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Third progressive report on
reliability analysis of drag dominated offshore platforms

Deterministic and reliability based single wave analysis methods for jack-up structures

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Abstract

Deterministic and probabilistic methods available for the analysis of jack-up platforms are inventoried. Traditional analysis methods are compared with the detailed finite element methods for problems associated especially in deep water environment and more dynamic sensitive. The overall purpose of this report is to give guidance for stochastic nonlinear dynamic analysis of elevated jack-up platforms under service conditions. The first step was achieved by single wave analysis of jack-up platform taking into account dynamic amplification factor from an equivalent single degree of freedom system.

A detailed model of jack-up hull structure was selected in order to gain an accurate model of whole structure. Limitation in Finite element program was the main reason that the leg structure has modelled by equivalent beam model up to the Still water Level. The detailed leg structures from the Still water Level are connected to the hull by a realistic deck-leg interface.

It is concluded that the effect of bending moments in the design of leg chords can not be ignored. From reliability analysis of leg chords it has been investigated that the most critical failures occur for the aft chord of port legs above the lower guide level. In spite of this fact, the probabilistic analysis will be accomplished if the dynamic analysis is performed for a detailed structure in a long term stochastic model for the wave loading. On the other hand, the full detailed model for leg structure gives an opportunity to dig in the sensivity analysis of chord and bracing elements below the sea water level. For this type of analysis further studies will be carried out by the nonlinear finite element program ANSYS.

1- Deterministic single wave analysis of jack-up platforms

1.1 Developments in design of jack-up platforms

Traditionary platforms in the sea had been build by the masonry and timber elements. The jack-up type of drilling units can be traced back to US patents of 1849 by Abraham Lincoln and further developments by Lewis's 1869 patent (see reference [15]). These type of jack-ups installed over the wooden ships and in many respects, these vessels were similar to modern life boats (life boats are self-propelled barges with more flexiblity in sway direction supported by rectangualr pads instead of spud cans [33]). Nevertheless, true predecessors of the modern jack-ups were introduced not until the later portion of the first half of this century. Looking to the process of improvement in platform construction, considerable developments can be observed in the second half of this century and this can be explained by the critical reasons as well as the oil problems, the political situation and sometimes for the development in the army.

The development in the construction of jack-ups has been lead to the application of independent legs instead of closed type legs with a large mat foundation. In 1965 R.L. LeTournea from LeTourneau Offshore Company introduced a guidance system in the pinion supports in order to impose a horizontal component of the load transfer due to the tooth pressure angle that directly imposes a moment in the leg chords. The further invention was intended to apply a fixation system with rack engaging members that engage, interdigitate and lock into preferably a number of the rack teeth of each leg [14,16]. By application of this system, the forces in the leg to be directed almost entirely as axial loading in the chords, except for the nominal shear loading due to wave and wind taking in the bracing.

One report by C.J. Mommaas and E.P. Blankestijn [14] has been described the development of new jack-up design philosophy. The report introduces a locking or fixation system which provides a clearance free mechanical connection between leg and hull in the design of MSC Type 3000 jack-up platform. The resulting design of CJ 46 jack-up for DYVI EPSILON location in North Sea has been compared with 3 other types of K-braced independent leg design with the pure economical reasons. The structures were 3 and 4 leg jack-ups with and without fixation system (thus in total 2 × 2 = 4 jack-ups). The conclusion was that in construction of 4 leg jack-up we need 30% more steels in legs comparing with the same 3 leg design. In addition the difference in the structural response for different leg shape designs as well as the circular closed, the square closed, the square (4 chord) truss and triangular (3 chord) truss structures provide the following conclusions:

"The triangular 3 chord leg is preferred for other types because :

1- It gives the lowest overturning moment which is governing factor for most of the structural elements.
2- It gives the lowest maximum leg reaction which is important in the selection of the

[1] Numbers in brackets designate References at end of paper.

jacking system and the preload capacity.
3- It shows the best cost/capability ratio."

1.2 Importance of dynamic response of jack-up platforms

In the first report on the reliability analysis of drag dominated offshore platforms, the developments in structural analysis of jack-ups has been outlined. Historically there are numerous methods of analyzing of jack-up platforms but unfortunately the results are sensitive to the method used. The initial static deterministic analysis method was based on the full size modelling of structural framework and its supports. For the evaluation of resulting loads of the rack teeth, the structural model was improved with the wave load members in the hull-leg interface models. Pseudo elements have been used to model the connection of hull and legs. In order to simulate the structural behavior with realistic model, the specific stiffness of members were replaced in the idealization of stiff members. The structural response in the traditional designs was not insured due to the ignoring of sway stiffness. In the first rough calculations, it has been assumed that the deck is clamped to the legs because its stiffness is much greater than the leg stiffness. In order to model the second order moment due to the deck weight, the so-called *P-Delta* effect was formulated in the structural idealized model of the jack-up. This has been implemented by the equivalent sway stiffness in the dynamic analysis of jack-ups contributed by P. Liu (1989) [5]. An alternative method has been suggested by N.D. Barltorp and A.J. Adams [6] with the calculation of average leg compressive force and evaluation of negative sway stiffness of the leg.

Table 1.1 Dynamic amplification of the extreme structural wave induced response ; Excerpted from [7]

Water depth (*m*)	**Typical natural period [1]**	**Typical dynamic amplification[2]**	**Typical dynamic vs. static response**
70	5	1.1	1.08
90	7	1.2	1.15
110	9	1.4	1.30
130	11	1.8	1.60

[1] 3-legged jack-up with lattice type legs
[2] Dynamic amplification is only applied to the amplitude value

Furthermore the practical interests in extreme seas are contributed from sensivity of drag loading and particularly the dynamically responsive. It should also be noted that the potential dynamic behavior of jack-up platform was recognized prior to 1982 [1]. The routine inclusion of the dynamic response in the design/assessment gets more effort with increasing the water depth and harsher environments. When the first natural period of the jack-up rig is below approximately 3 seconds then static deterministic "design wave" analysis are considered adequate for the structural assessment of the rig [2]. The extreme natural period for the calculation of deterministic loading is given equal to 5 seconds in DNV classification notes [3]. According to the simplified dynamic analysis procedure, typical dynamic amplification factor of the extreme structural wave induced response for excitation by harsh environments in jack-ups may be obtained by Table 1.1. In spite of small differences, the natural period of the jack-up in the first mode of vibration is an important indicator of the degree of dynamic response to be expected responses. This leads to the investigation of the dynamic response for most typical jack-up drilling rig in the elevated condition.

First developments for the dynamic analysis of jack-up platforms was initiated by the deterministic *quasi-static* method. Although the development of the quasi-static method was fundamentally empirical [4], but it assembles a suitable method to describe the physical behavior of the problem in practical conditions. As an alternative to the deterministic method, the quasi-static calculation may be used in accordance with the stochastic (probabilistic) distribution of wave loading.

1.3 Equivalent Linear single degree of freedom systems ; SDOF

This section deals with the response of a SDOF system with linear elastic stiffness characteristics. In simple terms the jack-up platform may be considered as a single degree of freedom system (SDOF), shown in Figure 1.1. Although the basic wave loading can not completely summaries in the trigonometric functions, it is practically useful to consider the behaviour of a single degree of freedom system with harmonic loading to verify the overall complete response of the jack-up dynamic system. As has been shown earlier, the dynamic effects have to evaluated for a proper model of jack-up structure and in order to reduce the complexity of reliability analysis procedure, the dynamic effects are evaluated by a *DAF* (Dynamic Amplification Factor) applied to the complex structure. For this purpose, only the first natural frequency of structure is needed to obtain the frequency ratio Ω as has been given by equation (1.4). Also the dynamic amplification factor has given by the frequency of excitation and thus have to be considered as a random variable. However first assume a periodic regular wave with a cosine phase angle applied to the system of linear mass-spring-damper (equivalent SDOF vibrational model in sway direction). The equation of motion is then:

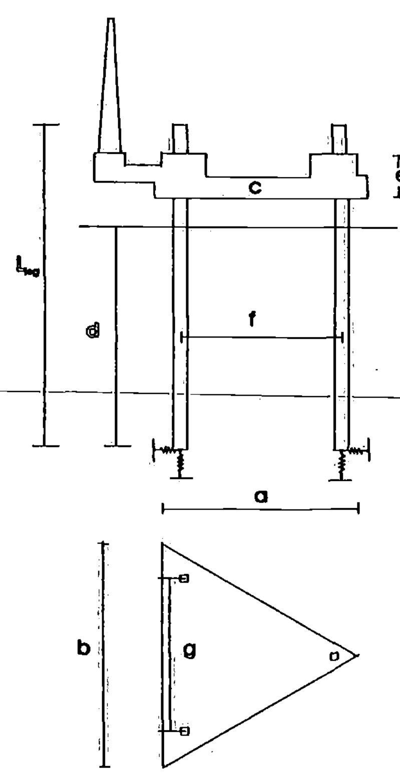


Figure 1.1 Schematic view of jack-up structure

$$M\ddot{x} + C\dot{x} + Kx = F_0 \cos(\omega t) \qquad (1.1)$$

where ;

M, C, K, x are mass, damping and stiffness of jack-up in sway direction.

and ;

