MSc Thesis

Fatigue analysis of subsea Jumper under external loads (steady current and earthquake)

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Pong Kosanunt

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Summary

In recent times, the oil and gas business has moved into unconventional reservoirs, especially in deep-water. One high-potential prospect was found in the deep-water area of Myanmar. A subsea production system pilot project is planned for implementation in this area.

One essential element of a subsea system is a “subsea jumper”. The main function is to interface between the subsea tree and subsea manifold. There are various subsea jumper configurations used in the market. This thesis focuses primarily on the U-inverse shape jumper as a fundamental shape which gives subsea jumper a flexible characteristic.

A subsea jumper that is used in a deep-water area is difficult to access for maintenance or repair. As such, it is essential to determine the reliability of a jumper system, especially thru fatigue failure analysis. The dynamic behaviors of a subsea jumper at various load conditions need to be taken into account during the design phase. In general, a subsea jumper system experiences many loads both internal and external, but there are only two key external loads, the steady sea current and earthquakes at designed area. These interesting factors are considered in this thesis.

In order to better understand the dynamic behaviors of a subsea jumper under load conditions, it is important to first analyze the dynamic characteristics of the jumper itself. A U-inverse shape jumper can be modelled by connecting three “pipe conveying fluid model” (or Euler Bernoulli Beam + internal flow effect). This is called a “subsea jumper model” or “Triple beam model”. This model gives the dynamic characteristics of a jumper in terms of “mode shape” and “natural frequency” in two vibration planes: inline and crossflow.

The dynamic behavior of a subsea jumper under a current load situation can be solved by using a wake oscillator model coupled with a subsea jumper model. The results show that a mild sea current is able to dramatically induce jumper oscillation. This phenomenon is called Vortex induced vibration (VIV). It can occur in both crossflow VIV and inline VIV; however, for both cases of VIV, a subsea jumper system is safe to operate under the designed current velocity (maximum current velocity is 0.832 m/s, based on a 100-year return value).

In an earthquake load condition, the subsea jumper model is coupled with an inertia load model (mass times acceleration). Two types of acceleration are considered in this thesis thru a sinusoidal model and simulation model. The first, sinusoidal model assumes that an earthquake is a continuous process with ground acceleration in a sinusoidal shape. It is used to analyze the dynamic behavior of a subsea jumper in terms of “seismic response spectra”. The second, a simulation model defines an earthquake in more realistic way by considering an earthquake as a shock of high magnitude in a small period. This model is more suitable for fatigue analysis.

It should be emphasized that a pure earthquake load is a rare occasion, as the current of the nearby seabed is always present. Thus, it is more helpful to investigate the fatigue lifetime of a jumper under a combination of earthquake effects and steady current.

The analysis results show that a subsea jumper can withstand up to 13,000 number of a high magnitude earthquake shock, over 7.5 Richter. However, during the designed lifetime of a subsea jumper there are typically only 600 shocks. Thus, one can conclude that a subsea jumper is safe against earthquakes in the designed area.
The designed subsea jumper may require changes if it is relocated to operate in another area with the presence of a stronger current velocity and/or earthquake conditions. Subsea jumper lifetime can be improved by designing dimensions and configurations to give natural frequencies out of the load range. This can be achieved by reducing the length of a jumper or increasing its diameter. Another method is to reduce the flow rate of the contained fluid. However, these methods may stimulate another problem if slug is present inside the jumper. Adjustments in flow rate or jumper dimension changes the impact period of slug at each bend of a jumper system. When slug impact load frequency is close to a natural frequency, there will be a dramatic response. Thus, considerations of slug should be taken into account for subsea jumper design, especially with any changes in dimension, configuration and flow rate. Lastly, other mitigation methods include a more robust material, controlling surface conditions and welding method.
## Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>EOM</td>
<td>Equation of motion</td>
</tr>
<tr>
<td>FDM</td>
<td>Finite different method</td>
</tr>
<tr>
<td>N-S</td>
<td>North-South direction</td>
</tr>
<tr>
<td>W-E</td>
<td>West-East direction</td>
</tr>
<tr>
<td>PDE</td>
<td>Partial different equation</td>
</tr>
<tr>
<td>R-K</td>
<td>Rung-Kutta method</td>
</tr>
<tr>
<td>VIV</td>
<td>Vortex induce Vibration</td>
</tr>
<tr>
<td>V/H</td>
<td>Ratio of earthquake amplitude Vertical to Horizontal ground motion</td>
</tr>
</tbody>
</table>
Notations

\( A \) \hspace{1cm} \text{Cross-section area of subsea jumper} \hspace{1cm} \text{[m}^2\text{]} \\
\( A_g \) \hspace{1cm} \text{Constant parameter in the Attenuation Law} \hspace{1cm} \text{[m/s}^2\text{]} \\
\( a \) \hspace{1cm} \text{Basquin specific parameter} \hspace{1cm} [-] \\
\( \hat{a}_g \) \hspace{1cm} \text{Amplitude of ground acceleration} \hspace{1cm} \text{[m/s}^2\text{]} \\
\( \bar{C}_{L0} \) \hspace{1cm} \text{Stationary lift coefficient} \hspace{1cm} [-] \\
\( C_{VD} \) \hspace{1cm} \text{Drag coefficient} \hspace{1cm} [-] \\
\( C_{VL} \) \hspace{1cm} \text{Fluctuation lift coefficient} \hspace{1cm} [-] \\
\( C_{Vv} \) \hspace{1cm} \text{Inline vortex force coefficient for subsea jumper} \hspace{1cm} [-] \\
\( C_{Vw} \) \hspace{1cm} \text{Crossflow vortex force coefficient for subsea jumper} \hspace{1cm} [-] \\
\( c_{sw} \) \hspace{1cm} \text{Hydrodynamic damping in still water per unit length} \hspace{1cm} \text{[Kg/(m \cdot s)]} \\
\( D \) \hspace{1cm} \text{Fatigue damage} \hspace{1cm} [-] \\
\( E \) \hspace{1cm} \text{Young modulus} \hspace{1cm} \text{[Pa]} \\
\( EI \) \hspace{1cm} \text{Bending stiffness} \hspace{1cm} \text{[Pa \cdot m}^4\text{]} \\
\( e \) \hspace{1cm} \text{Gap between cylinder body and boundary layer} \hspace{1cm} \text{[m]} \\
\( F_V \) \hspace{1cm} \text{Vortex force relative to flow velocity} \hspace{1cm} \text{[N]} \\
\( F_{VD} \) \hspace{1cm} \text{Vortex drag force relative to flow velocity} \hspace{1cm} \text{[N]} \\
\( F_{VL} \) \hspace{1cm} \text{Vortex lift force relative to flow velocity} \hspace{1cm} \text{[N]} \\
\( F_{Vv} \) \hspace{1cm} \text{Vortex inline force on subsea jumper relative to flow velocity} \hspace{1cm} \text{[N]} \\
\( F_{Vw} \) \hspace{1cm} \text{Vortex crossflow force on subsea jumper relative to flow velocity} \hspace{1cm} \text{[N]} \\
\( f_{\text{earthquake}} \) \hspace{1cm} \text{Earthquake load frequency} \hspace{1cm} \text{[Hz]} \\
\( f_o \) \hspace{1cm} \text{Natural frequency in still water} \hspace{1cm} \text{[Hz]} \\
\( f_s \) \hspace{1cm} \text{Vortex shedding frequency} \hspace{1cm} \text{[Hz]} \\
\( G \) \hspace{1cm} \text{Shear modulus} \hspace{1cm} \text{[Pa]} \\
\( ID \) \hspace{1cm} \text{Inner diameter of cylinder or subsea jumper} \hspace{1cm} \text{[m]} \\
\( J \) \hspace{1cm} \text{Polar Moment of inertia of area} \hspace{1cm} \text{[m}^4\text{]}
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k$</td>
<td>Basquin specific parameter</td>
<td>[-]</td>
</tr>
<tr>
<td>$L$</td>
<td>Length of subsea jumper</td>
<td>[m]</td>
</tr>
<tr>
<td>$M$</td>
<td>Bending moment</td>
<td>[N \cdot m]</td>
</tr>
<tr>
<td>$M_g$</td>
<td>Magnitude of earthquake</td>
<td>[Richter]</td>
</tr>
<tr>
<td>$M_f$</td>
<td>Mass of conveying fluid per unit length</td>
<td>[Kg/m]</td>
</tr>
<tr>
<td>$m$</td>
<td>Mass of subsea jumper system per unit length</td>
<td>[Kg/m]</td>
</tr>
<tr>
<td>$N$</td>
<td>Material fatigue life time in term of number of load cycle</td>
<td>[-]</td>
</tr>
<tr>
<td>$N_i$</td>
<td>Material fatigue life time accordance with stress level $i$</td>
<td>[-]</td>
</tr>
<tr>
<td>$n_i$</td>
<td>Number of stress cycle level $i$</td>
<td>[-]</td>
</tr>
<tr>
<td>$OD$</td>
<td>Outer diameter of cylinder or subsea jumper</td>
<td>[m]</td>
</tr>
<tr>
<td>$q$</td>
<td>Wake or vortex lift force coefficient</td>
<td>[-]</td>
</tr>
<tr>
<td>$R$</td>
<td>Distance to epicenter from earthquake location</td>
<td>[m]</td>
</tr>
<tr>
<td>$R_0$</td>
<td>Constant parameter in the Attenuation Law</td>
<td>[m]</td>
</tr>
<tr>
<td>$Re$</td>
<td>Reynolds number</td>
<td>[-]</td>
</tr>
<tr>
<td>$S$</td>
<td>Stress level</td>
<td>[Pa]</td>
</tr>
<tr>
<td>$St$</td>
<td>The Strouhal number</td>
<td>[-]</td>
</tr>
<tr>
<td>$U$</td>
<td>Relative fluid velocity to cylinder or subsea jumper motion</td>
<td>[m/s]</td>
</tr>
<tr>
<td>$U_f$</td>
<td>Conveying fluid velocity at steady state</td>
<td>[m/s]</td>
</tr>
<tr>
<td>$u$</td>
<td>Longitudinal deflection of subsea jumper</td>
<td>[m]</td>
</tr>
<tr>
<td>$V$</td>
<td>Fluid flow Velocity or Current velocity</td>
<td>[m/s]</td>
</tr>
<tr>
<td>$v$</td>
<td>Inline transverse deflection of subsea jumper</td>
<td>[m]</td>
</tr>
<tr>
<td>$v_f$</td>
<td>Kinematic Viscosity</td>
<td>[m$^2$/s]</td>
</tr>
<tr>
<td>$v_g$</td>
<td>Ground acceleration in inline direction</td>
<td>[m/s$^2$]</td>
</tr>
<tr>
<td>$v_s$</td>
<td>Poison ratio of steel</td>
<td>[-]</td>
</tr>
<tr>
<td>$w$</td>
<td>Crossflow transverse deflection of subsea jumper</td>
<td>[m]</td>
</tr>
<tr>
<td>$w_g$</td>
<td>Ground acceleration in crossflow direction</td>
<td>[m/s$^2$]</td>
</tr>
<tr>
<td>$x$</td>
<td>Axis coordinate</td>
<td>[-]</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td>Unit</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
<td>------</td>
</tr>
<tr>
<td>$y$</td>
<td>Axis coordinate</td>
<td>[-]</td>
</tr>
<tr>
<td>$z$</td>
<td>Axis coordinate</td>
<td>[-]</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Angle of the fluid velocity respect to inline axis</td>
<td>[rad]</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Angle of the relative fluid velocity respect to inline axis</td>
<td>[rad]</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>Tuning parameter of wake oscillator (Van der Pol equation)</td>
<td>[-]</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Density</td>
<td>[$Kg/m^3$]</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Bending stress at any arbitrary point on subsea jumper</td>
<td>[Pa]</td>
</tr>
<tr>
<td>$\omega_s$</td>
<td>Vortex shedding frequency</td>
<td>[rad/s]</td>
</tr>
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</table>
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Chapter 1: Introduction

1.1 General

Global energy demands continue to increase at a massive rate, especially for oil and gas products. In order to satisfy such needs, conventional reserves alone are insufficient. As such, oil and gas companies have started exploring and producing oil and gas product in unconventional reservoirs. One of the most promising unconventional reserves can be found in deep water. These sites have challenges both economically and technically that were insurmountable in the past. For the last few decades, however, with oil and gas prices gradually rising, there has been increased research that has resulted in gains regarding oil and gas recovery. Currently, oil and gas development fields in deep-water area are common around the world and increasingly venturing into even deeper areas.

Apart from corporate and private gains from exploring and producing oil and gas in ultra-deep areas, some developing counties have recently start investing on oil and gas business on domestic land. Several deep water prospects have been found with high developmental potential. One such interesting area is located in Myanmar, which is the main focus of this thesis.

Deep-water development fields generally use a subsea production system. One essential element of a subsea production system is a “subsea jumper”. Their main function is as an interface between the subsea tree and subsea manifold or other subsea infrastructures. Although subsea jumpers come in various types and shapes, the most common is a rigid subsea jumper in a U-inverse shape. This type offers an excellent flexible property to provide installation tolerance and operation expansion. However, this property also allows a subsea jumper to be sensitive to dynamic conditions from disturbance loads. These loads generally cause a fatigue failure on a subsea jumper system. A consequence of failure would be severe economic and environment loss.

There are many loads disturbance on a subsea jumper system. They can be categorized into internal loads and external loads. A common internal force is created by conveying fluid, while external disturbance loads are strongly dependent on the designed location. This thesis considers a deep-water field located in Myanmar. This area has a high possibility of earthquake occurrence since it is close to many earth faults. In addition, there is a strong current flow nearby seabed. As such, this thesis gives attention on dynamic behavior and fatigue lifetime while operating a subsea jumper system under two main external disturbances: a steady nearby seabed current and earthquakes. Both of these disturbances are able to induce drastic vibration on a subsea jumper that may cause fatigue failure to the jumper system.
1.2 Scope of thesis

The objective of this thesis is to provide a proof for the reliability of an operating subsea jumper under steady current and earthquake conditions. This task can be done by estimating the fatigue lifetime of a subsea jumper and comparing it with the designed operation time.

The thesis is presented in two main parts. The first part of the thesis begins with a “subsea jumper model”, based on a U-inverse configuration. This shape has one horizontal line connected to two vertical lines at both ends. All analysis throughout the thesis mainly focuses on the longest route subsea jumper which is used in market. The designed dimensions are 50 meters for horizontal line, 10 meters for vertical lines and 14 inches of a subsea jumper diameter. A subsea jumper model is established in the form of an equation of motion. After solving the subsea jumper model, mode shapes and its natural frequencies are known.

In order to understand the dynamic behavior of a subsea jumper under steady current or in VIV phenomenon, a “subsea jumper under a current load model” is established by coupling the subsea jumper model to a Van der Pol equation (or “wake oscillator model”). The wake oscillator model shows the characteristics of a fluctuation lift coefficient during VIV or wake-structure synchronization. The resultant motion of a subsea jumper can be obtained by solving a differential equation for the subsea jumper under current load model.

In an earthquake condition, it is able to establish a “subsea jumper under earthquake load model” by adding earthquake loads into the subsea jumper model. These earthquake loads can be represented by the inertia load due to ground acceleration. The ground acceleration is directly related to earthquake magnitude and distance from earthquake source. Similar to a steady current condition, the dynamic behavior of a subsea jumper during an earthquake can be acquired by solving a differential equation for the subsea jumper under earthquake load model.

In the last section for the first part of the thesis, the dynamic behaviors of a subsea jumper under a combination of current and earthquake conditions are analyzed. The combination model is constructed by integrating a wake oscillator model and inertia load from an earthquake in to subsea jumper model. It is called a “subsea jumper under combination load model”. One vital assumption is that current velocity is not influenced by the earthquake. It still presents as a steady current going past the subsea jumper. The dynamic behaviors under a combination of conditions are run in several scenarios in order to understand the couple effect between VIV and earthquake on a jumper system. In addition, the results of dynamic behavior appear in displacement time history. These are input data for fatigue analysis in the second part of the thesis.

It should be emphasized that all models of a subsea jumper under various conditions are indicated by a partial differential equation (PDE). Through the thesis, they are solved by numerical approximation with a finite differential method (FDM).

The second part of the thesis introduces fatigue analysis. The scope of the fatigue analysis focuses only on bending stress due to transverse motion. As such, the second part begins with a displacement-stress conversion method because fatigue failure is associated with cyclic stress on a structure. Later, fatigue lifetime is estimated based on accumulative fatigue or Miner’s rule. Finally, improvement of fatigue lifetime of a subsea jumper is discussed in regard to a steady current load and earthquake load.
Chapter 1: Introduction

1.3 Thesis outline

The current report is organized into two main parts in accordance with thesis objective, which comprise nine chapters. The first chapter presents a background together with thesis scope, essential objective and thesis outline. Then, four basic fundamental areas of knowledge are presented in the chapter 2 literature review: 1) general information of subsea jumper, 2) theory of vortex induce vibration due to current nearby seabed, 3) earthquake phenomenon and 4) fatigue analysis.

Part I presents the dynamic behavior of a subsea jumper in various conditions and is comprised of:

Chapter 3: The equation of motion for a subsea jumper, in U-inverse shapes, is provided for both crossflow and inline vibration. It is called a “subsea jumper model”. The dynamic characteristics of subsea jumper are provided in terms of mode shapes and natural frequencies.

Chapter 4: A subsea jumper under current load model is established by introducing a wake oscillator model into a subsea jumper model. With designed parameters for current near the seabed, the resultant motions of a subsea jumper under steady current are provided.

Chapter 5: A subsea jumper under earthquake load model is constructed by adding inertia load due to an earthquake into the subsea jumper model. Earthquake parameters at designed location are analyzed. With designed parameters, the dynamic responses of a subsea jumper are provided in seismic response spectra.

Chapter 6: An operating subsea jumper under steady current and earthquake is modelled by a subsea jumper under combination load model. The dynamic behaviors of a subsea jumper are analyzing in various combinations of conditions not only to use as input data for fatigue analysis but also gain a better understanding of couple effects from earthquakes and a steady current on a subsea jumper.

Part II relates to fatigue analysis, and is organized as follows:

Chapter 7: Fatigue lifetime estimation of subsea jumper using Miner's rule. This chapter also includes a displacement-stress conversion method and stress counting method. Subsea jumper lifetime on each extreme case is estimated. Finally, vital observations are given.

Chapter 8: Improvement methods to extend subsea jumper fatigue lifetime are proposed.

Chapter 9: Main conclusions and recommendations on dynamic behavior and fatigue analysis of subsea jumper are provided
2.1 Subsea jumper

Subsea jumpers are a key part of subsea field development. Their main function is typically as an interface between two subsea components. They also play an important role in providing installation tolerance and operation expansion. There are two types of subsea jumper. One is a rigid type, which is used to connect between different subsea structures such as a subsea well and manifolds. Another is a flexible type for connecting with PLEM/PLETS and riser bases. Figure 2.1 shows an arrangement of subsea equipment.

The configurations of subsea jumpers are typically a M-shape and a U-shape inverse shape as can be seen in Figure 2.2. This shape provides more flexibility during installation (positioning tolerance) and reduces additional stress during operation (thermal and pressure expansion). The end connection of a subsea jumper can be positioned either vertically or horizontally depending on interface equipment. Table 2.1 summarizes the advantages and disadvantages of a vertical and horizontal tie-in system.
Chapter 2: Literature Study

<table>
<thead>
<tr>
<th>Evaluation Issue</th>
<th>Horizontal Connection</th>
<th>Vertical Connection</th>
</tr>
</thead>
<tbody>
<tr>
<td>Required equipment for Installation</td>
<td>Complex Deployment system Relatively high for ROV performing task</td>
<td>Simple Deployment system Relatively low for ROV performing task</td>
</tr>
<tr>
<td>Duration of installation</td>
<td>Long</td>
<td>Short</td>
</tr>
<tr>
<td>Connector system and size</td>
<td>Connector is simple API/ANSI flange or clamp. Its weight is relatively low.</td>
<td>Connector system is rather complex thru use of a collet connector with an integral stroking mechanism. Weight is possibly up to several tons.</td>
</tr>
<tr>
<td>Required metrology and fabrication accuracy for installation purpose</td>
<td>Medium level of accuracy required. Connector and jumper spool can elastically deform for alignment.</td>
<td>High level of accuracy required since vertical connector has no allowance for correcting alignment.</td>
</tr>
<tr>
<td>Weather dependence for installation</td>
<td>Very low, since operation is independent of vessel motion.</td>
<td>Relatively high due to dependence on guidelines</td>
</tr>
</tbody>
</table>

Table 2.1: Comparison between vertical and horizontal tie-in for subsea jumper

The installation method for a subsea jumper is done simply by using only one crane operation with assistance from a ROV and guidewires. As the result, the installation of a subsea jumper is different from a marine pipeline as no pre-tension is required during installation. Figure 2.3 shows some subsea jumper installations from an underwater view. Typically, a subsea jumper will be installed while hanging between connected subsea components, and thus a gap is present between the jumper and sea bed. Its value depends on the size of mounted components, type of connection, jumper configuration and amount of sediment transportation. Typically, a minimum gap can be estimated between a subsea template and seabed as typically between 1 to 3 meters.

![Figure 2.3: Installation of subsea jumper](image)

The design load of a subsea jumper shall take dynamic loads into account during design phase. Dynamic loads can present in both internal loads and external loads. Internal loads are loads induced by the fluid contained inside the jumper, for example, fluid flow and slugging. External loads are loads induced by surrounding factors or the environment such as waves and current load, seismic load due to an earthquake, sediment transportation load and load induced by connected equipment. For the U shape, dynamic behaviors are sensitive to external loads that may lead to a fatigue problem during operation.

The design in sizing and shape of a subsea jumper is strongly dependent on a specific developed field. Even the same development field may need different subsea jumper shapes and sizes. However, each is comprised of at least one U-shape part as a basis element. It gives a flexible characteristic to subsea jumper. The sizing of subsea jumpers vary between 4 to 18 inches and length are between 5 to 50 meters.
Chapter 2: Literature Study

Material selection for a subsea jumper is an essential part of design. The choice of material is heavily dependent on the medium of the fluid inside, corrosion, constructability and cost. Subsea Jumpers are typically designed by using carbon steel as marine pipeline (standard grade B to grade X70).
2.2 Effect of steady-flow across stationary cylinders

The area of vibration on a cylinder body surrounded by a steady flow has been studied over many years. One of most powerful contribution comes from describing flow by using the Reynold Number, developed by Stoles (1851) and Reynolds (1883). Research by Strouhal (1878) found a relationship between flow velocity, diameter and frequency of vortex shedding for a tension string. Later, Prandtl (1904) and Karman (1977) put forth the concept of a boundary layer due to viscous action and the consequent development of the theory of a vortex street. These contributions lead our current understanding of interactions between a structure and fluid flow.

When considering steady flow with crossover on a cylinder body, the result downstream is two free shear layers separate from each side of the cylinder. In this case, these two separated shear layers are unstable. Each shear layer will roll up and circulate into large discrete vortices that form alternately on opposite sides of a cylinder. The process will repeat itself on the opposite sides. This phenomenon is called “alternate vortex shedding”. Figure 2.4 shows alternate vortex shedding in both an in-line and cross-flow response, as described by J. P. Kenny (1993).

The vortex velocity is larger than the ambient flow around with a consequence that the pressure in the vortex region is lower. Integrating pressure across a cylinder, results in a resultant force across the cylinder body as shown in Figure 2.5. Furthermore, a newly formed vortex is stronger than a previous and it is shed across on the opposite side. The flow field also changes in periodically in accordance with shedding frequency. As a results, the direction and magnitude of a resultant force changes periodically. The resultant force can be decomposed into lift and drag force that is expressed by using lift and drag force coefficient.
Before proceeding to approximating a lift and drag coefficient, it is important to be familiar with all important parameters that are used in studying steady flow across a cylinder and vortex induced vibration. These parameters are introduced here:

**Reynolds number** (Re)

The Reynolds number is a dimensionless parameter that represents the ratio between the inertia forces and friction forces acting on a body. The Reynolds number is used to classify dynamic flows. The Reynolds number can be expressed as:

\[ Re = \frac{V D_0}{v_f} \]  

Where, \( V \) is flow velocity, \( D_0 \) is a characteristic dimension of the body around the fluid flows and \( v_f \) is the kinematic viscosity coefficient of a fluid.

**The Strouhal number** (St)

The Strouhal number is a dimensionless parameter used to describe vortex shedding frequency. It can be expressed as:

\[ St = \frac{f_s D_0}{V} \]  

Where, \( V \) is flow velocity, \( D_0 \) is a characteristic dimension of the body around the fluid flows and \( f_s \) is vortex shedding frequency. The relationship between a Strouhal and Reynolds number are plotted in a graph as shown in Figure 2.6:

**Reduced Velocity** \((V_N)\)

Reduced velocity is the ratio of path length per cycle of flow to body width or diameter.

\[ V_N = \frac{\text{path length per cycle}}{\text{body width}} = \frac{V}{f_0 D_0} \]  

Where, \( f_0 \) is the natural frequency in still water. \( D_0 \) and \( V \) are characteristic dimension and flow velocity respectively.
As mentioned above, a dynamic flow field across a cylinder is considered. Note, in certain cases shedding frequency is equal to natural frequency. The values of reduced velocity will equal to 0.5 as in the following derivation:

\[ V_N = \frac{V}{f_0 \mu_0} = \frac{f_s \mu_0}{f_0 \mu_0} = \frac{1}{0.2} = 5 \]  

(2.4)

There are many studies and experiments that measure the lift and drag coefficient on a stationary cylinder. Figure 2.7 shows how the drag coefficient and lift coefficient vary with Reynolds number. One interesting observation is at a subcritical region, which is a very probable occurrence for marine pipeline and riser cases. The drag coefficient is almost a constant value of about 1.2, while the maximum lift coefficient is about 0.3 at the region.
2.3 Vortex Induce Vibrations (VIVs)

We look at here a case of steady flow past a cylinder which is free to move in crossflow direction. The relative velocity between flow velocity and structure needs to be taken into account, and results in a fluctuation of the lift force coefficient. The value of the lift force coefficient in the previous section (stationary case) is not valid to approximate the lift force. The fluctuating lift force coefficient has a significant value when shedding frequency is close to the natural frequency of a cylinder. The vortex-induced vibration of a free cylinder was studied by Khalak and Williamson (Khalak and Williamson 1999). They performed an experiment on a cylinder under steady flow, and the cylinder was allowed to move freely in a crossflow direction. A schematic diagram of the experiment is shown in Figure 2.8.

This experiment observes the cylinder response in a steady-state condition by recording the amplitude of vibration, frequency of vibration and shedding frequency. The experiment has been repeated by varying steady flow speed in order to capture any effect from vortex-vibration for a freely moving cylinder. Later, a similar experimental was set up by Jauvtis and Williamon (Jauvtis and Williamon, 2003). This time, the cylinder was allowed to move freely in both an inline and crossflow direction. Again, cylinder response and shedding frequency were observed. The results of both experiments are summarized in Figure 2.9. Dots (·) is experimental data from Khalak and Williamon, 1999, whereas Circles (°) indicate the later experimental by Jauvtis and Willamson, 2003.
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In Figure 2.9, the results of the experiment are shown in a dimensionless parameter. Amplitudes of vibration \( A_x \) and \( A_y \) are indicated by using a ratio of the amplitude of vibration to cylinder diameter and frequency response in respect to the natural frequency of the system. From the experimental data, the vibration of a freely moving cylinder can be distinguished into three regions as follows:

1) **At shedding frequency less than natural frequency,** \( V_N < 5 \)
   Vortex induced vibration does not exist in this region. Vortex shedding frequency is exactly the same as the frequency of a stationary cylinder. The amplitude of vibration is extremely low.

2) **At shedding frequency equal to natural frequency,** \( V_N = 5 \)
   Vortex induced vibration occurs at this stage. As shedding frequency and natural frequency start synchronizing, the shedding frequency no longer follows Strouhal's formula. This phenomenon is called "lock-in" or "synchronization".

3) **At shedding frequency larger than natural frequency,** \( 5 < V_N < 14 \)
   After entering synchronization, firstly, the frequency response of a cylinder is slowly increasing, but the values are still around the natural frequency of a system at the "upper branch". Continuing to reduce velocity until reaching lower branch, the response frequency becomes constant at a certain value. The lock-in vortices are synchronizing with the vibration of the structure and itself along the span of the cylinder. Again, shedding frequency no longer follows Strouhal’s formula.

4) **At shedding frequency larger than Natural Frequency,** \( V_N > 14 \)
   Once the reduced velocity is increased beyond the lock-in region, the vortex shedding again follows Strouhal’s law.

