Bio-inspired locomotion of a rotating cylinder pair

Guido Novati

10th of October, 2014
Bio-inspired locomotion of a rotating cylinder pair

Master of Science Thesis

For obtaining the degree of Master of Science in Mechanical Engineering at Delft University of Technology

Guido Novati

10th of October, 2014

P&E report number 2650
Summary

In this work, we developed bio-inspired reduced-order models of swimmers, consisting of a self-propelling pair of rotating cylinders. The aim of the project is twofold. First, simplified and non-deforming geometries can more easily be employed in small-scale robotic applications to solve relevant engineering problems. Second, they can serve as reduced physics models to efficiently simulate fluid-mediated interactions in schools of swimmers and perform learning studies involving multiple swimmers.

In the first half of the thesis, we investigate the self-propulsion regimes of a pair of counter-rotating cylinders. For low rotation rates, the cylinders behave like a vortex dipole and the flow is characterized by an elliptical closed streamline surrounding the cylinders. For intermediate rotation rates, the cylinders move in the opposite direction and each has a different set of closed streamlines. Further increasing the rotation rate, the motion of the pair becomes unstable. We systematically explore the phase space defined by the non-dimensional centre-to-centre distance and the rotational Reynolds number, and find inverted exponential correlations that describe the transition between states.

In the second half, we design three different locomotory modes of the cylinder pair, with few degrees of freedom, inspired by the movement of undulatory fish and jellyfish. The parameter spaces were explored with the CMA-ES stochastic optimization algorithm in order to find the best solutions in terms of maximum speed and efficiency. The undulatory fish-inspired motion achieves propulsion by shedding vorticity in a sequence of alternating sign vortices, similarly to its biologic counterpart. The jellyfish-inspired motion during each period sheds a vortex dipole, which generates a strong momentum flux in its wake and thus a large thrust, the definition of the swimming mode of jet-propelled oblate medusae.
First of all, I would like to thank Petros Koumoutsakos for inviting me to Zurich, giving me the opportunity of undertaking this project and for creating such a challenging and inspiring work environment at the CSE laboratory.

I’m extremely grateful to Wim van Rees for his precious guidance and collaboration, for all he taught me and his valuable comments on this thesis. I would also like to thank all my friends of the CSE laboratory, for all the time we spent together at the lab and at the pub.

I would like to thank Jerry Westerweel, for recommending me to work in Petros’ group in the first place and being the chairman of the thesis committee. I’m also very grateful to Daniel Tam. When I came back to Delft, I barged in his office unannounced with an half-written thesis and a lot of ideas, but he calmly gave me all the necessary indications to get everything done on time.

None of this would be possible without the support of my family, to whom this thesis is dedicated. I could never thank enough my parents and my sister for their patience with the scarcity of communications, and the trusting respect for my studies.

Most of all, I’m grateful to Costanza, my link to reality during the past two years.

I gratefully acknowledge the IDEA League for the scholarship that financially supported my stay in Zurich.

Delft, The Netherlands

10th of October, 2014

Guido Novati
## Contents

Summary iii

Acknowledgements v

List of Figures xiii

1 Introduction 1

2 Mathematical Methods 3
  2.1 Governing equations 3
  2.2 Remeshed particle method 4
    2.2.1 Remeshing 6
    2.2.2 Solid boundary conditions with Brinkman penalization 8
    2.2.3 Comparison with published results 9
  2.3 Evolution Strategy with Covariance Matrix Adaptation 12

3 Self-propulsion of a counter-rotating cylinder pair 17
  3.1 Introduction 17
  3.2 Problem setup 18
    3.2.1 Problem description 18
    3.2.2 Computational setup 19
  3.3 Results 20
    3.3.1 Effect of the Reynolds number for a fixed non-dimensional width $W^* = 2$ 20
    3.3.2 Effect of the non-dimensional width for a fixed Reynolds number $Re_T = 100$ 25
    3.3.3 Phase-space investigation 26
### 3.4 Discussion

- 3.4.1 Streamlines .................................................. 27
- 3.4.2 Tracer particles .............................................. 31
- 3.4.3 Effect of changing the angular velocity from steady-state ... 31
- 3.4.4 Instability ................................................... 32

### 3.5 Concluding remarks ........................................... 35

### 4 Optimal self-propulsion of simplified swimmers ............. 37

#### 4.1 Introduction .................................................. 37

#### 4.2 Problem description .......................................... 39

#### 4.3 Formulation of the optimization problem .................. 41
  - 4.3.1 Definition of the cost function ......................... 41
  - 4.3.2 Optimization variables .................................. 42
  - 4.3.3 Computational setup ..................................... 44

#### 4.4 Results ......................................................... 45
  - 4.4.1 Propulsion of a pair of cylinders with oscillating angular velocities ........................................ 45
  - 4.4.2 Propulsion of a pair of counter-rotating cylinders with pulsation along the mid-line ..................... 49
  - 4.4.3 Propulsion of a pair of counter-rotating cylinders at a fixed distance ........................................ 51

#### 4.5 Variations of the parameters ............................... 56
  - 4.5.1 Wavelike motion .......................................... 56
  - 4.5.2 Pulsating symmetric motion ............................. 57
  - 4.5.3 Rigid symmetric motion .................................. 59

#### 4.6 Discussion ..................................................... 61
  - 4.6.1 Comparison of the wavelike motion of the cylinder pair to anguilliform swimmers ......................... 62
  - 4.6.2 Comparison of the wave-like swimmers to the jet-like swimmers ............................................ 64

#### 4.7 Concluding remarks .......................................... 65

### 5 Conclusions & Outlook .......................................... 69

#### 5.1 Conclusions ................................................... 69

#### 5.2 Outlook ......................................................... 71

### References ....................................................... 73

### A Fitness function evolution for the self-propulsion optimization 79

### B Geometry and motion of the anguilliform swimmers .......... 85
List of Figures

2.1 The $M'_4$ interpolation kernel. ............................... 6

2.2 Evolution of the drag coefficient for $Re = 40-1000-3000$ for various resolutions on the left, the reference data is taken from [29], [32], and [44] respectively. On the right, wavelet-adapted grid for the highest resolution near the cylinder. The colors refer to the vorticity field. .......................... 10

2.3 On top, evolution of the drag (left) and lift (right) coefficient for $Re = 200$ at different angular to rectilinear speed ratios $\alpha$. The square-shaped data points indicate the reference solution by Chen et al. [10]. Below, vorticity field for $T = 24$. ............................... 11

3.1 Sketch of the problem setup ...................................... 18

3.2 Close-up of the initial wavelet-adapted grid with the two cylinders colored in red and blue respectively ............................... 20

3.3 Velocity of the counter-rotating cylinder pair at $W^* = 2$ at a series of Reynolds numbers up to $Re_\Gamma = 150$ (left). The simulations are stopped whenever the velocity does not change more than 0.03% in the last time unit, and extended with a dashed line to indicate the steady-state value. ............................... 21

3.4 Vorticity field of the counter-rotating cylinder pair at $W^* = 2$ at $Re_\Gamma = 50$ (top) and $Re_\Gamma = 150$ (bottom), at different times. The initial location of the cylinder pair is drawn with dashed black circles. ............................... 21

3.5 Velocity of the counter-rotating cylinder pair at $W^* = 2$ at a series of Reynolds numbers up to $Re_\Gamma = 750$. ............................... 22

3.6 Vorticity field of the counter-rotating cylinder pair at $W^* = 2$ at $Re_\Gamma = 500$, at different times. The initial location of the cylinder pair is drawn with dashed black circles. ............................... 23

3.7 Vorticity field of the counter-rotating cylinder pair at $W^* = 2$ at $Re_\Gamma = 750$, at different times. The initial location of the cylinder pair is drawn with dashed black circles. ............................... 24
<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.8</td>
<td>Force along the midline of the counter-rotating cylinder pair for $W^* = 2$ for a series of Reynolds numbers up to $Re_\Gamma = 625$.</td>
<td>24</td>
</tr>
<tr>
<td>3.9</td>
<td>Velocity of the counter-rotating cylinder pair at $Re_\Gamma = 100$ for a series of non-dimensional widths $W^*$. For the left plot, simulations are stopped whenever the velocity does not change more than 0.03% in the last time unit, and extended with a dashed line to indicate the steady-state value.</td>
<td>25</td>
</tr>
<tr>
<td>3.10</td>
<td>Transition between upward, dipole-like motion (upward triangles) and downward, jet-like motion (downward triangles) as a function of $Re_\Gamma = \Gamma/\nu$ and $W^* = W/D$. The crosses denote unstable motion.</td>
<td>26</td>
</tr>
<tr>
<td>3.11</td>
<td>Line-integral convolution [4] of the steady-state velocity field for $Re_\Gamma = 50$ (left) and $Re_\Gamma = 150$ (right) for $W^* = 2$, computed in the moving frame of reference of the cylinders. The cylinders are marked by dotted white lines, closed streamlines around the cylinders by dashed black lines and the colors refer to the vorticity field.</td>
<td>28</td>
</tr>
<tr>
<td>3.12</td>
<td>On the left, velocity of the cylinder pair at $Re_\Gamma = 10$ for a series of $W^<em>$. From blue to green: $W^</em> = [1.125, 1.25, 1.5, 1.75, 2.0, 2.25, 2.5, 3.0, 3.5, 4.0, 5.0]$. On the right, log-log plot of the steady-state velocity against the distance $W^*$ between the cylinders for various $Re_\Gamma$ compared to the velocity for a ideal vortex dipole. Dashed lines represent the hyperbolic interpolation.</td>
<td>28</td>
</tr>
<tr>
<td>3.13</td>
<td>Tracer particles advected by the steady-state velocity field ($Re_\Gamma = 50$ and 150, $W^* = 2$). The particles are seeded in closed streamlines around the counter-clockwise rotating cylinder (red), the clockwise rotating cylinder (blue), and in a strip upstream of the pair (black).</td>
<td>30</td>
</tr>
<tr>
<td>3.14</td>
<td>Tracer particles advected by the velocity field starting at rest for $Re_\Gamma = 50$, $Re_\Gamma = 150$, and $Re_\Gamma = 500$ ($W^* = 2$). The particles are seeded in an ellipsoidal strip upstream of the cylinders (black).</td>
<td>30</td>
</tr>
<tr>
<td>3.15</td>
<td>Velocity of the cylinder pair starting at $W^* = 2$ and $Re_{\Gamma,0} = 80$ on the left and $Re_{\Gamma,0} = 90$ on the right. The angular velocity is then changed over $T_F - T_0 = 10$ with a half-cosine wave.</td>
<td>32</td>
</tr>
<tr>
<td>3.16</td>
<td>Vorticity field of the counter-rotating cylinder pair at $W^* = 5$ for $Re_\Gamma = 300$ (top) and $Re_\Gamma = 500$ (bottom) at different times. The initial location of the cylinder pair is drawn with dashed black circles.</td>
<td>33</td>
</tr>
<tr>
<td>3.17</td>
<td>Vertical velocity of the cylinders with $W^* = 3$ (left) and $W^* = 5$ for Reynolds numbers around the instability.</td>
<td>33</td>
</tr>
<tr>
<td>3.18</td>
<td>Line-integral convolution field [4] of the velocity around the cylinders with $W^* = 5$ at $Re_\Gamma = 300$ (left) and $Re_\Gamma = 500$ (right), computed in the moving frame of reference of the cylinders. The cylinders are marked by dotted white lines, closed streamlines around the cylinders by dashed black lines and the colors refer to the vorticity field.</td>
<td>34</td>
</tr>
<tr>
<td>3.19</td>
<td>Displacement (left, non-dimensionalized with the diameter $D$) and velocity (right) for the cylinders with $W^* = 3$ and $Re_\Gamma = 1000$.</td>
<td>35</td>
</tr>
<tr>
<td>3.20</td>
<td>Vorticity field of the counter-rotating cylinders at $W^* = 3$ and $Re_\Gamma = 1000$.</td>
<td>35</td>
</tr>
</tbody>
</table>
4.1 On the left, variables of the wavelike motion: two amplitudes of the angular velocity $\Omega_R$ (right cylinder) and $\Omega_L$ (left cylinder), center-to-center distance $W$, and phase $\phi$ between the two harmonic functions. On the right, variables of the pulsating symmetric motion: pulsation amplitude $A$, minimal surface-to-surface distance $G$, angular velocity amplitude $\Omega$, and phase $\phi$ between the two harmonic functions.

4.2 On the left, number of revolutions of each cylinder with respect to the initial configuration $\theta/(2\pi)$ and angular velocity of the left cylinder compared to the in-vivo measurements (blue curve) by Nawroth et al. [39] of the angular velocity of jellyfish bell lobes. On the right, sketch of the rigid symmetric motion.

4.3 Illustration of the fastest wavelike motion. On the left, horizontal and vertical velocity (the dashed line indicates the average forward velocity over the last period). On the right, from the top: efficiency over a running window of one period, average and instantaneous inclination of the midline, and angular velocities of the cylinders.

4.4 Illustration of the most efficient wavelike motion. On the left, horizontal and vertical velocity on top (the dashed line indicates the average forward velocity over the last period). On the right, from the top: efficiency over a running window of one period, average and instantaneous inclination of the midline, and angular velocities of the cylinders.

4.5 Comparison of the fastest (left) and efficient (right) motions: the colours refer to the vorticity field and the dashed black lines denote the initial positions.

4.6 Illustration of the fastest pulsating symmetric motion. On the left, vertical velocity on top (the dashed line indicates the average forward velocity over the last period), the velocity of the pulsating motion along the mid-line of the cylinder, and the angular velocities of the cylinders. On the right we show the efficiency over a running window of one period.

4.7 Illustration of the efficient pulsating symmetric motion. On the left, vertical velocity on top (the dashed line indicates the average forward velocity over the last period), the velocity of the pulsating motion along the mid-line of the cylinder, and the angular velocities of the cylinders. On the right we show the efficiency over a running window of one period.

4.8 Comparison of the vorticity field around the efficient (top) and fastest (bottom) symmetric motions. The dashed lines denote the current location of the cylinders.

4.9 Kinematics of the fastest rigid symmetric motion for $W^* = 2.25$, case $F1$. On the left, velocity (dashed is the average over one period of motion), on the right, efficiency over a running window of one period.

4.10 Time-development of the vorticity field for case $F1$.

4.11 Kinematics of the rigid symmetric motion for case $F1$. On the left, vertical velocity (dashed is the velocity averaged over one period of motion), on the right, efficiency over a running window of one period.
4.12 Time-development of the vortex shed by the power stroke of the rigid symmetric motion for case $F_2$. The power stroke starts for $t_0 + T/10$ 54

4.13 Tracer particles are initialized in the closed streamline around the cylinders during the recovery stroke and passively advected by the flow for case $F_2$, highlighting the shedding of the vortex pair. 54

4.14 Kinematics of case $E$. On the left, vertical velocity (dashed is the velocity averaged over one period of motion), on the right, efficiency over a running window of one period. 55

4.15 Time-development of the vortex shed by the power stroke of the rigid symmetric motion for case $E$. 55

4.16 Effect of the variation of the optimization parameters with respect to the fastest wavelike motion on the speed, efficiency, and angular velocity (the latter averaged over the last 10 periods). The 6th panel shows the comparison of the trajectories with that of the optimum. 56

4.17 Effect of the variation of the optimization parameters with respect to the efficient wavelike motion on the speed, efficiency, and angular velocity (the latter averaged over the last 10 periods). The 6th panel shows the comparison of the trajectories with that of the optimum. 57

4.18 Effect of the variation of the optimization parameters on the speed and efficiency with respect to the fastest symmetric motion. 58

4.19 Effect of the variation of the optimization parameters on the speed and efficiency with respect to the efficient symmetric motion. 58

4.20 Effect of the variation of the peak rotational Reynolds numbers of the power stroke $Re_{\Gamma,1}$ and of the recovery stroke $Re_{\Gamma,2}$ with respect to the fastest (on top) and most efficient (below) symmetric rigid motion. 60

4.21 Effect of the variation of the rotational Reynolds numbers with respect to the fastest symmetric rigid motion with $W^* = 2.25$ on the speed and efficiency. 61

4.22 Comparison of anguilliform swimmer shapes A and B with the cylinder pair at the optimal width for fastest wavelike motion. 62

4.23 Comparison of the lateral (thin line) and forward (thick line) velocities (on the left) and efficiencies (on the right) of the wavelike swimmers. 63

4.24 Comparison of the vorticity field around the simplified swimmers (cylinder pair optimized for speed on the top and efficiency on the bottom), and anguilliform swimmer shapes A and B. 63

4.25 Passive tracer particles advected by the velocity field around the anguilliform swimmer shapes and wavelike motion of the cylinder pair optimized for speed. When the swimmers are moving at steady state, the particles are seeded in a rectangular stip located upstream of their trajectory and coloured in black if displaced by the velocity field. 64

4.26 Comparison of the propulsive performance of the rigid symmetric motion (jet-like motion) and the wavelike motion. The numerical data refers to the fastest solution for the wavelike motion and case $P$ of the rigid symmetric motion. 66
A.1 Evolution of the cost function and optimization variables against number of optimization iterations for the fastest horizontal motion. Blue dots correspond to current generation samples, the blue line denotes the current mean of the distribution, the green one corresponds to the best current solution, and the black one corresponds to the best solution ever. The bounds on the plots for the optimization variables correspond to the bounds chosen for the optimization.