The amplitude of the response can be represented in a dimension-less form by the dynamic amplification factor or *DAF*

$$DAF = \frac{amplitude\ of\ displacement}{equivalent\ static\ displacement} \tag{1.2}$$

$$DAF = \frac{1}{\sqrt{(1-\Omega^2)^2+(2\zeta\Omega)^2}} \tag{1.3}$$

where the frequency ratio (Ω) and the damping ratio (ζ) are equal to;

$$\Omega = \frac{\omega}{\omega_n} \tag{1.4}$$

and

$$\zeta = \frac{C}{C_c} \tag{1.5}$$

Or

$$\zeta = \frac{C}{2\sqrt{MK}} \tag{1.6}$$

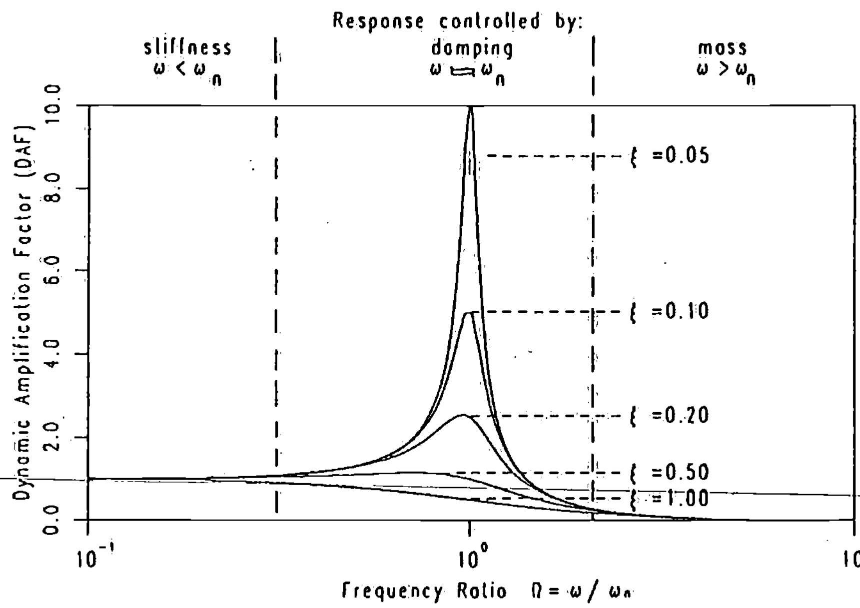


Figure 1.2 Verification of DAF with frequency ratio (Ω)

Knowing the variation of dynamic amplification factor (DAF) with frequency ratio (Ω), it can be used for the calculation of quasi-static loading. For low frequency loading ω, in comparison to the natural frequency, the response is essentially quasi-static and controlled by the stiffness. In the resonant area the response controlled by damping and for the high ω in comparison with the natural frequency the response is controlled by the mass (see Figure 1.2).

Due to the nonlinear stiffness (in structure and/or foundation), the natural frequency of the structure will shift its position as a function of load level. For large wave heights when the stiffness reduction is more pronounced, the shift towards the origin will be significant. In nonlinear stiffness model, the natural frequency and the dynamic amplification factor depends on the load level. The dynamic amplification factor below the original natural frequency will be increased due to the stiffness reduction (softening behaviour).

1.4 Selection of a parametric study

Although the fundamental issue of this report is to focus on the reliability analysis of jack-up platforms but in general the jack-up structure is represented by a *hull*, a *lattice leg* model (or may be a combination of lattice and *stick* model) and *leg-hull interface*. The hull structure may be represented as a collection of beam and mass elements with appropriate properties. The legs comprise either accurate models of three dimensional structure or stick modelling of legs with equivalent properties (Formulation of equivalent characteristics of legs has been given in classification notes e.g. Det Norske Vertias [3, 7] or Joint Industry Jack-up Committee [4, 8]). The leg-hull structure may be decided to model by the rigid connection when it has been equipped with a fixation system. As an alternative the behavior of leg-guide and leg-pinion subsystems can be modelled by the equivalent torsional springs at the connection points of upper and lower guides to the hull and legs of the platform. In these model some approximations should be imposed because in reality the combination of the leg-guide and leg-pinion sub-systems can not be done simply by summing up the reaction moments of both sub-systems for a certain inclination that is imposed at lower guide level [9,10,11].

For the development of the program with practical objects, a jack up structure is considered on the southeast coast of the Caspian Sea located at the north of the town Neka in Iran. The unit shall be capable of operating, all year around, in water depth ranging from *7.7 m* to *91.5m*. Technical specification has been given by *Rauma-Repola* offshore company which is the rig-builder of semi-submersibles, jack-ups, drillships and pipe-laying vessels from Finland. The structure is now constructed in sections on the specialist production line in the workshop, and the deck is completed onshore. The legs are fed from beneath, so high lifts are avoided. The rigs are launched totally complete.

The legs are designed as a lattice type frame work with three chords and tubular bracings. The chords of the legs have a construction with two gear racks on each chord and are manufactured of a high tensile steel. The three chords of every leg are interconnected by *K*-type tubular bracings. Free ends of the *K*-type tubular bracing are flame-cut to shape and size prior to welding to the chords.

In order to start the reliability analysis, the information from equivalent model is established for an adequate representation of jack-up structure. For a detailed description of the relevant parameters, the reader is referred to applied structural models that have been used in advanced studies of jack-ups based on FEM codes. A relative simple model of platform can be used to reflect the loading mechanisms and the most important dynamic characteristics (Figure 1.3). The overall leg truss work is idealized as a string of beam elements with equivalent stiffness properties. The computer model consist of 59 nodes and 71 elements. The leg-soil interactions is modelled by either clamped base constraints or linear springs in six DOF (Degree Of Freedom).

On the other hand, one may choose a relatively complex model of jack-up structure which includes the equivalent leg model for the legs until Still Water Level and the detailed model for the reserve parts of legs. Each deck level is composed of 366 plate elements and 108 beam elements and main and bottom deck structures are connected with side plates.

Anisotropy in two horizontal directions x, y for deck stiffness can be neglected because the equivalent thickness in longitudinal and transverse directions are approximately detremined by 5 *mm* [The stiffness of longitudinal angles ($150 \times 100 \times 10$ mm^3) have determined by 4 *mm* extra thickness and the angles in transverse direction could replace by 6.2 *mm* extra thickness in plate thickness]. It should be emphasized that the membrane and bending thikness for plate elements are different because plate elements of both deck form a box structure which has high level of bending stiffness. Although the total membrane thickness of plate element is taken by $8.5 + 5 = 13.5$ *mm*. but the bending stiffness shall be obtained by estimation of equivalent thickness with the same section modulus for combined bending stiffness of each deck plate.

The flanges and webs of each bulkhead are also modelled as plate elements. The jacking system and the legs (simplified model and detailed model) have modelled with 803 three-dimensional Timoshenko beam elements. The detailed leg model is only considered as three dimensional truss elements and a relatively complex geometrical deck-leg connection is

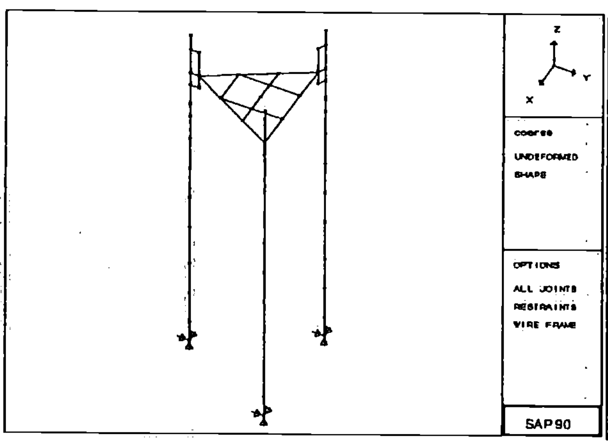


Figure 1.3 Stick model of Neka jack-up

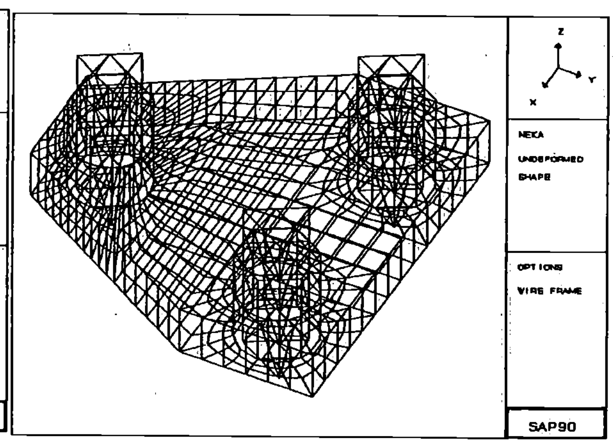


Figure 1.4 The detailed model Neka jackup

constructed for the deck-hull structure. Both finite element models are same until the Still water level and the overall structure of stick model and the detailed model of jack-up hull are shown in Figures 1.3 and 1.4 respectively. The characteristics of the jack-up platform are outlined in Table A.1 in appendix.

It will be very helpful to have pre-knowledge of the structure's eigen-value's characteristics before starting the reliability calculations. The eigenvalue and eigenvector analysis performed can be used to explain some of the amplification effects brought out by the dynamic computations. As it is shown in Table 1.2, principal natural frequency calculations, using the spring constant values at the spud tanks show three modes having a natural period in order of 3.0 - 6.5 seconds. The usual rig is equipped with a spud tank which is highly rigid at the bottom end of each leg. This spud tank is supposed to be the most rigid and uncertain part of the leg. It is obvious from calculation results shown in Table 1.2 that the spud tank causes the natural period to be slightly lower, due to the constrained rotation of the leg end at the sea floor. It was proposed that the natural frequency (or period) of an oil rig can be expected to be considerably affected by the constraint at the sea floor. In order to investigate this effect, three conditions involving support are taken into consideration. One is simple (pin, or hinge) support, the second is elastic support and the third is the fixed (or clamped) support. From the calculated natural frequency given in Table 1.2, the condition of simple support gives a frequency which is 44 ~ 45 % lower than the fixed support condition. It was found that when the jack-up was considered to be elastically supported, the computed natural period of the first two sway vibration modes might be adopted to the computed values by the practical formula given for the SDOF (simple degree of freedom system described in section 1.3), but the spring constant of sea floor obtained completely independent from the response of the structure due to the dynamic behavior. This has been discussed by Y. Hattori et al [21] due to the effect of local vibration of structure. The mode shape of an oil rig have divided to two types of vibrations, namely global vibration and local vibration. In the former, the motion of the platform predominates the vibration while in the latter the legs are frequently absorb the major part of vibration. For the first type, the SDOF can be adequately represented as the vibration mode but the local vibrations are not in order of magnitude by the SDOF system. In these local vibration modes the amplitude of platform is relatively small compared with that of legs and the mode is observed when the legs protrude above the platform in shallow water operation. Accordingly, the effective (or proper) spring constant of the sea floor is a very important factor, and it must be taken into account in calculating the dynamically sensitive behavior of an oil rig.

