



# SCALE SELECTION FOR WAVE MODELS

by

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ABSTRACT

The selection of scales for models with short waves, long waves and unidirectional current is discussed with particular emphasis on boundary layer motion and movement of sediment. The paper attempts to outline the present state of the art and to provide a framework for future research in the area of coastal sediment transport models at Queen's

"If I succeed in demonstrating with the model that the originally existing conditions can be reproduced typically; and if, moreover, by placing regulating works in the model, the same changes can be reproduced that were brought about by the training works actually built, then I am sure that I can take the third and most important step: namely, of investigating, with every promise of success, the probable effect of the projects that have been proposed .... "

- L.F. Vernon-Harcourt

Model technology has not advanced a great deal since L.F. Vernon-Harcourt, who continued the work of O. Reynolds made the above statement.

This paper has the purpose to advance the art of hydraulic modelling, be it ever so little beyond the above philosophical outlook.

- J.W. Kamphuis

#### ACKNOWLEDGEMENT

The author is greatly indebted to Dr. M.S. Yalin, Professor of Civil Engineering at Queen's University. It is through continuous discussions with Dr. Yalin, a personal friend and revered colleague that this paper has materialized. Many aspects of this paper have been directly inspired by his recent publication "Theory of Hydraulic Models" and by his insight into the theory of dimensions.



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SYMBOLS

$a$	wave orbital amplitude
$a_B$	value of 'a' at the bottom
$a_\delta$	value of 'a' at the top of the boundary layer
$A$	general property, dimensional dependent variable
$A_e$	area of roughness element
$C$	Chezy friction factor
$d$	depth of water
$D$	sediment particle diameter
$f$	function
$f$	Darcy Weisbach friction factor
$F$	force
$F_D$	drag force
$F$	Froude number
$g$	acceleration resulting from gravity
$H$	wave height
$k$	bottom roughness
$k_s$	sand grain roughness
$K$	eddy coefficient
$\ell$	general length parameter
$\ell$	as a subscript, refers to long waves and unidirectional flow
$L$	wave length
$m$	as a subscript, refers to the model
$m_k$	scale effect in $k$ resulting from $n_D \neq n$
$m_q$	scale effect in $q$ resulting from $n_D \neq n$
$n$	when not subscripted, general model scale

$n$	when subscripted, scale ( = prototype value / model value )
$N$	model distortion ( = $n_x / n$ )
$N_k$	distortion of total bottom roughness ( = $n_k / n$ )
$N_{k_s}$	distortion of sand grain roughness ( = $n_{k_s} / n$ )
$p$	pressure
$p$	as a subscript, refers to the prototype
$q$	sediment volume transported per unit width per unit time
$R$	hydraulic radius
$R$	Reynolds number
$s$	spacing of roughness elements
$s$	as a subscript, refers to short waves
$S$	slope of the free surface
$S_f$	surface slope caused by friction on the bed form
$S_{k_s}$	surface slope caused by friction on the grains
$t$	general time parameter
$T$	wave period
$u$	horizontal component of wave orbital velocity
$\hat{u}$	maximum value of $u$
$u_B$	value of $u$ at the bottom
$U$	velocity within the boundary layer
$U$	as a subscript, refers to the upper region
$U_L$	longshore current velocity
$\bar{U}$	average value of $U$ over the vertical
$U_\delta$	$U$ at the top of the boundary layer
$\hat{U}_\delta$	maximum value of $U_\delta$
$v_*$	shear velocity ( = $\sqrt{\tau_o / \rho}$ )

V	volume
w	sediment fall velocity
x	general horizontal space parameter
X	general horizontal space parameter within the boundary layer
$X_n$	dimensionless independent variable (in Eq. 3.2)
y	general horizontal space parameter
Y	dimensionless dependent variable (in Eq. 3.2)
$\overset{\text{vertical}}{Y}$	general horizontal space parameter within the boundary layer
z	general vertical space parameter measured upward from still water level
Z	general vertical space parameter within the boundary layer, measured upward from the bottom
$\alpha$	ripple coefficient
$\gamma_s$	underwater unit weight of sediment [ = ( $\rho_s - \rho$ ) g ]
$\delta$	boundary layer thickness
$\hat{\delta}$	maximum value of $\delta$
$\delta_L$	laminar boundary layer thickness expression
$\delta'_L$	actual thickness of the laminar boundary layer
$\delta_{vs}$	viscous sublayer thickness
$\Delta$	ripple height
$\epsilon$	lower limit of velocity distribution ( $\approx k_s / 30$ )
$\Lambda$	ripple length
$\mu$	dynamic viscosity of the fluid
$\nu$	kinematic viscosity of the fluid ( = $\mu / \rho$ )
$\xi$	drag coefficient
$\pi_A$	dimensionless dependent variable, dimensionless version of A
$\rho$	density of the fluid

$\rho_s$	density of the sediment
$\tau$	shear stress
$\tau_0$	bottom shear stress
$\tau_A$	additional shear stress
$\tau_f$	shear stress resulting from bed form
$\tau_{k_s}$	shear stress resulting from sand grain roughness
$\tau'$	effective shear stress
$\theta$	equilibrium beach slope

## 1. INTRODUCTION

The selection of model scales for hydraulic models involving wave motion is often done in a relatively arbitrary fashion. In this report a formal approach is discussed introducing the rationale related to the theory of dimensions.

It is this tool that is further used to determine scales for mobile bed wave models, an area of great uncertainty and which recently \* was described as relatively unknown by experts from some of the world's leading laboratories, in spite of a great deal of practical experimentation and model analysis.

Practical limitations are kept in mind throughout the report in order to enhance the practical value of the work.

The report has been written to serve two purposes. A framework for further study of modelling techniques presently underway at the Queen's University Coastal Engineering Laboratory was needed and therefore the report has the appearance of a state of the art paper. Secondly, the report forms the basic material for lectures on wave models, hence the didactic style.

\* Discussion at 12th Coastal Engineering Conference, Washington, September 1970.



## 2. GENERAL DESCRIPTION OF WAVE MOTION

Water waves as they occur in nature are usually classified in a number of different categories. One distinction that is often made is between short waves and long waves. Physically this distinction may be described as follows. For long waves, the vertical motion (accelerations, velocities and displacements) of particles is very small compared to the horizontal particle motion and can therefore be neglected. For short waves this vertical motion must be taken into account. Thus wind generated waves, both sea and swell, may be considered short waves, whereas tides, seiches and tsunamis are long waves. The distinction is of course academic since waves exist throughout the whole spectrum but it does help in classifying commonly occurring waves.

For wind generated waves or short waves a further distinction is usually made with respect to water depth. "Deep" water is considered to exist when the water particle motion resulting from wave action does not extend to the bottom. An approximation to this condition is given by the relation

$$\frac{d}{L} \geq 0.5 \quad (2.1)$$

where  $d$  is the water depth and  $L$  the wave length. Often a more practical lower limit for deep water is set at

$$\frac{d}{L} \geq 0.3 \quad (2.2)$$

The water depth is considered "shallow" when

$$\frac{d}{L} \leq 0.05 \quad (2.3)$$

and of "intermediate depth" when

$$0.3 \geq \frac{d}{L} \geq 0.05 \quad (2.4)$$

Again the distinction is rather academic and somewhat related to the wave theories that are traditionally used to describe wind generated waves.

It must further be realized that the boundary layer, the layer adjacent to the bottom within which shear is transferred from the bottom into the body of water, is a thin layer in the case of the wind generated wave, whereas it extends the full depth of the body of water for long waves. Since within the boundary layer the particle motion is predominantly horizontal, the above statement is consistent with the definition of long waves given earlier.

### 3. EQUATIONS FOR SHORT WAVE MOTION

#### 3.1 General Dimensional Analysis

Any property  $A$  of wave motion in a fluid may be described in its most general form by the following relationship:

$$A = f(H, T, d, k_s, \rho, \mu, g, x, y, z, t) \quad (3.1)$$

where  $H$  and  $T$  are the wave parameters - wave height and wave period,  $d$  and  $k_s$  are the fluid body parameters - depth of fluid and bottom roughness,  $\rho$  and  $\mu$  are the fluid parameters - density and viscosity,  $g$  is the acceleration resulting from gravity,  $x$ ,  $y$  and  $z$  are general space parameters ( $z$  is measured vertically up from the water surface) and  $t$  is the general time parameter (Fig. 3.1).

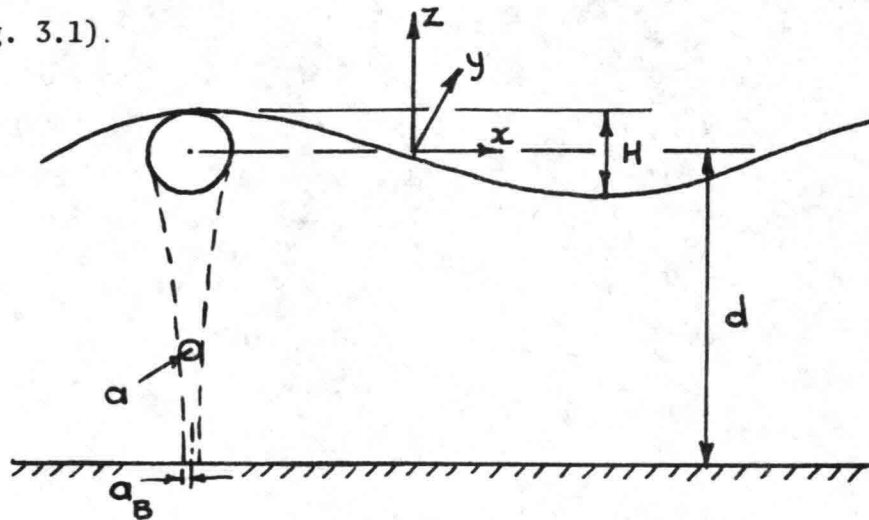


Fig. 3.1 Nomenclature for Upper Region

Equation 3.1 may be expressed in dimensionless form as

$$Y = \phi(X_1, X_2, X_3, \dots) \quad (3.2)$$

where  $Y$  and  $X_n$  are dimensionless variables.

The theory of dimensions and the method of dimensional analysis used to arrive at this new equation have been discussed in detail elsewhere (1,2,3)\* and some knowledge of the methods involved will be assumed.

Using  $T$ ,  $\rho$  and  $g$  as the repeating variables, Eq. 3.1 may be written in the dimensionless form of Eq. 3.2 as

$$\Pi_A = \phi_A \left( \frac{H}{gT^2}, \frac{d}{gT^2}, \frac{k_s}{gT^2}, \frac{\mu}{\rho g^2 T^3}, \frac{x}{gT^2}, \frac{y}{gT^2}, \frac{z}{gT^2}, \frac{t}{T} \right) \quad (3.3)$$

As example, consider wave length,  $L$ , to be the dependent quantity  $A$  in Eq. 3.1. Since  $L$  is independent of  $x$ ,  $y$ ,  $z$  or  $t$ , Eq. 3.1 may be rewritten in dimensionless form equivalent to Eq. 3.3 as

$$\Pi_L = \frac{L}{gT^2} = \phi_L \left( \frac{H}{gT^2}, \frac{d}{gT^2}, \frac{k_s}{gT^2}, \frac{\mu}{\rho g^2 T^3} \right) \quad (3.4)$$

Wavelength is such a common wave parameter that often it is used instead of  $T$  in Eq. 3.1. This results in another dimensionless relationship

$$\Pi_A = \phi'_A \left( \frac{H}{L}, \frac{d}{L}, \frac{k_s}{L}, \frac{\mu}{\rho L \sqrt{gL}}, \frac{x}{L}, \frac{y}{L}, \frac{z}{L}, t \sqrt{\frac{g}{L}} \right) \quad (3.5)$$

It may be noted that the same result can be achieved by

---

\* Superscripts refer to the references listed in the last chapter.

substituting Eq. 3.4 into Eq. 3.3. Yet another dimensionless function may be derived using  $H$ ,  $T$  and  $\rho$  as the repeating variables in Eq. 3.1. This yields

$$\Pi_A = \phi_A'' \left( \frac{d}{H}, \frac{k_s}{H}, \frac{uT}{\rho H^2}, \frac{gT^2}{H}, \frac{x}{H}, \frac{y}{H}, \frac{z}{H}, \frac{t}{T} \right) \quad (3.6)$$

Eqs. 3.3, 3.5 and 3.6 all mean the same, but use slightly different dimensionless variables.

Wave motion may be described as being composed of two definite layers, the upper region or the main body of fluid and the boundary layer, the region where viscous effects must be taken into account. As stated earlier, in the case of short waves, both these layers are present, while for long waves only the boundary layer need be considered since the boundary layer reaches the free surface.

### 3.2 Upper Region

Eqs. 3.3, 3.5 and 3.6 are valid for both the upper region and the boundary layer, however, in the case of short waves, for the upper region (above the boundary layer) certain simplifications may be made, based on the assumptions that the viscosity and the bottom roughness have negligible effect in this area. These simplifications result in

$$U\Pi_A = U\phi_A \left( \frac{H}{gT^2}, \frac{d}{gT^2}, \frac{x}{gT^2}, \frac{y}{gT^2}, \frac{z}{gT^2}, \frac{t}{T} \right) \quad (3.7)$$

$$U\Pi_A = U\phi_A' \left( \frac{H}{L}, \frac{d}{L}, \frac{x}{L}, \frac{y}{L}, \frac{z}{L}, t\sqrt{\frac{g}{L}} \right) \quad (3.8)$$

$$U^{\Pi_A} = U^{\phi_A} \left( \frac{d}{H}, \frac{gT^2}{H}, \frac{x}{H}, \frac{y}{H}, \frac{z}{H}, \frac{t}{T} \right) \quad (3.9)$$

where the subscript U refers to the upper region.

As an example, consider the horizontal component of orbital motion u. This may be expressed in a general dimensionless form, using Eq. 3.8, as

$$\Pi_u = \frac{u}{\sqrt{gL}} = \phi_u \left( \frac{H}{L}, \frac{d}{L}, \frac{x}{L}, \frac{y}{L}, \frac{z}{L}, t \sqrt{\frac{g}{L}} \right) \quad (3.10)$$

For comparison, from small amplitude wave theory, the expression for u for two dimensional motion is found to be

$$u = \frac{\pi H}{T} \frac{\cosh \left( \frac{2\pi d}{L} + \frac{2\pi z}{L} \right)}{\sinh \frac{2\pi d}{L}} \cos \left( \frac{2\pi x}{L} - \frac{2\pi t}{T} \right) \quad (3.11)$$

It may easily be shown that Eq. 3.11 is a particular expression of the general relationship represented by Eq. 3.10.