A.2 Evolution of the cost function and optimization variables against number of optimization iterations for the efficient vertical motion. Blue dots correspond to current generation samples, the blue line denotes the current mean of the distribution, the green one corresponds to the best current solution, and the black one corresponds to the best solution ever. The bounds on the plots for the optimization variables correspond to the bounds chosen for the optimization.

A.3 Evolution of the cost function and optimization variables against number of optimization iterations for the fastest vertical motion. Blue dots correspond to current generation samples, the blue line denotes the current mean of the distribution, the green one corresponds to the best current solution, and the black one corresponds to the best solution ever. The bounds on the plots for the optimization variables correspond to the bounds chosen for the optimization.

A.4 Evolution of the cost function and optimization variables against number of optimization iterations for the efficient vertical motion. Blue dots correspond to current generation samples, the blue line denotes the current mean of the distribution, the green one corresponds to the best current solution, and the black one corresponds to the best solution ever. The bounds on the plots for the optimization variables correspond to the bounds chosen for the optimization.
Chapter 1

Introduction

Natural swimmers achieve aquatic locomotion by displacement of fluid through periodic body deformations, such as mid-line undulations in fish or body expansion and contraction in jellyfish. These modes of swimming transfer momentum into the wake and generate unsteady locomotive forces that alternate periods of thrust and drag [11, 45, 49, 54]. Natural selection assures that the physical mechanisms underlying these forms of propulsion, although not necessarily optimal [51], are very efficient considering non-hydrodynamic constraints. For this reason natural swimmers are widely studied and used for the design of biomimetic swimming robots [34, 39, 48].

In this thesis we consider bio-inspired reduced-order models of swimmers, consisting of a self-propelling pair of rotating cylinders. These models are developed with a double intent. First, simplified and non-deforming geometries can more easily be employed in small-scale robotic applications to solve relevant engineering problems. Second, they can serve as reduced physics models to efficiently simulate fluid-mediated interactions in schools of swimmers and perform learning studies involving multiple swimmers. In fact, computational costs limit studies of fish schooling to potential models [47], however recent studies [19] demonstrated that the interaction of moving objects at finite Reynolds numbers is drastically different from the interaction in potential flow. Moreover, the interaction of viscous and inertial forces in the locomotion at moderate Reynolds number is not well understood [14]. The simple non-deforming geometries considered in this thesis, which can be coupled with body-fitted meshes or overlapping meshes, can be cheaply simulated and could allow schooling studies in viscous flow.

Rather than fixing the centre of mass and imposing a constant free stream velocity, the self-propulsion of the cylinders is due to the interaction with the surrounding fluid. For each model we prescribe the rotation rate or the relative motion of the cylinders, but the externally prescribed component of the motion has zero total linear and angular momentum all times.
In chapter 3, we consider the self-propulsion of two steadily counter-rotating cylinders linked together at a fixed distance by their centres. The propulsion of this system is obtained purely through viscous effects, as momentum is diffused from the surface of the cylinders. We systematically explore the phase space defined by the non-dimensional centre-to-centre distance and the rotational Reynolds number, and show the underlying fluid dynamics using streamlines and tracer particles.

In chapter 4, we present different parameterizations of periodic motions of two rotating cylinders with few degrees of freedom, that could produce both jet-like and wave-like flow patterns. The aim is the realization of self-propulsion states that can be compared to jellyfish-like and undulatory swimmers respectively. The reduced number of degrees of freedom of the models allows the relatively inexpensive application of optimization algorithms in order to find the fastest and most efficient combinations of motion parameters.

We approach the problem computationally: the simulations were performed with a wavelet-adapted remeshed vortex method [42, 43, 52] and the dynamic coupling between the fluid and the self-propelling bodies are done with the Brinkman volume penalization and projection approach [1]. The parameter space of the self-propelling test-cases is explored with the covariance matrix adaptation evolutionary strategy (CMA-ES), an efficient and robust stochastic optimization algorithm, able to handle noisy and multi-modal cost functions [21, 28]. The mathematical methods are discussed in chapter 2.
In this chapter we will introduce the remeshed vortex method as a scheme for direct numerical simulation of incompressible viscous flow and the Evolution Strategy with Covariance Matrix Adaptation (CMA-ES) optimization algorithm.

### 2.1 Governing equations

The Navier-Stokes (N.S.) equations is a set of partial differential equations (PDE) that describes the evolution of fluid flow inside a domain, given the initial and boundary conditions. All the simulations presented in this work consider the incompressible flow of a Newtonian fluid:

\[
\begin{align*}
\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} &= -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{u}, & \mathbf{x} \in \Omega \\
\nabla \cdot \mathbf{u} &= 0, & \mathbf{x} \in \Omega
\end{align*}
\]

(2.1)

where $\Omega$ is the fluid domain and the three independent fields are: the density $\rho$, the velocity $\mathbf{u}$, and the pressure $p$.

Equation 2.1 is also referred to as the velocity-pressure form of the N.S. equations. The velocity-vorticity formulation of the N.S. equations is obtained by taking the curl of equation 2.1:

\[
\frac{\partial \boldsymbol{\omega}}{\partial t} + (\mathbf{u} \cdot \nabla) \boldsymbol{\omega} = (\boldsymbol{\omega} \cdot \nabla) \mathbf{u} + \nu \nabla^2 \boldsymbol{\omega}, \quad \mathbf{x} \in \Omega
\]

(2.2)

Where $\boldsymbol{\omega} = \nabla \times \mathbf{u}$ is the vorticity field. The continuity equation for an incompressible flow ($\nabla \cdot \mathbf{u} = 0$) is identically satisfied by the introduction of the vector streamfunction:

\[
\mathbf{u} = \nabla \times \Psi
\]

(2.3)
Since the divergence of a curl is always zero. Taking the curl of equation 2.3 we obtain a relation between the vorticity and the streamfunction, such that the resultant flow field is divergence-free:

\[ \nabla \times \nabla \times \Psi = \nabla (\nabla \cdot \Psi) - \nabla^2 \Psi = \omega \]  

(2.4)

In order to further simplify the above equation, through Helmholtz decomposition, we can require the streamfunction vector \( \Psi \) to be solenoidal (divergence-free) and obtain:

\[ \nabla^2 \Psi = -\omega \]  

(2.5)

The velocity field is recovered through the streamfunction \( \Psi \) from the vorticity field by solving the Poisson problem (eq. 2.5) with appropriate boundary conditions.

Solving the velocity-vorticity formulation offers a number of advantages over solving the velocity-pressure form. First, vorticity has in general a more compact support than velocity\(^1\). Second, this formulation eliminates pressure from the unknowns in the equations. However, it requires the transformation of the velocity boundary conditions to vorticity form [12].

### 2.2 Remeshed particle method

Numerical techniques for the solution of the transport equations can be classified as Eulerian, Lagrangian, or mixed Lagrangian-Eulerian.

In the conventional Eulerian approach, the evolution of the system is solved on a stationary grid, employing discretization schemes such as finite volumes or finite elements. The discretization on a fixed grid of the advection term of the transport equations may cause numerical dissipation and oscillations if the conditions for stability are not met. Specifically, the necessary condition for the convergence of a finite difference method applied to hyperbolic PDE is known as the Courant-Friedrichs-Levy (CFL) condition, which for a one-dimensional convection equation discretized with a first order explicit scheme reads:

\[ CFL = \frac{\|u\|\infty \Delta t}{\Delta x} \leq 1 \]  

(2.6)

This implies that the time-step size \( \Delta t \) is related to the grid spacing \( \Delta x \), and, for advection-dominated problems, a finer spacial refinement constrains the time step more severely.

In Lagrangian methods, such as vortex methods, the transport equation is not solved on a static grid, but on moving computational elements. These computational elements

\(^1\)In an unbounded or periodic domain, vorticity is a local function of velocity, however velocity is a global function of the vorticity. Furthermore, in an incompressible flow, for which there are no density gradients, vorticity is only created at solid boundaries.
elements—or particles—carry the dependent quantities of the flow field and advective transport of these quantities is simulated simply by tracking the particles along their trajectory. For this reason, particle methods exhibit relaxed stability conditions for advection-dominated flows and are not subject to the linear CFL condition, the time-step being only related to the flow quantities rather than the grid spacing.

From a mathematical point of view, particle methods allow the approximation of smooth functions, such as the vorticity, by integrals that are discretized as Lagrangian computational elements called particles. A set of particles \( \{p\} \) have position \( \{x_p\} \), volume \( \{V_p\} \), and carry a vorticity with strength \( \{\omega_p\} \). The vorticity field can be recovered at every location of the domain with the reconstruction:

\[
\omega^h_\epsilon(x) = \sum_p \omega_p V_p \zeta_\epsilon(x - x_p) \tag{2.7}
\]

where \( \zeta_\epsilon \) is the reconstruction kernel characterized by smoothing length \( \epsilon \), which can be interpreted as the size of the particles, and \( h \) is the inter-particle distance.

These two parameters determine the two sources of error of the discretization. The kernel size \( \epsilon \) determines the physical scales of the flow quantities that can be resolved \[31\]. The associated mollification error is of the order of \( \mathcal{O}(\epsilon^r) \). The coefficient \( r \) denotes the order of the first non-vanishing moment of the reconstruction kernel \( \zeta_\epsilon \), hence the first source of error can be controlled by an appropriate choice of \( \zeta_\epsilon \). The inter-particle distance \( h \) determines the number of particles that support the spatial discretization. The associated error is linked to the accuracy of the numerical integration of the vorticity field through the reconstruction kernel and is of the order of \( \mathcal{O}(h/\epsilon)^m \), where \( m \) depends on the quadrature error. In order to ensure a sufficient sampling of \( \zeta_\epsilon \), the inter-particle distance must be smaller than the kernel support size \( (h/\epsilon < 1) \). This condition is referred to as the overlapping condition \[41\].

To increase the computational efficiency, vortex methods can be made to adapt the particles in the regions of the flow where an increased (or reduced) resolution is necessary (or sufficient) to adequately capture the gradients of the flow field, such as in boundary layers and wakes. The smoothing length \( \epsilon \) needs to be reduced to resolve smaller scales, and the inter-particle distance must be adapted accordingly in order to maintain the overlapping condition.

The time evolution of the system is simulated through the solution of ODEs, which are derived from the Lagrangian formulation of the Navier-Stokes equations. These equations determine the trajectories of the particles and the evolution in time of the carried quantities. These ODEs can be written as:

\[
\begin{align*}
\frac{dx_p}{dt} &= u(x_p, t) \\
\frac{d\omega_p}{dt} &= [\omega_p \cdot \nabla] u(x_p, t) + \nu \Delta \omega(x_p, t)
\end{align*}
\]
The equations require initial conditions, in addition to the no-slip condition in the presence of solid boundaries. Time integration according to these equations maintains the close link to the underlying physics of the fluid of the vortex method. In fact, the inviscid advection of vorticity-carrying particles is consistent with Kelvin’s circulation theorem, which, for an inviscid and incompressible fluid, states that the circulation in material elements is conserved.

2.2.1 Remeshing

The adaptivity of the computational elements comes at the expense of the loss of regularity in the particle distribution. As the particles are advected by the flow field the overlapping condition can be violated, leading to insufficient sampling of the kernel \( \zeta \), and thus the reconstructed vorticity field fails to converge to the exact solution.

To counter this problem, we introduce an underlying regular grid so that particles can be initialized at the grid point locations. After each time-step, the location of the distorted particle field is reprojected on a new set of particles and the carried quantities are recomputed by accurately interpolating the weights. The remeshing operation introduces additional numerical dissipation, but the dominant dissipative error of the scheme is introduced by the time discretization [30]. The choice of kernel depends on the nature of the problem: for the application to direct numerical simulation of turbulent flow, the use of an interpolating kernel would minimize the introduction of additional numerical diffusion. For compressible flow, however, an interpolating kernel would introduce spurious oscillations and shock waves near discontinuities in the density and velocity [2]. For the application to incompressible flow, the third-order accurate \( M'_4 \) function derived by Monaghan [35] is widely used. The carried quantities \( \{ \tilde{\omega} \} \) of the distorted particle set located at \( \{ \tilde{x} \} \) are

![Figure 2.1: The \( M'_4 \) interpolation kernel.](image-url)
interpolated to the normalized particle locations \( \{ x \} \). The new quantities \( \{ \omega \} \) are computed as:

\[
\omega(x_i) = \sum_{j=1}^{N} \tilde{\omega}_j \cdot M_4' \left( \frac{|x_i - \tilde{x}_j|}{h} \right)
\]

(2.9)

\[
M_4'(|\xi|) = \begin{cases} 
\frac{1}{2}(|\xi| - 1)(3|\xi|^2 - 2|\xi| - 2) & \text{if } |\xi| < 1, \\
-\frac{1}{2}(|\xi| - 1)(|\xi| - 2)^2 & \text{if } 1 \leq |\xi| < 2, \\
0 & \text{if } 2 \leq |\xi|. 
\end{cases}
\]

(2.10)

where \( h \) is the spacing of the underlying grid. An interpolation kernel with compact support limits the number of particles that contribute to the quantities at the nodes to those in its vicinity, which improves the speed of the computations, but decreases the accuracy of the interpolation. However, the third order accurate \( M_4' \) function conserves the 0\(^{th}\), 1\(^{st}\), and 2\(^{nd}\) order moments\(^2\) of the reconstructed field.

Besides guaranteeing convergence, the presence of an underlying regular grid introduces a number of benefits that increase the computational efficiency of the remeshed vortex method. First, the grid is used to efficiently evaluate differential operators, providing a significant advantage over pure particle methods that perform these operations directly on the particles [31]. Second, for an uniform Cartesian grid, fast Poisson solvers can be used to compute the velocity field. Third, a grid can be locally refined or compressed by employing wavelet-based adaptivity, adding or removing particles according to the local scales of the system. While a regular grid allows the use of FFT-based elliptic solvers [9], the different spacings of a multi-resolution grid reduce the efficiency of such solvers. For this reason, the computational method uses a multipole solver to reconstruct the velocity from the vorticity field, which has the advantage of naturally treating free-space boundary conditions [52]. The wavelet-based adaptivity introduces two additional parameters to the solver: the threshold for the grid refinement operation \( r_{tol} \) and the tolerance for the grid compression \( c_{tol} \).

In order to ensure the overlapping condition, assuming the remeshing operation is performed for each time step, the size of the time step is limited by the displacement of the particles. If \( C \) is a measure of the maximum distance between two neighbouring particles, for a one-dimensional case we have:

\[
\Delta t |\dot{h}| \leq C
\]

(2.12)

\(^2\)For a two-dimensional case the 0\(^{th}\), 1\(^{st}\) and 2\(^{nd}\) moments of a field are:

\[
M_0 = \sum_{i=1}^{N} w_i, \quad M_0^0 = \sum_{i=1}^{N} w_i x_i, \quad M_1^1 = \sum_{i=1}^{N} w_i y_i, \\
M_2^0 = \sum_{i=1}^{N} w_i x_i^2, \quad M_2^1 = \sum_{i=1}^{N} w_i x_i y_i, \quad M_2^2 = \sum_{i=1}^{N} w_i y_i^2
\]

(2.11)
where $|\dot{h}|$ is the rate of growth of the inter-particle distance. Since the constant $C$ depends on the kernel, which takes an argument that is normalized by the grid spacing $h$, it can be expressed as $C = C_1 h$. Moreover, $|\dot{h}|$ depends on the relative velocity of two adjacent particles $|\dot{h}| = |u_{i+1} - u_i| = h|\nabla u|$. From these considerations, for a general case, it follows:

$$\Delta t \leq C_1 \|\nabla \otimes u\|^{-1} \approx C_2 \|\omega\|^{-1}$$

(2.13)

This inequality is referred to as the Lagrangian CFL condition ($LCFL \equiv \Delta t\|\nabla \otimes u\|_\infty$) [17].

### 2.2.2 Solid boundary conditions with Brinkman penalization

In order to enforce the no-slip condition at the solid interface, we consider the Brinkman volume penalization [1]. The exchange of momentum between the fluid and the solid is represented by a local modification of the governing equations for the fluid. The solid is modelled as a porous medium and the fluid is extended inside the solid domain $\Sigma$. The Navier-Stokes equations with the Brinkman penalization term can be written as:

$$\begin{cases}
\frac{\partial u_\lambda}{\partial t} + (u_\lambda \cdot \nabla)u_\lambda = -\frac{1}{\rho} \nabla p_\lambda + \nu \nabla^2 u_\lambda + \lambda \chi_s (U_s - u_\lambda), & x \in \Sigma \\
\nabla \cdot u_\lambda = 0, & x \in \Sigma
\end{cases}$$

(2.14)

where $\lambda \gg 1$ is the penalization factor, and $\chi_s$ is the characteristic function of the solid. For $\lambda \to \infty$ the velocity of the fluid at the solid boundary will be $U_s$.

Furthermore, as shown by [5], for a finite value of $\lambda$, the error in the penalized solution is bounded by:

$$\|\tilde{u} - u_\lambda\| \leq C\lambda^{-\frac{1}{2}}\|\tilde{u}\|$$

(2.15)

where $\tilde{u}$ denotes the exact solution. However, a high value of $\lambda$ leads to a stiffer set of equations, and thus a strict limit on the time step. For moderate Reynolds numbers ($Re < 10^4$), having $\delta t \cdot \lambda \sim O(1)$ and $\lambda \in [10^4, 10^5]$, was found to be a good compromise between computational efficiency and accuracy [41, 17].

