Estimating the Tire-Road Friction Coefficient Based on Tire Force Measurements

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Master Thesis

TU Delft
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Based on Tire Force Measurements

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Master of Science Thesis
Delft, 8th July 2014

ME-BMD-AUT
Master: Mechanical Engineering
Track: Biomechanical Design
Specialisation: Automotive

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Estimating the Tire-Road Friction Coefficient Based on Tire Force Measurements

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Electronic stability control (ESC) has been shown to reduce the accident rate in a number of situations. Therefore, it is important to keep improving ESC and other control systems. One way to do this is by improving the available information on the state of the vehicle and its environment. The tire-road friction coefficient $\mu$ is particularly important, as it limits the maximum possible tire forces. Literature shows that systems like ESC, ABS, torque vectoring and adaptive cruise control all benefit from knowledge of $\mu$. In this study, a new estimation method is proposed to estimate $\mu$, using tire forces. The method works in the non-linear region of the tire and for combined slip conditions. For each tire, $\mu$ is estimated using an extended Kalman filter, together with a magic formula tire model. The algorithm then selects the estimate with the lowest uncertainty. The method is developed in Matlab, and evaluated with a CarSim vehicle model under braking, steady-state cornering and for a double lane change. The results show that this estimator is a promising first step. It succeeds to estimate $\mu$ when the tires are at their limit, but is less successful at lower slip levels. An explanation is that the effect of $\mu$ on the slip-slope is not present in CarSim. In the real world, a relationship exists between $\mu$ and the slip-slope, so that the estimator has the potential to work for lower levels of slip. Therefore, the proposed next step is to evaluate this estimation method on a real vehicle.

Keywords: tire-road friction coefficient, extended Kalman filter, on-line parameter estimation, vehicle dynamics

1 Introduction

For the last few years, over 1 million road accidents a year have been registered in the EU alone [1]. Research indicates that around 25% of accidents involve skidding of the vehicle, which increases to over 40% in case of fatal accidents [2, 3]. Over the last decades, different safety systems have been developed to prevent these accidents from happening. The best known examples are anti-lock braking system (ABS) and electronic stability control (ESC). Research suggests that ESC reduces the accident rate up to as much as 50% in different situations [4, 5]. These results demonstrate the importance to keep on improving safety systems such as ESC.

In order to develop a system like ESC it is important that the state of the vehicle can be estimated based on the available sensors. Current ESC systems use sensors to measure the yaw rate, wheel speed, steering angle and the lateral and longitudinal accelerations [6, 7]. Other values of interest, such as vehicle velocity, slip ratio, slip angle and the tire-road friction coefficient are not measured directly, and have to be estimated [7,8]. Improving the quality of these estimators can improve the performance of the control systems [8,9]. One value of particular interest is the tire-road friction coefficient $\mu$. For a single tire, $\mu$ is defined as:

$$\mu = \frac{\sqrt{(F_{x,\text{max}})^2 + (F_{y,\text{max}})^2}}{F_z}$$

in which $F_z$ is the vertical force, and $F_{x,\text{max}}$ and $F_{y,\text{max}}$ are the maximum achievable longitudinal and lateral tire forces respectively. Knowledge of $\mu$ is important since the maximum cornering, traction
and braking forces are all limited by its value. The tire-road friction coefficient $\mu$ is influenced by the road surface properties, but also by properties like the temperature, tire pressure and tire wear [10]. Many control systems can benefit from an accurate estimation of $\mu$, such as the above-mentioned ABS and ESC [9], torque vectoring [11] and adaptive cruise control (ACC) [10]. The advantage for an ABS system is clear. When $\mu$ is known, the system can directly brake until the limit, without the need for a cyclic algorithm. For both ESC and torque vectoring, the resulting control action is an additional yaw moment. For this, it is important to know where the limit is. For example in the ESC system developed in [7], $\mu$ is not known. Therefore the assumption is made that the tire limit is already reached, even if that is not the case. Finally, an ACC benefits from knowledge of $\mu$ as well. If its value is known, the required braking distance can be calculated. Based on this the ACC can then determine the optimal following distance.

In literature, several methods are proposed to estimate $\mu$. These algorithms can be divided into two categories: direct and indirect methods. The first group of methods measures the road surface directly by means of optical sensors [12, 13]. This has the advantage that $\mu$ can be estimated in all situations, even when the vehicle is not moving. In addition optical sensing makes it possible to estimate the friction of the road ahead. The disadvantage of this method is that careful tuning is required not only to interpret the road surface, but also the tire properties. Furthermore, this optical approach requires the implementation of new sensors in the vehicle.

The second category of methods estimate $\mu$ indirectly, by looking at the effects on the vehicle. This can be done in an active or a passive manner. In [14] a longitudinal force difference is generated between the left and right tires. By actively perturbing the system in this way, the road friction can be estimated. The drawback of this method is that you need to actuate the vehicle to do so, and thereby alter its dynamics. The final and largest group of estimation methods is purely based on indirect measurements of the vehicle state. Different solutions are proposed, such as looking at tire noise [15], tire deformation [12], wheel velocities [16], driveline torque [17, 18] or lateral or longitudinal slip [10, 19–23]. The main part of current research is done in this field of slip-based estimation methods. An overview of the different methods and their categorisation is shown in figure 1.

<table>
<thead>
<tr>
<th>Direct Methods</th>
<th>Indirect Methods</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optical Sensors</td>
<td>Active</td>
</tr>
<tr>
<td>Create Tire Force Difference</td>
<td>Acoustic Tire Noise</td>
</tr>
<tr>
<td>Tire Deformation</td>
<td>Longitudinal Slip</td>
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<tr>
<td>Wheel Velocities</td>
<td>Lateral Slip</td>
</tr>
<tr>
<td>Driveline Torque</td>
<td>Combined Slip</td>
</tr>
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</table>

**Figure 1:** Overview of the different methods from literature to estimate $\mu$. The dotted line shows the most common groups and subgroups.

It is important to note that these indirect methods will not work when the vehicle is stationary. When looking at the slip-based methods in particular, the slip ratio has to be above a certain threshold in order to do a reliable estimation of $\mu$ [23]. Furthermore, these methods have two other main limitations. The first problem is that most methods look either at pure lateral [24] or longitudinal dynamics...
This means that in order to estimate $\mu$, one has to assume that the force in the other direction is zero, which is not often the case in real-world driving scenarios. In [20] a method is proposed for combined operation, which shows promising results. Nevertheless, they conclude that the quality of their estimation still depends on the level and nature of the excitations, but also that there is still room for improvement concerning estimation delay.

A second limitation in much of the research is the focus on linear estimation, by looking only at the slip slope [19, 21, 22]. For low slip values, the tire force can be seen as a linear function of the slip, with a different slope for different values of $\mu$ [10]. An example of a tire curve can be seen in figure 2. Different solutions are proposed for the nonlinear region. In [19] the slip is assumed to be 100% as soon as the tire is in its nonlinear region, such that $\mu$ is equal to the normalised tire force $F_x/F_z$. However, this assumption will lead to inaccurate results in case the vehicle has not yet reached the limit in reality. In [22] the estimation algorithm is even completely deactivated during sharp bends, braking and skidding. Of course, the ability to estimate $\mu$ at low slip values is important, since the vehicle will be in that region more often. However, an accurate estimate of $\mu$ in the non-linear region is even more important, since this is the region in which systems like ABS and ESC are active. In order to provide such systems with an estimate of $\mu$, these linear methods are not the preferred option.

![Longitudinal Tire Curve](image)

**Figure 2:** An example of a longitudinal tire force-slip curve, consisting of both the linear and nonlinear region.

In this research a new method is proposed to estimate $\mu$. This method is developed to work in the nonlinear region and to combine both lateral and longitudinal friction estimation. The goal of this work is to accurately estimate the tire-road friction coefficient before the tire limits are reached by the vehicle. In order to do this, measurements of the tire forces will be used in addition to the more common information on the the slip ratio and slip angle. Until recently tire force measurements have only been possible by integrating sensors in the tires or by using a measurement rim. A promising novel technology, currently in development by SKF [25], is to measure these forces in the wheel hub bearing. By using advanced bearing models, the forces can be calculated based on measurements of the bearing deformation [26]. The availability of these force measurements is likely to improve the estimation of $\mu$ because of the direct relation between them.

In section 2, the complete estimation method will be explained in more detail. Next, the simulation results are presented in section 3. A discussion of the results follows in section 4. And finally, the conclusions are given in section 5, followed by suggestions for future steps in section 6.
2 Methods

In this work, a simulation study is carried out to develop and assess a method to estimate $\mu$. As explained, the tire-road friction coefficient $\mu$ is an important value, since it limits the tire forces as shown in the following equation:

$$\mu F_z = \sqrt{(F_{x,\text{max}})^2 + (F_{y,\text{max}})^2}$$  (2.1)

For this equation it is assumed that $\mu = \mu_x = \mu_y$. It is important to note that in reality $\mu$ can be different for the lateral and longitudinal direction, such that $\mu_y$ and $\mu_x$ can be defined separately. This is covered in more detail in section 2.3.1. The relation from equation 2.1 is commonly represented by the so-called friction circle, by which all tire forces are constrained. This circle is shown in figure 3.

![Figure 3: The friction circle representation for a single tire.](image)

The basic principle used here to estimate $\mu$ is a form of curve fitting. By fitting a tire model on the measured data in real time, $\mu$ can be determined. An advantage is that this method is suitable for non-linear models as well. A visual representation of this approach is shown in figure 4. In general, tire models describe the tire force as a function of the slip:

$$F = f(\kappa, \alpha)$$  (2.2)

in which $\alpha$ is the slip angle of the tire and $\kappa$ the longitudinal slip ratio, such that:

$$\kappa = \frac{\omega r - V_x}{V_x} \quad \text{and} \quad \alpha = \tan^{-1} \frac{V_y}{V_x}$$  (2.3)

in these equations $V_y$ and $V_x$ are the lateral and longitudinal velocity of the tire, $\omega$ the rotational wheel speed, and $r$ its effective radius. When $F$, $\alpha$ and $\kappa$ are all known, $\mu$ can be obtained from the tire model. Therefore, this work assumes the availability of the tire forces $F_x$, $F_y$ and $F_z$, for instance by means of load-sensing bearings [25]. The slip quantities $\alpha$ and $\kappa$ can't be measured directly, and must therefore be estimated. However, since the focus of this work is on estimating $\mu$, the estimated values for $\alpha$ and $\kappa$ are assumed to be known as well. An example of how to estimate $\kappa$ is given in [27] and possible estimation methods for $\alpha$ are proposed in [28,29]. Finally the road bank and inclination angles are assumed to be zero. In the following sections the design and implementation of this method is presented in more detail.
2.1 The Tire Model