Another important observation from the experiment regards the drag and lift force coefficient on a freely oscillating cylinder. The value of the drag and lift force coefficient does not match with the value obtained from a stationary rigid cylinder as described in section 2.2. When vortex-induced vibration takes places, the motion of the cylinder itself influences both coefficients. In an experiment from Khalak and William, the result was that a maximum lift coefficient on a freely oscillating cylinder is larger than a stationary case by around six times. As a result, it is necessary to use another force model when examining the vibration of a freely oscillating cylinder. One of the most effective models is a vortex-induced vibration model with a wake oscillator. This will be explained later in this chapter.
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2.4 Wake oscillator and couple force

A vortex-induced vibration phenomenon occurs when the shedding frequency enters synchronization with the natural frequency of a cylinder. This synchronization means that there is an interaction between structure motion and wake. Both factors directly influence vibration response. As a consequence, a vortex induced vibration model requires two equation of motion to describe the interaction phenomenon. One is an equation of motion for the structure of the system and the other is an equation of motion representing wake motion. Detail of a wake oscillator including a couple force (wake-cylinder system) will be given in this section.

A wake oscillator model is established by considering two essential characteristics of wake motion during a VIV process. A VIV process begins when fluid flows past a cylinder. The cylinder starts vibrating and its motion disturbs the flow field or wake behind. Modification of wake motion creates a change in force that causes vibration, which result in a change in the amplitude of vibration. This couple will repeat itself. The amplitude of vibration of a cylinder and wake grow together. This first characteristic is called, "self-excitation". The couple effect continues until reaching a steady-state or where there is no longer a change in the amplitude of the vibration. At a steady state of vibration there is a second characteristic called "self-limitation". According to these two vital characteristics of VIV, various wake oscillator models has been established. One well-known model is the Van der Pol oscillator. The equation of motion is expressed as follows:

\[ \ddot{q}(t) + \varepsilon \omega_s (q^2(t) - 1) \dot{q}(t) + \omega_s^2 q(t) = f(t) \]  

Where, \( q \) is wake or vortex lift force coefficient, \( \varepsilon \) is tuning parameter, \( f(t) \) is force induce wake motion or couple with cylinder motion and \( \omega_s \) is shedding frequency.

Self-excitation and self-limitation are governed by the second term in the equation of motion. This nonlinear term represents a damping of the system. To better understand this function, it can be distinguished by two parts: 1) positive damping, \( \varepsilon \omega_s q^2(t) \dot{q}(t) \) and 2) negative damping, \( -\varepsilon \omega_s \dot{q}(t) \). For positive damping, the amplitude of vibration will decrease with dissipating energy, which then results in self limitation. In negative damping an opposite phenomenon occurs where the amplitude of vibration is increasing. During a VIV process, wake motion starts from a small value, then the damping term result is negative. Wake motion grows until the motion is larger than one. The damping term then becomes a positive value. Then the amplitude of vibration is controlled until it reaches a steady state. The Van der Pol oscillator limits wake amplitude at 2 as shown in Figure 2.10. Another important feature that needs comment is the tuning parameter. This value corresponds to system behavior in a transient period. A large value in the tuning parameter makes a system reach a steady state faster.

![Wake motion following a Van der Pol oscillator model at same initial velocity 5m/s with different tuning parameters](image)

Figure 2.10: Wake motion following a Van der Pol oscillator model at same initial velocity 5m/s with different tuning parameters.
Wake motion during a VIV process has been described by the Van der Pol oscillator model. The next step is to couple the wake system and structural system, which here considers a freely moving cylinder. Firstly, the cylinder system is excited by wake motion. In lock-in region, VIV take place. The cylinder then experiences a fluctuating lift force. Wake motion can be used as an intermediate parameter to model the fluctuation lift coefficient as in the following expression:

\[ C_{VL} = \frac{C_{L0}}{2} \cdot q(t) \]  

(2.6)

Where, \( C_{VL} \) indicates fluctuation lift coefficient, \( C_{L0} \) is stationary lift coefficient and \( q(t) \) represent wake motion. However, there is one important comment regarding a freely moving cylinder. The cylinder will experience a relative velocity as a result of flow velocity and its motion. Figure 2.11 shows the cross-section of a cylinder in Cartesian coordinates and relative flow velocity, \( U \). As a result, the lift force and drag force cannot directly represent force on the X and Y axis. The vortex force is a resultant force due to pressure distribution on a cylinder surface as show in Figure 2.5. A special feature of VIV is that pressure distribution changes periodically. As a consequence, the resultant force changes in both magnitude and direction. This force can decompose into components by two methods, such as the lift and drag component and inline and crossflow component. Figure 2.12 shows fluid force in terms of lift and drag force with respect to relative velocity \( (F_{VL}, F_{VD}) \) and an inline and crossflow force component \( (F_{VX}, F_{VY}) \).

![Figure 2.11: Relative flow velocity diagram](image1)

![Figure 2.12: Fluid forces with relative velocity](image2)

Evaluating fluid force by a drag and lift force component is more practical and easier. This is because of the advantage of a mean drag and lift coefficient. However, the motion inline and crossflow are the main focus of this research. In order to take advantage of the drag and lift force coefficient, we need to find the expression of an inline and crossflow component in terms of a drag and lift force component. This can be achieved by a balancing force as shown in Figure 2.12. The expression of an inline and crossflow force \( (F_{VX}, F_{VY}) \) as function of lift and drag force \( (F_{VL}, F_{VD}) \) are written as follows:

\[ F_{VY} = F_{VD} \sin \beta + F_{VL} \cos \beta \]  

(2.7)

\[ F_{VX} = F_{VD} \cos \beta + F_{VL} \sin \beta \]  

(2.8)

Where, \( \beta \) is angle of the relative fluid velocity respect to axis X, \( \sin \beta = \frac{V_{Y}}{U} \) and \( \cos \beta = \frac{V_{X}}{U} \). The inline and crossflow coefficient can be expressed as a function of the drag and lift coefficient. Note: During VIV process, the lift coefficient is fluctuating which is related to wake motion as shown in equations 2.9 and 2.10:
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\[ C_{VV} = (C_{VD} \sin \beta + C_{VL} \cos \beta) \frac{u_v^2}{v^2} = \left( C_{VD} \sin \beta + \left( \frac{C_{Lo}}{2} \cdot q(t) \right) \cos \beta \right) \frac{u_v^2}{v^2} \]  \hfill (2.9)

\[ C_{VX} = (C_{VD} \cos \beta + C_{VL} \sin \beta) \frac{u_v^2}{v^2} = \left( C_{VD} \cos \beta + \left( \frac{C_{Lo}}{2} \cdot q(t) \right) \sin \beta \right) \frac{u_v^2}{v^2} \]  \hfill (2.10)

Similarly, inline and crossflow force can be evaluated by using the above coefficients. The expression is written as follows:

\[ F_{VV} = \frac{1}{2} \rho D_0 L V^2 C_{VV} \]  \hfill (2.11)

\[ F_{VX} = \frac{1}{2} \rho D_0 L V^2 C_{VX} \]  \hfill (2.12)

A cylinder system excited by wake motion is introduced by an inline and crossflow force. Secondly, wake motion excited by cylinder motion is considered. Recall the equation of motion of the wake oscillator, equation 2.5. Forcing term \( f(t) \) can be introduced by using cylinder motion. Both inline and crossflow motion of the cylinder influence wake motion. There are several ways to couple the systems. Here in this research, an acceleration couple is used to capture the lock-in region. The equation of motion of a wake oscillator can be expressed as follows:

\[ \ddot{q}(t) + \varepsilon \omega_s (q^2(t) - 1) \dot{q}(t) + \omega_s^2 q(t) = \frac{A}{OD} \left( \frac{\partial^2 Y}{\partial t^2} \cos \alpha - \frac{\partial^2 X}{\partial t^2} \sin \alpha \right) \]  \hfill (2.13)

Where, \( q(t) \) indicates wake motion, \( A \) is coupling coefficient, \( OD \) is diameter of cylinder, \( \alpha \) is angle of fluid flow velocity to x axis and \( \frac{\partial^2 Y}{\partial t^2} \) and \( \frac{\partial^2 X}{\partial t^2} \) are crossflow and inline acceleration of a cylinder.

In conclusion, a vortex-induced vibration model can be established by using this couple system. One reflects the cylinder system experiencing a fluid force as induced by wake motion. The other is a wake system where its motion is induced by the couple effect of cylinder motion.
2.5 Seabed effect on vortex shedding

In the previous section, vortex-induced vibration theory and model studies on a wall-free cylinder or its location was far from a boundary. It results with no effect of the boundary on vortex shedding. However, the boundary effect needs to be considered, especially for vortex-induced vibration of any system located near a seabed, for example with unburied marine pipeline systems. What are criteria can be used to distinguish between a wall-free system and wall-influences on vortex shedding?

There are many studies about the response of a cylinder under a steady disturbed flow in the vicinity of a boundary. The results of studies on cylinder response came to the same conclusions, and indicate that the response strongly depends on two factors: 1) Reynolds number, Re and 2) the ratio of the gap between a cylinder and wall to cylinder diameter, e/D. Figure 2.13 shows instantaneous non-dimensional vorticity contour plots for a constant Reynolds number at 3.6 X 10^6 with varying ratio gap and diameter, e/D at 0.1, 0.15, 0.3 and 1 near a boundary.

(a) e/D = 0.10: Vortex-shedding suppressed

(b) e/D = 0.15: Vortex shedding form and start the interaction with flat wall

(c) e/D = 0.30: Vortex shedding form and interaction with flat wall

(d) e/D = 1.00: Vortex shedding developed and interacting less with flat wall

Figure 2.13: The development of vortex shedding shown by instantaneous non-dimensional vorticity contour plots for constant Re and varying gap to diameter ratio (e/D)

In Figure 2.13, positive and negative vorticity are represented by the solid contour lines and dashed lines, respectively. There are three shear layers that appear in this phenomenon: two in the vicinity of a cylinder
and one at the flat wall. The interaction between these three shear layers strongly influence vortex shedding suppression and formation in the wake of a cylinder, which is strongly dependent on the gap between a wall and cylinder. The interaction of shear layers for each e/D values are summarized as follows:

Figure 2.13a, At e/D = 0.1, there is no mutual interaction between two shear layers at the cylinder. Both shear layers grow continuously and advect downstream without forming any vortices in the near wake of cylinder.

Figure 2.13b, At e/D = 0.15, two shear layers have started interacting each other. The vortices at the near wake of a cylinder are forming. However, it appears as an interaction between a shear layer at the lower part of the cylinder and shear layer at the flat wall. The vortex in the near-flat-wall region occurs as consequence in the lower part of a shear layer that destabilizes the wall shear layer.

Figure 2.13c, At e/D = 0.3, the upper shear layer of a cylinder starts interacting with the wall shear layer. As results, the group of vortices interact and form a lager vortex behind the cylinder.

Figure 2.13d, At e/D = 1.0, the vortex shedding are forming strongly by two shear layers from the cylinder with no longer interaction with the wall shear layer. At this region, it is safe to conclude that the phenomenon is a vortex-induced vibration of wall-free cylinder system.

In conclusion, vortex shedding is suppressed when e/D is smaller than a critical gap. Beyond the critical gap, vortex shedding begins to develop and becomes fully developed at e/D equal 1 as the influence of the wall vanishes.

Returning back to a consideration of a subsea jumper system as the main focus of this research, it is necessary to know whether the seabed has an influence on the jumper system. If so, then a minimum gap ratio for the jumper system shall be calculated. There are various diameter sizes for existing jumpers that vary between 4 to 18 inches. The distance from seabed to a subsea jumper can be approximated by the height of a connector at the manifold or subsea tree. This is because the main function of a subsea jumper is to connect between a subsea manifold and subsea Xmas-tree. The height of a connector typically varies between 4 to 10 meters. Thus, a minimum e/D of a subsea jumper equal $e/D = \frac{4 \text{ meter}}{18 \text{ inches}} = 8.75$. We can safely conclude that a seabed effect does not influence shedding of a subsea system.
2.6 Earthquake effect

An earthquake occurs as a result of movement of the earth crust or tectonic plate due to heat convection flow underneath. The majority of earthquakes or seismic activities can be described by the relative motion of tectonic plates. There are three common types of relative motion: 1) diverge boundary, two tectonic plates move apart from each other; 2) converge boundary or subduction, one plate crust moves downward into and underneath another crust; and 3) transform boundary, one tectonic plate moves horizontally pass other. All basic mechanisms of seismic activities are shown in Figure 2.14.

![Figure 2.14: Basic earthquake mechanisms A) diverge boundary, B) converge boundary and C) transform boundary](image)

During the process of tectonic plate movement, at a certain level, there is an accumulation of elastic energy at the edge of a plate that may release at the interaction point of two tectonic plates in a seismic propagation wave form. There are three types of seismic waves: 1) pressure-wave, 2) shear wave and 3) surface wave. A pressure wave causes motion inline of wave direction, whereas a shear wave causes motion in a transverse direction. Both seismic waves flow away though the earth and create a ground vibration and mainly damage buildings on ground. The point of release of the energy is called the “hypocentre”. The projection of this point to the earth surface is called the “epicentre”. Figure 2.15 shows a schematic diagram of a hypocenter and epicenter point.

![Figure 2.15: Hyocentre and Epicentre](image)
There are several methods to measure the magnitude of an earthquake. As such, there are several scales to describe the severity of an earthquake. One common unit is “Richter”. Richter represents energy release after an earthquake process. The maximum recorded value of Richter is 9. Table 2.2 shows the relation of Richter scale, earthquake effect and frequency of occurrence.

<table>
<thead>
<tr>
<th>Richter Magnitude</th>
<th>Description</th>
<th>Earthquake Effect</th>
<th>Frequency of Occurrence</th>
</tr>
</thead>
<tbody>
<tr>
<td>Less than 2.0</td>
<td>Micro</td>
<td>Micro-earthquakes, not felt</td>
<td>Millions per year</td>
</tr>
<tr>
<td>2.0 – 2.9</td>
<td>Minor</td>
<td>Generally, not felt, but recorded</td>
<td>Over 1 million year</td>
</tr>
<tr>
<td>3.0 – 3.9</td>
<td>Minor</td>
<td>Some people feel, indoor objects can shake</td>
<td>Over 100,000 per year</td>
</tr>
<tr>
<td>4.0 – 4.9</td>
<td>Light</td>
<td>Most people feel, indoor objects shake and fall to the ground</td>
<td>10,000 to 15,000 per year</td>
</tr>
<tr>
<td>5.0 – 5.9</td>
<td>Moderate</td>
<td>Damage to building which is not designed for earthquake</td>
<td>1,000 to 1,500 per year</td>
</tr>
<tr>
<td>6.0 – 6.9</td>
<td>Strong</td>
<td>Wide spread shaking far from epicenter, damage to buildings</td>
<td>100 to 150 per year</td>
</tr>
<tr>
<td>7.0 – 7.9</td>
<td>Major</td>
<td>Wide spread shaking in most areas</td>
<td>10 to 20 per year</td>
</tr>
<tr>
<td>8.0 – 8.9</td>
<td>Great</td>
<td>Wide spread damage in large area</td>
<td>About 1 per year</td>
</tr>
<tr>
<td>9.0 – 9.9</td>
<td>Great</td>
<td>Severe damage to most buildings</td>
<td>1 per 5-50 years</td>
</tr>
<tr>
<td>10.0 and over</td>
<td>Massive</td>
<td>Never recorded</td>
<td>Never recorded</td>
</tr>
</tbody>
</table>

Table 2.2: Relationship between Richter scale, earthquake effect and frequency of occurrence

Although the Richter scale is useful for reporting earthquake magnitude, it is also necessary to describe the intensity of an earthquake, which can be done by using the Mercalli scale. It indicates intensity based on structure damage. Ratings of the Mercalli scale vary from 1 to 7. Each value describes how much damage occurs to a building, however, this scale is not precise.

Both Richter and Mercalli are useful for risk analysis. In structure dynamic design, information on the duration and amplitude (maximum value) of the horizontal ground acceleration is required. The information of acceleration of ground motion can be measured by a special instrument called an “accelerogram”.

There is a simple model called, the Attenuation Law, which presents the relationship between the amplitude of horizontal ground motion and magnitude of an earthquake. The Attenuation Law can be written as follows:

$$\ddot{a}_g = A_g e^{0.8M_g(R + R_0)^{-2}}$$  \hspace{1cm} (2.14)

Where, $\ddot{a}_g$ is amplitude of ground acceleration in m/s^2 (sometime acceleration indicate in faction of gravity acceleration, g), $M_g$ is magnitude of earthquake according to Richter unit, R is distance to epicenter and constant $A_g$ and $R_0$ are 56 X 10^6 m/s^2 and 40 m, respectively.

Unlike wave and wind loads, earthquake frequency is relatively high and disordered. Dominant frequencies are 1 – 10 Hz. The duration of an earthquake is usually 5 – 40 seconds.
2.7 Dynamic model of structure under earthquake

General information regarding earthquakes was discussed in the previous section. For vibration analysis of a structure under seismic motion, it is necessary to develop a dynamic model of the structure. A dynamic model is introduced in this section.

A dynamic model of a structure under earthquake conditions can be developed easily by introducing an equation of motion for the system. In order to gain a better understanding of the main idea of a dynamic model, a simple single degree of freedom example is considered here. An example of a single degree of freedom system under seismic motion is illustrated in Figure 2.16. The system is comprised of a lumped mass structure m, weightless spring of stiffness K/2 in longitude, x direction while energy-dissipate mechanism present by damping C.

![Figure 2.16: Model of single degree of freedom structure under seismic conditions](image)

In Figure 2.16, there are three motions in the system: ground movement in x direction indicate by \(x_g(t)\); structure relative displacement to ground indicate by \(x(t)\); and total displacement \(x_{\text{total}}(t)\). The equation of motion of the system can be written as follows:

\[
mx_{\text{total}} + c\dot{x} + kx = 0 \quad (2.15)
\]

\[
m(\ddot{x} + \ddot{x}_g) + c\dot{x} + kx = 0 \quad (2.16)
\]

\[
m\ddot{x} + c\dot{x} + kx = -m\ddot{x}_g \quad (2.17)
\]

One can observe that the external-loading mechanism exerted on the structure is depending on \(\ddot{x}_g(t)\) or inertia force due to ground acceleration.
2.8 Fatigue analysis

Fatigue mechanisms have been studied since the 19th century. The first investigation in a laboratory was done by August Wohler. He found that a system can be damaged by a periodic single load when its amplitude is much more below the static strength of a system. This damage was later well-known as fatigue failure.

A general definition for fatigue is, "The process of progressive localized permanent structural change occurring in a material subjected to conditions which produce fluctuating stresses and strains at some point or points and which may culminate in cracks or complete fracture after a sufficient number of fluctuations."

Before going into a detailed discussion of fatigue analysis, several fatigue properties of a material should be introduced in terms of a fatigue curve (S-N curve) and fatigue limit. Fatigue properties are obtained by testing a specimen. Figure 2.17 shows an example of a S-N curve.

![S-N curve](image)

Figure 2.17: Fatigue test results of unnotched specimens of low-alloy steel (SAE)

An S-N curve is a result of a number of fatigue tests at different constant amplitude stress levels. The curve shows the relationship between stress-level, S and number of fatigue life, N in a log scale. The curve reflects, "Given a load to a material at a certain stress level, the material has a fatigue resistant in N number of load cycles. In other words, fatigue failure will occur if the load applied to the material reaches a certain number of N cycles". There are three main observations regarding a S-N curve.
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1. Upper horizontal asymptote line

When the stress amplitude reaches a tensile strength level, the material will fail at the first cycle. However, when the stress amplitude in a material is slightly smaller, the material can resist a few more cycles due to strain hardening. This phenomenon is shown as a result in an Upper horizontal asymptote line.

2. Middle curve (substantial range of S-N curve)

The advantage of log-scale plotting is that it allows us to obtain a linear relation between stress level, S and fatigue life, and N of a substantial part. The mathematically linear relation can be written as follows:

\[ S^k \cdot N = a \]  \hspace{1cm} (2.18)

Where, \( k \) and \( a \) are specific parameters which depend strongly on material and welding detail. This equation is also called the Basquin Relation. It will be used extensively for fatigue analysis in this research.

3. Lower horizontal asymptote line

Fatigue failure process begins with occurrence of a small crack at a micro-scale. If the load still continues to exert on a material, a crack will grow until the material breaks or there is fatigue failure. However, each material has a particular threshold for crack growth. If the stress level does not reach this threshold, the crack will not grow further, and so-called non-propagating failure and fatigue will not occur. The material threshold is called a fatigue limit as shown in the lower horizontal line. In a mathematic expression below the fatigue limit, the material has a fatigue life equal to infinity, \( N \rightarrow \infty \).

One of the main objectives of this research is to predict the fatigue life of a subsea jumper under external disturbance by steady current and earthquake. Fatigue lifetime can be simply calculated by the Miner Rule or thru linear cumulative damage. The main idea of Miner’s rule is to consider fatigue that arises due to cyclic load. Referring to the S-N curve, we found that at each certain stress level, a material has a capacity to resist fatigue failure equal to \( N \) cycle. When the applied load to a material has cycles less than \( N \) cycles, given \( n \) cycle, then the material accumulates damage or fatigue consuming equal to \( n/N \). Accordance to the Miner rule, a system under variable load amplitude will be damaged by fatigue at the moment that a summation of all fatigue life consumed equals to one or is express as follows:

\[ \sum \frac{n_i}{N_i} = 1 \]  \hspace{1cm} (2.19)

Where, \( n_i \) indicates number of cycle of stress level \( i \) and \( N_i \) is material fatigue lifetime accordance with stress level \( i \). The fatigue life consumption also called fatigue damage, \( D \).

Typically, fatigue analysis will be considering at certain design periods or design cycles. In order to predict total fatigue lifetime of any system, simply express as follows:-

\[ \text{Fatigue life time} = \left( \frac{1}{\sum \frac{n_i}{N_i}} \right) \cdot \text{Design period} = \left( \frac{1}{D} \right) \cdot \text{Design period} \]  \hspace{1cm} (2.19)
In order to predict fatigue lifetime according Miner’s Rule, it is necessary to calculate fatigue life consumption or fatigue damage. Fatigue damage consists of two components. The first component is material fatigue life capacity, which depends on material properties, and can be obtained from a S-N curve. Another component is the number of stress cycles at each certain level. The value of each stress cycle can be obtained by considering the stress-time history of a subsea jumper under external loads by using a cycling counting method.
2.9 Stress-time history

The fatigue problem deals with the level of stress in a material. However, dynamic behavior analysis of an equation of motion results in displacement over time of any arbitrary points in a system or displacement-time history. Conversion from displacement-time history to stress-time history is required as input information for fatigue analysis. Once stress-time history is obtained, the stress-counting method can be used to predict fatigue life. This section discusses the relationship of transverse displacement of a beam and its relative bending stress.

In the first consideration, the bending stress at any point on a beam along an x-axis can be calculated from local bending moment by the following expression:

\[ \sigma(x) = \frac{M(x)\gamma}{I_x} \]  

(2.20)

Where, \( \sigma \) indicate bending stress at any arbitrary point on a beam. \( M \) is bending moment, \( I \) is moment of inertia around neutral axis-X and \( y \) is distance to neutral axis of cross-section point. In case of piping, the maximum stress locates at the outer diameter or \( y \) equal outer to the pipe radius. The fatigue process occurs under a tensile strength level. It is reasonable to assume all processes are under an elastic deformation region. As a result, we found the relationship of a bending moment and transverse displacement of an Euler-Bernoulli beam as follow:-

\[ M(x, t) = EI \frac{\partial^2 w(x,t)}{\partial x^2} \]  

(2.21)

Where, \( EI \) is bending stiffness of a beam and \( w \) is a transverse motion of a beam.

Accordingly, the transverse motion at any point of a subsea jumper can be calculated from dynamic behavior analysis. Then, bending stress-time history can be derived by equation 2.20 and 2.21.
Part I

Dynamic behavior of subsea jumper under external disturbance

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Chapter 3

Subsea Jumper Model

Before analyzing the vibration problem in a subsea jumper, the characteristics of a subsea jumper itself needs to be understood. Typically, subsea jumper systems have at least two bends in a connection, which make their behavior different from a straight pipeline. This research focuses only on subsea jumpers of the U-inverse shape for a long route interfacing between subsea infrastructures. This shape comprises of three beams, two vertical lines and one long horizontal line. Although this configuration seems rather more complicated than a straight pipeline, a simple beam model can still be useful for modeling subsea jumper as the fundamental elements are similar and allow description of its dynamic behavior.

Subsea jumpers in operating conditions are also considered in this research. Thus, a simple Euler-Bernoulli Beam is insufficient to describe jumper behavior because it does not take internal fluid forces into account. As a result, this chapter begins with a model of an operating pipeline, which can also be called a “pipe conveying fluid model”. This model will be used extensively throughout this research.

Later on, a model of a subsea jumper is introduced, namely a “subsea jumper model”. It is established by using three pipe conveying fluid models connected together with 90-degree bends. Here an interface condition is required at the connection. The advantage of this model is that it provides accurate result of jumper motion because all lines in jumper system are influenced. It fully describes the motion of every section in a jumper system. In addition, the design of an end-connection of a subsea jumper is considered as a vertical tie-in or vertical clamp. Thus, the fixed-ends connection models a boundary condition. It is necessary to point out that two directions of motion are considered: crossflow direction and inline direction.

Based on the introduction of a subsea jumper model, subsea jumper characteristics can be defined by: 1) natural frequencies and 2) mode shape of vibration. This result is used for analyzing the vibration problem in the next chapter.
Chapter 3: Subsea Jumper model

3.1 Pipe conveying fluid model

The Euler Bernoulli beam model does well in describing the transverse motion of a pipeline. However, it is not sufficient to analyze an operating pipeline, which is the main focus of this thesis. While a pipeline is operating, internal fluid flows along the pipeline create internal loads in a transverse direction that act on the pipe wall. These loads influence the transverse motion of a pipeline. The internal loads become more important if a pipe contains a high-flow-rate fluid. As an operating subsea jumper is the main focused of this research, it is necessary to take internal fluid effect into consideration in the dynamic model.

There is considerable research looking into pipe conveying fluid models, and one of the most effective and simplest models was written by [Paidoussis 1998]. He modeled an internal flow by using a moving infinity long rod, while the pipeline uses a Euler Bernoulli beam model. It does well in describing the effect of internal fluid forces, especially for a uniform flow. Figure 3.1 shows a schematic diagram of an operating pipe and pipe conveying fluid model.

![Figure 3.1: Schematic diagram: left is pipeline containing uniform flow and right is a pipe conveying fluid model](image)

The internal fluid force in the transverse direction comprises of three elements: 1) centrifugal force, 2) coriolis force and 3) inertia force. The expressions of these three elements are shown in equation 3.1 to 3.3, respectively.

\[
F_{\text{centrifugat}} = M_f U_f^2 \frac{\partial^2 w}{\partial x^2} \quad (3.1)
\]

\[
F_{\text{coriolis}} = 2M_f U_f \frac{\partial^2 w}{\partial t \partial x} \quad (3.2)
\]

\[
F_{\text{inertia}} = M_f \frac{\partial^2 w}{\partial t^2} \quad (3.3)
\]

Where, \( M_f \) is mass per unit length of conveying fluid, \( U_f \) is internal fluid velocity in steady stage, \( w \) is transverse direction of piping along z-axis vary to time, \( t \). Establishing a pipe conveying fluid model can be simply achieved by adding three internal fluid forces into an Euler Bernoulli Beam model. This is expressed in equation 3.4:

\[
m \frac{\partial^2 w}{\partial t^2} + EI \frac{\partial^4 w}{\partial x^4} + M_f \left( U_f^2 \frac{\partial^2 w}{\partial x^2} + 2U_f \frac{\partial^2 w}{\partial t \partial x} + \frac{\partial^2 w}{\partial t^2} \right) = 0 \quad (3.4)
\]

Where, \( m \) is mass per unit length of pipe (or subsea jumper) and \( EI \) is bending stiffness of the piping.
3.2 Subsea jumper model (triple-beam model)

The previous section discussed a pipe conveying fluid model. However, it is insufficient for describing jumper motion due to the more complicated configuration of a jumper system. As a result, a subsea jumper model needs to further use three single beam models for two vertical lines and one horizontal line in a subsea system that are connected together at a rigid joint. In addition, the jumper system is connected to ground by a fixed-fixed connection. A schematic diagram of a triple-beam model is shown in Figure 3.2.

Figure 3.2: Schematic diagram of a triple-beam model

Figure 3.2 shows the transverse motion in two vibration planes as w represents transverse motion on a crossflow plane and v represents transverse motion on an inline plane. Considering the motion of two vibration planes, two sets of equation of motion need to be separately established. In this section, this begins with a crossflow vibration plane, then an inline vibration plane.

