Minimum drag control allocation for the Innovative Control Effector aircraft

*Optimal use of control redundancy on modern fighters*

A.R.J. Stolk

September 28, 2017
Minimum drag control allocation for
the Innovative Control Effector aircraft
Optimal use of control redundancy on modern fighters

MASTER OF SCIENCE THESIS

For obtaining the degree of Master of Science in Aerospace Engineering
at Delft University of Technology

A.R.J. Stolk

September 28, 2017
The undersigned hereby certify that they have read and recommend to the Faculty of Aerospace Engineering for acceptance a thesis entitled “Minimum drag control allocation for the Innovative Control Effector aircraft” by A.R.J. Stolk in partial fulfillment of the requirements for the degree of Master of Science.

Dated: September 28, 2017

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Summary

To increase agility and reduce observability, the next generation fighter aircraft is likely to feature a tailless design with control surfaces for pitch and yaw relocated on the main wing. One concept in this field is the Innovative Control Effector (ICE) aircraft, equipped with 11 control surfaces with overlapping functionality and two-directional thrust vectoring. Its complex aerodynamics and nonlinear coupled control characteristics however yield challenges in control law design.

First of all conventional (linear) modeling techniques and control allocation methods do not account for nonlinear input dynamics and interaction within the controls. In this thesis a (spline-based) incremental control allocation technique is proposed, that calculates the input increments based on the control demand increment, instead of the total inputs to achieve the total control demand. By locally linearizing the control effectiveness nonlinear input dynamics are properly handled while the control allocation problem can still be solved by well-known linear methods. On the ICE aircraft incremental control allocation clearly outperforms linear control allocation, as it shows lower allocation errors and superior rate tracking.

The second control challenge is that the number of inputs of the aircraft (13 effectors) exceeds the number of directions to control (3 rotations), so the control allocation problem is under-determined. Previous research propose including a secondary objective that provides uniqueness to the solution while it minimizes the control effort. Given the excessive number and diverse nature of the controls on the ICE aircraft however, it is considered that these general (fixed) objectives do not result in the most operationally attractive way of flying. This thesis therefore presents two control allocation objectives to minimize drag. One of them is based on penalizing the most resistant controls, while the other uses the onboard aerodynamic model. Both modes are tested in a general flight scenario and reduce the average drag by about 6.5% relative to a standard control allocation scheme. For practical applications the second mode is argued favorable because it is less sensitive to initial tuning and is considered to maintain its performance under a larger set of conditions.

The main conclusion is that mission-specific secondary objectives can indeed increase the operational performance of an aircraft. Future research should focus on developing objectives for other missions and on testing in more extreme regions of the flight envelope. The most important recommendation is to improve the ICE aerodynamic model, such that physically implausible phenomena are corrected. A higher fidelity model can substantiate the true potential of mission-specific control allocation for next generation fighter aircraft.
A.R.J. Stolk

Minimum drag control allocation for the Innovative Control Effector aircraft
Dear reader,

This thesis is the result of my graduation phase at the Delft University of Technology and the crown on 6 years of aerospace engineering study.

It was about one-and-a-half year ago that Mike Niestroy from Lockheed Martin visited the aerospace engineering faculty in Delft. The attenders were presented a simulation model of a next generation fighter aircraft concept with unusual aerodynamic properties. I was directly interested in the control side of this aircraft, and, when in October 2016 it was time for me to acquire a graduation assignment, the choice was easy.

Coen de Visser, my daily supervisor, and I decided to take a look into the physical impact of control design choices and to focus on mission-specific control allocation methods. After some preliminary work on various mission profiles, we chose a minimum drag objective for publication. The results are encouraging and I really hope my work contributes to a new generation of safer and more efficient aircraft.

I want to thank Mike Niestroy for giving me the opportunity to work with this engaging model. Many thanks to Coen de Visser for his guidance, feedback, and belief in a good outcome of the research, even when I turned skeptical. My thanks go also to my fellow 'ICE-men' for our interesting discussions about the topic and valuable input back and forth. I am sure many other students will join the club and make good use of our pioneering. Finally I want to thank my friends and family for their support and attempts to understand my passion for aircraft control.

Rob Stolk

Delft, September 2017
**Acronyms**

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<th>Description</th>
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<tr>
<td>AMS</td>
<td>Attainable moment set</td>
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<tr>
<td>AMT</td>
<td>All-moving wing tip</td>
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<td>CA</td>
<td>Control allocation</td>
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<tr>
<td>DRUD</td>
<td>Deployable rudder</td>
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<tr>
<td>FXP</td>
<td>Fixed-point</td>
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<tr>
<td>ICE</td>
<td>Innovative control effector</td>
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<tr>
<td>INCA</td>
<td>Incremental control allocation</td>
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<tr>
<td>LCA</td>
<td>Linear control allocation</td>
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<tr>
<td>LED</td>
<td>Leading edge down</td>
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<tr>
<td>LEF</td>
<td>Leading edge flap</td>
</tr>
<tr>
<td>LSP</td>
<td>Lower surface spoiler</td>
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<tr>
<td>MATV</td>
<td>Multi-axis thrust vectoring</td>
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<tr>
<td>MB</td>
<td>Model-based mode</td>
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<tr>
<td>NDI</td>
<td>Nonlinear dynamic inversion</td>
</tr>
<tr>
<td>PR</td>
<td>Prioritization mode</td>
</tr>
<tr>
<td>PTV</td>
<td>Pitch thrust vectoring</td>
</tr>
<tr>
<td>QP</td>
<td>Quadratic programming</td>
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<tr>
<td>RPI</td>
<td>Redistributed pseudo-inverse</td>
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<tr>
<td>SLS</td>
<td>Sequential least squares</td>
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<tr>
<td>SQP</td>
<td>Sequential quadratic programming</td>
</tr>
<tr>
<td>SSD</td>
<td>Spoiler slot deflector</td>
</tr>
<tr>
<td>ST</td>
<td>Standard mode</td>
</tr>
<tr>
<td>TED</td>
<td>Trailing edge down</td>
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<tr>
<td>TEU</td>
<td>Trailing edge up</td>
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<tr>
<td>WLS</td>
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<tr>
<td>YTV</td>
<td>Yaw thrust vectoring</td>
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### Greek Symbols

- $\alpha$: Aerodynamic angle of attack, deg
- $\beta$: Aerodynamic angle of sideslip, deg
- $\delta$: Control surface deflection, deg
- $\varepsilon$: Scaling factor
- $\lambda$: Scaling factor
- $\rho$: Air density, slug/ft$^3$
- $\tau$: Virtual input vector
- $\omega$: Angular rate vector, rad/s

### Roman Symbols

- $A_{x,y,z}$: Body accelerations, ft$^2$/s
- $A,Y,N$: Body forces (in aerodynamic model), lbf
- $B$: Control effectiveness matrix
- $b$: Wing span, ft
- $B^d$: B-form basis polynomial of degree $d$
- $b(x)$: Barycentric coordinate
- $C$: Dimensionless coefficient
- $\bar{c}$: Mean aerodynamic chord, ft
- $c$: B-coefficient vector
- $D$: Drag force, lbf
- $E$: Drag effectiveness matrix
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<td><strong>F</strong></td>
<td>Force vector, lbf</td>
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<td><strong>G</strong></td>
<td>Generalized inverse matrix</td>
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<tr>
<td><strong>g</strong></td>
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<td><strong>I</strong></td>
<td>Inertia matrix</td>
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<tr>
<td><strong>J</strong></td>
<td>Objective function</td>
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<tr>
<td><strong>L</strong></td>
<td>Lift force, lbf</td>
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<tr>
<td><strong>l, m, n</strong></td>
<td>Body moments, lbf·ft</td>
</tr>
<tr>
<td><strong>l_{tv}</strong></td>
<td>Thrust vectoring arm, ft</td>
</tr>
<tr>
<td><strong>M</strong></td>
<td>Moment vector, lbf·ft</td>
</tr>
<tr>
<td><strong>p, q, r</strong></td>
<td>Pitch, roll, and yaw rate rad/s</td>
</tr>
<tr>
<td><strong>Q</strong></td>
<td>Weighting matrix</td>
</tr>
<tr>
<td><strong>S</strong></td>
<td>Wing surface, ft²</td>
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<td><strong>T</strong></td>
<td>Thrust force, lbf</td>
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<tr>
<td><strong>u</strong></td>
<td>Input vector</td>
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<td><strong>u_p</strong></td>
<td>Preferred control input</td>
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<td><strong>u, v, w</strong></td>
<td>Body velocities, ft/s</td>
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<tr>
<td><strong>W</strong></td>
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<td><strong>x</strong></td>
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<td><strong>X, Y, Z</strong></td>
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**Subscripts**

- **a**: Aileron
- **c**: Command
- **e**: Elevator
- **lamt**: Left all-moving wing tip
- **lele**: Left elevon
- **llefi**: Left leading edge flap inboard
- **llefo**: Left leading edge flap outboard
- **lssd**: Left spoiler slot deflector
- **pf**: Pitch flaps
- **ptv**: Pitch thrust vectoring
- **r**: Rudder
- **ramt**: Right all-moving wing tip
rele Right elevon
rlefi Right leading edge flap inboard
rlefo Right leading edge flap outboard
rssd Right spoiler slot deflector
yttv Yaw thrust vectoring
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Chapter 1

Introduction

The future air combat environment demands low-observable super-maneuverable fighter aircraft with in-flight recovery capabilities. The first goal pushes research toward tailless configurations, as removal of the tail planes significantly decreases the radar cross-section. The absence of vertical and horizontal stabilizers also implies reduced stability, and thus in general increased maneuverability. The loss of control surfaces in the tail area necessitates the installation of unconventional control effectors that provide multi-axis moments. If applied redundantly, these innovative effectors can also contribute to the third goal: survivability.

An interesting research object in this field is the tailless fighter configuration developed under the Innovative Control Effector (ICE) program (Dorsett & Mehl, 1996; Dorsett et al., 1999). Recently a simulation model of this 65° leading edge swept delta wing configuration equipped with a large number of conventional and innovative control surfaces is released for research purposes (Niestroy et al., 2017). The aircraft is characterized by its relaxed directional stability, severe nonlinear dynamics, and the control surfaces show a high level of redundancy and nonlinear couplings. This yields the necessity of accurate nonlinear control laws combined with a smart control allocation approach.

An upcoming field in model-based aircraft control is the use of multivariate simplex spline models. It is considered that simplex splines have a much higher approximation power than ordinary polynomials, and that splines are specifically useful for identification of highly nonlinear systems with local irregularities. Spline-based control therefore is considered to be a very attractive tool for the ICE aircraft. Additionally, the excessive number of control surfaces and their overlapping functionality offers the potential to optimize mission-specific secondary objectives. Previous research included secondary objectives primarily to minimize the overall control effort, but changing the control allocation strategy to support different missions is less studied. The development of a nonlinear spline-based control technique that maximally exploits the effector redundancy given different mission profiles is the topic of the current research.

Minimum drag control allocation for the Innovative Control Effector aircraft

A.R.J. Stolk
1-1 Objective and research questions

The objective of this research is to improve the performance of tailless fighter aircraft with redundant innovative control effectors by developing and assessing a mission-specific spline-based control allocation technique. Although it is the aim to develop generally applicable technique, in this research it is to be applied to a simulation model of an aircraft developed under the Lockheed Martin Innovative Control Effector (ICE) program. The main research object of this project is therefore referred to as the 'ICE aircraft'.

This research project has three main internal goals. At first, the next generation fighter aircraft is characterized by a tailless design and a large number of control surfaces on the main wing. The stability and control dynamics of these type of aircraft are unknown and have to be investigated. Secondly spline-based identification and control is a relatively new research field, for which the ICE model forms an interesting test case due to its complex aerodynamics. At third little research is done to what extend mission-specific objectives in control allocation improve the operational performance of the aircraft over general (fixed) objectives. The mission for this study is to minimize the aerodynamic drag of the aircraft.

The three main goals of the project lead to the following research questions and sub-questions:

1. What are the control characteristics of the aircraft type under consideration according to the available simulation model?

2. What are recommended control allocation and system identification strategies for the aircraft?
   - What is the difference between control allocation algorithms in terms of objective fulfillment and computational complexity?
   - What is the difference between modeling techniques and in terms of model accuracy, allocation error, and tracking performance?

3. To what extend does mission-specificity in control allocation improve the operational performance of the aircraft?
   - How is a minimum drag profile translated into control allocation objectives?
   - How does minimum drag control allocation compare to general control allocation in terms of tracking performance and drag reduction?

The answer to the first question is a result of the investigation of the aircraft model and is described in chapter 3 of this thesis. The answer to the second question is introduced by the review on control allocation in chapter 4, and further substantiation chapter 6. Question 3 is answered in chapter 7 and can be considered the main scientific contribution of this work.
1-2 Approach

The research starts with a literature study on the ICE aircraft, nonlinear control theory, and control allocation in specific. At first general system dynamics are studied and nonlinear control of conventional aircraft is examined. Thereafter the characteristics of tailless multi-effecter aircraft are reviewed together with the aerodynamics of the innovative control surfaces of the ICE aircraft. The literature study ends with a survey on the topic of control allocation, exploring potential obstacles and proposed methods.

The research continues with studying some of the aforementioned methods in more detail, and applying them on the ICE aircraft simulation model. A comparison is done between the currently most common control allocation approach and the state-of-the-art technique. This involves both the form of the control allocation problem and the solving algorithms.

Finally the ability to increase operational performance of the aircraft by mission-specific control allocation is tested. Objectives to minimize drag are formulated and programmed in the simulation environment. Flight tests are performed and the results are compared. The validity of the results is assessed and a brief sensitivity analysis is carried out. Finally conclusions are drawn and recommendations for future research are formulated.

1-3 Thesis outline

To make the reader familiar with the topic, chapter 2 briefly explains the basics of aircraft control and defines the notation used in the remainder of this thesis. Chapter 3 subsequently treats the development and characteristics of the main object of this research: the Innovative Control Effector model. The fourth chapter gives a broad survey of control allocation and concludes the literature review.

In chapter 5 the simulation framework for this study is explained in detail. In chapter 6 the theory of (spline-based) incremental control allocation is introduced and its application on the ICE aircraft is assessed. Chapter 7 presents two minimum drag control allocation methods for the ICE aircraft, which forms the main contribution of this research. Finally in chapter 8 conclusions are drawn and recommendations for future research are given.

Appendices A and B contain the MATLAB code of the two control allocation algorithms used in this study. Appendix C includes additional plots of the simulations from chapter 7. Appendix D shows extra examples of mission-specific control allocation in a conceptual manner, and is a stepping stone for future research. In appendix E the scientific paper is included, which summarizes the most important work.
Chapter 2

Basics of aircraft control

Conventional aircraft are controlled using a small set of moving surfaces: the ailerons, elevator, and rudder. A convenient feature of these effectors is that there is only little cross-axis coupling: the elevator only affects pitch and aileron and rudder are used for lateral control. This leads to a very distinct system with three degrees of freedom and three control inputs. Note that the throttle is also a control input, but in this chapter only angular motions are treated, and speed control is left out.

In the early days, ailerons, elevator, and rudder were mechanically linked to the control column and pedals. With the rapid developments in the electronics industry in the past decades, computers became smaller and faster, and aircraft control slowly moved to the fly-by-wire principle: pilot inputs are measured electronically and the control surfaces are deflected via electronic actuation. The introduction of fly-by-wire opened up possibilities for the development of advanced control laws that improve stability and fault-tolerance. Aircraft control nowadays is mainly a matter of robust control algorithms together with smart model identification.

This chapter is meant as an introduction to aircraft dynamics and control, and to introduce the notation used in the remainder of this thesis. In the first section the derivation of the coefficients for stability and control is given. Section 2-2 treats the mathematics behind aircraft dynamics and in section 2-3 a general approach to model-based control of conventional aircraft is described.
2-1 Control and stability derivatives

In aerospace engineering, the dynamics of an aircraft are normally expressed using dimensionless coefficients for the aerodynamic forces and moments. In this research, a right-handed body axes system is used with $x_b$ pointing forward, $y_b$ pointing right (starboard), and $z_b$ pointing downwards. Clockwise moments about these axes are denoted by $l, m, n$. The coefficients in non-dimensional form are obtained by:

$$C_l = \frac{l}{\frac{1}{2} \rho V^2 S_b} \quad (2-1)$$

$$C_m = \frac{m}{\frac{1}{2} \rho V^2 S_c} \quad (2-2)$$

$$C_n = \frac{n}{\frac{1}{2} \rho V^2 S_b} \quad (2-3)$$

in which $\rho$ represents air density, $V$ the total airspeed, $S$ the wing surface, $b$ the wing span and $\bar{c}$ the mean aerodynamic chord.

Aerodynamic forces and moments are a function of aircraft states and inputs. Consider for example that the pitch moment is affected by the angle of attack ($\alpha$), pitch rate ($q$), and elevator deflection ($\delta_e$). If the terms in the denominator of Eq. 2-2 are constant, $C_m = f(m) = f(\alpha, q, \delta_e)$, and a first order Taylor expansion about an arbitrary state leads to (Mulder et al., 2013)

$$C_m = C_{m_0} + \frac{\partial C_m}{\partial \alpha} \alpha + \frac{\partial C_m}{\partial q} q + \frac{\partial C_m}{\partial \delta_e} \delta_e \quad (2-4)$$

The fraction terms are called stability and control derivatives, since they provide information about how a system reacts to for example increasing angle of attack or elevator deflection. This is discussed in more detail in chapter 3. For convenience $\frac{\partial C_m}{\partial \delta_e}$ is denoted as $C_{m_{\delta_e}}$, etc., so

$$C_m = C_{m_0} + C_{m_{\alpha}} \alpha + C_{m_{q}} q + C_{m_{\delta_e}} \delta_e \quad (2-5)$$

and similarly

$$C_l = C_{l_0} + C_{l_{\beta}} \beta + C_{l_p} \frac{p}{V} + C_{l_r} \frac{r}{V} + C_{l_{\delta_a}} \delta_a + C_{l_{\delta_r}} \delta_r \quad (2-6)$$

$$C_n = C_{n_0} + C_{n_{\beta}} \beta + C_{n_p} \frac{p}{V} + C_{n_r} \frac{r}{V} + C_{n_{\delta_a}} \delta_a + C_{n_{\delta_r}} \delta_r \quad (2-7)$$

in which $\beta$ is the angle of sideslip, $p$ and $r$ are the roll and yaw rate, and $\delta_a$ and $\delta_r$ are the aileron and rudder deflection respectively.

Please keep in mind that this formulation is a result of a first order Taylor expansion, so the coefficients are local derivatives. Their values differ for every state and the equations are only valid for small changes in state and control input.
2-2 Equations of motion

Most nonlinear systems can be represented by a set of ordinary differential equations, that is linear in the virtual input (Johansen & Fossen, 2013):

$$\dot{x} = f(x) + g(x) \tau$$  \hspace{1cm} (2-8)

in which $x$ is the system state and $\tau$ is a virtual control input. In mechanics for example, $\tau$ often represents forces and moments that act on the system. This virtual control input is in its turn a function of the system state and a physical control input $u$:

$$\tau = \Phi(x, u)$$  \hspace{1cm} (2-9)

Controlling such a system relies on inverting both relationships, as can be seen in figure 2-1. Whereas the system dynamics map $\tau \rightarrow x$, the motion controller should do the opposite. Similarly, control allocation involves the inversion of $u \rightarrow \tau$.

