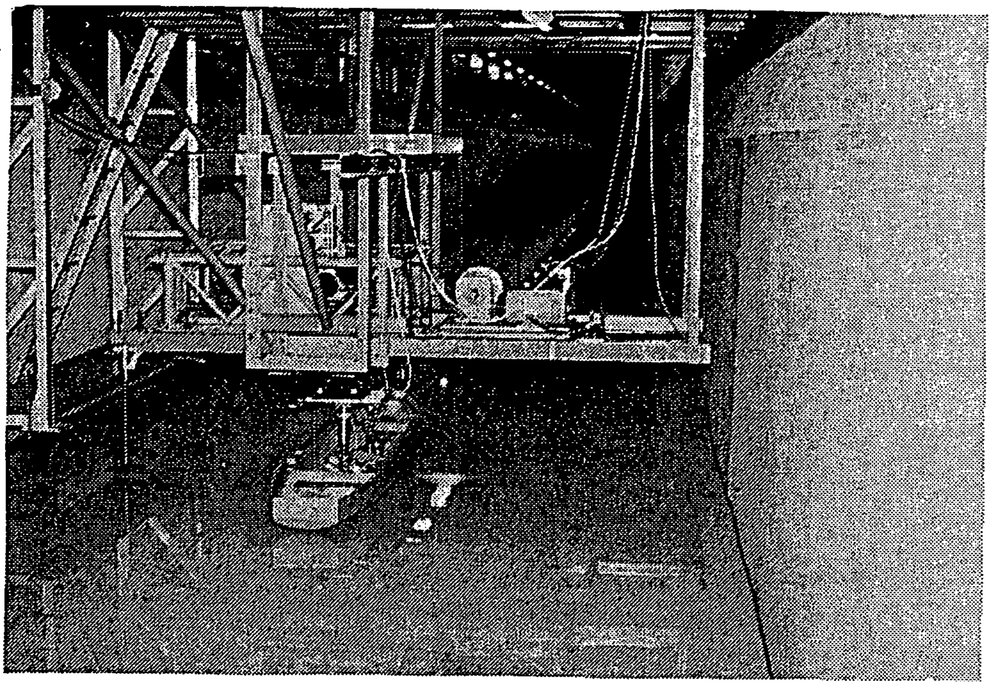


Figure 4. Experimental set up with PMM and ship model

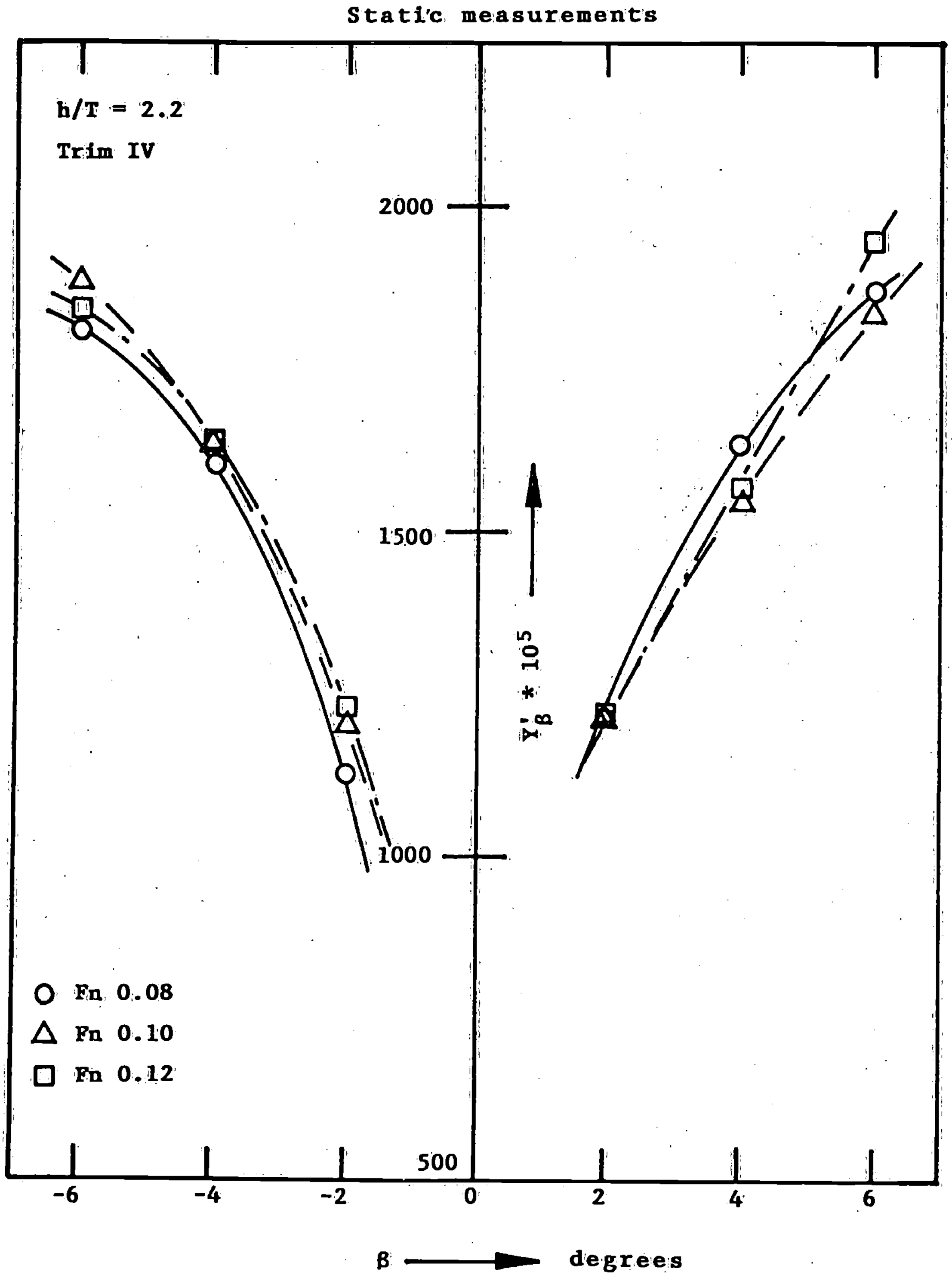


Figure 5. Drift force coefficient Y_β' as function of drift angle for h/T = 2.2 and trim condition IV.

