Communications on Hydraulic and Geotechnical Engineering

An Improved Settling Tube System For Sand

December 1986

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Report no. 86-4

Delft University of Technology
Department of Civil Engineering
Delft, The Netherlands
1986
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SUMMARY

General design aspects of settling tube systems for fall velocity analysis of sand are reviewed, with emphasize being placed on detection methods for particle arrival at the lower end of the settling column and the various sources of errors. This is followed by a detailed discussion on the development of an improved settling tube system employing an underwater balance.

In order to achieve a high degree of accuracy, stability and damping, the weighing system includes a feedback mechanism which consists of a solenoid-magnet unit and a differentiator; this yields a resolution of 10 μN. The overall accuracy of fall velocity is better than 3% (which takes into consideration concentration effect) and the precision is better than 4% (at a confidence level of 95%). Data acquisition and processing are performed by means of a microcomputer with a 12-bit A/D converter, sampling in the velocity domain.
1. **INTRODUCTION**

A settling tube is a device to determine the fall velocity distribution of particles in a sample. If particles are initially positioned at a common level near the top of a settling tube and, after being released, are stratified in the sedimentation fluid according to their respective settling velocities, then this type of analysis is described as layer (or line-start) method (Brezina 1972; Allen 1981). In sedimentology and hydraulic engineering the layer method is often applied for fall velocity analysis of sand, using water as sedimentation fluid.

At the lower end of the settling tube the arrival of the falling particles is sensed; thus the frequency distribution of the sample as a function of settling duration is obtained. It can be a distribution in terms of number of particles, particle volume, particle weight or projected particle area, depending upon the detection method used to determine particle arrival. The frequency distribution based on settling duration is converted into the fall velocity distribution using the settling height of the particles.

This paper described a settling tube system in which an improved detection method for particle arrival is used, i.e. an underwater balance with a feedback mechanism. Following a brief review of general design aspects, the choice of the improved detection method and its advantages over existing methods are examined. The incorporation of a feedback mechanism in the underwater balance results in an improvement of drift, noise and non-linearity by a factor 160 and an improvement of the reaction time of the weighing system by a factor 13. Also described briefly are the construction and the performance of the settling tube system.
It is intended not only to report that a high-accuracy instrument has been developed, but also to underline some general notions about design and description of a settling tube system.

Firstly, a proper choice of settling tube dimensions and of particle detection method requires an analysis of the errors due to sample release mechanism, particle interaction during settling and particle detection method.

The second notion worth underlining concerns the detection method. The quality of this subsystem is best described by quantities that are also used in customary theory of mechanical-electrical systems. It is not enough to analyze sediment samples, to convert the measured fall velocity distributions to size distributions and to compare these with corresponding size distributions obtained otherwise.
2. GENERAL DESIGN ASPECTS

In this paper sand denotes sediment particles having a density of about 2650 kg/m³ and a size which ranges between 0.06 and 2.0 mm, corresponding to a fall velocity in water between 3 and 300 mm/s. Often fall velocity is calculated from sieve data on particle size using experimental data on drag and an assumption on particle shape. However, size measured by wire cloth sieves is not a well defined property of irregular shaped grains. When a high accuracy of the fall velocity is required, it is better to apply a method that makes use of the fall velocity itself, i.e. settling tube analysis.

The choice of an appropriate detection method for particle arrival is a most important step in the design procedure of a settling tube system. Previously used methods vary substantially with reference to performance, cost and ease of operation. In order to judge whether the design criteria of accuracy and precision are satisfied, all systematic and random errors must be quantified.

2.1 Detection methods

Volume. - In the volumetric method, which is the oldest and the simplest, falling particles are deposited in an accumulation section at the lower end of the settling tube. The level of the top of the accumulating sediment is recorded visually, often using optical and mechanical aids. To achieve sufficient resolution the accumulation section has an inner diameter which is considerably smaller than that of the settling tube. The application of this method has been described by van Veen (1936), Emery (1938), Coiby & Christensen (1956), Poole (1957), Rukavina & Duncan (1970) and Vanoni (1975).
Weight. - In this method the weight of the particles that accumulate on a pan at the lower end of the settling tube is recorded as a function of time. The pan is suspended by wire, connected to a balance at the top of the settling tube (Doeglas & Brezesinska Smithuysen 1941; Doeglas 1946; Plankeel 1962; Biene et al. 1965; Sengupta & Veenstra 1968; Gibbs 1972; Rigler et al. 1981). The sediment weight can also be measured by means of strain gauges on a cantilever from which the pan is suspended (van Andel 1964; Felix 1969; Thiede et al. 1976; Flemming 1976; Flemming & Thum 1978; Anderson & Kurz 1979). Brezina (1972a, 1980) applies an underwater balance in which the pan is supported by leaf springs and the movement of the pan is measured by inductive displacement transducers.

Pressure. - A suspension of particles in a settling tube causes a piezometric head at a certain level that differs from the head in the case of clear water, the difference being directly proportional to the submerged weight of the particles that are present above the level of measurement. This weight decreases in compliance with the fall velocity distribution. The pressure method was first used by Mason (1949), and more recently by Zeigler et al. (1960), Schlee (1966), Brezina (1969), Channon (1971), Sanford & Swift (1971), Nelson (1976) and Daskaviraj (1981).