The focus of this chapter lies on the control of the aircrafts angular motion using the control surfaces. According to Eulers rotational equations, moments applied on a rigid body ($M$) are related to the inertia matrix ($\mathbf{I}$) and rotational velocity vector ($\omega$) by

$$M = \mathbf{I} \dot{\omega} + \omega \times \mathbf{I} \omega$$  \hspace{1cm} (2-10)

Rearranging Eq. 2-10, an explicit expression for the time derivative of the angular rate can be obtained:

$$\dot{\omega} = -I^{-1}(\omega \times I \omega) + I^{-1}M$$  \hspace{1cm} (2-11)

With the nondimensional coefficients from the previous section, this leads to

$$\begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = -I^{-1} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \times I \begin{bmatrix} p \\ q \\ r \end{bmatrix} + \frac{1}{2} \rho V^2 S \mathbf{I}^{-1} \begin{bmatrix} b \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} C_l \\ C_m \\ C_n \end{bmatrix}$$  \hspace{1cm} (2-12)

Given Eqs. 2-5 to 2-7, the vector of moment coefficients is a function of states and inputs:

$$\begin{bmatrix} C_l \\ C_m \\ C_n \end{bmatrix} = \begin{bmatrix} C_l_{0} + C_{l_p} \beta + C_{l_p} \frac{\rho V^2}{2} + C_{l_{e}} \frac{\rho V}{2} \alpha + C_{l_{a}} \delta_a + C_{l_{r}} \delta_r \\ C_{m_0} + C_{m_a} \alpha + C_{m_a} q + C_{m_e} \delta_e + C_{m_q} + C_{m_{a} \alpha} + C_{m_{a} \delta_a} + C_{m_{r} \delta_r} \\ C_{n_{0}} + C_{n_{p}} \beta + C_{n_{p}} \frac{\rho V^2}{2} + C_{n_{e}} \frac{\rho V}{2} \alpha + C_{n_{a}} \delta_a + C_{n_{r}} \delta_r \end{bmatrix}$$  \hspace{1cm} (2-13)

It is clear that Eqs. 2-12 and 2-13 have the form of Eqs. 2-8 and 2-9 respectively, with the vector of moment coefficients as the virtual control input $\tau$. Hence, the motion controller should be based on inverting Eq. 2-12 and control allocation involves inverting Eq. 2-13. This is the topic of the next section.
2-3 Nonlinear dynamic inversion

As the system maps \( u \to x \), the controller should calculate \( x_c \to u \). In the previous section, the system dynamics are divided using the virtual input \( \tau \). This makes it easy to also split up the control task into motion control \((x_c \to \tau_c)\) and control allocation \((\tau_c \to u)\). However, both relationships can be nonlinear, see Eqs. 2-8 and 2-9.

An often used technique to deal with motion dynamics like Eq. 2-8 is called Nonlinear Dynamic Inversion (NDI) (Snell et al., 1992). The control laws are based on inverting the nonlinear system of equations in order to derive an explicit expression for the control input. NDI makes use of a cascaded control architecture (see figure 2-2) with an inner loop controlling nonlinear dynamics and an outer loop linear controller.

Recalling Eq. 2-8
\[
\dot{x} = f(x) + g(x)\tau
\]
the control law is
\[
\tau_c = g^{-1}(x)[\dot{x}_c - f(x)]
\]
and \( x_c \) can be controlled by \( \dot{x}_c \) via a linear controller in the outer loop. In fact, \( \dot{x} \) in Eq. 2-15 could theoretically also have been \( \ddot{x} \) or any higher order derivative of the state.

Going back to the control of the rotational motions of an aircraft according to Eq. 2-12, it follows that \( f(x) \) includes the kinematics and \( g(x) \) is the relation between the moment vector and its dimensionless form:
\[
f(x) = -I^{-1}\left( \begin{bmatrix} p \\ q \\ r \end{bmatrix} \times I \begin{bmatrix} p \\ q \\ r \end{bmatrix} \right)
\]
\[
g(x) = \frac{1}{2} \rho V^2 S I^{-1} \begin{bmatrix} b & 0 & 0 \\ 0 & \bar{c} & 0 \\ 0 & 0 & b \end{bmatrix}
\]
resulting in the motion control law
\[
\begin{bmatrix} C_l \\ C_m \\ C_n \end{bmatrix}_c = \frac{I}{\frac{1}{2} \rho V^2 S} \begin{bmatrix} b & 0 & 0 \\ 0 & \bar{c} & 0 \\ 0 & 0 & b \end{bmatrix}^{-1} \left\{ \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix}_c + I^{-1} \left( \begin{bmatrix} p \\ q \\ r \end{bmatrix} \times I \begin{bmatrix} p \\ q \\ r \end{bmatrix} \right) \right\}
\]
Now the only thing that remains is the control allocation, i.e. mapping $\tau_c \rightarrow u$. It can actually be done rather straightforward here, since Eq. 2-13 can be written in the affine form $\tau = h(x) + Bu$:

$$
\begin{bmatrix}
C_l \\
C_m \\
C_n
\end{bmatrix} = 
\begin{bmatrix}
C_{l_t} + C_{l_q} \beta + C_{l_r} \frac{V}{r} + C_{l_r} \frac{V}{r} \\
C_{m_x} + C_{m_x} \alpha + C_{m_y} q \\
C_{m_x} + C_{m_x} \beta + C_{m_y} \frac{V}{r} + C_{m_y} \frac{V}{r}
\end{bmatrix} + 
\begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
0 & 0 & 0 \\
C_{l_t} & C_{l_t} & C_{l_t} \\
C_{m_x} & C_{m_x} & C_{m_x}
\end{bmatrix}
\begin{bmatrix}
\delta_a \\
\delta_e \\
\delta_r
\end{bmatrix}
$$

(2-19)

Writing the control input at front, the control allocation law is

$$
\begin{bmatrix}
\delta_a \\
\delta_e \\
\delta_r
\end{bmatrix} = 
\begin{bmatrix}
C_{l_t} & 0 & C_{l_t} \\
0 & C_{m_x} & 0 \\
C_{n_x} & 0 & C_{n_x}
\end{bmatrix}^{-1}
\begin{bmatrix}
C_l \\
C_m \\
C_n
\end{bmatrix} - 
\begin{bmatrix}
C_{l_t} + C_{l_q} \beta + C_{l_r} \frac{V}{r} + C_{l_r} \frac{V}{r} \\
C_{m_x} + C_{m_x} \alpha + C_{m_y} q \\
C_{m_x} + C_{m_x} \beta + C_{m_y} \frac{V}{r} + C_{m_y} \frac{V}{r}
\end{bmatrix}
\begin{bmatrix}
\delta_a \\
\delta_e \\
\delta_r
\end{bmatrix}
$$

(2-20)

An alternative to the two-step approach presented here is to combine motion control and control allocation in a single NDI step. Consider $\dot{x} = f(x) + g(x)\tau$ with $\tau = h(x) + Bu$ as in Eq. 2-19. The total system becomes

$$
\dot{x} = f(x) + g(x)h(x) + g(x)Bu
$$

(2-21)

with $\tilde{f}(x)$ including both kinematics and aerodynamics, and $\tilde{g}(x)$ representing the control effectiveness. With NDI from Eq. 2-15, a control law can be derived in a single step, and the resulting formula is in fact the same as Eqs. 2-18 and 2-20 combined. Although most literature on control of conventional aircraft consider this particular type of NDI, the applicability is limited. It should be clear that if $u$ cannot be separated from $\Phi(x,u)$, obtaining the required affine form for NDI is not possible. Also if the dimensions of $\tau$ and $u$ do not match, as is the case with e.g. over-actuated systems, the control effectiveness matrix $B$ is not square and $\tilde{g}(x)$ cannot be inverted.

On these grounds this research maintains the multi-stage control approach. The motion control law is actually the same for every aircraft, albeit with different (but constant) values for inertia, wing span, surface, and chord. It is control allocation that is oftentimes the most challenging part, since it relies on inconstant coefficients, and so thorough preliminary and/or online model identification is essential. Furthermore selecting the most convenient type of control allocation depends on the nature of the system, such as the presence of nonlinearities and cross-axis coupling. The next chapter therefore first describes the characteristics of the ICE aircraft, to return to the topic of control allocation in chapter 4.
Design of fighter aircraft ready for the future air combat environment leans basically on three pillars: maneuverability, survivability, and observability. Today's high agility requirements combined with fault-tolerance capabilities and low radar detection signature led to research on tailless multi-effector configurations.

The absence of a horizontal and vertical tailplane decreases the radar cross section significantly, improving 'stealth' features. Its influence on the handling qualities is twofold: maneuverability improves while stability reduces, characteristics that often go hand in hand. Stability issues and the lack of control surfaces in the tail area yield the urgency of a high number of control surfaces on the main wing. This redundant set of effectors can be used for active stabilization and is as well beneficial for fault-tolerance.

One development in the field of next generation aircraft with these properties is the Innovative Control Effector (ICE) study (Dorsett & Mehl, 1996; Dorsett et al., 1999). The first section of this chapter briefly describes the scope of the ICE study as well as the development of the model. The remainder of this chapter reveals the interesting aerodynamics of the aircraft. In section 4-2 the aerodynamic model is explained, while section 4-3 and 4-4 describe the stability and control derivatives.

3-1 Development

The Innovative Control Effector (ICE) study was a two-phased research started in 1993 by a cooperation of Lockheed Martin Tactical Aircraft Systems, Bihrle Applied Research, the Air Force's Wright Laboratory, and the Naval Air Warfare Center Aircraft Division. The primary goal was to identify and quantify the aerodynamics and performance of different low-observable tailless aircraft configurations with innovative control effectors. Innovative here means that they are never before applied on aircraft, or at least not in combination with each other. Two aircraft types were the subject of the research: a land-based 65° leading edge swept delta wing and a carrier-based canard-delta wing. This thesis only focuses on the first type, which is shown in figure 3-1.
The first phase of the ICE study concentrated on evaluating different configurations of the aircraft, that is, numerous combinations of effectors. While for pitch and roll control all configurations contained pitch flaps and trailing edge flaps (elevons), different types of control surfaces are investigated for yaw control. These included all-moving wing tips (AMT), spoiler slot deflectors (SSD), leading edge flaps (LEF), deployable rudders (DRUD) and lower surface spoilers (LSP). Additionally, research was done on the potential of thrust vectoring in pitch (PTV) and yaw (YTV) direction. It was considered though that YTV can only be applied in combination with aerodynamic control surfaces, due to high structural demands for thrust vectoring at high speeds. Also at low power settings, for example during the landing phase, thrust vectoring may not be powerful enough to obtain the required amount of control. Phase 1 of the ICE study relied for a large amount on previously collected wind tunnel data and in-house developed software to assess promising control concepts. The outcomes of the research, as it is documented in ADB212813 (Dorsett & Mehl, 1996), yields that the AMT is the most favorable effector for yaw control, followed by the SSD and the LEF.
For the second phase of the ICE study, the AMT and SSD concepts are selected for further analysis. Wind tunnel measurements on a scale model of the aircraft (figure 3-2) are conducted with varying sizes and locations of the AMTs and SSDs, with different velocities, angles of attack, angles of sideslip, and rotational rates. Also the aerodynamic interaction between the control surfaces is quantified. With the wind tunnel data the simulation model from phase 1 is updated and flying qualities of the aircraft are tested with and without multi-axis thrust vectoring (MATV). It turned out that the application of AMTs provided sufficient control power, and that the concept with only SSDs required additional yaw thrust vectoring at high angles of attack. In combination with MATV, all concepts showed very high maneuvering potential with respect to current fighter aircraft configurations. More information about ICE phase 2 can be found in ADB232172 (Dorsett et al., 1999).

A later publication on the ICE study (Addington & Myatt, 2000), introduced a concept with both AMTs, SSDs, and LEFs. The current research in fact considers the lay-out from Niestroy et al. (2017), with four leading edge flaps (inboard and outboard, both left and right) and MATV. The approximate locations of these effectors are shown in figure 3-3. The presence of such a wide range of control surfaces is expected to increase the possibilities of the aircraft and yields an interesting control problem. Before looking into how to use this effector redundancy, the remainder of this chapter focuses on the aerodynamics of the aircraft body and control surfaces.

**Figure 3-3:** Effectors of the ICE aircraft considered in this research (Dorsett & Mehl, 1996).
3-2 Aerodynamic model

The aerodynamic model of the ICE aircraft is more complex than that of a conventional aircraft as in Eqs. 2-5 to 2-7. The current model consists of 108 sub-coefficients over six degrees of freedom, and all sub-coefficients are nonlinearly dependent on multiple states. The moment coefficients are arranged as follows:

\[
C_l = C_{l1}(\alpha, M) + C_{l2}(\alpha, \beta, M) + C_{l3}(\alpha, M, \delta_{lele}, \delta_{issd}) - C_{l4}(\alpha, M, \delta_{rele}, \delta_{rssd})
- C_{l5}(\alpha, \beta, \delta_{lele}) + C_{l6}(\alpha, \beta, \delta_{rele}) - C_{l7}(\alpha, \beta, M, \delta_{lele}, \delta_{lefo})
+ C_{l8}(\alpha, \beta, M, \delta_{lele}, \delta_{lefo}) + C_{l9}(\alpha, \delta_{lefo}, \delta_{amt})
- C_{l10}(\alpha, \delta_{refo}, \delta_{ramt}) + C_{l11}(\alpha, \delta_{ramt}, \delta_{lele}) - C_{l12}(\alpha, \delta_{ramt}, \delta_{rele})
+ C_{l13}(\alpha, \delta_{rele}, \delta_{rsle}, \delta_{rsle}) + C_{l14}(\alpha, \beta, \delta_{ramt})
- C_{l15}(\alpha, \beta, \delta_{rsle}) + C_{l16}(\alpha, \beta, \delta_{rssd}) - C_{l17}(\alpha, \beta, \delta_{rssd})
\]

\[
\frac{pb}{2V}C_{l18}(\alpha, M) + \frac{rb}{2V}C_{l19}(\alpha, M)
\]

\[
C_m = C_{m1}(\alpha, M) + C_{m2}(\alpha, \beta, M) + C_{m3}(\alpha, M, \delta_{lele}, \delta_{issd}) + C_{m4}(\alpha, M, \delta_{rele}, \delta_{rssd})
+ C_{m5}(\alpha, \beta, \delta_{lele}) + C_{m6}(\alpha, \beta, \delta_{rele}) + C_{m7}(\alpha, \beta, M, \delta_{lele}, \delta_{lefo})
+ C_{m8}(\alpha, \beta, M, \delta_{lele}, \delta_{lefo}) + C_{m9}(\alpha, \delta_{lefo}, \delta_{amt})
+ C_{m10}(\alpha, \delta_{refo}, \delta_{ramt}) + C_{m11}(\alpha, \delta_{ramt}, \delta_{lele}) + C_{m12}(\alpha, \delta_{ramt}, \delta_{rele})
+ C_{m13}(\alpha, \delta_{rele}, \delta_{rsle}, \delta_{rsle}) + C_{m14}(\alpha, \beta, \delta_{amt})
+ C_{m15}(\alpha, \beta, \delta_{ramt}) + C_{m16}(\alpha, \beta, \delta_{rssd}) + C_{m17}(\alpha, \beta, \delta_{rssd})
\]

\[
+ \frac{qe}{2V}C_{m18}(\alpha, M)
\]

\[
C_n = C_{n1}(\alpha, M) + C_{n2}(\alpha, \beta, M) + C_{n3}(\alpha, M, \delta_{lele}, \delta_{issd}) - C_{n4}(\alpha, M, \delta_{rele}, \delta_{rssd})
- C_{n5}(\alpha, \beta, \delta_{lele}) + C_{n6}(\alpha, \beta, \delta_{rele}) - C_{n7}(\alpha, \beta, M, \delta_{lele}, \delta_{lefo})
+ C_{n8}(\alpha, \beta, M, \delta_{lele}, \delta_{lefo}) + C_{n9}(\alpha, \delta_{lefo}, \delta_{amt})
- C_{n10}(\alpha, \delta_{refo}, \delta_{ramt}) + C_{n11}(\alpha, \delta_{ramt}, \delta_{lele}) - C_{n12}(\alpha, \delta_{ramt}, \delta_{rele})
+ C_{n13}(\alpha, \delta_{rele}, \delta_{rsle}, \delta_{rsle}) + C_{n14}(\alpha, \beta, \delta_{amt})
- C_{n15}(\alpha, \beta, \delta_{ramt}) + C_{n16}(\alpha, \beta, \delta_{rssd}) - C_{n17}(\alpha, \beta, \delta_{rssd})
\]

\[
+ \frac{pb}{2V}C_{n18}(\alpha, M) + \frac{rb}{2V}C_{n19}(\alpha, M)
\]

with \(\delta\) the effector deflection and the abbreviations of the control surfaces given in table 3-1 in section 3-4.

The organization of the sub-coefficients is in fact for all directions the same. The fourth sub-coefficient for example always captures the interaction between the angle of attack, Mach number, right elevon deflection, and right SSD deflection. These relations are however not linear and no affine form exists, making it impossible to write the equations like \(\tau = h(\mathbf{x}) + Bu\). Furthermore, there is no single sub-coefficient that directly indicates the derivative w.r.t. a state or input, so the only way to get an indication of the stability and control derivatives is by adding up all sub-coefficients and examining the total contribution. This is the topic of the upcoming sections.
3-3 Stability

On conventional aircraft longitudinal and directional stability is assured by the horizontal and vertical stabilizers. Due to the absence of these components, tailless aircraft generally suffer from stability issues in pitch and yaw direction. This section gives a theoretical background for static stability of aircraft and looks into the stability derivatives of the ICE model.

Note that lateral stability is oftentimes not an issue with tailless aircraft. Section 3.9.1 of ADB232172 (Dorsett et al., 1999) mentions that the baseline configuration (i.e. with neutral control surface deflections) shows sufficient roll-damping throughout the entire $\alpha, \beta$-region.

Longitudinal stability

For most aircraft, the aerodynamic center of the main wing lies in front of the center of gravity. Increasing angle of attack causes a higher lift force through the aerodynamic center and will therefore cause a pitch up moment. For stability this is highly undesirable because a small upgust will cause a nose up movement, increasing the angle of attack even further, etc. Since also the fuselage has a destabilizing effect, see e.g. Mulder et al. (2013), conventional aircraft are equipped with a horizontal tailplane. Increasing $\alpha$ causes a higher lift force of the horizontal tailplane, but because its aerodynamic center lies aft of the center of gravity, the resulting moment is pitch down. The size and shape of the tailplane should be chosen to balance out the nose-up tendency of the fuselage and main wing.

The ICE aircraft however does not have a horizontal stabilizer, so it is interesting to look at its derivative of pitch moment coefficient with respect to angle of attack $C_{m\alpha} = \frac{\partial C_m}{\partial \alpha}$, see section 2-1). The slope of $C_{m\alpha}$ in different conditions is shown in figures 3-4 and 3-5. The plots in the ICE phase 2 documentation (Dorsett et al., 1999) show a similar response.

An increase of $\alpha$ should cause a negative (nose down) pitching moment, so the desired sign of $C_{m\alpha}$ is negative. As can be seen for the ICE aircraft, in subsonic flight the slope in the $C_{m\alpha}/\alpha$-plot is positive for $15^\circ < \alpha < 35^\circ$. The higher the airspeed, the smaller this unstable range. When suffering from sideslip, instability occurs at lower $\alpha$, but also the unstable range is smaller. In addition, it can be noticed that the relation between pitch moment and angle of attack is nonlinear and non-monotonic, whereas most conventional aircraft show a quasi-linear relationship (Mulder et al., 2013).

