Analyzing the Effect of Traffic Scenario Properties on Conflict Count Models

Sunil, Emmanuel; Þórðarson, Olafur; Ellerbroek, Joost; Hoekstra, Jacco

Publication date
2018

Document Version
Accepted author manuscript

Published in
2018 International Conference on Research in Air Transportation

Citation (APA)
Sunil, E., Þórðarson, O., Ellerbroek, J., & Hoekstra, J. (2018). Analyzing the Effect of Traffic Scenario Properties on Conflict Count Models. In *2018 International Conference on Research in Air Transportation: Barcelona, Spain, 2018*

Analyzing the Effect of Traffic Scenario Properties on Conflict Count Models

Emmanuel Sunil, Ólafur Þórðarson, Joost Ellerbroek and Jacco M. Hoekstra
Faculty of Aerospace Engineering, Delft University of Technology
Kluyverweg 1, 2629 HS, Delft, The Netherlands

***Abstract*—Decentralized en-route airspace concepts have been proposed by many studies to increase airspace safety and capacity. Most of these studies, including our own forays into this domain, have used fast-time simulation experiments to explore the benefits offered by decentralization. While simulations are indispensable during the initial design phase of any new airspace concept, the understanding gained using this approach can be difficult to generalize beyond the tested conditions. To address this issue, some researchers have presented analytical conflict count models to quantitatively analyze the effect of physical factors, such as traffic separation requirements, on the intrinsic safety of decentralized airspace concepts. However, the derivation of these models often make use of idealized assumptions regarding the behavior of traffic that do not always reflect realistic operations. To this end, this paper investigates the effect of these assumptions on the accuracy of the analytical conflict count models using targeted fast-time simulations of a direct-routing unstructured en-route airspace concept for a number of more realistic traffic patterns. The data collected from these simulations is also used to test so called 'model adjustments' that aim to relax the dependency of the models on the idealized traffic scenario assumptions. The results show that the assumptions do affect the accuracy of the analytical models, with some assumptions leading to a substantial under-estimation of conflicts. The results also show that the model adjustments increased accuracy for the more realistic scenarios to the levels previously found for the ideal traffic settings for all cases. Therefore, in addition to providing a physical understanding of the factors that affect airspace safety, the adjusted models can also be used as tools for practical airspace design applications.**

***Keywords*—Airspace safety; airspace design; conflict rate; conflict probability; gas models; BlueSky ATM simulator**

NOMENCLATURE

A	Airspace area
B	Airspace volume
C	Number of instantaneous conflicts
H	Height of airspace volume of interest
k	Model accuracy parameter
N	Number of instantaneous aircraft
p	Conflict probability
S	Separation requirement
t_l	Conflict look-ahead time
V	Aircraft Ground Speed
V_r	Relative Velocity
Z	Altitude
ρ	Traffic Density
ρ_{max}	Airspace Capacity
γ	Flight Path Angle of Climbing/Descending Aircraft
ψ	Aircraft heading
ε	Proportion of Cruising Aircraft
Subscripts:	
h	Horizontal
max	Maximum
min	Maximum
r	Relative
$total$	Total
v	Vertical

I. INTRODUCTION

The current system of Air Traffic Control (ATC) relies on a *centralized* control architecture. At its core, this system is heavily dependent on human Air Traffic Controllers (ATCos) to ensure safe separation between aircraft. While this system has served the needs of the air transportation industry thus far, the increasing delays and congestion reported in many parts of the world indicates that the centralized operational model is fast approaching saturation levels [1].

To cater for the expected future increases in traffic demand, several studies have proposed a transition to a *decentralized* traffic separation paradigm in en-route airspace [2]–[4]. In decentralized airspace, each individual aircraft is responsible for its own separation with all surrounding traffic. To facilitate decentralization, significant research has been made performed towards the development of automated airborne Conflict Detection and Resolution (CD&R) algorithms [5]. Some studies have also considered if such algorithms can be combined with alternate options for structuring air traffic to further increase safety and capacity over current operations [6].

It should be noted that most studies in this domain have used fast-time simulation experiments to test and analyze the performance of the algorithms and airspace design concepts that have been developed to implement decentralized control. Although fast-time simulations provide intuitive insights on the advantages and disadvantages of decentralized systems, they can be time consuming to develop, depending on the required level of realism. Furthermore, the results of such simulation studies are sometimes qualitative in nature, making it difficult to extrapolate their results beyond the specific conditions that have been tested.

To address this issue, some researchers have derived mathematical models to gain a more quantitative understanding of the interactions between aircraft in decentralized systems. When such methods are used to model the safety of an airspace design concept, they often make use of the so called 'gas modeling' approach [7]–[10]. This approach treats conflicts between aircraft similar to the collisions that occur between gas particles to determine instantaneous system wide-conflict counts as a measure of airspace safety. Because this approach uses measurable airspace parameters, such as traffic demand and separation requirements, as inputs, gas-models can be used to understand the factors that affect the safety of an airspace design concept.

However, to develop closed-form analytical expressions, many gas-models described in literature make use of idealized assumptions regarding the speed, heading, altitude and spatial/density distributions of traffic. Collectively, these four distributions describe the what is often referred to as a traffic scenario. Typically, the following four traffic scenario assumptions are used in the derivation of analytical gas-models:

- Equal ground speed for all aircraft
- Uniform heading distribution
- Uniform altitude distribution
- Uniform spatial/density distribution

In practice, however, a traffic scenario with these exact combination of properties is unlikely to occur. This raises the question of the accuracy of such models for more realistic traffic scenarios.

In this research, the accuracy of an analytical gas-model for an unstructured en-route airspace design concept will be tested for cases that do not respect the above ideal traffic scenario assumptions. This done by comparing model predictions to the results of four fast-time simulation experiments with varying speed, heading, altitude and spatial distributions. In each experiment, one traffic scenario assumption will be violated while respecting the other three assumptions. The data collected from these simulations is also used to test so called 'model adjustments' that aim to relax the dependency of the conflict count models on the idealized traffic scenario assumptions. The adjusted models use numerical methods to evaluate complex integrals that can not be solved analytically for non-ideal traffic scenarios.

This paper begins with a summary of important definitions, and an overview of the baseline analytical conflict count model in section II. Next, in section III, the effect of each traffic scenario assumption is analyzed, and a numerical approach is developed to adjust the model to handle non-ideal traffic scenarios. The design of the simulation experiments used to assess model accuracy is described in section IV. The results of the simulations are presented for both the baseline and the adjusted models in section V. Finally, the main conclusions of this study are summarized in section VI.

II. Background

This section presents the definitions and background material relevant to this study, including a summary of the baseline analytical conflict count model for unstructured airspace designs.

A. Conflicts vs. Intrusions

A conflict occurs if the horizontal and vertical distances between two aircraft are expected to be less the prescribed separation standards within a predetermined 'look-ahead' time. Conflicts are, therefore, predictions of *future* separation violations. Conflicts should not be confused with intrusions. Instead, intrusions, also referred to as losses of separation, occur when separation requirements are violated at the *present* time. This distinction between conflicts and intrusions is shown in Fig. 1.