Table 1.2 The calculated natural periods of the NEKA jack-up in Iran, considering the different rigidities at the sea floor (stick model)

Mode	Natural period (s)	Hinge support	Elastic support	Fixed support
First-mode sway, x Dir.		6.618	4.154	3.703
First-mode sway, y Dir.		6.469	4.090	3.636
First-mode torsion, z Dir		5.793	3.513	3.178

However the computed mass of the deck and stiffness of legs are obtained for the SDOF system as follows

$$M = 11306 \ tonnes \tag{1.7}$$

$$K = \frac{3EI}{L^3} = \frac{3 \times 2.0067 \times 10^8 \times 11.948}{(109)^3} = 5554.17 \ KN/m/leg \tag{1.8}$$

Assume that the deck is rigid and the horizontal deflection at the level of the deck is that of the sum of three cantilever beams having the same E, I and L as the legs. If the stiffness of a cantilever is obtained by equation (1.8), the natural period is roughly estimated by the multiplication of (2π) to the inverse of natural frequency. Thus

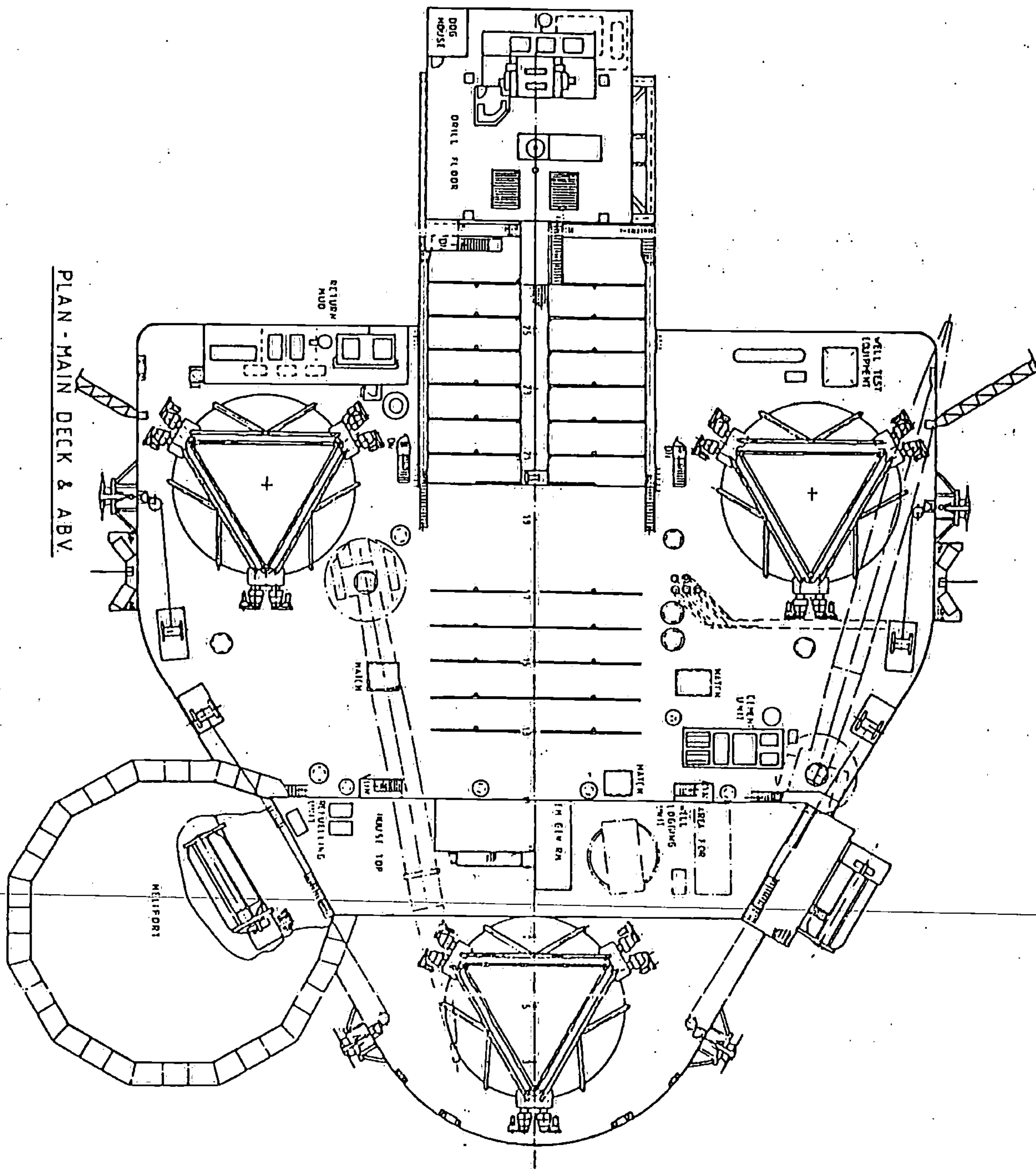


Figure 1.5 Plan of the multi-purpose jack-up for the Caspian Sea in Iran

$$T_n = 2\pi\sqrt{\frac{M}{K}} = 2\pi\sqrt{\frac{11306}{3\times 5554.17}} = 5.176 \text{ sec.} \tag{1.9}$$

In general, the estimation of natural period by approximate methods can be improved by taking into account the following parameters:

- the sway stiffness from deck flexibility,
- the negative sway stiffness from effect of leg compression
- the leg structural and hydrodynamical added mass with it's distribution in height.

It is important to investigate the characteristics of the vibration of jack-ups in practical situations rather than simplified discrete element systems. Practical estimation of natural periods has explained by Y. Hattori et al [21] in 1982. This was the first issue on the evaluations of natural periods taking into account the virtual mass of a leg vibrating in water, and the supporting condition of the sea bed, among others. The authors failed to obtain a resonance curve for the jack-up structure by the wind induced vibrations, because of the insufficiency of the exciting force. They applied the wire cutting method to an offshore platform (jack-up) for the first time. The wire cutting method is a simple method applying tension to structures by means of wire and releasing them suddenly to induced free vibration. A tug boat was employed to pull the jack-up in along its length by means of a wire. This method was enabled simple and short-term measurement of the natural period and the logarithmic decrement for global vibration of the self elevating jack-up. It might be useful to note that the logarithmic decrement for jack-up are between 0.2 and 0.3, which are 5 ~ 10 times greater than the logarithmic decrement for ship hull vibrations. Comparing the results with DnV rule (δ = 0.031 in air and δ = 0.126 in water), these results seems to be slightly larger, but they are assumed to be reasonable taking into account of the scattering of the measurements. It should be emphasized that before calculating the wave-caused dynamic response of a jack-up platform the logarithmic decrement must first be known.

In theory, detailed Finite Element Model is requested for the accuracy of the deck-leg interface model. For the type of tangential guiding system and the K-braced stiffness system, the approximate equivalent approach may give satisfactory results when compared to the detailed model. The approximate equivalent guide stiffness can be defined by distinguishing of two types of local leg deformations :

- local bending and shear deformation of the chords due to guide reactions
- bay deformation due to strain of horizontal and diagonal bracings.

If the guide reaction is imposed halfway a bay, the shear stiffness is governed by the horizontal displacement at the mid-point since bending is negligible. The corresponding stiffness is calculated by the equation (1.10) assuming that there are 4 pinions holding the leg rack teeth.

$$K_{shear} = \frac{4GA_s}{s} = \frac{4 \times (0.7718 \times 10^8)(576.09 \times 10^{-4})}{6} = 2.964 \times 10^6 \ \frac{KN}{m} \qquad (1.10)$$

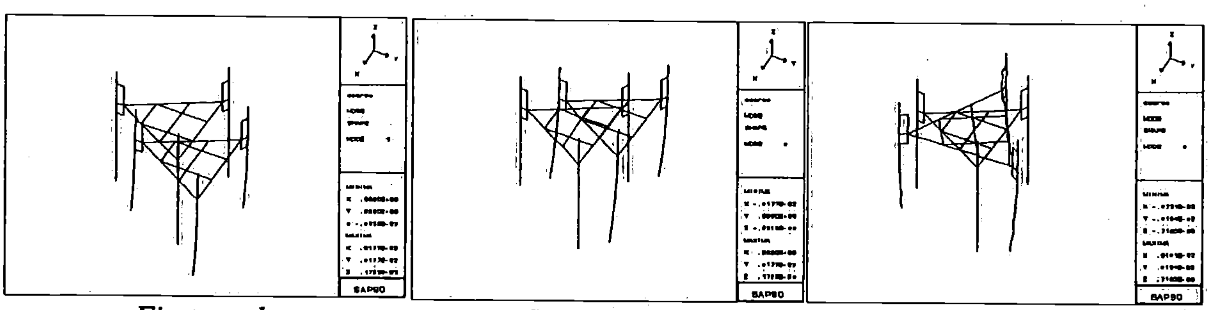
In order to determine the stiffness of bracings, assume that the horizontal of displacement of the chords due to the strain of the bracings is approximated by the eloganations of horizontal bracing projected in the direction of displacements induced by the local deformation of legs (the angle between horizontal plane with the guide reaction direction is taken 30° for the three chord configuration):

$$K_{brace} = -\frac{4EA_h}{h}\cot(30°) \tag{1.11}$$

$$K_{brace} = -\frac{4\times(2.0067\times10^{8})(327.23\times10^{-4})}{9.90}\cot(30°) = -4.595\times10^{6}\ \frac{KN}{m} \tag{1.12}$$

Two stiffnesses are in series, thus their flexiblities (1/stiffness) are additive.

$$K_t = \left(\frac{1}{k_{shear}}+\frac{1}{k_{brace}}\right)^{-1} = \left(\frac{1}{2.964\times10^{6}}-\frac{1}{4.595\times10^{6}}\right)^{-1} = 8.350\times10^{6}\ \frac{KN}{m} \tag{1.13}$$



First mode *Second mode* *Third mode*

Figure 1.6 Natural modes

It should be noticed that the first natural period of the jack-up structure is obtained by 6.618 *sec.* for simply supported case and 4.154 *sec.* for elastic support. The accurate natural period of the structure will depend on the sensivity of the total mode of vibration on leg-hull and leg-bottom interface models. In this stage it is clearly established that the dynamic analysis of the jack-up will be necessary and some care is required in the calculation of stiffnesses of the components constituting the interfaces of jack-up legs to hull structure and sea floor bed. The graphical presentation of the computed eigenvectors of the F.E.M. for the selected vibration modes of the simply supported platform are shown in Figure 1.7. As also discussed by Y. Hattori et al. [21], the highest three natural periods namely two sway modes and a torsional mode are very close to one another in operating condition. The natural periods beyond the fourth order mode are 7 ~ 10 times low as the first, and the frequencies higher than third order can be disregarded in calculating dynamic response in connection with wave action.

