Asymmetric current-phase relation due to spin-orbit interaction in semiconductor nanowire Josephson junction

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Abstract. We theoretically study the current-phase relation in semiconductor nanowire Josephson junction in the presence of spin-orbit interaction. In the nanowire, the impurity scattering with strong SO interaction is taken into account using the random matrix theory. In the absence of magnetic field, the Josephson current I and phase difference φ between the superconductors satisfy the relation of $I(\varphi) = -I(-\varphi)$. In the presence of magnetic field along the nanowire, the interplay between the SO interaction and Zeeman effect breaks the current-phase relation of $I(\varphi) = -I(-\varphi)$. In this case, we show that the critical current depends on the current direction, which qualitatively agrees with recent experimental findings.

Keywords: spin-orbit interaction, Josephson junction, semiconductor nanowire, Andreev bound state
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INTRODUCTION

Electrical spin manipulation is an important issue for spintronics. The spin-orbit (SO) interaction in narrow-gap semiconductors, such as InAs and InSb, is attractive in this context. Nanowires of such materials have a great potential for the application to the spintronic devices [1].

We consider effects of strong SO interaction in the nanowires on the Josephson current. In the Josephson junction, the quasiparticle states are formed by the Andreev reflections at the interfaces, which are called the Andreev bound states. The supercurrent flows through the Andreev bound states. The Josephson junctions of InAs and InSb nanowires were experimentally examined [2]. Recently, the critical current was studied in details when a magnetic field is applied along the nanowires [3].

In this paper, we investigate the Josephson current when the SO interaction and Zeeman effect coexist in the normal region. We consider the impurity scattering with SO interaction in the nanowire. In the absence of magnetic field, the Josephson current I and phase difference φ between the superconductors satisfy the relation of $I(\varphi) = -I(-\varphi)$. In the presence of magnetic field along the nanowires, the interplay between SO interaction and Zeeman effect results in (i) $I(\varphi) \neq -I(-\varphi)$ (anomalous Josephson effect) [4, 5], and (ii) the direction-dependence of critical current [5]. This is in qualitative agreement with recent experimental findings [3].

MODEL

A semiconductor nanowire of length L is connected to left and right superconductors. There are N conduction channels in the nanowire. The superconducting gap in the left (right) superconductor is $\Delta = \Delta_0 e^{i\varphi/2}$ ($\Delta = \Delta_0 e^{-i\varphi/2}$), whereas $\Delta = 0$ in the nanowire. φ is the phase difference between two superconductors.

We assume the diffusive regime for the nanowire, where $l \ll L$ with mean free path l. The impurity scattering with strong SO interaction in the nanowire is described in terms of the scattering matrices $\hat{S}_{\rm e}$ for electrons and $\hat{S}_{\rm h}$ for holes. To take into account a weak energy-dependence when L is comparable with the coherent length, we introduce a single resonant pole at ε_0 with resonant width Γ in the scattering matrix;

$$\hat{S}_{\rm e}(E) = \hat{S}_0 \left(\hat{1} - \frac{i\Gamma}{E - \varepsilon_0 + i\Gamma/2} \hat{P} \right) \hat{S}_0, \qquad (1)$$

where $\hat{P} = |\phi\rangle\langle\phi| + \hat{g}|\phi^*\rangle\langle\phi^*|\hat{g}^\dagger$ is the projection operator to the resonant state $|\phi\rangle$. $|\phi\rangle$ is given by a linear combination of N channels with random coefficients. $\hat{S}_0$ is given by the symplectic ensemble of random matrix theory. $\hat{S}_{\rm h}(E) = \hat{S}_{\rm e}^*(-E)$.

The orbital magnetization is neglected when a magnetic field is applied along the nanowire (x direction). For simplicity, we separate the scattering region described by $\hat{S}_\alpha$ ($\alpha = {\rm e,h}$) and the left and right transport regions with Zeeman effect $H_{\rm Z} = g_0 \mu_{\rm B} B \sigma_x/2$. g_0 (< 0) is the electron g-factor, $\mu_{\rm B}$ is the Bohr magneton, and σ_x is the x-component of the Pauli matrix. The left (right) region connects the scattering region and the left (right) interface. The Zeeman effect is considered as spin-dependent phases of quasiparticles accompanied by the transport through the left (right) transport region, $\pm\theta_{B{\rm L}}$ ($\pm\theta_{B{\rm R}}$). We introduce two parameters, $\theta_B = \theta_{B{\rm L}} + \theta_{B{\rm R}} = |g_0|\mu_B BL/(2\hbar v_{\rm F})$ and $\alpha_B = \theta_{B{\rm L}}/\theta_{B{\rm R}}$.

