Fast Frequency and Time Domain Integral Equation Modelling for Marine CSEM Applications

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To my kindest parents, Parisima and Kiumars and to my lovely wife Maryam.
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Symbols and Notations

- $E_k$ total electric field strength [Vm$^{-1}$]
- $E_i^k$ incident electric field strength [Vm$^{-1}$]
- $E_{sc}^k$ scattered electric field strength [Vm$^{-1}$]
- $E_{δ}^k$ electric field impulse response [Vm$^{-1}$]
- $H_j$ magnetic field strength [Am$^{-1}$]
- $J_k$ volume density of electric current [Am$^{-2}$]
- $J_e^j$ volume source density of electric current [Am$^{-2}$]
- $K_j^e$ volume source density of magnetic current [Vm$^{-2}$]
- $χ^σ$ electric contrast function [Sm$^{-1}$]
- $G$ scalar Green’s function [m$^{-1}$s$^{-1}$]
- $σ_{k,r}$ conductivity tensor [Sm$^{-1}$]
- $ε_{k,r}$ permittivity tensor [Fm$^{-1}$]
- $μ_{j,p}$ permeability tensor [Hm$^{-1}$]
- $δ_{k,r}$ Kronecker tensor [-]
- $i$ imaginary unit, $i^2 = -1$ [-]
- $x_m$ Cartesian coordinate [m]
- $k_m$ angular wave vector [m$^{-1}$]
- $s$ time-Laplace transform variable [s$^{-1}$]
- $t$ time coordinate [s]
- $f$ frequency [Hz]
- $δ_s$ skin depth [m]
- $ω$ angular frequency [rads$^{-1}$]
Symbols and Notations

- $\tau$  
  time period [s]

- $\Delta \tau$  
  time step [s]

- $erfc$  
  complementary error function [-]

- $H$  
  Heaviside step function [-]
Chapter 1

Introduction

1-1 Description of the research topic

Electromagnetic theory and its applications are an important tool for subsurface studying and exploration. Over the past few decades, a flurry of activities has been undertaken in order to explore new petroleum reserves. Many of them are focused on marine exploration. Even though hydrocarbon is produced from large reservoirs below shallow and deep water, still an immense area of the earth’s surface, which is covered by water, remained unexplored. Marine controlled-source electromagnetic (CSEM) method have the potential to detect petroleum, natural gas, gas hydrates and other resistive zones in a conductive background.

Although reflection seismic is the principal and most reliable geophysical tool for delineating subsurface structures based on the contrast in acoustic properties, there are geological terrains in which the interpretation of seismic data is difficult and can fail, such as regions dominated by strong scattering or high reflectivity, volcanic covers, complex carbonate areas, submarine permafrost, and salt layers and domes. CSEM can help tackle these problems. The seismic method is not particularly sensitive to the liquids in the pore space, because the acoustic properties of the liquids do not vary much. Given high-quality seismic and well data, and sophisticated seismic inversion and rock physics tools, it is sometimes possible to relate these seismic changes to saturation effects. In other situations, for example in chalk or carbonate reservoirs, seismic data can constrain the porosity of the subsurface, but determining the fluid content remains challenging. In contrast, the electric resistivity can change over one or two orders of magnitude depending on water or oil saturation. CSEM is therefore a good option in reducing the risks associated with the detection of hydrocarbon reservoirs and even more, discriminate fluids (e.g. Hoversten et al.)
(2006a), Harris et al. (2009), Gao et al. (2010) and DellAversana et al. (2011)). Furthermore, CSEM is a good option for frontier exploration into deep waters because deep water drilling is very expensive. Marine CSEM methods also have potential reservoir monitoring applications. As with oil and gas exploration, CSEM and seismic methods are complementary for this application, with the former being sensitive to the bulk volume of a resistor and the latter offering superior structural resolution. Hydrocarbon saturation volumes can be created by integrating porosity information derived from seismic inversions with water saturation data from CSEM inversions, both constrained by well logs. Some research examples can be found in Norman et al. (2008) and Andris and MacGregor (2011).

The main idea behind the controlled-source electromagnetic method (CSEM) for exploration is to detect and localize relatively thin high resistive bodies. Hydrocarbon reservoirs typically have a resistivity that is sometimes up to one hundred times higher than a water reservoir and surrounding lithology, such as shale and mudrock, and this is sufficient to generate an upward decaying electromagnetic field to be recorded by receivers located on the seabed. For instance, marine sediments saturated with saline water, have low resistivities (1-5 Ohm-m); by displacing the saline water with hydrocarbons the bulk resistivity of the reservoir will significantly increase (10-500 ohm-m). The marine CSEM method exploits this dramatic change in electrical resistivity to potentially delineate water-bearing formations from those containing hydrocarbons (Hoversten et al., 2006b).

The study of wave scattering at low frequencies was pioneered by Rayleigh (1871), however the earliest development work on seafloor controlled-source systems appears to be due to Drysdale (1924), who described an extensive program to measure the magnetic field and electric current around a submarine cable for use in a ship guidance system of World War 1 and his works show that many of the difficulties of working in the ocean have not changed over time. In an accompanying paper, Butterworth (1924), computed the fields around the cable and over an insulating seafloor for comparison to measurements. More recently, Bannister (1968), calculated the seafloor fields produced by an extended horizontal electric dipole (HED) source placed on the sea surface and a seafloor horizontal magnetic dipole (HMD) source, both with the purpose of determining the seabed conductivity. Coggon and Morrison (1970) modelled a seafloor vertical magnetic dipole (VMD) source with both electric and magnetic receivers, emphasizing exploration of the uppermost few hundred meters of the seabed. Strong sea surface effect while using marine controlled-source system in shallow water is also discussed by Coggon and Morrison (1970) and Chave and Cox (1982), which will be specially important on the continental shelves. Kaufman and Keller (1983) computed seafloor sounding curves for VMD and HED sources with a vertical magnetic field receiver.

In recent years, lots of research and activities are focused on special application of low frequency controlled-source electromagnetic sounding, which is the so-called seabed logging method, in order to explore for hydrocarbon reserves. Marine CSEM method has been significantly developed in theory, methodology, and instrumentation as a complimentary method besides seismic activities, specially in the aforementioned regions. Also, it can be used as a primary method to reduce the extent of the seismic acquisition area and it
can be applied as a time-lapse method by production monitoring. Early proposals to use the method for petroleum exploration, e.g., Chave et al. (1991), concentrated on relatively shallow water and exploration targets. The CSEM method was also developed for deep-water studies of the oceanic lithosphere (Cox (1981); Cox et al. (1986); Constable and Cox (1996)) and mid-ocean-ridges (Evans et al. (1994); MacGregor et al. (2001)). With the migration of hydrocarbon exploration into the deeper waters of the continental shelves, the marine CSEM method recently has become an important exploration tool for the hydrocarbon industry (e.g., Ellingsrud et al. (2002); Eidesmo et al. (2002); Johansen et al. (2005); Hesthammer and Boulaenko (2005)).

The marine CSEM method usually requires deploying receivers on the seafloor, to record signals which are emitted by a transmitter as a source, towed behind a vessel. The source generates current, that generates an electromagnetic field. The electromagnetic field diffuses through the earth. As the electromagnetic diffusion field passes through geological compartments with various resistivities, it will generate secondary electric currents, which subsequently generate electromagnetic fields that are measured by the receivers located on the seafloor. These data are processed to interpret the resistive structures below the seafloor. The spatial decay of the fields can be characterized by the skin depth equation,

\[ \delta_s = \sqrt{\frac{2}{\omega \mu \sigma}} \]

where \( \sigma \) represents the electrical conductivity of the medium in Sm\(^{-1}\), \( \omega \) is radial frequency in rads\(^{-1}\), \( \mu \) is magnetic permeability in Hm\(^{-1}\) and \( \delta_s \) is measured in m. As radial frequency increases, the skin depth decreases. Similarly, as conductivity increases, the skin depth decreases. To determine whether wave propagation dominates or diffusion, we can assess if \( \sigma \gg \omega \varepsilon \), where \( \varepsilon \) is the permittivity of the subsurface, then we have diffusion, otherwise wave propagation occurs. Therefore in a conductive medium, where permittivity is negligible, the low-frequency electromagnetic field behaves as diffusive. In diffusive fields, the wavenumbers are complex and cannot be represented by integrals of uniform plane waves with real angles of incidence. Several studies have raised interesting analogies between the electromagnetic diffusion equations and the seismic wave equations in layered media, such as Weidelt (1972); Levy et al. (1988); de Hoop (1996) and Kwon and Snieder (2010). Propagation of electromagnetic waves is described by Maxwell’s equations and includes displacement current.

1-2 Project objectives

According to the aforementioned historical development of CSEM, a fast and accurate forward modelling code will be useful for forward and inverse modelling. Interpretation of measured CSEM data is complicated because the fields come from all directions with substantial interference and are recorded by all receivers simultaneously. This is where forward modelling plays an important role to help understanding the modelled data with
various possible geological scenarios. Studying different scenarios requires a fast and accurate method, otherwise forward modelling for many different scenarios would not be feasible in practice. Therefore, the first objective of this thesis is to develop and accelerate the computational method of forward modelling.

For three-dimensional diffusive electromagnetic modelling problems, local methods seem to have outperformed global methods in terms of memory requirements and computational efforts. The main reason for pursuing integral equation methods for modelling is that for a large class of problems the modelling domain can be reduced to the target volume. For such problems integral equations are useful, because they are based on primary-secondary, or direct-scattered field separation and allow for several types of suitable approximations. Examples of forward and inverse scattering solutions using integral equations can be found in Abubakar and van den Berg (2004), Zhdanov et al. (2006) and Gribenko and Zhdanov (2007). A modification to the original CG method (Hestenes and Stiefel 1952) is an efficient way for solving integral equation problems (van den Berg 1984). An additional advantage in computational efficiency is achieved when the background medium can be chosen as a homogeneous space or a horizontally layered earth. Then the convolutional structure of the system matrix is exploited by using the FFT routine for fast computation of the discrete convolutions while the background medium is homogeneous Catedra et al. (1989) and Zwamborn and van den Berg (1992) and in case of a layered earth, a two-dimensional FFT exploits the convolutional structure in the two horizontal directions. The three-dimensional CG-FFT and related methods have been reported by many researchers, for examples Zhang and Liu (2001). For a more complete review, the reader is referred to Catedra et al. (1995).

For low frequencies and a relatively small volumetric contrast, the Born approximation (BA), which approximates the total internal electric field by the background field yields an extremely fast approximate solution (Born and Wolf (1980) and Alumbaugh and Morrison (1993)). Thus analysis of the Born approximation and the extended Born approximation (EBA) (Habashy et al., 1993) are of interest in solving three-dimensional problems. However, the extended Born approximation has much better accuracy than the Born approximation and has been successfully applied to a number of electromagnetic scattering problems (Abubakar and van den Berg (2000), Cui et al. (2004)). Other electromagnetic scattering approximations have been published during the recent years. Among them are the quasi-linear (QL) approximation (Zhdanov and Fang, 1996), the quasi-analytical (QA) approximation (Zhdanov et al., 2000), the high-order generalized extended Born approximation (Ho-GEBA) (Gao and Torres-Verdin, 2006), the multigrid quasi-linear approximation (MGQL) (Ueda and Zhdanov (2006), Ueda and Zhdanov (2008) and Endo et al. (2008)). In many cases the computation time can still be an issue, especially when these forward scattering solvers are used as an engine for inverse problems, so still more efficient approximate solutions are under investigation.

In this thesis we aim to present two results that can be useful for fast modelling algorithms. The first is to demonstrate the improved accuracy of an iterated version of the extended Born approximation, even with a reservoir consisting of two separated compartments. The
second is to show that the approximate results at the receiver level, which is usually the sea bottom, and also inside the reservoir are accurate. This iterative method is suitable as a modelling algorithm for solving the inverse scattering problem. It is worth to note that in cases where the scattered field only consists of inductive effects at low frequencies, the Born approximation works well and there is no need to use more complex methods with many terms (Habashy et al. 1993).

Reducing the dimensionality in the modelling effort may be useful in some situations by choosing the proper dimensions in the modelling. This is the second objective of the thesis. Two and half dimensional modelling, which is a fast method, can be even faster when using the integral equation method, and for a reservoir for which the plane of the 2.5D modelling is a plane of symmetry, 2.5D modelling is adequate for detection and surveillance studies. Feasibility studies and survey design are vital and crucial for a successful CSEM project. A different type of forward modelling can be used to provide understanding of the CSEM response from a complex geology. Using 2.5D CSEM modelling is fast and useful for sensitivity analysis of subsurface parameters. The size and depth of the target, hydrocarbon thickness, extent in cross-line dimension, frequency content of the emitted diffusive field and offsets will be required to resolve if the targets can be detected. At this stage, a decision can be made on whether or not the target reservoir is detectable and if CSEM is a suitable exploration tool capable of quantifying the prospective hydrocarbon reserves. From here an optimum survey can be designed, based on forward modelling results. Given the much shorter computation times, 2.5D modelling is an interesting option for several types of modelling studies.

In the last part of this thesis, we will discuss the increase of information content in the data by using frequency to time conversion methods. CSEM methods are generally divided into frequency-domain electromagnetic (FDEM) and time-domain (transient) electromagnetic (TDEM) method, depending on the waveform of the transmitted electrical current. We compare an analytical method to transform frequency-domain CSEM data back to the time domain with a numerical transformation. The analytical method exploits the fact that the kernel of the integral equation has a known behavior as a function of frequency and that the solution to the integral equation can be written as a sum of repeated applications of the kernel to the incident field. A set of expansion functions is found, which have analytically known time domain counterparts, which need only a limited number frequencies for the transformation back to time. We compare this analytical method, coined the diffusive expansion method, in CSEM applications with two other numerical methods, the Gaver-Stehfest method and an optimized form of the fast Fourier transformation method where the data is required at an optimum number of discrete frequency values such that the data at intermediate frequency values can be accurately obtained by interpolation. We end the thesis with conclusions and an outlook for marine CSEM.
1-3  Scientific originality and innovation

After a short introduction in chapter 1 and clarifying the statement of the problem that we aim to solve in this thesis, in chapter 2 we briefly discuss full solution of the 3D modelling problem using the iterative scheme of Conjugate Gradient Fast Fourier Transformation (CG-FFT) to solve the integral equations. The advantages of applying the CGFFT method to a class of large scale forward and scattering problems are outlined. Solutions of integral equations in three-layered earth CSEM application with an assumed reservoir is examined using CG-FFT. Later in this chapter the CG-FFT full solution will be used as a reference. In order to have a fast approach to solve the aforementioned CSEM problem, we introduce an approximate method for three-dimensional low frequency CSEM modelling using the integral equation (IE). We apply the method to a synthetic model in the marine CSEM exploration situation, where conductivity is different from the known background medium. For 3D configurations fast computational methods are relevant for both forward and inverse modelling studies. The Born approximation, extended Born approximation and iterative extended Born approximation are implemented and compared with the full solution of the conjugate gradient fast Fourier transformation method. These methods are based on an electric field domain integral equation formulation. We show how well the iterative extended Born approximation method performs in terms of both accuracy and speed with different configurations and different source positions.

With the help of this method sensitivity analysis using 3D modelling is possible in a timely manner, which is vital for CSEM applications. For forward modelling the solution at the sea bottom is of interest, because that is where the receivers are usually located. For inverse modelling, the accuracy of the solution in the target zone is important to obtain reasonably accurate conductivity values from the inversion using this approximate solution method. Our modelling studies investigate to which extent the iterative extended Born approximation method is fast and accurate for both forward and inverse modelling. Sensitivity analysis as a function of the source position and different reservoir sizes can validate the accuracy of the iterative extended Born approximation.

