PIV EXPERIMENTS ON THE FLOW INDUCED BY A SPHERE SEDIMENTING TOWARDS A SOLID WALL

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Master's Thesis
Abstract

The motion induced by gravity of solid spheres in a vessel filled with fluid has been investigated experimentally at Reynolds numbers in the range from 1-74 and Stokes numbers ranging from 0.2-17. Trajectories of the spheres have been measured with a focus on start-up behavior, and on impact with a horizontal wall. Two models have been investigated. The first describes the accelerating motion of the sphere. The second model predicts the distance from the wall at which the sphere starts decelerating.

The flow in the vicinity of the sphere was measured by means of PIV. The time scales and flow structures strongly depend on the Reynolds number. Measurements performed are in good agreement with simulations performed at the Kramers Laboratorium.
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<td>a</td>
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<td>$A_p$</td>
<td>particle surface area</td>
<td>$m^2$</td>
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<tr>
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<td>$n_{r2}$</td>
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<td>$N$</td>
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<td>pressure</td>
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<td>instantaneous Reynolds number of the particle</td>
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<td>radius of wave front at point of interest</td>
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<td>$R_r$</td>
<td>radius of wave front at reference point</td>
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<tr>
<td>$Re_s$</td>
<td>Reynolds number based on $v_s$</td>
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<td>$t$</td>
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<td>$t_{0}$</td>
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<td>$X$</td>
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<td>$z$</td>
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<td>angle</td>
<td>rad</td>
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<tr>
<td>$\delta Z$</td>
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<td>$\Delta$</td>
<td>diameter of a pixel</td>
<td>pix.</td>
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<td>$\Delta t$</td>
<td>time between two consecutive images acquired</td>
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<td>$\Delta x$</td>
<td>distance between top left of light sheet and right side of particle</td>
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<td>$\Delta_{\text{g}}$</td>
<td>distance between origin of cross-corr. plane and cross-corr. peak</td>
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<td>$\Delta z$</td>
<td>distance between light sheet and centre of mass of particle</td>
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<td>vortex position relative to sphere centre</td>
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<td>beam divergence at laser exit</td>
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<td>beam radius at reference point</td>
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<td>particle density</td>
<td>kg/m$^3$</td>
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<td>$\xi$</td>
<td>particle diameter to vessel width ratio</td>
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<td>$\chi$</td>
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### Miscellaneous

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<tr>
<td>$IA$</td>
<td>Interrogation Area</td>
</tr>
<tr>
<td>$PIV$</td>
<td>Particle Image Velocimetry</td>
</tr>
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<td>Transpose</td>
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Abstract

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Chapter 1

Introduction

1.1 General background

In many industrial processes, two-phase flows play an important role. A common kind of a two-phase flow is a particulate flow. This is a flow in which the dispersed phase is a solid in the form of particles and the continuous phase is a gas or a liquid. Such flows can be found in the fields of mechanical, petroleum, mining, chemical and nuclear engineering. Other examples of such flows can be found in natural processes like snow avalanches, erosion, sand storms, dune formation, etc.

Despite the prominence of this type of flows, its behaviour is not well understood. A large part of the industrial applications are designed empirically. Due to their prevalence, even small improvements in the performance of these technologies could have an important economic impact.

1.2 Micro-hydrodynamics in crystallisation

This research contributes to a larger project to improve the microscopic modelling of hydrodynamics in industrial crystallisers. In these crystallisers, particulate flows are very important, because the slurry in the crystalliser consists of solid particles, the crystals, dispersed in a liquid. To get a better insight in the process hydrodynamics, flow simulations of the crystalliser as a whole using the lattice-Boltzmann method have been performed, for more details see [Ten Cate et al., 2000]. These simulations only give information about the fluid flow on length scales much larger than the crystal size. However, as an important parameter in crystallisation is the desired crystal size, which is influenced by the rate at which crystal-crystal collisions occur, it is also important to obtain information about the processes that occur on the small length scales. Microscopic modelling deals with the modelling of these small scale phenomena in the crystalliser.

As the motion of single particles through a crystalliser is too complex to simulate numerically, a simplified approach is used to get insight in the hydrodynamics of single crystals. Approximately 100 spheres are put in a box in which turbulent flow
Chapter 1. Introduction

Simulations

- Crystalliser as a whole
- 100 spheres in box
- Simplification 1
- Simplification 2
- Single sphere in box
- Validation

Experiment

- Single sphere in box

Figure 1.1: Simplifications made for validating numerical simulations of a crystalliser.

is simulated. These spheres have Reynolds numbers ranging from 10 to 60. The flow conditions in the box are comparable to the flow conditions in the crystalliser. More information about those simulations can be found in [Ten Cate et al., 2000]. However, the question arises if the lattice-Boltzmann method is capable of accurately simulating the movement of spheres in a liquid. To check this, a model system is defined: a simulation of a single settling sphere in a box, again at Reynolds numbers ranging from 10 to 60 is done. Note that this is the most simple case of particulate flow. This simulation of a single sphere settling under gravity and colliding onto the bottom wall of a box is considered a good, first test case for the many-particle box simulations, see [Ten Cate et al., 2000]. This simple case has to be verified with experiments. As a summary, all simplifications made are shown in the left part of figure 1.1.

1.3 Objectives

The main objective of this research is to set up a series of experiments to obtain data that can be used for validating the single sphere simulation mentioned above. To do this, experimental set-ups are built with a box and a sphere having sizes that make it possible to use exactly the same geometry for simulations. This makes it possible to compare simulations and experiments directly, on a 1-on-1 scale, as illustrated in figure 1.1. For validation, measurements have been performed at Reynolds numbers ranging from 1 to 100, of two parameters:

- The trajectory of the sphere.
- The flow field in the fluid induced by the motion of the sphere. This flow field is measured using Particle Image Velocimetry (PIV).

It is desirable to use a particle and a fluid such that the particle to fluid density ratio is comparable to 1.4 (a typical value for crystals in crystallisers).
The second objective is to look into two models for the motion of the sphere. The first model describes the accelerating motion of the particle, the second predicts the distance from the bottom of the vessel at which it starts to decelerate before it hits the wall.

1.4 Outline of this thesis

In chapter 2 of this report the motion of a single particle in a fluid will be discussed. Theory about Particle Image Velocimetry needed for performing the experiments is given in chapter 3. In chapter 4 the experimental set-up used for trajectory measurements is presented. Results of trajectory measurements are discussed in chapter 5. In chapter 6 the used PIV measurement set-up is explained. The results of PIV measurements performed are given in chapter 7. Finally, in chapter 8 conclusions and recommendations concerning the research are given.
Chapter 2

Motion of a single particle in a fluid

2.1 Introduction

Consider a spherical particle hanging at rest in a rectangular vessel filled with liquid, see figure 2.1. If the density of the particle is larger than the density of the liquid, the particle starts falling down when it is released. Forces acting on the particle are the gravity force, Archimedes force and the drag force. These forces are illustrated in figure 2.1. The trajectory the particle travels can be divided into several parts. First, the forces acting on the particle will make it accelerate until a maximum velocity is reached. After this, the particle will (quickly) decelerate and come to rest on the bottom of the vessel, or it will bounce back from the bottom if its energy is large enough.

In the next sections, a more detailed treatment of the foregoing phenomena will be given. First, the equation of motion of the fluid is given. Second, the equation of motion of the particle in an infinite medium will be given. Third, steady state particle motion and unsteady particle motion will be discussed. Fourth, a simplified

![Figure 2.1: Particle in a rectangular vessel filled with liquid.](image)
equation of motion for the particle will be presented. Fifth, influences from the side-walls of the box will be discussed. Finally, criteria will be given, for bouncing back from the bottom wall of the vessel.

## 2.2 Equation of motion of the fluid

The liquid in the vessel is assumed to be Newtonian. The reason for this, is that the equations derived in the next paragraphs apply to Newtonian liquids. During this research project, glycerol-water-solutions and silicone oil are used. These liquids are Newtonian fluids. Newtonian liquids follow the law of Newton, which states:

$$ \tau = \mu_f (\nabla \mathbf{v}_f + \nabla \mathbf{v}_f^T) $$

where \( \tau \) is the shear stress, \( \mu_f \) is the dynamic viscosity of the fluid and \( \mathbf{v}_f \) is the velocity of the fluid. According to this equation, the shear stress is proportional to the strain rate \( \frac{1}{2}(\nabla \mathbf{v}_f + \nabla \mathbf{v}_f^T) \). Further, it is assumed, that the fluid is homogeneous and incompressible. As a consequence the density of the fluid is constant. The set of equations, that describe the flow of the fluid, are the continuity and Navier-Stokes equations, and in this case they simplify to:

$$ \nabla \cdot \mathbf{v}_f = 0 $$

and

$$ \frac{\partial \mathbf{v}_f}{\partial t} + (\mathbf{v}_f \cdot \nabla) \mathbf{v}_f = -\frac{1}{\rho_f} \nabla p + \frac{\mu_f}{\rho_f} \nabla \cdot \mathbf{v}_f + \mathbf{g} $$

where \( \rho_f \) is the density of the fluid, \( p \) is the pressure and \( \mathbf{g} \) is the gravitational acceleration.

## 2.3 Equation of motion of the particle

In this project, as well as in most other studies on related subjects, the geometrical dimensions and the density of the particle are supposed to remain constant. The particles are rigid spheres and have a smooth surface. The equation of motion for the particle can be obtained from a momentum balance following Newton’s second law:

$$ \rho_p V_p \frac{d(V_p)}{dt} = -\int \int_{A_p} p n dA_p + \int \int_{A_p} \tau \cdot n dA_p + \int \int_{V_p} g (\rho_p - \rho_f) dV_p $$

where \( V_p \) is the sphere volume, \( A_p \) is the sphere surface, \( v_p \) is the sphere velocity, \( \rho_p \) is the sphere density and \( n \) is the unit vector perpendicular to the surface of the sphere. The first term on the right-hand side of this equation is the local pressure of the fluid. The second term is the force of friction, which acts on the surface and is due to viscosity. Often these terms are combined to give one term, which is referred to as the drag force. In principle, the volume integral term comprises all external
field forces which act on the particle. In general, however, the external force field is given by the gravitational field only. Taking into account all assumptions mentioned above, this equation can be simplified to:

\[ V_p \rho_p \frac{dV_p}{dt} = V_p(\rho_p - \rho_f)g + F_D \]

(2.5)

where \( F_D \) is the drag force exerted on the particle. This drag force takes into account both the pressure and the shear effect.

## 2.4 Steady particle/fluid motion

The motion of the particle is called steady-state if the velocities of both the particle and the fluid are constant. If the particle velocity is different from the fluid velocity, the force exerted by the fluid on the particle depends on the relative velocity (or slip-velocity), \( u_r = v_f - v_p \). As mentioned in section 2.3, this force is called the drag force. Unfortunately, an expression for the drag force can not be derived analytically, even for a simple shape such as a sphere, except for extremely low Reynolds numbers. This is due to the fact, that exact solutions for the flow field are not available, except for the creeping flow region. Therefore, it proves useful to introduce an experimentally determined drag coefficient \( C_D \), valid for a single, smooth, non-rotating sphere in a steady, incompressible and unbounded flow field, in the expression of the drag-force. This coefficient is defined as the ratio of the drag force over the dynamic head of the flow and the frontal area of the particle. For spherical particles, the definition is:

\[ F_D = C_D \frac{1}{4} \pi D^2 \frac{1}{2} \rho_f (v_f - v_p)^2 \]

(2.6)

where \( D \) is the particle diameter. Since the drag may be positive or negative, depending on whether the fluid velocity is greater or smaller than the particle velocity, the square of the relative velocity is usually replaced by \((v_f - v_p)(v_f - v_p)\). The force is then always obtained with the correct sign. The drag coefficient is a function of the particle Reynolds number, \( Re_p \), which is defined as:

\[ Re_p = \frac{\rho_f D |v_f - v_p|}{\mu_f} \]

(2.7)

Results from measurements covering a wide range of experimental conditions can be found in the literature. From these measurements correlations for \( C_D \) as a function of Reynolds number have been determined by fitting equations to measured data. They can be found in most texts on fluid mechanics, for example [Janssen and Warmoeskerken, 1991]. The correlation for the drag coefficient used in this research is the expression which has been proposed by [Abraham, 1970]:

\[ C_D = \frac{24}{9.06^2} \left( \frac{9.06}{Re_p^{1/2}} + 1 \right)^2 \]

(2.8)

which is in good agreement with experiments (it has an average relative deviation of 7.6% and maximum relative deviations of +13 and -14%) for \( 0 \leq Re_p \leq 5000 \) [Abraham, 1970].
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2.5 Unsteady particle/fluid motion

When unsteady situations are considered, the problem grows more complex. A dimensional analysis, described by [Clift, 1978], gives insight into which parameters may influence the interaction between the fluid and the particle. In general, the (overall) drag coefficient of particles in unsteady motion is considered to depend on the following non-dimensional groups. First, the instantaneous Reynolds number should be mentioned, defined in equation (2.7). Second, the displacement modulus $M_D$, defined as the ratio of the displacement of the particle since it started to move and particle diameter, is an influencing parameter. The overall drag coefficient furthermore depends on the solid/fluid density ratio $\gamma$, defined as the ratio of particle density and fluid density. Last but not least, the acceleration modulus, $A_c$, is used as a non-dimensional parameter. In general, this acceleration modulus is defined by:

$$A_c = \frac{(v_f - v_p)^2}{D} \frac{d(v_f - v_p)}{dt}$$  \hspace{1cm} (2.9)

This number is a measure of the ability of the flow field around the particle to adjust to changing relative velocity. It stands for the ratio between forces due to convective acceleration ($\rho_f v_p \frac{\partial v_p}{\partial x} \sim \rho_f v_p^2/D$) and forces due to local acceleration ($\rho_f \frac{\partial v_p}{\partial t} \sim \rho_f \frac{dv_p}{dt}$). If the forces due to convective acceleration are small, compared to the forces due to local acceleration, the acceleration modulus will be low. Below a certain value of $A_c$ the contribution of the convective acceleration may be disregarded. For the situation of a particle moving in a liquid, the density ratio $\gamma$ is relatively low, i.e. up to a value of about 10. In that case, the unsteady motion can be characterised by a rapid change of Reynolds number with distance. The principal consideration in unsteady motion of this kind is the development of the flow pattern and associated entrainment of the fluid. Since the instantaneous flow pattern may have little resemblance with the fully-developed flow, the instantaneous drag may differ radically from and be even almost unrelated to the steady motion drag.