Photo extinction. - Till now this type of measurement, in which the interruption of a light beam by falling particles is sensed, is seldom applied to the analysis of sand-size sediment. It has been used by Taira & Scholle (1977).
2.2 Errors

The accuracy (systematic error) and the precision (random error) of the measured fall velocity distribution depend on several factors. These are:

i) initial position and initial velocity of the particles (sample introduction method),

ii) particle behaviour in the settling tube (wall effect, concentration effect),

iii) quality of the detection method for particle arrival at the lower end of the settling tube.

Errors due to the first factor can be significantly reduced by augmenting the length of the settling tube. Errors due to the second factor can be diminished by reduction of the average sediment concentration $c_1$ (the ratio between total particle volume and settling tube volume). However, if the required reduction can be achieved only by means of a smaller sample volume, then both the risk that the sample is not representative and the amount of labour involved in sample splitting increase. Besides, the demands on the accuracy of the detection method increase for small sample volumes. Because the errors depending on the second factor and the third factor are dominant they are examined more closely.

Concentration effect. - It has been shown that the error due to concentration effects increases with decreasing particle size and with increasing average sediment concentration $c_1$ (e.g. Brezina 1972a; Gibbs 1972; Kranenburg & Geldof 1974; Taira & Scholle 1977). It may in addition also depend upon the standard deviation of the particle size, the ratio between diameter and length of the settling tube and the temperature of the sedimentation fluid. A general analytic expression for the error as a function of these quantities is not known; some empirical results are discussed
later.

For the errors due to the detection method general analytic expressions can be given. These expressions are relevant to all four detection methods, although the notation is based on the weight method (some quantities are given in terms of weight).

Reaction time. - The relative error due to the reaction time of the measuring system is defined as the ratio between the delay time of the detection method (i.e. the time lag between input and output) and the settling duration of the particles \((L/w)\). For the weight method, however, the reaction time differs from the delay time \(\gamma_0\) because of a dynamic effect, viz. particle impact on the balance pan. This dynamic effect is estimated at \(-w/g\) (Appendix A). Then the relative error \(f_r\) due to the reaction time is given by

\[
f_r = \frac{w}{L} \left( \gamma_0 - \frac{w}{g} \right) \tag{1}
\]

in which \(g\) = acceleration of gravity,
\(L\) = length of settling tube,
\(w\) = fall velocity of particles,
\(\gamma_0\) = delay time of weighing system.

Equation 1 shows that \(f_r\) equals zero if \(w = g\gamma_0\). In the range \(0 < w < \frac{1}{2} (1 + \sqrt{2}) g\gamma_0\) the relative error \(f_r\) is less than \(g\gamma_0^2/4L\). Hence \(f_r\) can be optimized by adjustment of \(\gamma_0\).

Drift. - Aging of various components of the detection mechanism and temperature fluctuations give rise to drift in the output signal. The relative error \(f_d\) due to drift is described as

\[
f_d = \frac{\delta_1}{\dot{W} \dot{W}} \tag{2}
\]
in which $W$ = sample weight,
$\delta$ = drift expressed as equivalent sediment weight per unit of time.

Noise. - Noise in the output signal is induced by mechanical vibrations and electronics. It causes a relative error $f_n$ given by

$$f_n = \frac{\sigma_n}{W}$$

(3)

in which $\sigma_n$ = standard deviation of noise, expressed as equivalent sediment weight.

Non-linearity. - The non-linearity of the relation between sediment weight and output signal depends on the non-linearity of the components used in the detection mechanism. Defining $\Delta W$ as the maximum difference (in terms of sediment weight) between the actual, non-linear relation and its linear approximation, the expression for the relative error $f_1$ due to non-linearity reads

$$f_1 = \frac{\Delta W}{W}$$

(4)

For an existing settling tube system these general relations can be used to estimate the total error of the measured fall velocity. Alternatively, when it is necessary to design a settling tube system (in which sediment with a given range of characteristics can be analyzed with a desired accuracy and precision) then Equations 1 to 4 and an expression for the error due to concentration effect can be used to specify both the settling tube dimensions and the requirements which should be met by the detection method.
In addition, the choice of a detection method will also depend upon factors like suitability for routine analysis (nature of work by operator, data processing), requirements on operating conditions (temperature, electricity, mechanical disturbances) and cost.
3. SETTLING TUBE SYSTEM

3.1 Choice of detection method

At the very outset of the development of the Delft University Settling Tube (DUST for short), the volumetric method was discarded despite some obvious advantages, such as low cost and fewer requirements to operating conditions. There are three main reasons for rejecting the volumetric method. Firstly, the application of a relatively long and narrow accumulation section seems inconsistent with the attempt to reduce concentration effects on particle velocity. Secondly, the volumetric method requires the continuous attention of an operator during settling. Thirdly, the results of the analysis are burdened with human errors that are unpredictable.

