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SOME EFFECTS OF A VELOCITY GRADIENT
ON AIRCRAFT FLYOVER NOISE

by

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1. Introduction

Since the variations of temperature and wind velocity with height cause a different propagation speed at upper and lower parts of each wave front, the sound paths normally show a certain curvature [1]. In this paper some effects of this refraction of the sound paths on observed sound pressure levels from aircraft flyover noise have been investigated.

A main result from this study is that for typical atmospheric conditions occurring during daytime, noise reduction due to shadow formation is restricted to upwind directions.

Further, it appears that outside the shadow zone focusing effects have only a slight effect on ground observed sound pressure levels. On the other hand, this analysis demonstrates that in comparison with rectilinear sound propagation, at small elevation angles the presence of wind has a significant effect on the periodicity of reinforcements and cancellations from ground reflection effects.

These observations seem a valuable part in the analysis of noise problems around airports.

2. Curvature of sound paths

Suppose the temperature and wind velocity are functions only of height and that the wind has no vertical velocity component.

Let the sound travel in a z-x plane with the origin at the emission point of the source, located at a height h above the ground plane. If the positive z-coordinate is taken vertically downwards, the vertical and horizontal components of the propagation speed are given by

\[
\frac{dz}{dt} = c \sin \delta \tag{1}
\]

\[
\frac{dx}{dt} = c \cos \delta + u \tag{2}
\]

In these equations c is the speed of sound, u is the component of wind velocity in the x-direction and \( \delta \) is the angle the sound path makes with the horizontal. In equation (2) the positive sign denotes downwind directions and the negative sign upwind directions.
Snellius law of refraction requires the relation

\[
\frac{c_1}{\cos \delta_1} + U_1 = \frac{c}{\cos \delta} + U = \text{constant},
\]  

(3)

where the subscript 1 denotes the initial conditions of each sound ray at the emission point.

The radius of curvature of the sound ray is given by

\[
x = \frac{dr}{d\delta}
\]

(4)

where

\[
dr = \sqrt{(dx)^2 + (dz)^2}
\]

(5)

From equation (3) follows

\[
\frac{d\delta}{dr} = -\frac{\cos \delta}{c \sin \delta} \frac{dz}{dr} \left\{ \frac{dc}{dz} + \frac{du}{dz \cos \delta} \right\},
\]

(6)

and from equations (1), (2) and (5)

\[
\frac{dz}{dr} = -\frac{c \sin \delta}{\sqrt{c^2 + 2 \ c \ cos \delta + U^2}}
\]

(7)

Substitution of equation (7) into (6) yields

\[
x = -\sqrt{\frac{c}{\cos \delta} + U^2 + U^2 \tan^2 \delta} \frac{dc}{dz} + \frac{du}{cos \delta dz}
\]

(8)

Using equation (3), the radius of curvature in downwind direction can be expressed as
\begin{equation}
\begin{aligned}
 r &= - \frac{\sqrt{\left(\frac{c_1}{\cos \delta_1} + u_1^2 + u^2 \tan^2 \delta \right)}}{\frac{dc}{dz} + \frac{du}{dz} \cos \delta} \\
 r &= - \frac{\sqrt{\left(\frac{c_1}{\cos \delta_1} - u_1^2 + u^2 \tan^2 \delta \right)}}{\frac{dc}{dz} - \frac{du}{dz} \cos \delta}
\end{aligned}
\end{equation}

Likewise in upwind direction

3. Wind velocity

For an open area the variation of wind velocity with height is approximately as shown in Figure 1. For heights greater than \( h^* \) the wind velocity may be represented by the profile for a turbulent shear layer of a uniform wind over a flat plane

\begin{equation}
U = U^* + \frac{U^*}{K} \ln \frac{h-z}{h^*}, \text{ for } z \leq (h-h^*)
\end{equation}

where \( K \) is a constant of the order 0.4 and \( h^* \) is a characteristic height that depends on the roughness of the ground surface. The so-called friction velocity \( U^* \) at height \( h^* \) is determined by the magnitude of the velocity \( U_m \) measured at height \( h_m \)

\begin{equation}
U^* = \frac{U_m}{1 + \frac{1}{K} \ln \frac{h_m}{h^*}}
\end{equation}