The maximum velocity,  $\hat{u}$ , is obviously not a function of x, y and t, therefore

$$\Pi_{\hat{u}} = \frac{\hat{u}}{\sqrt{gL}} = \phi_{\hat{u}} \left( \frac{H}{L}, \frac{d}{L}, \frac{z}{L} \right) \quad (3.12)$$

The comparable expression for two dimensional motion using small amplitude wave theory may be derived from Eq. 3.11.

Similarly the orbit amplitude, a, may be expressed as

$$\Pi_a = \frac{a}{L} = \phi_a \left( \frac{H}{L}, \frac{d}{L}, \frac{z}{L} \right) \quad (3.13)$$

At the bottom of this upper region, i.e. at the top of the boundary layer, the particle orbits have degenerated into a horizontal motion. Here the horizontal velocity may be expressed as

$$\frac{u_B}{\sqrt{gL}} = \phi_{u_B} \left( \frac{H}{L}, \frac{d}{L}, \frac{x}{L}, \frac{y}{L}, t \sqrt{\frac{g}{L}} \right) \quad (3.14)$$

and from small amplitude theory the comparable expression is

$$u_B = \frac{\pi H}{T \sinh \frac{2\pi d}{L}} \cos \left( \frac{2\pi x}{L} - \frac{2\pi t}{T} \right) \quad (3.15)$$

The amplitude (semi-orbit length) of the bottom motion may be expressed as

$$\frac{a_B}{L} = \phi_{a_B} \left( \frac{H}{L}, \frac{d}{L} \right) \quad (3.16)$$

Small amplitude wave theory yields

$$a_B = \frac{H/2}{\sinh \frac{2\pi d}{L}} \quad (3.17)$$

which conforms with Eq. 3.16.

### 3.3 Boundary Layer

3.3.1 Dimensional Analysis - Within the boundary layer of thickness  $\delta$  the space co-ordinates are denoted by  $X, Y$  and  $Z$  as indicated in Fig. 3.2. Note that  $Z$  is measured upward from the bottom. Boundary layer velocities will be denoted by  $U$ . At the top of the boundary



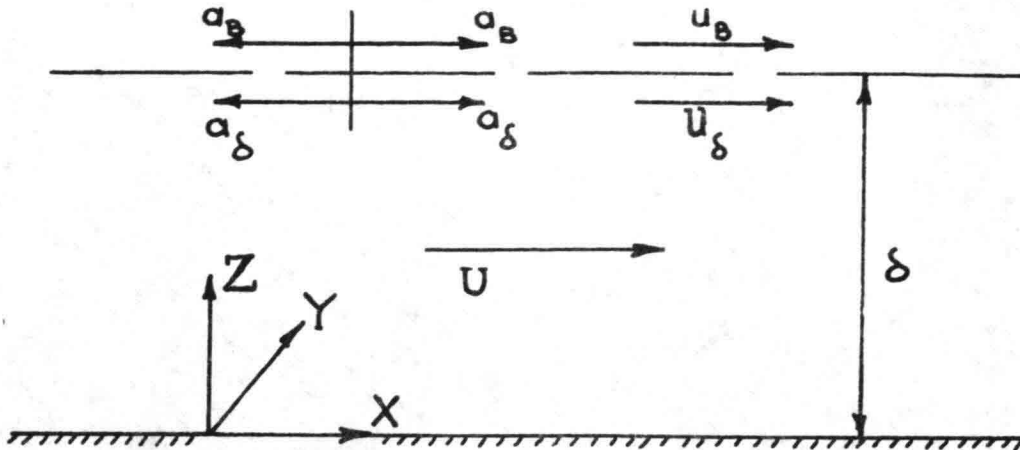


Fig. 3.2 Nomenclature for Boundary Layer

layer, the particle motion must conform with the motion outside the boundary layer. Therefore it is possible to write

$$\frac{U_\delta}{\sqrt{gL}} = \frac{U_B}{\sqrt{gL}} = \phi \left( \frac{H}{L}, \frac{d}{L}, \frac{x}{L}, \frac{y}{L}, t \sqrt{\frac{g}{L}} \right) \quad (3.18)$$

and

$$\frac{a_\delta}{L} = \frac{a_B}{L} = \phi \left( \frac{H}{L}, \frac{d}{L} \right) \quad (3.19)$$

Eq. 3.18 allows therefore a transfer of motion from the upper region to within the boundary layer and simply by imposing the correct horizontal velocity at the top of the boundary layer, it is possible to model the boundary layer separate from the upper region. Thus it is possible to write

$$A = f \left( U_\delta, k_s, \rho, \mu, X, Y, Z, t \right) \quad (3.20)$$

where the effect of gravity within the boundary layer is negligible.

In dimensionless form Eq. 3.20 becomes

$$\Pi_A = \phi \left( \frac{k_s}{U_\delta t}, \frac{\mu}{\rho U_\delta^2 t}, \frac{X}{U_\delta t}, \frac{Y}{U_\delta t}, \frac{Z}{U_\delta t} \right) \quad (3.21)$$

The concept expressed by Eq. 3.20 is very attractive but rather useless in practice since  $u$  is a function of both  $x$  and  $t$ . A common experimental facility used to model the boundary layer only is the oscillating water tunnel or equivalent equipment, where a body of water and a model bottom are displaced relative to each other. In this case the convective variation of  $u$  with  $x$  is not modelled and the total mass of water is moved with respect to the bottom as a function of time. A further simplification is often introduced by assuming  $u$  to vary sinusoidally with time and neglecting higher order effects such as mass transport (or introducing them separately). In this case

$$U_\delta = f(a_\delta, T) \quad (3.22)$$

and Eqs. 3.20 and 3.21 become

$$A = f(a_\delta, T, k_s, \rho, \mu, X, Y, Z, t) \quad (3.23)$$

$$\Pi_A = \phi' \left( \frac{k_s}{a_\delta}, \frac{\mu T}{\rho a_\delta^2}, \frac{X}{a_\delta}, \frac{Y}{a_\delta}, \frac{Z}{a_\delta}, \frac{t}{T} \right) \quad (3.24)$$

As an example, consider the shear stress. In the general case

$$\frac{\tau}{\rho U_{\delta}^2} = \phi_{\tau} \left( \frac{k_s}{U_{\delta} t}, \frac{\mu}{\rho U_{\delta}^2 t}, \frac{X}{U_{\delta} t}, \frac{Y}{U_{\delta} t}, \frac{Z}{U_{\delta} t} \right) \quad (3.25)$$

and in the case of the simplified (practical) model

$$\frac{\tau T^2}{\rho a_{\delta}^2} = \phi_{\tau}' \left( \frac{k_s}{a_{\delta}}, \frac{\mu T}{\rho a_{\delta}^2}, \frac{X}{a_{\delta}}, \frac{Y}{a_{\delta}}, \frac{Z}{a_{\delta}}, \frac{t}{T} \right) \quad (3.26)$$

As a further example consider the boundary layer thickness  $\delta$ , which is independent of  $Z$

$$\frac{\delta}{U_{\delta} t} = \phi_{\delta} \left( \frac{k_s}{U_{\delta} t}, \frac{\mu}{\rho U_{\delta}^2 t}, \frac{X}{U_{\delta} t}, \frac{Y}{U_{\delta} t} \right) \quad (3.27)$$

or in the simplified case

$$\frac{\delta}{a_{\delta}} = \phi_{\delta}' \left( \frac{k_s}{a_{\delta}}, \frac{\mu T}{\rho a_{\delta}^2}, \frac{X}{a_{\delta}}, \frac{Y}{a_{\delta}}, \frac{t}{T} \right) \quad (3.28)$$

It may be seen that  $\delta$  is a function of the horizontal space co-ordinates and time. The maximum boundary layer thickness  $\hat{\delta}$  may therefore be expressed as

$$\frac{\hat{\delta}}{U_{\delta} t} = \phi_{\delta}^{\wedge} \left( \frac{k_s}{U_{\delta} t}, \frac{\mu}{\rho U_{\delta}^2 t} \right) \quad (3.29)$$

or in the simple case

$$\frac{\hat{\delta}}{a_{\delta}} = \phi'_{\hat{\delta}} \left( \frac{k_s}{a_{\delta}}, \frac{\mu T}{\rho a_{\delta}^2} \right) \quad (3.30)$$

This definition of maximum boundary layer thickness is rather useful and allows rewriting of the awkward relationship, Eq. 3.21

$$\Pi_A = \phi \left( \frac{k_s}{\hat{\delta}}, \frac{\mu}{\rho U_{\delta} \hat{\delta}}, \frac{X}{\hat{\delta}}, \frac{Y}{\hat{\delta}}, \frac{Z}{\hat{\delta}}, \frac{t}{T} \right) \quad (3.31)$$

If it is understood that  $\delta$  is a certain defined boundary layer thickness, e.g. maximum thickness  $\hat{\delta}$ , then Eqs. 3.30 and 3.31 may be written in general as

$$\frac{\delta}{a_{\delta}} = \phi'_{\delta} \left( \frac{k_s}{a_{\delta}}, \frac{\mu T}{\rho a_{\delta}^2} \right) \quad (3.32)$$

and

$$\Pi_A = \phi \left( \frac{k_s}{\delta}, \frac{\mu}{\rho U_{\delta} \delta}, \frac{X}{\delta}, \frac{Y}{\delta}, \frac{Z}{\delta}, \frac{t}{T} \right) \quad (3.33)$$

From Eq. 3.32 it may be seen that  $\delta$  is a function only of relative roughness and viscosity and in the following subsections Eq. 3.32 is compared with common expressions derived elsewhere.

**3.3.2 Laminar Boundary Layer Equations.** - For the viscous boundary layer case, the roughness becomes unimportant, relative to the viscosity effect, and therefore as a limiting case, Eq. 3.32 may be restated as

$$\frac{\delta}{a_{\delta}} = \phi'_{\delta} \left( \frac{\mu T}{\rho a_{\delta}^2} \right) \quad (3.34)$$

From small amplitude wave theory, using the laminar dissipation function it may be seen theoretically (4, pp 81-87) that

$$\delta_L = \sqrt{\frac{\mu T}{\pi \rho}} \quad (3.35)$$

where  $\delta_L$  is an expression for the thickness of the laminar boundary layer. In actual fact if the boundary layer thickness is defined as the layer where

$$U \leq 0.99 U_\delta$$

then calling this thickness  $\delta'_L$

$$\delta'_L \approx 5 \delta_L \approx 5 \sqrt{\frac{\mu T}{\pi \rho}} \quad (3.36)$$

Either case may be shown to conform with Eq. 3.33.

**3.3.3 Rough Turbulent Boundary Layer Equations.** - For the rough turbulent boundary layer case, using Eq. 3.32

$$\frac{\delta}{a_\delta} = \phi' \left( \frac{k_s}{\delta} \right) \quad (3.37)$$

From boundary layer theory and experiment, the following relationships may be obtained. Flow may be classified as rough turbulent if the bottom roughness elements protrude through the viscous sublayer to such an extent that the flow is only a function of form drag on the roughness elements and no longer depends upon the viscosity within the boundary layer. For unidirectional flow, Schlichting<sup>(5, p 580)</sup> states empirically that this occurs when the roughness Reynolds number

$$\frac{v_* k_s}{\nu} > 70 \quad (3.38)$$

where  $v_*$  is the shear velocity

$$v_* = \sqrt{\frac{\tau_o}{\rho}}$$

and  $\nu$  is the kinematic viscosity,  $\mu/\rho$ .

Hydraulically smooth flow is defined to exist when the roughness elements lie wholly within the viscous sublayer layer, and at the same time do not deform the top of the viscous sublayer to the extent of disturbing the main flow. This is explained in detail by Yalin (17, p. 34)

$$\frac{v_* k_s}{\nu} < 5 \quad (3.39)$$

The range

$$5 < \frac{v_* k_s}{\nu} < 70 \quad (3.40)$$

is called the transition regime.

The well known expression (5, p 582) for the velocity distribution, in fully developed hydraulically rough turbulent flow in one direction, is

$$\frac{U}{v_*} = 2.5 \ln 30 \frac{Z}{k_s} = 5.75 \log \frac{Z}{k_s} + 8.5 \quad (3.41)$$

This expression is approximately applicable to oscillatory motion when  $\frac{\partial U}{\partial t} \rightarrow 0$ , i.e. near maximum velocity, under a wave crest or through. A comparison between Eqs. 3.25 and 3.41 indicates that the latter is a very simplified form of Eq. 3.25. At the top of the boundary

layer, i.e. at  $Z = \delta$ , Eq. 3.41 becomes

$$\frac{U_{\delta}}{v_*} = 2.5 \ln 30 \frac{\delta}{k_s} \quad (3.42)$$

For short waves the velocity distribution within the boundary layer is relatively unknown and Eq. 3.41 certainly is not applicable throughout the entire wave period. Only at times when  $\frac{\partial U}{\partial t} \rightarrow 0$ , could it approximate the true velocity distribution relatively closely. Until more accurate measurements show evidence to the contrary it has been usual (9,10,11) to assume the velocity distribution to be logarithmic as in Eq. 3.41.

