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EFFECTS OF FORWARD MOTION AND GROUND REFLECTION ON AIRCRAFT FLYOVER NOISE

by

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1. Introduction

Considerable attention is currently given to the effects of forward speed on ground observed noise levels from aircraft operations. A major problem in the interpretation of results from flyover measurements are the effects of ground reflected sound, which must be removed from the recordings before the directivity of the radiation may become visible.

In this respect, an analysis is presented of the sound radiation from a moving source and the effects of ground reflection on noise propagation. For this aim the radiation from an acoustic point monopole in the presence of a finite impedance plane is used as a reference to describe the features of the sound field.

2. Kinematics

The geometry of the problem is outlined in Figure 1, where the aircraft moves with constant subsonic speed in the x-direction at an altitude h above the ground (x-y plane). The aircraft is assumed to generate sound as a point source, and the atmosphere is considered to be quiet, isothermal and homogeneous. Then, from the source the sound travels rectilinear to the receiver, located at x, y, z, along a direct path R₁ and along a reflected path R₂.

Due to the forward motion of the source the sound observed at time t is emitted by the source at time \( t_e \)

\[
 t_e = t - R_1 / c, \tag{1}
\]

where c is speed of sound.

If the source is at \( X_s = 0 \) at time \( t = 0 \), it follows from Figure 1 that

\[
 R_1^2 = (x - V \ t + MR_1)^2 + y^2 + (z - h)^2 \tag{2}
\]

or

\[
 R_1 = \gamma^2 M (x-V \ t) + \gamma^2 \left\{ (x-V \ t)^2 + (y/\gamma)^2 + ((z-h)/\gamma)^2 \right\}^{1/2}, \tag{3}
\]
where \( \gamma^2 = (1 - M^2)^{-1} \) and \( M = V/c \) is flight Mach number.

Note that for subsonic speeds (\( M < 1 \)), only the positive sign in equation (3) is of significance (\( R_1 > 0 \)).

The reflected sound may be described by the concept of an image source, symmetric with respect to the real source across the reflecting plane. Then, if irregularities in the ground surface are assumed small when compared to the wavelengths of the sound, the reflection can be considered specular so that

\[
R_2^2 = (x - V \cdot t + M R_2)^2 + y^2 + (z+h)^2
\]  \hspace{1cm} (4)

or

\[
R_2 = \gamma^2 M (x - V \cdot t) + \gamma^2 \left\{ \left( (x - V \cdot t)^2 + (y/\gamma)^2 + ((z+h)/\gamma)^2 \right) \right\}^{1/2}
\]  \hspace{1cm} (5)

The reflected sound reaching the receiver at time \( t \) has covered a distance \( R_2 > R_1 \). This means that the reflected sound is emitted slightly earlier than the direct sound

\[
t_e - t^1_e = (R_2 - R_1)/c = \Delta R/c
\]  \hspace{1cm} (6)

An impression of the significance of the latter time difference can be obtained if the waves in the vicinity of the reflection point are considered to be plane waves. This assumption may be acceptable because the radius of curvature of the wave front near the ground is of the order of the distance between source and receiver, which under flyover circumstances is relatively large. Then, it easily can be shown that the path-length difference \( \Delta R \) is given by [1]

\[
\Delta R = 2 z \sin \psi,
\]  \hspace{1cm} (7)

where \( \psi \) is the angle of incidence of the reflected sound with the ground surface.

This result may indicate that at practical receiver heights the difference between both emission times and the corresponding displacement of the source (\( M \Delta R \)), in general, are negligible.
3. Velocity potential

Considered is a monopole source of strength $q_o$ with harmonic time dependence.

$$q(t) = q_o \sin \omega t = i q_o e^{-i\omega t} \quad \text{(8)}$$

According to reference [2], the wave equation for the sound pressure is of the form

$$\nabla^2 p - \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} = -\frac{3}{\partial t} q(t) \delta (x-vt) \delta (y) \delta (z-h) \quad \text{(9)}$$

Introducing the velocity potential $\phi(x, y, z, t)$ by

$$p = -\rho \frac{\partial \phi}{\partial t} \quad \text{(10)}$$

Then

$$\nabla^2 \phi - \frac{1}{c^2} \frac{\partial^2 \phi}{\partial t^2} = i \frac{q_o}{\rho} e^{-i\omega t} \delta (x-vt) \delta (y) \delta (z-h) \quad \text{(11)}$$

By use of the coordinate transformation

$$x' = \gamma^2 (x-vt)$$
$$y' = \gamma y$$
$$z' = \gamma z$$
$$h' = \gamma h$$
$$t' = \gamma^2 \left( t - \frac{V}{c^2} x \right) \quad \text{(12)}$$

equation (11) can be transformed to the following equation, which represents the equivalent radiation from a stationary source [2, 3]

$$\nabla^2 \phi - \frac{1}{c^2} \frac{\partial^2 \phi}{\partial t^2} = i \frac{q_o}{\rho} e^{-i\omega t'} \delta (x') \delta (y') \delta (z') \quad \text{(13)}$$