Below, roughness heights are given for flat surfaces and for a variety of different grass-covered surfaces, as obtained from experimental observations \([2]\). Also are listed values of \( U^* \) for a number of wind velocities \( U_m \) measured at a height of 2 metres above the ground.
<table>
<thead>
<tr>
<th>Type of surface</th>
<th>( h^* ) (m)</th>
<th>( U^* ) (m/s)</th>
<th>( U_m = 2 \text{ m/s} )</th>
<th>( U_m = 4 \text{ m/s} )</th>
<th>( U_m = 6 \text{ m/s} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>very smooth (mud flats, ice)</td>
<td>0.00001</td>
<td>0.06</td>
<td>0.13</td>
<td>0.19</td>
<td></td>
</tr>
<tr>
<td>thinly grown grass up to 0.1 m high</td>
<td>0.007</td>
<td>0.13</td>
<td>0.26</td>
<td>0.40</td>
<td></td>
</tr>
<tr>
<td>densely grown grass up to 0.1 m high</td>
<td>0.023</td>
<td>0.16</td>
<td>0.33</td>
<td>0.49</td>
<td></td>
</tr>
<tr>
<td>thinly grown grass up to 0.5 m high</td>
<td>0.050</td>
<td>0.20</td>
<td>0.39</td>
<td>0.59</td>
<td></td>
</tr>
<tr>
<td>densely grown grass up to 0.5 m high</td>
<td>0.090</td>
<td>0.23</td>
<td>0.46</td>
<td>0.69</td>
<td></td>
</tr>
</tbody>
</table>

The variable velocity gradient follows from equation (11) as

\[
\frac{dU}{dz} = - \frac{U^*}{K} \frac{1}{(h-z)} \quad \text{for } Z < (h-h^*)
\] (13)

Very near the ground surface the gradient (13) may be inaccurate because of the laminar sublayer adjacent to the surface. Therefore, for heights lower than \( h^* \) the velocity gradient is assumed to be constant and equal to

\[
\frac{dU}{dz} = -\frac{U^*}{h^*}
\] (14)

For the case of an isothermal atmosphere equations (9) and (10) become

\[
r = -\sqrt{\frac{C_1}{\cos \delta_1} \left( \pm \frac{U_1}{U} \right)^2 + U^2 \tan^2 \delta} \quad \frac{du}{dz} \cos \delta
\] (15)
Due to the varying wind gradient and wind velocity the curvature of the sound paths varies with height. This means that an exact determination of the path lengths only can be obtained by numerical integration or by use of simplifying assumptions with regard to the curvatures. Figure 2 shows a qualitative picture of the sound radiation predicted by equation (15).

Considered is a typical wind structure of an increasing velocity with increasing height. In this case the sound rays are bent towards the ground in downwind direction and upwards in upwind direction. Upwind there will be a shadow zone, into which no direct sound can penetrate.

4. Temperature gradient

In an adiabatic atmosphere, as is usually the case during daytime, the air temperature decreases with increasing height above the ground. Under standard-atmospheric conditions the temperature gradient is regarded as a constant [3]. In this case the speed of sound can be written as

\[ c = c_0 \sqrt{1 + \frac{\lambda}{T_0} (z-h)}, \]  

(16)

where \( \lambda = 0.0065 \) K/m is the temperature gradient and \( T_0 = 288.15 \) K and \( c_0 = 340.294 \) m/s are the temperature and speed of sound at sea level, respectively.