The simplified form of the boundary layer equations

$$\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial X} = - \frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 U}{\partial z^2}$$

may, when neglecting the small convective term, be written as

$$\frac{\partial U}{\partial t} = - \frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{1}{\rho} \frac{\partial \tau}{\partial z} \quad (3.43)$$

Further it may be stated from small amplitude wave theory that

$$\frac{\partial U_{\delta}}{\partial t} = - \frac{1}{\rho} \frac{\partial p}{\partial x}$$



and combining these equations, the pressure gradient  $\frac{\partial p}{\partial x} = \frac{\partial p}{\partial x}$ , may be eliminated to give

$$\frac{\partial}{\partial t} (U_\delta - U) = - \frac{1}{\rho} \frac{\partial \tau}{\partial z} \quad (3.44)$$

Integration of Eq. 3.44 yields

$$\frac{\tau_o}{\rho} = \int_{\epsilon}^{\delta} \frac{\partial}{\partial t} (U_\delta - U) dz \quad (3.45)$$

where  $\epsilon$  is the lower limit of applicability of the velocity distribution. Jonsson<sup>9</sup> performs this integration for  $\epsilon = \frac{k_s}{30}$  and arrives at the same answer as if  $\epsilon = 0$ . An expression for  $(U_\delta - U)$ , the velocity deficit may be derived from Eqs. 3.41 and 3.42

$$\frac{(U_\delta - U)}{v_*} = 2.5 \ln \frac{\delta}{z} \quad (3.46)$$

which is valid in the neighbourhood of

$$\frac{\partial U}{\partial t} = \frac{\partial U_\delta}{\partial t} = 0.$$

Substitution of Eq. 3.46 into Eq. 3.45 gives

$$\frac{\tau_o}{\rho} = \int_0^{\delta} \frac{\partial}{\partial t} (2.5 v_* \ln \frac{\delta}{z}) dz$$

where  $\epsilon$  is assumed to be zero. This may be rewritten as

$$\frac{\tau_o}{\rho} = 2.5 \frac{\partial}{\partial t} \int_0^{\delta} v_* \ln \frac{\delta}{z}$$

$$\frac{\tau_o}{\rho} = 2.5 \frac{\partial}{\partial t} \left[ v_* \left( -z \ln \frac{\delta}{z} + z \right) \right]_0^{\delta}$$

$$\frac{\tau_o}{\rho} = 2.5 \frac{\partial}{\partial t} (v_* \delta) \quad (3.47)$$

and this equation is strictly speaking only valid in the neighbourhood of  $\frac{\partial U}{\partial t} = 0$ . Substitution of Eq. 3.42 into Eq. 3.47 yields

$$\frac{(0.4)^2 U_{\delta}^2}{\ln^2 30 \frac{\delta}{K_s}} = 2.5 \frac{\partial}{\partial t} \left( \delta \frac{0.4 U_{\delta}}{\ln 30 \frac{\delta}{K_s}} \right) \quad (3.48)$$

Simplifying Eq. 3.22 (to be written at a definite value of  $\delta$ , i.e. at a definite time) to

$$U_{\delta} = \text{cst} \left( \frac{a_{\delta}}{T} \right)$$

and substituting into Eq. 3.48 gives an expression which can at best only be considered a gross approximation

$$\frac{\text{cst} \frac{a_{\delta}^2}{T^2}}{\ln^2 30 \frac{\delta}{K_s}} = \text{cst} \frac{\delta \frac{a_{\delta}}{T}}{T \ln 30 \frac{\delta}{K_s}}$$

or

$$\frac{a_\delta}{\delta} = \text{cst} \left( \ln 30 \frac{\delta}{k_s} \right) \quad (3.49)$$

This expression corresponds to the one derived by Jonsson (9, Eq. 4.16) and conforms with Eq. 3.37, but the number of simplifying assumptions made in this comparison must be noted carefully!

The criterion for existence of hydraulically rough turbulent flow under short waves is stated as follows. The data are very scant and the thinking behind the representation is by no means complete. Several experimental investigations have been performed and Li <sup>(6)</sup> states that the flow may be considered hydraulically rough when

$$\frac{k_s}{\delta_L} > 0.25 \quad (3.50)$$

while it is smooth for

$$\frac{k_s}{\delta_L} < 0.15 \quad (3.51)$$

To ensure turbulent flow within the boundary layer an additional critical condition is usually given and Brebner, Askew and Law <sup>(7)</sup> indicate that

$$\frac{\hat{U}_\delta k_s}{\nu} > 100 \quad (3.52)$$

where  $\hat{U}_\delta$  signifies the maximum value of  $U_\delta$ .

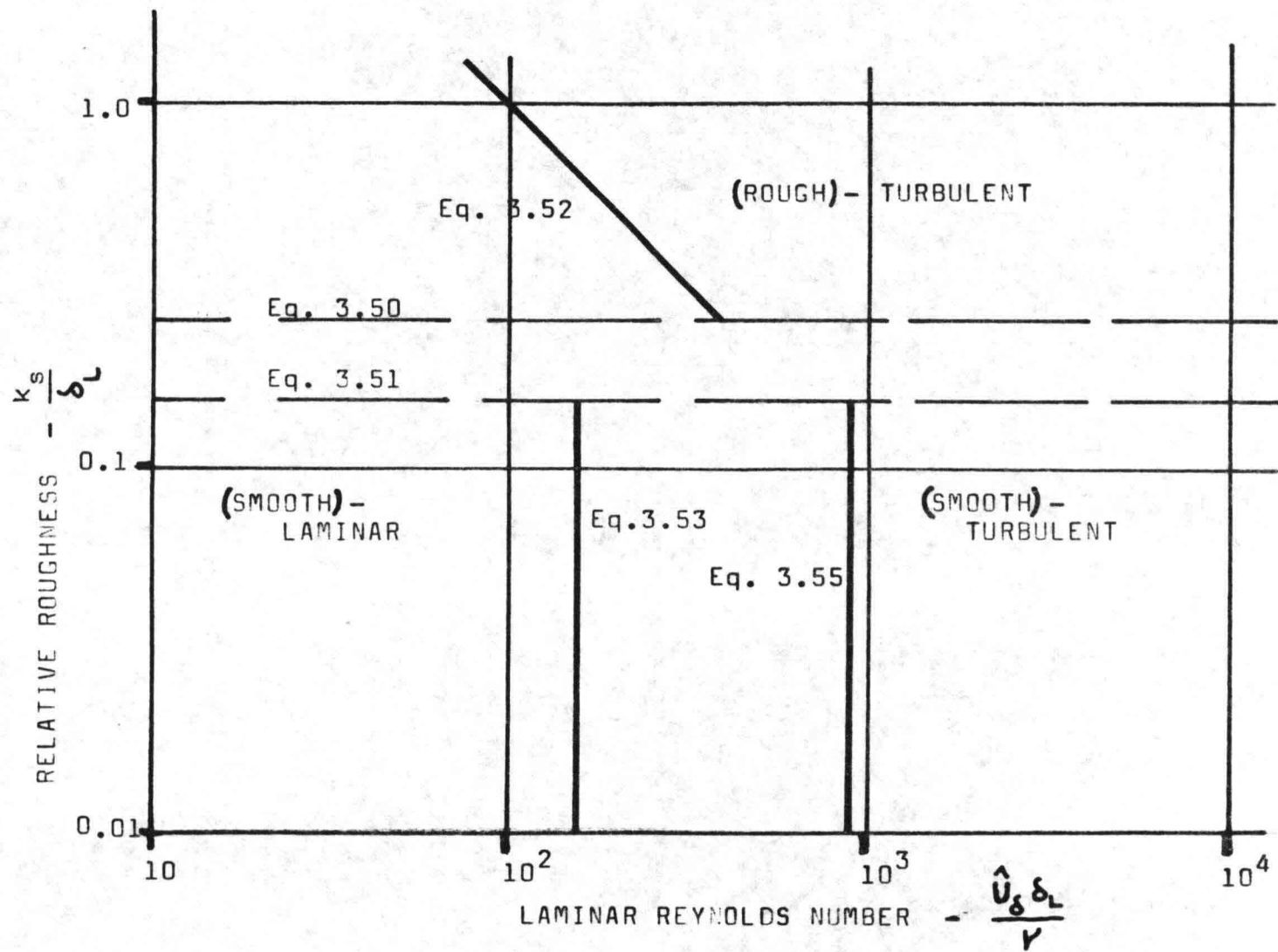


FIGURE 3 - BOUNDARY LAYER FLOW REGIMES

Kalkanis <sup>(11)</sup> using Li's equipment expresses approximate agreement with this equation.

Collins <sup>(12)</sup> found that on a smooth bottom laminar flow conditions exist until

$$\frac{\hat{U}_\delta \delta_L}{\nu} = 160 \quad (3.53)$$

Kajiura <sup>(8)</sup> states that for smooth boundaries the flow is laminar if

$$\frac{\hat{U}_\delta \delta_L}{\nu} < 35 \quad (3.54)$$

while it is turbulent for

$$\frac{\hat{U}_\delta \delta_L}{\nu} > 920 \quad (3.55)$$

Using the above equations Fig. 3 has been drawn up outlining the approximate locations of boundary layer flow regimes. These must be further determined by additional research.

#### 4. SCALE SELECTION FOR SHORT WAVE MODELS

##### 4.1 General

Dimensionless relationships, such as those written in Chapter 3 are valid for both model and prototype, in fact if the model is dynamically similar to the prototype all the dimensionless ratios are identical. The first such ratio in Eq. 3.3 is  $H/gT^2$ . If model and prototype values are denoted by subscripts  $m$  and  $p$  respectively, then for dynamically similar models

$$\left( \frac{H}{gT^2} \right)_m = \left( \frac{H}{gT^2} \right)_p \quad (4.1)$$

or

$$\frac{H_m}{g_m T_m^2} = \frac{H_p}{g_p T_p^2} \quad (4.2)$$

If the scale,  $n$ , is defined as the ratio of the prototype value over the model value in order to obtain convenient integer values, rather than fractions, for the major scales, then

$$n_H = \frac{H_p}{H_m}, \quad n_T = \frac{T_p}{T_m}, \quad n_g = \frac{g_p}{g_m} \quad (4.3)$$

From Eq. 4.2 it may be seen that

$$n_H = n_g n_T^2 \quad (4.4)$$

Because  $n_g$  may usually be approximated by one, Eq. 4.4 may be

simplified as

$$n_H = n_T^2 \quad (4.5)$$

Eq. 4.5 is therefore one scale law that governs design of a proper dynamically similar model. Using Eqs. 3.3, 3.5 and 3.6 in a similar fashion, the following scale laws may be derived for short wave models.

$$n_H = n_L = n_d = n_{k_s} = n_x = n_y = n_z = n \quad (4.6)$$

where  $n$  is the general model scale, which may be freely chosen.

Using the notation of Eq. 3.2, it may be seen that the dimensionless ratios  $X_3$  and  $X_4$  of Eq. 3.6 both give scale laws with respect to the wave period  $T$  :

$$n_T n_\mu = n_\rho n_H^2 \quad \text{and} \quad n_T^2 n_g = n_H \quad (4.7)$$

Since for practical models  $n_g = n_\mu = n_\rho = 1$ , these expressions present two different relations between  $n_T$  and  $n_H$ . This is a familiar situation.  $X_3$  is a Reynolds number while  $X_4$  is a Froude number, leading to the well known classification of Reynolds number models, neglecting the effect of  $g$  and therefore of  $X_4$ , and Froude number models neglecting the effect of viscosity. The two ratios  $X_3$  and  $X_4$  are mutually exclusive if  $n_g = n_\rho = n_\mu = 1$  and a choice must be made. The choice obviously depends on the relative importance of viscosity or gravity. As discussed in Chapter 3, outside the boundary layer in the upper region



viscosity, and therefore  $X_3$ , becomes unimportant. Eq. 3.6 neglecting  $X_3$  then yields

$$n_T = n_H^{1/2} = n^{1/2} = n_t \quad (4.8)$$

This is in agreement with Eq. 3.9 which was especially written for the upper region and where viscosity does not appear. Within the boundary layer, however,  $X_3$  is important. But  $X_4$  is also important here because conditions within the boundary layer result from the motion outside the boundary layer. Therefore it does not appear possible to model both the upper flow and the boundary layer flow simultaneously. Fortunately there is one exception: the common case of fully established rough turbulent boundary layer flow. In this case the viscosity term becomes negligible throughout the boundary layer and the upper region causing Eq. 4.8 to be valid for this type of model, not only outside, but also within the boundary layer.

If viscosity is important within the boundary layer, the problem is more difficult. One solution is to model the boundary layer separately. This means that the orbital motion outside the boundary layer is modelled correctly as suggested in Eqs. 3.20 to 3.24, thus obviating concern about gravity and consequently about  $X_4$ .

#### 4.2 Upper Region Models

For the upper region the following scale laws may be derived from Eqs. 3.7 to 3.9.

$$n_H = n_L = n_d = n_x = n_y = n_z = n \quad (4.9)$$

$$n_t = n_T = n^{1/2} \quad (4.10)$$

Further from Eqs. 3.10 and 3.13

$$n_u = n^{1/2} \quad n_a = n \quad (4.11)$$

The type of model dealt with in this section is the usual two or three dimensional wave model, where little concern is expressed about the effects of bottom boundary layers. In this type of model, often large plan sizes must be modelled in a limited laboratory space. At the same time the vertical scale must be small enough so that wave heights and bottom contours can be modelled and measured with sufficient accuracy. This introduces the concept of distortion, i.e. the plan scales are greater than the vertical scales.

The condition expressed in Eq. 3.1, is very general and under certain circumstances this condition may be relaxed in order to allow distortion. For instance, a refraction pattern is not affected by  $x$  and  $y$ . Thus, for models where refraction is the major consideration,  $n_x$  and  $n_y$  need not be the same as  $n_z$ , the scale that determines the refraction pattern. In general, for a model where distortion is acceptable Eqs. 4.9 to 4.11 may be rewritten as

$$\begin{aligned} n_H &= n_L = n_d = n_z = n_a = n \\ n_u &= n_t = n_T = n^{1/2} \\ n_x &= n_y = Nn \end{aligned} \quad (4.12)$$

where  $N$  is the model distortion

$$N = \frac{n_x}{n_z} = \frac{n_y}{n_z} \quad (4.13)$$

Both  $N$  and  $n$  may be freely chosen, which could be coined as two "degrees of choice".  $n$  is normally determined by the laboratory size and the area to be modelled.  $N$  is usually a function of the accuracy of field measurements of depth and wave height, model accuracies of the same quantities, minimum depths required in certain model areas and maximum slopes that can be used without causing additional effects such as separation and vortices. It might appear that this is an easier model to design since there are two "degrees of choice". But the model is now based on a number of additional trade-offs, necessary in order to achieve this extra choice. These must be carefully evaluated for each model. As an example consider a refraction-diffraction model. If two "degrees of choice", i.e. a distorted model, are insisted upon, the total wave field consisting of the model diffraction pattern and the model refraction pattern does not correspond to the prototype wave field. Thus the effect of this discrepancy must be evaluated and, in the light of this, the "degrees of choice" are determined - either one (undistorted), or two (distorted).

#### 4.3 Boundary Layer Models

In many cases the velocities within the boundary layer are important and must be modelled correctly. Sediment transport models fall into this category.

4.3.1 Rough Turbulent Boundary Layer. - If the boundary layer flow is fully developed rough turbulent in both model and prototype, viscosity is negligible and Eqs. 4.12 and 4.13 are valid.

From Eqs. 3.21, 3.24 and 3.33 it may be seen that

$$n_{\delta} = n_a = n_{k_s} = n_z = n \quad (4.14)$$

while for a distorted model

$$n_x = n_y = n_X = n_Y = Nn \quad (4.15)$$

In practice it is often impossible to satisfy  $n_{k_s} = n$ , because the model particles need to be too small or the model bottom too smooth. Usually  $n_{k_s} \leq n$ . Choosing a different  $n_{k_s}$  has some consequences which should be investigated. A different value of  $n_{k_s}$  will have little influence on the motion in the upper region. In the boundary layer, however, the influence will be felt. From physical concepts it may therefore be argued that if  $n_{k_s} \neq n$

$$n_{\delta} \neq n \quad (4.16)$$

while

$$n_a = n_z = n \quad (4.17)$$

Not modelling  $k_s$  to scale represents another "degree of choice" and again constitutes certain further limitations imposed on the model. The ratio of  $n_{k_s}/n$  may be defined as the bottom roughness distortion  $N_{k_s}$ . When a larger roughness is used (usual case),  $N_{k_s} < 1$ . To gain some insight into the consequences

of distorting the bottom roughness the following approximation is made

$$\ln 30 A \approx \text{cst } (A)^{1/6} \quad \text{for} \quad 2 \leq A \leq 100 \quad (4.18)$$

where  $A$  is simply the argument of the logarithmic function.