We describe the configuration for both the forward source and forward scattering problems, which are in a three-layered medium, consisting of air, sea and ground, with a scattering object as a model for a reservoir. Modelling efficiency can also be increased enormously when we reduce the dimensions of the model to one-dimensional or two-and-a-half dimensional leading to fast codes. In Chapter 3, we look into the question in what model configurations two-and-a-half dimensional modelling is a good choice for modelling a three-dimensional reservoir response to the diffusive EM field. We use integral equation methods for one-dimensional, two-and-a-half dimensional and three-dimensional CSEM modelling. We follow up how much one-dimensional and two-and-a-half dimensional integral equation modelling overestimate the result in comparison with three-dimensional modelling of the marine controlled-source electromagnetic method. Then we apply the method to a synthetic model in the marine controlled-source electromagnetic exploration situation where...
conductivity is different from the known background medium. We implemented two-and-a-half dimensional modelling by using three-dimensional source and two-dimensional reservoir and compared the results with one-dimensional and three-dimensional modelling. It is shown how the two-and-a-half dimensional method performs, in terms of both accuracy and speed, with different configurations and different source positions. The next step in Chapter 3 is dedicated to the sensitivity analysis of one-dimensional, two-and-a-half dimensional and three-dimensional modelling. This analysis has been done for a symmetrically placed reservoir and the in-line acquisition configuration. It is investigated how the accuracy of 2.5D modelling in comparing with 3D modelling depends on the configurations. Variables for sensitivity analysis are function of different reservoir sizes in cross-line dimensions, different reservoir thickness, a range of frequencies, different depth and moving source position.

In Chapter 4 we compare a quasi-analytical method to transform frequency-domain CSEM data back to the time-domain with a numerical transformation. The quasi-analytical method exploits the fact that the kernel of the integral equation has a known behavior as a function of frequency and that the solution to the integral equation can be written as a sum of repeated applications of the kernel to the incident field. A set of expansion functions is found, which have analytically known time domain counterparts, and that need only a few frequencies for the transformation back to the time. We evaluate the accuracy of this function in different environments and different methods such as the Gaver-Stehfest method. Finally we compare this analytical method for CSEM application with a numerical method containing of a combination of optimized fast Fourier transformation and the cubic hermite interpolation.

In the last chapter (5), we discuss general conclusions and outlook. In the appendix we list the space-frequency-domain expressions in terms of Fourier-Bessel transforms for the Green functions that are used as kernels for the integral equations.

1-4 General description

In this thesis formulations are given in a Cartesian coordinate system, which is useful in applied geophysics where the surface of the earth can be considered flat. The coordinate system is set up by three mutually perpendicular base vectors \{\hat{i}_1, \hat{i}_2, \hat{i}_3\} of unit length each and origin \(O\). In the indicated order, the base vectors form a right-handed system. The subscript notation for Cartesian vectors and tensors is used, except when explicitly specified. The subscript notation applies to repeated lower-case Latin subscripts which range over the values 1, 2 and 3. To denote the horizontal coordinates only, Greek subscripts are used to which the values 1 and 2 are to be assigned. When changing from Latin to Greek subscripts, their corresponding equivalents are used. Appropriately, the position is also specified by the vector \(\mathbf{x} = x_m \hat{i}_m\). \(\overline{\mathbb{D}}\) denotes the complement of the domain \(\mathbb{D}\) in \(\mathbb{R}^3\).
Chapter 2

3D Modelling and Approximations

Fast and Accurate Three-Dimensional Controlled-Source Electromagnetic Modelling

ABSTRACT

We would like to discuss a solution for diffusive electromagnetic fields which is a fast approximate method for three-dimensional low frequency controlled-source electro-magnetic modelling using the integral equation (IE) method. We apply the method to a synthetic model in a typical marine controlled-source electromagnetic scenario, where conductivity and permittivity are different from the known background medium. For 3D configurations, fast computational methods are relevant for both forward and inverse modelling studies. Since this problem involves a large number of unknowns, it has to be solved efficiently to obtain results in a timely manner, without compromising accuracy. For this reason, the Born approximation, extended Born approximation and iterative extended Born approximation are implemented and compared with the full solution of the conjugate gradient fast Fourier transformation method. These methods are based on an electric field domain integral equation formulation. It is shown here how well the iterative extended Born approximation method performs in terms of both accuracy and speed with different configurations and different source positions. The improved accuracy comes at virtually no additional computational cost. With the help of this method, it is now possible to perform sensitivity analysis using 3D modelling in a timely manner, which is vital for controlled-source electromagnetic applications. For forward modelling the solution at the sea bottom is of interest, because that is where the receivers are usually located. For inverse modelling, the accuracy of the solution in the target zone is important to obtain reasonably accurate conductivity values from the inversion using this approximate solution method.
Our modelling studies show that the iterative extended Born approximation method is fast and accurate for both forward and inverse modelling. Sensitivity analysis as a function of the source position and different reservoir sizes validate the accuracy of the iterative extended Born approximation.

2-1 Introduction

For three-dimensional diffusive electromagnetic modelling problems, local methods seem to have outperformed global methods in terms of memory requirements and computational efforts. The main reason for pursuing integral equation methods for modelling is that, for a large class of problems, the modelling domain can be reduced to the target volume. For such problems integral equations are useful, because they are based on primary-secondary, or direct-scattered field separation and allow for several types of suitable approximations. The integral equation uses the unperturbed field as a kernel multiplying the unknown perturbation on one side, with the source of the perturbation on the other side. Fast forward modelling algorithms are especially important for solving a parametric inverse problem. Examples of forward and inverse scattering solutions using integral equations can be found in Abubakar and van den Berg (2004), Zhdanov et al. (2006) and Gribenko and Zhdanov (2007). A modification to the original conjugate gradient method (Hestenes and Stiefel 1952) is an efficient way for solving integral equation problems (van den Berg 1984). An additional advantage in computational efficiency is achieved when the background medium can be chosen as a homogeneous space or a horizontally layered earth. Then, the convolutional structure of the system matrix is exploited by using the FFT routine for fast computation of the discrete convolutions while the background medium is homogeneous (Catedra et al. 1989, Zwamborn and van den Berg 1992) and in case of a layered earth, a two-dimensional FFT exploits the convolutional structure in the two horizontal directions. The three-dimensional conjugate gradient fast Fourier transform (CG-FFT) and related methods have been reported by many researchers, for example Zhang and Liu (2001). For a more complete review, the reader is referred to Catedra et al. (1995).

For low frequencies and a relatively small volumetric contrast, the Born approximation (BA), which approximates the total internal electric field by the background field yields an extremely fast approximate solution (Born and Wolf 1980 and Alumbaugh and Morrison 1993). Thus analysis of the Born approximation and the extended Born approximation (EBA) Habashy et al. 1993 are of interest in solving three-dimensional problems. However, the extended Born approximation has much better accuracy than the Born approximation, and has been successfully applied to a number of electromagnetic scattering problems, Abubakar and van den Berg 2000 Cui et al. 2004 Tehrani and Slob 2008.

Other electromagnetic scattering approximations have been published in recent years. Among them are the high-order generalized extended Born approximation (Ho-GEBA) (Gao and Torres-Verdin 2006), the quasi-analytical (QA) approximation (Zhdanov et al. 2000), the quasi-linear (QL) approximation (Zhdanov and Fang 1996) and the multigrid
quasi-linear approximation (MGQL) (Ueda and Zhdanov 2006 Ueda and Zhdanov 2008 Endo et al. 2008). In many cases the computational time can still be an issue, especially when these forward scattering solvers are used as an engine for inverse problems, so still more efficient approximate solutions are under research.

In this paper we aim to present two results that can be useful for fast modelling algorithms. The first is to demonstrate improved accuracy of an iterative version of the extended Born approximation, even with a reservoir consisting of two separated compartments. The second is to show that the approximate results are accurate both at the receiver level (which is usually the sea bottom) and inside the reservoir. This iterative method is suitable as a modelling algorithm for solving the inverse scattering problem.

It is worth to note that in cases where the scattered field only consists of inductive effects at low frequencies, the Born approximation works well and there is no need to use more complex methods with many terms to converge (Habashy et al. 1993).

In the following section, we give the theoretical details for the Born and the extended Born approximations. We formulate the iterative extended Born approximation (IEBA) through an integral equation for the electric field. First, we formulate the integral equation representation of the electric field everywhere in space. Next, we discuss the iterative extended Born approximation and we compare the Born and extended Born approximations and its iterative version with the full solution obtained by the CG-FFT method. The numerical solution of EBA using CG-FFT has already been confirmed by analytical solutions (Liu et al. 2001). It has been done here, for different reservoir geometries and different source positions.

2-2 Theory

We describe the configuration for both the forward source and forward scattering problem, which are depicted in Figure 2-1.

It shows a three-layered medium, consisting of air, sea and ground, with a scattering object as a model for a reservoir. Let $\mathbb{D}_1$ be the domain occupied by the sea-layer. In this medium horizontal electric dipole (HED) sources are present in the bounded domain $\mathbb{D}^c$, while the domain $\mathbb{D}_2$ is the lower half-space. The domain of the scattering object, is denoted $\mathbb{D}^{sc}$. Electromagnetic fields are shown by the arrows schematically. The incident field from the source to receivers in $\mathbb{D}_1$, is called case 1. The incident field from source to the reservoir, is called case 2 and the scattered field from the scattering object (reservoir) to the receiver, is called case 3.

We use the subscript notation for Cartesian vectors and tensors. The analysis is carried out in the Laplace transform domain, where the transform parameter $s$ is taken to be either real and positive or purely imaginary.
The method we follow is based on Höhmann (1975) for a homogeneous earth and Wannamaker et al. (1984) for a layered earth. There are quite some papers afterward such as Michalski and Zheng (1990).

We decompose the total electric field inside the reservoir $E^i_k(x, x^S, s)$ into the incident field $E^i_k(x, x^S, s)$ and the scattered field $E^sc_k(x, x^S, s)$,

$$E^i_k(x, x^S, s) = E^i_k(x, x^S, s) + E^sc_k(x, x^S, s). \quad (2-1)$$

The index $k$ denotes the vectors components that range over the values 1, 2 and 3. The receivers and source positions are shown by $x$ and $x^S$, respectively.

The incident field, can be calculated as,

$$E^i_k(x, s) = \int_{x^S \in \mathbb{D}^c} G^{EJ}_{kr}(x, x^S, s)J^e_r(x^S, s)d^3x^S \quad (2-2)$$

where $x \in \mathbb{D}^c$ and $G^{EJ}_{kr}(x, s)$ denotes the Green’s function for electric field generated by an electric current. The first subscript denotes the vector component of the electric field, while the second subscript denotes the vector component of the electric current source and $J^e_r$ is the volume density of electric current source.

The equation shows that the diffusive electromagnetic field from a source with known parameters in a known medium can be calculated in all space once the fields radiated by appropriate point sources have been calculated.

Now we formulate the forward scattering problem. We investigate the scattering of diffusive electromagnetic fields by a contrasting domain of bounded extent present in an unbounded embedding. Let $\mathbb{D}^ac$ be the bounded domain occupied by the scatterer and let $\sigma^{ac}(x)$ be its conductivity. The embedding exterior to $\mathbb{D}^c$ is denoted $\bar{\mathbb{D}}^ac$ and has a conductivity $\sigma(x)$. 

![Figure 2-1: Schematic diagram of Diffusive fields present in a three horizontally stratified earth layer configuration for both forward source and scattering problems.](image-url)
To arrive at the integral equations for the unknown field strengths inside the scatterer, we confine the position of observation to the domain of the scatterer \( x \in I_{D}^{sc} \), we obtain the integral equation for the electric scattered field in homogeneous layered earth as,

\[
E_{sc}^{k}(x, s) = \int_{x' \in I_{D}^{sc}} G^{EJ}_{kr}(x, x', s) J_{sc}^{s}(x', s) d^{3}x',
\]

(2-3)

where the contrast source function \( J_{sc}^{s}(x', s) \) is given by

\[
J_{sc}^{s}(x', s) = \chi_{\sigma}(x') E_{r}(x', s),
\]

(2-4)

in which the electric contrast function is given by

\[
\chi_{\sigma}(x') = \sigma_{sc}(x') - \sigma(x').
\]

(2-5)

From this system of integral equations the total diffusive electric field \( E_{k} \) in the scatterer domain can be obtained by,

\[
E_{k}(x, s) = E_{i}^{k}(x, x_{S}, s) + \int_{x' \in I_{D}^{sc}} G^{EJ}_{kr}(x, x', s) \chi_{\sigma}(x') E_{r}(x', s) d^{3}x',
\]

(2-6)

where \( x \in I_{D}^{sc} \).

We can solve this integral equation of the second kind through reducing the integral equation to a linear system of algebraic equations, then discretizing this system and approximating the unknown total electric field.

Once the total field has been found for all points inside the reservoir, we can compute the total field at the receiver,

\[
E_{k}^{r}(x, s) = E_{i}^{r}(x, x_{S}, s) + E_{sc}^{k}(x, x_{S}).
\]

(2-7)

The incident field can be found from Eq. (2), whereas the scattered field can be written as follows,

\[
E_{sc}^{k}(x, x', s) = \int_{x' \in I_{D}^{sc}} G^{EJ}_{kr}(x, x', s) \chi_{\sigma}(x') E_{r}(x', s) d^{3}x'.
\]

(2-8)

where \( x \in I_{1} \).

For low frequencies, small contrasts and a scattering domain that is small relative to the skin-depth, it has been shown that approximating the total internal electric field by the background field is a good approximation (Habashy et al. 1993). The scattered field is then computed at low computational cost. Analysis of the Born and extended Born approximations are therefore of interest. In cases where the scattered field only consists of inductive effects at low frequencies, the Born approximation works well and there is no need to use more complex methods with many terms to converge.
If we consider $E_k(0)(\mathbf{x}, s)$ as the total field inside the reservoir then the Born approximation gives

$$E_k(0)(\mathbf{x}, s) = E_i^k(\mathbf{x}, s), \quad (2-9)$$

which means the total field strength inside the scatterer (reservoir) equals the background field (incident field). Then we are using the Born approximation (BA).

On the other hand if we consider $E_k(0)(\mathbf{x}, s)$ as an initial guess, we can write any order approximation of the total diffusive electric field as,

$$E_k^n(\mathbf{x}, s) = E_i^k(\mathbf{x}, s) + \int_{\mathbf{x'} \in \mathbb{D}^{sc}} G^{EJ}_{kr}(\mathbf{x}, \mathbf{x'}, s) \chi^{\sigma}(\mathbf{x'}) E_r^{(n-1)}(\mathbf{x'}, s) d\mathbf{x'}, \quad (2-10)$$

for $n = 1, 2, 3, \ldots$.

Convergence criteria for the iterative Born approximation (IBA) are given in (de Hoop 1991). In Eq. (2-10) every iteration requires a full matrix-vector multiplication in a numerical implementation.

The extended Born approximation is based on the dominant contribution of the integral equation at locations where the Green’s function is singular, leading to

$$E_s(0)(\mathbf{x}, s) = M_{sk}^{-1}(\mathbf{x}, s) E_i^k(\mathbf{x}, s), \quad (2-11)$$

where $M_{sk}^{-1}$ is defined as,

$$M_{sk}^{-1}(\mathbf{x}, s) M_{kr}(\mathbf{x}, s) = \delta_{sr} \quad (2-12)$$

and

$$M_{kr}(\mathbf{x}, s) = \delta_{kr} - K_{kr}(\mathbf{x}, s) \quad (2-13)$$

where

$$K_{kr}(\mathbf{x}, s) = \int_{\mathbf{x'} \in \mathbb{D}^{sc}} G^{EJ}_{kr}(\mathbf{x}, \mathbf{x'}, s) \chi^{\sigma}(\mathbf{x'}) d^3\mathbf{x'}. \quad (2-14)$$

Similar to the IBA, but keeping the concept of EBA in every iteration, we get for the general iterative form as,

$$E_k^n(\mathbf{x}, s) = E_i^k(\mathbf{x}, s) + K_{kr}(\mathbf{x}, s) E_r^{(n-1)}(\mathbf{x}, s). \quad (2-15)$$

for $n = 1, 2, 3, \ldots$.