**Crossflow vibration:** In order to describe the crossflow transverse motion of a jumper in a u-inverse shape, the longitude motion of each beam needs to fulfill the kinetic and kinematic relation at a rigid joint, which is a connection point between a horizontal beam and vertical beam. Figure 3.3 shows a schematic diagram of the relationship between transverse and longitude motions for crossflow vibration.

Figure 3.3: Kinematic relation of crossflow vibration at a rigid joint
Chapter 3: Subsea Jumper model

Before going further into detail about the equation of motion for crossflow vibration, the general expression for the longitude motion of a beam along x-axis is introduced. It is written as follows:

\[ m \frac{\partial^2 u}{\partial t^2} + EA \frac{\partial^2 u}{\partial x^2} = 0 \]  

(3.5)

Where, \( m \) is mass per unit length of pipe, \( E \) is modulus elastic of pipeline, \( A \) is cross-section area of piping. One should be noted that equation 3.5 is a general form of longitude motion of a beam. It is expressing a longitude motion, \( u \) along general x-axis while there are two axis considered here, \( x_1 \) and \( x_3 \) for a vertical lines whereas \( x_2 \) for a horizontal line.

It is obvious that the equation of motion for crossflow vibration of a subsea jumper shall be comprised of two motions: 1) transverse motion and 2) longitude motion. Then, a set of equation of motions for crossflow vibration can be established as follows:

**Transverse motion (crossflow):**

- **Vertical left beam**
  \[ m \frac{\partial^2 w_1}{\partial t^2} + EI \frac{\partial^4 w_1}{\partial x_1^4} + M_f \left( U_f \frac{\partial^2 w_1}{\partial x_1^2} + 2 U_f \frac{\partial^2 w_1}{\partial t \partial x_1} + \frac{\partial^2 w_1}{\partial t^2} \right) = 0 \]  
  (3.6)

- **Horizontal beam**
  \[ m \frac{\partial^2 w_2}{\partial t^2} + EI \frac{\partial^4 w_2}{\partial x_2^4} + M_f \left( U_f \frac{\partial^2 w_2}{\partial x_2^2} + 2 U_f \frac{\partial^2 w_2}{\partial t \partial x_2} + \frac{\partial^2 w_2}{\partial t^2} \right) = 0 \]  
  (3.7)

- **Vertical right beam**
  \[ m \frac{\partial^2 w_3}{\partial t^2} + EI \frac{\partial^4 w_3}{\partial x_3^4} + M_f \left( U_f \frac{\partial^2 w_3}{\partial x_3^2} + 2 U_f \frac{\partial^2 w_3}{\partial t \partial x_3} + \frac{\partial^2 w_3}{\partial t^2} \right) = 0 \]  
  (3.8)

**Longitude motion:**

- **Vertical left beam**
  \[ m \frac{\partial^2 u_1}{\partial t^2} + EA \frac{\partial^2 u_1}{\partial x_1^2} = 0 \]  
  (3.9)

- **Horizontal beam**
  \[ m \frac{\partial^2 u_2}{\partial t^2} + EA \frac{\partial^2 u_2}{\partial x_2^2} = 0 \]  
  (3.10)

- **Vertical right beam**
  \[ m \frac{\partial^2 u_3}{\partial t^2} + EA \frac{\partial^2 u_3}{\partial x_3^2} = 0 \]  
  (3.11)

The set of equation of motion is valid for any arbitrary points on a jumper; however, not for a rigid joint and end connections. Then, the only remaining task is to complete a mathematic expression of the triple-beam model for crossflow vibration, by analyzing interface condition and boundary condition. The expression of interface conditions of crossflow vibration for a rigid connection is written as follows:

**Displacement balance**

- **Left connection**
  \[ w_1 = u_2 \quad \text{and} \quad u_1 = -w_2 \]  
  (3.12)

- **Right connection**
  \[ w_3 = -u_2 \quad \text{and} \quad u_3 = w_2 \]  
  (3.13)
Angular balance

Left connection
\[ \frac{\partial w_2}{\partial x_2} = \frac{\partial w_1}{\partial x_1} \]  \hspace{1cm} (3.14)

Right connection
\[ \frac{\partial w_2}{\partial x_2} = \frac{\partial w_3}{\partial x_3} \]  \hspace{1cm} (3.15)

Moment balance

Left connection
\[ \frac{\partial^2 w_2}{\partial x_2^2} = \frac{\partial^2 w_1}{\partial x_1^2} \]  \hspace{1cm} (3.16)

Right connection
\[ \frac{\partial^2 w_2}{\partial x_2^2} = \frac{\partial^2 w_3}{\partial x_3^2} \]  \hspace{1cm} (3.17)

Force balance (disregarding the change of momentum of the conveying fluid)

Left connection
\[ EI \frac{\partial^3 w_2}{\partial x_2^3} = -EA \frac{\partial u_1}{\partial x_1} \quad \text{and} \quad EI \frac{\partial^3 w_1}{\partial x_1^3} = EA \frac{\partial u_2}{\partial x_2} \]  \hspace{1cm} (3.18)

Right connection
\[ EI \frac{\partial^3 w_2}{\partial x_2^3} = EA \frac{\partial u_3}{\partial x_3} \quad \text{and} \quad EI \frac{\partial^3 w_3}{\partial x_3^3} = -EA \frac{\partial u_2}{\partial x_2} \]  \hspace{1cm} (3.19)

The expression of interface condition for crossflow direction gives a relationship between longitude motion and transverse motion on crossflow. In order to satisfy this condition, it is necessary to assume that longitude motion is only related to crossflow motion, but it does not affect inline vibration.

Finally, the boundary condition that is considered as fixed-fixed ends can be expressed as follows:

At end connection of vertical beam on left side is
\[ w_1 = 0 \quad \text{and} \quad u_1 = 0 \quad \text{and} \quad \frac{\partial w_1}{\partial x_1} = 0 \]  \hspace{1cm} (3.20)

At end connection of vertical beam on right side is
\[ w_3 = 0 \quad \text{and} \quad u_3 = 0 \quad \text{and} \quad \frac{\partial w_3}{\partial x_3} = 0 \]  \hspace{1cm} (3.21)
**Chapter 3: Subsea Jumper model**

**Inline vibration:** transverse motion on each beam is sufficient to describe the interface condition at rigid joints. However, it is rather more complicated than a crossflow vibration case due to angular motion in the beam. Figure 3.4 shows the kinetic and kinematic relationship between inline motions of connected beams.

![Diagram showing Inline vibration and its components](image)

**A.** Inline Transvers motion of subsea jumper on each beam

**B.** Kinetic and kinematic relation of horizontal beam (right)

**C.** Kinetic and kinematic relation of vertical beam (right)

**D.** Kinetic and kinematic relation of horizontal beam (left)

**E.** Kinetic and kinematic relation of vertical beam (left)

*Figure 3.4: Kinetic and kinematic relation of inline vibration at a rigid joint*
Chapter 3: Subsea Jumper model

According to Figure 3.4, it is obvious that the equation of motion for inline vibration of a subsea jumper contains only a transverse motion. Thus, a set equation of motions for inline vibration can be expressed as follows:

\[
\begin{align*}
\text{Vertical left beam} & : \quad m \frac{d^2v_1}{dt^2} + EI \frac{d^4v_1}{dx_1^4} + M_f \left( U_f^2 \frac{d^2v_1}{dx_1^2} + 2U_f \frac{d^2v_1}{dx_1^2} + \frac{d^2v_1}{dx_1^2} \right) = 0 \\
\text{Horizontal beam} & : \quad m \frac{d^2v_2}{dt^2} + EI \frac{d^4v_2}{dx_2^4} + M_f \left( U_f^2 \frac{d^2v_2}{dx_2^2} + 2U_f \frac{d^2v_2}{dx_2^2} + \frac{d^2v_2}{dx_2^2} \right) = 0 \\
\text{Vertical right beam} & : \quad m \frac{d^2v_3}{dt^2} + EI \frac{d^4v_3}{dx_3^4} + M_f \left( U_f^2 \frac{d^2v_3}{dx_3^2} + 2U_f \frac{d^2v_3}{dx_3^2} + \frac{d^2v_3}{dx_3^2} \right) = 0
\end{align*}
\]

(3.22) 
(3.23) 
(3.24)

Similar to crossflow vibration, the above equations of motion are valid for any arbitrary points on a subsea jumper system; however, it is not applicable for both rigid joint and end connections. Thus, interface conditions and boundary conditions need to be fulfilled.

In order to obtain interface conditions at a rigid joint, kinetic (displacement and angular) and kinematic (moment and force) relations between the horizontal and vertical beam need to be known. Each relationship can be considered as in Figure 3.4. Firstly, the displacement relation at the interface condition is unsophisticated. Both the horizontal beam and vertical beam have the same transverse displacement at a rigid joint as shown in Figure 3.4A. Then,

\[
\text{Displacement balance} \quad v_1 = v_2 \quad \text{and} \quad v_2 = v_3
\]

(3.25)

The angular relation between the horizontal and vertical beam can see clearly in Figure 3.4B and 3.4C. The bending angle of the horizontal beam is related to the torsion angle of the vertical beam. It is the same as the vertical beam as the bending angle of the vertical beam is related to the torsion angle of the horizontal beam.

\[
\text{Angular balance: Left connection} \quad \frac{d}{dx_2} \quad \text{and} \quad \frac{d}{dx_2} \quad (3.26)
\]

\[
\text{Right connection} \quad \frac{d}{dx_3} \quad \text{and} \quad \frac{d}{dx_3} \quad (3.27)
\]

The interface condition for moment balance at a rigid joint for a horizontal beam is obtained by balancing torsion on the horizontal beam to bending moment at the end of the vertical beam as shown in Figure 3.4B. By definition, torsion at both ends shall be same, but in different sign. In addition, the resultant torsion on a horizontal beam shall be considered as the relative torsion angle between both ends. Thus, the interface condition of moment balance for a horizontal beam can be written as follows:

\[
\text{Moment balance} \quad EI \frac{d^2v_3}{dx_3^2} = - \frac{JG}{L_H} \left( v_1 \right)
\]

(3.28)

\[
\text{Moment balance} \quad EI \frac{d^2v_3}{dx_3^2} = - E \frac{d^2v_1}{dx_1^2}
\]

(3.29)

Where, I is area moment of inertia unit in \( m^4 \) \( (I = \frac{\pi}{64} (OD^4 - ID^4)) \), for hollow cylinder, E indicates Young’s modulus, J is polar moment of inertia of an area unit in \( m^4 \) \( (J = \frac{\pi}{32} (OD^4 - ID^4)) \), for hollow cylinder, G is shear modulus. It relates to Young’s modulus by using poison ratio \((v_3 + 1) \cdot 2G = E\)
In the case of a vertical beam, it is different because one end of the connection is fixed. As a consequence, the torsion angle at a rigid joint already represents the absolute torsion angle. Furthermore, the bending moment at the horizontal end is double the torsion on a vertical beam. Thus, the interface condition for a vertical beam can be expressed as follows:

**Moment balance**

Left connection

\[ EI \cdot \frac{\partial^2 v_2}{\partial x_2^2} = -\frac{JG}{L_V} \left( -\frac{\partial v_2}{\partial x_2} \right) \]  
(3.30)

Right connection

\[ EI \cdot \frac{\partial^2 v_2}{\partial x_2^2} = \frac{JG}{L_V} \left( -\frac{\partial v_2}{\partial x_2} \right) \]  
(3.31)

In force balance, one can see that shear force at the end of each beam are in balance. As a result, Equation 3.32 and Equation 3.33 indicate the interface condition of force balancing as follows:

**Force balance**

Left connection

\[ \frac{\partial^3 v_2}{\partial x_2^3} = \frac{\partial^3 v_1}{\partial x_1^3} \]  
(3.32)

Right connection

\[ \frac{\partial^3 v_2}{\partial x_2^3} = \frac{\partial^3 v_3}{\partial x_3^3} \]  
(3.33)

The last remaining task is to introduce the boundary condition as a fixed-fixed end. This can be written as follows:

At end connection of vertical beam on left side is

\[ v_1 = 0 \quad \text{and} \quad \frac{\partial v_1}{\partial x_1} = 0 \quad \text{and} \quad \theta_{\text{torsion},1} = 0 \]  
(3.34)

At end connection of vertical beam on right side is

\[ v_3 = 0 \quad \text{and} \quad \frac{\partial v_3}{\partial x_3} = 0 \quad \text{and} \quad \theta_{\text{torsion},3} = 0 \]  
(3.35)

In summary, a triple-beam model can be expressed by a set of equation of motions. Here, we considered transverse motion in two planes, one in a crossflow plane and one in an inline plane. Crossflow vibration for a triple-beam model requires longitude motion along a beam length to fulfill the interface condition. The model of crossflow vibration is comprised of: 1) equation of motion for any arbitrary point (equation 3.6 to 3.11), 2) interface condition (equation 3.12 to 3.19) and 3) boundary condition (equation 3.20 and 3.21)

In inline vibration, the model is established by transverse motion and torsion. However, only the equation of motion in terms of inline transverse is sufficient for the model. The triple-beam model for inline vibration is comprised of: 1) equation of motion for any arbitrary point (equation 3.22, 3.23 and 3.24), 2) interface condition (equation 3.25 to 3.33) and 3) boundary condition (equation 3.34 and 3.35).
3.3 Natural frequency and mode shape of subsea jumper

As mentioned earlier, only a subsea jumper system in a U-inverse shape with a long horizontal line is considered here. However, there are still many varieties for material properties and dimensions for a jumper system. These parameters need analysis in terms of jumper dynamics characteristics. In this section, first we present all design parameters for subsea jumper characteristic analysis. All design parameters for a subsea jumper system are summarized in Table 3.1.

<table>
<thead>
<tr>
<th>Design Parameter: Dimension and Material Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of horizontal beam ( (L_2) ) &amp; 50 m</td>
</tr>
<tr>
<td>Length of vertical beam ( (L_1 \text{ and } L_3) ) &amp; 10 m</td>
</tr>
<tr>
<td>Outer diameter ( (OD) ) &amp; 0.35 m (or 14 inches)</td>
</tr>
<tr>
<td>Wall thickness &amp; 0.015 m</td>
</tr>
<tr>
<td>Piping material &amp; Carbon Steel Grade X70</td>
</tr>
<tr>
<td>Young's Modulus ( (E) ) &amp; 210 GPa</td>
</tr>
<tr>
<td>Pipe density ( (\rho_{\text{steel}}) ) &amp; 7800 Kg/m(^3)</td>
</tr>
<tr>
<td>Poison Ratio ( (\nu_{\text{steel}}) ) &amp; 0.3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Design Parameter: Operation Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oil flow rate &amp; 3 MM barrel per day</td>
</tr>
<tr>
<td>Oil density ( (\rho_{\text{oil}}) ) &amp; 790 Kg/m(^3)</td>
</tr>
</tbody>
</table>

Table 3.1: Design parameters for subsea jumper system

Based on the design parameter in Table 3.1, the characteristics of a subsea jumper system can be analyzed. Full explanations and solutions are provided in Appendix A. Here, only the results of analysis are shown. The natural frequencies value for the first five modes for cross flow vibration and an example of mode shape for the first-three modes of a triple-beam model are shown in Table 3.2 and Figure 3.5, respectively.

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>1st ( \text{Natural frequency (rad/s)} )</th>
<th>2nd ( \text{Natural frequency (rad/s)} )</th>
<th>3rd ( \text{Natural frequency (rad/s)} )</th>
<th>4th ( \text{Natural frequency (rad/s)} )</th>
<th>5th ( \text{Natural frequency (rad/s)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2.3168</td>
<td>7.7500</td>
<td>16.3080</td>
<td>27.8288</td>
<td>42.0663</td>
</tr>
<tr>
<td></td>
<td>0.3687</td>
<td>1.2334</td>
<td>2.5955</td>
<td>4.4291</td>
<td>6.6951</td>
</tr>
</tbody>
</table>

Table 3.2: Natural frequencies for first-five modes of cross flow vibration of a subsea jumper model
Chapter 3: Subsea Jumper model

Figure 3.5: Example for first five mode shapes of crossflow vibration of a subsea jumper model
The natural frequencies value for the first five modes for *inline vibration* and example of mode shape for the first-three modes of a triple-beam model are shown in Table 3.3 and Figure 3.6, respectively.

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
<th>4th</th>
<th>5th</th>
</tr>
</thead>
<tbody>
<tr>
<td>Natural frequency (rad/s)</td>
<td>0.9695</td>
<td>4.1207</td>
<td>9.0072</td>
<td>15.5404</td>
<td>23.7233</td>
</tr>
<tr>
<td>Natural frequency (Hz)</td>
<td>0.1543</td>
<td>0.6558</td>
<td>1.4335</td>
<td>2.4733</td>
<td>3.7755</td>
</tr>
</tbody>
</table>

*Table 3.3: Natural frequencies for first-five modes of inline vibration of a subsea jumper model*

In conclusion, the natural frequencies value of subsea jumper systems including both inline and crossflow vibration are shown in Table 3.4.
### Table 3.4: Natural frequencies of subsea jumper

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>Natural Frequency (Hz)</th>
<th>Type of Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>0.1543</td>
<td>Inline</td>
</tr>
<tr>
<td>2nd</td>
<td>0.3687</td>
<td>Crossflow</td>
</tr>
<tr>
<td>3rd</td>
<td>0.6558</td>
<td>Inline</td>
</tr>
<tr>
<td>4th</td>
<td>1.2334</td>
<td>Crossflow</td>
</tr>
<tr>
<td>5th</td>
<td>1.4335</td>
<td>Inline</td>
</tr>
<tr>
<td>6th</td>
<td>2.4733</td>
<td>Inline</td>
</tr>
<tr>
<td>7th</td>
<td>2.5955</td>
<td>Crossflow</td>
</tr>
<tr>
<td>8th</td>
<td>3.7755</td>
<td>Inline</td>
</tr>
</tbody>
</table>
Chapter 4

Dynamic Behavior under Steady Current

In the previous chapter, characteristics of a subsea jumper system were derived in terms of natural frequencies and their mode-shapes. This chapter will analyze vibration problems for a subsea jumper under current flow. The current velocity profile varies along with water depth and the subsea jumper under consideration has a maximum difference in elevation of only 10 meters. It is a rational assumption that current velocity is a steady current and uniform flow. The steady current near the seabed has a relatively low velocity, but it can cause significant vibration on the piping system or subsea jumper through the creation of vortex-induced vibration.

As discussed so far, VIV occurs when the subsea jumper starts moving. Its motion disturbs fluid flow, resulting in a fluctuating lift force coefficient. The maximum lift coefficient on freely oscillating cylinder is much larger than for a stationary case. As a result, this chapter focuses on the dynamic behavior under steady current with a focus on the VIV phenomenon. A vortex-induced vibration phenomenon can be represented by a wake oscillator model. The dynamic model for a subsea jumper under VIV is constructed by coupling a subsea jumper model with a wake oscillator model. It is called a “subsea jumper under current load model”. This model is introduced in first section. Since vortex-induced vibration is a couple-effect or interaction phenomenon, the model results in a couple of equations of motion between the subsea jumper system and wake motion.

It should be noted that VIV will occur only in a lock-in region or when the shedding frequency is close to the natural frequency of the jumper. Since current velocity relates to shedding frequency directly according to the Strouhal equation, the current velocity becomes the most influential parameter for VIV analysis. Current velocity near the seabed is different in various locations. This research considers a South-East Asia location. Unfortunately, there is no measurement data available for this area of interest. As such, current velocity information of a nearby location is used for the analysis. Other designed parameters are introduced in section 4.2.

The results of the subsea jumper under current load model are provided together with important aspects that are discussed in the last section.
4.1 Subsea jumper under current load model

A subsea jumper system can use a "subsea jumper model" for modelling introduced in section 3.2. It uses a three pipe conveying model connected with a 90 degree bend. The connections at the bends are rigid joints as an interface condition, whereas end-connections are fixed-fixed as a boundary condition. The VIV phenomenon can be described using a wake oscillator model as represented in section 2.3.

Consideration of the subsea jumper model and wake oscillator can establish a "subsea jumper under current load model" by coupling both models. The general form of a VIV model can be written in a set of equation of motion as follows:

**Wake oscillator model:**

\[
\ddot{q}(t) + \varepsilon \omega_s (q^2(t) - 1)\dot{q}(t) + \omega_s^2q(t) = \frac{A}{D} \left( \frac{\partial^2 w}{\partial t^2} \cos \alpha - \frac{\partial^2 v}{\partial t^2} \sin \alpha \right) 
\]

\[
C_{vw} = \left( C_{VD} \sin \beta + \left( \frac{C_{L0}}{2} \cdot q(t) \right) \cos \beta \right) \frac{U^2}{V^2}
\]

\[
C_{vv} = \left( C_{VD} \sin \beta + \left( \frac{C_{L0}}{2} \cdot q(t) \right) \cos \beta \right) \frac{U^2}{V^2}
\]

Where, is \( w \) crossflow motion whereas \( v \) is inline motion. This set of equations is valid for any arbitrary points on a subsea jumper. The interface condition and boundary condition are required at the connection point and end point. Their expressions are exactly the same as stated in section 3.2.
4.2 Design parameters for current and fluid properties

One of the most important parameters for VIV analysis on a subsea jumper is current velocity. The measurement data is required in order to obtain more accuracy and to reflect a more realistic response of operation of a subsea jumper under steady current flow at a designed location.

An area in South-East Asia is the main focus of this research. Unfortunately, the only available current information is an average current velocity at 0.2 m/s. The current velocity of a nearby location was used to analyze jumper motion. Table 4.1 shows available measurement data of a current near seabed which has the same current velocity at most percentages of occurrence in the same area in South-East Asia as 0.2 m/s. This information will be used as guideline in probabilistic design.

<table>
<thead>
<tr>
<th>Type of Data</th>
<th>Percentage Exceedance</th>
<th>Percentage Occurrence</th>
<th>Current Velocity (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measurement current data</td>
<td>100.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>97.77</td>
<td>0.23</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>78.46</td>
<td>19.31</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td><strong>46.19</strong></td>
<td><strong>32.27</strong></td>
<td><strong>0.20</strong></td>
</tr>
<tr>
<td></td>
<td>23.61</td>
<td>22.58</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>12.51</td>
<td>11.10</td>
<td>0.30</td>
</tr>
<tr>
<td></td>
<td>6.72</td>
<td>5.79</td>
<td>0.35</td>
</tr>
<tr>
<td></td>
<td>3.70</td>
<td>3.02</td>
<td>0.40</td>
</tr>
<tr>
<td></td>
<td>1.34</td>
<td>2.36</td>
<td>0.45</td>
</tr>
<tr>
<td></td>
<td>0.34</td>
<td>1.00</td>
<td>0.50</td>
</tr>
<tr>
<td></td>
<td>0.00</td>
<td>0.34</td>
<td>0.55</td>
</tr>
<tr>
<td>Extreme current data (based on probabilistic calculation)</td>
<td>100 years</td>
<td>3.42e-04</td>
<td>0.8320</td>
</tr>
<tr>
<td></td>
<td>10 years</td>
<td>3.42e-03</td>
<td>0.7202</td>
</tr>
<tr>
<td></td>
<td>5 years</td>
<td>8.85e-03</td>
<td>0.6687</td>
</tr>
<tr>
<td></td>
<td>1 year</td>
<td>3.42e-02</td>
<td>0.5658</td>
</tr>
</tbody>
</table>

Table 4.1: Measurement data of current velocity near seabed for reference

In Table 4.1, there are two types of data. First are current measurements, resulting in a probability of occurrence and probability of exceedance at each current velocity level. The maximum current velocity is measured at 0.55 m/s.

The second type is extreme current data. This was obtaining by probabilistic calculation as a return period. For instance, a 100 year return period means that an extreme current velocity at 0.832 m/s will be occur once every 100 years. These values are used for structure design. A more extreme case is selected for a more severe service. For subsea jumper calculation, a 100 year return period satisfies as the most severe case. Thus, in analysis of VIV for a subsea jumper, current velocity should not exceed 0.832 m/s in a 100 year return. Nevertheless, measurement data is still interesting for fatigue analysis. For example, current velocity at 0.2 m/s has the highest possibility of occurrence.

Other design parameters are summarized in Table 4.2. Additional design parameter for a subsea jumper system is given in Table 3.1.
### Design Parameters: Current and Fluid Properties

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sea water density ($\rho_{\text{sea}}$)</td>
<td>1025 Kg/m³</td>
</tr>
<tr>
<td>Stationary lift coefficient ($C_{LO}$)</td>
<td>0.3842</td>
</tr>
<tr>
<td>Stationary drag coefficient ($C_{VD}$)</td>
<td>1.1856</td>
</tr>
<tr>
<td>Strouhal number ($St$)</td>
<td>0.2</td>
</tr>
<tr>
<td>Wake coupling coefficient</td>
<td>1.2</td>
</tr>
<tr>
<td>Wake tuning parameter ($\varepsilon$)</td>
<td>0.7</td>
</tr>
</tbody>
</table>

*Table 4.2: Design parameters for current and fluid properties*
4.3 Dynamic behavior of subsea jumper under steady current

The previous section gave us a set of equations for a subsea jumper under current load model. Solving them gave subsea jumper motion in both a crossflow and inline direction. One of most effective solving methods is the numerical finite difference method. Details of this solution approach is given in Appendix B. In this section, only the results of subsea jumper motion by using triple-beam model are provided.

The main objective of this research is to analyze the fatigue lifetime of a subsea jumper under an external disturbance load. Thus, a displacement-time series of the jumper in various conditions is needed as input for fatigue analysis. However, based on VIV theory, we know that the amplitude of vibration during synchronization and non-synchronization are very different. It is important to understand when VIV or synchronization starts. In the other words, which current velocity creates a VIV phenomenon for a subsea jumper. This can be achieved by calculation of how the amplitude of vibration varies with current velocity. The results are shown in Figure 4.1 for the middle point of a horizontal beam.

![Figure 4.1: Amplitude vibration of horizontal beam on (i) crossflow direction and (ii) inline direction against percentage of possibility of exceedance on each current velocity](image-url)
Chapter 4: Dynamic Behavior under Steady Current

In Figure 4.1, it is obvious that synchronization takes place at 0.275 m/s for inline VIV and it enters crossflow VIV at 0.68 m/s, which corresponds to natural frequency of the first and second mode of a subsea jumper as shown in Table 3.4. These two current velocities are interesting for fatigue analysis since it creates a maximum transverse motion inline and at crossflow, respectively. It should be noted again that VIV can occur at higher current velocity; however, the designed current velocity in this research is limited to a 100 year return period value of 0.832 m/s.

As a consequence, the dynamic behavior of a subsea jumper will be the focus in two differently design cases. Design case A uses a current velocity at 0.275 m/s, which is the lock-in frequency for inline VIV. Whereas, design case B, considers a current velocity at 0.68 m/s.

The dynamic behavior of a subsea jumper for each design case results in transverse displacement varies with time or “displacement-time history”. This is used as input for fatigue analysis later in this research. We will focus only on the middle point of each line for both an inline and crossflow direction.

DESIGN CASE A: Dynamic behavior of subsea jumper at current velocity, 0.275 m/s

The results of displacement time-history are presented for a vertical left beam, horizontal beam and vertical right beam corresponding to wake motion at each point.

A. Midpoint of vertical left beam
   a. Amplitude of crossflow vibration of vertical left beam and vortex lift force coefficient (q)

   ![Crossflow transverse displacement time-series of a vertical left line and vortex lift force coefficient at current velocity 0.275 m/s](image)

   ![Inline transverse displacement time-series of a vertical left line at current velocity 0.275 m/s](image)

   Figure 4.2: Crossflow transverse displacement time-series of a vertical left line and vortex lift force coefficient at current velocity 0.275 m/s

   Figure 4.3: Inline transverse displacement time-series of a vertical left line at current velocity 0.275 m/s:
   - Left graph shows vibration during t=0s to 200s and right graph shows vibration during t=200s to 2000s
B. Midpoint of horizontal beam
   a. Amplitude of crossflow vibration of horizontal beam and vortex lift force coefficient (q)

![Crossflow transverse displacement time-series of a horizontal line and vortex lift force coefficient at current velocity 0.275 m/s](image)

![Inline transverse displacement time-series of a horizontal line at current velocity 0.275 m/s: Left graph shows vibration during t=0s to 200s and right graph shows vibration during t=200s to 2000s](image)

C. Midpoint of vertical right beam
   a. Amplitude of crossflow vibration of vertical right beam and vortex lift force coefficient (q)

![Crossflow transverse displacement time-series of a vertical right line and vortex lift force coefficient at current velocity 0.275 m/s](image)
b. Amplitude of inline vibration of vertical right beam

Figure 4.7: Inline transverse displacement time-series of a vertical right line at current velocity 0.275 m/s: Left graph shows vibration during t=0s to 200s and right graph shows vibration during t=200s to 2000s
DESIGN CASE B: Dynamic behavior of subsea jumper at current velocity, 0.68 m/s

The results of displacement time-history are presented for a vertical left beam, horizontal beam and vertical right beam corresponding to the wake motion at each point.