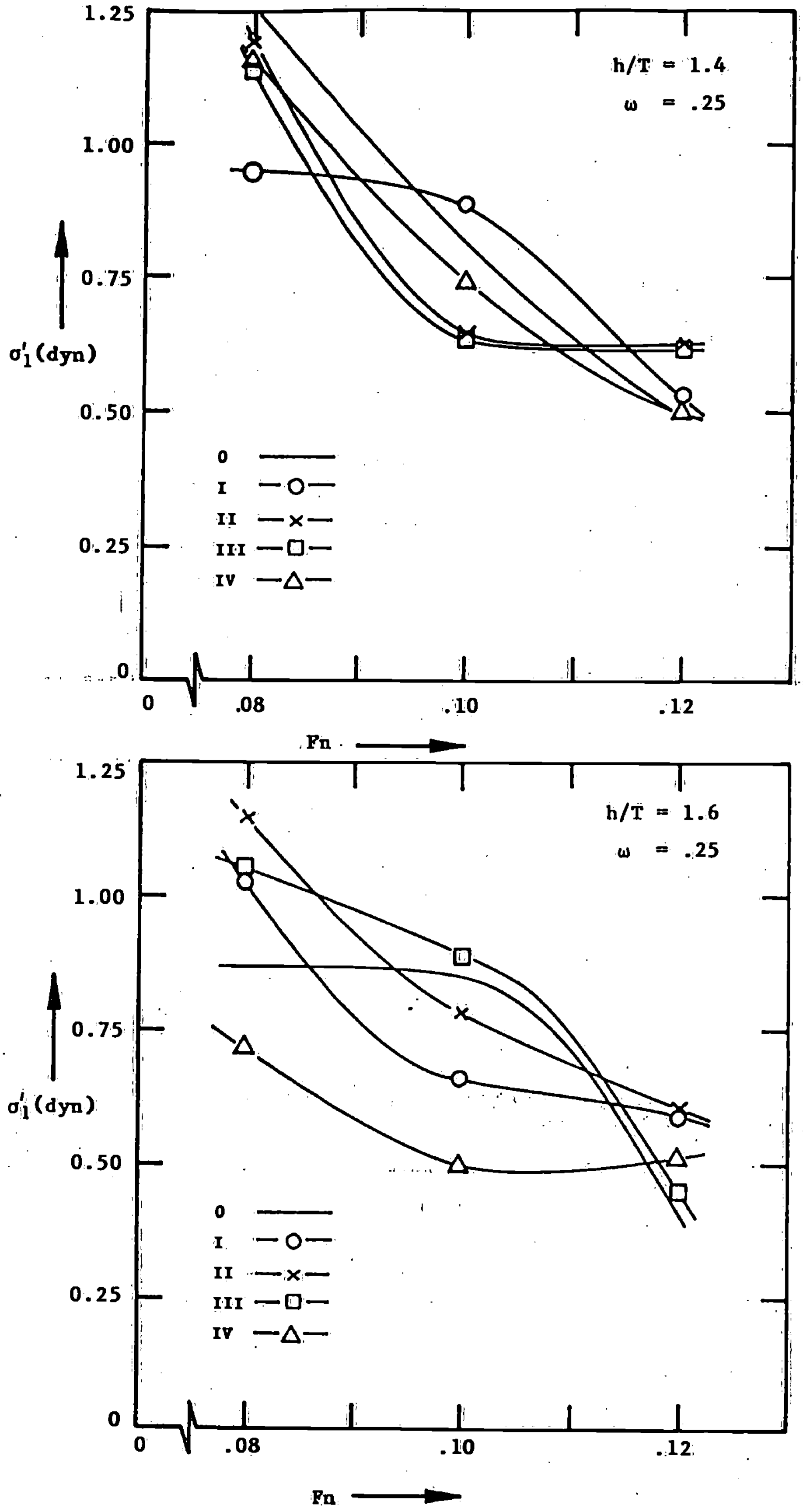


Figure 6. The dynamic stability root σ_1(dyn) as function of forward speed for h/T = 1.4 and 1.6.

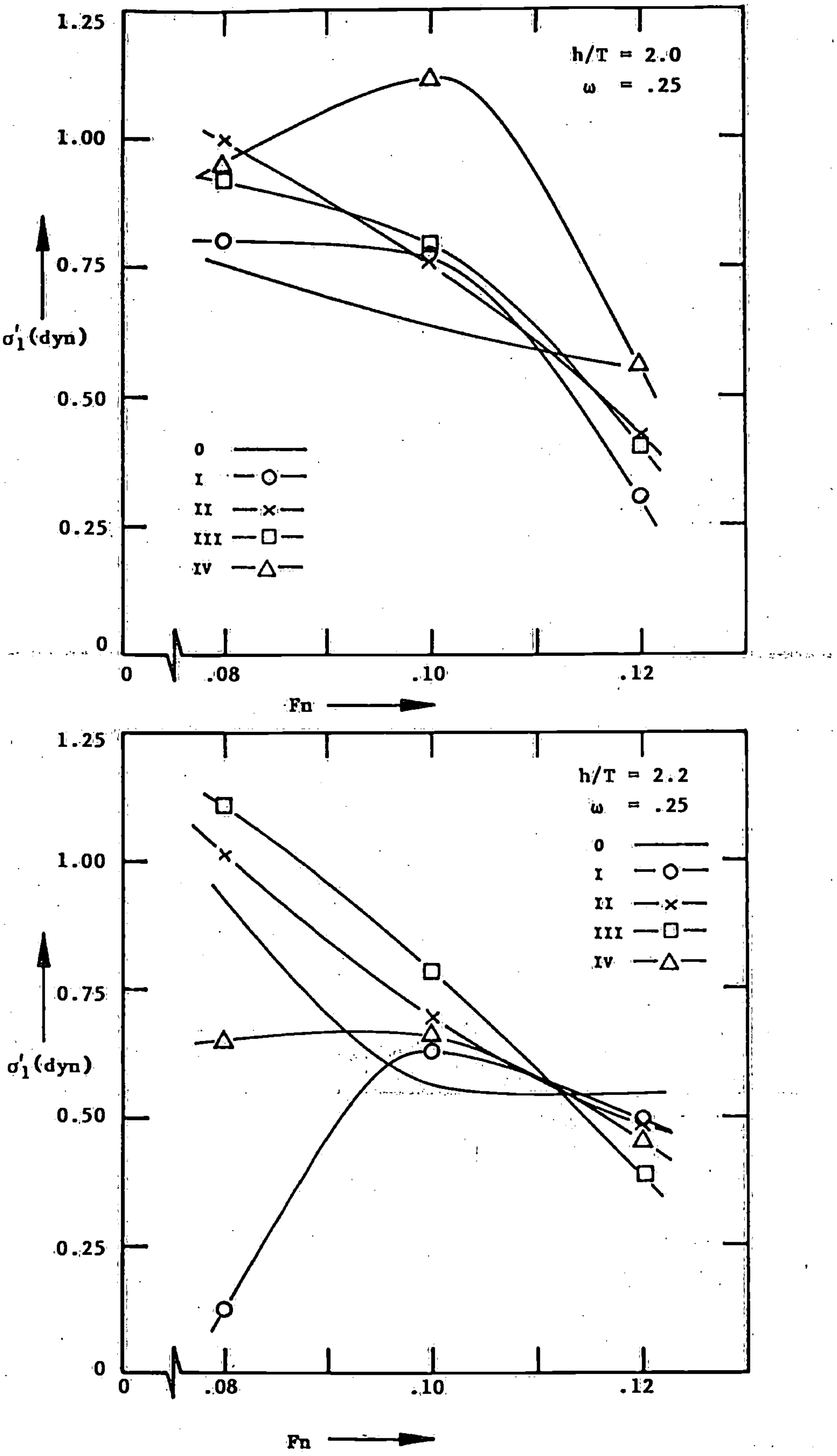


Figure 7. The dynamic stability root σ_1(dyn) as function of forward speed for h/T = 2.0 and 2.2.