Equation 2-15 does not represent a proper series expansion and we can not expect convergence to the true solution, but we do expect improvement in the first few iterations.

An error reduction of 0.1% is used as a stopping criterion.

The Born approximation is computed at the cost of zero iterations and EBA is computed at the cost of one iteration compared to the full solution with a large number of iterations. The iterative extended Born approximation requires a computational cost of just one iteration of the operator just as the EBA, then we need only local updates.
2-3 Method of numerical implementation

As we mentioned in the last section, and shown in Figure 2-1, the solution consists of four steps. For the solution at the receiver level we need to compute the direct incident field from source to the receivers in the water layer. For solving the integral equation we need to compute the incident field at every point in the scattering domain.

We compute the background Green’s functions using standard Fourier-Bessel transformations. For our three-layered background medium the wave-number frequency-domain solution is known in closed form and can be found in textbooks, e.g. Chew 1999.

We compute the total electric field inside the scattering domain by solving the weak form of the integral equation as described in detail in Zwamborn and van den Berg 1992.

The last step would be the computation of the total electric field at the receiver level.

Let the size of the reservoir be discretized as a rectangular block of \( N_I \times N_J \times N_K \) elements, each of volume \( \Delta V = \Delta x_1 \Delta x_2 \Delta x_3 \) where \( \Delta x_k \) denotes the step-size in coordinate \( x_k \). Then introducing counters \( I, J, K \) for the discretized spatial coordinates \( x_1, x_2, x_3 \), we can rewrite Eq. (2-11) as,

\[
E_p^{(0)}(I, J, K) = M_{pq}^{-1}(I, J, K)E_q^{(1)}(I, J, K),
\]

(2-16)

where the inverse of the \( 3 \times 3 \) matrix \( M_{pq} \) is obtained as

\[
M_{pq}^{-1}(I, J, K) = \frac{\varepsilon_{qkl} \varepsilon_{pmn} M_{kn}(I, J, K) M_{ln}(I, J, K)}{\varepsilon_{ijk} \varepsilon_{lmn} M_{il}(I, J, K) M_{jm}(I, J, K) M_{kn}(I, J, K)}.
\]

(2-17)

where \( \varepsilon_{ijk} \) is Levi-Civita tensor, and

\[
M_{kr}(I, J, K) = \delta_{kr} - K_{kr}(I, J, K)
\]

(2-18)

with

\[
K_{kr}(I, J, K) = \sum_{I'=1}^{N_I} \sum_{J'=1}^{N_J} \sum_{K'=1}^{N_K} G_{kr}(I - I', J - J', K, K') \chi(I', J', K') \Delta V.
\]

(2-19)

The details of the discretization are specified in Zwamborn and van den Berg 1992 and are not important here. Finally Eq. (2-15) is given by,

\[
\]

(2-20)

From Eq. (2-20) it can be seen that the iteration is a point-wise product of the \( 3 \times 3 \) matrix \( K_{kr} \) and the previous estimate of the total electric field, while \( K_{kr} \) has already been computed to obtain the initial estimate of Eq. (2-16).
2-4 Numerical results

2-4-1 Accuracy of the iterative extended Born approximation and the number of iterations

We use the configurations depicted in Figure 2-2 for the three-dimensional numerical examples.

Figure 2-2 shows a layered earth of air, sea and ground with a reservoir model. The background conductivity in the ground is 1 S/m and the reservoir’s conductivity is 0.02 S/m. Air and sea have conductivity of 0 S/m and 3 S/m, respectively. The source is located above the center of the reservoir and situated 25 m above the sea bottom, whereas the receivers are spread in the area of $8 \times 16$ km$^2$.

Figure 2-2: Diffusive fields present in a three media configuration with one reservoir.

The water depth is 1 km and the reservoir is located at the depth of 1 km below the sea bed. The dimensions of the reservoir are $4000 \times 2000 \times 250$ m$^3$. A single frequency of 1 Hz is used in this example.

Here we investigate the accuracy of the approximations at the receiver level. Later we investigate the accuracy of the method at the reservoir level, where we need high accuracy when we want to use the code for inverse modelling.

In all the examples presented in this study, we only show the horizontal electric field components. Figure 2-3 shows the comparison of the full solution of conjugate gradient fast Fourier transform (CG-FFT), the Born approximation and the iterative extended Born approximation, together with the horizontal electric field in absence of the reservoir, for the configuration depicted in Figure 2-2. In Figure 2-3 the cross plots are along the source in the $x_1$-direction. We have used 800 receivers along the $x_1$-axis with a spacing of 20 m, but we only show the stretch of 800 m with maximum effect of the reservoir on the data. It can be seen that the iterative extended Born approximation (IEBA) agrees well with the CG-FFT method.

Now we zoom in to investigate the accuracy of the iterative extended Born approximation in more detail and analyze the number of required iterations. Figure 2-4 shows a detailed
Figure 2-3: 3D total electric field at the receiver level while we have one assumed reservoir and source is located 25 m above the sea bed in the middle of the reservoir at top. Full solution of CG-FFT is compared with Born approximation, iterative extended Born approximation and the case we have no reservoir.

The green solid line shows the Born approximation result. The black dashed line shows the extended Born approximation result that is more accurate than the Born approximation. Results from the iterative extended Born approximation are shown as a purple solid line using 1 iteration, blue dashed line with 2 iterations and cyan solid line with 7 iterations. Table I shows the average errors in percentage for the different approximations relative to the CG-FFT method results, depicted in Figure 2-4. The errors in the total fields are small, because the reservoir response is small compared to the background response.

At 7 iterations the result saturates and remains constant when the number of iterations is increased. It means there is no improvement in accuracy. As we can see, the iterative method can improve the results, without significantly increasing the computation time as explained in the previous section. In this example we discretized the reservoir by $128 \times 32 \times 8$ points, for such a number of points the CG-FFT method takes 311 s to be computed using Matlab programming language, whereas EBA and 7 iterations of IEBA take 19 s and 84 s.
(a) Full solution of CG-FFT is compared with Born approximation, extended Born approximation and iterative extended Born approximation with different iterations.

(b) Full solution of CG-FFT is compared with extended Born approximation and iterative extended Born approximation with different iterations in more details.

Figure 2-4: 3D total electric field at the receiver level while we have one assumed reservoir and source is located 25 m above the sea bed in the middle of the reservoir at top.
2-4 Numerical results

<table>
<thead>
<tr>
<th></th>
<th>BA</th>
<th>EBA</th>
<th>IEBA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Error $E_1^t$ [%]</td>
<td>35</td>
<td>12</td>
<td>8.7</td>
</tr>
<tr>
<td>Error $E_1^{sc}$ [%]</td>
<td>86</td>
<td>29.6</td>
<td>21</td>
</tr>
</tbody>
</table>

Table 2-1: Average error in percentage for approximations Vs. full solution for the total field and the scattered field. IEBA has been done by 7 iterations.

to be computed, respectively, on a laptop with 2.80 GHz dual core and 3.48 GB of RAM. If we increase the number of points, we will see a larger difference in computation time between the CG-FFT method and the approximations.

2-4-2 Sensitivity of the iterative extended Born approximation to the source position and the reservoir configuration

In Figure 2-5 we show another example, similar to the previous one but this time two separated reservoirs are modelled instead of one. It is worth noting that this example can be considered as a case with a reservoir with separated compartments.

The dimensions of each of the reservoirs are $1650 \times 2000 \times 250$ m$^3$. They are located at the same depth level as in the previous example and have a distance of 700 m. The same configuration and parameters of the last example are applied for this case.

This example lets us evaluate the accuracy of the iterative extended Born approximation for different sizes of the reservoir in comparison with the previous example in section 3.1. Furthermore, we can evaluate the accuracy of the iterative extended Born approximation when we have several resistors close to each other.

In Figure 2-6 we evaluate and validate the accuracy of the iterative extended Born approximation with two different reservoirs, as depicted in Figure 2-5. In this plot the results come from the case where the source is located 25 m above the sea bed centered...
between the two reservoirs. We can compare the responses of the horizontal electric field at the receiver level with the situation when there is no reservoir. The iterative extended Born approximation result agrees very well with the full solution, even better than in the case shown in Figure 2-3. This occurs because now the reservoir is smaller.

![Figure 2-6: 3D total electric field at the receiver level while we have two assumed reservoirs and source is located 25 m above the sea bed in the middle of two reservoirs at top. Full solution of CG-FFT is compared with Born approximation, iterative extended Born approximation and the case we have no reservoir.](image)

With Figures 2-7a, 2-7b, 2-8a and 2-8b we aim to show the sensitivity of the method for both aforementioned examples as a function of the horizontal source position along the $x_1$-axis. Figure 2-7a shows the scattered 3D electric field at the receiver level at the right-hand side of the source. In this case we have one reservoir and the source is again located 25 m above the sea bed in the middle of the reservoir. The iterative extended Born approximation result (red curve) is compared with the electric field in absence of the reservoir (dashed blue curve).

We show the schematic configuration of the sea surface, sea bed, location of the source and the reservoir as a box in the upper right corner of Figure 2-7a. To have an idea regarding the sensitivity of the method in terms of different source positions and different reservoir configurations, we compare different reservoir responses. In Figure 2-7a the biggest difference in amplitude between two reservoir response curves, is 0.25 on a logarithmic scale.
(a) One single reservoir; source is located 25 m above the sea bed at top of the middle of the reservoir.

(b) One single reservoir; source is located 25 m above the sea bed at top of the left edge of the reservoir.

**Figure 2-7:** Iterative extended Born approximation is compared with electric field in absence of the reservoir for the 3D electric scattered field at the receiver level.
(a) Two adjacent reservoirs; source is located 25 m above the sea bed at top of the middle of the reservoirs.

(b) Two adjacent reservoirs; source is located 25 m above the sea bed at top of the left edge of the left reservoir.

Figure 2-8: Iterative extended Born approximation is compared with electric field in absence of the reservoir for the 3D electric scattered field at the receiver level.
Figure 2-7b shows the 3D scattered electric field at the receiver level with a single reservoir and the source is located 25 m above the sea bed above the left edge of the reservoir, which is 2 km away from the center of the configuration at the left-hand side. The iterative extended Born approximation result is compared with the electric field in absence of the reservoir. In this case the biggest reservoir response has 0.45 amplitude difference on logarithmic scale.

Figure 2-8a shows the 3D scattered electric field at the receiver level with two separated reservoirs and the source is located 25 m above the sea bed above the middle of the reservoirs. The iterative extended Born approximation result is compared with electric field in absence of the reservoir and the biggest reservoir response has 0.21 amplitude difference on logarithmic scale.

Figure 2-8b shows the 3D scattered electric field at the receiver level with two separated reservoirs and the source located 25 m above the sea bed at the left edge of the left reservoir, which means 2 km away from the center of the configuration at the left-hand side. The iterative extended Born approximation result is compared with the electric field in absence of the reservoir and the biggest reservoir response has 0.23 amplitude difference on logarithmic scale.

It can be seen in Figure 2-7b where we have the source at the edge of the reservoir we have more response compared to the others in Figures 2-7a, 2-8a and 2-8b.

To have more detail the configuration is shown in Figure 2-9. Figure 2-9a shows that when the source is located at the left edge of one big reservoir we obtain a stronger response, than if the source is located above the middle of the reservoir, as can be seen in Figure 2-9b. In case of two reservoirs when the source is in zero offset we obtain the minimum response (Fig. 2-9d).

Responses in the configuration shown in Figure 2-9c give a bit higher magnitude in comparison with the configuration in Figure 2-9d because the edge effect enhance the reflection return of the reservoir, while the direct incident field is weaker at large offsets. However, the responses are less strong than the cases of having a single big reservoir. We can observe that the accuracy of the IEBA decreases with increasing the size of the reservoir. Table II shows different amplitudes of IEBA response with no reservoir response for different source positions.

<table>
<thead>
<tr>
<th></th>
<th>IEBA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>One reservoir</td>
</tr>
<tr>
<td></td>
<td>Source at top</td>
</tr>
<tr>
<td>Amp. diff. with no reservoir</td>
<td>0.25</td>
</tr>
</tbody>
</table>

**Table 2-2**: Different source position response of the reservoir compared with the case of no reservoir for IEBA
3D Modelling and Approximations

(a) One single reservoir; source is located 25 m above the sea bed at top of the left edge of the reservoir.

(b) One single reservoir; source is located 25 m above the sea bed at top of the middle of the reservoir.

(c) Two adjacent reservoirs; source is located 25 m above the sea bed at top of the left edge of the left reservoir.

(d) Two adjacent reservoirs; source is located 25 m above the sea bed at top of the middle of the reservoir.

Figure 2-9: Iterative extended Born approximation for 3D electric scattered field at the receiver level is compared with electric field in absence of the reservoir in more details.

2-4-3 Accuracy of extended Born approximation and iterative extended Born approximation at the reservoir level

Up to now the accuracy of the iterative extended Born approximation has been demonstrated at the receiver level. In order to use the method for inverse modelling, the accuracy must be high at the reservoir level as well. The extended Born approximation
2-4 Numerical results

(a) Scattered field responses of full solution and approximation at the reservoir level for one single reservoir.

(b) CG-FFT vs. IEBA at the reservoir level.

Figure 2-10: Scattered field at the reservoir level and its error in percentage approximated by IEBA.
has already been proposed and successfully used in the inverse problems for buried objects Torres-Verdin and Habashy (2001). Also high order solutions have been implemented successfully for low frequency inversion of 3D buried objects Cui et al. (2006). Now we investigate the accuracy of the iterative extended Born approximation. In Figures 2-10a and 2-10b we can see that IEBA gives accurate results at the reservoir level along the $x_1$-axis, at $x_2 = 0$, and $x_3$ is 1150 m below the sea floor.

Figure 2-10a shows the scattered field responses of the full solution and the approximation at the reservoir level for one single reservoir.

In Figure 2-10b we can see the normalized difference in percent; between the CG-FFT result and the iterative extended Born approximation result along the in-line shown in Figure 2-10a. The normalized error is on average less than four percent, while the maximum error is less than ten percent. From these results we conclude that the IEBA method can be used for inverse modelling.

2-5 Conclusions

We have proposed an iterative extended Born approximation (IEBA) to compute 3D diffusive electromagnetic field configurations. The method is based on the integral equation formulation. We have shown that the iterative extended Born approximation method gives better results than the extended Born approximation, also when we have two adjacent scattering objects. The improved accuracy comes at virtually no additional computational cost. With IEBA we seem to have reduced one of the major problems in three-dimensional diffusive electromagnetic modelling, which is the high cost of computation time. With the help of this method sensitivity analysis, which is vital for controlled-source electromagnetic application, is possible in a timely manner using 3D modelling for simple background configurations. We have used the advantage of computational speed to do sensitivity analysis as a function of the source position and different reservoir sizes that validated the accuracy of the IEBA. We have also shown that this method can be a good candidate for inverse modelling, because it gives quite accurate electric field results inside the reservoir that will allow for accurate conductivity estimations inside the reservoir.
Chapter 3

Applicability of 1D and 2.5D mCSEM modelling

ABSTRACT

We present two-and-a-half dimensional (2.5D) and three-dimensional (3D) integral equation modelling of the marine controlled-source electromagnetic method. We apply the method to a synthetic model in the marine controlled-source electromagnetic exploration situation where conductivity is different from the known background medium.

