To Professor Gerritsma with compliments of T F Ogilvie

SHIP SLAMMING AND SUPERSONIC AIRFOIL FLOW

by T. F. Ogilvie
1962

INTRODUCTION

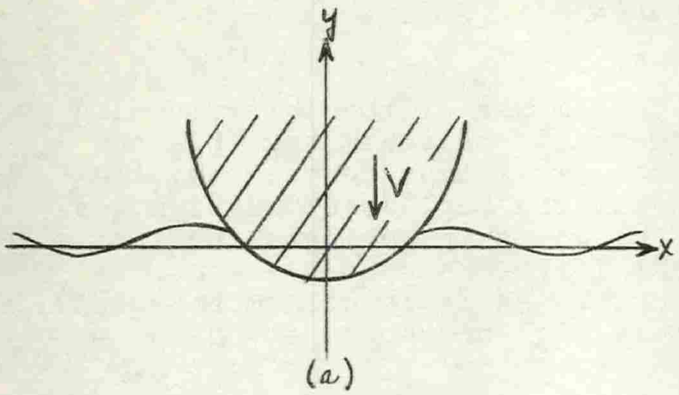
In most analytical treatments of the ship slamming problem, the water is considered to be incompressible and nonviscous. A potential problem is thus formulated, the potential being required to satisfy Laplace's Equation, as well as boundary conditions on the ship hull and the free surface. One result is that if the tangent plane at the bottom of the ship is horizontal (as it is with most ships), the theory predicts infinitely high pressures at the instant of impact.

In order to correct this result, it is necessary to drop the assumption of incompressibility. In this process an already difficult problem would seem to become utterly intractable. However, the time scale for compressibility effects is so grossly different from the time scale for inertial and gravitational effects that some useful results can be discovered.

This report consists of an elaboration of some suggestions by Professor R. Timman (Technological University, Delft, Netherlands) for treating the problem of a body impacting on a compressible fluid. The idealizations are rather great. However, the working out of a more realistic model is fairly straightforward, especially if a high speed computer is available. The methods are well-known to students of supersonic flow theory.

Essentially, Professor Timman's suggestion is this: If the problem is linearized in an appropriate way, and if the effect of the ship's hull is replaced by a condition that the fluid has a downward vertical velocity on a section of the free surface, then in two dimensions the problem can be made mathematically equivalent to the problem of steady supersonic flow over a lifting surface. The latter problem has been solved for some years. In fact, most of Chapter Six of G.N. Ward's book, Linearized Theory of Steady High-Speed Flow, (Cambridge, 1955) is devoted to just this problem.

In the following sections, the linearization is carried out, and then Ward's procedure is adapted to the slamming problem. Finally, some remarks are included on how to introduce other effects that have been ignored. The treatment is two-dimensional throughout.



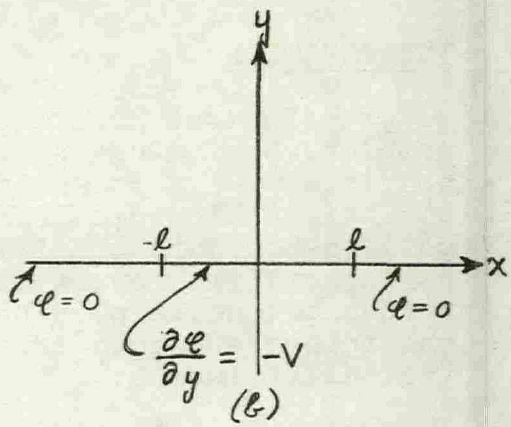


Figure 1. The physical problem and its mathematical idealization.

THE LINEARIZATION

The most drastic assumption made is that the ship can be considered to produce a constant vertical velocity of the fluid at the free surface in the region, $-\ell < x < \ell$, for all time $t > 0$. (It will be mentioned later how some improvement can be made on this. In particular, we can have $\ell = \ell(t)$ and $V = V(t)$.)

Since the fluid is considered as nonviscous and it starts from a state of rest, the motion is irrotational. Thus, the fluid velocity can be represented as the gradient of a potential,

$$\underline{v} = \nabla \varphi,$$

defined everywhere in the space occupied by the fluid. The function $\varphi(x,y,t)$ satisfies the equation:

$$c^2 \nabla^2\varphi - \varphi_{tt} = \frac{\partial}{\partial t} (\nabla\varphi)^2 + \nabla\varphi \cdot \left[(\nabla\varphi \cdot \nabla)\nabla\varphi\right].$$

(See Ward, op. cit., Section 1.4). c is the velocity of sound.

Let the equation of the free surface be

$$y - Y(x,t) = 0, \quad \text{for } |x| > \ell.$$

Then there are two conditions to be satisfied on this surface:

$$\varphi_y - Y_x \varphi_x - Y_t = 0 \tag{1}$$

$$gY + \varphi_t + \frac{1}{2} (\nabla \varphi)^2 = 0 \tag{2}$$

These conditions are derived, for example, in Water Waves, by J.J. Stoker (Interscience, 1957) and in Surface Waves, by J.V. Wehausen and E.V. Laitone, Handbuch der Physik, Vol. IX (Springer Verlag, 1960). The first is a kinematic condition, and the second is Bernouli's Equation, a dynamic condition. g is the acceleration due to gravity.

To account for the effect of the ship, let:

$$\frac{\partial \varphi}{\partial y} = -V \text{ on } y = 0, \ -\ell < x < \ell, \ \ t > 0.$$

For $t < 0$, we require $\varphi = |\nabla \varphi| = 0$ everywhere.