Thus Eq. 3.49 may be expressed as

$$\frac{a_\delta}{\delta} \approx \text{cst} \times \left( \frac{\delta}{k_s} \right)^{1/6} \quad (4.19)$$

which yields

$$\delta \approx \text{cst} \times a_\delta^{6/7} k_s^{1/7} \quad (4.20)$$

and therefore

$$n_\delta \approx n^{6/7} n_{k_s}^{1/7} = n N_{k_s}^{1/7} < n \quad (4.21)$$

The boundary layer thickness is therefore increased as a result of the larger bottom roughness.

It may be noted that the boundary layer thickness is distorted by the seventh root of the roughness distortion, i.e. the boundary layer thickness is not seriously affected.

For bottom shear Eqs. 3.25 and 3.26 indicate that

$$n_\tau = n \quad (4.22)$$

This may be compared with the expression derived by Kajiura <sup>(8)</sup>, who assumes that the shear stress may be expressed as

$$\tau = K \frac{\partial U}{\partial Z}$$

where  $K$  is a type of eddy coefficient. He states that in the outer layer

$$K = f(U_\delta, \delta_L)$$

yielding

$$n_\tau = n_{U_\delta} n_{\delta_L} \frac{n_U}{n_Z} = n$$

which is the same as Eq. 4.22.

For the case of roughness distortion one can only rely on physical equations applicable within the boundary layer. Eq. 3.47 yields

$$n_{V_*} \approx \frac{n_\delta}{n_t} = n^{1/2} N_{k_s}^{1/7} \quad (4.23)$$

and therefore

$$n_\tau \approx n N_{k_s}^{2/7} \quad (4.24)$$

The velocity scales may be derived as follows. Using Eqs. 3.41, 3.42 and 3.46 it may be seen that for  $N_{k_s} = 1$

$$n_U = n_{U_\delta} = n_U = n^{1/2} \quad (4.25)$$

but in the case of roughness distortion, Eq. 3.42 yields

$$n_{U_\delta} = n_{V_*} \frac{n_\delta^{1/6}}{n_{k_s}^{1/6}} = n^{1/2} \quad (4.26)$$

This is as might be expected since  $U_\delta = u_B$ . Eq. 3.41 yields

$$n_U = n_{V*} \frac{n^{1/6}}{n_{k_s}} = n^{1/2} N_{k_s}^{-1/42} \quad (4.27)$$

This equation obviously does not apply either very close to the bottom or at  $Z = \delta$ . But in any case the correction factor is small. Therefore it may be stated that

$$n_U \approx n^{1/2} \quad (4.28)$$

In summary, the model scales for fully developed rough turbulent oscillatory boundary layer flow are

$$\begin{aligned} n_H &= n_L = n_d = n_z = n_Z = n_a = n \\ n_u &= n_{U_\delta} = n_t = n_T = n^{1/2} \\ n_U &= n^{1/2} N_{k_s}^{-1/42} \approx n^{1/2} \\ n_x &= n_y = n_X = n_Y = Nn \\ n_\delta &\approx n^{6/7} n_{k_s}^{1/7} = n N_{k_s}^{1/7} \\ n_\tau &\approx n N_{k_s}^{2/7} \\ n_{V*} &\approx n^{1/2} N_{k_s}^{1/7} \end{aligned} \quad (4.29)$$

4.3.2 Laminar Boundary Layer. - In the unusual case where the prototype boundary layer and the model boundary layer are both laminar, the viscosity cannot be ignored but the bottom roughness becomes immaterial, Eq. 3.34. The boundary layer region must be modelled separately as discussed earlier. Eqs. 3.21, 3.24, 3.25, 3.26 and 3.33 apply and from these equations, the following general scale laws may be derived.

$$\begin{aligned} n_\delta &= n_a = n_X = n_Y = n_Z = n \\ n_t &= n_T = n^2 \\ n_{U_\delta} &= \frac{1}{n} \\ n_\tau &= \frac{1}{n^2} \end{aligned} \quad (4.30)$$

The last expression agrees with Yalin (13, Eq. 4.13)

$$\tau_o = \text{cst} \frac{\hat{\mu} U_\delta}{\delta} \left[ \sin \left( \frac{2\pi t}{T} + \epsilon \right) + \cos \left( \frac{2\pi t}{T} + \epsilon \right) \right]$$

and with the work of Iwagaki (14)

$$\hat{\tau}_o = \frac{\rho g H \sqrt{2\nu}}{g \sinh kd} \left( \frac{\pi}{T} \right)^{3/2}$$

Note that for this case, within reasonable limits, bottom roughness distortion does not affect the scaling laws.

The separation of the boundary layer region from the upper region can only be achieved in an oscillating water tunnel, or equivalent equipment, with the restriction that convective accelerations



are not modelled. It may be seen from Eq. 4.30 that for laminar boundary layers the period scale is equal to the model scale squared. Even for very small model scales, the period scale mounts rapidly. This type of model must therefore necessarily have a scale close to unity, modelling the boundary layer almost at full scale, since physically it is impossible to work with too large a period scale. Also a large period scale would result in a turbulent boundary layer within the model and this is obviously undesirable.

When scaling down a prototype turbulent boundary layer, the possibility exists that the model boundary layer flow becomes laminar. While the prototype Reynolds number is in the turbulent range, the model Reynolds number is in the laminar range and therefore viscous effects cannot be ignored. The boundary layer may be modelled separately as described above. If this is not done, then the period scale is forced to be equal to  $n^{1/2}$  by the upper layer flow. This causes a Reynolds number scale which may be derived from Eq. 3.24

$$n_R = \frac{n_T}{n_{a_\delta}^2} = n^{-3/2} \ll 1 \quad (4.31)$$

Thus the Reynolds number becomes greatly exaggerated, e.g. if  $n = 25$ ,  $n_R = \frac{1}{125}$ . This is an impossible situation and in fact the model boundary layer will remain turbulent. The model results, however, are open to question. This phenomenon is even enhanced when  $k_s$  is not modelled to scale.

To gain some insight into the effect of modelling a laminar boundary layer in combination with the upper layer flow, suppose that the model boundary layer, in spite of Eq. 4.31, is laminar. Eq. 3.35

then yields

$$n_{\delta} = n_T^{1/2} = n^{1/4} \quad (4.32)$$

This indicates that for these conditions the model boundary layer thickness is greatly exaggerated and approaches the prototype boundary layer thickness.

The reduction of a turbulent boundary layer to a laminar range is rather common. Eq. 4.31 indicates that the model boundary layer will likely remain turbulent and this condition is very difficult to identify and should be avoided or the boundary layer should be modelled separately.

**4.3.3 Boundary Layer Models of Scale 1.** - Several times in the preceding the oscillating water tunnel or equivalent equipment has been mentioned. It has been shown that this equipment is a valid method of reproducing laminar boundary layers. It has also been shown that these models are operated at  $n \approx 1$ . The usefulness of this equipment goes far beyond laminar boundary layers, however. The turbulent boundary layer can also be modelled to almost prototype scale. This circumvents the problem of roughness distortion and facilitates measurement within the boundary layer. For mobile bed models to be discussed later prototype material may be used. The disadvantage of this type of equipment is that convective accelerations are not modelled.

## 5. EQUATIONS FOR LONG WAVES AND UNIDIRECTIONAL FLOW

### 5.1 Dimensional Analysis

For long waves it is only necessary to consider the boundary layer region since the boundary layer occupies the whole depth of the flow. Therefore

$$A = f ( U, T, d, k_s, \rho, \mu, g, X, Y, Z, t ) \quad (5.1)$$

which may be written in dimensionless form as

$$\pi_A = \phi_A \left( T \sqrt{\frac{g}{d}}, \frac{k_s}{d}, \frac{\mu}{\rho U d}, \frac{U}{\sqrt{g d}}, \frac{X}{d}, \frac{Y}{d}, \frac{Z}{d}, t \sqrt{\frac{g}{d}} \right) \quad (5.2)$$

Because the flow in both model and prototype is almost always rough turbulent (ensured in the design of such a model), the Reynolds number effect is negligible and

$$\pi_A = \phi_A \left( T \sqrt{\frac{g}{d}}, \frac{k_s}{d}, \frac{U}{\sqrt{g d}}, \frac{X}{d}, \frac{Y}{d}, \frac{Z}{d}, t \sqrt{\frac{g}{d}} \right) \quad (5.3)$$

### 5.2 Equations

Eqs. 3.38 and 3.41 may be considered valid, i.e.

$$\frac{v_* k_s}{\nu} > 70 \quad (3.38)$$

and

$$\frac{U}{v_*} = 2.5 \ln 30 \frac{Z}{k_s} \quad (3.41)$$

For long waves and unidirectional flow it is usually easier to deal with average velocities over the vertical (not time average)

$$\begin{aligned}
 \frac{\bar{U}}{v_*} &= \frac{1}{d} \int_0^d \left( 2.5 \ln 30 \frac{z}{k_s} \right) dz \\
 &= \frac{2.5 k_s}{30 d} \left[ 30 \frac{z}{k_s} \ln 30 \frac{z}{k_s} - 30 \frac{z}{k_s} \right]_0^d \\
 &= 2.5 \left( \ln 30 \frac{d}{k_s} - 1 \right) \\
 \frac{\bar{U}}{v_*} &= 2.5 \ln 11 \frac{d}{k_s} \tag{5.4}
 \end{aligned}$$

This equation may be written in terms of the Chezy friction coefficient (C) and the Darcy Weisbach friction factor (f) as

$$\frac{\bar{U}}{v_*} = \frac{C}{\sqrt{g}} = \sqrt{\frac{8}{f}} = 2.5 \ln 11 \frac{d}{k_s} \tag{5.5}$$

Shear stress may be computed using the equation of motion

$$\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial X} = - \frac{1}{\rho} \frac{\partial p}{\partial X} + \nu \frac{\partial^2 U}{\partial Z^2} \tag{5.6}$$

In the case of long waves and unidirectional flow

$$- \frac{1}{\rho} \frac{\partial p}{\partial X} \approx - gS \tag{5.7}$$

Where S is the slope of the free surface. Neglecting convective inertia, Eq. 5.6 may therefore be rewritten as

$$\frac{\partial U}{\partial t} = -gS + \frac{1}{\rho} \frac{\partial \tau}{\partial z} \quad (5.8)$$

Integration may be performed on Eq. 5.8 to yield

$$\frac{\tau_o}{\rho} = \int_{\epsilon}^d \left( \frac{\partial U}{\partial t} + gS \right) dz \quad (5.9)$$

$$= \frac{\partial}{\partial t} \int_{\epsilon}^d U dz + gS \int_{\epsilon}^d dz$$

Assuming  $\epsilon \approx 0$  this yields

$$\frac{\tau_o}{\rho} = \frac{\partial}{\partial t} (\bar{U}d) + gSd \quad (5.10)$$

$$\frac{\tau_o}{\rho} = 2.5 \frac{\partial}{\partial t} \left( v_* d \ln 11 \frac{d}{k_s} \right) + gSd \quad (5.11)$$

It may be seen that the shear stress has two components; one resulting from the velocity and one resulting from the slope of the water surface.

## 6. SCALE SELECTION FOR LONG WAVES

For long waves, the vertical accelerations are negligible. Another statement which means the same is that the boundary layer occupies the entire depth of the water, or

$$\delta \equiv d \quad (6.1)$$

Assuming rough turbulent flow in both the model and the prototype (always assured in the design of such a model) and assuming that the model is distorted by a distortion  $N$  (almost always a necessity in order to model miles of plan and feet of depth simultaneously)

$$n_d = n_z = n \quad (6.2)$$

while

$$n_x = n_y = Nn \quad (6.3)$$

Since it is absolutely essential to model the length of the long wave correctly in plan, e.g. water level elevations in a tide or flood model must be geographically correct,

$$n_L = n_x = Nn \quad (6.4)$$

This represents the crucial difference between long wave and short wave models. However, accepting this "degree of choice" means that Eq. 5.3 is no longer satisfied completely, since this equation assumes no distortion.

The Froude criterion must still be satisfied and therefore

$$n_U^- = n_U = n_d^{1/2} = n^{1/2} \quad (6.5)$$

but the time scale and therefore the period scale must be derived from

$$dU = \frac{dX}{dt}$$

and therefore

$$n_t = \frac{n_X}{n_U} = Nn^{1/2} = n_T \quad (6.6)$$

Using Eq. 5.4 and assuming that

$$\ln 11 A \approx \text{cst } A^{1/8} \quad \text{for } 10 < A < 100,000 \quad (6.7)$$

it is possible to write

$$n_U^- = n_{V*} \left( \frac{n_d}{n_{k_s}} \right)^{1/8} \quad (6.8)$$

and  $n_{k_s}$  and  $n_{V*}$  are therefore related by

$$n_{V*} = \frac{n^{1/2}}{n^{1/8}} n_{k_s}^{1/8} = n^{1/2} N_{k_s}^{1/8} \quad (6.9)$$

or

$$n_t = n N_{k_s}^{1/4} \quad (6.10)$$

From the expression for shear, Eq. 5.11, it may be seen that  
for  $S = 0$

$$n_{v*}^2 = \frac{n_{v*} n_d}{n_t} \frac{n_d^{1/8}}{n_{k_s}^{1/8}}$$

or

$$n_{v*} = n^{1/2} N^{-1} N_{k_s}^{-1/8} \quad (6.11)$$

Thus for proper modelling of the shear Eq. 6.11 and Eq. 6.9 lead to

$$n^{1/2} N_{k_s}^{1/8} = n^{1/2} N^{-1} N_{k_s}^{-1/8}$$

or

$$N_{k_s} = \frac{1}{N^4} \quad (6.12)$$

For the case of zero velocity Eq. 5.11 yields

$$n_{v*} = n^{1/2} N^{-1/2} \quad (6.13)$$

and the required bottom roughness distortion may be derived as

$$n^{1/2} N_{k_s}^{1/8} = n^{1/2} N^{-1/2}$$

or

$$N_{k_s} = \frac{1}{N^4}$$

which is the same as eq. 6.12.

Substitution of Eq. 6.12 into Eq. 6.10 yields

$$n_t = n N^{-1} \quad (6.14)$$



a necessary condition for the velocities and current patterns to be modelled correctly.

From Eq. 6.12 it is also possible to define the bottom roughness scales required. Using Eqs. 5.5 and 6.7

$$n_C = N_{k_s}^{-1/8} = N^{1/2} \quad \text{and} \quad n_f = N^{-1} \quad (6.15)$$

The first expression is identical to the ones derived by Bijker (15,V3-1) and Yalin (1, p 119).