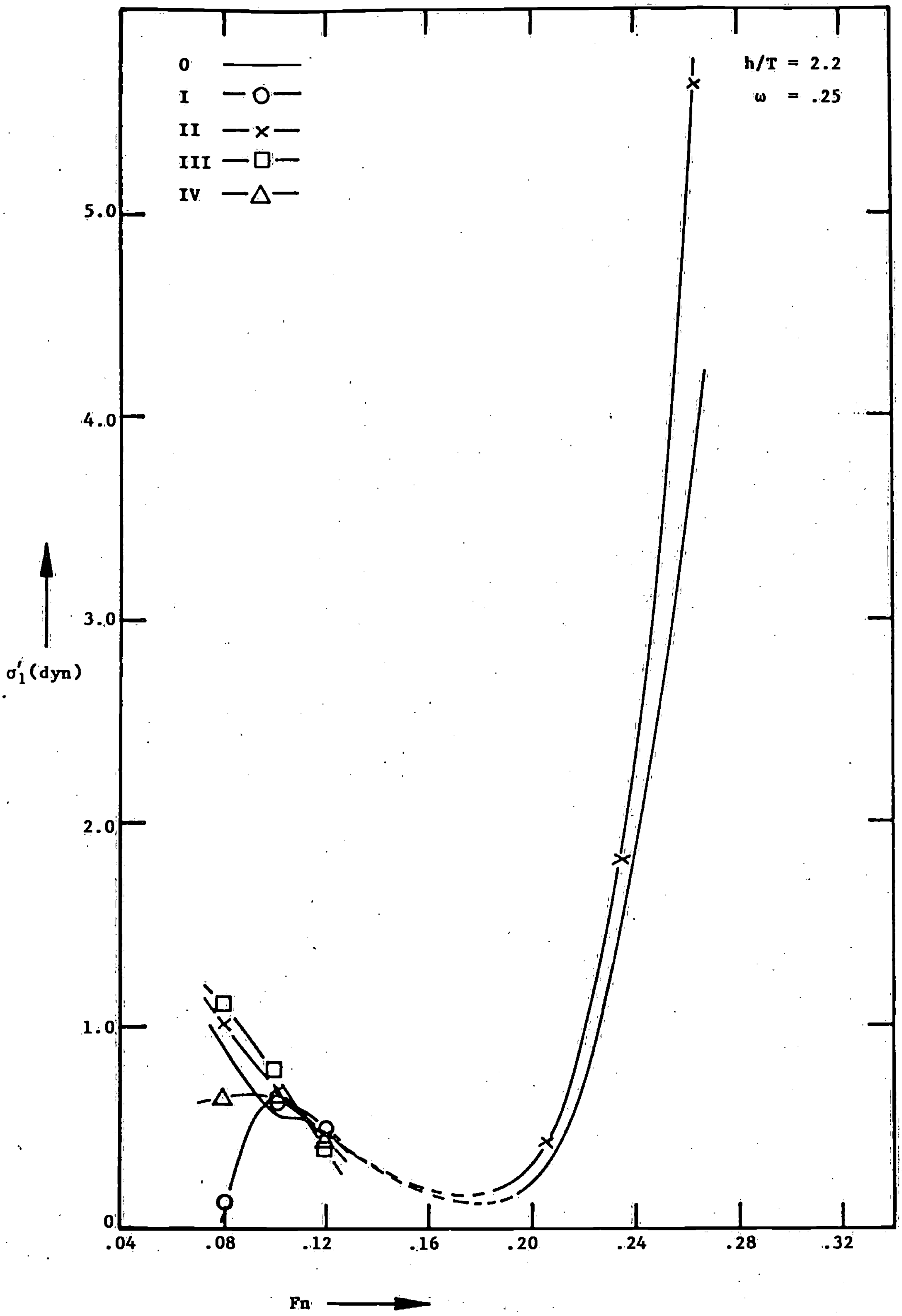


Figure 8. The dynamic stability root σ_1(dyn) as function of forward speed for h/T = 2.2.

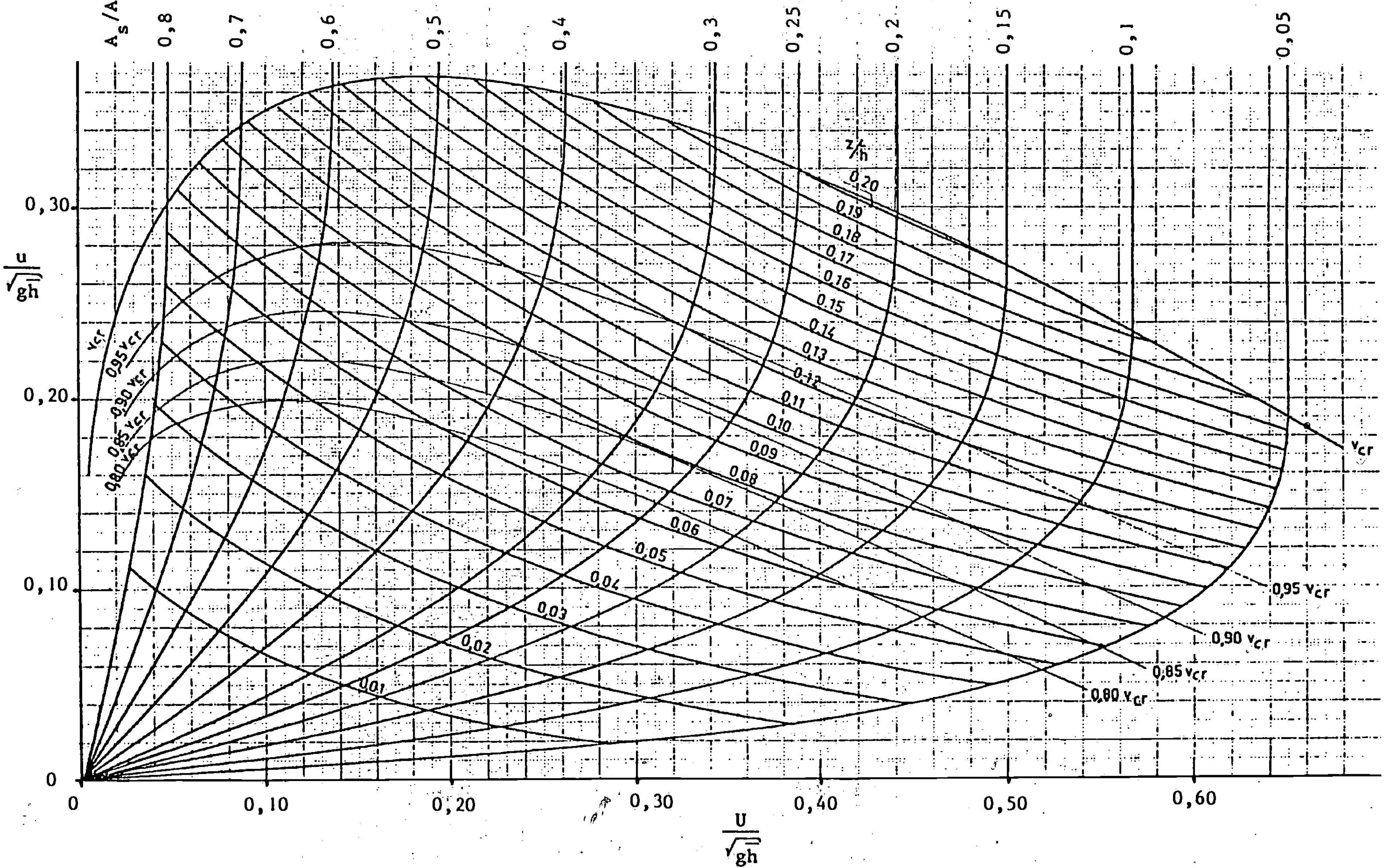


Figure 9. Diagram for speeds and waterlevel related to ships in a channel (Schijf, $\alpha = 1.1$).

Table 1. Main dimensions of ship, model and propeller.

		Ship	Modelnr. 280
Length over all	L_{OA}	132.000 m	3.300 m
Length between perpendiculars	L_{PP}	126.100 m	3.152 m
Beam	B	22.700 m	.567 m
Draught (even keel)	T	5.685 m	.142 m
Displacement		8874 ton	138.660 kg
LCB forward of $L_{PP}/2$		-2.870 m	-0.720 m
Radius of gyration for yaw (adjusted for each condition)	k_{zz}		0.250 L_{PP}
Propeller NSMB - Serie B-380			
Diameter	D		80 mm
Pitch ratio	P_O/D		0.90
Blade area ratio of propeller	A_E/A_O		0.80
Number of blades	z		3
Number of propellers			3
Model scale			1:40

Table 2. Testconditons.

			Ship	Model
Trimcondition	0 (even keel)			
	I (trim by the bow)		0.375 m	0.0094 m
	II (trim by the bow)		0.750 m	0.0188 m
	III (trim by the bow)		1.125 m	0.0281 m
	IV (trim by the stern)		0.375 m	0.0094 m
Speeds	Fn = 0.08		5.5 knots	0.445 m/s
	0.10		6.8 knots	0.556 m/s
	0.12		8.2 knots	0.667 m/s
	Fn = 0.205	for h/T = 2.2 and cond. 0 and 1 only	14 knots	1.140 m/s
	0.234		16 knots	1.300 m/s
	0.262		18 knots	1.460 m/s
Oscillation-frequencies		ω	0.04 rad/s	0.25 rad/s
			0.08 rad/s	0.50 rad/s
			0.12 rad/s	0.75 rad/s
Waterdepth/draught ratio		h/T	1.4	1.4
			1.6	1.6
			2.0	2.0
			2.2	2.2
Drift angles for static tests		β	± 2 degrees	± 2 degrees
			± 4 degrees	± 4 degrees
			± 6 degrees	± 6 degrees

Table 3. Friction correction and number of revolutions.

speed			Friction correction	number of revolutions			
				h/T			
Fn	Ship	Model	R_A in N	1.4	1.6	2.0	2.2
0.080	5.5 kn.	0.455 m/s	0.479	6.25	6.45	6.95	7.10
0.100	6.8 kn.	0.556 m/s	0.706	8.00	8.10	8.55	8.95
0.120	8.2 kn.	0.667 m/s	0.969	9.80	10.1	10.1	10.5
0.205	14 kn.	1.140 m/s	2.476**	-	-	-	26*
0.234	16 kn.	1.300 m/s	3.120**	-	-	-	26*
0.262	18 kn.	1.460 m/s	3.827**	-	-	-	26*

* maximum number of revolutions and insufficient thrust.
** nominale value for deep water.