The first step for this method is to select a suitable tire model. The model has to incorporate non-linear tire dynamics, has to be differentiable, and must be able to describe combined slip conditions. Based on these requirements the magic formula tire model [30] is selected. A more detailed motivation of this choice is given in appendix A. The basic form of the magic formula for pure slip conditions is given by:

\[ F_{x0} = D \sin \left( C \tan^{-1} \left( B \kappa - E \left( B \kappa - \tan^{-1} (B \kappa) \right) \right) \right) \]  

(2.4)

This same formula can be used for the lateral case, by replacing \( F_x \) with \( F_y \) and \( \kappa \) with \( \tan \alpha \). However, in order to explain the method, only the longitudinal case is examined here. It can be seen that equation 2.4 contains a set of parameters: \( B, C, D \) and \( E \). These parameters are dependent on the physical properties of the tire, but also on the dynamic state of the vehicle. However, to avoid using an overly complicated model for the estimation algorithm, the steady-state parameters are used for estimation. More details on this derivation are given in appendix C. From the given parameters, \( D \) represents the peak value of the tire force, since the output of the sine function lies between -1 and 1. Therefore it is possible to say \( \mu = D / F_z \). Furthermore, when the model is linearised for zero slip the equation simplifies to:

\[ F_{x0} = BCD \kappa \]  

(2.5)

Therefore the slope at the origin is represented by \( BCD \). An interesting observation is that this slope is also a function of \( D \). This means that in this model, an influence of \( \mu \) could already be reflected in the slip slope. Finally, \( C \) represents the shape of the curve and the asymptotic value for complete sliding, and \( E \) the curvature around the peak. The influence of the different parameters on the shape of the curve can be seen visually in figure 5. When the force and slip information is available, this model can be used as a parameter estimation problem, in which \( B, C, D \) and \( E \) have to be estimated. However, from figure 5 it can be seen that the influence of \( C \) and \( E \) on the shape of the tire curve is only minor. Therefore these parameters can be chosen beforehand, so that only \( B \) and \( D \) have to be estimated, as also suggested in [31].

In order to also account for combined slip conditions the equations must be extended. When there
Figure 5: The influence of the model parameters $D$, $BCD$, $C$ and $E$ on the shape of the longitudinal tire curve. Only one of these value is varied at a time.

is longitudinal slip, such that $\kappa \neq 0$, the lateral tire curve will be influenced, and it’s maximum value will decrease. The same thing is true for the influence of $\alpha$ on the longitudinal tire curve. To account for this effect, the longitudinal tire model is modified with a cosine function of the lateral slip. This means that when $\alpha = 0$ the equation returns the same tire force as the equation for pure slip conditions. However, when there is lateral slip present, the equation will return a lower value for $F_x$. The equation is given by:

$$F_x = G_{xa}F_{xo}$$ (2.6)

In this equation, $F_{xo}$ refers to equation 2.4, which is now corrected for combined slip by the function $G_{xa}$. In [30], both basic and extended equations are given for these weighting functions $G_{xa}$. The basic weighting functions $G_{xa}$ are given by:

$$G_{xa} = \cos(C_{xa} \tan^{-1}(B_{xa} \tan \alpha))$$ (2.7)

In which $B_{xa}$ and $C_{xa}$ are two additional coefficients that depend on the properties of the tire. The complete equations for combined lateral and longitudinal slip are:

$$F_x = \cos(C_{xa} \tan^{-1}(B_{xa} \tan \alpha)) \cdot D_x \sin\{C_x \tan^{-1}(B_x \kappa - E_x (B_x \kappa - \tan^{-1}(B_x \kappa)))\}$$ (2.8)

$$F_y = \cos(C_{ya} \tan^{-1}(B_{ya} \kappa)) \cdot D_y \sin\{C_y \tan^{-1}(B_y \tan \alpha - E_y (B_y \tan \alpha - \tan^{-1}(B_y \tan \alpha)))\}$$ (2.9)

As stated earlier, the goal of this work is to estimate the tire-road friction coefficient, before the maximum force is reached by the tire. This means that the tire model must be able to be fitted, before the
peak is reached. This might be possible by looking at the linear region, since then the tire force can be seen as a linear function of the slip, with a different slope for different values of $\mu$ [10]. However, more information can be retrieved when fitting data to the non-linear region of the curve. For example, [23] shows that the slip ratio has to be above a certain threshold in order to do a reliable estimation of the tire-road friction coefficient.

### 2.2 The Parameter Identification Method

Next, a suitable mathematical method is selected to estimate these tire model parameters $B$ and $D$, in real-time. A well-known method for model fitting, as visualised in figure 4, is the least squares approach. This method is also suitable for non-linear problems [32]. The least squares approach can be seen as an optimisation problem with the form:

$$\min_x f(x) = \min_x \sum_{i=1}^{n} e_i^2(x)$$  \hspace{1cm} (2.10)

In this equation, $e$ is a vector with the errors between the last $n$ measured values, and the corresponding model outputs. By setting $n$, the sensitivity of the algorithm can be changed. When $n$ is very small, the estimator will have a short response time. However, this will also increase the sensitivity to disturbances. By setting $n$ to a higher value, the system becomes less sensitive to these changes and disturbances. Unfortunately, this basic least squares approach is not appropriate for online estimation, since it is very computationally expensive. The reason for this is that for each iteration, the data from the last $n$ time steps must be processed [33].

Instead of this non-recursive approach, a recursive method must be used. According to table 1.2 in [33], there are two possible identification methods for on-line estimation of non-linear systems. A parameter estimation approach or an extended Kalman filter. However, the first of these two methods is only usable for non-linear systems which are linear in the parameters, which is not the case for the magic formula. Therefore the extended Kalman filter (EKF) is selected here.

#### 2.2.1 The Extended Kalman Filter

The EKF is an extension of the standard Kalman filter for non-linear systems [34]. Originally the goal of the Kalman filter is to estimate the state of the system. However, the method can also be modified for parameter estimation as described in [33, 35]. More detailed information on the EKF and its usage for parameter estimation is given in appendix B. The complete system is described by:

$$x_{k+1} = f(a_k, u_k) + w_k$$  \hspace{1cm} (2.11)

$$a_{k+1} = a_k + \xi_k$$  \hspace{1cm} (2.12)

$$z_k = h(a_k) + v_k$$  \hspace{1cm} (2.13)

For this system, the parameters $a_k$ can be estimated by means of the following equations:

**Time update:**

$$a_{k+1} = a_k$$  \hspace{1cm} (2.14)

$$P_{k+1} = S_k$$  \hspace{1cm} (2.15)
Measurement update:

\[ K_k = P_k^{-\frac{1}{2}} H_k^T (H_k P_k^{-\frac{1}{2}} H_k^T + R_k)^{-1} \]  

\[ \hat{a}_k = \hat{a}_k^- + K_k (z_k - h(\hat{a}_k^-)) \]  

\[ P_k = (I - K_k H_k) P_k^- \]  

Where:

\[ H_k = \frac{\partial h(\hat{a}_k)}{\partial \hat{a}} \]  

In these equations, \( P_k \) is the error covariance matrix, \( R_k \) the measurement noise covariance matrix and \( S_k \) the process noise covariance matrix for the parameters. Since the filter is now used for parameter estimation instead of state estimation, \( S_k \) can be seen as an indication for the amount of variation in the parameters. Furthermore, \( a_k \) is a vector with the parameters to be estimated, \( v_k \) the measurement noise, \( z_k \) a measurement, and \( h \) the function which relates the measurement to the parameters.

### 2.3 Combining Individual Wheels

The tire model presented in section 2.1 and the estimation method given in section 2.2 are the most important building blocks of the complete \( \mu \)-estimation algorithm. The EKF is used to estimate the parameters of the magic formula tire model, such that \( h_k \) is given by equation 2.8 or 2.9 and the parameter vector \( a_k \) is \{\( D \), \( B \)\}. Three additional steps are taken to complete the algorithm. The first is to combine lateral and longitudinal estimation, which result in one \( \hat{\mu} \) for each tire. The second step is to develop a measure for the uncertainty of the estimation, and finally a strategy is proposed to combine the information from the individual wheels. These steps are described in section 2.3.1, 2.3.2 and 2.3.3.

These steps are based on the assumption that the estimated tire-road friction coefficient \( \hat{\mu} \) will become more accurate when the used friction \( \mu_{\text{used}} \) approaches the friction limit \( \mu \). Where \( \mu_{\text{used}} \) is defined as:

\[ \mu_{\text{used}} = \sqrt{\frac{F_x^2 + F_y^2}{F_z}} \]  

This assumption is based on literature where several studies have indeed shown that the estimation is more accurate when \( \mu_{\text{used}} \) is closer to \( \mu \) [20, 23]. With this, an indication of the uncertainty can be developed, which is then used for the combination of the different wheel estimates. Even though the estimates for the individual wheels are combined, at the basis of this estimation approach each tire is treated as a separate system. This means that when this last step is omitted, \( \mu \) will be estimated for all tires individually. Therefore, the method also has the potential to detect split-\( \mu \) surfaces.

#### 2.3.1 Combined Lateral and Longitudinal Estimation

The tire model is defined by two separate equations for the lateral and longitudinal dynamics, which are stated in equation 2.8 or 2.9. Until now \( \mu \) has been defined as a single value. However, two different values can be distinguished. With the formula for \( F_z \), \( \mu_x \) can be estimated, whereas \( \mu_y \) can be estimated based on the equation for \( F_y \). These are two different values, dependent on the lateral and longitudinal properties of the tire. When \( \mu_x \) and \( \mu_y \) are not the same, the circle from figure 3 will have a different shape, representing an ellipse instead of a circle. Pacejka suggests in [36] that \( \mu_x \) and \( \mu_y \) may indeed differ from each other. Nevertheless, both [37] and [38] state that a circle is an accurate estimation for the friction ellipse. Therefore \( \mu \) is defined here as \( \mu = \mu_x = \mu_y \). Still, it should be noted that with minor changes this algorithm is also suited for estimating \( \mu_x \) and \( \mu_y \) independently.
Because of this relationship, both the lateral and longitudinal dynamics can be used to estimate a single combined $\mu$ value. In the ideal situation, both the lateral and the longitudinal estimator will give an accurate result. However, there will be situations where one of the estimates can be more uncertain than the other. When the vehicle is for example driving straight ahead, $\mu_y$ cannot be estimated accurately. In this situation the algorithm should place more emphasis on the longitudinal friction estimation. This is confirmed by the assumption that $\hat{\mu}$ will be more accurate when $\mu_{used}$ approaches $\mu$. Based on these observations, a combination strategy is proposed which is shown in figure 6. In this algorithm the lateral and longitudinal estimates are weighted by the current tire forces $F_x$ and $F_y$.

This is done by multiplying $\hat{\mu}_x$ with $|F_x|/(|F_x| + |F_y|)$ and $\hat{\mu}_y$ with $|F_y|/(|F_x| + |F_y|)$. The advantage of this weighting is that these weighted values can now simply be added together, to have an unbiased value for $\hat{\mu}$. When $F_x > F_y$ more emphasis is placed on $\hat{\mu}_x$ and vice versa. By using both lateral and longitudinal information, an accurate estimate for $\mu$ can be obtained in more driving situations.