The function $\chi_s$ individuates the geometry by taking unitary value inside the obstacle and a zero value outside:

$$\chi_s(d) = \begin{cases} 1 & \text{if } d < -\epsilon, \\
\frac{1}{2} \left[ 1 + \cos \left( \frac{\pi (d + \epsilon)}{2\epsilon} \right) \right] & \text{if } -\epsilon \leq d \leq \epsilon, \\
0 & \text{if } d > \epsilon.
\end{cases}$$

(2.16)

where $d$ is the signed distance from the body surface (negative inside), and $\epsilon$ is the mollification length. This length also defines the size of the smallest features of the
solid that can be resolved. Previous work by [17] found that, for moderate Reynolds numbers ($Re < 10^4$), $\epsilon$ should allow the mollified characteristic function to span 4-5 grid points, while still remaining a small fraction ($< 1\%$) of the characteristic length of the solid.

We can obtain the velocity-vorticity formulation of the Navier Stokes with Brinkman penalization by taking the curl of equation 2.14:

$$\frac{\partial \omega_\lambda}{\partial t} + u_\lambda \cdot \nabla \omega_\lambda = (\omega_\lambda \cdot \nabla) u_\lambda + \nu \nabla^2 \omega_\lambda + \lambda \times (U_s - u_\lambda) \chi_s$$

(2.17)

The velocity $U_s$ of the solid object having center of mass located at $x_{CM}$ is decomposed into three contributions:

$$U_s = U_t + \Omega_s \times (x - x_{CM}) + U_{DEF}$$

(2.18)

where $U_t$ is the translation velocity, which, along with the angular velocity $\Omega_s$, describes the rigid body motion that arises from the hydrodynamic forces exchanged with the fluid. The deformation velocity $U_{DEF}$ is externally imposed.

The forces acting on the solid body are computed by integrating the penalization term

$$F = \lambda \int_\Sigma \chi_s (u_\lambda - U_s) \, dx$$

(2.19)

More information on the computational method can be found in [17, 41, 43, 52].

2.2.3 Comparison with published results

Flow past impulsively started cylinder

The flow past an impulsively started cylinder is one of the most common benchmark unsteady separated flow problems. The cylinder moves at a speed $U$ in the $x$-direction and the Reynolds number is defined as $Re = DU_\infty/\nu$ where $D$ is the diameter of the cylinder and $\nu$ is the kinematic viscosity of the fluid. The dimensional time $t$ is non-dimensionalized as $T = 2U_\infty t/D$. The simulations were carried out in a domain $\Sigma = [0, 1] \times [0, 1]$, the velocity scale is set as $U = 0.1$, and the diameter of cylinder is 0.005. The viscosity of the fluid is obtained as $\nu = \Gamma/Re_T$. For the fluid solver, the time-step is constrained by $LCFL = 0.05$, the refinement and compression thresholds are $r_{\text{tol}} = 10^3$, $c_{\text{tol}} = 10^4$ and the penalization factor is $\lambda = 10^4$.

Here we compare the evolution in time of the drag coefficient $C_D = 2F_x/(DU_\infty^2)$ experienced by the cylinder for various Reynolds numbers $Re = 40 - 1000 - 3000$ to the results obtained by [29], [32], and [44] respectively. Figure 2.2 shows the results obtained for several spatial resolution, measured by the number of grid points over the radius of the cylinder $ppr = ER \cdot D/2$. For $Re = 40$, the resolution $ppr = 41$ was
Figure 2.2: Evolution of the drag coefficient for $Re = 40 - 1000 - 3000$ for various resolutions on the left, the reference data is taken from [29], [32], and [44] respectively. On the right, wavelet-adapted grid for the highest resolution near the cylinder. The colors refer to the vorticity field.
Figure 2.3: On top, evolution of the drag (left) and lift (right) coefficient for $Re = 200$ at different angular to rectilinear speed ratios $\alpha$. The square-shaped data points indicate the reference solution by Chen et al. [10]. Below, vorticity field for $T = 24$.

enough have a discrepancy with respect to the reference solution that was always inferior to 3%, while the lowest resolution $ppr = 5$ yielded an error of 20%. For $Re = 1000$, the discrepancy between the best resolution $ppr = 163$ and the reference solution is at all times less than 2.5%, while up to 17.5% for the lowest resolution $ppr = 20$.

Flow past impulsively started steadily rotating cylinder

A second prototypical test-case for unsteady fluid-structure interaction is the flow past an impulsively started steadily rotating cylinder. This flow problem is closely linked to the other cases considered throughout the thesis, and for this reason serves an useful benchmark for the flow solver. Here we confront our results to the results
obtained by Chen et al. [10], who studied the unsteady flow separation for different angular velocities of the cylinder.

Chen et al. [10] studied the development of the flow past an impulsively started steadily rotating cylinder for $Re = 200$. The cylinder moves at a speed $U$ in the $x$-direction and rotates with angular velocity $\Omega = \alpha U/a$. The Reynolds number is defined as $Re = 2aU/\nu$, where $a$ is the cylinder radius and $\nu$ is the kinematic viscosity of the fluid. The dimensional time $t$ is non-dimensionalized as $T = U_\infty t/a$. The cylinder experiences both a drag and a lift force. The lift ($C_D$) and drag ($C_L$) coefficients are defined as:

$$
C_D = \frac{F_x}{1/2aU_\infty^2}, \quad C_L = \frac{F_y}{1/2aU_\infty^2}
$$

(2.20)

The fluid scales and solver parameters are maintained from the impulsively started cylinder. The effective resolution is set at $ER = 32768$, so that there are over 80 grid points across the radius of the cylinder, which during the previous test was found to be a good compromise between accuracy and computational costs for $40 \leq Re \leq 3000$.

Figure 2.3 shows the obtained results compared with the ones from Chen et al. [10]. The largest discrepancy of 8% was obtained in the drag coefficient for $\alpha = 1$, but otherwise the results were consistent both quantitatively and in the trends. The figure also shows the vorticity plots for $T = 24$ that can be compared to the ones presented in [10].

### 2.3 Evolution Strategy with Covariance Matrix Adaptation

In this section we present the Evolution Strategy with Covariance Matrix Adaptation (CMA-ES), which over many benchmark problems has been demonstrated to be an efficient and robust algorithm suited to handle noisy, non linear, and multi-modal cost functions [20, 23, 51].

Evolutionary algorithms take inspiration from the process by which nature chooses its solutions in order to solve optimization and learning problems. The main strengths of evolutionary algorithms are their adaptability to specific problems, robustness and scalability. In this work, the optimization problems will require the solution of fluid mechanics problems which often have multiple local optima, discontinuities and numerical noise, which are not differentiable and finite differences are not trustworthy. These issues make fluid mechanics problems unsuited for gradient-based optimization algorithms [33]. On the other hand, fluid mechanics problems involve expensive function evaluations, making evolutionary algorithms, which require large numbers of iterations, more expensive than gradient-based methods.
The cost functions considered in this thesis require expensive simulations, but can be computed independently. For this reason, the function evaluations can be distributed across multiple hosts. In this work, following Gazzola et al. [21], the communication between the optimizer and the individual solvers is performed through two files: one containing the parameters of the evaluation and the other containing the corresponding value of the cost function. This, together with the fact that the cost function is treated as a black-box by the CMA-ES, allows great generality and flexibility, making the optimization framework independent from the problem-specific application.

The Evolution Strategy with Covariance Matrix Adaptation was initially proposed by Hansen and Ostermeier in 2001 [24]. The algorithm operates by sampling a multivariate Gaussian distribution with evolving mean and covariance matrix. The evolution of the Covariance matrix around the optimum is analogous to the successive approximations of the Hessian matrix typical of quasi-Newton methods [22]. For each iteration, $\ell$ candidate solutions are generated. Choosing a small size $\ell$ of the population leads to faster convergence, but a large population size helps avoiding local optima.

In this work we employed the algorithm as presented by Hansen and Kern in [23, 27]. The optimization algorithm starts with the generation of an initial population of function evaluations with parameters $X^{(0)} = \{x_1^{(0)}, \ldots, x_{\ell}^{(0)}\}$ according to:

$$x_k^{(0)} \sim \mathcal{N}\left(\langle x \rangle^{(0)}, \sigma^{(0)} I\right), \quad k = 1, \ldots, \ell$$  \hspace{1cm} (2.21)

where $\mathcal{N}(m, C) \in \mathcal{S} \subseteq \mathbb{R}^n$ is a normally distributed random vector in the $n$-dimensional search space with mean $m$ and covariance matrix $C$. The initial value for the mean of the distribution $\langle x \rangle^{(0)}$ and the initial standard deviation for each parameter $\sigma^{(0)}$ are defined according to the problem.

The $\ell$ cost function evaluations of generation $g$ are selected according to:

$$x_k^{(g)} \sim \mathcal{N}\left(\langle x \rangle^{(g)}_w, \sigma^{(g)} C^{(g)}\right), \quad k = 1, \ldots, \ell$$  \hspace{1cm} (2.22)

where $\langle x \rangle^{(g)}_w = \sum_{i=1}^{\mu} w_i x_i^{(g-1)}$ is the recombination point, which is the weighted mean of the $\mu$ best offspring of generation $g - 1$ such that $\sum_{i=1}^{\mu} w_i = 1$. The index notation $i : \ell$ denotes the $i$-th best out of the $\ell$ evaluations. This weighted recombination discussed in [24] is considered more natural than the intermediate recombination obtained by setting all $w_i = 1/\mu$.

The evolution of the mutation parameters is split between the adaptation of the covariance matrix $C^{(g)}$ and the adaptation of the global step size $\sigma^{(g)}$. The covariance
matrix is updated according to:

\[
C^{(g+1)} = (1 - c_{\text{cov}}) \cdot C^{(g)} + c_{\text{cov}} \cdot \left( \frac{1}{\mu_{\text{cov}}} p_c^{(g+1)} \otimes p_c^{(g+1)} \right) + \left( 1 - \frac{1}{\mu_{\text{cov}}} \right) \sum_{i=1}^{\mu} \left[ \frac{w_i}{\sigma(g)^2} \left( x_{i,\ell}^{(g)} - \langle x \rangle_{w}^{(g)} \right) \otimes \left( x_{i,\ell}^{(g)} - \langle x \rangle_{w}^{(g)} \right) \right] \tag{2.23}
\]

where \( p_c^{(g+1)} \) is the evolution path:

\[
p_c^{(g+1)} = (1 - c_c)p_c^{(g)} + H_{\sigma}^{(g+1)} \sqrt{c_c(2 - c_c)} \frac{\mu_{\text{eff}}}{\sigma(g)} \left( x_{i,\ell}^{(g)} - \langle x \rangle_{w}^{(g)} \right)
\]

where \( \mu_{\text{eff}} = (\sum_{i=1}^{\mu} w_i^2)^{-1} \) and \( H_{\sigma}^{(g+1)} \) is defined as:

\[
H_{\sigma}^{(g+1)} \begin{cases}
1 & \text{if } \frac{\|p_{\sigma}^{(g+1)}\|}{\sqrt{1 - (1 - c_c)^2}} < (1.5 + \frac{1}{n - 0.5}) E(||N(0, I)||), \\
0 & \text{else.}
\end{cases}
\] (2.25)

where the expected value \( E(||N(0, I)||) \approx \sqrt{n}(1 - \frac{1}{4n} + \frac{1}{21n^2}) \). The parameter \( c_{\text{cov}} = \min(1, 2\mu_{\text{eff}}/n^2) \) determines the learning rate for the adaptation of the covariance matrix \( C \), whereas \( c_c \) is the equivalent for the evolution path \( p_c \).

The adaptation of the global step size \( \sigma^{(g+1)} \) is updated according to:

\[
\sigma^{(g+1)} = \sigma^{(g)} \cdot \exp \left( \frac{c_{\sigma}}{d_{\sigma}} \left( \frac{\|p_{\sigma}^{(g+1)}\|}{E(||N(0, I)||)} - 1 \right) \right)
\]

The damping parameter \( d_{\sigma} \) controls the change in magnitude of \( \sigma \) and prevents the population from converging prematurely. The conjugate evolution path \( p_{\sigma}^{(g+1)} \) is also iteratively updated as:

\[
p_{\sigma}^{(g+1)} = (1 - c_{\sigma})p_{\sigma}^{(g)} + \sqrt{c_{\sigma}(2 - c_{\sigma})} B^{(g)} (D^{(g)})^{-1} (B^{(g)})^T \frac{\mu_{\text{eff}}}{\sigma(g)} \left( x_{w}^{(g+1)} - \langle x \rangle_{w}^{(g)} \right)
\]

which requires the principal component decomposition of the covariance matrix \( C^{(g)} = B^{(g)} (D^{(g)})^{-1} (B^{(g)})^T \). The initial direction for the evolution paths is random selection \( p_{\sigma}^{(0)} = p_c^{(0)} = 0 \). The standard values for the other parameters of the algorithm are:

\[
\ell \approx n\sqrt{n}, \quad \mu = \ell/2, \quad w_i = \text{const} \cdot (\ln(\mu + 1) - \ln(i)), \quad c_{\sigma} = \frac{\mu_{\text{eff}} + 2}{n + \mu_{\text{eff}} + 3}
\]

\[
d_{\sigma} = \max \left( 1, \sqrt{\frac{\mu_{\text{eff}}}{n + 2}} \right) + c_{\sigma}, \quad c_c = \frac{4}{n + 4}, \quad \mu_{\text{cov}} = \mu_{\text{eff}}
\]

\[
c_{\text{cov}} = \frac{1}{\mu_{\text{cov}}} \left( \frac{2}{(n + \sqrt{2})^2} + \left( 1 - \frac{1}{\mu_{\text{cov}}} \right) \min \left( 1, \frac{2\mu_{\text{eff}} - 1}{(n + 2)^2 + \mu_{\text{eff}}} \right) \right)
\]

\[
(2.28)
\]
These parameters are described more thoroughly in [24].

The population evolves over the iterations until a stopping criterion has been met. This can be either a tolerance in the cost function or a measure of the diversity in the current generation, sign that the algorithm is unlikely to further explore the parameter space.
Chapter 3

Self-propulsion of a counter-rotating cylinder pair

3.1 Introduction

In this chapter we consider the self-propulsion regimes of two steadily counter-rotating cylinders linked together at a fixed distance by their centres. This system can be characterized by two parameters: the non-dimensional width and the rotational Reynolds number.

In a similar problem, the flow over a pair of fixed counter rotating cylinder was recently investigated numerically by Chan et al. [7]. They demonstrated that, for a range of geometry parameters, there exist specific combinations of rotation and inflow Reynolds numbers that achieve the complete suppression of the unsteady vortex shedding, and even zero or negative drag. In the present self-propelled setting, however, the incoming flow is a result of the motion of the cylinder pair, and therefore the inflow Reynolds number can not be controlled independently from the rotation rate.

Similar results were found for a torus rotating about its centreline by Moshkin et al. [36]. The axisymmetric flow past a fixed torus was studied for a range of incoming flow Reynolds numbers and geometry parameters in order to find the critical rotation rate that resulted in zero drag, where the chosen rotation direction was such that the inner side of the torus had a surface velocity opposite to the incoming flow. A second zero-drag solution was found by the same author in [37] with opposite rotation direction with respect to the previous study. By fixing the centre of mass of the torus, the zero-drag solutions of these studies are limited to steady-state solutions.

We approach the problem numerically using a 2D multiresolution remeshed vortex method (as seen in section 2.2). We systematically vary the relevant parameters and
report the behaviour of the system while moving through this phase-space. Using streamlines and passive tracer particles we show the underlying fluid dynamics and report on the high Reynolds number instabilities.

3.2 Problem setup

3.2.1 Problem description

We consider the problem sketched in figure 3.1: two counter-rotating cylinders that are linked together at a fixed distance by their centres so as to move as a single rigid body. The cylinder pair is defined by the distance between the centres of the two cylinders, \( W \), their diameter \( D \), and mass \( m = \pi D^2/4 \). The cylinders rotate with equal and opposite angular velocity \( \Omega \) leading to a surface velocity of magnitude \( U_\theta = \Omega D/2 \). Throughout this section, the left cylinder will be given a positive (counter-clockwise) angular velocity. Furthermore, we obtain the circulation around a cylinder as

\[
\Gamma = \pi D U_\theta = \frac{1}{2} \pi D^2 \Omega
\]  

(3.1)

The fluid is incompressible, has kinematic viscosity \( \nu \) and density \( \rho_f \), which is equal to the density of the immersed body \( \rho_s = \rho \) eliminating any effect of gravity. For this problem, the Reynolds number is defined as the ratio between the circulation \( \Gamma \) around a cylinder and the viscosity of the fluid \( \nu \):

\[
Re_\Gamma = \frac{\Gamma}{\nu}
\]  

(3.2)

An alternative Reynolds number based on the angular velocity \( Re_\Omega = \Omega D^2/\nu = 2Re_\Gamma/\pi \) can be easily obtained from \( Re_\Gamma \). The geometry of the problem is defined by the scaling factor \( W^* \) as the ratio between the distance between the two centres \( W \) and the diameter \( D \). These two numbers, the rotational Reynolds number and geometry parameter, uniquely define the scaling similarity of the problem.
The dimensional time $t$ is rendered non-dimensional using the time required for one revolution of the cylinders:

$$T = t/T_{rev} = \frac{\Omega}{2\pi} \frac{t}{(\pi D)^2}$$  \hfill (3.3)

Moreover, any dimensional velocity $u$ is rendered non-dimensional by the surface velocity:

$$U = \frac{u}{U_\theta} = \frac{2}{\Omega D} u = \frac{\pi D}{\Gamma} u$$  \hfill (3.4)

At the beginning of the simulation, the angular velocity is ramped up over one rotation of the cylinders with a quarter sine wave:

$$\Omega_L = \begin{cases} \Omega \sin \left( \frac{1}{2} \pi T \right) & \text{if } T < 1 \\ \Omega & \text{if } T \geq 1 \end{cases}$$  \hfill (3.5)

$$\Omega_R = -\Omega_L$$  \hfill (3.6)

We consider the immersed boundary method with Brinkman penalization to enforce the no-slip condition. Hence, the forces acting on the solid body are computed from the velocity field by integrating the penalization term:

$$F = \lambda \int_{\Sigma} \chi_s (U_s - u_\lambda) \, dx$$  \hfill (3.7)

The force along the mid-line $f$ is non-dimensionalized with the characteristic dimensions of the system:

$$F = \frac{T_{\text{reg}}^2}{mD} f = \frac{4\pi}{\rho D U_\theta^2} f$$  \hfill (3.8)

### 3.2.2 Computational setup

The simulations were carried out in the computational domain $\Sigma = [0, 1] \times [0, 1]$, the center of mass of the two cylinders is initially located at $(0.5, 0.5)$ and the line that connects the two centres is initially horizontal. The two cylinders having the diameter $D = 0.005$ rotate around their centres in opposite directions with surface velocity $U_\theta = 0.1$.