Table 4. Non-dimensional derivatives.

$$Y_\beta{}' = \frac{Y_\beta}{\frac{1}{2}\rho L^2 U^2}$$

$$N_\beta{}' = \frac{N_\beta}{\frac{1}{2}\rho L^3 U^2}$$

$$(m - Y_{\dot{v}})' = \frac{m - Y_{\dot{v}}}{\frac{1}{2}\rho L^3}$$

$$Y_v{}' = \frac{Y_v}{\frac{1}{2}\rho L^2 U}$$

$$N_{\dot{v}}{}' = \frac{N_{\dot{v}}}{\frac{1}{2}\rho L^4}$$

$$N_v{}' = \frac{N_v}{\frac{1}{2}\rho L^3 U}$$

$$(I_{zz} - N_{\dot{r}})' = \frac{I_{zz} - N_{\dot{r}}}{\frac{1}{2}\rho L^5}$$

$$N_r{}' = \frac{N_r}{\frac{1}{2}\rho L^4 U}$$

$$Y_{\dot{r}}{}' = \frac{Y_{\dot{r}}}{\frac{1}{2}\rho L^4}$$

$$(m - Y_r)' = \frac{m - Y_r}{\frac{1}{2}\rho L^3}$$

Table 5a

$h/T = 1.4$

Coefficients * 10^5

$Fn = 0.08$

LOAD-CONDITION	ω	0	I	II	III	IV
$+Y_\beta'$	-	2450	2069	2216	2213	2391
$+N_\beta'$	-	677	780	812	855	592
$(m-Y_{\dot{v}})'$	.25 .50 .75	2101 2174 2454	2053 2181 2453	2116 2283 2443	2107 2168 2426	2053 2136 2462
$-Y_v'$	.25 .50 .75	2427 2950 5115	2167 2912 4917	2298 2727 5020	2271 2849 4976	2470 2896 5444
$-N_{\dot{v}}'$	25 .50 .75	-66 -58 -52	-74 -48 -46	-57 -13 -44	-91 -57 -47	-121 - 86 - 66
$-N_v'$	.25 .50 .75	680 736 887	752 875 1038	852 900 1051	842 900 1144	564 667 795
$(I_{zz}-N_{\dot{r}})'$	.25 .50 .75	255 203 224	300 226 244	265 221 246	248 234 230	190 181 209
$-N_r'$	.25 .50 .75	-247 -260 -233	-199 -250 -245	-209 -298 -238	-160 -310 -279	-167 -230 -225
$-Y_{\dot{r}}'$	.25 .50 .75	435 229 148	442 228 149	470 237 162	459 258 155	401 194 137
$(m-Y_r)'$	.25 .50 .75	62 324 470	89 385 449	98 394 506	244 295 421	128 362 455
σ_1' (stat.)	.25 .50 .75	1.247 1.615 1.355	.964 1.531 1.359	1.178 1.811 1.368	1.149 1.727 1.586	1.174 1.564 1.346
σ_2' (stat.)	.25 .50 .75	- .920 -1.166 -1.177	- .770 -1.060 -1.042	- .784 -1.066 -1.128	- .868 -1.041 -1.090	- .922 -1.211 -1.145
σ_1' (dyn.)	.25 .50 .75	1.249 1.620 1.331	.944 1.523 1.321	1.193 1.824 1.320	1.134 1.710 1.565	1.152 1.583 1.324
σ_2' (dyn.)	.25 .50 .75	- .910 -1.365 -2.169	- .814 -1.388 -2.089	- .804 -1.261 -2.151	- .889 -1.287 -2.114	- .960 -1.421 -2.289

Table 5b

h/T = 1.4 Coefficients * 10^5 Fn = 0.10

LOAD-CONDITION	ω	0	I	II	III	IV
$+Y_\beta'$	-	2408	2134	2278	2225	2366
$+N_\beta'$	-	631	749	801	811	600
$(m-Y_{\dot{v}})'$	.25	2216	2191	2143	2093	2108
	.50	2140	2212	2166	2183	2183
	.75	2542	2572	2540	2682	2625
$-Y_v'$	.25	2258	2346	1983	2170	2270
	.50	2985	2712	2691	2758	2847
	.75	4936	4785	4737	4920	4507
$-N_{\dot{v}}'$	.25	-61	-69	-56	-95	-118
	.50	-65	-51	-60	-61	- 80
	.75	-77	-81	-72	-92	- 89
$-N_v'$	.25	680	780	812	809	518
	.50	797	904	941	967	699
	.75	1025	1139	1129	1280	925
$(I_{zz}-N_{\dot{r}})'$	.25	283	267	376	305	248
	.50	198	232	222	249	192
	.75	222	241	235	239	214
$-N_r'$	.25	-168	-134	-172	- 72	-154
	.50	-232	-283	-233	-282	-239
	.75	-258	-263	-257	-288	-247
$-Y_{\dot{r}}'$	.25	540	539	626	642	459
	.50	275	273	326	363	289
	.75	191	222	230	216	193
$(m-Y_r)'$	.25	- 31	134	- 97	159	- 76
	.50	241	206	301	219	142
	.75	439	432	462	431	430
σ_1' (stat.)	.25	.789	.889	.623	.627	.763
	.50	1.486	1.546	1.507	1.534	1.500
	.75	1.438	1.430	1.486	1.541	1.411
σ_2' (stat.)	.25	- .739	- .699	- .600	- .659	- .724
	.50	-1.083	- .930	-1.023	- .928	- .981
	.75	-1.079	- .970	-1.048	- .972	-1.031
σ_1' (dyn.)	.25	.818	.885	.646	.633	.741
	.50	1.544	1.584	1.559	1.579	1.528
	.75	1.502	1.466	1.504	1.604	1.471
σ_2' (dyn.)	.25	- .663	- .760	- .483	- .643	- .726
	.50	-1.297	-1.142	-1.167	-1.108	-1.154
	.75	-1.982	-1.872	-1.885	-1.857	-1.775

Table 5c

h/T = 1.4 Coefficients * 10^5 Fn = 0.12

LOAD-CONDITION	ω	0	I	II	III	IV
$+Y_\beta'$	-	2496	2252	2305	2291	2502
$+N_\beta'$	-	691	807	832	824	625
$(m-Y_{\dot{v}})'$	.25	2180	2190	2060	2193	2166
	.50	2157	2147	2167	2285	2253
	.75	3003	2955	2919	3017	2822
$-Y_v'$	.25	2187	2249	2441	2078	2428
	.50	2752	2737	3044	2609	2919
	.75	5462	5357	5113	4964	5641
$-N_{\dot{v}}'$	.25	- 77	- 96	-138	-108	-112
	.50	- 74	- 77	- 83	- 65	- 87
	.75	-149	-149	-140	-162	-172
$-N_v'$	.25	667	785	856	819	574
	.50	831	949	1005	993	748
	.75	1352	1352	1364	1507	1092
$(I_{zz}-N_{\dot{r}})'$	.25	314	312	372	309	257
	.50	206	256	237	245	191
	.75	228	247	235	219	196
$-N_r'$	.25	- 77	- 72	-177	- 64	- 83
	.50	-176	-211	-203	-273	-187
	.75	-259	-181	-198	-244	-198
$-Y_{\dot{r}}'$	.25	707	701	663	786	578
	.50	338	370	391	386	338
	.75	277	327	319	287	264
$(m-Y_r)'$	.25	59	63	-146	97	23
	.50	223	177	245	107	124
	.75	331	423	497	448	397
σ_1' (stat.)	.25	.479	.543	.626	.593	.520
	.50	1.216	1.183	1.306	1.469	1.257
	.75	1.366	1.103	1.262	1.461	1.261
σ_2' (stat.)	.25	- .658	- .523	- .534	- .501	- .688
	.50	-1.040	- .904	- .942	- .830	- .944
	.75	- .883	- .872	- .944	- .898	- .985
σ_1' (dyn.)	.25	.496	.531	.625	.621	.503
	.50	1.279	1.216	1.344	1.531	1.299
	.75	1.507	1.169	1.355	1.640	1.359
σ_2' (dyn.)	.25	- .566	- .531	- .573	- .449	- .688
	.50	-1.115	-1.061	-1.177	- .914	-1.070
	.75	-1.703	-1.694	-1.708	-1.627	-1.906