The choice between the three remaining measuring principles is determined primarily by which method has the least drift and noise. With regard to noise, the optical method has the advantage of being insensitive to mechanical vibrations. However, it yields a fall velocity distribution in terms of projected particle area which is not a well defined quantity in case of particles with irregular shape. Furthermore, information based on particle volume is of immediate relevance to sediment transport technology. Hence weight and pressure measurements seem more appropriate.

Available descriptions of the pressure method in general do not include specifications of drift and noise. From some measurements by the authors using a standard differential pressure transducer it appears that drift disqualifies the pressure method. This applies especially to fine and very fine sand for which the samples have to be small due to the
concentration effect. In case of coarse-grained sand the pressure signal shows deviations from the static value; so far as known this dynamic effect has not been investigated (note the analogy with the particle impact for the weight method).

In contrast to the pressure method, the weight method has the advantage that feedback can be used. In general a feedback mechanism greatly improves the performance of a system which becomes evident in its response, stability and accuracy. Then the performance of such a system is mainly determined by the components in the feedback mechanism which have to be accurate, linear and stable.

Often the weighing pan is suspended by threads from the top of the settling tube. Then vibrations of the threads will raise the noise level. These vibrations may arise from particle impact on the pan and from external mechanical disturbances (caused by engines, traffic, etc.). Hence a different suspension method is preferred.

The DUST system described in this paper has an underwater balance including a feedback mechanism.

3.2 System configuration

The DUST system is shown in Figure 1. The acrylic tube has an inner diameter of 0.175 m and a length of 1.65 m. Hence the effective volume of the sedimentation fluid equals 39.7 litres. The overall height of the system, including the platform, is 2.82 m. The balance housing is an acrylic block with a cavity which is almost spherical. This near spherical construction improves the reduction of vibrations of the weighing pan induced via the walls of the housing. The settling tube system is replaced on a heavy platform with four air springs for
Fig. 1.— Schematic diagram of Delft University Settling Tube system; 1. sample introduction device, 2. settling tube, 3. housing of underwater balance, 4. weighing pan, 5. heavy platform, 6. air spring, 7. glycerine.

additional reduction of mechanical vibrations.

The dry weight of the sediment on the balance pan must not exceed about 0.7 N. When its limit is reached the pan can be cleaned by rotating the balance. The sand particless are
collected in the funnel beneath the balance housing and are discharged by opening the tap at the bottom.

3.3 Underwater balance with feedback

The critical feature of the DUST system is the underwater balance (Fig. 2). It consists of:

i) a weighing pan with an air chamber to provide buoyancy,
ii) a special construction of springs only allowing axial displacement of the weighing pan,
iii) two inductive transducers to measure displacements,
iv) a solenoid-magnet unit for feedback and taring.

The suspension of the weighing pan resembles the construction described in the patent registered by Brezina (1972b). The DUST balance, however, has coupling rods between the springs; these rods diminish the sensitivity of the balance to vibrations.

The weighing system must have a fast, critically damped response; drift, noise and non-linearity must be small. In order to meet these requirements the balance is provided with a solenoid-magnet unit with which feedback is applied (Fig. 3). The differentiator in the feedback section is used to stabilize the system; neither oscillations nor overshoot occur when the system is critically damped. The imperfections of the balance are reduced by the factor (see Appendix B)

\[ r = 1 + \frac{A}{C} \]  

(5)

where \( A \) = amplification factor transducer-Wheatstone bridge,  
\( C \) = spring constant,  
\( G \) = transfer function driver-solenoid system.
Fig. 2.—Underwater balance; 1. weighing pan, 2. vertical axis, 3. leaf springs, 4. coupling rod, 5. inductive displacement transducer, 6. sensing plate, 7. magnet, 8. solenoid support.
Fig. 3.-Block diagram of weighing system including feedback section.

For the DUST underwater balance the factor $r$ equals 160, so both the internally generated noise and drift and the non-linearity (arising between the points $P_1$ and $P_2$, see Fig. 3) are substantially reduced. However, the influence of external disturbances will not decrease through application of feedback.

Fluctuation of the water temperature causes a change of the buoyancy of the weighing pan. Therefore such fluctuation is an external source of drift and it must be prevented.

Mechanical vibrations induced on the weighing pan, e.g. via the ground, can be considered as externally generated noise. A proper construction of the balance housing and a platform on air springs, damped in glycerine, substantially reduce the sensitivity to mechanical vibrations.

Another important characteristic of the balance is its natural frequency $\omega_0$, which is given by

$$\omega_0 = \sqrt{\frac{C}{M}}$$

(6)
in which $M = \text{inertial mass of the balance.}$

For a fast responding system the inertial mass $M$ must be kept to a minimum. However, $M$ is determined not only by the mass of the moving parts of the balance, but also by the virtual mass due to the acceleration in water. The virtual mass $M_v$ of a pan with diameter $D$ can be estimated roughly by

$$M_v = c \rho D^3$$

(7)

in which $c = \text{constant,}$

$\rho = \text{density of water.}$

The constant $c$ has been measured with a test version of the DUST system ($D = 0.12 \text{ m}$) and is about 1. In the present system the balance pan has a diameter of 0.186 m, so Equation 7 yields an estimated virtual mass of 6.4 kg. Since the moving parts of the DUST balance have a mass of about 0.8 kg, it is evident that the total inertia mass of the balance is primarily determined by the virtual mass. This is corroborated by measurements. Since the spring constant $C$ equals 170 N/m and the natural frequency $\omega_0$ (without feedback) equals 4.6 rad/s, the inertial mass of the DUST balance turns out to be 8 kg. (Table 1).