2. Reliability based single wave analysis of jack-up platforms

2.1 Probabilistic distribution of loads on jack-up structure

A jack-up structure has to be sustain for the specific loading condition in most unfavourable case. A realistic model for any type of loading is taken in reliability study. The starting point is the (wave, wind or gravity) force equation F which includes a number of variables with different probability distributions. The probabilistics of loads are calculated in two ways:

1- In this formulation, only the mean and standard deviation of force is given in the limit state function and the probabilistics of basic variables on for example force equation as well C_D, C_M and so on are transformed to the probabilistics of force effect. The mean and standard deviation of any type of loading is determined by a so-called mean value first order second moment method taken into account the mean and covariance of all variables in force equation. The method enables to predict the probabilistic of loading with an accurate force equation but approximate transformation for the original basic variables.

2- In second method, the transformation of basic variables of force equation is not a necessary task and the force equation is directly used in the limit state function. Also the probabilistics of loading variables are included in reliability analysis but the method overcomes to the difficulties in the approximation of forces with equivalent normal distributions. The difficulties are arising from the original distributions, especially if the extremes of loads are aimed to obtain in the reliability analysis process. This method is used in the calculation of loading in the present study.

2.2 Functional loads

The functional loads illustrated previously by the fixed loads, live loads and operational loads. For an accurate analysis, all these load sets on deck structure shall distribute by estimation of pressures at entire nodes of finite element model. The total permanent load that has to be supported by jack-up structure is generally the sum of the self weights of many individual structural elements. For this reason, such loads are represented by normal probability distribution according to that the weight of individual structural elements can be assumed normally distributed [28]. The implementation of gravity loads is adopted by using the **POTENTIAL** code in the SAP90 finite element program. In the present stage, all these functional load sets are considered uniformly distributed and have modelled by self weight of plate elements in negative z direction. The leg weights are also modelled by the unit weight of elements in detailed part and by an equivalent weight in the idealized sections.

2.3 Wave loads

The wave loading on jack-up legs are calculated by use of Morison equation. Due to the leg distances, the total forces on the legs are composed of three loads F_1 , F_2 and F_3 in uni-directional long crested wave model. Each of leg forces are contributed from the lumped forces on leg parts and can be projected in two horizontal directions x and y. Thus the relation between nodal lumped forces and total force for leg is given by

$$F_i = \sum_{j=lumped\ nodes} f_{ji} \qquad \forall i = 1,2,3 = leg\ number \tag{2.1}$$

In which f_{ji} denotes the wave force on leg number i for lumped joint j. Thus by this way, the internal forces on elements are correlated to the external loads for three legs. By performing an analysis for each of three loads F_1 , F_2 and F_3 , the response of each element and its probabilistic characteristics are evaluated and the internal forces for elements are obtained from the structural analysis results knowing the ratio's a_1, a_2 and a_3 :

$$S = a_1 F_1 + a_2 F_2 + a_3 F_3 \tag{2.2}$$

where a_1, a_2 and a_3 are the ratio's of internal forces in the element to the total applied forces on legs F_1 , F_2 and F_3 .

Consider the simplest wave theory (linear Airy wave theory) for the computation of wave kinematics. Basically the common wave theories are used in order to determine the wave velocity potential Φ. The velocity potential is determined in term of wave characteristics by the following expression

$$\Phi = \frac{gH}{2\omega} \frac{\cosh ks}{\cosh kd} \cos\Theta \tag{2.3}$$

where

$$s = z + d$$
$$\Theta = \omega t - k.x$$

By substitution of wave kinematics in the Morison equation, the wave force for an element with unit length is obtained by the following equation

$$f_i(t) = \rho \frac{\pi}{4} D^2 C_M \dot{u}_x + \frac{1}{2} \rho D C_D |u_x| u_x \tag{2.4}$$

where i = 1,2,3 indicates the leg number. Substitution of water particle velocity and accelerations in the Morison equation, the wave force model for the element (with unit length) is calculated form

$$f_i(t) = \rho g k \frac{\cosh ks}{\cosh kd} [\frac{\pi}{4} D^2 C_M (a.\cos\Theta) + \frac{1}{2} D C_D |\frac{(gk a \sin\Theta)}{\omega} \frac{\cosh ks}{\cosh kd}| \frac{(a \sin\Theta)}{\omega}] \tag{2.5}$$

Integrating this equation with respect to the length of vertical idealized cylinder, the wave forces on the structure are obtained by

$$F_i(t) = F_{Mi}(t) + F_{Di}(t) \tag{2.6}$$

and for an element located in the range of $s \geq s_0$ we have

$$F_{Mi}(t) = \frac{\rho\, g(\sinh ks - \sinh ks_0)}{2\cosh kd} \cdot \frac{\pi}{4} D^2 . C_M H . \cos\Theta \tag{2.7}$$

$$F_{Di}(t) = \frac{\rho\, g^2 k^2}{8\,\omega^2 \cosh^2 kd} . \left(\frac{s - s_0}{2} + \frac{\sinh ks . \cosh ks - \sinh ks_0 . \cosh ks_0}{2k}\right) . D\, C_D H^2 . \sin^2\Theta \tag{2.8}$$

In evaluation of hydrodynamic forces, the wave forces are lumped at the nodes of idealized element. For random variables, the probabilistic distributions are used in the formulation of limit state functions including both inertia and drag effects. Because the structure can not accepted the external load at the sea bed connection, the hydrodynamic forces are determined from half length of lowest leg structure. The integration is performed from this half length level (i.e. $s_0 = 4$ m) up to the surface of sea. In order to perform the reliability analysis, 4 differrent wave directions have implemented in the Finite Element analysis. The proposed wave directions are given by

1) Wave direction in 30° counterclockwise is considered from the positive x axis.
2) Wave direction is considered parrallel to the positive x axis.
3) Wave direction is considered parrallel to the negative y axis.
4) Wave direction is considered parrallel to the positive y axis.

For each type of loading, the effect of phase between the structure and wave loading is examined on the resultant wave forces. It has been concluded that the wave position which gives the maximum total horizontal force depends on the wave direction and other numerical variables in the Morison equation. For each direction (1 ~ 4 above) the corresponding phase and the position of wave crest with respect to the central line (Center of Gravity) of the structure are summurized in Table 2.1.

Table 2.1 Effect of position of wave crest for different wave directions

Design wave number	Phase = ω.t	Position of wave crest
1	1.450	0.051 L*
2	1.257	0.050 L
3	0.846	0.448 L
4	1.668	0.048 L

* L is the wave length, for the Capian Sea environment (L=357 m).

For any direction, the wave forces F_1, F_2 and F_3 for the maximum total force are obtained from equations (2.7) and (2.8) knowing the phase due to the distanes between legs. This means that the wave forces in the three legs have considered with additinal phase due to the effect of leg distances. For leg 1, it is assumed that the wave is in phase ($x = 0$) and for two other legs the phase depends on the wave direction as given below (note that leg 2 is positioned in negative x and positive y directions but leg 3 is positioned in positive x, y coordinates as shown in Figure 1.3):

Table 2.2 Effect of leg distance in the wave force model

Design wave number	1		2		3		4	
Leg number	2	3	2	3	2	3	2	3
Phase = k*x	0.037	0.564	-0.304	0.304	-0.601	-0.601	0.601	0.601

* k is the wave number, for the Caspian Sea environment (k = 0.0176 m^{-1}).

2.4 Wind loads

The wind loads for jack-up have been presented by different loads distributed on the hull depth, the legs above the Still Water Level, the derrick structure and so on. Except for the leg parts, the wind loads for other parts are assembled together and it has been assumed that the wind loads are acting equally at the main deck and bottom deck levels. The total wind load acting on deck structure in Caspian Sea environment is equal to 4417 KN and in North Sea environment is equal to 2271.59 KN. It is assumed that the wind load to be proportional to the projected area and from equation (2.43), the equivalent wind velocity is obtained for each parts of jack-up structure (items in Table 2.1 and 2.2). The equivalent wind speed is determined from

$$v_{e.q.} = \sqrt{\frac{2F_{total}}{A_{e.q.}\,\rho\, C_s}} \tag{2.9}$$

where $A_{e.q.}$ is the total area of the each part of jack-up structure subjected to the wind loads.

The three parameter Weibull distribution is used to extrapolate the predicted wind speeds in long term design of jack-up.

3. Strength criteria for structural elements, reliability analysis

3.1 Introduction

In order to obtain the maximum benefit from any method of reliability analysis, an understanding of the strength characteristics of materials is essential. The total strength of an offshore platform depends on many factors, hier the attention is given for the yielding, buckling and the effect of bendings on the strength of beam and plate elements.

Among of all offshore structures, a jack-up platform behaves in a markedly fashion in the range of different type of elements as well as beam elements for lattice legs, plate elements for hull structure, and more strangely the connection of hull to leg structure through the guiding system and elevating systems. This makes it difficult to establish an unit criteria for collapse of whole structure and thus in most of designs, the hull structure is modeled by a rigid body. The additional rise in the leg forces due to the hull rigidity may result in a conservative design. The uncertainty in the hull modelling, was an initiate to investigate different type of FE models namely the stick model and the detailed model.

For any type of element in a jack-up platform, the strain-stress relationship have to be known and this is based on the basic models for stress-strain characteristics of the steel elements. The more convinient experiment in the structural design is the uni-axial test of beam elements in tension or compression. The stress-strain relations for steels have sumurized to four models in term of post failure behaviour of materials:

- The elastic, perfect plastic model which assumes the linear relationship between stress and strain up to limit point and purely plastic behaviour after this point.