A. Midpoint of vertical left beam
   a. Amplitude of crossflow vibration of vertical left beam and vortex lift force coefficient (q)

   ![Figure 4.8: Crossflow transverse displacement time-series of a vertical left line and vortex lift force coefficient at current velocity 0.68 m/s](image)

   ![Figure 4.8: Crossflow transverse displacement time-series of a vertical left line and vortex lift force coefficient at current velocity 0.68 m/s](image)

   b. Amplitude of inline vibration of vertical left beam

   ![Figure 4.9: Inline transverse displacement time-series of a vertical left line at current velocity 0.68 m/s: Left graph shows vibration during t=0s to 200s and right graph shows vibration during t=50s to 2000s](image)
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B. Midpoint of horizontal beam
   a. Amplitude of crossflow vibration of horizontal beam and vortex lift force coefficient (q)

   ![Figure 4.10](image1.png)

   Figure 4.10: Crossflow transverse displacement time-series of a horizontal line and vortex lift force coefficient at current velocity 0.68 m/s

   b. Amplitude of inline vibration of horizontal beam

   ![Figure 4.11](image2.png)

   Figure 4.11: Inline transverse displacement time-series of a horizontal line at current velocity 0.68 m/s: Left graph shows vibration during t=0s to 200s and right graph shows vibration during t=50s to 2000s

C. Midpoint of vertical right beam
   a. Amplitude of crossflow vibration of vertical right beam and vortex lift force coefficient (q)

   ![Figure 4.12](image3.png)

   Figure 4.12: Crossflow transverse displacement time-series of a vertical right line and vortex lift force coefficient at current velocity 0.68 m/s
b. Amplitude of inline vibration of vertical right beam

Figure 4.13: Inline transverse displacement time-series of a vertical right line at current velocity 0.68 m/s: Left graph shows vibration during t=0s to 200s and right graph shows vibration during t=50s to 2000s
4.4 Discussions

According to the resulting motion of a jumper under a steady current from the previous section, several important aspects are considered as follows:

4.4.1 Synchronization or lock-in phenomenon

Theoretically, vortex-induce vibration occurs once a jumper and wake is synchronizing or with lock-in. At the lock-in stage, shedding frequency is close to the natural frequency of any orthogonal mode of vibration. This causes fluctuation on a lift force coefficient subsequent to high vibration.

A subsea jumper system has an orthogonal mode at a low frequency, especially for the in-line plane mode as seen in Table 4.3 for the first five modes of vibration both inline and crossflow (reference to calculation results in chapter 3, Table 3.4).

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>Natural Frequency (Hz)</th>
<th>Mode Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>0.1543</td>
<td>Inline</td>
</tr>
<tr>
<td>2nd</td>
<td>0.3687</td>
<td>Crossflow</td>
</tr>
<tr>
<td>3rd</td>
<td>0.6558</td>
<td>Inline</td>
</tr>
<tr>
<td>4th</td>
<td>1.2334</td>
<td>Crossflow</td>
</tr>
<tr>
<td>5th</td>
<td>1.4335</td>
<td>Inline</td>
</tr>
</tbody>
</table>

Table 4.3: Natural frequencies of a subsea jumper

Since all natural frequencies of subsea jumper are known so expected current velocity can cause VIV to system can be estimated by equaling the shedding frequency to natural frequencies of system. Based on Strouhal formula, current velocity can be calculated. The results of calculation in first five modes of crossflow are shown again in Table 4.4.

<table>
<thead>
<tr>
<th>Model</th>
<th>Vibration Mode</th>
<th>Natural Frequency (Hz)</th>
<th>Current Velocity (m/s)</th>
<th>Mode Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subsea jumper model</td>
<td>1</td>
<td>0.1543</td>
<td>0.27</td>
<td>Inline</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.3687</td>
<td>0.66</td>
<td>Crossflow</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.6558</td>
<td>1.16</td>
<td>Inline</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>1.2334</td>
<td>2.19</td>
<td>Crossflow</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>1.4335</td>
<td>2.54</td>
<td>Inline</td>
</tr>
</tbody>
</table>

Table 4.4: Lock-in current velocity calculation

In Table 4.4, we obtained the expected velocity that causes VIV for the first five modes. However, the measurement data results in a maximum current velocity at 0.55 m/s and maximum extreme data (probabilistic calculation) result at 0.832 m/s. Thus, only the first two modes of the subsea jumper can be expected to have VIV under a steady current condition, one inline VIV at 0.27 m/s and one crossflow VIV at 0.66 m/s.

The results of dynamic behavior for a subsea jumper shown in Figure 4.1 satisfied the following VIV theory. Inline VIV occurs once the current velocity enters a locking region at 0.275 m/s whereas the crossflow VIV starts once current velocity reaches 0.68 m/s. In conclusion, VIV phenomenon of a jumper system can exist at an extremely low velocity, unlike in cases of a riser or marine pipeline.

Returning to an observation of the results of the displacement time series, in design case A, the inline VIV takes place at the moment that current velocity reaches 0.275 m/s, where the shedding frequency is close to the natural frequency of inline vibration. The amplitude of inline vibration significantly increases compared to outside the lock-in region. It is interesting that the amplitude of crossflow vibration is also
increasing which is influenced by inline vibration. However, crossflow motion is very low and can be neglected. Opposite, in design case B, crossflow VIV occurs when the shedding frequency is close to the natural frequency in the 2nd mode of the subsea jumper or 1st mode of crossflow vibration. This results in dramatic crossflow vibration.

Increasing current velocity inside the lock-in region, vibration still results in a high amplitude until current velocity goes beyond the lock-in region. The amplitude vibration then drops to a stationary case or VIV vanishes. The results in shown in Figure 4.1 represent the VIV circumstance complies with the freely moving cylinder theory as introduced in chapter 2. As the result, it can conclude that a lock-in phenomenon occurs for both inline and cross flow vibration if the shedding frequency is close to one of the natural frequency vibrations of a jumper.

4.4.2 Resultant motion

Accordingly, to VIV occurs for both inline and crossflow vibration. The resultant motion should be considered separately. When shedding frequency is outside the lock-in region of crossflow VIV, current velocity is below 0.68 m/s. The resultant of transverse motion in a crossflow direction is extremely low, around $2 \times 10^{-4}$ m (or equal $5 \times 10^{-4}$ in dimensionless amplitude, w/OD). As a result, it can neglect the motion. Stationary cylinder theory can be applied to calculate a hydrodynamic force. Nonetheless, when entering synchronization, the amplitude of vibration is significantly high, $0.096$ m (or equal $0.27$ in dimensionless amplitude, w/OD) which is caused by a fluctuation of the lift force coefficient. The stationary cylinder is no longer valid for analysis. A wake oscillator model works properly in this case. Similarly to crossflow vibration, inline VIV take place when current velocity enters a lock-in region. The amplitude of inline vibration becomes significantly high at, $1.5 \times 10^{-5}$ m (or equal $4.4 \times 10^{-5}$ in dimensionless amplitude, v/OD).

Even so, the amplitude of inline vibration is extremely low compared to crossflow vibration. It cannot be dismissed out of fatigue analysis because it can occur at a low velocity which has a high probability of occurrence in the current velocity as indicated in Figure 4.1. This may result in high damage to fatigue lifetime.

According to the results of the displacement time series, one can observe that the amplitude of transverse vibration of a vertical beam is extremely low compare to motion of horizontal beam.

Another observation, inline vibration will oscillate around mean values due to drag force. The magnitude of mean displacement of inline vibration depends strongly on current velocity. Increasing current velocity increases mean value. Moreover, the location of the highest amplitude of vibration is at the middle point of a horizontal line in every design case. Meanwhile vertical lines result in low amplitude in a transverse motion. This is because of the geometry of a subsea jumper. A U-inverse shape with a long horizontal line is considered here, thus the horizontal section is the dominant part for vibration in both an inline and a crossflow direction.

4.4.3 Transient motion duration

For both inline and crossflow transverse motion, the displacement-time series of the two design cases show a transient and steady state. The system grows to a steady state condition faster when increasing current velocity. In design case A, a steady state starts at 1500 seconds, but the steady state of case B is at 50 seconds by approximation.

Inside the inline lock-in region, the displacement-time history of inline and crossflow motion at a transient condition consists of 2 phases; first, the amplitude of vibration is decreasing until reaching a certain point,
then amplitude increases again in the second phase until reaching a steady-state. This is because effect of self-limitation and self-excitation of the wake oscillator model.
Another important external disturbance on a subsea jumper is an earthquake. When earthquakes occur offshore, it influences a subsea jumper system thru ground vibration, scour transportation, turbidity current, fault rapture, etc. These create both direct and indirect hazards for a subsea jumper system. This research only focuses on ground vibration effects.

While earthquake may appear to a normal person to be a rarely occurring phenomenon, in fact, instrument detects an estimated 500,000 earthquakes per year. However, only 20 percent are reported as their magnitude is large enough to be felt. The possibility of occurrence and magnitude of an earthquake depends strongly on location, as indicated by a seismic map or earth fault map. Locations that are closer to an earth fault have more severe earthquakes in magnitude. Typically, the magnitude of an earthquake is reports with the Richter scale. However, ground acceleration is an important parameter to analyze the dynamic behavior of structure, and thus, the conversion of earthquake magnitude to ground acceleration is required and can be done thru the Attenuation Law.

For offshore structure design, it is necessary to understand the characteristics of seismic activities in a designed area. This research considers the Andaman Ocean, South-East Asia where a new deep-water field development project is located.

The main objective of this chapter is to analyze the dynamic behavior of a subsea jumper in an earthquake and determine the most severe earthquake frequency. This information will be used as a design parameter for a combination load effect of earthquake and steady current.

This chapter begins with introduction of a “subsea jumper model under earthquake load model” by using an equation of motion for a subsea jumper model coupled with an earthquake load model. An earthquake model is modelled by an inertia load (mass time ground acceleration). There are two ground acceleration models used in this thesis. Firstly, ground acceleration is varying with time in a sinusoidal function. This model represents an earthquake as a continuous ground motion. In the second model, ground motion is modelled with more accuracy and realistically by using a “simulation model”. This model is characterized by comparing it with measurements of seismic data.

All design parameters are introduced in the second section including the conversion method for defining the acceleration of ground motion from earthquake magnitude. Then, the results of the dynamic behavior of a subsea jumper during an earthquake are presented with both earthquake frequency response and displacement-time history. The final section provides some important observations.
5.1 Subsea jumper under earthquake load model

As discussed previously, an earthquake induces ground vibration by releasing energy from the earth’s crust in the form of a propagated seismic wave. Ground vibration due to an earthquake is a natural phenomenon and should be considered as a scholastic process. In structure design, the ground vibration is commonly considered in three directions: 1) horizontal in N-S, 2) horizontal in W-E and 3) vertical direction. Here, we use inline and crossflow direction to represent the horizontal direction of ground vibration instead of N-S and W-E. A subsea jumper system experiences earthquake forces in terms of inertia force in the same direction as ground vibration. Figure 5.1 shows the direction of ground motion that induced an inertia force to the subsea jumper system.

![Figure 5.1: Schematic diagram of a subsea jumper system against ground motion](image)

In reality, horizontal and vertical ground motions have different magnitudes. A common perception is that the vertical component is lower than the horizontal components. The ratio of vertical to horizontal acceleration, V/H, is assumed to be equal to 2/3 as originally proposed by [Newmark et al., 1973].

One can observe that inertia force adding into subsea jumper system is related to 2 transverse motion planes: inline plane and crossflow plane. Horizontal ground motion along the jumper direction and vertical ground motion influences the transverse motion of a jumper in a crossflow plane. Whereas, horizontal ground motion in a perpendicular direction to the jumper induces motion in inline vibration. As a result, a model for a subsea jumper in an earthquake can be simply achieved by adding inertia force terms into the equation of motion of a triple-beam model on each plane as demonstrated in chapter 3.2. The set of equations of motion for each ground motion direction can be expressed as follows:
Vertical ground motion is related to crossflow vibration of jumper.

For vertical left beam
\[ m \frac{\partial^2 w_1}{\partial t^2} + EI \frac{\partial^4 w_1}{\partial x_1^4} + M_f \left( U_f \frac{\partial^2 w_1}{\partial x_1^2} + 2U_f \frac{\partial^2 w_1}{\partial x_1 \partial t} + \frac{\partial^2 w_1}{\partial t^2} \right) + c_{sw} \frac{\partial w_1}{\partial t} = 0 \]  \hspace{1cm} (5.1)

For horizontal beam
\[ m \frac{\partial^2 w_2}{\partial t^2} + EI \frac{\partial^4 w_2}{\partial x_2^4} + M_f \left( U_f \frac{\partial^2 w_2}{\partial x_2^2} + 2U_f \frac{\partial^2 w_2}{\partial x_2 \partial t} + \frac{\partial^2 w_2}{\partial t^2} \right) + c_{sw} \frac{\partial w_2}{\partial t} = 0 \]  \hspace{1cm} (5.2)

For vertical right beam
\[ m \frac{\partial^2 w_3}{\partial t^2} + EI \frac{\partial^4 w_3}{\partial x_3^4} + M_f \left( U_f \frac{\partial^2 w_3}{\partial x_3^2} + 2U_f \frac{\partial^2 w_3}{\partial x_3 \partial t} + \frac{\partial^2 w_3}{\partial t^2} \right) + c_{sw} \frac{\partial w_3}{\partial t} = 0 \]  \hspace{1cm} (5.3)

Horizontal ground motion along jumper direction is related to crossflow vibration of jumper.

For vertical left beam
\[ m \frac{\partial^2 v_1}{\partial t^2} + EI \frac{\partial^4 v_1}{\partial x_1^4} + M_f \left( U_f \frac{\partial^2 v_1}{\partial x_1^2} + 2U_f \frac{\partial^2 v_1}{\partial x_1 \partial t} + \frac{\partial^2 v_1}{\partial t^2} \right) + c_{sw} \frac{\partial v_1}{\partial t} = -m \frac{\partial^2 v_g}{\partial t^2} \]  \hspace{1cm} (5.4)

For horizontal beam
\[ m \frac{\partial^2 v_2}{\partial t^2} + EI \frac{\partial^4 v_2}{\partial x_2^4} + M_f \left( U_f \frac{\partial^2 v_2}{\partial x_2^2} + 2U_f \frac{\partial^2 v_2}{\partial x_2 \partial t} + \frac{\partial^2 v_2}{\partial t^2} \right) + c_{sw} \frac{\partial v_2}{\partial t} = -m \frac{\partial^2 v_g}{\partial t^2} \]  \hspace{1cm} (5.5)

For vertical right beam
\[ m \frac{\partial^2 v_3}{\partial t^2} + EI \frac{\partial^4 v_3}{\partial x_3^4} + M_f \left( U_f \frac{\partial^2 v_3}{\partial x_3^2} + 2U_f \frac{\partial^2 v_3}{\partial x_3 \partial t} + \frac{\partial^2 v_3}{\partial t^2} \right) + c_{sw} \frac{\partial v_3}{\partial t} = -m \frac{\partial^2 v_g}{\partial t^2} \]  \hspace{1cm} (5.6)

Horizontal ground motion perpendicular to jumper direction is related to inline vibration of jumper.

For vertical left beam
\[ m \frac{\partial^2 v_1}{\partial t^2} + EI \frac{\partial^4 v_1}{\partial x_1^4} + M_f \left( U_f \frac{\partial^2 v_1}{\partial x_1^2} + 2U_f \frac{\partial^2 v_1}{\partial x_1 \partial t} + \frac{\partial^2 v_1}{\partial t^2} \right) + c_{sw} \frac{\partial v_1}{\partial t} = -m \frac{\partial^2 v_g}{\partial t^2} \]  \hspace{1cm} (5.7)

For horizontal beam
\[ m \frac{\partial^2 v_2}{\partial t^2} + EI \frac{\partial^4 v_2}{\partial x_2^4} + M_f \left( U_f \frac{\partial^2 v_2}{\partial x_2^2} + 2U_f \frac{\partial^2 v_2}{\partial x_2 \partial t} + \frac{\partial^2 v_2}{\partial t^2} \right) + c_{sw} \frac{\partial v_2}{\partial t} = -m \frac{\partial^2 v_g}{\partial t^2} \]  \hspace{1cm} (5.8)

For vertical right beam
\[ m \frac{\partial^2 v_3}{\partial t^2} + EI \frac{\partial^4 v_3}{\partial x_3^4} + M_f \left( U_f \frac{\partial^2 v_3}{\partial x_3^2} + 2U_f \frac{\partial^2 v_3}{\partial x_3 \partial t} + \frac{\partial^2 v_3}{\partial t^2} \right) + c_{sw} \frac{\partial v_3}{\partial t} = -m \frac{\partial^2 v_g}{\partial t^2} \]  \hspace{1cm} (5.9)

Where, \( w \) is crossflow motion and \( v \) is inline motion whereas \( w_g \) is crossflow ground and \( v_g \) is inline ground motion. \( c_{sw} \) represents a damping force coefficient in still water, in unit N/(m/s)/m. This set of equations is valid for any arbitrary point on a subsea jumper. In order to complete a set of equations of motion, the interface and boundary condition of each vibration plane are required at the connection point and end point as stated in section 3.2.

Three important observations should be raised. First, the amplitude of vertical acceleration is equal to two-thirds of the horizontal ground motion acceleration amplitude. Second, this model is simplified by assuming there is no effect of an earthquake to the longitude motion of a subsea jumper system. Third, a damping term needs to be introduced in the equation of motion. This is because there is an effect of surrounding seawater in term of hydrodynamic damping. One should emphasize that the damping term is absent in the current subsea jumper model because hydrodynamic damping is already incorporated in the wake oscillator model. Still water damping is considered in accordance with [Venugopal's damping model]. This can be expressed as follows:

\[
C_{sw} = \frac{\alpha \rho o \Omega_D^2}{2} \left( \frac{2 \sqrt{T}}{\sqrt{\alpha \rho o}} + K_{sw} \left( \frac{W_g}{\rho o} \right)^2 \right), \quad \text{Re}_o = \frac{\omega \cdot \Omega_D^2}{v_f}.
\]  \hspace{1cm} (5.10)

This model contains two terms. The first term corresponds to skin friction according to Stoke’s law whereas the second term is a pressure-dominated force. Where, \( \omega \) is vibration frequency [rad/s], \( \rho \) is fluid
Chapter 5: Dynamic Behavior during an earthquake

density \( [Kg/m^3] \), OD is diameter of cylinder, \( W_0 \) is amplitude of transverse vibration, \( K_{sw} \) is a fitting curve factor in constant value, 0.25 and \( v_f \) is the kinematic viscosity of a fluid \([m^2/s]\).

In reality, earthquake is a natural phenomenon so ground acceleration should be described as a scholastic function. An earthquake model represents a highly complex function and measurement data is necessary to validate the model. However, defining the most accurate earthquake model is not the main objective of this research. As such, two simplified models for ground acceleration are introduced here. These models satisfactorily describe the general characteristics of a subsea jumper under an earthquake condition. Firstly, an idealistic ground vibration model is assumed to be a simple sinusoidal function. This model assumes a ground motion as continuous process by presenting it in a sinusoidal function. It is used to characterize the amplitude response spectra of a jumper during an earthquake. The expression of the first model (called "sinusoidal earthquake model" in this research) is written as follows:

\[
\text{Inline ground acceleration} \quad v_g(t) = \tilde{v}_g \sin(\omega_g t) \tag{5.11}
\]

\[
\text{Crossflow ground acceleration} \quad w_g(t) = \hat{w}_g \sin(\omega_g t) \tag{5.12}
\]

Where, \( \tilde{v}_g \) and \( \hat{w}_g \) presents a maximum amplitude of earthquake in inline and crossflow direction, respectively. \( \omega_g \) is an earthquake frequency. Typically, dominant frequencies are 1 – 10 Hz.

Secondly, ground vibration is modelled to be more accurate. It is based on natural characteristics of an earthquake. Typically, an earthquake process contains many short period ground motions, called “shock”. Each ground motions begins at small amplitude and grows until reaching a maximum amplitude. After that the amplitude will decrease and vanish. Thus, the second model shall represent both an increasing period and a decreasing period. This model is used to analyze the dynamic behavior of a subsea jumper in a more accuracy way. The second model (called "simulated earthquake model") can be expressed in mathematically as follows:

\[
\text{Inline ground acceleration} \quad v_g(t) = \tilde{v}_g \cdot e^{-at} \cdot (b \cdot t) \cdot \sin(\omega_g t) \cdot \left( \text{Heaviside}(t) - \text{Heaviside}(t - t_{eq}) \right) \tag{5.13}
\]

\[
\text{Crossflow ground acceleration} \quad w_g(t) = \hat{w}_g \cdot e^{-at} \cdot (b \cdot t) \cdot \sin(\omega_g t) \cdot \left( \text{Heaviside}(t) - \text{Heaviside}(t - t_{eq}) \right) \tag{5.14}
\]

Where, \( a \) represents control parameter for decreasing amplitude period whereas \( b \) indicates control parameter for increasing amplitude period. Earthquake period is indicated by \( t_{eq} \). The duration of earthquake is usually 5 – 40 seconds by approximation. Figure 5.2 shows a comparison of a simulated earthquake model with measurement data.

![Figure 5.2: Comparison ground acceleration from measurement data (left) and simulated earthquake model (right)](image)
5.2 Design parameters for earthquake

As discussed in the previous section, an earthquake load can be simply defined in term of inertia load. The inertia load comprises of two components, mass of system itself and ground acceleration due to earthquake. Here, the mass of a system is the subsea jumper plus contained fluid. The ground acceleration term is defined by amplitude and frequency.

Considering amplitude acceleration of earthquake, most of the earthquake data are reported in earthquake magnitude by Richter unit. Figure 5.3 shows a fault map source in Thailand and Myanmar compared to the location of the new field development project.

![Figure 5.3: Fault sources in Thailand and Myanmar [USGS, documentation for Southeast Asia Seismic Hazard map]](image)

In Figure 5.3, the location of the new field development is indicated by a green circle. There are two mains faults close to this location, namely, Three Pogodaz fault, source 6 and Red River fault, source 10. Table 5.1 shows measured earthquake data corresponding to the fault map source.
Chapter 5: Dynamic Behavior during an earthquake

<table>
<thead>
<tr>
<th>Design Parameter: Earthquake Magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnitude of earthquake source 6 $(M_{g,1})$</td>
</tr>
<tr>
<td>Magnitude of earthquake source 10 $(M_{g,2})$</td>
</tr>
</tbody>
</table>

Table 5.1: Measurement earthquake magnitude data

Converting from magnitude to amplitude of acceleration can be achieved by Attenuation Law, as present in section 2.6. The expression is presented again as follows:

$$\hat{a}_g = A_g e^{0.8M_g(R + R_0)^{-2}} \quad (5.15)$$

Where, $\hat{a}_g$ is amplitude of ground acceleration in m/s$^2$ (sometime acceleration indicate in faction of gravity acceleration, $g$), $M_g$ is magnitude of earthquake according to Richter unit, $R$ is distance to epicenter and constant $A_g$ and $R_0$ are $56 \times 10^6$ m/s$^2$ and 40 m, respectively. Table 5.2 represents corresponding maximum amplitude to earthquake source from Table 5.1. The chosen design parameter is considering the most severity case for large magnitude earthquake model, thus designed peak ground acceleration is a combination of two sources.

<table>
<thead>
<tr>
<th>Design Parameter: Corresponding Earthquake Amplitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum magnitude of source 6 $(M_{g,1})$</td>
</tr>
<tr>
<td>Maximum magnitude of source 10 $(M_{g,2})$</td>
</tr>
<tr>
<td>Distance source 6 $(R_1)$</td>
</tr>
<tr>
<td>Distance source 10 $(R_2)$</td>
</tr>
<tr>
<td>Peak ground acceleration source 6 $(\hat{a}_{g,1})$</td>
</tr>
<tr>
<td>Peak ground acceleration source 10 $(\hat{a}_{g,2})$</td>
</tr>
<tr>
<td>Design peak ground acceleration $(\hat{a}_g)$</td>
</tr>
</tbody>
</table>

Table 5.2: Corresponding earthquake amplitude

Another vital parameter is earthquake frequency. It is a scholastic parameter. Typically, dominant frequencies are $0 - 10$ Hz. In reality, an earthquake process has random and fluctuation frequencies. However, in this research made assumption of singular earthquake frequency during earthquake process. Thus, subsea jumper system shall be analyzing base on dominant earthquake frequencies for all direction in order to getting better understand of its behavior under earthquake condition. The duration of earthquake is usually $5 - 40$ seconds. Here 25 seconds is assumed. Finally, the damping force coefficient of still water, $c_{sw}$ can be calculated according to equation 5.7. The related parameters for damping term are summarized together with other designed parameters in Table 5.3.
### Chapter 5: Dynamic Behavior during an earthquake

#### Design Parameter: Earthquake Condition

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design Peak ground acceleration</td>
<td>$\bar{a}_g$ = 3.63 m/s² (0.37 g)</td>
</tr>
<tr>
<td>Design earthquake frequencies</td>
<td>$f_{earthquake}$ = 0 - 10 Hz</td>
</tr>
<tr>
<td>Control earthquake parameter</td>
<td>$a$ = 0.45</td>
</tr>
<tr>
<td></td>
<td>$b$ = 1</td>
</tr>
<tr>
<td>Earthquake Duration</td>
<td>25 sec</td>
</tr>
</tbody>
</table>

#### Design Parameter: Damping Coefficient

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sea water density $\rho_{sea}$</td>
<td>1025 Kg/m²</td>
</tr>
<tr>
<td>Sea water kinematic viscosity $\nu_{sea}$</td>
<td>$1.83 \times 10^{-6}$ m²/s</td>
</tr>
<tr>
<td>Fitting curve factor $K_{sw}$</td>
<td>0.25</td>
</tr>
</tbody>
</table>

*Table 5.3: Earthquake design parameters*
5.3 Dynamics behavior of subsea jumper under earthquake

In section 5.1, solutions of subsea jumper under earthquake load model are transverse motions in inline and crossflow direction. They are influenced by three earthquake directions, two horizontals and one vertical. Similarly as subsea jumper under current steady model, solving method is a finite different analysis. The detail of solving step is given in appendix B.

This section begins with a seismic response spectrum investigation. It is useful information because it provides displacement amplitude of subsea jumper in steady-state condition varies on dominant earthquake frequencies, 0 – 10 Hz. This result gives more insight into subsea jumper sensitivity to an earthquake condition. Seismic response spectra are calculated by using “sinusoidal earthquake model”.

Seismic response spectra on each earthquake direction; vertical earthquake, horizontal earthquake (in crossflow direction) and horizontal earthquake (in inline direction), represented in figure 5.4, 5.5 and 5.6 respectively. In addition, each figure provides a comparison of seismic response spectra at the middle point of a vertical and a horizontal line.

![Figure 5.4: Seismic response spectra for vertical earthquake: (i) vertical line and (ii) horizontal line](image-url)
Figure 5.5: Seismic response spectra for horizontal earthquake along to jumper direction (crossflow direction):
(i) vertical line and (ii) horizontal line
Figure 5.6: Seismic response spectra for horizontal earthquake perpendicular to jumper direction (inline direction): (i) vertical line and (ii) horizontal line

Figure 5.4, 5.5 and 5.6 give results of seismic response spectra of subsea jumper under each earthquake direction. It is obvious that different direction of ground motion result in different frequency response of jumper vibration. Before going beyond into detail, one should note that only one graph represents the vertical line. This is because main objective of seismic response is only to compare sensitivity between horizontal line and vertical line of subsea jumper under an earthquake load. In addition, amplitudes of vibration on left and right vertical line are more or less same. Thus, only amplitude motion of vertical left line is plotted in seismic response spectra.

A seismic response spectrum of subsea jumper under vertical ground motion is given in Figure 5.4. Seismic responses of horizontal line and vertical line have a same trend. The maximum peak amplitude of jumper displacement occurs at low earthquake frequency, 0.36 Hz. This earthquake frequency is close to natural frequency of 1st mode crossflow vibration of jumper. Other peak amplitudes are found when earthquake frequency reaches jumper natural frequencies in mode 3rd, 5th, 7th and so on.

Seismic response spectra of jumper under horizontal earthquake in crossflow direction, shows in Figure 5.5. One can see that responses of vertical line and horizontal line have different profile; however, they
found peak of vibration in same frequency. Each peak frequency is related to crossflow natural frequency of subsea jumper. In horizontal line, the maximum peak displacement occurs at low frequency that is close to natural frequency of 1st mode of crossflow vibration, 0.36 Hz. Next peak amplitudes occur at frequency closed to mode 2nd, 3rd, 4th etc. Nevertheless, peak of odd mode (mode 3rd, 5th, 7th) are higher than even mode (mode 2nd, 4th, 6th etc.). In vertical line, it does not show maximum peak of vibration, obviously. Amplitude response of odd modes results in same amplitude; in range 0.002 to 0.003 whereas amplitude of even modes has a lower amplitude; in range 0.001 to 0.002.