In a distorted model, all slopes are magnified by the distortion  $N$ , i.e.

$$n_S = N^{-1} \quad (6.16)$$

where  $S$  is the surface slope. This causes all velocities to be exaggerated and in order to maintain Eq. 6.5 a very large value of shear must be introduced. This may be seen when comparing Eq. 6.14 with Eq. 4.24, i.e. the shear for a long wave model is approximately  $N$  times the shear for a short wave model.

This requirement for additional shear may be satisfied either by using additional bottom roughness, or by introducing roughness in the form of vertical strips or rods. In any case, with a view to the subsequent discussion on sediment transport it is important to distinguish between actual bottom shear and the "shear" introduced by additional vertical roughness elements.

If roughness strips or rods are used in order to satisfy Eqs. 6.14 and 6.15, the drag on each roughness element may be

expressed as

$$F_D = \xi \rho A_e \bar{U}_m^2$$

where the subscript  $m$  refers to the model, where  $\xi$  is a drag coefficient which is a function of the shape of the roughness elements and where  $A$  is the cross sectional area of a roughness element facing the flow. If the spacing of the elements is  $s$ , then each causes a "shear"

$$\tau_m = \frac{F_D}{s} = \xi \frac{\rho A_e \bar{U}_m^2}{s}$$

To satisfy Eq. 6.14, ignoring for the moment all bottom shear,

$$\frac{\tau_p}{\tau_m} = \frac{\tau_p}{\xi \frac{\rho A_e \bar{U}_m^2}{s}} = \frac{n}{N}$$

Therefore

$$\frac{A_e}{s} = \frac{N}{\xi} \left( \frac{v_*}{\bar{U}} \right)_p^2 \quad (6.17)$$

where the subscript  $p$  refers to the prototype.

Using Eq. 5.5 this may be related to the roughness coefficients as

$$\frac{A_e}{s} = \frac{N}{\xi} \frac{g}{C_p^2} = \frac{N}{\xi} \frac{f_p}{8} \quad (6.18)$$

Eq. 6.18 may be used as a first estimate of necessary additional roughness required in the model. It is evident that Eq. 6.18 assumes that the bottom shear is negligible with respect to the additional roughness introduced and therefore Eq. 6.18 will give an overestimate of the number of roughness elements required. In any case to compute the additional roughness, whether bottom roughness (Eq. 6.12) or vertical roughness elements, Eq. 6.18 may be used. This value of additional roughness, needs considerable adjustment during the calibration stage of the model study.

If roughness strips or rods are used, what value must be assigned to  $v_*$  and  $\tau$  where it is understood that these values apply to bottom shear only? If it is assumed that the extra roughness elements do not interfere with what is basically a logarithmic velocity profile in the model, caused by actual bottom roughness, then Eqs. 6.9 and 6.10 may be used inferring that

$$n_{v_*} = n^{1/2} N_{k_s}^{1/8} \quad \text{and} \quad n_\tau = n N_{k_s}^{1/4} \quad (6.19)$$

where  $v_*$ ,  $\tau$  and  $N_{k_s}$  refer to actual bottom shear and roughness distortion.

In summary then it may be said that for long wave and unidirectional flow models under fully developed rough turbulent flow conditions

$$\begin{aligned} n_d &= n_z = n \\ n_x &= n_y = n_L = Nn \\ n_U^- &= n_U = n^{1/2} \end{aligned} \quad (6.20)$$

$$\begin{aligned}
n_t &= n_T = Nn^{1/2} \\
N_{k_s} &= N^{-4} \\
n_\tau &= nN^{-1} \\
n_C &= N^{1/2} \\
n_f &= N^{-1} \\
n_S &= N^{-1}
\end{aligned}
\tag{6.20}$$

(cont'd)

These equations apply when all roughness is supplied at the bottom.

If vertical roughness elements are used  $N_{k_s}$  is a "degree of choice" and not equal to  $N^{-4}$ . In this case

$$\begin{aligned}
n_C &= N^{1/2} \\
n_f &= N^{-1} \\
n_\tau &= n N_{k_s}^{1/4}
\end{aligned}
\tag{6.21}$$

## 7. COMBINING SHORT WAVES, LONG WAVES AND UNIDIRECTIONAL FLOW

At times a single model is required to solve problems related to short waves, long waves and unidirectional flow. For this type of model, the flow must necessarily be rough turbulent.

It is assumed that the vertical scale is synonymous with the general scale  $n$ . For short waves Eq. 4.29 gives the required modelling laws while for long waves and unidirectional flow Eqs. 6.20 and 6.21 may be used.

If the two types of waves and the unidirectional flow are combined, the following scales are self evident from Eqs. 4.29 and 6.20.

$$\begin{aligned} n_H &= n_d = n_z = n_Z = n_a = n \\ n_x &= n_y = n_X = n_Y = Nn \\ n_u &= n_{U_\delta} = \ell^{n_U} = \ell^{n_U} \approx s^{n_U} = n^{1/2} \end{aligned} \quad (7.1)$$

The extra subscript  $\ell$  refers to long waves and unidirectional flow while the  $s$  refers to short waves.

Two scales of wave length are found

$$s^{n_L} = n \quad \ell^{n_L} = Nn \quad (7.2)$$

as well as two time scales

$$s^{n_t} = s^{n_T} = n^{1/2} \quad \ell^{n_t} = \ell^{n_T} = Nn^{1/2} \quad (7.3)$$

To model the long wave correctly using bottom roughness only

$$\ell N_{k_s} = N^{-4} \quad (7.4)$$

Since there is no similar roughness requirement for the short wave model (a conclusion reached by Le Mehauté <sup>(18)</sup> also) this value of bottom roughness could be used in a combined model. This means that

$$s^{n_\delta} = n \ell N_{k_s}^{1/7} = n N^{-4/7} \quad (7.5)$$

i.e.  $s^{n_\delta}$  is greater than the geometrically similar  $s^{n_\delta}$  by a factor  $N^{4/7}$ . Thus it would appear that in those parts of the model where wave action is of prime importance the larger bottom roughness required by the long wave condition would distort the wave model boundary layer thickness excessively.

Therefore vertical roughness elements must be used. The wave boundary layer thickness will then be much closer to normal, but it must be kept in mind that diffraction takes place around each of these bars. This solution is to be preferred, however, since the roughness element dimensions are usually very small compared to the wave dimensions. For this case it is possible to use Eq. 6.21, where  $N_{k_s}$  is the same for long waves and short waves. This leads to

$$s^{n_\tau} = n N_{k_s}^{2/7} \quad \text{and} \quad \ell n_\tau = n N_{k_s}^{1/4} \quad (7.6)$$

To achieve the correct wave-current interaction these two shear stress scales must obviously be the same and it may be seen that for

small values of  $N_{k_s}$  the difference between the two shear stress scales is small and within the range of approximations taken in Eqs. 4.18 and 6.7.

Other scales obtained from Eqs. 4.29, 6.20 and 6.21 are

$$s_{\delta}^{n_{\delta}} = n N_{k_s}^{1/7} \quad (7.7)$$

and

$$n_C = N^{1/2} \quad n_f = N^{-1} \quad (7.8)$$

The last scales are a result of the analysis for long waves or unidirectional current. Bijker <sup>(15)</sup>, however, correctly points out that Eq. 7.8 is a necessary condition in three dimensional models of purely short waves, because of the presence of wave generated currents.

## 8. EQUATIONS FOR A MOBILE BED

Sediment transport along the bottom may be described by the following parameters, Yalin (1, Ch. 6),

$$A = f(\rho, \mu, D, \rho_s, g, \ell, v_*) \quad (8.1)$$

where  $\mu$  and  $\rho$  are the fluid density and viscosity,  $D$  and  $\rho_s$  are the particle diameter and density,  $g$  is the acceleration due to gravity,  $\ell$  is a typical length and  $v_*$  is the shear velocity  $= \sqrt{\tau_o/\rho}$ ,  $\tau_o$  being the shear stress at the bed.

Because gravity affects the system only through submerged unit weight it is common to use  $\rho_s - \rho$  instead of  $\rho_s$ . Also

$$\gamma_s = (\rho_s - \rho)g \quad (8.2)$$

may be substituted for  $g$ . This results in the following dimensionless relationship

$$\Pi_A = \phi \left( \frac{v_* D}{\nu}, \frac{\rho v_*^2}{\gamma_s D}, \frac{\rho_s - \rho}{\rho}, \frac{\ell}{D} \right) \quad (8.3)$$

The first two dimensionless variables are the  $X$  and  $Y$  axes of the Shields diagram, shown in Fig. 4, grain size Reynolds Number and the Shields parameter.



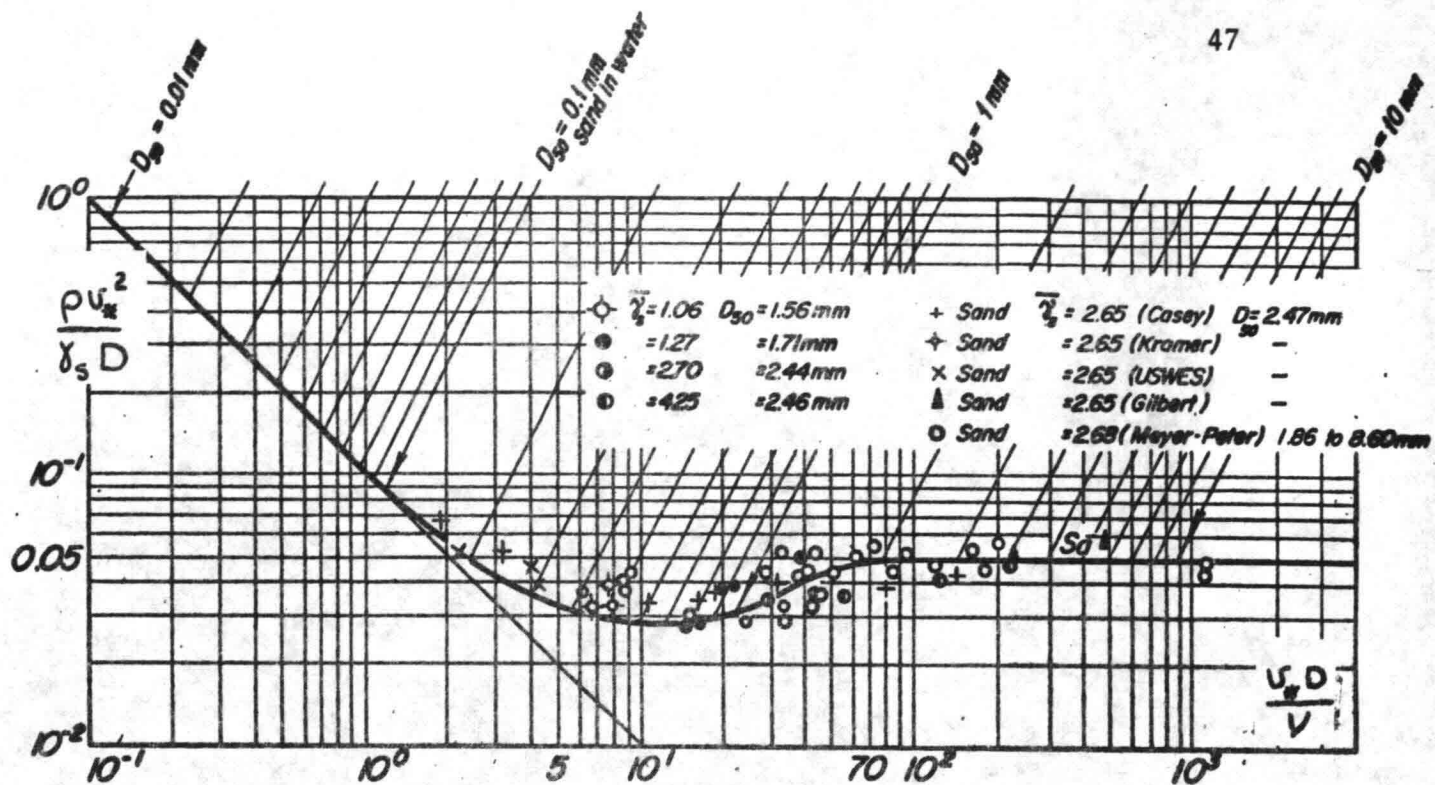


FIGURE 4 SHIELDS DIAGRAM

For unidirectional flow, Yalin (1, Ch. 6) has discussed the values of  $v_*$  and how the bottom shear is a combination of several components. If the bed form is flat,  $v_*$  is readily determined and easily varied in a definable fashion since

$$v_* = (gRS)^{1/2} \quad (8.4)$$

where  $R$  is the hydraulic radius of the flow and  $S$  the slope of the water surface. If bed forms are present

$$S = S_{k_s} + S_f \quad (8.5)$$

where  $S_{k_s}$  is the slope caused by the friction on the actual grains

and  $S_f$  is the slope caused by the bed form. Experimentally the following two expressions have been derived

$$S_{k_s} = \frac{F}{(2.5 \ln 11 \frac{d}{k_s})^2} \quad (8.6)$$

$$S_f = \frac{1}{2} \frac{\Delta^2}{\Lambda d} F \quad (8.7)$$

where  $F$  is the Froude number of the flow

$$F = \frac{\bar{U}^2}{gd_s}$$

Eq. 8.6 may be derived directly from Eq. 5.5. Eq. 8.7 may be found in Yalin (1, p167).

Under waves  $v_*$  is not an easy variable to use and it would be convenient if Eq. 8.3 could be rewritten as

$$s_{\Pi_A} = \phi_s \left( \frac{U_\delta D}{v}, \frac{\rho U_\delta^2}{\gamma_s D}, \frac{\rho_s - \rho}{\rho}, \frac{a_\delta}{D}, \right) \quad (8.8)$$

Even for long waves and unidirectional current a more convenient form of Eq. 8.3 would be

$$s_{\Pi_A} = \phi_L \left( \frac{\bar{U} D}{v}, \frac{\rho \bar{U}^2}{\gamma_s D}, \frac{\rho_s - \rho}{\rho}, \frac{d}{D} \right) \quad (8.9)$$

Both  $a_\delta$  and  $U_\delta$  are functions of other variables as indicated in Eqs. 3.13 and 3.14 and therefore Eq. 8.8 is a short form for more extensive equations involving the wave parameters  $H$ ,  $T$ ,  $L$ ,  $x$ ,  $y$ ,  $z$  and  $t$ .

