Gas kinetic traffic flow modelling including continuous driver behaviour models

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Abstract

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Traffic flow models are used to analyse and predict traffic flows on road networks. Macroscopic traffic flow models assume traffic density, traffic volume and speed as variables and use relatively simple equations, such as the conservation equation (traffic volume equals density times speed) and a decreasing function expressing speed as function of density. These models are usually solved by simple analytical techniques. On the other hand microscopic traffic flows models assume detailed submodels for the behaviour of driver/vehicle combinations and the interaction in terms of free-driving, car-following and lane changing. Microscopic traffic flow models are usually solved by time incremental simulation. However, both macroscopic and microscopic appear to have problems in representing different types and phase of congested traffic flows. For a review of existing traffic flow models we refer to Tampère (2001), Hoogendoorn & Bovy (2001) or Helbing (2001).

Recent years have shown an increasing interest in the application of gas kinetic traffic flow models (Hoogendoorn, 1999; Helbing 1997; Klar & Wegener, 1999). They bridge the gap between macroscopic and microscopic models by combining aggregate traffic flow variables with assumptions on the interaction between driver/vehicles units. Interestingly, they appear to be able to represent different types and phases of congested traffic flows (Helbing, 1998; Treiber et al., 1999).

A drawback of gas kinetic traffic flow models up till now has been the discrete nature of the interaction modelling between driver/vehicles units. This paper addresses the inclusion of continuous adaptive models for driver/vehicle behaviour in gas kinetic traffic flow models. The paper is part of a research project that studies the potential impact of Advanced Driver Assistance (ADA) systems in congested traffic flows. The demand for including continuous driver models was induced by the need to incorporate different assumptions for car following and more refined driver behavioural assumptions. In this respect the main issue in building macroscopic traffic flow models for currently non-existing traffic flows (like flows with all types of ADA systems) is to replace the empirical relation between equilibrium speed and the density with a relationship based on available knowledge of the driving behaviour, that is gained through driving simulator experiments and field operational tests with a limited number of instrumented vehicles.
2 The theoretical basis: (Gas-) kinetic traffic flow equations

Gas kinetic traffic flow models were first proposed by Prigogine (1961) and are based upon the analogy between gas flows and traffic flows. Where in the former case the dynamics are governed by interacting gas particles the latter deals with interacting vehicles. In order to realistically describe traffic flows the specification of the vehicle interactions must obviously differ from that of gas particles, the main difference being that drivers do not behave according to physical laws. So far no examples exist where the continuous adaptive nature of human drivers is taken into account. We propose a way in which this can be accomplished, however without affecting the basic theoretical foundations of (gas-) kinetic modelling that is presented in this chapter.

The basis of the derivation in this paper is the phase-space density (PSD): $\rho(t, S)$. The PSD is a generalisation of the well-known traffic density $k$ that is interpreted in a statistical manner. In statistical mechanics, the density $k$ (in veh/m) is defined by: $k(t,x)$-dx is the expected number of vehicles at time $t$ in the incremental roadway length dx. Similarly the PSD $\rho(t, S)$-dS is the expected number of vehicles at time $t$ that are in state $S$. The definition of $\rho$ is further completed by defining the dimensions of the state $S$. The most simple definition of the PSD that is a generalisation of the density is obtained by including the speed $v$ as a dimension within the state $S$. Thus with $S=(x,v)$ the PSD $\rho(t, S)$-dS is the expected number of vehicles at time $t$ with speed $v \in [v,v+dv)$ and location $x \in [x,x+dx)$. Analogously we could include arbitrary number of dimensions in $S$.

In statistical mechanics (e.g. the description of gas kinetics) a generic dynamic equation for the kinetics of the PSD is determined (see for example Hoogendoorn, 1999):

$$
\frac{\partial \rho}{\partial t} + \nabla_S \cdot \left( \rho \frac{dS}{dt} \right) = \left( \frac{d\rho}{dt} \right)_{\text{events}}
$$

(1)

In this equation the product operator $\cdot$ denotes the inner product and the Nabla operator $\nabla_S$ for the state vector $S$ with dimensions $(s_1, s_2, \ldots, s_n)$ is defined by:

$$
\nabla_S = \left( \frac{\partial}{\partial s_1}, \frac{\partial}{\partial s_2}, \ldots, \frac{\partial}{\partial s_n} \right)
$$