Finally, Figure 5.6 represents seismic response spectra of jumper under horizontal earthquake in inline direction. Seismic frequency response is distributing in same manner as in vertical beam. Peak amplitude of motion occur when earthquake frequency equal to jumper natural frequency in mode 1st, 3rd, 5th and so on. However, peak frequency is obvious different from vertical vibration due to natural frequency of crossflow and inline of a subsea jumper are different. In seismic spectrum response, the maximum peak amplitude is found at earthquake frequency equal 0.15 Hz. It has same value with 1st mode of inline vibration.

Regarding the main objective of this research is to investigate a fatigue life-time. The horizontal line is more interesting than the vertical line because it vibrates at higher amplitude. In addition, the middle point of horizontal line is given highest amplitude of transverse motion which results in highest bending stress. Thus, displacement time series of middle point of horizontal line of subsea jumper at maximum peak amplitude of each earthquake direction are interesting.

Consequence, there are three interesting design cases according to three earthquake directions. Firstly, design case A is to investigate a displacement time series of a subsea jumper under vertical ground motion at earthquake frequency equal 0.36 Hz. Later, displacement time series of jumper under horizontal ground motion in crossflow direction which earthquake frequency equal 0.36 Hz is shown in design case B. Finally, design case C represents for horizontal earthquake in inline direction at earthquake frequency equal to 0.15 Hz.

In addition, seismic response spectra provides seismic sensitivity characteristic of a subsea jumper to earthquake. On the other words, one can expect which earthquake frequency creates most severe motion of a subsea jumper. However, a seismic response spectrum is considering a motion at steady state by assuming ground motion is pure harmonic function which is an ideal case. However, the nature of an earthquake has a characteristic of shock load, a small period with a high magnitude. Thus, a simulation ground motion model as introduce in beginning of chapter are applied in each design cases in order to obtain more information to investigate fatigue life-time of a subsea jumper.
**DESIGN CASE A:** Vertical earthquake at frequency equals 0.36 Hz

A.1 Sinusoidal model

Displacement time history at midpoint of horizontal line (crossflow vibration)

![Figure 5.7: Crossflow transverse displacement time-series of a horizontal line during vertical earthquake at frequency 0.36 Hz (sinusoidal model)](image)

A.2 Simulation model

Displacement time history at midpoint of horizontal line (crossflow vibration)

![Figure 5.8: Crossflow transverse displacement time-series of a horizontal line during vertical earthquake at frequency 0.36 Hz (simulation model)](image)
DESIGN CASE B: Horizontal earthquake (crossflow direction) at frequency equals 0.36 Hz

B.1 Sinusoidal model

Displacement time history at midpoint of horizontal line (crossflow vibration)

![Figure 5.9: Crossflow transverse displacement time-series of a horizontal line during horizontal earthquake at frequency 0.36 Hz (sinusoidal model)](image)

B.2 Simulation model

Displacement time history at midpoint of horizontal line (crossflow vibration)

![Figure 5.10: Crossflow transverse displacement time-series of a horizontal line during horizontal earthquake at frequency 0.36 Hz (simulation model)](image)
DESIGN CASE C: Horizontal earthquake (inline direction) at frequency equals 0.15 Hz

C.1 Sinusoidal model

Displacement time history at midpoint of horizontal line (crossflow vibration)

Figure 5.11: Inline transverse displacement time-series of a horizontal line during horizontal earthquake at frequency 0.15 Hz (sinusoidal model)

C.2 Simulation model

Displacement time history at midpoint of horizontal line (crossflow vibration)

Figure 5.12: Inline transverse displacement time-series of a horizontal line during horizontal earthquake at frequency 0.15 Hz (simulation model)
Chapter 5: Dynamic Behavior during an earthquake

5.4 Discussions

5.4.1 Peak amplitude of vibration occurrence

A subsea jumper system is modelled here by using a subsea jumper model or a triple-beam model that is a continuous system. In theory, the sensitivity of continuous system response is related directly to two main characteristics of loads. One is a load frequency and other one is distribution of load along continuous system. The system will result in a maximum peak amplitude response only when load frequency is close to one of the natural frequency of system and load distribution along system is the same shape with orthogonal shape on that mode.

By assumption, the load of an earthquake is an inertia load that is equal to mass times ground motion acceleration. The ground acceleration varies on time domain only, but not in space domain. Thus, it has a constant profile along beam axis, $x_2$-axis for horizontal beam and $x_1$ and $x_3$ axis for vertical beams. In the other words, the load distribution has a symmetric profile. In accordance with the response of continuous system, only most symmetric mode of vibration expects to observe a peak amplitude response. Referring to seismic response spectra for cross flow and inline vibration in Figure 5.4, 5.5 and 5.6, one can see that the result complies with theory. Significant peak amplitudes of vibration have been always found, especially on horizontal line in symmetric mode.

5.4.2 Maximum peak amplitude of vibration occurrence

A maximum amplitude response will occur when load has a characteristic same with mode shape and natural frequency. First we consider inline vibration in Figure 5.6. Spectra response of a subsea jumper under horizontal ground vibration (inline) shows maximum peak amplitude at earthquake 0.15 Hz which equal to 1st mode of natural frequency in inline vibration. Referring to the mode shape of inline vibration in chapter 3.4, one can see the first orthogonal mode shape is closest to a load distribution profile since all vertical and horizontal lines are distributed in phase of a shape with a symmetric profile.

For crossflow vibration of a subsea jumper can consider vertical and horizontal ground motion separately. In vertical ground motion, load only applied into horizontal line whereas no load exert to vertical beam by assumption, thus horizontal line is dominant and induce vibration of vertical line by coupling at the interface joint. The most identical mode shape of horizontal line to earthquake load is first mode. Then, maximum peak amplitude exists at first mode of crossflow vibration. This high response induces vibration to vertical beam because it has same phase of vibration. Thus, vertical line also result highest amplitude at first mode.

For horizontal earthquake (crossflow direction) is different from vertical earthquake. In this case, an earthquake load exerts to the vertical line by horizontal ground motion. As a consequence, the vertical lines are dominant of vibration. Referring to mode shape of vertical line in chapter 3, vertical line has no purely symmetric and asymmetric mode. Thus, all earthquake frequencies are close to one of crossflow natural frequency result in a high response. Vibration of vertical lines induces a motion of a horizontal line at the interface connections. As a result, horizontal line is induced by vertical load at both ends. These induced loads are almost symmetric because both vertical lines vibrate at slightly different amplitude. According to mode shape of the horizontal line are in both purely symmetric shape (odd mode) and purely asymmetric shape (even mode). Thus, horizontal beam has maximum amplitude in 1st mode and result in high response in odd mode. This is because it has the most symmetric and in phase distribution according to induced load by vertical lines. In addition, horizontal line also results in high peak when earthquake frequency close to even mode because of not purely symmetric load applied at connection joint by vertical lines. Not only vertical beam induce vibration of horizontal line, horizontal motion has an influence to vertical line also. One can observe the result on Figure 5.5 that; vertical line has amplitude of vibration in odd mode greater than even modes.
One important observation has been made on amplitude of vibration. In all earthquake directions, the amplitude response of the horizontal line is greater than the vertical line. As such, the motion of the horizontal line is interesting for fatigue analysis for earthquake condition.

5.4.3 Subsea jumper sensitivity to earthquake

Typically, measurement data of earthquake show dominant frequencies in range 1 – 10 Hz. One can expect results of seismic spectra in this chapter to cover highest possible earthquakes in the world. Seismic response spectra give important information of jumper behavior to an earthquake load. Considering amplitude response, it is possible to conclude that a subsea jumper is more sensitive in inline ground motion than crossflow ground motion.

In case of inline vibration, 1st orthogonal mode is most sensitive to earthquake since three lines are moving in phase themselves and inertia load. Thus, it results in high amplitude response. However, it is different from crossflow vibration.

In case of crossflow vibration, horizontal line and vertical lines have different characteristics. Horizontal line is straight forward. Its first orthogonal mode is most similar to load distribution along horizontal line. As such this mode is most sensitive for horizontal beam.

Referring to chapter 3, the mode shapes of a subsea jumper were defined. One can observe that both vertical lines are not purely symmetric and asymmetric. In case load applied direct to vertical line as in horizontal earthquake (crossflow direction). It responds sensitively when earthquake load frequencies are close to one of its natural frequencies. However, it is not valid for vertical earthquake which load applied to horizontal beam. Vertical lines will vibrate accordance with motion of horizontal line. Thus, it results in high response when horizontal line vibrates dramatically.

5.4.4 Ground acceleration model

In this research, there are two ground acceleration models. First is sinusoidal model, it is simplified ground motion by mean of harmonic function. This model is well defining amplitude response of earthquake or earthquake response spectra as see result in Figure 5.4, 5.5 and 5.6. However, this model is based an ideal earthquake by assuming as a continuous process.

The second model is a simulation model or shock model. Its expression is established by measured ground acceleration data. Typically, each earthquake phenomenon contains many shocks. Each shock has a characteristic as increasing in first phase until reach maximum magnitude. Then it decreases to small amplitude value. Total duration of each shock is less than 40 seconds. This model does well in describing a more realistic earthquake phenomenon. Thus, it is used to simulate a motion of a subsea jumper under earthquake in fatigue life time analysis.
Chapter 6: Dynamic Behavior under Combination Effect

Chapter 6

Dynamic Behavior under Combination Effect

In chapter 4 and 5, the dynamic behaviors of a subsea jumper were analyzed separately in a steady current condition and an earthquake condition. In reality, there is a possibility that both situations will occur at the same time. Thus, a combination load situation should be applied to a subsea jumper system in order to understand the dynamic behavior of a subsea jumper. These results will be used as input data for fatigue analysis in the next chapter. This is the first objective of this chapter.

The second objective of this chapter is to understand the effect of an earthquake in a VIV process. In reality, an earthquake influences the current flow by changing hydrodynamic pressure in a nearby seabed consequence the current flow gradient and velocity fluctuate. Nevertheless, this thesis is examined a simplified combination effect by assuming that the earthquake has no disturbance on steady current velocity. In the other words, the current is assumed to be a constant and uniform flow while an earthquake is occurring. Although we assume no change in the current flow during an earthquake, it is interesting to investigate whether an earthquake can induced a VIV phenomenon when current velocity does not reach the locking region and/or whether an earthquake can deconstruct the VIV process.

This chapter begins with a combination effect model for subsea jumper system under an earthquake load and a steady current load. It is called a “subsea jumper under combination load model”. This model is simply established by adding two external load models: 1) wake oscillator model and 2) inertia model into a subsea jumper model. Later in this chapter, results and discussion will be provided in accordance with design parameters for each design case.
6.1 Subsea jumper under combination load model

Main purpose of a subsea jumper under combination load model is to describe the transverse responses of a subsea jumper under steady current and earthquake. Figure 6.1 shows a schematic diagram of a subsea jumper in combination load situation.

The subsea under combination load model is simply established by combining three models; 1) subsea jumper model, that uses to analyze transverse motion of a subsea jumper system, 2) wake oscillator model that uses to describe VIV phenomenon and 3) inertial load model that represents an earthquake load. This model is a set of equation of motion for subsea jumper in transverse motion in both inline and crossflow vibration. However, there are three mains assumptions on combination effect model as follow:-

1. Current velocity assumes to be constant and non-disturbance during earthquake process. Steady current flow past subsea jumper in inline direction.
2. Earthquake phenomenon creates ground movement in three directions simultaneously. Ground motion comprise of 1) vertical ground motion, 2) horizontal ground motion in inline plane and 3) horizontal ground motion in crossflow plane.
3. Inline and crossflow vibration are coupled by wake oscillator model whereas vertical beam and horizontal beam are coupled by interface condition.

The mathematic expression of a subsea under combination load model is in the form of an equation of motion. It can be considered as occurring in two vibration planes as crossflow and inline. It is written as follows:
Chapter 6: Dynamic Behavior under Combination Effect

Crossflow vibration plane

For vertical left beam
\[ m \frac{\partial^2 w_x}{\partial t^2} + EI \frac{\partial^4 w_x}{\partial x_1^4} + M_f \left( U_f^2 \frac{\partial^2 w_x}{\partial x_1^2} + 2U_f \frac{\partial^2 w_x}{\partial t \partial x_1} + \frac{\partial^2 w_x}{\partial t^2} \right) = \frac{1}{2} \rho D V^2 C_v w_x - m \frac{\partial^2 w_x}{\partial t^2} \tag{6.1} \]

For horizontal beam
\[ m \frac{\partial^2 w_x}{\partial t^2} + EI \frac{\partial^4 w_x}{\partial x_2^4} + M_f \left( U_f^2 \frac{\partial^2 w_x}{\partial x_2^2} + 2U_f \frac{\partial^2 w_x}{\partial t \partial x_2} + \frac{\partial^2 w_x}{\partial t^2} \right) = \frac{1}{2} \rho D V^2 C_v w_x - m \frac{\partial^2 w_x}{\partial t^2} \tag{6.2} \]

For vertical right beam
\[ m \frac{\partial^2 w_x}{\partial t^2} + EI \frac{\partial^4 w_x}{\partial x_3^4} + M_f \left( U_f^2 \frac{\partial^2 w_x}{\partial x_3^2} + 2U_f \frac{\partial^2 w_x}{\partial t \partial x_3} + \frac{\partial^2 w_x}{\partial t^2} \right) = \frac{1}{2} \rho D V^2 C_v w_x - m \frac{\partial^2 w_x}{\partial t^2} \tag{6.3} \]

Inline vibration plane

For vertical left beam
\[ m \frac{\partial^2 v_1}{\partial t^2} + EI \frac{\partial^4 v_1}{\partial x_1^4} + M_f \left( U_f^2 \frac{\partial^2 v_1}{\partial x_1^2} + 2U_f \frac{\partial^2 v_1}{\partial t \partial x_1} + \frac{\partial^2 v_1}{\partial t^2} \right) = \frac{1}{2} \rho D V^2 C_v v_1 - m \frac{\partial^2 v_1}{\partial t^2} \tag{6.4} \]

For horizontal beam
\[ m \frac{\partial^2 v_2}{\partial t^2} + EI \frac{\partial^4 v_2}{\partial x_2^4} + M_f \left( U_f^2 \frac{\partial^2 v_2}{\partial x_2^2} + 2U_f \frac{\partial^2 v_2}{\partial t \partial x_2} + \frac{\partial^2 v_2}{\partial t^2} \right) = \frac{1}{2} \rho D V^2 C_v v_2 - m \frac{\partial^2 v_2}{\partial t^2} \tag{6.5} \]

For vertical right beam
\[ m \frac{\partial^2 v_3}{\partial t^2} + EI \frac{\partial^4 v_3}{\partial x_3^4} + M_f \left( U_f^2 \frac{\partial^2 v_3}{\partial x_3^2} + 2U_f \frac{\partial^2 v_3}{\partial t \partial x_3} + \frac{\partial^2 v_3}{\partial t^2} \right) = \frac{1}{2} \rho D V^2 C_v v_3 - m \frac{\partial^2 v_3}{\partial t^2} \tag{6.6} \]

Wake oscillator model

\[ \ddot{q}(t) + \varepsilon \omega_q (q^2(t) - 1) \dot{q}(t) + \omega_q^2 q(t) = \frac{A}{D} \frac{\partial^2 w}{\partial t^2} \cos \alpha - \frac{\partial^2 v}{\partial t^2} \sin \alpha \tag{6.7} \]

\[ C_{vw} = \left( C_{vd} \sin \beta + \frac{C_{lo}}{2} \cdot q(t) \cos \beta \right) \frac{\dot{q}^2}{\dot{v}^2} \tag{6.8} \]

\[ C_{vv} = \left( C_{vd} \sin \beta + \frac{C_{lo}}{2} \cdot q(t) \cos \beta \right) \frac{\dot{v}^2}{\dot{v}^2} \tag{6.9} \]

This set of equation is valid any arbitrarily points on subsea jumper. Interface and boundary condition for both crossflow vibration and inline vibration are required at the connection points and end points as stated in section 3.2. Two idealistic cases for an earthquake condition are assumed as the same as described in chapter 5. One described by sinusoidal model is assumed that an earthquake is a continuous process. The profile of ground acceleration is a harmonic shape. The expression of "sinusoidal earthquake model" is presented again as follows:

Inline ground acceleration
\[ v_g(t) = \ddot{\bar{v}}_g \sin(\omega_g t) \tag{6.10} \]

Crossflow ground acceleration
\[ w_g(t) = \ddot{\bar{w}}_g \sin(\omega_g t) \tag{6.11} \]

Where, \( \ddot{\bar{v}}_g \) and \( \ddot{\bar{w}}_g \) presents a maximum amplitude of earthquake in inline and crossflow direction, respectively. \( \omega_g \) is an earthquake frequency. Another model describes an earthquake by a simulation model. It simulates an earthquake as “shock”, earthquake process has a small period with peak amplitude. The expression of “simulation earthquake model” is written as follows:

Inline ground acceleration
\[ v_g(t) = \ddot{\bar{v}}_g \cdot e^{-at} \cdot (b \cdot t) \cdot \sin(\omega_g t) \cdot \left( \text{Heaviside}(t) - \text{Heaviside}(t - t_{eq}) \right) \tag{6.12} \]

Crossflow ground acceleration
\[ w_g(t) = \ddot{\bar{w}}_g \cdot e^{-at} \cdot (b \cdot t) \cdot \sin(\omega_g t) \cdot \left( \text{Heaviside}(t) - \text{Heaviside}(t - t_{eq}) \right) \tag{6.13} \]

Where, \( a \) and \( b \) are control parameters.
6.2 Design parameters for combination effect (VIV and earthquake)

Before going further into details of design parameters for a combination effect, one should emphasize the main objectives of this chapter. Then, all designed parameters are selected based on objectives of this chapter. There are two main objectives as follow:

1. Analyze dynamic behavior of a subsea jumper in most severe case of a combination effect and result in displacement time history. It can be used as input data for fatigue analysis in next the part of the thesis.
2. Understand induce effect of earthquake on VIV phenomenon which are:
   a. Does an earthquake induce VIV if the earthquake has a ground motion frequency that is the same as shedding frequency in lock-in region?
   b. Does an earthquake demolish the VIV phenomenon on a subsea jumper?

For the first objective, design parameters shall be selected in order to create most severe transverse motion on a subsea jumper. However, designed parameters should be also in high possibility of occurrence. For current velocity, one can recall result in chapter 4. Subsea jumper responses a high amplitude when VIV takes place. There are two possible current velocities that create a VIV phenomenon: 1) current velocity equal 0.275 m/s create an inline-VIV and 2) current velocity equal 0.68 create a crossflow-VIV. Thus, these current velocities are selected as the designed current velocities. For earthquake, sensitivity of jumper to an earthquake frequency is obtaining from seismic response spectra in chapter 5. Here, earthquake frequencies assume to be different on each ground motion plane. For inline ground motion, earthquake frequency equal to 0.15 Hz is selected whereas ground motion in horizontal crossflow direction and vertical direction, an earthquake frequency of 0.36 Hz is selected. In addition, the earthquake model used a simulation model because it gives a more realistic result than a sinusoidal model. In addition, it is interesting to observe the dynamic behavior of a subsea jumper when earthquake load suddenly exerts on jumper system while subsea jumper motion has already synchronized with the wake.

All design parameters for combination effect accordance with first objective are shown in Table 6.1.

<table>
<thead>
<tr>
<th>Design Parameter: Current</th>
<th>Design Parameter: Earthquake</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design current velocity</td>
<td>Design peak ground acceleration</td>
</tr>
<tr>
<td>( V )</td>
<td>( \ddot{a}_g )</td>
</tr>
<tr>
<td>0.275 m/s (Inline VIV)</td>
<td>3.63 m/s² (0.37 g)</td>
</tr>
<tr>
<td>0.68 m/s (Crossflow VIV)</td>
<td></td>
</tr>
<tr>
<td>Design earthquake frequencies</td>
<td>Design earthquake frequencies</td>
</tr>
<tr>
<td>( f_{\text{earthquake}_1} )</td>
<td>( f_{\text{earthquake}_2} )</td>
</tr>
<tr>
<td>0.15 Hz (inline ground motion)</td>
<td>0.36 Hz (Crossflow ground motion)</td>
</tr>
</tbody>
</table>

Table 6.1: Design parameters of a combination load for design case A and B

For the second objective, we first determine on the chapter objective 2a, one should ensure that there is no VIV occurrence from current flow. A design current velocity shall be selected outside lock-in region. As a result, current velocity that is equal to 0.4 m/s is selected. For earthquake condition, its frequency shall characterize as same as lock-in frequency which is 0.15 Hz for inline VIV and 0.36 Hz for crossflow VIV. Thus, design case, C and D are constructed on different design earthquake frequencies. In addition, a
“sinusoidal model” used to describe a ground motion. The designed parameters for design case C and D is shown in Table 6.2.

<table>
<thead>
<tr>
<th>Design Parameter: Current</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design Current velocity</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Design Parameter: Earthquake</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design peak ground acceleration</td>
</tr>
</tbody>
</table>

| Design earthquake frequencies (\( f_{\text{earthquake,1}} \)) | 0.15 Hz (inline ground motion) |
| Design earthquake frequencies (\( f_{\text{earthquake,2}} \)) | 0.36 Hz (Crossflow ground motion) |

**Table 6.2: Design parameters of a combination load for design case C and D**

Finally, design case E is created in order to understand an objective 2b. In this situation, VIV phenomenon shall be first created, then a subsea jumper system is disturbed by earthquake that has a frequency as same as lock-in frequency. A “sinusoidal model” is used to investigate this situation. Furthermore, crossflow VIV is considered because it reaches steady state much faster than inline VIV. As such, the designed parameters are current velocity at 0.68 m/s and earthquake frequency equal 0.36 Hz. All design parameters are summarized in Table 6.3.

<table>
<thead>
<tr>
<th>Design Parameter: Current</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design Current velocity</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Design Parameter: Earthquake</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design peak ground acceleration</td>
</tr>
</tbody>
</table>

| Design earthquake frequencies (\( f_{\text{earthquake}} \)) | 0.36 Hz |

**Table 6.3: Design parameters of a combination load for design case E**

One should be noted design case E is different from case B by using different ground acceleration model. In case B, earthquake load is gradually decreasing and die out from a subsea jumper system. Besides design case E, earthquake load is applied to system continuously.

Other general design parameters are summarized in Table 6.4.

<table>
<thead>
<tr>
<th>Design Parameter: Current and Fluid Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sea water density ( (\rho_{\text{sea}}) )</td>
</tr>
<tr>
<td>Stationary lift coefficient ( (C_{L0}) )</td>
</tr>
<tr>
<td>Stationary drag coefficient ( (C_{VD}) )</td>
</tr>
<tr>
<td>Strouhal number ( (St) )</td>
</tr>
<tr>
<td>Wake coupling coefficient</td>
</tr>
</tbody>
</table>
### Chapter 6: Dynamic Behavior under Combination Effect

<table>
<thead>
<tr>
<th>Design Parameter: Earthquake condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wake tuning parameter $(\varepsilon)$</td>
</tr>
<tr>
<td>Control earthquake parameter $(\alpha)$</td>
</tr>
<tr>
<td>$(b)$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Design Parameter: Dimension and Material Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of horizontal beam $(L_2)$</td>
</tr>
<tr>
<td>Length of vertical beam $(L_1$ and $L_3)$</td>
</tr>
<tr>
<td>Outer diameter $(OD)$</td>
</tr>
<tr>
<td>Wall thickness</td>
</tr>
<tr>
<td>Piping material</td>
</tr>
<tr>
<td>Young’s modulus $(E)$</td>
</tr>
<tr>
<td>Pipe density $(\rho_{steel})$</td>
</tr>
<tr>
<td>Poison ratio $(\nu)$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Design Parameter: Operation Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oil flow rate</td>
</tr>
<tr>
<td>Oil density $(\rho_{oil})$</td>
</tr>
</tbody>
</table>

**Table 6.4: Other general Design parameter of combination effect**
6.3 Dynamics behavior of subsea jumper under combination effect

Similar manner as done in previous chapter, dynamic motion of a subsea jumper system under combination effect is obtained by solving a subsea jumper under combination load model in section 6.1. The dynamic behavior results in term of a displacement-time history. The detail of solving method is given in appendix B.

In the previous section, the research objectives are introduced in order to selected proper design parameters. Each objective can be achieved by creating an appropriated circumstance. Thus, five design cases were established in order to accomplish the research objectives.

Firstly, a displacement time series of a subsea jumper under combination loads of current and earthquake is the first goal in this chapter in order to use as input data for fatigue analysis. Fatigue life-time of a subsea jumper should be considered in most critical case and also has high possibility of occurrence. Two design cases are initiated as Design case A, that is a situation of subsea jumper system under inline VIV and earthquake condition whereas combination of crossflow VIV and earthquake is analyzed in Design case B.

Besides, it is important to understand the influence of an earthquake to a vortex induce vibration phenomenon. According to a “subsea jumper under combination load model”, an earthquake load has an indirect effect to a wake. An earthquake first induces a subsea jumper to oscillate. Subsequence the wake is induced by subsea jumper movement. As such, two interesting issues are raised. The first one is to understand whether an earthquake can induce both subsea jumper and wake to enter synchronization or a VIV phenomenon. Thus, design case C and design Case D were constructed. Both design cases use same design current velocity but design earthquake frequencies are different. The second one is to understand whether earthquake can demolish a VIV phenomenon, then design case E was established by considering a combination effect of crossflow-VIV and earthquake load at the same frequency as lock-in crossflow frequency.

In the previous chapter, the highest resultant motions of a subsea jumper under steady current or earthquake was found at horizontal beam in every case. The dynamic behavior of a combination load is expected similar results as seen in steady current and earthquake condition. Thus, a dynamic response at middle of horizontal line is observed and given results in both inline and crossflow vibration whereas response of vertical lines are neglected in the results here.
Chapter 6: Dynamic Behavior under Combination Effect

DESIGN CASE A:
Design current: Steady current velocity at 0.275 m/s
Design earthquake: Simulation earthquake model with amplitude 3.63 $m/s^2$

Crossflow ground vibration frequency is 0.36 Hz.
Inline ground vibration frequency is 0.15 Hz.

A. Inline vibration at midpoint of horizontal beam

![Figure 6.2: Inline transverse displacement time-series at middle of horizontal beam in design case A](image)

B. Crossflow vibration at midpoint of horizontal beam

![Figure 6.3: Crossflow transverse displacement time-series at middle of horizontal beam in design case A](image)

C. Vortex lift coefficient at midpoint of horizontal beam

![Figure 6.4: Vortex lift coefficient at middle of horizontal beam in design case A](image)
DESIGN CASE B:

Design current: Steady current velocity at 0.68 m/s

Design earthquake: Simulation earthquake model with amplitude 3.63 $m/s^2$

Crossflow ground vibration frequency is 0.36 Hz.

Inline ground vibration frequency is 0.15 Hz.

A. Inline vibration at midpoint of horizontal beam

![Figure 6.5: Inline transverse displacement time-series at middle of horizontal beam in design case B](image)

B. Crossflow vibration at midpoint of horizontal beam

![Figure 6.6: Crossflow transverse displacement time-series at middle of horizontal beam in design case B](image)

C. Vortex lift coefficient at midpoint of horizontal beam

![Figure 6.7: Vortex lift coefficient at middle of horizontal beam in design case B](image)
Chapter 6: Dynamic Behavior under Combination Effect

DESIGN CASE C:

Design current: Steady current velocity at 0.4 m/s

Design earthquake: Sinusoidal earthquake model with amplitude $3.63 \, m/s^2$

Crossflow ground vibration frequency is 0.15 Hz.

Inline ground vibration frequency is 0.15 Hz.

A. Inline vibration at midpoint of horizontal beam

![Figure 6.8: Inline transverse displacement time-series at middle of horizontal beam in design case C](image)

B. Crossflow vibration at midpoint of horizontal beam

![Figure 6.9: Crossflow transverse displacement time-series at middle of horizontal beam in design case C](image)

C. Vortex lift coefficient at midpoint of horizontal beam

![Figure 6.10: Vortex lift coefficient at middle of horizontal beam in design case C](image)
D. Inline and crossflow motion in cross section plane of horizontal beam

Figure 6.11: Inline and crossflow motion in cross section plane of horizontal beam in design case C
Chapter 6: Dynamic Behavior under Combination Effect

DESIGN CASE D:

Design current: Steady current velocity at 0.4 m/s

Design earthquake: Sinusoidal earthquake model with amplitude $3.63 \ m/s^2$

Crossflow ground vibration frequency is 0.36 Hz.