The problem can be linearized intuitively or systematically. I prefer the latter, although many people will obtain the same result more quickly by an intuitive argument. To effect the linearization, nondimensionalize all quantities as follows. Set

$$x = 2\ell\xi \ ;$$

$$y = 2\ell\eta \ ;$$

$$t = \frac{2\ell}{c_0} \tau \ ;$$

$$\varphi(x,y,t) = 2\ell V \varphi^*(\xi, \eta, \tau).$$

The fourth of these implies that fluid velocities are expected to be of the order of magnitude of V (rather than, say, of c), and the third implies that we are interested in what happens very quickly after impact. (c_0 is the acoustic speed in the undisturbed fluid). Now substitute the new variables into the conditions above, express all terms as functions of the ratio V/c_o, and assume that V/c_o is very small. Then the coefficients of the lowest power of V/c_o can be set equal to zero to give a linearized approximation. It will also be assumed that

$$\frac{2\ell g}{V^2} = O(1)$$

as $V/c_o \longrightarrow 0$, so that the gravitational term in Bernouilli's Equation will be lost in the first approximation. The results are

$$\varphi^*_{\xi\xi} + \varphi^*_{\eta\eta} - \varphi^*_{\tau\tau} = 0 \quad \text{in } y < 0,\ t > 0;$$

$$\left.\begin{aligned} Y_\tau &= 0 \\ \varphi^*_\tau &= 0 \end{aligned}\right) \quad \begin{aligned} &\text{on } y - Y(x,t) = 0, \\ &|x| > \ell,\ t > 0. \end{aligned}$$

Initially, $Y(x,t) = 0$ and $\varphi(x,0,t) = 0$. Thus, in the linearized version, the last conditions above imply that

$$\varphi^* = 0 \quad \text{on } y = Y = 0, \text{ for } |x| > \ell,\ t > 0.$$

In terms of the original variables, the problem is now as follows:

$$\nabla^2\varphi - \frac{1}{c^2}\varphi_{tt} = 0 \quad \text{in } y < 0;$$

$$\varphi_y = -V, \quad \text{on } y = 0,\ -\ell < x < \ell,\ t > 0;$$

$$\varphi = 0, \quad \text{on } y = 0, \quad |x| > \ell,\ t > 0;$$

$$\varphi = |\nabla\varphi| = 0, \text{ everywhere for } t < 0.$$

Here c has the value c_o, that is, the sound speed in calm water, but the subscript will be omitted from here on.

From the condition that $\varphi = 0$ on $y = 0$, we can continue the potential function into the upper half space as a function odd in y. Then $\frac{\partial \varphi}{\partial y}$ is even in y, while $\frac{\partial \varphi}{\partial x}$ and $\frac{\partial \varphi}{\partial t}$ are odd.

THE EQUIVALENT SUPERSONIC FLOW PROBLEM

Figure 2 depicts the problem in a three-dimensional x,y,t, space.

Suppose now that we consider a three-dimensional, steady flow problem. Replace t by z, and let $\frac{1}{c^2} = M^2 - 1$. The wave equation above becomes:

$$\varphi_{xx} + \varphi_{yy} - (M^2 - 1)\ \varphi_{zz} = 0,$$

which is the linearized differential equation for steady, supersonic, irrotational flow in the z-direction. M is the Mach number of the undisturbed flow.

The boundary conditions are unchanged in the new problem. Physically they are equivalent to imposing a given vertical component of velocity on the plane of a "wing" and requiring no horizontal velocity component in the $y = 0$ plane outside of the "wing."

The problem may now be compared directly with Chapter 6 of Ward's book. In particular, Fig. 2 here is essentially the same as Ward's Figure 6.2, if our t is replaced by z. We follow Ward closely in the following.

The wave equation for our problem is, of course, a well-studied equation. If we choose a point (x_1, y_1, t_1), we can pass through it a conical surface

$$c^2(t - t_1)^2 - (x-x_1)^2 - (y-y_1)^2 = 0\ ,$$

which is a <u>characteristic</u> <u>surface</u> of the differential equation. The interior of the cone for $t < t_1$ is the dependence domain of (x_1, y_1, t_1), i.e. the phenomena occurring at x_1, y_1 at time t_1 depend only on disturbances at points and times x, y, t such that

$$ct < ct_1 - \sqrt{(x-x_1)^2 + (y-y_1)^2} .$$

The dependence domain for (x_1, y_1, t_1) is shown in Fig. 2 as far back as $t = 0$. The rest of the cone, extending to the right but not shown in Fig. 2, bounds the influence domain of (x_1, y_1, t_1).

