WAVE-INDUCED ANTISYMMETRIC RESPONSE OF A FLEXIBLE SHIP IN AN IRREGULAR SEAWAY

by

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Summary

Calculated results are presented for a flexible container ship travelling at 26 knots in a long crested irregular bow sea. Representative mean square spectral densities are given for lateral shear force, lateral bending moment and twisting moment. The influence of allowing for warping stiffness is examined and comparisons are made with results obtained using conventional theory based on the assumption of hull rigidity.

1. Introduction

Unified dynamic theories have been given for both symmetric [1,2] and antisymmetric [2,3] responses of flexible hulls travelling in a sinusoidal seaway. From such theories it has been shown that the mode shapes and natural frequencies of the dry hull and the resonance frequencies of the wet hull play important parts in determining the form of the responses at any position in the hull. These responses can be found by considering the response amplitude operators (RAO) of the structure when it is excited by regular sinusoidal waves of unit amplitude.

The theory of symmetric response has received the more attention and has been extended so as to encompass transient responses of a hull in waves as well as the spectral responses to a known irregular seaway [2,4,5].

The antisymmetric theory, being the more complicated [6-8] and requiring detailed structural data (which is rarely available), still requires refinement in matters of detail rather than of general approach. The indications are that, for a container ship, results depend very sensitively on the allowance made for warping stiffness [2,3] and it is therefore highly desirable that the allowance for, and data on, warping should be refined.

In this paper, the container ship [2,3] is assumed to travel at 26 knots in bow seas (i.e. with a heading angle of $\chi = 135^\circ$). The antisymmetric response amplitude operators are calculated for lateral shear force, lateral bending moment and twisting moment at different points (i.e. $1/4$, $1/2$, $3/4$) along the hull. Comparisons are made to illustrate the influence of warping. The calculated data are used to derive response spectra for the ship operating in an irregular long-crested seaway. Statistical properties defined in terms of the moments of the individual spectra are thus derived. Where conventional techniques permit it, a further comparison is made between the response spectra and statistics of the flexible hull on the one hand and those based on the assumption of hull rigidity as in conventional analysis on the other.

2. Basic theory

The responses of the flexible hull to regular sinusoidal waves of unit amplitude and frequency of encounter $\omega$ are governed by the equation [2]

$$\ddot{\xi} + A(\omega)\dot{\xi} + B(\omega)\dot{\eta} + C\eta = \pm \omega v \sin \omega t .$$

The square matrices $A$, $B$, $C$ contain terms of both structural and hydrodynamic origin and are neither symmetric nor positive definite; they depend on the frequency of encounter through the hydrodynamic contributions. The hull is idealised as a non-uniform free-free beam and the hydrodynamic data are derived from an appropriate strip-theory [9,10]. The column matrix $\xi(t)$ contains the generalised principal coordinates of the dry hull and $\eta$ is a column matrix of the generalised wave excitation forces due to waves of unit amplitude approaching the ship at some arbitrary heading angle $\chi$ ($=180^\circ$ for head seas). All the matrices are of order $N$, determined by the number of principal coordinates used in the analysis.

The responses of lateral shear force $V(x,t)$, lateral bending moment $M(x,t)$ and twisting moment $T(x,t)$ at any position $x$ from the stern are

$$V(x,t) = \sum_{r=3}^{N} V_r(x)p_r e^{-i\omega t},$$

$$M(x,t) = \sum_{r=3}^{N} M_r(x)p_r e^{-i\omega t},$$

$$T(x,t) = \sum_{r=3}^{N} T_r(x)p_r e^{-i\omega t},$$

where $V_r(x)$, $M_r(x)$, $T_r(x)$ are the characteristic functions of shear force, bending moment and twisting moment respectively corresponding to the $r$th principal mode of the dry hull. It will be noted that

$$V_r(x) = 0 = M_r(x) = T_r(x)$$

for $r = 0$ (sway), $r = 1$ (yaw) and $r = 2$ (roll).
The response amplitude operators of these responses may be derived in the usual manner. Thus the shear force operator associated with the frequency of encounter \( \omega_e \) is

\[
|V(x, \omega_e)| = \left| \sum_{k=1}^{N} V(x) p_k \right|
\]

Similar relationships exist for the other two responses.

