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A real-time nonlinear MPC for extreme lateral stabilization of passenger vehicles

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***Abstract*—Loss of lateral stability remains a major cause of road accidents in recent years. Further improvement of passenger vehicle's active safety requires a more efficient utilization of the tire-road friction. Nonlinear model predictive control (NMPC) is expected to fulfill such a role, as the nonlinear characteristics of the vehicle are included and the control input is optimized. However, the computational load can be excessive for onboard hardware, which hinders the NMPC from practical implementation. To tackle the problem, this study proposes a method to improve the computational efficiency in NMPC. The proposed solution consists of an explicitly stored look-up table for generating initial guesses and an online optimization component. The look-up table is based on the offline solution of a hybrid MPC controller. Through the simulation with multibody vehicle model, impressive control performance has been observed, as the vehicle can be stabilized from a side-slip angle of up to 0.5 rad.**

***Index Terms*—vehicle control, MPC**

I. Introduction

Striving towards the ultimate active safety of passenger vehicles has never stopped. Despite the wide application of current active safety systems like anti-lock braking system (ABS) and electronic stability control (ESC), road accident is still a major cause of unnatural deaths. According to traffic data in the US, loss of lateral stability still contributed to 20-30% of total road fatalities [1]. New actuators have given the necessary degrees of freedom but the problem of coordinating them for optimal utilization of frictional forces remains unsolved. Quick development of computing hardware and numerical optimization algorithms has widened the application of NMPC. As a powerful method to incorporate nonlinearities in the plant and coordinate multiple control inputs, NMPC is expected to contribute greatly to the enhancement of active safety. However, model accuracy comes at the cost of computational complexity, which can prevent the implementation of NMPC in systems with quick dynamics like the passenger vehicle. Borrelli et al [2] first used nonlinear MPC to control the front steering angle alone and pointed out the problem in computational complexity. In the follow-up study [3], the researchers added the braking torques on individual wheels into the control inputs and hence the simulation became too time-consuming for controller tuning. A recent study [4] claimed to have achieved real-time NMPC control of torque-vectoring using a commercial toolbox. However, the numerical quality is not satisfactory, as the final cost reached 28% higher than the actual optimum in some cases. Simplifying the model of dynamics can reduce the computational burden. Beal and Gerdes [5] developed an affine-force-input model for controlling the front steering with MPC. The lateral force on the front tires was adopted directly as the control input and the rear tires were considered linear. Yet such simplifications are not valid for limit-handling cases. To handle the nonlinearities, Falcone et al [6] looked into the linear time-varying (LTV) modeling. LTV-MPC assumes linear dynamics through the prediction horizon and changes model parameters every time step. As a result, the optimization problem was reduced to quadratic programming like linear MPCs. Unfortunately, such an approach is not applicable for long-term prediction because, throughout the prediction horizon, the dynamics may vary greatly from where it is linearized. Hybrid MPC is another possible option and has been investigated by Di Cairano et al [7]. A hybrid model can be utilized to approximate nonlinear dynamics with linear sections and thus yields moderate complexity and sub-optimal control.

Apart from changing the modeling techniques, the optimization process itself can be improved as well. Tondel [8] has attempted with explicit MPC which aims to move intensive computation offline, using the multi-parametric programming technique. This technique has also been adopted by Di Cairano et al [7] to enable real-time experiments. Explicit MPC performs only tree-search operation and evaluation of affine functions online. The downside is, the storage of offline solution demands plenty of memory. Around 5,000 space partitions are needed for approximating the hybrid MPC solution [7] and the number grows to 11,277 when a nonlinear model was used [8]. Nevertheless, these studies limited the state variables in a rather narrow range and the prediction horizons were short.

Improvements are achievable within the optimization process itself as well. It generally believed that the initial guess (i.e. starting point) plays an important role in numerical optimization. An initial guess is usually required as a manual input or randomly generated. For non-convex problems, multiple random initial guesses are necessary for raising the chance of finding the global optimum, which has no theoretical guarantee. It can be a reasonable choice to use the roughly stored optimal solution as the initial guess. Such idea can be

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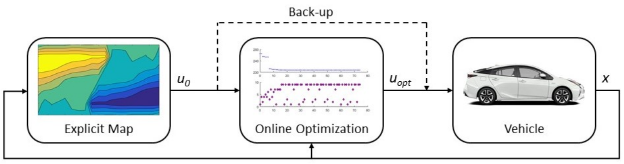

Fig. 1. Schematic drawing of the proposed control system. The explicit map first generates an initial guess to warm-start the online optimization. Then the input is optimized and eventually sent to the vehicle.

observed from Zeilinger et al [9], where the implementation of MPC in large-scale linear systems was accelerated, by means of generating initial guesses with explicit piecewise-affine (PWA) functions and limiting the iterations in online optimization.