Inline ground vibration frequency is 0.36 Hz.

A. Inline vibration at midpoint of horizontal beam

![Figure 6.12](image1.png)

**Figure 6.12:** Inline transverse displacement time-series at middle of horizontal beam in design case D

B. Crossflow vibration at midpoint of horizontal beam

![Figure 6.13](image2.png)

**Figure 6.13:** Crossflow transverse displacement time-series at middle of horizontal beam in design case D

C. Vortex lift coefficient at midpoint of horizontal beam

![Figure 6.14](image3.png)

**Figure 6.14:** Vortex lift coefficient at middle of horizontal beam in design case D
D. Inline and crossflow motion in cross section plane of horizontal beam

![Graph showing inline and crossflow motion](image)

Figure 6.15: Inline and crossflow motion in cross section plane of horizontal beam in design case D
DESIGN CASE E:

Design current: Steady current velocity at 0.68 m/s

Design earthquake: Sinusoidal earthquake model with amplitude 3.63 m/s²

Crossflow ground vibration frequency is 0.36 Hz.

Inline ground vibration frequency is 0.36 Hz.

A. Inline vibration at midpoint of horizontal beam

![Figure 6.16: Inline transverse displacement time-series at middle of horizontal beam in design case E](image)

B. Crossflow vibration at midpoint of horizontal beam

![Figure 6.17: Crossflow transverse displacement time-series at middle of horizontal beam in design case E](image)

C. Vortex lift coefficient at midpoint of horizontal beam

![Figure 6.18: Vortex lift coefficient at middle of horizontal beam in design case E](image)
D. Inline and crossflow motion in cross section plane of horizontal beam

Figure 6.19: Inline and crossflow motion in cross section plane of horizontal beam in design case E
Chapter 6: Dynamic Behavior under Combination Effect

6.4 Discussions

There are several important observations made from the dynamic responses of a subsea jumper as follows:

6.4.1 Resultant motion of subsea jumper under a combination effect

In design case A and B, a subsea jumper was firstly induced by steady current flow at lock-in velocity for inline VIV and crossflow VIV, respectively. Later, the system was disturbed by adding an earthquake load. The design earthquake load is an earthquake shock load. It exerts on a subsea jumper system in a short period, and then dies out from the system.

The dynamic response of design case A and B show that VIV process is fully developed in the first phase. A subsea jumper oscillated due to VIV. Here, it is called “VIV oscillation”. In the second phase, an earthquake shock was instantaneously added on a jumper. Consequence VIV oscillation was disturbed. The amplitude of vibration of a jumper increased until reaching the maximum point and then decreased gradually. After a certain period, the jumper motion became “VIV oscillation” again. Thus, one can conclude that earthquake shock is unable to deconstruct a VIV process. However, it creates a high response of subsea jumper in short period. This circumstance is interesting to investigate in terms of fatigue life time and is addressed in chapter 7.

6.4.2 Couple effect of VIV process and idealistic earthquake

As discussed in previously, the second objective of this chapter is “to understanding couple effect of VIV process and idealistic earthquake”. This was achieved by constructing design case C, D and E.

All three design cases (case C, D and E) use a sinusoidal model to characterize ground acceleration. This model assumes that earthquake is a continuous process with one frequency. It is obviously an idealistic model because nature of earthquake is a non-continuous phenomenon and scholastic process. However, it is still interesting to understand an effect of sinusoidal ground vibration to VIV process on a jumper system.

In order to investigate whether VIV process can be induced by an idealistic earthquake, design case C and D were established. In both situations, a subsea jumper system experience by steady current, velocity 0.4 m/s. According to results from chapter 4, lock-in current velocity for inline VIV is 0.275 m/s and for crossflow VIV is 0.68 m/s. As such, current velocity, equal 0.4 m/s, is unable to create a vortex induce vibration phenomenon on a subsea jumper. An idealistic earthquake, that has a design frequency equal to lock-in frequency, was also added on a jumper system in order to examine effect of an earthquake to VIV process.

The result from design case C shows that, subsea jumper oscillates at 0.15 Hz in inline direction as same as wake which is oscillating at 0.15 Hz. As a consequence, one can conclude that VIV phenomenon does occur by idealistic earthquake induction. There is another interesting aspect about amplitude of vibration. In the comparison of amplitude of vibration between purely idealistic earthquake (chapter 5) and combination load effect (case C), we found the amplitude of vibration in a combination case C is less than purely earthquake case. In conclusion, a hydrodynamic damping in flow condition is higher than in still water condition.
In design case D, a subsea jumper was induced by a constant current at 0.40 m/s and idealistic earthquake at frequency 0.36 Hz. The result shows that a subsea jumper oscillates at 0.36 Hz for crossflow vibration as same frequency as wake oscillation. It seems a crossflow VIV occurs because subsea jumper and wake are synchronizing. Further consideration on amplitude of crossflow vibration, a jumper oscillates at dimensionless amplitude 2.7 (or equal 0.95 m) by approximation. This amplitude is close to a combination dimensionless amplitude of 1) vertical earthquake, 0.27, 2) horizontal earthquake in crossflow direction, 2.4 and 3) crossflow VIV, 0.05 m. Thus, we can conclude that, crossflow VIV can be induced by an idealistic earthquake. In addition, the resultant amplitude of crossflow vibration under combination loads can be approximated by superposition. On the other words, these external loads are related in linear relation which is compiled to thesis assumption for combination effect.

In design case E, VIV process allowed to develop together with earthquake process. Earthquake load was modelled by sinusoidal function. Resultant motion of case E shows VIV process does not disturb by earthquake. Crossflow VIV is still process continuously. One can see that, results of design case E and D were more or less the same. There is a slight difference in amplitude of vibration because both design cases have a different damping force coefficient due to difference current velocity.
Part II

Fatigue life time Investigation

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Chapter 7

Fatigue Analysis

The main objective of this chapter is to introduce a fatigue analysis on subsea jumper system. There are various conditions that have already analyzed in previous chapters; however, a summary for all design cases, which are interesting for fatigue analysis, are presented here. This summary is provided in section 7.1.

Subsea jumper motions under various conditions are obtained by solving the "subsea jumper model" as illustrated in part I of this research. However, these results cannot be used directly as input for fatigue analysis because fatigue analysis associates with periodic stress in a structure. It is necessary to convert the transverse motion of a subsea jumper into bending stress. A conversion method was previously introduced in chapter 2. We review this again in section 7.2.

In order to estimate the fatigue life-time of a subsea jumper, Miner’s rule is used though this research. The main idea of Miner’s rule is that “Each structure has a fatigue capacity at each certain stress level. The ratio of number of load exert on structure to its fatigue capacity is called “Fatigue damage. Fatigue failure will occur on structure once accumulated fatigue damage equals one. Thus, fatigue damage has a linear relation with fatigue lifetime”. For a subsea jumper under external disturbances, fatigue damage due to earthquake and steady current is the main focus for calculating fatigue lifetime of a subsea jumper.

In accordance with Miner’s rule, a stress counting method was selected to analyze fatigue damage on a subsea jumper together with S-N curve. The result of fatigue damage on each design case is shown in section 7.3. Later section 7.4 presents results of fatigue lifetime based on Miner’s rule. Finally, some important aspects are discussed in section 7.5.
7.1 Fatigue design case

Although several design cases were investigated in Part I of this research, not all the cases are interesting in fatigue analysis. The main interest of fatigue lifetime estimation is the most critical and high possibility of occurrence. Thus, one can consider four cases separately.

Steady current situation

Current flow in a nearby seabed must be considered in subsea jumper design because it is rational to assume that there is flow past the subsea jumper system continuously with an infinite long period. Current velocity near the seabed is extremely low. Based on measurement data at a South East Asia location, the dominant range of velocity is 0.05 to 0.55 m/s, whereas the designed current velocity can reach 0.832 m/s based on probabilistic estimation (100 year return period). One can safely state that current velocity does not exceed 0.85 m/s. In chapter 4, the dynamic response of a jumper varies was analyzed within this design current velocity range. There are two current velocities that create a high jumper response due to a VIV phenomenon. The first creates an inline VIV at current velocity equal to 0.275 m/s and the second creates a crossflow VIV at a current velocity equal to 0.68 m/s. It is interesting to estimate fatigue lifetime based on inline VIV and cross flow VIV. As such, design case A considers subsea jumper under steady current purely at current velocity 0.275 m/s. Additionally, current velocity, 0.68 m/s is investigated in design case B

Combination situation

In reality, a subsea jumper rarely experience only an earthquake load because current flow is always present in the nearby seabed. Thus, the effect of an earthquake on a subsea jumper system should be considering together with current flow effect. This is called a “combination situation”.

Earthquake is an occasional phenomenon or a non-continuous phenomenon. It generally consists of many shocks, up to ten-thousand shocks per one earthquake. Each shock has a small period of ground vibration. As such, a sinusoidal model is not suitable for fatigue lifetime estimation. A “shock model” is recommended to simulate an earthquake load as a more realistic model. The result of a combination effect is given in terms of a subsea jumper lifetime reduction per one earthquake shock. 

According with seismic spectra in chapter 5, the critical earthquake frequencies on each vibration plane are known. Inline vibration plane has a critical earthquake frequency equal 0.15 Hz and crossflow vibration has a critical earthquake frequency equal 0.36 Hz. In order to create the most severe situation, an idealistic case is designed here, by applying the critical earthquake frequency on each vibration plane.

In addition, current effect should enhance the effect of an earthquake in order to obtain the most severe vibration on a subsea jumper. As such, lock-in current velocities are applied to the subsea jumper system. These velocities will create a VIV phenomenon on the jumper. As such, there are two designed current velocities, a combination situation constructed by adding earthquake into design case A and design case B and they are called design case C and design case D, respectively.
In conclusion, there are four design cases considered in this chapter. Table 7.1 shows a summary of all the design cases for the fatigue analysis. Only the main parameters are introduced in the table. Other design parameters are exactly the same as in the analysis in the previous chapter.

<table>
<thead>
<tr>
<th>Design Case</th>
<th>Condition</th>
</tr>
</thead>
</table>
| **A**       | Steady current: $V = 0.275 \text{ m/s}$  
              | Earthquake: No consideration |
| **B**       | Steady current: $V = 0.68 \text{ m/s}$  
              | Earthquake: No consideration |
| **C**       | Steady current: $V = 0.275 \text{ m/s}$  
              | Earthquake: Simulation model  
              | Inline earthquake frequency: 0.15 Hz  
              | Crossflow earthquake frequency: 0.36 Hz |
| **D**       | Steady current: $V = 0.68 \text{ m/s}$  
              | Earthquake: Simulation model  
              | Inline earthquake frequency: 0.15 Hz  
              | Crossflow earthquake frequency: 0.36 Hz |

Table 7.1: Summary of fatigue design cases
7.2 Bending stress-time history conversion

In this thesis, fatigue failure deals with bending stress on a subsea jumper. All results of the displacement time series of each design case that are obtained from dynamic behavior analysis, are required to convert into stress time history. Figure 7.1 shows an example of the relationship between transverse motion and bending stress in a crossflow vibration plane. This relation is valid for any arbitrary point along the horizontal line of a jumper system. One should note that it has the same relation for an inline vibration but the moment and stress plane are different. Thus, the expression only provides for a crossflow vibration.

\[
\sigma(x_2, t) = \frac{M(x_2, t) y}{I_{x_2}} = EI \frac{\partial^2 w(x_2, t)}{\partial x_2^2} \cdot \frac{y}{I_{x_2}} \quad (7.1)
\]

Where, \(\sigma\) indicate a bending stress at any arbitrarily point on horizontal beam. \(M\) is bending moment, \(EI\) is bending stiffness of beam, \(w\) is a transverse motion of beam. \(I\) is moment of inertia around neutral axis \(-X_2\) and \(y\) is distance to neutral axis of cross-section point. The maximum stress of pipe locates at the outer diameter or \(y\) equal outer pipe radius.

Again, we approximate the second order differential term by finite differential method. Estimation can be done by the following expression. Solving this equation, we obtain stress time history at each certain point.

\[
\frac{\partial^2 w(x_2, t)}{\partial x_2^2} = \frac{w_{i-1}(t) - 2w_i(t) + w_{i+1}(t)}{(\Delta x)^2} \quad (7.2)
\]

In accordance with the results of dynamic behavior of a subsea jumper in part I, we found that the middle point of horizontal beam is the most critical part of a subsea jumper system under steady current, earthquake and in combination effect. This is because it results in the highest oscillation for all cases. As such, stress time history conversion is applied only this point.
Chapter 7: Fatigue Analysis

7.3 Fatigue damage calculation

Accumulation of fatigue is called “Fatigue damage”. It comprises of two main components according to equation 2.19:

\[
\text{Fatigue Damage} = \sum_{i} n_i a_i \quad (7.3)
\]

The first component, \( n_i \), is number of cycles on each stress level occurs while loads apply to system. This can be achieved by counting the stress range on stress time series of each cyclic load. Second component, \( N_i \), is materials lifetime. This indicates a fatigue capacity of each material by representing the number of cycles on each stress level. The calculation method for two main components is given in this section.

7.3.1 S-N curve

This component can be approximated by S-N curve or Basquin Relation equation.

\[
S^k \cdot N = a \quad (7.4)
\]

Where, \( N \) is fatigue capacity unit in cycles, \( S \) indicates stress level, \( k \) and \( a \) are specific parameters which depend strongly on material and welding detail. Design fatigue parameter used in this research based on the available project data. In addition, the design values assume to be the same in every design case. A summary of fatigue design values are given in Table 7.2.

<table>
<thead>
<tr>
<th>Design Parameter: Dimension and Material Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
</tr>
<tr>
<td>K</td>
</tr>
<tr>
<td>Fatigue designed life time</td>
</tr>
</tbody>
</table>

Table 7.2: Design parameters for fatigue analysis

One should be noted that S-N curve is constructed by a fatigue experiment. In the laboratory, applied cyclic load on a specimen is typically a zero mean load. In other words, this load varies around zero level and causes a zero mean stress level on a specimen. It is the same case as seen in a crossflow VIV where the jumper oscillates around a zero mean displacement. As the result, stress in jumper is obviously a zero mean stress level.

Unlike an inline VIV case, a subsea jumper is firstly pushed by drag force. Then it vibrates around non-zero mean displacement. This situation creates non-zero mean stress level on the subsea jumper. Stress amplitude alone is insufficient to predict fatigue lifetime by using S-N curve as a consequence in the original S-N curve is not valid for inline VIV.

In this situation, mean stress level and stress amplitude are necessary to predict equivalent stress amplitude in order to be compatible with S-N curve. Thus a correlation method is required to find this equivalent stress amplitude before using S-N curve to predict fatigue lifetime. The most widely accepted
method is [Goodman,1899]. This method provides relations between non-zero mean stress level and equivalent stress level on S-N curve. The expression is written as follows:

\[
\frac{S_a}{S_e} + \frac{S_m}{S_u} = 1
\]  

(7.4)

Where, \(S_a\) represents stress amplitude, \(S_m\) indicates a mean stress level and \(S_u\) is ultimate stress of material. The equivalent stress level or Stress life fatigue, \(S_e\) can be calculated if mean-stress and stress amplitude are known.

### 7.3.2 Counting method

A stress range counting method is widely used for fatigue life assessment of machine components or structures, especially under constant amplitude stress. According to results in part I of this thesis, one can expect that the bending stress of subsea jumper under steady current and/or earthquake condition, is mostly a constant amplitude. Thus, a stress range counting method works properly for this thesis.

The function of a stress counting method is extraction of the number of cycles from load, strain or stress under different amplitude and mean level. These results allow one to calculate expected fatigue lifetime according to Miner’s rule.

In addition, designed stress cycles that are captured by counting method should cover the whole stress range profile. It is fortunately found in the results of displacement time series that have an constant amplitude profile. It can also be found in stress time series. As a consequence, design stress cycle duration can be captured in a short period.

In general, fatigue lifetime estimation considers periodic stress in a material at steady state condition. It is straightforward for design case A and B, which are under steady current only. Cyclic stress is counted when VIV oscillation is fully developed. For inline VIV, a steady state starts at 1500 seconds whereas it starts at 50 seconds in crossflow VIV.

For design case C and D, an earthquake is exerted into the system. The results of displacement time series are analyzed in chapter 6. We know that the response of a jumper due to a sudden earthquake load is only in over a short period; however, it has high amplitude. Later, the jumper motion becomes a steady state condition as same as in a VIV case. In these cases, the counting method captures the stress range at a transient period in order to understanding the relation of fatigue lifetime on one shock of earthquake.

### 7.3.3 Fatigue damage result

There are three type of data provided on each design case namely: 1) bending stress conversion, 2) histogram of stress counting and 3) fatigue damage result according to equation 7.3. One should note that only results of fatigue damage on each design case are provided in this chapter. Details of fatigue damage calculation are provided in Appendix C.
Chapter 7: Fatigue Analysis

DESIGN CASE A:

Design current: Steady current velocity at 0.275 m/s

Design earthquake: No consideration

A. Bending stress conversion
   a. Inline vibration

   ![Figure 7.2: Displacement-stress conversion of inline vibration for design case A](image)

   ![Figure 7.3: Displacement-stress conversion of crossflow vibration for design case A](image)

b. Crossflow vibration
B. Stress counting result

![Histogram of inline stress counting for designed case A](image)

![Histogram of crossflow stress counting for designed case A](image)

Figure 7.4: Histogram of stress counting for design case A

C. Fatigue damage calculation for inline motion and crossflow motion.

<table>
<thead>
<tr>
<th>Fatigue damage per cycles</th>
<th>Inline Vibration</th>
<th>Crossflow Vibration</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.44E-18</td>
<td>6.71E-12</td>
</tr>
</tbody>
</table>

Table 7.3: Fatigue damage of design case A for inline vibration and crossflow vibration
DESIGN CASE B:

Design current:  Steady current velocity at 0.68 m/s

Design earthquake:  No consideration

A. Bending stress conversion
   a. Inline vibration

   ![Figure 7.5: Displacement-stress conversion of inline vibration for design case B](image)

b. Crossflow vibration

   ![Figure 7.6: Displacement-stress conversion of crossflow vibration for design case B](image)
B. Stress counting result

![Histogram of inline stress counting for designed case B](image1)

![Histogram of crossflow stress counting for designed case B](image2)

Figure 7.7: Histogram of stress counting for design case B

C. Fatigue damage calculation for inline motion and crossflow motion.

<table>
<thead>
<tr>
<th>Fatigue damage per cycles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inline Vibration</td>
</tr>
<tr>
<td>Crossflow Vibration</td>
</tr>
</tbody>
</table>

Table 7.4: Fatigue damage of design case B for inline vibration and crossflow vibration
DESIGN CASE C

Design current:  Steady current velocity at 0.24 m/s

Design earthquake:  Simulation earthquake model with amplitude 3.63 $m/s^2$

Crossflow ground vibration frequency is 0.34 Hz.

Inline ground vibration frequency is 0.13 Hz.

A. Bending stress conversion
   a. Inline vibration

![Image of inline vibration conversion](image1)

*Figure 7.8: Displacement-stress conversion of inline vibration for design case C*

b. Crossflow vibration

![Image of crossflow vibration conversion](image2)

*Figure 7.9: Displacement-stress conversion of crossflow vibration for design case C*
B. Stress counting result

Figure 7.10: Histogram of stress counting for design case C

C. Fatigue damage calculation

<table>
<thead>
<tr>
<th></th>
<th>Fatigue damage per cycles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inline Vibration</td>
<td>7.84E-05</td>
</tr>
<tr>
<td>Crossflow Vibration</td>
<td>2.28E-06</td>
</tr>
</tbody>
</table>

Table 7.5: Fatigue damage of design case C for inline vibration and crossflow vibration
Chapter 7: Fatigue Analysis

**DESIGN CASE D**

Design current: Steady current velocity at 0.68 m/s

Design earthquake: Simulation earthquake model with amplitude 3.63 $m/s^2$

Crossflow ground vibration frequency is 0.34 Hz.

Inline ground vibration frequency is 0.13 Hz.

A. Bending stress conversion
   a. Inline vibration

   ![Figure 7.11: Displacement-stress conversion of inline vibration for design case D](image)

b. Crossflow vibration

   ![Figure 7.12: Displacement-stress conversion of crossflow vibration for design case D](image)
B. Stress counting result

![Histogram of inline stress counting for designed case D](image1)

![Histogram of crosflow stress counting for designed case D](image2)

**Figure 7.13: Histogram of stress counting for design case D**

C. Fatigue damage calculation for inline motion and crossflow motion.

<table>
<thead>
<tr>
<th></th>
<th>Fatigue damage per cycles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inline Vibration</td>
<td>7.43E-05</td>
</tr>
<tr>
<td>Crossflow Vibration</td>
<td>2.65E-06</td>
</tr>
</tbody>
</table>

**Table 7.6: Fatigue damage of design case D for inline vibration and crossflow vibration**
7.4 Fatigue life time analysis

Expected fatigue lifetime can be achieved by using Miner’s rule. Theory of Miner’s rule is “Any system under variable load amplitude will damage by fatigue at moment accumulation of fatigue life fatigue damage equal to one or express as follows:

$$\sum D_i = 1$$  \hspace{1cm} (7.5)

In a previous section, fatigue damage per stress cycle, \(D_i\) was calculated on each design case. Thus, expected fatigue lifetime can simply be calculated with the following equation:

$$Fatigue\ life\ time = \frac{1}{D_i} \cdot Design\ period$$ \hspace{1cm} (7.6)

The results of fatigue lifetime estimation for design case A and B are provided in Table 7.7 and 7.8 respectively.

### DESIGNED CASE A:

<table>
<thead>
<tr>
<th></th>
<th>Inline Vibration</th>
<th>Crossflow Vibration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fatigue damage</td>
<td>1.44E-18 cycles</td>
<td>Fatigue damage</td>
</tr>
<tr>
<td>Fatigue capacity</td>
<td>6.95E+17 sec</td>
<td>Fatigue capacity</td>
</tr>
<tr>
<td>Fatigue life estimation</td>
<td>1.39E+20 day</td>
<td>Fatigue life estimation</td>
</tr>
<tr>
<td></td>
<td>1.61E+15 day</td>
<td>Fatigue life estimation</td>
</tr>
<tr>
<td></td>
<td>4.40E+12 years</td>
<td>Fatigue life estimation</td>
</tr>
<tr>
<td>Fatigue life time estimation of jumper under inline VIV</td>
<td>1.39E+20 sec</td>
<td>Fatigue life time estimation of jumper under crossflow VIV</td>
</tr>
</tbody>
</table>

Table 7.7: Fatigue lifetime estimation of subsea jumper system in design case A

### DESIGNED CASE B:

<table>
<thead>
<tr>
<th></th>
<th>Inline Vibration</th>
<th>Crossflow Vibration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fatigue damage</td>
<td>6.58E-16 cycles</td>
<td>Fatigue damage</td>
</tr>
<tr>
<td>Fatigue capacity</td>
<td>1.52E+15 sec</td>
<td>Fatigue capacity</td>
</tr>
<tr>
<td>Fatigue life estimation</td>
<td>7.60E+16 day</td>
<td>Fatigue life estimation</td>
</tr>
<tr>
<td></td>
<td>8.80E+11 day</td>
<td>Fatigue life estimation</td>
</tr>
<tr>
<td></td>
<td>2.41E+09 years</td>
<td>Fatigue life estimation</td>
</tr>
<tr>
<td>Fatigue life time estimation of jumper under crossflow VIV</td>
<td>7.60E+16 day</td>
<td>Fatigue life time estimation of jumper under crossflow VIV</td>
</tr>
</tbody>
</table>

Table 7.8: Fatigue lifetime estimation of subsea jumper system in design case B

In case of a combination between current and earthquake effect, it is suitable to give results in terms of a reduction of fatigue lifetime per one main earthquake shock as provided in Table 7.9.

<table>
<thead>
<tr>
<th>Design case</th>
<th>Fatigue lifetime reduction (percent per one earthquake)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Inline Vibration</td>
</tr>
<tr>
<td>C</td>
<td>0.0078 %</td>
</tr>
<tr>
<td>D</td>
<td>0.0074 %</td>
</tr>
</tbody>
</table>

Table 7.9: Fatigue lifetime reduction per one main earthquake shock
7.5 Discussions

7.5.1 Vortex induce vibration

Displacement of subsea jumper results in high amplitude of vibration while VIV is taking place. Regarding equation 7.1, it creates high bending stress on subsea jumper that causes fatigue lifetime reduction. According to design current velocity, it is possible to induce subsea jumper system into VIV phenomenon when current velocity reaches 0.275 m/s, for inline VIV and 0.68 m/s, for crossflow VIV. Thus, it is interesting to analyze fatigue lifetime of a jumper system under inline and crossflow VIV conditions. These conditions were established in design case A and B.

Design case A presents inline VIV on subsea jumper. Fatigue lifetime was estimated by Miner’s rule. Fatigue lifetime estimation results in 9.45E+05 years, which is much longer than designed lifetime of a jumper, 30 years.

In design case B, fatigue of subsea jumper in crossflow VIV is analyzed and results in 12.5 years which is lower than the designed lifetime by three times. However, one should note that the results of fatigue analysis means that the subsea jumper is able to withstand a crossflow VIV for 12.5 years. A crossflow VIV required current velocity equal 0.68 m/s which is extremely high. It has a possibility of occurrence of only 0.008. In the other words, it has a chance to occur only hundred days in thirty years of subsea jumper designed lifetime. As such, it is possible to conclude that subsea jumper is safe to operate from current load in the new development field in Myanmar. Nevertheless, the same designed subsea jumper may not suitable for other locations that have a higher current velocity.

In addition, the results of fatigue lifetime estimation from both design case A and B show crossflow VIV is more severe than inline VIV from a fatigue point of view.

7.5.2 Combination effect (steady current and earthquake)

The dynamic behavior of subsea jumper under combination effect was discussed in chapter 6. It shows that earthquake shock and steady current has no couple effect based on a “subsea jumper under combination load model” (subsea jumper model couple with wake oscillator and inertia load). An earthquake disturbs the motion of a jumper only over a short period. After the earthquake vanishes, subsea jumper resumes into steady state condition again. Here, only two cases of combination effect which inline and crossflow VIV take place are under consideration. They are presented in design case C and D respectively.

A transverse motion and bending stress are related in accordance with equation 7.1. A peak amplitude of motion results in peak of stress level on subsea jumper. Both design cases show the same results. At the moment earthquake is occurring, the subsea jumper responds with high motion that leads to high bending stress. After the earthquake is removed from system, the jumper returns to a VIV oscillation condition again. In these two design cases, only the disturbance period during an earthquake is captured and analyzed for fatigue lifetime effect.

Designed case C presents a combination effect of inline VIV and earthquake. Cyclic stress range during earthquake period was counted and used to calculate fatigue damage of a jumper which results in 7.84E-05 for inline vibration. This means one main earthquake occurrence during inline VIV reduces subsea
jumper fatigue lifetime equals to 0.0078%. Similarly in designed case D, combination effect of crossflow VIV and earthquake, fatigue damage equals to 7.43E-05 leading to fatigue lifetime reduction by 0.0074% per one main earthquake occurrence.

According to the results of the combination effect, one can see that earthquake load can have a dominant effect on a subsea jumper system. Both inline VIV and crossflow VIV have a slightly different in lifetime reduction. Response in inline direction of combination effect under inline VIV is greater than combination load under crossflow VIV. In contrast, response in crossflow direction of combination effect under crossflow VIV is more severe than inline VIV case. Thus, one can conclude that combination loads from an earthquake has a dominant influence on a subsea jumper system no matter how strong the current flow.

Nevertheless, an analysis of combination effect in this chapter only considered the main earthquake or maximum ground motion amplitude in the earthquake process. In general, an earthquake process contains numerous ground vibrations that have smaller amplitude than the main earthquake and are called “foreshocks” if they occur before the main earthquake and called “aftershock” if they occur after the main earthquake. An earthquake is usually comprised of thousand shocks to ten thousand shocks. As such, number of shocks (foreshocks and aftershocks) should be taken into account in fatigue lifetime analysis. If it is assumed that every shock has the same amplitude of vibration as the main earthquake, then a subsea jumper can resist a combination effect as in design case C and D for 13,000 shocks; however, one should be emphasis that shocks are generally smaller than main earthquake. In other words, the subsea jumper system should have more capacity than the provided results.

According to Table 2.2, frequency of occurrence of earthquake, we found that earthquake magnitude between 7.0 to 7.9 Richter has a possibility of occurring only 10 - 20 times per year. Thus, the designed earthquake shock is expected to be found only 600 times over the designed lifetime of a subsea jumper. One can conclude that earthquake from a nearby fault does not influence fatigue failure to a subsea jumper at the new development field in the Andaman Sea.
Chapter 8

Fatigue Improvement

As discussed so far, the results of the dynamic response of subsea jumper under external disturbances and its corresponding fatigue lifetime need be analyzed. Only two external loads are considered in this thesis, those from earthquakes and a steady current.