Figure 6: The complete algorithm to estimate $\mu$ for a single tire.

2.3.2 Estimation Uncertainty

Nevertheless there will still be situations in which the uncertainty of $\hat{\mu}$ is an issue. Therefore an attempt is made to quantify this uncertainty. This value will be calculated in parallel with the estimation algorithm itself, so that a better interpretation of the results is possible. Keeping in mind the assumption that $\hat{\mu}$ will be more accurate when $\mu_{used}$ approaches $\mu$, two situations can be described:

1. The first case is when $\mu_{used} < \hat{\mu}$, which is by far the most common situation, given the nature of the estimation. Because the expectation is that $\hat{\mu}$ will be more accurate when $\mu_{used}$ is closer to $\mu$, it is also expected that $\hat{\mu}$ will be more accurate when $\mu_{used}$ is closer to $\hat{\mu}$ itself. Therefore an indication for the uncertainty $q$ can be in the form of $q = \hat{\mu} - \mu_{used}$.

2. The second case is when $\mu_{used} > \hat{\mu}$. It should be noted that this situation is an exception. Since $\hat{\mu}$ is the maximum value of the fitted model, the measured data on which this estimate is based, will in general have a lower value. However, this situation can occur in case of sudden changes in the tire forces or the road surface. In this case $\hat{\mu}$ should at least converge to $\mu_{used}$. Therefore the uncertainty for this case can for example be defined as $q = \mu_{used} - \hat{\mu}$.

These two situations can be combined in one single equation which gives an indication of the uncertainty of the estimation:

$$q = q_n + \frac{|\hat{\mu} - \mu_{used}|}{\max(\hat{\mu}, \mu_{used})}$$  \hspace{3cm} (2.21)

The measure for the uncertainty is normalised here, so that the uncertainty term returns zero in case $\mu_{used} = \hat{\mu}$. Furthermore, an additional term $q_n$ is introduced in the equation, which represents the
constant uncertainty, for instance due to inaccuracies in the tire model. Depending on the results, this coefficient can also be tuned to a negative value. For instance when the results show that for all situations an accurate estimate can already be given when \( \mu_{\text{used}} = 0.8 \cdot \hat{\mu} \). In that case \( q_n \) can be set to \(-0.2\). In the case of a negative value for \( q_n \) the equation has to be modified to prevent \( q \) from becoming negative. During the estimation, \( q \) will be calculated in real time, to give an indication of the trustworthiness of the result.

### 2.3.3 Combined Estimation for all Wheels

Finally, the estimators for the different wheels are combined to improve the reliability of the global estimate \( \hat{\mu} \). This is important for a number of situations. For example, when the vehicle is accelerating in a straight line. In this case only the driven wheels will have tire forces. Therefore estimation is impossible for the non-driven wheels.

By combining the information from the individual wheels, it is still possible to estimate a value \( \hat{\mu} \) for the complete vehicle. The most basic combination strategy would be taking the mean value of \( \hat{\mu}_{\text{fl}}, \hat{\mu}_{\text{fr}}, \hat{\mu}_{\text{rl}} \), and \( \hat{\mu}_{\text{rr}} \), in which the indices denote the wheel position from front-left (fl) to rear-right (rr). When again looking at the example of straight line acceleration, this will improve the estimate compared to that of the non-driven wheels. However, the estimate is also likely to be worse than that of the driven wheels.

In the equation 2.21, a measure was given for the uncertainty \( q \). This uncertainty can be calculated for each wheel individually. By using these values, a smarter combination strategy is proposed. Instead of taking the mean of the different wheels, the wheel with the lowest uncertainty is selected. A diagram is shown in figure 7. In this diagram, \( \text{min\_index} \) returns the index with the lowest uncertainty, which is then used to switch between the different values for \( \hat{\mu} \). To prevent excessive switching when two uncertainty values are very close, a delay is built into the \( \text{min\_index} \) function. Only when the difference in uncertainty between two wheels changes above a certain threshold, the output of \( \text{min\_index} \) is updated. The logic is as follows:

\[
\begin{align*}
\text{IF} & \quad q_s - q_k > x \\
& \quad s = k
\end{align*}
\]

(2.22)

In this, \( s \) and \( k \) are indices which denote the wheel position from front-left (fl) to rear-right (rr). The value of \( k \) is the index for the wheel with the lowest uncertainty at the current time step. The previously saved index is represented by \( s \). If \( q_s - q_k \) is above a defined threshold \( x \), \( s \) will be updated with the value of \( k \).

![Diagram](image)

*Figure 7: A global \( \hat{\mu} \) is defined based on the estimated values for individual tires.*
2.4 Implementation in Matlab and CarSim

In the previous sections the estimation method is described. The next step is to implement this method in such a way that the performance can be evaluated. This can be done either by using simulations or by implementing the method on a real vehicle. The advantages of a simulation study are the simplicity and freedom to design, and execute your experiments. This is much more complicated for real-world driving scenarios. Since this research is about the development of the basic concept, a simulation study is performed here. If the results are satisfactory, the estimator can be evaluated and validated on a real vehicle as a next step. In this work, the estimator is developed in Matlab/Simulink, and evaluated in combination with CarSim. The following sections explain how these tools are used to develop and evaluate the estimation algorithm. More details about the used models are given in appendix C.

2.4.1 Matlab and Simulink

The global structure is developed in Simulink, and can be seen in figure 8. This block diagram consists of three parts. The signal generator, the vehicle model and the $\mu$-estimator itself. The reference value for $\mu$ is set in Simulink and given as an input to the vehicle model. The vehicle model is defined within the second block. This vehicle is not modelled within Matlab itself but instead a model is provided by CarSim, which is described in section 2.4.2. Using the value for $\mu$, CarSim provides information on the state of the vehicle back to Simulink. In this case the tire forces are provided, as well as the slip angle and slip ratio for each wheel. Based on this state information, $\mu$ is estimated in the third block. The content of this estimator block is presented in the previous sections. First, the values of $\hat{\mu}$ for each individual wheel, are provided by the block diagram shown in figure 6. Next, a global value for $\hat{\mu}$ is chosen as shown in figure 7. Finally, all signals are combined into one output vector, so that the results can be evaluated in Matlab.

![Figure 8: This figure gives an overview of the simulation setup in Simulink. In this diagram, the vehicle model is provided by CarSim.](image)

2.4.2 CarSim

CarSim is used to provide the vehicle model. It is possible to integrate the CarSim models into Matlab/Simulink. Inputs for the vehicle model can then be set from Simulink, while selected model outputs can be returned back to Simulink. In this way, the estimation algorithm can be developed in Matlab/Simulink, and tested on the detailed vehicle model as provided by CarSim.

The basic CarSim model is a 15 degrees of freedom (DOF) multibody vehicle model [39]. But next to that, CarSim also has models available for the tires, the full powertrain, road and environment parameters, and driver behaviour. As an accurate alternative to real-world experiments, this complete modelling of vehicle, driver and environment is used here to evaluate the performance of the estimation algorithm. In this study a rear-wheel drive (RWD) vehicle with a sprung mass of 1020 kg is simulated. The used tire model is the magic formula Pacejka model. The details of the manoeuvres and trajectories are set in CarSim as well. The only external input to CarSim is the reference signal for
2.5 Performance Evaluation

In the previous sections, the entire method to estimate $\mu$ is presented. In order to evaluate this method, different scenarios will be selected and executed by using CarSim. This software is capable of simulating both open-loop and closed-loop driving manoeuvres. Open-loop manoeuvres can be seen as pre-defined signals for the throttle, the brakes and the steering wheel, acting on the vehicle. On the other hand, the closed-loop manoeuvres are based on path-following and speed control, by using a preview of the upcoming road. It is important to note that for all experiments the initial value of $\hat{\mu}$ will be set to 1. By using this information, two different situations can be distinguished for the conducted experiments:

1. In the first situation, $\mu$ is set to a constant value lower than one. Since the initial value for $\hat{\mu}$ will always be one, this means that there is an estimation error at the beginning of the experiment. The results of this type of experiment will show under which circumstances $\hat{\mu}$ will be able to converge to $\mu$.

2. For the second type of experiments, $\mu$ will not remain constant. Instead its value will vary over time. The results of these experiments will give a better understanding about the ability of the algorithm to track changes in $\mu$ for different situations.

Three different scenarios are selected in order to look at different facets of the developed method. A Braking action, driving around a circular track, and a double lane change manoeuvre.

2.5.1 Braking Action

The first scenario is a pure braking action. This experiment is designed as an open-loop manoeuvre where the vehicle will be coasting at a speed of 120 km/h, without steering input. At $t = 5$ a step braking input will be given. The tire-road friction coefficient $\mu$ will be set to a constant value, so that the convergence can be assessed, as discussed earlier. The details of this scenario are summarised in figure 9. It is interesting to look at this braking action, because of its relevance for ABS. If the algorithm succeeds to estimate $\mu$ accurately here, then this estimator can be used to improve ABS systems.

<table>
<thead>
<tr>
<th>Friction coefficient</th>
<th>Initial speed</th>
<th>Speed</th>
<th>Braking Steps</th>
<th>Steering</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu = 0.7$</td>
<td>120 km/h</td>
<td>coasting</td>
<td>3 MPa, 2.4 MPa, and 1.8 MPa at $t=5$</td>
<td>none</td>
</tr>
</tbody>
</table>

*Figure 9: Details for the pure braking manoeuvre.*

2.5.2 Circular Track

In the next manoeuvre the vehicle is driving around a circular track. Three different scenarios will be assessed. An overview is given in figure 10. With these scenarios, the performance for pure lateral slip, as well as combined slip will be evaluated. By driving in a circle, a constant lateral acceleration can be maintained. This is a convenient way to test the performance of the estimator for different levels of $\mu_{\text{used}}$. In order to do this, the friction coefficient is varied over time. In the first scenario, the
manoeuvre is executed with a constant velocity, in order to assess the performance of the estimator for a lateral load case.

The second and the third scenario represent combined slip conditions. For the second scenario, $\mu$ is kept constant, but the vehicle speed is increased during the manoeuvre. Because now both lateral and longitudinal accelerations are present, the convergence of the estimator is evaluated here for combined slip conditions. This scenario is an example where an ESC system could become active around the vehicle limit. Therefore the performance of the $\mu$-estimator during this test is of particular interest. The conditions for the final circular track experiment are the same, but now $\mu$ is varied as well during the manoeuvre.

![Circular Track Diagram](image)

**Figure 10:** Details for the three different scenarios on a circular track.

### 2.5.3 Double Lane Change

Finally, the double lane change (DLC) is evaluated. The double lane change is a more dynamic manoeuvre than the previous two. This manoeuvre is often used to assess vehicle behaviour, for instance by evaluating the performance of ESC systems [40]. Here, a double lane change steering input is given for different initial speeds. During this test, $\mu$ is kept at a constant value of 0.4. The details for this manoeuvre are given in figure 11.