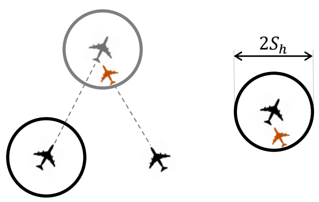
As mentioned earlier, this paper is concerned with the accuracy of *conflict* count models. Therefore, the rest of this paper only deals with aspects that are relevant to conflicts.

B. The Unstructured Airspace Design Concept

This study uses an Unstructured Airspace (UA) design as a 'test-bench' to measure the accuracy of the 'gas-modeling' approach of quantifying airspace safety. As the name suggests, no constraints are imposed on aircraft motion in UA. Instead, this simplest form of en-route airspace design focuses on maximizing overall system efficiency. Therefore, aircraft are free to use direct horizontal routes, as long as such routing is not obstructed by weather or static obstacles. Similarly, aircraft can also fly with preferred speeds and at optimum altitudes, based on their performance capabilities and trip distances. By offering greater freedom to aircraft operators, UA has been found to result in a more uniform distribution of traffic, both horizontally and vertically, reducing traffic concentrations and ensuing delays [3].

C. Gas Modeling Approach for Estimating Conflict Counts

This work uses the 'gas-model' approach to quantify the intrinsic safety of an airspace design concept. Gas models compute the number of instantaneous conflicts in a given volume of airspace as a measure of intrinsic safety. Here, the notion of intrinsic safety considers the ability of an airspace design concept to prevent conflicts from occurring solely due to the constraints that it imposes on traffic motion, i.e., without the aid of tactical conflict resolution systems.



(a) Conflict (b) Intrusion

Fig. 1. The difference between intrusions and conflicts, displayed here for the horizontal plane. Here, S_h is the horizontal separation requirement.

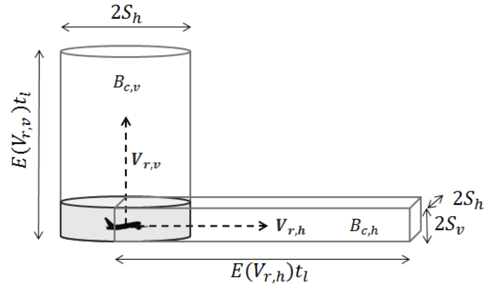


Fig. 2. Volume searched for conflicts by an aircraft in 3D airspace

Gas models compute the number of instantaneous conflicts as a product of two factors, namely the number of combinations of two aircraft, and the conflict probability between any two aircraft. In essence, the number of combinations of two aircraft is the maximum number of conflicts that can occur, since multi-aircraft conflicts, i.e., conflicts involving more than two aircraft, can also be decomposed into a series of two-aircraft conflicts. The conflict probability, on the other hand, scales down the number of combinations so that only those aircraft that are within range each other and those with intersecting trajectories are counted as conflicts. Thus, the basic equation used by all gas models can be written out in words as:

$$\text{No. Inst. Conflicts} = \begin{array}{c}\text{No. of Combinations of 2 Aircraft} \\ \times \\ \text{Conflict Probability Between 2 Aircraft}\end{array} \tag{1}$$

D. Baseline Analytical Conflict Count Model for UA

The following paragraphs summarize the analytical conflict count model for UA. The analytical model is derived while assuming equal ground speeds for all aircraft, as well as uniform heading, altitude and spatial distributions of traffic. The full derivation of these equations cane be found in [10].

As stated in section II-C, gas models compute the number of instantaneous conflicts, C, as a product of the number of combinations of two aircraft, and the conflict probability between any two aircraft, p. For UA, the number of combinations can be computed directly using the binomial theorem, since this airspace design imposes no constraints on the motion of aircraft. Therefore for UA, C can be expressed as [9], [10]:

$$C = \frac{N\,(N-1)}{2}\;p \tag{2}$$

Here, N is the total number of instantaneous aircraft in the airspace of all flight phases. To model p, it is necessary to consider the process of conflict detection. This study considers the so called 'state-based' conflict detection algorithm, which is the method used by most studies on decentralized control. In state-based CD, aircraft search for conflicts within a volume of airspace in front of them. In essence, this involves a 4D extrapolation of aircraft position vectors, assuming constant velocity vectors. If traffic density is uniform, and if aircraft are uniformly distributed in altitude, it can be shown that p is equal to the ratio between the volume of airspace searched for conflicts, B_c, and the total volume of the airspace under consideration, B_{total}. For mathematical convenience, B_c can be decomposed into its horizontal and vertical components, see Fig. 2. Using this approach, p as can be expressed as [10]:

$$p = \frac{B_{c,h} + B_{c,v}}{B_{total}} = \frac{4\;S_h S_v\;\mathbf{E}\left(V_{r,h}\right)t_l + \pi S_h^2\;\mathbf{E}\left(V_{r,v}\right)t_l}{B_{total}} \tag{3}$$

Here, S_h and S_v are the horizontal and vertical separation requirements, and t_l is the CD 'look-ahead' time. $\mathbf{E}\left(V_{r,h}\right)$

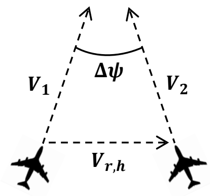


Fig. 3. The relationships between velocity, V, relative velocity, V_r, and heading difference $\Delta\psi$ for two arbitrary aircraft

and $\mathbf{E}(V_{r,v})$, are the horizontal and vertical components of the expected relative velocity of all aircraft pairs. The expected relative velocity can be considered equivalent to the weighted average of the relative velocity of all aircraft pairs in the airspace, taking into account the heading and speed distributions of all aircraft. If all aircraft are assumed to have equal speeds, and if aircraft headings are assumed to be uniformly distributed between 0^o-360^o, then it is possible to show that [10]:

$$\mathbf{E}(V_{r,h}) = \frac{4V}{\pi} \quad (4a)$$

$$\mathbf{E}(V_{r,v}) = V\sin(\gamma)\left(1-\varepsilon^2\right) \quad (4b)$$

Here, V is aircraft ground speed, γ is the flight path angle of climbing/descending aircraft, and ε is the proportion of cruising aircraft in the airspace. Note that $\gamma = 2.8^o$ and $\varepsilon = 0.82$ for all traffic scenarios in this work.

III. Traffic Scenario Adjusted Conflict Count Models

As mentioned before, the baseline analytical conflict count model makes use of idealized assumptions of the speed, heading, altitude and spatial distributions of aircraft. If these assumptions are not respected, usage of the analytical models is expected to lead to inaccurate conflict count predictions. By analyzing the assumptions, and where they affect the equations, this section proposes numerical adjustments to increase the accuracy of the models for more realistic traffic scenarios.

The accuracies of the baseline and adjusted models are to be determined in this study by comparing model predictions to conflict counts obtained using fast-time simulation experiments. Therefore the derivations of the model adjustments use the same traffic scenario distributions as used in the experiments. Moreover, the model adjustments are derived separately for each of the four traffic scenario assumptions. Therefore each model adjustment applies to the case where only one of the four ideal traffic scenario assumptions is violated, while respecting the other three assumptions.

A. Ground Speed Distribution Adjustment

An important step in the deviation of conflict count models is the computation of the expected relative velocity between aircraft, $\mathbf{E}(V_r)$, as this is needed to determine the conflict probability between aircraft, p, see (3). The derivation of an analytical expression for $\mathbf{E}(V_r)$ assumes that all aircraft fly with the same ground speed. Because the flight path angles for aircraft in en-route airspace are relatively small, this assumption mainly affects the calculation of the expected *horizontal* relative velocity between aircraft, $\mathbf{E}(V_{r,h})$, see (4a).