3. The seaway and its statistical properties

The mean square spectral density of the shear force response (say) at position \( x \) is given by the familiar input-output relationship of random process theory \([11]\). It is

\[
\phi_{VV}(x, \omega_e) = |V(x, \omega_e)|^2 \phi(\omega_e),
\]

where \( \phi(\omega_e) \) is the wave encounter spectrum of the seaway. We shall use an ITTC wave spectrum, i.e. a spectrum of the form

\[
\phi(\omega) = \frac{A}{\omega^5} \exp\left(-\frac{B}{\omega^4}\right)
\]

where \( A = 8.1 \times 10^{-3} g^2 \), \( B = 3.11 h_{1/3}^4 \) and \( h_{1/3} \) is the significant wave height measured in metres. The corresponding encounter spectrum is

\[
\phi(\omega_e) = \phi(\omega) \left| 1 - \frac{2\bar{U}}{g} \cos x \right|
\]

with

\[
\omega_e = \omega - \frac{\bar{U} \omega^2}{g} \cos x,
\]

where \( \omega \) is the absolute frequency of the waves and \( \bar{U} \) is the mean forward speed of the ship. These spectra – the wave spectrum and the encounter spectrum – are illustrated in Figure 1 for a significant wave height \( h_{1/3} = 5 \) m, \( \bar{U} = 13.38 \) m/s and \( \chi = 135^\circ \).

The \( n \)th statistical moment, \( m_n(x) \), of the shear force response spectrum at position \( x \) is given by

\[
m_n(x) = \frac{1}{\Delta} \int_0^{\Delta} \omega^n \phi_{VV}(x, \omega_e) d\omega_e
\]

for \( n = 0, 1, 2, \ldots \). Various statistical indices may be found on the basis of such moments \([2,11]\). Thus the average period of wave upcrossing is

\[
\bar{T}(x) = 2\pi \sqrt{\frac{m_0(x)/m_2(x)}{m_4(x)/m_6(x)}}
\]

and it will be noted that this quantity varies along the length of the hull.

4. Computations

4.1. General description

The principal dimensions of the container ship to be considered are \([3]\):

\[
L = 281.0 \text{ m}, \quad B = 32.26 \text{ m}, \quad T = 12.2 \text{ m} \quad \text{and} \quad \Delta = 67150 \text{ tonne}.
\]

Results will be presented, giving antisymmetric responses at positions \( x = l/4, l/2, 3l/4 \) measured from the stern of the vessel travelling at 13.38 m/s in long crested bow seas (i.e. with \( \chi = 135^\circ \)).

The hull was divided into 50 sections for both the structural and the hydrodynamic calculations. Two dimensional hydrodynamic properties were determined using multiparameter conformal transformations \([12]\) together with the strip theory of Salvesen, Tuck and Faltinsen \([9]\). The effects of bilge keels and of the rudder were ignored.

In calculations for the dry hull the modal damping factors were taken as

\[
\nu_3 = 0.010, \quad \nu_4 = 0.012, \quad \nu_5 = 0.015, \quad \nu_6 = 0.019, \quad \nu_7 = 0.024, \quad \nu_8 = 0.030, \quad \nu_9 = 0.037
\]

in the absence of any reliable data on the subject.

Calculations were made in which warping stiffness was excluded (so that \( C_1 = 0 \)) and included (\( C_1 \neq 0 \)).
Figure 2. Shear force, transverse bending moment and twisting moment response amplitude operators for $x = 1/4$ in a ship travelling at 13.38 m/s in regular sinusoidal bow waves, i.e. $\chi = 135^\circ$. 
Figure 3. Shear force, transverse bending moment and twisting moment response amplitude operators for $x = 1/2$ in a ship travelling at 13.38 m/s in regular sinusoidal bow waves, i.e. $\chi = 135^\circ$. 

**Diagram a**

**Diagram b**

**Diagram c**
Table 1 illustrates the influence of warping on the dry hull natural frequencies and wet hull resonance frequencies. The highest modal index taken in the calculations of wave induced antisymmetric response was generally $N = 6$, though results are included for which $N = 9$.

A comparison was made between predictions made on the basis of rigid and flexible body theories. This relates to lateral bending moment and twisting moment at the three positions along the hull. The rigid body results were obtained using program UCLARM [1], which is a variant of program SCORES [13].

### 4.2. Response amplitude operators

Figures 2 and 3 illustrate the three response amplitude operators at $x = l/4$ and $l/2$ respectively. The frequency range $\omega_e$ covers the first three resonances for $C_1 \neq 0$ and five resonances for $C_1 = 0$. In these calculations the highest modal index used was $N = 6$. The differences between the predictions for $C_1 = 0$ and $C_1 \neq 0$ are clearly visible especially in the regions of resonance.

