A MODEL ON THE WILLINGNESS TO PAY FOR DRIVER INFORMATION

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1. INTRODUCTION

Increasing congestion emphasizes the need for a more efficient use of existing road infrastructure. Driver information systems (DIS) are generally considered as a tool to support this objective. Examples of DIS that already are or will be soon generally available are network status messages through television, radio and telephone (available at home or work places), in-car route guidance systems, variable message signs, etc.

Drivers generally have limited and uncertain knowledge about the travel times in a network. This is especially true for dynamically changing network conditions with recurrent and non-recurrent congestion. The trip pattern therefore is not as efficient as it could be if perfect information about network conditions would have been available. DIS can give substantial individual benefits to drivers by enabling the usage of shorter or quicker alternatives and by circumventing congested network parts. In addition, DIS can offer societal benefits. It can lead to better usage of existing capacities through a more efficient distribution of travel demand over time and space, leading to less congestion, less exhausts, etc.

Earlier research has shown that driver misperceptions about network conditions lead to both excess travel time and excess travel distance relative to the fastest or shortest routes (see [5], [6] and [7]). Reported percentages vary between six percent ([5]) and fifteen percent ([6]). Although this puts an upper limit to the individual benefits of proposed information systems, it does not mean that these improvements will be reached if such systems are introduced. Also the question what the effects on route choice are, needs to be answered.

In this paper we focus on the problem of predicting the fraction of travellers that uses an information system as a function of the costs of this service. The cost component is not necessarily limited to a monetary sacrifice but can also consist of the extra effort of tuning in to a radio or TV station, or processing the presented information.

We suppose that drivers before starting their trip can receive perfect information about the exact travel times in the network for the period they intend to travel. If they make use of the supplied information they will choose the subjective least cost route (subjective utility maximization). Non-informed drivers have a probability distribution for the expected route travel times being summations of corresponding expected link travel times. In principle, the deviation of the expected travel time distribution from

307
the real travel time (location and variance) is individual-specific. Also non-informed drivers are assumed to behave as utility maximizers, however in our approach they have to make their route choice decisions under uncertainty (risk behaviour). The question now is to estimate the probability that an individual driver will acquire travel time information given his prior pre-trip travel time expectations and given the cost of the information service. From these probabilities the fraction of informed travellers can be derived.

It is repeatedly asserted that the size of this fraction is of paramount importance to the effectiveness of DIS [9]. We will first introduce the problem using a simplified network. A discrete choice model will be proposed for individual information acquisition behaviour (Sections 2 and 3). After having derived expressions for a single driver we enlarge the analysis to a population of drivers with different prior travel time perceptions and beliefs (section 4). The next step then is a generalization of these expressions to a general network (section 5). A simulation-based algorithm will be proposed and explained to calculate the fraction of informed drivers, that is the fraction of drivers that will acquire information through the use of DIS service. To show the behavioural properties of the developed model as well as the computational aspects of the algorithm results of a simulation exercise will be discussed (section 6).

The notation used in this paper is summarized on a separate sheet in section 8.

2. ROUTE CHOICE UNDER UNCERTAINTY

As an illustration, assume a simple, single OD-pair two-route network, see figure 1. The historical average of travel time on route $k$ between $O$ and $D$ is denoted by $C_k$. Before starting the trip individual $n$ has its own perceptions of the route travel times.

Uncertainty of individual travellers about travel times is expressed by defining a function that describes the extent of belief for every possible path travel time value, better known as the Bayesian belief. For maximum simplicity a normal distribution is assumed with a mean $C_{kn}$ and a variance proportional with $C_{kn}$ with a constant parameter $\beta$. So from the perspective of individual $n$ the prior probability distribution of the experienced travel time for path $k$, $C^{exp}_{kn}$, is:

$$C^{exp}_{kn} \sim \mathcal{N}(C_{kn}, \beta C_{kn})$$

Parameter $\beta$ addresses the level of uncertainty about travel times. The individually perceived prior expectation $C_{kn}$ in this Bayesian belief is defined by:

$$C_{kn} = C_k + \xi_{kn}$$

The variable $\xi_{kn}$ accounts for individual misperceptions and other individual factors influencing the disutility or generalized cost of a route. To keep the model as simple as possible it is assumed that the variance of $\xi_{kn}$ is proportional to the value of $C_k$ with some parameter $\alpha$. This is in accordance with [2] and [4].

$$\xi_{kn} \sim \mathcal{N}(0, \alpha C_k)$$

Parameter $\alpha$ relates to the heterogeneousness of perceptions and preferences within the population of drivers.