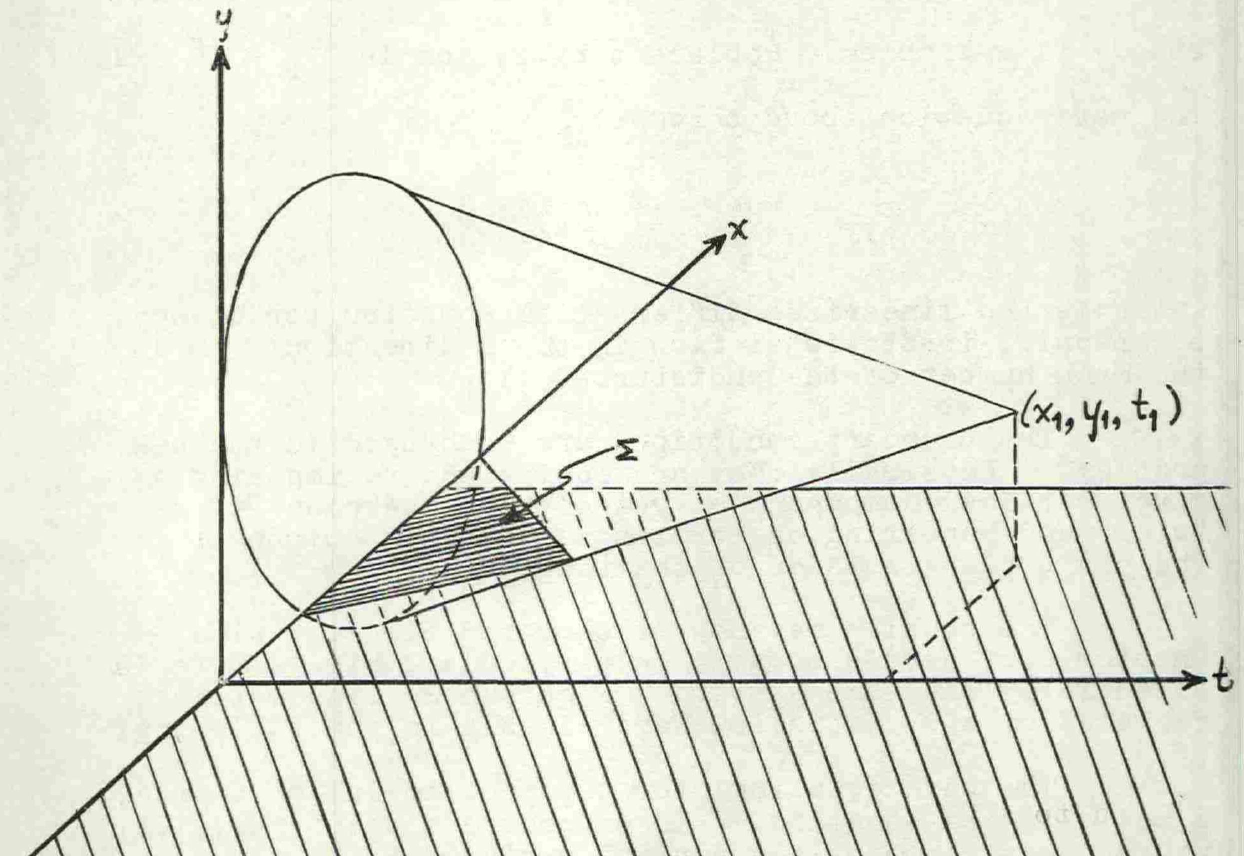


Figure 2. The dependence domain of the point (x_1, y_1, t_1)

Any disturbance at x_1, y_1 at time t_1 will have effects at a point x,y only for times t such that

$$ct > ct_1 + \sqrt{(x-x_1)^2 + (y-y_1)^2} .$$

These facts follow directly from study of the wave equation, although their physical interpretation is rather obvious.

From the boundary conditions and a relation quite analogous to Green's Theorem, the following formula can be deduced for the potential:

$$\varphi(x_1, y_1, t_1) = -\frac{c}{\pi} \iint_{\Sigma} \left(\frac{\partial \varphi}{\partial y}\right)_{y=0} \frac{dt\,dx}{\sqrt{c^2(t-t_1)^2 - (x-x_1)^2 - y_1^2}}, \quad (3)$$

where Σ is that portion of the y = 0 plane which is within the domain of dependence of (x_1, y_1, t_1) and for which $t > 0$. (See Ward, Section 6.3). As long as Σ includes only a region in which $-\ell < x < \ell$, this is the final solution for the potential, since $\left(\frac{\partial \varphi}{\partial y}\right)_{y=0}$ is known over this surface. However, for larger times t_1, the region Σ includes part of the y = 0 plane which is outside of $-\ell < x < \ell$, where $\left(\frac{\partial \varphi}{\partial y}\right)_{y=0}$ is not known, and the solution is more involved. However, as will be shown presently, the early stages of this solution are still fairly simple.

For the very first stage, in which Σ is contained in $-\ell < x < \ell$, the solution can be obtained by a simple argument, without performing the above integration (although that is not difficult either). At such a point, (x_1, y_1, t_1), no information has arrived indicating the bounds of the strip, $-\ell < x < \ell$. That is, the disturbances at the ends have not been felt. Then clearly the local behavior must be the same as if the whole x-axis were subjected to the same boundary condition, $\frac{\partial \varphi}{\partial y} = -V$. In such a case, there would be no variation of quantities with x, and the differential equation would be simply: $\varphi_{yy} - \frac{1}{c^2}\varphi_{tt} = 0$. The general solution of this equation is

$$\varphi(y,t) = \varphi_1(ct - y) + \varphi_2(ct + y).$$

But φ_1 must be identically zero* for $y < 0$ and $\varphi_2 \equiv 0$ for $y > 0$. Consider only $y < 0$, so that $\varphi = \varphi(ct + y)$. At $y = -0$, $\varphi_y = \varphi'(ct - 0) = -V$, which is constant for all $t > 0$. Thus $\varphi'(ct + y) = -V$ for all (y,t) being considered, and $\varphi(ct + y) = -V \cdot (ct + y)$.

SOLUTION FOR SMALL VALUES OF TIME

The information desired from the solution will all be contained in φ_t, evaluated on the surface $y = 0$, $-\ell < x < \ell$, $t > 0$. Therefore, in Equation (3), set $y_1 = 0$. Then

$$\varphi(x_1, +0, t_1) = -\frac{c}{\pi} \iint_{\Sigma} \left(\frac{\partial \varphi}{\partial y}\right)_{y=0} \frac{dt\,dx}{\sqrt{c^2(t-t_1)^2-(x-x_1)^2}} . \quad (4)$$

Σ is that part of $y = 0$ for which

$$ct < ct_1 - \left|x - x_1\right| .$$

Now perform a transformation of coordinates to the characteristic variables in the x-t plane. (See Ward, Section 6.5.) Let

$$\xi = ct_1 - x_1 ;$$

$$\eta = ct_1 + x_1 ;$$

$$\xi' = ct - x ;$$

$$\eta' = ct + x .$$

Also, let

$$N(\xi', \eta') = -\frac{1}{2\pi} \left(\frac{\partial \varphi}{\partial y}\right)_{y=0} ;$$

$$\varphi(\xi, \eta) = \varphi(x_1, +0, t_1) .$$

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* Otherwise effects would be observed at (x,y,t) arising from disturbances in the domain of influence of (x,y,t).