Two-and-a-half dimensional modelling is implemented using a point source and a two-dimensional reservoir, and the results are compared with point source responses from one-dimensional and three-dimensional reservoir models. These methods are based on an electric field domain integral equation formulation. It is shown how the 2.5D method performs, in terms of both accuracy and speed with different configurations.

We compare the results from 1D, 2.5D and 3D modelling, for a symmetrically placed reservoir and the in-line acquisition configuration, as a function of different reservoir sizes in the cross-line direction, thickness, and for different frequencies and depths. Depending on the model’s parameters 2.5D modelling can be considered as an accurate and fast method for marine controlled-source electromagnetic acquisition optimization and interpretation. The biggest amplitude difference between 2.5D and 3D models occur when the source is located in the middle above the reservoir. They are less than 10% if the thickness of the reservoir is one fifth of skin depth of the embedding or less, and also if depth of the reservoir is two times the skin depth of the embedding or more. In this chapter supportive examples with different configurations are discussed, where the 2.5D results lead to an optimistic detection estimate. Phase differences are even smaller and the 2.5D solution can be used to assess the ability to detect the reservoir with a given acquisition configuration.
1 Introduction

Since the last few years we have seen a rapid development in modelling capabilities with different approaches in the field of controlled-source electromagnetic (CSEM) methods in general, and for seabed logging in particular. In the seventies and eighties of the previous century a large volume of modelling papers have been published, covering 1D, 2D, 2.5D and 3D implementations of both local and global methods. An early example is Chave and Cox (1982) who analyzed the 1D method. Stoyer and Greenfield (1976) who presented a 2.5D finite-difference formulation, while Lee (1978) used a finite-element method to solve 2.5D problem. For three-dimensional problems the early example is Raiche (1974), Hohmann (1975) and Weidelt (1975) who all used integral equation formulations. Local methods in the seventies modeled 3D structures for MT purposes and one of the first finite difference solution was presented by Zhdanov et al. (1982). Most of these methods were developed for finding conductors in a relatively resistive embedding, today we are also interested in thin resistive bodies in a conductive embedding.

For this reason there are some recent examples of using 2.5D integral equation modelling (Abubakar et al., 2006), finite-element CSEM modelling (Li and Key (2007) and Kong et al. (2008)), and finite-difference CSEM modelling (Abubakar et al., 2008).

We focus here on the question in what model configurations 2.5D modelling is a good choice for modelling a real reservoir. We use integral equation methods for 1D, 2.5D and 3D CSEM modelling. A three-dimensional integral equation is easily transformed to a 2.5D formulation by assuming the scattering object is of infinite extent in the cross-line direction. In the plane containing the vertical axis and the line of acquisition, the problem can be solved for a small number of different cross-line wave numbers and using FFT to map the solution to the desired plane. We investigate a 2D reservoir type scattering object for which the plane containing the vertical axis and the line of acquisition is the plane of symmetry. For such a configuration we expect the best results compared to the full three-dimensional solution.

To run many forward models, in practice one-dimensional solution is one of the standard methods (Løseth and Ursin, 2007; Morris, 2008; Chave, 2009) in application of CSEM. Particularly for sensitivity analysis for the optimization of acquisition configuration and parameters, and also determining the optimum frequency range, because three-dimensional modelling can be expensive. In spite of the fact that there are some approximations such as extended Born approximation (Habashy et al., 1993), Quasi-linear approximation (Zhdanov and Fang, 1996), Quasi-analytical approximation (Zhdanov et al., 2000), high-order extended Born approximation (Cui et al., 2004) and the iterative extended Born approximation (Tehrani and Slob, 2010) that speed up three-dimensional CSEM modelling, there are other options, such as 2.5D model approximation. Working with 2.5D modelling methods, is much faster and, depending on configuration, almost as accurate as three-dimensional modelling. This can be useful when we would like to carry out acquisition parameter optimization for reservoir detection before planning an actual
survey. Comparing 2.5D modelling results with 3D modelling results in terms of different reservoir sizes and depths, and for different frequencies can be a guide for interpreters to predict and understand the responses of these different modelling results for a source along the receiver line. Results from modelling the reservoir as a horizontal layer (1D) are included in the examples for illustration.

2 Theory

We investigate when a reservoir can be modeled as a volume of finite thickness (1D model), as a volume of two finite dimensions (2D model) or as a three-dimensional volume. We compare three-dimensional modelling results to similar results using a layered code and a 2.5D code. In all models we use the same acquisition configuration for the sources and receivers. We show the three different model configurations in Figure 3-1. For all models a single line of data is acquired with an in-line horizontal electric dipole source in the ocean and the resulting in-line electric field is measured by receivers located on the ocean bottom. The background model consists of three layers; the upper half-space is air, the sea is modelled as a layer of finite thickness, and ground is modeled as a homogeneous half-space. A high-resistivity volume is located in the lower half-space. This high-resistivity body is a 3D volume, as shown in Figure 3-1(a). We model this configuration also with a 2D volume, which is the same as the three-dimensional volume in in-line direction and depth, but with infinite length in the cross-line direction, as shown in Figure 3-1(b). Considering both horizontal dimensions as infinite a 1D volume, or horizontal layer, of finite thickness results, which is depicted in Figure 3-1(c).

The models for 1D and 3D scattering objects are well-described in the literature, for example Constable and Weiss (2006) and Weiss and Constable (2006). For 2D models in configurations with 3D sources and receivers we briefly describe our numerical algorithm. We describe all equations in the frequency-domain and use subscript notation. The summation convention applies to repeated lower case Latin subscripts, which take on the values 1, 2, and 3. Whenever convenient we use underline to indicate vectors.

The total electric field vector $\mathbf{E}_t(x, \omega)$ is decomposed in the incident field vector $\mathbf{E}_i(x, \omega)$ and the scattered field vector $\mathbf{E}_s(x, \omega)$, in which $\omega$ is frequency. We can then describe the total electric field as the sum of these two. In 2.5D modelling we reduce dimensions of the target to 2D and let the source and receivers remain in 3D. In order to do that we convert the cross-line dimension $x_2$ from space domain to wave number domain $k_2$ and then compute the field for a small number of $k_2$ values. Once the scattered and incident fields in the receivers are known for all required $k_2$ values we sum over $k_2$ to map the fields to the receiver line at $x_2 = 0$. The electric field integral equation can be written as,
Applicability of 1D and 2.5D mCSEM modelling

(a) 3D reservoir as an assumed scatterer.

(b) 2D reservoir as an assumed scatterer.

(c) 1D reservoir as an assumed scatterer.

Figure 3-1: Schematic diagram of Diffusive fields present in a three horizontally stratified earth layer configuration for both forward source and scattering problems.
\[ \tilde{E}_k^s(x_1, k_2, x_3, \omega) = \tilde{E}_k(x_1, k_2, x_3, \omega) - \int_{\mathbb{D}_{sc}} \chi^s(x'_1, x'_3) \times \tilde{G}_{kr}^E(x_1 - x'_1, k_2, x_3, x'_3, \omega) \tilde{E}_r(x'_1, k_2, x'_3, \omega) dx'_1 dx'_3, \]  

where \( x_1 \) and \( x_3 \) are the locations where the total electric field has to be determined and \( x'_1 \) and \( x'_3 \) the secondary source locations in the cross-sectional plane. \( G_{kr}^{EJ}(x_1 - x'_1, k_2, x_3, x'_3, \omega) \) denotes the Green’s function for electric field generated by an electric current for a 1D background medium. The subscripts denote the vector components of the electric field according to the summation convention. The conductivity contrast function is given by,

\[ \chi^s(x'_1, x'_3) = \sigma^{sc}(x'_1, x'_3) - \sigma(x'_1, x'_3). \]  

Let \( \mathbb{D}^{sc} \) be the bounded domain occupied by the scatterer and let \( \sigma^{sc}(x) \) be its conductivity. The embedding exterior to \( \mathbb{D}^{sc} \) is denoted \( \mathbb{D}_{sc}^{c} \) and has a conductivity \( \sigma(x) \). \( \mathbb{D}_1 \) indicates the sea layer.

Equation (3-1) must be solved for the total electric field inside \( \mathbb{D}^{sc} \). When the total electric field is known inside the reservoir, the scattered electric field can be computed at the receivers \( (x^R_R) \) located on the ocean bottom.

\[ \hat{E}^{sc}_k(x^R_1, k_2, x^R_3, \omega) = \int_{\mathbb{D}_{sc}} \chi^s(x'_1, x'_3) \tilde{G}_{kr}^E(x_1^R - x'_1, k_2, x_3^R, x'_3, \omega) \tilde{E}_r(x'_1, k_2, x'_3, \omega) dx'_1 dx'_3, \]  

where \( x^R_R \in \mathbb{D}_1 \). This is done for each \( k_2 \) value separately. After this step the scattered field at the receiver line can be computed for each position by summing over all \( k_2 \) values from

\[ \hat{E}^{sc}_k(x^R_1, x^R_2, x^R_3, \omega) = \frac{1}{2\pi} \int_{k_2 = -\infty}^{\infty} e^{-ik_2x^R_2} \hat{E}^{sc}_k(x^R_1, k_2, x^R_3, \omega) dk_2 \]  

which then solves the 2.5D problem. In order to sum over \( k_2 \) we do inverse Fourier transformation from \( x^R_2 = 0 \). Only positive \( k_2 \) values are considered. For non-zero \( x_2 \), the sum can be taken including the Fourier kernel for the specific values of \( x_2 \).

Because of the nonlinear behavior of the field around the target all the computations are carried out with logarithmic spacing for \( k_2 \) (Mitsuhata, 2000). The map to \( x_2 = 0 \) is obtained using cubic hermite interpolation (Fritsch and Carlson, 1980) to the linear scale and summing the results, similar to mapping frequency-domain results to the time-domain as shown in Mulder et al. (2008). We need a few \( k_2 \) values depending on the configuration to solve the 2.5D problem, typically the number of points varies between 10 to 20 values, also
Figure 3-2: A test model in the layered media in which the source is 25 m above the seabed and the water depth is 1000 m. The resistivity of the scatterer is 0.02 S/m which is located 1000 m below seabed, with the size of 4000×2000×250 m.

shown in Mitsuhata (2000) and Abubakar et al. (2006). The discretization used is similar to the one adopted in Zwamborn and van den Berg (1992).

Based on above theory and with the usage of different model configurations we will investigate the applicability of 2.5D modelling through numerical examples described in the next section.

3 Numerical results

As a numerical example, three different configurations of the layered earth with an assumed reservoir is considered. The first contains a three-dimensional reservoir (Fig. 3-1(a)), another one contains a two-dimensional reservoir (Fig. 3-1(b)) and the third incorporates a one-dimensional reservoir (Fig. 3-1(c)).

It is worth noting that these 1D, 2.5D and 3D integral equation codes were compared with known multigrid code introduced by Mulder (2006) and the results from both codes differ less than 1%.

For the sensitivity analysis we first assign a standard model configuration as a reference model. Then changes will be initiated to the model parameters for further analysis. Given the simple model configuration, we use a reservoir that is not very large in terms of its size and also in terms of its conductivity contrast to the surroundings, and with moderate depth, so that the result can be useful for more practical configurations. As a reference for the three-dimensional model, the configuration depicted in Figure 3-2 is used.

Figure 3-2 shows the layered earth with a reservoir. The background conductivity is 1 S/m and the reservoir’s conductivity is 0.02 S/m. Air and sea have conductivity of 0 S/m and 3 S/m, respectively. The source is located at 25 m above the sea bottom centrally positioned above the reservoir, and receivers are spread symmetrically in the area of 16
The water depth is 1000 m and the top of the reservoir is located at 1000 m below the sea bottom. The dimensions of the reservoir are $4000 \times 2000 \times 250$ m$^3$. We use a single frequency of 1 Hz in this example. We note that higher frequencies will give a stronger relative reservoir response, but these higher frequencies cannot be used for deep targets. For the 2.5D configuration all dimensions are the same as in the 3D configuration, except for the cross-line dimension of the reservoir, which is extended from 2 km in the 3D problem to infinity in the 2.5D problem.

In these examples 3D reservoir is discretized by $128 \times 64 \times 8$ points. The 3D code is written in Matlab and Fortran and the computation time is 98.8 s to compute the scattered field at the receivers for the 3D reservoir. The 2D reservoir is discretized in $128 \times 8$ points and 16 $k$-values are used to compute the field at $x_2 = 0$. The 2.5D modelling code takes 2.8 s to compute the scattered field at the receiver line using only Matlab code. All computations are done on a standard laptop computer with a 2.80 GHz dual core CPU and 3.48 GB of RAM.

### 3.1 Comparison of 2.5D modelling results with 1D and 3D modelling results

In this section we would like to find out how close the 2.5D model results are relative to the 3D model results at the receivers on the sea bottom.

![Figure 3-3: 3D electric scattered field at the receiver level while we have an assumed reservoir is compared with 2.5D modelling of the electric scattered filed at the same configuration.](image-url)
In Figure 3-3 the 3D scattered electric field at the receiver level (blue curve) is compared with 2.5D scattered electric field (red curve) at the same locations. From this result we can observe that the 2.5D modelling result is close to the result of 3D modelling, although there are some small differences in the two response curves.

Figure 3-4: The electric fields are shown above where the blue curve shows the received signal in absence of the reservoir, green one is the horizontal electric field response of the 1D reservoir, red curve response of the 3D modelling and black curve shows the result of 2.5D modelling which are all compared with the others.

This is of interest for studying acquisition optimization because 2.5D modelling is much faster than 3D modelling. Only when cross-line side effects of the reservoir become of interest in model studies, we should use the 3D model.

Figure 3-4 shows the comparison of the total electric field response from 3D, 2D and 1D.
reservoirs, and without reservoir, for an electric dipole centered over the 3D reservoir and when the in-line component of the electric field is measured. By comparing Figures 3-4(a) and 3-4(b) it can be seen that for the 3D reservoir model there is not much qualitative difference between the responses from a 2D reservoir and the 3D reservoir. From Figure 3-4(c) it can be seen that the 2D reservoir response is only slightly larger than the 3D reservoir response in the offset range where the reservoir can be detected. In this example Figure 3-4(d) shows that a 1D reservoir has a much different response than the 2D and 3D reservoirs for offsets larger than 4 km, but still acceptable within the offset range of interest however it is not as close as 2.5D reservoir response to the 3D reservoir response.

In the later modelling study we will investigate that in which offsets 1D reservoir response can be closer to 3D reservoir response than 2.5D reservoir response. In general 1D model seems less attractive than the 2D model to be used for a 3D model, if the receiver line is the plane of the symmetry, perpendicular to the middle of the reservoir. Therefore 2.5D modelling is of interest for studying acquisition optimization because it is much faster than 3D modelling. Only when cross-line side effects of the reservoir become of interest in model studies, we should use the 3D model.

![Normalized amplitude and phase difference of the total horizontal electric field](image)

**Figure 3-5:** Normalized amplitude and phase difference of the total horizontal electric field regarding 3D modelling, 2.5D modelling and 1D modelling.

To compare the differences in the responses of 1D, 2D and 3D reservoirs as shown in Figure 3-4, we show the logarithm of the amplitude responses of the total horizontal electric field of these three reservoirs normalized by the response without reservoir, as a function of source-receiver offset in Figure 3-5(a). The red curve represents the normalized amplitude of the 2D reservoir (hence from the 2.5D model), which is very similar to the response of the 3D reservoir shown in blue and at the worst case it has less that 20% amplitude difference. Although the 2D reservoir response is bigger than 3D reservoir response due to
infinite extension in the cross-line direction in 2.5D modelling, they are quite close to each other with small amplitude difference. Therefore in this example 2.5D modelling is a good substitute for 3D modelling. The 1D response is not in agreement, specially from 4 km offset as shown by the green curve. In Figure 3-5(b) the phase angle of the normalized 3D, 2D and 1D reservoirs responses are shown. This is equal to the phase difference between the response with reservoir and the response without reservoir. The phase angle for 3D modelling and 2.5D modelling shows a very close match (at the worst case less that 10\% phase difference), while the 1D modelling result starts to be completely different from 3 km offset onwards.