In the absence of any travel-time information service, individual route choice will be
governed by the joint prior probability distribution of \( \{C^n_{\text{exp}} \} \). This is a case of
decision making under uncertainty. Various choice mechanisms are possible:

- a- selection of the route with the largest subjective probability of being the
  shortest
- b- selection of the route with the lowest expected travel time
- c- randomized choice, in which the probability of selecting route alternative \( k \) is
  proportional to the subjective probability for this alternative to be the shortest

Another subdivision can be made by distinguishing between risk neutral, risk prone
and risk averse travellers, see [10]. Risk neutral drivers will choose the route with the
largest subjective probability of being the shortest. Risk prone travellers are willing
to choose routes with a low probability of being the shortest if such a route choice
incidentally leads to high travel time gains. Risk averse drivers on the other hand
prefer routes with a low risk of large travel time losses. It is beyond the scope of this
paper to deal with all these classes of choice behaviour. In the following we will
confine our contemplations to the risk neutral travel behaviour combined with
minimization of expected travel time.

For our two route network case this means that route choice will be determined by the
difference of the two probability distributions \( C^n_{\text{exp}} \) and \( C^n_{\text{exp}} \). If these distributions
are normal, the difference is again normal, see figure 2. So in our example it holds that:

\[
C^n_{\text{exp}} - C^n_{\text{exp}} \sim N(C^n_{\text{exp}} - C^n_{\text{exp}}, \alpha(C^n_{\text{exp}} + C^n_{\text{exp}}))
\]

(4)

Using this symmetrical probability distribution causes choice mechanisms -a- and -b-
to be equivalent. The risk neutral traveller will opt for route 1 as long as \( C^n_{\text{exp}} > C^n_{\text{exp}} \) or

\[
\int_{-\infty}^{0} p_{C^n_{\text{exp}} - C^n_{\text{exp}}} \, dx < \frac{1}{2}
\]

respectively.

3. INFORMATION ACQUISITION AT INDIVIDUAL LEVEL

Now let us assume that drivers can acquire travel time information at some cost
because the use of a DIS requires investing time, effort or money. The available
information from a DIS in general will affect location and shape of the joint
distribution of \( C^n_{\text{exp}} \). Information will lead to shifts in \( C^n_{\text{exp}} \) as well as reductions in
uncertainty. A driver is assumed to acquire extra information only if the expected
utility of a reduced travel time exceeds the disutility of acquiring this information.

It is assumed that information is available at constant costs \( C^n_{\text{info}} \) and that for person
\( n \), the perceived expected travel time in case of DIS usage is \( C^n_{\text{informed}} \). Since \( C^n_{\text{informed}} \)
is a subjective quantity it can be viewed on as a function of perceptions of an
individual. These are in turn characterized by \( \{C^n_{\text{informed}}\} \) and \( \beta \). In absence of DIS usage,
the prior expectation of the travel times on route \( k \) is \( C^n_{\text{exp}} \). The combined acquisition
and route choice decision process of travellers can now be described with a random
utility model. The utility of the \( k \)th route alternative for person \( n \) consists of the
expected travel times and the costs for acquiring information. The dimension of the
utility is chosen in such a way that the parameter value for travel times vanishes. If
no information is acquired, the utility is defined as:
\[ U_{kn} = -C_{kn} \]

Let alternative 0 denote the case of obtaining information. For this alternative, the utility for person n is given by:

\[ U_{kn} = -C_{n}^{\text{Informed}} - \gamma C_{n}^{\text{Info}} \]

A parameter \( \gamma \) for the contribution of \( C_{n}^{\text{Info}} \) to the utility remains to be specified. The value of \( \gamma \) is related to the value of time.

The possibility of acquiring extra information has intentionally been labelled as a separate route alternative. Although the decision to acquire extra information implies that a route will be travelled of which the physical characteristics are not known in advance, travellers do have a perception of the travel time needed for this alternative, see figure 3.

The uncertainty that might cause a traveller to acquire extra information was expressed in equation (1).

For the sake of argument, we assume that a perfect information system exists, that can tell the user the travel time that will be experienced at each route alternative at cost \( C_{n}^{\text{Info}} \). Furthermore, it is assumed that a traveller operates fully rational, and uses the expected utility as a decision criterion.