In this study, we present an NMPC controller that coordinates the braking and steering action to stabilize the vehicles lateral motion (Fig. 1). The combination of these control inputs utilizes tire-road friction efficiently so that the range of state where stabilization is achievable can be expanded. In addition, a warm-start method has been proposed to reduce computation time. By approximating the nonlinear characteristics with PWA formulation, a hybrid MPC solution can be calculated offline and then serve as the initial guess to the NMPC solution online. On top of that, a quick local optimization process is adopted to find the optimal control input. Of course, the initial guess itself is already a sub-optimal control input that can be used as a backup solution in case that the online optimization fails.

The paper is organized as follows. The derivation of PWA tire model and hybrid 2-track model of vehicles lateral dynamics are covered in section II, where the tuning of hybrid MPC is also included. Then the warm-start nonlinear MPC is explained in section III. Section IV introduces the test of control performance using multibody simulation. Conclusions and future directions are pointed out in Section V.

II. Hybrid MPC for Explicit Initial Guess

A. PWA tire model

PWA modeling of tire behavior is the core of the hybrid vehicle model, as the tire forces contribute greatly to the nonlinear vehicle dynamics. Based on Magic Formula, the PWA tire model incorporates the saturation of longitudinal and lateral forces and the combined-slip behavior. Originally, the Magic Formula uses longitudinal slip κ and wheel slip angle α as variables to determine longitudinal and lateral forces. These forces are also influenced by parameters including vertical load F_z, camber angle γ, inflation pressure p_i, etc. In this study, the influence of p_i and γ is neglected and F_z on each tire are assumed constant for hybrid modeling. Since F_x is directly exploited in the control inputs, one can transform F_y into a function of α and F_x under certain conditions. Given a certain α, F_x is a function of κ. As long as κ does not exceed a certain limit, F_x is monotonic to κ. Therefore, the inverse function exists in this range of κ and its value can be approximated

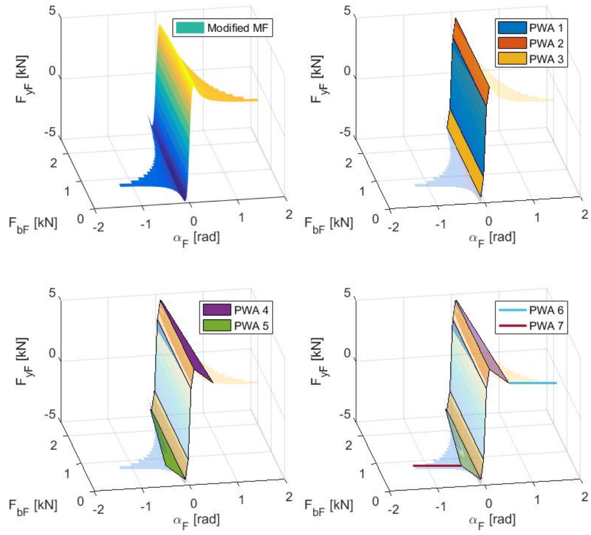

Fig. 2. PWA modelling of the characteristics of tire's lateral force generation. The upper-left subplot shows the modified Pacejka's model. The planar sections represents the modes in the PWA model and are separated by color.

by means of interpolation. The modified tire model can be visualized as a 3-D surface and the PWA tire model can then be derived by approximating the surface with planar sections. These planar sections are expressed with:

$$
\begin{aligned}
& F_{yij} = a_{ij}\alpha_i + b_{ij}F_{bi} + c_{ij} \\
& d_{ijk}\alpha_i + e_{ijk}F_{bi} + g_{ijk} \leq 0 \\
& i \in \{FL, FR, RL, RR\} \\
& j \in \{1, 2, \cdots N_{modes}\} \\
& k \in \{1, 2, \cdots N_{constraints}\}
\end{aligned} \tag{1}
$$

In this study, it has been determined that 7 sections are required to obtain a proper quality of approximation. The sections are visualized in Fig. 2 and they represent the 7 modes in the PWA tire model. Mode 1-5 are each defined within 4 linear constraints and mode 6 and 7 are defined on a straight line section.