The solution of this equation can be written as the sum of the velocity potential of the direct sound and the velocity potential of the reflected sound, the latter multiplied by the plane wave reflection coefficient $Q_p$.

$$\phi(x, t') = -i \gamma^2 \frac{q_o}{\rho 4\pi} \left\{ \frac{e^{-i\omega (t'-\frac{r_1}{c})}}{r_1} + Q_p \frac{e^{-i\omega (t'-\frac{r_2}{c})}}{r_2} \right\} , \quad \text{(14)}$$
where \( r_1 = \left\{ x'^2 + y'^2 + (z'-h')^2 \right\}^{\frac{1}{2}} \) and \( r_2 = \left\{ x'^2 + y'^2 + (z+h')^2 \right\}^{\frac{1}{2}} \).

An expression for the reflection coefficient follows from the requirement that the complex ratio between the sound pressure at the surface and the air velocity into the surface, equals the acoustic impedance \( Z \) of the surface.

From ground-to-ground noise propagation studies in reference [4] and [5], it appears that the boundary condition of a locally reacting surface is an adequate assumption, which term denotes the condition that lateral propagation through the ground is absent or can be ignored. In this case the acoustic impedance is independent of the angle of incidence \( \psi \), and the boundary condition in terms of the velocity potential becomes

\[
\frac{1}{c} \frac{\partial \Phi}{\partial t} - \frac{Z}{\rho c} \frac{\partial \Phi}{\partial z} = 0, \text{ at } z = 0
\]

(15)

The transformation (12) changes this boundary condition to

\[
\frac{1}{c} \frac{\partial \Phi}{\partial t'} - \frac{m}{\rho c} \frac{\partial \Phi}{\partial x'} - \frac{Z}{\rho c} \frac{1}{\gamma} \frac{\partial \Phi}{\partial z'} = 0, \text{ at } z' = 0
\]

(16)

Substitution of (14) into (16) and assuming plane waves \( (1/r^2 << 1/r) \) gives the resulting reflection coefficient as

\[
Q_p = \frac{Z \frac{\gamma'}{\gamma} + h'}{(r_2 + x' m) \frac{\gamma'}{\gamma} + (r_2 + x' m)}
\]

(17)

Transforming back to the initial variables in the physical space yields

\[
\begin{align*}
    r_1 &= \gamma^2 \left\{ (x - vt)^2 + \left( \frac{y}{\gamma} \right)^2 + \left( \left( \frac{z - h}{\gamma} \right) \right)^2 \right\}^{\frac{1}{2}} \\
    r_2 &= \gamma^2 \left\{ (x - vt)^2 + \left( \frac{y}{\gamma} \right)^2 + \left( \left( \frac{z + h}{\gamma} \right) \right)^2 \right\}^{\frac{1}{2}}
\end{align*}
\]

(18)
and

\[
\begin{align*}
    t' - \frac{r_1}{c} &= t - \frac{R_1}{c} \\
    t' - \frac{r_2}{c} &= t - \frac{R_2}{c}
\end{align*}
\]

(19)

where \( R_1 \) and \( R_2 \) are given by (3) and (5).

Thus

\[
\phi(R, t) = -i \gamma^2 \frac{Q_0}{\rho 4\pi} \left\{ \frac{e}{r_1} - i\omega(t - \frac{R_1}{c}), \frac{e}{r_2} - i\omega(t - \frac{R_2}{c}) \right\}
\]

(20)

where

\[
Q_p = \frac{Z}{\rho c} \frac{Z + h}{R_2} - 1 = \frac{Z}{\rho c} \sin \psi - 1
\]

(21)

\[
\frac{Z}{\rho c} \frac{Z + h}{R_2} + 1 = \frac{Z}{\rho c} \sin \psi + 1
\]

4. Sound pressure

From equations (10) and (20) the sound pressure is obtained as

\[
p(t) = \gamma^2 p_1 \left\{ \frac{e}{r_1} \left( 1 - \frac{1}{c} \frac{dr_1}{dt} \right) + Q_p \frac{e}{r_2} \left( 1 - \frac{1}{c} \frac{dr_2}{dt} \right) \right\}
\]

(21)

\[
= \gamma^2 p_1 \left\{ \gamma^2 \left( \frac{r_1}{R_1} \right)^2 \frac{e}{R_1} - i\omega(t - \frac{R_1}{c}) \right\} + \gamma^2 \left( \frac{r_2}{R_2} \right)^2 Q_p \frac{e}{R_2} - i\omega(t - \frac{R_2}{c}) \right\}
\]

(22)

where \( p_1 = q_0/4\pi \) represents the amplitude of the sound pressure at unit distance from the source.