From the natural frequency $\omega_0$, which indicates how fast a system will respond, a more appropriate quantity can be derived, i.e. the delay time $\gamma_0$ of the balance (Appendix B). Feedback reduced the delay time by $\sqrt{\tau}$, which equals 13 for the DUST balance.

Usually, the natural damping of the balance, which is due to internal friction in springs and water, is too small, so the balance will show an oscillatory response to a step-wise input signal. The quality factor $Q$ is a measure for the damping (Appendix B). If $Q > 0.5$ then the output signal of the balance will have overshoot or oscillations. A differentiator in the
feedback circuit (Fig. 3) provides the possibility to influence the damping of the balance. By adjusting the time constant of the differentiator the damping can be made critical ($Q = 0.5$; fastest response without overshoot or oscillation).

The presence of the solenoid-magnet unit in the DUST balance provides two additional advantages. Firstly, the balance can be tared electronically. The maximum taring capacity of the balance is about 0.7 N dry weight, which is equivalent with a mass of 70 g. Secondly, the response of the balance can be checked and adjusted by means of an electronic signal (a square wave for the damping and a triangular wave for the delay time). The response of the balance, both without feedback and with feedback (critically damped), is shown in Figure 4a, b. Figure 4c shows a check on the delay time.

Measurements with an accuracy of 0.1% do not exhibit any deviation from linearity between full scale input and output, hence non-linearity is less than 0.1% F.S.

Table 1 summarizes the characteristics of the weighing system of the DUST.
Effective inertial mass\(^+\)  \( M = 8 \) kg
Spring constant  \( C = 170 \) N/m
Coefficient of natural damping  \( k = 5.3 \) Ns/m
Amplification factor  \( A = 1.8 \times 10^6 \) V/m
Natural frequency (no feedback)  \( \omega_o = 4.6 \) rad/s
Quality factor (no feedback)  \( Q = 7 \)
Natural frequency (feedback)  \( \omega_{of} = 58 \) rad/s
Quality factor (feedback)  \( Q_f = 0.5 \)
Transfer function solenoid  \( G = 15 \times 10^{-3} \) N/V
Reduction factor  \( r = 160 \)
Time constant of differentiator  \( \tau = 35 \times 10^{-1} \) s
Delay time  \( \gamma_o = 35 \times 10^{-3} \) s
Non-linearity (feedback)  \( \Delta W/W < 0.1 \) % F.S.
Short-term drift (feedback, 4 minutes)  \( \delta = 2 \times 10^{-b} \) N/s
Long-term drift (feedback, 1 hour)  \( \delta = 4 \times 10^{-6} \) N/s

\(^+\) The effective inertial mass is composed of the mass of the balance pan and the magnet and of the virtual mass in consequence of the acceleration in water.

Table 1.—Characteristics of the DUST weighing system.
3.4 **Sample introduction mechanism**

The introduction mechanism at the top of the settling tube is of the venetian blind type with 6 mm wide, rotating lamellae (Fig. 5; also see Brezina (1969, 1980)). When the lamellae are closed, sediment is spread evenly over the lamellae; this effectively minimizes concentration effects. The sample is released by activating a solenoid that opens the lamellae. Subsequently, the lamellae vibrate during a certain time (adjustable between 0 and 10 s) to ensure the release of all
particles.

Ideally, the initial position of all particles is in the same horizontal plane (homogeneous distribution) and the initial particle velocities equal the terminal fall velocities. In practice this ideal situation can only be approximated. It is required now that the distance between the introduction device and the position where a particle attains its terminal velocity is small when compared to the length of the settling tube. This means that the width of the lamellae should be small in comparison with the settling tube length, i.e. small differences between initial vertical positions. In the DUST settling tube the error due to the differences between initial and terminal particle velocity is smaller than 1%. However, for particles larger than 2 mm or a settling tube length smaller than 1.65 m this error may be larger. The concave shape of the upper side of the lamellae contributes to the reduction of the error due to the velocity difference.

Fig. 5.—Sample introduction device.
3.5 Platform with air springs

Mechanical vibrations distort measurements made with the underwater balance. Since the resolution of the balance is of the order of 10 μN, which corresponds with displacements of the order of 10 nm, mechanical vibrations have to be kept at a minimum. A concrete platform placed on four air springs reduces the vibrations induced via the ground. The cut-off frequency of the air springs is 3.5 Hz (independent of the inertial mass of the platform), hence vibrations with a frequency above 3.5 Hz will be reduced with 12 dB/octave.