(2)

By substitution of (2) in (1) we find as the basic kinetic equation of traffic flows:

$$
\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial s_1} \left( \rho \frac{ds_1}{dt} \right) + \frac{\partial}{\partial s_2} \left( \rho \frac{ds_2}{dt} \right) + \cdots + \frac{\partial}{\partial s_n} \left( \rho \frac{ds_n}{dt} \right) = \left( \frac{d\rho}{dt} \right)_{\text{events}}
$$

(3)

continuous changes

discrete changes
motion of the vehicles with the flow and autonomous changes $\frac{ds_i}{dt}$ of the individual state variables $s_i$ contained in $S$ (induced by the driver). The traffic flow model is theoretically specified if all relevant continuous or discrete processes in traffic flow are mathematically described and included in (3). We will give examples of specifications for these two types of processes further on.
3 Building aggregate traffic flow models

Aggregate traffic flow models describe the evolution in both space (for simplicity unidimensional: x) and in time t of aggregate traffic flow variables. Usually the aggregate variables density \( k \), the vehicle-averaged speed \( V \) and/or the flow intensity \( q = k \cdot V \) are taken as the model variables, since these can be derived directly from traffic flow measurements. This type of model is commonly referred to as ‘second-order’ macroscopic traffic flow model. Some authors have also proposed the use of the speed variance \( \theta \) as an aggregate variable in traffic flow, yielding a ‘third-order’ model.

It can be shown that the dynamic equations for these aggregate variables are actually included in the more general equation (3), since \( k(t,x) \), \( V(t,x) \) and \( \theta(t,x) \) are in fact the 0th, 1st and 2nd order velocity moments of the probability density function \( \rho(t,S) \), after elimination of all dimensions of \( S \) (except for the longitudinal position \( x \)). Their dynamic equation can be found using the so-called ‘method of moments’. Before we extend the classical derivation of aggregate traffic flow models in order to explicitly include detailed knowledge about individual driver behaviour, we illustrate the method of moments using a simple example.

Let us consider only two dimensions for \( S \): the location \( x \) and the speed \( v \). In order to find the dynamic equation for \( k(t,x) \), the 0th moment of the probability density function \( \rho \) we multiply both RHS and LHS of equation (3) by 1 and integrate over \( v \) to eliminate this dimension:

\[
\int_{v=-\infty}^{v=\infty} \left( \frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x} \left( \rho \frac{dx}{dt} \right) + \frac{\partial}{\partial v} \left( \rho \frac{dv}{dt} \right) \right) dv \bigg|_{v=0} = \int_{v=-\infty}^{v=\infty} \left( \frac{d\rho}{dt} \right) dv \tag{4}
\]

Using the definitions:

\[
k(t,x) \equiv \int_{v=-\infty}^{v=\infty} \rho(t,x,v) dv \tag{5}
\]

\[
V(t,x) \equiv \int_{v=-\infty}^{v=\infty} \frac{v \cdot \rho(t,x,v)}{k(t,x)} dv \tag{6}
\]

the fact that the order of integration and differentiation can be altered for independent variables, e.g.:

\[
\int_{v=-\infty}^{v=\infty} \frac{\partial}{\partial x} \left( v \cdot \rho(t,x,v) \right) dv = \frac{\partial}{\partial x} \left( \int_{v=-\infty}^{v=\infty} v \cdot \rho(t,x,v) dv \right) \tag{7}
\]

and the partial integration (in which we use the fact that \( \rho(t,x,\infty) = 0 \)):
we find:

\[ \frac{\partial k}{\partial t} + \frac{\partial k \cdot V}{\partial x} = 0 \]  

(9)

For the right hand side of this equation we have assumed that \( \left( \frac{d\rho}{dt} \right)_{\text{event}} \) only contains transitions between different speeds (so that an increase of \( \rho \) at a certain speed is compensated by an equally large decrease of \( \rho \) at another speed) and no emergence or dissipation of new vehicles. When integrating over all speeds these ‘internal’ events cancel out and the integral equals zero. The latter would not be the case when we treated for example lane changes or entrances from ramps as discrete events in our model, so that an increase of \( \rho \) at a certain speed is not compensated for by an equally large decrease of \( \rho \) at another speed.