The effective resolution of the domain is set at $ER = 32768$, so that there are over 80 grid points across the radius of each cylinder, and the time-step criterion is $LCFL = 0.05$. The refinement and compression thresholds are $r_{\text{tol}} = 10^3$, $c_{\text{tol}} = 10^4$ and the penalization factor is $\lambda = 10^4$. The viscosity of the fluid is obtained as $\nu = \Gamma/ReT$. 
3.3 Results

Here we show first the result of varying of $Re_{\Gamma}$ while keeping $W^* = 2$, then varying $W^*$ while fixing $Re_{\Gamma} = 100$. Subsequently we show the characteristic behaviour of the cylinders as a function of both $Re_{\Gamma}$ and $W^*$.

3.3.1 Effect of the Reynolds number for a fixed non-dimensional width $W^* = 2$

The plot in figure 3.3 shows the vertical velocity of the cylinder pair for different Reynolds numbers at a fixed non-dimensional width of $W^* = 2$. The lowest Reynolds number investigated was $Re_{\Gamma} = 10$, below which the computational cost becomes prohibitive due to the time-step restriction of the diffusion term.

For Reynolds numbers up to $Re_{\Gamma} = 85$ the cylinders move upwards. From $Re_{\Gamma} = 10$, if we increase the rotational Reynolds number we observe an increasing steady-state velocity, but also an increase of the number of rotations required to reach steady-state. For $Re_{\Gamma} = 90$, the cylinders accelerate upwards for the first five revolutions but then the acceleration gradually changes direction so that the cylinders eventually move in downward direction and reach a negative steady-state velocity. Further increasing the Reynolds number, the initial upwards acceleration lasts fewer rotations and the cylinder pair reach steady-state in shorter times. Also, the downward steady-state velocity increases with the Reynolds number.

Figure 3.4 shows the vorticity fields for the two typical solutions around the transition. The structure of the vorticity field around each cylinder at $Re_{\Gamma} = 50$ shows a $\pi$-rotational symmetry and the main component of the flow in the proximity of
3.3 Results

Figure 3.3: Velocity of the counter-rotating cylinder pair at $W^* = 2$ at a series of Reynolds numbers up to $Re_\Gamma = 150$ (left). The simulations are stopped whenever the velocity does not change more than 0.03\% in the last time unit, and extended with a dashed line to indicate the steady-state value.

Figure 3.4: Vorticity field of the counter-rotating cylinder pair at $W^* = 2$ at $Re_\Gamma = 50$ (top) and $Re_\Gamma = 150$ (bottom), at different times. The initial location of the cylinder pair is drawn with dashed black circles.
the cylinders is a rotational motion. In contrast, at \( R_{\Gamma} = 150 \) the highest vorticity magnitudes are on the inside of the cylinder pair, near their surfaces, and a clear wake is visible corresponding to a strong flow in between the cylinders.

Figure 3.5 shows the vertical velocity of the cylinders for rotational Reynolds numbers between \( R_{\Gamma} = 100 \) and \( R_{\Gamma} = 750 \). The steady-state downward vertical velocity increases with the Reynolds number up until \( R_{\Gamma} = 500 \), while the steady-state is achieved in fewer rotations until \( R_{\Gamma} = 250 \). Further increasing the Reynolds number, we observe oscillations in the velocity curve and a longer initial transient. At \( R_{\Gamma} = 625 \) the motion becomes fully unstable, with strong vertical velocity fluctuations reaching both negative and positive values. The cylinders do not reach a steady-state or a periodic pattern of vertical velocity within the first 130 revolutions.

Figure 3.6 shows the vorticity field around the cylinders for \( R_{\Gamma} = 500 \). Until \( T = 32 \), we observe a build-up of vorticity around the slowly descending cylinder pair. After \( T = 32 \), the vorticity build-up has gathered enough strength and is gradually shed as a vortex dipole, rapidly accelerating the cylinders downwards. As soon as the vortex dipole is fully detached, subsequent vorticity is not shed, but is build-up around the cylinders again, causing deceleration. At \( T = 48 \), a second–weaker–vortex pair begins shedding. The intermittent vortex shedding corresponds to the velocity fluctuations observed in the velocity plot of figure 3.5. The fact that the cylinders do not stop after the shedding of the first two vortices, but only decelerate, means that eventually the shedding is sustained continuously rather than in discrete pairs. For \( R_{\Gamma} = 500 \), the cylinder pair reaches a state of approximate steadiness after the first 60 rotations.
3.3 Results

Figure 3.6: Vorticity field of the counter-rotating cylinder pair at $W^* = 2$ at $Re_\Gamma = 500$, at different times. The initial location of the cylinder pair is drawn with dashed black circles.

However, the interaction with the wake and intermittent vortex shedding can lead to an unstable solution. For example, figure 3.7 shows the vorticity field around the cylinders for $Re_\Gamma = 750$ every 24 rotations. In this case, after the first vortex dipole is shed, the build-up of vorticity around the cylinders halts the pair. The subsequent intermittent vortex shedding and interaction with the wake causes wide oscillations in the velocity, as observed in Fig. 3.5, making the cylinders unable to achieve steady self-propulsion. The last panel (bottom left) of figure 3.7 shows the vorticity field at $T = 168$, where we observe that second vortex dipole was shed and the acceleration of the pair is again halted by the interaction with the wake.

Figure 3.8 shows the force along the mid-line for the whole range of Reynolds numbers reported so far; negative values of the force denote that the two cylinders are pushed together, positive values denote traction. We observe that, for $Re_\Gamma$ below the transition threshold, this force has a positive value during the initial transient, gradually decreasing and changing direction so that the mid-line is under compression at steady-state. The magnitude of this force increases very weakly with the Reynolds number. For Reynolds numbers above the transition to downward motion, the behaviour of the force is similar to that of the vertical velocity: the force that pulls the cylinders apart increases in magnitude for increasing values of $Re_\Gamma$, with growing oscillations that reach instability for $Re_\Gamma = 625$.

We should note that, even tough the upward moving cases generally reach greater velocity magnitudes than the downward moving ones, the tensile force along the mid-line for $Re_\Gamma \geq 90$ has a magnitude that is six times or greater of that for $Re_\Gamma \leq 75$. In fact, figure 3.4 shows that the vorticity field for upward moving
**Figure 3.7:** Vorticity field of the counter-rotating cylinder pair at $W^* = 2$ at $Re_{\Gamma} = 750$, at different times. The initial location of the cylinder pair is drawn with dashed black circles.

**Figure 3.8:** Force along the midline of the counter-rotating cylinder pair for $W^* = 2$ for a series of Reynolds numbers up to $Re_{\Gamma} = 625$. 
3.3 Results

Figure 3.9: Velocity of the counter-rotating cylinder pair at $Re_\Gamma = 100$ for a series of non-dimensional widths $W^\ast$. For the left plot, simulations are stopped whenever the velocity does not change more than 0.03% in the last time unit, and extended with a dashed line to indicate the steady-state value.

solutions has a more compact support and a reduced intensity than that of the downward moving solutions, implying that less of the unperturbed flow is displaced by the motion of the cylinders, leading to reduced forces being exchanged between the fluid and the body.

### 3.3.2 Effect of the non-dimensional width for a fixed Reynolds number $Re_\Gamma = 100$

Figure 3.9 shows the vertical velocity curves obtained by fixing the Reynolds number $Re_\Gamma = 100$ while varying the geometry parameter $W^\ast$. We find that the cylinders move upwards for $W^\ast \leq 1.75$ and downwards for $W^\ast \geq 2$, suggesting that the transition between the two propulsion regimes occurs at higher Reynolds numbers when decreasing the spacing between the cylinders. Furthermore, the upward vertical velocity decreases when increasing the distance $W^\ast$ up to the transition to downward motion. The discontinuity between the two motions is thus different from the previous case, where we found greater vertical velocities when increasing the Reynolds numbers up to the transition threshold. Conversely, increasing the spacing above $W^\ast = 2$ results in greater downward velocities, similarly to increasing $Re_\Gamma$ above transition in the previous case.

On the right plot in figure 3.9 we present the transition to unstable motion. Increasing the non-dimensional width past $W^\ast = 3$, we observe a longer initial transient and oscillation in the vertical velocity; these oscillations lead to instability for $W^\ast = 9$. In particular, we observe that for $W^\ast = 10$, after initial oscillations, the system achieves stable upwards motion, whereas for $W^\ast = 15$ the system achieves stable
downwards motion.

3.3.3 Phase-space investigation

To quantify the transition between upwards and downwards motion, as well as between stable and unstable motion, we performed a series of computations varying $Re_\Gamma$ and $W^*$, manually choosing the points in the phase space. Each computation is run until steady-state or a determined trend is noted. The flow case is determined unstable if, during the first 100 cylinder revolutions, we observe wide oscillations in the vertical velocity to both positive and negative values. We cannot exclude that such system could escape the instability and settle into either downward or upward steady-state motion over a longer observation window. Hence, we can characterize the stability based on whether the oscillations of the vertical velocity during the initial transient are limited as not to cause upward velocities. The extreme cases, towards very narrow or very large widths, could not be simulated due to computational restrictions \(^1\), however for any width within $1.25 \leq W^* \leq 10$ we found the upward/downward transition occurring at a finite Reynolds number.

\(^1\)For very narrow non-dimensional widths $W^*$, a greater effective resolution is required to capture the gradients along the mid-line. For very large non-dimensional widths, on the other hand, a longer simulation time was required due to the increased number of rotations before steady-state is reached.
Likewise, in the same range, there always existed a Reynolds number that led to an unstable solution.

Figure 3.10 shows the explored phase space with coloured areas denoting the region of stability for upward motion, downward motion and the unstable motion. The boundaries between these regions are characterized by an exponential correlation of the form:

$$Re_\Gamma(W^*) = \alpha \exp \left[ \left( \frac{\beta}{W^* - 1} \right)^\gamma \right]$$

(3.9)

For the first transition $Re_{\Gamma,\text{trans.}}(W^*)$, the observed critical Reynolds number is defined as the average between the largest $Re_\Gamma$ resulting in upward motion and the smallest $Re_\Gamma$ resulting in downward motion. For the transition to instability $Re_{\Gamma,\text{stab.}}(W^*)$, the observed threshold is defined as the smallest $Re_\Gamma$ resulting in unstable motion. In the latter case, we observed some stable–downward moving–solutions for $Re_\Gamma - W^*$ combinations beyond those of an unstable point. The parameters were found with a least-squares fit that minimizes the relative error $[(Re_{\Gamma,\text{observed}} - Re_{\Gamma,\text{fit}}) / Re_{\Gamma,\text{observed}}]^2$. For the transition between upwards and downwards motion we obtained the following correlation:

$$Re_{\Gamma,\text{trans.}}(W^*) = 6.7 \exp \left[ \left( \frac{17.5}{W^* - 1} \right)^{1/3} \right]$$

(3.10)

For the transition between stable and unstable motion we obtained:

$$Re_{\Gamma,\text{stab.}}(W^*) = 13 \exp \left[ \left( \frac{56}{W^* - 1} \right)^{1/3} \right]$$

(3.11)

Both correlations have a vertical asymptote for $W^* \to 1$, which implies the impossibility of obtaining unstable or downward motion as the surfaces of two cylinders approach contact. Furthermore, both correlations have an horizontal asymptote at $Re_\Gamma = 0$ for $W^* \to \infty$.

3.4 Discussion

In this section we present the analysis of the different flow regimes. First we show the streamlines for each stable motion using a Line Integral Convolution [4], and we show the flow patterns visualized with tracer particles. Then, we discuss the stability of the two steady flow patterns to changes in the cylinders’ angular velocity. Lastly, we present a closer inspection of the flow patterns that can occur in the unstable regime.

3.4.1 Streamlines

Figure 3.11 shows the visualization of the velocity field around the cylinders at steady-state using the Line Integral Convolution (LIC) technique. This technique
Figure 3.11: Line-integral convolution [4] of the steady-state velocity field for $Re_T = 50$ (left) and $Re_T = 150$ (right) for $W^* = 2$, computed in the moving frame of reference of the cylinders. The cylinders are marked by dotted white lines, closed streamlines around the cylinders by dashed black lines and the colors refer to the vorticity field.

Figure 3.12: On the left, velocity of the cylinder pair at $Re_T = 10$ for a series of $W^*$. From blue to green: $W^* = [1.125, 1.25, 1.5, 1.75, 2.0, 2.25, 2.5, 3.0, 3.5, 4.0, 5.0]$. On the right, log-log plot of the steady-state velocity against the distance $W^*$ between the cylinders for various $Re_T$ compared to the velocity for a ideal vortex dipole. Dashed lines represent the hyperbolic interpolation.

uses the velocity vector field to advect white noise and leads to an output which can be seen as a space-filling streamline plot [4]. We show the LIC field for both $Re_T = 50$ and $Re_T = 150$ for the same width $W^* = 2$ in the frame of reference of the cylinder pair’s center of mass.
The solution for $Re_T = 50$ is illustrative of flow pattern in case of steady upward motion. The streamlines around the cylinder pair form a virtual elliptical body, showing a region of circulating fluid travelling upwards with the cylinders. Moreover, the streamlines closely resemble those for an ideal vortex dipole in potential flow. In the problem formulation where the cylinders are fixed, the existence of a stable elliptic virtual body for certain rotation rates was first predicted by Chan and Jameson [8]. The result was also validated with a high-order spectral difference method and experimental results in a water channel using digital particle image velocimetry by Chan et al. [7].

From these considerations it follows that, in the moving frame of reference, the flow between the cylinders moves in opposite direction of the incoming flow, which is deflected and accelerated by the rotation on the outside of the pair. Hence, the force along the mid-line, as seen in figure 3.8, is compressive because the flow between the cylinders has a greater velocity than the flow on the outer side.

In figure 3.12 we compare the velocity of the cylinder pair at low Reynolds numbers (steady upward motion) to the velocity of the potential flow solution for a vortex dipole. The velocity of the vortex dipole is given by $u = \Gamma/(2\pi W) = 0.5 \frac{U_0}{W^*}$, where we expressed the circulation $\Gamma$ through the corresponding surface velocity for a cylinder of diameter $D$. The relation between non-dimensional velocity and the non-dimensional width for a vortex dipole can thus be written as $U = 0.5/W^*$. A similar relation ($U = \alpha/W^*$) was found for the velocity of the cylinder pair as a function of $W^*$, with the coefficient $\alpha$ depending on the Reynolds number. For $Re_T = 5, 10, 20, 30$ we obtained $\alpha = 0.41, 0.45, 0.485, 0.518$ respectively. The similarity in the coefficient $\alpha = 0.5$ for the ideal vortex dipole to the coefficients obtained for the low-$Re_T$ solutions suggests that there is not only a qualitative similarity in the solutions, but also a quantitative one.

The streamlines plot for $Re_T = 150$ is illustrative of a typical solution for Reynolds numbers just above the transition to steady downward motion. In this case, we observe a closed, tear-shaped, virtual body around each cylinder. The incoming flow travels both around and in between the cylinders: while the flow on the outer side is decelerated and pulled through the mid-line, causing a tensile force that would pull them apart. The flow driven in the middle is accelerated, reminding of a jet-like flow structure. Furthermore, the cylinders initially move upwards even for $Re_T > Re_{T,trans}$ because the flow through the mid-line can be accelerated against the still weak inflow. As the cylinders gain momentum, however, the virtual elliptic body is separated in two sets of closed streamlines due to the flow-through. At that point, the flow is accelerated through the mid-line rather than deflected to the outer sides and the cylinders gradually change direction of motion.

These considerations justify the function chosen for the correlation between upward and downward stable motion (eq. 3.10). As the non-dimensional width $W^*$ increases, it becomes more difficult for the rotation of the cylinders to reverse the flow in between the cylinders, preventing the upward motion and we have $Re_{T,trans} \rightarrow 0$ for $W^* \rightarrow \infty$. Moreover, as the surfaces of the cylinders approach contact ($W^* \rightarrow 1$)
Figure 3.13: Tracer particles advected by the steady-state velocity field ($Re_\Gamma = 50$ and $150$, $W^* = 2$). The particles are seeded in closed streamlines around the counter-clockwise rotating cylinder (red), the clockwise rotating cylinder (blue), and in a strip upstream of the pair (black).