For the drag in creeping flow, the convective acceleration term in the Navier Stokes equation is negligible. Although this results in a considerable simplification of the problem, solution is still complicated and requires a thorough mathematical analysis. For details, see [Talman, 1994] or [Althusius, 1997].

Equation of Odar and Hamilton

For the drag at Reynolds numbers higher than that of creeping flow, no theoretical expression is available. [Odar and Hamilton, 1964] suggested an expression for the drag force. By making use of empirical coefficients they proposed the following equation:

$$F_D = C_D \frac{1}{4} \pi D^2 \frac{1}{2} \rho_f |v_r| v_r + C_A \frac{1}{6} \pi D^3 \rho_f \frac{dv_r}{dt} + C_H \frac{1}{4} D^2 \sqrt{\pi \mu_f \rho_f} \int_0^t \frac{dv_r}{d\tau} d\tau$$ \hspace{1cm} (2.10)

where $C_D$ is the steady drag coefficient, $C_A$ is the (empirical) added-mass coefficient and $C_H$ is the (empirical) history coefficient. The second term on the right is usually
2.6. Simplified equation of motion

called the added-mass term, the third the Basset force or history term. Based on measurements with a sphere oscillating in a viscous liquid, [Odar, 1966] gave the following empirical formulas for these coefficients:

\[ C_A = 1.05 - \frac{0.066}{Ac^2 + 0.12} \quad (2.11) \]
\[ C_H = 2.88 + \frac{3.12}{(Ac + 1)^3} \quad (2.12) \]

Note that \( \gamma \) is not included in these expressions. While these expressions were derived from experiments with spheres in harmonic motion, [Odar, 1966] found, again experimentally, that the above expressions were equally applicable to the situation of a sphere falling in a liquid. However, the opposite has been demonstrated in later experiments by other researchers, who have found several empirical relationships for different experimental conditions, see [Talman, 1994]. From this the conclusion must be drawn that, unfortunately, the correlations for \( C_A \) and \( C_H \) are not universally valid, so that each problem requires its own expression, or perhaps different parameters are needed to describe the problem correctly.

2.6 Simplified equation of motion

The problem with equation (2.10) is that the integral makes it difficult to get an analytical solution for the equation of motion. To get an analytical solution, [Ferreira and Chhabra, 1998] entirely neglect the history term and consequently put \( C_H = 0 \). They also put \( C_A = 1/2 \), which is the value for the added-mass coefficient for irrotational flow. According to Ferreira, this approximation is consistent with previous studies in this field. The equation of motion becomes:

\[ m \frac{dv_p}{dt} = mg \left( 1 - \frac{\rho_f}{\rho_p} \right) - \frac{1}{8} \pi D^2 \rho_f v_p^2 - \frac{1}{12} \pi D^3 \rho_f \frac{dv_p}{dt} \quad (2.13) \]

This equation can be solved analytically. The result can be found in Appendix A.

The significance of the Basset-force diminishes with increasing density of the sphere in comparison with the fluid, i.e. \( \rho_p \gg \rho_f \). An order of magnitude estimation for this force can be made from equation (2.10) by assuming \( \frac{dv_p}{dt} \) constant:

\[ F_{D,Basset} \simeq C_H \frac{1}{4} D^2 \sqrt{\pi \mu_f \rho_f} \frac{dv_p}{dt} \int_0^t \frac{dt}{\sqrt{t - \tau}} \quad (2.14) \]

Now an estimation for \( C_H \) has to be made. For unsteady, rectilinear movement of a sphere at low Reynolds numbers, \( C_H = 6 \). [Odar and Hamilton, 1964] determined \( C_H \) for a sphere in oscillatory motion at high Reynolds numbers. They found, that \( C_H = 6 \) is a maximum. [Dohmen, 1968] found values of \( C_H \) around 1 for a falling sphere. Here a value of \( C_H = 1 \) is used. After integration, equation (2.14) becomes:

\[ F_{D,Basset} \simeq \frac{1}{4} D^2 \sqrt{\pi \mu_f \rho_f} \frac{dv_p}{dt} 2 \sqrt{t} \quad (2.15) \]
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The value for \( t \) can be experimentally estimated as the time it takes for the sphere to reach steady state falling velocity \( v_s \). The factor \( \frac{dn_t}{dt} \) can then be estimated by calculating \( \frac{v_s - v_{0t}}{t - t_0} = \frac{v_s}{t} \) if the particle starts from rest at \( t = 0 \).

2.7 The wall effect

The theory for steady and unsteady motion of a sphere in a fluid as discussed in the foregoing sections, was derived for an infinite medium. Since in the experiments the fluid is bounded by the walls of the vessel, it is necessary to investigate possible wall effects. In general it may be said, that if the diameter of the sphere becomes appreciable with respect to the diameter of the vessel in which it is settling, the walls of the vessel will exert an additional retarding effect, resulting in a lower steady state particle velocity than would be the case in an infinite medium. It is brought about by the upwards counterflux of fluid which balances the downward flux of the solid and that of the entrained down flow [Di Felice, 1996]. The smaller the area available for the counterflux, i.e. the smaller the the vessel cross sectional area compared to the particle size, the more important the phenomenon is. The magnitude of this effect also depends on the Reynolds number. In case of creeping flow the wall effects are larger than for the situation of intermediate or high Reynolds number flow around the sphere. For intermediate flow (\( Re_p \) roughly 1-100) around a single settling sphere at steady state falling velocity, [Di Felice, 1996], found the following empirical correlation for a sphere falling in a cylindrical tube:

\[
\left( \frac{v_{s+\text{corr}}}{v_s} \right) = \left( \frac{1 - \frac{v_s}{v_s}}{1 - 0.33\xi} \right)^\alpha
\]

\[
\alpha = \frac{3.3 + 0.085 Re_v}{1 + 0.1 Re_v}
\]

where \( v_{s+\text{corr}} \) is the terminal velocity corrected for wall effects, \( v_s \) is the terminal settling velocity in an infinite medium, \( \xi \) is the particle to the tube diameter ratio and \( Re_v \) is the Reynolds number based on the terminal velocity in an infinite medium. It has to be noted however, that this correlation is not very accurate, especially for small values of \( \xi \), because reported experimental evidence for intermediate Reynolds number flow are scarce. The equation mentioned above is derived for a tube, but it is also valid for a rectangular vessel, if for \( \xi \) the particle diameter to vessel width ratio is used [Althuisius, 1997].

2.8 Wall impact

Several things happen when a sphere approaches the bottom of a square vessel:

- there is a change in flow-field, and consequently, a change of drag on the sphere.
- fluid is pushed out of the gap between the sphere and the vessel.
- added mass of the sphere is changed, due to the approach of the wall.
2.8. Wall impact

- if the flow is Stokes flow, asymmetry of the flow is developed.

When the sphere approaches the wall, it decelerates. This phenomenon has been investigated by [Gondret et al., 1999]. He studied rigid spheres approaching a wall moving in a viscous fluid. In this case its kinetic energy is dissipated by viscous forces as it approaches the wall. For Stokes flow, [Brenner, 1961] has calculated the viscous force of resistance $F_{visc}$ acting on the sphere moving at the velocity $v_p$ into a fluid of viscosity $\mu_f$ at a distance $h$ between its bottom apex and the wall,

\[ F_{visc} = 3\pi \mu_f D v_p \zeta, \]

where $\zeta = \zeta(h/D)$ is a correction to Stokes’ law given by an infinite series. There is no correction when the sphere is far away from the wall, so $\zeta \to 1$ when $h/D \to \infty$. When the distance gets infinitely small, $\zeta \to D/h$ when $h/D \to 0$. The rate of approach is asymptotically slow and the sphere does not rebound. The approach of the wall of the sphere in Stokes flow has been tested experimentally by [Gondret et al., 1999] and the results are in good agreement with Brenner’s theory. This makes it possible to calculate the trajectory of the sphere, and to predict at which height $h_c$ the sphere starts ‘feeling’ the wall, i.e. when it starts decelerating. For larger Reynolds numbers, fluid inertia needs to be considered as well. A quick estimate of the magnitude of $h_c$ can be made by comparing the time of diffusion for the momentum on the length scale $h$, $t_{diff} = h^2 \rho_f/\mu_f$ and the time for the particle advection on the same length scale, $t_{adv} = h/v_{s^+}$. A critical distance $h_c$ can be defined as the height at which these two times are equal:

\[ \frac{h_c}{D} = \frac{1}{Re_{s^+}} \]  

(2.18)

Where $Re_{s^+}$ is the Reynolds number of the particle based on $v_{s^+}$. Hence the distance from the wall at which the sphere starts decelerating, is smaller when $Re_{s^+}$ is larger.

For the intermediate flow regime, the region between Stokes and turbulent flow, rebound may occur. This depends on the part of the particle kinetic energy that will be dissipated in the fluid and by deformation of both the particle and the vessel wall. If the elastic deformation of the sphere and the vessel wall is large enough, rebound occurs. The relevant parameter for the bouncing transition is the Stokes number:

\[ St = \frac{1}{9} \frac{\rho_p D v_{s^+}}{\mu_f} \]  

(2.19)

This dimensionless number characterises particle inertia relative to viscous forces. The critical Stokes number $St_c$, which defines the non-bouncing to bouncing transition, was found to be around 20 by experiments with a steel sphere falling in a liquid towards a glass wall [Gondret et al., 1999].

A correlation for $St_c$ derived from experiments with different particles in water is reported by [Zenit and Hunt, 1999]:

\[ St_c = \frac{e^{-\pi} \rho_f}{C_0^2 \rho_p} \]  

(2.20)

where $C_0$ is a proportionality constant. Values of $C_0$ used are 0.05, 0.03 and 0.02. Comparisons between this model and experiments show qualitative agreement [Zenit
and Hunt, 1999]. The model predicts that when the density ratio increases, the critical Stokes number decreases. Hence, as the particle is lighter, the viscous forces become more dominant and therefore the critical Stokes number increases.
Chapter 3

PIV Theory

One of the objectives of this research is to measure the flow field around a sphere falling in a liquid and colliding with a wall using Particle Image Velocimetry (PIV). In this chapter, theory needed for designing and operating a PIV experimental set-up is given. Also the data analysis procedure is explained.

3.1 Principle of Particle Image Velocimetry

PIV is a technique to measure the velocity field in a fluid flow. A plane is illuminated with a sheet of laser light. The flow is seeded with small tracer particles, that scatter light when they are in the laser sheet. Pictures are taken from consecutive illuminations. From these pictures, the instantaneous velocity distribution of the flow field is reconstructed by measuring the distance travelled by groups of particles between two consecutive illuminations. The pictures taken are divided in small areas. Using an interrogation method, the distance travelled by the particles in these areas can be determined. When this distance is divided by the time between the two consecutive illuminations, the velocity of the fluid is known. This procedure will be explained in the following sections by discussing the following topics:

- Optical system
- Seeding
- Laser sheet
- Image acquisition
- Analysis of images
- Spurious vectors

In these sections also some choices that are made with respect to the laser sheet and the way of acquiring and processing images are explained.
Chapter 3. PIV Theory

3.2 Optical system

It is important to obtain pictures with clear and sharp particle images. A schematic representation of the light sheet and optical system for imaging tracer particles in a planar cross section of a flow is shown in figure 3.1. It is assumed, that the system consists of an aberration-free, thin circular lens with a focal length $f$ and a diameter $D_{app}$. The object distance $Z_0$ and image distance $z_0$ satisfy the geometrical lens law:

$$\frac{1}{Z_0} + \frac{1}{z_0} = \frac{1}{f} \tag{3.1}$$

The image magnification is given by:

$$M = \frac{z_0}{Z_0} \tag{3.2}$$

The particles in the object plane are illuminated with a thin light sheet from a coherent light source, for example a laser with wavelength $\lambda$. It is important that all observed particles are in focus. This condition is satisfied if the thickness of the light sheet is smaller than the the object focal depth $\delta Z$ of the imaging lens, as given by [Adrian, 1991]:

$$\delta Z = 4 \left( 1 + \frac{1}{M} \right)^2 \frac{f^2 \lambda}{D_{app}^2} \tag{3.3}$$

3.3 Seeding

The flow has to be seeded with particles, that have to obey four criteria:

- The particles must be neutrally buoyant. If they are not, they will be displaced with a gravitationally induced sedimentation velocity $U_g = \frac{d^2}{18 \mu_f} g (\rho_r - \rho_f)$ (calculated for very low Reynolds numbers [Raffel et al., 1998]).
3.4 Laser sheet

- The particle diameter must be small compared to the length scales that are to be investigated.