Note that equation (9) is the well-known ‘conservation of vehicles’ law that is always part of an aggregate traffic flow model. It states that a spatial change of the flow \( k \cdot V \) can only exist if this is compensated by an equally large but opposite change in time of the density \( k \).

We now repeat this procedure for the 1\textsuperscript{st} moment, by multiplication of equation (3) by \( v \) and consecutive integration over all speeds \( v \). In the derivation we use the definition:

\[ \theta(t,x) = \int_{v=-\infty}^{v=\infty} (v - V(t,x))^2 \cdot \frac{\rho(t,x,v)}{k(t,x)} dv \]  

(10)

and partial integration, for example:

\[ \int_{v=-\infty}^{v=\infty} v \cdot \left( \frac{\rho \cdot dv}{dt} \right)_v \]  

\[ = \left. v \cdot \rho \cdot \frac{dv}{dt} \right|_{v=-\infty}^{v=\infty} - \int \rho \cdot \frac{dv}{dt} dv \]  

(11)

We use the notation \( \langle y \rangle_{s_i} \) to denote the expected value of \( y \) when averaging over all \( s_i \). In (11) this has been applied to the continuous acceleration term, defining the expected acceleration:

\[ \langle \frac{dv}{dt} \rangle_v = \int_{v=-\infty}^{v=\infty} \frac{\rho}{k} \cdot \frac{dv}{dt} dv \]  

(12)
Finally we find:

\[
\frac{\partial V}{\partial t} + V \frac{\partial V}{\partial x} = \frac{1}{k} \int_{v=-\infty}^{v=\infty} v \left( \frac{d\rho}{dt} \right)_{\text{event}} \, dv + \left\langle \frac{dv}{dt} \right\rangle_v - \frac{1}{k} \frac{\partial (k \cdot \theta)}{\partial x} \tag{13}
\]

The convection term in the LHS of this equation models that speed is a property vehicles carry along with the flow. The interpretation of this term is as follows. Consider a location with average speed \( V \) and a positive spatial derivative of the speed (convective term positive). This means that in time slower traffic from upstream will flow towards this location and faster traffic will flow out with speed \( V \). This will decrease the average speed on this location (time derivative of \( V \) negative).

Also in the RHS of (13) we find a convective term, often referred to as the ‘pressure’ term (in analogy with a similar term in gas kinetics). Although interpreted by many authors as a term describing the anticipation of drivers, it follows directly from the (gas-) kinetic approach that in fact it models a convective process (Hoogendoorn, 1999; Treiber et al., 1999). This can be understood by considering a short stretch of road (length \( dx \)) with a homogeneous density \( k \) and increasing speed variance \( \theta \) (derivative of \( k \cdot \theta \) positive). Suppose for simplicity that \( \theta = 0 \) at \( x \) and has some positive value \( \theta \) at \( x+dx \). This means that on the upstream boundary of the cell traffic flows in with individual speeds \( v \) exactly equal to \( V \). At the downstream boundary individual speeds are spread around \( V \). The faster vehicles (\( v>V \)) will leave this cell earlier than the slower vehicles (\( v>V \)) and are replaced by vehicles with speed \( V \). Therefore the average speed in the cell will decrease (negative derivative of \( V \)). This effect is purely due to ‘transport of speed’ by the vehicles in the flow, what we call convection, and not by any intervention by the driver (no anticipation).

This leaves two other terms in (13) that will contain the acceleration / deceleration behaviour of drivers. In all traffic flow models that have used the kinetic approach so far the acceleration has been modelled in the continuous term, while deceleration has been modelled as a discrete event. Acceleration on the individual driver level is then modelled as:

\[
\frac{dv}{dt} = \frac{w-v}{\tau} \tag{14}
\]

with \( w \) the desired speed of the driver. Assuming the same relaxation time \( \tau \) for all vehicles and defining the average desired velocity \( W \) analogously to (6) this yields as expected acceleration:

\[
\left\langle \frac{dv}{dt} \right\rangle_v = \int_{v=-\infty}^{v=\infty} \frac{1}{k} \frac{w-v}{\tau} \, dv = \frac{W-V}{\tau} \tag{15}
\]