Figure 3.14: Tracer particles advected by the velocity field starting at rest for $Re_\Gamma = 50$, $Re_\Gamma = 150$, and $Re_\Gamma = 500$ ($W^* = 2$). The particles are seeded in an ellipsoidal strip upstream of the cylinders (black).
3.4 Discussion

, less of the incoming flow can be pulled through the mid-line, but the cylinders deflect it sideways, making the upward motion more stable ( \( Re_{\Gamma,\text{trans.}} \to \infty \) for \( W^* \to 0 \)).

3.4.2 Tracer particles

To further illustrate the differences in the flow patterns for downward and upward steady motion, we seed the flow with passive tracer particles that are advected according to the local velocity field.

In figure 3.13, we show the evolution of particles that have been seeded in the flow when a steady motion was achieved for both \( Re_{\Gamma} = 50 \) and \( Re_{\Gamma} = 150 \) at the same width \( W^* = 2 \). The particles confirm the observations made based on the streamlines. For the lower Reynolds number, corresponding to upwards motion, the surrounding flow does not penetrate the closed region of fluid surrounding the cylinder pair, instead moving around it. At higher Reynolds number, the particles are pulled between the two cylinders in a jet-like flow.

Figure 3.14 shows the evolution of the tracer particles seeded in the flow from the initial conditions of rest. While the seeded particles for \( Re_{\Gamma} = 50 \) and \( Re_{\Gamma} = 150 \) are advected similarly to the steady-state case, the frames at the bottom show the abrupt transition to steady state for \( Re_{\Gamma} = 500 \). In this case, the cylinder pair initially moves slowly downwards, together with a greater virtual mass of particles which is shed after approximately \( T = 40 \) rotations, corresponding to the shedding of the vortices as seen in figure 3.6.

3.4.3 Effect of changing the angular velocity from steady-state

In this section we discuss the effect of changing the angular velocity \( \hat{\Omega}(T) \) of the cylinders at steady state. In the following, the non-dimensionalizations are performed with the initial angular velocity \( \Omega \). The simulations are carried out with an initial rotational Reynolds number \( Re_{\Gamma,0} \) until \( T = T_0 = 200 \) is reached. Then, the angular velocity is smoothly changed over \( T_F - T_0 = 10 \) with a half-cosine wave\(^2\) until the rotational Reynolds number reaches \( Re_{\Gamma,F} \).

\(^2\)The transition between the initial angular velocity \( \Omega \) and the new one is determined by the two rotational Reynolds numbers: before the transient \( Re_{\Gamma,0} \) and after the transient \( Re_{\Gamma,F} \):

\[
\hat{\Omega}(T) = \begin{cases} 
\Omega, & T \leq T_0 \\
\Omega \left\{ 1 + \frac{1}{2} \left( \frac{Re_{\Gamma,F}}{Re_{\Gamma,0}} - 1.0 \right) \left[ 1 - \cos \left( \frac{\pi (T - T_0)}{T_F - T_0} \right) \right] \right\}, & T_0 < T < T_F \\
\Omega \frac{Re_{\Gamma,F}}{Re_{\Gamma,0}}, & T \geq T_F 
\end{cases}
\]  

(3.12)
Figure 3.15: Velocity of the cylinder pair starting at $W^* = 2$ and $Re_{\Gamma,0} = 80$ on the left and $Re_{\Gamma,0} = 90$ on the right. The angular velocity is then changed over $T_F - T_0 = 10$ with a half-cosine wave.

Figure 3.15 shows the evolution of the vertical velocity starting from $Re_{\Gamma,0} = 80$ (left) and $Re_{\Gamma,0} = 90$ (right) which are respectively above and below $Re_{\Gamma,\text{trans.}}(W^* = 2)$ . For the first case, we observe that a gradual increase in the rotational Reynolds number from an already upward-moving solution, extends the stability domain for the dipole-like flow pattern. On the other hand, for a solution that is just above the transition to jet-like motion like $Re_{\Gamma,0} = 90$, a gradual decrease in the angular velocity causes an immediate switch to dipole-like propulsion. The experiment was repeated for $T_F - T_0 = 1$ and 100, and obtained the same terminal velocities. These results imply that near the transitional rotation rate the dipole-like state is a stable configuration, whereas the downward-moving state is more sensitive to disturbances.

3.4.4 Instability

Figure 3.17 shows solutions for increasing values of the Reynolds number at the fixed widths $W^* = 3$ and $W^* = 5$. For $W^* = 3$, we observe the first instability at $Re_{\Gamma} = 500$, but the vertical velocity for the $Re_{\Gamma} = 300$ solution already shows wide oscillations indicating incipient instability. For $W^* = 5$ we observe the first instance of instability at $Re_{\Gamma} = 200$. However, at $Re_{\Gamma} = 300$ and $Re_{\Gamma} = 500$ the cylinder pair escapes the initial instability and settles into steady upward and downward motion respectively.

Figure 3.16 shows the vorticity fields for $Re_{\Gamma} = 300$ and $Re_{\Gamma} = 500$ illustrating the transition from unstable to stable motion. We observe that, for higher values of the Reynolds number, the boundary layer on each of the cylinders surface starts
3.4 Discussion

Figure 3.16: Vorticity field of the counter-rotating cylinder pair at $W^* = 5$ for $Re_T = 300$ (top) and $Re_T = 500$ (bottom), at different times. The initial location of the cylinder pair is drawn with dashed black circles.

Figure 3.17: Vertical velocity of the cylinders with $W^* = 3$ (left) and $W^* = 5$ for Reynolds numbers around the instability.
to develop an instability that grows as it is convected around the surface. Eventually the instabilities cause the boundary layer vorticity to detach and as it is shed into the wake, the cylinders accelerate in the opposite direction. The rate of growth of the instability is likely governed by the Reynolds number, but the initial perturbation that triggers the start of the instability growth is responsible for the direction in which the vorticity is eventually shed. The large uncertainty in this initial perturbation translates into an uncertain direction of motion of the cylinder pair.

These two stable motions are illustrated in figure 3.18 by the LIC field. The downward motion obtained for \( Re_T = 500 \) is comparable to that seen in figures 3.11 and 3.13. In the solution for \( Re_T = 300 \), we also observe tear-shaped virtual bodies around each cylinder. However, it is the flow on the outer side that is accelerated by the cylinders rotation, while the flow close to the inner sides is decelerated and pulled towards the outer sides. This consideration also explains the horizontal asymptote in the correlation for the transition between stable and unstable motion: as the non-dimensional width approaches infinity, the wakes around each cylinder are independent from each other, eliminating any fluid mediated interaction. For this reason, each motion becomes equally possible.

A quasi-periodic motion was found for \( W^* = 3 \) and \( Re_T = 1000 \), as shown in figure 3.19. A vortex pair is periodically shed that propels incrementally forward and then attracts the cylinder pair backward. Each time the cylinders slow down, the boundary-layer instability builds up anew until it sheds again.
3.5 Concluding remarks

In this chapter we presented the dynamics of an essential self-propulsive system consisting of a pair of counter-rotating cylinders. The cylinders are linked together by their centres so that they move as a single rigid body responding to the forces acting on their boundary.

For low rotation rates, the cylinders behave as a vortex dipole, the flow being characterized by an elliptical virtual body moving along with the cylinders, identified by closed streamlines. As in the flow pattern of an ideal vortex pair, the external fluid is pushed around this elliptical region and the propulsive force comes from the acceleration of the flow on the outer side of the cylinders’ mid-line. Moreover, an inversely proportional relation was found between the velocity and the circulation of the cylinders, reinforcing the similarity to the ideal vortex pair.

Increasing the rotation rate shows a transition to a new state where the cylinders move in the opposite direction to a vortex dipole. In contrast to the previous
case, there is no single virtual body, but each of the cylinders has a different set of closed streamlines. Furthermore, the external flow is pulled in between the cylinders, accelerated and expelled like a jet. The point of transition shows an inverted exponential dependency of the rotation rate based on the width between the cylinders, with larger widths requiring smaller rotation rates to move from the vortex-dipole-type to the jet-type flow.

Moreover, we performed an initial characterization of the stability of these two flow patterns by varying the angular velocity of the cylinders from a solution at steady-state. We observed that near the transitional rotation rate the upward-moving state is stable, acting similarly to an attractor for dynamical systems, whereas the downward-moving state is more sensitive to disturbances. This preliminary analysis indicates the presence of hysteresis effects that could be more thoroughly investigated in the future.

Increasing further the rotation rate, the motion of the cylinder pair becomes unstable and we observe wide oscillations in the vertical velocity. We observed that the evolution of the instability can lead to aperiodic or periodic velocity patterns, and the cylinders can eventually escape the instability and settle in either upward or downward motion. An inverted exponential correlation was found to describe the transition between stable downward motion and the onset of instability. The instability occurs at higher rotation rates when the cylinders are separated by a smaller width, and at lower rotation rates when the increased width reduces the fluid-mediated interactions, making either direction of motion equally probable.
Chapter 4

Optimal self-propulsion of simplified swimmers

4.1 Introduction

In this chapter, we present simplified models with few degrees of freedom that could produce both jet-like and wave-like motions that result in self-propulsion states that can be compared to jellyfish-like and undulatory swimmers respectively. The reduced number of degrees of freedom allows the relatively inexpensive application of optimization algorithms in order to find the fastest and most efficient combinations of motion parameters. The chosen optimization algorithm is the evolutionary strategy with covariance matrix adaptation (CMA-ES) discussed in section 2.3.

An abstraction of the undulatory motion of anguilliform swimmers as three linked ellipses was analysed by Eldredge [14, 15]: a vortex method was used in order to solve the two-dimensional flow at different Reynolds numbers, but a comparison of the three-links swimmer with anguilliform swimmers was beyond the scope of their works. Moreover, with the same two-dimensional three-linked ellipses abstraction, it was found in [16] that, like for natural fish, it was possible for the fish-like system to be passively propelled in the unstable wake of a cylinder. A comparison of the three-linked swimmer with in-vivo measurements of a leech was done by Kajtar et al. [26]. A simple relationship for the speed of the three-linked swimmer was obtained by changing the amplitude and frequency of the motion, and the relationship was compared favourably with the velocity of the leech as measured by Taylor [46]. These works suggest that the abstraction of the undulating motion at intermediate Reynolds numbers as two-dimensional linked bodies could be a reasonable approximation.

Wilson et al. [53] also used a symmetric array of linked rigid bodies in order to abstract jellyfish-like swimming. The relative trajectories of five markers along the
jellyfish body were approximated by the motion of rigid linked ellipsoidal bodies. The length of each body was taken as the average distance between two markers and the oscillation of the relative angle was fitted with a sinusoidal curve. The efficiency and speed of the motion when substituting linked elements with prescribed kinematics with passively responsive elements was compared to the fully prescribed case. It was found that, by replacing the terminal hinge with a passive torsional spring, the efficiency could be improved and an optimal stiffness for the spring was obtained that maximized the performance of propulsion. Simplified jellyfish swimming was also modelled as a two dimensional impermeable bell-shaped line by Herschlag et al. [25] in order to explore how the intermediate Reynolds numbers affect efficiency of propulsion for oblate and prolate jellyfish and how that sets physical limits on the motion of natural swimmers. In an axisymmetric formulation of the same bell-shaped line abstraction, the effects of shape and motion parameters was studied by Peng et al. [40], finding that prolate jellyfish can achieve faster propulsion than oblate swimmers at the cost of the efficiency. Contrary to these studies, in the present work we analyse non-deforming geometries whose utilization to propel robotic applications could results in simpler systems.

CMA-ES has already been applied to the optimization of the self-propulsion of swimmers at intermediate Reynolds numbers in several previous studies. Kern et al. [28] studies the motion of a three-dimensional anguilliform swimmer. The mid-line curvature was parameterized and the optimization algorithm was applied in order to find the most efficient and fastest solutions. The optimal escape response for larval fish was investigated by Gazzola et al. [20]: it was found that the motion that maximizes the distance travelled from an initially still swimmer is the C-start, which is in agreement with \textit{in-vivo} experiments. The optimal shape for fast and efficient three-dimensional anguiliform swimmers was investigated by Van Rees et al. [51], also, a sensitivity analysis of the optimal shapes was performed in order to separate the effects on propulsion of single morphological features. The optimal solutions found by these studies, while sharing features with those found in nature, indicate that engineered swimmers can potentially outperform natural swimmers, not being constrained by the same limitations.

Applications of other optimization algorithms to fluid-structure interaction problems include the use of a gradient-based algorithms. For example, in Tuncer et al. [50] a steepest ascent algorithm was used in order to optimize the thrust and the efficiency of propulsion for flapping airfoils. The motion of the airfoil was composed of a sinusoidal plunge and a pitching motion and the chosen optimization variables were the amplitudes and the two motions. It was found that, while the fastest motion was obtained for large plunge and pitch amplitudes, the most efficient motion was obtained by reducing the amplitudes in order to prevent the formation of leading edge vortices.
4.2 Problem description

We investigate the self-propulsion of a model swimmer consisting of a pair of rotating identical cylinders. In the following, we will refer to the prescribed rotation and relative displacement of the cylinders as motion, which, in the present self-propelled setting, has zero total linear and angular momentum.

We consider three sets of parameters that describe possible motions of the cylinder pair that will be referred to as wavelike motion, pulsating symmetric motion and rigid symmetric motion and will be introduced in this section. In order to identify the optimal parameters for speed and efficiency of the motion, we use a stochastic optimization algorithm: the evolutionary strategy with covariance matrix adaptation (CMA-ES). The computational methods and code were extensively verified in [19, 41, 43] and used for fluid-solid optimization problems in [20, 21, 51].

Wavelike motion

The parameters that define the first two motions are illustrated in figure 4.1. For the wavelike motion, we take inspiration from the undulating motion of anguilliform swimmers: we discretize the mid-line kinematics into the angular velocity of two cylinders of diameter $D$, linked at a fixed distance $W$, rotating at different rates. The angular velocity of the cylinders is defined by two harmonics of amplitude $\Omega_L$ (left cylinder) and $\Omega_R$ (right cylinder), with period $\mathcal{T}$ and offset by a phase $\phi$:

$$\Omega_L(t) = \Omega_L \sin \left[ \frac{2\pi}{\mathcal{T}} t + \phi \right]$$ (4.1)

$$\Omega_R(t) = \Omega_R \sin \left[ \frac{2\pi}{\mathcal{T}} t \right]$$ (4.2)
Pulsating symmetric motion

For the pulsating symmetric motion we take inspiration from the motion of jellyfish: the distance between the two cylinders extends and contracts harmonically with amplitude $A$, the cylinders rotate in opposite directions with the magnitude oscillating with amplitude $\Omega$. In the pulsating motion, the centres of the cylinders have a minimum distance $G > D$ and there is a phase $\phi$ between the harmonic function that determines the width and the harmonic function that determines the angular velocity:

$$\Omega_L(t) = -\Omega_R(t) = \Omega \sin \left[ 2\pi \frac{t}{T} + \phi \right] \quad (4.3)$$

$$W(t) = G + A \left[ 1 + \cos \left( 2\pi \frac{t}{T} \right) \right] \quad (4.4)$$

These relations describe the two cylinders starting off at maximum extension with no pulsating velocity.

Rigid symmetric motion

In chapter 3 we considered the self-propulsion of a pair of steadily counter-rotating cylinders and identified two stable flow patterns. In this chapter we are interested in rigid-body motions comparable with finite-deformation propulsion techniques. For this reason we use a periodic function for the angular velocity such that the cylinders can switch between the two stable flow patterns while also having periodic deformation, defined as the angle $\theta$ with respect to the initial configuration.

For the time-equation of the angular velocity we took inspiration from the in-vivo measurements by Nawroth et al. [39] of the angular velocity of jellyfish bell lobes. Figure 4.2 shows the considered rigid symmetric motion, which, like the jellyfish stroke cycle, is divided in a power stroke and a recovery stroke. The time-dependent angular velocity depends on two parameters: the peak angular velocity during the power stroke $\Omega_1$ and the peak angular velocity during the recovery stroke $\Omega_2$. In the case where $\Omega_1 > \Omega_2$ the angular velocity follows:

$$\Omega_L(t) = \Omega(t) = \begin{cases} \frac{\Omega_1 - \Omega_2}{2} - \frac{\Omega_1 + \Omega_2}{2} \cos \left( 2\pi \frac{t}{T_1} \right) & \text{if } t < T_1 = \frac{2\Omega_2}{\Omega_1 + \Omega_2}, \\ -\frac{\Omega_2}{2} & \text{if } T_1 \leq t \leq T \end{cases}$$

(4.5)

$$\Omega_R(t) = -\Omega(t)$$

(4.6)

Where $T$ is the period of motion. This function was chosen as it allows to recover a sinusoidal function for $\Omega_1 = \Omega_2$, it is continuously differentiable, and it satisfies the constraint of finite deformation:

$$\int_N^{(N+1)T} \Omega(t)dt = 0 \quad (4.7)$$
4.3 Formulation of the optimization problem

As shown in figure 4.2, equation 4.5 for the angular velocity is shifted in time as to let the motion start with $\Omega(0) = 0$ just before the power stroke. This was found to reduce the time required for the average velocity to reach a constant value.

4.3 Formulation of the optimization problem

In this section we will discuss the definition of cost function, the optimization variables, the parameters of the optimization algorithm, and the parameters of the flow solver.