Conclusively, at least for a some part of the flight envelope there is no static longitudinal stability. This induces the necessity of artificial stabilization, meaning that a flight computer actively steers the aircraft to maintain its attitude.
Figure 3-4: Pitch coefficient vs. angle of attack.
\( \beta = 0^\circ, \omega = 0 \text{ rad/s}, \text{ all } \delta = 0^\circ \)

Figure 3-5: Pitch coefficient vs. angle of attack.
Mach 0.5, \( \omega = 0 \text{ rad/s}, \text{ all } \delta = 0^\circ \)
Directional stability

Comparable to the response to increasing angle of attack, it is interesting to examine the aircraft’s response to increasing angle of sideslip $\beta$. The belonging stability derivative is called the weathercock stability coefficient $C_{n\beta}$ (Mulder et al., 2013). It is convenient that the aircraft has the tendency to rotate about the top axis in order to reduce this sideslip. For stability, an increasing sideslip angle should make the aircraft yaw right, hence the preferred sign of $C_{n\beta}$ is positive. On conventional aircraft, the wing and fuselage have a small negative contribution to the static directional stability, but this is compensated by a dominant positive contribution from the vertical tailplane.

![Figure 3-6: Yaw coefficient vs. angle of sideslip. Mach 0.5, $\omega = 0$ rad/s, all $\delta = 0^\circ$](image)

It is known that tailless aircraft suffer from ‘relaxed’ yaw stability. For the ICE aircraft, the directional stability is shown in figure 3-6 for three different angles of attack. It is observed that only in high $\alpha, \beta$-regions the slope is positive, but that in a large part of the flight envelope $C_{n\beta}$ is negative. This conclusion is in accordance with section 3.9.1 of the ICE phase II report (Dorsett et al., 1999). Hence, directional stability should be artificially obtained by active yaw steering. Which effectors are suitable for this purpose is studied in the next section.
### 3-4 Control surfaces

As is mentioned in the beginning of this chapter, instead of the three conventional control surfaces (aileron, elevator, rudder), the ICE aircraft configuration contains eleven independently controllable control surfaces, combined with thrust vectoring in pitch and yaw direction. This makes a total of thirteen effectors, of which the abbreviations and deflection limits are given in table 3-1.

<table>
<thead>
<tr>
<th>Effector</th>
<th>Abbr.</th>
<th>Limits, deg</th>
<th>Positive deflection</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Left leading edge flap (LEF) inboard</td>
<td>llefi</td>
<td>0 / 40</td>
<td>LED</td>
</tr>
<tr>
<td>2 Left leading edge flap (LEF) outboard</td>
<td>llefo</td>
<td>-40 / 40</td>
<td>LED</td>
</tr>
<tr>
<td>3 Left all-moving wing tip (AMT)</td>
<td>lamt</td>
<td>0 / 60</td>
<td>TED</td>
</tr>
<tr>
<td>4 Left elevon</td>
<td>lele</td>
<td>-30 / 30</td>
<td>TED</td>
</tr>
<tr>
<td>5 Left spoiler slot deflector (SSD)</td>
<td>lssd</td>
<td>0 / 60</td>
<td>TEU / LED</td>
</tr>
<tr>
<td>6 Pitch flap</td>
<td>pf</td>
<td>-30 / 30</td>
<td>TED</td>
</tr>
<tr>
<td>7 Right leading edge flap (LEF) inboard</td>
<td>rlefi</td>
<td>0 / 40</td>
<td>LED</td>
</tr>
<tr>
<td>8 Right leading edge flap (LEF) outboard</td>
<td>rlefo</td>
<td>-40 / 40</td>
<td>LED</td>
</tr>
<tr>
<td>9 Right all-moving wing tip (AMT)</td>
<td>ramt</td>
<td>0 / 60</td>
<td>TED</td>
</tr>
<tr>
<td>10 Right elevon</td>
<td>rele</td>
<td>-30 / 30</td>
<td>TED</td>
</tr>
<tr>
<td>11 Right spoiler slot deflector (SSD)</td>
<td>rssd</td>
<td>0 / 60</td>
<td>TEU / LED</td>
</tr>
<tr>
<td>12 Pitch thrust vectoring</td>
<td>ptv</td>
<td>-15 / 15</td>
<td>down</td>
</tr>
<tr>
<td>13 Yaw thrust vectoring</td>
<td>ytv</td>
<td>-15 / 15</td>
<td>left</td>
</tr>
</tbody>
</table>

The main advantage of having an over-actuated system is that there are often multiple ways to achieve a certain command, so secondary objectives can be involved to increase the performance of the controller. Also for fault-tolerance, having redundant effectors is beneficial, as they can take over each others task in case of malfunctioning. However, allocating the control commands is not as simple as it is for a conventional aeroplane. This section treats the characteristics of all control surfaces of the ICE aircraft from a control point of view. Detailed information about the origin and the design of the control surfaces can be found in the original ICE documentation (Dorsett & Mehl, 1996; Dorsett et al., 1999).

#### Pitch flaps

The only two surfaces that are not independently controllable are the pitch flaps. These flaps are located left and right of the jet nozzle and are ganged, which means that they move simultaneously. Because these flaps deflect symmetrically, this is the only control input that affects just one direction of rotation, as can be seen in figure 3-8.

The pitch flaps cause a pure pitching moment, just like the elevator on conventional aircraft, which is a convenient property for control allocation. Due to the location at the center rear of the aircraft however, the control effectiveness of the pitch flaps is heavily dependent on aircraft states and deflections of other control surfaces (see $C_{m_{13}}$ in Eq. 3-2):

$$C_{m_{spf}} = f(\alpha, M, \delta_{pf}, \delta_{lssd}, \delta_{rssd})$$

(3-4)
3-4 Control surfaces

Elevons

The elevons are located outboard of the pitch flaps and can both separately be deflected. The elevons cause moments about all three axes of rotation, as is visible in figure 3-9. By treating the elevons as ailerons though, so ganging the left and right elevon oppositely, moments in pitch and yaw can approximately be canceled out, as shown in figure 3-10. However, coupling the elevon deflections reduces the flexibility of control allocation.

The elevons are located downstream of the spoiler slot deflectors, so the control effectiveness of the elevons significantly suffers from the SSD angle, see for example figure 3-11. With a left SSD deflection of 30° or more, the lines are almost horizontal, indicating that $C_{l,ele}$, $C_{m,ele}$, and $C_{n,ele}$ are about zero. Just like for the pitch flaps, the control effectiveness functions are complicated and involve both aircraft states and control surface deflections, e.g. according to $C_{l3}$ and $C_{l11}$ in Eq. 3-1:

$$C_{l,ele} = f(\alpha, M, \delta_{ele}, \delta_{ssd}, \delta_{lamt}) \quad (3-5)$$

Spoiler slot deflectors

Just in front of the elevons, the spoiler slot deflectors (SSDs) are located. These effectors actually consist of two surfaces: an upper surface spoiler and a lower surface deflector. In closed condition, the two surfaces cover a slot in the aft of the wing. Once deflected (max 60°), the slot is opened, increasing drag and decreasing lift. Deflecting the left SSD for example results in roll and yaw moments to the left, and a pitch up moment due to reduced lift on the rear part of the wing.

Remark that the upper spoiler is hinged on the upstream edge and deflects TEU, like a conventional spoiler, while the lower deflector is hinged on the downstream edge and deflects LED. The hinge moments of the two surfaces counterbalance each other to a large extend, requiring less actuator power than conventional spoilers.

It can be observed from figures 3-8 to 3-15 that the SSDs provide more yaw power in low angle of attack regions than any other control surface. At higher angles of attack, the SSDs lose their yaw power, and additional thrust vectoring may be required, as is mentioned in section 3-1. Though at higher angles of attack opening the SSDs is useful to recover flow over the trailing edge surfaces. It is notable that, as the primary task of the SSDs is increasing drag, their use generally requires higher power settings. Moreover, according to Dorsett & Mehl (1996) and J. F. Buffington (1999), SSD deflection increases the radar cross-section much more than the other controls. A final disadvantage is that deflection of the SSDs heavily influences the control effectiveness of the trailing edge control surfaces, as is visualized in e.g. fig 3-11. This forms a severe challenge in control system design.

All-moving wing tips

At the aft corners of the delta wing, all-moving wing tips are present. Both can deflect separately from 0° to 60° TED (see figure 3-7) and are primarily installed for their yaw power at high angles of attack. Compare for example figures 3-12 and 3-13: whereas the SSDs lose their yaw power at high angles of attack, $C_{n,laamt}$ is about constant over all values of $\alpha$. 

Minimum drag control allocation for the Innovative Control Effector aircraft

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In phase I of the ICE study (Dorsett & Mehl, 1996), the AMTs are ranked first with respect to control power, integration impact, and low observability in use. From a control allocation perspective, the roll and pitch response cause difficulties though. It can be seen that the sign of $C_{t\delta_{lamt}}$ at a given $\delta_{lamt}$ is a strong nonlinear function of $\alpha$. The non-monotonic character of the control effectiveness functions requires advanced control allocation techniques, which is addressed in chapter 4.

**Leading edge flaps**

On the front edge of the aircraft, four leading edge flaps are present. The inboard leading edge flaps (LEFs) can deflect up to 40° (LED), while the outboard LEFs can deflect 40° upwards and downwards. Moment generation by leading edge flaps is much less significant than by other control surfaces, as can be observed from figures 3-14 and 3-15.

However, the leading edge flaps can for example be used to improve longitudinal stability, as can be seen in figure 3-16. With both leading edge flaps deflected maximally downwards, longitudinal stability (negative $C_{m\alpha}$) can be extended up to $\alpha = 20^\circ$. Additionally, leading edge flaps can significantly decrease the value of $C_{m\alpha}$, repressing the aircrafts nose-up tendency between $\alpha = 20^\circ$ and $\alpha = 40^\circ$. Furthermore, leading edge flaps are commonly used in the take-off and landing phase (J. F. Buffington, 1999).

Since the outboard LEF is directly located downstream the inboard LEF, as can be seen in figure 3-3, there is a lot of aerodynamic interaction between the two. This makes the control effectiveness of the outboard flaps a strong function of the inboard LEF deflection.
Figure 3-8: Moment coefficients vs. pitch flaps deflection.
\[ \alpha = \beta = 0^\circ, \ \omega = 0 \text{ rad/s}, \ \text{all other } \delta = 0^\circ \]

Figure 3-9: Moment coefficients vs. left elevon deflection.
\[ \alpha = \beta = 0^\circ, \ \omega = 0 \text{ rad/s}, \ \text{all other } \delta = 0^\circ \]

Figure 3-10: Moment coefficients vs. oppositely coupled elevon deflection.
The elevons behave like ailerons, producing roll only.
Mach 0.5, \[ \alpha = \beta = 0^\circ, \ \omega = 0 \text{ rad/s}, \ \text{all other } \delta = 0^\circ \]
Figure 3-11: Moment coefficients vs. left elevon deflection.
SSDs decrease the effectiveness of the elevons.
Mach 0.5, $\alpha = \beta = 0^\circ$, $\omega = 0$ rad/s, all other $\delta = 0^\circ$

Figure 3-12: Moment coefficients vs. left SSD deflection.
SSDs lose yaw power at high angles of attack.
Mach 0.5, $\beta = 0^\circ$, $\omega = 0$ rad/s, all other $\delta = 0^\circ$

Figure 3-13: Moment coefficients vs. left AMT deflection.
AMTs provide constant yaw power, but the roll & pitch response is non-monotonic.
Mach 0.5, $\beta = 0^\circ$, $\omega = 0$ rad/s, all other $\delta = 0^\circ$
3-4 Control surfaces

Figure 3-14: Moment coefficients vs. left inboard LEF deflection.
Mach 0.5, $\beta = 0^\circ$, $\omega = 0$ rad/s, all other $\delta = 0^\circ$

Figure 3-15: Moment coefficients vs. left outboard LEF deflection.
Mach 0.5, $\beta = 0^\circ$, $\omega = 0$ rad/s, all other $\delta = 0^\circ$

Figure 3-16: Pitch coefficient vs. angle of attack, varying leading edge flap deflection.
Mach 0.5, $\beta = 0^\circ$, $\omega = 0$ rad/s, all other $\delta = 0^\circ$
Innovative Control Effector aircraft

A.R.J. Stolk

Minimum drag control allocation for the Innovative Control Effector aircraft
Chapter 4

Review of control allocation

With the modular design from chapter 2, the regulation task of the control system is separated from the actuator selection task. After the motion controller calculates the virtual control command for the desired state, the control allocator relates this to the set of physically controllable variables. In other words, control allocation (CA) is the process of distributing a certain control demand over the control inputs.

The crux of CA can be illustrated by the elementary example from Härkegård (2003). Consider that two inputs $u_1$ and $u_2$ act on a system, and that for the desired motion a total input $u_1 + u_2 = 1$ is required. A broad range of solutions to this problem exist, for example $u_1 = 1$ and $u_2 = 0$, or vice versa. One can also evenly spread the command over the two inputs ($u_1 = u_2 = 0.5$), or choose a less obvious solution such as $u_1 = 11$, $u_2 = -10$. Smart CA relies on picking the most favorable combination given the current circumstances.

This chapter provides a literature review on the topic of CA. Section 4-1 revisits the general control problem from chapter 2 and discusses potential challenges. Sections 4-2 and 4-3 describe the CA problem from different angles, and list a variety of available methods and algorithms. Section 4-4 surveys existing research on CA of tailless aircraft, whereas section 4-5 treats spline-based control. Finally section 4-6 gives some final remarks and concludes the literature review.

4-1 Challenges

Recall from chapter 2 that in a modular control system, allocation is based on inverting

$$\tau = \Phi(x, u)$$

with $\tau \in \mathbb{R}^m$ and $u \in \mathbb{R}^p$. In some cases, the virtual command is linear in the control inputs, giving the affine form

$$\tau = B(x)u \quad \text{or} \quad \tau = h(x) + B(x)u$$

(4-2)
in which $B(x) \in \mathbb{R}^{m \times p}$ is called the control effectiveness matrix. Now if $p = m$, solving $u$ for a given $\tau$ simply relies on inverting the square matrix $B(x)$:

$$u = B(x)^{-1} \tau$$

or

$$u = B(x)^{-1}(\tau_c - h(x)) \quad (4-3)$$

Control allocation becomes more complicated for over-actuated systems, in which $p > m$. In these situations the control effectiveness matrix is not square and cannot be inverted to find the unique solution. In fact, the problem has less equations than unknowns, so an infinite set of solutions for $u$ exists, as long as $B(x)$ is not rank-deficient (Johansen & Fossen, 2013). It can be imagined though that in practice not all of these solutions are efficient or even feasible. As discussed in a later section, CA is therefore oftentimes formulated as an optimization problem, incorporating secondary objectives to guide the selection of the best feasible solution. There are however some practical issues to keep in mind when formulating the CA problem.

In physical systems, actuators have constraints in the form of position and rate limitations. This means that in fact $u \in U$, in which $U$ is the subset of $\mathbb{R}^p$ regarding the limits of the actuators. If CA does not account for these limits, it might occur that unfeasible commands to the actuators are given. If any actuator reaches its limit, one or more elements of $\tau$ do not correspond with $\tau_c$, and this might lead to a completely different motion than desired. Actuator limits therefore have to be included in CA at the beforehand, such that the command is for example downgraded if it is beyond the capabilities of the system.

Another obstacle in CA are the actuator dynamics. Actuators are generally mass-spring-damper systems, characterized by a second-order lag response. For the ICE aircraft for example, the trailing edge control surface dynamics are represented by

$$\frac{\delta}{\delta_c}(s) = 40 \cdot \frac{400}{(s + 40)(s + 100)}$$

Whereas position and rate limits can be relatively easily incorporated in CA design, it is common for actuator dynamics to exclude them. Instead, a separate module after CA can be used to handle actuator dynamics.

In the beginning of this section, the non-affine form of Eq. 4-1 is simplified to Eq. 4-2 under the assumption that the virtual command is linear in the control input. At first instance, it may look like this assumption approximately holds for at least some effectors of the ICE aircraft, see for example figure 3-8. It has to be acknowledged though that the effectiveness of a control surface is heavily affected by the state of the others, recall Eqs. 3-4 and 3-5. These inter-effector aerodynamics cause serious challenges in CA.

Additionally, there are control surfaces on the ICE aircraft of which the control effectiveness is not linear at all, see e.g. figure 3-9. A way to still use the affine formulation is by linearizing the control effectiveness function about the current degree of deflection $u_0$ (Härkegård, 2003):

$$\tau = h(x, u) \approx h(x, u_0) + \frac{\partial h(x, u_0)}{\partial u}(u - u_0)$$

$$\approx h(x, u_0) + B(x, u_0)(u - u_0) \quad (4-4)$$
As long as the function is monotonic and the sampling rate of the control system is high, this is a sufficient way to overcome nonlinear mappings. However, the linearization can be misleading in case of non-monotonic nonlinearities, as shown in figure 4-1. Consider that the current $\delta_{\text{amt}} = 40^\circ$, at which the local slope is negative. Now in order to decrease $C_l$, the controller increases effector deflection, while in fact decreasing the deflection has more potential to reach low $C_l$-values.

![Figure 4-1: Example of a non-monotonic relationship: roll effectiveness of the left AMT. Remark that the sharp corners are a result of low resolution data in the aerodynamic model. Physically seen a discontinuous control derivative function is implausible.](image)

An obvious solution to this problem would be narrowing the actuator limits such that only the monotonic range of an effector is used. Such a measure is not desirable though, since it severely affects the operating range of some control surfaces, and degrades the potential of the aircraft. More elegant ways to deal with these kind of behavior reversals and nonlinear mappings in general are proposed by Doman & Oppenheimer (2002). Calculating the local derivative and correcting for the global effectiveness provides a sufficient approximation of the actual behavior, while clipping logic is used to deal with non-monotonic mappings. Including higher order approximations of the control effectiveness in the CA problem can also overcome non-monotonic nonlinearities.

### 4-2 Generalized inverse

A large amount of CA techniques that can cope with one or more of the obstacles listed in the previous section have been developed. Some extensive surveys exist with rather complete overviews of available methods and implementation schemes, for example Bodson (2002),
Härkegård (2003), Ahmad (2009), and the more recent Johansen & Fossen (2013). The aim of this research is not to describe all methods in detail again, but rather to give an overview of the advantages and disadvantages of certain approaches. This section introduces simple generalized inverse CA methods for over-actuated systems.