Using in equation (16) the approximation

\[ \sqrt{1 + \frac{\lambda}{T_0} (z-h)} = 1 + \frac{\lambda}{2T_0} (z-h), \]  

(17)

also the speed of sound becomes a linear function of height

\[ c = c_0 + \frac{\lambda c_0}{2T_0} (z-h) = c_0 + \frac{dc}{dz} (z-h), \]  

(18)

where the gradient of the speed of sound is

\[ \frac{dc}{dz} = \frac{\lambda c_0}{2T_0} = 0.00384 \frac{m/s}{m} \]  

(19)
In the absence of wind equations (9) and (10) reduce to

\[ r = - \frac{C_1}{\cos \delta \frac{dc}{dz}} \]  

(20)

This equation indicates that in a quiescent atmosphere the radius of curvature of each sound ray simply is a constant. Since \( r = \frac{dR}{d\delta} < 0 \), the rays are refracted upwards symmetrical with respect to the vertical axis through the emission point of the source.

The geometrical construction of the solution (20) is outlined in Figure 3. From this Figure it can be seen that the centres of the circular paths lie at a level \( z = - \frac{C_1}{\frac{dc}{dz}} \), at which height the speed of sound would be zero.

5. Shadow zone formation

An important effect of upward bending of the sound rays is the creation of a shadow zone. This region is determined by the sound ray which just strikes the ground surface (Figure 2).

Figure 3 indicates that in the case of a thermal gradient only, the horizontal distance \( X \) between the emission point and the point on which the limiting ray touches the ground, is given by

\[ \left( \frac{C_1}{\frac{dc}{dz}} + h \right)^2 = X^2 + \left( \frac{C_1}{\frac{dc}{dz}} \right)^2 \]  

(21)

or

\[ X = \sqrt{h(h + \frac{2C_1}{\frac{dc}{dz}})} \]  

(22)

Insertion of equation (18) yields

\[ X = \sqrt{h\left(\frac{2C_0}{\frac{dc}{dz}} - h\right)} = \sqrt{h\left(\frac{4T_0}{\lambda} - h\right)} \]  

(23)

Since \( h \ll \frac{4T_0}{\lambda} \), equation (23) can be approximated by

\[ X = \sqrt{\frac{4T_0}{\lambda} h} \]  

(24)
The corresponding elevation angle is given by

$$\psi = \tan^{-1} \frac{h}{X} = \tan^{-1} \sqrt{\frac{Ah}{4T_0}}$$

(25)

In Figure 4, distance X and angle \(\psi\) as a function of source height are presented, using \(T_0 = 288.15\ K\) and \(\lambda = 0.0065\ K/m\). In this Figure also the location of the shadow zone in a combined field of temperature and wind gradient is given. The effects of wind result from numerical integration of the pathlengths. Considered are wind velocities measured at a height of 2 m above a grass-covered surface. The wind gradients are determined according to equations (11), (13) and (18). Obviously, the distance to the shadow zone strongly decreases with increasing wind velocity.

It is clear that only the component of the wind velocity in the direction of the sound rays affects the refraction. In this connection, it is often reported that in certain downwind directions the temperature gradient may balance the effect of wind [4]. However, equation (14) indicates that even for very low wind velocities the wind gradient may exceed the temperature lapse rate in a large degree.

Then, independent of the magnitude of wind velocity and source height, directly downwind the distance to the shadow zone will recede to infinity. In other words, as shown in Figure 5, for the typical daytime conditions it seems more realistic to expect that only in the half plane in upwind direction shadow zone formation occurs.

Sound energy is diffracted into the shadow zone and scattered into it by turbulence. Measurement data in Reference [5] indicate that the excess attenuation inside the shadow zone increases rapidly with increasing distance to the shadow boundary and tends to level off to an approximately constant value.