![Double Lane Change Diagram](image)

**Figure 11:** Details for the double lane change.

<table>
<thead>
<tr>
<th>Friction coefficient</th>
<th>Initial speeds</th>
<th>Speed</th>
<th>Braking</th>
<th>Steering</th>
</tr>
</thead>
<tbody>
<tr>
<td>varying $\mu$-profile</td>
<td>85 km/h, 75 km/h and 65 km/h</td>
<td>constant</td>
<td>none</td>
<td>$R = 100m$</td>
</tr>
<tr>
<td>$\mu = 0.5$</td>
<td>60 km/h</td>
<td>increasing from 60-90 km/h</td>
<td>none</td>
<td>$R = 100m$</td>
</tr>
<tr>
<td>varying $\mu$-profile</td>
<td>60 km/h</td>
<td>increasing from 60-90 km/h</td>
<td>none</td>
<td>$R = 100m$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Friction coefficient</th>
<th>Initial speeds</th>
<th>Speed</th>
<th>Braking</th>
<th>Steering</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu = 0.4$</td>
<td>105 km/h and 85 km/h</td>
<td>constant</td>
<td>none</td>
<td>open-loop profile</td>
</tr>
</tbody>
</table>
3 Results

In this section the simulation results are presented. The results are given for the different manoeuvres described in section 2.5. For all experiments the initial value of $\hat{\mu}$ is set to 1. In section 3.5 the influence of the initial value $\mu_0$ on the estimator performance is shown. All other results in this section, consist of two plots. In the first plot, $\mu$ and $\hat{\mu}$ are plotted together. Furthermore this plot contains the lateral used friction of one of the tires. The tire which is plotted, is the one with the maximum value of $\mu_{y,\text{used}}$ during that manoeuvre. Next to that, the longitudinal used friction is plotted for the tire with the maximum value of $\mu_{x,\text{used}}$. These two lines give an indication of what the vehicle is doing, but also how close to the limit the vehicle has to be for the estimation to be successful.

In the bottom graph of each result the absolute error $|\mu - \hat{\mu}|$ is plotted, together with the estimator uncertainty as defined in section 2.3.2. By plotting the uncertainty and the real error together, an indication is given of the validity of the uncertainty calculation. Finally the root mean square error (RMSE) of the estimator is given in the title of this plot. This value can be used to compare the error for different runs of the same manoeuvre.

3.1 Braking Action

First the results are given for the braking scenario as described in section 2.5.1. This manoeuvre is executed for three different values of braking pressure, in order to see the performance for different levels of excitation. The first result is shown in figure 12, for a braking pressure of 3MPa. This is a severe braking action, during which the limit is reached. This means that the wheels lock, resulting in full sliding. It can be seen that $\mu$ can be estimated very accurately in this situation. At $t = 6.22 \text{s}$ The
Figure 13: Estimator performance during straight line coasting, when a braking action of 2.4MPa is applied from $t = 5$ onwards.

Figure 14: Estimator performance during straight line coasting, when a braking action of 1.8MPa is applied from $t = 5$ onwards.
vehicle reaches the limit of 0.7. At this point \( \hat{\mu} \) has converged to 0.82. When \( t = 6.38 \) s, \( \hat{\mu} \) has converged to 0.7. This means that there is a delay of 0.16 s.

When there is less braking, the estimate also becomes less accurate. This can be seen in figure 13 and 14, which represent a 2.4MPa and a 1.6MPa braking action respectively. In figure 13 the vehicle is braked up to the limit, but the limit is not reached. It can be seen that this has a negative impact on the estimation result when compared with the stronger braking shown in figure 12. In figure 14 an even lower braking action is applied. It can be seen that the algorithm does not succeed to estimate \( \mu \) in this scenario. The maximum value reached by \( \mu_{x,fl,used} \) is 0.52, while \( \hat{\mu} \) converges to 0.88.

### 3.2 Steady-State Circle

Next, the estimator is evaluated during steady-state cornering as described in section 2.5.2. This manoeuvre is executed at three different speeds, and with a changing \( \mu \)-profile. In the first result, shown in figure 15, the speed is set to 85 km/h. It can be seen that the estimator works very well in this scenario. The \( \mu \)-profile is estimated accurately, and without large delays. It can be seen that the used lateral friction is such that the limit is reached when \( \mu \) decreases to 0.5. Another observation is that the used lateral friction increases when \( \mu \) changes back up to 0.9. Since this is the line for the front-left tire, this means that the vehicle is understeering. What is happening here is that the path-following algorithm from CarSim tries to maintain its trajectory, even though the vehicle is running wide. It tries to do so by increasing the steering angle even further, resulting in more understeer. The same happens with the longitudinal slip. Since the tires are already at their limit, additional acceleration due to the speed controller, results in more wheel spin. Because of this, the longitudinal friction is increasing as well during the manoeuvre. So, for this high slip situation the estimator works very well.

*Figure 15: Estimator performance while driving at a circular track with a radius of 100 meters, at a constant speed of 85 km/h.*
Figure 16: Estimator performance while driving at a circular track with a radius of 100 meters, at a constant speed of 75 km/h.

Figure 17: Estimator performance while driving at a circular track with a radius of 100 meters, at a constant speed of 65 km/h.
The slip levels are much lower for a speed of 75 km/h at the same circular track. The results for this scenario are shown in figure 16. Here the limit is not reached. Compared with the previous experiment it can be seen that the longitudinal used friction therefore stays at a lower level. Nevertheless, the estimator is still able to track most of the \( \mu \)-profile. Between \( t = 8 \) and \( t = 12 \), the mean value of \( \hat{\mu} \) is 0.58, which is close to the real value of 0.5. An interesting observation is that the increase in \( \mu \) after \( t = 12 \) is also estimated correctly, even though the uncertainty increases as well. Finally the manoeuvre is repeated with a speed of 75 km/h. This result is shown in figure 17. Here the mean value of \( \hat{\mu} \) between \( t = 8 \) and \( t = 12 \) is 0.75, but still the estimator is able to track the step back to 0.9.

### 3.3 Combined Slip

Now the same circular track is used for two combined slip cases, as described in section 2.5.2. It can be seen that the algorithm still performs well under these conditions. For the first case, shown in figure 18, \( \mu \) is kept at a constant value of 0.5 during the manoeuvre, while the speed is increased from 60-90 km/h. Because of this, not only the longitudinal, but also the lateral tire forces will increase. The reason for this is that for a given radius of the trajectory, the lateral acceleration will increase when the forward velocity increases. It can be seen that for this combined load case, the estimator is still able to converge to the real value of \( \mu \), but not before the level of used friction has converged to the limit.

The second combined load case is shown in figure 19. This is exactly the same scenario, but now the tire-road friction coefficient follows the same profile as in section 3.2. It can be seen that \( \mu_{\text{used}} \) increases during the manoeuvre, and when it reaches the real friction coefficient, \( \hat{\mu} \) converges to \( \mu \) as well.

![Figure 18: Estimator performance while driving at a circular track with a radius of 100 meters. During the manoeuvre, the vehicle is accelerated from 60-90 km/h.](image)
Figure 19: Estimator performance while driving at a circular track with a radius of 100 meters. During the manoeuvre, the vehicle is accelerated from 60-90 km/h.

Figure 20: Estimator performance during a double lane change. This open-loop manoeuvre is executed here with a speed of 105 km/h.
3.4 Double Lane Change

The final scenario that is evaluated is the double lane change. This manoeuvre is often used for the assessment of vehicle dynamics, and is described in section 2.5.3. Here, the manoeuvre is performed at 105 km/h and 85 km/h, for which the results are shown in figure 20 and figure 21 respectively. Like for all tests, the initial value for $\hat{\mu}$ is set to 1. The real friction coefficient is set here to 0.4.

![Double Lane Change, V = 85 km/h](image)

**Figure 21:** Estimator performance during a double lane change. This open-loop manoeuvre is executed here with a speed of 85 km/h.

During this manoeuvre the algorithm does not succeed to converge. During cornering, when the tire forces increase, the error $|\mu - \hat{\mu}|$ decreases. However, when the vehicle returns to steady state, the error increases back again. Still, the estimator becomes more accurate when $\mu_{used}$ increases, even in this combined slip scenario. This is what is expected, and can also be seen from the similarity between the signals for the uncertainty and the error itself. Additional heuristic rules might improve the algorithm for these dynamic driving situations. For example by updating $\hat{\mu}$ only when the uncertainty $q$ is below a certain threshold. This topic will be described in more detail in section 4.

3.5 Influence of the Initial Value

The initial value of an estimator can be an important influence on the performance of the estimator. Since a non-linear system is estimated, based on derivatives, the algorithm could in theory return a local solution. In figure 22 the result is shown for different initial values $\hat{\mu}_0$, between 0.2 and 1.5. It can be observed that for this manoeuvre, the influence of the initial value on the final result is only minor. The estimator quickly converges, after which the estimate is mainly determined by the vehicle behaviour, and no longer by its initial value.
Varying Initial Value - Circle, $R = 100\text{m}$, $V = 70\text{ km/h}$

Figure 22: Estimator performance for different initial values. In the left graph, the first second is shown in more detail.

4 Discussion

In this work a method has been developed to estimate the tire-road friction coefficient $\mu$. When looking at the results, it can be seen that in most cases an accurate estimate of $\mu$ is only obtained when the tire limits are reached, so that $\mu - \mu_{\text{used}}$ reaches zero. For the application of this estimator in systems like ABS or ESC, that is already interesting information. However, the goal of this study was to have an accurate estimate $\hat{\mu}$, already when the tires are not yet at their limit, meaning $\mu - \mu_{\text{used}} > 0$. Knowing this information before reaching the limit, might improve the performance and response time of such safety systems even further. It can be seen from the results that, in general, the algorithm does not succeed to do so. Only in some cases, such as in figure 16, $\mu$ is tracked accurately even though the used friction is at a lower level. In this discussion, the results will be evaluated, and suggestions for possible improvements will be given. First, the influence of the basic model fitting on the results will be discussed, after which the algorithm which combines the information from the individual wheels will be evaluated.

4.1 The Basic Principle

The goal of this study was to estimate $\mu$ before it the tire limit is reached. Because this information seems to be available from the non-linear shape of the tire curve, the hypothesis was that this is indeed possible. Of course it has to be noted that the parameters of the tire curve are different for different tires, making it either easier or harder to distinguish the tire curves for different values of $\mu$. In figure 23, the steady-state, pure slip tire curves are plotted for three different values of $\mu$, with the exact parameters from the CarSim tire model used in this study. Around zero slip, the curves are overlapping, because they are still within the linear region, but for increasing slip the curves diverge. A first observation which can be made, is that when the peak of the curve increases, the curvature around the peak also increases, making the non-linear region more pronounced. Therefore, the higher $\mu$ is, the sooner it should be possible to obtain a good estimate.