Then

$$\varphi(\xi,\eta) = \iint\limits_{\Sigma} \frac{N(\xi',\eta')\,d\xi'\,d\eta'}{\sqrt{(\xi-\xi')(\eta-\eta')}} ,$$

where Σ is the region

$$\xi' \le \xi , \quad \eta' \le \eta .$$

Figure 3 shows some of the lines ξ = const., η = const. on the x - t plane. The particular lines $\xi = 0$, $\eta = 0$ are those passing through the point x = t = 0.

The potential is even in x, and so we concern ourselves with finding it in the region $0 < x < \ell$, $t > 0$. We proceed step-by-step through the regions marked I, III, IV, V in Fig. 3. It will be necessary also to consider region II, although we are not really interested in the solution there. To go beyond region V introduces further difficulty, and a simple analytical result does not seem possible. However, procedures are available for obtaining numerical results, and it is here that a computer would be useful. We shall not consider such problems here. It will be seen presently that the boundary between IV and V is a significant one; here, at $t = 2\ell/c$, the pressure is zero for $-\ell < x < \ell$. For later times, the pressure magnitude will be less than the previous maximum.

REGION I

The region of integration, Σ, is shown in Fig. 4a. This case has already been discussed above, where it was shown that $\varphi(x_1, -0, t_1) = -Vct_1$. The same result follows from the integral formula, (4):

$$\varphi(\xi,\eta) = \frac{V}{2\pi} \int_{-\eta}^{\xi} \frac{d\xi'}{\sqrt{\xi-\xi'}} \int_{-\xi'}^{\eta} \frac{d\eta'}{\sqrt{\eta-\eta'}}$$

$$= \frac{V}{\pi} \int_{-\eta}^{\xi} d\xi' \sqrt{\frac{\xi+\eta}{\xi-\xi'}}$$

$$= \frac{V}{2}(\xi+\eta) = Vct_1 .$$

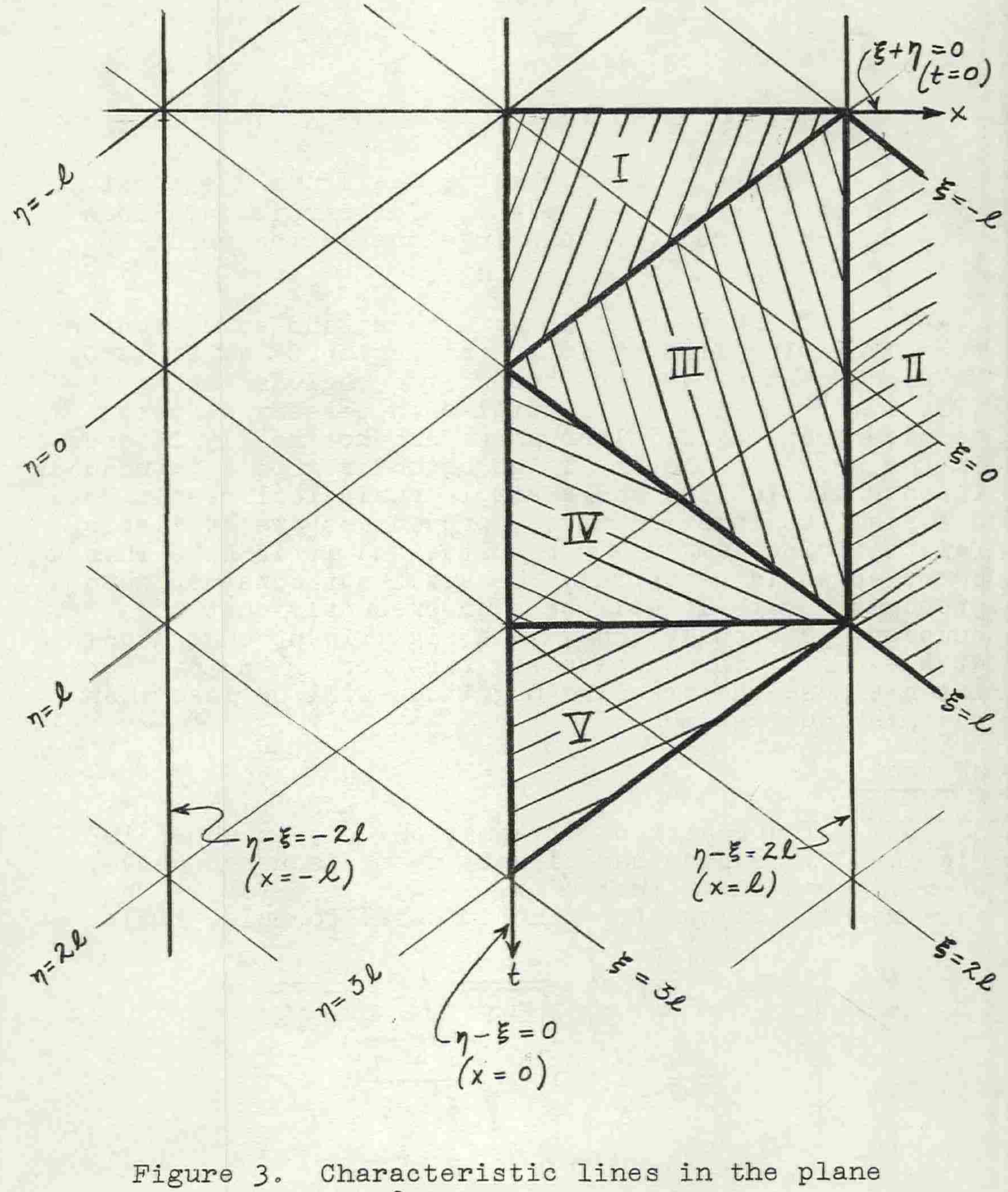

Figure 3. Characteristic lines in the plane y = 0.