In CSEM data interpretation the phase attribute can play an important role since the information provided by phase can give more details about the target and therefore improve our geological model (Orange et al., 2009). Figure 3-6 shows the phases of the in-line horizontal total electric field for the model with 3D reservoir, 2D reservoir, and without reservoir. Since phase is changing between $-\pi$ and $+\pi$ we have phase wrapping. The phase is unwrapped in Figure 3-6. At the edges of the reservoir, at 2 km offsets, the slope of the phase changes for both 3D (dashed blue line) and 2.5D (solid red line) modelling compared to the response in absence of the reservoir (dashed black line).

![Figure 3-6: In-line total electric field phase, recorded at the sea bottom, for 2D reservoir, 3D reservoir and without reservoir. The phase is unwrapped.](image)

Now we would like to analyze the sensitivity of 1D, 2.5D and 3D modelling for two single frequencies (0.2 and 1 Hz) in different offsets in terms of amplitude versus offset, normalized amplitude and phase difference.
3 Numerical results

Figure 3-7(a) shows amplitude versus offset for single frequency of 1 Hz when the source is located above the middle of the reservoir. In Figures 3-7(b) and 3-7(c) normalized amplitude and phase difference are shown. Now the question is what will be the responses when the source is in different locations? Figures 3-8, 3-9, 3-10 and 3-11 show the results when the source is moved to the left hand side of the reservoir at 1 km, 2 km, 3 km and 4 km away from the middle, respectively.

We realized that when the source is located at -1 km up to -3 km away from the middle of the reservoir (Figs. 3-8, 3-9, 3-10), in the interesting part of the signal where the amplitude is above noise level \(10^{-14} \text{VA}^{-1} \text{m}^{-2}\), 1D can be closer than 2.5D, specifically at the edge of the reservoir. For instance at 3 km horizontal in-line distance to middle of the reservoir when the source offset is -2 km (Fig. 3-9(b)), normalized amplitude difference between 1D reservoir and 3D reservoir is 2.2 that is smaller than the difference between 2D reservoir and 3D reservoir which is 2.8. But in other cases when the source offset is 0 km (Fig. 3-7(b)) or -4 km (Fig. 3-11(b)), normalized amplitude difference between 1D reservoir and 3D reservoir is 1.1 and 2.9 respectively and normalized amplitude difference between 2D reservoir and 3D reservoir is 1.05 and 1 respectively. In near offset and far offset (Figs 3-7 and 3-11) 2.5D is always better. This is due to the surface charge when the current density is normal to the interface separating two media which is the cross-line edge of the reservoir. Having an infinite edge surface for 2D reservoir gives a huge surface charge. In Fig. 3-18 we also observe that how small is the difference between 2D and 3D reservoir when the diffusion field is along the in-line edge of the reservoir.

On the other hand when the source is directly over the edge of the 2D body, the galvanic surface charge effect of the infinite edge of the reservoir adds strongly to the observed response, giving a much larger amplitude to the 2D response than the 3D and 1D models. That this is a galvanic effect is supported by the observation that it only affects the 2D amplitude response and not the phase. When the source is positioned away from the edge of the reservoir, the galvanic effect of the lateral 2D edge is diminished due to its scaling with distance from the edge, hence the 2D response is closer to the 3D response.

Phase angles from 2.5D modelling are much better than the one from 1D modelling at all offsets (Figs. 3-7(c), 3-8(c), 3-9(c), 3-10(c)).

In the next step we did the same comparison, but this time with single frequency of 0.2 Hz (Figs. 3-12, 3-13, 3-14, 3-15 and 3-16). In this case 2.5D modelling gives better results and performs mostly better than 1D modelling, however again at the cross-line edge 1D modelling can be better, for the receivers located at edge and up to about 2 km away from the edge (Fig. 3-14).

Also in Figure 3-15 where source is located at -3 km away from the middle of the reservoir we can notice, we have more accurate results from 1D modelling in the receivers located at the edge up to about 1.5 km away from the edge of the reservoir.

Using 0.2 Hz frequency gives less accurate phase attribute but in general phase attribute is giving good results in 2.5D modelling, which indicates phase angle can be an accurate tool, while using 2.5D modelling with different frequencies and for different source locations.
The main difference between 2D reservoir and 3D reservoir for a source above the edge of the reservoir is in fact that 2D reservoir has infinite long edge in cross-line direction. By decreasing the frequency the thickness of the reservoir effectively decreases that also reduces the effective area. For the same reason, at fixed frequency the same behaviour is seen for a physically thinner reservoir.

Figure 3-17 shows that if we have a thinner reservoir, in this case 100 m thickness, even when the source is located at the edge of the reservoir and a single frequency of 1 Hz is used, amplitude (Fig. 3-17(a)), normalized amplitude (Fig. 3-17(b)) and phase difference (Fig. 3-17(c)) of 2D reservoir, are getting more closer to 3D reservoir. For even more realistic example we considered a reservoir with 20 m thickness that corroborated our conclusions, for which reason these results are not shown. For the relatively thin resistors expected of most hydrocarbon reservoirs (<100 m), the difference between 2.5D and 3D responses is almost insignificant.

The effect of having a receiver line at the in-line edge of the reservoir instead of over the middle is shown in Figure 3-18, that in comparison with Figure 3-7(a), in both, source is located above the middle of the reservoir but in figure 3-18 receivers are along the inline edge and not the crossline edge. For an \( x_1 \)-directed source and computing the \( x_1 \)-directed electric field at the receivers, the receivers are above the middle of the reservoir (Fig 3-7(a)) or above the \( x_1 \)-directed (inline) edge (Fig 3-18) for which the \( x \)-component of the electric field behaves smoothly. This means we do not have the big effect of the component of the field normal to the edge. But instead we have \( y \)-component. For the \( x_1 \)-directed source above the \( y \)-directed (crossline) edge, the strongest electric field component, the \( x \)-component, is normal to the edge leading to large differences between the electric fields for a source above the middle of the reservoir and a source above the crossline edge. Because of the relative large crossline offset at near inline offsets we can see bigger differences at near inline offsets in figure 3-18, compared to figure 3-7(a). For large inline offset, the fixed crossline offset becomes smaller and we see less differences.
Figure 3-7: (a) Amplitude versus offset for 1D, 2D, 3D and when there is no reservoir, (b) Anomalous responses for the 1D, 2D and 3D reservoirs normalized by the no-reservoir response, (c) Phase difference. Frequency is 1 Hz.

Figure 3-8: (a) Amplitude versus offset for 1D, 2D, 3D and when there is no reservoir, (b) Anomalous responses for the 1D, 2D and 3D reservoirs normalized by the no-reservoir response, (c) Phase difference. Frequency is 1 Hz.
Figure 3-9: (a) Amplitude versus offset for 1D, 2D, 3D and when there is no reservoir, (b) Anomalous responses for the 1D, 2D and 3D reservoirs normalized by the no-reservoir response, (c) Phase difference. Frequency is 1 Hz.

Figure 3-10: (a) Amplitude versus offset for 1D, 2D, 3D and when there is no reservoir, (b) Anomalous responses for the 1D, 2D and 3D reservoirs normalized by the no-reservoir response, (c) Phase difference. Frequency is 1 Hz.
**Figure 3-11:** (a) Amplitude versus offset for 1D, 2D, 3D and when there is no reservoir, (b) Anomalous responses for the 1D, 2D and 3D reservoirs normalized by the no-reservoir response, (c) Phase difference. Frequency is 1 Hz.

**Figure 3-12:** (a) Amplitude versus offset for 1D, 2D, 3D and when there is no reservoir, (b) Anomalous responses for the 1D, 2D and 3D reservoirs normalized by the no-reservoir response, (c) Phase difference. Frequency is 0.2 Hz.
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**Figure 3-13:** (a) Amplitude versus offset for 1D, 2D, 3D and when there is no reservoir, (b) Anomalous responses for the 1D, 2D and 3D reservoirs normalized by the no-reservoir response, (c) Phase difference. Frequency is 0.2 Hz.

**Figure 3-14:** (a) Amplitude versus offset for 1D, 2D, 3D and when there is no reservoir, (b) Anomalous responses for the 1D, 2D and 3D reservoirs normalized by the no-reservoir response, (c) Phase difference. Frequency is 0.2 Hz.
3 Numerical results

Figure 3-15: (a) Amplitude versus offset for 1D, 2D, 3D and when there is no reservoir, (b) Anomalous responses for the 1D, 2D and 3D reservoirs normalized by the no-reservoir response, (c) Phase difference. Frequency is 0.2 Hz.

Figure 3-16: (a) Amplitude versus offset for 1D, 2D, 3D and when there is no reservoir, (b) Anomalous responses for the 1D, 2D and 3D reservoirs normalized by the no-reservoir response, (c) Phase difference. Frequency is 0.2 Hz.
Figure 3-17: (a) Amplitude versus offset for 1D, 2D, 3D and when there is no reservoir, in this case the thickness of the reservoir is 100 m, (b) Anomalous responses for the 1D, 2D and 3D reservoirs normalized by the no-reservoir response, (c) Phase difference. Frequency is 1 Hz.

Figure 3-18: Anomalous responses for the 1D, 2D and 3D reservoirs normalized by the no-reservoir response, while source is located in 0 offset and receiver line are along in-line edge of the reservoir. Frequency is 1 Hz.
3.2 Sensitivity analysis of 2.5D CSEM modelling

Here we investigate how the accuracy of 2.5D modelling depends on the configurations. It is shown that in some scenarios it is quite reasonable to use 2.5D modelling instead of 3D modelling. However the differences between 2.5D model responses and 3D model responses can be increased in some other cases.

In practice, working with 1D, 2.5D, or 3D modelling, no matter if it is used for feasibility study, survey planning, or interpretation, can be more efficient and successful if the knowledge about the limitations and strength of the methods are available.

The numerical examples are done using simple models and configurations to understand the fundamental behaviour of these modelling methods, for instance how the electric field is changing with frequency, size, depth and thickness. Although in more complex scenarios, different responses for the electric field are expected, the same fundamental behaviour is also expected to be conformed.

For example, in spite of different responses due to different shapes of the reservoir, they will all show the same trend and effect in their responses while moving them in depth, and assuming other parameters unchanged.

In general a simple model as an index can help the interpreter to observe if the change in the electric field is caused by different added complexity to the lithology or that can be due to different size, or depth or different ranges of the frequencies or other parameters.

Figure 3-19 shows the normalized amplitude differences of 2.5D and 3D modelling in percentage, as a function of the reservoir cross-line size. Since the background does not change, the curves show the difference in 3D and 2D reservoir responses. We expect to see less difference between 2.5D and 3D modelling results with increasing the reservoir size in
cross-line direction, but we are interested to get a quantitative estimate. In this example when we have a reservoir with 4000 m width, we observe a small difference between 2.5D modelling and 3D modelling, but with decreasing the width of the reservoir from 4000 m to 2000 m and 1000 meter the average difference in the mid offset range, from 3 km to 5 km, increases from about 4% to 15% and 47%, respectively, in a nonlinear way. As a matter of fact, the errors at large in-line offsets are smaller for reservoirs that are large in the cross-line direction.

Now let us investigate the accuracy of the 2.5D modelling as a function of reservoir depth. In order to do this analysis we use our reference configuration which is depicted in Figure 3-2. But different depths of the reservoir will be assumed. We have used depths to top of the reservoir of 800 m, 1000 m, and 1200 m below the seabed. In Figure 3-20 we can see the normalized difference curves as a function of offset of 2.5D and 3D modelling in percentage, for the three depths of the reservoir.

We show the difference from the middle of the reservoir (0 on the x-axis) up to the 8 km offset. For the limited offset range from 3 km to 5 km, the response of the 2D reservoir located at a depth of 800 m below sea bottom has an average difference with the 3D reservoir response of 24%, whereas for 1000 m depth we find an average difference of 15%, and for the reservoir at 1200 m depth we observe an average amplitude difference of just below 10%. Although a shallow reservoir is geometrically closer to a 2D reservoir than a deep reservoir, the cross-line edge effects of the 3D reservoir are clearly stronger for a shallow reservoir than for a deep one.

In CSEM applications, the emitted diffusive field can have different frequencies depending on the acquisition parameters. In the case of different frequencies the difference of 2.5D and 3D modelling would be different as well. Figure 3-21 shows that in the offset range

**Figure 3-21**: Difference between 2.5D and 3D modelling results for different frequency of the emitted field.

**Figure 3-22**: Difference between 2.5D and 3D modelling results for different thickness of the reservoir.
from 3 km to 5 km the difference between a 3D reservoir and a 2D reservoir decreases with decreasing frequency. It means with decreasing the frequency the response from 2.5D modelling would resemble to the 3D modelling result. So in the case of deeper targets for which lower frequencies can be used 2.5D modelling will give better results than 3D modelling.

The hydrocarbon thickness is another parameter that is of interest for studying the effectiveness of 2.5D modelling. Figure 3-22 shows the difference of 2.5D and 3D modelling results for different thicknesses of the reservoir. The red curve shows the difference in percentage between 2.5D and 3D modelling when the thickness of the reservoir is 100 m. The dashed black line shows the case when the thickness of the reservoir is 250 m and the blue curve shows the case when the thickness of the reservoir is 500 m. Here we observe an increasing difference between 2D reservoir and 3D reservoir responses with increasing reservoir thickness.

We continue by looking into more details and emulate some cases by changing the source-receiver offset and also using different frequencies. We will investigate the effects of varying depth of the reservoir and its thickness as well.

Figure 3-23 shows the difference between 2.5D and 3D modelling results for different frequencies. Each figure represents the result for different source offset. When the source is in the middle above the reservoir (Fig. 3-23(a)) we have less difference between 2.5D and 3D modelling. The biggest difference is when the source is located at the edge of the reservoir (Fig. 3-23(c)), and it can be seen in a wide range of offsets. When the source is at -1 km offset from the middle of the reservoir (Fig. 3-23(b)) in comparison with zero offset source location we find bigger difference between 2.5D and 3D modelling, in a wider range in near offset. In case the source offset is 3 km it is located outside the reservoir and 1 km away from the edge. We still observe a large difference between 2.5D and 3D modelling responses but of course less than when the source is at cross-line edge (Fig. 3-23(d)). And again by going far away (Figs. 3-23(e)) we will have less difference in general.

In all cases in lower frequencies, smaller difference exists, however the difference occurs in a wider range in terms of offset in compare with using higher frequencies. This implies, using lower frequencies gives better approximation of a 3D reservoir, using 2D reservoir model. Figure 3-24 shows how the depth of the reservoir can make a difference between 2.5D modelling and 3D modelling in different offsets, for a single frequency of 1 Hz.

In all cases we have the biggest difference for a shallow reservoir, which is due to the infinite width of the 2D reservoir in cross-line direction, however, we showed in Figure 3-5, when the reservoir is thin the difference is indeed small even if we put the source at the edge of the reservoir and using single frequency of 1 Hz.

In the case, source is in zero offset (Fig. 3-24(a)) we have the smallest difference between 2.5D and 3D modelling compared to a source located at the edge of the reservoir (Fig. 3-24(b)) that has the biggest response, and to a source in far offset at -4 km of the middle of the reservoir(Fig. 3-24(c)).
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Figure 3-23: This figure shows difference between 3D and 2.5D modelling results for different frequencies. The responses are also compared with no-reservoir case.
Figure 3-24: Difference between 2.5D and 3D modelling results for different depth and different source offset.
4 Interpretation and discussions

In the aforementioned numerical examples we considered a single source and a line of receivers but based on reciprocity we can easily change source and receiver while interpreting the plots. Of course, due to small vertical distance between the source and the receivers in this study, the result would be slightly different by changing the source and receiver but it does not affect the interpretation of the results and its consequent conclusion. Therefore if the reader would like, it can be reasonably assumed to consider one single receiver in the middle above the reservoir and imagine sources in different locations. This can make the story closer to a commercial case interpretation.