As an example, the longitudinal case is considered here. To see if it would be possible to predict $\mu$ for this specific tire model, one important question is asked, which is: For a given level of $\mu_{\text{used}}$, what are the different values of the slip ratio $\kappa$, corresponding to $\mu = 1$, $\mu = 0.7$ and $\mu = 0.5$? In table 1, these
slip values are given for selected levels of \( \mu_{\text{used}} \). The first observation is that the values are indeed different, albeit very minor. Therefore in a theoretical ideal situation, already at \( \mu_{x,\text{used}} = 0.4 \), \( \mu \) can be estimated for the cases \( \mu = 1 \), \( \mu = 0.7 \) and \( \mu = 0.5 \), based on the corresponding tire slip values. To better understand these values, the formula for the slip ratio \( \kappa \) is recalled, which has been given in equation 2.3. Two example cases are given:

1. When \( \mu_{x,\text{used}} = 0.4 \), and \( V_x = 100 \text{km/h} \), the wheel speed \( \omega r \) will be 102.9 km/h for \( \mu = 0.5 \) and 102.4 km/h for \( \mu = 0.7 \). Now, when the used friction changes to \( \mu_{x,\text{used}} = 0.5 \), the wheel speed \( \omega r \) will be 107.7 km/h for \( \mu = 0.5 \) and 103.3 km/h for \( \mu = 0.7 \).

2. When \( \mu_{x,\text{used}} = 0.6 \), and \( V_x = 100 \text{km/h} \), the wheel speed \( \omega r \) will be 104.7 km/h for \( \mu = 0.7 \) and 103.6 km/h for \( \mu = 1 \). Now, when the used friction changes to \( \mu_{x,\text{used}} = 0.7 \), the wheel speed \( \omega r \) will be 110.8 km/h for \( \mu = 0.7 \) and 104.5 km/h for \( \mu = 1 \).

It can immediately be seen that the difference in slip, for two values of \( \mu \), is very small, but increases substantially when one of the values is reached by \( \mu_{\text{used}} \). This partially explains why the estimator succeeds when \( \mu_{\text{used}} = \mu \), but not when its just below \( \mu \). Nevertheless, when only looking at these numbers, one could expect that the algorithm should be able to predict \( \mu \) also in the latter case, since this is a simulation study only. The reason that this is not the case, is that the tire model used in CarSim is more complicated than the tire model used by the estimator. Even though both are based on the magic formula tire model, the parameters of the model are simplified in the estimator for steady-state

<table>
<thead>
<tr>
<th>( \mu_{x,\text{used}} = 0.4 )</th>
<th>( \mu = 1.0 )</th>
<th>( \mu = 0.7 )</th>
<th>( \mu = 0.5 )</th>
<th>( \mu_{x,\text{used}} = 0.5 )</th>
<th>( \mu = 1.0 )</th>
<th>( \mu = 0.7 )</th>
<th>( \mu = 0.5 )</th>
<th>( \mu_{x,\text{used}} = 0.6 )</th>
<th>( \mu = 1.0 )</th>
<th>( \mu = 0.7 )</th>
<th>( \mu = 0.5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu_{x,\text{used}} = 0.4 )</td>
<td>0.024</td>
<td>0.036</td>
<td>0.047</td>
<td>( \mu_{x,\text{used}} = 0.5 )</td>
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<td>0.094</td>
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</tr>
<tr>
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<td>0.094</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>( \mu_{x,\text{used}} = 0.6 )</td>
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<td>0.046</td>
<td>( \mu_{x,\text{used}} = 0.7 )</td>
<td>0.047</td>
<td>0.065</td>
<td>-</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \mu_{x,\text{used}} = 0.7 )</td>
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<td>0.108</td>
<td>-</td>
<td>( \mu_{x,\text{used}} = 0.7 )</td>
<td>0.067</td>
<td>0.132</td>
<td>-</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Selected coordinates from the tire curves in figure 23. The values in the table are the slip ratios and slip angles which correspond with the given \( \mu \) and \( \mu_{\text{used}} \).

Figure 23: Longitudinal tire curves for \( \mu = 1, \mu = 0.7 \) and \( \mu = 0.5 \).
conditions. This is explained in section 2.1. Therefore dynamic effects, such as the relaxation length, are not modelled in the estimator, and will result in small estimator errors. Because small differences in slip can already result in big differences of $\hat{\mu}$, the algorithm does not succeed to predict $\mu$ before it is reached because of these dynamic effects. This will be even more true for a real vehicle, where a lot of additional disturbances and sensor noise are introduced into the system.

Another drawback of this estimation method is that it relies on some pre-set parameters for the tire model. This means that the accuracy of the estimator will be influenced by changes in the tires, such as the tire pressure, wear, or even temperature. By making the tire model more accurate to account for additional dynamic effects, the number of parameters will increase even further. However, this is not suggested, since this will increase the sensitivity to changes in the tire as well. Still the results are promising. The friction coefficient $\mu$ is estimated very accurate for high levels of used friction, such as in figure 12 and figure 15. Next to that, all results show that changes in $\mu$ are also detected when the limit is not reached, albeit less accurate. The uncertainty value which is developed in this study gives a good real-time insight into the quality of the estimator and is used in real-time to support the estimator itself.

### 4.1.1 Influence of the Friction Coefficient on the Slip-slope

The previous part of the discussion suggests that the performance of the estimator is likely to decrease when implemented on a real vehicle. There is however also a reason why estimating $\mu$ might actually be more successful in the real world. When looking back at the CarSim tire curves in figure 23, an observation can be made. For the different values of $\mu$, the slip-slope in the linear region is exactly the same. This is not the expected behaviour. Like stated in the introduction, this slip-slope is also used in literature to estimate $\mu$ [19, 21, 22]. A thorough evaluation of this topic is given in [10], which states that for low slip values, the tire force can be seen as a linear function of the slip, with a different slope for different values of $\mu$. Table 1 in [10] shows an overview of literature which supports this claim.

This means that in CarSim $\mu$ is only used to scale the tire curve, whereas in reality the slope of the tire curve will also be different for different road surfaces. This effect is not implemented in CarSim. In [19], this difference in slope is already enough to estimate $\mu$ when the vehicle is still in the linear region for a real-world study. This suggests that the implementation of $\mu$ in CarSim might actually be a big factor which impacts the results of this study in a negative way.

### 4.2 Combination Logic

In this second part of the discussion the proposed combination algorithm is evaluated. In this work, the information from all wheels is combined to have an accurate global estimate of $\mu$ for the complete vehicle. This is achieved by selecting the wheel with the lowest uncertainty. In general it can be said that this algorithm works well. However, since this part of the estimator is based on heuristic rules, there is potential for further improvements by learning from the results.

A first suggestion could be to move away from a continuously updated estimation. Currently, the estimator is updated each time step, based on the state of the vehicle. This also happens when the uncertainty $q$ is high, which is likely to result in a wrong estimate. A possible improvement could be to store the previous value of $\hat{\mu}$, and only update its value when $q$ is below a certain threshold, like for instance 0.5. This will especially improve the result for the double lane change manoeuvre which is shown in figure 20 and figure 21. For example, the improvement for the double lane change at 105 km/h is shown in figure 24.
A second point of possible improvement is the combination of lateral and longitudinal estimates. These are now combined as a function of the tire forces. This means that when the lateral and longitudinal tire forces are equal, the average of the lateral and longitudinal estimate is taken. However, when there is only a small amount of longitudinal slip, and severe lateral slip, the longitudinal estimate will still contribute to the combined estimate. It is true that its contribution will be minor, but its estimate can also be very inaccurate due to the low level of slip. For instance, the small peaks in figure 15, around $t = 12s$, are caused by the longitudinal estimate of a front tire. Since the vehicle is rear-wheel drive and the brakes are not applied here, this is a highly uncertain estimate. However, at the same time it must be noted that when the estimate would be more accurate for lower slip levels, the proposed combination method might still be the best. This might be the case in reality. As discussed in 4.1.1, there is an effect of $\mu$ on the slip-slope, which is likely to improve estimation at lower slip values.

Finally, in the ideal situation, the value of $\mu$ is estimated accurately for each wheel individually. This can potentially be done with the current algorithm. In fact $\hat{\mu}$ is already estimated for each wheel individually. In the combination algorithm, the wheel with the lowest uncertainty is selected as the final estimate. Another possibility could be to return a separate $\hat{\mu}$ value for each wheel. Then, when the uncertainty for one of the wheels is above a defined threshold, the value for that specific wheel can still be updated based on wheels with a higher certainty. It is possible to make this work for split-$\mu$ surfaces as well. In that case information from the wheels on the left cannot be combined with estimates from the right side, and vice versa.

5 Conclusions

In this work a new method is proposed to estimate $\mu$, based on tire forces. To do this an extended Kalman filter is used to fit a tire model to both the longitudinal and the lateral dynamics of each wheel. The goal of this method was to predict the value of $\mu$, before $\mu_{used}$ has reached $\mu$. Several systems such as ABS and ESC [9], torque vectoring [11] and adaptive cruise control (ACC) [10] are likely to improve when the tire-road friction coefficient is known.

The estimator has been implemented in Matlab/Simulink, and a simulation study is performed in cooperation with CarSim. Next the performance is analysed for different scenarios. The estimation algorithm succeeds to estimate $\mu$ if the tires are at their limits. When $\mu_{used}$ is below $\mu$, the estimator
does no longer succeed to track $\mu$ accurately. Nevertheless, the algorithm is still able to track changes in $\mu$ to a certain extent. It is also shown that the estimator succeeds to work for a wide range of initial values $\hat{\mu}_0$. Furthermore, the algorithm works in the non-linear region, and is modified to also work in combined slip conditions.

The reason that it is difficult to estimate $\mu$ at lower levels of used friction is because the tire curves are very close together for lower levels of slip. Estimation in these situations is even less likely to succeed in real-world scenarios, due to additional noise and disturbances. This suggests that this method is not very promising for real-world implementation. However, the results are most likely hampered by the implementation of the tire-road friction coefficient in CarSim. It is shown that CarSim neglects the effect of $\mu$ on the slip-slope. This makes it more difficult to distinguish between tire curves at low slip levels. In reality, the slip-slope is different for different values of $\mu$. Therefore the difference between tire curves for different values of $\mu$ will be more pronounced in the real-world. Other studies have already succeeded to estimate $\mu$ from this slip-slope during real-world experiments. This means that even though there will be more noise and disturbances in the real system, the predictability of $\mu$ might still be better when compared with the results of this study.

The proposed method does not only contain the estimation step itself, but also a heuristic algorithm which determines the uncertainty of the estimates, and returns the value with the lowest uncertainty. This works well and the uncertainty value is a realistic description of the state of the estimator. Additional gains could be made by further improving on this heuristic logic. This can for instance be done by changing from a continuous estimator to an algorithm where the estimate is only updated if the uncertainty is below a predefined value.