Table 6a

h/T = 1.6 Coefficients * 10^5 Fn = 0.08

LOAD-CONDITION	ω	0	I	II	III	IV
$+Y_\beta'$	-	1740	1748	1652	1855	1900
$+N_\beta'$	-	655	640	628	601	539
$(m-Y_{\dot{v}})'$	.25 .50 .75	1914 1999 2161	1939 1976 2103	1834 1967 2112	1990 1985 2150	1865 1922 2084
$-Y_v'$	.25 .50 .75	1817 2375 3470	1795 2150 3354	2086 2085 3195	1893 2289 3379	1973 2484 3414
$-N_{\dot{v}}'$	.25 .50 .75	-139 - 77 - 46	-93 -51 -38	-91 -57 -41	-77 -39 -31	-62 -44 -39
$-N_v'$	.25 .50 .75	347 539 703	603 603 786	596 547 717	675 690 838	547 623 667
$(I_{zz}-N_{\dot{r}})'$	.25 .50 .75	158 174 207	263 198 224	203 194 210	237 206 220	251 189 204
$-N_r'$	.25 .50 .75	-105 -274 -258	-201 -275 -275	-154 -290 -282	-149 -315 -300	-116 -287 -275
$-Y_{\dot{r}}'$	.25 .50 .75	447 225 136	476 221 141	463 255 146	508 246 145	401 219 129
$(m-Y_r)'$	.25 .50 .75	102 319 505	19 361 533	141 359 547	138 388 554	136 345 522
σ_1' (stat.)	.25 .50 .75	1.154 1.964 1.633	1.047 1.786 1.610	1.221 1.913 1.741	1.000 1.917 1.723	.720 1.851 1.667
σ_2' (stat.)	.25 .50 .75	- .593 - .956 -1.053	- .627 - .990 -1.071	- .678 - .930 -1.034	- .703 -1.019 -1.081	- .827 -1.059 -1.121
σ_1' (dyn.)	.25 .50 .75	.866 1.851 1.566	1.023 1.736 1.594	1.147 1.826 1.704	1.051 1.949 1.766	.718 1.874 1.664
σ_2' (dyn.)	.25 .50 .75	- .716 -1.217 -1.761	- .657 -1.158 -1.767	- .853 -1.107 -1.689	- .699 -1.212 -1.753	- .856 -1.328 -1.798

Table 6b

$h/T = 1.6$

Coefficients * 10^5

$Fn = 0.10$

LOAD-CONDITION	ω	0	I	II	III	IV
$+Y_\beta'$	-	1718	1741	1525	1800	2029
$+N_\beta'$	-	603	667	628	676	604
$(m-Y_{\dot{v}})'$	.25	2008	1936	1809	2055	1949
	.50	1797	1972	1952	1987	1887
	.75	2194	2136	2150	2182	2134
$-Y_v'$	.25	1794	1803	1762	1696	1986
	.50	2291	2156	2256	2300	2424
	.75	3087	2853	2886	3018	2945
$-N_{\dot{v}}'$	.25	-128	-97	-115	-78	-82
	.50	- 80	-60	- 68	-49	-61
	.75	- 64	-52	- 53	-42	-48
$-N_v'$	.25	413	584	569	652	565
	.50	577	652	641	717	634
	.75	749	791	775	844	671
$(I_{zz}-N_{\dot{r}})'$	.25	138	271	282	258	287
	.50	170	210	185	205	187
	.75	199	224	207	211	211
$-N_r'$	.25	- 79	-100	-161	-120	- 81
	.50	-225	-223	-234	-268	-247
	.75	-283	-300	-308	-317	-293
$-Y_{\dot{r}}'$	.25	529	606	538	601	565
	.50	299	310	316	324	286
	.75	188	197	198	198	198
$(m-Y_r)'$	.25	- 8	69	- 23	124	69
	.50	221	284	296	302	234
	.75	431	435	474	536	438
σ_1' (stat.)	.25	1.059	.725	.836	.892	.514
	.50	1.714	1.500	1.738	1.780	1.689
	.75	1.759	1.709	1.876	1.933	1.726
σ_2' (stat.)	.25	- .358	- .520	- .483	- .582	- .661
	.50	- .906	- .890	- .816	- .911	-1.027
	.75	- .946	- .973	- .898	-1.030	-1.083
σ_1' (dyn.)	.25	.851	.661	.784	.887	.499
	.50	1.678	1.455	1.690	1.770	1.684
	.75	1.772	1.711	1.886	1.960	1.717
σ_2' (dyn.)	.25	- .472	- .572	- .604	- .556	- .661
	.50	-1.163	-1.058	-1.110	-1.112	-1.198
	.75	-1.505	-1.435	-1.462	-1.534	-1.465

h/T = 1.6 Table 6c Fn = 0.12

Coefficients * 10^5

LOAD-CONDITION	ω	0	I	II	III	IV
$+Y_\beta'$	-	1796	1717	1691	1881	1908
$+N_\beta'$	-	622	677	585	634	580
$(m-Y_{\dot{v}})'$	.25	2054	2057	2016	2103	1922
	.50	2078	1909	2001	2049	1909
	.75	2287	2218	2191	2278	2176
$-Y_v'$	.25	1815	1882	1982	1827	1827
	.50	2252	2225	2208	2241	2101
	.75	2793	3023	3008	2874	3002
$-N_{\dot{v}}'$	.25	-153	-113	-112	-87	-71
	.50	- 74	- 88	- 69	-52	-67
	.75	- 75	- 68	- 71	-60	-61
$-N_v'$	.25	494	648	625	665	569
	.50	587	723	619	718	559
	.75	794	863	809	898	730
$(I_{zz}-N_{\dot{r}})'$	.25	286	320	298	313	329
	.50	179	203	193	184	200
	.75	200	218	208	211	188
$-N_r'$	.25	- 74	-111	-109	- 65	-112
	.50	-169	-193	-196	-224	-191
	.75	-270	-282	-279	-286	-296
$-Y_{\dot{r}}'$	.25	612	678	695	757	633
	.50	362	341	388	346	350
	.75	248	257	274	272	223
$(m-Y_r)'$	.25	19	15	- 9	- 4	- 20
	.50	164	183	194	228	143
	.75	338	374	393	457	337
σ_1' (stat.)	.25	.473	.623	.613	.427	.515
	.50	1.358	1.362	1.430	1.669	1.291
	.75	1.682	1.676	1.696	1.754	1.886
σ_2' (stat.)	.25	-.449	-.438	-.431	-.388	-.580
	.50	-.749	-.800	-.753	-.858	-.855
	.75	-.868	-.878	-.871	-.950	-.954
σ_1' (dyn.)	.25	.412	.591	.606	.451	.517
	.50	1.295	1.349	1.414	1.705	1.263
	.75	1.728	1.703	1.761	1.860	1.922
σ_2' (dyn.)	.25	- .512	- .504	- .511	- .356	- .552
	.50	- .923	- .997	- .947	- .988	- .941
	.75	-1.243	-1.378	-1.383	-1.333	-1.397

