A new method to convert unleveled marine seismic data to leveled split-spread data

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SUMMARY

During marine seismic data acquisition, the data are shot in the so-called end-on configuration, which means that the receivers are positioned only at one side from the source. In marine seismic data processing, the missing offsets at the other side from the source are often reconstructed by applying physical reciprocity, i.e., source- and receiver positions are interchanged. As a result, source- and receiver depths are interchanged as well, leading to discontinuities in source- and receiver depths at the zero-offset position. This can lead to errors when spatially dependent operators are applied to the data. In this paper, a method is presented to level source- and receiver depths to the same depth before physical reciprocity is applied. Since the source- and receiver depths are then equal, these discontinuities are not present and any spatially dependent operator can be applied to the data correctly.

INTRODUCTION

In marine seismic data processing, many techniques applied to the data employ operators that are spatially dependent. These types of operators can be applied to the data either in the spatial Fourier domain via a multiplication, or in the spatial domain via a convolution summation. Either to transform the data to the spatial Fourier domain, or to calculate the convolution, an integration of the measured pressure wavefield is involved where the integration paths for the horizontal coordinates are from minus infinity to infinity. Specifically, to apply any spatially dependent operator to common shot data, receiver data are required from minus infinity to infinity (see Fig. 1). Similarly, to apply any spatially dependent operator to common receiver data, source data are required from minus infinity to infinity (Fig. 2). The configurations depicted in Figs. 1 and 2 are known as split-spread configurations. During marine data acquisition, the data are shot in the so-called end-on configuration, i.e., the receivers are positioned only at one side from the source position, shown in Fig. 3. Obviously, by padding zeros at the unknown receiver positions, large numerical errors are introduced since these zeros are only due to practical acquisition limitations and are not related to any geological configuration. In practice, often physical reciprocity is used to obtain split-spread data, which states that source- and receiver positions can be interchanged, i.e., source positions become receiver positions and receiver positions become source positions, expressed as

$$p(x^S|x^R, z) = p(x^R|x^S, z),$$

where $z$ is the transform parameter specific for the one-sided temporal Laplace transformation, and $x^R = x_1^R + x_2^R i_1 + x_3^R i_3$ denotes the position of the receiver, and $x^S = x_1^S + x_2^S i_1 + x_3^S i_3$ denotes the position of the source, in a Cartesian coordinate system that is set up by two mutually perpendicular base vectors $\{i_1, i_3\}$ of unit length and origin $O$. Physical reciprocity can be derived directly from Rayleigh's field reciprocity relation (see Fokkema and van den Berg, 1993). By applying physical reciprocity, the data are mirrored around the zero-offset axis to reconstruct the receiver positions at the side from the shot position where the receiver data are missing (see Fig. 4). However, by interchanging source- and receiver positions, source- and receiver depths are interchanged as well. As a result, for the recorded receiver positions, the receivers are at source depth $x_3^R$, and the source is at receiver depth $x_3^S$. Figure 4 shows the source- and receiver depths for one common shot gather, after physical reciprocity is applied. It is noted that both source- and receiver depths are discontinuous at the zero offset position. These discontinuities in source- and receiver depths can lead to errors when spatially dependent operators are applied to the data, since these types of operators require constant source- and receiver depths over the entire integration interval. In many processing sequences, this error is neglected, by assuming that

$$P(x^S, z^S|x^R, z^R) \approx P(x^R, z^R|x^S, z^S).$$

The validity of Eq. (2) depends very much on the geological configuration. For example, in the case of plane, horizontally layered configurations the error is zero, since in this case interchanged source- and receiver depths result in equal data. However, for laterally varying configurations the error will, in general, not be equal to zero. In this paper, we present a method to level source- and receiver depths, before physical reciprocity is applied to obtain split-spread data. After leveling the source- and receiver depths, the discontinuities are no longer present and any spatially dependent operator can be applied correctly.

DERIVATION OF THE INTEGRAL EQUATION

We consider the actual marine configuration (see Fig. 5), in which the waterlayer $D_w$ and the earth geology $D_g$ are distinguished. The waterlayer is bounded by the water surface $z_3 = 0$ and boundary $\partial D_w$. The top of the geology is denoted by $z_3, \text{max}$. The acoustic wavefield $\{p, \phi, \theta\}$ is generated by sources, located at position $x^S$, and measured by receivers, located at position $x^R$. The watersurface is assumed to behave as a perfect compliant reflector. The waterlayer $D_w$ is assumed to be homogeneous with material constants $[\rho_w, \mu_w]$. The wave speed in the waterlayer is $c_w$.