A disadvantage of air springs is the presence of a resonance peak since air springs are underdamped (Q = 10). This means that frequencies around 3.5 Hz will be amplified by a factor of 10. Nevertheless, air springs are preferred to metal springs because the amplitude of the resonance peak of the latter is 50 times larger than that of the former (which gives rise to a 50 times larger gain of ground vibrations at the resonance frequency).

To reduce the resonance peak, the platform is placed in a container filled with a viscous fluid (glycerine). By adjusting the distance between the underside of the platform and the bottom of the container the damping can be varied. Although this damping construction is not ideal, the overall reduction of the vibrations is about 30 dB RMS-value measured in full band width.
4. **Performance**

The precision of the DUST system is determined by drift and noise. The accuracy is limited by the concentration effect due to hindered settling and settling convection. During the measurements the water temperature should be constant since any fluctuation will cause a change of the buoyancy of the weighing pan (drift) as well as a variation in the viscosity of the water, which affects particle velocity.

Errors due to the three phenomena mentioned above will be discussed. The errors are divided into a systematic part and a random part, i.e. accuracy and precision, respectively. If the systematic error is known, it can be corrected; for the random error this is impossible. However, the latter can be reduced by averaging repetitive measurements.

4.1 **Drift and noise**

The drift during the measurement determines the minimum sample weight that can be measured for a given precision (see Eq. 2). During 4 minutes, i.e. the settling time of 0.1 mm particles, the drift turns out to be less than 2 μN. For instance, if the required precision is 1% then the minimum dry weight of a sample of 0.1 mm particles will be 200 μN. For larger particles the influence of drift lessens because settling time decreases.

The other limiting factor is the noise level in the output signal due to electronic noise and mechanical vibration. A proper design of the electronics will make the contribution of the electronic noise sufficiently small; in this connection the application of feedback is recalled. Mechanical vibrations will be reduced by the platform on air springs damped in glycerine. In general the noise should be negligible compared to the level of
the output signal that corresponds to the sample weight (see also Eq. 3).

For small samples an additional reduction of the noise induced by mechanical vibrations may be necessary. This can be accomplished by means of a low-pass electronic filter. Then, however, the delay time of the system increases. In consequence, the maximum particle velocity (diameter) that can be measured with the required precision decreases.

In order to improve the signal to noise ratio (Eq. 3) and to minimize the error due to the reaction time (Eq. 1) the DUST system included an adjustable low-pass filter. As a result the delay time can be varied between 56 ms and 2 s.

In general the larger the sample the greater the signal to noise ratio (i.e. the precision) will be. However, for very large samples the accuracy of the measurements is reduced because of the concentration effect.

4.2 Concentration effect

Due to settling convection and hindered settling the fall velocity of particles in a sample will be different from the velocity of free falling particles (e.g. Brezina 1972a; Kranenburg & Geldof 1974). Results of some preliminary measurements on these concentration effects are presented. In the measurements three sieved fractions of sand (nominal sieve aperture ratio \( \sqrt{2} : 1 \)) have been used, each with four different sample weights. For each combination of sieve fraction and sample weight five analyses have been run, yielding values for the mean fall velocity \( \bar{\bar{w}} \) (Table 2). For each series of five analyses the average value of \( \bar{\bar{w}} \) and the standard deviation, indicated by \( \bar{\bar{w}} \sigma \) and \( \sigma_\sigma \) respectively, have been calculated; the latter is a measure for the precision of \( \bar{\bar{w}} \). Using Student's t-distribution with a 95% confidence interval
<table>
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<tr>
<th>sample weight W (N)</th>
<th>mean settling velocity ( \bar{w} ) (mm/s) of sample no.</th>
<th>average velocity ( \bar{w}_5 ) (mm/s)</th>
<th>standard deviation ( \sigma_5 ) (mm/s)</th>
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<tr>
<th>sample weight W (N)</th>
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Table 2.—Experimental results of mean fall velocity of sieved sand particles settling in water (\( \theta = 19^\circ C \)).
(t_{0.975} = 2.776) the precision of the mean fall velocity turns out to be better than 4%.

Figure 6 shows the variation of \( \bar{w}_5 \) and its confidence interval with sample weight \( \bar{w} \) for each sieve fraction. Extrapolation of the curve to \( \bar{w} = 0 \) gives the fall velocity of the sieve fraction when concentration effects are absent. Due to the scatter of particle size within a sieve fraction the fall velocity has to be interpreted as the average of the fall velocity of each single particle in the fraction. The scatter of particle size within a sieve fraction is relatively small, hence the relationship between fall velocity and particle size is assumed to be linear within a fraction. Then the fall velocity found by extrapolation (\( \bar{w}_0 \)) can also be interpreted as the fall velocity of a single particle with a size that is equal to the mean size of the particles in the sieve fraction.