The goal of control allocation is to find an input vector $u$ to obtain the command $\tau_c$, which is in its simplest form given by

$$ Bu = \tau_c $$

(4-5)

with $B$ whether or not dependent on $x$. For over-actuated systems, $B$ is not square and generally has a non-trivial null space, so an infinite number of vectors $u$ satisfy Eq. 4-5 for any arbitrary $\tau_c$. A convenient way to guide the selection of the ‘best’ of these solutions is the introduction of an auxiliary goal. The function of such a secondary objective is twofold: it provides uniqueness to the solution, while it exploits the excess control power to increase the performance of the system. A common example in optimization-based CA is the minimization of the weighted least squares of the control effort:

$$ \min_u (u - u_p)^T W (u - u_p) $$

(4-6)

subject to $Bu = \tau_c$

with $W$ a weighting matrix and $u_p$ the preferred state of the actuators. When $B$ has full rank, the solution is given by (Johansen & Fossen, 2013)

$$ u = (I - GB)u_p + G\tau_c $$

(4-7)

in which $I$ is the identity matrix and $G$ is the generalized inverse given by

$$ G = W^{-1}B^T(BW^{-1}B^T)^{-1} $$

If $W = I$ and $u_p = 0$, the solution to the CA problem

$$ \min_u \frac{1}{2} u^T u $$

subject to $Bu = \tau_c$

is given by

$$ u = B^+\tau_c $$

(4-9)

in which $B^+ = B^T(BB^T)^{-1}$, known as the Generalized Moore-Penrose inverse, often called the pseudo-inverse (Prasad & Bapat, 1992). Unconstrained CA of over-actuated systems with this specific objective function can thus be solved by elementary linear algebra, comparable to CA with a square $B$-matrix. However, in physical systems actuators have constraints, meaning that $u$ from Eq. 4-9 may be beyond the capabilities of the actuators ($U$). From another point of view, it can be said that $\tau_c$ lies outside the Attainable Moment Set (AMS), the collection of $\tau$-vectors that can be reached given the current state and actuator limits. To cope with these constraints, some improvements on pseudo-inverse CA are proposed.
4-2 Generalized inverse

4-2-1 Direct allocation

One way of making the solution of Eq. 4-9 feasible is of course by clipping all elements of \( \mathbf{u} \) that violate a constraint at their admissible limits:

\[
\tilde{\mathbf{u}} = \text{Proj}_U(\mathbf{u}) \tag{4-10}
\]

However, this does not mean that \( \tilde{\mathbf{r}} = B\tilde{\mathbf{u}} \) has the same direction as \( \mathbf{r}_c \), so a different moment than desired might be obtained.

A safer alternative would be to preserve the direction of \( \mathbf{r}_c \) by scaling the total vector \( \mathbf{u} \) from Eq. 4-9, such that all elements are within the limits

\[
\max_{0 \leq \lambda \leq 1} \lambda \quad \text{subject to } \lambda \mathbf{u} \in U \tag{4-11}
\]

This does not automatically mean that the generated moment by \( \lambda \mathbf{u} \) is the maximum achievable in that direction though. Another approach would be to calculate the AMS mentioned earlier, and scale \( \mathbf{r}_c \) such that it is at the border of the AMS (Durham, 1993):

\[
\max_{0 \leq \lambda \leq 1} \lambda \quad \text{subject to } \lambda \mathbf{r}_c \in \text{AMS} \tag{4-12}
\]

Now there theoretically exists a feasible \( \mathbf{u} \) for \( \lambda \mathbf{r}_c \), but the chances are small that it is found directly by the unconstrained pseudo-inverse \( \mathbf{u} = B^+ \lambda \mathbf{r}_c \), so still some other CA method is required.

4-2-2 Redistributed pseudo-inverse

Another way to treat unfeasible actuator commands is by recalculating the pseudo-inverse of the unsaturated part of the previous solution, called redistributed pseudo-inverse (RPI) (Virnig & Bodden, 1994). It begins by projecting the unfeasible solution from the pseudo-inverse on the admissible actuator settings \( U \) giving \( \tilde{\mathbf{u}} \), as in Eq. 4-10. Now the saturated elements from this solution \( (\tilde{\mathbf{u}}_S) \) are removed from the CA problem, as well as the corresponding part of the control effectiveness matrix (\( B_S \)). The unsaturated inputs \( (\tilde{\mathbf{u}}_U) \) are set free to solve

\[
B_U \mathbf{u}_U = \mathbf{r}_c - B_S \tilde{\mathbf{u}}_S \tag{4-13}
\]

using the pseudo-inverse again. Most research propose the iterative variant, in which Eqs. 4-10 and 4-13 are repeated until a feasible solution to \( \mathbf{r}_c = B \mathbf{u} \) is found, or no further improvement is possible. RPI is also known as the cascaded pseudo-inverse or cascaded generalized inverse (Ahmad, 2009).

RPI is easy to implement, and it is generally an effective way to generate a feasible solution in constrained CA. However, the method does not guarantee that the best \( \mathbf{u} \) is found with respect to minimizing the allocation error \( (\mathbf{r}_c - B \mathbf{u}) \), as is shown by Bodson (2002).

4-2-3 Daisy chaining

An alternative to RPI is Daisy Chaining, in which the use of a certain set of effectors is prioritized (Johansen & Fossen, 2013). Consider the set of actuators with the highest priority
to be \( u_1 \) with control effectiveness \( B_1 \). Using the pseudo-inverse, the solution for a certain \( \tau_c \) is found by

\[
  u_1 = B_1^+ \tau_c \quad (4-14)
\]

If one or more elements of \( u_1 \) saturates, the control inputs are projected on the limits (see Eq. 4-10) and CA continues with the second group of actuators:

\[
  u_2 = B_2^+(\tau_c - B_1 \hat{u}_1) \quad (4-15)
\]

This is repeated until a feasible solution is found, or no effector groups are left. However, daisy chaining shows sub-optimal performance, since an entire group is frozen as soon as one of the effectors saturates. The remaining (unsaturated) effectors in a frozen group could theoretically have been used to further minimize the allocation error.

### 4-3 Multi-objective optimization

Adequate control allocation involves three important aspects: **feasibility**, **deficiency**, and **sufficiency** (J. F. Buffington, 1999). Feasibility means that the solution lies within the capabilities of the system, deficiency yields that if a command is not achievable, it is degraded to the maximum attainable, and sufficiency is the feature that there exists only one unique solution to the problem.

As was illustrated in section 4-2, generalized-inverse methods do not guarantee all of these aspect. In fact, the role of the objective function (\( J = \frac{1}{2} u^T u \)) is to provide sufficiency, but the solution to the pseudo-inverse is not always feasible, and the other methods do not guarantee deficiency. Another approach would be to do mixed optimization of the allocation error and control effort, giving the scheme proposed by Bodson (2002):

\[
  \min_u J = (1 - \varepsilon) \|Bu - \tau_c\| + \varepsilon \|u - u_p\|
  \quad \text{subject to } u \in \mathbb{U} \quad (4-16)
\]

in which scaling factor \( \varepsilon \ll 1 \) is used to indicate the importance of error minimization over control minimization. \( \|\cdot\| \) is some norm of degree \( p \), defined as (Härkégård, 2003)

\[
  \|u\|_p = \left( \sum_{i=1}^{m} |u_i|^p \right)^{1/p} \quad \text{for } 1 \leq p \leq \infty \quad (4-17)
\]

Mixed optimization in fact includes feasibility, deficiency and sufficiency in a single step. Feasibility is guaranteed by the constraint that \( u \) should be within its limits. Deficiency is achieved by the first term of the objective function, minimizing the error between \( Bu \) and \( \tau_c \). Sufficiency is obtained by the secondary objective (\( \|u - u_p\| \)), making sure that only one unique solution exists.

Johansen & Fossen (2013) propose a more advanced formulation of the mixed optimization problem, minimizing slack variables \( s \) and introducing weighting matrices \( Q \) and \( W \) to prior-
itize certain control directions and effectors respectively:

\[
\min_{u,s} \|Qs\| + \|Wu\| \tag{4-18}
\]

subject to

\[
\begin{align*}
\tau_c &= Bu + s \\
u &\in \mathbb{U}
\end{align*}
\]

\(Q\) and \(W\) can be varied per mission, but in general \(W \ll Q\) since minimizing the allocation error is the primary objective. Härkegård (2003) describes a similar optimization scheme. J. F. Buffington (1999) suggests using a scaling factor to make sure the eventual \(\tau\) preserves the direction of \(\tau_c\):

\[
\min_{u,\lambda} \|Wu\| - \lambda \tag{4-19}
\]

subject to

\[
\begin{align*}
Bu &= \lambda \tau_c \\
u &\in \mathbb{U} \\
0 \leq \lambda \leq 1
\end{align*}
\]

Different weighting matrices \(W\) are developed for different missions, such as minimum drag, minimum wing load, and minimum radar signature. Also a weighting matrix is suggested for system identification purposes, that triggers the use of all effectors in order to estimate their effectiveness in-flight.

Two commonly used norms as defined in Eq. 4-17 are \(l_1\)- and \(l_2\)-optimal control (Härkegård, 2003), requiring linear and quadratic programming methods respectively. Regarding CA, the difference in behavior is that \(l_2\) tends to spread the control demand over the actuators, while \(l_1\) saturates the actuators with the highest effectiveness first. The 1-norm is often used in older research due to the limited computational demand of solving a linear program over a quadratic one. However, today’s control systems are easily capable of solving a quadratic programs in real-time as well, provided efficient algorithms.

### 4-3-1 Fixed-point algorithm

An effective method to solve weighted \(l_2\)-optimal control allocation problems is the fixed-point (FXP) algorithm from Burken et al. (2001). With \(u_p = 0\), Eq. 4-16 becomes

\[
J = (1 - \varepsilon)\|Bu - \tau_c\|_2 + \varepsilon\|u\|_2 \tag{4-20}
\]

subject to \(u \in \mathbb{U}\). A fixed-point iteration involves

\[
u_{k+1} = \text{sat} \left[ (1 - \varepsilon)\eta B^T \tau_c - (\eta M - I)u_k \right] \tag{4-21}
\]

with

\[
M = (1 - \varepsilon)B^T B + \varepsilon I \\
\eta = 1/\|M\|_2
\]

and sat is the saturating function clipping \(u_{k+1}\) with respect to \(\mathbb{U}\). The algorithm can be seen as a gradient search method with step size \(\eta\).
The fixed-point method is simple and guarantees to find the optimum, but the number of iterations required depends heavily on the current $u$ and $\tau_c$. For real-time implementation some degree of sub-optimality has to be accepted due to a maximum number of iterations per time step. To decrease the number of iterations required, the authors suggest to start the iterations each time step with an initial input $u_0$ equal to the optimum solution from the previous time step.

### 4-3-2 Active set methods

Active set methods solve optimization problems with inequality constraints by solving a sequence of equality constrained problems (Härkegård, 2003). Each iteration, some inequality constraints are treated as equality constraints, forming the working set $W$. Consider the constraint $u \in U$ rewritten as follows:

$$ C u \geq U $$ \quad (4-22)

with

$$ C = \begin{pmatrix} I & -I \end{pmatrix}, \quad U = \begin{pmatrix} u \bar{u} \end{pmatrix} $$

in which $u$ and $\bar{u}$ represent lower and upper position and rate limits of the actuators. Each iteration, a subset of $u$ is saturated, such that equality constraints are present for $u_{i0}$. The other inequality constraints are disregarded. The working set at the optimum is known as the active set of the solution (Härkegård, 2002).

For the $l_1$ case, active set CA can be performed using the simplex algorithm, as described in Bodson (2002). This algorithm is based on stepping among the boundaries of the search space, by alternating saturation of the inputs, in order to find corners with a lower cost function value. The author tested the algorithm on an C-17 aircraft model, which has 16 conventional control surfaces, and an older version of the ICE model with 11 effectors. Mixed optimization problems with linearized $B$-matrices are solved in less than a millisecond, making it well suitable for real-time implementation in control systems. However, measures have to be taken against cycling, a phenomenon in which the algorithm revisits the same coordinates over and over without converging to the optimum.

Active set solvers for quadratic programming ($l_2$) are extensively researched by Härkegård (2002). The approach is the same as the simplex method, but it considers a sequential least squares (SLS) or weighted least squares (WLS) formulation of the CA problem. The combined objective function in WLS form for example is

$$ J = \|Q(Bu - \tau_c)\|_2^2 + \|W(u - u_p)\|_2^2 $$

$$ = \left( \sum_{i=1}^{n+m} \left\{ \left( \frac{QB}{W} \right)_i u - \left( \frac{Q\tau_c}{Wu_p} \right)_i \right\}^2 \right)^{1/2} \quad (4-23) $$

Algorithms for both SLS and WLS form are presented and compared with the redistributed pseudo-inverse (RPI) and the fixed-point (FXP) algorithm in an aircraft control example. It is claimed that active set methods are well suited for CA purposes since they benefit from a good estimation of the optimal active set. Because the optimization problem is considered
to not change much between two sampling instants, the active set of the previous time step is often a good starting point for the new iterations. It is found that both the SLS and WLS algorithms indeed find the optimum within a few iterations, indicating that only a small number of changes in the working set occur. The computation times of the active set methods and RPI and FXP are within the same order of magnitude, but the active set methods produce better quality solutions.

4-3-3 Interior-point methods

Both linear and nonlinear CA problems can also be solved using interior-point methods. The idea behind these optimization techniques is that constraints are rewritten as smooth (logarithmic) barrier functions. These barriers form an additional term in the objective function, giving infinitely high penalties if any constraints are violated. The solver therefore only considers points (solutions) in the interior feasible space, giving it the name of the method.

Petersen & Bodson (2005) describe various interior-point algorithms for multi-objective linear-programming problems in aircraft CA. Two main techniques included are primal-dual path following and predictor-corrector path following algorithms. Some small refinements to these algorithms are proposed to suit the problem under consideration. Test cases are the aforementioned C-17 aircraft model and ICE model. In both simulations, the methods performed well, but is has to be mentioned that a linear state-space model of the aircraft is used.

The claimed advantage of interior-point methods is that they progress uniformly to the optimum, while active set methods progress along the boundary of the feasible space. Although interior-point methods converge slower than active set methods, they find a better estimate if stopped prematurely. For real-time implementation with a fixed allowable number of iterations, the interior-point method could be favorable. An important remark by the authors is that interior-point algorithms can also be applied to nonlinear programming problems, such that nonlinear cost functions or nonlinear effector models can be included, at the cost of higher computational demands.

4-3-4 Metaheuristic methods

The methods above are all based on starting at one initial point and stepping through the feasible space to minimize the value of the objective function. A fundamentally different way of thinking about optimization problems is based on evolution theory. Metaheuristic methods start with an initial population of feasible solutions, of which the coordinates are decoded by so-called genes. The value of the objective function of each coordinate is considered to be the fitness of that specific solution. The fittest solutions are likely to be close to the optimum, and form the basis for the new generation. Techniques like mutation and crossover are applied to widen the search and escape local optima. Examples of metaheuristic optimization algorithms are genetic algorithms, simulated annealing, and ant colony optimization. A broad survey on this topic can be found in Bianchi et al. (2009).

Guo et al. (2014) propose the ant colonization and differential evolution algorithms for CA of over-actuated systems with non-monotonic nonlinearities. Silimarly, J. Ma et al. (2009) developed a differential evolution strategy for CA of an over-actuated aircraft. A clear advantage of
these optimization techniques is that they can solve very complex problems with little design effort, as long as the computational resources are sufficient. The objective function can be any nonlinear function, so the effector model does not have to be linear, although Guo et al. (2014) still propose piecewise linearization of the control effectiveness curve. As J. Ma et al. (2009) claim, it is also one of the very few ways to deal with non-monotonic mappings as described in section 4-1. There are two large drawbacks that make metaheuristic methods not suitable for aircraft control though. At first, the number of iterations required to obtain a certain degree of accuracy of the final solution is hard to predict, making these methods less attractive for real-time applications. Additionally, metaheuristic methods are almost impossible to certify for use in aerospace applications, as it cannot guarantee to come up with a suitable solution in a finite number of iterations. The stochastic nature of the optimization means that the solution found is dependent on the set of random variables generated. While other optimization methods are more systematic, metaheuristic search is partially trial-and-error and hence not suitable for safety-critical systems such as aircraft.

4-4 Research on comparable aircraft

The first publication on CA with the ICE model as subject is from J. M. Buffington (1997). The formulation from Eq. 4-19 is enhanced with prioritized control law command limiting, axes prioritization, and actuator prioritization. A linear state-space model of the ICE aircraft at Mach 0.4 and 15000 ft is used. The author states that the algorithm used is theoretically capable of solving complex objectives, but a linear objective is used to reduce computational requirements. A later publication by the same author (J. F. Buffington, 1999) presents a more detailed description of the modular control system design for the ICE aircraft. Weighted linear and sequential linear programs for CA are proposed, and the different weighting matrices mentioned in the introduction of section 4-3 are established. This in fact allows the operator to change the CA strategy depending on the current mission without changing the actual flight control algorithms.

Eberhardt & Ward (1999) suggest an indirect adaptive modular control system architecture for the ICE aircraft, with online parameter estimation. RPI is chosen for CA, since multi-objective methods are considered to be too computationally expensive for real-time implementation. Davidson et al. (2001) also propose a cascaded pseudo-inverse technique for real-time adaptive CA, but introduces an adaptation to deal with control interactions and nonlinearities. Control effectiveness as a function of command magnitude and first order control interaction effects are included by

$$\tau = Bu + B_2|u| + \sum_{j=1,i=1}^{m,m} B_{ij}|u_j|u_i$$  \hspace{1cm} (4-24)

This was one of the first attempts to take the complex dynamics of the aircraft into account. With respect to model-free CA the work of Calise et al. (2001) provides some interesting theories. A neural-network-based direct adaptive control technique is proposed for application on an over-actuated tailless canard-delta wing fighter configuration. The suggested control method works well in piloted simulation and flight tests with the X-36 aircraft. Using neural-networks for control is essentially a 'black-box' approach, since the controller develops its own
representation of the system, which is captured in the weights of the neural connections. The obvious advantage of this approach is that there is no need for preliminary or online parameter identification, and that the controller can quickly adjust to new dynamics, e.g. due to in-flight failures. However, it is very unlikely that this kind of nontransparent controllers will be used in the aerospace industry, due to the same certification issues as mentioned in section 4-3-4.

C. Ma & Wang (2009) implemented a multi-objective nonlinear programming method for CA of the ICE aircraft. Of all CA techniques available, it is considered that only optimization-based methods are able to include every desired aspect, such as dealing with nonlinear mappings, deficiency and sufficiency measures, and choice of secondary objectives. The objective function consists of a summation of functions of $\mathbf{u}$, with weights depending on the importance of the corresponding objective. Sequential quadratic programming (SQP) is used for solving the sub-problems in each iteration. The controller shows appropriate angle tracking in all flight profiles (maximum drag, maximum lift, etc.) but no statements about the computational complexity are made.

Mingxing et al. (2013) propose a similar approach with SQP to solve a nonlinear CA problem. The nonlinear control effectiveness function is approximated with a third order polynomial to come up with the explicit form

$$\Delta C_j = \sum_{i=1}^{8} a_i u_i + \sum_{i=1}^{8} b_i u_i^2 + \sum_{i=1}^{8} c_i u_i^3 + k_i$$

with $j = l, m, n$ and $a_i$, $b_i$, $c_i$, and $k_i$ are the coefficients of the polynomial approximation. The third order polynomial is a sufficiently accurate representation for the purpose, but does not include interactions between different effectors. The approach requires more calculations than that of older research, but unfortunately no statements about the computation time is made. It is interesting to identify whether the current generation computers is able to process the suggested algorithms in real-time.

4-5 Spline-based control allocation

Essentially all control allocation methods are model-based and require an internal representation of the system, either to invert or to search through. A topic not specifically treated so far is where this model comes from and how to obtain the control effectiveness functions. It should be clear that the better the internal model, the higher the potential of the controller. In chapter 3 is explained that the ICE aircraft has complex nonlinear aerodynamics, so it crucial that the controller is aware of all nonlinear mappings, local phenomena, and inter-effector aerodynamics.