An earthquake phenomenon contains one main earthquake and numerous small period ground vibrations, with up to thousand to ten-thousand instances of “shocks”. Both the main earthquake and shocks influence and cause the subsea jumper to oscillate in a high amplitude motion. However, the resultant bending stress does not exceed subsea jumper yield stress. While the subsea jumper system does not fail due to ductile failure, fatigue failure plays an important role in the subsea jumper system due to the large number of shocks per one earthquake phenomenon.

A steady current is considered as a continuous phenomenon. It is a main factor for fatigue lifetime of a subsea jumper. Fatigue failure of the designed subsea jumper under a mild current condition, with a current velocity below 0.68 m/s is impossible. However, results are different in more a severe current condition, with current velocity over 0.68 m/s. This condition causes a crossflow VIV, which leads to fatigue lifetime over a shorter than designed lifetime. Thus, fatigue lifetime needs to be improved. This chapter presents several methods to improve subsea jumper fatigue lifetime.

8.1 Subsea jumper dimensions

The dynamic behavior of a subsea jumper under external loads results in high motion and bending stress when its natural frequency is close to load frequency and load distribution along the jumper that is similar to one of jumper mode shape. In this section, we focus mainly on the natural frequency factor.

Subsea jumper natural frequencies are influenced by subsea jumper dimensions. Here, we focus on two main parameters: 1) diameter and 2) length of subsea jumper. Thus, one way to control the dynamic behavior of a subsea jumper is to design subsea jumper dimensions in order to obtain a natural frequency far from external load frequencies, such as for an earthquake load and VIV lock-in region. Both loads have characteristics in low frequencies. An earthquake load has a dominant frequency in a range 0-10 Hz, whereas the designed steady current load has a dominant frequency below 0-0.5 Hz.

In theory, selecting proper subsea jumper dimensions can prevent a resonance phenomenon for both earthquake load and current load. However, subsea jumper, especially in a U-inverse case, also has a low natural frequency in the first orthogonal modes. Thus, it is difficult to select jumper dimensions in order to obtain natural frequencies higher than 10 Hz (earthquake load frequency range); however, this is possible for a current load. As such, increasing the natural frequency of a subsea jumper so that it is larger than designed current load frequencies will improve subsea jumper fatigue lifetime.

In this section, three different dimension of subsea are compared their fatigue lifetime against each other. For design case A, a subsea jumper has a dimension exactly the same as the basis designed in this thesis, with a 50 meter length in the horizontal beam and diameter equal to 14 inches. In design case B, a
subsea jumper also has 50 meter length, but a diameter equal to 20 inches. Finally, a subsea jumper is designed with only a 30 meter length with a 14 inch diameter in design case C. Table 8.1 provides a summary of each set of design dimensions and their corresponding natural frequencies in first mode of vibration and Figure 8.1 shows a comparison between the natural frequencies of a subsea jumper on each designed dimension and the current load frequencies range.

<table>
<thead>
<tr>
<th>Design Case</th>
<th>$L_H$ (m)</th>
<th>$L_V$ (m)</th>
<th>Dia (inch)</th>
<th>Natural Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>50</td>
<td>10</td>
<td>14</td>
<td>0.1543</td>
</tr>
<tr>
<td>B</td>
<td>50</td>
<td>10</td>
<td>20</td>
<td>0.3611</td>
</tr>
<tr>
<td>C</td>
<td>30</td>
<td>10</td>
<td>14</td>
<td>0.4607</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.3687</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.6202</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.1580</td>
</tr>
</tbody>
</table>

Table 8.1: Natural frequency of first mode of vibration for different dimensions

In Figure 8.1, one can see that the natural frequency of each design case for the 1st mode of inline vibration changes significantly. The first natural frequency of inline vibration in case B and C are outside the measurements of the current data region. This results in a much lower possibility of inline VIV occurrence than case A. This is because a higher load frequency (higher current velocity) has a lower possibility of occurrence as a consequence of a lower possibility of synchronization between the current load and subsea jumper system. Thus, one can conclude that the fatigue lifetime of a subsea jumper is improved in view of inline VIV.
Similarly, one can expect that design case B and C have longer fatigue lifetime than designed case A in view of crossflow VIV. This is because the first natural frequencies of crossflow vibration in design case B and C are outside of the 100 year return period of extreme current data. As such, VIV from crossflow vibration will not occur. In addition, we know from chapter 7 that crossflow VIV has a dominant effect on the fatigue lifetime of a subsea jumper. Thus, design case B and C can significantly improve the fatigue lifetime of a subsea jumper. Table 8.2 shows a comparison of the fatigue lifetime on each design case at lock-in frequency of crossflow VIV for design case A or current velocity at 0.68 m/s.

<table>
<thead>
<tr>
<th>Design Case</th>
<th>Fatigue Lifetime of Jumper under Steady Current 0.68m/s (year)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>12.5</td>
</tr>
<tr>
<td>B</td>
<td>67,467</td>
</tr>
<tr>
<td>C</td>
<td>2,379,022</td>
</tr>
</tbody>
</table>

Table 8.2: Fatigue lifetime comparison for different dimensions

In this aspect, one recommendation for the subsea jumper system design is to avoid long routing for subsea jumpers because external loads near the seabed have a dominant frequency at low values. A longer subsea jumper creates lower natural frequency of the system that is closer to load frequency and where a resonance phenomenon can occur. As such, a longer subsea jumper would have a lower fatigue lifetime in the same environment. For situations where a long-route subsea jumper is required to interface between subsea infrastructures because they are located far away from each other, it is recommended that the subsea jumper system be broken up into several jumper spools and connected by a subsea hub as shown in Figure 8.2.

Figure 8.2: Recommended jumper routing design for long interfacing subsea equipment broken into shorter spool in order to avoid inline and crossflow VIV due to steady current near the seabed

### 8.2 Subsea jumper configuration

Although there are several configurations for subsea jumpers used in the oil and gas industry, this thesis only considers a symmetric U-inverse shape (left and right vertical beams have same length). In this section, we consider the effect of an asymmetric configuration of a subsea jumper on dynamic behavior and fatigue lifetime. Three configurations are considered in this section. Each subsea jumper is 14 inches in diameter. Figure 8.3 shows a configuration of a symmetric and asymmetric subsea jumper in a U-
Chapter 8: Fatigue Improvement

inverse configuration and Table 8.3 presents the corresponding natural frequency of the first mode of vibration for each configuration.

![Figure 8.3: Configuration of subsea jumper (A) Symmetric U-inverse shape and (B) Asymmetric U-inverse shape](image)

<table>
<thead>
<tr>
<th>Design Case</th>
<th>$L_{VL}$ (m)</th>
<th>$L_{HI}$ (m)</th>
<th>$L_{VR}$ (m)</th>
<th>First inline mode of vibration</th>
<th>First mode of crossflow vibration</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>10</td>
<td>50</td>
<td>10</td>
<td>0.1543</td>
<td>0.3687</td>
</tr>
<tr>
<td>B</td>
<td>5</td>
<td>50</td>
<td>10</td>
<td>0.2099</td>
<td>0.3891</td>
</tr>
<tr>
<td>C</td>
<td>1</td>
<td>50</td>
<td>10</td>
<td>0.2613</td>
<td>0.4080</td>
</tr>
</tbody>
</table>

Table 8.3: Natural frequency of first mode of vibration for different configurations

Similar to the previous section, a higher order of asymmetry in a jumper configuration results in a higher natural frequency for the subsea jumper system. Thus, the dynamic behavior of a subsea jumper is improved by shortening vertical line. This results in a longer fatigue lifetime, especially in crossflow VIV condition.

8.3 Internal flow

An internal fluid has a great effect on the dynamic behavior of subsea jumper. The effects of internal flow were described in an equation of motion for a subsea jumper model in chapter 3. A conveying fluid creates three internal forces on a jumper system. These forces contain two main parameters: fluid mass and flow velocity. As such, the natural frequency of a subsea jumper can be adjusted by designing the flow rate of the internal flow or changing inside fluid. Table 8.4 shows a comparison table for the natural frequency of a subsea jumper with various flow rates of crude oil inside.

<table>
<thead>
<tr>
<th>Design Case</th>
<th>Flow Rate (MMbbl/day)</th>
<th>Natural Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>First inline mode of vibration</td>
<td>First mode of crossflow vibration</td>
</tr>
<tr>
<td>A</td>
<td>3</td>
<td>0.1543</td>
</tr>
<tr>
<td>B</td>
<td>2</td>
<td>0.2441</td>
</tr>
<tr>
<td>C</td>
<td>1</td>
<td>0.2840</td>
</tr>
</tbody>
</table>

Table 8.4: Natural frequency of first mode of vibration on different flow rate of crude oil

Similar to the previous section, one can see that a lower flow rate of internal fluid results in a higher natural frequency. Thus, increasing production by increasing flow rate might reduce the fatigue lifetime of a subsea jumper due to current load because the decreasing natural frequency of a subsea jumper creates a higher probability of crossflow VIV occurrence.
Although this research assumes that the internal fluid is a single phase of crude oil which flows in a uniform pattern with a constant flow rate, the subsea jumper is generally conveying a multiphase flow. In some field development, slug flow can also be experienced and is another concern for fatigue failure. Slug flow can create a cyclic shock load to the subsea jumper system by attacking an elbow or connection joint of a jumper. In subsea jumper design, one should ensure that the period of shock load by slug flow is not close to the natural period of a subsea jumper in order to prevent a high dynamic response from the subsea jumper.

The period of shock load can be approximated by the length of each section of a jumper system and slug velocity. Slug velocity varies with operation flow rate; typically, its velocity is assumed to be equal to internal production flow. As such, the adjustment of production flow rate effects the period of shock load which might induced fatigue failure on a subsea jumper. Table 8.5 represents a summary of periods of shock load on the vertical and horizontal line with various operation flow rates and corresponding schematic diagram is provided in Figure 8.4. In addition, the calculation is based on the original dimension and configuration of subsea jumper as used through the thesis.

<table>
<thead>
<tr>
<th>Design Case</th>
<th>Flow Rate (MMbbl/day)</th>
<th>Vuleral line Period (s)</th>
<th>Vertical line frequency (Hz)</th>
<th>Horizontal line Period (s)</th>
<th>Horizontal line frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>3</td>
<td>0.15</td>
<td>6.63</td>
<td>0.75</td>
<td>1.32</td>
</tr>
<tr>
<td>B</td>
<td>2</td>
<td>0.22</td>
<td>4.42</td>
<td>1.13</td>
<td>0.88</td>
</tr>
<tr>
<td>C</td>
<td>1</td>
<td>0.45</td>
<td>2.21</td>
<td>2.26</td>
<td>0.44</td>
</tr>
</tbody>
</table>

Table 8.5: Periods of slug load vary with various operation flow rates

Apart from adjustment of the internal flow rate, subsea configuration and dimension also have a great effect on fatigue failure due to slug load. This is because changes in the length of horizontal and vertical cause a shift of slug load frequency, which might be close to that of natural frequency.

According to section 8.1 and 8.2, the fatigue lifetime of subsea jumper can be improved by shortening the length of the horizontal and vertical section. However, the natural frequency of the new adjustment of subsea jumper length needs to be checked with shifting frequency of shock load due to slug flow. One should emphasize that slug load has the greatest influence on crossflow vibration accordance with the load direction. Table 8.6 presents a comparison of slug load frequency with closest crossflow natural frequency of each designed subsea jumper. This calculation is based on a crude oil flowrate of 3 million barrels per day.
Chapter 8: Fatigue Improvement

<table>
<thead>
<tr>
<th>Design Case</th>
<th>$L_{VL}$ (m)</th>
<th>$L_{H}$ (m)</th>
<th>Slug Load Frequency (Hz)</th>
<th>Closest Crossflow Natural Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>10</td>
<td>50</td>
<td>6.63</td>
<td>1.32</td>
</tr>
<tr>
<td>B</td>
<td>5</td>
<td>50</td>
<td>13.26</td>
<td>6.63</td>
</tr>
<tr>
<td>C</td>
<td>1</td>
<td>50</td>
<td>66.30</td>
<td>6.63</td>
</tr>
<tr>
<td>D</td>
<td>10</td>
<td>30</td>
<td>6.63</td>
<td>2.21</td>
</tr>
</tbody>
</table>

Table 8.6: Comparison of slug load frequency with closest crossflow natural frequency of each design case

One can see from Table 8.6 that design case 2 and 3 are definitely not recommended because two natural frequencies of crossflow vibration are close to the slug load frequency and may cause severe crossflow vibration in a subsea jumper system.

8.4 Material selection

In general, a subsea jumper spool is typically made from carbon steel grade B to grade X70; however, more robust materials are an alternative choice to improve fatigue lifetime for a subsea jumper system. Higher fatigue resistance material has a higher capacity to resist cyclic load exerted on the system. Material selection thus needs to be considered along with other factors, for example economic, construction and installation method, property of fluid contain inside piping, surrounding or environment effect to subsea jumper, etc.

8.5 Subsea jumper fabrication

Guidance on fatigue design for marine pipeline and risers are provided in BS7608:1993 and DNV-RP-C203, DNV-RP-C203 and addresses specifically girth welding in a pipeline which categorized into five classifications for fatigue design of girth welds. These are as follows:

- Class C: Double-sided welds subsequently ground flush and proven to be free from significant welding flaws.
- Class D: Down-hand double-sided welds except those made by submerged arc welding (SAW)
- Class E: SAW and positional welds made by any process.
- Class F: Single-side welds made on permanent backing.
- Class F2: Single-side welds made without backing, concerning over joint misalignment and poor weld root conditions.

Each classification has a different fatigue behavior that can represent in S-N curve based on BS7608. Figure 8.7 shows S-N curve of class C to F. One can see that fatigue capacity decrease from C to F for the same stress level. Thus, selection of welding joint for a jumper spool is essential to the fatigue lifetime of a subsea jumper system.
8.6 Surface treatment

In general, fatigue failure or fatigue cracks start at the surface of a material. As such, surface condition plays an important role for fatigue lifetime capacity of a subsea jumper system. Sources of fatigue problem that relate to surface condition consist of corrosion pits, fretting corrosion, nicks and dents. Good surface treatment can improve the fatigue capacity of a subsea jumper system, especially for high-cycle load fatigue.

Surface treatment can be done during the production process of material or during construction. The purpose of surface treatment are: 1) protection against corrosion, 2) improvement of fatigue properties, 3) ratification of a poor surface quality, 4) improved wear resistance and 5) surface appearance. Thus, quality control of surface treatment during production and construction is essential to the fatigue lifetime of system.

Nowadays there are a variety of methods for surface treatment applied to a material. Surface treatment methods are mostly associated with three properties of material surface layers, namely 1) fatigue resistance surface layer material, 2) surface roughness and 3) residual stress in a surface layer. After completion of surface treatment on material, a structure can have an improved fatigue lifetime and increased material fatigue limit.
Chapter 9

Conclusions

There are two main parts in this thesis. Part I deals with the dynamic behavior of a subsea jumper under external loads: a steady current and earthquake. A subsea jumper model together with wake-oscillator model (steady current load) and inertia load model (earthquake) are used to simulate crossflow and inline vibration of a subsea jumper in U-inverse shape. The simulation provides the resultant motion of a jumper in terms of displacement time series. These are used as input data for fatigue analysis which is the topic of Part II.

9.1 Main conclusions

Part I: Dynamic behavior of subsea jumper under external loads

It is found that a U-inverse configuration has a direct influence on the dynamic behavior of a subsea jumper, especially in terms of natural frequency. Its first natural frequency is extremely low and corresponded to inline vibration. In addition, the dimensions of a subsea jumper are also a main factor: a longer route and smaller diameter cause a lower natural frequency.

The dynamic behavior of a subsea jumper under current flow can be simulated by using a "subsea jumper under current load model". This model is established by coupling a subsea jumper model with wake oscillator model. It results the dynamic responses of a jumper in terms of a response spectrum for both inline and crossflow vibration. As such, lock-in current velocity (or lock-in frequency) can be predicted. An inline VIV is found at current velocity equals to 0.275 m/s whereas current velocity equals to 0.68 m/s induced a crossflow VIV. The measurement data shows that higher current velocity has a lower possibility of occurrence. As a result, inline VIV has higher possibility of occurrence than cross VIV. However, the amplitude of vibration of subsea jumper under crossflow VIV is larger than inline VIV. Thus, both inline and crossflow VIV are significant for subsea jumper lifetime investigation.

The dynamic behavior of a subsea jumper during earthquake can be simulated by using a "subsea jumper under earthquake load model". The earthquake load is modeled by using an inertia load model (mass times ground acceleration). There are two ground acceleration models used to describe earthquakes in different ways. The first model assumes an earthquake as a continuous process and modeling ground acceleration as a simple sinusoidal function. It is called "sinusoidal model". This model is used to analyze a seismic spectrum which results in terms of the amplitudes of vibration for three different earthquake directions: one vertical and two horizontals. The spectrum shows that subsea jumper is most sensitive to horizontal ground vibration in the inline direction. The second model is considered an earthquake load as shock load. It is called "simulation model". This is modelled by simulating an earthquake characteristic from measurement data of ground acceleration. This model provides a more realistic dynamic response of a subsea jumper under earthquake load.

To consider an effect of combination of loads from a steady current and earthquake, the dynamic behavior of a subsea jumper is analyzed by using a "subsea jumper under combination load model". This model is constructed by coupling: 1) subsea jumper model. 2) wake oscillator and 3) inertia load. However, two earthquake models are used for different objectives as follows:
Chapter 9: Conclusion

- Analysis of the dynamic behavior of a subsea jumper under combination loads for fatigue analysis. An earthquake is a non-continuous process, it is reasonable to consider this in a simulation model. The model provides a more realistic response of a subsea jumper under one earthquake shock.

- Analysis of the dynamic behavior of subsea jumper under combination loads for study the effect of earthquakes to VIV phenomenon. In this study, earthquake assumes to be a continuous process. A sinusoidal model is used. The results of analysis show that an idealistic earthquake does not demolish the VIV phenomenon. However, it can induce a VIV phenomenon on a subsea jumper if the ground motion oscillates at the same frequency as the lock-in frequency.

Part II: Fatigue analysis

In this research, the fatigue lifetime of a subsea jumper is estimated under four different situations:

- Design case A: inline VIV condition
- Design case B: crossflow VIV condition
- Design case C: inline VIV and one shock of earthquake
- Design case D: crossflow VIV and one shock of earthquake

In design case A and B, subsea jumper experienced only a steady current. The results of fatigue lifetime estimation indicate that crossflow VIV is the most critical effect to subsea jumper lifetime. However, a subsea jumper of a U-inverse shape is able to withstand a current load for a 30-year design lifetime in the Andaman sea area.

In design case C and D, the subsea jumper experienced combination external loads. The result of fatigue analysis indicates that an earthquake is the dominant influencing factor on subsea jumper lifetime, especially in the inline direction. One shock from an earthquake is able to reduce subsea jumper lifetime by 0.008% under any current flow velocity. In other words, a subsea jumper of a U-inverse shape can withstand an estimated 13,000 shocks. According to measurement data, extreme magnitude earthquakes occur only 600 times during the design lifetime of jumper. As such, subsea jumpers are safely used in operations under a combination of effects in the Andaman sea area.

The designed subsea jumper may need an improvement if it is relocated to operate in another area where there is a stronger current velocity and/or earthquake conditions. Subsea jumper lifetime can be improved by designing its dimension and configuration in order to give natural frequencies that are out of the load range. This can be achieved by reducing the length of a jumper or enlarging its diameter. Another method is to reduce the flow rate of the contained fluid. These methods can prevent a resonance phenomenon between the subsea jumper and external loads, especially the current load. However, these improvement methods may stimulate another problem if slug is present inside the jumper. Adjusting the flow rate or jumper dimension changes the impact period of slug load in a jumper system. The new slug load frequency may close to one of natural frequencies of a subsea jumper. The slug could induce in a dramatic vibration of the jumper. Thus, the slug issue should be taken into account during subsea jumper design, especially when dimension, configuration and flow rate are changed. Other methods include using more robust material, controlling surface conditions and various welding methods.
Chapter 9: Conclusion

9.2 Recommendations

Although a subsea jumper model together with wake oscillator model and inertia load model can describe the dynamic motion of a subsea jumper in various conditions, there are many assumptions made especially in a combination load situation. Thus, possible next steps would be to: 1) add a nonlinear couple effect between earthquakes and current into the combination model, for example, an earthquake creates changes in current velocity and flow pattern around the subsea jumper, and 2) consider indirect effects of earthquake on a subsea jumper such as turbidity flow, scour transportation, liquid fraction and land slide.

Another recommendation concerns on subsea jumper modelling. The design of a subsea jumper throughout this thesis only considered a U-inverse configuration. This means that the end-connection of the designed subsea jumper is connected to subsea equipment by a clamped-connection. As a consequence, the boundary condition of the subsea jumper is considered as a fixed-fixed connection or clamped connection. Figure 9.1 presents an example of subsea jumper in a U-inverse shape and its corresponding designed boundary condition.

Figure 9.1: End connection of subsea jumper in U-inverse shape and corresponding boundary condition

Apart from a U-inverse configuration, one common shape for a subsea jumper is the M-shape as shown in Figure 9.2. This configuration has a U-shape section at the middle of a jumper system. If the dynamic behavior of U-shape section is only interesting, one can apply a "subsea jumper model" for analysis. However, the boundary condition as fixed-fixed connection is not applicable since the U-shape section attaches to jumper lines at ends instead of connecting direct to subsea equipment as seen in a U-inverse shape. As such, the boundary condition should be modelled by liner spring and rotational spring. Figure 9.2 presents a U-shape section in a M-shape jumper and a corresponding boundary condition.

Figure 9.2 End connection of subsea jumper in M-shape and correspond boundary condition
Final recommendation is to improve the measurement data. Lack of information at designed location is a key problem on load modelling, especially earthquake load. One can see that only an earthquake magnitude is available for dynamic behavior and fatigue analysis in this thesis. As such, earthquake load model can be modelled by graphical simulation method or “simulation model”. This model is only given a general characteristic of earthquake in time domain but it does not provide a frequencies characteristic of earthquake load. If ground acceleration data are measured directly at site, it is possible to model the ground acceleration model in more accurate method, for example using Fourier-transform. This modelling method provides a load characteristic in both time domain and frequencies domain consequence it results in a more accurate and realistic response of subsea jumper.
Appendix

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Appendix A

Eigen Value Problem

A subsea jumper model is established in terms of the forth orders partial differential equation (PDE). The solutions of the model reflect the dynamic characteristics of a jumper: mode shapes and natural frequencies. Appendix A gives the solving detail by using a finite different method (FDM). FDM is selected because it is a simple and efficient method to solve PDE equation.

FDM is only a numerical approximating method for solving PDF equation. Therefore, it is necessary to understand an analytical method as a fundamental approach to solve an Eigen value problem. Here, a modal analysis method is introduced.

The first section of this appendix provides an example of the “Modal analysis” on a pipe conveying fluid model as a simple model. Later, an example of FDM applied on a pipe conveying fluid is provided for comparison purpose. A detail of FDM applied to a subsea jumper model that is provided in the last section.

This research considers dynamic behaviors of a jumper system in inline vibration and crossflow vibration separately. Thus, there is no mode shape of a jumper results in a combination shape of inline and crossflow vibration.

A.1 Eigen value problem of a pipe conveying fluid model

A.1.1 Analytical method (modal analysis)

A pipe conveying fluid model can be written in form of an equation of motion as follows:

\[ m \frac{\partial^2 w}{\partial t^2} + EI \frac{\partial^4 w}{\partial x^4} + M_f \left( U_f \frac{\partial^2 w}{\partial x^2} + 2U_f \frac{\partial^2 w}{\partial t \partial x} + \frac{\partial^2 w}{\partial t^2} \right) = 0 \]  \hspace{1cm} (A.1)

The mode shapes and natural frequencies of the model can be obtained by using the modal analysis method. This method considers a pipe conveying model as an undamped system in free vibration. As such, all damping and force terms shall be firstly removed. The pipe conveying model contains one damping component that is represented in terms of Coriolis force, \( 2U_f \frac{\partial^2 w}{\partial t \partial x} \). As a consequence, the equation of motion is reduced as follows:

\[ EI \frac{\partial^4 w}{\partial x^4} + (m + M_f) \frac{\partial^2 w}{\partial t^2} + M_f \left( U_f \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial t^2} \right) = 0 \]  \hspace{1cm} (A.2)

\[ EI \frac{\partial^4 w}{\partial x^4} + (m + M_f) \frac{\partial^2 w}{\partial t^2} + M_f U_f \frac{\partial^2 w}{\partial x^2} = 0 \]  \hspace{1cm} (A.3)

Here, the boundary conditions of a pipe conveying fluid model are considered as fixed-fixed connection. The modal analysis is provided in following steps:
Appendix A: Eigen Value Problem

**Step 1:** Substituting the general solution of the transverse motion in harmonic form, \( w(x, t) = W(x) \exp(i \omega t) \) into equation of motion. The equation of motion results in only space domain as follows:

\[
EI \frac{d^4w}{dx^4} + M_f \frac{d^3w}{dx^3} - (m + M_f) \omega^2 W = 0
\]  
(A.4)

**Step 2:** According to the equation of motion in space domain, we can find the general solution in terms of a space dependent. Here, the maple program is used to establish the general solution. The general solution can be written as follows:

\[
W(x) = \text{coeff } BC \text{ coeff } BC \text{ coeff } BC \text{ coeff } BC
\]  
(A.5)

**Step 3:** Substituting the general solution of space dependent term into the boundary conditions. Here, the fixed-fixed connection is considered. The boundary condition comprise of four equations as follows:

\[
W(0) = W'(0) = 0 \text{ and } W(L) = W'(L) = 0
\]  
(A.6)

**Step 4:** Arranging the equations of the boundary conditions into 4 linear algebraic equations with respect constant parameters: \( C_1, C_2, C_3 \) and \( C_4 \). This set of equation can be written in matrix form.

\[
\begin{bmatrix}
\text{coeff } BC1 & \text{coeff } BC1 & \text{coeff } BC1 & \text{coeff } BC1 \\
\text{coeff } BC2 & \text{coeff } BC2 & \text{coeff } BC2 & \text{coeff } BC2 \\
\text{coeff } BC3 & \text{coeff } BC3 & \text{coeff } BC3 & \text{coeff } BC3 \\
\text{coeff } BC4 & \text{coeff } BC4 & \text{coeff } BC4 & \text{coeff } BC4
\end{bmatrix}
\begin{bmatrix}
C_1 \\
C_2 \\
C_3 \\
C_4
\end{bmatrix} =
\begin{bmatrix}
0 \\
0 \\
0 \\
0
\end{bmatrix}
\]  
(A.7)

**Step 5:** The frequency equation or characteristic equation can be found by setting the determining of the coefficient matrix equal to zero. Consequence the natural frequency on each mode can be solved.

**Step 6:** The constant parameters cannot be solved directly because there are five unknowns: constant values \( C_1, C_2, C_3, C_4 \) and natural frequency respect to 4 algebra equation. As a consequence, these constant parameters shall be formatted in term of ratio form. This can be done by dividing any three algebra equations by a remaining algebra equation. For example, the ratio constant parameter respect to \( C_4 \), the ratio constant parameter can be written as \( \frac{C_1}{C_4}, \frac{C_2}{C_4}, \frac{C_3}{C_4} \). As a result, constant parameters can be solved. The mode shapes can also be obtained by substituting a corresponding natural frequency in to the general solution of space domain.

A.1.2 Finite different method (FDM)

FDM is used to solve a differential equation by replacing all derivative terms with the differential quotients. In this section, a pipe conveying model is still analyzed. The equation of motion is introduced again as follows:

\[
EI \frac{d^4w}{dx^4} + m \frac{d^2w}{dt^2} + M_f \left( U_f^2 \frac{d^2w}{dx^2} + \frac{d^2w}{dt^2} \right) = 0
\]
Appendix A: Eigen Value Problem

The pipe conveying fluid model is considered as a continuous system. In order to apply a FDM, the system needs to be discretized into N+4 nodes as see in Figure A.1.