B. Hybrid planar vehicle model

The hybrid model of the vehicle dynamics is based on the planar vehicle model which incorporates the generation of yaw moment by braking. The yaw rate r and body slip angle β are the measures of lateral motion and forward velocity v_x is the measure of longitudinal motion.

$$
\begin{aligned}
\dot{r} &= \frac{1}{I_z}\left(F_{yF}l_F - F_{yR}l_R + (F_{xR} - F_{xL})\frac{B}{2}\right) \\
\dot{\beta} &= \frac{1}{m}(F_{yF} + F_{yR}) - r \\
\dot{v}_x &= \frac{1}{m}(F_{xL} + F_{xR})
\end{aligned} \tag{2}
$$

where, the force components are as explained by:

$$
\begin{aligned}
F_{yF} &= (F_{yFL}+F_{yFR})\cos\delta - (F_{bFL}+F_{bFR})\sin\delta \\
F_{yR} &= F_{yRL}+F_{yRR} \\
F_{xL} &= -F_{bFL}\cos\delta - F_{yFL}\sin\delta - F_{bRL} \\
F_{xR} &= -F_{bFR}\cos\delta - F_{yFR}\sin\delta - F_{bRR}
\end{aligned}
\tag{3}
$$

Several simplifications are required to enable hybrid formulation. The vertical load on each tire is assumed constant as well as the forward velocity (fixed at 20 m/s). The transformation effect of the front steering angle is neglected and the wheel slip angles are approximated with their tangent values.

$$
\begin{aligned}
\tilde{\alpha}_{FL} &= \tilde{\alpha}_{FR} = \delta - \frac{rl_F}{v_x} + \beta \\
\tilde{\alpha}_{RL} &= \tilde{\alpha}_{RR} = \frac{rl_R}{v_x} - \beta
\end{aligned}
\tag{4}
$$

Therefore, the lateral dynamics is linear with respect to lateral forces and braking forces.

$$
\begin{aligned}
\dot{r}_h &= \frac{1}{I_z}\left(\tilde{F}_{yF}l_F - \tilde{F}_{yR}l_R + \left(\tilde{F}_{xR}-\tilde{F}_{xL}\right)\frac{B}{2}\right) \\
\dot{\beta}_h &= \frac{1}{m}\left(\tilde{F}_{yF}+\tilde{F}_{yR}\right) - r_h
\end{aligned}
\tag{5}
$$

And since the lateral forces are also PWA functions of state variables and control inputs, the planar model derived here is indeed a PWA model. As each tire has 7 modes of behavior, 4 independent tires originally result in 2,401 possible combinations. Because the wheel slip angles of both wheels on the same axle are equal, both wheels are in the identical mode. Thus each axle has 7 modes and the vehicle has 49 modes of lateral dynamics. For implementing hybrid MPC, the model is then discretized with a sampling time of 50 ms.

C. Specifications of Hybrid MPC

The hybrid MPC controller finds the optimal input by optimizing a cost function:

$$
J = \sum_{i=1}^{N_p}\left(y_{k+i}^T Q y_{k+i} + \mathbf{u}_{\mathbf{k+i-1}}^{\mathbf{T}}\mathbf{R}\mathbf{u}_{\mathbf{k+i-1}}\right) \tag{6}
$$

In this case, the input and output are given by:

$$
\begin{aligned}
y &= \alpha_R(r,\beta,v_x) \\
\mathbf{u} &= [F_{bFL}, F_{bFR}, F_{bRL}, F_{bRR}, \delta]^T
\end{aligned}
\tag{7}
$$

The cost function emphasizes the wheel slip angle of the rear tires. It can be approximated with the linear combination of state variables, as has been mentioned above. The choice is based on the fact that smaller wheel slip angle on the rear axle is beneficial for quicker lateral stabilization. Because the actuators are not fast-responding when compared to the sampling rate of the controller, the control horizon should be much shorter than the prediction horizon. And for the safety-critical situation, the weight of system output should be significantly larger than the weights of control inputs. The parameters are tuned with the hybrid model and the eventual choices are given in Table I. The resulted MIQP problem can be solved by multiple commercial software and Gurobi optimizer was chosen in this study. The build of hybrid model and formulation of hybrid MPC problem was supported by MPT Toolbox [10].