In order to interpret 1D and 2.5D modelling results, we need to consider at what frequency we are working and also we need to take into account the source and receiver locations relative to the reservoir. Depth, thickness and width of the reservoir play a role in choosing the method for modelling.

1D modelling amplitude response is closer to 3D modelling amplitude response than 2.5D modelling amplitude response in near (Figures 3-9(a) and 3-10(a)) or mid-range (Figure 3-8(a)) offsets when the source is located around the in-line edge of the reservoir, whose thickness is larger than one fifth of the skin depth of the embedding.

The reason that 2.5D modelling fails when the source is around the cross-line edge of the reservoir is that infinite edge of the reservoir gives high response and this response is growing with increasing the thickness of the reservoir. In the same way since frequency is sensitive to the edge therefore 2.5D-to-3D differences increase with frequency when the source is around the cross-line edge of the 2D reservoir. Although in higher frequencies we have the differences in narrower offset range which is around the edge but bigger in magnitude. For instance if the source is located at -2 km but we are looking at the response at 6 km then using higher frequencies gives closer 2.5D-to-3D response than using lower frequencies (Fig. 3-23(c)).

In most configurations 2.5D modelling results are very close to 3D modelling results and provide an attractive alternative to 3D modelling.

Phase angle is an accurate tool while using 2.5D modelling with different frequencies and source locations, because it is always very close to the phase angle obtained from 3D modelling.

These results can be used as fundamental and basic rules, no matter what the complexity of the configuration is, to see when we can use 1D modelling or 2.5D modelling safely with high reliability and when we better use 3D modelling.
5 Conclusions

Modelling efficiency is increased enormously when a reservoir can be modelled as a 2D structure, leading to fast codes. This is especially so for a reservoir for which the plane of modelling is the plane of symmetry, although maps to receivers in other planes is not much more difficult. We have shown that, for sources and receivers in the plane of symmetry, 2.5D modelling is adequate for detection and surveillance studies.

When the reservoir is thin, about one fifth of the skin depth of the embedding or less, the amplitude difference between the 3D reservoir response and that of the corresponding 2D reservoir response is very small. Even when the source is located at the edge of the reservoir, which is where the largest error occurs using a 2D reservoir model compared to a 3D reservoir model, the responses are still very close to each other as shown in Fig. 3-17.

Using lower frequencies gives closer results of 2.5D modelling to the 3D modelling in near and mid offsets. The phase of the in-line electric field component is almost independent of the cross-line reservoir model size. The change in the slope of the phase occurs at the same location independent whether the reservoir is modelled as a 3D or a 2D body. The phase of the data from models where the reservoir is modelled as a 1D layer are not sufficiently accurate to be used for detectability studies of 3D reservoirs.

We have demonstrated the sensitivity of the model results to several model parameters. With increasing target width and depth, decreasing target thickness, and decreasing frequency of operation, the amplitudes of 2.5D modelling results show decreasing differences to the amplitudes of 3D modelling results.

One-dimensional modelling would be of interest only in the mid offset range, together with the condition that the reservoir is thicker than one fifth of the skin depth of the embedding. Phase angles from 2.5D modelling at all offsets and frequencies are pretty match with 3D modelling which is not the case for 1D modelling.

At the end we deduced that 1D model responses are significantly different than 3D responses at long offsets, 2.5D modelling is sufficiently close to 3D modelling in most model configurations, except when the transmitter is right over the edge of the reservoir.

Given the much shorter computation times, 2.5D modelling is an interesting option for several types of modelling studies, including acquisition configuration optimization and data interpretation.
Chapter 4

Quasi-Analytical Frequency-to-Time Conversion Method

ABSTRACT

Frequency-to-time transformations are of interest to controlled-source electromagnetic methods when time-domain results have to be estimated from field measurements or when time-domain data are inverted for a subsurface resistivity model by numerical frequency-domain modelling at a selected, small number of frequencies while the data misfit is determined in the time-domain. We propose an efficient, Prony-type method of suitably chosen basis functions for which the time-domain equivalents are known. Diffusive fields are characterized by an exponential part whose argument is proportional to the square root of frequency and a part that is polynomial in integer powers of the square root of frequency. Data at a limited number of frequencies suffice for the transformation back to the time. We demonstrate that when the frequency-domain data is accurately approximated by the basis functions, the time-domain result is also accurate. We further show that this method is accurate over a wider time range than other methods and always has the correct late-time asymptotic behaviour. We finally show that the method works well for data computed for layered and three-dimensional subsurface models.
1 Introduction

Controlled-source electromagnetic (CSEM) methods are generally divided into frequency-domain electromagnetic (FDEM) and time-domain (transient) electromagnetic (TDEM) methods, depending on the waveform of the transmitted electrical current. In the FDEM method, we measure the electromagnetic response caused by a time-periodic source current. The FDEM and TDEM fields are related by the Fourier transformation, and the TDEM field can thus be obtained as the Fourier transform of FDEM data when recorded at a sufficient number of frequencies.

Modelling is a vital part of the interpretation in diffusive electromagnetic field data. For modelling in time, we can use fast frequency-domain modelling codes (Wannamaker et al. 1984; Alumbaugh et al. 1996; Abubakar and van den Berg 2004; Zhdanov et al. 2006; Mulder et al. 2008; Tehrani and Slob 2010) and then convert the results to the time-domain with a suitable numerical method. An efficient transformation method will allow for the use of a limited number of important frequencies, thereby reducing the modelling effort during the inversion of time-domain data when the data fit is evaluated in the time-domain (Druskin and Knizhnerman, 1994; Wirianto et al., 2011).

The Gaver-Stehfest method (Gaver 1966; Stehfest 1970) is one example of a numerical inverse Laplace transformation and used by J.H. Knight and A. Raiche (1982), but only works when the Laplace-domain data are available at machine precision. Therefore, it is only applicable to solutions that are exactly known in the Laplace domain, but for which no closed-form expression exists in the time-domain. There is a number of methods for computing time-domain electromagnetic data from frequency-domain data. One of these is the Discrete Fourier Transformation, which can be combined with cubic Hermite interpolation (Mulder et al., 2008). This leads to a transformation method requiring data at a limited number of optimally spaced frequencies, but the linear (equidistant) FFT usually requires over $10^5$ points, which still is expensive if transients are needed at many receiver positions. The logarithmic FFT (Talman, 1978; Haines and Jones, 1988) is much faster than the linear FFT. Slob et al. (2010) provide an example for the diffusive electric field in a VTI half-space. However, the method suffers from instabilities that are not easily avoided. The decay spectrum method (Newman et al., 1986) is another option. In this method, the time-domain EM signal is considered as a combination of a number of exponentially damped functions with unknown coefficients. The exponential damping factors are also unknown and must be chosen carefully. After solving for the coefficients in the frequency-domain, the time-domain EM signal follows as a series of basis functions. The results of this Prony-type method are reportedly subjective and the solutions are unstable at late times. Because the method expands the field in terms of damped exponentials, the late-time asymptote always has exponential decay.

None of the existing methods is considered suitable for accurate frequency-to-time transformation at a large number of receiver positions. We therefore continue with the method proposed in Slob and van den Berg (1999) and investigate if it works for arbitrary models, including the earth’s surface and three-dimensional models. We call it the Diffusion
Expansion Method (DEM). It utilizes the possibility to expand the electromagnetic field into a sequence of diffusive basis functions with a known frequency dependence, but with unknown diffusion times that must be chosen based on the data to be transformed. It is therefore a Prony-type method, like the decay spectrum method, but with the advantage that the late-time behaviour is always correct. The expansion functions as a function of frequency have analytically known time-domain equivalents. Hence, once the expansion coefficients in the frequency-domain have been solved, the transient result is known by inspection for impulse, step, and ramp responses, while they can be obtained to any desired precision by numerical integration for more complicated source time functions.

To obtain a quantitative measure, we first investigate how well the Gaver-Stehfest, FFT, and DEM methods perform on problems where the time-domain solution is known exactly. We then compare the FFT method with DEM results for a three-layered medium and a 3-D model, using numerical solutions.

2 Theory

2.1 Numerical Methods for Frequency-Domain to Time-Domain Conversion

Modelling of the transient electromagnetic field for CSEM applications can be performed in an efficient way by first solving the problem in the frequency-domain and then obtaining the time-domain solution by a suitable numerical methods such as the decay spectrum method or the Fourier transformation. The Fourier transformation method is straightforward and fast if the number of frequencies can be kept small (Mulder et al., 2008; Slob et al., 2010). This method has been tested against exact solutions for some simple problems and a realistic marine example in Mulder et al. (2008). They showed that a limited number of frequencies suffices to provide time-domain solutions, combining piecewise-cubic Hermite interpolation to minimize the number of frequency points where data should be computed, followed by fast Fourier transformation of the data after interpolation to an equidistant (linear) frequency range. However, the time window where the obtained solution is accurate is limited and cannot be extended by incorporating more frequencies.

2.2 Quasi-Analytical Method for Frequency-Domain to Time-Domain Conversion

Slob and van den Berg (1999) showed on scattered electromagnetic data from an object in a homogeneous embedding that the electric and magnetic fields can be expanded in diffusive decay functions as a sum of polynomials in \( \sqrt{\omega} \), each multiplied by functions that decay proportional to \( \exp(-2\sqrt{\omega}\tau) \). Each power of \( \sqrt{\omega} \) has an unknown constant coefficient. The parameter \( \tau \) is in principle unknown, but depends on the measurement
configuration and the subsurface heterogeneity. The minimum $\tau$-value can be estimated from the high-frequency behaviour and the range of $\tau$-values can span one or two decades depending on the subsurface complexity and the time window of interest.

The simplest way to implement this method is to determine a minimum and maximum value for $\tau$ and divide the time range up in $(k - 1)$ equidistant time steps, $\tau_k = \tau_{\text{min}} + (k - 1)\Delta\tau$, $\tau_{\text{min}}$ being the smallest diffusion time and $\Delta\tau$ denotes the time step. Another straightforward way is to use an equidistant diffusion time step on a logarithmic axis. The maximum value of $\tau$ depends on the time window of interest, because the expansion functions decay exponentially for $\tau > t$. On the other hand the smallest value of $\tau$ can be found from the slope of high-frequency logarithmic decay curve in the data. The smallest value of $\tau$ can always be estimated from the configuration if distance and conductivity are known.

The electric field impulse response can be expanded as

$$
\hat{E}^\delta(R, i\omega) \approx \sum_{k=1}^{K} \sum_{j=0}^{J} \alpha_{k,j} \hat{F}^{(j)}(\tau_k, i\omega),
$$

where $\hat{F}^{(j)}(\tau_k, i\omega) = (i\omega)^{j/2} m^j \exp(-2\sqrt{i\omega \tau_k})$. The maximum power of $\sqrt{i\omega}$ depends on the type of source and can be taken as $J = 3$ for an electric current source and $J = 2$ for a magnetic current source. For a step current source function we should replace $\hat{F}^{(j)}(\tau, i\omega)$ by $\hat{F}^{(j-2)}(\tau, i\omega)$. The maximum number of diffusion times, $K$ can be chosen in relation to the number of frequencies to ensure an overdetermined system of equations can be solved. Because the coefficients $\alpha_{k,j}$ are strictly real, equation (4-1) is solved simultaneously for the real and imaginary parts of the electric field.

The time-domain equivalents of $\hat{F}^{(j)}(\tau, i\omega)$ are available in recursive form

$$
F^{(j)}(\tau, t) = \frac{\sqrt{\tau}}{t} F^{(j-1)}(\tau, t) - \frac{j}{2t} F^{(j-2)}(\tau, t).
$$

With the two functions,

$$
F^{(-2)}(\tau, t) = \text{erfc}(\sqrt{\tau/t}) H(t), \quad F^{(-1)}(\tau, t) = \frac{\exp(-\tau/t)}{\sqrt{\pi t}} H(t),
$$

where erfc denotes the complementary error function. Functions for any other value of $j$ can be obtained by combining equations (4-2) and (4-3). We have implemented the expansion given in equation (4-1) for the transformation back to time of the electric field impulse response. It is worth noting that although the air-wave related field, which is represented by modified Bessel functions, is not properly accounted for, it can still be approximated as a series of exponential diffusion functions. We therefore expect that the method will still perform well in the presence of a strong air-wave related field.
3 Numerical results

3.1 Half-space configurations

The first example is a VTI half-space below a non-conductive half-space. The source and receiver are located at the surface between the two half spaces and their distance is given by $x^R - x^S = (2000, 0, 0) \text{ m}$. The horizontal and vertical conductivities are $\sigma^{(h)}, \sigma^{(v)} = \{0.1, 0.025\} \text{ S/m}$. The frequency- and time-domain responses are known in closed form for the $x$-component electric field generated by an $x$-component electric dipole source. We can therefore compare exact results with numerical results obtained with the Gafer-Stehfest method (GSM), the adaptive FFT (FFT), and the diffusion-expansion method (DEM). No specific modifications are needed to apply the DEM for an anisotropic medium, because the anisotropy is accounted for by choosing the proper range of $\tau$-values.

For the adaptive FFT method, 39 frequencies, unevenly spaced on a logarithmic scale between $10^{-3}$ and $10^3 \text{ Hz}$, are necessary to retrieve accurate data at any other frequency by interpolation within the range and extrapolation for frequencies smaller than 1 mHz. The data are interpolated and extrapolated with $2^{21}$, or about 2 million points, using a frequency interval $\Delta f \approx 2.38 \times 10^{-4} \text{ Hz}$, corresponding to a time step of $\Delta t = 0.1 \text{ ms}$ in the time-domain. The same 39 frequencies are used in the DEM, with $K = 4$ and $J = 3$. To account for the impulse response that occurs in the data for a source and receiver located at the interface, we set $\tau_1 = 0$. For $k = 1$, we choose $J = 0$. Then $\tau_2 = 32.7 \text{ ms}$, which is estimated from the high-frequency behavior, whereas $\tau_4 = 10\tau_2$ and $\tau_3 = (\tau_2 + \tau_4)/2$ is the midpoint between $\tau_2$ and $\tau_4$. The maximum normalized error in the DEM result is computed as the amplitude of the maximum difference, normalized by the amplitude of the data,

$$R_{\text{max}}^l = \max \left( \frac{|E^{\text{DEM}}(\omega) - E(\omega)|}{|E(\omega)|} \right)$$

and produces $1.2 \times 10^{-5}$. The normalized global root-mean-square error is the average error over all 39 frequencies used in the computation,

$$\text{RMSE} = \sqrt{\frac{\sum_\omega |E^{\text{DEM}}(\omega) - E(\omega)|^2}{\sum_\omega |E(\omega)|^2}}$$

and is $5.8 \times 10^{-6}$. From these results, we obtain a time window where the local normalized error is less than $5 \times 10^{-2}$ given by $2.2 \times 10^{-3} \leq t \leq 100 \text{ s}$, spanning almost five decades. The results for all methods are displayed in Figure 4-1, where it can be seen that the FFT result suffers from numerical saturation, but the normalized error is less than $5 \times 10^{-2}$ in the window $4 \times 10^{-3} \leq t \leq 1 \text{ s}$, still spanning almost three decades. From this result, we observe that also the GSM produces very good results when the frequency-domain function is known explicitly.
Quasi-Analytical Frequency-to-Time Conversion Method

Figure 4-1: Electric-field impulse response for a source and receiver on the surface between a VTI half-space and a non-conductive half-space. Shown is the exact solution together with results from GSM, FFT, and DEM.