The deceleration is then modelled as discrete interactions in which it is assumed that faster drivers catching up with slower traffic immediately adapt their speed to that of the slower predecessor. In the model this means that any encounter of fast and slow
\[-\left(\begin{array}{c}
\frac{d\rho(t,x,w)}{dt} \\
\end{array}\right)_{\text{deceleration}} = \left(\begin{array}{c}
\frac{d\rho(t,x,v)}{dt} \\
\end{array}\right)_{\text{deceleration}}\] (16)

The models are then further specified by defining the probabilities of these deceleration events. The models of Prigogine (1961), Paveri-Fontana (1975), Helbing (1997), Hoogendoorn (1999), Klar & Wegener (1998) all differ in the assumptions made for the specification of the discrete deceleration interaction. We will not elaborate any of these models further. We point out that in general all these models can finally be cast into the following form in which the two competing processes of acceleration and deceleration are combined in what is usually called the ‘equilibrium speed’ \(V^e(k)\). The continuous and discrete speed change terms in equation (13) can then be summarised as:

\[\frac{V^e(k) - V}{\tau} = \frac{W + \frac{\tau}{k} \int_{\nu}^{\nu_{\text{event}}} \left(\frac{d\rho}{dt}\right)_{\text{event}} dv - V}{\tau}\] (17)

This shows the merit of the kinetic modelling approach: the equilibrium speed that occurs in many traffic flow models (e.g. the model of Payne, 1971) is analytically derived from the individual acceleration and deceleration behaviour.

However, the individual driver behaviour is only poorly modelled in these models. Because of the discrete character of the deceleration process only the conditions in which deceleration occurs (e.g. only if no lane change is possible, only with vehicles within a certain interaction range, ...) and not the continuous adaptive process that is usually assumed on the individual driver level. Furthermore, the number and effect of the interactions must be exaggerated because the deceleration term is not substituted by but must compensate for the acceleration term that will always predict a non-negative acceleration. Finally from a theoretical point of view the different treatment of acceleration and deceleration would only be justified if the time scale for deceleration were negligible with respect to that of acceleration. This is not the case.

For this reason equation (17) is seldom used to analytically establish the equilibrium speed. In most cases the equilibrium speed is directly derived from empirical traffic data. An example of an empirically derived equilibrium speed density relation and the corresponding speed flow relation is depicted in figure 2.
4 Continuous adaptive individual driver behaviour in aggregate traffic flow models

We will now formulate an aggregate traffic flow model that allows a direct transformation of smooth adaptive driver behaviour into one acceleration term for equation (13). For this purpose we do not consider any discrete acceleration or deceleration events. The discrete speed change term therefore equals zero. Instead we will elaborate the continuous speed change term and include all acceleration and deceleration behaviour in the expected acceleration (equation (12)).

We consider the speed distribution at $x_\alpha$ and the vehicle and speed distribution immediately downstream of $x_\alpha$. Note that this implicitly guarantees that behaviour is only influenced by traffic conditions downstream (anisotropy property). Suppose that a vehicle $\alpha$ with speed $v_\alpha$ is present at location $x_\alpha$. We now consider every possible position $x_\beta (=x_\alpha+s_\alpha)$ and speed $v_\beta$ of the predecessor $\beta$ of this vehicle. Every time we calculate:

(i) the probability $p(s_\alpha \mid x_\alpha, v_\alpha) \cdot p(v_\beta \mid x_\alpha, v_\alpha, s_\alpha)$ that the predecessor is at a distance $s_\alpha$ of vehicle $\alpha$ driving with speed $v_\beta$;

(ii) the acceleration $a_\alpha(x_\alpha, v_\alpha, s_\alpha, v_\beta)$ of vehicle $\alpha$ in that case.