Table 7a

$h/T = 2.0$ Coefficients * 10^5 $Fn = 0.08$

LOAD-CONDITION	ω	0	I	II	III	IV
$+Y_\beta'$	-	1397	1219	1305	1347	1483
$+N_\beta'$	-	508	526	599	568	369
$(m-Y_{\dot{v}})'$	.25 .50 .75	1825 1751 1773	1807 1760 1780	1774 1756 1771	1822 1770 1799	1842 1762 1807
$-Y_v'$	.25 .50 .75	1572 1810 2264	1544 1921 2167	1583 1828 2151	1496 1841 2148	1540 1845 2215
$-N_{\dot{v}}'$	.25 .50 .75	-71 -17 -11	-73 -19 -14	-65 - 7 - 4	-39 - 4 2	-99 -58 -36
$-N_v'$	.25 .50 .75	448 456 534	502 504 557	524 507 557	512 517 606	367 377 464
$(I_{zz}-N_{\dot{r}})'$	.25 .50 .75	223 175 194	232 171 203	252 192 199	216 189 209	171 158 192
$-N_r'$	.25 .50 .75	-102 -293 -308	-103 -305 -330	-147 -291 -354	-108 -334 -358	-119 -297 -294
$-Y_{\dot{r}}'$	.25 .50 .75	452 230 133	482 217 136	589 248 132	502 243 143	423 224 131
$(m-Y_r)'$	.25 .50 .75	156 407 541	172 403 541	170 437 560	139 410 538	116 372 545
σ_1' (stat.)	.25 .50 .75	.825 2.146 1.997	.866 2.269 2.039	1.109 2.099 2.258	.995 2.296 2.156	.953 2.132 1.793
σ_2' (stat.)	.25 .50 .75	- .612 - .925 -1.022	- .549 - .843 - .927	- .545 - .902 -1.000	- .546 - .886 - .972	- .645 - .930 -1.010
σ_1' (dyn.)	.25 .50 .75	.756 2.065 1.957	.801 2.195 1.996	.995 1.965 2.182	.917 2.210 2.125	.947 2.124 1.826
σ_2' (dyn.)	.25 .50 .75	- .694 -1.117 -1.459	- .674 -1.178 -1.403	- .666 -1.131 -1.409	- .614 -1.115 -1.732	- .668 -1.112 -1.408

Table 7b

$h/T = 2.0$ Coefficients $* 10^5$ $Fn = 0.10$

LOAD-CONDITION	ω	0	I	II	III	IV
$+Y_\beta'$	-	1347	1366	1287	1330	1366
$+N_\beta'$	-	518	529	547	570	479
$(m-Y_{\dot{v}})'$	.25 .50 .75	1846 1728 1792	2002 1737 1792	1878 1773 1788	1846 1758 1806	1792 1802 1821
$-Y_v'$	.25 .50 .75	1523 1904 2104	1469 1969 2056	1418 1891 2016	1424 1835 2140	1675 1830 2130
$-N_{\dot{v}}'$	.25 .50 .75	-67 -38 -17	-33 -33 -15	-54 -11 - 9	-49 -22 - 7	-146 - 54 - 38
$-N_v'$	.25 .50 .75	499 496 525	556 546 545	535 552 545	554 565 618	452 410 486
$(I_{zz}-N_{\dot{r}})'$	.25 .50 .75	278 168 184	290 188 197	254 188 199	254 187 203	162 160 186
$-N_r'$	.25 .50 .75	-114 -247 -339	-140 -254 -341	- 96 -258 -354	-107 -291 -366	-124 -218 -310
$-Y_{\dot{r}}'$	.25 .50 .75	540 287 179	562 316 192	569 313 132	578 314 198	545 295 185
$(m-Y_r)'$	.25 .50 .75	34 250 460	39 286 488	126 302 560	67 282 503	116 279 470
σ_1' (stat.)	.25 .50 .75	.671 1.925 2.260	.764 1.825 2.165	.790 1.911 2.210	.818 2.082 2.281	1.179 1.783 2.028
σ_2' (stat.)	.25 .50 .75	- .464 - .797 - .924	- .463 - .810 - .939	- .480 - .773 - .966	- .444 - .784 - .921	- .516 - .795 - .925
σ_1' (dyn.)	.25 .50 .75	.640 1.858 2.216	.769 1.786 2.129	.760 1.854 2.159	.790 2.030 2.256	1.117 1.681 1.988
σ_2' (dyn.)	.25 .50 .75	- .542 -1.062 -1.295	- .493 -1.091 -1.276	- .528 -1.048 -1.322	- .483 -1.018 -1.318	- .630 -1.004 -1.293

Table 7c

h/T = 2.0 Coefficients * 10^5 Fn = 0.12

LOAD-CONDITION	ω	0	I	II	III	IV
$+Y_\beta'$	-	1284	1317	1342	1353	1463
$+N_\beta'$	-	544	545	571	660	523
$(m-Y_{\dot v})'$	.25	1889	1889	1861	1904	1900
	.50	1784	1780	1790	1795	1792
	.75	1831	1821	1810	1832	1860
$-Y_v'$	.25	1535	1273	1450	1516	1374
	.50	1724	1758	1614	1786	1780
	.75	1980	1940	1981	2003	2073
$-N_{\dot v}'$	.25	-77	-60	-73	-82	-104
	.50	-36	-31	-20	-34	- 63
	.75	-17	-18	-14	-20	- 41
$-N_v'$	.25	558	532	562	642	448
	.50	477	517	496	610	437
	.75	522	538	561	633	509
$(I_{zz}-N_{\dot r})'$	.25	335	406	308	310	249
	.50	185	195	192	209	178
	.75	180	185	192	201	174
$-N_r'$	.25	-102	- 33	- 40	6	- 62
	.50	-218	-199	-215	-223	-176
	.75	-311	-318	-335	-341	-295
$-Y_{\dot r}'$	.25	712	933	759	776	658
	.50	349	367	381	290	363
	.75	240	243	251	256	237
$(m-Y_r)'$	.25	24	42	61	179	110
	.50	127	186	187	208	181
	.75	386	418	424	430	392
σ_1' (stat.)	.25	.578	.305	.454	.464	.609
	.50	1.614	1.493	1.652	1.529	1.412
	.75	2.211	2.207	2.258	2.258	2.111
σ_2' (stat.)	.25	-.363	-.264	-.310	-.362	-.450
	.50	-.631	-.679	-.681	-.746	-.729
	.75	-.826	-.858	-.873	-.884	-.905
σ_1' (dyn.)	.25	.554	.305	.425	.404	.556
	.50	1.517	1.421	1.549	1.451	1.306
	.75	2.139	2.152	2.197	2.180	2.059
σ_2' (dyn.)	.25	- .446	- .257	- .345	- .401	- .447
	.50	- .840	- .876	- .808	- .940	- .879
	.75	-1.145	-1.146	-1.169	-1.174	-1.182

Table 8a

h/T = 2.2 Coefficients * 10^5 Fn = 0.08

LOAD-CONDITION	ω	0	I	II	III	IV
$+Y_{\beta}'$	-	1369	1338	1351	1214	1172
$+N_{\beta}'$	-	608	622	568	561	512
$(m-Y_{\dot{v}})'$	.25 .50 .75	1763 1701 1732	1746 1717 1730	1682 1715 1712	1770 1665 1696	1863 1744 1738
$-Y_v'$	.25 .50 .75	1466 1870 2160	1424 1745 2133	1407 1891 2088	1510 1647 2108	1714 1926 2325
$-N_{\dot{v}}'$	.25 .50 .75	-67 -27 -20	-57 -16 -14	-83 -24 -14	-31 -15 - 9	-12 1 -16
$-N_v'$	.25 .50 .75	484 460 501	472 463 531	428 469 505	539 449 536	498 444 424
$(I_{zz}-N_{\dot{r}})'$	.25 .50 .75	230 187 209	247 205 211	225 184 207	217 191 210	284 189 202
$-N_r'$	.25 .50 .75	-144 -286 -307	- 40 -296 -318	-164 -331 -352	-147 -329 -345	-110 -306 -314
$-Y_{\dot{r}}'$	.25 .50 .75	414 222 127	437 238 133	468 236 138	450 236 135	491 232 129
$(m-Y_r)'$	.25 .50 .75	116 403 534	- 86 399 517	101 392 523	142 417 547	124 349 517
σ_1' (stat.)	.25 .50 .75	1.025 2.052 1.920	3.689 1.973 1.970	1.137 2.291 2.126	1.170 2.251 2.092	.724 2.090 1.955
σ_2' (stat.)	.25 .50 .75	- .603 - .957 -1.064	-1.664 - .918 -1.034	- .588 - .910 -1.020	- .554 - .875 - .970	- .496 - .781 - .916
σ_1' (dyn.)	.25 .50 .75	.922 1.884 1.791	.127 1.806 1.850	1.011 2.164 2.033	1.107 2.110 2.009	.649 1.974 1.822
σ_2' (dyn.)	.25 .50 .75	- .669 -1.180 -1.425	- .282 -1.091 -1.404	- .649 -1.165 -1.379	- .678 -1.075 -1.421	- .721 -1.145 -1.475