Increasing both the conductivity values by an order of magnitude to $\sigma^{(h)}, \sigma^{(v)} = \{1, 0.25\}$ S/m and putting the source and receivers inside the VTI half-space will produce a strong air-wave related field, because the source and receiver are below the surface with a relatively large horizontal offset. We put the source at $x^S = (0, 0, 75)$ m and the receiver at $x^R = (2000, 0, 125)$ m. For the adaptive FFT method, 73 frequencies, unevenly spaced on a logarithmic scale between $10^{-3}$ Hz and $10^3$ Hz, are necessary to retrieve the data accurately by interpolation at any other frequency. Again, the data are interpolated or extrapolated with $2^{21}$ points using a frequency interval $\Delta f \approx 2.38 \times 10^{-4}$ Hz. The same 73 frequencies are used in the DEM, with $\tau_{min} = 12.6$ ms and $\tau_{max} = 11.4$ s, and we chose a logarithmic spacing to accommodate the large differences in diffusion times between the air-wave related field and the direct field. We set $K = 21$ and $J = 2$. The maximum normalized error in the DEM result is $R_{l_{max}} = 1.2 \times 10^{-4}$, up to $f = 125$ Hz, while it grows exponentially to $R_{l_{max}} = 0.16$ at $f = 1$ kHz. The normalized global root-mean-square error $RMSE = 1.7 \times 10^{-6}$ over all 73 frequencies.

From this result, we obtain the time where the local normalized error is less than $5 \times 10^{-2}$, given by $2 \times 10^{-3} \leq t \leq 650$ s and spanning just over five decades. The results for all methods are shown in Figure 4-2, where it can be seen that the FFT result suffers from numerical saturation, while the normalized error is less than $5 \times 10^{-2}$ in the window $10^{-2} \leq t \leq 2$ s, spanning only two decades. This can be extended to earlier times by
including higher frequencies, up 10 kHz, in the adaptive FFT, leading to computing the field at 85 frequency values and a time window where the local error is below the threshold of \(5 \times 10^{-2}\) down to 3 ms, while the upper limit is unchanged. On the other hand, we have also tested DEM with 39 frequencies evenly spaced on the logarithmic frequency axis in the same range as before. Then, using the same parameters for \(K, J, \tau\), we find that the range where DEM produces time-domain data with an error less than 5% runs from 1.2 ms to 750 s, which amounts to almost six decades.

![Impulse Response at offset: 2000m](image)

**Figure 4-2:** Electric-field impulse response for a source and receiver below the surface in the VTI half-space below a non-conductive half-space. Shown is the exact solution given together with results from GSM, FFT, and DEM.

We conclude that DEM produces very accurate time-domain data with a smaller number of frequency data points than necessary for FFT, over a much wider time range than the FFT method provides. The FFT method will work independently of the given subsurface geometry, while for DEM this remains to be demonstrated, which we will do next.

### 3.2 Three-layer configuration

Next, we consider a three-layer example for which no explicit frequency-domain or time-domain solution exists. We make the step from known to unknown time-domain solutions and, based on what was established in the above examples, we can assume that the late-time behaviour is accurate. Therefore, we trust that the time-domain DEM results are accurate
when the frequency-domain solution is fitted by the DEM with a small RMSE. Now, we apply the method to data obtained from modelling a three-layered earth configuration. The upper half-space is non-conductive, while the conductivity of the second layer and lower half-space are given by $\sigma_1 = 3 \, S/m$ and $\sigma_2 = 1 \, S/m$, respectively. The second layer has a thickness of 1 km, an $x_1$-directed electric dipole source is placed 25 m above the bottom interface and the receiver measures the $x_1$-directed electric field at the bottom interface at a 1-km offset in the $x_1$-direction and at zero offset in the $x_2$-direction. We show the time-domain result of DEM and FFT in Figure 4-3.

To obtained the DEM result we used $K = 10$, $J = 2$ and logarithmic spacing for $\tau$ between $\tau = 0.61 \, s$ and $\tau = 311 \, s$. We used 33 frequencies logarithmically spaced between 1 mHz and 100 Hz. The global normalized error in the data fit was $RMSE = 6 \times 10^{-7}$. For FFT the same frequency range was used and data at 113 frequency values were necessary for accurate interpolation. The same number of points were used for the linear FFT as described above. From this result we see that adaptive FFT requires more than three times the number of data points for a result that is accurate over a much smaller time window than obtained with DEM.

![Figure 4-3: Time-domain results obtained with the DEM and the adaptive FFT for $E_1$, generated by $J_{1}^{e}$ with 20km offset in the $x_1$-direction in a three-layered earth configuration.](image-url)
3.3 Three-layer model with a resistive 3-D body

The three-layer model is modified to have shallower water and to include a three-dimensional resistive body, mimicking a hydrocarbon reservoir. The background conductivity in the lower half-space is 0.5 S/m, air has zero conductivity and the sea water 3 S/m, while the resistive body has a conductivity of 0.02 S/m. The source is located 100 m above the sea bottom and a receiver is placed on the sea bed. The water depth is 200 m. A resistive body has dimensions $4 \times 4 \times 0.2 \text{ km}^3$ and its top is located at 400 m below the sea bed (Fig. 4-4).

![Figure 4-4: A three-layer earth model including an assumed reservoir.](image)

The $x$-component of the electric field, generated by an $x_1$-directed electric dipole, is computed at an offset of 1 km in the $x_1$-direction and at zero offset in the $y$-direction. Frequency-domain data in this 3-D model were generated by a finite-volume code (Mulder, 2006). The time-domain solution was obtained by using 61 frequencies, unequally spaced on a logarithmic axis, followed by cubic interpolation to a linear frequency axis using $10^6$ points and a FFT, as described by Mulder et al. (2008).

For the DEM, we used 31 logarithmically spaced frequencies, ranging from 1 mHz to 1 kHz. We let $K = 10$ and $J = 2$ and set $\tau_{\text{min}} = 0.135$ s and $\tau_{\text{max}} = 25\tau_{\text{min}}$. With these settings, we obtained a global $\text{RMSE}$ of $1.8 \times 10^{-4}$. Figure 4-5 depicts the time-domain result for the DEM together with the result from the FFT. The two results agree very well in the time range from 20 ms to 2 s, but the FFT result at early time has a zero-crossing and even becomes negative before 8 ms, while it suffers from noise after 2 s. The DEM solution seems to be correct over the whole plotted time range.
4 Conclusions

The Diffusion Expansion Method seems a good candidate for CSEM frequency-to-time conversion of data for any kind of subsurface model and survey configuration. The expansion contains the unknown expansion coefficients, multiplied by expansion functions whose time-domain equivalents are known. The latter consist of a polynomial term, with integer powers of the square root of frequency, as well as an exponential function, whose argument is proportional to the square root of frequency. The proportionality factor contains a diffusion time that is unknown and must be estimated from the data. We have shown that this can be achieved by considering the high-frequency behavior of the data.

Because the frequency-domain expansion functions have analytically known time-domain equivalents, solving for the expansion coefficients is sufficient to obtain the transient field for different type for source time functions. If the electromagnetic field is generated by a step-response, the electromagnetic field that would have been generated by impulse or ramp source time functions can be obtained directly from the step response. Hence, it is straightforward to obtain the Earth’s impulse response from step-response data.

From the results obtained on functions that are known exactly in both the frequency- and time-domain, we found that the DEM produces accurate time-domain results when the
frequency-domain fit is accurate. In the VTI half-space example with a strong airwave related signal, time-domain results were obtained with an error below 5% over a time window spanning almost six decades. Given the results obtained in the three-layer model and in the one with an additional three-dimensional resistive body, we believe that accuracy of the frequency-domain data fit ensures an accurate time-domain result. In the examples, we found an error less than 5% in the time window spanning four decades.

The results obtained with the DEM seem more accurate over a wider time window than those obtained with a FFT, while the number of frequency values at which the frequency-domain data should be known is smaller, which means that modelling 3-D data can be done at a smaller number of frequency values. The computation of each transient requires less time with the DEM than with the FFT, because for the FFT, typically in the order of $10^6$ points, computed by cubic interpolation to an equally spaced frequency axis, should be used, while the DEM result is obtained after inverting a matrix of typical size $30 \times 30$, followed by multiplying the expansion coefficients with the transient expansion functions. The most time consuming part can be to find the proper range of diffusion time values that are input into the expansion functions. In our experience, this is not too difficult, but of course each human interaction requires time. Once a proper diffusion time range is found for one receiver, it can be modified in an automated way for adjacent receivers. We found that the solution is quite robust for changes in the diffusion time range.
Chapter 5

General conclusions and outlook

The objectives of this dissertation are elucidated in the introduction. The first objective was to develop a fast approach for three-dimensional modelling of the marine controlled-source electromagnetic method and accelerate the computation via approximations. The second objective was started with the same inspiration via two-and-a-half dimensional modelling and continued by sensitivity analysis of one-dimensional, two-and-a-half dimensional and three-dimensional modelling. After modelling in the frequency-domain, the third goal was developing a fast and reliable method to convert the controlled-source electromagnetic response from frequency-domain to time-domain, which is performed through a quasi analytical method.

In this chapter we reflect on these objectives and give general conclusions and an outlook.

1 Conclusions

We divide the conclusions on the objective of the thesis in three categories: conclusions on the fast and accurate three-dimensional modelling and approximations, on the two-and-a-half dimensional technique, and on the frequency to time conversion.

1.1 Fast and accurate three-dimensional modelling and approximations

We discussed the aspects of full solution for 3D forward modelling using the iterative scheme of Conjugate Gradient Fast Fourier Transformation (CG-FFT) to solve the integral equations in three-layered earth controlled-source electromagnetic application with an assumed reservoir. The advantages of applying the CG-FFT method to a class of large scale forward
and scattering problems are outlined in Sarkar et al. (1986). Fast and accurate forward modelling code may find application in forward and inverse modelling. Therefore the first objective of this thesis was to develop and accelerate the computational method.

For three-dimensional diffusive electromagnetic modelling problems, local methods seem to have outperformed global methods in terms of memory requirements and computational efforts. The main reason for pursuing integral equation methods for modelling is that for a large class of problems the modelling domain can be reduced to the target volume. For such problems integral equations are useful, because they are based on primary-secondary, or direct-scattered field separation and allow for several types of suitable approximations.

We discussed several approaches based on electric field domain integral equation (EFIE) formulation, to model three-dimensional low frequency controlled-source electromagnetic diffusive field, which are fast approximation methods. They represent a compromise between efficiency and accuracy. We proposed an iterative extended Born approximation (IEBA) based on the integral equation formulation to compute 3D diffusive electromagnetic field configurations.

For three-dimensional configurations fast computational methods are relevant for both forward and inverse modelling studies. The Born approximation, extended Born approximation and iterative extended Born approximation are implemented and compared with the full solution of the conjugate gradient fast Fourier transformation method. We have used the advantage of computational speed to do sensitivity analysis as a function of the source position and different reservoir sizes that validated the accuracy of the IEBA.

We summarize the most important outcomes below:

- Iterative extended Born approximation method improves the accuracy of extended Born approximation up to about 30%, in modelling three-dimensional diffusive electromagnetic fields, at virtually no additional computational cost.

- With the help of this method sensitivity analysis, which is vital for controlled-source electromagnetic application, is possible in a timely manner using 3D modelling.

- It is also shown that this method can be a good candidate for inverse modelling, because it gives quite accurate electric field results inside the reservoir that will allow for accurate conductivity estimations within the reservoir. Our modelling studies show that the iterative extended Born approximation method is fast and accurate for both forward and inverse modelling.

### 1.2 Two-and-a-half dimensional modelling

Modelling efficiency and speed can also be increased enormously when we reduce the dimensions of the model to one dimension or two-and-a-half dimensions, that led us to a fast code. In chapter 3 of this dissertation, we found that reducing the dimensionality in
1 Conclusions

the modelling effort is useful in some situations by choosing the proper dimensions in the modelling.

We looked into the question in what model configurations two-and-a-half dimensional modelling is a good choice for modelling a real reservoir. We implemented two-and-a-half dimensional modelling, which consists of point sources and point receivers in a two-dimensional earth model. We compared the results with one-dimensional and three-dimensional modelling. These methods are based on an electric field domain integral equation formulation. Two-and-a-half dimensional modelling, which is a fast method, can be even faster using integral equation method.

We applied the method to a synthetic model in the marine controlled-source electromagnetic exploration situation where conductivity is different from the known background medium. We showed the results of the sensitivity analysis comes from one-dimensional, two-and-a-half dimensional and three-dimensional modelling for a symmetrically placed reservoir and the in-line acquisition configuration as a function of different reservoir sizes in cross-line dimensions and thickness, different frequency content of the emitted diffusive field and offsets, different depth and also the moving source position.

We summarize the most important outcomes below:

- Modelling efficiency is increased enormously when a reservoir can be modelled as a 2D structure. We have shown that, for sources in the plane of symmetry, 2.5D modelling is adequate for detection and surveillance studies. Using 2.5D CSEM modelling is fast and allows sensitivity analysis of reservoir properties to estimate reservoir detectability.

- Modelling results show that CSEM method is sensitive to several model parameters which are depth, target size and frequency. With increasing target width and depth, decreasing target thickness, and decreasing frequency of operation, the amplitudes of 2.5D modelling results show decreasing differences to the amplitudes of 3D modelling results. In this case even if the source is located at the edge of the reservoir, which is where the largest error occurs using a 2D reservoir model compared to a 3D reservoir model, the responses are still very close to each other.

- The phase of the in-line electric field component is almost independent of the cross-line reservoir model size. The change in the slope of the phase occurs at the same location independent whether the reservoir is modelled as a 3D or a 2D body. When the reservoir is modelled as a 1D layer the phase of the data are not sufficiently accurate to be used for detectability studies of 3D reservoirs.

- One-dimensional modelling would be of interest only in the mid offset range, together with the condition that the reservoir is thicker than one fifth of the skin depth of the embedding. We deduced that 1D modelling is inadequate at long offsets, 2.5D modelling is sufficiently close to 3D modelling in most model configurations, except when the transmitter is right over the edge of the reservoir.
• Given the much shorter computation times, 2.5D modelling is an interesting option for several types of modelling studies, including acquisition configuration optimization and data interpretation. At this stage a decision can be made on whether or not the target reservoir is detectable and if CSEM is a suitable exploration tool capable of quantifying the prospective hydrocarbon reserve since feasibility study and survey design are vital and crucial for a successful CSEM measurement campaign. 2.5D forward modelling results provide a quick understanding of the CSEM response through different geological scenarios and can lead to an optimum survey design.

1.3 Frequency to time conversion

Using frequency-domain to time-domain conversion can increase information content in the data. In chapter 4 of this thesis, we compared a quasi analytical method to transform frequency-domain controlled source electromagnetic data back to the time-domain with an analytical and numerical transformations. The quasi analytical method exploits the fact that the kernel of the integral equation has a known behavior as a function of frequency and that the solution to the integral equation can be written as a sum of repeated applications of the kernel to the incident field. A set of expansion functions is found, which have analytically known time-domain counterparts, which need only a limited number of frequencies for the transformation back to the time.

We evaluated the accuracy of this function in different scenarios and for different methods. Finally we compared the diffusive expansion method in CSEM application with a numerical method consisting of a combination of optimized fast Fourier transformation and the cubic hermite interpolation.

The expansion contains the unknown expansion coefficients, multiplied by expansion functions whose time-domain equivalents are known. The latter consist of a polynomial term, with integer powers of the square root of frequency, as well as an exponential function, whose argument is proportional to the square root of frequency. The proportionality factor contains a diffusion time that is unknown and must be estimated from the data. We have shown that this can be achieved by considering the high-frequency behavior of the data.