To understand how the equal ground speed assumption affects the calculation of $\mathbf{E}(V_{r,h})$, it is useful first consider the magnitude of the horizontal relative velocity between two arbitrary aircraft, $V_{r,h}$, see Fig. 3. If both aircraft are assumed to have equal speeds, i.e., if $V_1 = V_2 = V$, then the geometry between V_1, V_2 and $V_{r,h}$ becomes an isosceles triangle. Therefore, $V_{r,h}$ can be computed simply as:

$$V_{r,h\ baseline} = 2\ V\ \sin\left(\frac{|\Delta\psi|}{2}\right) \quad (5)$$

Since all aircraft are assumed to have equal speeds, (5) states that the only factor that causes variations in $V_{r,h}$ between different aircraft pairs is the absolute heading difference between two aircraft, $|\Delta\psi|$. Consequently, to compute $\mathbf{E}(V_{r,h})$, the baseline analytical model integrates (5) over all possible values of $|\Delta\psi|$:

$$\mathbf{E}(V_{r,h})_{baseline} = \int_0^{\alpha} V_{r,h}\ P(|\Delta\psi|)\ \ d\Delta\psi \quad (6)$$

By using the above equation to compute $\mathbf{E}(V_{r,h})$, the baseline analytical model essentially assumes that $\mathbf{E}(V_{r,h})$ is only dependent one probability density function, i.e., that of the absolute heading difference between any two aircraft in an airspace, $P(|\Delta\psi|)$. Evaluation of (6) for UA, while assuming a uniform heading distribution of all aircraft between 0^o-360^o, results in the simple analytical expression for $\mathbf{E}(V_{r,h})$ given by (4a).

However, for real-life operations, all aircraft in a given volume of airspace are unlikely to fly with equal ground speeds. This can be due to several reasons including the fact that different aircraft types have different optimum cruising speeds. If aircraft are not assumed to fly with equal ground speeds, then the model for $\mathbf{E}(V_{r,h})$ must be adjusted to take into account the actual speed (and heading) distributions of all aircraft in the airspace.

To begin the derivation of the ground speed adjusted model for $\mathbf{E}(V_{r,h})$, it is once again useful to first consider the computation of $V_{r,h}$ for an arbitrary pair of aircraft, see Fig. 3. If $V_1 \neq V_2$, the cosine rule needs to be used to rewrite (5) as:

$$V_{r,h\ adjusted} = \left(V_1^2 + V_2^2 - 2V_1V_2\cos(\Delta\psi)\right)^{1/2} \quad (7)$$

Since the values of $V1$, $V2$ and $|\Delta\psi|$ can be different for every pair of aircraft in the airspace, to compute the ground speed adjusted version of (6), it is necessary to integrate (7) over all possible values of velocity and absolute heading difference, while taking into account the probability density functions of velocity magnitudes and absolute heading differences, $P(V_1)$, $P(V_2)$ and $P(|\Delta\psi|)$:

$$\mathbf{E}(V_{r,h})_{adjusted} = \int_{V_1}\int_{V_2}\int_0^{\alpha} V_{r,h}(V_1, V_2, \Delta\psi)\, P(|\Delta\psi|)\, P(V_1)\, P(V_2)\ d\Delta\psi\, dV_1\, dV_2 \quad (8)$$

Due to the complexity of (8), the ground speed model adjustment can only be determined numerically. This has been performed for the four speed distributions displayed in Fig. 4,

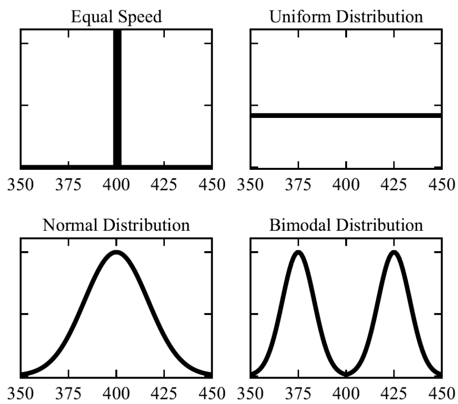


Fig. 4. Probability density functions for the four speed distributions used for fast-time simulations

TABLE I
EFFECT OF SPEED DISTRIBUTION ON THE EXPECTED HORIZONTAL RELATIVE VELOCITY, KTS

Baseline (Equal)	Uniform	Normal	Bimodal
509	512	507	509

while assuming a uniform distribution of aircraft headings (as stated previously, the model adjustments consider the effect of one scenario assumption at a time). Here the equal speed case corresponds to the assumption used by the baseline analytical model. The physical interpretation for the other three distributions corresponds to the hypothetical distributions of different aircraft types in a particular volume of airspace. For example, a bimodal speed distribution can occur if there are two dominant aircraft types in an airspace, e.g. 737s and A320s.

The numerically computed values for $\mathbf{E}\left(V_{r,h}\right)$ using the ground speed model adjustment, given by (8), are listed in Table I. Here it can be seen that varying the ground speed distribution has no significant effect on $\mathbf{E}\left(V_{r,h}\right)$, since all considered distributions have the same mean speed of 400 kts, see Fig. 4. Therefore the equal ground speed assumption is not expected to have a significant impact on the accuracy of the baseline analytical conflict count model for UA.

B. Heading Distribution Adjustment

As implied by (8), in addition to the speed distribution of aircraft, the expected horizontal relative velocity between aircraft, $\mathbf{E}\left(V_{r,h}\right)$, is also affected by the probability density function of the absolute heading difference between aircraft, $P\left(|\Delta\psi|\right)$. The baseline analytical model for UA assumes a uniform distribution of aircraft headings between 0^o-360^o. For this ideal case, it can be shown that $P\left(|\Delta\psi|\right)$ takes a triangular shape between 0^o-360^o [9]:

$$P\left(|\Delta\psi|\right)_{uniform} = \frac{1}{\pi}\left(1 - \frac{\Delta\psi}{2\pi}\right) \tag{9}$$

Logically, (9) should only be used to evaluate (8) when aircraft headings are uniformly distributed between 0^o-360^o. However, the simplified expression for $\mathbf{E}\left(V_{r,h}\right)$ used by the baseline analytical conflict count model, given by (4a), assumes a uniform heading distribution of aircraft, regardless of the actual heading distribution specified in given traffic scenario. Therefore using (4a) to compute $\mathbf{E}\left(V_{r,h}\right)$ is expected to reduce model accuracy for all cases other than for a uniform distribution of aircraft headings.