Equation (12) then becomes:

$$
\left\langle \frac{dv}{dt} \right\rangle = \int_{v_\alpha=-\infty}^{v_\alpha=\infty} \int_{x_\alpha=0}^{x_\alpha=\infty} \int_{v_\beta=-\infty}^{v_\beta=\infty} \rho(t, x_\alpha, v_\alpha) \cdot p(s_\alpha \mid x_\alpha, v_\alpha) \cdot \int \int p(v_\beta \mid x_\alpha, v_\alpha, s_\alpha) \cdot a_\alpha(x_\alpha, v_\alpha, s_\alpha, v_\beta) dv_\beta \cdot ds_\alpha \cdot dv_\alpha
$$

(18)

For a full specification of the traffic flow model we need to define the acceleration behaviour and the probabilities $p$ in this acceleration integral. For the acceleration $a_\alpha(x_\alpha, v_\alpha, s_\alpha, v_\beta)$ any individual driver model can be used. For the purpose of illustrating the procedure we test a simple specification of individual driver behaviour in the remainder of this paper. However, the assumptions about the individual behaviour can be as complex as necessary. The only practical limitation is that the integral expression (18) has to be approximated numerically for the purpose of numerical simulation, which might become a complex procedure if one makes complex assumptions about individual driver behaviour. The model considered in the next chapters consists of:

- a simple car following rule
- and a rule deciding when to change lanes.

For the probabilities in equation (18) we use the aggregate traffic variables $k$, $V$ (and $\theta$) and additional assumptions about the type of distribution for individual speeds and for headways between vehicles. Moreover we also assume that these distributions are mutually independent. This is an important simplification, since in reality the speeds of successive vehicles and the following distance are all correlated; see for example Helbing (2001).
measurements from field experiments with individual drivers, from simulation with stochastic individual driver models etc might be useful sources to define $\theta$. If a dynamic equation for the speed variance is used (in so-called third or higher order models, obtained by applying the method of moments with multipicator $\nu^2$) the dynamic value of $\theta$ is simply available. Note that we need the speed distribution twice in the integral (18), once for the speed distribution of $\alpha$ and once for the speed distribution of $\beta$. This also implies that in the former case we evaluate $V$ and $\theta$ at $x_\alpha$, while for $\beta$ we have to use the values at $x_\beta = x_\alpha + s_\alpha$. This non-locality can be modelled explicitly by evaluating $V$ and $\theta$ at $x_\beta$ (Treiber et al., 1999) or $V(t, x_\alpha + s_\alpha)$ (and analogously the variance) can be approximated by:

$$V(t, x_\alpha + s_\alpha) = V(t, x_\alpha) + s_\alpha \frac{\partial V(t, x_\alpha)}{\partial x}$$

(19)

Note how anticipation to conditions downstream is implicitly accounted for in the acceleration integral in both methods for modelling the non-locality. This means that, similarly to Treiber and co-workers, we do not need to introduce anticipation or viscosity terms separately in our aggregate traffic flow model, as is the case for example with the models of Helbing (1997) and Hoogendoorn (1999). Note that in the simulations presented in this paper we have not yet included the anticipation according to (19). This is a subject of present work.

For the headway distribution we use the fact that the inverse of the density $k(t,x)$ equals the average headway $\langle s_\alpha \rangle$ on that location. For the type and variance of the distribution we need additional assumptions. This is subject of ongoing research. In the simulation results in this paper we have used a lognormal distribution (uncorrelated to the speed distributions $v_\alpha$ or $v_\beta$) and simply approximated the variance of the headway distribution by $\frac{1}{k(t,x_\alpha)^b}$ with $b=0.6$. Here again it can be necessary to use the density at some distance ahead of vehicle $\alpha$ in the parameters of the headway distribution. This would also be an implicit element of anticipation to traffic conditions ahead. So far we have not elaborated this. The relatively coarse assumptions about headway distribution suffice for the proof of concept, but for the model to be valid they will need further refinement.
5 Aggregate traffic flow model for a 1-lane road with and without overtaking using a simple car following model

In this example we consider a simple car following model, as it is used in the MIXIC microsimulation model (van Arem, 1997). The acceleration of vehicle $\alpha$ is determined from:

$$a_{\alpha}(x_\alpha, v_\alpha, s_\alpha, v_\beta) = \min \left( \frac{w_\alpha - v_\alpha}{\tau_w}, \frac{s_\alpha - s_\alpha^0(v_\alpha)}{\tau_s}, \frac{v_\beta - v_\alpha}{\tau_v} \right)$$

(20)

$$s_\alpha^0(v_\alpha) = s_0 + s_1 \cdot v_\alpha + s_2 \cdot v_\alpha^2$$

(21)

Furthermore the acceleration is limited between a minimal and maximal possible acceleration (here: -5 and 3.5 m/s$^2$). According to this model the individual tries to maintain a desired speed $w_\alpha$ whenever possible. That is unless the acceleration for car following is more restrictive. This acceleration is meant to maintain a desired distance $s_\alpha^0(v_\alpha)$ while at the same time annihilating the relative speed with the predecessor. The desired headway is a quadratic function of the driving speed. The parameters of this microscopic model were calibrated for Dutch motorway data in the non-congested regime. The parameters are listed in Table 1.