Because the frequency-domain expansion functions have analytically known time-domain equivalents, solving for the expansion coefficients is sufficient to obtain the transient field for different types of source time functions. If the electromagnetic field is generated by a step-current that is switched on, the electromagnetic field that would have been generated by impulse or ramp source time functions can be obtained directly from the step response. Hence, it is straightforward to obtain the Earth’s impulse response from step-response data.

We summarize the most important outcomes below:
• The Diffusion Expansion Method seems a good candidate for CSEM frequency-to-time conversion of data for any kind of subsurface model and survey configuration. From the results obtained on functions that are known exactly in both the frequency- and time-domains, we found that the DEM produces accurate time-domain results when the frequency-domain fit is accurate. Even with strong airwave related signal.

• The results obtained with the DEM are more accurate over a wider time window than those obtained with FFT, while the number of frequency values at which the frequency-domain data should be known is smaller, which means that modelling 3-D data can be done at a smaller number of frequency values.

• The computation of each transient requires less time with the DEM than with the FFT, because for the FFT, typically in the order of $10^6$ points, computed by cubic interpolation to an equally spaced frequency axis, should be used, while the DEM result is obtained after inverting a matrix of typical size $30 \times 30$, followed by multiplying the expansion coefficients with the transient expansion functions.

We dedicate the last part of this chapter to the potential future research and outlook:

### 2 Outlook for future research

In the course of this project, we identified various areas where further research is needed. The areas of further research include the following:

• Studies are required to implement iterative extended Born approximation into an inversion code which is vital for CSEM interpretation that needs an accurate approximate solution since this process is very time consuming.

• Quality studies must be carried out by using the approximations for more complex models in terms of anisotropy, complex surrounding lithology and sea bed topography in shallow water to be more useful in real world.

• It would be useful to investigate how iterative extended Born approximation works for other applications such as ground penetrating radar and seismic waveform modelling in a 3D Earth. It can bring an effective step forward for applications of Born approximation widely used in different wave propagation studies.

• Due to recent activities on field development using CSEM, sensitivity analysis using iterative extended Born approximation is recommended for oilfield infrastructure (pipelines, cables, steel templates, etc., installed on the seafloor during the life of a producing reservoir) as an efficient time-lapse study.
• Since CSEM inversion is a time consuming process therefore 2.5D modelling can be a great help. It would be recommendable, running similar sensitivity analyses using 2D reservoir and 3D sources in an inversion process, considering anisotropy and noises.

• Real data application can be another important issue for further study to ensure the applicability and robustness of these algorithms.
Appendix A

The Method of Solution

1 Numerical evaluation of the integrals

The electric field vector in the three-layered earth problem can be derived explicitly in the wave number domain and then be transformed back to the spatial domain by carrying out the inverse Fourier transformation (Ward and Hohmann, 1987). We consider a three-layered medium and number the layers (air-sea-ground) from top to the bottom as 0, 1 and 2, respectively. Layer 1 has a thickness $d$. Each layer has different parameters, $\mu$ and $\sigma$. A coordinate system is defined such that the origin is located at the interface of layer 0 and layer 1. A unit-strength $x_1$-directed electric current source is located at coordinate $x^s = (x_1^s, x_2^s, x_3^s)$, $0 < x_3^s < d$.

We compute the electric field vector anywhere in the water layer or in the lower half-space generated by an $x_1$-directed electric dipole source in water layer.

The electric field in the water layer is necessary as the incident field measured at the receivers, given by,

$$ E^{i}_{\alpha;1}(\mathbf{x}, \mathbf{x}^s, \omega) = \hat{G}^E_{\alpha\beta}(\mathbf{x}, \mathbf{x}^s, \omega) \hat{J}_\beta^e(\omega), \quad (A-1) $$

and the electric field in the lower half-space is necessary as the incident electric field to solve the integral equation, given by,

$$ E^{i}_{k;2}(\mathbf{x}, \mathbf{x}^s, \omega) = \hat{G}^E_{kr}(\mathbf{x}, \mathbf{x}^s, \omega) \hat{J}_r^e(\omega), \quad (A-2) $$

In the water layer the $x_1$-directed electric field will be,
the reflection coefficients are given by $M$ and the multiple generators are $A$.

The total depth is given by
\[ \Gamma_1 = (k_1^2 + k_2^2 + \eta_1 \zeta)^{-\frac{1}{2}}, \]

and the multiple generators are
\[ M^T = 1 + r_0^T r_1^T \exp(-2\Gamma_1 d) \quad \text{and} \quad M^E = 1 + r_0^E r_1^E \exp(-2\Gamma_1 d), \]

the reflection coefficients are given by
\[ r_0^T = \frac{\eta_1 \Gamma_0 - \eta_0 \Gamma_1}{\eta_1 \Gamma_0 + \eta_0 \Gamma_1}, \quad r_1^T = \frac{\eta_2 \Gamma_1 - \eta_1 \Gamma_2}{\eta_2 \Gamma_1 + \eta_1 \Gamma_2}, \]

\[ r_0^E = \frac{\eta_1 \Gamma_0 - \eta_0 \Gamma_1}{\eta_1 \Gamma_0 + \eta_0 \Gamma_1}, \quad r_1^E = \frac{\eta_2 \Gamma_1 - \eta_1 \Gamma_2}{\eta_2 \Gamma_1 + \eta_1 \Gamma_2}. \]
and
\[ r_{0}^{TE} = \frac{\Gamma_{0} - \Gamma_{1}}{\Gamma_{0} + \Gamma_{1}}, \quad r_{1}^{TE} = \frac{\Gamma_{1} - \Gamma_{2}}{\Gamma_{1} + \Gamma_{2}}. \]

The solution for third layer or lower half-space where the assumed reservoir can be located is,
\[
\hat{G}_{11;2}^{EJ}(x, x^{s}, \omega) = -\frac{1}{2\pi} \int_{0}^{\infty} (F_{1}(\kappa) + F_{2}(\kappa)) \exp(\Gamma_{2}(x_{3} - d)) J_{0}(\kappa r^{s}) \kappa d\kappa + \\
\frac{\cos(2\phi)}{2\pi} \int_{0}^{\infty} (F_{1}(\kappa) - F_{2}(\kappa)) \exp(\Gamma_{2}(x_{3} - d)) J_{2}(\kappa r^{s}) \kappa d\kappa, \quad (A-7)
\]

\[
\hat{G}_{21;2}^{EJ}(x, x^{s}, \omega) = \frac{\sin(2\phi)}{2\pi} \int_{0}^{\infty} (F_{2}(\kappa) - \frac{1}{2} F_{1}(\kappa)) \exp(-\Gamma_{2}(x_{3} - d)) J_{2}(\kappa r^{s}) \kappa d\kappa, \quad (A-8)
\]

\[
\hat{G}_{31;2}^{EJ}(x, x^{s}, \omega) = \frac{1}{2\pi} \int_{0}^{\infty} \kappa \cos(\phi) F_{3}^{s}(\kappa) \exp(-\Gamma_{2}(x_{3} - d)) J_{1}(\kappa r^{s}) \kappa d\kappa, \quad (A-9)
\]

where
\[
F_{1} = \frac{\zeta(1 + r_{1}^{TE})}{2\eta_{1} M_{TE}^{TM}} (\exp(-\Gamma_{1}(d - x_{3}^{s})) + r_{0}^{TE} \exp(-\Gamma_{1}(d + x_{3}^{s}))), \quad (A-10)
\]

\[
F_{2} = \frac{\Gamma_{1}(1 + r_{1}^{TM})}{2\eta_{1} M_{TM}} (\exp(-\Gamma_{1}(d - x_{3}^{s})) + r_{0}^{TM} \exp(-\Gamma_{1}(d + x_{3}^{s}))), \quad (A-11)
\]

and
\[
F_{3} = \frac{(1 + r_{1}^{TM})}{2\eta_{2} M_{TM}^{TM}} (\exp(-\Gamma_{1}(d - x_{3}^{s})) + r_{0}^{TM} \exp(-\Gamma_{1}(d + x_{3}^{s}))), \quad (A-12)
\]

The computation of Fourier-Bessel integrals are performed numerically for every receiver location separately using Gauss type quadrature rules available in standard mathematical libraries. It has been done in Fortran programming language and then using Matlab programming language the results are regridded from cylindrical coordinate to cartesian coordinate.


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Summary

In this study we developed algorithms for fast frequency and time domain integral equation modelling for marine controlled-source electromagnetic (CSEM) applications.

Solutions of integral equations for CSEM applications in a three-layered earth with an assumed reservoirs is examined using the conjugate gradient fast Fourier transformation (CG-FFT) method, which is used as a reference. For 3D configurations fast computational methods are relevant for both forward and inverse modelling studies. The Born approximation, extended Born approximation, and iterative extended Born approximation are implemented and compared with the full solution of the CG-FFT, even with a reservoir consisting of two separated compartments. We also showed that the approximate results are accurate at the receiver level, which is usually the sea bottom, and inside the reservoir. This iterative method is suitable as a modelling algorithm for solving the inverse scattering problem as well. These methods are based on an electric field domain integral equation formulation. With the help of this method sensitivity analysis using 3D modelling is possible in a timely manner, which is vital for CSEM applications. Our modelling studies investigate to which extent the iterative extended Born approximation method is fast and accurate for forward modelling and could be used for inverse modelling. Sensitivity analysis as a function of the source position and different reservoir sizes validates the accuracy of the iterative extended Born approximation.

We also looked into the question in what model configurations two-and-a-half dimensional modelling is a good modelling choice for three-dimensional reservoir response to the diffusive EM field. It is investigated how the accuracy of 2.5D modelling compares with 3D modelling depending on the configurations. Because 2.5D modelling is much faster than 3D modelling, proper use of 2.5D CSEM modelling provides sensitivity analysis of subsurface parameters for many different scenarios in a limited amount of time. Depending on size and depth of the target, hydrocarbon thickness, and extension in cross-line dimension, the required frequency content of the emitted diffusive field and the number of offsets must be determined to assess if the target can be detected. At this stage a decision can be made...
on whether or not the target reservoir is detectable. From here an optimum survey can be designed based on the forward modelling results. Similarly, in this way the detectability from time-lapse monitoring data of changes in the reservoir due to production can be assessed.

Finally we investigated how frequency-domain data can be efficiently transformed to the time-domain. CSEM methods are generally divided into frequency-domain electromagnetic (FDEM) and time-domain (transient) electromagnetic (TDEM) methods, depending on the waveform of the transmitted electrical current. We compared a quasi-analytical method to transform frequency-domain CSEM data back to the time-domain with a numerical transformation. The quasi-analytical method exploits the fact that the kernel of the integral equation has a known behavior as a function of frequency and that the solution to the integral equation can be written as a sum of repeated applications of the kernel to the incident field. A set of expansion functions is found, which have analytically known time domain counterparts, which need only a limited number of frequency values for the transformation back to time. We compared this quasi-analytical method, coined the Diffusive Expansion Method in CSEM applications with two other numerical methods, Gaver-Stehfest method and an optimized form of the fast Fourier transformation method where the data is required at an minimum number of discrete frequency values such that the data at intermediate frequency values can be accurately obtained by interpolation. We found that the Diffusion Expansion Method is a good candidate for CSEM frequency-to-time conversion of data for any kind of subsurface model and survey configuration.
Samenvatting

In dit onderzoek hebben we algoritmen ontwikkeld voor een snelle modelleren van frequentie- en tijdsdomein integraalvergelijkingen voor toepassingen van mariene electromagnetische methoden met gecontroleerde bronnen (CSEM). Oplossingen van integraalvergelijkingen voor CSEM toepassingen in een drielagige aarde met een verondersteld reservoir wordt onderzocht met behulp van de geconjugeerde gradinten Fast Fourier Transformatie (CG-FFT) methode, die als referentie oplossing wordt gebruikt. Voor 3D configuraties, snelle rekenmethoden zijn relevant voor zowel de voorwaarts als de inverse modellerings studies. De Born benadering, uitgebreden Born benadering en iteratief uitgebreide Born benadering zijn uitgevoerd en vergeleken met de volledige oplossing van de CG-FFT methode, zelfs met een reservoir bestaande uit twee gescheiden compartimenten. De benaderende resultaten bleken nauwkeurig te zijn bij de ontvangers, die meestal op de zeebodem zijn geplaatst, en in het reservoir. Deze iteratieve methode is ook geschikt als modelleringsalgoritme voor oplossen van inverse verstrooiingsprobleem. De methoden zijn gebaseerd op een elektrisch veld domeinintegraalvergelijking-formulering. Met behulp van deze methode is een snelle gevoeligheidsanalyse met gebruik van 3D modelering mogelijk, wat essentieel is voor CSEM toepassingen. Onze modelleringsstudies onderzoeken in welke mate de iteratieve uitgebreide Born benadering snel en accuraat is voor voorwaartse modelleren, zodat ze kunnen worden gebruikt voor inverse modelleren. Gevoeligheidsanalyse, als functie van de bron positie en verschillende reservoir maten, bevestigt de nauwkeurigheid van de iteratieve uitgebreid Born benadering.

We hebben ook gekeken naar de vraag in welk model configuraties twee-en-een-half dimensionale modelleren een goede keuze is voor het modelleren van de responsie van een driedimensionaal reservoir. De nauwkeurigheid van de resultaten van 2.5D modellen zijn vergeleken met die van 3D resultaten, afhankelijk van de configuratie. Aangezien 2.5D modelleren veel sneller is dan 3D modelleren, biedt het juiste gebruik van 2.5D CSEM modelleren de gevoeligheidsanalyse van ondergrondse parameters voor verschillende scenarios in een beperkte tijd. Afhankelijk van de lengte, diepte, en dikte van het reservoir, en
de breedte ervan (loodrecht op meetrichting), de benodigde frequentie inhoud van de uitgezonden diffuse veld, en het aantal offsets moet worden bepaald om te beoordelen of een reservoir kan worden gedetecteerd. In dit stadium kan een beslissing worden genomen of het reservoir kan worden opgespoord met CSEM. Op basis hiervan kan een optimaal onderzoek worden. Ook kan op deze wijze de detecteerbaarheid uit time-lapse monitoringgegevens van veranderingen in het reservoir als gevolg van de productie worden beoordeeld.

Tot slot is onderzocht hoe gegevens in het frequentie-domein efficiënt terug te transformeren zijn naar het tijd-domein. CSEM methoden zijn in het algemeen verdeeld in frequentie-domein elektromagnetische (FDEM) en tijd-domein (voorbijgaande) elektromagnetische (TDEM)-methode, afhankelijk van de golfvorm van het verzonden elektrische signaal. We vergeleken een quasi-analytische methode, de Diffusie Expansie Methode, met twee andere numerieke methoden. De quasi-analyse methode gebruikt het feit dat de kernfunctie van de integraalvergelijking een bekend gedrag heeft als functie van de frequentie en de oplossing van de integraalvergelijking kan worden geschreven als de som van herhaalde toepassingen van de kernfunctie op het incident veld. Een reeks van ontwikkelfuncties wordt zo gevonden, waarvan de tijd-domein tegenhangers bekend zijn. Hierdoor is slechts een beperkt aantal frequenties nodig voor de transformatie terug naar de tijd. We vergeleken deze quasi-analytische methode met de Gaver-Stehfest methode en een optimale vorm van de snelle Fourier transformatie waarbij de gegevens vereist zijn op een minimaal aantal discrete frequentie waarden, zodanig dat de gegevens op iedere willekeurig andere frequentie waarde nauwkeurig kan worden verkregen door interpolatie. We vonden dat de Diffusie Expansie Methode een goede kandidaat is voor CSEM frequentie naar tijd conversie van gegevens voor ieder vorm van ondergrond model en meetconfiguratie.
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