Concerning the data acquisition decision, the traveller now has two options:

I. **No information acquisition.** As explained before, the traveller will be assumed to select the route alternative with the maximum expected utility. The expected utility will be the maximum of \( \{-C_{kn}\} \).

II. **Acquire extra information at cost \( C_{n}^{\text{Info}} \).** As a result of the information acquisition the traveller will know the exact utilities of all alternatives. Subsequently, the maximum utility alternative will be chosen.

From the perspective of the traveller, the expected utility of option (II) is:

\[ E[\max\{-C_{kn}^{\text{exp}}\}] - \gamma C_{n}^{\text{Info}} \]

The expected utility with information minus the expected utility without information can be interpreted as the Expected Value of Perfect Information (EVPI), see also [11], so:

\[ \text{EVPI}_{n} = E[\max\{-C_{kn}^{\text{exp}}\}] - \max\{-C_{kn}\} \]

Again, using the perspective of the traveller, \( C_{kn}^{\text{exp}} \) is a random variable with mean \( C_{kn} \) and variance equal to \( \beta C_{kn}^{2} \). Therefore \( \text{EVPI}_{n} \) can be shown to have the following properties:

- **a-** \( \text{EVPI}_{n} \) is strictly positive
- **b-** \( \text{EVPI}_{n} \) is an increasing function of the variances of \( C_{kn}^{\text{exp}} \), i.e., an increasing function of \( \beta \).
- **c-** the value of \( \text{EVPI}_{n} \) increases if the values of \( C_{kn} \) are moved in the direction of \( \min\{C_{kn}\} \). The maximum is reached if all alternatives have equal expected utilities.
- **d-** extending the choice set with an alternative with an expected utility lower than the best available, increases the value of \( \text{EVPI}_{n} \).

These properties coincide with the intuitive view; information is always worth
"something", the more uncertain one is about the situation, the more information can be of use, and finally the more the available alternatives are competing, the more important is the availability of information.

As far as the potential user of the information system is concerned, a decision will be made to use the DIS only if the utility gain of the EVPI exceeds the disutility of using the system

\[ E[\max\{-C_{kn}^{\exp}\}] - \max\{-C_{kn}\} > \gamma C^{\text{info}} \]  (9)

For analytical evaluation of equation (8) it is necessary to have a probability distribution of \( \max\{-C_{kn}^{\exp}\} \) available.

Since such a distribution can not be derived if normal distributions are assumed for \( C_{kn}^{\exp} \), two possibilities remain:

I. Replace the normal distributions with better behaved distributions. For example, if travel times of all alternatives are assumed independent and distributed with equal variance, the normal distribution in (1) may be replaced with Gumbel distributions. Using the properties of a Gumbel distribution, see [1], equation (8) can be shown to change in (\( \mu \) is the scale parameter in a Gumbel distribution):

\[ EVPI_n = \mu \ln (\Sigma_k \text{exp} \mu C_{kn}) - \max\{-C_{kn}\} \]  (10)

II. Evaluate (8) through simulation. Since the distribution of \( \{C_{kn}^{\exp}\} \) is known, the expected value of \( \max\{-C_{kn}^{\exp}\} \) can be approximated by the average of outcomes of a number of experiments. Every experiment involves taking the minimum of a number of random variables, where the joint distribution of these variables is defined by (1).

If speed is of secondary importance, the second option is preferred, because it matches best with the proposed model. Another advantage of the simulation approach is that it can easily be adapted to deal with other information contents than perfect information on all routes. The simulation results presented in section 6 are therefore based on this method. For larger scale applications, the practicability of approximation 8 can be tested.

4. CALCULATING THE PROPORTION OF INFORMED DRIVERS

In the previous section we modelled in what situations a traveller will use an information system. Since this concerns only one traveller, this is still a binary choice. In this section the previous model will be extended to a population of travellers, permitting different individual perceptions of travel times. The probability distributions of these perceptions follow from equations (2) and (3).

To predict the fraction of travellers that use the information service, a simulation approach is followed. This simulation is a straightforward application of the individual model. In order to apply the model to general networks, the simulation model is defined in terms of link travel times rather than route travel times. This enables us to include effects of partly overlapping routes. The principle of the method however remains unaltered.
5. EXTENSION TO A GENERAL NETWORK

In a general network routes and route travel times are built from links and corresponding link travel times. Routes can partly overlap on common series of links, a feature that causes statistical interdependencies between routes that has not been dealt with in the previous sections. In addition multiple OD pairs are now present.