Furthermore we will assume that drivers who have to brake for a predecessor will change lanes if they have the opportunity to do so. For that purpose we consider the probability that in the adjacent lane there is a gap and this gap is large enough for the vehicle to fit into. We define the immediate lane change probability as:

$$p_{\text{overtake}}(v_\alpha) = p(\text{no car in adjacent lane}) \cdot p(s_{\alpha, \text{adjacent}}^0 \geq \text{required})$$

$$= (1 - p(\text{car in adjacent lane})) \cdot p(s_{\alpha, \text{adjacent}}^0 \geq c \cdot s_\alpha^0(v_\alpha))$$

$$= (1 - k(t, x_\alpha) \cdot s_0) \cdot p(s_{\alpha, \text{adjacent}}^0 \geq c \cdot s_\alpha^0(v_\alpha))$$

(22)

The probability of being adjacent to a car in the adjacent lane indeed equals the expected total roadway length occupied by vehicles per unit of roadway length. Furthermore we assume that a car that is not adjacent to another in the target lane will change lanes only if the gap is large enough, which we define as a multiple of the own desired headway (parameter $c$). In the simulations presented in this paper we used $c=1.5$.

We apply the overtaking probability only if the vehicle is impeded by slower traffic. This means that only if the acceleration according to (20) is negative, we replace this acceleration by:

$$(1 - p_{\text{overtake}}(v_\alpha)) \cdot a_{\alpha}(x_\alpha, v_\alpha, s_\alpha, v_\beta) + p_{\text{overtake}}(v_\alpha) = 0 = (1 - p_{\text{overtake}}(v_\alpha)) \cdot a_{\alpha}(x_\alpha, v_\alpha, s_\alpha, v_\beta)$$

(23)

This means that we assume that a vehicle that can change lanes will not accelerate nor
The traffic flow model derived in the previous chapters is only useful if it can be evaluated numerically. To this end the integral expression (18) has to be solved. Since it turns out that the integral only depends on the present values of $V(t,x)$ and $k(t,x)$ we can solve it for any possible combination of these variables and store it in a look-up table prior to numerically solving the coupled partial differential equations (PDE) (9) and (13). Instead of performing the numerical integration during the solution of the PDE's we can now simply interpolate in this table, which substantially increases the computational speed of the PDE solver.

Another advantage of this procedure is that the acceleration look-up table obtained in this way gives insight in the behaviour of the model. Figure 1 shows the acceleration as a function of $k$ and $V$ for the case with (a) and without overtaking opportunity (b). As expected, combinations of low density and low speeds allow a sharp acceleration, nearly equal to the maximum possible acceleration. On the other hand high density with high speeds leads to a sharp expected deceleration. Interesting information is contained in the cross-section of this acceleration surface with the plane $a_e=0$, marked as the curved lines in figure 1. Indeed, this line of $(V,k)$ combinations are combinations in which the expected acceleration is zero, which means that the average speed is maintained. This is by definition an equilibrium situation. So, the zero-acceleration or equilibrium line (figure 2a) plays the same role as the equilibrium speed $V^e(k)$ in equation (17). The only difference is that non-equilibrium situations have accelerations that are not necessarily linearly dependent on the speed difference with the equilibrium speed, as was the case in (17). Also, in more complex model specifications the acceleration might depend on more than the two variables $V$ and $k$, so that the equilibrium states have a more complex relation than the two dimensional $V^e(k)$ curve.

We have transformed the equilibrium line of speed versus density (figure 2a) into a flow versus density curve (figure 2b). It appears that the difference between the models without and with overtaking opportunities is mainly a higher equilibrium speed for the model with overtaking. This is because some vehicles are able to maintain their speed by changing lane instead of braking for slower traffic. As a result the capacity (highest flow rate) of the model with overtaking is higher. At very high density suitable gaps are hardy available so that lane changing does not occur in either model; therefore the curves approach each other again.