For each sieve fraction the variation in the relative error \( \varepsilon \), which is defined as \( (\bar{w}_5 - \bar{w}_0)/\bar{w}_0 \), with sample weight \( \bar{w} \) or average concentration \( c_1 \) is shown in Figure 7. It appears that for a given sample weight the magnitude of \( \varepsilon \) depends on particle size, which is in accord with the findings of Gibbs (1972) and Taira & Scholle (1977), and with the results of some measurements in a test version of DUST. The results given in Table 2 and Figures 6 and 7 are based on a limited number of measurements; for a more critical analysis of concentration effects a comprehensive series of measurements is presently being analyzed.
Fig. 6. - Mean velocity versus sample weight. Water temperature 19°C. a. d = 125-150 m, b. d = 350-420 m, c. d = 710-850 m.
Fig. 7.-Error due to concentration effect versus sample weight for three sieve fractions. Water temperature 19°C.

4.3 **Sample size**

The sample weight used in an analysis is a compromise between the error $\epsilon$ due to concentration effect and the signal to noise ratio SNR. The relationship between sample weight $W$ and particle size $d$ is shown in Figure 8 for two values of $\epsilon$ (viz. 1% and 5%) and for two values of SNR (viz. 72 dB and 60 dB, i.e. 4000 and 1000, respectively). In the measurements on which this relationship is based the delay time $\tau_0$ was set to 1% of the settling time which varies with particle diameter.

When a sample with an estimated mean particle size is to be analyzed, first the maximum acceptable value of $\epsilon$ and the minimum acceptable value of SNR are chosen. Then the curves of Figure 8 can be used to estimate the range of sample weight which satisfies both requirements. For particles larger than about 0.1 mm it is possible to choose the sample weight such that the concentration
effect $\varepsilon$ is less than 1\% and the signal to noise ratio is better than 72 dB. For coarse to very coarse sand the upper limit of the sample range is not determined by the chosen value of $\varepsilon$ but is determined by the weighing capacity of the underwater balance (0.2 N).

Fig. 8.-Diagram for selection of appropriate sample weight. $\gamma_0$ is equal to 1\% of settling time.

Besides the systematic error due to the concentration effect there are also systematic errors caused by the delay time and particle impact, the introduction mechanism and the non-linearity of the weighing system. Excluding concentration effects the total systematic error of the DUST system is about 2\%.

In general the error due to drift is relatively large for small particles (i.e. long settling times) as well as for small sample weights. Since the recommended sample weight for 0.1 mm particles is 1.5 mN (in case of $\varepsilon = 1\%$ and SNR = 72 dB, see Fig. 8) a drift of 2 $\mu$N gives rise to an error which is less than 0.2\%. So even for a most unfavourable combination of particle size and sample weight the error due to drift is very small.
4.4 Influence of water temperature

During a sediment analysis the water temperature should be kept constant since temperature fluctuation will cause a change of buoyancy of the weighing pan and a change of the fall velocity of the particles.

Since the relation between the density of water and water temperature is non-linear, there exists an optimum value of the water temperature at which the variation in the buoyancy is virtually zero. In the DUST system the weighing pan is made of hard PVC (expansion coefficient $2.4 \times 10^{-4}/^\circ C$) and the optimum temperature is about $23^\circ C$. A few degrees below and above this optimum the variation in the buoyancy is of the order of $+0.1$ and $-0.1$ mN/°C, respectively.

In the temperature range of $16^\circ C$ to $30^\circ C$ the rate of change of buoyancy varies between $+0.75$ mN/°C and $-0.75$ mN/°C. Assuming temperature to change $2^\circ C$ in 8 hours (ordinary laboratory conditions) the maximum drift during 4 minutes will be about $13 \frac{mN}{m}$. Hence for 0.1 mm particles with a sample weight of 1.5 mN the error due to temperature fluctuation is less than 1% within the temperature range between $16^\circ C$ and $30^\circ C$. Beyond this range a temperature control system may be necessary, depending upon the requirements on accuracy and precision.

The change of fall velocity resulting from temperature variation is primarily brought about by viscosity, the density variation of sand and water being negligible. For 2 mm particles viscosity causes the fall velocity to change $0.5%/^\circ C$, for 0.1 mm particles the velocity varies $2%/^\circ C$ (at a water temperature of $20^\circ C$). Assuming, as before, temperature to vary $0.02^\circ C$ during 4 minutes the fall velocity of 0.1 mm particles changes about $0.04\%$, which is of course negligible.
More important is the effect of a vertical gradient of water temperature in the settling tube. Without precautions the temperature difference between top and bottom of the tube may run to 1°C which gives rise to an uncertainty of the order of 1% in the settling velocity.

In summary, provided that sample weight exceeds a certain minimum (see Fig. 8), temperature effects can be neglected if a) the variation of water temperature is limited to 2°C in 8 hours, b) the water temperature is in the range of 16°C to 30°C, and c) vertical temperature gradients are kept to a minimum by mixing occasionally.
5. **Data Acquisition and processing**

The output signal of the underwater balance is the (apparent) weight of the sand particles resting on the weighing pan as a function of time. For the conversion of the resulting distribution of settling duration to the required fall velocity distribution a microcomputer system, including a 12-bit AD converter, is connected to the settling tube. By means of a programmable timer the output is sampled at equal velocity intervals.