The influence of the dry hull mode shapes is shown by the different magnifications at the resonance at the two positions along the hull. For example, the curve for $C_1 \neq 0$ and $x = l/2$ shows a marked resonance of the shear force at $\omega_e = 2.05$ rad/s although no such resonance is apparent at $x = l/4$. This is due to the relative magnitudes of $V_3(l/2)$ and $V_3(l/4)$; whereas $V_3(l/2) \neq 0$, the value of $V_3(l/4)$ is small [3].

Comparison of the results for lateral bending moment based on the assumption of rigidity show limited agreement with those of the dynamical theory outside the regions of wet hull resonance. The discrepancies arise because the former, more rudimentary analysis is unable to account for dynamic effects. A large discrepancy occurs around $\omega_e = 1.0$ rad/s and this may be reduced for the $C_1 \neq 0$ case by increasing the modal summation to $N = 9$ as shown in Figure 4. Although the dry hull natural frequencies and wet hull resonance frequencies of these three additional modes differ greatly from 1.0 rad/s, the modes concerned contribute significantly to the RAO. In fact it has been shown previously [7] that the component $M_3(x)\dot{p}_3(t)$ makes the largest contribution of the additional terms.
and does so because mode \( r = 7 \) has two nodes in both bending and twist and falls in a sequence of twist dominated modes.

A comparison of the results for the response operator of twisting moment with those obtained with the conventional rigid body theory reveals that there is reasonable agreement in the region of roll resonance (\( \omega_r = 0.17 \text{ rad/s} \)) but that large discrepancies then begin to appear. This is especially true in the region of \( \omega_r = 1.0 \text{ rad/s} \), the dominant region of the wave encounter spectrum shown in Figure 1. An increase in the summation index \( N \) proves to have little effect now, either for \( C_1 = 0 \) or \( C_1 \neq 0 \).

The existence of such a discrepancy may be attributed to the low natural frequencies of the hull (see Table 1) and the dominance of twisting in the modes [3,7]; this twisting produces a very substantial contribution to the twisting moment, mainly through the component \( T_3(x)p_3 \). This is something which the rigid body theory is unable to cope with and, since the corresponding natural frequencies are low, the encounter spectrum magnifies its influence very considerably. It is evident that conventional techniques are dangerously optimistic in such circumstances.

### 4.3. Statistical properties

The response spectra in Figures 5 and 6 correspond to the response amplitude operators shown in Figures 2 and 3 respectively, for the sea spectra of Figure 1. The discrepancies discussed previously between predictions based on the flexible and the rigid hull theories are both magnified and distorted by the shape of the wave encounter spectrum. Again the discrepancy in the lateral bending moment response spectra can be reduced to some extent by taking \( N = 9 \) in the modal summation, as shown in Figure 7.

Table 2 gives the root mean square values of the response (i.e. \( \sqrt{m_0} \)), while Table 3 shows the average upcrossing periods at the three positions along the hull.

### Table 2

<table>
<thead>
<tr>
<th>Response</th>
<th>Theory</th>
<th>( l/4 ) ( N=6 )</th>
<th>( l/4 ) ( N=9 )</th>
<th>( l/2 ) ( N=6 )</th>
<th>( l/2 ) ( N=9 )</th>
<th>( 3l/4 ) ( N=6 )</th>
<th>( 3l/4 ) ( N=9 )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Shear force (MN)</strong></td>
<td>( C_1 \neq 0 )</td>
<td>2.03</td>
<td>3.04</td>
<td>1.82</td>
<td>1.97</td>
<td>2.26</td>
<td>2.97</td>
</tr>
<tr>
<td></td>
<td>( C_1 = 0 )</td>
<td>2.88</td>
<td>2.88</td>
<td>2.62</td>
<td>2.65</td>
<td>3.06</td>
<td>3.04</td>
</tr>
<tr>
<td><strong>Twisting moment (MNm)</strong></td>
<td>( C_1 \neq 0 )</td>
<td>62.27</td>
<td>63.42</td>
<td>101.61</td>
<td>103.11</td>
<td>39.10</td>
<td>32.44</td>
</tr>
<tr>
<td></td>
<td>( C_1 = 0 )</td>
<td>92.16</td>
<td>92.16</td>
<td>139.84</td>
<td>140.46</td>
<td>61.10</td>
<td>59.51</td>
</tr>
<tr>
<td>Rigidity</td>
<td>body</td>
<td>19.14</td>
<td>12.25</td>
<td></td>
<td></td>
<td>8.53</td>
<td></td>
</tr>
<tr>
<td><strong>Bending moment (MNm)</strong></td>
<td>( C_1 \neq 0 )</td>
<td>94.22</td>
<td>114.28</td>
<td>167.63</td>
<td>254.42</td>
<td>88.00</td>
<td>123.46</td>
</tr>
<tr>
<td></td>
<td>( C_1 = 0 )</td>
<td>93.47</td>
<td>103.65</td>
<td>235.15</td>
<td>239.52</td>
<td>93.79</td>
<td>95.57</td>
</tr>
<tr>
<td>Rigidity</td>
<td>body</td>
<td>144.90</td>
<td>272.39</td>
<td></td>
<td></td>
<td>118.37</td>
<td></td>
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</table>
Figure 5. Shear force, transverse bending moment and twisting moment spectra for $x = l/4$ in a ship travelling in a long crested bow seaway of significant height $h_{1/3} = 5$ m.
Figure 6. Shear force, transverse bending moment and twisting moment spectra for $x = 1/2$ in a ship travelling in a long crested bow seaway of significant height $h_{1/3} = 5$ m.
Figure 7. Transverse bending moment spectra for $x = \frac{1}{4}$ and $x = \frac{1}{2}$, evaluated taking $N = 9$.