Using the approximation \((R_1/r_1)^2 \approx (R_2/r_2)^2\), equation (22) reduces to

\[
p(t) = p_1 \left( \frac{r_1}{R_1} \right)^2 \left\{ \frac{e}{R_1} \left( 1 - \frac{1}{c} \frac{dr_1}{dt} \right) + Q_p \frac{e}{R_2} \right\}
\]

(23)
From equations (18), (3) and Figure 1, one obtains

\[ r_1 = R_1 - \gamma^2 M(x - vt) = R_1 - \gamma^2 M(R_1 \cos \theta - M R_1) = R_1 \gamma^2 (1 - M \cos \theta), \]  

(24)

where \( \theta \) is the emission angle of the sound.

Insertion of (24) into (23) yields

\[ p(t) = \frac{P_1}{(1 - M \cos \theta)^2} \left\{ \frac{e^{-i\theta(t - \frac{R_1}{c})}}{R_1} + \frac{e^{-i\theta(t - \frac{R_2}{c})}}{P R_2} \right\} \]  

(25)

Equation (25) shows that the sound pressure can be expressed in terms of the radiation geometry as measured with respect to the emission position of the source. Apparently, only for a source position where the sound is emitted at overhead (\( \theta = 90^\circ \)) the sound pressure equals the sound pressure from a stationary source (\( M = 0 \)).

\[ p(t) = P_1 \left\{ \frac{e^{-i\theta(t - \frac{R_1}{c})}}{R_1} + \frac{e^{-i\theta(t - \frac{R_2}{c})}}{P R_2} \right\} \]  

(26)

According to equation (25) the sound pressure has the phase \( w(t - R/c) \). Defining the concept of frequency as the time derivative of the phase, the angular frequency becomes

\[ \omega_c = \frac{\text{d}}{\text{d}t} w(t - R/c) = w(1 - \frac{1}{c} \frac{dR}{dt}) \]  

(27)

With the expressions for \( (1 - \frac{1}{c} \frac{dR}{dt}) \) in (23) and (24) and \( w = 2\pi f \), the general Doppler formula is obtained

\[ f_c = \frac{f}{1 - M \cos \theta} \]  

(28)

Obviously, as the sound pressure in (25), also the observed frequency \( f_c \) is simply determined by the instant the sound reaching the receiver was radiated from the source.
5. Sound pressure level

A sound level meter at the receiver position will display the sound pressure level according to

\[ L_p = 10 \log_{10} \left( \frac{P_e}{P_{e_0}} \right)^2 \]  \hspace{1cm} (29)

where \( P_e \) is the r.m.s. value of the sound pressure given by (25) and \( P_{e_0} \) is the reference sound pressure of \( 2 \cdot 10^{-5} \text{ N/m}^2 \).

Thus, for a monopole the modification caused by convection to the sound pressure level of a stationary source is

\[ \Delta L_{PC} = 10 \log \frac{1}{(1 - M \cos \theta)^4} = -40 \log_{10} (1 - M \cos \theta) \]  \hspace{1cm} (30)

This expression indicates that depending on Mach number, the effect of forward motion can give a significant contribution to the directivity of the radiation from real aircraft noise sources. For example, at a Mach number \( M = 0.5 \) the sound pressure level varies from a 12 dB rise at \( \theta = 0^\circ \) to a 7 dB decrease at \( \theta = 180^\circ \).

From equation (25) it also follows that at a given emission angle the difference between the sound pressure level measured at the receiver and the sound pressure level at the receiver which would occur in the free field, is given by

\[ \Delta L_{Pg} = 10 \log_{10} \left\{ 1 + \left( \frac{R_1}{R_2} \right)^2 \left| Q_p \right|^2 + 2 \frac{R_1}{R_2} \left| Q_p \right| \cos (k \Delta R + \beta) \right\} \]  \hspace{1cm} (31)

where \( k = \omega c / c \) is the propagation constant in air.

To include a change of amplitude as well as phase on reflection in (31) a complex notation for the reflection coefficient is used

\[ Q_p = \left| Q_p \right| e^{i \beta} \]  \hspace{1cm} (32)
From equation (31) it appears that the effects of the ground are mainly determined by the angle of incidence and frequency dependent reflection coefficient. Hence, depending on the amplitude of $Q_p$, the reflected sound can change the sound pressure level by any value between a 6 dB rise and negative infinity.

Equation (21) provides a representation of the reflection coefficient in terms of the acoustic impedance of the ground surface. In turn, a prediction of ground reflection effects occurring above a particular test site requires knowledge of the impedance over the frequency range of interest.

However, the results from impedance measurements so far show a large variance in data scattering [6]. Further, these data are not available for all the types of ground surfaces used for aircraft noise testing. Therefore, future research must include the development of methods to determine the impedance values in direct conjunction with the actual noise measurements.

6. References


FIGURE 1: Geometry of source-receiver coordinate system.