The use of  $U_\delta$  and  $\bar{U}$  instead of  $v_*$  is justified when Eqs. 8.8 and 8.9 are to be used for deriving model scales, as long as

$$n_{U_\delta} \approx s n_{v_*} \quad \text{and} \quad n_{\bar{U}} \approx \ell n_{v_*} \quad (8.10)$$

Using Eqs. 4.23 and 4.26 it may be seen that

$$n_{U_\delta} = s n_{v_*} N_{k_s}^{-1/7} \quad (8.11)$$

which indicates that substitution of  $U_\delta$  for  $v_*$  in Eq. 8.3 is not unreasonable. For long waves and unidirectional current, Eqs. 6.5 and 6.19 yield

$$n_{\bar{U}} = n_{v_*} N_{k_s}^{-1/8} \quad (8.12)$$

indicating again that the approximation is reasonable, as long as vertical roughness elements are used to keep  $n_{\bar{U}} = n^{1/2}$ . If all roughness is bottom roughness

$$n_{\bar{U}} = n_{v_*} N^{1/2} \quad (8.13)$$

in which case  $\bar{U}$  cannot be substituted for  $v_*$ .

## 9. SCALE SELECTION FOR SHORT WAVE MODELS WITH A MOBILE BED

### 9.1 General

In order to model the boundary layer and the wave motion both simultaneously and correctly, it was seen that the boundary layer motion must be fully developed rough turbulent in case of the prototype as well as the model.

Yalin (1,p224) has demonstrated that for proper sediment transport scaling  $v_*$  should be used rather than  $U_\delta$  and  $U$  as suggested in Chapter 8. The following model scales may be derived from Eq. 8.3, using  $a_\delta$  as the typical length for models of short waves and assuming  $n_v = n_\rho = 1$

$$n_D = \frac{1}{n_{v_*}} \quad (9.1)$$

$$n_{\gamma_s} = \frac{n_{v_*}^2}{n_D} \quad (9.2)$$

$$n_{\rho_s - \rho} = n_\rho = 1 \quad (9.3)$$

$$n_D = n_{a_\delta} = n \quad (9.4)$$

Eqs. 9.1 and 9.2 ensure that both model and prototype fall on the same point on the Shields diagram. Valenbois <sup>(21)</sup> combines these into

$$n_D = n_{\gamma_s}^{-1/3}$$

Eq. 9.3 states that the density scales for the fluid and the submerged sediment must be the same. Since  $n_\rho$  is usually equal to unity, this leaves the impression that the only proper model material is the material found in the prototype, a very restricting concept. But Yalin <sup>(1,p162)</sup> states that this is only of importance when considering the motion of individual grains. When mass movement of bed form and discharge of material is of interest, this very stringent scale law can be relaxed.

If the mobile bed is flat, i.e. without bed forms such as ripples, then from Eq. 4.23 it may be seen that Eqs. 9.1 and 9.2 become

$$n_D = n^{-1/2} N_{k_s}^{-1/7} \quad (9.5)$$

$$n_{\gamma_s} = n^{3/2} N_{k_s}^{3/7} \quad (9.6)$$

In most sediment transport models the bed is not flat. For this case, assumptions must be made as to the relative influence of grain size roughness and bed form roughness. The grain size roughness appears in all expressions of velocity distribution used so far. Is it possible to replace this simply by the total roughness or must it be assumed that the boundary layer follows the bed form so that the velocity distribution remains a function of grain size roughness only? If the latter assumption is made, Eqs. 9.5 and 9.6 apply as they are written. If the assumption is made that the bed form roughness is responsible for the boundary layer velocity distribution, then

$$n_D = n^{-1/2} N_k^{-1/7} \quad (9.7)$$

$$n_{Y_S} = n^{3/2} N_k^{3/7} \quad (9.8)$$

where  $N_k$  refers to the total roughness, bed form and grain size combined. Both assumptions are incorrect and the preference for either one or the other depends to a great extent on the nature of the bed form. For long ripples or dunes Eqs. 9.5 and 9.6 would be more applicable, whereas for short ripples of small height perhaps Eqs. 9.7 and 9.8 should be used. In the following Eqs. 9.5 and 9.6 using  $N_{k_s}$  will be called Assumption I and Eqs. 9.7 and 9.8 using  $N_k$  will be called Assumption II.

It is well known from observations for unidirectional flow

with rippled beds that the sediment movement is not related to the total bed shear but only to a portion of it. Bijker <sup>(15)</sup> and others introduce here a ripple factor,  $\alpha$ , so that

$$\tau' = \alpha \tau \quad (9.9)$$

where  $\tau'$  is the effective shear stress, i.e. the shear stress that moves the sediment. In how far this applies to the ripples under waves remains to be seen and in any case, it is unlikely that the other properties such as growth of bed form, bed form dimensions, etc. are dependent on the same  $\alpha$ . Furthermore, it is found that in most cases  $n_\alpha \approx 1$ . Therefore the ripple factor has only a small effect on the scaling problem and this approach becomes synonymous to Assumption II.

When Eqs. 9.4, 9.5 or 9.7 are examined, there appears to be a conflict in the value of  $n_D$ . Since  $n > 1$  Eq. 9.4 results in  $n_D < 1$  while Eq. 9.5 and 9.7 yields  $n_D > 1$ . This is a conflict which must be resolved for all mobile bed models.

## 9.2 Large Grain Sizes

One obvious solution occurs when the grain size Reynolds number both in model and prototype is in the turbulent range. From the Shields diagram it may be inferred that this occurs approximately when

$$\frac{v_* D}{\nu} > 100$$

For this case the grain size Reynolds number effect can be neglected

and Eqs. 4.29, 9.2 and 9.4 yield

$$\begin{aligned} n_D &= n_{a_\delta} = n \\ n_{\gamma_s} &= \frac{n_{v_s}^2}{n_D} = \frac{n N_k^{2/7}}{n} = N_k^{2/7} \end{aligned} \quad (9.10)$$

where it is understood that  $N_k$  refers to total roughness or grain size roughness, depending on the assumption made. If  $N_{k_s}$  is used (Assumption I) the problem becomes relatively simple. Since  $k_s$  varies with  $D$ ,  $N_{k_s}$  is likely very close to unity. Thus

$$n_{\gamma_s} \approx 1 \quad (9.11)$$

and the resulting model bed consists of prototype sand material, but of much smaller grain size. If  $N_k$  is used, however, (Assumption II) the problem is more complex.  $N_k$  is a function of the grain size diameter, determined by Eq. 9.9 and the model wave conditions which cause the bed form. At present little is known about the variation of bed form below waves and research is underway at Queen's University to determine this. It is very likely that  $N_k$  is a function of  $n_{\gamma_s}$ , which means that an iterative procedure may need to be followed in order to select the correct  $n_{\gamma_s}$  even after expressions for  $N_{k_s}$  have been determined experimentally.

Preliminary test results indicate that both ripple height  $\Delta$  and ripple length  $\Lambda$  are functions of  $a_\delta/D$  primarily, with some effect of  $\gamma_s$ . Thus if Eq. 9.9 is satisfied,  $N_{k_s}$  could still have



a value close to one, as long as  $a_\delta/D$  does not change a great deal within the model and prototype (see discussion in next section). Therefore ripple sizes scale down approximately by the model scale and Eq. 9.11 is valid. In this case Eq. 9.3 is also satisfied and both mass movement and movement of individual grains are modelled correctly.

There are obvious lower limits to this type of model. Problems arise when the model boundary layer becomes smooth and laminar. Also, when sand size particles are modelled by clay size particles the condition in Eq. 9.9 must be dropped. This leaves the physical restriction that

$$n_D < n$$

and therefore

$$n_{\gamma_s} > 1$$

resulting in a situation similar to the one described more extensively in the next section.

### 9.3 Smaller Grain Sizes

In most cases the flow around the individual grains is not turbulent and an assumption must be made as to the relative importance of Eqs. 9.1 and 9.4. If Eq. 9.4 is considered to be more important, then one quickly arrives at the situation that  $D_m$  becomes too small, as described above. This leads directly to the obvious choice. Since it is almost impossible to satisfy Eq. 9.4, especially for the smaller grain sizes, Eq. 9.1 must be considered, leaving the misrepresentation of Eq. 9.4 to scale effect, where

scale effect is defined as inaccuracies resulting directly from not adhering to certain scale laws.

As an example the total bed roughness  $k$ , may be considered since it influences the value of  $N_k$  which, under Assumption II, may be used eventually for scale selection. This is a problem presently under investigation at Queen's University.

$$\pi_k = \frac{k}{D} = \phi_k \left( \frac{v_* D}{\nu}, \frac{\rho v_*^2}{\gamma_s D}, \frac{\rho_s - \rho}{\rho}, \frac{a_\delta}{D} \right) \quad (9.12)$$

Satisfying the scale laws of Eqs. 9.1 and 9.2 and recognizing that for mass motion Eq. 9.3 need not be considered, it may be stated in very simplified terms that

$$n_k = m_k n_D = \frac{m_k}{n^{1/2} N_{k_s}^{1/7}} \quad (9.13)$$

for Assumption I, while for Assumption II

$$n_k = m_k n_D = \frac{m_k}{n^{1/2} N_k^{1/7}} = \frac{m_k^{7/8}}{n^{5/16}} \quad (9.14)$$

Here  $m_k$  is the scale effect with respect to  $k$ , resulting from Eq. 9.4 not being satisfied.

Yalin (1,p226) and present research at Queen's University indicate that in models ripple height and length are indeed functions of  $\frac{a_\delta}{D}$  and that therefore  $\frac{k}{D}$  must also be a function of this parameter (1,p227). But  $a_\delta$  is also a function of water depth and

therefore in a single model there are many values of  $m_k$ , each corresponding to a different depth. Yalin states that one can only design for one value of  $m$  but the use of a single value of  $m_k$  is rather dangerous since this means that the model is designed for one distortion of roughness with respect to  $D$ . The factor  $m_k$  must therefore be thoroughly investigated and that is the reason for the present work carried out at Queen's University to come to a more fundamental understanding of this matter.

If Assumption II is made the scale effect  $m_k$  is found back in other scale relationships which include  $N_k$ .

For instance Eq. 4.29 yields

$$n_\tau = n N_k^{2/7} = n^{5/8} m_k^{1/4} \quad (9.15)$$

$$n_\delta = n N_k^{1/7} = n^{13/16} m_k^{1/8}$$

and using Eqs. 9.7 and 9.8 it may be seen that

$$n_D = \frac{1}{n^{5/16} m_k^{1/8}} \quad (9.16)$$

$$n_{\gamma_s} = n^{15/16} m_k^{3/8}$$

The smaller powers of  $m_k$  indicate that the influence of the scale effect is not very serious for  $n_\delta$  and  $n_D$ , but the actual value of  $m_k$  can conceivably be quite large since for small waves  $\Delta$  and  $\Lambda$

are direct functions of  $a_\delta/D$ , while for prototype waves this relationship may be decoupled and  $\Delta$  and  $\Lambda$  may be independent of  $a_\delta/D$ .

For Assumption I the comparable equations are the ones used earlier

$$n_\tau = n N_{k_s}^{2/7} \quad (4.24)$$

$$n_\delta = n N_{k_s}^{1/7} \quad (4.21)$$

$$n_D = n^{-1/2} N_{k_s}^{-1/7} \quad (9.5)$$

$$n_{\gamma_s} = n^{3/2} N_{k_s}^{3/7} \quad (9.6)$$

The sediment transport scale may also be derived and it may be seen that

$$\frac{q}{v_* D} = \phi_q \left( \frac{v_* D}{\gamma}, \frac{\rho v_*^2}{\gamma_s D}, \frac{\rho_s - \rho}{\rho}, \frac{a_\delta}{D} \right) \quad (9.19)$$

where  $q$  is the volume of material transported per unit width, per unit time. This, by analogy to Eqs. 9.12 to 9.14, may be expressed as

$$n_q = m_q n_{v_*} n_D = m_q \quad (9.20)$$

Thus if Eq. 9.4 is not satisfied the sediment transport scale is not equal to one. Also  $m_q$  is again a function of depth and therefore  $n_q$  is not constant throughout the model. Bijker and Svasek<sup>(16,p4)</sup> clearly state that it is of the utmost importance that the sediment

transport scale be constant throughout the model and that model design should take this into account (16,p8). Again it is only possible to design for a single value of  $m_q$  and the proper choice is of extreme importance and worthy of additional study.

It must furthermore be noted that the model distortion which constituted a degree of choice for fixed bed models must now be chosen more carefully. Since many models involve beaches, the model equilibrium slope must correspond to the prototype equilibrium slope so that they are both subjected to the same conditions. If the equilibrium beach slope is denoted by  $\theta$  then the above condition may be stated as

$$N = n_{\theta}^{-1} \quad (9.21)$$

The actual physical meaning of  $\theta$ , for instance, what portion of the equilibrium beach is to be used, as well as the feasibility of using an equation similar to Eq. 9.21 at all, are at present also under investigation at Queen's University.

#### 9.4 Summary

The scale relationships for short wave models with a mobile bed are as follows:

$$\begin{aligned} n_H &= n_L = n_d = n_z = n_Z = n_a = n_{a\delta} = n \\ n_u &= n_{u\delta} = n_t = n_T = n^{1/2} \\ n_x &= n_y = n_X = n_Y = Nn \\ N &= n_{\theta}^{-1} \end{aligned} \quad (9.22)$$

For models with large grain size Reynolds number

$$n_D = n$$

$$n_{\gamma_s} = N_{k_s}^{2/7} \quad \text{or} \quad n_{\gamma_s} = N_k^{2/7} \quad (9.23)$$

If  $N_{k_s}$  or  $N_k$  are near unity

$$n_{\gamma_s} \approx 1$$

$$n_\rho \approx n_{\rho_s} \quad (9.24)$$

$$n_\delta = n_\tau \approx n$$

However, these models quickly become physically impossible forcing  $n_D < n$ . For the more usual models with grain size Reynolds numbers below the fully turbulent range,

$$n_D = n^{-1/2} N_{k_s}^{-1/7} \quad \text{or} \quad n_D = n^{-5/16} m_k^{-1/8}$$

$$n_{\gamma_s} = n^{3/2} N_{k_s}^{3/7} \quad \text{or} \quad n_{\gamma_s} = n^{15/16} m_k^{3/8}$$

$$n_k = m_k n^{-1/2} N_{k_s}^{-1/7} \quad \text{or} \quad n_k = m_k^{7/8} n^{-5/16} \quad (9.25)$$

$$n_\tau = n N_{k_s}^{2/7} \quad \text{or} \quad n_\tau = n^{5/8} m_k^{1/4}$$

$$n_\delta = n N_{k_s}^{1/7} \quad \text{or} \quad n_\delta = n^{13/16} m_k^{1/8}$$

$$n_q = m_q$$

In each case the first expression is valid if Assumption I is made while the second expression refers to Assumption II.