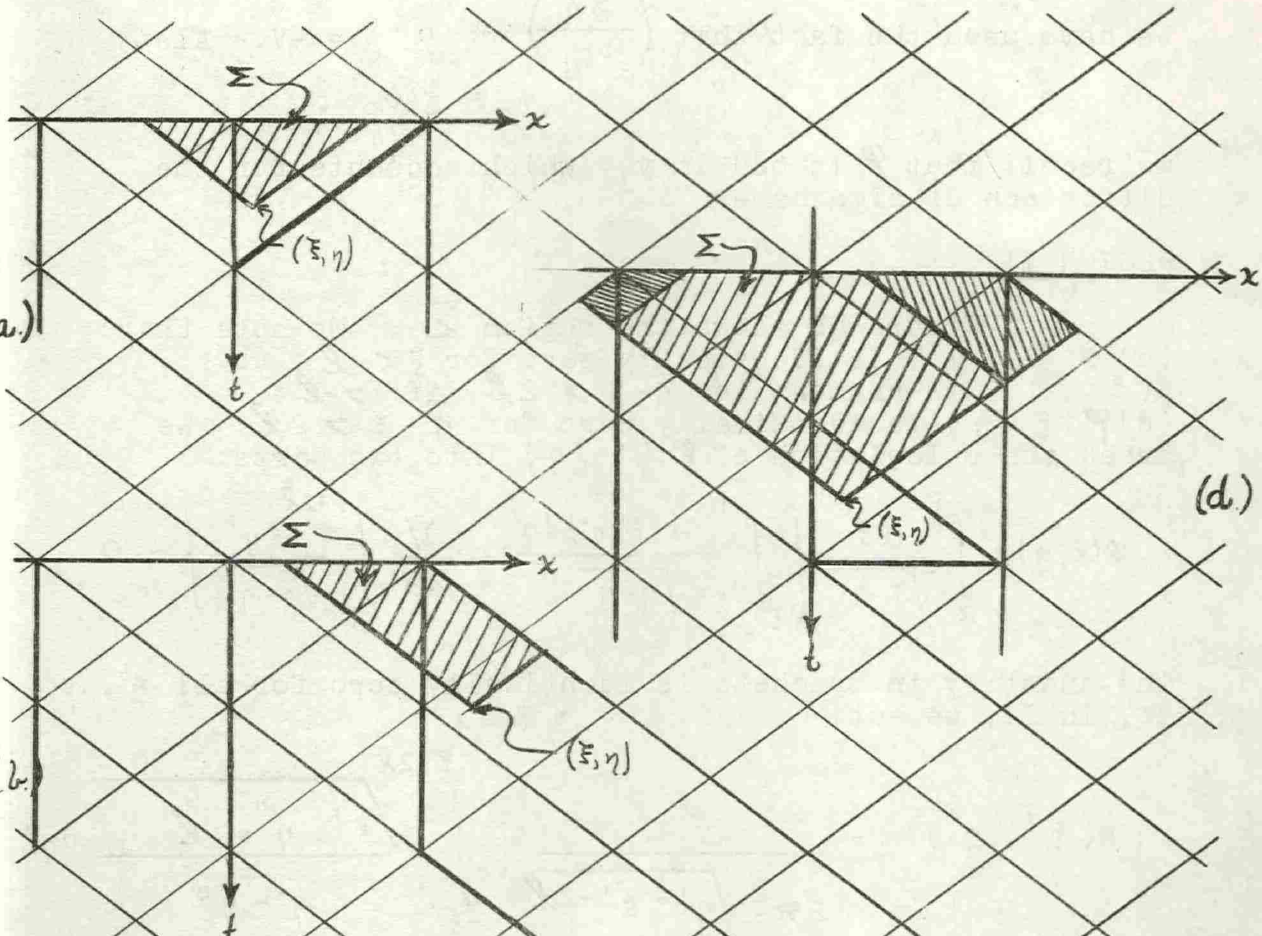

Figure 4. Domains of integration for calculating the potential, $\varphi(\xi, \eta) = \varphi(x_1, +0, t_1)$.

We have used the fact that $\left(\frac{\partial\varphi}{\partial y_1}\right)_{y_1=\pm 0} = -V$. Also, we recall that φ is odd in y_1, which accounts for the difference of sign here.