The input parameters for the sampling program are the settling tube length $L$, the water temperature $\theta$, the maximum and minimum fall velocity to be expected in the sample ($w_{\text{max}}$ and $w_{\text{min}}$, respectively), and the required velocity sampling interval. Sampling is started $L/w_{\text{max}}$ seconds after release and stopped at the time $L/w_{\text{min}}$. Meanwhile the data points are plotted on a monitor screen which gives the opportunity for an almost simultaneous check. Data can be stored permanently on a floppy disk. In addition, a hard copy of the velocity distribution (cumulative and/or density curve) and of its mean, standard deviation and skewness can be provided.

The results of the analysis can also be expressed in terms of particle size. For the conversion of fall velocity to either nominal diameter or sedimentation diameter a general and accurate analytic expression is used (Slot 1984; in this reference an incorrect numerical value for the constant $C_0$ is mentioned, $C_0$ should be 0.0125).
6. Conclusions

A settling tube system with an underwater balance utilizing a feedback mechanism is a suitable device for measuring the fall velocity distribution of sand particles settling in water. The fall velocity is measured with an accuracy better than 2%. The sample weight has to be chosen such that both the error due to concentration effects and the signal to noise ratio are acceptable. It turns out that even for 0.1 mm particle it is possible to keep the error due to concentration effects down to 1% and to attain a signal to noise ratio of 72 dB. Then the overall accuracy (i.e. the systematic part of the total error) is better than 3%.

The precision, which is the random part of the total error, appears to be better than 4% at a 95% confidence level; if required the precision can be improved by averaging repetitive measurements.

The system includes a microcomputer which facilitates signal processing and calculation of the fall velocity and size distribution of the particles.
7. **Acknowledgements**

We wish to thank Mr. A.M. den Toom who has been particularly helpful in finding practical solutions to design problems. For the construction of the mechanical components of the DUST-system we are indebted to Mr. A. van Gent and his colleagues of the workshop of the Department of Civil Engineering.
Appendix A. Particle Impact

The output signal of the balance is determined both by the weight and by the impact of the particles. The impact of a single particle causes a momentary increase of the output signal with respect to its static level due to the particle weight. The impact of many particles gives rise to a persistent upward shift of the output signal. This mean effect can be interpreted as a change in the time lag between input and output signal. It is calculated assuming that

i) particle arrival time is Poisson distributed with intensity parameter $\lambda$,

ii) the number of particle impacts per second $\lambda$ is constant in the time interval $0 < t < T$, beyond this interval $\lambda$ equals zero,

iii) particle impact on the balance pan is perfectly inelastic,

iv) each particle has the same mass $m$, while $m \ll M$ (the inertial mass of the balance).

The expression $E(x(t))$ of the displacement $x(t)$ of a critically damped balance with delay time $\gamma_0 (= 2/\omega_o)$ is then given by

$$
\frac{E(x(t))}{\alpha \lambda} = \frac{\omega}{g} - \gamma_o + t \left( \frac{\omega}{g} - \gamma_o + \left( \frac{2\omega}{g\gamma_0} - 1 \right) t \right) \exp \left( - \frac{2E}{\gamma_0} \right) \tag{A1}
$$

for $0 \leq t \leq T$

and

$$
\frac{E(x(t))}{\alpha \lambda} = \left( \frac{\omega}{g} - \gamma_o + \left( \frac{2\omega}{g\gamma_0} - 1 \right) (t - T) \right) \exp \left( - \frac{2}{\gamma_0} (t - T) \right) + \frac{2\omega}{g\gamma_0} - 1 \right) \left( 2t\omega + T \right) \text{ for } t > T \tag{A2}
$$
in which \( a = mg \gamma_o^2/4M \). Figure A1 shows the effect of particle impact when \( \lambda \) is constant. For a perfect balance with \( \gamma_o = 0 \) and for \( w = 0 \), the output coincides with the input. For a real balance with \( \gamma_o > 0 \) the output is shown both without particle impact (\( w = 0 \)) and with particle impact (\( w > 0 \)). In Figure A2 the output signal is shown for various values of \( w \). During the interval \( 3\gamma_o \leq t \leq T \) the effect of the discontinuity in the input at \( t = 0 \) can be neglected and the output runs parallel to the input (with a certain time delay). From equation A1 it follows that in the interval \( 3\gamma_o \leq t \leq T \)

\[
\frac{E(x(t))}{a\lambda} = \frac{w}{\lambda} - \gamma_o + t \tag{A3}
\]

Therefore the reaction time, which is the sum of delay time and time effect of particle impact, equals \( \gamma_o + \frac{w}{\lambda} \). Since the actual deposition of particles on the balance pan proceeds gradually (no discontinuities), it is allowed to approximate the output signal by straight line segments and to neglect the effect of the discontinuities at \( t = 0 \) and \( t = T \).

---

**Fig. A1.** Effect of particle impact.
Fig. A2.-Effect of particle impact for various values of \( w \).
Appendix B. Effect of Feedback

Transfer function.— A balance is a second-order system. Its transfer function $H(\omega)$ reads (Fig. 3)

$$H(\omega) = \frac{A B(\omega)}{A (-M\omega^2 + jk\omega + C)^{-1}}$$  \hspace{1cm} (B1)

in which $A = \text{amplification factor transducer-Wheatstone bridge}$,
$B = \text{transfer function of balance}$,
$C = \text{spring constant}$,
$j = \sqrt{-1}$,
$k = \text{coefficient of natural damping}$,
$M = \text{inertial mass of balance}$.