For the network definition the notation of [12] is adopted. Let the directed graph \( G = (N, A) \) represent a transportation network where \( N \) is a set of nodes and \( A \) is a set of directed arcs. Let \( e = [e_h] \) be a vector that contains the travel times for the arcs \( A \). Let \( A = [a_{hk}] \) denote the link-path incidence matrix, where \( ahk \) is 1 if path \( k \) uses link \( h \) and 0 elsewhere. If \( C = [C_h] \) defines the travel times at route level, clearly \( C \) satisfies:

\[
C_{hk} = \sum_a a_{hk} c_h
\]

or in matrix notation, using the prime character as the transpose operator:

\[
C = A' e
\]

The equivalent of (1), (2) and (3) at link level are:

\[
c_{hn}^{\text{exp}} \sim \text{N}(c_{hn}, \beta c_{hn})
\]

\[
c_{hn} = c_h + e_{hn}
\]

\[
e_{hn} \sim \text{N}(0, \alpha C_h)
\]

Then as a result of equations (12) and (13), the joint distribution of the vector \( C_n^{\text{exp}} = [c_{hn}^{\text{exp}}] \) can be shown to have a multivariate normal distribution (see [8]). From equation (12) it follows that the expected value is \( A' e_n \), and that the variance-covariance matrix equals \( \alpha A'C_n' A \), where \( C_n \) is a matrix with the elements of \( e_n \) on its diagonal:

\[
C_n^{\text{exp}} \sim \text{MVN}(A' e_n, \alpha A'C_n' A)
\]

The quantitative evaluation of expression (9) is only possible through simulation. For each OD-pair a number of sets of link travel times is drawn from the distribution defined by (14) and (15). Subsequently the decision whether or not to acquire extra information is solved by a second simulation (see figure 4).

Compared to the traditional probit network assignment model, the number of operations has increased with a factor \( N_m^2 \). To maintain computational feasibility, a route enumeration method is used, see e.g. [3]. All important feasible routes are generated by a repeated process of sampling random travel times and shortest route calculations. Every time a new route is found, it is added to the extended assignment map \( A \). \( N_m^2 \) complete sets of (correlated) travel times at route level can be found by multiplying the matrix \( A' \) with a matrix \( C^{\text{link}} = [c_{hn}^{\text{exp}}] \), where \( c_{hn}^{\text{exp}} \) is the sampled travel time for link \( h \) in experiment \( i \).

\[
\text{C}_{\text{route}} = A' \; C^{\text{link}}
\]

The element on row \( k \) and column \( i \) of \( C_{\text{route}} \) now contains the experienced travel time on route \( k \) during the \( i \)th experiment. Finding the fastest path for a certain OD-pair now reduces to picking the smallest values of \( C_{\text{route}} \) from the rows that correspond with this OD-pair. Since this procedure is the direct equivalent of a "real" shortest path calculation, the correlations between the elements are automatically correct, and a
Choleski factorization as described in [13] is not needed.

6. SIMULATION RESULTS
To demonstrate the feasibility of the model and to test the model for counter intuitive results, a series of simulation experiments is done. A grid network as shown in figure 5 is used. The travel time on the horizontal and vertical links is assumed two and one unit respectively. The OD-pair from node 1 (upper left) to node 20 (down right) is analysed. For this OD-pair not less than 35 route alternatives exist. All of these alternatives have an equal traveltime of 11 units.

On average, 145 repetitions of the route enumeration method were sufficient to generate all of these alternatives. For various values of $\alpha$, $\beta$ and $C_{\text{info}}$ the fraction of informed travellers was calculated. The value of $\gamma$ was left out of consideration. The number of repetitions $N^{\text{inf}}$ and $N^{\text{no}}$ were set to 170. This means that 28900 shortest path calculations are necessary for the calculation of one point in figure 6 or 7.

The results are shown in figure 6 and 7. Figure 6 shows the relation between the share of informed travellers and the costs of the information service for various values of the heterogeneity of perception, $\alpha$, while $\beta$ is constant at 0.3. Figure 7 shows the same relationship but now for different values for the level of uncertainty, $\beta$, while $\alpha$ is constant at 0.3.