To conclude, it can be said that this research is a first step in a promising direction. The proposed non-linear estimation method, based on tire slip and tire forces succeeds to estimate $\mu$ accurately at the limit. Furthermore, it also has the potential to do so for lower slip levels. When the effects of $\mu$ on the slip-slope are also incorporated, the algorithm will potentially work both in the linear and the non-linear region. In the end this will lead to an all-round method to estimate and predict the tire-road friction coefficient $\mu$ for a wide range of driving scenarios.

6 Future Work

The most important next step would be to implement the developed method on a real vehicle. Three things are required for this. First of all, the vehicle must have the ability to measure the tire forces $F_x$, $F_y$, and $F_z$. Next to that, estimates for the slip ratios $\kappa$, and slip angles $\alpha$ must be available to the algorithm. Next to these requirements on the vehicle, it is also important to know the real value for $\mu$. This value has to be available in order to determine the accuracy of the estimator. The challenge is that this value is not only dependent on the road surface, but also on other environmental factors, such as the temperature, and the properties of the tire itself. Therefore, to have an accurate reference value of $\mu$, the specific tire must be analysed on that specific road surface first. Next, when this reference value for $\mu$ is available, the estimation result can be compared with the measured value of $\mu$.

Moving away from simulations will introduce new challenges, such as sensor noise and other disturbances in the system. These uncertainties will influence the quality of the estimator, which means additional filtering steps might be required. The estimator has to be modified to be able to cope with this new situation, but also to work in the linear region when the slip-slope will change, based on $\mu$. Like discussed, this effect is not present in CarSim, so that a real-world implementation will have ad-
ditional potential to predict \( \mu \) for lower levels of slip. In order to look at the influence of the slip-slope, a relation must be established between the Pacejka parameters \( B, C \) and \( D \) and the road surface. This must be done based on experimental data from different road surfaces. When the slip-slope, which is reflected in the Pacejka parameters \( B, C \) and \( D \), can be defined as a function of \( \mu \), the current algorithm will be able to estimate \( \mu \) both in the linear and non-linear region of the tire.

In section 4, different potential improvements are suggested for the algorithm, mainly in the area of the combination logic. Further research is necessary to determine the optimal solution. Like discussed, the best solution might be different for the current CarSim model and the real-world. Therefore it is suggested that this research is conducted after the implementation on a real vehicle.

These steps will hopefully result in an all-round \( \mu \)-estimator, which gives accurate results in many driving scenarios. Of course, only knowing \( \mu \) is not enough to improve the performance, functionality or safety of the vehicle. It's value needs to be implemented and used by the different systems which will benefit from knowledge of \( \mu \). When the estimation method is developed successfully, these systems, such as ABS, ESC, ACC and torque vectoring must be modified to use the knowledge of \( \mu \) to their full benefit, so that their performance will improve.
A Selecting a Tire Model

In this appendix the choice for the magic formula tire model is motivated. A tire model calculates the tire forces based on the state of the vehicle. Tire models can be divided into analytical and semi-empirical models. Analytical models are based on a physical interpretation of the tire. On the other hand, semi-empirical models are based on a curve-fitting approach. These models depend on non-physical parameters in order to define the shape of the force-slip relationship. In general, tire models often describe the steady state behaviour only. There are however also some dynamical models which also incorporate time-related effects such as the relaxation length \[41\].

In the next sections, three different tire models are presented, discussed and compared. Only models which are able to model combined slip have been selected. This is something which is not available in all tire models. Other models such as the semi-empirical Burckhardt model \[42\] and the dynamical LuGre model as proposed in \[43\] do not describe combined slip conditions, and are therefore not included in this comparison. Finally the tire model is selected which is best suited for online friction estimation.

A.1 The Dugoff Tire Model

First, the Dugoff tire model is discussed. This analytical model is presented in \[44\], and altered for combined slip in \[45\]. In this simplified model the effects of camber and turn slip are neglected. Furthermore a uniform vertical pressure distribution is assumed in the tire. The equations for pure slip conditions are given by:

\[
F_x = C_\sigma \sigma_x f(\lambda) \quad \text{and} \quad F_y = C_\alpha \sigma_y f(\lambda)
\]

\[
\lambda = \frac{\mu F_z (1 + \kappa)}{2 \sqrt{(C_\sigma \sigma_x)^2 + (C_\alpha \tan \alpha)^2}}
\]

\[
f(\lambda) = \begin{cases} 
\lambda (2 - \lambda) & \text{if } \lambda < 1 \\
1 & \text{if } \lambda \geq 1 
\end{cases}
\]

in which:

\[
\sigma_x = \frac{\kappa}{1 + \kappa}, \quad \sigma_y = \frac{\tan \alpha}{\kappa + 1} \quad \text{and} \quad \kappa = \frac{\omega r - V_x}{V_x}
\]

In which \(C_\sigma\) and \(C_\alpha\) represent the longitudinal and lateral tire stiffness and \(\alpha\) the slip angle of the tire. Furthermore, \(\omega\) is the rotational wheel speed, and \(r\) the effective radius of the wheel. The basic principle of this simplified model is that the linear tire force is calculated, which is then modified using the shape function \(f(\lambda)\). When the combination of \(F_x\) and \(F_y\) is smaller than \(\mu F_z/2\), the model remains in the linear region. Above that the tire force converges to \(\mu F_z\). It is important to note that these equations are only valid for pure lateral, or pure longitudinal slip. In \[45\] a combined slip Dugoff model is developed, which is given below.

\[
F_x = \begin{cases} 
C_\sigma \sigma_x & \text{if } \lambda \geq 1 \\
C_\sigma \sigma_x \frac{\mu_{\text{res}}}{\mu_{\text{used}}} & \text{if } \lambda < 1 
\end{cases}
\]

\[
F_y = \begin{cases} 
C_\alpha \sigma_y & \text{if } \lambda \geq 1 \\
C_\alpha \sigma_y \frac{\mu_{\text{res}}}{\mu_{\text{used}}} & \text{if } \lambda < 1 
\end{cases}
\]
The advantage of this model is that it does not require a large number of parameters. And since it is an analytical model, these parameters can be derived in an intuitive way. The drawback of this method is that the gradient cannot be computed directly, since there are different equations for sliding and pre-sliding. Furthermore, the model is a very simplified representation. Therefore the shape of the tire curve is limited, such that a peak in the tire curve cannot be modelled.

A.2 The Brush Model

In this section the brush model is presented, as proposed in [30]. The brush model is an analytical tire model, meaning that it only depends on physical parameters of the tire. In the brush model the tire is described as a series of elastic elements, which can deflect in contact with the road. The following equations use a parabolic vertical pressure distribution. This has an advantage, since modelling based on a uniform pressure distribution results in an asymptotic behaviour in which full slip is not really reached [46]. The equations for pure lateral and pure longitudinal slip for this case is given by:

\[
\begin{align*}
F_x &= \begin{cases} 
\mu F_z (1 - \lambda^3_x) \text{sgn} \sigma_x & \text{if } |\sigma_x| \leq \frac{1}{\theta_x} \\
\mu F_z \text{sgn} \sigma_x & \text{if } |\sigma_x| > \frac{1}{\theta_x}
\end{cases} \\
F_y &= \begin{cases} 
\mu F_z (1 - \lambda^3_y) \text{sgn} \alpha & \text{if } |\tan \alpha| \leq \frac{1}{\theta_y} \\
\mu F_z \text{sgn} \alpha & \text{if } |\tan \alpha| > \frac{1}{\theta_y}
\end{cases}
\end{align*}
\] (A.8)

in which:

\[
\lambda_x = 1 - \theta_x |\sigma_x|, \quad \lambda_y = 1 - \theta_y |\tan \alpha|, \quad \theta_x = \frac{2c_{px} l^2}{3\mu F_z} \quad \text{and} \quad \theta_y = \frac{2c_{py} l^2}{3\mu F_z}
\] (A.10)

In these equations \( l \) is the half-length of the contact patch, and \( c_{px} \) and \( c_{py} \) the longitudinal and lateral stiffness per length unit respectively. \( 1/\theta_x \) and \( 1/\theta_y \) represent the slip at which complete sliding is reached. From this point the force is simply put equal to the force limit \( \mu F_z \). Next to these equations for pure longitudinal and lateral slip, the model can also be defined for combined slip conditions [30]. For this, the lateral and longitudinal tire stiffness are assumed to be equal such that \( c_p = c_{px} = c_{py} \). This leads to:

\[
\begin{align*}
F_x &= F \frac{\sigma_x}{\sigma} \quad \text{and} \quad F_y = F \frac{\sigma_y}{\sigma}
\end{align*}
\] (A.11)

in which:

\[
F = \begin{cases} 
\mu F_z (1 - \lambda^3) \text{sgn} \sigma & \text{if } |\sigma| \leq \frac{1}{\theta} \\
\mu F_z \text{sgn} \sigma & \text{if } |\sigma| > \frac{1}{\theta}
\end{cases} \\
\sigma = \sqrt{\sigma_x^2 + \sigma_y^2}, \quad \lambda_x = 1 - \theta |\sigma| \quad \text{and} \quad \theta = \frac{2c_p l^2}{3\mu F_z}
\] (A.13)
In fact this model is very comparable with the Dugoff model presented earlier, and has the same advantages, and disadvantages. Nevertheless this representation is more realistic, because of the more advanced vertical pressure distribution [46].

A.3 The Magic Formula

As opposed to the previous models, the magic formula is a semi-empirical model. This means that the model is based on real measurement data. However, this also means that the parameters of the model are not physical properties of the tire. This model is developed by Pacejka, and described in [30, 36]. The general form of the magic formula is valid both for pure longitudinal and lateral slip. It is given by:

\[ y = D \sin\left[ C \tan^{-1}\left( B x - E \left( B x - \tan^{-1}(B x)\right)\right)\right] \quad (A.14) \]

This same formula is used for the lateral case, with \( y = F_y \) and \( x = \tan \alpha \) as well as the longitudinal case with \( y = F_x \) and \( x = \kappa \). Furthermore this formula contains a set of parameters: \( B, C, D \) and \( E \). These parameters are defined as functions of constants, but also depend on the vehicle state. Next, the equations for combined slip are given.

\[ F_x = G_{xa} F_{xa} \quad \text{and} \quad F_y = G_{yk} F_{yo} \quad (A.15) \]

In these equations \( F_{xa} \) and \( F_{yo} \) represent the tire forces for pure slip conditions. Which are then corrected for combined slip by the functions \( G_{xa} \) and \( G_{yk} \). In [30], both basic and extended equations are given for these weighting functions. The basic equations are given by:

\[ G_{xa} = \cos(C_{xa} \tan^{-1}(B_{xa} \tan \alpha)) \quad (A.16) \]

\[ G_{yk} = \cos(C_{yk} \tan^{-1}(B_{yk} \kappa)) \quad (A.17) \]

In which \( B_{xa}, C_{xa}, B_{yk} \) and \( C_{yk} \) are additional coefficients. Such that the complete equations for combined slip are:

\[ F_x = \cos(C_{xa} \tan^{-1}(B_{xa} \tan \alpha)) \cdot D_x \sin\left[ C_x \tan^{-1}\left( B_x \kappa - E_x \left( B_x \kappa - \tan^{-1}(B_x \kappa)\right)\right)\right] \quad (A.18) \]

\[ F_y = \cos(C_{yk} \tan^{-1}(B_{yk} \kappa)) \cdot D_y \sin\left[ C_y \tan^{-1}\left( B_y \tan \alpha - E_y \left( B_y \tan \alpha - \tan^{-1}(B_y \tan \alpha)\right)\right)\right] \quad (A.19) \]

Unlike the analytical models discussed before, this model describes pre-sliding and sliding in just one equation. This means that the model is differentiable, which is an advantage. Furthermore, the magic formula model is a very accurate representation of actual tire dynamics. But this comes at a cost, since the model depends on a lot of parameters which might be difficult to obtain. However, since the goal of this study is to estimate the tire-road friction coefficient, the influence of several of these parameters can be neglected, as proposed in [31].