- The elasic, harden material which again assumes the linear relationship between stress and strain up to limit point and the linear hardening behaviour after this point.

- The elastic, nonlinear hardening which assumes the linear relationship between stress and strain up to limit point and then the non-linear hardening.

- The full non-linear model [Ramberg-Osgood] (1943), in which the initial slope of the curve takes the value of Young's modulus E at $\sigma = 0$, and it decreases monotonically with increasing load.

In the present work, the material behaviour is considered as perfect elastic material behaviour using the results of linear Finite element analysis programme SAP90. In the next fase, the classical bilinear kinematic hardening material behaviour is adopted through the plastic option of nonlinear Finite element program ANSYS. Let to start with the limit state functions which can be used in the plastic analysis of structures.

3.2 Limit states for beam elements

This section provides guidance on the selection of limit state functions for structural elements in jack-up structures. Consider the tubular members which are frequently used in offshore structures and a large number of researches have been directed to the behaviour of tube elements in static and dynamic range. Almost all of tubular members have subjected to combined effects of axial loading and bending moment because in reality the behaviour of actual member is geometrically and materially imperfect. The follwoing illustrates limit state methodology for a member in combined level of axial and moment distribution.

1) Plasticity limit

A tubular member in full plastic stress is considered in combined effect of axial loading and bending (Fig. 3.1). The stress resultants of section can be given by plastic moment in tension zone and axial plastic loading in compression zone, we have

$$M=4R^2\sigma_y t\sin\beta \qquad (3.1)$$

$$N=2Rt(\pi-2\beta)\sigma_y \qquad (3.2)$$

in which σ_y is the yield stress in simple tension and compression. Eliminating the angle β between above two equations leads to

$$M=4R^2\sigma_y t\cos\left(\frac{N}{4Rt\sigma_y}\right) \qquad (3.3)$$

If we define the plastic moment by M_P and plastic axial loading by N_P

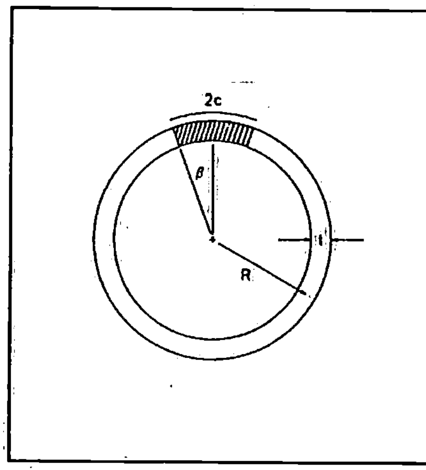


Figure 3.1 A tubular section

$$M_P=4tR^2\sigma_Y \qquad (3.4)$$

$$N_P=2\pi Rt\sigma_Y \qquad (3.5)$$

then the relation (3.3) reduces to the simple form

$$\frac{M}{M_P}=\cos\left(\frac{\pi}{2}\frac{N}{N_P}\right) \qquad (3.6)$$

In above formulation, the bending moment M is composed of two bending moment in directions of y and z (i.e. M_Y and M_Z). The limit state function for plastic yields to the following equation:

$$\Gamma_P = \frac{|\sqrt{M_y^2 + M_z^2}|}{M_P} - \cos\left(\frac{\pi}{2}\frac{N}{N_P}\right) = 0 \tag{3.7}$$

For other sections, similare expressions can be expressed for the interaction of axial load and moment. For a rectangular cross section it is shown that the plastic state of stress is defined as

$$\Gamma_P = X_P - \left\{\frac{M}{M_P} + \left(\frac{N}{N_P}\right)^2\right\} \tag{3.8}$$

where X_P , N_P and M_P are the model uncertainty variable for plasticity, the section plastic modulus in axial loading and the plastic bending moment for rectangulare section. In most cases, the nonlinear interaction relation is replaced by a linear interaction curve as follows

$$\Gamma_P = X_P - \left\{\frac{M}{M_P} + \frac{N}{N_P}\right\} \tag{3.9}$$

For a rectangular section, it has been found that the numerical values for probability of failure due to the two criteria are identical when *safety margin* is extremely large or small, i.e. the bending moment or axial force is dominant. In between, the linear failure relation predicts conservative probabilities of failure [Morutso (1986)].

It is clear that in this analysis, the effect of shearing stresses on plastic strength is assumed to be negligible. Thus two internal shearing stresses and one twisting moment do not affect the full plastic strength interaction relationship.

2- Buckling limit

In recent years, the theoritical and experimental strength formula's for circular tubular sections have modified subjected to the combined effect of compression and bending. S. Timoshenko [42] established the concept of Reduced Young's Modulus in order to derive the buckling strength of elements beyond the elastic limit. In Reduced Young's theory, the stress in cancave side of column cross section increases in accordance with inelastic or tangent modulus and that the stresses on the convex side decreases with the elastic Young's Modulus the theory (this theory has been developed by B. de Saint Venant in his notes to the famous Navier's book: " Resume des lecons," 1864). The section allowed to the strain reversal of the partially plasticized cross section. With the strain reversal effect, the maximum load capacity of column is increased and theoritically the reduced Young modulus predicts the exact critical load for ideal columns. However, the initial imperfications has an dramatic decresing effect on the load carrying capacity of column. In real conditions, the the increasing effect of strain reversal on the maximum load capacity of column is uasually offset by the decreasing effect of initial imperfections. Thus Yura et al. proposed the application of Tangent modulus load that is slightly below the maximum load carrying capacity of column [53]. Furthermore the effect of residual stresses on the column strength can be predicted easily by the modified tangent modulus. Against this background, the tangent Young Modulus has been accepted by reference codes as well as Columns Research Council (CRC) and AISC design specifications (1969).

The upper bound stress for the buckling in plastic limit and by using the yield stress, a modified slenderness parameter is defined by

$$\lambda = \frac{1}{\pi}\sqrt{\frac{\sigma_Y}{E}}\,\frac{KL}{r} \tag{3.10}$$

Where K is the *effective length factor* and r is the *radius of Gyration*. A column will fail as a result of elastic buckling if $\lambda > 1$ or will yield if $\lambda \leq 1$. This criteria was based on theoritical investigations while later a series of tests at Lehigh University showed that the the initial imperfections, the elsto-plastic behaviour and the residual stresses reduce the critical load of columns (Chen et al.) [47,50].

The linear interaction relationship between axial loading and bending moment is the basic criteria for the buckling failure of beam columns (Galambos 1988) [45]:

$$\Gamma_B = X_B - \{\frac{N}{N_B} + \frac{C_{m_z} M^0_{z_P}}{M_{z_P}[1-(\frac{N}{N_{E_z}})]} + \frac{C_{m_y} M^0_{y_P}}{M_{y_P}[1-(\frac{N}{N_{E_y}})]}\} = 0 \tag{3.11}$$

where X_B is a model uncertainty variable, N_B is the ultimate axial capacity of beam column in the absense of bending moments and M_{zp} and M_{yp} are the ultimate plastic bending moments in the absence of axial load, reduced for the possible prsence of lateral torsional buckling if necessary (when $P = 0$). Note that the z-direction corresponds to bending about the strong z axis and M_z^0, M_y^0 are the maximum applied first order moments. The in-plane bending is obtained by setting $M_y = 0$, provided that the bending is about the strong axis. An amplification factor $[1/(1 - N/N_B]$ for the maximum moment at the the center of a plastic beam column have been used by Chen and Atsuta [50]. Also for the unsymmetric case of end moments a reduction factor C_m is applied to the largest end moments M_A and M_B ($M = C_m M_A$). By an improvement for the particular case of hollow circular and square box sections suggested by Piallai (1970) is (see also Galambos 1988, Bazant 1991, Chen and Atsuta 1976) [43,45,50]

$$\Gamma_B = X_B - \{\frac{N}{N_B} + C[\frac{C_{m_z} M^0_{z_P}}{M_{z_P}[1-(\frac{N}{N_{E_z}})]} + \frac{C_{m_y} M^0_{y_P}}{M_{y_P}[1-(\frac{N}{N_{E_y}})]}]\} = 0 \tag{3.12}$$

in which C is given by the following relationship.

$$C = \frac{(e_x^2 + e_y^2)^{\frac{1}{2}}}{e_x + e_y} \tag{3.13}$$

The reduction factor C_{my} and C_{mz} are determined by elastic analysis and Euler buckling load for perfect column in any particular y and z directions. For braced frames, the following relations have given in AISC code (excerpted from Wang and Salmon 1979) [45]

$$C_m = 0.6 - 0.4\frac{M_B}{M_A} \geq 0.4 \quad , \quad N_E = \frac{\pi^2 EA}{(\frac{KL}{r})^2} \tag{3.14}$$

In which M_B/M_A is positive when the member is bent in double curvature and negative when it is bent in reverse curvature.