In Fokkema and van den Berg (1993), the total measured pressure wavefield is split into an incident and a scattered wavefield, i.e.,

$$p(x^R|x^S, z) = p^{\text{inc}}(x^R|x^S, z) + p^{\text{scat}}(x^R|x^S, z),$$

in which the incident pressure wavefield is the wavefield in absence of the earth geology $D_g$, but in presence of the free-surface. The scattered wavefield is decomposed into up- and downward wavefield constituents, i.e.,

$$p^{\text{scat}}(x^R|x^S, z) = p^{\text{down}}(x^R|x^S, z) + p^{\text{down}}(x^R|x^S, z),$$

where

$$F(p^{\text{down}}(x_1, x_2, z|x^S, z) = \frac{\exp(-\alpha x_1 x_3^{1/2} \gamma \chi)}{2 \sinh(\alpha x_3^{1/2} \gamma \chi)} F(p^{\text{up}}(x_1, x_2, z|x^S, z))$$

for $0 < z_2^{1/2} < z_3, \text{min}$. 

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and

\[ F\{ρ^b(x_1, x_2|x_0^b, s)\} = \frac{\exp(\text{i}kx_0^b)}{2\sinh(\text{i}kx_0^b)} F\{ρ^{\text{act}}(x_1, x_2^b|x_0^b, s)\}, \]

for \(0 < x_0^b < x_{2,\text{min}}\). \hspace{1cm} (6)

in which \(F\) denotes the forward spatial Fourier transform with respect to the horizontal receiver coordinate \(x_1\), defined by

\[ F\{\tilde{u}(j\omega_1, x_2)\} = \int_{\mathbb{R}} \exp(j\omega_1 x_1) \tilde{u}(x_1, x_2, s) dA, \]

with a related definition for the inverse transform \(F^{-1}\). Here, \(\omega_1\) is the angular-slowness, \(\omega_1\) is the real transform parameter, and

\[ \Gamma = \left( \frac{1}{c_0^2} + \omega_1^2 \right)^{1/2}, \text{ for } \text{Re}(\Gamma) > 0. \]

Adding Eqs. (5) and (6), and taking \(x_0 = x_0^b\), we arrive at an expression for the desired scattered pressure wavefield \(ρ^{\text{act}}(x_1, x_2^b|x_0^b, s)\), measured at receivers at depth \(x_2^b\), in terms of the scattered pressure wavefield, measured at receiver depth \(x_0^b\):

\[ F^{-1}\left\{ \frac{\sinh(\text{i}kx_0^b 2)}{\sinh(\text{i}kx_2^b 2)} F\{ρ^{\text{act}}(x_1, x_2^b|x_0^b, s)\} \right\} = ρ^{\text{act}}(x_1, x_2^b|x_0^b, s) \]

for \(0 < x_0^b < x_{2,\text{min}}\), and \(x_1 \in \mathbb{R}. \hspace{1cm} (9)\)

Note that the factor

\[ \frac{\sinh(\text{i}kx_0^b 2)}{\sinh(\text{i}kx_2^b 2)} = \exp(\text{i}\Gamma(x_0^b - x_2^b)) \]

does not remain bounded for \(x_0^b > x_0^b\) when \(\omega_1 \to \infty\). Therefore, equation (9) is only useful for \(x_0^b < x_2^b\).

For the case that \(x_0^b > x_2^b\), a similar equation can be derived to level the source depth \(x_0^b\) to the receiver depth \(x_2^b\). It is given by

\[ F^{-1}\left\{ \frac{\sinh(\text{i}kx_2^b 2)}{\sinh(\text{i}kx_0^b 2)} F\{ρ^{\text{act}}(x_0^b, x_2^b|x_2^b, s)\} \right\} = ρ^{\text{act}}(x_0^b, x_2^b|x_2^b, s) \]

for \(0 < x_2^b < x_0^b < x_{2,\text{min}}\), and \(x_2 \in \mathbb{R}. \hspace{1cm} (11)\)

where the factor

\[ \frac{\sinh(\text{i}kx_2^b 2)}{\sinh(\text{i}kx_0^b 2)} = \exp(\text{i}\Gamma(x_2^b - x_0^b)) \]

remains bounded for \(x_0^b > x_2^b\).

As mentioned before, during the data acquisition the data are shot in end-on configuration, which means that \(ρ^{\text{act}}(x_0^b|x_2^b)\) is only known in domain \(\mathcal{D}_s\), defined by

\[ \mathcal{D}_s = \{ x_0^b \in \mathbb{R}, x_2^b \in \mathbb{R}|x_0^b \leq x_2^b \}. \hspace{1cm} (13)\]