A.4 Comparison

In the previous sections three different tire models were presented. Their most important properties are compared in table 2.

It can be seen that the magic formula can achieve the highest accuracy, but at the same time also requires the most parameters. Nevertheless, it has to be noted that some of those parameters are unimportant for a friction estimation application [31]. On the other hand, the analytical models are more simple, and intuitive to work with, requiring only information on the tire stiffness.
### Table 2: A qualitative comparison between the different tire models, specifically aimed at on-line friction estimation. The number of parameters represents the required parameters, beside $\mu$ itself, to calculate $F_x$ for both pure longitudinal slip, and combined slip conditions.

<table>
<thead>
<tr>
<th></th>
<th>Dugoff model</th>
<th>Brush model</th>
<th>Magic formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accuracy</td>
<td>+/-</td>
<td>+</td>
<td>++</td>
</tr>
<tr>
<td>Number of Parameters, pure</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Number of Parameters, combined</td>
<td>1</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>Analytical</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Differentiable</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
</tbody>
</table>

The major drawback of the Dugoff and Brush model is that their equations are not directly differentiable, since a separate equation is given for the linear and non-linear region. It is important to note that the size of these regions is a function of $\mu$. Therefore, when the goal is to estimate $\mu$, this value is not always known. This means that the algorithm cannot be sure which equation to use. Since estimation methods often depend on the gradient of the function to be estimated, the single equation is a major advantage of the magic formula. Based on this comparison the magic formula is selected for the estimation method proposed in this work.

### B Kalman Filter Parameter Estimation

In this appendix the equations for the extended Kalman filter (EKF) are given. The EKF is an extension of the standard Kalman filter for non-linear systems [34]. Originally the goal of the Kalman filter is to estimate the state of the system. However, the method can also be modified for parameter estimation as described in [33, 35]. First the standard form of the EKF is given, which is used for state estimation. Next its derivation for state estimation is given as well as its application in this study.

#### B.1 The Extended Kalman Filter

First, the standard equations of the EKF are given, as stated in [47]. These equations are used to estimate the state of a system with the form:

\[
x_{k+1} = f(x_k, u_k) + w_k \tag{B.1}
\]

\[
z_k = h(x_k) + v_k \tag{B.2}
\]

In which $x_k$ is the state vector, $z_k$ a measurement, $u_k$ the control input, and $w_k$ and $v_k$ the process and measurement noise. For this system, the state can be estimated by means of the following equations:

**Time update:**

\[
\dot{x}_k^- = f(\dot{x}_k, u_k, 0) \tag{B.3}
\]

\[
P_k^- = A_k P_k A_k^T + Q_k \tag{B.4}
\]

**Measurement update:**

\[
K_k = P_k^- H_k^T (H_k P_k^- H_k^T + R_k)^{-1} \tag{B.5}
\]

\[
\dot{x}_k = \dot{x}_k^- + K_k (z_k - h(\dot{x}_k^-)) \tag{B.6}
\]

\[
P_k = (I - K_k H_k) P_k^- \tag{B.7}
\]
Where:

$$A_k = \frac{\partial f(\hat{x}_k, u_k)}{\partial x} \quad \text{and} \quad H_k = \frac{\partial h(\hat{x}_k)}{\partial x}$$  \hspace{1cm} (B.8)

In these equations, $P_k$ is the error covariance matrix, $R_k$ the measurement noise covariance matrix and $Q_k$ the process noise covariance matrix.

### B.2 Combined State and Parameter Estimation

A solution to estimate model parameters with this method, is to augment the state with a parameter vector $a_k$ [33], such that:

$$x_{a,k+1} = \begin{bmatrix} x_{k+1} \\ a_{k+1} \end{bmatrix} = \begin{bmatrix} f(x_k, a_k, u_k) \\ a_k \end{bmatrix} + \begin{bmatrix} w_k \\ \xi_k \end{bmatrix}$$  \hspace{1cm} (B.9)

Where $x_{a,k}$ is the state vector $x_k$ augmented with the parameter vector $a_k$. It can be seen that the parameters are modelled as constant values. However, the model incorporates a disturbance $\xi_k$. Because of this, the values for $a_k$ will still be adjusted during the estimation. In case of pure state estimation, the predicted error covariance is given by:

$$P_{k+1}^- = A_k P_k A_k^T + Q_k$$  \hspace{1cm} (B.10)

This is the error covariance, which is defined as:

$$P_{k+1}^- = E\{ (\hat{x}_{k+1} - x_{k+1})(\hat{x}_{k+1} - x_{k+1})^T \}$$  \hspace{1cm} (B.11)

The same derivation now must be made for $a_k$. Such that:

$$P_{k+1}^- = \begin{bmatrix} E\{ (\hat{\xi}_{k+1} - x_{k+1})(\hat{\xi}_{k+1} - x_{k+1})^T \} \\ E\{ (\hat{\xi}_{k+1} - x_{k+1})(\hat{\xi}_{k+1} - x_{k+1})^T \} \end{bmatrix}$$  \hspace{1cm} (B.12)

In order to derive this equation, $a_{k+1}$ and $\hat{a}_{k+1}$ are needed. $a_{k+1}$ is defined in equation B.9. Since the noise $\xi_k$ is assumed to be zero mean, the parameter estimates are updated as $\hat{a}_{k+1} = a_k$. This results in:

$$E\{ (\hat{a}_{k+1} - a_{k+1})(\hat{a}_{k+1} - a_{k+1})^T \} = E\{ \xi_k \xi_k^T \} = S_k$$  \hspace{1cm} (B.13)

Where $S_k$ is defined as the process noise covariance matrix for the parameters. As explained, this noise is modelled in order to allow $a_k$ to change during estimation. Therefore, setting this value influences the rate of change for $a_k$. By combining the final equations for $P_{k+1}^-$ we get:

$$\begin{bmatrix} A_k P_{a,k} A_k^T + Q_k \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ S_k \end{bmatrix}$$  \hspace{1cm} (B.14)

### B.2.1 The complete equations for state and parameter estimation

Finally these modifications are applied to the complete equations for the EKF as given in section B.1. Now the system equations are defined as:

$$x_{k+1} = f(x_k, a_k, u_k) + w_k$$  \hspace{1cm} (B.15)

$$a_{k+1} = a_k + \xi_k$$  \hspace{1cm} (B.16)

$$z_k = h(x_k, a_k) + v_k$$  \hspace{1cm} (B.17)
For this system, the state and the parameters can be estimated by means of the following equations:

**Time update:**

\[
x_{a,k+1} = f(x_k, a_k, u_k)
\]

\[
P_{k+1}^- = A_k P_{a,k} A_k^T + Q_k
\]

**Measurement update:**

\[
K_k = P_k^- H_k^T (H_k P_k^- H_k^T + R_k)^{-1}
\]

\[
\hat{x}_{a,k} = \hat{x}_{a,k}^- + K_k (z_k - h(\hat{x}_k, \hat{a}_k))
\]

\[
P_k = (I - K_k H_k) P_k^-
\]

Where:

\[
A_k = \frac{\partial f(\hat{x}_k, \hat{a}_k, u_k)}{\partial x_a}
\] and \[
H_k = \frac{\partial h(\hat{x}_k, \hat{a}_k)}{\partial x_a}
\]

**B.3 Parameter Estimation Only**

In this work the goal is to estimate the parameters of the magic formula tire model. In this model the tire forces are a function of the tire slip, and therefore not differential equations. In this case the state estimation is irrelevant, and only estimation of the parameters is of interest. Therefore the equations can be simplified for this specific task. The system is now defined as:

\[
x_{k+1} = f(a_k, u_k) + w_k
\]

\[
a_{k+1} = a_k + \xi_k
\]

\[
z_k = h(a_k) + v_k
\]

For this system, the parameters $a_k$ can be estimated by means of the following equations:

**Time update:**

\[
a_{k+1} = a_k
\]

\[
P_{k+1}^- = S_k
\]

**Measurement update:**

\[
K_k = P_k^- H_k^T (H_k P_k^- H_k^T + R_k)^{-1}
\]

\[
\hat{a}_k = \hat{a}_k^- + K_k (z_k - h(\hat{a}_k^-))
\]

\[
P_k = (I - K_k H_k) P_k^-
\]

Where:

\[
H_k = \frac{\partial h(\hat{a}_k)}{\partial a}
\]

This is the form in which the EKF is used in this work.
C CarSim Model Details

In this section more details are given on the vehicle and tire models from CarSim, which are used to assess the performance of the developed estimator. CarSim is a software package which is able to simulate vehicle behaviour in great detail. The used models, as well as the parameters of the models are discussed in section C.1. Section C.2 shows how the tire model is parametrised in the Estimator.

C.1 CarSim Model

The basic CarSim model is a 15 degrees of freedom (DOF) multibody vehicle model. But next to that, CarSim also has models available for the tires, the full powertrain, road and environment parameters, and driver behaviour. The choice of the vehicle parameters is mostly arbitrary, since the estimation method for $\mu$ is not dependent on the layout of the vehicle. In this study the CarSim model for a small B-class sports car is selected. In table 3 the relevant model parameters are shown.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sprung Weight</td>
<td>1020 kg</td>
</tr>
<tr>
<td>Weight Distribution</td>
<td>50/50 front/rear</td>
</tr>
<tr>
<td>Wheelbase</td>
<td>2.33 m</td>
</tr>
<tr>
<td>Suspension</td>
<td>Independent</td>
</tr>
<tr>
<td>Layout</td>
<td>Rear-wheel drive (RWD)</td>
</tr>
<tr>
<td>Engine</td>
<td>150 kW</td>
</tr>
<tr>
<td>Transmission</td>
<td>CVT</td>
</tr>
<tr>
<td>Differential</td>
<td>Open rear differential</td>
</tr>
<tr>
<td>Tire Model</td>
<td>Pacejka 5.2 model</td>
</tr>
</tbody>
</table>

Table 3: Properties of the selected CarSim vehicle model.

