On the Scaling of Sediment Transport in the Nearshore

Introduction

This paper investigates the scaling of sediment transport by waves in the nearshore. Scaling of sediment is often needed in laboratory wave flumes where spatial dimensions are about a factor 10 smaller. Generally, for the physical model, normal sand is chosen with a relatively small diameter. Scaling relations are consulted to translate the spatial dimension and sediment flux to prototype values. Often, the correct scaling of processes, such as transport modes and turbulence regimes, are ignored. However, when the objective is to study intra wave sediment transport it is vital to ensure that the laboratory model has the same flow and transport regimes as in the prototype. Since a great range of artificial sediment is available, the objective here is to find suitable sediment properties to model intra wave sediment transport.

Scaling Relationships

The water motion in short waves is mainly determined by gravitational and inertial forces. In prototype and model the gravitational force is the same. Therefore, the inertial forces should also be the same in prototype and model. This leads to the following scale relation for short waves (known as the Froude scale):

\[ n_H = n_L = n_T^2 = n_t^2 = n_u^2 = n_h \]

where
- \( n \) is the ratio of the prototype value over the model value of the index parameter
- \( H \) is wave height
- \( L \) is wave length
- \( T \) is wave period
- \( t \) is time
- \( u \) is flow velocity
- \( h \) is water depth.

Scaling according to the Froude scale leads to the correct reproduction of wave steepness, shoaling, refraction and diffraction.

The spatial dimensions of the wave flume (i.e. water depth) and the specifications of the wave maker (i.e. wave height) lead to a general spatial scale. Short wave properties (i.e. wave period) are further obtained by using the general spatial scale in combination with the Froude scale. For practical purposes the wave flume is usually filled with natural water which leads to
\[ n_p = n_v = 1. \]  

where  
\( \rho \) is water density  
\( \nu \) is kinematic viscosity

This leaves only two other scalable variables: the sediment diameter and density.

At present there are various papers and reference books that deal with scaling sediment. Usually, they describe a set of dimensionless numbers and parameters representing processes and magnitudes. Having the same values for the numbers in prototype and model promotes similitude and reduces scale effects. In the following, the dimensionless numbers and parameters are summarized, elaborated on, and rewritten as scale relations.

The flow conditions near the bed can be divided in laminar, smooth turbulent and rough turbulent. These regimes can be predicted with the grain size Reynolds number

\[ \text{Re} = \frac{u_d}{\nu} \]  

which can be expressed in the scale relation

\[ n_{Re} = n_{u_s} n_{\nu}^{-1} \]  

where

\( u_s \) is shear velocity  
\( d \) is grain diameter

The densimetric Froude number (named Shields number from hereon)

\[ \theta = \frac{\rho u_s^2}{\gamma d} \]  

is important for mobile beds and leads to the scale relation

\[ n_{\theta} = n_{u_s}^2 n_{\nu}^{-1} n_d^{-1} \]  

where

\( \rho_s \) is sediment density  
\( \gamma \) is submerged unit weight of sediment = \((\rho_s - \rho)g\) where \( g \) is the gravitational constant.

If this number is not preserved it is possible that there is no movement of sediment at all. This parameter is also a good indicator for the different transport regimes which are divided in saltating motion, rippled bed and sheet flow.

During half a wave cycle sediment is picked up, brought in suspension and settles again. The suspended sediment distribution over the water depth is strongly related to the relative settling velocity (van Rijn 1993)
which Kamphuis (1991) introduced as a scale relation

\[ n_v = n_w n_u^{-1} \]  

where \( w_s \) is settling velocity of sediment. A physical interpretation of the parameter is the ratio of the settling velocity and turbulence generated by bed friction (expressed with the friction velocity). The settling velocity can be approximated using Hallermeier’s relationship (Hallermeier, 1981)

\[ w_s = \gamma^{0.7} d^{1.1} \rho^{-0.7} \nu^{0.4} \]

which is convenient to rewrite as a scale relation

\[ n_w = n_r^{0.7} n_d^{1.1} n_p^{-0.7} n_v^{-0.4} . \]

The Dean number

\[ D_{w_s} = \frac{H/T}{w_s} \]

is another popular scale relation

\[ n_{D_{w_s}} = n_h n_T^{-1} n_{w_s}^{-1} . \]