Table 3
Average upcrossing period for the responses (The rigid body results are independent of the modal index $N$)

<table>
<thead>
<tr>
<th></th>
<th>theory</th>
<th>$\frac{1}{4}$</th>
<th>$\frac{1}{2}$</th>
<th>$\frac{3}{4}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$N = 6$</td>
<td>$N = 9$</td>
<td>$N = 6$</td>
<td>$N = 9$</td>
</tr>
<tr>
<td>Shear force</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$C_1 \neq 0$</td>
<td>5.59</td>
<td>5.87</td>
<td>4.63</td>
<td>4.69</td>
</tr>
<tr>
<td>$C_1 = 0$</td>
<td>4.56</td>
<td>4.55</td>
<td>4.26</td>
<td>4.23</td>
</tr>
<tr>
<td>Twisting moment</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$C_1 \neq 0$</td>
<td>5.01</td>
<td>4.64</td>
<td>4.18</td>
<td>4.26</td>
</tr>
<tr>
<td>$C_1 = 0$</td>
<td>4.26</td>
<td>4.22</td>
<td>4.18</td>
<td>4.12</td>
</tr>
<tr>
<td>Rigid body</td>
<td>5.94</td>
<td>5.67</td>
<td></td>
<td>5.52</td>
</tr>
<tr>
<td>Bending moment</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$C_1 \neq 0$</td>
<td>5.38</td>
<td>5.63</td>
<td>5.58</td>
<td>6.03</td>
</tr>
<tr>
<td>$C_1 = 0$</td>
<td>4.70</td>
<td>4.59</td>
<td>4.23</td>
<td>4.23</td>
</tr>
<tr>
<td>Rigid body</td>
<td>6.10</td>
<td>6.24</td>
<td>6.24</td>
<td>5.66</td>
</tr>
</tbody>
</table>
5. Conclusions

Computed results are presented for a container ship travelling in a seaway. The ship may be assumed to be either flexible or rigid and the corresponding results are contrasted. Further comparisons are given illustrating the marked influence of warping stiffness in the analysis of the flexible ship.

Significant differences are found between results obtained with the ‘rigid’ and ‘flexible’ theories. The discrepancies lie in the region of the dominant wave encounter frequency at about $\omega_w = 1.0$ rad/s. Whereas the discrepancy could be reduced substantially for the lateral bending moment by allowing more contributions in the modal summations, this leads to little improvement of agreement where twisting moment is concerned. Any increase in the structural damping coefficients of the flexible ship theory or modification of the roll damping in the rigid body analysis influences only the magnitudes of the resonances and the discrepancies in this frequency region remain.

The explanation of these discrepancies is to be found in the nature of the vessel itself. This ship has low natural frequencies when dry and low resonance frequencies when wet and the lowest of these latter coincide with the dominant wave encounter frequency. The rigid body analysis cannot make allowance for this. This means that conventional rigid body theory fails for ships, like the one under discussion, with low natural frequencies.

The ‘rigid ship’ theory produces low results. These can be matched for a flexible ship, but only if it possesses high natural frequencies when dry, and resonance frequencies when wet – frequencies lying well above the region of dominant waves in the encounter spectrum. This was indicated by modifying the properties of the ship in question by making it more rigid, i.e. by neglecting the actual flexibility of the ship.

The conclusion to which this investigation leads is that conventional ‘rigid ship’ theory may fail hopelessly to predict the responses of a perfectly normal hull in a seaway. While, to be sure, the ‘flexible ship’ approach certainly needs improvement in points of detail – and again attention is drawn specially to warping stiffness – this is a problem that confronts naval architects. No amount of ‘improvement’ is likely to render the rigid ship approach trustworthy.

References