Most of these parameters are self-explanatory. For the transmission a continuous variable transmission (CVT) is selected. The reason for this is that in this way, continuous acceleration signals can be generated over a wide range of velocities. By using a CVT the acceleration will not be interrupted by gear changes, making it easier to set a continuous reference signal. In the next subsections, the complete CarSim tire model are given.

C.1.1 CarSim Longitudinal Tire Model

The equations used by the CarSim tire model are given by Pacejka in [30]. Here, the Pacejka parameters, such as $B$, $C$, $D$ and $E$, are not defined as constants, but are dependent on the physical properties of the tire and the dynamic state of the vehicle. Where the parameters in CarSim are zero, the equations are simplified. For the longitudinal case, the resulting model is stated below. Since this model is not directly part of this work, its equations are neglected for the nomenclature in appendix E. More background information on the equations can be found in [30].

$$F_x = G_{xa}F_{xa}$$  \hspace{1cm} (C.1)

$$F_{xa} = D_x \sin \left( C_x \tan^{-1} \left( B_x \kappa - E_x \left( B_x \kappa - \tan^{-1} (B_x \kappa) \right) \right) \right)$$  \hspace{1cm} (C.2)

$$G_{xa} = \cos \left( C_x \tan^{-1} \left( B_{xa} \alpha_s - E_{xa} \left( B_{xa} \alpha_s - \tan^{-1} (B_{xa} \alpha_s) \right) \right) \right) / G_{sxa}$$  \hspace{1cm} (C.3)

$$G_{sxa} = \cos \left( C_{xa} \tan^{-1} \left( B_{xa} S_{ixa} - E_{xa} \left( B_{xa} S_{ixa} - \tan^{-1} (B_{xa} S_{ixa}) \right) \right) \right)$$  \hspace{1cm} (C.4)

$$\alpha_s = \alpha^* + S_{ixa}$$  \hspace{1cm} (C.5)
The parameters in this model are defined as:

\[
C_x = p_{C_x1} \tag{C.6}
\]
\[
D_x = \mu_x F_z \tag{C.7}
\]
\[
E_x = p_{E_x1} + p_{E_x2} df_z + p_{E_x3} df_z^2 \tag{C.8}
\]
\[
B_x = F_z \left( p_{K_x1} + p_{K_x2} df_z \right) \exp \left( p_{K_x3} df_z \right) / (C_x D_x) \tag{C.9}
\]

And the combined slip parameters:

\[
B_{xa} = r_{Bx1} \cos \left( \tan^{-1} (r_{Bx2} \kappa) \right) \tag{C.10}
\]
\[
C_{xa} = r_{Cx1} \tag{C.11}
\]
\[
E_{xa} = r_{Ex1} + r_{Ex2} df_z \tag{C.12}
\]
\[
S_{Hyx} = r_{Hy1} \tag{C.13}
\]

C.1.2 CarSim Lateral Tire Model

The model for the lateral tire dynamics is closely related. There are however some differences. The equations are stated below. In these equations \( \gamma \) represents the camber angle.

\[
F_y = G_{yx} F_{yo} \tag{C.14}
\]
\[
F_{yo} = D_y \sin \left( C_y \tan^{-1} (B_y a - E_y (B_y a - \tan^{-1} (B_y a))) \right) + S_{Vy} \tag{C.15}
\]
\[
G_{xy} = \cos \left( C_{xy} \tan^{-1} (B_{xy} \kappa_{xy} - E_{xy} (B_{xy} \kappa_{xy} - \tan^{-1} (B_{xy} \kappa_{xy}))) \right) / G_{yko} \tag{C.16}
\]
\[
G_{yko} = \cos \left( C_{xy} \tan^{-1} (B_{xy} S_{Hyxy} - E_{xy} (B_{xy} S_{Hyxy} - \tan^{-1} (B_{xy} S_{Hyxy}))) \right) \tag{C.17}
\]
\[
\kappa_s = \kappa + S_{Hyxy} \tag{C.18}
\]
\[
\alpha = \alpha^s + S_{Hyxy} \tag{C.19}
\]

The parameters in this model are defined as:

\[
C_y = p_{Cy1} \tag{C.20}
\]
\[
D_y = \mu_y F_y \tag{C.21}
\]
\[
E_y = p_{Ey1} + p_{Ey2} df_z \left( 1 - (p_{Ey3} + p_{Ey4} \sin \gamma) \text{sgn}(\alpha) \right) \tag{C.22}
\]
\[
B_y = p_{Ky1} F_z \left( 2 \tan^{-1} \left( F_z / (p_{Ky2} F_z) \right) \right) \left( 1 - p_{Ky3} \sin^2 \gamma \right) / (C_y D_y) \tag{C.23}
\]
\[
S_{Vy} = \left( p_{Vy3} + p_{Vy4} df_z \right) \sin \gamma \tag{C.24}
\]
\[
S_{Hy} = p_{Hy3} \sin \gamma \tag{C.25}
\]

And the combined slip parameters:

\[
B_{yk} = r_{By1} \cos \left( \tan^{-1} (r_{By2} (\alpha^s - r_{By3})) \right) \tag{C.26}
\]
\[
C_{yk} = r_{Cy1} \tag{C.27}
\]
\[
E_{yk} = r_{Ey1} + r_{Ey2} df_z \tag{C.28}
\]
\[
S_{Hyxy} = r_{Hy1} + r_{Hy2} df_z \tag{C.29}
\]

C.1.3 CarSim Tire Model Parameters

In this subsection the CarSim parameter values are given for the just described model. As a final equation, \( df_z \) is defined as:

\[
df_z = \frac{F_z - F'_{zo}}{F'_{zo}} \tag{C.30}
\]

All parameters are shown in table 4.
Table 4: Numerical values of all parameters of the Pacejka tire model, as implemented in CarSim.

C.2 Simplifications for the Estimator Tire Model

When looking at the complete CarSim tire model, as described in section C.1.1-C.1.3, it is obvious that it is too complicated to implement directly into the estimator. Nevertheless, the equations are stated in this appendix, to understand how the internal model of the estimator is derived. To do this, several simplifications have been made to the model parameters. As a first simplification the basic equations for combined slip are used. In [30], both basic and extended equations are given for these weighting functions. The basic equations are given by:

\[ G_x = \cos(C_x \tan^{-1}(B_x \tan \alpha)) \]  
\[ G_y = \cos(C_y \tan^{-1}(B_y \kappa)) \]  

Next to that, all parameters are simplified for steady state conditions. This means that \( df_z, \kappa \) and \( \alpha \) are set to zero for the parameter equations. Furthermore the influence of the camber angle on the lateral tire model equations is neglected. With these modifications, the parameters simplify to:

Table 5: Numerical values of the simplified tire model parameters, as used in the estimator, for the longitudinal and lateral tire model respectively.

D Glossary

This list gives a complete overview of the abbreviations used throughout this work.

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABS</td>
<td>Anti-lock Braking System</td>
</tr>
<tr>
<td>ACC</td>
<td>Adaptive Cruise Control</td>
</tr>
<tr>
<td>CVT</td>
<td>Continuous Variable Transmission</td>
</tr>
<tr>
<td>DLC</td>
<td>Double Lane Change</td>
</tr>
<tr>
<td>DOF</td>
<td>Degrees of Freedom</td>
</tr>
</tbody>
</table>
E Nomenclature

In this section all symbols used in this work are listed. For clarity, the nomenclature for the complete CarSim vehicle model stated in section C.1 is omitted. More information on these equations is found in [30]. The variables which are defined for individual wheels are noted in this nomenclature with the subscript \( fl \). This represents the front-left wheel, and can be replaced with \( fr \), \( rl \) and \( rr \) for the front-right, rear-left and rear-right wheel respectively.

- \( \alpha \): Lateral slip angle for a single tire
- \( \alpha_{fl} \): Lateral slip angle for the front-left tire
- \( a_k \): Vector containing the parameters to be estimated by the EKF
- \( \hat{a}_k \): Vector containing the predicted parameters by the EKF
- \( B_x \): Parameter for the longitudinal Pacejka model
- \( B_y \): Parameter for the lateral Pacejka model
- \( B_{xa} \): Parameter for the longitudinal combined slip Pacejka model
- \( B_{yx} \): Parameter for the lateral combined slip Pacejka model
- \( c_{px} \): Longitudinal stiffness per unit length for the brush model
- \( c_{py} \): Lateral stiffness per unit length for the brush model
- \( C_{\alpha} \): Lateral stiffness for the Dugoff model
- \( C_{\sigma} \): Longitudinal stiffness for the Dugoff model
- \( C_x \): Parameter for the longitudinal Pacejka model
- \( C_y \): Parameter for the lateral Pacejka model
- \( C_{xa} \): Parameter for the longitudinal combined slip Pacejka model
- \( C_{yx} \): Parameter for the lateral combined slip Pacejka model
- \( D_x \): Parameter for the longitudinal Pacejka model
- \( D_y \): Parameter for the lateral Pacejka model
- \( e \): Vector containing estimation error values
- \( E_x \): Parameter for the longitudinal Pacejka model
- \( E_y \): Parameter for the lateral Pacejka model
- \( F_x \): Longitudinal tire force
- \( F_{xo} \): Longitudinal tire force for pure slip
- \( F_{x,fl} \): Longitudinal tire force for the front-left tire
- \( F_{x,max} \): Maximum longitudinal tire force for a given \( \mu \)
- \( F_y \): Lateral tire force
- \( F_{yo} \): Lateral tire force for pure slip
- \( F_{y,max} \): Maximum lateral tire force for a given \( \mu \)
- \( F_z \): Vertical tire force
- \( G_{xa} \): Longitudinal weighing function for combined slip
- \( G_{yk} \): Lateral weighing function for combined slip
- \( h_k \): Function to be estimated by the EKF
- \( H_k \): Gradient of \( h_k \) with respect to \( a_k \)
- \( \kappa \): Longitudinal slip ratio for a single tire
- \( \kappa_{fl} \): Longitudinal slip ratio for the front-left tire
Kalman gain of the EKF
Half-length of the contact patch
Theoretical longitudinal tire slip
Theoretical lateral tire slip
Variable used by the Brush tire model
Error covariance matrix of the EKF
Predicted error covariance matrix of the EKF
Uncertainty value of the estimated value of $\mu$
Uncertainty value for the front-left tire
Constant in the equation for $q$
Process noise covariance matrix for the EKF
Effective rolling radius of the wheel
Radius of the road trajectory
Measurement noise covariance matrix for the EKF
Time in seconds
Tire-road friction coefficient
Estimated tire-road friction coefficient
Tire-road friction coefficient of the front-left wheel
Longitudinal tire-road friction coefficient
Lateral tire-road friction coefficient
The amount of friction used by the tires
Internal variable of the Dugoff tire model
System control input
Longitudinal velocity
Lateral velocity
Measurement noise of the EKF
Rotational wheel velocity
State vector estimated by the EKF
State vector, augmented with $\alpha_k$ estimated by the EKF
Parameter noise vector of the EKF
Transition point to the sliding region of the Dugoff and brush models
Measurement for the EKF

References