Usually, the parameter is thought of as the ratio of settling time and the wave period. An alternative interpretation of this parameter is the ratio of turbulence generated by wave breaking and the settling velocity. Turbulent energy from wave breaking scales with the ratio of wave height over wave period. This can be shown by modeling the wave energy dissipation of a breaking wave in analogy of a hydraulic jump (Battjes 1975, Fredsoe & Deigaard, 1992)

\[ k \propto H^2 \left( \frac{g}{hT} \right)^{2/3} . \]

Rewriting the relation as a scale relation and substituting the Froude scale relation \( n_h = n_T^2 \) yields

\[ n_k \propto n_h^2 n_h^{-2/3} n_T^{-2/3} \rightarrow n_k \propto n_h^2 n_T^{-4/3} n_T^{-2/3} \rightarrow n_k 0.5 \propto n_h n_T^{-1} . \]

Therefore, the Dean number can also be expressed as

\[ n_{D_{w_s}} = n_k^{0.5} n_{w_s}^{-1} . \]

So basically, the relative settling velocity and the dean number are both ratios between turbulence and settling velocity but focus on different processes. Turbulence by bed friction depends on the sediment diameter (bottom roughness) and hydraulic conditions where turbulence by wave breaking depends only on the hydraulic conditions. This suggests that the Dean number should be used when the area of interest is dominated by wave breaking.

The relative density of sediment is important when considering particle accelerations. Horizontal pressure gradients and inertia forces can have a
significant impact on the total force balance of a sediment particle. This impact can be expressed with the parameter \( S \) (named Sleath number from hereon)

\[
S = \frac{\rho u}{(\rho_s - \rho) g T} \left( \frac{2\pi}{\rho} \right).
\]  

which is the ratio of inertial forces and gravitational forces acting on individual grains of sediment (Zala, Flores, & Sleath, 1998). High Sleath numbers (\( S > 0.2 \)) suggests that sediment will start to move earlier than predicted by the Shields curve. In addition, the mode of transport can become different and sediment will start to move as a block, also known as plug flow (Madsen, 1974; Foster, 2006). Rewriting the Sleath number in a scale relation and substituting the Froude scale relation \( n_u = n_T \) yields

\[
n_s = \rho n_s n_\gamma^{-1} n_T \rightarrow n_s = \rho n_s n_\gamma^{-1} n_T \rightarrow n_s = \rho n_\gamma^{-1}
\]

suggesting that the scale relation only depends on sediment density.

Another similarity parameter is the relative length parameter which is the ratio of a typical length (i.e. wave amplitude) over the sediment diameter

\[
\frac{\lambda}{d}.
\]

Scale effects from dissimilarity of the relative length parameter are unclear. Mogdridge and Kamphuis (1972) conducted experiments with various sediment densities and indicated that bedform patterns depend on the relative length parameter. On the other hand, Nielsen (1992) showed that ripple steepness is predominantly related to the Shields number but this does not account for the ripple height. The bed porosity is directly related to the relative length parameter which is important for wave energy absorption by porosity effects (Kamphuis, 1991). Due to the nontransparent relation between scale effects and the relative length scale, it will only be used qualitatively. The scale relation would be

\[
n_{\lambda/d} = n_\lambda n_d^{-1}.
\]

To complement the set of scale relations, the relation between flow velocity and friction velocity is defined according to Jonsson (1966)

\[
u_u^* = 0.5 f_w u^2
\]

where \( f_w \) is the friction factor. This gives the scale relation

\[
n_u = n_{f_w}^{0.5} n_u
\]

Now the only variable that still needs to be expressed in short wave and sediment properties is the friction factor. This expression depends on the flow regime which is assumed rough turbulent in both prototype and model. This assumption should be checked with the use of e.g. diagrams by Jonsson (1966). The most well-known relation for the friction factor in the rough turbulent regime is given by Swart (1974):
\[ f_w = \exp \left[ -5.997 + 5.213 \left( \frac{a}{k_s} \right)^{-0.194} \right] \quad [22] \]

where

- \( a \) is the horizontal excursion of the orbital motion at the bottom
- \( k_s \) is the effective bed roughness height (i.e. \( k_s = 3d \)).