- To follow the fluid motion correctly, the particle relaxation time \( \tau_p = \frac{d^2}{18 \mu} \) has to be small [Raffel et al., 1998]. This implies that the particle diameter must be small.

- The light scattered by the particles has to be as intense as possible. For that reason, the difference between the refractive index of the particles and the fluid has to be as large as possible. Because the intensity of the light scattered by particles between 1 and 100 \( \mu m \) linearly increases with increasing particle volume, see [Adrian, 1991], the size of the particles must not be too small.

3.4 Laser sheet

For illumination of the particles, a laser sheet is used. A laser makes it possible to get a high light intensity in a small volume, which ensures a detection of the particles. Different ways of making the sheet are:

- Sweeping sheet

- Continuous sheet

- Pulsating sheet

A large sweeping sheet can be made using a rotating polygon mirror. A small continuous sheet can be made using one or more lenses. The essential element for generating a continuous light sheet is a cylindrical lens. When using a laser with a sufficiently small beam diameter and divergence, for example an Argon-ion laser, one cylindrical lens can be sufficient to generate a light sheet of appropriate shape. However, often additional spherical lenses are used.

For pulsating sheets, often NdYAG lasers are used. This type of lasers is able to generate a pulsating sheet with high energy and frequency. However, it is also possible to make a pulsating sheet using a continuous (for example Argon-ion) laser. To do this, in front of the laser-exit a chopper has to be installed, which chops the laser beam periodically. A disadvantage of this method is the loss of effective laser-power when the chopper is closed.

For the experiments performed in this research, a small sheet is sufficient. Evaluation of the data is done using multi-frame/single exposure PIV (see section 3.5). For these reasons, a continuous light sheet has been chosen. Another advantage of using a continuous sheet is that it is possible to use all laser power available.
3.5 Image acquisition

When discussing image acquisition, two topics are important: the recording method and the camera. A choice has to be made for a recording method. There are two distinctive fundamentally different recording methods: single frame/multi-exposure PIV and multi-frame/single exposure PIV. A schematic picture of these two methods is shown in figure 3.2. The principal distinction is that without additional effort the first method does not retain information on the temporal order of the illumination pulses, giving rise to a directional ambiguity in the recovered displacement vector. In contrast, multi-frame/single exposure PIV recording inherently preserves the temporal order of the particle images and hence is the method of choice if a fast camera is available. Also in terms of evaluation, this method is much easier to handle. For these reasons, the choice is made to use multi-frame/single exposure PIV in this research.

A camera is used for registration of the illuminated particles in the light sheet. Both a digital CCD camera and a photographic camera can be used. As analysis of the pictures using a computer requires digital pictures, a big advantage of using a CCD camera is digital registration. Another advantage of CCD compared to photographic cameras is the high frequency at which images can be taken. However, this frequency is inversely proportional to the spatial resolution of the camera (e.g., when the number of pixels of the CCD array is increased by a factor 4, the maximum capturing frequency is 4 times lower). Most conventional CCD cameras have low resolution compared to photographic film, for example 512x512 pixels compared to 3000x4500 lines. A disadvantage of CCD cameras compared to photographic cameras is the amount of light needed. CCD cameras generally need more scattered light for imaging of tracer particles. As a consequence, more laser power is required when using a CCD camera. In general, a high recording frequency is needed when using the multi-frame/single exposure PIV recording method. For this reason, a CCD camera is used in this research.
3.6 Analysis of images

3.6.1 Analysis of displacement

To analyse digital PIV images, an interrogation method is used. The images are divided in small areas, the interrogation areas (IA’s), as shown in figure 3.3. In this research, the multi-frame/single exposure PIV recording method is used. In this case, analysis of images is performed by cross-correlation. The cross-correlation between two interrogation windows out of two consecutive recordings is calculated. The vector $\Delta s$ between the origin of the cross-correlation plane and the cross-correlation peak resulting from this calculation is a measure for the average displacement of the particles in the IA. As the time $\Delta t$ between two consecutive frames and the magnification factor of the images are known from the measurement, the velocity $v_f$ of the fluid corresponding to this IA can easily be calculated:

$$ v_f = \frac{\Delta s}{M\Delta t} \quad (3.4) $$

3.6.2 Calculation of the cross-correlation

Consider an image of $N \times N$ pixels. Two coordinates $i, j$ determine the place of a pixel with intensity $I_{i,j}$. The spatial average of this region is defined as:

$$ \bar{I} = \frac{1}{N^2} \sum_{i=1}^{N} \sum_{j=1}^{N} I_{i,j} \quad (3.5) $$

This spatial average is used in calculating the image cross-correlation for two IA’s, $I'_{i,j}$ and $I''_{i,j}$ for a shift over $[r,s]$, see [Westerweel, 1993]:

$$ \hat{R}_{II}[r,s] = \frac{1}{N^2} \sum_{i}^{N-|r|} \sum_{j}^{N-|s|} (I'_{i,j} - \bar{I})(I''_{i+r,j+s} - \bar{I}) \quad (3.6) $$

with:

$$ \sum_{i}^{N-|r|} = \begin{cases} \sum_{i=1}^{N-r} & \text{for } r \geq 0 \\ \sum_{i=1-r}^{N} & \text{for } r < 0 \end{cases} \quad (3.7) $$
3.6.3 Estimation of fractional displacement

The particle image displacement is given by the location of the cross-correlation peak with respect to the origin. Provided there are sufficient matching particle image pairs, the cross-correlation peak is defined as the maximum of $R_{II}[r,s]$, located at $[m_0,n_0]$. The actual displacement $(r_D,s_D)$ can be written as:

$$ r_D = (m_0 + \epsilon_m)\Delta \quad \text{and} \quad s_D = (n_0 + \epsilon_n)\Delta $$

where $m_0$ and $n_0$ are defined as the nearest integer displacements, and $\epsilon_m$ and $\epsilon_n$ as the fractional displacements, which both have a value between -0.5 and 0.5 pixels. $\Delta$ is the size of a pixel. The fractional displacement can be estimated using a three-point estimator. This is a function that estimates the position of a peak by fitting a function (e.g. a Gaussian function or a parabola) to the cross-correlation function. It is assumed that $R_{II}[r,s]$ is circularly symmetric and separable in $r$ and $s$ and that $\epsilon_m$ and $\epsilon_n$ are statistically orthogonal: $\text{cov}(\epsilon_m, \epsilon_n) = 0$. This makes it possible to deal with the fractional displacement in the two directions as if they are independent. The Gaussian peak fit estimator gives the best result for the fractional displacement [Westerweel, 1993]. This estimator is given by:

$$ \hat{\epsilon}_G = \ln R_{-1} - \ln R_{+1} $$

where $\hat{\epsilon}_G$ is the Gaussian fractional displacement estimator, $R_0$ is the highest value of the peak and $R_{-1}$ and $R_{+1}$ are the values adjacent to the highest value. The displacement estimation error is a function of the displacement [Westerweel, 1997]. This can be seen in figure 3.4. In this figure the RMS measurement error for the displacement in pixel units for a $32 \times 32$ pixel IA in cross-correlation is given, for particle images with a diameter $d_r$ of 2 and 4 pixels. The dots are the results from
3.7. **Spurious vectors**

Monte-Carlo simulations and the solid lines are analytical results [Westerweel, 1993]. From this figure the measurement error is approximately constant if the displacement due to the flow is larger than 0.5 pix, but for displacements smaller than 0.5 pix, the error is linear with displacement. Therefore, the absolute measurement accuracy for particle images with a diameter of 4 pix is approximately 0.1 pix for displacements larger than 0.5 pix and the relative measurement accuracy is approximately 17% for displacements smaller than 0.5 pix.

3.6.4 **Optimization of correlation**

To get optimal results from the PIV analysis, [Keane and Adrian, 1990] carried out Monte-Carlo simulations to determine the requirements for the experimental parameters to yield optimal performance of the PIV analysis. They recommend the following criteria:

- The number of particles per interrogation area should be at least 15
- The particle-image displacement in the direction perpendicular to the light sheet ("out-of-plane" displacement) should be less than 1/4 of the thickness of the light sheet
- The in-plane displacement of the particle images should be about or less than 1/4 of the size of the interrogation area
- The velocity difference over the interrogation area should be at most 5% of the mean velocity

3.7 **Spurious vectors**

In PIV one often finds that measurement results contain a number of "spurious" vectors. These vectors deviate un-physically in magnitude and direction from nearby "valid" vectors. In general these vectors originate from IA’s containing insufficient particles. In practise, the number of spurious vectors in a PIV data set is relatively low (typically less than 5% for data with good signal to noise ratio). However, their occurrence is more or less inevitable: even in carefully designed experiments there is a probability that an interrogation yields a spurious vector.

Spurious vectors can be detected using the local median test [Westerweel, 1994]. In this test, the median of the length of all the neighbouring (max. 8) vectors is used to determine the residual of every displacement vector. From a histogram of these residuals the cut-off residual is determined. All vectors having a larger residual than the cut-off residual are removed. After this removal, the resulting empty spaces can be filled by vectors determined from the local mean displacement of the neighbouring vectors. In this research, Cleanvec [Solof and Meinhart, 1999] is used for both the removals and the replacements.


3.8 Estimation of vorticity

After velocity vector fields have been obtained and spurious vectors have been replaced, the out-of-plane component of the vorticity of the flow field can be calculated. In this research, the vorticity estimate for point \((i, j)\) is calculated using a circulation estimate around the neighbouring eight points [Raffel et al., 1998]:

\[
(\omega_Z)_{i,j} = \frac{\Gamma_{i,j}}{4\Delta X\Delta Y}
\]  

(3.10)

\[
\Gamma_{i,j} = \frac{1}{2} \Delta X[(v_{fx})_{i-1,j-1} + 2(v_{fx})_{i,j-1} + (v_{fx})_{i+1,j-1}] + \frac{1}{2} \Delta Y[(v_{fy})_{i+1,j-1} + 2(v_{fy})_{i+1,j} + (v_{fy})_{i+1,j+1}]
\]

\[
- \frac{1}{2} \Delta X[(v_{fx})_{i+1,j+1} + 2(v_{fx})_{i,j+1} + (v_{fx})_{i-1,j+1}] - \frac{1}{2} \Delta Y[(v_{fy})_{i-1,j+1} + 2(v_{fy})_{i-1,j} + (v_{fy})_{i-1,j-1}]
\]  

(3.11)

where \(\Delta X\) and \(\Delta Y\) are the distances between the mesh points in a PIV data set in the x- and y-direction.
Chapter 4

Experimental set-up for sphere trajectory measurements

In this chapter, first a global description is given of the measurement setup for sphere trajectories. Second, a more elaborate description of some parts of the setup is given. Third, image processing needed for extracting place-time graphs of the sphere from raw measurement data will be explained.

4.1 Measurement setup

In figure 4.1 a schematic representation of the measurement setup is shown. The main part of this setup is a rectangular vessel. On the bottom of the vessel lies a 19 mm thick glass plate. The vessel is filled with a fluid, in this case a glycerol-water mixture of a certain viscosity and density. In the top of the vessel a glass sphere is hanging at a release mechanism. The sphere is positioned in the centre of the vessel. The distance between the top of the sphere and the top of the glass plate is 130 mm. This sphere is illuminated by a 750 W spotlight. A dimming glass plate

![Figure 4.1: Schematic picture of the experimental set-up used for trajectory measurements: (1) Rectangular vessel, (2) Glass plate, (3) Fluid, (4) Glass sphere, (5) Release mechanism, (6) Spotlight, (7) Dimming glass plate, (8) Camera.](image-url)
Chapter 4. Experimental set-up for sphere trajectory measurements

is placed between the spotlight and the vessel to make the light of this lamp more homogeneous. In front of the vessel, a digital camera is placed to take pictures of the dark sphere against a white background.

A measurement is made by starting capturing images with the digital camera. Starting the camera triggers the release mechanism; the sphere is released and will start falling down until it reaches the bottom of the vessel and comes to rest. During the settling, images of the falling sphere are taken at high frequency. When the measurement is finished, a film that can be saved to computer disk has been obtained. Using a C-routine in SCIL-image, an image processing package [Scilimage, 1998], a place-time datafile of the sphere can be extracted from the film. Thus, information of the trajectory of the sphere as a function of time is obtained. In the next section, more will be said about:

- Rectangular vessel
- Spheres
- Fluid
- Release mechanism
- Digital camera

4.2 Detailed description of some components

4.2.1 Rectangular vessel

The experiments are performed in a transparent perspex rectangular vessel. The inner dimensions are: \(d \times w \times h = 134 \times 134 \times 185\) mm. It is filled with liquid to a height of 165 mm. At the bottom lies a glass plate of \(d \times w \times h = 120 \times 120 \times 19\) mm. (This plate has been inserted because the initial idea was to do bouncing experiments with the sphere. However, it turned out that at interesting Reynolds numbers no rebound occurs.)

4.2.2 Spheres

The spheres used in this research are non-transparent black glass beads from Brooks flow-meters. They are used in flow-meters, which implies that they may be considered as perfectly spherical. Spheres of four different diameters are used. The diameter was determined experimentally using a slide-rule: 9.53, 6.35, 5.55 and 3.18 mm. The mass of each sphere was measured using a substitution balance (type Mettler B5). Dividing by the volume of the sphere results in the density \(\rho_p\) of the spheres:

\[
\rho_p = (2.55 \pm 0.02) \cdot 10^3 \ kg \cdot m^{-3}
\]
4.2. Detailed description of some components

4.2.3 Fluid

The fluids used in this experiment are glycerol-water mixtures. They behave as Newtonian liquids. The more glycerol is contained in the mixture, the larger the dynamic viscosity is. A wide range of viscosities can be created. For example: a solution of 5 weight-% glycerol has a viscosity of 1.143 mPas at 293 K and 100 weight-% glycerol has a viscosity 1499 mPas at 293 K, see [Janssen and Warmoeskerken, 1991]. An advantage of using glycerol-water solutions, is that they are cheap. A disadvantage is the temperature sensitivity of the viscosity. For example: a 80 weight-% glycerol-water solution has a viscosity of 62.0 mPas at 293 K, but a viscosity of 45.86 mPas at 298 K. This is a decrease of 3.2 mPas/K. This decrease can be a problem when doing experiments throughout the day: the viscosity of the fluid changes.