Parameter	N_c	N_p	Q_y	R_{Fbi}	R_δ
Value	1	8	100	1	4

TABLE I
PARAMETERS OF HYBRID MPC.

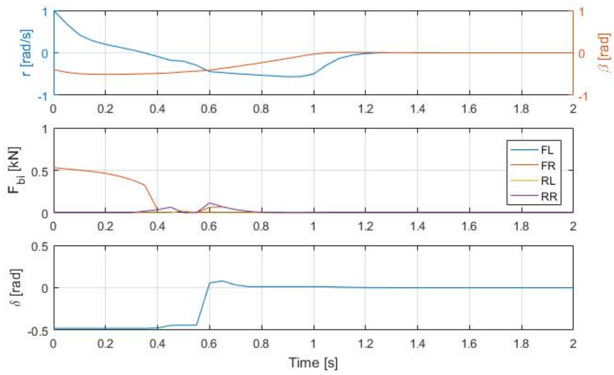

Fig. 3. Closed-loop behavior of the hybrid model and MPC.

The validity of hybrid MPC has been examined with closed-loop simulation (Fig. 3). Starting from the initial state of $xh0 = [1, 0:4]^T$, the controller stabilized the vehicles lateral motion within 1.2 seconds. The steering and braking action are utilized comprehensively to generate yaw moment.

Next, a look-up table (Fig. 4) has been generated by solving the hybrid MPC problem at a set of initial states. The initial states lay on a uniform grid with the yaw rate varying from -1.8 to 1.8 rad/s and body slip angle from -0.6 to 0.6 rad and the step length is 0.2 for both state variables. Consequently, the hybrid MPC is solved at 133 initial states and in total 931 floating-point numbers are needed to contain the information. The look-up table will be used to warm-start the nonlinear MPC, which will be introduced in the next section.

III. WARM-START NONLINEAR MPC

A. Nonlinear planar vehicle model

In the nonlinear MPC, the online optimization of control inputs is performed, starting from the initial guess given by the explicit map. The nonlinear planar vehicle model is adopted for prediction, where the tire forces are determined with Magic Formula. Here, the effect of varying vertical load on the force generation is taken into account in the prediction.

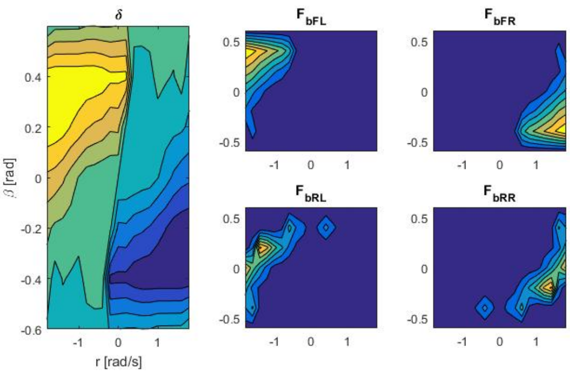

Fig. 4. The explicit control solution calculated with hybrid MPC. For steering, the yellow area means positive steering angle of the front wheels.

$$\begin{aligned}
F_x &= G_x F_{x0} \\
F_y &= G_y F_{y0} \\
F_{x0} =& D_x \sin(C_x \tan^{-1}(B_x \kappa \\
&- E_x(B_x\kappa - \tan^{-1}(B_x\kappa)))) \\
F_{y0} =& D_y \sin(C_y \tan^{-1}(B_y \alpha \\
&- E_y(B_y\alpha - \tan^{-1}(B_y\alpha)))) \\
G_x &= \cos(C_\alpha \tan^{-1}(B_\alpha \alpha \\
&- E_x(B_\alpha\alpha - \tan^{-1}(B_\alpha\alpha)))) \\
G_y &= \cos(C_\kappa \tan^{-1}(B_\kappa \kappa \\
&- E_\kappa(B_\kappa\kappa - \tan^{-1}(B_\kappa\kappa))))
\end{aligned} \tag{8}$$