For unbraced beam columns, the reduction factor C_m is found by the following formula in the most unfavorable condition (Z.P. Bazant and L. Cedolin) [43].

$$C_m = 1 - 0.18\frac{N}{N_E} \tag{3.15}$$

The ultimate axial compression load is given by CRC specification in two regions of elastic and inelastic buckling limit. For $\sigma_u \leq 0.5\ \sigma_y$ it follows the Euler hyperbola and for $\sigma_u >$ $0.5\ \sigma_y$ it is assumed to have the shape of a parabola that has a horizontal tangent at $\lambda \to 0$ and is tangent to the Euler hyperbola. The tangent point is found to be $\sigma_u = 0.5\ \sigma_y$, and formulation of Inelastic buckling is given by proportional limit σ_P. The constant C is corresponding to the slenderness ratio in two regions seperated by slenderness ratio for stress equal to the proprtional limit σ_P

$$C = \pi\sqrt{\frac{E}{\sigma_P}} \tag{3.16}$$

$$\frac{N_B}{N_P} = 1 - \frac{1}{2}(\frac{KL}{Cr})^2 \qquad \frac{KL}{r} < C \qquad \textit{Inelastic Buckling} \tag{3.17}$$

$$\frac{N_B}{N_P} = \pi^2 E/\,(\frac{KL}{r})^2\,\sigma_Y \qquad \frac{KL}{r} \geq C \qquad \textit{Elastic Buckling} \tag{3.18}$$

The above formula's for ultimate axial compression load was given based on the Allowable Stress Design mehod. New trends for the load and resistance factor design adopted by the following relationship for the compressive strength of perfect columns:
For fabricated steel tubular members, Chen and Toma proposed two computed curve (A) and

$$\frac{N_B}{N_y} = 0.658^{\lambda^2} \qquad \lambda < 1.5 \tag{3.19}$$

$$\frac{N_B}{N_y} = \frac{0.877}{\lambda^2} \qquad \lambda \geq 1.5 \tag{3.20}$$

(B) [47]. The computed curve (A) is applicable for tubes and is given by the following formula:

$$\frac{N_B}{N_y} = 1 - 0.091\,\lambda - 0.22\,\lambda^2 \qquad for\ 0 \leq \lambda \leq 1.41 \tag{3.21}$$

$$\frac{N_B}{N_y} = 0.15 + \frac{0.834}{\lambda^2} \qquad for\ 1.41 \leq \lambda \leq 2 \tag{3.22}$$

The model uncertainty for both yielding and buckling is typically chosen by μ_{Xm} = 1. to 1.5 and V_{Xm}= 0.1 to 0.3 according to P.K. Das and D. Faulkner [54]. In the following, the interaction formula (3.12) is adopted in the ultimate buckling analysis of tubular elements where the ultimate buckling strength in absence of bending moments (N_B) are given by equations (3.21) and (3.22).

3.3 Limit states for hull plate buckling

a) Plate buckling in stiffened panels: One of benefits of using a detailed deck model is the reliability based retropective strength assements of hull structure. The most important failure mode for the deck plates is the buckling mode theoritically the buckling strength of thin or thick plates has been studied by several authors (see for example S. Timoshenko [42] pp. 324). The ultimate strength of plating between longitidunals of the jack-up hull, is essentially "long plate". The plate buckling strength of this long plates are obtained with the following assumptions (see Figure 3.2):

1) All plates are simply supported,
2) The long edges are constrained to remain straight but they are free to rotate in plane.
3) Plate compressive strngth is a function of slenderness ratio β while β is defined as

$$\beta = \left(\frac{b}{t}\right)\sqrt{\frac{\sigma_Y}{E}} \tag{3.23}$$

where t is the plate thickness and b is the stiffener spacing.

With this assumption, A.K. Thayamballi et al. (1987) [32] formulate the classical bifurcation buckling strength of thin perfectly flat plates using the elastic solution of Bryan (1891) as follows

$$\text{for } \beta > 2.7 \qquad \frac{\sigma_B}{\sigma_Y} = \frac{3.6}{\beta^2} \tag{3.24}$$

and

$$\text{for } \beta \leq 2.7 \qquad \frac{\sigma_B}{\sigma_Y} = 1 - 0.069\,\beta^2 \tag{3.25}$$

Equation (3.24) is Bryan's elastic solution, while equation (3.25) is an approximate correction for inelastic effects. In literature, many other formula's have been obtained for the ultimate compressive strength of panel plates. Conley et al. (1963) developed a mathematical formualtions based on von Karman (1932) studies and Winter (1947) proposed the effect of residual stress on the stiffness of panels [36,45]

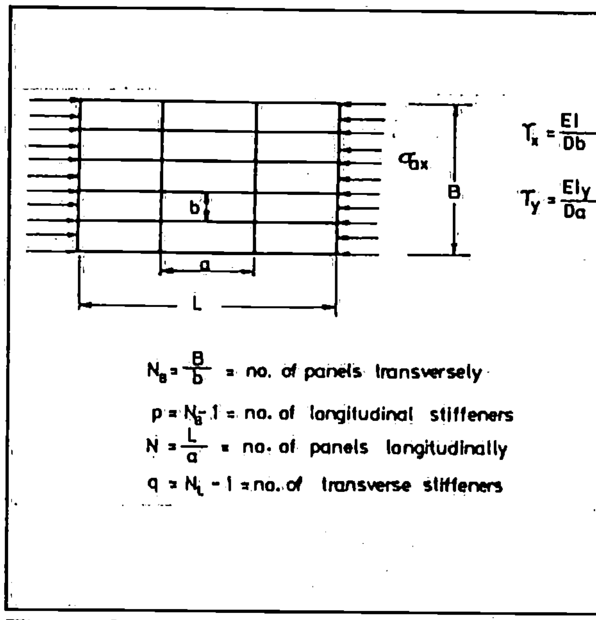


Figure 3.2 Stiffened plates

$$\text{for } \beta > 1 \qquad \frac{\sigma_B}{\sigma_Y} = \frac{1.82}{\beta}\sqrt{\frac{\sigma_e}{\sigma_Y}} - \frac{0.82}{\beta^2} \tag{3.26}$$

where σ_e is the Euler elatic buckling stress for the effective plate-stiffner cross section. For values of $\beta \leq 1$ there would be no loss of effectiveness of plate and

$$\text{for } \beta \leq 1 \qquad \frac{\sigma_B}{\sigma_Y} = 1 \tag{3.27}$$

Faulkner gives a similar expression for the mean plate ultimate strength with constants 2 and 1 respectively while $\sqrt{\sigma_e/\sigma_Y}$ has been ommitted in the first expression of equation (3.26) [32]. The combined effect of compression and shear in the ultimate strength of plates has been reveiwed by Hughes (1987) and Galambos (1988) [36,45].

<u>b) Overall buckling in stiffened panels:</u> For stiffened plates, the instability can occur due to the overall buckling of stiffeners or the local buckling of the stiffeners develops the nodal lines and furhter buckling is formed in the plate panels. For short stiffened panels, the Euler buckling column equation is used (elastic buckling) while for long stiffenned panels, the failure stress is dependent of the panel length. This effect occurs for large values of the panel aspect ratio $\Pi = L/B$ and for small values of stiffener rigidity relative to the plating. Sharp

(1969), formulated the effect of additinal rigidity on the slenderness ratio as follows

$$\frac{L}{r} = C_{\Pi}\frac{a}{r} = \frac{C_{\Pi}\, a}{\sqrt{\dfrac{I_s}{(A_s + b_e t)}}} \tag{3.28}$$

In which A_s is the stiffener area, b_e and t are the effective width and stiffener thickness and C_{Π} is given by

$$C_{\Pi} = \frac{1}{\Pi}\sqrt{\frac{\gamma_x}{2(1+\sqrt{1+\gamma_x})}} \qquad \vee \qquad C_{\Pi} = 1 \tag{3.29}$$

whichever is less. In above equation γ_x is the ratio of the flexural rigidity of the combined section to the flexural rigidity of the plating

$$\gamma_x = \frac{EI_s}{Db} = \frac{12(1-\nu^2)I_s}{bt^3} \tag{3.30}$$

In order to find a relation between the state of stress in original plate and in the plate with effective width b_e, equation of Van Korman is implied for both cases. Substition of proper boundary conditions, the following relationship is obtained

$$\frac{b_e}{b} = 1.9\frac{t}{b}\sqrt{\frac{E}{\sigma_e}} \tag{3.31}$$

where σ_e is the Euler stress of effective edge and it reaches the maximum value when the effective width would reach its smallest possible value.

The critical stress in stiffener has propsed by Faulkner in elstic and post-buckling range by the following equations;

$$for \quad \frac{\sigma_E}{\sigma_Y} \geq 0.5 \qquad \frac{\sigma_e}{\sigma_Y} = 1 - 0.25 \times \frac{\sigma_Y}{\sigma_E} \tag{3.32}$$

$$for \quad \frac{\sigma_E}{\sigma_Y} \leq 0.5 \qquad \frac{\sigma_e}{\sigma_Y} = \frac{\sigma_E}{\sigma_Y} = \frac{\pi^2 E}{(\frac{l}{r})^2 \sigma_Y} \tag{3.33}$$

where the equaivalent (L/r) is obtained from equation (3.28).

The critical stress is also related to the overall buckling stress by application of statics In order to find the proper unknown outset of edge stress σ_b, a repeated procedure must be

$$\sigma_b = \sigma_e \frac{(b_e t + A_s)}{(bt + A_s)} \tag{3.34}$$

performed as follows

1. Assume some initial value for b_e.
2. Calculate the Radius of gyration by equation (3.28).
3. Calculate the critical value of axial stress in stiffener from equations (3.32) or (3.33).
4. Using this value of σ_e recalculate b_e from equation (3.31).
5. Repeat steps 2 until b_e has converged.
6. Calculate σ_b from equation (3.34).

To account the effects of residual stress, the effective flange length is multiplaied by a constant R. The reduction factor R is given by the following relationships

$$for \quad \beta > 1 \qquad R = 1 - [\frac{2\eta_R}{(\frac{s}{t}) - 2\eta_R}][\frac{\beta^2}{2\beta - 1}]\frac{E_t}{E} \tag{3.35}$$

$$for \quad \beta \leq 1 \qquad R = 1 - [\frac{2\eta_R}{(\frac{s}{t}) - 2\eta_R}]\frac{E_t}{E} \tag{3.36}$$

where the ratio of E_t/E is given by

$$for \quad \beta < 2.7 \qquad \frac{E_t}{E} = [\frac{3.6\beta^2}{(13.1 + 0.25\beta^4)}]^2 \tag{3.37}$$

$$for \quad \beta \geq 2.7 \qquad \frac{E_t}{E} = 1 \tag{3.38}$$

The term η indicates the width of the tensile residual stress zone, and is obtained between η_R = 3 and 4.5 (Faulkner et al) [32].

c) Numerical DATA: Using the geometrical and physical data for our case study, the following slenderness ratio and buckling stress are obtained:

$$\beta = \frac{0.635}{8 \times 10^{-3}} \sqrt{\frac{5 \times 10^5}{2.0067 \times 10^8}} = 3.96 \tag{3.39}$$

The plate buckling stress is found by the elastic Bryan solution for simply supported plate In order to find the overall buckling stress, the repeated procedure described in section *(b)*

$$\sigma_B = \frac{3.6}{\beta^2}\sigma_Y = \frac{3.6}{(3.96)^2}5.10^5 = 114784.21 \ \frac{KN}{m^2} \tag{3.40}$$

above is used. Starting with the effective width of $b_e = 0.8\ b$, by three iteration the convergance is achieved and the effective width of stiffened plate and the critical stress are found by

$$b_e = 0.6 \times b = 0.6 \times 0.635m = 0.38m \quad , \quad \sigma_e = \frac{\pi^2 E}{(\frac{l}{r})^2} = \frac{\pi^2 E}{78.08^2} = 324825 \ \frac{KN}{m^2} \tag{3.41}$$

where $a = 2.5\ m$ is the distance of transverse stiffeners and r is the radius of gyration for stiffeners. The overall buckling stress is found by equation (3.34)

$$\sigma_{ob} = \frac{0.635 \times 8 \times 10^{-3} + 24 \times 10^{-4}}{0.38 \times 8 \times 10^{-3} + 24 \times 10^{-4}} \times 324825 = 220500 \ \frac{KN}{m^2} \tag{3.42}$$

The overall buckling stress is greater than the plate buckling stress and the requirement that overall buckling not precede plate buckling appears in a more explicit form.