At the end of each series of measurements, the viscosity of the fluid is measured, to decrease the influence of temperature change. This is done using a Contraves Rheomat 115 rotational viscometer. The density of the fluid is determined using a 100 ml flask and a Mettler balance.

4.2.4 Release mechanism

A schematic picture of the release mechanism is given in figure 4.2. The main part of the release mechanism is a Pasteur pipette (length 23 cm). The large opening of the pipette (diameter \( \approx 5 \text{ mm} \)) is both connected to the vacuum net of the laboratory and an electronic valve which stands directly in contact with the surroundings. The sphere is hanging at the small (diameter \( \approx 1 \text{ mm} \)) opening of the pipette, sucked by the vacuum. A valve is inserted between the pipette and the vacuum net to control the pressure drop over the sphere. A pressure gauge is put between the pipette and the vacuum net, to get an idea of the pressure-drop. When the electronic valve is opened by the trigger signal, the vacuum vanishes. The sphere is released.
Chapter 4. Experimental set-up for sphere trajectory measurements

Figure 4.3: Steps to determine the centre of mass of a sphere from raw data. (a) Raw image after contrast-stretch (by eye, the raw image without contrast-stretch looks the same), (b) Image a after background subtraction, inverting and another contrast-stretch, (c) Image b after thresholding.

4.2.5 Digital camera

During this experiment two CCD (Charge Coupled Device) cameras are used. The first one is a Dalsa CA-D6-0512W-BCEW camera of 512 x 512 pixels and a maximum capturing frequency $f_{\text{max}}=250$ Hz, the second a Dalsa CA-D6-0256W-ECEW camera of 256 x 256 pixels with $f_{\text{max}}=1000$ Hz. The pixel-size is 10 x 10 $\mu$m for both cameras. The camera used is connected to a computer running a data-acquisition program. With this program it is possible to adjust camera settings, like recording frequency and number of frames to acquire.

For trajectory measurements two lenses are used. The accelerating and steady state motion is measured with a magnification factor $M=0.038$ using a lens with a focal length of 16 mm and a minimum f-number (ratio of focal length and aperture diameter or the opening of the diaphragm of the lens) of 1.4, trajectories close to the wall with $M=0.14$ using a Nikon lens of focal length of 55 mm and a minimum f-number of 2.8.

4.3 Image processing

In this research, the image processing package SCIL-Image is used to find the centre of mass of the sphere on recorded films. The images consist of a number of points in a two-dimensional plane. Each pixel is assigned a grey-value according to the intensity of the on-falling light. This grey-value is in the range of 0 (black) and 255 (white). In this section, the steps needed to find the centre of mass are explained.

An example of an image is shown in figure 4.3a. This picture is a contrast-stretched version of the raw measurement data. Contrast-stretching is performed to obtain an optimum contrast between objects in an image and its background. The entire range of existing grey-values in the new image has been rescaled to the maximum possible values 0-255, while all other pixels have been multiplied using the same factor as
4.3. Image processing

used for the highest pixel value. In the recordings resulting from the experiments the particle is dark and the background is light. Contrast stretching has assigned a value of 255 to the background.

It is desirable to remove objects other than the sphere, for example the vessel wall, from the image before processing. This is done using background subtraction. A recorded background image is subtracted from the image. To get an image of a white sphere on a black background, the image is inverted. Finally a second contrast-stretch is performed on the image. This results in an image of a white sphere on a dark background, as shown in figure 4.3b.

The next step is to divide the image into a number of objects, containing the sphere, and a background. This is done using image segmentation. One of the most simple ways to perform this segmentation is thresholding. Pixels with grey-values equal to or higher than the adjusted threshold value are made object pixels (value 1) and pixels with grey-values below the threshold values are made background values (value 0). The result is a binary image shown in figure 4.3c: the pixels are either 0 or 1. In general, the main problem using this type of segmentation is to find a suitable threshold value. For this purpose, a histogram of grey-values may be constructed. This shows that the background of the images have pixel values that do not exceed the value of 5. Therefore, the threshold value has been chosen just above this value, at a value of 10. This way a clear distinction between object and background has been obtained.

After this the objects in the image have to be labelled: every object is given a number. The sphere is selected and the x- and y-coordinate of the centre of mass are determined. Using a C-routine, the whole film is processed automatically and the result, the trajectory the sphere, is written to a data-file.

4.3.1 Calibration

The last thing that requires attention is that the place coordinates of the sphere stored in the data file are in pixels. By calibration of the recordings the absolute values of the coordinates can be calculated. This calibration is performed by placing a ruler at the place where the sphere is falling. By taking an image (figure 4.4), the area to which a pixels corresponds (m x m) is determined.

To check if there is image distortion, the distance between the 11 and 12, 5 and 6 and the 0 and 1 cm mark on the ruler was determined. These are 39, 39 and 40 pixels, respectively. This indicates that image distortion can be neglected: there is no large difference between the area to which a pixel corresponds in the top, middle and bottom of the field of view of the camera.
Figure 4.4: Example of a picture of a ruler used for calibration of trajectory measurements.
Chapter 5

Results and discussion of trajectory measurements

In this chapter, results of sphere trajectory measurements will be presented. First, the accuracy and reproducibility of the used measurement method is discussed. Second, results of measured trajectories are shown and derived quantities, for example velocity, will be calculated. Third, some final remarks will be made.

5.1 Accuracy and reproducibility

The accuracy and reproducibility of the measurements is discussed in this section. First, spatial and grey-value discretization are discussed. Second, the positioning of the camera is explained. Third, attention is given to the release moment of the sphere. Fourth, errors in measured and calculated quantities are given. Fifth, the reproducibility of the measurements is discussed.

Every recorded image consists of a limited number of pixels (512 x 512 or 256 x 256). The diameter of the sphere calculated from the recordings should be equal to the real diameter. For example, in a measurement the diameter for the 6.35 mm sphere was 24 or 25 pixels. The area to which a pixel corresponds was 0.26 x 0.26 mm$^2$, resulting in sphere diameters of 6.24 and 6.50 mm. Because of the good resemblance between measured and real sphere diameters, the size of the error due to spatial discretization is estimated 1 pixel. For the measurements of acceleration and steady-state trajectories this results in errors relative to sphere diameter of 3%, 5%, 5% and 9% for sphere diameters of 9.53, 6.35, 5.55 and 3.18 mm, respectively. For the experimental conditions of the wall impact measurements, this error is 3%.

In section 4.3 it has been described how on recorded films the place of the sphere is determined. It is reasonable that a change of threshold level (between certain levels) for a spherical particle on a dark background should have no influence on the determined centre of gravity of the particle. For this reason, some recordings have also been processed using a threshold value of 50 instead of 10. There was observed no difference between the position of the centre of gravity. Since the error in the
experimentally determined diameter of the sphere is of the order of 1 pixel, the error in the determined centre of gravity of the sphere will be less than 1 pixel.

If the camera is not positioned exactly horizontally, deviations will occur in the coordinates of the particle position. A spirit-level was placed on top of the camera, to position it horizontally. To align the camera more accurate, a plummet was hung in the vessel. The camera was tilted until the plummet was imaged exactly on one vertical line of pixels. This means that the camera is aligned with the direction of gravitational acceleration.

Due to variations in the measurement process the sphere was not always released at the same time. On average, the sphere was released 4 frames after the camera started acquiring frames, with a deviation of 2 frames. This difference in release moment can be compensated for by applying a small time-shift to the resulting place-time graph.

Inaccuracies of some measured and calculated quantities are given in table 5.1. The inaccuracy of $\rho_f$ is mainly due to the inaccuracy of using the flask. In $\mu_f$ the inaccuracy is due to the measurement error of the viscosimeter. In $v_s$ it is due to the error in the used correlation for the drag curve. The inaccuracy in $v_{s+}$ mainly originates from performing linear regression to calculate this velocity from the data. In $Re_s$ the main influence is again the inaccuracy of the used drag curve to calculate $v_s$.

The acceleration to steady-state measurements were reproduced four times. The approach of wall measurements were reproduced two times. The results for $Re_s = 74.2$ and $Re_s = 1.5$ are shown in figures 5.1 and 5.2. Plots have been made of $h/D$ against $t$, where $h$ is the distance between the bottom apex of the sphere and the bottom of the vessel and $t$ is time. In figure 5.1 all trajectories overlap. In figure 5.2 there is also little difference in trajectory. This indicates that these experiments were reproducible. All other cases measured were also reproducible, because the trajectories overlapped as well.
5.1. Accuracy and reproducibility

Figure 5.1: Accelerating motion of sphere until steady-state for $Re_s=74.2$.

Figure 5.2: Approach of bottom of vessel for $Re_s=1.5$.
Chapter 5. Results and discussion of trajectory measurements

Table 5.2: Accelerating motion and steady-state: definition of cases.

<table>
<thead>
<tr>
<th>$D$ (mm)</th>
<th>$\mu_f$ (mPas)</th>
<th>$\rho_f$ (kg/m$^3$)</th>
<th>$v_s$ (cm/s)</th>
<th>$Re_s$ (-)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.18</td>
<td>50.3</td>
<td>1205</td>
<td>8.8</td>
<td>6.7</td>
</tr>
<tr>
<td>5.55</td>
<td>50.3</td>
<td>1205</td>
<td>18.7</td>
<td>24.8</td>
</tr>
<tr>
<td>6.35</td>
<td>52.0</td>
<td>1205</td>
<td>21.6</td>
<td>31.8</td>
</tr>
<tr>
<td>9.53</td>
<td>52.0</td>
<td>1205</td>
<td>33.6</td>
<td>74.2</td>
</tr>
</tbody>
</table>

Figure 5.3: Accelerating motion of a sphere until steady-state for 4 different Reynolds numbers.

5.2 Trajectories: Accelerating motion to steady-state

The accelerating motion of a sphere to steady-state has been measured for four different Reynolds numbers. $Re_s$ was varied by changing sphere diameter and keeping fluid viscosity and density constant. The definition of cases measured is given in table 5.2. In figure 5.3 plots have been made of $h/D$ against $tv_s/D$. By scaling this way, the trajectories get a slope of approximately -1 at steady-state settling velocity.

Before the results are discussed further, it has to be checked if the measured steady-state velocities make sense. This is done by comparing the measured maximum falling velocities $v_{s+}$ in the vessel to the calculated theoretical steady-state velocities $v_s$ in an infinite medium. The results are shown in table 5.3. The highest measured velocity in the vessel is smaller than the theoretical value. This is to be expected due to the wall effect. The ratio $v_{s+}/v_s$ is a measure for the wall effect as defined in section 2.7. However, care has to be taken when interpreting $v_s$. It has been calculated using the empirically determined drag coefficient $C_D$. The drag curve
5.2. Trajectories: Accelerating motion to steady-state

Table 5.3: Check of particle settling velocity.

<table>
<thead>
<tr>
<th>Re_s (-)</th>
<th>( v_{s^+} ) (cm/s)</th>
<th>( v_s ) (cm/s)</th>
<th>( \frac{v_{s^+}}{v_s} ) (-)</th>
<th>( \frac{v_{s^+}}{v_s} )_{corr} (-)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.7</td>
<td>8.2</td>
<td>8.8</td>
<td>0.93</td>
<td>0.96</td>
</tr>
<tr>
<td>24.8</td>
<td>16.6</td>
<td>18.7</td>
<td>0.89</td>
<td>0.96</td>
</tr>
<tr>
<td>31.8</td>
<td>18.8</td>
<td>21.6</td>
<td>0.87</td>
<td>0.95</td>
</tr>
<tr>
<td>74.2</td>
<td>29.4</td>
<td>33.6</td>
<td>0.88</td>
<td>0.94</td>
</tr>
</tbody>
</table>

Figure 5.4: Accelerating motion of sphere for \( Re_s = 31.8 \). Together with model of Ferreira et al. used in this thesis, see section 2.4, has an average inaccuracy of 7.6%. This means that the average inaccuracy in \( v_s \) is around 4%.

The calculated ratio of \( \frac{v_{s^+}}{v_s} \) can be compared with this ratio predicted by the wall effect correlation of Di Felice as mentioned in section 2.7. This correlation has been calculated and the result is shown in the last column of table 5.3. The correlation gives a higher value for \( \frac{v_{s^+}}{v_s} \)_{corr} than the measured value. If the inaccuracy of the drag-curve is taken into account upper boundaries for \( \frac{v_{s^+}}{v_s} \) are 0.96, 0.92, 0.90 and 0.91 for increasing Reynolds number, respectively. Now for the \( Re_s = 6.7 \) measurement the predicted value is the same as the measured result.

It is clear that the correlation proposed by Di Felice underestimates the wall effect for the cases measured. This is possibly due to the fact that the correlation is not very accurate, especially for small values of particle to vessel width ratio.

Ferreira model

In figure 5.4, a measurement result of the accelerating motion of a sphere together
Chapter 5. Results and discussion of trajectory measurements

Table 5.4: Order of magnitude estimation of the Basset force.