The complement of domain \(\mathcal{D}_s\) is denoted as \(\mathcal{D}_0\). Since the source- and receiver depths are equal for the desired scattered pressure wavefield, we may write

\[ ρ^{\text{act}}(x_0^b, x_2^b|x_2^b, s) = S\{ρ^{\text{act}}(x_0^b, x_2^b|x_2^b, s)\} \]

where

\[ S\{ρ^{\text{act}}(x_0^b, x_2^b|x_2^b, s)\} = \frac{1}{2} ρ^{\text{act}}(x_0^b, x_2^b|x_2^b, s) + \frac{1}{2} ρ^{\text{act}}(x_2^b, x_0^b|x_2^b, s), \]

for \(x_2^b = x_0^b, \hspace{1cm} (14)\)

in which the operator \(S\) enforces the desired scattered pressure wavefield \(ρ^{\text{act}}(x_0^b|x_2^b)\) to be a symmetric (even) function around the axis \(x_2^b = x_0^b\). As a result, the unknown quantity \(ρ^{\text{act}}(x_0^b|x_2^b)\) has to be known only in domain \(\mathcal{D}_s\). In \(\mathcal{D}_0\), \(ρ^{\text{act}}(x_0^b|x_2^b)\) can directly be obtained from Eq. (14). Applying Eq. (14) to equations (9) and (11) respectively, we arrive at the integral equations

\[ F^{-1}\left\{ \frac{\sinh(\text{i}kx_0^b 2)}{\sinh(\text{i}kx_2^b 2)} F\{S\{ρ^{\text{act}}(x_0^b, x_2^b|x_2^b, s)\}\} \right\} = ρ^{\text{act}}(x_1, x_2^b|x_0^b, s) \]

for \(0 < x_0^b < x_2^b < x_{2,\text{min}}\), and \(n_1 \in x_0^b, x_2^b \in \mathcal{D}_s. \hspace{1cm} (15)\)

to level the receiver depth \(x_2^b\) to the source depth \(x_0^b\), and

\[ F^{-1}\left\{ \frac{\sinh(\text{i}kx_2^b 2)}{\sinh(\text{i}kx_0^b 2)} F\{S\{ρ^{\text{act}}(x_0^b, x_2^b|x_0^b, s)\}\} \right\} = ρ^{\text{act}}(x_0^b, x_2^b|x_2^b, s) \]

for \(0 < x_2^b < x_0^b < x_{2,\text{min}}\) and \(n_1 \in x_2^b, x_1 \in \mathcal{D}_s, \hspace{1cm} (16)\)

to level the source depth \(x_0^b\) to the receiver depth \(x_2^b\). Note that both integral equations are to be solved only for \(x_0^b, x_2^b \in \mathcal{D}_s\), and hence sufficient information is available.

Both integral equations derived can be written in operator notation as

\[ \text{Lu} = f. \hspace{1cm} (17)\]

In Fukuhara and Van den Berg (1993), different solution methods are discussed to solve integral equations of this form. It is shown that when the adjoint operator \(L^*\) can be found such that

\[ \langle \theta(x_1^b|x_0^b, s), \bar{u}(x_1^b|x_0^b, s) \rangle = \langle L^* u(x_1^b|x_0^b, s), \bar{u}(x_1^b|x_0^b, s) \rangle, \hspace{1cm} (18)\]

in which the inner product of two functions \(u\) and \(\theta\) is defined as

\[ \langle u(x_1^b|x_0^b, s), \theta(x_1^b|x_0^b, s) \rangle = \int_{x_1^b \in \mathbb{R}} \int_{x_0^b = -\infty}^{x_0^b} \bar{u}(x_1^b|x_0^b, s) \theta^*(x_0^b|x_1^b, s) dA dA, \hspace{1cm} (19)\]

a very efficient conjugate-gradient type of iterative scheme can be derived to solve integral equation. In the next section, the adjoint operator \(L^*\) is derived using definitions (18) and (19).

**DERIVATION OF THE ADJOINT OPERATOR**

In the following derivations, we consider the case that \(x_0^b < x_2^b\). Defining the window function \(\chi\) as

\[ \chi(x_0^b|x_2^b) = \begin{cases} 1 & \text{for } x_0^b, x_2^b \in \mathcal{D}_s, \\ 0 & \text{for } x_0^b, x_2^b \in \mathcal{D}_0, \end{cases} \]

repeated application of Parseval's theorem leads to

\[ \langle u, Lu \rangle = \int_{x_0^b \in \mathbb{R}} \int_{x_2^b \in \mathbb{R}} \chi(x_0^b|x_2^b) \theta(x_0^b|x_2^b, s) \]

\[ \left[ F^{-1}\left( \frac{\sinh(\text{i}kx_0^b 2)}{\sinh(\text{i}kx_2^b 2)} F\{S\{\tilde{u}(x_1, x_0^b, s)\}\} \right) \right]^{*} dA dA. \hspace{1cm} (19)\]
\[
= \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F \left\{ \chi(z_1|x_1^0) \delta(z_1|x_1^0, s) \right\} \cdot \sinh(\alpha(x_1^0)) \frac{\sinh(\alpha(x_1^0 - x_2^0))}{\sinh(\alpha(x_1^0 + x_2^0))} \cdot d\alpha dA
\]