REGION II

Figure 4b shows the region Σ. We note that: (a) $N(\xi', \eta')$ is identically zero for $\xi < -\ell$; (b) $N(\xi', \eta')$ is unknown for $\eta - \xi > 2\ell$, if $\xi > -\ell$; (c) $\varphi(\xi, \eta)$ is identically zero for $\eta - \xi > 2\ell$. We break the calculation of $\varphi(\xi, \eta)$ into two parts:

$$\varphi(\xi,\eta) = \int_{\ell}^{\xi} \frac{d\xi}{\sqrt{\xi-\xi'}} \left\{ \int_{\xi'+2\ell}^{\eta} \frac{N(\xi',\eta')\,d\eta'}{\sqrt{\eta-\eta'}} + \frac{V}{2\pi} \int_{-\xi'}^{\xi'+2\ell} \frac{d\eta'}{\sqrt{\eta-\eta'}} \right\} = 0.$$

The quantity in brackets is identically zero for all ξ', η if, in II, we set:

$$N(\xi', \eta') = -\frac{V}{2\pi^2\sqrt{\eta'-\xi'-2\ell}} \int_{-\xi'}^{\xi'+2\ell} \frac{\sqrt{\xi'-\eta''+2\ell}}{\eta'-\eta''}\, d\eta''.$$

Actually we do not use this expression for $N(\xi', \eta')$; all we need is the knowledge that the sum of the integrals in brackets above can be set identically equal to zero in the region:

$$\ell > \xi' > -\ell, \quad \eta > \xi' + 2\ell.$$

REGION III

Σ is shown in Fig. 4c, where it is the union of the two shaded areas. From the result in Region II, the small shaded area in the corner contributes nothing, so that the solution for φ is:

$$\varphi(\xi,\eta) = \frac{V}{2\pi} \int_{\eta-2\ell}^{\xi} \frac{d\xi'}{\sqrt{\xi-\xi'}} \int_{-\xi'}^{\eta} \frac{d\eta'}{\sqrt{\eta-\eta'}}$$

$$= Vct_1\left[\frac{1}{2} - \frac{1}{\pi}\sin^{-1}\frac{ct_1+2x_1-2\ell}{ct_1}\right] + \frac{2V}{\pi}\sqrt{(\ell-x_1)(ct_1+x_1-\ell)}.$$

REGION IV

The problem here is similar to that in Region III. A result similar to that of Region II may be proved for the left side of the figure, and then it is seen that the two small shaded areas in Fig. 4d contribute nothing to the integral over Σ. The solution is:

$$\varphi(\xi,\eta) = \frac{V}{2\pi}\left\{\int_{\eta-2\ell}^{-\xi+2\ell}\frac{d\xi'}{\sqrt{\xi-\xi'}}\int_{-\xi'}^{\eta}\frac{d\eta'}{\sqrt{\eta-\eta'}} + \int_{-\xi+2\ell}^{\xi}\frac{d\xi'}{\sqrt{\xi-\xi'}}\int_{\xi-2\ell}^{\eta}\frac{d\eta'}{\sqrt{\eta-\eta'}}\right\}$$

$$= \frac{V}{\pi}\left\{2\sqrt{(\ell+x_1)(ct_1-x_1-\ell)} + 2\sqrt{(\ell-x_1)(ct_1+x_1-\ell)} + ct_1\left[\sin^{-1}\frac{-ct_1+2x_1+2\ell}{ct_1} - \sin^{-1}\frac{ct_1+2x_1-2\ell}{ct_1}\right]\right\}.$$

REGION V

When we extend the arguments used in Regions III and IV for eliminating parts of Σ from consideration, we find in Region V (see Fig. 4e) that we use the contribution of Σ' twice. Thus, we must subtract this contribution an extra time, and we obtain:

$$\varphi(\xi,\eta) = \frac{V}{2\pi}\left\{\int_{\eta-2\ell}^{\xi}\frac{d\xi'}{\sqrt{\xi-\xi'}}\int_{\xi-2\ell}^{\eta}\frac{d\eta'}{\sqrt{\eta-\eta'}} - \int_{2\ell-\xi}^{\eta-2\ell}\frac{d\xi'}{\sqrt{\xi-\xi'}}\int_{-\xi'}^{\xi-2\ell}\frac{d\eta'}{\sqrt{\eta-\eta'}}\right\}$$

$$= \frac{V}{\pi}\left\{2\sqrt{(\ell+x_1)(ct_1-x_1-\ell)} + 2\sqrt{(\ell-x_1)(ct_1+x_1-\ell)} - ct_1\left[\sin^{-1}\frac{ct_1+2x_1-2\ell}{ct_1} - \sin^{-1}\frac{-ct_1+2x_1+2\ell}{ct_1}\right]\right\}.$$

We npte that this is the same expression as that obtained in Region IV.

PRESSURE ON THE BODY

Bernoulli's Equation for unsteady flow of a compressible fluid is

$$\varphi_{t_1} + gy_1 + \frac{1}{2}(\nabla\varphi)^2 + \int^{p}\frac{dp'}{\rho(p')} = 0 .$$

In the linearized model, $\rho(p')$, can be replaced by its mean value, and the integral term becomes simply p/ρ .

When the whole equation is linearized, we are left with

$$p = -\rho \varphi_{t_1} \quad .$$

Under the restrictions of the linearized theory, this holds throughout the fluid, in general, and on the surfaces $y_1 = \pm 0$, in particular.

We have calculated $\varphi(x_1, +0, t_1)$ for a range of t_1, and so we can now directly calculate the pressure in that range. Since our interest is in the lower half-space, i.e., $y_1 < 0$, we again call attention to the fact that $\varphi(x_1, +0, t_1) = -\varphi(x_1, -0, t_1)$, and then we use the previously obtained formulae to find $p(x_1, -0, t_1)$, for $-\ell < x_1 < \ell$:

$$\text{I:} \quad p(x_1, -0, t_1) = \rho Vc \quad ;$$

$$\text{III:} \quad p(x_1, -0, t_1) = \rho Vc \left[\frac{1}{2} - \frac{1}{\pi} \sin^{-1} \frac{ct_1 + 2x_1 - 2\ell}{ct_1} \right] ;$$

$$\text{IV; V:} \quad p(x_1, -0, t_1) = \rho Vc \left[\frac{1}{\pi} \sin^{-1} \frac{-ct_1 + 2x_1 + 2\ell}{ct_1} - \frac{1}{\pi} \sin^{-1} \frac{ct_1 + 2x_1 - 2\ell}{ct_1} \right].$$

All of the angles defined here are to be taken between $-\pi/2$ and $+\pi/2$.