<table>
<thead>
<tr>
<th>Reₜ</th>
<th>t (s)</th>
<th>F_D, Basset (µN)</th>
<th>F_D, Basset, stationary (µN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.7</td>
<td>0.3</td>
<td>10</td>
<td>0.05</td>
</tr>
<tr>
<td>24.8</td>
<td>0.25</td>
<td>70</td>
<td>0.06</td>
</tr>
<tr>
<td>31.8</td>
<td>0.25</td>
<td>106</td>
<td>0.06</td>
</tr>
<tr>
<td>74.2</td>
<td>0.2</td>
<td>415</td>
<td>0.07</td>
</tr>
</tbody>
</table>

Table 5.5: Approach of bottom of vessel: Definition of cases. Particle used: D=3.175 mm.

<table>
<thead>
<tr>
<th>µ_f (mPas)</th>
<th>ρ_f (kg/m³)</th>
<th>v_s (cm/s)</th>
<th>Reₜ (-)</th>
</tr>
</thead>
<tbody>
<tr>
<td>122.3</td>
<td>1218</td>
<td>4.6</td>
<td>1.5</td>
</tr>
<tr>
<td>76.0</td>
<td>1219</td>
<td>6.7</td>
<td>3.4</td>
</tr>
<tr>
<td>54.6</td>
<td>1215</td>
<td>8.4</td>
<td>5.9</td>
</tr>
<tr>
<td>41.7</td>
<td>1215</td>
<td>10.0</td>
<td>9.1</td>
</tr>
<tr>
<td>26.1</td>
<td>1198</td>
<td>13.0</td>
<td>18.8</td>
</tr>
</tbody>
</table>

with the model of Ferreira as described in section 2.6 is shown. The Ferreira model is in qualitative agreement with measurements: it predicts approximately the same shape of particle trajectory as measured. However, it predicts higher sphere velocities. Several reasons exist for this. First, the wall effect is not included in this model. Second, the added-mass-coefficient used is 0.5 for irrotational flow. Third: the Basset-force is not included in this model. An estimation of this force can be made using equation 2.15. The results of these calculations are given in table 5.4. It is clear that it is not allowed to neglect this force under these circumstances: it has a magnitude of around 6% of the steady-state drag force acting on the particle in an infinite medium. Reasons for this are already mentioned in section 2.6. The Basset force diminishes with increasing particle to fluid density ratio. In this case this ratio is low (around 2). The Basset force is also proportional to the square root of the viscosity of the fluid. In this research fluids with relatively high viscosity have been used. If the experiments would have been performed in water, the Basset-force would have been smaller and the Ferreira model would have been in better agreement with the measured trajectory.

5.3 Trajectories: Wall impact

For 5 different Reynolds numbers trajectories of a sphere approaching the bottom of the vessel have been measured. The definition of cases measured is given in table 5.5. Reₜ was varied by varying the fluid viscosity. In figure 5.2 it can be seen that the
sphere decelerates as it approaches the wall. The question arises how the approach to the bottom wall is influenced by Reynolds number. For this reason, a plot for different Reynolds numbers has been made in figure 5.5. Clearly, the sphere starts to decelerate for larger \( h/D \) when \( Re_\infty \) gets smaller. This is in agreement with the model of [Gondret et al., 1999] which has been described in section 2.8.

To get another view of the approach of the sphere to the bottom wall, the trajectories have been differentiated. For the two lowest Reynolds numbers this has been done using a least squares method [Raffel et al., 1998]. This is equivalent to fitting a straight line through 5 data points and taking the slope of this line as the derivative. The other trajectories are differentiated using an ordinary central difference scheme. In figure 5.6 plots were made of \( v_p \) against \( h/D \). This graph gives a different interpretation. If the slope of the curves in the graph is not zero, the particle is decelerating. This can clearly be seen for the \( Re_\infty = 1.5 \) measurement: the plot is already slightly curved for \( h/D = 4 \), indicating that the particle is already decelerating at this height. The \( Re_\infty = 18.8 \) measurement however shows a flat plot until approximately \( h/D = 1 \): the particle starts to decelerate at a much smaller value of \( h/D \).

It is interesting to determine the force acting on the sphere as a function of height. For this the acceleration of the sphere has to be determined from the data. However, the data is not accurate enough to calculate the second derivative.

To get an indication of the size of this force, a function is fitted. To do this, a non-dimensional velocity \( v^* \) is defined as \( v_p/v_s \) and \( h^* \) is defined as \( h/D \). The following function was used:

\[
v^*(h^*) = \frac{ah^*}{1 + bh^*}
\]

The coefficients \( a \) and \( b \) were determined using non-linear regression. The results are given in table 5.6. As an example, the result for \( Re_\infty = 5.9 \) is shown in figure 5.7.
Chapter 5. Results and discussion of trajectory measurements

Figure 5.6: Velocity of the sphere as a function of h/D for different Re_s.

Table 5.6: Result for fit coefficients.

<table>
<thead>
<tr>
<th>Re_s</th>
<th>a</th>
<th>inaccuracy a</th>
<th>b</th>
<th>inaccuracy b</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>2.81</td>
<td>0.05</td>
<td>3.06</td>
<td>0.06</td>
</tr>
<tr>
<td>3.4</td>
<td>3.12</td>
<td>0.09</td>
<td>3.12</td>
<td>0.12</td>
</tr>
<tr>
<td>5.9</td>
<td>10.0</td>
<td>0.6</td>
<td>10.5</td>
<td>0.7</td>
</tr>
<tr>
<td>9.1</td>
<td>18.6</td>
<td>0.9</td>
<td>18.7</td>
<td>0.9</td>
</tr>
<tr>
<td>18.8</td>
<td>49.2</td>
<td>5.5</td>
<td>50.5</td>
<td>5.8</td>
</tr>
</tbody>
</table>
5.4 Critical Stokes number measurements

The fit looks reasonable. It shows the appropriate limit-behaviour. From this fitted function, the qualitative shape of the force acting on the particle as a function of height can be calculated by differentiation using the chain rule \( \frac{dv_p}{dt} = \frac{dv_p}{dh^*} \frac{dh^*}{dt} \):

\[
F(h^*) = m \frac{dv_p}{dt} = m \frac{v_p^2}{D} \frac{d}{dh^*} \left( \frac{1}{2} v^*(h^*) \right)^2
\]

The result of this calculation for \( Re_s = 5.9 \) is given in figure 5.8. The result is in agreement with what can be expected. When the sphere is far away from the wall, it is falling at steady-state velocity. There is no net force acting on the particle. When it starts approaching the bottom of the vessel, the force increases until it reaches a maximum value. Finally the force decreases to zero when the particle is at rest.

5.4 Critical Stokes number measurements

An attempt has been made to measure the critical Stokes number of a glass sphere colliding on the glass bottom of the vessel. By varying fluid viscosity the Stokes number of the particle was changed. To judge whether or not a rebound occurred the films resulting from the measurement were studied by eye. The results are shown in table 5.7. It follows that \( St_c \), and thus the critical Stokes number based on the velocity of the sphere in an infinite medium lies between 9.75 and 13.4. This result is not very accurate. However, it is in agreement with the value of 20 resulting from measurements performed by [Gondret et al., 1999], because these measurements were also not very accurate and a steel instead of a glass sphere has been used. The correlation derived by [Zenit and Hunt, 1999] in section 2.8 gives values \( St_c = 8.1, 22.6 \) and 50.83 for the three different fit parameter values mentioned. As the measured values are in this range, they are not contradicting this correlation. To do more accurate measurements of \( St_c \), a more sophisticated measurement method.
Figure 5.8: Force acting on the particle as a function of height. Derived by calculating the derivative of the fit of function to $Re_z=5.9$ data.

Table 5.7: Measurement results for determination of critical St-number. Particle used: $D=6.35$ mm.

<table>
<thead>
<tr>
<th>$\mu_f$ (mPas)</th>
<th>$\rho_f$ (kg/m³)</th>
<th>$v_s$ (cm/s)</th>
<th>$Re_z$ (-)</th>
<th>$St_z$ (-)</th>
<th>Rebound?</th>
</tr>
</thead>
<tbody>
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<td>51.4</td>
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<td>13.4</td>
<td>Certainly</td>
</tr>
<tr>
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<td>28.9</td>
<td>86.1</td>
<td>20.5</td>
<td>Certainly</td>
</tr>
</tbody>
</table>
is needed, because the detection of the bouncing-to-no-bouncing transition by eye is not accurate.

5.5 Final remarks

- Reproducible sphere trajectories for validation of simulations have been obtained.

- The model of Gondret et al. is in qualitative agreement with the results. The lower the Reynolds number is, the earlier the sphere starts to decelerate as it approaches the wall.

- The model of Ferreira et al. is in qualitative agreement with the results. There is no quantitative agreement with the results. Causes are the wall effect and the fact that it is not allowed to neglect the Basset force.

- Critical Stokes number measurements did not yield accurate results. A more sophisticated measurement set-up is needed to measure this parameter more accurately.
Chapter 6

PIV experimental set-up and measurement method

To obtain information of the flow field of a falling sphere for 1-on-1 comparison with simulations, PIV experiments are performed. In this chapter, first the measurement set-up for the flow field combined with the sphere trajectory is presented. At first sight this setup has many similarities with the setup for sphere trajectory measurements, but there are so many changes that the PIV set-up will be explained from scratch. After this a more detailed description of some parts of the setup will be given. Then image processing needed for extracting place-time data of the sphere will be explained. Finally the software used for PIV processing will be discussed.

6.1 Measurement setup

In figure 6.1 the measurement setup is given. The main part is a rectangular glass vessel filled with silicon oil seeded for PIV measurements with small tracer particles. In the top of the vessel a nylon sphere is hanging at the release mechanism. The sphere is positioned in the centre of the vessel, the distance between the top of the sphere and the bottom of the vessel is 120 mm. The tracer particles in the fluid are illuminated with a continuous light sheet from an Argon-ion laser using a cylindrical and a convex lens. This light sheet enters the vessel from beneath, using a mirror. In front of the vessel a digital camera is positioned to take pictures of the flow field. This camera is connected to a computer.

A measurement is made by starting capturing pictures with the digital camera. This triggers the release mechanism; the sphere is released and will start falling down until it reaches the bottom of the vessel and comes to rest. During sphere settling images are taken of the flow, resulting in a film of the moving sphere and tracer particles. From this film instantaneous flow fields are extracted using PIV software. The sphere trajectory is extracted using a C-routine in SCIL-Image.
6.2 Detailed description of some components

The release mechanism of this set-up is exactly the same as described in chapter 4 on particle trajectory measurements. In the next section, more will be said about:

- Rectangular vessel
- Fluid
- Seeding
- Camera
- Sphere
- Laser
- Sheet formation optics

6.2.1 Rectangular vessel

The experiments are performed in a window glass rectangular vessel. The inner dimensions of this vessel are \( d \times w \times h = 100 \times 100 \times 160 \text{ mm} \). To get a rigid vessel, the bottom is made of a 19 mm thick glass plate. It is filled with liquid to a height of 140 mm.

6.2.2 Fluid

The fluid used in this experiment is silicone oil. This fluid behaves as a Newtonian liquid. The advantage of using silicone oil compared to Glycerol-water solutions is its small change of viscosity with temperature (e.g. smaller than 1 mPas/K for 50 mPas oil at 293 K). Dow Corning DC200 silicone oils of 50 and 350 mPas are used. They are mixed to get intermediate viscosities.

6.2.3 Seeding

The seeding used are hollow glass spheres with a density of 1.055 g/cm³. The mean diameter is 11.2 \( \mu \text{m} \) with a standard deviation of 6.6 \( \mu \text{m} \) (Coulter-counter measurement). For a typical fluid used during this research with \( \rho_f = 960 \text{ kg/m}^3 \) and \( \mu_f = 50 \text{ mPas} \), the gravitationally induced settling velocity of the seeding (see section 3.3) \( U_g = 1.3 \cdot 10^{-7} \text{ m/s} \). This means that the particles are displaced 0.5 mm per hour, negligible for the experiments performed. The relaxation time \( \tau_p = 1.5 \cdot 10^{-7} \text{ s} \). This type of seeding is also used in water, where this time would be larger: \( 7.0 \cdot 10^{-6} \text{ s} \). Therefore, the relaxation time is small enough for the experiments performed.
Chapter 6. PIV experimental set-up and measurement method

6.2.4 Camera

The cameras available are already mentioned in section 4.2.5. For PIV a high spatial resolution is needed. Therefore the camera with 512 x 512 pixels is used. Two camera lenses were used: A Nikon lens of focal length 50 mm and a minimum f-number of 1.4 to get a magnification factor M=0.12 and a Nikon lens of focal length 35 mm and a minimum f-number of 2.0 to get M=0.17.

6.2.5 Sphere

Compared to trajectory measurements, the additional PIV criteria as described in section 3.6.4 have to be satisfied. This imposes a criterion on the maximum settling velocity of the sphere. To derive this criterion, the following numbers are used. A typical size of an interrogation area is 32 x 32 pixels. Using a 50% overlap for the interrogation areas a velocity field of 31 x 31 vectors can be determined using the 512² camera. To obtain vector fields with reasonable resolution, the number of vectors to be determined on a line of the length of one sphere diameter has to be around 10.

Now, consider a sphere in a field of view of three times its diameter, as depicted in figure 6.2. It is assumed that the maximum fluid velocity in the vessel is equal to the settling velocity of the sphere. This implies, according to the PIV criteria, that the sphere may fall a distance of \( \frac{3}{4} D \) between two consecutive frames. This distance is equal to \( \frac{3D}{16} = \frac{3D}{64} \). This distance may be travelled in \( \frac{1}{f_c} \) seconds where \( f_c \) is the frequency of the camera which is 250 Hz at maximum. Thus, the criterion for the settling velocity of the sphere of \( v_{st} < \frac{3Df_c}{64} \) is obtained.