Comparing to the longitudinal stresses for the top and bottom hull structure, it has been found that even for the plate elements with concentrated transverse loadings, the compressive stresses are in order of one tenth of the buckling stress. Thus in the operational or survival condition, the buckling mode of hull plate is not a dominant failure mode. However in some storm cases, the water may enter to the hull and the additional water pressures increase the plate stresses above the ultimte limit. This may happen near the deck hatches where the stress concentration is maximum value. One report describes the loss of jackup structure by a storm in North Sea environment when the structure was in the tow condition. In that cases, the ultimte limit strength of hull girders, the strength of stiffened plate panels should also considered in the reliability analsyis.

3.4 Realibility analysis with FORM and Monte Carlo methods

3.4.1 Monte Carlo and Importance Sampling methods

One way of studying a structural model is the application of numerical hypothesis with the physical and mathematical problems. For most of nonlinear formulations in mathematics a closed form solution can not be found and numerical methods with iterative solutions are used in a successful step by step skeleton.

Numerical methods can be approved by a wide sense of *simulation* or by a narrow sense of *Monte Carlo* method. Simulation has defined by a way of performing sampling experiments on the model of system while the response of system is usually a very complicated process over time. In Monte Carlo (simulation), stochastic or deterministic variables are sampled from probability distributions that essentially independent random variables uniformaly distributed over the unit interval [0,1].

The oldest and simplest methods in Monte Carlo simulation is the *Crude* or *Classical* Monte Carlo method. In this method, for each random variable a random number can be generated giving a specified combination of random variables. Note that the variables are statisticaly independent otherwise the uncorrelated basic variables should be found by a method proposed in the first report. The structural failure is supposed to occur if the limit state function is negative for this specified combination of variables. Mathematical estimation of failure probability requires the integration of joint probability density function in the region of basic variables where the limit state function is less than zero:

$$P_r(z \leq 0) = \int .. \int_{Z<0} f(x_1, x_2, .., x_N) . \delta x_1 . \delta x_2 ... \delta x_N \tag{3.43}$$

In evaluation of integral (3.43), the zero-one indicator based Monte Carlo Simulation is used in space of original x variables. An indicator function is equal to one when the argument is true (the limit stae function is less than zero) and otherwise zero (when the limit state function is positive). Equation (3.43) can be rewritten as

$$P_r(z \leq 0) = \int_{x_1=-\infty}^{\infty} .. \int_{x_N=-\infty}^{\infty} I_z . f(x_1, x_2, .., x_N) . \delta x_1 \delta x_2 .. \delta x_N \tag{3.44}$$

By repeating this procedure for the number of simulations N, an approxiamte estimate of probability of failure is obtained by

$$P_r(Z<0) = \frac{1}{N} \sum_{i=1}^{N} I_{z_i} \tag{3.46}$$

where I_z is the number of times that the failure is encountered ($Z < 0$). If during the simualtion, the mean and standard deviation of limit state function computes for the reliability function, the *adjusted Monte Carlo* method follows an alternative answer for the probability of failure.

Itagaki et al. (1979) [55] proposed the application of extermal distributions of the original distributions to reduce the computation time. In the same report, they proposed application of conditional probability of load or strength variables as a second technique for reducing the computational time. It has been observed that as long as the failure criteria are defined and the conditions of $Z \leq 0$ are desired, the first method has very advantage that the complexity of the problem has almost no effects on the computation of probability. For a two story frame structure with 19 limit state function, the computation time is reduced from *800 sec.* to *70 sec.* with application of extermal distribution instead of classical Monte Carlo simulation.

Theoritically, Monte Carlo simulation may yield accurate solution if the sample size is sufficiently large. However, if the failure probability of the problem under consideration is very small, the dircet Monte Carlo Simulation is almost impracticable owing to the huge amount of computation inevitably required. Therefore, it is really necessary combining Monte

Carlo Simulation with *variance reduction techniques* to reduce the sample size. While several methods has been investigated in literature (see R.Y. Rubinstein, 1981) [56], among of the variance reduction techniques, the Importance Sampling method is explained in details.

The basic idea of Importance Sampling is to concentrate the distribution of the sampling points in the region of most 'importance', i.e. the part that mainly contributes to the failure probability. Similare to the all variance reduction techniques, the method can not be gain any information if nothing is known about the problem. For getting started, the FORM analysis or the classical Monte Carlo can be used to indicate the failure domain of any random variable. In classical method, the random varibles are selected from entire distribution but in Importance Sampling method, the 'shifted' distribution of original probability distributions are attemped to use. The failure probability in Integral form follows (as equation (3.44))

$$P_r(z \le 0) = \int_{x_1=-\infty}^{\infty} .. \int_{x_N=-\infty}^{\infty} [I_z \frac{f(x_1,x_2,..x_N)}{g(x_1,x_2,..x_N)}].g(x_1,x_2,..x_N).\delta x_1.\delta x_2 ... \delta x_N \qquad (3.46)$$

where $g(x_1,x_2,..,x_n)$ is the importance sampling density function. In this formulation, the integral is shifted to the region of importance function where the joint probability distribution is maximum. If the *weight function* is defined as the ratio of original joint probability function to the importance sampling joint probability function, the equivalent of equation (3.45) can be rewritten as

$$P_r(z \le 0) = \frac{1}{N}\sum_{i=1}^{N} (I_{z_i}.\frac{f_i}{g_i}) = \frac{1}{N}\sum_{i=1}^{N} (I_{z_i}.\omega_i) \qquad (3.47)$$

where N is the number of simulations. The importance sampling probability density function g_i may be chosen as a normal distribution, a uniform distribution and the extreme maxima or minima distribution. In evaluation of accurate probability of failure, a normal distribution has the advantage that its mean value can be selected at the point of a tailed distribution function. Other type of distributions may be used in accurate calculation of mean and standard deviation of limit state function.

3.4.2 Failure in buckling or yielding of chords

The response of chord elements in jack-up structure is dominated by axial force taken into account the possible bending moments. The loads in the chords are highly dependent on the actual direction and position of the wave with respect to the rig. It is necessary to determine the maximum stress level in the chord of interest with the combined vertical reaction and bending moments. In this awy the worst way direction and position is found.

The worst wave direction is often head sea direction and this is also the case for the example platform considered here. Based on the numerical results of FEM method, the maximum compressive chord stress occurs in the forward chords of two aft legs.

To find the maximum stressed members in jack-up legs, two distinct sections are studied in the categerry of datailed modellling part and idealized modelling part. In premilinary analysis with FORM package, no iteration was found within Max Iteration of MI = 100. In FORM program, the starting point for iteration is the mean value for each random variable and in Defult option the program tries to find a design point belonging to the probabilities of exceedence of:

10^{-4} for the maximum value: Xmax;
$1 - 10^{-4}$ for the minimum value: Xmin.

It has been observed that for two basic variables of wind velocity and wave height, the design points are outside of the above boundaries and the following boundary values for Xstart and Xmin are used for two mentioned basic variables

Table 3.1 The boundries of design points for wind velocity and wave height

Variable	Xstart	Xmin
Wind velocity (v_R)	65.71 m/s	0.099 m/s
Wave height (H)	18.18 m	0

1) The detailed model:

In the deck-leg connection we have modelled the legs with space farme lattice type structure. The maximum load capacity of these elements are identical and thus by performing a series of FORM calculations the following elements are selected with the highest probability of failure. The reliability index for these elements are in the interval $\beta_{min} + \Delta\beta_1$ where $\beta_{min} =$ 4.640 is the reliability index for most probable failure element 881 and $\Delta\beta_1 = 3.0$ is the tolerance for most important failure elements in buckling or tension adopted from Thoft Christensen et al.