Figure 5 presents some numerical results for $x_1/\ell = 0, 1/4, 1/2, 3/4$. As mentioned above, it is seen that $p(x_1, -0, t_1)$ changes sign as t_1 passes the value $2\ell/c$. This is seen to be generally true from formulae IV and V above.

VALIDITY OF THE SOLUTION; EXTENSIONS

Physically it is apparent that the mathematical problem posed above must approach a steady state. Mathematically in the solution for Region V above (see Fig. 4e), we see that the first integral is taken over a domain shaped like a parallelogram; neither the shape nor size of this domain nor the integrand changes if we increase the value of t_1. Further changes in the value of φ come about only through the addition of integrals over domains farther removed from (x_1, t_1), and, because of the form of the integrand, these have lesser effect on the value of the potential. These statements imply then that the most important compressibility effects are over at a time of, say, $t_1 = 4\ell/c$.

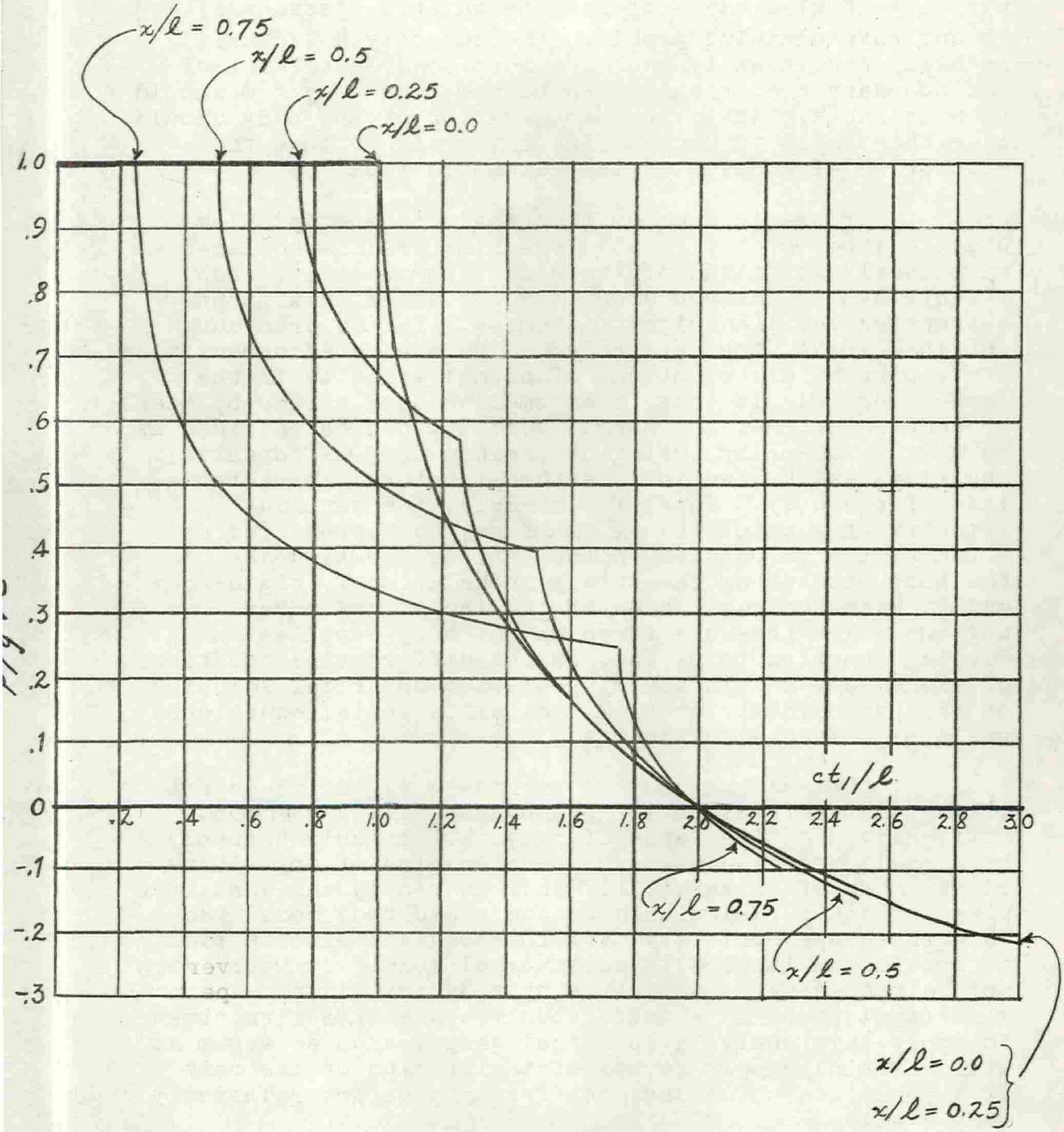

Figure 5. Pressure as a function of time, for several values of x_1, and $y_1 = -0$.

The most important check on the validity of the present approach is to compare this time with other time scales in the problem. For example, when $t_1 = 4\ell/c$, the body itself will have entered the water a distance $4\ell(\frac{V}{c}.)$ In any ship slamming problem, the quantity 4 V/c will probably be extremely small, and so the replacement of the boundary condition on the body by one on y = 0 should be reasonable. Also, the deceleration of the body should be rather small in such a time interval, so that the assumption of constant V is reasonable too.