Figure 6 shows that as the heterogeneity of perception, $\alpha$, of the population increases, the number of people that is willing to pay for extra information decreases. The reason for this is that according to the network definition all routes are objectively equivalent, which implies a potentially high value of information. Introducing misperceptions and other factors leads to less competitiveness between the alternatives and therefore reduces the value of information. This matches properly with the third property of the EVPI, described in section 3.

Figure 7 shows that less people are willing to pay for information if the level of uncertainty, $\beta$, decreases, which is in accordance with the second property of the EVPI as mentioned in section 3.

The graphs in figures 6 and 7 are less smooth than expected. This may be due to an insufficient number of iterations and the large number of route alternatives. However, increasing the number of iterations leads to unacceptable computation times. Figures 6 and 7 therefore also show the limitations of the simulation approach.
7. CONCLUSIONS

A model was presented in which a travellers decision whether or not to acquire extra traffic information is modelled as a part of the route choice problem. In this model the data acquisition decision depends on the expected value of perfect information (EVPI). In practice, such a model can be used to predict public acceptance and user response to new traveller information services.

Numerical results are obtained using simulation techniques. To make this possible within a reasonable amount of time, a route enumeration method was used. The simulation results match with intuitive expectations. As far as future research is concerned, there are many possibilities to extend or improve this model. To mention a few:

- **a**- By employing a maximum likelihood estimator, model parameters can be estimated.
- **b**- The speed of execution of the model can be improved by using approximations for some probability distributions in the model.
- **c**- The model can be adapted to special situations in which the offered information is incomplete or imperfect.

8. NOTATION

**Constants**

- \( A = [a_{hk}] \) Link-path incidence matrix
- \( e = [e_h] \) Link travel times (historical average)
- \( C = [C_k] \) Path travel times (historical average)
- \( C^{\text{info}} \) Cost of acquiring extra information
- \( N^{\text{out}} \) Number of repetitions in probit simulation model (outer loop)
- \( N^{\text{in}} \) Number of repetitions in probit simulation model (inner loop)

**Random variables**

- \( e_{hn} \) Error in perceived travel time on link \( h \) for person \( n \)
- \( e_n = [e_{hn}] \) Expectations of link travel times of person \( n \)
- \( e_{kn} \) Error in perceived travel time on route \( k \) for person \( n \)
- \( C_n = [C_{kn}] \) Expectations of route travel times of person \( n \)
- \( C_n^{\text{Informed}} \) Expected travel time for informed traveller \( n \)
- \( U_k \) Utility of \( k \)th route alternative for person \( n \)
- \( e_{n,\text{exp}} = [c_{hn,\text{exp}}] \) Experienced link travel times of person \( n \)
- \( C_{n,\text{exp}} = [c_{kn,\text{exp}}] \) Experienced route travel times of person \( n \)
- \( C^{\text{link}} \) Matrix of randomized experienced travel times at link level
- \( C^{\text{route}} \) Matrix of randomized experienced travel times at route level

**Model parameters**

- \( \alpha \) Parameter for heterogeneity of population
- \( \beta \) Parameter for level of uncertainty of decision makers
- \( \gamma \) Value of time of time parameter
REFERENCES


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Figures

Figure 1: Sample network

Figure 2: Choice between two route alternatives with uncertain travel times
Figure 3: The data-acquisition decision as a part of the route choice problem

```plaintext
set FRACTION=0
for r=1:N^{out}
    Draw a sample C_{n} of individual travel time perceptions at link level
    Determine the route alternative with the shortest expected travel time, \min(C_{kn})
    set C_{n, Informed} = 0
    for s=1:N^{in}
        Draw a sample of experienced travel times at link level, C_{n}^{Exp}
        Determine the shortest experienced travel time at route level, \min(C_{n}^{Exp}).
        set C_{n, Informed} = C_{n, Informed} + \min(C_{n}^{Exp}) / N^{in}
    end
    if C_{n, Informed} + \gamma C_{Info} < \min(-C_{kn})
        set FRACTION = FRACTION + 1 / N^{out}
    end
end
```

Figure 4: Algorithm to determine the fraction of informed travellers
Figure 5: Sample network

Figure 6: Fraction of Informed Travellers as a Function of Information Cost, 
$\beta=0.3$, $\alpha=0.15, 0.3, 0.45, 0.6$
Figure 7: Fraction of Informed Travellers as a Function of Information Cost, 
$\alpha=0.3, \beta=0.15, 0.3, 0.45, 0.6$