To ensure high model accuracy for other heading distributions, the appropriate function for $P\left(|\Delta\psi|\right)$ should be used when numerically evaluating (8). In this study, the four heading distributions pictured in Fig. 5 are used. Here, the uniform heading distribution matches the assumption made by the analytical model. Normal and ranged-uniform heading distributions represent traffic scenarios with one, or a range of, predominant aircraft headings. These two distributions can be an example of traffic moving towards oceanic airspace. On the other hand, the bimodal distribution is used to simulate scenarios with head-on traffic. This could be representative of the pattern between the east and west coast of the United States. For these cases, the following expressions describe $P\left(|\Delta\psi|\right)$:

$$P(|\Delta\psi|)_{normal} = \frac{\sqrt{2}}{\sigma\sqrt{\pi}} e^{-\frac{\Delta\psi^2}{2\sigma^2}} \tag{10a}$$

$$P(|\Delta\psi|)_{bimodal} = \frac{1}{2\sqrt{2\pi\sigma^2}} e^{\frac{(\Delta\psi-\pi)^2}{2\sigma^2}} + \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{\Delta\psi^2}{2\sigma^2}} \tag{10b}$$

$$P(|\Delta\psi|)_{ranged-uniform} = \frac{4}{\alpha^2}(\alpha - 2\Delta\psi) \tag{10c}$$

Table II displays the $\mathbf{E}\left(V_{r,h}\right)$ values for the four heading distributions pictured in Fig. 5. These values were computed by numerically evaluating (8) using the appropriate expressions for $P\left(|\Delta\psi|\right)$, while assuming equal speeds for all aircraft. This table indicates no significant differences between the uniform and bimodal distributions. However, $\mathbf{E}\left(V_{r,h}\right)$ is significantly lower for the normal and ranged-uniform heading distributions when compared to the uniform case. Therefore the baseline analytical model is expected to over-estimate conflict counts when aircraft headings normally or ranged-uniformly distributed.

C. Altitude Distribution Adjustment

The conflict probability between any two arbitrary aircraft, p, is computed for the baseline analytical model as the ratio between the volume of airspace searched for conflicts by an aircraft, B_c, and the total volume of the airspace B_{total}, see (3). This formulation for p assumes that conflicts are equally likely in all parts of the airspace. However, if aircraft are not spread uniformly in the vertical direction, then it is logical that aircraft in busier altitudes are more likely to experience conflicts than aircraft in less dense altitudes.

To take into account the vertical distribution of traffic in a given scenario, the model adjustment for aircraft altitude distributions introduces a new variable p_v. This variable considers the effect of the altitude distribution on p, and it can be calculated as [7]:

$$p_v = \int_{Z_{min}}^{Z_{max}} P_z(h) \int_{h+S_v}^{h-S_v} P_z(u)\ du\ dh \tag{11}$$

Here, Z_{min} and Z_{max} are the minimum and maximum altitudes of the airspace volume of interest and P_z is the probability density function for aircraft altitudes. Additionally, h is the altitude variable for an aircraft i, and u is the altitude variable for an aircraft j, where aircraft i and aircraft j are two arbitrary aircraft in the airspace that could potentially conflict with each other. In essence the inner integral in the above equation considers the probability that aircraft j is located within the vertical separation requirement of aircraft i, and the outer integral evaluates the probability that aircraft i is located within the upper and lower altitudes of the airspace volume of interest. Because both aircraft i and j are assumed to be located within the same airspace, the same probability density function of altitudes can be used for both aircraft.

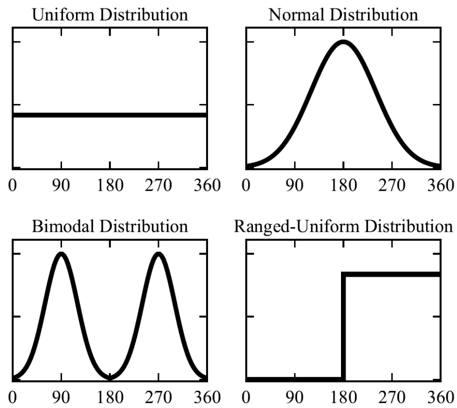


Fig. 5. Probability density functions for the four heading distributions used for fast-time simulations

TABLE II
EFFECT OF HEADING DISTRIBUTION ON THE EXPECTED HORIZONTAL RELATIVE VELOCITY, KTS

Baseline (Uniform)	Bimodal	Normal	Ranged-Uniform
509	485	395	370

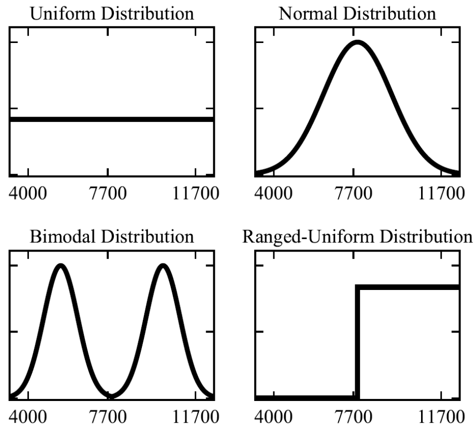


Fig. 6. Probability density functions for the four altitude distributions used for fast-time simulations

TABLE III
EFFECT OF ALTITUDE DISTRIBUTION ON THE CONFLICT PROBABILITY BETWEEN AIRCRAFT, p_v

Baseline (Uniform)	Normal	Bimodal	Ranged-Uniform
0.171	0.289	0.289	0.342

Using (11), it is possible to derive the expression used by the baseline analytical conflict count model for p, given by (3). This can be done by first evaluating (11) for the case where altitudes are uniformly distributed:

$$p_{v,\ uniform} = \frac{1}{Z_{max} - Z_{min}} = \frac{1}{H} \tag{12}$$

Here, H is the height of the airspace volume of interest. The above expression was implicitly used during the derivation of (3):

$$p_{uniform} = \frac{4\ S_h S_v\ \mathbf{E}\left(V_{r,h}\right) t_l + \pi S_h^2\ \mathbf{E}\left(V_{r,v}\right) t_l}{A_{total}} \cdot \frac{1}{H} \tag{13}$$

Here, the relationship between the area and the volume of a shape with a constant cross-section is used, i.e., $B_{total} = A_{total}\ H$. From the above derivation of p for the uniform altitude distribution case, it is clear that:

$$p = \frac{4\ S_h S_v\ \mathbf{E}\left(V_{r,h}\right) t_l + \pi S_h^2\ \mathbf{E}\left(V_{r,v}\right) t_l}{A_{total}} \cdot p_v \tag{14}$$

Here, p_v should be computed using (11) while taking into account the actual altitude distribution in a given traffic scenario. As for the other model adjustments, (11) can be evaluated numerically for cases where the altitude distribution is non-uniform. This approach has been taken for the four altitude distributions considered in this study, see Fig. 6. As for the other scenario properties, the uniform distribution corresponds to assumption used by the baseline analytical model. Normally distributed altitudes represent the case where most aircraft prefer to cruise within a narrow range of flight levels, a situation that is representative of current en-route operations over continental airspace. A similar explanation can also be applied to the bimodal distribution when considering a mix of turbo-prop and jet aircraft; a set of lower altitudes for turbo-props, and a set of higher altitudes for jets. Finally, the ranged-uniform case approximates the preference of long distance flights over oceanic airspace to use only high altitude flight levels to minimize fuel burn.

The numerically computed values of p_v for the four considered altitude distributions are listed in Table III. Unlike the heading distribution of aircraft, this table shows that the uniform distribution of altitudes leads to the lowest conflict probability between aircraft. Therefore, the baseline analytical model is expected to under-estimate conflict counts for cases with non-uniform altitude distributions.