4.3.1 Definition of the cost function

The cost function to be minimized can be defined with the goal of fast propulsion or efficient propulsion. The fastest motion is identified by maximizing the average velocity $\bar{U}$ over one period. The cost function is defined as:

$$f_{vel} = -\bar{U} = -\frac{1}{T} \int_{N}^{(N+1)T} u(t) \cdot n(t) dt$$  \hspace{1cm} (4.8)$$

where $u(t)$ is the velocity of the cylinder pair, $N$ is the number of periods allowed for the transient before starting the measurement, and $n(t)$ is the desired swimming direction, which is tangent to the average mid-line inclination in the last period for the wave-like motion and orthogonal for the symmetric motions.
The most efficient motion is the one that maximizes the Froude efficiency: the ratio between the useful energy and the sum of the input and the useful energy [51]. The cost function is defined as:

$$f_{\text{eff}} = -\frac{E_{\text{useful}}}{E_{\text{input}} + E_{\text{useful}}} = -\frac{m\bar{U}^2/2}{\frac{1}{\tau} \int_{N+1}^{N+1} P_{\text{input}} dt + m\bar{U}^2/2}$$  \hfill (4.9)

where $m$ is the mass of the cylinders and $\bar{U}$ is the average velocity, $P_{\text{input}}$ is the instantaneous power transferred to the fluid by the cylinders:

$$P_{\text{input}} = \int_{\partial \Omega} (\mathbf{n} \cdot \sigma \cdot \mathbf{u}) dS$$  \hfill (4.10)

where $\Omega$ is the volume of the cylinders. Applying Gauss theorem to equation 4.10 yields:

$$P_{\text{input}} = \int_{\Sigma \setminus \Omega} \nabla \cdot \sigma \cdot u \ dV = \int_{\Sigma \setminus \Omega} (u \cdot (\nabla \cdot \sigma) + \sigma : (\nabla u)) \ dV$$  \hfill (4.11)

where $\Sigma$ is the domain. The first term can be rewritten in terms on the velocity field using the N.S. equation in absence of body forces: $\rho \frac{D\mathbf{u}}{Dt} = \nabla \cdot \sigma$. The second term can be expanded with the definition of the stress tensor $\sigma = -pI + \tau$:

$$P_{\text{input}} = \int_{\Sigma \setminus \Omega} \left( \rho \mathbf{u} \cdot \frac{D\mathbf{u}}{Dt} - p \nabla \cdot \mathbf{u} + \tau : \nabla \mathbf{u} \right) \ dV$$  \hfill (4.12)

$$= \int_{\Sigma \setminus \Omega} \left( \rho \frac{D \mathbf{u}}{Dt} \mathbf{u}^2 + \tau : \nabla \mathbf{u} \right) \ dV$$  \hfill (4.13)

where we made use of the incompressibility of the velocity field. For a Newtonian fluid the constitutive equation for the shear stress tensor $\tau = \mu(\nabla \mathbf{u} + (\nabla \mathbf{u})^T)$:

$$P_{\text{input}} = \frac{d}{dt} \int_{\Sigma \setminus \Omega} \rho \frac{u^2}{2} \ dV + \mu \int_{\Sigma \setminus \Omega} (\nabla \mathbf{u} + (\nabla \mathbf{u})^T) : \nabla \mathbf{u} \ dV$$  \hfill (4.14)

For the wavelike case, we begin the integration of the cost function after $N = 34$ periods. For the symmetric pulsating case we begin the integration of the cost function after $N = 9$, and for the rigid symmetric case after $N = 3$ periods. The choice of observation window was made after initial manual investigation of the time required to reach steady state.

### 4.3.2 Optimization variables

The optimization algorithm operates on the variables of the problem in their non-dimensional form.
Wavelike motion

For the wavelike motion, the variables, as shown in figure 4.1, are the amplitudes $\tilde{\Omega}_L$ and $\tilde{\Omega}_R$ of the harmonics that define the angular velocity of the two cylinders, the phase $\phi$ between them, and the distance between the two centres $W$. The angular velocity amplitudes are non-dimensionalized as:

$$
\Omega^*_L = \frac{\tilde{\Omega}_L T}{2\pi}, \quad \Omega^*_R = \frac{\tilde{\Omega}_R T}{2\pi}
$$

These variables measure the number of revolutions during an interval $T$ of a steadily rotating cylinder rotating with angular velocity $\tilde{\Omega}$. The distance between the centres is non-dimensionalized with the diameter $W^* = W/D$. Furthermore, for simplicity of interpreting the results of the optimizer, the phase $\phi$ will be reported as $\phi/\pi$.

Pulsating symmetric motion

Similarly, for the pulsating symmetric motion, the amplitude of pulsation and the minimum distance between the two centres are non-dimensionalized with the diameter, the amplitude of the harmonic for the angular velocity of the cylinders is non-dimensionalized with the period of the motion, and the phase between the two functions is reported as $\phi/\pi$:

$$
G^* = \frac{G}{D}, \quad A^* = \frac{A}{D}, \quad \Omega^* = \frac{\tilde{\Omega} T}{2\pi}
$$

In order to compare the dynamic scales of the systems, these parameters can be expressed through different definitions of the Reynolds numbers. For both the wavelike and the pulsating symmetric motions, a rotational Reynolds number can be defined as:

$$
Re_\Omega = \frac{\Omega D^2}{\nu} = \frac{\Omega^* 2\pi D^2 / T}{\nu}
$$

For the pulsating symmetric motion, we can additionally define a Reynolds number of pulsation:

$$
Re_A = \frac{V_A D}{\nu} = \frac{A 2\pi D / T}{\nu} = \frac{A^* 2\pi D^2 / T}{\nu}
$$

Where $V_A$ is the amplitude of the pulsation velocity.

Rigid symmetric motion

For consistency with chapter 3, the optimization variables for the rigid symmetric motion will be reported as $Re_{\Gamma,1} = \frac{\pi \tilde{\Omega}_1 D^2}{\nu}$ (the peak rotational Reynolds number of the power stroke), $Re_{\Gamma,2} = \frac{\pi \tilde{\Omega}_2 D^2}{\nu}$ (the rotational Reynolds number during the recovery stroke), and $W^* = W/D$. 
Moreover, it is conventional with parameter-space explorations to fix the physical scales of the system, including the time-scale. However, during an initial attempt at optimizing the rigid symmetric motion, we chose to fix the period $T = 1$, but were unable to find a solution with reliable propulsion. This will be discussed in section 4.5.3.

Based on the observations of chapter 3, the cylinders propel with flow features similar to that of a vortex dipole or to a jet through the mid-line depending on the rotational Reynolds number $Re_\Omega$, and thus on $\Omega$. However, the cylinder pair settles into either state only after a transient, measured with the non-dimensional time which is equal to the number of revolutions of each cylinder.

Since we are looking for a self-propulsion regime that alternates between the two states, $\bar{\Omega}_2$ is limited below a certain value, which is the maximum Reynolds number that allows steady upward–vortex-like–propulsion. However, by limiting the time scale and having a limit on the maximum $\bar{\Omega}_2$, the number of rotations during a stroke could be not enough for the flow to switch from one regime to the other.

For this reason, as shown in figure 4.2, we define the period of motion $T$ as the time required by the cylinder to perform $N_{\text{revs}} = 10$ counter clockwise rotations during the power-stroke and 10 clockwise rotations during the recovery stroke, and thus it depends on $\bar{\Omega}_1$ and $\bar{\Omega}_2$ through equation 4.5. The choice of $N_{\text{revs}}$ was made based on the order of the number of rotations before steady state for a counter-rotating cylinder pair as observed in chapter 3. The effect of $N_{\text{revs}}$ on the performance is discussed in section 4.5.3. This parameterization, on the other hand, does not allow the definition of a fixed time-scale to compare the self-propelled velocities achieved by the motion. The period $T$ can be expressed as a function of the angular velocities $\bar{\Omega}_1$ and $\bar{\Omega}_2$ and the number of revolutions $N_{\text{revs}}$, for $\bar{\Omega}_1 > \bar{\Omega}_2$, as:

\[
T = \frac{2\pi N_{\text{revs}}/\bar{\Omega}_2}{\alpha \arccos (-\alpha) + \sin (\arccos (-\alpha))}, \quad \alpha = \frac{\bar{\Omega}_1 - \bar{\Omega}_2}{\bar{\Omega}_1 + \bar{\Omega}_2}
\]

(4.19)

4.3.3 Computational setup

There are two relevant sets of Reynolds numbers: one set arises from the scales resulting from the optimization variables, the other based on the resulting self-propulsion. For both the wavelike and the pulsating motions, we maintain constant three independent scales of the system: the diameter of the cylinders $D = 0.01$, the period of the motion $T = 1$, and the fluid viscosity $\nu = 2 \cdot 10^{-6}$. For this reason, we have that the Reynolds numbers of the optimization are equal to a constant multiplied by an optimization parameter:

\[
Re_\Omega \approx 314 \, \Omega^*, \quad Re_A \approx 314 \, A^*
\]

(4.20)

For the rigid pulsating motion, as discussed in the previous section, we keep the number of rotations $N_{\text{revs}} = 10$ constant, and we let the period of the motion $T$ vary
4.4 Results

4.4.1 Propulsion of a pair of cylinders with oscillating angular velocities

In this section we discuss the wavelike motion of the cylinders, optimized for speed and efficiency of propulsion. The evolution during the optimization of the fitness evaluations and the corresponding parameters are shown in Appendix A.

The fastest configuration is characterized by an asymmetry in the amplitude of the angular velocities of the two cylinders. While the right cylinder’s amplitude reaches...
Figure 4.3: Illustration of the fastest wavelike motion. On the left, horizontal and vertical velocity (the dashed line indicates the average forward velocity over the last period). On the right, from the top: efficiency over a running window of one period, average and instantaneous inclination of the midline, and angular velocities of the cylinders.

the upper optimization bound of $\Omega^* = 5$, which corresponds to $Re_{\Omega,R} = 1570$, the optimal left cylinder’s amplitude is a third of that of the right one, $Re_{\Omega,L} = 527$. The phase between the two harmonics is $0.42\pi$, meaning that when one cylinder switches direction of rotation the other one is close to the maximum rotational velocity. Lastly, the distance between the two centres has an internal optimum for $2.25D$.

Figure 4.3 illustrates the kinematics and the efficiency of the fastest wavelike motion. Similar to undulatory swimmers, the lateral velocity oscillates over the period of motion around the zero value, while the forward velocity, after an initial transient, hovers around a constant value $\langle U \rangle = -1.0023(D/T)$, which corresponds to $Re_U \approx 50$. Specifically, the lateral velocity oscillations are in phase with the angular velocity of the right, more strongly rotating, cylinder. On the other hand, the forward velocity oscillates with a frequency that is twice that of the cylinders’ rotation, each peak roughly corresponding to a peak in the angular velocity of either the left or the right cylinder. The figure also shows the efficiency, as defined in equation 4.9, integrated over a moving time window of one period $T$. The propulsive efficiency reaches around 1.1% at steady-state. Moreover, we show the inclination with respect to the average angle of the mid-line at steady-state: the cylinder pair, during the initial acceleration, rotates clockwise by roughly $0.4\pi$ before oscillating around a fixed angle. At steady-state, the instantaneous inclination is in phase with $\Omega_R$. 

The efficient motion differs from the fastest for the lower rotation rates (roughly a fourth of that of the fastest cylinder of the fastest motion) and for the near-symmetry in the rotation rates of the two cylinders. In fact, we have $\Omega^*_L \approx 1.28$ and $\Omega^*_R \approx 1.36$, corresponding to $Re_{\Omega,L} = 402$ and $Re_{\Omega,R} = 427$. Moreover, the optimal phase between the two angular velocities is $0.5\pi$, and the width between the two centres reaches the lower bound $W = 1.5D$ during the optimization. In fact, a smaller distance between the two cylinders, besides reducing the moment of inertia, increases the fluid mediated interactions, allowing propulsion with reduced magnitudes of the angular velocity and thus decreasing the dissipated energy.

Figure 4.4 illustrates the kinematics and the efficiency of the most efficient wave-

<table>
<thead>
<tr>
<th>Case</th>
<th>$\Omega^*_L$</th>
<th>$\Omega^*_R$</th>
<th>$W^*$</th>
<th>$\phi/\pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fastest</td>
<td>1.68</td>
<td>5</td>
<td>2.25</td>
<td>0.42</td>
</tr>
<tr>
<td>Efficient</td>
<td>1.28</td>
<td>1.36</td>
<td>1.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Start</td>
<td>0.5</td>
<td>1.5</td>
<td>3</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Table 4.1: Optimal solutions for the wavelike motion. We report also the start search point for the optimization for speed. The initial point for the optimization for efficiency was the fastest solution.
like motion. Again, the vertical velocity oscillates around the zero value, with
the same frequency as the motion, but this time it is in phase with the angular
velocity of the left cylinder. The horizontal velocity approaches steady state at
\( \langle U \rangle = -0.5476(D/T) \) or \( Re_U \approx 27.38 \), half of that of the fastest case. Again, the
oscillations occur at twice the frequency of the motion. Moreover, the propulsive
efficiency reaches 2.8\% at steady state, almost three times that of the fastest mo-
tion. Similarly to the other solution, the mid-line of the cylinder pair rotates by
roughly 0.4\( \pi \) during the initial transient.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{vorticity_fields.png}
\caption{Comparison of the fastest (left) and efficient (right) motions: the colours refer to the vorticity field and the dashed black lines denote the initial positions.}
\end{figure}

Figure 4.5 shows the comparison of the vorticity fields for the fastest motion (on
the left) and the efficient motion (on the right), both at steady-state. The frames
are taken every \( 5T/4 \) so that each frame shows the vorticity field after one fourth
of the period of motion plus a constant leftward displacement. For the fastest
solution, the left cylinder alternatingly sheds counter-clockwise rotating vortices
below and clockwise rotating vortices above the right cylinder. This vorticity affects
the oscillation of the cylinders’ mid-line, but the wake mainly results from the vortex
shedding behind the right cylinder. In both cases, the vorticity is shed with double
the frequency of the cylinders’ angular velocities. The efficient motion has a much
more compact vorticity, and the wake is located above and below, rather than
directly behind, the cylinders.
4.4 Results

Figure 4.6: Illustration of the fastest pulsating symmetric motion. On the left, vertical velocity on top (the dashed line indicates the average forward velocity over the last period), the velocity of the pulsating motion along the mid-line of the cylinder, and the angular velocities of the cylinders. On the right we show the efficiency over a running window of one period.

4.4.2 Propulsion of a pair of counter-rotating cylinders with pulsation along the mid-line

In this section we discuss the pulsating symmetric motion of the cylinders, optimized for speed and efficiency of propulsion. The evolution during the optimization of the fitness evaluations and the corresponding parameters are shown in Appendix A.

The optimization for the fastest motion reaches the upper bound for the amplitude of the angular velocity of the cylinders, but finds an internal optimum for the amplitude of the pulsating motion. Specifically, the solution was found for \( Re_\Omega = 1570 \) and \( Re_A = 804 \). The phase between the two harmonics gives an optimum for \( \pi = \phi \), meaning that when the pulsating velocity reaches a maximum, so does the rotation rate. Also, the minimum distance between the cylinders reaches a value \( G = 9.5D \) which is very close to the upper bound.

Figure 4.6 shows the kinematics and the efficiency of the fastest solution. At steady state, the vertical velocity oscillates with twice the frequency of the motion around

<table>
<thead>
<tr>
<th>Case</th>
<th>( A^* )</th>
<th>( \Omega^* )</th>
<th>( G^* )</th>
<th>( \phi/\pi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fastest</td>
<td>1.5</td>
<td>5</td>
<td>9.5</td>
<td>1.0</td>
</tr>
<tr>
<td>Efficient</td>
<td>0.58</td>
<td>0.41</td>
<td>6.37</td>
<td>1.24</td>
</tr>
<tr>
<td>Start</td>
<td>1</td>
<td>2</td>
<td>6</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Table 4.2: Optimal solutions for the pulsating symmetric motion. We report also the start search point for the optimization for speed. The initial point for the optimization for efficiency was the fastest solution.
Figure 4.7: Illustration of the efficient pulsating symmetric motion. On the left, vertical velocity on top (the dashed line indicates the average forward velocity over the last period), the velocity of the pulsating motion along the mid-line of the cylinder, and the angular velocities of the cylinders. On the right we show the efficiency over a running window of one period.

\[ \langle U \rangle = -7.3081(D/T), \text{ or } Re_U = 365. \] The locations of the maxima in the vertical velocity coincide with either a minimum or a maximum in the angular velocity of the cylinders. The propelling force can be easily explained with the Kutta–Joukowski theorem: during the motion, the surface velocity on the upward-facing side of the cylinders always has the same direction as the incoming flow caused by the pulsation along the mid-line. This method of propulsion, even if optimized for speed, allows the motion to achieve an efficiency of 5%, greater than for the most efficient wavelike motion. In fact, the solution acquires 50 times the kinetic energy although at a cost of 11 times the input energy.

The most efficient solution is internal to the optimization bounds: the optimum is found for \( Re_\Omega = 128, Re_A = 182, G = 6.4D, \) and \( \phi = 1.24\pi. \) The pulsating motion and the angular velocity of the cylinders are out of phase. In fact, the Kutta–Joukowski lift that explains the propulsion of the cylinders is obtained for inviscid flow: the reduced Reynolds numbers that define the most efficient solution suggest that viscosity effects affect the optimal phase between the cylinders. Moreover, a wide gap between the two cylinders is chosen by the optimizer for both cost functions, implying that a reduced interaction between the wakes behind each cylinder is beneficial to the efficiency.