In general, the aerodynamic model of an aircraft is based on wind tunnel experiments, computational fluid dynamics analysis, or in-flight measurements. The current simulation model is built on a large amount of multi-dimensional data tables, and the aerodynamic coefficients are obtained by linear or cubic interpolation of wind tunnel data mentioned in chapter 3. For the remainder of this thesis, this is seen as the ‘real’ system. For control purposes however, the internal model is preferred to be a mathematical expression of some form. Previous research mainly considered ordinary polynomial models to estimate the actual behavior of the aircraft. The benefit thereof is that the control effectiveness at a certain state can be
calculated analytically by the partial derivative of the polynomial with respect to the input. The approximation power of ordinary polynomials however is proportional to their degree, so to accurately capture highly nonlinear behavior and local irregularities, a very high order polynomial is required.

A brighter approach is to split up the flight envelope and define lower-order polynomials at each part. This is the concept of model identification with multivariate simplex splines, as described by De Visser (2011). Splines are functions consisting of multiple polynomials defined on adjacent triangular bases: simplices. It is considered that splines have a higher approximation power than ordinary (global) polynomials, especially for highly nonlinear systems, at the cost of a higher number of coefficients. The disadvantage of a spline-based model is that finding the control derivatives requires some mathematical tricks. The splines are defined on barycentric coordinates instead of Cartesian coordinates, and are thus not expressed in terms of the physical states and inputs. Another obstacle is the fact that no global formula of the system exists, so the control derivative function is only locally valid and has to be recalculated once moved out of the current simplex.

Spline-based Nonlinear Dynamic Inversion (SNDI) and CA is already researched by Tol et al. (2014). The authors come up with two theorems to calculate the Jacobian and the Hessian of a B-form simplex polynomial with respect to the physical states and inputs. Consider the barycentric coordinate \( b(x) = (b_0, b_1, \cdots, b_n) \) which can be expressed as an affine function of the physical states as

\[
    b(x) = [a_1 \ a_2 \ \cdots \ a_n]_t_j \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + k = A_{t_j} x + k \quad (4-26)
\]

with \( t_j \) the current simplex. Please refer to Tol et al. (2014) for the full definition. Now the partial derivative of the B-form basis polynomial of degree \( d \) with respect to physical state \( x_i \) is given by the multivariable chain rule:

\[
    \frac{\partial B^d_i(b(x))}{\partial x_i} = \frac{\partial B^d_i(b(x))}{\partial b_0} \frac{\partial b_0}{\partial x_i} + \frac{\partial B^d_i(b(x))}{\partial b_1} \frac{\partial b_1}{\partial x_i} + \cdots + \frac{\partial B^d_i(b(x))}{\partial b_n} \frac{\partial b_n}{\partial x_i} \quad (4-27)
\]

which is in short

\[
    \frac{\partial B^d_i(b(x))}{\partial x_i} = \begin{bmatrix} \frac{\partial B^d_i(b(x))}{\partial b_0} \\ \frac{\partial B^d_i(b(x))}{\partial b_1} \\ \vdots \\ \frac{\partial B^d_i(b(x))}{\partial b_n} \end{bmatrix} = a_i^T \nabla_b B^d_i(b(x)) \quad (4-28)
\]

in which \( a_i \) is the \( i \)th column of \( A_{t_j} \) and \( \nabla_b \) the gradient with respect to \( b \). The B-form simplex polynomial is the product of the basis polynomials and the B-coefficients \( c \):

\[
    p^{t_j}(b(x)) = B^d_{t_j}(b(x)) c^{t_j} \quad (4-29)
\]

Its Jacobian is then given by

\[
    \nabla_x p^{t_j}(b(x)) = A^T_{t_j} \nabla_b B^d_i(b(x)) c^{t_j} \quad (4-30)
\]
An expression for the Hessian is derived similarly.

The authors propose a linear, successive linear, and nonlinear control allocation method. The first one linearizes the control effectiveness about the current control input using the aforementioned expression for the Jacobian, such that

\[
\tau = p(u_0) + \begin{bmatrix}
\nabla_{u} f_1(u_0) \\
\vdots \\
\nabla_{u} f_m(u_0)
\end{bmatrix} \Delta u
\] (4-31)

This equation captures nonlinear input dynamics in a linear form and thus can the input increment \( \Delta u \) be calculated using well-known methods from the preceding sections. The successive linear method does the same for a number of feasible \( u_0 \) and picks the solution with the lowest allocation error. The nonlinear method includes the full nonlinear control effectiveness function, after which a second-order approximation of the objective function is obtained with the Jacobian and Hessian, and solved with the Levenberg-Marquardt algorithm.

In Tol et al. (2014) a high fidelity F-16 aircraft model is used as the test case and the results are promising. Throughout the entire flight envelope, excellent tracking is achieved, significantly outperforming ordinary polynomial-based control techniques in highly nonlinear regions. An interesting detail is that the successive linear and nonlinear control allocator perform about equally well, while the first one is computationally much less expensive.

### 4-6 Concluding remarks

The first section of this chapter pointed out several challenges in CA, such as effector redundancy, actuator limitations, and nonlinear mappings. The simplest CA technique for over-actuated systems makes use of the pseudo-inverse, although this method requires a linear control effectiveness function and does not take actuator limits into account. Some refinements to the basic pseudo-inverse are proposed, that adapt the unfeasible solution to suit the constraints, while still remaining computationally inexpensive. The currently most common method for CA on over-actuated systems is the redistributed pseudo-inverse from section 4-2-2.

It is however believed that, especially for highly over-actuated systems, performance can be increased by quadratic programming (QP) CA. This genre was long considered to be too computationally expensive for real-time application, but the enormous increase in computational power in the past decade makes this well possible nowadays. The freedom to specify the relative importance of every objective, and the guarantee of feasibility, deficiency, and sufficiency (recall section 4-3) are the main advantages, and make multi-objective CA an attractive tool for this research. Given consulted literature, a weighted least-squares (WLS) formulation of the CA problem with variable weighting matrices is the best starting point to include mission-specificity. The active set QP algorithm as described by Härkegård (2002) is the most promising quadratic solver for this purpose, because the dimension of the problem is rather small, and it is proven to converge to the optimal solution in just a few iterations.

Finally, aircraft control stands or falls with the quality of the internal system model. System identification nowadays is commonly based on data tables with parameters of a linear input
Review of control allocation

model. This method assumes an input affine aerodynamic model like Eq. 4-2. The moments due to effector deflections here are modeled by a linear parameter - the control effectiveness - times the magnitude of the deflection. This method is comprehensive because the number of parameters is small and it can be easily obtained from any complex aerodynamic model, but the control effectiveness is a fixed value over the entire range of the effector, so it assumes linear input dynamics and no aerodynamic interaction within the controls.

For nonlinear systems it is therefore more useful to define a higher order model with affine polynomial functions of the inputs. Due to the highly nonlinear dynamics and local irregularities present in the ICE aerodynamic model, ordinary polynomials are not considered capable of ensuring the required model accuracy, so a multivariate simplex spline model of the ICE aircraft is developed and documented in Van der Peijl (2017). The benefit of this model is that the Jacobian with respect to the inputs can be calculated locally, such that a precise value of the control effectiveness at any state and input \( u_0 \) is obtained. This leads to the incremental input model introduced in Eqs. 4-4 and 4-31, and as long as the increments are small, this technique properly handles nonlinear input dynamics and the effect of other control surfaces. For that reason, after the development of the simulation framework in chapter 5, the theory of incremental control allocation is further explained and a comparison with linear CA for the ICE aircraft is given in section 6. Based on the conclusions from this chapter the recommended CA method for the aircraft is established. In chapter 7 the ultimate goal of the research, a minimum drag control allocation mode, is pursued. Conclusions upon the entire thesis are then given in chapter 8.
Before evaluating different control allocation methods, the simulation framework is defined. The flowchart of the control process is given in figure 5-1. It is clear that in this setup the control allocation module forms the central part: it accepts a control demand $\tau_c$ to allocate, and parameters that provide real-time information about the system, whereas its output $u$ is fed to the aircraft model. Simulation of the aircraft dynamics results in the state vector $x$, which is fed back to the modules and the cycle continues.

The simulations in this research are done in a MATLAB/Simulink environment. The model runs at 100 Hz, so the time steps are 0.01 seconds. The development and implementation of the motion controller, system identification module, and aircraft model are described in the following sections. The process of control allocation is the topic of later chapters.

### 5-1 Aircraft model

The establishment of the aerodynamic model of the ICE aircraft is already described in chapter 3, but a full simulation model of an aircraft includes more than just an aerodynamic database.
Figure 5-2 shows the block diagram of the aircraft model, of which the implementation in MATLAB/Simulink is documented in Niestroy et al. (2017).

![Block diagram of the aircraft model.](image)

The aerodynamic model is based on wind tunnel measurements with a scale model of the aircraft. The data consists of dimensionless coefficients $C_A$, $C_Y$, $C_N$, $C_l$, $C_m$, $C_n$ such that the behavior can easily be scaled with its dimensions, recall section 2-1. Thrust and thrust vectoring is not included in the aerodynamic model, so its contribution to the forces and moments is added separately. A right handed body axis system is used with the force $X$ pointing forward, $Y$ pointing right, and $Z$ pointing downward:

$$
X = -C_A \cdot \frac{1}{2} \rho V^2 S + T \cdot \cos(\delta_{ptv}) \cos(\delta_{gtv})
$$

$$
Y = C_Y \cdot \frac{1}{2} \rho V^2 S + T \cdot \cos(\delta_{ptv}) \sin(\delta_{gtv})
$$

$$
Z = -C_N \cdot \frac{1}{2} \rho V^2 S - T \cdot \sin(\delta_{ptv}) \cos(\delta_{gtv})
$$

$$
l = C_l \cdot \frac{1}{2} \rho V^2 Sb
$$

$$
m = C_m \cdot \frac{1}{2} \rho V^2 S \bar{c} - T \cdot l_{tv} \sin(\delta_{ptv}) \cos(\delta_{gtv})
$$

$$
n = C_n \cdot \frac{1}{2} \rho V^2 Sb - T \cdot l_{tv} \cos(\delta_{ptv}) \sin(\delta_{gtv})
$$

in which $T$ is the thrust force and $l_{tv}$ the arm of thrust vectoring.

The original simulation model does not include actuator dynamics and limits, so these are added prior to the aerodynamic model and thrust vectoring by transfer functions, rate limits, and position limits in that order. The position limits are given in table 3-1. Rate limits of the leading edge flaps are 40 deg/sec, and that of trailing edge flaps and thrust vectoring are 150 deg/sec. The dynamics of the leading edge flaps are represented by

$$
\frac{\delta}{\delta_c}(s) = 18 \cdot \frac{400}{(s + 18)(s + 100)}
$$

and of all other effectors by

$$
\frac{\delta}{\delta_c}(s) = 40 \cdot \frac{400}{(s + 40)(s + 100)}
$$

with $\delta$ the real deflection and $\delta_c$ the commanded deflection. Because the gain of both transfer functions is 4, actuator control consists of a gain of 1/4. No more detailed actuator controller is applied for now.

1Thrust vectoring has circular limits. In control allocation pitch and yaw thrust vectoring limits are set to 10.6°, such that the combined deflection cannot exceed 15°.

A.R.J. Stolk Minimum drag control allocation for the Innovative Control Effector aircraft
5-2 Motion control

Control of over-actuated systems is generally separated into control allocation and motion control, recall chapter 2. For the ICE aircraft a multi-loop architecture is chosen to control the aerodynamic angles, their derivatives, body rates, and body angular accelerations, see figure 5-3.

The inner-loop rate controller is based on the NDI law derived in section 2-3, that calculates the required moment coefficients to obtain the commanded angular accelerations:

\[
\begin{bmatrix}
C_l \\
C_m \\
C_n
\end{bmatrix}
= \frac{I}{2\rho V^2 S}
\begin{bmatrix}
b & 0 & 0 \\
0 & \tilde{c} & 0 \\
0 & 0 & b
\end{bmatrix}
^{-1}
\begin{bmatrix}
\dot{p} \\
\dot{q} \\
\dot{r}
\end{bmatrix}_c
+ I^{-1}
\begin{bmatrix}
p \\
q \\
r
\end{bmatrix}
\times I
\begin{bmatrix}
p \\
q \\
r
\end{bmatrix}
\tag{5-4}
\]

\[
[\dot{p} \dot{q} \dot{r}]_c^T
\]

is set by a PID-based linear controller with gains given in table 5-1.

An aerodynamic angle outer loop is implemented to control the angles of roll, attack, and sideslip. The NDI control law is derived in Tol et al. (2014) and reads as follows:

\[
\begin{bmatrix}
p \\
q \\
r
\end{bmatrix}_c
= g_\phi(x)\begin{bmatrix}
\phi \\
\dot{\phi} \\
\beta \\
\dot{\beta}
\end{bmatrix}_c
- f_\phi(x)
\]

(5-5)

with

\[
f_\phi = 0
\]

\[
g_\phi = \begin{bmatrix}
1 & \sin \phi \tan \theta & \cos \phi \tan \theta
\end{bmatrix}
\]

\[
g_\alpha = \frac{1}{u^2 + w^2}[u(A_x + g \cos \theta \cos \phi) - w(A_x - g \sin \theta)]
\]

\[
g_\alpha = \begin{bmatrix}
\frac{-uv}{u^2 + w^2} & 1 & \frac{-uv}{u^2 + w^2}
\end{bmatrix}
\]

\[
f_\beta = \frac{1}{\sqrt{u^2 + w^2}}\left[\frac{-uv}{V^2}(A_x - g \sin \phi) + \left(1 - \frac{v}{V}\right) \times (A_y + g \sin \phi \cos \theta) - \frac{vw}{V^2}(A_x + g \cos \phi \cos \theta)\right]
\]

\[
g_\beta = \frac{w}{\sqrt{u^2 + w^2}}\begin{bmatrix}
0 & -w
\end{bmatrix}
\]

Minimum drag control allocation for the Innovative Control Effector aircraft

A.R.J. Stolk
Table 5-1: Linear controller (PID) gains.

<table>
<thead>
<tr>
<th></th>
<th>P</th>
<th>I</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p, q, r$</td>
<td>10</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\phi, \alpha$</td>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$\beta$</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

in which $u, v, w$ are the body velocities, $g$ is the gravitational acceleration, and $A_x, A_y, A_z$ are the body accelerations. The gains for the linear outer-loop controller are given in table 5-1.

Remark that the gains in table 5-1 are experimentally obtained and might not be optimal under all circumstances. For a more robust controller further tuning is required, and probably flight envelope protection has to be incorporated. This is out of the scope of the current study.

5-3 System identification

Control allocation requires, in addition to the commanded moment set by the motion controller, parameters of the system. The original ICE model contains no analytic functions, it is purely based on linear or cubic interpolation of data points. For control purposes the onboard model has to consist of affine functions of the inputs, as is explained in section 4-1. Three onboard models that provide the required CA parameters are developed and implemented.

5-3-1 Linear model

An in older research commonly used method is to assume linear input dynamics, recall Eq. 4-1:

$$
\tau = h(x) + B_{lin}(x)u
$$

$h(x)$ then consists of the aerodynamics with all effectors at their zero position. The moment contribution of an effector is captured by the control effectiveness times the total deflection. Such a model for the ICE aircraft is developed in the following form:

$$
C_\ell = C_{\ell 0} + C_{\ell p} \frac{pb}{2V} + C_{\ell r} \frac{rb}{2V} + \sum_{i=1}^{13} (C_{\ell i} u_i)
$$

$$
C_m = C_{m 0} + C_{m q} \frac{qc}{2V} + \sum_{i=1}^{13} (C_{m i} u_i)
$$

$$
C_n = C_{n 0} + C_{n p} \frac{pb}{2V} + C_{n r} \frac{rb}{2V} + \sum_{i=1}^{13} (C_{n i} u_i)
$$

in which $C_{\ell 0}$ etc. cover the mainframe aerodynamics and $C_{\ell p}$ etc. are the moment sub-coefficients with respect to rotational rates. Finally $C_{\ell i}$ etc. are the control derivatives defined as $\frac{\partial C_\ell}{\partial u_i}$. Together these sub-coefficients form the control effectiveness matrix $B_{lin}$.

The values of the sub-coefficients are established from the original ICE aerodynamic model by the following steps:
Sub-coefficients $C_l(\alpha, \beta, M)$ etc. are calculated by evaluating the entire original model on a three-dimensional grid of $\alpha$, $\beta$, and $M$-values. All other states ($V, p, q, r$) and inputs are zero. The range of the grid is $\alpha = [-5, 90]$ deg, $\beta = [-30, 30]$ deg, and $M = [0.3, 2.16]$ with a total of $48 \times 15 \times 10$ grid points.

The part covering the moments generated by rotational rates is already an affine function of $p, q, r$, so coefficients $C_{lp}(\alpha, \beta, M)$ etc. can directly be calculated from evaluation of the particular lookup tables on the $\alpha, \beta, M$-grid.

The control derivatives are obtained by the central difference theorem. Double evaluations of the complete model at the $\alpha, \beta, M$-grid with one input slightly above and below zero gives the sensitivity of the moment coefficients to that particular input. Consider that the control derivative toward the $i$-th input is calculated, while all other inputs are zero, then

$$\frac{\partial C}{\partial u_i}(\alpha, \beta, M) = \frac{C(\alpha, \beta, M, u = 0, u_i^+) - C(\alpha, \beta, M, u = 0, u_i^-)}{u_i^+ - u_i^-} \quad (5-8)$$

Accurate derivatives are found with $u_i^+ = 0.1^\circ$ and $u_i^- = -0.1^\circ$, smaller step sizes are not necessary.

The sub-coefficients are stored in 41 four-dimensional $(\alpha, \beta, M, C)$ data tables. In between the grid points the data is linearly interpolated. Thrust vectoring is not a part of the aerodynamic model, so its control derivatives are added later on:

$$\frac{\partial C_m}{\partial \delta_{ptv}} = -l_{tv} \frac{T}{1/2\rho V^2 S \bar{c}} \frac{\pi}{180} \quad \frac{\partial C_n}{\partial \delta_{vtv}} = -l_{tv} \frac{T}{1/2\rho V^2 S_b} \frac{\pi}{180} \quad (5-9)$$

The term $\frac{\pi}{180}$ is the result of the small angle approximation. Originally $m = -l_{tv} * T * \cos(\delta_{ptv}) \sin(\delta_{ptv}) \approx -l_{tv} \sin(\delta_{ptv})$ with $\delta_{ptv}$ in radians. Because the deflections are measured in degrees the term $\frac{\pi}{180}$ is added.

5-3-2 Spline model

To include nonlinearities in the inputs a higher order polynomial model is required, exponentially increasing the number of coefficients with the previous technique. A more effective model of the ICE aerodynamics is developed with use of multivariate simplex splines, documented in Van der Peijl (2017). Splines are low order polynomials defined on small parts of the domain, and are known for their high approximation power. Spline-based control allocation is already researched (Tol et al., 2014) and makes use of input increments instead of calculating the total inputs, recall sections 4-5 and 4-6 and Eq. 4-31:

$$\tau = \tau_0(x, u_0) + B_{inc}(x, u_0) \Delta u \quad (5-10)$$

With the theory provided in section 4-5 $\tau_0$ and $B_{inc}$ can be calculated in real-time by simply filling in the current states and inputs in the spline (derivative) functions. Because thrust vectoring is not part of the aerodynamic model, it is modeled according to the equations in table 5-2.
Table 5-2: Thrust vectoring model in system identification.