Now, for example consider a glass sphere (as used for trajectory measurements) of diameter 9.53 mm with a density of 2550 kg/m³ falling in silicone oil with a density of 960 kg/m³ and a viscosity of 50 mPas. The maximum settling velocity in an infinite medium is 0.41 m/s, \( Re_s=75 \). Clearly, the criterion for the settling velocity is not fulfilled: \( 0.41 > 0.11 = \frac{3D}{64} \cdot \frac{0.00953 \cdot 250}{64} \). Doing the PIV measurement with this equipment using a glass sphere is not possible. For this reason, a different particle is used.
6.2. Detailed description of some components

The sphere used in this research is a non-transparent Nylon sphere obtained from IBS Bearing Services Rotterdam. The diameter of the sphere is 15 mm, with a tolerance of 12.2 μm. The density of the sphere was determined using a substitution balance (Type Mettler B5) and dividing by the volume of the sphere. The result for the density of the sphere \( \rho_p \) was:

\[
\rho_p = (1.12 \pm 0.01) \times 10^3 \text{kg} \cdot \text{m}^{-3}
\]

Using the same fluid as in the previous example, this sphere has a maximum settling velocity in an infinite medium of 0.14 m/s, \( Re_s = 39 \). The criterion for the settling velocity is fulfilled: 0.14 < 0.18.

A problem encountered when doing the first test measurements was that the sphere reflected too much light. The fluid and seeding were illuminated by light scattered from the sphere. This problem was solved by making the bottom of the sphere black using a Staedtler Lumocolor 318 permanent black marker.

6.2.6 Laser

For the formation of the light sheet a 4W Argon-ion laser from Spectra-Physics (2016-05) is used. This laser can operate in two modes:

- Single-line mode; only one color 488 nm (blue), is used. The maximum laser-power is 1.4W.
- Multi-line mode; all the available colors of the laser are used. The maximum laser-power is 4.0W.

For PIV it is important to have enough light to illuminate the seeding particles. For this reason, the multi-line mode is used.

6.2.7 Sheet formation optics

To convert the laser beam into a light sheet a cylindrical lens and a convex lens are used. This can be seen in figure 6.3. This light sheet will have a Gaussian intensity distribution. In section 6.2.4 two camera lenses are given. The 35 mm lens has a higher minimum f-number than the 50 mm lens and therefore requires more light for imaging tracer particles. The light sheet has a larger light intensity when the convex lens has a larger focal distance. For this reason, a cylindrical lens with focal distance \( f_1 = -12.0 \text{ mm} \) is used in combination with the 35 mm camera lens and a cylindrical lens with focal distance \( f_1 = -6.0 \text{ mm} \) in combination with the 50 mm camera lens. The convex lens has a focal length \( f_2 = 500 \text{ mm} \). This lens is used to compensate for the divergence of the sheet created by the cylindrical lens. This lens also converges the sheet in the waist position. At the waist position the beam spot size of the laser sheet is minimal. The radius of the beam is defined as the distance from the middle of the beam to the position where the intensity is a factor \( 1/e^2 \) of the intensity in the middle of the laser beam. Two times the radius of the beam
is defined as the thickness. At the waist position the intensity per unit volume is
the largest and the thickness of the sheet is almost constant. The waist position is
therefore chosen as the measurement position. To calculate the waist of the laser
sheet at the measurement position, the optical path between the exit of the laser tube
and the measurement position is described by a series of matrices (see appendix B).
In these calculations the distance between the spherical lens and the bottom of the
vessel is the only variable. By changing this distance, the measurement position can
be made equal to the waist position. The laser sheet at the waist position will also
have a width which is much larger than the thickness. The width of the sheet can be
calculated using the same theory the waist is calculated with. The only difference
is the orientation of the waist.

The sheet enters the vessel at the bottom, not from the side. An advantage of
this choice is that the distance travelled by the light to the measuring position is
as short as possible, minimising disturbing reflections. A disadvantage is, that the
sphere blocks the light sheet: it is not possible to measure the flow field of the fluid
in the shadow the sphere.

6.3 Image processing

In this section image processing needed for determination of the sphere position in
PIV recordings is explained. For more general information on image processing see
section 4.3.

6.3.1 Sphere position

As the sphere is made black at its bottom, the procedure of thresholding and de-
termination of the centre of mass of the sphere (see section 4.3) does not work. For
this reason the top of the sphere is left white. This white spot appears clearly on
6.3. Image processing

Figure 6.4: Detection of sphere position. (a) Raw PIV-recording with sphere. (b) Part of recording after a kuwahara-filter has been applied. (c) Result of thresholding image b. (d) Magnification of the top of the sphere.

the recorded images. A remaining problem however, is that the edge of the sphere is not sharp due to motion blur. To determine the top of the sphere, the following steps are made. First a strip which contains the top of the sphere is taken from the recorded image. Then an edge-preserving kuwahara-filter (size 13 × 13 pixels) [Scilimage, 1994] is applied to sharpen the edge. The result can be seen in figure 6.4b. Finally, a normal threshold is applied to the image. This results in a picture at which the top of the sphere can be detected (see figure 6.4c). The top of the sphere is magnified in figure 6.4d. The y-coordinate of the top can be detected by locating the minimum y-value for which a pixel exists. The x-coordinate can not be detected so easily: there can be more x-pixels having the same y-coordinate. This problem is so lved by taking the average x-pixel coordinate of all pixels existing for the y-coordinate. However, the x-coordinate determined this way, was not very stable. A difference of 5 pixels between two consecutive frames could exist. Assuming only vertical motion, a more stable x-coordinate could be obtained by taking the average of all x-coordinates determined from all frames of a film. This coordinate is accurate enough for it’s purpose: removing the spurious vectors behind the sphere as will be described in section 6.4.

6.3.2 Calibration

For calibration of PIV the measurements, a rectangular grid with known grid-spacing is put into the vessel. By taking an image (see figure 6.5), the number of pixels/cm is determined.

To check if there is image distortion, the distance between the most left and most right dot on the top row, middle rows, and the bottom row of the grid is measured. These distances are equal: 426 pix. This indicates that there is no image distortion.
6.4 PIV processing

For analysis of the acquired PIV images PIVWare, a software package developed by J. Westerweel, is used. The images are processed using interrogation areas of a size of $32 \times 32$ pixels with a 50% overlap, using a unit weight kernel and a Gaussian peak fit estimator for estimation of the fractional displacement (see section 3.6.3). This results in vector fields of $31 \times 31 = 961$ velocity vectors.

As a consequence of the fact that the light sheet is entering the vessel from beneath, there is no light behind the sphere, see figure 6.4a. This means that there are no illuminated tracer particles behind the sphere: there is no signal for measurement of velocity vectors. The same holds for the part of the images where the sphere is depicted. The result is that the vector fields determined by the PIV software contain spurious vectors in these two regions. These vectors are removed using a program which cuts out these two regions using information of the sphere position determined as described in section 6.3.1.
Chapter 7

Results and discussion of PIV measurements

In this chapter, results of PIV measurements are presented. First, the cases that have been measured will be defined. After this, the accuracy of the results will be discussed. Then a qualitative comparison between the results of different Reynolds numbers will be made. Next, quantitative results obtained from the data will be shown. Finally, a comparison with simulations performed at the laboratory will be given.

7.1 Definition of Cases

For 5 different Reynolds numbers the trajectory and flow field of the falling sphere have been measured. The definition of the cases that have been measured are given in table 7.1. P is the laser power used and \( t_{illumination} \) the time the shutter of the camera was opened for taking each frame. The Reynolds numbers have been varied by varying the viscosity of the fluid. As the structures of the flow are smaller when \( R_{es} \) is larger, two different fields of view (FOV's) have been used to capture the flow with sufficient resolution. A schematic picture of the fields of view used for the measurements is given in figure 7.1. For the measurement at \( R_{es}=32 \) and

<table>
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<th>( \mu_f ) (mPas)</th>
<th>( \rho_f ) (kg/m(^3))</th>
<th>( v_s ) (cm/s)</th>
<th>( R_{es} ) (-)</th>
<th>( f^c ) (Hz)</th>
<th>P (W)</th>
<th>( t_{illumination} ) (% of frame time)</th>
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</tbody>
</table>

Table 7.1: PIV measurements: definition of cases.

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Chapter 7. Results and discussion of PIV measurements

11.6 The trajectory was divided into four parts labelled h1, h2a, h2b and h3 using a field of view of 32 \times 32 \text{ mm}. These measurements are called the high-resolution measurements. For \( \text{Re}_s = 4.4, 4.1 \) and 1.5 the trajectory was divided in three parts labelled l1, l2 and l3 using a field of view of 45 \times 45 \text{ mm}. These measurements are called the low-resolution measurements. Every case was measured two times.

It can be seen in table 7.1 that two measurements with \( \text{Re}_s \) close to 4 have been done. This is because a mistake with the illumination of the tracer particles was made. The measurements of \( \text{Re}_s = 4.4 \) in FOV’s l2 and l3 resulted in data with bad signal to noise ratio. For this reason the measurement was repeated. This is the measurement with \( \text{Re}_s = 4.1 \). However, this time mistakenly the file containing the measurement data for FOV l1 was overwritten. Good results were obtained for FOV’s l2 and l3. By combining the results for \( \text{Re}_s = 4.1 \) and 4.4, flow field results for \( \text{Re}_s \) around 4 in every FOV were obtained. Sphere trajectory measurements succeeded for both \( \text{Re}_s = 4.4 \) and 4.1.

7.2 Accuracy

In this section the accuracy of the PIV measurements is discussed. The accuracies of \( \rho_f, \mu_f, \nu_x, \nu_{x+}, \text{Re}_s \) and the release moment of the sphere are the same as for trajectory measurements. These have already been discussed in section 5.1.

Thickness of the laser sheet

To do reliable PIV measurements, it is important to have a well defined laser sheet. To check if the theory of [Yariv and Yeh, 1984] gives a good prediction of the thickness of the laser sheet, the thickness and intensity distribution of the light...
7.2. Accuracy

![Graph showing fit of a Gaussian curve to a normalised experimental intensity distribution. The arrow indicates the distance from the centre at which the intensity is reduced with a factor \( \frac{1}{2} \).](image)

The accuracy of the laser sheet thickness in the vessel using the theory described in Appendix B is given in figure 7.3. The calculated laser sheet radius as a function of the distance from the vessel bottom for the measurements done in FOV 12 is shown. The minimal sheet thickness was 178 \( \mu m \) at waist position. The average sheet thickness in the FOV was 189 \( \mu m \). The width of the sheet was 2.6 cm. Calculated thicknesses for the other FOV’s showed similar results.

**Sphere position and rotation of the sphere**

The accuracy of the sphere position determined using the method described in 6.3.1 was estimated by comparing this algorithm to the case when the sphere is lying at the bottom of the vessel. In this case the exact position of the sphere is known. The deviation between the estimated and real position of the sphere was 3 pix. at
Chapter 7. Results and discussion of PIV measurements

Figure 7.3: Half the laser sheet thickness calculated as a function of the distance to the vessel bottom for the measurements done in FOV 12 (indicated by the dotted lines).

Figure 7.4: Calculation of out-of-plane movement of sphere from raw data. The raw data has been inverted to be able to indicate $\Delta x$.

maximum. Relative to sphere diameter this results in accuracies of 1% and 2% for the high and low resolution measurements, respectively.

As the theory given in chapter 2 is only valid for a non-rotating sphere, it was checked by eye if the sphere rotates during settling. No rotation was seen on the films studied.

Out-of-plane movement of the sphere

During measurement, the centre of mass of the sphere should move straight down through the light sheet. Whether this was the case can be checked from the measurement data using the procedure illustrated in figure 7.4. In this figure, a top view of the sphere is depicted. The light sheet can be represented by a line because its thickness is much smaller than the diameter of the sphere. If the sheet points through the centre of mass of the sphere, $\Delta x$ is 0. However, if the sphere falls with
7.2. Accuracy

its centre of mass out of the light sheet, $\Delta z$ is not 0. $\Delta z$ can be estimated from the measurement data as the distance between the right side of the sphere and the point where the shadow behind the sphere changes to the light sheet. Now the out-of-plane distance of the sphere $\Delta z$ can be calculated from $\cos(\alpha) = 1 - \frac{2\Delta z}{D}$ and $\Delta z = \frac{D \sin(\alpha)}{2}$. For the measurements performed, $\Delta z$ was estimated to be 2 pixels at most. For a minimum sphere diameter of 180 pixels this results in $\Delta z = 18$ pix, or 10% of sphere diameter. This is reasonable, because during measurement the position of the sphere in the sheet was only checked by eye.

Cross-correlation of two interrogation areas

A raw picture of a measurement was already shown in figure 6.4a. To get an idea of particle image size, two IAs have been cut from the data. The result is shown in figure 7.5a and 7.5b. The particle images are sharp and most particle images have a size of 2-4 pixels, except for the big particle just above the middle of the picture. The result of a cross-correlation between these two IAs is given in figure 7.5c. A clear correlation peak is obtained, indicating a good signal to noise ratio of the data.

Accuracy of velocity vectors

As can be seen from a typical measurement result shown in figure 7.6, there are large differences in fluid velocity: close to the sphere velocities are much higher than two diameters away from the sphere. This has impact on the accuracy of the measurements, see section 3.6.3. Far away from the sphere, particle image displacements were less than 0.5 pix, resulting in a relative accuracy of the measured velocity vectors of 17%. Close to the sphere the particle image displacements were around 7 pixels, resulting in a relative accuracy of 2%.