Figure 4.7 shows the data referring to the efficient motion. The motion achieves a propulsive efficiency of almost 20% and a speed of \( \langle U \rangle = 1.1(D/T), Re_U = 55, \) approximately the same speed as the fastest wavelike motion. At steady-state, the velocity has a period that is double of that of the motion but is out of phase with both the pulsation and the angular velocity.

Figure 4.8 shows the vorticity field at steady-state for both the most efficient and
4.4 Results

the fastest motions. For the efficient motion, we observe two separate wakes, each with separate areas of positive and negative vorticity. The fastest solution presents well defined vortices being shed during the pulsating motion.

4.4.3 Propulsion of a pair of counter-rotating cylinders at a fixed distance

In this section we present the results for the rigid symmetric motion. We consider three sets of parameters. The first, case $F1$, is an optimization for speed with fixed $W^* = 2.25$, which is the same width as the fastest rigid wavelike motion. The second, case $F2$, is the fastest solution within the optimization bounds with variable $W^*$. The third, case $E$, is the rigid motion optimized for efficiency of propulsion.

Figure 4.9 shows the velocity and efficiency of case $F1$, which was obtained for $Re_{\Gamma,1} = 205$ and $Re_{\Gamma,2} = 43.5$. The cylinder pair accelerates forward during the
Figure 4.9: Kinematics of the fastest rigid symmetric motion for $W^* = 2.25$, case $F1$. On the left, velocity (dashed is the average over one period of motion), on the right, efficiency over a running window of one period.

Figure 4.10: Time-development of the vorticity field for case $F1$.

power stroke (negative peaks of the velocity), but the momentum is lost as soon as the recovery stroke begins. During the recovery stroke, consistently with the results from chapter 3 since $Re_{t,2} < Re_{t,trans.}(W^* = 2.25)$, the cylinder begins again the forward acceleration, which is continued during the power stroke. As shown in figure 4.10, during the power stroke the cylinder accelerates forward and there is a vorticity build-up in its wake. However, when the cylinders begin the recovery stroke, the vorticity is convected back through the mid-line in front and around the pair. Effectively the motion is unable to escape its own wake, which halts the propulsion. In fact the average forward velocity of the motion only reaches
4.4 Results

Figure 4.11: Kinematics of the rigid symmetric motion for case $F1$. On the left, vertical velocity (dashed is the velocity averaged over one period of motion), on the right, efficiency over a running window of one period.

$Re_U = 1.55$. This is due both to the low Reynolds number of the recovery stroke and to the choice of the number of revolutions during each stroke $N_{revs} = 10$, as we will see in section 4.5.3.

Figure 4.11 shows the velocity and efficiency for the fastest rigid symmetric motion, case $F2$, which was found at the lower bound of the distance between the two cylinders $W^* = 1.5$, the upper bound of the rotational Reynolds number for the power stroke $Re_{\Gamma,1} = 4000$, and $Re_{\Gamma,2} = 250$. Again, the velocity peaks correspond to the power stroke but, unlike the previous case, the forward momentum is maintained during the recovery stroke. In fact, while we observe a sharp decrease in the velocity as soon as the power stroke begins, during the power stroke the cylinders accelerate forward and the velocity only gradually decreases during the recovery. In this case we have that the angular velocity during the power stroke is faster than the observed threshold for the dipole-like propulsion of the counter-rotating cylinder pair ($Re_{\Gamma,2} > Re_{\Gamma,trans}(W^* = 1.5$)), which might explain the slow decrease in velocity during the recovery stroke. Still, the high value of $Re_{\Gamma,2}$ allows the motion to be faster decreasing the time between power strokes, and thus increases the velocity. The average forward velocity can be compared to that of the previous optimization problems through the Reynolds number based on the average forward velocity. In this case we have $Re_U \approx 38$, approximately 20% lower than that of the fastest wavelike motion.

Figure 4.12 shows the vorticity field at different time intervals. We observe that the power stroke is characterized by the build-up and subsequent shedding of the vorticity around the cylinders. The vorticity is shed as a vortex pair travelling in the
Figure 4.12: Time-development of the vortex shed by the power stroke of the rigid symmetric motion for case $F_2$. The power stroke starts for $t_0 + T/10$.

Figure 4.13: Tracer particles are initialized in the closed streamline around the cylinders during the recovery stroke and passively advected by the flow for case $F_2$, highlighting the shedding of the vortex pair.

opposite direction of the pair, which accelerates the cylinders forward. Moreover, the sign of the vortex pair is consistent with the starting vortex ring formed during the contraction phase (i.e. power stroke) of the swimming cycle of oblate medusan jellyfish, as observed by Dabiri et al. [13].

Figure 4.13 shows tracer particles illustrating the vortex shedding. During the recovery stroke, in fact, the flow is similar to the dipole-like flow pattern of the counter rotating cylinder pair, which develops a closed streamline of fluid travelling with the pair. The tracer particles were initialized inside the observed closed streamlines for a solution at the time instant during the recovery when the velocity was varying the least. The figure shows that the cylinders, during the power stroke, shed part

<table>
<thead>
<tr>
<th>Case</th>
<th>$Re_{\Gamma,1}$</th>
<th>$Re_{\Gamma,1}$</th>
<th>$W^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1</td>
<td>205</td>
<td>43.5</td>
<td>2.5</td>
</tr>
<tr>
<td>F2</td>
<td>4000</td>
<td>250</td>
<td>1.5</td>
</tr>
<tr>
<td>E</td>
<td>471</td>
<td>105</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Table 4.3: Optimal solutions for the rigid symmetric motion.
4.4 Results

Figure 4.14: Kinematics of case $E$. On the left, vertical velocity (dashed is the velocity averaged over one period of motion), on the right, efficiency over a running window of one period.

Figure 4.15: Time-development of the vortex shed by the power stroke of the rigid symmetric motion for case $E$.

of the fluid moving along with the pair.

Figure 4.14 shows the most efficient rigid symmetric motion, case $E$. Also this solution was obtained for $W^* = 1.5$, but we obtain an internal optimum for the stroke parameters: $Re_{\Gamma,2} = 471$ and $Re_{\Gamma,2} = 105$. Again, the peaks in the velocity correspond to the power stroke, but, similarly to the first case for $W^* = 2.25$, the momentum partly lost at the onset of the recovery stroke. The average velocity Reynolds number Figure 4.15 shows the vorticity field at different times. Again we observe the vorticity build-up around the pair at the beginning of the power stroke that is shed backward as a vortex pair.
4.5 Variations of the parameters

In this section we show the effect of variations of the parameterization variables on the propulsive performance of the cylinder pair.

4.5.1 Wavelike motion

Figures 4.16 and 4.17 report the effect of the variation of the optimization parameters on the propulsion of the wavelike motion. Besides the horizontal speed and efficiency we report the averaged angular velocity over the last 10 periods of motion and normalized with respect to the maximum value between all the variations. This quantity was introduced as an indication of the angular stability of the motion. As can be observed from the 6th panel of the figures, in fact, depending on the parameters, the trajectory of propulsion is not always rectilinear after the initial transient.

For the fastest motion (figure 4.16), we see that most of the propulsion comes from the right cylinder, the speed of the motion being reduced only by reducing the angular velocity of the right cylinder and not by the angular velocity of the left one. The width of the pair determines the stability of the motion: if the cylinders

Figure 4.16: Effect of the variation of the optimization parameters with respect to the fastest wavelike motion on the speed, efficiency, and angular velocity (the latter averaged over the last 10 periods). The 6th panel shows the comparison of the trajectories with that of the optimum.
are too close to each other they do not reach stable horizontal propulsion. The same result is obtained by increasing either of the angular velocities. In fact, the fifth panel shows the effect of proportionally varying the amplitude of both angular velocities. If the multiplier is less than one, the reduced velocity of the right cylinder causes a decrease speed of the motion and an increase in efficiency, meaning that the loss of kinetic energy is compensated by a reduction in energy expenditure. If the multiplier is greater than one, both speed and efficiency decrease due to the instability of the trajectory.

For the efficient motion (figure 4.17), both angular velocities affect the propulsive performance to a similar extent, and have similar magnitudes. Again, the width of the pair affects the horizontal stability, but the reduced amplitudes of the two angular velocities allow stable motion for $W^* = 1.5$. Different from the previous case is the near-symmetry of the performance with respect to the phase around $\phi = 0.5\pi$.

### 4.5.2 Pulsating symmetric motion

Figures 4.18 and 4.19 report the effect of the variation of the optimization parameters of the pulsating symmetric motion on the propulsion of the cylinder pair. In
Figure 4.18: Effect of the variation of the optimization parameters on the speed and efficiency with respect to the fastest symmetric motion.

Figure 4.19: Effect of the variation of the optimization parameters on the speed and efficiency with respect to the efficient symmetric motion.
this case, due to the symmetry of the problem, the trajectory was always purely vertical.

For the fastest motion, the vertical velocity together with the efficiency, approaches zero if the pulsating amplitude decreases to zero, but plateaus if $A$ increases past $1.5D$. This cannot be directly explained with the Kutta-Joukowski theorem, which states that the lift force on the cylinders should linearly increase with the pulsating velocity. On the other hand, a wider pulsation increases the size of the wake behind the motion, and hence the dissipated energy, which is the reason why the pulsation amplitude does not strictly increase the velocity. The velocity linearly increases with $\Omega^*$, while the efficiency linearly decreases. The former result can be easily explained by equating at steady state the Kutta-Joukowski lift ($F_L \propto \rho U \Gamma$) and the drag equation ($F_D \propto \frac{1}{2}\rho DU^2$), which is a good approximation for $Re_U \gg 1$. The gap between the two cylinders during the pulsation has the least effect on the performance.

For the efficient motion we observe similar trends as for the fastest motion. The linear relationship between efficiency, speed, and angular velocity amplitude are recovered for higher values of $\Omega^*$, but both the efficiency and the speed go to zero for $\Omega^* \rightarrow 0$. While for the fastest case the phase $\phi = \pi$ maximizes both the speed and the efficiency of the motion, the most efficient solution is found for $\phi = 1.25\pi$ and the speed is increased by decreasing the phase to $\phi = 1.1\pi$.

### 4.5.3 Rigid symmetric motion

Figure 4.20 reports the effect of the stroke parameters $Re_{\Gamma,1}$ and $Re_{\Gamma,2}$ on the propulsive performance for the fastest and most efficient rigid motions.

For the fastest motion, the speed of propulsion increases with power stroke Reynolds number $Re_{\Gamma,1}$, and the optimum was found at the upper bound of the optimization space. In fact, increasing the angular velocity leads to the shedding of a vortex pair with greater intensity in the wake of the motion, which increases the acceleration during the power stroke. On the other hand, we observe that the efficiency decreases with $Re_{\Gamma,1}$, meaning that the gain in kinetic energy is less than the additional dissipated power due to the greater velocity gradients in the flow. The optimum for the Reynolds number of the recovery stroke was found for $Re_{\Gamma,2} \approx 250$, but we observe that for constant $Re_{\Gamma,1}$ there is a local maximum of the efficiency for $Re_{\Gamma,2} \approx 200$. In fact, as we observed for the counter-rotating cylinder pair in chapter 3, the velocity of the cylinders in the dipole-like configuration has a weak dependence on the rotational Reynolds number, especially for rotation rates near the transition to jet-like motion. Hence, a lower rotation rate achieves a similar forward velocity while decreasing the dissipated energy.

For the most efficient motion we observe similar trends. The speed increases monotonically with $Re_{\Gamma,1}$, but the optimal balance between dissipated energy and average kinetic energy is found for $Re_{\Gamma,1} \approx 471$. For the peak rotational Reynolds number
Figure 4.20: Effect of the variation of the peak rotational Reynolds numbers of the power stroke $Re_{\Gamma,1}$ and of the recovery stroke $Re_{\Gamma,2}$ with respect to the fastest (on top) and most efficient (below) symmetric rigid motion.

The effect of the recovery stroke we observe again two local maxima for the efficiency and, at a greater $Re_{\Gamma,2}$, for speed. These two maxima are shifted by about 100 from the ones observed with respect to the fastest motion, implying that the dipole-like flow pattern around the cylinder pair is made more stable by the faster motion. The reason of this shift might be found either in the vorticity distribution, or in the different $Re_U$ that affects the interaction of the pair with its own wake.

Figure 4.21 shows the effect of the number of revolutions during one period of motion $T$ with respect to the fastest motion for $W^* = 2.25$. The rigid motion is performed with the same stroke parameters, except for the number of revolutions $N_{\text{revs}}$. The vorticity plots are taken for a solution moving at a constant average $U$ at equal non-dimensional time intervals. In order to compare the velocities with varying period of motion, the graph in figure 4.21 non-dimensionalises the forward velocity with the average angular velocity $\langle \Omega \rangle$.

The effect of the period, or the number of rotations, on the ability of the cylinder
pair to propel is clear: a longer period allows the cylinder pair to impose a pattern to the flow that pushes the fluid backwards, freeing the pair from the influence of its own wake. For example, even the fastest stroke parameters found by the optimizer for \( W^* = 2.25 \), did not lead to efficient propulsion. On the other hand, just by doubling \( N_{\text{revs}} \) the pair achieves stable forward velocity.

4.6 Discussion

In this section we will first present a comparison of the wavelike motions of the cylinder pair and anguilliform swimmers and then compare the wave-like and the jet-like motions.
4.6.1 Comparison of the wavelike motion of the cylinder pair to anguilliform swimmers

The first parameterization leads to a wave-like optimal motion for the cylinders which results in stable horizontal propulsion. Here we compare this self-propulsion to that of a two-dimensional anguilliform swimmer. The details on the simulations of anguilliform swimmers can be found in the work of Gazzola, Van Rees et al. [18, 20, 21, 51].

Even though the phenomenology of the two swimmers is very different, we are interested in simple models that can be cheaply simulated (e.g. with the employment of a body fitted mesh or overlapping meshes), achieve comparable self-propelled velocities, and produce qualitatively similar wakes. These characteristics would make it possible to use this simple model to efficiently simulate fluid-mediated interactions in schools of swimmers and perform learning studies involving multiple swimmers. Moreover, simplified and non-deforming geometries can have engineering applications for small-scale robotic applications.

In figure 4.22, we show the comparison between the rotating cylinder pair and two shapes of anguilliform swimmers. Shape A has the same area as the fastest wavelike shape and its head has the same size as one of the cylinders. Shape B reproduces the geometry of zebrafish larvae of age 5 days post-fertilization, which swim in the same range of Reynolds numbers (in the range from 10 to 900 [38]) as the fastest motion. The shape was used in the work of Gazzola et al. [20] and in turn was inspired by the experimental observations of Muller et al. [38]. A brief description of the motion and shape of the anguilliform swimmers is given in appendix B.

Figure 4.23 compares the propulsive performance of the two anguilliform swimmers to that of the optimized motions for the cylinder pair. The velocities are reported non-dimensionalized with the length of the swimmer $L$ and the period of motion. We have $L = 0.0325$ for all shapes except for the most efficient motion of the cylinder pair which has $L = 0.025$.

The most remarkable result is that the simplified model can reach similar velocities as the anguilliform swimmers. In fact we observe that shape A, which was designed to have the same mass and bluff shape as the cylinder pair to be a fairer comparison,
Figure 4.23: Comparison of the lateral (thin line) and forward (thick line) velocities (on the left) and efficiencies (on the right) of the wavelike swimmers.

Figure 4.24: Comparison of the vorticity field around the simplified swimmers (cylinder pair optimized for speed on the top and efficiency on the bottom), and anguilliform swimmer shapes A and B.

achieves the lowest forward velocities, while the difference in the speed of the fastest simplified swimmer and shape B is only of 15%. However, the propulsive efficiencies of the cylinder pair is much lower than that of the anguilliform swimmers, shape A being twice as efficient as the most efficient motion of the pair. Considering that the volume of the shapes is the same, but the speed of shape A is lower than for the cylinder, the anguilliform swimmers are able to move through the fluid with lower energy required.

Figure 4.24 shows a comparison of the vorticity field for each of the swimmers mov-
Figure 4.25: Passive tracer particles advected by the velocity field around the anguilliform swimmer shapes and wavelike motion of the cylinder pair optimized for speed. When the swimmers are moving at steady state, the particles are seeded in a rectangular stip located upstream of their trajectory and coloured in black if displaced by the velocity field.

At steady state. As expected, the vorticity distributions around the cylinder pair and that around the anguilliform swimmers are qualitatively different. However, in the wake of the propulsion, for the fastest motion we observe the same alternatingly shed vortices as observed for the anguilliform swimmers.

A visual comparison between the wakes behind the swimmers with tracer particles is shown in figure 4.26. The particles highlight the different mechanisms of propulsion between the two swimmers. For the fastest wavelike motion, the rotation of the cylinders transfers momentum into the flow through shear, hence the particles are advected along the surface of the cylinders until the stagnation point and then released into the wake. On the other hand, the anguilliform swimmers push fluid backwards through the deformation of the body. The body deforms as a travelling wave, hence the particles contained inside one bend of the body move along the swimmer until they are released inside a coherent vortex structure by the flapping of the tail.