The feedback circuit has a transfer function $B(\omega)$ given by

$$B(\omega) = G (1 + \omega^2)$$  \hspace{1cm} (B2)

in which $G = \text{transfer function of driver-solenoid system}$,
$\tau = \text{time constant of differentiator}$.

When the feedback circuit is included in the balance, then the transfer function $H_x(\omega)$ of the weighing system becomes

$$H_x(\omega) = \frac{-M\omega^2 + j(G\tau + k)\omega + G + \frac{C}{A}}{A}$$  \hspace{1cm} (B3)

Reduction factor.— Feedback reduces the influence of all internally generated parasitic phenomena such as drift, noise and non-linearity. The extent of this positive effect can be demonstrated as follows.

A deviation $\Delta H(\omega)$ of the transfer function $H(\omega)$ of the balance without feedback mechanism (due to temperature
fluctuation, aging or otherwise) causes a deviation $\Delta H_F(\omega)$ of the transfer function of the balance with feedback. The deviation $\Delta H_F(\omega)$ is given by

$$\frac{\Delta H_F(\omega)}{H_F(\omega)} = \left| \frac{1}{1 + R(\omega)H(\omega)} \right| \frac{\Delta H(\omega)}{H(\omega)} \quad (B4)$$

Equation B4 shows that the influence of parasitic properties of the balance is reduced by a factor $r$ which is defined as

$$r = |1 + \beta(\omega)H(\omega)| \quad (B5)$$

Also Equations B4 and B5 show that the application of feedback is meaningful only if $|S(\omega)H(\omega)| \gg 1$.

For slowly changing disturbances with a frequency smaller than the cut-off frequency of $H(\omega)$ the reduction factor $r$ can be calculated for $\omega = 0$. Substitution of Equation B1 and B2 into Equation B5 yields a simple expression for the reduction factor

$$r = 1 + G \frac{A}{C} \quad (B6)$$

For noise the reduction factor must be calculated over the bandwidth of interest (0 ... cut-off frequency of $H_F(\omega)$). However, for frequencies smaller than the cut-off frequency of $H(\omega)$ Equation B6 provides a good approximation.

It is emphasized that the components in the feedback circuit must be at least a factor $r$ more stable and more accurate than the other components of the balance.

Damping.- The quality factor $Q$ is a measure for the damping of a system. The balance without feedback mechanism has a transfer function according to Equation B1 and a quality factor
\[ Q = \frac{1}{k} \sqrt{CM} \]  

(B7)

If \( Q > 0.5 \) then the system will show oscillation or overshoot in its response to a step function and its amplitude spectrum will have a peak at the natural frequency \( \omega_0 \). The magnitude of this peak is \( Q \) times larger than the amplitude for \( \omega = 0 \). If \( Q = 0.5 \), then the system is critically damped and it will have the fastest response without overshoot.

By means of a differentiator in the feedback section (Fig. 3) the response can be made critical. For a system with feedback the expression for the quality factor \( Q_f \) can be derived from Equation B3. It reads

\[ Q_f = \frac{1}{\frac{1}{\tau} + \frac{k}{A}} \sqrt{\frac{A}{A} \left( \frac{C}{A} + G \right)} = \frac{\sqrt{1 + \frac{AG}{C}}}{1 + \frac{AG}{k}} Q \]  

(B8)

By adjustment of the time constant \( \tau \) the quality factor \( Q_f \) can be made equal to 0.5.

Delay time.— Every physical system has a certain delay in its response to an input. Often the undamped natural frequency \( \omega_0 \) is used to describe this delay. For a balance without feedback \( \omega_0 \) follows from

\[ \omega_0 = \sqrt{\frac{C}{M}} \]  

(B9)

A balance with feedback has an undamped natural frequency \( \omega_{0f} \) given by

\[ \omega_{0f} = \sqrt{\frac{A}{M} \left( \frac{G}{A} + \frac{C}{A} \right)} = \sqrt{1 + \frac{AG}{C}} \omega_0 \]  

(B10)

For a settling tube system the delay time \( \gamma_0 \) is a more appropriate quantity to describe the delay of the response. It is defined as
the time lag between the output of the system and the input when the input signal is changing linearly. For a second order system the delay time follows from

$$\gamma_o = \frac{1}{Q \omega_o}$$  \hspace{1cm} (B11)

In order to estimate the influence of feedback on the delay time, it is assumed that the balance without feedback mechanism is also critically damped. Then it is found that the delay time $\gamma_{of}$ of the balance with feedback mechanism is given by

$$\gamma_{of} = \frac{\gamma_o}{\sqrt{1 + AG/C}}$$  \hspace{1cm} (B12)

Hence a feedback mechanism will reduce the delay time by a factor $\sqrt{1 + AG/C}$, which is identical with $\sqrt{\tau}$. 
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