\[
= \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \sinh(\alpha(x_1^0)) \cdot \sinh(\alpha(x_1^0 - x_2^0)) \cdot \sinh(\alpha(x_1^0 + x_2^0)) \cdot d\alpha dA
\]

\[
F \left\{ \chi(z_1|x_1^0) \delta(z_1|x_1^0, s) \right\} \cdot \delta(x_1^0|x_2^0, s) \cdot d\alpha dA
\]

\[
= \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S(P^{-1}) \cdot \left\{ \sinh(\alpha(x_1^0)) \cdot \sinh(\alpha(x_1^0 - x_2^0)) \right\} \cdot d\alpha dA
\]

\[
= \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S(P^{-1}) \cdot \left\{ \sinh(\alpha(x_1^0)) \cdot \sinh(\alpha(x_1^0 - x_2^0)) \right\} \cdot d\alpha dA
\]

\[
(2L, \theta, \phi) \quad \text{for} \quad x_1^0, x_2^0 \in \mathbb{R}^3.
\]

The adjoint operator \( L^* \), defined by Eq. (20), can now be used in any iteration scheme for selfadjoint operators \( LL^* \) to solve integral equation (15). Equivalently to Eq. (20), an adjoint operator can be derived to solve Eq. (16).

**NUMERICAL EXAMPLE**

To illustrate the effect of leveling source- and receiver depths, we applied a method to remove the surface-related multiples from the data that employs spatially dependent operators (see Van Borselen, 1991) to synthetic data from a finite, rigid strip, embedded in a homogeneous half-space (see Fig. 7). In Fig. 8, an end-on common shot gather is shown, before the removal of the free-surface-related multiples. The shot record was shot above the center of the strip at depth \( z_2^0 = 5 \text{ m} \), and measured by receivers placed at depth level \( z_1^0 = 10 \text{ m} \). Figure 9 shows the result after multiple removal using end-on data, i.e., using only the negative offsets of every shot gather. After applying physical reciprocity to reconstruct the positive offsets, the integral equation was solved, disregarding the discontinuities in source- and receiver depths. Note that although the multiples are suppressed significantly still multiple energy is present in the output results of the multiple removal scheme. Furthermore, numerical artifacts are introduced. Next, we applied our method to level the source depth \( z_1^0 \) from 5 m to receiver depth \( z_1^0 = 10 \text{ m} \). As a result, after processing the data, the sources and receivers are all located at depth level 10 m, i.e., \( z_2^0 = z_1^0 = 10 \text{ m} \). Figure 10 shows the center input shot record after leveling the source depths to 10 m. Figure 11 shows the result after multiple removal using the leveled data. Note that a much better suppression of the multiples is achieved and that no numerical artifacts are introduced by the leveling scheme.

**CONCLUSIONS**

The conventional procedure of interchanging source- and receiver positions (application of physical reciprocity) to reconstruct the missing offsets in marine seismic data processing, introduces discontinuities in source- and receiver depths. These discontinuities can lead to errors when spatially dependent operators are applied to the data. A method is presented to level source- and receiver depths before physical reciprocity is applied to obtain the missing offsets. For equal source- and receiver depths these discontinuities are not present and any spatially dependent operator can be applied correctly. The numerical example presented in this paper illustrates that indeed a much better performance of the spatially dependent operator to remove surface-related multiples is obtained when applied to data for which source- and receiver depths are leveled to one constant depth. Future research will concentrate on incomplete data sets, as missing near- and intermediate traces, and missing shot records.

**REFERENCES**


**Fig. 1** Split-spread common shot gather

\[ x = x_S \]

\[ x = x_R \]

**Fig. 2** Split-spread common receiver gather

\[ x = x_S \]

\[ x = x_R \]

**Fig. 3** End-on spread common shot gather

\[ x = x_S \]

\[ x = x_R \]

**Fig. 4** Physical reciprocity: Interchanging of source- and receiver positions

\[ x_1^R = x_1^S \]
Split-spread conversion

**Fig. 5** Discontinuity in source- and receiver depths after application of physical reciprocity

\[ \Delta x_3 = x_3^S \]

\[ \Delta x_3 = x_3^R \]

**Fig. 6** The actual marine configuration

\[ x_3 = 0 \]

\[ x_3^S = 5 \text{ m} \]

\[ x_3^R = 10 \text{ m} \]

\[ x_3 = 100 \text{ m} \]

**Fig. 7** The finite rigid strip configuration

**Fig. 8** End-on input shot gather shot above center of strip

**Fig. 9** Output after multiple-removal using unleveled end-on data

**Fig. 10** Leveled end-on input shot gather shot above center of strip

**Fig. 11** Output after multiple-removal using leveled end-on data