There is another time scale in such problems which may be more critical: the time scale associated with local vibrations of the body. Fortunately, such effects can be incorporated into the solution without disrupting the linearization scheme. In the procedure outlined above, the integrals can be evaluated numerically for arbitrary distributions of normal velocity in the impact region. If this is accomplished by a step-by-step procedure in time, the normal velocity can be retained as an unknown quantity, being determined from differential equations which also include the structural characteristics of the body. Thus, for example, the vertical velocity at a point in $-\ell < x < \ell$ may be represented as a sum of the velocities in the various normal modes of the body (including the purely translational, rigid-body mode), with the amplitudes of the individual modes unknown. The pressure force on the body supplies the forcing function to be used in the differential equation of each mode, and in the step-by-step numerical solution of the hydrodynamic problem, the differential equations would be solved simultaneously.

Unless the body is extremely rigid, it is not likely that this vibration phenomenon will cause much difficulty. A time lapse of, say, $4\ell/c$ would probably be a small fraction of the natural period of any vibrational mode of interest. If this is really the case, the pressure can be calculated on the rigid body model and integrated appropriately over the body and also in time to provide an impulse to each normal mode. It would then not be necessary to solve the hydrodynamical and structural problems simultaneously. When the time comes to apply this analysis to actual ships, such an argument will certainly apply to modes of vibration of the ship as a whole, in which case the frequencies are relatively very small.

Another complication which can be handled to some extent (with a computer) is the following: How does one account for the fact that ships do not have rectangular cross-sections? Obviously, as a ship bow impacts and then immerses, the width of the free surface which is broken increases rapidly. In order to have a tractable problem, let us retain the assumption that the body acts on the water by imparting a constant vertical velocity to the water on $y = 0$, $-\ell < x < \ell$, but now we let ℓ be a function of time. This problem is in principle no more difficult than that already solved. In fact, Ward actually treats this more general problem. Figure 6 depicts the situation in the x-y-t space, corresponding to Fig. 2 for the simpler problem. So long as the body has a horizontal tangent plane at the bottom, the solution proceeds exactly as before. The role of the vertices of the rectangular region in Fig. 2 is now played by the points at which the characteristics in the x-t plane are tangent to the shaded region.

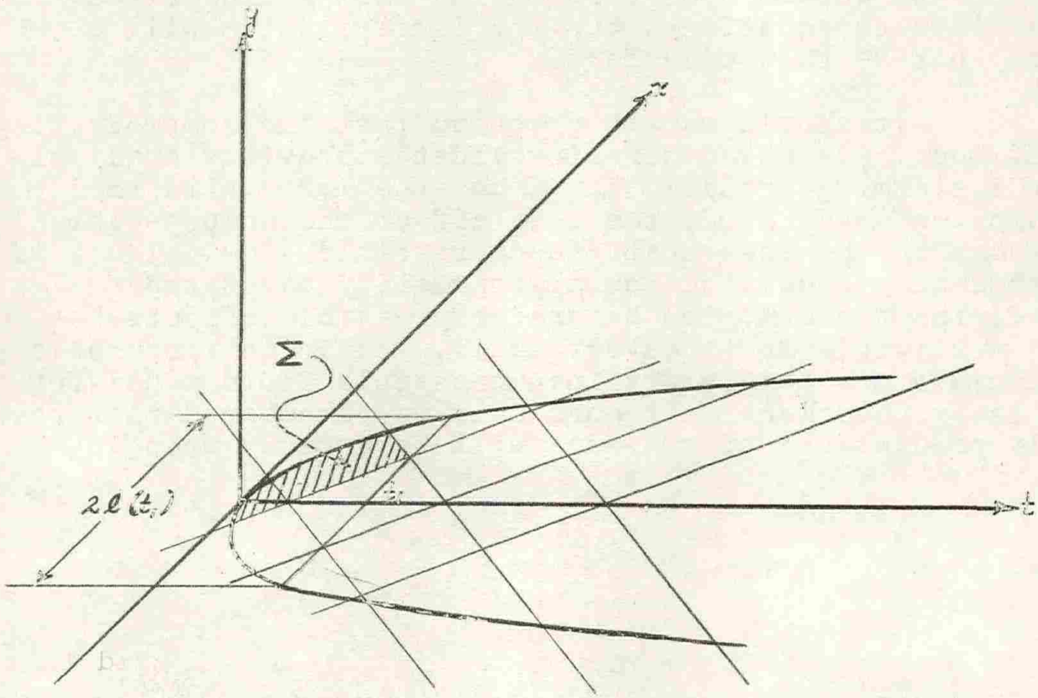

Figure 6. The region of integration, Σ, in the case that $\ell = \ell(t_1)$. Several characteristics of each set are shown.

Of course, the true boundary condition for this problem would specify the component of velocity normal to the body, not a component normal to the x-axis. To the extent that this distortion of the formulation propagates errors, the above statement of the boundary value problem is invalid. However, tractability seems to require that the boundary condition be stated on the x-axis, and in this kind of mathematical problem one cannot generally specify derivatives in a direction non-normal to a surface. So, there seems to be no way out of specifying the vertical velocity component on the x-axis. It may be possible to find a distribution of vertical velocity which is better than the constant distribution.

Finally, there remains the purely incompressible, free surface, hydrodynamic problem. After a short time lapse, the assumptions made above become invalid; in particular, the free surface boundary condition is not really $\varphi = 0$, but the equations (1) and (2). It may be possible to obtain some information from a linearized form of these equations, but the linearization will certainly not be that used here.

It should not be presumed that the compressible fluid model discussed here invalidates previous analyses of the slamming problem. In fact, the more valid the assumptions made here, the less effect the compressible flow has on the subsequent incompressible flow. Thus, if the boundary condition can appropriately be stated on $y = 0$, the fluid motion before compressible effects become negligible must be very small, and it is reasonable to formulate a completely incompressible fluid model for the later phenomena. It must only be remembered that the early predictions of pressure will be grossly wrong.