Steady-state load transfers in both longitudinal and lateral direction are included, in order to capture how the body accelerations influence tire behaviors.

$$\begin{aligned}
M_{roll} &= a_y\left(m_{uF}\left(h_{cg} - h_{rF}\right) + m_{uR}\left(h_{cg} - h_{rR}\right)\right) \\
M_{pitch} &= a_x\left(m_{uF} + m_{uR}\right) h_{cg} \\
\Delta F_{zFL} &= \frac{l_R}{2L} mg - \frac{K_{rF}}{K_{rF} + K_{rR}} \frac{M_{roll}}{B} - \frac{M_{pitch}}{2L} \\
\Delta F_{zFR} &= \frac{l_R}{2L} mg + \frac{K_{rF}}{K_{rF} + K_{rR}} \frac{M_{roll}}{B} - \frac{M_{pitch}}{2L} \\
\Delta F_{zRL} &= \frac{l_F}{2L} mg - \frac{K_{rR}}{K_{rF} + K_{rR}} \frac{M_{roll}}{B} + \frac{M_{pitch}}{2L} \\
\Delta F_{zRR} &= \frac{l_F}{2L} mg + \frac{K_{rR}}{K_{rF} + K_{rR}} \frac{M_{roll}}{B} + \frac{M_{pitch}}{2L}
\end{aligned} \tag{9}$$

The cost function and control and prediction horizon are identical to those of hybrid MPC, except that the approximation has been discarded.

$$\begin{aligned}
\alpha_{FL} = \alpha_{FR} &= \delta - \tan^{-1}\left(\frac{r l_F}{v_x} + \beta\right) \\
\alpha_{RL} = \alpha_{RR} &= \tan^{-1}\left(\frac{r l_R}{v_x} - \beta\right)
\end{aligned} \tag{10}$$

B. Local optimization algorithm

Although the gradient of the objective function can be calculated numerically, such calculation is time-consuming and thus contradicts with the purpose of accelerating the optimization process. Therefore, a gradient-free method has been implemented. The algorithm combines perpendicular search method with varied-step line search, which tries to evaluate the objective function as few times as possible. The step length will be scaled down several times as the line search goes on and hence yields higher approaching speed at the beginning and good accuracy when in the neighborhood of the optimum or the constraints.

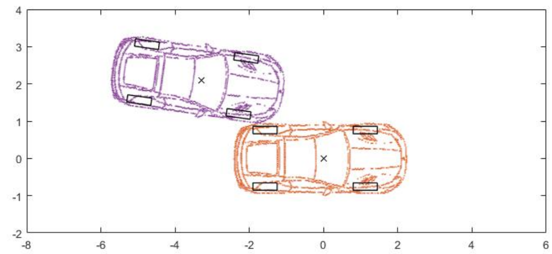

Fig. 5. Demonstration of simulated scenario where the purple vehicle collided with the red vehicle (controlled).

IV. Simulation with multibody model

A. Simulation Setup

The simulation of nonlinear MPC is performed on a roll-axis model built with SimScape. The modeled lateral dynamics has been finely validated using experimental data [11]. The electro-hydraulic braking system is used to enable the active generation of braking force without pedal input. The transmission of the electric signal and hydraulic pressure are modeled as a load-dependent delay plus a second-order response. The controller's performance can be better demonstrated when the vehicle is directly forced into side-slip motion. This can be observed when an excessive lateral force is exerted on the rear end of the vehicle. The potential causes include a collision on the highway when one of the drivers accidentally moves off its original lane and collides with the car in the neighboring lane (Fig. 5).

B. Simulation results

The controller has been tested at two velocities. Starting at 14 m/s (50 km/h), the vehicles trajectory after the impact is shown in Fig. 6. The trajectory from uncontrolled case shows that the impact is sufficiently large to destabilize the vehicle. With the NMPC on the other hand, the stabilization is quick that the vehicle's lateral displacement is limited when stable motion has been recovered. The vehicle states and the controller's actions are shown in Fig. 7 and Fig. 8. The impact immediately raises r to 1.5 rad/s, causing β to increase negatively. Without control, the loss of stability is obvious. The explicit and NMPC control, steering and braking actions are utilized comprehensively to counter the spinning. Direct adoption of the explicitly stored initial guess is already

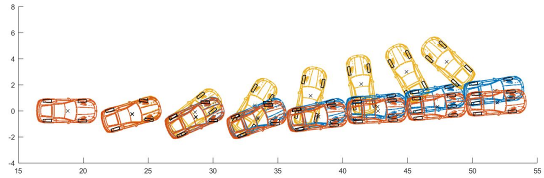

Fig. 6. Trajectory of the vehicle after the external impact. The initial speed is 50 km/h. Yellow for no control, blue for controlling with initial guess and orange for controlling with NMPC.