4.6.2 Comparison of the wave-like swimmers to the jet-like swimmers

Of the propulsion methods considered in this chapter, the one that achieves both the highest velocities and efficiencies is the pulsating symmetric motion. In fact, it is the only one that does not rely on purely shear-based mechanism to transfer
momentum to the flow, which could make it more similar to propulsion of jellyfish. The fastest solution is able to reach an average velocity of 7.3 diameters per period and an efficiency of 5%, while the most efficient solution reaches an average velocity of 1.14 diameters per period and an efficiency of 19%. Furthermore, the parameterization has the shortest transient to reach these averaged quantities, making it an interesting candidate for the application to small-scale robotic applications.

The other two motions have in common that the geometry of the swimmer, the distance between the two cylinders, is fixed in time. This constraint reduces the performances of the two motions, as all momentum transfer to the fluid occurs through shear, but at the same simplifies the system.

The optimization for speed reached the upper rotational Reynolds number bound for both the wavelike and the rigid symmetric motions. Since the bounds of the two problems were different, we manually picked a new set of stroke parameters for the rigid symmetric motion in order to compare it to the fastest wavelike motion. This case is named case \( P \) and is defined by \( Re_{\Gamma,1} = 2467, Re_{\Gamma,2} = 200 \) and \( W^* = 1.5 \), such that the peak rotation rate during the power stroke is equal to the amplitude of the angular velocity of the right cylinder in the fastest wavelike motion. This set of parameters is close to the average between the fastest \( (F2) \) and the most efficient \( (E) \) cases. Since the two periods of motion are different, but the maximum angular velocity is the same, the time is non-dimensionalized as \( t \cdot \Omega_{\text{max}}/(2\pi) \) and the velocity as \( u \cdot 2/(\Omega_{\text{max}}D) \).

Figure 4.26 compares the anguilliform fish-inspired wave-like motion to the jellyfish-inspired jet-like motion. The former is represented by the fastest solution of the wave-like motion, which achieves propulsion by shedding vorticity in a sequence of alternating sign vortices, which is common in thrust-producing wakes. The latter is represented by case \( P \) of the rigid symmetric motion, which during each period sheds a vortex dipole, which generates a strong momentum flux in its wake and thus a large thrust. This is also the definition of the swimming mode of jet-propelled medusae, which are distinct from rowing medusae [40]. The two motions have very similar swimming performances. The average forward velocity of the jet-like motion is lowered by the fact that every period some momentum is lost when switching from power stroke to recovery stroke. On the other hand, the momentum flux in the wake as a stable vortex-dipole makes the motion more efficient, requiring only a fifth of the input energy compared to the fastest wavelike motion.

### 4.7 Concluding remarks

In this chapter we presented different parameterizations of the motion of a pair of cylinders inspired by the movement of undulatory fish and jellyfish. The parameter space was explored with the CMA-ES stochastic optimization algorithm in order to find the best solutions in terms of maximum speed and efficiency.
In the first parameterization, the wavelike motion, the cylinders are kept at a fixed distance and are made to rotate with two independent sinusoidal functions with different amplitudes and phases. The parameters were optimized looking for motions that propel along the mid-line of the pair. We showed the fastest and most efficient solutions within the bounded parameter space and compared them to the propulsion of anguilliform swimmers. The considered anguilliform swimmer geometry was that of larval zebrafish, which naturally swim in the same range of Reynolds numbers as the parameter space, and a modified geometry which has the same area and length as the fastest obtained wavelike motion. Both the cylinders and the swimmers shed vorticity in a sequence of alternating sign vortices, which is common in thrust-producing wakes. Moreover, the cylinder pair achieves similar forward velocities to the swimmers’, but the propulsion is less efficient. The mechanism for the transfer of momentum to the flow was found to be qualitatively different for the cylinder pair and the swimmers: the first is based on shear forces, the second
4.7 Concluding remarks

on pushing the fluid downstream via deformation of the solid boundary.

In the second parameterization, the pulsating symmetric motion, the distance between the two cylinders pulsates with a sinusoidal function, and at the same time the cylinders rotate in opposite direction with a sinusoidal function. The parameters were optimized in order to achieve locomotion along the normal direction to the mid-line. The two optimal parameter sets were the most efficient and fastest solutions overall for the cylinder pair. In fact, this was the only motion that does not rely on a purely shear-based mechanism to transfer momentum to the flow. Specifically, it was observed that the propelling force can be explained with the Kutta–Joukowski theorem.

With the third parameterization, the rigid symmetric motion, we applied the existence of two stable propulsion states for counter rotating cylinders in order to design a mode of locomotion with periodic deformation, rather than the cylinders steadily rotating from the initial configuration. The angular velocity of the cylinders followed a periodic function inspired by measurements of the angular velocity of jellyfish bell lobes by Nawroth et al. [39]. We showed that the length of the period determines the ability of the cylinders to switch between the two flow patterns of the counter-rotating cylinders, and thus determines whether efficient propulsion is possible. For the optimized cases, the pair sheds a vortex dipole after each power stroke, which generates a strong momentum flux in its wake and thus a large thrust, similarly to the swimming mode of oblate medusae [13]. Furthermore, even though some momentum is lost when switching from power stroke to recovery stroke, the rigid symmetric motion was shown to be more efficient than the other shear-based locomotory mode, the wavelike motion.
In this work we considered different locomotory modes of a pair of rotating cylinders in a two-dimensional, viscous, incompressible flow.

The simulations were performed with a wavelet-adapted remeshed vortex method and the dynamic coupling between the fluid and the self-propelling bodies was done with the Brinkman volume penalization and projection approach. The parameter space of the self-propelling test-cases was explored with the covariance matrix adaptation evolutionary strategy (CMA-ES), an efficient and robust stochastic optimization algorithm, able to handle noisy and multi-modal cost functions.

5.1 Conclusions

Counter-rotating cylinder pair

In the first part of the thesis we discussed the self-propulsion of a pair of identical counter-rotating cylinders linked together at a fixed distance by their centres of mass. We showed that the self-propulsion of a counter-rotating cylinder pair is a very interesting stability problem where the end state of the system varies nonlinearly with the parameters of the problem and depends on the initial conditions.

We explored the phase space defined by the two independent non-dimensional parameters of the system: the rotational Reynolds number and the ratio of the center-to-center distance over the diameter. For low rotation rates, the cylinders behave like a vortex dipole and we showed that the streamline pattern around the cylinders form an elliptic virtual region of recirculating flow that moves with the cylinders. From the perspective of the cylinder pair, the incoming flow is pushed around the elliptical region and the propulsive force comes from the acceleration of the flow on
the outer side of the cylinders’ mid-line. Moreover, an inversely proportional relation was found between the velocity and the circulation of the cylinders, reinforcing the similarity to the ideal vortex pair.

Increasing the rotation rate, the cylinders change the direction of motion. The elliptic virtual body is substituted by two separated sets of closed streamlines, the external flow is pulled through the mid-line, accelerated, and expelled in a jet-like manner. The transitional rotation rate between these two states shows an inverted exponential dependency on the width between the centres of mass, with larger width requiring smaller rotation rates to switch from dipole-like to jet-like flow.

Moreover, we performed an initial characterization of the stability of these two flow patterns by varying the angular velocity of the cylinders from a solution at steady-state. We observed that near the transitional rotation rate the upward-moving state is stable, acting similarly to an attractor for dynamical systems, whereas the downward-moving state is more sensitive to disturbances.

For greater rotation rates, the motion of the cylinder pair becomes unstable leading to oscillations in the velocity. The cylinders can escape the interaction with the vorticity in their wake and settle in either upward or downward steady motion, or behave in a quasi-periodic or fully unpredictable unsteady way. Also in this case, an inverted exponential correlation was found to predict the transition between stable downward and the onset of instability. The instability occurs at lower rotation rates when the cylinders are separated by a larger width.

Bio-inspired self-propulsion of two rotating cylinders

In the second half of the thesis we discussed three different locomotory modes of the cylinder pair, inspired by the movement of undulatory fish and jellyfish. These studies aimed at using the cylinder pair as a reduced physics model for the different types of swimmers. The parameter space of the motions was explored with the CMA-ES stochastic optimization algorithm in order to find the best solutions in terms of maximum speed and efficiency.

In the first parameterization, the wavelike motion, the cylinders are kept at a fixed distance and are made to rotate with two independent sinusoidal functions with different amplitudes and phase-shifts. The parameters were optimized looking for motions that propel along the mid-line of the pair. We showed the fastest and most efficient solutions within the bounded parameter space and compared them to the propulsion of anguilliform swimmers. The considered anguilliform swimmer geometry was that of larval zebrafish, which naturally swim in the same range of Reynolds numbers as the parameter space, and a modified geometry which has the same area and length as the fastest obtained wavelike motion. Both the cylinders and the swimmers shed vorticity in a sequence of alternating sign vortices, which is common in thrust-producing wakes. Moreover, the cylinder pair achieves similar forward velocities to the swimmers’, but the propulsion is less efficient. The
mechanism for the transfer of momentum to the flow was found to be qualitatively different for the cylinder pair and the swimmers: the first is based on shear forces, the second on pushing the fluid downstream via deformation of the solid boundary.

In the second parameterization, the pulsating symmetric motion, the distance between the two cylinders oscillates with a sinusoidal function, and at the same time the cylinders counter-rotate with rotation rates specified by a sinusoidal function. The parameters of the motion were optimized in order to find optimal motions that achieve velocity along the normal to the mid-line. The obtained parameter set lead to the most efficient and fastest propulsion of the cylinder pair. In fact, this was the only motion that does not rely on a purely shear-based mechanism to transfer momentum to the flow. Specifically, it was observed that the propelling force can be explained with the Kutta–Joukowski theorem.

With the third parameterization, the rigid symmetric motion, we applied the existence of two stable propulsion states for counter rotating cylinders in order to design a mode of locomotion with periodic deformation, rather than the cylinders steadily rotating from the initial configuration. The angular velocity of the cylinders follows a periodic function inspired by the angular velocity of jellyfish bell lobes [39] and is characterized by the peak angular velocity during the power stroke and the peak angular velocity during the recovery stroke. These two stroke parameters were optimized in order to find the fastest and most efficient combinations for propulsion. We showed that the length of the period determines the ability of the cylinders to switch between the two flow patterns of the counter-rotating cylinders, and thus determines whether efficient propulsion is possible. For both solutions, the pair sheds a vortex dipole after each power stroke, which generates a strong momentum flux in its wake and thus a large thrust, similarly to the swimming mode of jet-propelled medusae [40]. In fact, the sign of the vortex pair is consistent with the starting vortex ring formed during the contraction phase (i.e. power stroke) of the swimming cycle of oblate medusan jellyfish [13]. The motion has very similar swimming performances to the wavelike-motion. The average forward velocity of the jet-like motion is lowered by the fact that every period some momentum is lost when switching from power stroke to recovery stroke. On the other hand, the momentum flux in the wake as a stable vortex-dipole makes the motion more efficient.

### 5.2 Outlook

In this section we will indicate some of the directions for future research on the topics of the present study.

One interesting development for the counter-rotating cylinder pair would be the systematic analysis of the hysteresis effects of the problem. For example, one could analyse how the history of variations in the angular velocity of the cylinders can affect the resulting steady-state.
The bio-inspired motions of the cylinder pair could be used as reduced physics model to efficiently simulate fluid-mediated interactions in schools of swimmers and perform learning studies involving multiple swimmers. In fact, computational costs limit studies of fish schooling to potential models [47], however recent studies [19] demonstrated that the interaction of objects at finite Reynolds numbers is drastically different from the interaction in potential flow. The simple non-deforming geometries considered in this thesis, which can be coupled with body-fitted meshes or overlapping meshes, can be cheaply simulated and could allow schooling studies in viscous flow. Future research could be aimed at investigating the self-propulsion of the swimmers in the presence of ambient vorticity, or at simulating maneuvring. Moreover, the propulsive performance of the symmetric motions of the cylinder pair, both with and without pulsation along the mid-line, could be compared to two-dimensional jellyfish-like geometries, similar to those studied by Bergmann and Iollo [3], at comparable Reynolds numbers.

Furthermore, the study can be broadened to three-dimensions. For example, expanding on the counter-rotating cylinder pair problem, a preliminary study with a reduced flow resolution (ppr = 8, limited by the cost of the simulation) was performed on a pair of identical spheres, counter-rotating around a direction that is normal to the connecting line between the two centres of mass, for $W^* = W/D = 2$ and a few rotational Reynolds numbers. At the lower rotational Reynolds numbers ($Re_T = 25, 150$) the spheres moved, with respect to the angular velocity of the spheres, in the same direction as a vortex dipole, on the other hand, at the higher rotational Reynolds number ($Re_T = 500$), the velocity had switched sign. This indicates consistency between the two-dimensional and the three dimensional cases. In particular, for the intermediate rotational Reynolds number $Re_T = 150$, the two-dimensional cylinder pair moved with a jet-like flow pattern, in the opposite direction to the pair of counter-rotating spheres. In a second experiment, we analysed two prolate spheroids, rotating around the major axis, having the ratio between the distance of the centres of mass and the minor axis equal to 2. It was observed that, for the same rotational Reynolds number $Re_T = 150$, if the major axis is much longer than the minor axis, the same direction of self-propulsion is obtained as for the cylinder pair, hence recovering the two-dimensional result. If further analysed, the third dimension adds new complexities to the vorticity structures of the fluid mediated interaction, bringing the study closer to applications for robotic propulsion.


Fitness function evolution for the self-propulsion optimization

In this work, we used the optimization algorithm of evolutionary strategy with covariance matrix adaptation (CMA-ES) in order to find optimal propulsive motion of simplified swimmers. We considered three test-cases, all explained in further detail in chapter 4: horizontal motion, vertical pulsating motion, and vertical rigid motion. This appendix contains the output of the optimization algorithm for each of these cases. We show the evolution of the fitness function and the corresponding values of the optimization parameters against the number of iterations.
Figure A.1: Evolution of the cost function and optimization variables against number of optimization iterations for the fastest horizontal motion. Blue dots correspond to current generation samples, the blue line denotes the current mean of the distribution, the green one corresponds to the best current solution, and the black one corresponds to the best solution ever. The bounds on the plots for the optimization variables correspond to the bounds chosen for the optimization.
Figure A.2: Evolution of the cost function and optimization variables against number of optimization iterations for the efficient vertical motion. Blue dots correspond to current generation samples, the blue line denotes the current mean of the distribution, the green one corresponds to the best current solution, and the black one corresponds to the best solution ever. The bounds on the plots for the optimization variables correspond to the bounds chosen for the optimization.
Fitness function evolution for the self-propulsion optimization

Figure A.3: Evolution of the cost function and optimization variables against number of optimization iterations for the fastest vertical motion. Blue dots correspond to current generation samples, the blue line denotes the current mean of the distribution, the green one corresponds to the best current solution, and the black one corresponds to the best solution ever. The bounds on the plots for the optimization variables correspond to the bounds chosen for the optimization.
Figure A.4: Evolution of the cost function and optimization variables against number of optimization iterations for the efficient vertical motion. Blue dots correspond to current generation samples, the blue line denotes the current mean of the distribution, the green one corresponds to the best current solution, and the black one corresponds to the best solution ever. The bounds on the plots for the optimization variables correspond to the bounds chosen for the optimization.
Fitness function evolution for the self-propulsion optimization
Appendix B

Geometry and motion of the anguilliform swimmers

Simplified models of self-propulsion were compared to simulations of two-dimensional anguilliform swimmers. While a detailed description of the simulations can be found in the work of Gazzola, Van Rees et al. [18, 20, 21, 28, 51], here we briefly discuss the geometry and the equation for swimming motion of anguilliform swimmers.

The geometry of the swimmers is defined by the half-width $w(s)$ as a function of the arc length along the mid-line:

$$w(s) = \begin{cases} 
  w_h \sqrt{1 - \left( \frac{s_h - s}{s_h} \right)^2} & \text{if } 0 \leq s < s_h, \\
  (w_t - w_h) \left[ 3 \left( \frac{s - s_h}{s_t - s_h} \right)^2 - 2 \left( \frac{s - s_h}{s_t - s_h} \right)^3 \right] + w_h & \text{if } s_h \leq s < s_t, \\
  w_t \left[ 1 - \left( \frac{s - s_t}{L - s_t} \right)^2 \right] & \text{if } s_t \leq s < L 
\end{cases}$$  

where $L = 0.0325$ is the length of the swimmer.

Shape A has the same area as the fastest horizontal shape and its head has the same size as one of the cylinders. These two constraints impose that the shape parameters should be:

$$w_h = s_h = D/2 = 0.005, \quad w_t = 0.4w_h, \quad s_t = 0.0133$$  

Shape B reproduces the geometry of zebrafish larvae of age 5 days post-fertilization, which swim in the same range of Reynolds numbers as the optimized motions of the simple models presented in chapter 4. The shape was used in the work of Gazzola.
et al. [20] and in turn was inspired by the experimental observations of Muller et al. [38]. The corresponding parameters are:

\[ w_h = 0.0635L, \quad s_h = 0.0862L, \quad w_t = 0.4w_h, \quad s_t = 0.3448L \quad (B.3) \]

The swimming motion is based on the work of Carling et al. [6] and was used in the work of Van Rees et al. [51] and Kern et al. [28]. The motion is defined by imposing the lateral deformation of the swimmer’s mid-line \( y(s, t) \):

\[ y(s, t) = 0.125L \frac{0.03125 + s/L}{1.03125} \sin \left[ 2\pi \frac{s}{L} - \frac{t}{T} \right] \quad (B.4) \]

where \( t \) is the time and \( T \) is the swimming period.