### 9.5 Substitutions for $v_*$

As noted in the previous chapter,  $v_*$  is a difficult quantity to work with and although  $v_*$  has been conveniently eliminated from the scaling problem, the actual experimentation with the model still depends on the measurement of  $v_*$  as expressed in Eq. 8.3. Eq. 8.11 indicates that for short waves, substitution of  $U_\delta$  for  $v_*$  is not an unreasonable approximation and therefore Eq. 8.8 may be used as an approximation of Eq. 8.3. Further if the wave motion is sinusoidal, Eq. 8.8 may be simplified to

$$s\Pi_A = \phi'_s \left( \frac{\hat{U}_\delta D}{v}, \frac{\rho \hat{U}_\delta^2}{\gamma_s D}, \frac{\rho_s - \rho}{\rho}, \frac{a_\delta}{D} \right) \quad (9.26)$$

and since for sinusoidal motion

$$\hat{U}_\delta = \text{cst} \left( \frac{a_\delta}{T} \right)$$

$$s\Pi_A = \phi''_s \left( \frac{a_\delta D}{vT}, \frac{\rho a_\delta^2}{\gamma_s T^2 D}, \frac{\rho_s - \rho}{\rho}, \frac{a_\delta}{D} \right) \quad (9.27)$$

It must be recognized that the above dimensionless quantities are not entirely constant, for instance:

$$n_{X_1} = \frac{n_{a_\delta} n_D}{n_v n_T} = \frac{n}{n^{1/2}} \cdot \frac{1}{n^{1/2} N_{k_s}^{1/7}} = N_{k_s}^{-1/7} \quad (9.28)$$

or

$$n_{\chi_1} = \frac{n}{n^{1/2} n^{5/16} m_k^{1/8}} = n^{3/16} m_k^{-1/8} \quad (9.29)$$

### 9.6 Distortion of N

At times it is suggested in the literature, e.g. Goddet and Jaffry <sup>(20)</sup> that the wave height can be distorted to force proper sediment transport conditions by waves, i.e.

$$n_H = N_H n \quad (9.30)$$

since an increase in wave height has little influence on the wave refraction patterns. It must be borne in mind, however, that the position of the breaker is influenced by this distortion and therefore the effects on the model results of this "degree of choice" must be studied carefully. Goddet and Jaffry <sup>(20)</sup> suggest that

$$N_H = N^{1/4} \quad (9.31)$$

is permissible.



# 10. SCALE SELECTION FOR LONG WAVE AND UNIDIRECTIONAL FLOW MODELS WITH MOBILE BED

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For long waves and unidirectional flow similar reasoning may be used as in Chapter 9. Using Eq. 8.3 and using the water depth  $d$  as a typical length, Eqs. 9.1, 9.2 and 9.3 are valid while Eq. 9.4 becomes

$$n_D = n_d = n \quad (10.1)$$

Again, if the mobile bed is flat, i.e. without any bed form  $v_*$  in Eqs. 9.1 and 9.2 is as expressed in Eq. 6.14 and these equations may be rewritten as

$$n_D = n^{-1/2} N^{1/2} \quad (10.2)$$

$$n_{\gamma_s} = n^{3/2} N^{-3/2} \quad (10.3)$$

Eqs. 10.2 and 10.3 are identical to those derived by Le Mehauté (18,p1091) for both short wave and long wave and unidirectional models! When Eqs. 10.2 and 10.3 are multiplied together, Goddet and Jaffry's (20) expression may be derived as well as Bijker's expression (15,V3-3) assuming  $n_\alpha$ , the ripple factor scale, to approximate one. It must be recognized that model distortion introduces a requirement for additional roughness. Eqs. 10.2 and 10.3 presuppose that all this additional roughness is added in the form of bottom roughness. Also, once again, there is a conflict between Eqs. 10.1 and 10.2 with respect to  $n_D$ . Eq. 10.2 is the basis for a common expression for model distortion developed by Yalin (1,p235). From experiment it has been found that

$$C \sim \left( \frac{d}{D} \right)^{1/5} \quad (10.4)$$

Using Eqs. 6.15 and 10.2 it is possible to derive

$$N = n^{1/2} \quad (10.5)$$

or

$$n_X = n_Z^{3/2} \quad (10.6)$$

Yalin (1,p236) has plotted values for model studies performed at Wallingford which substantiate Eq. 10.6. It may be noted in passing that Eq. 10.6 may be derived from Lacey's regime equations. These equations are based on erodible channels in an identical soil medium where the smaller channels form "models" of the larger channels. Le Mehauté (18) calls this the natural distortion and states it is only valid when prototype material is used in the model, i.e. when  $n_D$  and  $n_Y$  in Eqs. 10.2 and 10.3 are equal to unity. The above development, however, indicates that this distortion can be generally accepted for all long wave models.

Most sediment transport problems do not present a flat bed. In this case, if the model is distorted, additional roughness may be added in the form of vertical roughness elements. Thus the total model roughness is the sum of grain size roughness, bed form roughness and artificial roughness, i.e.

$$\tau = \tau_{k_s} + \tau_f + \tau_A \quad (10.7)$$

The use of vertical roughness elements in sediment transport models may be open to question. Roughness strips will cause substantial scour in their immediate vicinity and also, roughness strips must be present in the original bottom before erosion has taken

place. It is felt, however, that the addition of bottom roughness causes unacceptable conditions since the additional roughness will alter the bed forming process and causes exaggerated wave bottom boundary layers. The additional vertical roughness must undoubtedly consist of a close grid of small elements in order to bring the local scour problems to a minimum.

The shear acting on the bottom particles and causing sediment movement surely excludes the artificial roughness. Therefore it is a function of the actual bottom roughness distortion  $N_{k_s}$  or  $N_k$  depending on whether Assumption I or II is used. Eqs. 9.1, 9.2 and 6.21 give

$$n_D = n^{-1/2} N_{k_s}^{-1/8} \quad \text{or} \quad n_D = n^{-1/2} N_k^{-1/8} \quad (10.8)$$

$$n_{\gamma_s} = n^{3/2} N_{k_s}^{3/8} \quad \text{or} \quad n_{\gamma_s} = n^{3/2} N_k^{3/8} \quad (10.9)$$

in addition to Eqs. 9.3 and 10.1. As before, if Eq. 10.1 is ignored

$$n_k = m_k n_D = m_k n^{-1/2} N_{k_s}^{-1/8} \quad \text{or} \quad n_k = m_k^{8/9} n^{-1/3} \quad (10.11)$$

where  $m_k$  is the scale effect. Once again it is only possible to design a model for a single value of  $m_k$ , while in fact  $m_k$  is a function of the variation in the water depth  $d$ . Again the left hand equations refer to Assumption I while the right hand ones refer to Assumption II. For the latter assumption, the determination of  $N_k$  is rather difficult.

Some indication as to the value of  $N_k$  may be obtained from

using Eqs. 8.6 and 8.7. It has been assumed that Eq. 8.6 applies to both model and prototype, therefore it is not unreasonable to assume that Eq. 8.7 applies to both also. This means that

$$n_{S_{k_s}} = \frac{n_F}{n_{1n}^2 \frac{d}{k_s}} \approx \frac{1}{\left(\frac{n_d}{n_{k_s}}\right)^{1/4}} \quad (10.12)$$

But  $\tau = \rho g R S$

$$\text{Therefore } n_{\tau_{k_s}} \approx n_{S_{k_s}} n_d = \frac{n_{k_s}^{1/4}}{n_d^{1/4}} n_d = n_{k_s}^{1/4} \quad (10.13)$$

which is the same as Eq. 6.10.

In a similar fashion

$$n_{S_f} = \frac{n_{\Delta}^2}{n_{\Lambda} n_d} n_F = \frac{n_{\Delta}^2}{n_{\Lambda} n_d} \quad (10.14)$$

leading to

$$n_{\tau_f} \approx \frac{n_{\Delta}^2}{n_{\Lambda}} \quad (10.15)$$

Yalin (17,p232) indicates that  $\Lambda$  is a function of  $d$  and independent of  $D$ , i.e.  $k_s$ , while  $\frac{\Delta}{\Lambda}$  depends slightly on excess shear but may be approximated as a constant for model and prototype with considerable sediment transport. Thus

$$n_{\tau_f} = \frac{n_{\Delta}^2}{n_{\Lambda}^2} n_{\Lambda} = n_d = n \quad (10.17)$$

i.e. the bed form shear scales down with the model scale. Further, if  $\Lambda$  is independent of grain size, its value will not be affected by neglecting Eq. 10.1. Therefore  $m_\Lambda$  will be unity. If  $\frac{\Delta}{\Lambda}$  is approximately constant and a function of excess shear, it will not be greatly affected by Eq. 10.1 and therefore  $m_{(\Delta/\Lambda)}$  is unity. This would tend to indicate that  $m_{k_f}$  would be close to one, where  $m_{k_f}$  is the scale effect on the bed form. Since the bed form is usually considerably larger than the grain size and under Assumption II it is the combined effect that causes the velocity profile it may be said that  $m_k$  is close to unity causing

$$n_k \approx n^{-1/3} \quad (10.18)$$

$$N_k \approx n^{-4/3} \quad (10.19)$$

A more careful study of this phenomenon is certainly warranted.

The scale effect of Eq. 10.11 may be substituted into Eqs. 6.19, 10.8 and 10.9 to give

$$n_\tau = n N_{k_s}^{1/4} \quad \text{or} \quad n_\tau = n^{2/3} m_k^{2/9} \quad (10.20)$$

$$n_D = n^{-1/2} N_{k_s}^{-1/8} \quad \text{or} \quad n_D = n^{-1/3} m_k^{-1/9} \quad (10.21)$$

$$n_{\gamma_s} = n^{3/2} N_{k_s}^{3/8} \quad \text{or} \quad n_{\gamma_s} = n m_k^{1/3} \quad (10.22)$$

and comparable expressions for  $m_k = 1$  may be deduced from Eq. 10.20 to 10.22.

In parallel to Eqs. 9.19 and 9.20 the scale for sediment transport is

$$n_q = m_q$$

In summary, the scale relationships governing long wave and unidirectional flow mobile bed models may be written as

$$\begin{aligned} n_d &= n_z = n \\ n_x &= n_y = n_L = Nn \\ n_{\bar{u}} &= n_U = n^{1/2} \\ n_t &= n_T = Nn^{1/2} \\ n_C &= N^{1/2} \\ n_f &= n_S = N^{-1} \end{aligned} \tag{10.23}$$

while when vertical roughness elements are used

$$n_{\tau} = n N_{k_s}^{1/4} \tag{10.24}$$

Depending on whether Assumption I or II is used (right or left hand equations below respectively)

$$\begin{aligned} n_D &= n^{-1/2} N_{k_s}^{-1/8} & \text{or} & & n_D &= n^{-1/3} m_k^{-1/9} \\ n_{Y_s} &= n^{3/2} N_{k_s}^{3/8} & \text{or} & & n_{Y_s} &= n m_k^{1/3} \end{aligned} \tag{10.25}$$

$$n_k = m_k n^{-1/2} N_{ks}^{-1/8} \quad \text{or} \quad n_k = m^{8/9} n^{-1/3}$$

$$n_\tau = n N_{ks}^{1/4} \quad \text{or} \quad n_\tau = n^{2/3} m_k^{2/9} \quad (10.25)$$

(cont'd)

$$n_q = m_q$$

Again, inherent in the above derivation under Assumption II is the assumption that  $n_\alpha = 1$ , i.e. the ripple factor in the model approaches that in the prototype.



# 11. MOBILE BED MODELS FOR COMBINED SHORT WAVE, LONG WAVE AND UNIDIRECTIONAL FLOW

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From the foregoing discussion it appears that to combine short waves with the other two, vertical additional roughness elements are necessary in distorted models. In previous studies Bijker <sup>(15)</sup>, Goddet and Jaffry <sup>(20)</sup> and Le Méhauté <sup>(18)</sup> inherently assume that all the shear necessary for the long wave model portion is supplied on the bottom, hence their scale relationships are variations of

$$n_{\tau} = n N^{-1} \quad (6.14)$$

$$n_D = n^{-1/2} N^{1/2} \quad (10.2)$$

$$n_{\gamma_s} = n^{3/2} N^{-3/2} \quad (10.3)$$

This results, however, in an undue distortion of the short wave boundary layer

$$n_{\delta} = n N^{-4/7} \quad (7.6)$$

Bijker <sup>(15)</sup> has performed his experiments without additional roughness. This is the method used for most wave-current models. For this case he correctly suggests that in order to achieve the correct wave-current interaction, the long wave or unidirectional current velocities must be exaggerated. The bottom shear stresses, resulting from waves and currents, must be the same, i.e.



$$s^{n_{V*}} = \ell^{n_{V*}} \quad (11.1)$$

This means from Eqs. 4.23, 4.26, 5.5 and 6.15

$$s^{n_{V*}} = n_{U\delta} N_{k_s}^{1/7} = \frac{n_U}{n_C} = \frac{n_U}{N^{1/2}} = \ell^{n_{V*}}$$

or

$$n_U = N^{1/2} N_{k_s}^{1/7} n_{U\delta} \quad (11.2)$$

Bijker's relation (15-V3-2) is slightly more simplified

$$n_U = N^{1/2} n_{U\delta} \quad (11.3)$$

This is also the relationship derived by Goddet and Jaffry (20).

The following scale relationships may be deduced from Eqs. 9.22 and 9.25, and 10.23 to 10.25

$$n_H = n_d = n_z = n_Z = n_a = n_{a\delta} = n$$

$$n_X = n_Y = N_X = N_Y = Nn$$

$$N = \frac{n_X}{n_z} = \frac{n_Y}{n_Z} = \frac{n_X}{n_Z} = \frac{n_Y}{n_Z} = n_\theta^{-1}$$

$$s^{n_u} = s^{n_{U\delta}} = \ell^{n_U} = \ell^{n_U} \approx s^{n_U} = n^{1/2} \quad (11.4)$$

$$s^{n_L} = n \quad \ell^{n_L} = Nn$$

$$s_t^n = s_T^n = n^{1/2} \quad \ell_t^n = \ell_T^n = Nn^{1/2}$$

$$n_C = N^{1/2} \quad (11.4)$$

(cont'd)

$$n_f = n_S = N^{-1}$$

These scales are based on the assumption that additional roughness is supplied to cause

$$n_C = N^{1/2}$$

If no additional roughness is supplied,  $U$  must be exaggerated with respect to  $U_\delta$  as outlined in Eq. 11.3.