h/T = 2.2

Table 8b

Coefficients * 10^5

Fn = 0.10

LOAD-CONDITION	ω	0	I	II	III	IV
$+Y_\beta'$	-	1353	1321	1302	1213	1205
$+N_\beta'$	-	541	601	546	554	546
$(m-Y_{\dot{v}})'$	.25	1795	1770	1707	1727	1694
	.50	1729	1715	1662	1667	1777
	.75	1750	1761	1727	1719	1742
$-Y_v'$	.25	1375	1366	1364	1333	1408
	.50	1764	1740	1822	1779	1777
	.75	2029	1988	1993	1992	1976
$-N_{\dot{v}}'$	.25	-80	-77	-97	-61	-68
	.50	-36	-40	-57	-33	- 2
	.75	-26	-16	-20	-10	-18
$-N_v'$	.25	447	514	447	506	453
	.50	459	477	462	494	428
	.75	498	515	492	522	460
$(I_{zz}-N_{\dot{r}})'$	.25	292	342	271	286	318
	.50	189	206	183	193	197
	.75	201	205	199	206	199
$-N_r'$	.25	-112	-173	-131	-143	-156
	.50	-223	-239	-244	-258	-229
	.75	-334	-322	-365	-373	-331
$-Y_{\dot{r}}'$	.25	516	477	553	574	552
	.50	284	296	311	313	312
	.75	178	184	188	191	183
$(m-Y_r)'$	.25	16	- 77	0	29	- 25
	.50	281	260	322	316	269
	.75	463	485	470	461	448
σ_1' (stat.)	.25	.612	.659	.766	.837	.733
	.50	1.635	1.629	1.829	1.868	1.684
	.75	2.067	2.055	2.266	2.262	2.100
σ_2' (stat.)	.25	-.463	-.430	-.431	-.428	-.413
	.50	-.824	-.793	-.838	-.787	-.716
	.75	-.954	-.958	-.929	-.879	-.875
σ_1' (dyn.)	.25	.560	.628	.694	.786	.663
	.50	1.527	1.491	1.699	1.752	1.520
	.75	1.991	1.939	2.179	2.182	1.982
σ_2' (dyn.)	.25	- .509	- .488	- .498	- .494	- .545
	.50	-1.015	- .992	-1.085	-1.057	- .979
	.75	-1.280	-1.261	-1.267	-1.266	-1.240

Table 8c

$h/T = 2.2$

Coefficients * 10^5

$Fn = 0.12$

LOAD-CONDITION	ω	0	I	II	III	IV
$+Y_{\beta}'$	-	1333	1335	1301	1192	1220
$+N_{\beta}'$	-	559	608	547	536	504
$(m-Y_{\dot{v}})'$	.25 .50 .75	1831 1722 1775	1841 1770 1768	1690 1694 1735	1873 1721 1737	1733 1738 1747
$-Y_{v}'$	.25 .50 .75	1328 1668 1965	1373 1633 1920	1434 1509 1890	1470 1559 1881	1267 1669 1940
$-N_{\dot{v}}'$	.25 .50 .75	-85 -64 -39	-96 -42 -28	-124 - 63 - 33	-56 -31 -18	-67 -41 -23
$-N_{v}'$	.25 .50 .75	488 430 487	511 464 510	498 402 480	580 468 512	478 428 472
$(I_{zz}-N_{\dot{r}})'$	.25 .50 .75	342 198 191	338 227 208	314 200 187	380 175 201	352 193 189
$-N_{r}'$	.25 .50 .75	-118 -171 -289	- 98 -140 -312	- 75 -199 -338	- 72 -212 -354	- 90 -173 -314
$-Y_{\dot{r}}'$	.25 .50 .75	597 349 229	640 366 249	699 381 236	720 338 245	651 348 236
$(m-Y_{r})'$	.25 .50 .75	34 156 390	23 225 379	62 181 383	20 221 395	23 157 355
σ_1' (stat.)	.25 .50 .75	.582 1.274 1.954	.553 1.090 1.979	.543 1.440 2.251	.405 1.744 2.220	.473 1.305 2.082
σ_2' (stat.)	.25 .50 .75	-.448 -.681 -.887	-.383 -.712 -.872	-.392 -.685 -.868	-.317 -.683 -.807	-.392 -.636 -.804
σ_1' (dyn.)	.25 .50 .75	.544 1.139 1.850	.494 .940 1.856	.487 1.287 2.154	.390 1.633 2.148	.453 1.197 2.004
σ_2' (dyn.)	.25 .50 .75	- .470 - .852 -1.177	- .433 - .850 -1.139	- .460 - .799 -1.150	- .400 - .852 -1.143	- .423 - .851 -1.154

Table 9a

$h/T = 2.2$ Coefficients * 10^5 Fn = .205

LOAD-CONDITION	ω	0	I	II	III	IV
$+Y_\beta'$	-	1281		1277		
$+N_\beta'$	-	523		599		
$(m-Y_{\dot{v}})'$	.25 .50 .75	1988 1926 2579		2034 1987 2664		
$-Y_v'$	.25 .50 .75	1308 1548 2774		1285 1474 2841		
$-N_{\dot{v}}'$	.25 .50 .75	-165 -105 -219		-219 -130 -280		
$-N_v'$	.25 .50 .75	488 515 758		598 575 861		
$(I_{zz}-N_{\dot{r}})'$	.25 .50 .75	604 326 183		544 358 150		
$-N_r'$	.25 .50 .75	-89 -46 5		-93 -26 -42		
$-Y_{\dot{r}}'$	.25 .50 .75	1294 587 562		1406 686 571		
$(m-Y_r)'$	.25 .50 .75	53 11 243		6 36 415		
σ_1' (stat.)	.25 .50 .75	.343 .290 .445		.436 .262 .877		
σ_2' (stat.)	.25 .50 .75	-.292 -.324 -.456		-.199 -.261 -.616		
σ_1' (dyn.)	.25 .50 .75	.321 .261 .425		.434 .223 .979		
σ_2' (dyn.)	.25 .50 .75	-.314 -.427 -.673		-.201 -.331 -.870		

h/T = 2.2

Table 9b
Coefficients * 10^5

Fn = .234

LOAD-CONDITION	ω	0	I	II	III	IV
$+Y_\beta'$	-	1238		1276		
$+N_\beta'$	-	493		578		
$(m-Y_{\dot{v}})'$	.25	2177		2229		
	.50	2008		1957		
	.75	865		981		
$-Y_v'$	.25	1326		1352		
	.50	2066		1963		
	.75	2703		2868		
$-N_{\dot{v}}'$	.25	-412		-449		
	.50	-340		-337		
	.75	- 64		-112		
$-N_v'$	.25	424		507		
	.50	282		300		
	.75	-102		- 44		
$(I_{zz}-N_{\dot{r}})'$	.25	31		- 27		
	.50	179		190		
	.75	271		274		
$-N_r'$	.25	-173		-180		
	.50	-154		-168		
	.75	-364		-381		
$-Y_{\dot{r}}'$	.25	1951		2139		
	.50	797		839		
	.75	259		304		
$(m-Y_r)'$	.25	54		110		
	.50	361		293		
	.75	410		348		
σ_1' (stat.)	.25	1.636		1.968		
	.50	1.099		1.207		
	.75	1.782		1.795		
σ_2' (stat.)	.25	- .169		- .166		
	.50	- .532		- .486		
	.75	-1.460		-1.264		
σ_1' (dyn.)	.25	1.502		1.819		
	.50	.850		.886		
	.75	1.215		1.269		
σ_2' (dyn.)	.25	- .193		- .183		
	.50	- .784		- .720		
	.75	-3.089		-2.803		