Figure 6 shows a summary chart of an idealized attenuation function, derived from measurements at near horizontal sound propagation in straight upwind direction. These data were obtained in Reference [5] for wind velocities of 3 to 20 mph and for four octave bands ranging between 300 and 4800 Hz, and a ground cover height of 1 ft. The source and receiver were located at a height of 12 and 5 ft, respectively.
6. Reflection index

Assuming a quiescent and isothermal atmosphere, wherein rectilinear sound propagation occurs, the so-called reflection index, being the difference between the sound pressure measured above a reflecting ground surface and the sound pressure level in free space, is given by [6, 7].

\[
\Delta L_{pg} = 10 \log_{10} \left\{ 1 + \left( \frac{1}{R_2} \right)^2 |Q|^2 + 2 \frac{1}{R_2} |Q| \cos(KA + \phi) \right\}, \tag{26}
\]

where \( R_1 \) is the direct travel distance between source and receiver, \( R_2 \) the distance along the path reflected from the ground, \( Q = |Q|e^{i\phi} \) the reflection coefficient and \( K \) the propagation constant in air.

Contrary to the mode of propagation assumed in equation (26), the curvature of the sound paths resulting from a velocity gradient causes an increment of the lengths \( R_1 \) and \( R_2 \), which may affect the reflection index.

Some results of computations, obtained by numerical integration of the pathlengths, are presented in Figures 7 and 8. Figure 7 shows the reflection index as a function of receiver height at a given frequency for two elevation angles. As in Figure 4, a wind velocity measured at a height of 2 m above a grass-covered surface is considered. Figure 8 shows corresponding values of the reflection index as a function of frequency at a given receiver height.

The main conclusions from these computations can be summarized as follows:
- Temperature effects are small when compared to the influence of wind.
- The effect of a velocity gradient on the amplitude of the reflection index may be negligible.
- A curvature of the sound paths certainly affects the periodicity of reinforcements and cancellations. The importance of this effect increases with increasing frequency, increasing receiver height and decreasing elevation angle.
7. Focusing effects

In References [8] and [9], it is reported that for ground-to-ground noise propagation where the sound rays make small angles to the horizontal, divergence and convergence effects occur for both refraction and reflection in the presence of a velocity gradient. In the following, the significance of these focusing effects for the case of flyover noise is considered.

Refraction - In order to specify the problem elementary bundles of sound rays travelling in upwind direction are considered. Figure 9 shows, in the plane of propagation, the extreme rays of bundles with and without velocity gradient. Since the same amount of energy is confined to the bundles, the sound intensity depends on their cross-sectional areas. Along the bundles the angular width is constant, but the angular height of the refracted bundle changes by the slightly different curvatures of the rays. Therefore, at the receiver the ratio of the amplitude of the sound pressure of the refracted rays to the amplitude of the non-refracted rays is equal to the square root of the ratio of the lateral spacings.

A description of the modification of the sound pressure levels caused by refraction can be made by regarding the radius of curvature a constant. The variables in equation (8) may be determined at the half of the vertical distance between source and receiver.

Then, from the geometrical construction in Figure 10, it follows that in the case of upwind propagation the ratio of lateral spacings at the receiver comes near

\[
\frac{a}{a_0} = \frac{2 \{r_2 \sin(\delta_2 - \psi) - r_1 \sin(\delta_1 - \psi)\} \sin(\delta_1 - \psi)}{2r_1 \sin(\delta_1 - \psi) \Delta \delta_1},
\]

(27)

where \( \delta_2 = \delta_1 + \Delta \delta_1 \).

Using further the approximation

\[
r_2 = r_1 \cos \frac{\delta_1}{\delta_2},
\]

(28)
the following expression is obtained

$$\frac{a}{a_0} = \cos \delta_1 \cos \psi (1 + \tan^2 \delta_1)$$  \hspace{1cm} (29)$$

Thus, the difference between the sound pressure level measured with and without velocity gradient, is given by

$$\Delta L_{p_c} = 10 \log_{10} \frac{a_0}{a} = 10 \log_{10} \{\cos \delta_1 \cos \psi (1 + \tan^2 \delta_1)\}^{-1}$$  \hspace{1cm} (30)$$

For a radiation geometry defined by $\Delta h$ and $\psi$, the angle $\delta_1$ is given by the relation

$$\delta_1 = \psi + \sin^{-1} \left\{ \frac{\Delta h}{2 r_1 \sin \psi} \right\}$$  \hspace{1cm} (31)$$

Numerical calculations on the magnitude of $\Delta L_{p_c}$ are made for several source heights and for various angles of incidence. However, the results indicate that for air-to-ground sound propagation focusing effects are negligible, since the deviations from rectilinear sound propagation are less than 0,5 dB.