Valid data-yield

After removing the spurious vectors behind the sphere as described in section 6.4, an amount of around 10 spurious vectors per frame remains. This is only 1% per frame. Compared to the typical value of 5% given in section 3.7, this is very little. Most likely this is due to the fact that the measurements are performed in a laminar flow with little out-of-plane motion of the tracer particles, resulting in high signal to noise ratios.

Convection

A peculiar problem occurred for the $Re_z=32$ measurement. When the sphere was lying at rest at the bottom of the vessel, the laser illuminated the bottom of the sphere, warming up the fluid. This resulted in free convection of the fluid, and consequently possible incorrect measurement of the liquid flow velocity. The solution to this problem was to insert a chopper between the laser and the first small mirror.
Figure 7.5: Cross-correlation between two IA's from measured data. (a) IA at time $t$, (b) IA at time $t + \Delta t$. (c) Cross-correlation plane.
7.2. Accuracy

The chopper was only opened during measurements. This reduced most of the influence of natural convection on the measurements, probably because there was no warm-up of the fluid by the laser light before the start of the measurements. However, to get an idea of the magnitude of the velocities induced by the convection, the steady-state convection flow was filmed. An estimation of the velocity of the fluid was made. In points of the fluid, the magnitude of the convection flow and the magnitude of the flow due to the movement of the sphere were compared. The first magnitude was always at least two orders of magnitude smaller than the second. Therefore, it is assumed that there is no problem regarding the accuracy of the measurements.

Reproducibility

Every measurement case was reproduced one time. For the $Re_s = 1.5$ experiment the reproducibility was rigorously checked. The result for the measured trajectories of the sphere is shown in figure 7.7. The two trajectories overlap. From this result a reproducible experiment can be concluded.

To demonstrate that the flow field measurements are reproducible, two velocity fields for the sphere at the same height $h/D = 1$ from two different measurements are shown in figure 7.8. The velocity fields are (almost) the same and the place and shape of the vortex is the same.

In section 7.2 it was concluded that the sphere falls with its centre of mass through the light sheet. The streamlines in figure 7.8 are almost closed. This indicates that the flow is 2-dimensional.
Chapter 7. Results and discussion of PIV measurements

Figure 7.7: Sphere trajectories resulting from the 3 different field of views for \( \text{Re}_s=1.5 \). Every trajectory has been measured 2 times.

Figure 7.8: Flow fields of two different runs for \( \text{Re}_s=1.5 \) and \( h/D=1 \).
7.3. Results

7.3.1 Trajectories

The results of trajectory measurements for 4 different $Re_s$ are given in figure 7.9. Trajectories of all FOV's belonging to the same measurement case have been pasted behind each other in order to obtain almost complete sphere trajectories. Just like in section 5.2, $v_{s+}$ is compared to $v_s$. The result is shown in table 7.2. $v_{s+}$ is always smaller than $v_s$, as is to be expected due to the wall effect. Results of calculations for this effect estimated by the correlation of Di Felice (see section 2.7) are also given in table 7.2. In these cases the correlation overestimates the wall effect. However, the wall effect was underestimated for the measurements described in section 5.2. This is another indication that the correlation of Di Felice is not very accurate.
Chapter 7. Results and discussion of PIV measurements

7.3.2 Flow pattern of a sphere

Many studies of the flow pattern of a sphere investigate the case where the sphere is fixed and the fluid is flowing past the sphere: the frame of reference is a coordinate system attached to the sphere. A typical flow pattern is given in figure 7.10a. However, in this research the sphere is falling through a stagnant fluid: the frame of reference is the laboratory. A typical flow pattern for this case is given in figure 7.10b. An important difference between these two cases is the position of the vortex. In the frame of reference of the sphere it lies downstream and behind the sphere; in the frame of reference of the laboratory it lies next to the sphere.

7.3.3 Flow field shortly before wall impact

The way the fluid in the vessel behaves just before wall impact is very different for different Reynolds numbers. The meaning of the Reynolds number in the sense that it is a measure for inertial forces vs. viscous forces can clearly be seen from the measurements. In figure 7.11, four vector fields from FOV's II and h1 with the sphere at a distance 0.15D from the bottom wall are given. For $Re_s=1.5$, the flow is almost at rest when the sphere is at this height. However, for $Re_s=32$, the flow is certainly not at rest: fluid is strongly pushed out of the gap between the sphere and the vessel wall. This can be explained by the Reynolds number. For low Reynolds numbers, there is more dissipation due to viscosity in the fluid than for higher Reynolds numbers. Consequently, the fluid tends to come to rest earlier for smaller Reynolds numbers. The time scales and flow structures of the fluid for a sphere hitting a wall strongly depend on $Re_s$. This is also illustrated by figures 7.11b and 7.11c.
7.3. Results

Figure 7.11: Flow field for sphere at $h/D=0.15$. (a) $Re_s=32$, (b) $Re_s=11.6$, (c) $Re_s=4.1$, (d) $Re_s=1.5$
Chapter 7. Results and discussion of PIV measurements

7.3.4 Comparison of the flow field in the middle and bottom of the vessel

The flow field changes significantly when the sphere approaches the bottom wall of the vessel, see figure 7.12. The position of the vortex relative to the centre of mass of the sphere changes during this process: it moves slightly upwards. This is probably due to the fact that fluid is pushed out between the sphere and the wall. This fluid has to recirculate along the bottom wall of the vessel. Room is needed for this process, the fluid is pushed slightly upwards, and consequently the vortex is moved upwards. This can be seen even more clearly when comparing figure 7.12 with figure 7.11d, where the vortex has moved even further away from the centre of mass of the sphere.

7.3.5 Development of the flow field of the accelerating sphere

To see what happens to the flow when the sphere is released, three snap-shots at times 0.05, 0.32 and 0.51s after sphere release of the $Re_s=4.1$ measurement are shown in figure 7.13. First the instantaneous Reynolds number of the sphere is small and the flow is Stokes flow. The flow pattern is symmetric with respect to a horizontal line through the centre of mass of the sphere as can be seen in figure 7.13a. The sphere accelerates and its velocity increases. After this, the vortex moves to the right, away from the sphere. This can be seen in figure 7.13b. The vortex moves also a bit to the back of the sphere, as can be seen in 7.13b and 7.13c.
Figure 7.13: Three snap-shots of accelerating motion of a sphere for $Re_s=4.1$. Note that the distance of $y/D=0$ to the bottom of the vessel is 5.3D. (a) 0.05 s after release, (b) 0.32 s after release, (c) 0.51 s after release.
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7.3.6 Vorticity at a height of 0.045 D above the wall

[Harada et al., 1998] performed 2-dimensional simulations of a cylinder approaching a solid wall. A result of vorticity contours for $h/D = 0.045$ and $\rho_p/\rho_f = 2.48$ for $Re_s = 37.6$ is shown in figure 7.14a. Using the method as described in section 3.8, the vorticity of the $Re_s = 32$ measurement of this research has been calculated, see figure 7.14b. These two results are in qualitative agreement: the shape of the two results for the vorticity contours is approximately the same. Of course a very good agreement is not to be expected, since 2- and 3-dimensional flow have a different flow pattern and $Re_s$, $\rho_p/\rho_f$ and $h/D$ are different for the two cases compared.

7.3.7 Time series of fluid velocity in a monitor point

So far, only instantaneous velocity fields at a certain time after the sphere was released or for a certain value of $h/D$ have been given. To obtain information about the fluid flow as a function of time, a monitor point has been chosen in the FOV’s $l_1$ and $h_1$, at a position $D$ to the right of the top of the sphere when it lies at the bottom of the vessel, see figure 7.15. Time series for the magnitude of the velocity of the fluid flow in the monitor point for $Re_s = 1.5$ and $Re_s = 32$ measurements are given in figure 7.16. The arrow indicates the moment the sphere comes to rest at the bottom of the vessel.

These time series give information about the time it takes for the fluid to come...
7.3. Results

Figure 7.15: Position of monitor point used to extract time series from the measurement data.

Figure 7.16: Magnitude of the fluid velocity in the monitor point. The arrow indicates the moment the sphere comes to rest at the bottom of the vessel. (a) $Re_s = 1.5$, (b) $Re_s = 32$
Chapter 7. Results and discussion of PIV measurements

7.3.8 Time series of vortex position

It is also possible to extract time series of the position of the sphere vortex as a function of time. A result for $Re_e = 4.1$ is given in figure 7.17. Instantaneous flow fields of this case have already been given in figure 7.13. $\Delta x_v$ and $\Delta y_v$ are the x- and y-distance of the middle of the vortex to the centre of the sphere, see figure 7.18. During acceleration, $\Delta x_v$ and $\Delta y_v$ increase.
7.4 Comparison with simulations performed at this laboratory

In figure 7.19, the position of the vortex as a function of vertical sphere position is given for the wall-approach of the \( Re_s = 1.5 \) measurement. In this case, the vortex moves away from the centre of the sphere as \( h/D \) becomes smaller. Especially \( \Delta y_v \) increases more than a factor 3. This is most likely due to the fact that the fluid pushed out of the gap between the sphere and the bottom wall of the vessel pushes the vortex upwards when it recirculates along this wall (see also section 7.3.4).

7.4 Comparison with simulations performed at this laboratory

In this section simulations of the model system performed at our laboratory by A. Ten Cate are compared with the measurements.

First, the trajectories of the four different cases are compared. Second, a time series of the fluid velocity in the monitor point is compared. Finally, a comparison between two instantaneous flow fields is made.

Comparison of trajectory

Measured and simulated sphere trajectories are given in figure 7.20. Both the accelerating motion and the approach of the bottom wall of the sphere are in good agreement: the difference between simulations and measurements is 5% of the sphere diameter at maximum. However, the simulations tend to slightly underestimate the steady-state velocity of the sphere. In figure 7.21 the sphere velocity as a function of time is given for the \( Re_s = 11.6 \) case. The measurement data have been differentiated with the least square method [Raffel et al., 1998] already used in section 5.3.
Chapter 7. Results and discussion of PIV measurements

Figure 7.20: Comparison between measured and simulated sphere trajectories.

Figure 7.21: Comparison between measured and simulated sphere velocity for $Re_s = 11.6$. 
7.4. Comparison with simulations performed at this laboratory

Figure 7.22: Comparison between measured and simulated time serie of velocity magnitude in monitor point for \( Re_s = 32 \). The arrow indicates the moment of sphere impact.

This comparison also shows good agreement. The velocity of the sphere as a function of time is correctly predicted by the simulations, just like the deceleration as the sphere approaches the bottom wall.

**Comparison of a time serie**

A comparison between measured and simulated results of a velocity magnitude time series in the monitor point as defined in section 7.3.8 for the \( Re_s = 32 \) case is given in figure 7.22. There is good agreement. The simulations give the correct trend of fluid velocity. The two peaks are simulated at the right time. Also the decay of the fluid flow after sphere impact is described correctly. However, the simulation overestimates the magnitude at \( t = 1.1s \) and underestimates the magnitude at \( t = 1.3s \).

**Comparison of instantaneous flow fields**

A 1-on-1 comparison between simulated and measured flow fields is made in figures 7.23 and 7.24 for the cases \( Re_s = 1.5 \) and \( Re_s = 32 \) in the FOV’s h1 and h1, respectively. There is good agreement between simulations and experiments. The velocity magnitude is predicted well. Also streamlines and vortex position are well predicted.
Chapter 7. Results and discussion of PIV measurements

Figure 7.23: 1-on-1 comparison between simulations (left) and measurements (right) of fluid velocity magnitude and streamlines for Reₐ = 1.5 for h/D = 0.15

Figure 7.24: 1-on-1 comparison between simulations (left) and measurements (right) of fluid velocity magnitude and streamlines for Reₐ = 32 for h/D = 1.5
7.4. Comparison with simulations performed at this laboratory

Table 7.3: Comparison of \( \frac{\nu}{\nu_*} \) between simulations (sim.) and measurements (meas.) at points \( \frac{x}{D}, \frac{y}{D} \). The points probed are illustrated in figure 7.25.

<table>
<thead>
<tr>
<th>( \left( \frac{x}{D}, \frac{y}{D} \right) )</th>
<th>( \frac{\nu}{\nu_*} ) sim. (-)</th>
<th>( \frac{\nu}{\nu_*} ) meas. (-)</th>
<th>deviation of sim. from meas. (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0.9,0.1)</td>
<td>0.33</td>
<td>0.37</td>
<td>-11</td>
</tr>
<tr>
<td>(0.9,0.2)</td>
<td>0.50</td>
<td>0.43</td>
<td>16</td>
</tr>
<tr>
<td>(0.9,0.4)</td>
<td>0.40</td>
<td>0.41</td>
<td>-2</td>
</tr>
<tr>
<td>(0.9,0.6)</td>
<td>0.26</td>
<td>0.25</td>
<td>4</td>
</tr>
<tr>
<td>(1.2,0.1)</td>
<td>0.15</td>
<td>0.14</td>
<td>7</td>
</tr>
<tr>
<td>(1.2,0.2)</td>
<td>0.24</td>
<td>0.21</td>
<td>14</td>
</tr>
<tr>
<td>(1.2,0.4)</td>
<td>0.27</td>
<td>0.26</td>
<td>4</td>
</tr>
<tr>
<td>(1.2,0.6)</td>
<td>0.24</td>
<td>0.21</td>
<td>14</td>
</tr>
<tr>
<td>(1.5,0.1)</td>
<td>0.08</td>
<td>0.06</td>
<td>33</td>
</tr>
<tr>
<td>(1.5,0.2)</td>
<td>0.12</td>
<td>0.10</td>
<td>20</td>
</tr>
<tr>
<td>(1.5,0.4)</td>
<td>0.15</td>
<td>0.14</td>
<td>7</td>
</tr>
<tr>
<td>(1.5,0.6)</td>
<td>0.15</td>
<td>0.14</td>
<td>7</td>
</tr>
</tbody>
</table>

It is also possible to compare the instantaneous fluid velocity of simulations and measurements in a number of points. This has been done for the \( Re_s = 32 \) case. The results are shown in table 7.3. The points at which the data was probed are given in figure 7.25. For the three points at \( \frac{x}{D} = 0.9 \) and \( \frac{y}{D} = 1.2 \), the deviation between measurements and simulations is approximately 10%. For the three points at \( \frac{x}{D} = 0.15 \), the difference is approximately 20%.