The form of this expression is not convenient to rewrite as a scale relation. Therefore, the formula is approximated with

\[ f_w = 0.078 \left( \frac{a}{k_s} \right)^{-0.3} \quad [23] \]

which has a correlation of 0.98 with Swart’s relation for the range \( 100 \leq a/k_s \leq 3000 \). Rewriting the relation in a scale relation and substituting the Froude scale relation \( n_u = n_h \) yields

\[ n_{f_u} = n_h^{-0.3} n_d^{0.3} \quad [24] \]

Therefore, the relation between friction velocity and flow velocity can be written as

\[ n_{u^2}^u = n_h^{-0.3} n_d^{0.3} n_{u^2} \quad [25] \]

and in combination with the Froude scale relation \( n_{u^2}^u = n_h \) leads to

\[ n_{u^2}^u = n_h^{0.7} n_d^{0.3} \quad [26] \]

For a given general scale \( n_h \), the scale relations can be plotted as a function of relative density and relative grain diameter. The lines in Figure 1 indicate where the scale relation is equal to 1 (corresponding to preserving the scale number or parameter). From the figure it can be seen that at some point the lines cross and some scale relations are simultaneously equal to 1. The scaling rules that correspond to these points will be discussed in the next sections.
Figure 1. Scaling relations equal to 1 as a function of relative sediment density and relative grain diameter. The general spatial scale is set equal to 10. Green star indicates sediment properties of the case study. To maintain overview, the figure does not include the Sleath number which would be a vertical line at \( n_{p_{\text{sediment}}} = 1 \).

**Bed-load Dominated Model**

The present laboratory study is conducted for research into onshore bar migration in the nearshore which occurs when there is almost no wave breaking on the bar. Therefore, the scale ratios of the Reynolds number, Shields number and relative settling velocity should ideally be equal to 1 (this point is indicated in Figure 1). In addition, applying the following conditions and substitutions to the scale ratios:

- usage of natural water \( n_p = n_v = 1 \)
- substitution of hydraulic scale ratios into the water depth ratio using the Froude scale relation \( n_H = n_L = n_r^2 = n_i^2 = n_u^2 = n_h \)
- substitution of the friction velocity \( n_u^2 = n_h^{0.7} n_d^{0.3} \)

results in

\[
   n_{Re} = n_u n_d n_v^{-1} = 1 \rightarrow n_h^{0.7} n_d^{2.3} = 1
\]
This indicates that the Reynolds number can be preserved by selecting a grain diameter equal to \( n_d = n_h^{0.3} \). Surprisingly, the Shields number and the relative settling velocity are both preserved when \( n_\gamma = n_d^{-3} \). Following these relations results in a model where at least three processes are properly scaled:

- turbulence regime in the wave boundary layer
- mobility of sediment by bed friction
- settling of suspended sediment.

Scale effects may occur by not maintaining the Sleath number and relative length parameter. Scaling according to the bed-load model described above, will result in a mismatch of the Sleath number equal to \( n_s = n_h^{-0.9} \). It is challenging to assess the corresponding scale effect since the number is only partially validated. Nonetheless, it is fair to conclude that the mobility of sediment can be larger than predicted due incorrect scaling of the relative density. The mismatch of the relative length \( n_{\lambda/d} = n_h^{1.3} \) suggests that ripple geometry will not be scaled correctly and the model bed porosity will be relatively larger (Kamphuis 1985).

**Suspension Dominated Model**

Turbulent energy generated by wave breaking is a factor 10 larger (or more) than turbulent energy generated by bottom friction. Therefore, it would be appropriate to use the Dean number for scaling instead of the relative settling velocity in the presence of wave breaking. It is possible to preserve the Dean number together with the Shields number (this point is indicated in Figure 1).

Using natural water \( n_p = n_\gamma = 1 \) and the Froude scale relation \( n_h = n_\gamma^2 = n_h \), the Dean number scale relation can be written as

\[
n_D = n_h n_\gamma^{-1} n_w^{-1} = 1 \rightarrow n_d^{1.1} = n_h^{0.5} n_\gamma^{-0.7}.
\]  

[30]

Applying the same procedure for the Shields number and using relation for the friction velocity \( n_u = n_h^{0.7} n_d^{0.3} \) leads to

\[
n_\theta = n_u^{-1} n_\gamma^{-1} n_d^{-1} = 1 \rightarrow n_d^{0.7} = n_h^{0.7} n_\gamma^{-1}.
\]  

[31]

Substitution of the two relations above results in

\[
n_\gamma = n_h^{0.7}
\]  

[32] and

\[
n_d \approx 1.
\]  

[33]

Following these relations results in a model where at least two processes are properly scaled:
• mobility of sediment by bed friction
• settling of suspended sediment.