<table>
<thead>
<tr>
<th>C</th>
<th>∂C/∂ptv</th>
<th>∂C/∂ytv</th>
</tr>
</thead>
<tbody>
<tr>
<td>C_X</td>
<td>(\frac{T}{\rho V^2 S}) cos(δ_{ptv}) cos(δ_{ytv})</td>
<td>(-\frac{T}{\rho V^2 S}) sin(δ_{ptv}) cos(δ_{ytv})</td>
</tr>
<tr>
<td>C_Y</td>
<td>(\frac{T}{\rho V^2 S}) sin(δ_{ptv}) cos(δ_{ytv})</td>
<td>(-\frac{T}{\rho V^2 S}) sin(δ_{ptv}) sin(δ_{ytv})</td>
</tr>
<tr>
<td>C_Z</td>
<td>(-\frac{T}{\rho V^2 S}) sin(δ_{ptv}) cos(δ_{ytv})</td>
<td>(-\frac{T}{\rho V^2 S}) sin(δ_{ptv}) sin(δ_{ytv})</td>
</tr>
<tr>
<td>C_m</td>
<td>(-\frac{T}{\rho V^2 S}) cos(δ_{ptv}) sin(δ_{ytv})</td>
<td>(-\frac{T}{\rho V^2 S}) cos(δ_{ptv}) sin(δ_{ytv})</td>
</tr>
<tr>
<td>C_n</td>
<td>(\frac{T_l}{\rho V^2 S_b}) cos(δ_{ptv}) sin(δ_{ytv})</td>
<td>(-\frac{T_l}{\rho V^2 S_b}) cos(δ_{ptv}) sin(δ_{ytv})</td>
</tr>
</tbody>
</table>

5-3-3 Exact model

The most exact parameters however are obtained from the original ICE aerodynamic model directly. Instead of storing coefficients for a fixed \(\alpha, \beta, M\)-grid and linearizing the input dynamics about their zero state (see section 5-3-1), the coefficients can be evaluated on-the-fly at the \(x, u_0\), and local control effectiveness can be evaluated in real-time through the central difference theorem, e.g.

\[
\frac{\partial C}{\partial u_i}(x, u_0) = \frac{C(x, u_0, u_i^+) - C(x, u_0, u_i^-)}{u_i^+ - u_i^-} \quad (5-11)
\]

with \(u_i^+ = u_{0,i} + 0.1^\circ\) and \(u_i^- = u_{0,i} - 0.1^\circ\) ensuring sufficient accuracy. Thrust vectoring is again modeled following table 5-2.

The method above gives the most accurate information, but requires each time step 22 evaluations of the original aerodynamic model in all directions. This makes it for real applications probably less attractive than the spline model. More research has to be done on the exact computational demands of both methods.

5-3-4 Graphic comparison

For the linear model, the control effectiveness of the inputs is considered linear and independent of other inputs. Figure 5-4 shows that this is a fair approximation in some cases, but figure 5-5 shows that sometimes the valid range is limited. Moreover figure 5-6 shows that interaction within the controls is a serious concern, and that the linear model gives misleading control allocation information.

On the other hand the spline model is much more accurate and accounts for nonlinearities and other effectors. The spline graphs in figures 5-4 to 5-6 are similar to the exact model so the control effectiveness estimation is precise. Moreover the functions can be mathematically differentiated, such that the local control effectiveness can be calculated easily by filling in the current states and inputs. This makes it suitable for incremental control allocation, which is the topic of chapter 6.
Figure 5-4: Roll moment coefficient vs. left elevon deflection.

Figure 5-5: Roll moment coefficient vs. left AMT deflection.
Figure 5-6: Roll moment coefficient vs. left elevon deflection. Left SSD deflection is 20 deg.
Chapter 6

Incremental control allocation

Most literature on control allocation assumes dynamics that are linear in the input, recall chapter 4. Except for metaheuristic methods, all algorithms are based on solving the linear control allocation (LCA) problem in the form

\[ \text{Find } \mathbf{u} \leq \mathbf{u} \leq \mathbf{\bar{u}} \text{ to minimize } B_{lin} \mathbf{u} - (\mathbf{\tau}_c - h(\mathbf{x})) \]  

(6-1)

The applicability of this form is limited though, as it assumes that the control effectiveness of an input is constant over its entire range, and unaffected by the other inputs. As seen in chapter 3 this is not a valid assumption for the innovative control effectors. The dilemma is that on the one hand the linear form is preferred because it is widely researched and computationally efficient, but on the other hand severe nonlinearities cannot be neglected. The solution to this, incremental control allocation, is introduced in section 6-1. The CA problem statement and solving methods are described in section 6-2. In section 6-3 the performance on the ICE aircraft is compared with that of LCA. This chapter ends with a small conclusion in section 6-4.

6-1 Concept

The philosophy of incremental control allocation (INCA) is that input increments, i.e. the deviations from their current positions, are calculated instead of the total inputs. These increments are calculated based on the control demand increment: the difference between the commanded moments and the current moments. The approach is described by Härgard (2003) (recall section 4-1) and spline-based INCA is introduced by Töl et al. (2014) (recall section 4-5). Consider that the moment coefficient \( \mathbf{\tau} \) consists of the current value \( \mathbf{\tau}_0 \) plus a contribution from input increments:

\[ \mathbf{\tau} = \mathbf{\tau}_0(x, \mathbf{u}_0) + B_{inc}(x, \mathbf{u}_0)\Delta \mathbf{u} \]  

(6-2)

\[ \mathbf{u} = \mathbf{u}_0 + \Delta \mathbf{u} \]
In Eq. 6-2 $B$ is the incremental control effectiveness matrix containing local control derivatives. In fact $B$ is the Jacobian of the control directions w.r.t. the inputs evaluated at the current state and inputs:

$$
B_{inc}(x, u_0) = \begin{bmatrix}
\frac{\partial C_1}{\partial u_1}(x, u_0) & \cdots & \frac{\partial C_1}{\partial u_m}(x, u_0) \\
\frac{\partial C_m}{\partial u_1}(x, u_0) & \cdots & \frac{\partial C_m}{\partial u_m}(x, u_0) \\
\frac{\partial C_1}{\partial u_1}(x, u_0) & \cdots & \frac{\partial C_m}{\partial u_m}(x, u_0)
\end{bmatrix}
$$

(6-3)

The incremental $B$-matrix thus accounts for nonlinear input dynamics as well as aerodynamic interactions.

Figure 6-1 shows an elementary example of INCA to deal with nonlinear input dynamics. As long as input increments are small, the local linearization ‘follows’ the nonlinear curve, making it possible to use nonlinear input dynamics in combination with a linear solver. Of course local derivatives can be misleading in case of large increments, but since the update frequency of the controller is 100 Hz, and the rate limit of the effectors is 150 deg/s or less, the maximum increment each time step is only 1.5 deg. This is well within the valid range of the local control effectiveness estimation.

The advantage of INCA becomes even more clear if effector interactions are looked at. See figure 6-2 in which the elevon effectiveness is shown when the SSD deflection is 10 deg. The decrease in effectiveness due to upstream effectors is not taken into account by LCA, resulting in large estimation errors. INCA handles this properly as can be seen in the right figure.

One important feature of the ICE model not taken into account for now are non-monotonic nonlinearities in the input dynamics. Functions such as shown in section 4-1 can be misleading if control effectiveness is evaluated beyond the slope reversal. Although for now this
phenomenon is not encountered in practice, it is worthwhile a more detailed analysis in future research.

6-2 Algorithm trade-off

In sections 4-2 and 4-3 various algorithms for LCA are listed. The problem stated in Eq. 6-1 can be solved by for example the generalized inverse

\[ \mathbf{u} = (\mathbf{I} - \mathbf{G}_{\text{lin}}\mathbf{B}_{\text{lin}})\mathbf{u}_p + \mathbf{G}_{\text{lin}}(\mathbf{\tau}_c - h(x)) \]  

(6-4)

or by quadratic programming (QP) solving the WLS form

\[
\min_{\mathbf{u} \leq \mathbf{u} \leq \mathbf{u}} \left\| \begin{pmatrix} \mathbf{Q}_{\text{lin}} & \mathbf{Q} \end{pmatrix} \mathbf{u} + \begin{pmatrix} \mathbf{Q}h(x) - \mathbf{\tau}_c \end{pmatrix} \right\|_2
\]

(6-5)

\[
\mathbf{u} = \max(\mathbf{u}_0 - \dot{\mathbf{u}}, \mathbf{u}_{\text{min}}) \\
\overline{\mathbf{u}} = \min(\mathbf{u}_0 + \dot{\mathbf{u}}, \mathbf{u}_{\text{max}})
\]

The INCA problem reads as follows:

Find \( \mathbf{u} \leq \Delta \mathbf{u} \leq \overline{\mathbf{u}} \) to minimize \( B_{\text{inc}} \Delta \mathbf{u} - (\mathbf{\tau}_c - \mathbf{\tau}_0) \)  

(6-6)

which is not very different from the LCA problem in Eq. 6-1, so the exact same solvers can be used. The RPI for example can just as well calculate the input increment, but keep in mind to subtract the current input from the preferred input since \( \mathbf{u}_p = \mathbf{u}_0 + \Delta \mathbf{u}_p \) so \( \Delta \mathbf{u}_p = \mathbf{u}_p - \mathbf{u}_0 \). The first solution is then given by

\[ \Delta \mathbf{u} = (\mathbf{I} - \mathbf{G}_{\text{inc}}\mathbf{B}_{\text{inc}})(\mathbf{u}_p - \mathbf{u}_0) + \mathbf{G}_{\text{inc}}(\mathbf{\tau}_c - \mathbf{\tau}_0) \]  

(6-7)
after which the solution is clipped on the incremental boundaries and the remainder of the problem is solved with the unsaturated inputs, see section 4-2-2. In WLS optimization form, the INCA problem is

\[
\min_{\Delta u \leq \Delta u} \left\| \left( QB_{inc} \right) \Delta u + \left( Q(\tau_0 - \tau_c) \right) W(u_0 - u_p) \right\|_2
\]

\[
\Delta u = \max(\dot{u}, u_{\text{min}} - u_0)
\]

\[
\Delta u = \min(\dot{u}, u_{\text{max}} - u_0)
\]

In chapter 4 is already argued that optimization-based CA with active set QP is favorable over RPI because of the design freedom and better quality solutions. This hypothesis is tested with a brief simulation. The simulation takes place in the framework from chapter 5 at Mach 0.6 and with the exact onboard model from 5-3-3. A predefined set of joystick inputs \((p_c, q_c, r_c)\) is used consisting of separate roll, pitch, and yaw commands and a combined roll-pitch maneuver. The MATLAB-codes of both algorithms are in appendices A and B and the following settings are used:

\[
Q = \text{diag}(10000, 10000, 10000)
\]

\[
W = \text{diag}(1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)
\]

\[
u_p = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]^T
\]

Figure 6-3 shows the response to the rate commands with both algorithms. The QP method clearly outperforms the RPI algorithm and tracks the roll and pitch commands very well. Roll and pitch response with the RPI algorithm is slower and the commanded value is most cases not fully achieved. The yaw command is challenging for both algorithms, but again the QP method proofs to be more stable and maintains zero roll and pitch rates. The yaw rate overshoot is a result of the lack of yaw damping due the absence of a vertical tail. Tweaking of the linear outer-loop gains may enhance the yaw response.

The source for the improper tracking performance of the RPI algorithm is that it is not able to perfectly minimize the control allocation error, see figure 6-4. The allocation error is defined as the amount of commanded moment that is not achieved by the proposed input, so it is a direct measure of how well the algorithm finds a way to meet the control demand. With the RPI algorithm the allocation error is never exactly zero, whereas with the QP algorithm the allocation error is minimized to zero quickly. This zero allocation error is also a good sign for mission-specific control allocation: it means that there is room to optimize a secondary objective.

From figure 6-5 can be deduced that the number of iterations to converge to the final solution is higher for the QP method than for the RPI method though. The computation times per iteration are comparable for both algorithms, approximately 0.2 ms on a 1.6 GHz processor. On average the RPI algorithm is about three times as fast, but this does not justify the improper rate tracking performance. In worst cases QP is still fast enough for real-time implementation on a modern computer. Moreover RPI lacks the ability to specify the importance of allocation error minimization over control minimization, and to introduce very specific secondary objectives. In conclusion QP is without any doubt the favorable INCA solver.
Figure 6-3: Rate tracking performance with RPI and QP algorithms.
Figure 6-4: Allocation error with RPI and QP algorithms.

Figure 6-5: Number of required iterations per time step with RPI and QP algorithms.
6-3 Comparison with linear control allocation

Similar simulations as in the preceding section are carried out to compare the performance of INCA on the ICE aircraft with that of LCA. The simulation framework and inputs are the same as for the previous tests. Naturally LCA is used in combination with the linear onboard model from section 5-3-1, and INCA is applied with the exact model from section 5-3-3 (INCA-E) and spline model from section 5-3-2 (INCA-S). The solving algorithm is active set QP with the settings from Eq. 6-9. Figure 6-6 shows the response to the rate commands with the three methods.

The first roll command is appropriately tracked by both LCA and INCA. In this maneuver the pitch flap, elevon, AMT, and SSD deflections were small (<10 deg) and in the quasi-linear range. In the pitch maneuver the SSD deflections become larger and the inter-effector relations play a serious role, which is not accounted for by LCA. This leads to inaccurate tracking response, but the robustness of the closed-loop controller prevents the aircraft from destabilizing. The yaw command however results in such large SSD deflections (>15 deg) that control effectiveness of trailing edge surfaces is completely misjudged by LCA, recall figure 6-2. The aircraft destabilizes and the subsequent maneuver is completely missed. Figure 6-7 shows the response when the yaw maneuver is skipped, but also in the final combined roll-pitch maneuver the response of LCA is sub-optimal.

On the other hand INCA-E and INCA-S track the commands very well. Figure 6-8 explains the significant improvement in performance. The estimation errors are defined as follows:

\[
\text{Estimation error LCA} = h(x) + B_{\text{lin}}(x)u - \tau_{\text{real}}(x,u)
\]

\[
\text{Estimation error INCA} = \tau_0(x,u_0) + B_{\text{inc}}(x,u_0)\Delta u - \tau_{\text{real}}(x,u_0 + \Delta u)
\]

with \(\tau_0\) and \(B\) going into the allocator and \(u\) or \(\Delta u\) the proposed solution. INCA differentiates itself from LCA in that it adds the estimated moment increments due to input increments and not the entire moment due to the entire inputs. In general \(\Delta u < u\) so the potential error of INCA is already smaller, but additionally \(B_{\text{inc}}\) is more accurate than \(B_{\text{lin}}\).

Summarizing, LCA oftentimes 'thinks' that its solution results in the control demand, but in reality its extrapolation of the global control effectiveness is misleading and the obtained moments are unexpected. INCA has a much better picture of reality and can very accurately insinuate the generated moments that belong to the proposed solution. INCA-S shows larger estimation errors than INCA-E, but that is primarily a result of incorrect \(\tau_0\) information at negative angles of attack. In the cruise part from 8s to 15s in figure 6-8 the spline model shows a small bias, but control effectiveness estimation with INCA-S is still much better than with LCA, so it shows more stable rate tracking performance.
Figure 6-6: Rate tracking performance with LCA, exact INCA, and spline-based INCA.
Figure 6-7: Rate tracking performance with LCA, exact INCA, and spline-based INCA. The yaw command is skipped.
Figure 6-8: Estimation error of moment coefficients. The yaw command is skipped.
6-4 Concluding remarks

In this chapter the theory of incremental control allocation (INCA) is described and the results of preliminary tests are shown. The idea of INCA is that the input increments are calculated based on the increment of the control demand and with use of a local control effectiveness estimation. The benefit over linear control allocation (LCA) is that nonlinearities are taken into account without increasing the complexity of the CA problem.

Preliminary tests on INCA with two different solvers prove that the RPI algorithm comes up with sub-optimal solutions, resulting in poor rate tracking. In combination with a QP solver, INCA is able to minimize the allocation error in the majority of the time steps, significantly improving the handling qualities. The zero allocation error also implies that access control power can achieve secondary objectives. The most important benefit of QP is that the user can determine the ratio of importance between the primary and secondary objective, and that a wider variety of secondary objectives can be incorporated. This is the topic of the next chapter.

Comparison of INCA with LCA makes clear that the ICE aircraft has too nonlinear input dynamics to assume them linear. With LCA the controller calculates inputs that in reality do not achieve the command to satisfactory degree. A direct result is unacceptable rate tracking performance and unstable behavior. The spline model is accurate enough for satisfactory control performance, at least for the maneuvers tested. Robustness of the feedback controller makes the difference between flight with the spline model and the exact model barely notable, despite the fact that the moment estimations are not exact. The spline model is put further to the test in the next chapter, where it is also used to estimate forces.
Chapter 7

Minimum drag objective

So far control allocation is tested on the ICE aircraft with a fixed secondary objective. It is in fact an approach that is taken in most previous research to minimize the overall control effort via the objective

$$\min \|u\|_2$$

(7-1)

so the full WLS INCA problem becomes $^1$

$$\min_{\Delta u \leq u} \left\| \begin{pmatrix} QB \\ W \end{pmatrix} \Delta u + \begin{pmatrix} Q(\tau_0 - \tau_c) \\ W(u_0 - u_p) \end{pmatrix} \right\|_2$$

(7-2)

$$Q = \text{diag}(10000, 10000, 10000)$$

$$W = \text{diag}(1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$$

$$u_p = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]^T$$

Although this method comes up with a convenient set of inputs, the goal of the secondary objective is to provide uniqueness of the solution and to prevent illogical effector use, but the focus is less on its physical impact. For aircraft with four or five control surfaces this way of thinking is understandable, but given the excessive number and diverse nature of the controls on the ICE aircraft, it is considered that more thorough secondary objectives can yield a significant improvement of the operational performance. This chapter proposes two control allocation strategies to reduce the aerodynamic drag of the aircraft in sections 7-1 and 7-2. In sections 7-3 and 7-4 the results of simulations with two different onboard models are shown. The chapter ends with a discussion.

7-1 Effector prioritization

J. F. Buffington (1999) first experimented with exploiting the redundancy of the ICE aircraft using mission-specific $W$-matrices, recall sections 4-3 and 4-4. The idea is that a higher $W$-matrix

$^1$The subscript $\text{inc}$ is omitted from now on.
value penalizes the deflection of an effector, such that the use of the others is prioritized. To minimize the wing loading the most outboard effectors (the AMTs) were penalized, whereas to reduce the radar cross-section the SSDs were given the highest $W$-values.

The same philosophy can be applied for decreasing aerodynamic drag. The weighing matrix has to penalize the most resistant controls, such that their use is avoided if the control power is sufficient. It is known that in case of the ICE aircraft, the SSDs generally cause the most drag, and the leading edge flaps and thrust vectoring the least. This is substantiated by figure 7-1 showing the axial forces generated by each effector at Mach 0.5. For the control surfaces this data comes directly from the aerodynamic model, and the axial force of thrust vectoring is considered as the loss of forward thrust:

$$C_A = C_T \cdot (1 - \cos \delta_{pte} \cos \delta_{gte}) \quad (7-3)$$

in which $C_T = \frac{T}{\frac{1}{2} \rho V^2 S}$ is the thrust coefficient. At low angles of attack the axial force is approximately the drag force. The following weighing is chosen:  

$$Q = \text{diag}(20000, 20000, 20000) \quad (7-4)$$

$$W = \text{diag}(1, 1, 5, 5, 10, 5, 1, 1, 5, 5, 10, 1, 1)$$

$$u_p = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]^T$$

![Figure 7-1: Axial force component by deflection of the effectors.](image)

Although the values in Eq. 7-4 are based upon the analysis above and iterative experiments, they might not be the most suitable in every single situation. This however is inherent to fixed control allocation objectives. The $Q$-values are chosen such that the relative importance of the secondary objectives is about equal between Eqs. 7-2 and 7-4.