Figure 7.25: Points probed for the results given in table 7.3.
Chapter 8

Conclusions and recommendations

In this chapter, the conclusions of this research and recommendations for further research are presented.

8.1 Conclusions

- Two experimental set-ups have been developed. The first one for measuring sphere trajectories, the second for simultaneously measuring sphere trajectories and the flow field due to the sphere using PIV. Trajectories and flow fields that can be used for validation of simulations have been measured for Reynolds and Stokes numbers ranging from 1-74 and 0.2-17, respectively. Both the trajectories and the flow fields were measured reproducibly.

- The time scales and flow structures due to the sphere when it approaches the bottom wall of the vessel strongly depend on the Reynolds number. Simulations are in good agreement with measurements. Sphere trajectories are predicted with deviations smaller than 5%. Flow patterns, vortex positions and time series extracted from simulations and measurements show good resemblance. The difference between simulations and experiments of the magnitude of the fluid velocity in points in the fluid is approximately 10% (close to the sphere) to 20% (further away from the sphere).

- The model of Ferreira et al. is based on integrating the equations of motion taking into account the added-mass term. The prediction of this model for the sphere trajectory during acceleration and steady state settling is in qualitative agreement with the results. There is no quantitative agreement with the results. There are two reasons for this. First, the wall effect is not included in the model. The measured steady state sphere settling velocities are 4%-13% smaller than the predicted velocities. The correlation of Di Felice to estimate the size of this wall effect does not give accurate results: deviations with measurements up to 24% occur. Second, the model neglects the Basset force, which has an estimated average magnitude during acceleration of 5-7% of the steady state drag force acting on the sphere.
• The model of Gondret et al. for predicting the distance between the bottom apex of the sphere and the bottom wall of the vessel for which the sphere starts decelerating is in qualitative agreement with the results: when the Reynolds number decreases, this distance increases.

### 8.2 Recommendations

• In the present experimental set-up, the laser sheet enters the vessel from the bottom. This choice was made to study the wall impact of the sphere without disturbing reflections of the light sheet in the walls of the glass vessel. However, for future experiments it would be interesting to change the set-up in such a way that the light sheet enters the vessel from the side. This way also the wake of the sphere can be investigated.

• The critical Stokes number determines if the sphere bounces back from the bottom wall of the vessel. Measurements of this parameter did not yield accurate results. A more sophisticated measurement set-up is needed to measure this number more accurately.
Appendix A

A solution for the equation of motion of the sphere

In this appendix the solution of equation (2.13), which describes the accelerating motion of a vertically falling sphere in incompressible Newtonian media, as derived by [Ferreira and Chhabra, 1998], is given. The solution presented is the solution Ferreira derives for large body to fluid density ratio.

The drag curve used is of the type:

$$C_D = \left( \frac{A}{Re^{1/2}} + B \right)^2$$  \hspace{1cm} (A.1)

For this thesis, $A = \sqrt{24}$ and $B = \sqrt{24}/9.06$ are used. The criterion Ferreira uses for large body to fluid density ratio can be written as:

$$Re_s > \frac{A^2}{4B^2} (\sqrt{2} - 1)^2$$  \hspace{1cm} (A.2)

For the drag curve used this corresponds to $Re_s > 3.52$. The Reynolds number can be written as:

$$Re_p = \frac{\rho_f u_p D}{\mu_f}$$  \hspace{1cm} (A.3)

Substituting these two equations in equation (2.13) gives:

$$m' \frac{dv_p}{dt} = mg' - (\nu_p \frac{1}{2} + \kappa v_p)^2$$  \hspace{1cm} (A.4)

where

$$m' = m \left( 1 + \frac{\rho_f}{2\rho_p} \right)$$  \hspace{1cm} (A.5)

$$g' = g \left( 1 - \frac{\rho_f}{\rho_p} \right)$$  \hspace{1cm} (A.6)
Appendix A. A solution for the equation of motion of the sphere

\[ l = \frac{A}{2} \left( \frac{\pi \mu_f D}{2} \right)^{1/2} \]  
\[ \kappa = \frac{B}{2} \left( \frac{\pi \rho_f D^2}{2} \right)^{1/2} \]

A complete analytical solution was obtained using the change of variable:

\[ u = v_p^{1/2} \]

Which enables (A.4) to be integrated to obtain the time \( t \) and distance \( \chi \) travelled by the sphere as a function of its velocity \( v_p \). The expressions which were obtained for \( t \) and \( \chi \) were non-dimensionalized as:

\[ X = \frac{\rho_f}{D\rho_p \left( 1 + \frac{\rho_f}{2\rho_p} \right)} \chi \]
\[ T = \frac{\rho_f v_s}{D\rho_p \left( 1 + \frac{\rho_f}{2\rho_p} \right)} t \]

The final result is:

\[ X = X_0 + \frac{8}{3B^2} \left\{ \ln \left[ \frac{-\Delta_+^{3/2} + \frac{1}{P^{4(\Delta_+ - 1)}}}{Q_+^{4(\Delta_+ - 1)^2} Q_+^{4(\Delta_+ - 1)}} \right] - \frac{-3\Delta_+^2 + 7}{2(\Delta_+^2 - 1)(\Delta_+^2 - 2)^{1/2}} \arctan(S) \right\} \]
\[ T = \frac{8(\Delta_+ - 1)^2}{3B^2} \left\{ \ln \left[ \frac{P^{\Delta_+^{3/2} - 1}}{Q_+^{4(\Delta_+ - 1)} Q_+^{4(\Delta_+ - 1)}} \right] - \frac{1}{2(\Delta_+^2 - 1)(\Delta_+^2 - 2)^{1/2}} \arctan(S) \right\} \]

where:

\[ P = \frac{1 + \frac{\nu_p}{\nu_s} + \frac{2}{\Delta_+ - 1} \left[ 1 + \left( \frac{\nu_p}{\nu_s} \right)^{1/2} \right]}{1 + \frac{\nu_p}{\nu_s} + \frac{2}{\Delta_+ - 1} \left[ 1 + \left( \frac{\nu_p}{\nu_s} \right)^{1/2} \right]} \]
\begin{align*}
S &= \frac{\Delta_+ - 1}{(\Delta_+^2 - 2)^{1/2}} \cdot \frac{\left(\frac{v_p}{v_s}\right)^{1/2} - \left(\frac{v_u}{v_s}\right)^{1/2}}{1 + \frac{1}{\Delta_+^2 - 2} \left[1 + (\Delta_+ - 1) \left(\frac{v_p}{v_s}\right)^{1/2}\right] \left[1 + (\Delta_+ - 1) \left(\frac{v_p}{v_u}\right)^{1/2}\right]} \\
\Delta_+ &= \left(1 + \frac{4\kappa}{\ell^2 (mg')^{1/2}}\right)^{1/2} \\
Q_{++} &= \frac{1 + \frac{\Delta_+ - 1}{\Delta_+ + 1} \left(\frac{v_p}{v_s}\right)^{1/2}}{1 + \frac{\Delta_+ - 1}{\Delta_+ + 1} \left(\frac{v_u}{v_s}\right)^{1/2}} \\
Q_{+0} &= \frac{1 - \left(\frac{v_p}{v_s}\right)^{1/2}}{1 - \left(\frac{v_u}{v_s}\right)^{1/2}} \\
v_s &= u_{++}^2 \\
m &= \frac{1}{6} \pi D^3 \rho_p
\end{align*}
Appendix B

ABCD-law for Gaussian beams

Yariv and Yeh [Yariv and Yeh, 1984] describe a transformation algorithm for light beams that have a Gaussian intensity profile, i.e. a laser beam. This transformation can be used to calculate the the radius of the light beam and the radius of curvature of its phase front after it crossed different media, for example lenses and interfaces with different indices of refraction.

First a complex beam parameter at a reference point (in this thesis the output of the laser tube is chosen) $q_r$ has to be calculated from the radius of curvature of the phase front and the radius of the light beam:

$$\frac{1}{q_r} = \frac{1}{R_r} - i \frac{\lambda}{\pi \omega_r^2 n_r}$$

(B.1)

where $R_r$ is the radius of curvature of the phase front at the reference point, $\lambda$ is the wavelength of the light, $\omega_r$ is the beam radius at the reference point, and $n_r$ is the refraction index of the medium at the reference point. For determination of the complex beam parameter at the reference point, $R_r$, $\lambda$, $\omega_r$ and $n_r$ have to be known. From the specifications of the laser $\omega_r$ and the beam divergence $\theta_d$ can be obtained. However, $R_r$ has to be calculated from $\lambda$, $n_r$, $\theta_d$ and $\omega_r$:

$$\omega_0 \approx \frac{\lambda}{\pi \theta_d n_r} \text{ for } \theta_d \ll \pi$$

(B.2)

$$z = \frac{\pi \omega_0^2 n_r}{\lambda} \sqrt{\frac{\omega_r^2}{\omega_0^2} - 1}$$

(B.3)

$$R_r = z \left[ 1 + \left( \frac{\pi \omega_0^2 n_r}{\lambda z} \right)^2 \right]$$

(B.4)

where $\omega_0$ is the radius of the beam at waist position (the position where the beam has the smallest radius $\omega_0$), $z$ is the distance from the laser exit $z=z_1=0$ and $\theta_d$ is the beam divergence at the laser exit.

The ABCD-law describes what happens when the beam intersects several optical components:

$$q_t = \frac{A_T q_r + B_T}{C_T q_r + D_T}$$

(B.5)
Appendix B. ABCD-law for Gaussian beams

Table B.1: Transformation matrices for optical elements

<table>
<thead>
<tr>
<th>component</th>
<th>Matrix</th>
</tr>
</thead>
</table>
| straight section: length 1 (m) | \[
|                             | \begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix} \] |
| thin lens: focal length f (m) | \[
|                             | \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix} \] |
| flat interface: refractive indices \(n_{r1}, n_{r2}\) | \[
|                             | \begin{pmatrix} 1 \\ 0 \end{pmatrix} \] \[
|                             | \begin{pmatrix} n_{r1} \\ n_{r2} \end{pmatrix} \] |

This equation relates \(q_r\) at \(z=z_1\) to \(q_i\) at a point of interest \(z=z_2\). The \(A_T, B_T, C_T\) and \(D_T\) are the elements of a matrix which describes the optical path between \(z_1\) and \(z_2\). This matrix is the ordered product of all optical elements between \(z_1\) and \(z_2\), starting with the last element the beam encounters. In table B.1 the transformation matrices for the optical elements and media used in this research are described. In practice, \(q_r\) is the original beam parameter and \(q_i\) is the beam parameter after a series of components.

In the manual of the laser the waist and angular beam spread of the laser beam at the exit of the laser can be found. These parameters are a function of the different wavelengths at which the laser operates. In this research the laser is used in multi-line mode. An intense wavelength in the spectrum of the laser is the 588 nm blue line. For this reason, the beam divergence of 0.25 mrad and the radius curvature of 682.0 \(\mu m\) of this wavelength are used as a reference case. Substitution of these values in equations B.2, B.3 and B.4 and using \(n_r=1.00\) for air, gives the radius of curvature of the phase front at the exit of the laser tube. The laser beam is now described at the exit of the laser. With equation B.5 and the values of the parameters of the optical elements along the path given in table B.2, the complex beam parameter at the point of interest can be calculated. Finally, the beam radius and the radius of curvature can be calculated from \(q_i\) with:

\[
\frac{1}{q_i} = \frac{1}{R_i} - i \frac{\lambda}{\pi \omega_0^2 n_i} \tag{B.6}
\]

where \(n_i\) is the refractive index of the medium at the point of interest.
Table B.2: Parameters of the optical elements of the PIV setup

<table>
<thead>
<tr>
<th>optical element</th>
<th>parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>distance laser to first lens</td>
<td>$f_1 = -6.0$ or $-12.0$</td>
</tr>
<tr>
<td>length of straight section</td>
<td>$f_1 + f_2$</td>
</tr>
<tr>
<td>thin lens</td>
<td>$f_2 = 500$</td>
</tr>
<tr>
<td>straight section length</td>
<td>variable</td>
</tr>
<tr>
<td>bottom of vessel</td>
<td>$n_{r1}=1.0$, $n_{r2}=1.50$</td>
</tr>
<tr>
<td>- interface</td>
<td>length=19.0</td>
</tr>
<tr>
<td>- straight section length</td>
<td>$n_{r1}=1.50$, $n_{r2}=1.4$</td>
</tr>
<tr>
<td>- interface</td>
<td>length=set =&gt; measurement position</td>
</tr>
</tbody>
</table>
Bibliography


Dohmen, W., 1968, De instationaire stromingsweerstand van bollen: Master’s Thesis, Delft University of Technology.


Bibliography