D. Spatial Distribution Adjustment

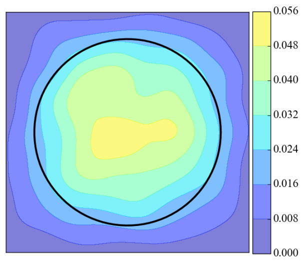
As stated in section III-C, the conflict probability model for the baseline analytical model assumes that conflicts are equally likely in all parts of the airspace. However, if the spatial/density distribution of aircraft is not uniform, then more conflicts can be expected in areas with higher traffic densities.

To deal with the effect of so called traffic density 'hot-spots' on conflict counts, the model adjustment for the spatial distribution discretizes the airspace into a number of areas with uniform traffic densities. The total conflict count can then be determined as the summation of conflict counts in each sub-area, while taking into account the interactions between with different the sub-areas:

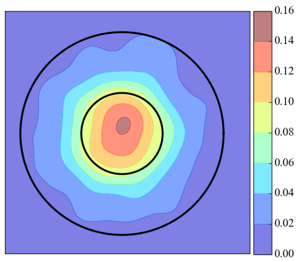
$$\begin{aligned} C = &\sum_{i=1}^{n} \frac{N_{area_i}\left(N_{area_i} - 1\right)}{2} p_{area_i} \quad + \\ &\sum_{i=1}^{n} \sum_{j=1}^{n} N_{area_i} N_{area_j} p_{area_{i,j}} \end{aligned} \tag{15}$$

Here, N_{area_i} is the number of aircraft in area i, and p_{area_i} is the conflict probability in area i. Because this method assumes a constant, or near constant, density in each sub area, p_{area_i} can be computed using (3), while taking into account the size of area i. A similar procedure can be followed for calculating $p_{area_{i,j}}$, except in this case the sum of the sizes of areas i and j should be used. Furthermore, the first term of (15) considers the number of conflicts in each the sub-area, while the second term considers conflicts that occur as a result of interactions between aircraft in different sub-areas. A discretization into sub-areas can be performed using clustering algorithms. Once the number of sub-areas is determined, recursive programming can be used to implement (15).

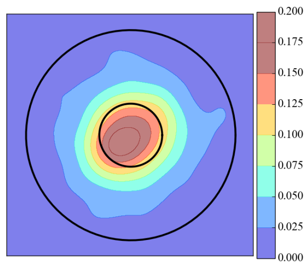
Heat-maps for the three spatial distributions considered in this study are pictured in Fig. 7. Here it can be seen that the traffic density is relatively uniform for the baseline scenario within the cylindrical 'experiment region', while the other two cases contain density hot-spots at the center of the airspace, with a radius of 55 NM and 40 NM, respectively. Such hot-spots can occur for real-life at the merge point of several



(a) Baseline/Uniform



(b) Hot-Spot 1, Radius = 55 NM



(c) Hot-Spot 2, Radius = 40 NM

Fig. 7. Traffic density heat-maps for the three spatial distributions used for fast-time simulations. The outer circle represents the boundary of the 'experiment region'. The inner circle represents the boundary of the hot-spot area.

traffic streams. Because the baseline analytical model assumes a constant density distribution, it is expected to under-estimate the number of conflicts for the hot-spot conditions, with the highest number of conflicts expected for hot-spot 2.

IV. Fast-Time Simulation Design

Four fast-time simulation experiments were performed within the context of an unstructured airspace design concept to investigate the accuracies of the baseline and adjusted conflict count models for traffic scenarios with varying speed, heading, altitude and spatial distributions. This section describes the design of these experiments.

A. Simulation Development

1) Simulation Platform
The BlueSky open-source ATM simulator, developed at TU Delft using the Python programming language, was used as the simulation platform in this research. BlueSky has many features, including the ability to perform batch simulations using all CPU cores. For a full account of BlueSky capabilities, the refer to [11].

2) Conflict Detection
As stated before, state-based CD was used in this study, see section II-D. In this study, a look-ahead time of 5 minutes, as well as separation requirements of 5 nautical miles horizontally and 1000 ft vertically, were used. It should be noted that CD was performed assuming perfect knowledge of aircraft states. This is in line with the findings of a recent study that concluded that ADS-B characteristics have little effect on the performance of state-based CD [12].

B. Traffic Scenarios

1) Testing Region and Flight Profiles
A large three-dimensional en-route sector was used as the physical environment for traffic simulations, see Fig. 8. In the horizontal plane, the sector had a square-shaped cross-section of 400 x 400 NM. In the vertical dimension, the sector is divided into two parts; a 'transition zone' with a height of 4000 ft for climbing and descending traffic, and a 'cruising zone' with a height of 7700 ft. Fig. 8 also shows the horizontal and vertical flight profiles of an example flight. Because the simulations are performed within the context of an Unstructured Airspace (UA) design, see section II-B, aircraft use direct-horizontal routes.

As no traffic was simulated outside the simulated sector, aircraft near the edges of the 'simulation region' are unlikely to get into conflicts. To solve this issue, a smaller cylindrical 'experiment region' was defined in the center of the 'simulation region'. The resulting gap between the experiment and simulation regions ensures that aircraft within the experiment region are surrounded by traffic in all directions. Correspondingly, only aircraft within the experiment region, and only conflicts with closest points of approach within the experiment region, were used to assess the accuracy of the conflict count models.

2) Traffic Demand Scenarios
Five traffic demand scenarios of increasing density, ranging between 5-100 aircraft per 10,000 NM^2 in the simulation region, were used for all four experiments. Note that this is more than twice the maximum traffic density of 32 aircraft per 10,000 NM^2 in the upper airspace (>18,000 ft) over the Netherlands in 2017 (computed using logged ADS-B data). Additionally, it should be noted that for each traffic demand condition, five repetitions, representing five random initial conditions, were tested.

Traffic scenarios were generated with a duration of 2 hrs, consisting of a 1 hr traffic volume buildup period, and a 1 hr logging period during which the traffic density was held constant at the required level.

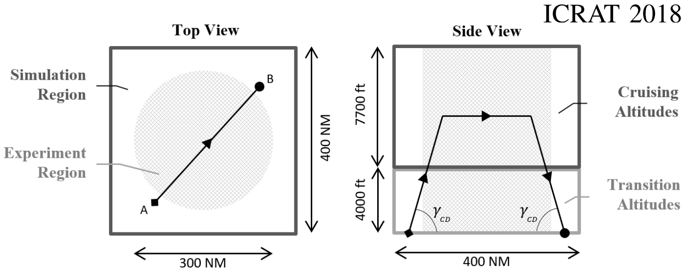


Fig. 8. Top and side views of the simulation's physical environment

C. Independent Variables

Although five traffic demands, and five repetitions per demand were used for all experiments, the specific traffic patterns used varied between the four simulation experiments. This is because each experiment focuses on analyzing the effect of one particular traffic scenario distribution on the accuracies of the baseline and adjusted conflict count models. The follow paragraphs describe the traffic speed, heading, altitude and spatial distributions used by each experiment.

1) Ground Speed Experiment
In this experiment, four different distributions were used to specify the ground speeds of aircraft, see Fig. 4. All speed distributions have a mean speed of 400 kts. Additionally, the scenarios of this experiment used uniform heading, altitude and spatial distributions. This resulted in a total of 100 simulation runs, involving over 250,000 flights.