---

The order of the effectors is given in table 3-1.
7-2 Model-based

It is of course also possible to drop the regular CA formulation with \( W \) and \( u_p \), and to introduce an analytic expression for the objective. To minimize drag the CA problem becomes

\[
\min_{\mathbf{u} \leq \Delta \mathbf{u} \leq \mathbf{u}} \|Q(B\Delta \mathbf{u} + \mathbf{\tau}_0 - \mathbf{\tau}_c)\|_2 + \|C_D(x, \mathbf{u})\|_2
\]

(7-5)

In order for the minimum drag objective to fit in the incremental WLS form, the drag coefficient first has to be expressed as an affine function of \( \Delta \mathbf{u} \). That is achieved as follows. The drag coefficient is defined as the negative \( X \)-force coefficient in the aerodynamic frame. The force coefficients in the aerodynamic frame are calculated by the following transformation:

\[
\begin{bmatrix}
\frac{\partial C'_X}{\partial \mathbf{u}} \\
\frac{\partial C'_Y}{\partial \mathbf{u}} \\
\frac{\partial C'_Z}{\partial \mathbf{u}}
\end{bmatrix} = \begin{bmatrix}
\cos \alpha & 0 & \sin \alpha \\
0 & 1 & 0 \\
-\sin \alpha & 0 & \cos \alpha
\end{bmatrix} \begin{bmatrix}
\frac{\partial C_X}{\partial \mathbf{u}} & -\frac{\partial C_T}{\partial \mathbf{u}} \\
\frac{\partial C_Y}{\partial \mathbf{u}} & 0 \\
\frac{\partial C_Z}{\partial \mathbf{u}} & 0
\end{bmatrix}
\]

(7-6)

The thrust coefficient has to be subtracted to make sure drag only consists of control surface resistance and the loss of thrust in forward direction due to thrust vectoring, see Eq. 7-3. Given Eq. 7-6

\[ C_D = -C'_X = -C_X \cos(\alpha) \cos(\beta) - C_Y \cos(\alpha) \sin(\beta) - C_Z \sin(\alpha) + C_T \cos(\alpha) \cos(\beta) \]

(7-7)

with \( C_X(x, \mathbf{u}), C_Y(x, \mathbf{u}), \text{ and } C_Z(x, \mathbf{u}) \) in the body frame as recorded in the models from section 5-3. The drag coefficient is a linear combination of the force coefficients in the body frame, so the drag derivative w.r.t an input is

\[
\frac{\partial C_D}{\partial u_i} = -\frac{\partial C'_X}{\partial u_i} \cos(\alpha) \cos(\beta) - \frac{\partial C'_Y}{\partial u_i} \cos(\alpha) \sin(\beta) - \frac{\partial C'_Z}{\partial u_i} \sin(\alpha)
\]

(7-8)

The derivatives toward all inputs are combined in the drag effectiveness matrix

\[
E(x, \mathbf{u}_0) = \left[ \frac{\partial C_D}{\partial u_1} \bigg|_{x, \mathbf{u}_0}, \ldots, \frac{\partial C_D}{\partial u_m} \bigg|_{x, \mathbf{u}_0} \right]
\]

(7-9)

and so the incremental drag formula reads

\[ C_D = C_{D_0}(x, \mathbf{u}_0) + E(x, \mathbf{u}_0)\Delta \mathbf{u} \]

(7-10)

In principle this is not different from the way incremental control allocation is derived for the moment coefficients in the previous chapter. The full model-based minimum drag CA problem is now as follows:

\[
\min_{\mathbf{u} \leq \Delta \mathbf{u} \leq \mathbf{u}} \left\| \left( \begin{bmatrix}
Q & \mathbf{E}
\end{bmatrix} \Delta \mathbf{u} + \begin{bmatrix}
Q \mathbf{\tau}_0 - \mathbf{\tau}_c
\end{bmatrix} \right) \right\|_2
\]

(7-11)

\[ Q = \text{diag}(5, 5, 5) \]

The secondary objective in Eq. 7-11 does not 'know' the exact minimum drag position of the effectors. Rather the incremental nature implies a gradient decent search toward this position.
The magnitude of each $E$-value represents the amount of resistance per unit increment of the particular effector, while the sign of the $E$-value indicates if the minimum drag position is higher or lower than the current position. If $Q$ is chosen too low, drag is considered more important than the allocation error, and the effectors are indeed driven toward their minimum drag position in a couple of time steps, but the aircraft destabilizes. Appropriate $Q$-values are found to be in the range of 5 to 10.

At first instance the method described looks to be related to a control minimization objective with $W = \text{diag}|E|$ and $u_p$ the minimum drag position. Though $W$ is in that scenario treated as the global drag effectiveness from $u_p$ to $u_0$, while $|E|$ actually represents the local resistance about $u_0$. Moreover finding the minimum drag position each time step for each effector requires a lot of computations, making the approach from Eq. 7-11 far more useful.

Please also note that the problem from Eq. 7-11 has only 3 parameters (3 $Q$-values), whereas Eq. 7-4 has 29 (3 $Q$-values, 13 $W$-values, and 13 $u_p$-values). Hence the model-based objective requires less tuning, for it gets most information through system identification. This makes the method less sensitive to preliminary decisions, but more to modeling errors.

### 7-3 Results with exact model

So far in this chapter three CA modes are suggested: the standard mode (ST) from Eq. 7-2, the prioritization mode (PR) from Eq. 7-4, and the model-based mode (MB) given in Eq. 7-11. All modes are in the INCA form and solved with active set QP according to chapter 6. In this section the system information comes from the exact onboard model from section 5-3-3.

The performance of the three modes is assessed in a mixed flight scenario. The flight test consists of moderate maneuvering with roll and sideslip (aiming) commands at Mach 0.6 at 600 ft. After 5 seconds of stabilization a 5 second left turn is taken in a bank angle of -30 degrees. Between 15 and 20 seconds a -20 degrees angle of sideslip command is given. The mission ends with a steep right turn of half a minute in a bank angle of 60 degrees. It is known that especially the latter maneuver is important in air combat, but often limited by the available power. Drag reduction in this scenario increases the odds over the competitor.

For assessment altitude is kept constant through an outer loop controlling the angle of attack. Constant thrust is set to 6500 lbf. No climbing maneuvers are included in the simulation, because it is found in a climb situation most drag, both from the mainframe and from the effectors, is lift-induced drag. For a given angle of attack it is possible to reduce drag by placing control surfaces parallel to the airflow, but this will be accompanied by a decrease of lift. To compensate for that, a higher angle of attack has to be flown and the drag reduction in the end is not significant.

The attitude response, airspeed, altitude, and aerodynamic drag are shown in figures 7-2 to 7-5. The flight track and plots of the effector positions can be found in appendix C. The attitude, airspeed, and altitude are direct outputs of the simulation model. Drag is calculated with the measured body forces $X, Y, Z$, thrust $T$, and aerodynamic angles $\alpha, \beta$ through the same transformation as in equation 7-7. The values of average drag are given in table 7-1. For clarity of the graphs turbulence in the environmental model (figure 5-2) is turned off, but drag values of simulations with turbulence are also given in table 7-1.
Figure 7-2: Aerodynamic angles drag during flight with exact model. ST = standard mode, PR = prioritization mode, MB = model-based mode.

Figure 7-3: Airspeed and altitude during flight with exact model. ST = standard mode, PR = prioritization mode, MB = model-based mode.
Figure 7-4: Drag during flight with exact model.
ST = standard mode, PR = prioritization mode, MB = model-based mode.

Figure 7-5: Drag reduction during flight with exact model.
ST = standard mode, PR = prioritization mode, MB = model-based mode.
7-3 Results with exact model

Figure 7-6: Effector use during steady part of the aiming maneuver, measured at 19s. ST = standard mode, PR = prioritization mode, MB = model-based mode.

Figure 7-7: Effector use during steady part of the right turn, measured at 35s. ST = standard mode, PR = prioritization mode, MB = model-based mode.
Table 7-1: Results of flight with exact model.
ST = standard mode, PR = prioritization mode, MB = model-based mode.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Average drag [lbf]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>turbulence off</td>
</tr>
<tr>
<td>ST</td>
<td>6462.6</td>
</tr>
<tr>
<td>PR</td>
<td>6041.2 (−6.52%)</td>
</tr>
<tr>
<td>MB</td>
<td>6046.1 (−6.44%)</td>
</tr>
</tbody>
</table>

It is clear from figures 7-2 to 7-4 that under constant bank angles (5s – 10s and 25s – 55s) the PR-mode and MB-mode reduce drag with respect to the ST-mode. Also sideslip commands (15s – 20s) are well tracked by these modes, while reducing drag with about 8% relative to the standard mode. The difference in average drag values between the PR-mode and MB-mode is from table 7-1 not significant, but from figure 7-5 it may be clear that the PR-mode is beneficial during the dynamic part of the maneuver, whereas the MB-mode reduces more drag in the steady parts. Overall the PR-mode performs slightly better, though in turbulence it is the other way around. Notice that in cruise flight drag reduction is not significant, which is the reason that climb maneuvers are not investigated for now.

It is interesting to look how the minimum drag modes accomplish the drag reduction. Figures 7-6 and 7-7 show the effector positions in the steady part of the sideslip maneuver (at 19s) and right turn (at 35s) respectively. Some specific choices are clear. Whereas the ST-mode uses primarily the left SSD in a sideslip, the PR-mode selects yaw thrust vectoring and the left outboard LEF, and the MB-mode chooses yaw thrust vectoring and the right inboard LEF. To suppress sideslip in the right turn the ST-mode again uses the yaw power of the left SSD, and the other modes choose thrust vectoring. Effector usage in the dynamic parts of the mission can be read from the figures in appendix C.

It looks like the most drag reduction can be achieved by just keeping the SSDs closed. A simulation with only this constraint already shows a drag reduction of almost 5%. At low angles of attack the SSDs are indeed a poor choice for yaw control because they generate a lot of drag and degrade the effectiveness of the elevons. Though in extreme situations extra yaw power can be necessary, and additionally the SSDs are very useful at high angles of attack to recover airflow over the trailing edge flaps.

7-4 Results with spline model

In the previous simulations the exact onboard model from section 5-3-3 is used, so $\tau_0$, $B$, $C_{D_0}$, and $E$ used in control allocation are the exact values. To study the sensitivity of the modes to model inaccuracies the same mission if flown with the spline model from section 5-3-2 onboard. The aerodynamic angles, airspeed, altitude, and drag plots are given in figures 7-8 to 7-11. Additional plots can be found in appendix C. Average drag values are listed in table 7-2.
Figure 7-8: Aerodynamic angles drag during flight with spline model. ST = standard mode, PR = prioritization mode, MB = model-based mode.

Figure 7-9: Airspeed and altitude during flight with spline model. ST = standard mode, PR = prioritization mode, MB = model-based mode.
Figure 7-10: Drag during flight with spline model. 
ST = standard mode, PR = prioritization mode, MB = model-based mode.

Figure 7-11: Drag reduction during flight with spline model. 
ST = standard mode, PR = prioritization mode, MB = model-based mode.
Table 7-2: Results of flight with spline model.

ST = standard mode, PR = prioritization mode, MB = model-based mode.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Average drag [lbf]</th>
<th>turbulence off</th>
<th>turbulence on</th>
</tr>
</thead>
<tbody>
<tr>
<td>ST</td>
<td>6523.2</td>
<td>6588.8</td>
<td></td>
</tr>
<tr>
<td>PR</td>
<td>6174.4 (−5.35%)</td>
<td>6218.8 (−5.62%)</td>
<td></td>
</tr>
<tr>
<td>MB</td>
<td>6307.6 (−3.30%)</td>
<td>6362.4 (−3.44%)</td>
<td></td>
</tr>
</tbody>
</table>

From figures 7-8 and 7-9 it can be concluded that the tracking performance is similar to when an exact model is used (figures 7-2 and 7-3). Hence the spline model is accurate enough for decent operation, at least for the kinds of maneuvers tested. Also the drag values of the ST-mode and PR-mode are comparable.

However from table 7-2 and figure 7-11 it can be deduced that the performance of the MB-mode is notably degraded by the use of the spline model. The reason for this is that an imperfect model sometimes provides misleading (drag-)information to the controller. See for example figure 7-12. According to the spline model the deflection of the left AMT can be a couple of degrees without producing much drag. In reality however the drag is significantly higher, up to 8% at $\delta_{\text{lamt}} = 4$ deg. The aerodynamic model consists of 108 sub-models, each of them with this kind of inaccuracies, adding up to the loss in performance of the MB-mode. The PR-mode does not use this information, so its drag reduction capability is less affected by the quality of the model.

![Drag coefficient graph](image)

**Figure 7-12:** Drag coefficient vs. left AMT deflection at Mach 0.6 and $\alpha = 3$ deg.
Table 7-3 shows the results if the real aircraft dynamics were based on the spline model. Then the original model is inaccurate and the spline model is ‘exact’, i.e. the roles are reversed. It is clear that again the MB-mode is favorable if the onboard model is precise and the PR-mode if it is not.

Table 7-3: Results of flight with the aircraft dynamics based on the spline model. The original model is now inaccurate and the spline model exact.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Average drag [lbf]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>original model</td>
</tr>
<tr>
<td>ST</td>
<td>6300.1</td>
</tr>
<tr>
<td>PR</td>
<td>5875.3 (-6.74%)</td>
</tr>
<tr>
<td>MB</td>
<td>6018.1 (-4.48%)</td>
</tr>
</tbody>
</table>

7-5 Discussion

From tables 7-1 and 7-2 it can be concluded that mission specific control allocation can indeed optimize operational performance, in this case drag reduction. The advantages are most apparent in the steady states of a turn or aiming maneuver, and in the roll-in and roll-out phase. In cruise flight the drag reduction is marginal. The validity of the comparison of drag values is proven by figures 7-2, 7-3, 7-8, and 7-9. Under all three modes the same maneuver is flown: altitude is kept constant and the aerodynamic angle tracking exactly matches. Each test the thrust setting is the same, but the final airspeeds of the PR-mode and MB-mode are higher than that of the ST-mode. This indicates that the aircraft encountered less resistance along the way. When thrust for the PR-mode and MB-mode is set to 6100 lbf the final airspeeds are comparable (with the exact model), underlining these modes make the aircraft more fuel efficient.

The simulations described are performed at Mach 0.6 at an altitude of 600 ft. Simulations of the same maneuvers in other conditions prove that the performance is maintained, see figures 7-13 and 7-14. The PR-mode turns out to be more sensitive to Mach number than the MB-mode. The influence of altitude is small. The maneuvers are not very suitable for supersonic flight or high altitudes, so the ability of drag reduction in this part of the flight envelope is a topic for future research. Figure 7-15 shows the drag reduction in a right turn with various bank angles. The sharper the turn, the more drag is reduced. It is also clear that the MB-mode outperforms the PR-mode, primarily in the steady part as is argued earlier. Steeper bank angles than 60 degrees can be flown, but the aircraft easily destabilizes. Better tuning of the outer loops and flight envelope protection is required for further testing.

All in all, both minimum drag modes are able to reduce the operation drag of the ICE aircraft, and from section 7-4 it turns out that the PR-mode produces better solutions if the onboard model lacks accuracy, as was expected. But does this make the PR-mode the preferred technique in practical applications? As mentioned earlier the MB-mode has a lot less parameters to be manually tuned than the PR-mode. Moreover the performance of the MB-mode turns out to be rather robust to the choice of Q-values, as long as the importance of the primary objective is clear ($Q \geq 5$). The reason for this is that in most situations,
7-5 Discussion

Figure 7-13: Drag reduction at various Mach numbers with exact model. Altitude is 10000 ft. Maneuver is the same as in previous sections. ST = standard mode, PR = prioritization mode, MB = model-based mode.

Figure 7-14: Drag reduction at various altitudes with exact model. Mach 0.6. Maneuver is the same as in previous sections. ST = standard mode, PR = prioritization mode, MB = model-based mode.

Figure 7-15: Drag reduction at various bank angles with exact model. Mach 0.6 at 600 ft. Maneuver consists of 5 seconds cruise, 20 seconds turn, and 5 seconds cruise. ST = standard mode, PR = prioritization mode, MB = model-based mode.
especially in the steady parts of the mission, the allocation error can easily be minimized to zero, so the relative importance of the secondary objective does not play a role. Practically speaking control allocation does not have to make a concession between primary and secondary objective, because the primary objective can be achieved to the fullest, leaving enough excess power for the secondary objective whatever $Q$-value.

On the other hand, the PR-mode is very sensitive to the choice of its parameters. In fact the entire power of PR-mode relies on the iterative establishment of $W$-values. For example a simulation with a slightly different $W = \text{diag}(1, 1, 5, 5, 5, 1, 1, 5, 5, 5, 2, 2)$, shows that the drag reduction capability is more than halved: $-3.10\%$ against $-6.52\%$ originally. The $W$-matrix in this study is specifically optimized for the maneuvers performed, but in an other mission it can behave differently. This vulnerability of the PR-mode is inherent to its design and makes the MB-mode a preferable option in systems of which the dynamics are accurately mapped.

A final point of attention is that the model now assumed to be ’real’ is in some parts very illogical. Linear interpolation of sparse data results in implausible input dynamics, see the plots in section 3-4. Multivariate simplex splines are not able to capture such sharp functions, which explains most of the inaccuracy of the spline model. In reality though, the aircraft is expected to feature smoother input dynamics, and the spline model will not be that imprecise. The drop in performance of the MB-mode by the use of the spline model is thus exaggerated in this study.
In this thesis the development and testing of mission-specific control allocation (CA) for the Innovative Control Effector (ICE) aircraft are presented. The ICE aircraft is a tailless fighter concept equipped with 13 control effectors. It features highly complex aerodynamics with longitudinal and directional instability in some parts of the flight envelope and nonlinear input dynamics. Because of the lack of a vertical tail plane, spoiler slot deflections (SSDs) are installed on the main wing to provide the necessary yaw power. The disadvantage of these control surfaces is their location upstream some trailing edge flaps, severely affecting the control effectiveness of these neighbors.

The fact that the aircraft is highly over-actuated and the inputs are nonlinear and interacting makes the CA problem complex. With a robust and well-tuned feedback controller linear control allocation (LCA) works to a certain extend, but the generated solutions are sub-optimal and handling qualities are poor. Incremental control allocation (INCA) is therefore proposed to deal with the nonlinear inputs and aerodynamic interaction. Combined with an active set quadratic programming (QP) solver, INCA is proven to significantly increase the rate tracking performance without complicating the CA process. INCA parameters can be obtained from the non-affine aerodynamic model directly, see section 5-3-3, but this requires many evaluations of the 108 look-up tables. A more logical choice is to model the aerodynamics with multivariate simplex-splines, such that the control derivatives are analytic functions of states and inputs. The developed spline model of the ICE aircraft is accurate enough for decent handling qualities, at least for the maneuvers tested. The rate tracking response of spline-based INCA is comparable to that of INCA with the exact model.