Table 9c

h/T = 2.2 Coefficients * 10^5 Fn = .263

LOAD-CONDITION	ω	0	I	II	III	IV
$+Y_\beta'$	-	1498		1545		
$+N_\beta'$	-	466		533		
$(m-Y_{\dot{v}})'$	.25	1165		1205		
	.50	1153		1190		
	.75	1025		1014		
$-Y_v'$	.25	1437		1494		
	.50	1612		1670		
	.75	1475		1542		
$-N_{\dot{v}}'$	.25	-290		-354		
	.50	-146		-160		
	.75	- 75		- 87		
$-N_v'$	.25	394		472		
	.50	360		423		
	.75	312		370		
$(I_{zz}-N_{\dot{r}})'$	.25	-123		-211		
	.50	215		211		
	.75	213		213		
$-N_r'$	.25	- 73		-157		
	.50	- 98		-101		
	.75	-132		-176		
$-Y_{\dot{r}}'$	.25	1213		1323		
	.50	688		769		
	.75	449		480		
$(m-Y_r)'$	.25	- 65		-131		
	.50	- 51		- 29		
	.75	- 54		- 45		
σ_1' (stat.)	.25	4.184		6.050		
	.50	.789		.952		
	.75	.888		1.215		
σ_2' (stat.)	.25	-9.064		- .133		
	.50	- .448		- .395		
	.75	- .771		- .792		
σ_1' (dyn.)	.25	3.740		5.633		
	.50	.665		.790		
	.75	.774		1.057		
σ_2' (dyn.)	.25	-.102		-.143		
	.50	-.603		-.529		
	.75	-.912		-.935		

Table 10a. Critical- and returnspeeds, fall of waterlevel.

	U_m	$h/T = 1.4$	$h/T = 1.6$	$h/T = 2.0$	$h/T = 2.2$
h		0.199	0.227	0.284	0.312
$A_c = B_o * \bar{h}$		0.8049	0.9256	1.167	1.285
$\bar{h} = A_c/B_o$		0.190	0.218	0.275	0.303
$\sqrt{g\bar{h}}$		1.365	1.462	1.642	1.724
A_s/A_c		0.094	0.082	0.065	0.058
$V_{cr}/\sqrt{g\bar{h}}$		0.58	0.60	0.63	0.65
Fn = 0.08	0.445	$U_m/\sqrt{g\bar{h}} = 0.326$ $U_m < V_{cr}$ $Z/\bar{h} = 0.021$ $U/\sqrt{g\bar{h}} = 0.042$ Z = 0.004 m u = 0.057 m/s $\frac{u+U_m}{\sqrt{gL}} = 0.09$	$U_m/\sqrt{g\bar{h}} = 0.304$ $U_m < V_{cr}$ $Z/\bar{h} = 0.016$ $U/\sqrt{g\bar{h}} = 0.033$ Z = 0.003 m u = 0.048 m/s $\frac{u+U_m}{\sqrt{gL}} = 0.09$	$U_m/\sqrt{g\bar{h}} = 0.271$ $U_m < V_{cr}$ $Z/\bar{h} = 0.011$ $U/\sqrt{g\bar{h}} = 0.022$ Z = 0.003 m u = 0.036 m/s $\frac{u+U_m}{\sqrt{gL}} = 0.09$	$U_m/\sqrt{g\bar{h}} = 0.258$ $U_m < V_{cr}$ $Z/\bar{h} = 0.0086$ $U/\sqrt{g\bar{h}} = 0.018$ Z = 0.003 m u = 0.031 m/s $\frac{u+U_m}{\sqrt{gL}} = 0.09$
Fn = 0.10	0.556	$U_m/\sqrt{g\bar{h}} = 0.407$ $U_m < V_{cr}$ $Z/\bar{h} = 0.038$ $U/\sqrt{g\bar{h}} = 0.062$ Z = 0.007 m u = 0.085 m/s $\frac{u+U_m}{\sqrt{gL}} = 0.12$	$U_m/\sqrt{g\bar{h}} = 0.380$ $U_m < V_{cr}$ $Z/\bar{h} = 0.028$ $U/\sqrt{g\bar{h}} = 0.047$ Z = 0.006 m u = 0.069 m/s $\frac{u+U_m}{\sqrt{gL}} = 0.11$	$U_m/\sqrt{g\bar{h}} = 0.338$ $U_m < V_{cr}$ $Z/\bar{h} = 0.018$ $U/\sqrt{g\bar{h}} = 0.031$ Z = 0.005 m u = 0.051 m/s $\frac{u+U_m}{\sqrt{gL}} = 0.11$	$U_m/\sqrt{g\bar{h}} = 0.323$ $U_m < V_{cr}$ $Z/\bar{h} = 0.014$ $U/\sqrt{g\bar{h}} = 0.025$ Z = 0.004 m u = 0.031 m/s $\frac{u+U_m}{\sqrt{gL}} = 0.11$
Fn = 0.12	0.067	$U_m/\sqrt{g\bar{h}} = 0.489$ $U_m < V_{cr}$ $Z/\bar{h} = 0.067$ $U/\sqrt{g\bar{h}} = 0.094$ Z = 0.013 m u = 0.128 m/s $\frac{u+U_m}{\sqrt{gL}} = 0.14$	$U_m/\sqrt{g\bar{h}} = 0.456$ $U_m < V_{cr}$ $Z/\bar{h} = 0.046$ $U/\sqrt{g\bar{h}} = 0.067$ Z = 0.010 m u = 0.098 m/s $\frac{u+U_m}{\sqrt{gL}} = 0.14$	$U_m/\sqrt{g\bar{h}} = 0.406$ $U_m < V_{cr}$ $Z/\bar{h} = 0.028$ $U/\sqrt{g\bar{h}} = 0.042$ Z = 0.008 m u = 0.069 m/s $\frac{u+U_m}{\sqrt{gL}} = 0.13$	$U_m/\sqrt{g\bar{h}} = 0.387$ $U_m < V_{cr}$ $Z/\bar{h} = 0.023$ $U/\sqrt{g\bar{h}} = 0.034$ Z = 0.007 m u = 0.059 m/s $\frac{u+U_m}{\sqrt{gL}} = 0.13$

Table 10b. Critical- and returnspeeds, fall of waterlevel.

	U_m	$h/T = 2.2$
h		0.312
$A_c = B_o * \bar{h}$		1.285
$\bar{h} = A_c/B_o$		0.303
$\sqrt{gh}$		1.724
A_s/A_c		0.058
$V_{cr}/\sqrt{gh}$		0.65
Fn = 0.205	1.140	$U_m/\sqrt{gh} = 0.661$ $U_m > V_{cr}$ $Z_{cr}/h \approx 0.173$ $u_{cr}/\sqrt{gh} = 0.230$ $Z = 0.065$ m $u_{cr} = 0.397$m/s $\frac{u_{cr}+U_m}{\sqrt{gL}} = 0.276$
Fn = 0.234	1.300	$U_m/\sqrt{gh} = 0.754$ $U_m > V_{cr}$ $Z_{cr}/h = 0.180$ $u_{cr}/\sqrt{gh} = 0.230$ $Z = 0.085$ m $u_{cr} = 0.397$m/s $\frac{u_{cr}+U_m}{\sqrt{gL}} = 0.305$
Fn = 0.262	1.460	$U_m/\sqrt{gh} = 0.847$ $U_m > V_{cr}$ $Z_{cr}/h = 0.180$ $u_{cr}/\sqrt{gh} = 0.230$ $Z = 0.103$ m $u_{cr} = 0.397$m/s $\frac{u_{cr}+U_m}{\sqrt{gL}} = 0.334$