2) Heading Experiment
For the heading experiment, simulations were repeated for the four different heading distributions shown in Fig. 5. Each heading distribution was combined with uniform altitude and spatial distributions. Furthermore, this experiment used an equal ground speed of 400 kts for all aircraft. This resulted in a total of 100 simulation runs, using over 250,000 flights.

3) Altitude Experiment
The altitude experiment considered the effect of the four altitude distributions displayed in Fig. 6 on conflict counts. For this experiment, the ground speed of all aircraft equaled 400 kts, while the altitude and spatial distributions of traffic were uniform. This resulted in a total of 100 simulation runs, with over 250,000 flights.

4) Spatial Experiment
The final experiment considered the effect of the spatial distribution of traffic on conflict counts. Therefore, the simulations were performed for three different spatial distributions, see Fig. 7. All spatial distributions were combined with traffic scenarios that had a uniform distribution of headings and altitudes, and with an equal ground speed of 400 kts for all traffic. This resulted in a total of 75 simulation, using almost 200,000 flights.

D. Dependent Variables

To determine the accuracy of both baseline and adjusted conflict count models, model predictions were compared to actual conflict counts logged during the simulations. Model accuracy was quantified by introducing a model accuracy parameter, k, as illustrated below:

$$\text{No. of Inst. Conflicts} = \text{Gas Model} \times k$$

From the above, it can be seen that k acts as a constant scaling parameter to the models. The value of k is determined by fitting the models to the simulation data in a least-square sense. A value of k close to 1 indicates high model accuracy, while $k < 1$ and $k > 1$ indicates model over- and under-estimation of simulation data, respectively. For easy analysis of the results, model accuracy was also computed as a percentage by comparing the fitted k value to a reference value of 1.

V. Results and Discussion

The results of the fast-time simulation experiments are presented and discussed in this section. The analysis is concerned with the effect of traffic scenario distributions on the accuracy of the analytical and adjusted conflict count models. As stated previously, the results are valid for a direct-routing Unstructured Airspace (UA) design.

A. Accuracy of the Analytical Model for Ideal Traffic Scenario

Before studying the effect of the four traffic scenario distributions on model accuracy, it is useful to consider the performance of the baseline analytical conflict count model for the 'ideal' traffic scenario. For the ideal case, all traffic scenario properties match the assumptions made during the derivation of the analytical model, i.e., all aircraft are assumed to fly with equal ground speeds, with uniform heading, altitude and spatial traffic distributions. The corresponding results are shown in Fig. 9. In this figure, the solid line represents the prediction of the analytical model, whereas the scatter points display the raw simulation data.

From Fig. 9 it can be clearly seen that the baseline analytical model, given by (2), closely approximates both the shape and the scaling of the relationship between the number of instantaneous aircraft (input) and the number of instantaneous conflicts (output). In fact, using the method described in section IV-D, the accuracy of the baseline analytical model was computed to be 97.6% for ideal traffic scenarios.

As implied in the above paragraph, the accuracy of a model depends on its ability to correctly predict both the *shape* and the *scaling* of the relationships between its input and output parameters. For gas-models, the shape of the model is dependent on the modeling of the number of combinations of two aircraft, i.e. the first component of gas models, see (1). The scaling, on the other hand, is dependent on the computation of the conflict probability between aircraft, i.e, the second component of gas models, see (1), since conflict probability, as defined in this paper, is independent of traffic density.

The model adjustment derivations described in section III have showed that all traffic scenario properties only affect the computation of the conflict probability between aircraft for UA, i.e., the scaling of the model. As such, the number of combinations of aircraft, or equally the shape of gas models, is not affected by traffic scenario distributions. For this reason, the analysis that follows for each of the four simulation experiments will focus on whether the analytical models can correctly compute the conflict probability between aircraft, or equally, the *scaling* of the number of combinations of two aircraft. Moreover, if the analytical models can't predict the scaling correctly, then the analysis will consider if the model adjustments derived in section III can be used to improve accuracy for non-ideal traffic scenarios.

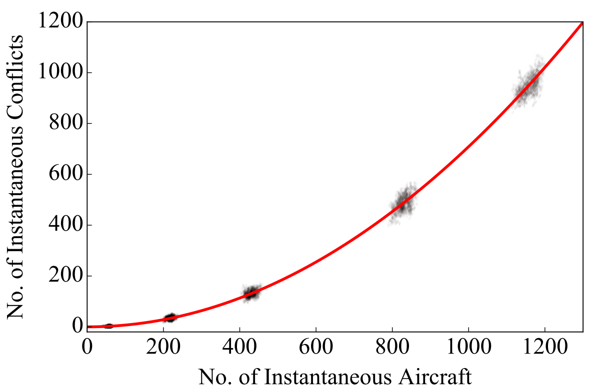


Fig. 9. Simulation results (scatter points) and analytical model prediction (solid line) for the ideal traffic scenario

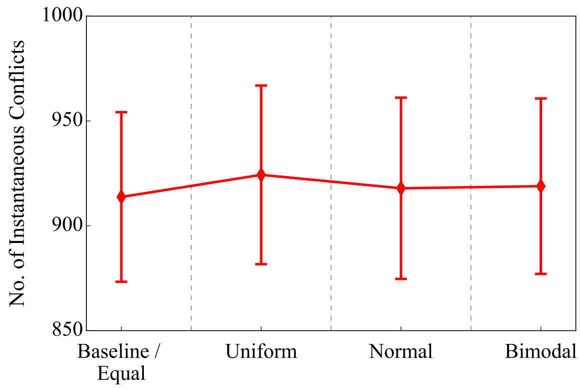


Fig. 10. Means and 95% confidence intervals of conflict counts at the highest traffic demand condition for the ground speed experiment

TABLE IV
MODEL ACCURACY GROUND SPEED EXPERIMENT

	Baseline Equal	Uniform	Normal	Bimodal
Analytical	1.024 (97.6%)	1.026 (97.4%)	1.026 (97.4%)	1.020 (97.9%)
Adjusted	1.024 (97.6%)	1.022 (97.7%)	1.029 (97.0%)	1.019 (98.0%)

B. Ground Speed Experiment

The analytical conflict count model assumes that all aircraft fly with equal ground speed. Since this assumption deviates from realistic operations, the ground speed experiment investigated the sensitivity of the analytical model to this assumption by repeating traffic simulations for the four speed distributions displayed in Fig. 4.

1) Effect of Speed Distribution on Conflict Counts

Before evaluating the effect of the equal speed assumption on model accuracy, it is useful to compare the actual conflict count results for the four simulated speed distributions. To this end, Fig. 10 displays the means and the 95% confidence intervals of the number of conflicts logged at the highest traffic demand condition for all speed distributions. Here it can be seen that speed distribution has a negligible effect on conflict counts for UA. This invariance of conflict counts with ground speed distribution can be explained by the fact that the same average ground speed is used by all four distributions. Therefore, this result indicates that the shape of the speed distribution does *not* affect the conflict probability between aircraft; instead, conflict probability is only affected by the magnitude of the average ground speed of all aircraft.