Table 3.2 Probability of failure for chord elements in detailed model

Element Number	Elevation from SWL	Probability of failure (P_F)	Reliability Index (FORM)
879	From -1 up to 5m	2.576E-7	5.021
880	From -1 up to 5m	1.646E-6	4.652
881	From -1 up to 5m	1.740E-6	4.640
882	From -1 up to 5m	1.223E-8	5.577
883	From -1 up to 5m	2.048E-10	6.251
884	From -1 up to 5m	5.890E-11	6.443
885	From -1 up to 5m	1.166E-8	5.586

886	From -1 up to 5m	5.933E-11	6.442
887	From -1 up to 5m	2.291E-10	6.233
924	From 5 up to lower Guide level (8.7 m)	2.405E-7	5.034
925	From 5 up to 8.7m	1.576E-6	4.661
926	From 5 up to 8.7m	1.671E-6	4.649
927	From 5 up to 8.7m	1.175E-8	5.584
928	From 5 up to 8.7m	1.911E-10	6.262
929	From 5 up to 8.7m	4.837E-11	6.473
930	From 5 up to 8.7m	1.126E-8	5.592
931	From 5 up to 8.7m	4.924E-11	6.470
932	From 5 up to 8.7m	2.189E-10	6.241
1149	From lower guide level 8.7 up to 11m	2.187E-7	5.052
1150	From 8.7 to 11 m	1.406E-6	4.684
1151	From 8.7 to 11 m	1.490E-6	4.672
1152	From 8.7 to 11 m	1.059E-8	5.603
1153	From 8.7 to 11 m	1.776E-10	6.273
1154	From 8.7 to 11 m	4.519E-11	6.483
1155	From 8.7 to 11 m	1.014E-8	5.610
1156	From 8.7 to 11 m	4.596E-11	6.480
1157	From 8.7 to 11 m	2.033E-10	6.252

Contribution due to the dynamic amplification factor is taken into account by equivalent single degree of freedom system as discussed in section 1. The frequency ratio for design wave is found by ratio of frequancies of applied wave model to the first natural frequency of the structure.

$$\Omega = \frac{\omega}{\omega_n} = \frac{T_n}{T} = \frac{5.176}{15.6} = 0.332 \tag{3.48}$$

And the damping ratio ζ is assumed to be 5% in this study (including hydrodynamic damping, foundation damping, and structural damping). Thus the load from wave and current is multiplied with the DAF given by

$$DAF = \frac{1}{\sqrt{(1-\Omega^2)^2 + (2\zeta\Omega)^2}} = 1.123 \tag{3.49}$$

In order to see the validity of excluding the effect of bending moments in chord designs, the complete limit state functions for tension and compression elements are used. For tension elements, the original limit state function (3.35) has failed in estimation of failure probability and the calculation was stopped because a zero value of probability of failure was obtained for all tension elements. The limit state function for tension elements has modified from the original statement with the aim that the trigenometric limit state function is reduced to an equivalent algebric function. The following limit sate function was found by the *arccos* function of original limit state function.

$$\Gamma_P = \arccos\left(\frac{|\sqrt{M_Y^2 + M_z^2}|}{M_P}\right) - \frac{\pi}{2}\frac{N}{N_P} = 0 \tag{3.50}$$

Taking into account the effect of bending moments, the probability of failures and reliability indices are listed in Table 3.3 for the same elements in Table 3.2.

Table 3.3 Probability of failure for most probable failed chord elements (the limit state function includs the effect of bending moments)

Element Number	P_F (IM)	P_F (FORM)	Reliability Index (FORM)
879	5.870E-7	4.920E-7	4.895
880	3.350E-6	3.227E-6	4.511
881	4.210E-6	3.344E-6	4.504
882	2.520E-8	2.540E-8	5.449
883	5.740E-10	6.888E-10	6.059
884	2.800E-10	2.671E-10	6.209
885	2.550E-8	2.427E-8	5.457
886	2.680E-10	2.634E-10	6.212
887	7.270E-10	7.483E-10	6.045
924	4.650E-6	4.434E-6	4.443
925	2.130E-5	1.813E-5	4.130
926	2.450E-5	1.831E-5	4.128
927	1.560E-7	1.299E-7	5.151

928	9.470E-9	8.163E-9	5.647
929	5.290E-9	5.245E-9	5.723
930	1.440E-7	1.241E-7	5.159
931	4.770E-9	5.043E-9	5.730
932	7.950E-9	8.189E-9	5.647
1149	4.800E-6	4.373E-6	4.446
1150	2.570E-5	1.739E-5	4.140
1151	2.570E-5	1.791E-5	4.133
1152	1.180E-7	1.093E-7	5.183
1153	6.660E-9	5.999E-9	5.700
1154	4.320E-9	3.672E-9	5.783
1155	1.110E-7	1.040E-7	5.192
1156	4.780E-9	3.549E-9	5.789
1157	6.110E-9	5.969E-9	5.701

Table 3.3 indicates that the bending moments have an important factor on the probability of failure for all elements investigated. This amplification in failure probabilities due to the interaction of axial stress with bending stresses has been increased from 1.9 to 77.2 for element numbers 879 and 1156 respectively. This ratio is increased with the elevation of elements from sea bed bottom and it takes it's maximum value when the element is coming close to the guide level. Above the lower guide level, the effect of bending moments is significant and the maximum failure proability occurs for the aft chord of port leg (element 1150 or 1151 with P_f = 2.570E-5).

Application of Importance Sampling is also presented in Table 3.3. The importance sampling density function is the normal probability distribution. This distribution is centered around the point in the space of basic variables having the maximum joint probaility function. Two Importance Sampling coefficients for wind velocity and wave height are used with mean values given in Table 3.1 for X_{start} and arbitrary standard deviation with $\sigma_v = 150\ m/s$ and $\sigma_H = 20\ m$ for wind velocity and wave height respectively. Using 10000 simulations, the results can be upgraded and a better estimate of probaility of failure is found by the Importance Sampling using the Design point (ISPUD).

2) The idealized leg model

Below the water level, the leg structure has modelled by the idealized structure. The stiffness characteristics of idealized structure has discussed in section 2 and hier the ultimate strength of idealized structure is established in order to perform the reliability analysis.

The pertinent parameters for each leg have obtained in either case of tension and compression. Let to start with the maximum capacity of leg model in tension which can be obtained as follows:

$$I_y = I_z = \frac{1}{2} A_{ch} h^2 = 7.145 \ m^4 \tag{3.51}$$

$$N_P = (3A_c)\sigma_y = 3(2\pi Rt + 2LW)\sigma_y = = 218715 \, KN \tag{3.52}$$

$$M_{PY} = \sigma_Y \times (2\pi Rt \frac{h\sqrt{3}}{2}) = 505029.76 \ KN.m \tag{3.53}$$

$$M_{PZ} = \sigma_y \times 2\pi Rth = 583158.14 \, KN.m \tag{3.54}$$

If the member is in the compression, the ultimate limit state formula is again equation (3.13) where the compressive strength of the compound column is obtained by equation (3.20) or (3.21). The only factors are the effective length of lattice structure and its slenderness ratio which has been studied by Bleich (1952) and can be obtained by the following relationships

$$for \quad K\frac{L}{r} > 40 \qquad \acute{K} = K\sqrt{1 + \frac{300}{(\frac{KL}{r})^2}} \tag{3.55}$$

$$for \quad \frac{KL}{r} \leq 40 \qquad \acute{K} = 1.1K \tag{3.56}$$

For the lattice structure, the ratio $KL/r = 20.14 < 40$ and the coefficient of buckling length is obtained by $K' = 0.88$, and the radius of Gyration is given by $r_y = r_z = 4.041$. Finally the slenderness ratio is found by

$$\lambda = \frac{1}{\pi\sqrt{2.0067 \times 10^8}} \frac{0.88 \times 6}{4.041} \sqrt{\sigma_y} = 2.936 \times 10^{-5} \sqrt{\sigma_y} \tag{3.57}$$

Substitution of this formula in equation (3.19) gives the ultimate compressive strength of idealized columns as

$$N_U = 0.43743 \, \sigma_Y \times (0.658)^{(8.62 \sigma_y^2 \times 10^{-10})} \tag{3.58}$$

The Euler buckling load for the compound elements are evaluated by following equation Table 3.4 indiactes the results of Importance Sampling (IM) method and FORM analsyis for idealized model of leg structure. The maximum probability of failure ($P_f = 7.660 \times 10^{-7}$)

$$P_E = \frac{\Pi^2 EI}{(\acute{K}L)^2} = 446682.827 \quad MN \tag{3.59}$$

corresponds to the element 877 located in the elevation of 1 *m* till 4 *m* below the water level. The element 877 is one of aft legs located just below the lower guide level of structure. The probabilities of failure and reliability indices for other important compononts are listed below.

Table 3.4 Probability of failure for most probable failure elements in idealized model

Element Number	P_F (IS)	P_F (FORM)	Reliability Index (FORM)
852	0	0	15.72
853	2.610E-12	2.402E-12	6.912
854	2.180E-12	1.919E-12	6.994
855	0	0	15.75
856	1.980E-14	1.758E-10	6.275
857	1.570E-10	1.243E-10	6.329
858	6.050E-15	8.028E-15	7.680
859	2.260E-9	2.244E-9	6.442
860	1.690E-9	1.505E-9	5.932
861	3.950E-11	4.495E-11	6.484
862	2.140E-8	1.850E-8	5.505
863	1.180E-8	1.200E-8	5.581
864	9.630E-10	9.841E-10	6.001
865	6.880E-8	6.357E-8	5.283
866	4.560E-8	4.011E-8	5.367
867	6.920E-9	7.042E-9	5.673
868	1.720E-7	1.694E-7	5.101
869	1.060E-7	1.045E-7	5.192
870	2.670E-8	2.858E-8	5.428
871	5.190E-7	3.791E-7	4.946
872	2.800E-7	2.284E-7	5.044

873	6.980E-8	6.524E-8	5.279
874	6.790E-7	6.363e-7	4.844
875	3.850E-7	3.706E-7	4.948
876	9.230E-8	8.372E-8	5.233
877	7.660E-7	7.435E-7	4.813
878	4.680E-7	4.374E-7	4.918

4. Conclusions

For a typical jack-up platform, probabilities of failure have been calculated by the FORM and ISPUD methods. The failure mode considered is collapse of a chord in compression or tension and the buckling of hull structure.

The structural modelling takes into account the dynamic effects with an equivalent SDOF system. For the structural model, the head sea direction was found most critical in the response and effects of bending moments on the chord designs has evaluated. The failure probabilities for idealized model devaite considerably from the failure probability for the detailed model and it has been concluded that a detailed model for leg structure will give more accurate failure probabilities for chords.

The most important parameters in the reliability analysis have been found by the wave height and wind velocity. The only other uncertain parameter was the drag coefficient and the remaining parameters can be taken as deterministic variables in static analysis. However for dynamic analysis, there is no evidance that which parameters are most important.

For evaluation of extreme response, it is necessary that the nonlinear analysis is performed by a nonlinear time domain simulation. The next step is the selection of an appropraite probabilistic model to match the extreme response of jack-up based on short term sea state. The Long term response for a jack-up structure has to be derived by a contribution of several short term response analysis.

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