The electrons (holes) are converted to the holes (electrons) by the Andreev reflection. The energy levels $\varepsilon_n(\varphi)$ of Andreev bound states, called the Andreev levels, are

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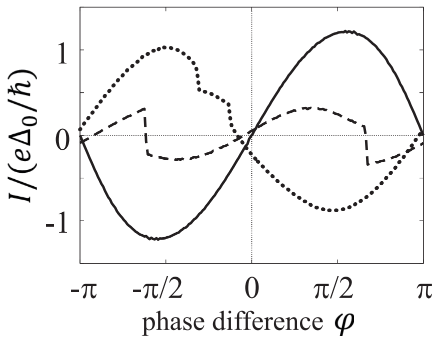

FIGURE 1. Numerical results of the Josephson current as a function of phase difference φ between the superconductors. The strength of Zeeman effect in magnetic field is $\theta_B = 0$ (solid line), 0.35π (broken line), and 0.53π (dotted line).

calculated by the combination of the scattering matrix, $\hat{S}_\text{e}$ and $\hat{S}_\text{h}$, the spin-dependent phase, $\pm\theta_B$, and the phases due to the Andreev reflection. The Josephson current is evaluated as

$$I(\varphi) = -\frac{e}{\hbar}\sum_n \frac{d\varepsilon_n}{d\varphi}, \quad (2)$$

where the summation in Eq. (2) is taken over all the positive Andreev levels.

CALCULATED RESULTS

In the following, we set $N = 3$, $\alpha_B = \sqrt{2}$, $\varepsilon_0 = 0.5\Delta_0$, and $\Gamma = 3\Delta_0$.

Figure 1 shows the Josephson current $I(\varphi)$ as a function of phase difference φ between the superconductors. When $\theta_B = 0$ (solid line), the Josephson current approximately behaves as $I(\varphi) \propto \sin\varphi$. The SO interaction splits the Andreev levels when $\varphi \neq 0, \pm\pi$ [6, 7]. The Josephson current satisfies the relation of $I(\varphi) = -I(-\varphi)$. In consequence, the current vanishes at $\varphi = 0$. In the presence of magnetic field, the Zeeman effect also splits the Andreev levels. The interplay between the SO interaction and Zeeman effect gives rise to $I(\varphi) \neq -I(-\varphi)$. When $\theta_B = 0.35\pi$ (broken line), $I(\varphi) \sim \sin(2\varphi)$ (higher order Josephson effect) with discontinuity at $\varphi = 0.67\pi$ and -0.62π. The sudden change of $I(\varphi)$ corresponds to the zero points of the lowest Andreev level. With $\theta_B = 0.53\pi$ (dotted line), $I(\varphi) \sim \sin(\varphi + \pi)$, which is similar to a character of the π-state. In Fig. 1, the dotted line clearly shows an anomalous Josephson current $I(\varphi = 0) \neq 0$.

Next, we examine the critical current. The critical current corresponds to the maximum I_c^+ or minimum values I_c^- of $I(\varphi)$ in Fig. 1. Figure 2 shows I_c^+ and $|I_\text{c}^-|$ as functions of magnetic field (θ_B). I_c^+ and $|I_\text{c}^-|$ mean the critical current in $+x$ and $-x$ directions, respectively. We find the oscillation of critical currents with increase in θ_B, which is similar to the feature of 0-π transition. In Fig. 2, $I_\text{c}^+ \neq |I_\text{c}^-|$ in the magnetic field, which means that the critical current depends on the current direction. This calculated result qualitatively explains the recent experimental finding [3]. In the absence of SO interaction, I_c^+ and $|I_\text{c}^-|$ are identical to each other (not shown). Therefore, the direction dependence of critical current could detect the SO interaction in the normal region.

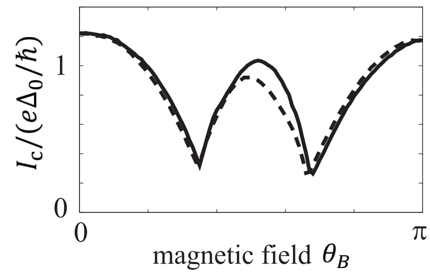

FIGURE 2. Numerical results of critical current I_c as a function of magnetic field (strength of Zeeman effect, θ_B). For the current in x ($-x$) direction, I_c is indicated by solid (broken) line.

CONCLUSIONS

We have theoretically investigated the Josephson current when the SO interaction and Zeeman effect coexist in the semiconductor nanowire. The impurity scattering with SO interaction is considered using the symplectic ensemble of random matrix theory, whereas the Zeeman effect in a magnetic field along the nanowire is taken into account by spin-dependent phases. In the absence of magnetic field, the current-phase relation satisfy $I(\varphi) = -I(-\varphi)$. In the presence of magnetic field, the relation of $I(\varphi) = -I(-\varphi)$ does not hold, and the anomalous Josephson current is induced. In addition, we have found that the critical current depends on the current direction. This is in qualitative agreement with recent experimental result.

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