2) Effect of Speed Distribution on Model Accuracy

The accuracy results for the ground speed experiment are listed in Table IV for both the analytical and adjusted models. Here it should be noted that the analytical model was evaluated assuming equal speeds for all aircraft, regardless of the actual distribution used in the simulation. As evidenced by Fig. 10, the accuracy of the analytical model is unaffected by speed distribution, and it remains very high for all conditions. This trend was hypothesized during the derivation of the ground speed model adjustment, where it was found that the expected horizontal relative velocity was largely unaffected by speed distribution, see Table I. Nonetheless, the ground speed adjustment, which is also shown in Table IV to have produced high model accuracies, can be useful for scenarios where the speed distribution is non-equal *and* the heading/altitude/spatial distribution is also non-uniform.

C. Heading Experiment

The heading experiment considered the accuracy of the analytical conflict count models for the four heading distributions pictured in Fig. 5.

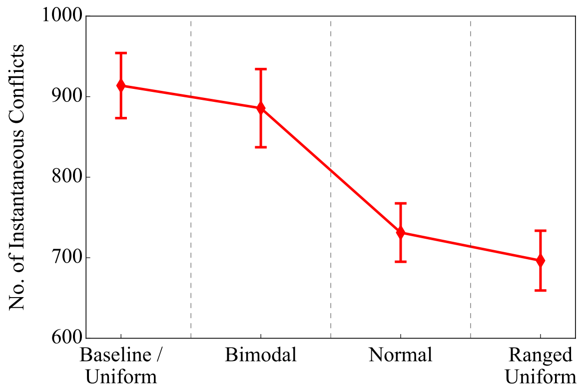


Fig. 11. Means and 95% confidence intervals of conflict counts at the highest traffic demand condition for the heading experiment

TABLE V
MODEL ACCURACY HEADING EXPERIMENT

	Baseline Uniform	Bimodal	Normal	Ranged Uniform
Analytical	1.024 (97.6%)	1.004 (99.5%)	0.812 (76.8%)	0.768 (69.9%)
Adjusted	1.024 (97.6%)	1.041 (96.0%)	0.982 (98.1%)	0.974 (97.3%)

1) Effect of Heading Distribution on Conflict Counts
Figure 11 displays the effect of heading distribution on conflict counts. Here it can be seen that conflict counts are the highest when aircraft headings are uniformly distributed, i.e., for the distribution assumed by the baseline analytical model. Although there are no significant differences between the uniform and bimodal distributions, the normal and ranged uniform distributions led to substantially lower conflict counts. Moreover, the relative safety differences between the four heading distributions match the relative differences between the expected horizontal relative velocity between aircraft, see Table II. These results strongly indicate that the heading distribution of traffic can have an effect on the intrinsic safety of UA; the magnitude of this effect depends on the shape of the distribution used.

2) Effect of Heading Distribution on Model Accuracy
The model accuracy results for the heading experiment are listed in Table V. As suggested by Fig 11, the analytical model, which assumes a uniform heading distribution for all simulation conditions, over-estimates conflict counts when the actual headings in the simulation followed normal and ranged-uniform distributions (k<1). In addition to indicating the accuracy of the models, the 'k' values for the analytical model can also be used to compute the relative differences between conditions; for example, Fig. 11 shows that the conflict count for the normal distribution condition is approximately 0.8 times lower than the count for the uniform condition, since k≈0.8 for the normal distribution, and k≈1 for the uniform condition.

Table V also indicates that the inaccuracies of the analytical model can be effectively compensated for by using the adjusted model; the numerically adjusted model increases model accuracy for the normal and ranged-uniform heading distributions to the level found using the analytical model for the uniform case. This implies that that the model adjustment for heading distributions, derived in section III-B, correctly determines the effect aircraft headings on the conflict probability between aircraft.

D. Altitude Experiment

To evaluate the effect of altitude distribution conflict count model accuracy, in this experiment, traffic simulations were conducted for the four altitude distributions pictured in Fig. 6

1) Effect of Altitude Distribution on Conflict Counts
Figure 12 clearly shows that the altitude distribution has a large impact on conflict counts for UA. In contrast to the heading experiment, the uniform altitude distribution, corresponding to the setting assumed by the baseline analytical model, led to the lowest number of conflicts. On the other hand, the ranged-uniform condition led to highest number of conflicts of all tested distributions. Furthermore, the differences between the studied conditions closely match with the hypothesized effect of altitude distribution on conflict probability, see Table III.

2) Effect of Altitude Distribution on Model Accuracy
The effect of altitude distribution on model accuracy is shown in Table VI. The table indicated that the analytical model significantly under-estimates conflict counts (k>1) for all non-uniform altitude distributions. This result is unsurprising given the fact that the uniform altitude distribution was noted above to lead to the lowest number of conflicts of all studied conditions. But, model accuracies for the non-uniform conditions were found to be very high when the predictions of the adjusted conflict count model were compared with logged simulation data. This indicates that the impact of altitude distribution on conflict probability can be effectively taken into account using the corresponding model adjustment given by (11) and (14).

E. Spatial Experiment

While the previous experiment considered the effect of the vertical distribution of traffic, this experiment investigated the effect of the horizontal distribution of traffic, on the accuracy of the analytical conflict count model. Correspondingly, simulations were conducted for the three spatial distributions displayed in Fig. 7.

1) Effect of Spatial Distribution on Conflict Counts
Unlike the other experiments, error-bars are not used to compare the conditions of this experiment. This is because the scenarios with hot-spots resulted in much higher traffic densities due to the fact that the hot-spot scenarios, by their very nature, aim to create traffic concentrations within the 'experiment region'. Because such density differences are not visible with error bars, the spatial conditions can be compared using Fig. 13. In this figure, the simulation data (scatter points) is plotted together with the corresponding model *fits* (solid lines). The higher traffic densities for the hot-spot conditions can be easily seen; the highest traffic demand scenario resulted in approximately 1200 instantaneous aircraft for the uniform spatial distribution, while it is greater than 1400 instantaneous

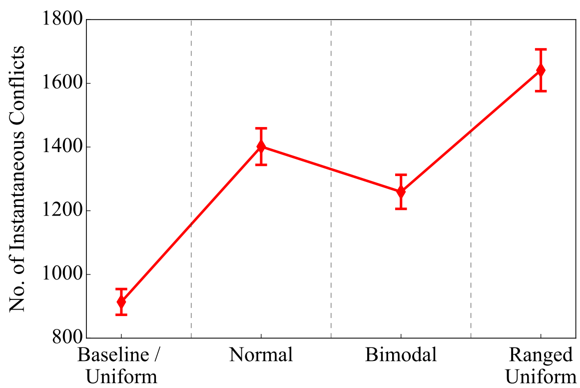


Fig. 12. Means and 95% confidence intervals of conflict counts at the highest traffic demand condition for the altitude experiment

TABLE VI
MODEL ACCURACY ALTITUDE EXPERIMENT

	Baseline Uniform	Normal	Bimodal	Ranged Uniform
Analytical	1.024 (97.6%)	1.569 (63.7%)	1.416 (70.6%)	1.576 (63.4%)
Adjusted	1.024 (97.6%)	1.102 (90.6%)	0.994 (99.4%)	0.957 (95.5%)