**Reflection** - In consequence of the fact that for curved sound rays the reflection process involves different deviations for different angles of incidence, an additional modification of the sound pressure in waves reflected from the ground may occur.

The latter effect is illustrated in Figure 11, where two curved sound rays, separated at the emission point by an angle $\Delta \delta_1$, are reflected from the ground. Under the circumstances of upwind and reflection from a flat surface, the upper ray of the incident sound is refracted an additional angle $\Delta \alpha$ after the lower ray has reached the surface. Since by symmetry, the angular height between the rays is increased $2 \Delta \alpha$ at reflection their lateral spacing at the receiver, is given by

$$a = R_{21} \Delta \delta_1 + R_{22} (\Delta \delta_1 + 2 \Delta \alpha),$$  \hspace{1cm} (32)$$

where $R_{21}$ and $R_{22}$ are the portions of the total pathlengths before and after reflection, respectively.
If the radius of curvature in the vicinity of the reflection point is denoted \( r_0 \), it follows from Figure 11 that the angle \( \Delta \alpha \) can be expressed as

\[
\Delta \alpha = \frac{R_{21} \Delta \delta_1}{r_0 \tan \psi},
\]

(33)

where the angle of incidence \( \psi \) may be approximated by

\[
\psi = \tan^{-1} \left( \frac{h + \frac{m}{s}}{2} \right).
\]

(34)

Substitution of equation (34) into (32) yields

\[
a = \left\{ R_{21} + R_{22} + \frac{2R_{21} R_{22}}{r_0 \tan \psi} \right\} \Delta \delta_1
\]

(35)

In the absence of the reflecting plane, the final spacing would be

\[
a_0 = (R_{21} + R_{22}) \Delta \delta_1
\]

(36)

Hence, the ratio of cross-sectional areas with and without refraction becomes

\[
\frac{a}{a_0} = 1 + \frac{2 R_{21} R_{22}}{(R_{21} + R_{22}) r_0 \tan \psi}
\]

(37)

Consequently, the reflection index given by equation (26) must be written as

\[
\Delta L_{pg} = 10 \log \left\{ 1 + \left( \frac{R_1}{R_2} \right)^2 \left| \frac{2}{a} \right| + 2 \frac{R_1}{R_2} \left| \frac{a_0}{a} \right| \cos (k \Delta R + \psi) \right\}
\]

(38)

In Figure 12, the ratio (37) as a function of angle of incidence is plotted. This Figure shows that at small angles the ratio of cross-sectional areas \( a/a_0 \) is greater than unity so that in that region the sound pressure tends to decrease.

Clearly, in downwind direction the angular separation is decreased at reflection and the opposite effect on the sound pressure is predicted. However, the occurrence of this classic phenomenon strongly depends on the flatness of the ground surface. Further, predictions are not accurate
because of the inherent inaccuracy of the velocity profile very near the ground. Therefore, the extent to which the foregoing expressions can be applied to practical outdoor situations is subject to experiment.

7. References


Fig. 1. Typical wind profile.
Fig. 2. Refraction of sound rays by wind.
Fig. 3. Refraction of sound rays by temperature gradient.
Figure 4: Significance of shadow region.
Figure 6: Excess attenuation inside shadow zone
Fig. 7. Effect of velocity gradient.

$h = 100$ m; $T_0 = 293$; $u = 4$ m/s; $f = 500$ Hz

$\psi = 95^\circ$

$\psi = 20^\circ$
Fig. 8. Effect of velocity gradient.
Fig. 9. Divergence of rays on refraction.
Fig. 10. Refraction of sound rays in uniform velocity gradient.
Fig. 11. Divergence of curved rays on reflection at a flat surface.
wind velocity 4 m/s
at height of 2 m

Figure 12: Focusing effect on reflection in upwind direction