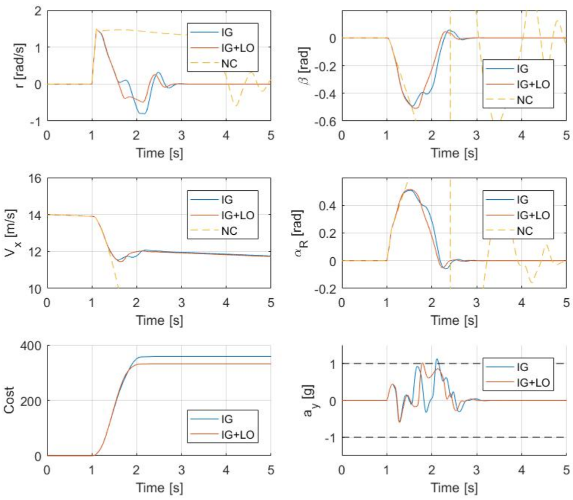

Fig. 7. Vehicle states according to simulation with an initial speed of 50 km/h. The lateral impact was exerted at T=1.0s. NC for no control, IG for controlling with initial guess and IG+LO for controlling with NMPC.

capable of recovering the vehicle, while further optimization reduced the stabilization time by 11.3%. In addition, online optimization also saved control effort significantly. The peak braking force was reduced by 63.6% and 41.3% on the front- and rear-right wheel, respectively. The simulation was also performed at an initial speed of 35 m/s (126 km/h), which is close to the speed limits on the highway in the Netherlands and several other countries. The corresponding results are shown in Fig. 9 and Fig. 10. The results show similar trends. The reduction in stabilization time is 23.3% in this case.

C. Real-time simulation

The warm-start strategy is expected to achieve real-time implementation. For the proof of such capability, the simulation using a multi-core dSPACE platform has been performed. The multibody model and the controller were compiled on two different cores on the DS1006 processor board (2.8GHz quad-core, 1GB DDR2 SDRAM). The multibody model runs at 100 Hz while the controller runs at 20 Hz. As suggested by recorded data (Fig. 11), the processor is able to return the optimized control input in less than 0.165 ms. Compared to the sampling time of the system, which is 50 ms, the proposed

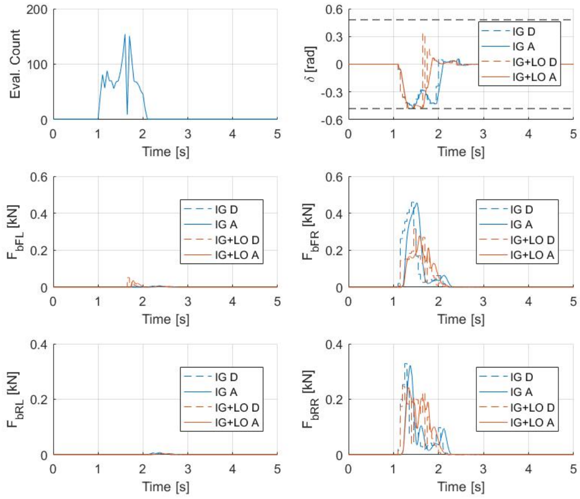

Fig. 8. Controller's action according to simulation with an initial speed of 50 km/h. The lateral impact was exerted at T=1.0s. NC for no control, IG for controlling with initial guess and IG+LO for controlling with NMPC. D for control demand and A for actual response of the actuators.