From the kinetic wave theory of Lighthill & Whitham (1955) we know that a wave front moves with a speed equal to the slope of the line connecting the two corresponding equilibrium points on the fundamental diagram of (equilibrium) flow against density and that small disturbances move with a wave speed equal to the tangent of this curve. From figure 2b it can be read that our model – although based on a simple car-following relationship that was only calibrated for free flowing traffic – has a low density region with forward moving waves (i.e. travelling in the direction of traffic) and a high density region with backward moving waves, as is required for traffic flow models.
It is clear that the shape of the curves is qualitatively comparable to that of the empirical curve by Kerner that is shown for comparison. Yet there are some clear quantitative differences that will have their impact on congestion formation and wave propagation. Note for instance that at high densities the curve deviates from the empirical one. This may be a result from the fact that the car following relation (20) and corresponding parameters of table 1 that are used in this example were borrowed from a microsimulation model that was only calibrated for non-congested traffic (speeds near 100 km/h, see van Arem et al. 1997). Dijker et al. (1997) have shown that a substantial change in the acceleration law (by modification of the desired headway in (21) for low driving speeds) is necessary in order to provide better representation of congestion. This is another subject of ongoing research.

Altogether we can conclude that it is encouraging that even with some important simplifications (no anticipation according to (19), uncorrelated speed and headway distributions) the properties of the aggregate flow model and those of the individual driver model are qualitatively closely linked and the first calculations agree qualitatively with well-known empirical and theoretical fundamental relationships.
In the previous chapter we analysed the traffic flow properties of the newly developed model based on the equilibrium line of the acceleration surface. However, this line only indicates the equilibrium states and not the temporary deviations from that equilibrium. For comparison we have simulated the model of Payne under the same initial conditions. The model of Payne uses a similar dynamic speed equation as our new model:

\[
\frac{\partial V}{\partial t} + V \cdot \frac{\partial V}{\partial x} = V^{''}(k) - \frac{c_0^2}{\tau} \frac{k}{V} \frac{\partial k}{\partial x}
\]

(24)

Whereas our model is:

\[
\frac{\partial V}{\partial t} + V \cdot \frac{\partial V}{\partial x} = \left( \frac{dv}{dt} \right) - \frac{1}{k} \frac{\partial (k \cdot \theta)}{\partial x}
\]

(25)

with equation (18) for the expected acceleration and for the speed variance we use (with Philips, 1979):

\[
\theta = \theta^e(k) = \theta_0 \left( 1 - \frac{k}{k_{max}} \right)
\]

(26)

For comparability we have chosen \( \sqrt{\theta_0} = c_0 = 15 \text{m/s} \) and \( k_{max} \) is common to both models since we use for the equilibrium speed in (24) (with Leutzbach, 1988):

\[
V^{e}(k) = V_0 \left( 1 - \left( \frac{k}{k_{max}} \right)^{n_1} \right)^{n_2}
\]

(27)

Other parameter values are: \( k_{max} = 0.2 \text{ veh/m} \), \( n_1 = 1.4 \), \( n_2 = 4 \) and \( V_0 = 33.3 \text{ m/s} \).

The numerical scheme that is applied here is a moving mesh finite volume solver. The technique is robust for shock waves and by redistribution of the computational points in the x-direction a limited number of discretisation points for the x-direction suffices to allow a stable and fast simulation. The solver was implemented in Matlab with additional functionality for easy analysis of traffic flow models and is freely available (Van Dam, 2002).

The problem that was simulated is a fictitious situation of a ringroad of 10 km long where traffic leaving the road at x=10000 m re-enter at the origin x=0 m. The initial condition is a sinusoidal density distribution (period = 10 km) with an average of 30 veh/km and amplitude of 10 veh/km.