From the scale relations it appears the suspension model has similar scale effects as the bed-load model (but slightly less magnitude). The mismatch of the Sleath number is $n_S = n_h^{-0.7}$ and of the relative length parameter is $n_{s,ld} = n_h$. Considering scale effects due to mismatch in sediment density and grain diameter, similar conclusions can be applied to both models.

**Case Study Bed-load Model**

The case study consists of a wave flume experiment. The purpose of the case study is to see whether the sediment behaves as expected according to the bed-load model and to assess the scale effects. The scaling rules of the bed-load model are applied for the physical model in the wave flume. In the flume an onshore sandbar migration needs to be simulated which corresponds more or less to the onshore sandbar migration that occurred during the DUCK-94 field experiment (Elgar 2001).

During the onshore sandbar migration at DUCK-94 Shields numbers higher than 0.8 occurred suggesting no ripples on the bar. The sediment transport over the bar should be bed-load dominated, suspended sediment should settle swiftly.

For the model a general scale of $n_h = 10$ is chosen which is imposed by wave flume dimensions and the available amount of sediment. The sediment properties for the model are based on the bed-load model scaling rules. The necessary sediment properties are approached as close as possible depending on availability. See Figure 1 for the position of the relative sediment density and grain diameter. This resulted in sediment with a density of 1200 kg*m$^{-3}$ and a grain diameter of 540 µm (the prototype grain diameter is 220 µm).

The hydraulic conditions for the model can be estimated by applying the Froude scale for the prototype conditions. Similitude of the wave period could not be obtained due to limitations of the wave maker and the wave flume. Therefore, the wave period in the model was chosen relatively smaller than in the prototype. Nevertheless, the Shields numbers over the bar in the model were still higher than 0.8.

The initial profile in the flume has a well-defined bar similar to the profile found at the DUCK-94 experiment. Due to the small available amount of sediment, the general slope of profile (1:15) in the model is much steeper than the prototype slope (1:80).
During the experiment, the bar moved onshore as can be seen in Figure 2. No ripples where found on the bar and sheet flow was observed frequently. Suspended sediment was only observed close to the bottom.

Scale effects where visible. Energetic monochromatic waves tend to liquefy the upper 2 cm of the bed after approximately 10 waves. This was not found with irregular and bichromatic waves.

![Graph showing water depth vs. distance from wave board.](image)

**Figure 2.** Bottom evolution in the wave flume during irregular waves. Solid line is initial profile. Dashed line is profile after 5 hours. Water depth at the toe of the slope is 0.5 m.

**Discussion**

For the bed-load model the friction factor is important. In the description above the flow regime is assumed rough turbulent for prototype and model. Similar results are obtained if the flow regime is assumed laminar or smooth turbulent. Using the expression for the friction factor that corresponds with these regimes also results in a bed-load model where the Reynolds number, Shields number and the relative settling velocity are preserved. The scaling rules will slightly change.
Present literature suggests that preserving the Dean number results in geometrically undistorted models. To the author’s knowledge, this has only been confirmed for models with natural sand and does not necessarily apply for lightweight sediment. Hughes (1993) mentioned that distorted model laws converge for models that are geometrically undistorted and use natural sand. Strangely, when the density of sediment is included in the distorted model law of Vellinga (1986) there is no convergence with the Dean number for a geometrically undistorted model with lightweight sediment.

In present literature bedforms are often related to the Shields parameter. These relations are often derived from experiments with natural sand and do not necessarily apply for lightweight sediment. Nonetheless, preserving the Shields number suggests preserving the ripple steepness (Van Rijn 1993; Nielsen 1992). But the relations of lightweight sediment to ripple height and ripple length remain unclear to the author.

**Conclusions**

Lightweight sediment gives the opportunity to preserve several similarity parameters. This is valid for the combination of the Reynolds number, Shields number and relative settling velocity (referred to as bed-load model). Also combination of the Dean number with the Shields number is possible (referred to as suspended model).

A wave flume experiment was conducted using the bed-load model. Transport regimes observed for the bed-load model agreed well with the expectations based on the Shields number and relative settling velocity. The morphological evolution of the coastal profile in the wave flume was similar to that observed in nature.

Both models will have similar scale effects since their mismatch in the Sleath number and relative length parameter is about the same. There is a large chance that liquefaction of the bed occurs under energetic monochromatic waves. This has not been evident for irregular and bichromatic waves. Kamphuis () suggests that the energy absorption by the bed is probably larger due to the increased bed porosity.

**Acknowledgements**

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