The Partial differential equations of the pipe conveying fluid model can be expressed by using space discretization. The general expression for the first to forth order of partial differential in space domain can be expressed as follow:

\[
\frac{\partial w(x_i,t)}{\partial x} = \frac{w_{i+1}(t) - w_{i-1}(t)}{2(\Delta x)}
\]

\[
\frac{\partial^2 w(x_i,t)}{\partial x^2} = \frac{w_{i+1}(t) - 2w_i(t) + w_{i-1}(t)}{(\Delta x)^2}
\]

\[
\frac{\partial^3 w(x_i,t)}{\partial x^3} = \frac{-w_{i-2}(t) + 2w_{i-1}(t) - 2w_{i+1}(t) + w_{i+2}(t)}{2(\Delta x)^3}
\]

\[
\frac{\partial^4 w(x_i,t)}{\partial x^4} = \frac{w_{i-2}(t) - 4w_{i-1}(t) + 6w_i(t) - 4w_{i+1}(t) + w_{i+2}(t)}{(\Delta x)^4}
\]

The next step is to replace all space differential terms by space discretization. This results in a change equation of motion in terms of a partial differential equation to ordinary differential equation respect to time. The equation of motion of any arbitrary node can be written as follow:

\[
EI \left(\frac{w_{i+1}(t) - 4w_{i}(t) + 6w_{i-1}(t) - 4w_{i+2}(t) + w_{i+4}(t)}{(\Delta x)^4}\right) + (m + M_f) \frac{\partial^2 w}{\partial t^2} + M_f U_f^2 \left(\frac{w_{i+1}(t) - 2w_i(t) + w_{i-1}(t)}{(\Delta x)^2}\right) = 0 \quad (A.8)
\]

\[
M \frac{\partial^2 w}{\partial t^2} + A \left(w_{i-2}(t) - 4w_{i-1}(t) + 6w_i(t) - 4w_{i+1}(t) + w_{i+2}(t)\right) + B \left(w_{i+1}(t) - 2w_i(t) + w_{i-1}(t)\right) = 0 \quad (A.9)
\]

\[
M \frac{\partial^2 w}{\partial t^2} + Aw_{i-2}(t) + (B - 4A)w_{i-1}(t) + (6A - 2B)w_i(t) + (B - 4A)w_{i+1}(t) + Aw_{i+2}(t) = 0 \quad (A.10)
\]

Where \( M = m + M_f \), \( A = EI / (\Delta x)^4 \) and \( B = M_f U_f^2 / (\Delta x)^2 \).

The node \( N_0 \) and \( N_{n+1} \), are considered as boundary nodes. As such, the boundary conditions as fixed-fixed connections are applied for these two nodes. The boundary conditions can be written in terms of boundary nodes and adjacent nodes as follows:

At Node 0

\[
W(0) = w_0(t) = 0 \text{ and } W'(0) = \frac{w_5(t) - w_{-1}(t)}{2(\Delta x)} = 0; w_{-1}(t) = w_5(t) \quad (A.11)
\]
Appendix A: Eigen Value Problem

At Node N+1

\[ W(L) = w_{N+1}(t) = 0 \text{ and } W'(0) = \frac{w_{N+2}(t) - w_N(t)}{2(\Delta x)} = 0; w_N(t) = w_{N+2}(t) \]  

(A.12)

The equation of motion for boundary nodes can be established by substituting the boundary conditions which are expressed in equation A.11 and A.12 into equation of motion. As a result, the expression equations for node 1, node 2, node N and node N+1 can be found as follows:

**Node 1:** substitute \( w_0(t) = 0 \) and \( w_1(t) = w_1(t) \)

\[
M \frac{\partial^2 w}{\partial t^2} + (6A - 2B)w_1(t) + (B - 4A)w_2(t) + Aw_3(t) = 0 \\
M \frac{\partial^2 w}{\partial t^2} + (7A - 2B)w_1(t) + (B - 4A)w_2(t) + Aw_3(t) = 0
\]  

(A.13)

**Node 2:** substitute \( w_0(t) = 0 \)

\[
M \frac{\partial^2 w}{\partial t^2} + (B - 4A)w_1(t) + (6A - 2B)w_2(t) + (B - 4A)w_3(t) + Aw_4(t) = 0
\]  

(A.14)

**Node N:** substitute \( w_{N+1}(t) = 0 \) and \( w_N(t) = w_{N+2}(t) \)

\[
M \frac{\partial^2 w}{\partial t^2} + Aw_{N-2}(t) + (B - 4A)w_{N-1}(t) + (6A - 2B)w_N(t) + (B - 4A)w_{N+1}(t) + Aw_{N+2}(t) = 0 \\
M \frac{\partial^2 w}{\partial t^2} + Aw_{N-2}(t) + (B - 4A)w_{N-1}(t) + (7A - 2B)w_N(t) = 0
\]  

(A.15)

**Node N-1:** substitute \( w_{N+1}(t) = 0 \)

\[
M \frac{\partial^2 w}{\partial t^2} + Aw_{N-3}(t) + (B - 4A)w_{N-2}(t) + (6A - 2B)w_{N-1}(t) + (B - 4A)w_N(t) + Aw_{N+1}(t) = 0 \\
M \frac{\partial^2 w}{\partial t^2} + Aw_{N-3}(t) + (B - 4A)w_{N-2}(t) + (6A - 2B)w_{N-1}(t) + (B - 4A)w_N(t) = 0
\]  

(A.16)

After replacing all space discretizing terms into equation of motion, the ordinary differential equation can be solved by composing all system from node 1 to node N in matrix form.

\[
[M][\ddot{w}] + [K][w] = 0
\]  

(A.17)

The eigenvalue can be found by solving characteristic equation and mode shape can be solving by using

\[
[[K] - \omega^2[M]] = 0
\]  

(A.18)
A.2 Eigen value problem of a subsea jumper model

A subsea jumper model can be solved in same manner as a pipe conveying model. Only two main different are 1) number of equation is larger because it comprises of three beam connected to each other and 2) interface conditions are required to define at each connection of beam.

In section A.1.2, general equation of any arbitrary points including end points of a pipe conveying fluid model are defined. As a subsea jumper model is considered as a set of connecting pipe and has the same boundary conditions as a pipe conveying fluid model. Thus, these general equations in section A.1.2 are also applicable for a subsea jumper model for arbitrary points and end connections. However, connection points or interface nodes between vertical beams and horizontal beam still need to define in order to complete mass and spring matrix which are used for solving Eigen problem of a subsea jumper. As the results, the detail of defining interface equations of connection nodes is only provided in this section.

This research considers the dynamic behaviors of subsea jumper in inline and crossflow vibration separately. Thus interface condition shall be defined for each vibration plane. First interface condition of crossflow vibration presents follow with inline vibration.

A.2.1 Interface condition of crossflow vibration

The schematic diagram represents relation of vertical and horizontal beam at interface nodes or rigid joints on crossflow vibration is introduced again in Figure A1.2.

In order to implement FDM on a subsea jumper model, a jumper needs to be discretized in to several nodes. At connection points are required extra four nodes, or called ghost nodes. Thus, a subsea jumper in U-inverse shape uses eight ghost nodes to describe the interface conditions. Figure A1.3 shows a node discretization for subsea jumper.
The general expression of the interface nodes are established by using ghost nodes. The mathematic expression of ghost nodes can be found by solving interface condition equation. The interface conditions of crossflow vibration are provided again as follow.

**Displacement balance**

Left connection \( w_1 = u_2 \) and \( u_1 = -w_2 \) \( (A.19) \)

Right connection \( w_3 = -u_2 \) and \( u_3 = w_2 \) \( (A.20) \)

**Angular balance**

Left connection \( \frac{\partial w_2}{\partial x_2} = \frac{\partial w_1}{\partial x_1} \) \( (A.21) \)

Right connection \( \frac{\partial w_2}{\partial x_2} = \frac{\partial w_3}{\partial x_3} \) \( (A.22) \)

**Moment balance**

Left connection \( \frac{\partial^2 w_2}{\partial x_2^2} = \frac{\partial^2 w_1}{\partial x_1^2} \) \( (A.23) \)

Right connection \( \frac{\partial^2 w_2}{\partial x_2^2} = \frac{\partial^2 w_3}{\partial x_3^2} \) \( (A.24) \)

**Force balance** (disregarding the change of momentum of the conveying fluid)

Left connection \( EI \frac{\partial^3 w_2}{\partial x_2^3} = -EA \frac{\partial u_2}{\partial x_2} \) and \( EI \frac{\partial^3 w_1}{\partial x_1^3} = EA \frac{\partial u_1}{\partial x_1} \) \( (A.25) \)

Right connection \( EI \frac{\partial^3 w_2}{\partial x_2^3} = EA \frac{\partial u_2}{\partial x_2} \) and \( EI \frac{\partial^3 w_3}{\partial x_3^3} = -EA \frac{\partial u_2}{\partial x_2} \) \( (A.26) \)

The mathematic expression for each ghost node can be obtained in similar way as introduced in a pipe conveying model. The interface nodes can be expressed by substituting the ghost nodes into the general equation of motion at interface nodes. The results of general equation of each interface nodes are provided as follows:
Appendix A: Eigen Value Problem

**Node N1**: the general equation can be written as follows:

\[ M \frac{\partial^2 X}{\partial t^2} + AX_{N1-2}(t) + (B - 4A)X_{N1-1}(t) + (6A - 2B + (C_2 + C_1)A)X_{N1}(t) + (B - 4A)X_{N1+1}(t) + AX_{N1+2}(t) = 0 \]

**Node N2**: the general equation can be written as follows:

\[ M \frac{\partial^2 X}{\partial t^2} + AX_{N2-2}(t) + (B - 4A)X_{N2-1}(t) + (6A - 2B + (C_2 + C_1)A)X_{N2}(t) + (B - 4A)X_{N2+1}(t) + AX_{N2+2}(t) = 0 \]

The final step is to replace all space discretizing terms into equation of motion, the ordinary differential equation can be solved by composing all system from node 1 to node N in matrix form.

\[ [M][\ddot{\omega}_i] + [K][\omega_i] = 0 \]

The natural frequencies can be found by solving characteristic equation while he mode shapes can be analyzed by solving a following equation.

\[ [[K] - \omega^2[M]] = 0 \]

Solutions of first five mode shape and corresponding natural frequencies of a subsea jumper in U-inverse shape for crossflow vibration are provided in chapter 3.

**A.2.1 Interface condition of inline vibration**

The schematic diagram represents inline vibration of each line is introduced again in Figure A1.4.

![Schematic diagram for inline vibration at rigid joint](image)

Figure A.4: Schematic diagram for inline vibration at rigid joint

The method of implementation a FDM on a subsea jumper model in inline vibration is similar to crossflow vibration. A jumper needs to be discretized in to several nodes. At connection points are required extra four nodes, or called ghost nodes. Thus, a subsea jumper in U-inverse shape uses eight ghost nodes to describe the interface conditions. Figure A1.5 shows a node discretization for subsea jumper.
The general expression of the interface nodes are established by using ghost nodes. The mathematic expression of ghost nodes can be found by solving interface condition equation. The interface conditions of inline vibration are provided again as follows:

**Displacement balance**

\[ v_1 = v_2 \quad \text{and} \quad v_2 = v_3 \]  
(A.27)

**Angular balance**

\[ \theta_{\text{torsion,2}} = -\frac{\partial v_1}{\partial x_1} \quad \text{and} \quad \theta_{\text{torsion,2}} = -\frac{\partial v_3}{\partial x_3} \]  
(A.28)

\[ \theta_{\text{torsion,1}} = -\frac{\partial v_2}{\partial x_2} \quad \text{and} \quad \theta_{\text{torsion,3}} = -\frac{\partial v_2}{\partial x_2} \]  
(A.29)

**Moment balance**

Horizontal beam

\[ EI \frac{\partial^2 v_3}{\partial x_3^2} = -\frac{J_G}{L_H} \left( \frac{\partial v_3}{\partial x_3} - \frac{\partial v_1}{\partial x_1} \right) \]  
(A.30)

\[ EI \frac{\partial^2 v_1}{\partial x_1^2} = -EI \frac{\partial^2 v_1}{\partial x_1^2} \]  
(A.31)

Vertical left beam

\[ EI \cdot \frac{\partial^2 v_2}{\partial x_2^2} = -\frac{J_G}{L_Y} \left( -\frac{\partial v_2}{\partial x_2} \right) \]  
(A.32)

Vertical right beam

\[ EI \cdot \frac{\partial^2 v_2}{\partial x_2^2} = \frac{J_G}{L_Y} \left( -\frac{\partial v_2}{\partial x_2} \right) \]  
(A.33)

**Force balance**

Left connection

\[ \frac{\partial^3 v_2}{\partial x_2^3} = \frac{\partial^3 v_1}{\partial x_1^3} \]  
(A.34)

Right connection

\[ \frac{\partial^3 v_2}{\partial x_2^3} = \frac{\partial^3 v_3}{\partial x_3^3} \]  
(A.35)
Appendix A: Eigen Value Problem

The mathematic expression for each ghost node can be obtained in similar way as introduced in a pipe conveying model. The interface nodes can be expressed by substituting the ghost nodes into the general equation of motion at interface nodes. The results of general equation of each interface nodes are provided as follows:

**Node N1**: the general equation can be written as follows:

\[
M \frac{\partial^2 X}{\partial t^2} + AX_{N1-2}(t) + (-B)X_{N1-1}(t) + (2A - 2B)X_{N1}(t) + (-B)X_{N1+1}(t) + AX_{N1+2}(t) = 0
\]

**Node N1-1**: the general equation can be written as follows:

\[
M \frac{\partial^2 X}{\partial t^2} + AX_{N1-3}(t) + (B - 4A)X_{N1-2}(t) + (5A - 2B)X_{N1-1}(t) + (B - 2A)X_{N1}(t) = 0
\]

**Node N1+1**: the general equation can be written as follows:

\[
M \frac{\partial^2 X}{\partial t^2} + (B - 2A)X_{N1}(t) + (5A - 2B)X_{N1+1}(t) + (B - 4A)X_{N1+2}(t) + AX_{N1+3}(t) = 0
\]

**Node N2**: the general equation can be written as follows:

\[
M \frac{\partial^2 X}{\partial t^2} + AX_{N2-2}(t) + (-B)X_{N2-1}(t) + (2A - 2B)X_{N2}(t) + (-B)X_{N2+1}(t) + AX_{N2+2}(t) = 0
\]

**Node N2-1**: the general equation can be written as follows:

\[
M \frac{\partial^2 X}{\partial t^2} + AX_{N2-3}(t) + (B - 4A)X_{N2-2}(t) + (5A - 2B)X_{N2-1}(t) + (B - 2A)X_{N2}(t) = 0
\]

**Node N2+1**: the general equation can be written as follows:

\[
M \frac{\partial^2 X}{\partial t^2} + (B - 2A)X_{N2}(t) + (5A - 2B)X_{N2+1}(t) + (B - 4A)X_{N2+2}(t) + AX_{N2+3}(t) = 0
\]

The final step is to replace all space discretizing terms into equation of motion, the ordinary differential equation can be solved by composing all system from node 1 to node N in matrix form.

\[
[M][\ddot{w}] + [K][w] = 0
\]

The natural frequencies can be found by solving characteristic equation while he mode shapes can be analyzed by solving a following equation.

\[
[[K] - \omega^2[M]] = 0
\]

The solutions of first five mode shape and corresponding natural frequencies of a subsea jumper in U-inverse shape for inline vibration are provided in chapter 3.
Appendix B

Numerical Method

A numerical method is used to solve the dynamic behavior of subsea jumper and fatigue analysis in this thesis. A finite difference method is selected for space discretization as discussed in Appendix A. The procedure of space discretization on partial differential equation (or a subsea jumper model) and Eigen problem solving method are already presented for both inline and crossflow vibration. As the resulted, dynamic characteristic of a subsea jumper are known. In addition, the expression of any arbitrary nodes of a subsea jumper model, including interface node and boundary node, are obtained. However, in order to obtain the dynamic behavior of subsea jumper under various conditions only space dependent terms of solution of a subsea jumper model is insufficient. The time dependent terms need to be defined. This appendix gives a detail of solving method for time dependent terms. A numerical method by ODE45 is used to approximate time dependent terms in this thesis.

Appendix B begins with programming flow chart for well understanding a thesis approach. Then, a detail of time domain solution is introduced. The final section all subprograms (Matlab) which used in this thesis are listed.

B.1 Programming flow chart

It is necessary to understand the overview of thesis approach or programming flow chart. Figure B.1 shows a programming flow chart at the beginning step until obtaining the answers in the final step. The Matlab program is used extensively thru this research.

The first step is to establish an equation of motion for transverse motion of a subsea jumper in terms of differential equations. A set of equations is divided into inline vibration and crossflow vibration which are mainly different at interface conditions. Then, the differential equation can be approximated by replacing the derivative terms with the differential quotients or called “Finite differential method” or “space discretization” as see in flow chart. After this step, there are two separated paths: the first path is to solve an Eigen values problem which is already described in appendix A. The second path is to solve the dynamic responses of subsea jumper under various conditions by using ode45, built-in code in Matlab program. It results in transverse motion of subsea jumper at any time steps. After dynamic responses of subsea jumper are known, they can be converted into a stress-time series at any arbitrary points by using numerical approximation. The results of conversion are used as input data for fatigue lite time estimation. The approach of displacement and stress conversion was provided in chapter 7.2.

In accordance with Figure B.1, there are three main steps which use a numerical approximation; 1) Eigen value solution, 2) ode45 solver and 3) displacement-stress conversion. However, ode45 solver is only provided in Appendix B.
Appendix B: Numerical Method

Figure B.1: Programming flow chart or thesis approach
B.2 Time domain solution

A numerical method uses to approximate a solution of (first order) differential equation with initial condition. The simplest method is Euler’s method. It is multiplying the derivative with time step as written as follows:

\[ y_{j+1} = y_j + \frac{dy}{d\tau}_{\tau = j} \Delta \tau, \quad \frac{dy}{d\tau}_{\tau = j} = f(\tau_j, y_j) \]

By known the initial condition at time \( j \) and first derivative function, \( f \), then solution of next time step, \( y_{j+1} \), is obtained. This method can also be described in Figure B.2.

![Figure B.2: Euler’s method for approximation differential equation](image)

However, the current thesis use MATLAB’s standard solver in built-in function \texttt{ode45} for ordinary differential equation (ODEs). This function approximates a variable time step by implementing a Rung-Kutta method (R-K method). The R-K method is given a more accurate result compare to Euler’s method. This is because it considers the slope of several points with in time interval \( \Delta \tau \). The order of R-K method indicated the number of points for slope approximation. One general form of R-K method is 4\(^{th}\) order which can be expressed as follows:

\[ y_{j+1} = y_j + \frac{1}{6} (K_1 + 2K_2 + 2K_3 + K_4) \Delta \tau \]

\[ K_1 = f(x_i, y_j) \]
\[ K_2 = f(x_i + \frac{1}{2} h, y_j + \frac{1}{2} k_1 h) \]
\[ K_3 = f(x_i + \frac{1}{2} h, y_j + \frac{1}{2} k_2 h) \]
\[ K_4 = f(x_i + h, y_j + k_3 h) \]

One important remark should be made before using an \texttt{ode45} as it is designed to handle only first order time derivative. The first order time derivative can be written in a following form.

\[ \frac{dx}{dt} = f(t, x), \quad x(t_0) = x_0 \]
Appendix B: Numerical Method

Here, \( t \) is a time variable, \( x \) is a space vector to be found and \( f \) represents transverse motion of \( t \) and \( x \). In addition, an initial condition is required to solve time domain solution as given \( x = x_0 \) at time \( t_0 \).

According a subsea jumper model contains the second order of time derivative in time domain for crossflow equation, inline equation and wake equation. In order to use ode4 solver, it is necessary to introduce three more parameters in order to describe the second order of time derivative in term of the first derivative equation. The set of first order derivative equation are written as follows:

Crossflow equation:

\[
\frac{dw}{dt} = r = f_1(t, w) \\
\frac{dr}{dt} = \frac{d^2w}{dt^2} = \frac{df(t, w)}{dt} = f_2(t, w)
\]

Inline equation:

\[
\frac{dv}{dt} = s = f_3(t, v) \\
\frac{ds}{dt} = \frac{d^2v}{dt^2} = \frac{df(t, v)}{dt} = f_4(t, v)
\]

Wake equation:

\[
\frac{dq}{dt} = u = f_5(t, q) \\
\frac{du}{dt} = \frac{d^2q}{dt^2} = \frac{df(t, q)}{dt} = f_6(t, q)
\]

The ODE45 function gives a solution in matrix form. The matrix consists of 1) Rows provide a solution of time step which used an ODE45 for approximation and 2) Column indicates solution of each node at any instant time. Each node contains 6 variables which are divided into three main parts, crossflow, inline and wake. Each part contains a solution (transverse displacement, \( w \) and \( v \) and life force coefficient, \( q \)) and its first derivative. Figure B.3 presents the structure of solution matrix.

![Figure B.3: Structure of equation in time solution in matrix form](image)
Appendix B: Numerical Method

B.3 Matlab codes

Matlab program is mainly used in this report. List of Matlab code comprise of subprograms as follow:

<table>
<thead>
<tr>
<th>Codes Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subsea_jumper_inputdata</td>
<td>Input code of all designed parameters. Including subsea jumper, earthquake and current parameter</td>
</tr>
<tr>
<td>Modeshape_CF</td>
<td>Code for calculation natural frequency and mode shape in crossflow vibration plane</td>
</tr>
<tr>
<td>Modeshape_IN</td>
<td>Code for calculation natural frequency and mode shape in inline vibration plane</td>
</tr>
<tr>
<td>Review_mode_shape_CF</td>
<td>Code for plotting mode shape of subsea jumper in crossflow vibration plane</td>
</tr>
<tr>
<td>Review_mode_shape_IN</td>
<td>Code for plotting mode shape of subsea jumper in inline vibration plane</td>
</tr>
<tr>
<td>wakeoscillatormodel</td>
<td>Code for calculate displacement time series of subsea jumper under steady current</td>
</tr>
<tr>
<td>EQSinusodialmodel</td>
<td>Code for calculate displacement time series of subsea jumper under earthquake by using sinusoidal model</td>
</tr>
<tr>
<td>EQSimulationmodel</td>
<td>Code for calculate displacement time series of subsea jumper under earthquake by using simulation model or shock model</td>
</tr>
<tr>
<td>Combinationmodelsin</td>
<td>Code for calculate displacement time series of subsea jumper under combination load by using sinusoidal model for earthquake load</td>
</tr>
<tr>
<td>Combinationmodelsim</td>
<td>Code for calculate displacement time series of subsea jumper under combination load by using simulation model for earthquake load</td>
</tr>
<tr>
<td>ConversionDS.M</td>
<td>Code for convert displacement time series to bending stress time series</td>
</tr>
<tr>
<td>Extractstress.M</td>
<td>Code for extract interesting period of stress for using as input for fatigue estimation</td>
</tr>
<tr>
<td>Countingmethod.M</td>
<td>Code for evaluation value of bending stress at peak point on interesting period, these values will be used as input for counting number stress range.</td>
</tr>
</tbody>
</table>

Table B.1: List of subprogram
Appendix C

Fatigue Damage Calculation

A fatigue damage calculation can be obtained by using equation 7.3. Its expression is written again as follows:

\[ Fatigue\ Damage = \sum \frac{n_i}{N_i} \]  \hspace{1cm} (C.1)

The first component, \( n_i \), is number of cycles on each stress level occurs while loads apply to system. This can be achieved by counting stress range on stress time series on each cycle of load. The results of counting method are provided in Histogram in section 7.3. The second component, \( N_i \), is materials life time. It indicates fatigue capacity of each material by representing number of cycles on each stress level. This component can be approximated by S-N curve or Basquin Relation equation as following expression.

\[ S^k \cdot N = a \]  \hspace{1cm} (C.2)

Where, \( N \) is fatigue capacity unit in cycles, \( S \) indicates stress level, \( k \) and \( a \) are specific parameters which depend strongly on material and welding detail. Designed fatigue parameter use in this research based on available project data. All designed values are same in every design cases. A summary of fatigue design parameters is given in table C.1

<table>
<thead>
<tr>
<th>Design Parameter: Dimension and Material Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>a \hspace{1cm} 6.3 \times 10^{11} \hspace{1cm}</td>
</tr>
<tr>
<td>K \hspace{1cm} 3 \hspace{1cm}</td>
</tr>
<tr>
<td>Fatigue designed life time \hspace{1cm} 31 \hspace{1cm} years</td>
</tr>
</tbody>
</table>

Table C.1: Design parameters for fatigue analysis

As discussed in chapter 7, S-N curve is constructed from a fatigue experiment in a laboratory. A specimen is applied under cyclic load which is typically a zero mean load. In other words, the cyclic load varies around zero level and causes a zero mean stress level on the specimen. It is comparable to a crossflow VIV condition that the subsea jumper oscillates around zero mean displacement. As a result, bending stress in jumper is a zero mean stress level.

Unlike inline VIV situation, a subsea jumper is firstly pushed by drag force. Then it vibrates around non-zero mean displacement. This situation creates non-zero mean stress level on subsea jumper. Stress amplitude alone is insufficient to predict a fatigue lifetime by using an S-N curve as cross flow VIV. In this situation, mean stress level and stress amplitude are necessary to predict equivalent stress amplitude in order to be compatible with S-N curve. Thus a correlation method is required to find the equivalent stress amplitude before using S-N curve. The most widely accepted method is [Goodman,1899]. This method provides relations between non-zero mean stress level and equivalent stress level on S-N curve. The expression is written as follows:
Appendix C: Fatigue Damage Calculation

\[
\frac{S_a}{S_u} + \frac{S_m}{S_u} = 1
\]  

(C.3)

Where, \( S_a \) represents stress amplitude, \( S_m \) indicates a mean stress level and \( S_u \) is ultimate stress of material. The equivalent stress level or Stress life fatigue, \( S_e \) can be calculated when mean-stress and stress amplitude are known.

Fatigue Damage of each design case in chapter 7 can be estimated by substituting two components, \( n \) and \( N \) at each stress level. The fatigue damage of each design case is provided in following tables:

**DESIGN CASE A:**

- Fatigue damage of inline vibration in design case A

<table>
<thead>
<tr>
<th>Stress Range (Mpa)</th>
<th>S-N Data</th>
<th>Fatigue Damage</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Stress Range</strong></td>
<td><strong>S-N Data</strong></td>
<td><strong>Fatigue Damage</strong></td>
</tr>
<tr>
<td>( S_a )</td>
<td>( S_e )</td>
<td>( n )</td>
</tr>
<tr>
<td>1.97E-03</td>
<td>1.99E-03</td>
<td>2.5</td>
</tr>
<tr>
<td>2.02E-03</td>
<td>2.04E-03</td>
<td>3</td>
</tr>
<tr>
<td>2.07E-03</td>
<td>2.09E-03</td>
<td>2.5</td>
</tr>
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</tr>
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<td>2.21E-03</td>
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</tr>
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</tr>
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<td>2.31E-03</td>
<td>2.33E-03</td>
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</tr>
<tr>
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</tr>
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</tr>
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<tr>
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<td>3.01E-03</td>
<td>6.5</td>
</tr>
</tbody>
</table>

| Fatigue damage | 1.44E-18 |

Table C.2: Fatigue damage of inline vibration in fatigue design case A
- Fatigue damage of crossflow vibration in design case A

<table>
<thead>
<tr>
<th>Stress Range</th>
<th>S-N Data</th>
<th>Fatigue Damage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sa (Mpa)</td>
<td>n</td>
<td>Sa (Mpa)</td>
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</tr>
</tbody>
</table>

Table C.3: Fatigue damage of crossflow vibration in fatigue design case A
Appendix C: Fatigue Damage Calculation

DESIGN CASE B:

- Fatigue damage of inline vibration in design case B

<table>
<thead>
<tr>
<th>Stress range (MPa)</th>
<th>Se (MPa)</th>
<th>n</th>
<th>S-N data</th>
<th>Se (MPa)</th>
<th>N</th>
<th>Fatigue damage n/N</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.22E-03</td>
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Table C.4: Fatigue damage of inline vibration in fatigue design case B
Appendix C: Fatigue Damage Calculation

- Fatigue damage of crossflow vibration in design case B

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| Fatigue Damage | 1.26E-07 |

Table C.5: Fatigue damage of crossflow vibration in fatigue design case B
### Appendix C: Fatigue Damage Calculation

#### DESIGN CASE C:

- Fatigue damage of inline vibration in design case C

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| Fatigue damage | 7.84E-05 |

Table C.6: Fatigue damage of inline vibration in fatigue design case C
Appendix C: Fatigue Damage Calculation

- Fatigue damage of crossflow vibration in design case C

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Fatigue damage: 2.28E-06

Table C.7: Fatigue damage of crossflow vibration in fatigue design case C
### DESIGN CASE D:

- Fatigue damage of inline vibration in design case D

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| Fatigue damage | 7.43E-05 |

Table C.8: Fatigue damage of inline vibration in fatigue design case D
Fatigue damage of crossflow vibration in design case D

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A.V. Metrikine, *Dynamics, Slender Structures and an Introduction to Continuum Mechanics*, (2006), Faculty of Civil Engineering and Geosciences, Delft University of Technology.


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