Figures 3 a and b and 4 a and b show the solutions after 500 seconds for the Payne (reference) model and the new model respectively. It is clear that qualitatively the two
simulations are equivalent: the wave has travelled downstream (towards $x=10000$ m) and has re-entered at the origin. While travelling downstream the upstream front of the wave has steepened and the downstream side gradually declines (expansion wave), which is consistent with the Lighthill & Whitham kinetic wave theory and confirms our findings from the previous chapter. Quantitatively the solutions are different in that the wave speed is different, the solution of the new model is smoother and the speed level is higher for the Payne model. However, since the simulations were only meant to illustrate the qualitative similarities we have not calibrated the new model for optimal fit to the Payne model, which we seek to improve in the future by including more refined driver behaviour in our model.
We have analytically derived a macroscopic traffic flow model from specifications on
the individual driver level by applying the gas-kinetic traffic flow modelling
approach. In contrast to earlier models we have treated acceleration and deceleration
both as continuous adaptive processes. The procedure implicitly accounts for the
anisotropy of information flow in traffic, for anticipation behaviour of drivers and for
the finite space requirement of vehicles, as long as these properties have been
specified at the level of the individual driver behaviour.

As an illustration of the procedure a simple car-following model with and without
overtaking opportunities is implemented and its macroscopic counterpart is
analytically derived. However, the procedure is potentially applicable to more
complex individual driver behaviour specifications. It turns out that even with the
simple microscopic model we can reproduce qualitatively the relevant properties of
traffic flow dynamics and wave propagation.

The simulation results presented in this paper are encouraging and prove the strength
of the approach. Still a lot of refinements need to be made, among others:

• the incorporation of anticipation by drivers
• relaxation of some assumptions needed to analytically solve the model, especially
  the assumption of mutually independent speed and headway distributions have to
  be replaced by appropriately correlated distributions
• specifying more refined driver behaviour models, like lane changing and adaptive
  speed choice during (mandatory) lane changing, the influence of congestion
  formation and driver psychology on the expected acceleration (and thus on traffic
  flow dynamics)
• calibration and validation of the model, including stability analysis and analysis of
  congestion formation and propagation (e.g. stop and go waves).

We believe that including more refined driver behavioural models and linking them to
traffic flow dynamics using this approach will increase significantly the insight in the
process of congestion formation and propagation and the role of the driver therein.
After all, dynamic changes in driver psychology are hypothesised as one of the major
explanatory factors of congested traffic that are still lacking in existing traffic flow
models (e.g. Zhang, 1999; Daganzo, 2002; Smulders et al. 2000).
9 Acknowledgements

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Figure 1a: acceleration surface of the simple car-following model with overtaking; the curve is the intersection with the plane $dv/dt=0$: the equilibrium line

Figure 1b: acceleration surface of the simple car-following model without overtaking; the curve is the intersection with the plane $dv/dt=0$: the equilibrium line
Figure 2 a and b: Left: example of an empirically derived speed-density relation (dash-dot cyan line) and results of the new model with (blue full line) and without overtaking (red dashed line); Right: the corresponding speed-flow relations (right). The empirically derived model is due to Kerner & Konhäuser (1993)

Figure 3 a and b: simulation output of the Payne model
Figure 4 a and b: simulation output of the new model

Table 1: parameter values used in the modelling example

<table>
<thead>
<tr>
<th>$w_\alpha$</th>
<th>Desired speed</th>
<th>$35$</th>
<th>m/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_w$</td>
<td>Relaxation time for the desired speed</td>
<td>$2.5$</td>
<td>$s$</td>
</tr>
<tr>
<td>$\tau_s$</td>
<td>Relaxation time for the distance error</td>
<td>$3.33$</td>
<td>$s$</td>
</tr>
<tr>
<td>$\tau_v$</td>
<td>Relaxation time for the speed error</td>
<td>$0.588$</td>
<td>$s$</td>
</tr>
<tr>
<td>$s_0$</td>
<td>Desired headway at standstill</td>
<td>$8$</td>
<td>m</td>
</tr>
<tr>
<td>$s_1$</td>
<td>Linear speed factor of the desired headway</td>
<td>$0.25$</td>
<td>$s$</td>
</tr>
<tr>
<td>$s_2$</td>
<td>Quadratic speed factor of the desired speed</td>
<td>$0.02$</td>
<td>$s^{2/m}$</td>
</tr>
</tbody>
</table>
References


Helbing, D., (1997), Verkehrsdynamik, Neue physikalische Modellierungskonzepte, Springer Verlag, Berlin


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