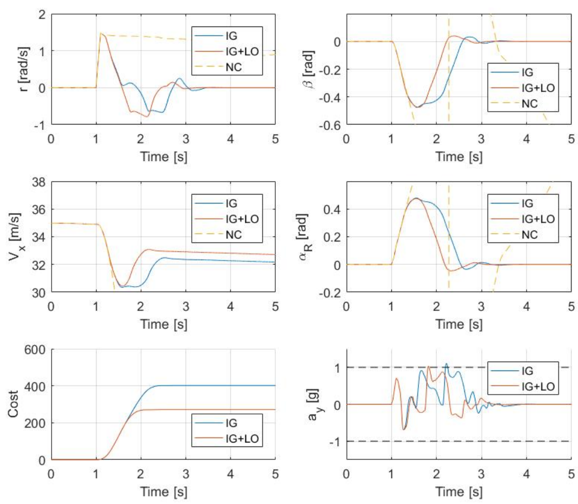

Fig. 9. Vehicle states according to simulation with an initial speed of 126 km/h. The lateral impact was exerted at T=1.0s. NC for no control, IG for controlling with initial guess and IG+LO for controlling with NMPC.

method has succeeded in enabling real-time implementation of the nonlinear MPC.

V. Conclusions

In this paper, we proposed a method for accelerating nonlinear MPC, in order to enable its real-time implementation in the control of passenger vehicles. The method combines an explicit map of initial guess with a quick local optimization algorithm. The initial guess is generated with a hybrid MPC based on a PWA model of vehicle dynamics. The PWA model not only captures the nonlinear characteristics of the tire's

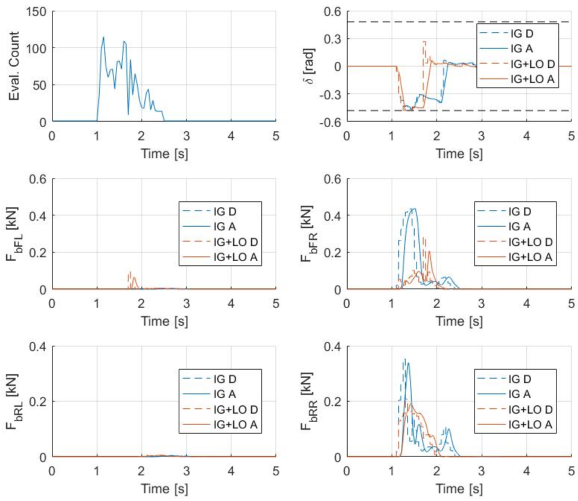

Fig. 10. Controller's action according to simulation with an initial speed of 126 km/h. The lateral impact was exerted at T=1.0s. NC for no control, IG for controlling with initial guess and IG+LO for controlling with NMPC. D for control demand and A for actual response of the actuators.

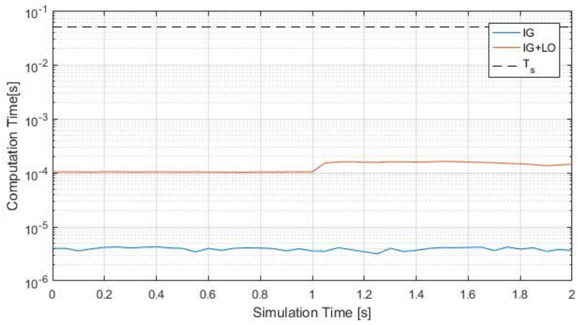

Fig. 11. Computation time recorded from dSPACE experiment. Blue line represents the time needed for returning the initial guess with the look-up table and the orange line represents the time for returning the optimized control input. Dashed line is the sampling time of the system.

lateral and longitudinal forces alone but also the combined-slip behavior. Starting from the initial guess, the control input is further optimized online according to a more accurate nonlinear model. The method eventually proves to be highly powerful regarding the stabilization performance in extreme situations. The controller can stabilize the vehicle lateral motion with a large side slip angle and at both low and high speed. The experiment on dSPACE platform suggested a promising capability of real-time implementation.

By modifying the cost function, the NMPC controller can be extended for tracking reference yaw rate, which is a key part of path-following control in autonomous vehicles. By also tracking a reference body-slip angle, the controller can possibly achieve combined path and attitude control. The framework of exploiting hybrid model to improve computational efficiency in nonlinear MPC may also be implemented in other systems with quick and nonlinear dynamics. Nevertheless, an advanced state estimator is required to allow the NMPC to work properly, as the current solution is not robust against measurement noise. The variation in load distribution can cause a mismatch between the vehicle and its model, which may also affect the control performance.

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