For Assumption I, i.e. velocity distribution is a function of grain size

$$\begin{aligned} s_D^n &= n^{-1/2} N_{k_s}^{-1/7} & \ell_D^n &= n^{-1/2} N_{k_s}^{-1/8} \\ s_{\gamma_s}^n &= n^{3/2} N_{k_s}^{3/7} & \ell_{\gamma_s}^n &= n^{3/2} N_{k_s}^{3/8} \\ s_k^n &= s_k^m n^{-1/2} N_{k_s}^{-1/7} & \ell_k^n &= \ell_k^m n^{-1/2} N_{k_s}^{-1/8} \\ s_\tau^n &= n N_{k_s}^{2/7} & \ell_\tau^n &= n N_{k_s}^{1/4} \\ s_\delta^n &= n N_{k_s}^{1/7} \\ s_q^n &= s_q^m & \ell_q^n &= \ell_q^m \end{aligned} \quad (11.5)$$

For Assumption II, i.e. velocity distribution is a function of the bed form and grain size

$$\begin{aligned}
 s_D^n &= n^{-5/16} s_k^{m_k - 1/8} & l_D^n &= n^{-1/3} m_k^{-1/9} \\
 s_{\gamma_s}^n &= n^{15/16} s_k^{3/8} & l_{\gamma_s}^n &= n m_k^{1/3} \\
 s_k^n &= s_k^{7/8} n^{-5/16} & l_k^n &= m_k^{8/9} n^{-1/3} \quad (11.6) \\
 s_\tau^n &= n^{5/8} s_k^{1/4} & l_\tau^n &= n^{2/3} m_k^{2/9} \\
 s_\delta^n &= n^{13/16} m_k^{1/8} \\
 s_q^n &= s_q^m & l_q^n &= l_q^m
 \end{aligned}$$

It may be seen that although Eqs. 11.5 and 11.6 give slightly different scales, the short wave and long wave scales are quite close and relatively similar so that if either one is chosen, the other will not be very wrong, as long as the additional roughness is supplied by vertical roughness elements.

## 12. TIME

The question of time scales is complex and for a complicated model study the time scales for sediment transport are determined by observation of the model. An attempt will be made in this chapter to outline the various time scales involved in a model.

In Eq. 11.4 two time scales are given

$$s^{n_t} = s^{n_T} = n^{1/2} \quad \text{and} \quad \ell^{n_t} = \ell^{n_T} = Nn^{1/2} \quad (11.4)$$

These are the time scales used for the wave motion, the forcing mechanism in a mobile bed model. For the sediment transport, the list in Eq. 8.3 may be extended using the general time parameter  $t$ , resulting in

$$\Pi_A = \phi \left( \frac{v_* D}{v}, \frac{\rho v_*^2}{\gamma_s D}, \frac{\rho_s - \rho}{\rho}, \frac{\ell}{D}, \frac{v_* t}{D} \right) \quad (12.1)$$

giving a time scale

$$n_t = \frac{n_D}{n_{v_*}} \quad (12.2)$$

But Eq. 9.1 states

$$n_D = n_{v_*}^{-1}$$

Therefore

$$i^{n_t} = n_D^2 = \frac{1}{n_{v_*}^2} = \frac{1}{n_\tau} \quad (12.3)$$

where  $n_t$  is a function of Assumption I or II and whether or not roughness strips are used (Eqs. 6.14, 11.5 and 11.6). This may be considered as the theoretical time scale for motion of individual grains and bed forms and is therefore denoted by  $i n_t$ . Depending on  $n_t$ , which in any case is greater than one, it may be seen that  $i n_t$  is a fraction

$$i n_t \ll n_t$$

When dealing with erosion or deposition, it is the volume eroded or deposited which determines the time scale.

$$n_V = n_x n_y n_z = N^2 n^3 \quad (12.4)$$

where  $V$  is the volume.

But

$$\frac{q}{v_* D} = \phi_q \left( \frac{v_* D}{v}, \frac{\rho v_*^2}{\gamma_s D}, \frac{\rho_s - \rho}{\rho}, \frac{l}{D}, \frac{v_* t}{D} \right) \quad (12.5)$$

and

$$n_q = m_q n_{v_*} n_D = m_q \quad (12.6)$$

where  $q$  is the sediment transport in volume/unit time/unit width, and  $m_q$  is the scale effect in  $q$  because

$$n_l \neq n_D$$

Note that  $m_q$  is different for the short wave and long wave portions of the model. Therefore, the time scale for erosion or deposition is

$$e_t^n = \frac{n_V}{n_Y n_q} = \frac{N^2 n^3}{N n m_q} = \frac{N n^2}{m_q} \quad (12.7)$$

Similar time scales may be derived for other model transport phenomena such as movement of sand waves, transport of tracer materials, etc.

From Eqs. 11.4, 12.3 and 12.7 it may be noted that

$$e_t^n > l_t^n > s_t^n > i_t^n \quad (12.8)$$

This is very fortunate. Changes in bed formation, erosion and accretion, are very long term processes. Because  $e_t^n$  is so large, it is possible to perform model studies on these phenomena in reasonable time.

### 13. BREAKERS AND LONGSHORE CURRENTS

Since sediment motion in a coastal mobile bed model is brought about mainly by the agitation of sediment in the breaker zone, it is essential that conditions in this area are modelled correctly. The breaker position will be correct if  $n_H = n_d$  and the refraction pattern will be correct if  $n_L = n_d$  while the beach upon which the waves act will be modelled correctly if  $N = n_\theta^{-1}$ . All these conditions are incorporated in Eq. 11.4. However, if a simplified mechanism is envisioned, in which the waves stir up the material which is subsequently transported by the wave orbital motion and the longshore current, generated by the wave action, it is essential that

$$\frac{n_w}{s n_u} = 1 \quad \text{and} \quad \frac{n_w}{n_{U_L}} = 1 \quad (13.1)$$

where  $U_L$  is the longshore current velocity and  $w$  is the fall velocity of the sediment particles.

Yalin (17,p69) shows  $\frac{wD}{v}$  to be a function of  $\frac{\gamma_s D^3}{\rho v^2}$ . This is also stated by Valembois (21).

For laminar motion  $(\frac{wD}{v} < 1; \frac{\gamma_s D^3}{\rho v^2} < 18)$

$$w = \text{cst} \frac{\gamma_s D^2}{\mu} \quad (13.2)$$

while for fully turbulent motion  $(\frac{wD}{v} > 10^3; \frac{\gamma_s D^3}{\rho v^2} > 10^5)$

$$w^2 = \text{cst} \frac{\gamma_s D}{\rho} \quad (13.3)$$

But Eqs. 11.5 and 11.6 indicate that

$$n_{\gamma_s} = n_D^{-3} \quad (13.4)$$

Therefore

$$\frac{n_{\gamma_s} D^3}{\rho v^2} = 1 \quad (13.5)$$

which indicates that

$$\frac{n_{wD}}{v} = 1 \quad (13.6)$$

for proper reproduction of fall velocity, or

$$n_w = n_D^{-1} \quad (13.7)$$

This relation has also been derived by Bonnefille (22).

Yalin (17,p71) also demonstrates that if the  $X$  and  $Y$  parameters on the Shields diagram are the same for model and prototype, i.e. Eqs. 9.1 and 9.2 are satisfied, then

$$\frac{w}{v_*} = \text{cst} \quad (13.8)$$



This means that

$$n_w = n_{v*} = n_D^{-1} \quad (13.9)$$

which is the same as Eq. 13.7.

The above argumentation is based on spherical particles but could be extended to particles of any shape, as long as the shape factors in model and prototype are similar.

The conditions expressed in Eq. 13.1 must now be checked. The longshore current velocity  $U_L$  is generated by the waves and many formulas are proposed for the generation of longshore currents, e.g. Fan and Le Méhauté (19,p22) and preliminary investigation into this area at Queen's University indicates that many formulations will fit laboratory results adequately but cannot describe field results. For small angles of approach, the single factor that influences the longshore current velocity most is the wave height. Model tests seem to indicate that

$$U_L = f(H^{1/2}) \quad (13.10)$$

Therefore it may be assumed that

$$n_{U_L} = n_H^{1/2} = n^{1/2} \quad (13.11)$$

and from Eqs. 11.4, 11.5, 11.6, 13.7 and 13.11 it may be seen that Eq. 13.1 is approximately satisfied. It is not very fruitful to pursue this line of thought any further until additional research has shown more clearly what drives the longshore current velocity.

14. FOOD FOR THOUGHT

Throughout this paper the problem of the proportion of total shear going into sediment transport has been touched upon as a basic criterion for similarity in sediment transport. It has been suggested that the prototype and model points must fall on the same location of the Shields diagram. It is obvious that if the model falls below the Shields curve for initiation of motion, while the prototype falls above, the model will be useless, but it is not entirely clear if Eqs. 9.1 and 9.2 are satisfied, that the model will represent the prototype correctly. The reason for this would be basically a difference in bed form between the model and the prototype. If the bed form is identical,  $\alpha$ , the proportion of shear going into sediment transport will be the same and the model results should represent the prototype. If the bed form is different,  $\alpha$  must be taken into account and the parallel scale laws may be developed from

$$n \frac{\alpha \rho v_*^2}{\gamma_s D} = n \frac{\alpha^{1/2} v_* D}{v} = 1 \quad (14.1)$$

Present sediment transport relationships for unidirectional flow are usually presented in a form related to

$$\frac{q}{v_* D} = f \left( \frac{\rho v_*^2}{\gamma_s D} \right) \quad (14.2)$$

i.e. a simple version of Eq. 9.19. The relationships apply to the

turbulent region of the Shields curve and it is understood that  $v_*$  is not the total shear. Therefore, to extend this system using the present terminology, it is possible to write

$$\frac{q}{\alpha^{1/2} v_* D} = f \left( \frac{\alpha^{1/2} v_* D}{v}, \frac{\rho \alpha v_*^2}{\gamma_s D}, \frac{\rho_s - \rho}{\rho}, \frac{l}{D} \right) \quad (14.3)$$

The relationship between  $Y$  and  $X_1$  and  $X_2$  in Eq. 14.3 can be represented by a modified Shields diagram with axes  $X_1$  and  $X_2$  projected into a third dimension thus presenting a surface as shown in Fig. 5.

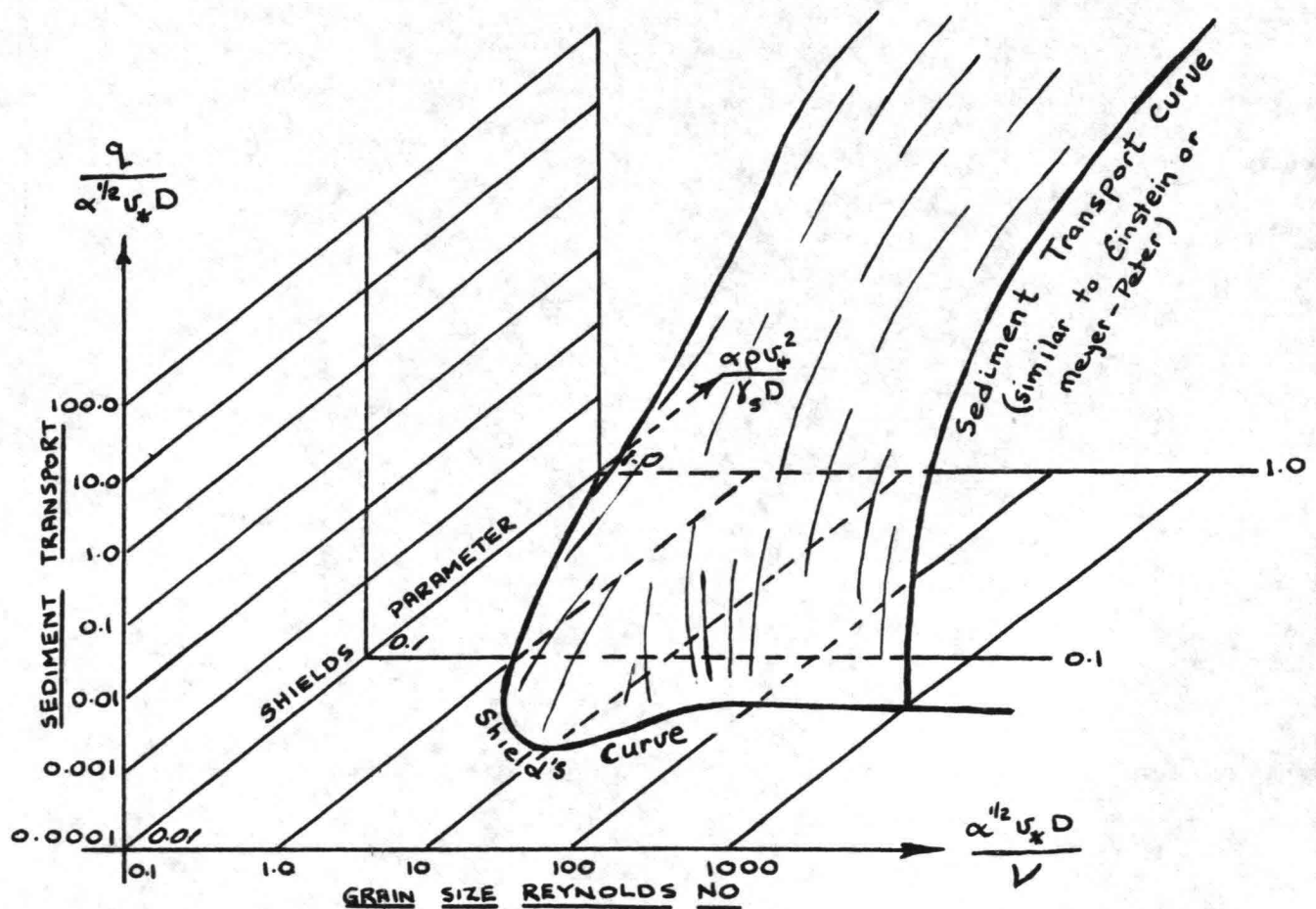


Figure 5 - SEDIMENT TRANSPORT SURFACE

This yields the more complete scaling laws

$$n_{\alpha}^{1/2} n_{v*} n_D = 1$$

$$n_{\alpha} n_{v*}^2 = n_{\gamma_s} n_D \quad (14.4)$$

$$n_q = n_{\alpha}^{1/2} n_{v*} n_D$$

or

$$n_{\alpha}^{1/2} n_{v*} = \frac{1}{n_D}$$

$$n_{\gamma_s} = \frac{n_{\alpha} n_{v*}^2}{n_D} = \frac{1}{n_D^3} \quad (14.5)$$

$$n_q = 1$$

The above equations assume no scale effect in  $q$  as a result of

$$n_{\ell} \neq n_d$$

Eqs. 14.5 indicate that the derivations used in this paper are limited to  $n_{\alpha} = 1$ . The problem associated with the determination of  $\alpha$  and  $n_{\alpha}$  is of course the largest single problem in sediment transport study. It should be of prime concern, not only to the model builder,

but also to the sediment transport student in general. Under oscillatory waves, where no dunes appear, the problem may be relatively simple and is very worthy of intensive investigation.

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