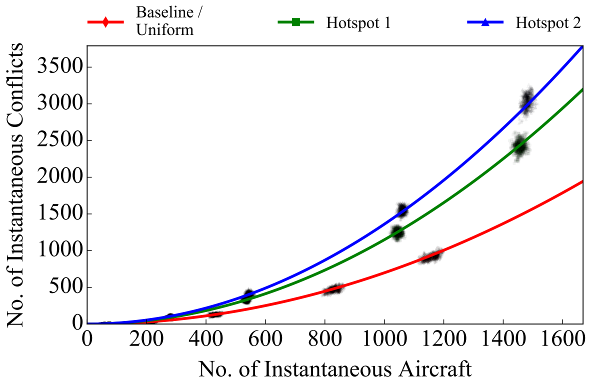


Fig. 13. Simulation results (scatter points) and model fits (solid line) for the spatial experiment

TABLE VII
MODEL ACCURACY SPATIAL EXPERIMENT

	Baseline Uniform	Hot-Spot 1	Hot-Spot 2
Analytical	1.024 (97.6%)	1.724 (57.9%)	2.077 (48.1%)
Adjusted	1.024 (97.6%)	0.906 (89.6%)	1.017 (98.2%)

aircraft for the hot-spot conditions. Furthermore, Fig. 13 indicates that the hot-spots led to a significantly higher number of conflicts, with hot-spot 2 resulting in the highest conflict counts of all traffic scenario distributions considered in this work. This implies that the spatial distribution of traffic can have a significant effect on conflict probability, and therefore on airspace safety.

2) Effect of Spatial Distribution on Model Accuracy
As expected, the model accuracy results for the spatial experiment, listed in Table VII, indicate that the analytical model grossly under-estimated the number of conflicts for the hot-spot conditions (k>1). On the other hand, model accuracy is significantly improved for the adjusted models. For instance, the accuracy for hot-spot 2 increases from 48.1% for the analytical model to 98.2% for the adjusted model. Although the adjusted model also improved accuracy considerably for hot-spot 1, it did so to a lesser extent than for hot-spot 2. This suggests that the model adjustment procedure for the spatial distribution is, while effective, less robust than the adjustments derived for the other traffic scenario properties.

VI. CONCLUSIONS

This paper investigated the effect of traffic scenario properties on the accuracy of gas model inspired analytical conflict count models. These analytical models are derived using idealized assumptions regarding the speed, heading, altitude and spatial distributions of traffic. The sensitivity of the analytical models to these assumptions was evaluated using four fast-time simulations experiments within the context of an unstructured airspace design concept. Data from these simulations is also used to test numerical 'model adjustments' that aim to relax the dependency of the conflict count models on the idealized traffic scenario assumptions. The following conclusions can be drawn:

- As found by previous research, the accuracy of the analytical model was found to be very high for ideal traffic scenarios that respect all scenario related modeling assumptions.
- However, the conflict probability between aircraft was incorrectly predicted by the analytical models when non-ideal heading, altitude and spatial traffic distributions were used. Consequently, the analytical models could predict the shape, but not the correct scaling, of the relationship between traffic density and instantaneous conflict counts for non-ideal settings of these three traffic scenario properties.
- Although the magnitude of the error between model predictions and simulation results depends on the specific distributions tested for each scenario property, for the studied conditions, it was found that the spatial distribution had the largest negative impact on the accuracy of the analytical models.
- On the other hand, the ground speed distribution did not significantly impact the accuracy of the analytical models. This result indicates that the shape of the speed distribution does not affect the conflict probability between aircraft; instead, conflict probability is only affected by the magnitude of the average ground speed of all aircraft.
- The numerical model adjustments derived in this work were found to increase model accuracy for all non-ideal traffic scenario distributions to the levels found with the analytical model for the ideal traffic scenarios. Consequently, the adjusted models can be used to accurately predict conflict counts for any traffic scenario, as long as the shapes of the underlying distributions are known or can be determined empirically.
- This research focused exclusively on conflict count models for an unstructured airspace design concept. To further increase the scope of this line of research, it is recommended to also develop analogous conflict count model adjustments for other airspace design concepts, most notably layered airspace designs. This will be the focus of future research.

ACKNOWLEDGMENTS

The research contained in this paper was performed as part of Ólafur Þórðarson's MSc thesis at TU Delft. This research would not been possible without the efforts of the open-source community that continually contributes to the development of the BlueSky ATM simulator. BlueSky can be downloaded from https://github.com/ProfHoekstra/bluesky.

REFERENCES

[1] Performance Review Commission, "Eurocontrol Performance Review Report 2016," Eurocontrol, Tech. Rep. PRR2016, 2016.
[2] J. M. Hoekstra, R. N. H. W. van Gent, and R. C. J. Ruigrok, "Designing for safety: the 'free flight' air traffic management concept," *Reliability Engineering & System Safety*, vol. 75, no. 2, pp. 215–232, Feb. 2002.
[3] J. M. Hoekstra, R. C. J. Ruigrok, and R. N. H. W. van Gent, "Free Flight in a Crowded Airspace?" in *Proceedings of the 3rd USA/Europe Air Traffic Management R&D Seminar*, Naples, Jun. 2000.
[4] M. Ballin, J. Hoekstra, D. Wing, and G. Lohr, "NASA Langley and NLR Research of Distributed Air/Ground Traffic Management," in *AIAA Aircraft Technology, Integration, and Operations (ATIO) Conference, AIAA-2002-5826*, 2002.
[5] J. K. Kuchar and L. C. Yang, "A Review of Conflict Detection and Resolution Modeling Methods," *IEEE Transactions on Intelligent Transportation Systems*, vol. 1, pp. 179–189, 2000.
[6] E. Sunil, J. Ellerbroek, J. Hoekstra, A. Vidosavljevic, M. Arntzen, F. Bussink, and D. Nieuwenhuisen, "Analysis of Airspace Structure and Capacity for Decentralized Separation Using Fast-Time Simulations," *Journal of Guidance, Control, and Dynamics*, vol. 40, no. 1, pp. 38–51, 2017.
[7] S. Endoh, "Aircraft collision models," Ph.D. dissertation, Massachusetts Institute of Technology, Flight Transportation Laboratory, 1982. [Online]. Available: http://dspace.mit.edu/handle/1721.1/68072
[8] M. R. Jardin, "Analytical Relationships Between Conflict Counts and Air-Traffic Density," *Journal of Guidance, Control, and Dynamics*, vol. 28, no. 6, pp. 1150–1156, 2005.
[9] Hoekstra, J., Maas, J., Tra, M., and Sunil, E., "How Do Layered Airspace Design Parameters Affect Airspace Capacity and Safety?" in *Proceedings of the 7th International Conference on Research in Air Transportation*, Jun. 2016.
[10] Sunil, E., Ellerbroek, J., Hoekstra, J.M., and Maas, J., "Three-Dimensional Conflict Count Models for Unstructured and Layered Airspace Designs," *submitted to Transportation Research Part C: Emerging Technologies*, Dec. 2017.
[11] J. Hoekstra and J. Ellerbroek, "BlueSky ATC Simulator Project: an Open Data and Open Source Approach," in *Proceedings of the 7th International Conference on Research in Air Transportation*, Jun. 2016.
[12] T. Langejan, E. Sunil, J. Ellerbroek, and J. Hoekstra, "Effect of ADS-B Characteristics on Airborne Conflict Detection and Resolution," in *Proccedings of the 6th Sesar Innovation Days*, 2016.