Since the aircraft is highly over-actuated and the nature of the controls is diverse, it is likely that ordinary control allocation objectives do not steer well toward operationally attractive solutions. As an example of mission-specific control allocation two secondary objectives to minimize drag are proposed. The first drag minimization mode (prioritization, PR) penalizes the deflection of the most drag-causing effectors, such that the use of low-resistant effectors is preferred. In the mission flown, the PR-mode reduces drag with about 6% relative to the standard control allocation mode. The reduction is primary the result of using thrust vectoring instead of the SSDs for yaw control. The performance of the PR-mode is relatively
insensitive to the accuracy of the onboard model, yet it is vulnerable to the choice of the initial parameters. Slight changes in the parameters or a different flight mission may largely affect the drag reduction capabilities. The second drag minimization mode (model-based, MB) incorporates the drag function in the CA problem and uses the excess control power to force the effectors to their minimum drag position. If the onboard model is precise the performance is comparable to that of the PR-mode, as it reduces drag with about 6.5%. With an inaccurate onboard model however the drag reduction is significantly less. In return the number of initial parameters is much less and the performance is rather insensitive to these parameters.

In conclusion, incremental spline-based control allocation with a quadratic programming solver is clearly the recommended approach for next generation high-performance tailless aircraft. Effector prioritization can be an effective way to achieve secondary goals, but its efficiency depends on the initial tuning. For practical application the technique is questionable because acceptable performance cannot be guaranteed over the entire flight envelope, especially not with one fixed set of parameters. On the other hand model-based objectives in CA design are theoretically substantiated and expected to maintain its performance in a much wider range of circumstances. The prerequisite is that the onboard model accurately estimates the actual behavior of the aircraft.

In this thesis the focus was on minimum drag control allocation for the ICE aircraft, but with the theory provided in chapter 7 other objectives can easily be defined as well. The most obvious one is a maximum lift mode, of which the theory and some preliminary tests are included in appendix D. This mode increases drag extremely, but in take-off cases where thrust is not an issue and the runway length is short, it is a useful mode. Other objectives discussed in appendix D are a maximum lift-over-drag mode and an input excitation mode that can be used for in-flight system identification. In future research the CA problem can be extended to a full 6 degrees-of-freedom problem, with thrust as the 14th input and a size $6 \times 14$ control effectiveness matrix. Furthermore a full incremental control approach can be experimented with, to cancel out some of the model inaccuracies. In that case incremental control allocation (INCA) is combined with incremental nonlinear dynamic inversion. The idea is that required moment increments are calculated based on the required rate acceleration increment (Eq. 2-15 becomes $\Delta \tau_c = g^{-1}(x)[\dot{x}_c - \dot{x}]$) and that $\Delta u$ is computed with $\Delta \tau_c$ through INCA. This method requires no calculation of $\tau_0$, so the potential errors are smaller.

Future research has to point out the performance of this approach.

Coming back to drag reduction, tests have to be done in other, more extreme, parts of the flight envelope in order to substantiate the true potential of model-based minimum drag CA. This may require better tuning of the outer loops and/or flight envelope protection methods as well. The most important recommendation is to improve the ICE aerodynamic model, such that physically implausible phenomena are corrected. Some characteristics shown in chapter 3 are unrealistic, primarily because of the linear interpolation of sparse data. A higher fidelity model can expose the real opportunities of spline-based modeling and mission-specific control for next generation fighter aircraft.
This appendix includes the MATLAB code for the redistributed pseudo-inverse (RPI) algorithm for control allocation.

For simulation all matrices have to be bounded in size. Therefore \( m_{\text{free}} \), the number of free inputs, is bounded by the loop in lines 30 - 32 before it can be used to make the identity matrix in line 35.

```matlab
function [u, iter] = rpi(B, tau, umin, umax, up, imax)
    % This algorithm solves the control allocation problem
    % Find u to minimize Bu - tau, subj. to umin <= u <= umax

    m = length(umin);
    u = pinv(B)*tau + (eye(m) - pinv(B)*B)*up;

    i_min = u < umin; u(i_min) = umin(i_min);
    i_max = u > umax; u(i_max) = umax(i_max);
    i_sat = (i_min | i_max);
    i_free = ~i_sat;

    % Calculate allocation error
    err = tau - B*u;

    % Repeat as long as there are free inputs and the error > threshold
    while any([i_min ; i_max]) & iter < imax & any(i_free) & abs(err) > 10^{-10};
        iter = iter + 1;
        B_sat = B(:, i_sat);
        B_free = B(:, i_free);
        m_free = sum(i_free);
```
if m_free > m
    m_free = m;
end

u(i_free) = pinv(B_free)*(tau - B_sat*u(i_sat)) + ...
            (eye(m_free) + pinv(B_free)*B_free)*up(i_free);

i_min = u < umin; u(i_min) = umin(i_min);
i_max = u > umax; u(i_max) = umax(i_max);
i_sat = (i_sat | i_min | i_max);
i_free = ~i_sat;

err = tau - B*u;
end
This appendix includes the MATLAB code for the active set quadratic programming algorithm for control allocation. The algorithm is a variation on the WLS allocation algorithm retrieved from http://research.harkegard.se/, see Härkegård (2003).

Please note two important differences. The initial solution (line 10) is set to zero rather than to the optimal solution of the previous time step, because the increment is calculated and not the entire input. Accordingly the initial working set (line 11) is chosen empty instead of equal to the previous optimal working set. Secondly the pseudo-inverse is chosen as the internal unconstrained solver (line 22) instead of the original QR-decomposition method. QR-decomposition is computationally cheaper, but can cause difficulties if the rank of the first matrix of the combined objective (A) is not full. This is the case with the objective from section 7-2.

```matlab
function [u, iter] = wls_simple(A,b,umin,umax,imax)

m = length(umin);
u = zeros(m,1);
WS = zeros(m,1);
i_free = WS==0;
d = b - A*u;

for iter = 1:imax
    % -------------------------------------------------- --
    % Compute optimal perturbation vector of free inputs

```
% Active set quadratic programming algorithm

A_free = A(:, i_free);
p_free = pinv(A_free)*d;
p = zeros(m,1);
p(i_free) = p_free;

% Check feasibility
u_opt = u + p;
infeasible = (u_opt < umin) | (u_opt > umax);

if ~any(infeasible(i_free))
    % Feasible, check for optimality with lagrangian multipliers
    u = u_opt;
d = d - A_free*p_free;
lambda = WS.*(A'*d);

    if lambda >= -eps
        % Optimum found, bail out
        return;
    else
        % Optimum not found, remove most negative lambda from WS
        [lambda_neg, i_neg] = min(lambda);
        WS(i_neg) = 0;
        i_free(i_neg) = 1;
    end

else
    % Not feasible, find primary bounding constraint

    % Compute distances to the different boundaries
    dist = ones(m,1);
i_min = i_free & p<0;
i_max = i_free & p>0;
dist(i_min) = (umin(i_min) - u(i_min))./p(i_min);
dist(i_max) = (umax(i_max) - u(i_max))./p(i_max);

    % Proportion of p to travel
    [alpha, i_alpha] = min(dist);

    % Update point and residual
    u = u + alpha*p;
d = d - A_free*alpha*p_free;

    % Add corresponding constraint to working set
    WS(i_alpha) = sign(p(i_alpha));
    i_free(i_alpha) = 0;
end
Appendix C

Additional plots

This appendix shows additional plots of the simulations from chapter 7. Figures C-1 and C-2 show the 2-dimensional flight paths of the simulations with the exact onboard model (see section 7-3) and the onboard spline model (see section 7-4) respectively. Figures C-3 to C-15 show the effector positions during flight with the exact model and figures C-16 to C-28 show the effector positions during flight with the spline model.

Remark: ST = standard mode, PR = prioritization mode, MB = model-based mode.

Figure C-1: Flight paths with the exact model. With the PR-mode and MB-mode the airspeeds increase relative to the ST-mode, explaining the different turn radii. With a lower thrust setting for the PR-mode and MB-mode the paths are more alike.
**Figure C-2**: Flight paths with the spline model. The difference in airspeeds is less than with the exact model, and the PR-mode already drifts a little to the right in the aiming maneuver. The difference in turn radii is therefore less obvious than in the previous figure.

**Figure C-3**: Left inboard LEF deflection during flight with exact model.

**Figure C-4**: Left outboard LEF deflection during flight with exact model.
Figure C-5: Left AMT deflection during flight with exact model.

Figure C-6: Left elevon deflection during flight with exact model.

Figure C-7: Left SSD deflection during flight with exact model.

Figure C-8: Pitch flap deflection during flight with exact model.
Figure C-9: Right inboard LEF deflection during flight with exact model.

Figure C-10: Right outboard LEF deflection during flight with exact model.

Figure C-11: Right AMT deflection during flight with exact model.

Figure C-12: Right elevon deflection during flight with exact model.
**Figure C-13:** Right SSD deflection during flight with exact model.

**Figure C-14:** Pitch thrust vectoring during flight with exact model.

**Figure C-15:** Yaw thrust vectoring during flight with exact model.

**Figure C-16:** Left inboard LEF deflection during flight with spline model.
Figure C-17: Left outboard LEF deflection during flight with spline model.

Figure C-18: Left AMT deflection during flight with spline model.

Figure C-19: Left elevon deflection during flight with spline model.

Figure C-20: Left SSD deflection during flight with spline model.
Figure C-21: Pitch flap deflection during flight with spline model.

Figure C-22: Right inboard LEF deflection during flight with spline model.

Figure C-23: Right outboard LEF deflection during flight with spline model.

Figure C-24: Right AMT deflection during flight with spline model.
Figure C-25: Right elevon deflection during flight with spline model.

Figure C-26: Right SSD deflection during flight with spline model.

Figure C-27: Pitch thrust vectoring during flight with spline model.

Figure C-28: Yaw thrust vectoring during flight with spline model.
Appendix D

Preliminary tests on other objectives

With the theory from this thesis more objectives than just the minimum drag example can be formulated. Specific W-matrices to express secondary goals, such as minimum wing load and minimum radar cross-section, are already described and tested by J. F. Buffington (1999). This appendix shows some preliminary results on two other objectives, to encourage future research on new CA modes.

Maximum lift

The most obvious objective is to maximize the lift coefficient $C_L$. Given Eq. 7-6 the lift coefficient is defined as

$$C_L = -C_Z' = C_X \sin(\alpha) \cos(\beta) + C_Y \sin(\alpha) \sin(\beta) - C_Z \cos(\alpha) - C_T \sin(\alpha) \cos(\beta) \quad (D-1)$$

and correspondingly

$$\frac{\partial C_L}{\partial u_i} = \frac{\partial C_X}{\partial u_i} \sin(\alpha) \cos(\beta) + \frac{\partial C_Y}{\partial u_i} \sin(\alpha) \sin(\beta) - \frac{\partial C_Z}{\partial u_i} \cos(\alpha) \quad (D-2)$$

The incremental lift formula is then

$$C_L = C_{L_0}(x, u_0) + E_L(x, u_0) \Delta u \quad (D-3)$$

with

$$E_L(x, u_0) = \left[ \frac{\partial C_L}{\partial u_1} \bigg|_{x, u_0}, \ldots, \frac{\partial C_L}{\partial u_m} \bigg|_{x, u_0} \right]$$

The CA solvers consider a minimization problem, so to maximize lift one can minimize the difference between $C_L$ and its preferred value. It is best to choose an unachievable, but not too large $C_L$-value here, e.g. 1. The CA problem becomes

$$\min_{\Delta u \leq \Delta u \leq \Delta u} \left\| \begin{pmatrix} Q B \\ E_L \end{pmatrix} \Delta u + \begin{pmatrix} Q (\tau_0 - \tau_c) \\ C_{L_0} - 1 \end{pmatrix} \right\|_2 \quad (D-4)$$

$$Q = \text{diag}(100, 100, 100)$$
Preliminary tests on other objectives

$Q$-values are larger than previously, because the magnitude of the secondary objective is larger than for the minimum drag mode (average $C_{D_0} \approx 0.01$ whereas $C_{L_0} - 1 \approx -0.9$).

Figure D-1: Lift coefficient vs. angle of attack.

Figure D-2: Drag coefficient vs lift coefficient.
With this objective it turns out to be possible to increase lift for a given angle of attack while maintaining steady flight (zero angular rates). The obtained $C_L$-values at Mach 0.6 with thrust set to 20000 lbf for some angles of attack are given in figure D-1. The objective makes sure pitch flaps are maximally deflected downward, AMTs 10 degrees upward, and elevons 12 degrees upward. Surprisingly pitch thrust vectoring is pointing upward, so in fact providing a force downward, but this is to compensate for the roll moment from the pitch flaps. From the $E$-values can be read that the lift effectiveness of the trailing edge flaps is higher than that of thrust vectoring, so the deflections mentioned above obtain the moment equilibrium with maximum lift.

Though the drag increase with this mode is extreme, and the drag coefficient for a given lift coefficient is worse with the maximum lift objective, see figure D-2. In flight it is better to fly a higher angle of attack to increase lift than to use this mode. However, if increasing angle of attack is impossible, for example during take-off run, and enough thrust is available, the maximum lift mode can be useful. More research has to be done to optimize the method and to identify applications. Also the function of the LEFs has to be looked at, since according to literature these effectors are beneficial in the take-off phase.

For other purposes it might also be beneficial to specifically maximize the lift-over-drag ratio. The coefficient is calculated as follows:

$$\frac{C_{L/D}}{C_D} = \frac{C_L}{C_D} = -\frac{C'_Z}{C'_X}$$

with $C'_Z$ and $C'_X$ calculated from body forces according to Eq. 7-6. However, obtaining the derivatives with respect to the inputs is troublesome, because $C_{L/D}$ is not longer a linear combination of the body force coefficients. Simply dividing $\frac{\partial C_L}{\partial u_i} = \frac{\partial C_D}{\partial u_i}$ cancels out the dependency on the inputs. Hence a maximum lift-over-drag objective requires different mathematical operations to obtain the lift-over-drag effectiveness matrix $E_{L/D}$. This might be a topic for future research.

**In-flight system identification**

The ultimate goal for the ICE aircraft control system would be to make it adaptive, so that online system identification estimates the system parameters given the current circumstances. In case of effector damage or sensor faults it can be imagined that the predefined parameters are not valid anymore, and that accurate control requires real-time updates. An adaptive control system literally adapts to the new situation and guarantees the best possible control under all circumstances.

For in-flight system identification it is necessary to excite the system, meaning that it has to deviate from a steady state. For conventional aircraft this can be achieved by a couple of pitch, roll, and yaw maneuvers, but for the ICE aircraft it is not ensured that all effectors are used in that case. A system identification CA mode is therefore developed, that changes the preferred state of the effectors randomly. Every 0.1 seconds one effector (sequentially)
receives a new preferred state, which is kept until the round is finished:

\[ u_{p1} = [n^1, 0, 0, \ldots, 0]^T \times (u_{\text{max}} - u_{\text{min}}) + u_{\text{min}} \]
\[ u_{p2} = [n^1, n^2, 0, \ldots, 0]^T \times (u_{\text{max}} - u_{\text{min}}) + u_{\text{min}} \]
\[ \vdots \]
\[ u_{p13} = [n^1, n^2, n^3, \ldots, n^{13}]^T \times (u_{\text{max}} - u_{\text{min}}) + u_{\text{min}} \]
\[ u_{p14} = [n^{14}, n^2, n^3, \ldots, n^{13}]^T \times (u_{\text{max}} - u_{\text{min}}) + u_{\text{min}} \]
\[ u_{p15} = [n^{14}, n^{15}, n^3, \ldots, n^{13}]^T \times (u_{\text{max}} - u_{\text{min}}) + u_{\text{min}} \]

with \( u_{pi} \) the preferred input and \( n_i \) a random number given at the \( i \)th (10 Hz) time step. \( u_{\text{min}} \) and \( u_{\text{max}} \) scale the random number from 0 to 1 to the effector range.

The INCA problem is formulated as in Eq. 6-8 with

\[ Q = \text{diag}(100000, 100000, 100000) \]
\[ W = \text{diag}(1, 1, 1, 1, 1, 1, 1, 1, 1, 1) \]

The mode is compared with the standard allocation scheme with

\[ Q = \text{diag}(10000, 10000, 10000) \]
\[ W = \text{diag}(1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1) \]
\[ u_p = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0]^T \]

The purpose of this system identification (SysID) mode is that dynamics at certain effector positions are explored that would otherwise be unidentified. Figure D-3 shows the mean exploration of the range of the effectors when the aircraft is subjected to the roll, pitch, and yaw inputs from chapter 6 at Mach 0.6 with turbulence. It is clear that the standard mode only explores during maneuvers, whereas the SysID mode also explores in steady parts. Command tracking is less accurate but still stable, as can be seen in figure D-4.

In cruise flight, with zero rate commands, the difference is enormous. The standard CA mode uses less than 1% of the effector range, even in turbulence. In 20 seconds the SysID mode visits almost 70% of the total range, while tracking performance is about equally good, see figures D-5 and D-6.

Figures D-7, D-8, and D-9 show some of the effector deflections with respect to their preferred state. For the fast effectors it is clear that the preferred states can be approximately achieved while the inputs in total maintain the moment equilibrium. The leading edge flap dynamics are too slow for this mode, so they are not able to achieve the preferred state in one round (1.3 seconds). This leads to a dilemma: on the one hand identification has to be done as quick as possible, but on the other hand effectors have to be given time to achieve extreme deflections. A recommendation for succeeding work is to investigate appropriate time steps. Furthermore the exploration of the total range does not say anything about the identification of effector interactions, so this is also a topic for future research. With an improved system identification CA mode, fault-tolerant adaptive control comes one step closer, enhancing survivability of both fighter and civilian aircraft.
Figure D-3: Exploration of the effector range during maneuvers.

Figure D-4: Rate tracking during maneuvers.
Preliminary tests on other objectives

Figure D-5: Exploration of the effector range in cruise flight.

Figure D-6: Rate tracking in cruise flight.
Figure D-7: Right elevon deflection in cruise flight.

Figure D-8: Pitch flap deflection in cruise flight.

Figure D-9: Right outboard LEF deflection in cruise flight.
Appendix E

Scientific paper

This appendix includes the paper on the main scientific contribution of the research.
Minimum drag control allocation for the Innovative Control Effector aircraft

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Delft University of Technology, Delft, 2600 GB, The Netherlands

The Innovative Control Effector model is a tailless delta-wing aircraft concept equipped with 11 control surfaces with overlapping functionality and two-directional thrust vectoring. The high level of redundancy makes it an interesting object for research on mission-specific control allocation. A (spline-based) incremental control allocation approach is proposed to deal with nonlinear input functions and aerodynamic interaction between multiple control surfaces. The control allocation task is formulated as a weighted least squares problem with variable secondary objectives. Two control allocation modes to minimize drag are proposed and assessed in a general flight scenario. With both modes the average drag is reduced by about 6.5% relative to a standard control allocation scheme. Sensitivity analysis points out that one mode is vulnerable to the choice of initial parameters, whereas the other is primarily sensitive to the accuracy of the onboard model. Improvement of the ICE aerodynamic model is necessary to substantiate the true potential of mission-specific control allocation for next generation aircraft.

Nomenclature

\[ A, Y, N \quad \text{Forces along the body axes (aerodynamic model), lbf} \]
\[ B \quad \text{Control effectiveness matrix} \]
\[ b \quad \text{Wing span, ft} \]
\[ C \quad \text{Dimensionless coefficient} \]
\[ \bar{c} \quad \text{Mean aerodynamic chord, ft} \]
\[ D \quad \text{Drag force, lbf} \]
\[ E \quad \text{Drag effectiveness matrix} \]
\[ I \quad \text{Inertia matrix} \]
\[ l_{tv} \quad \text{Thrust vectoring arm, ft} \]
\[ l, m, n \quad \text{Moments along the body axes, lbf·ft} \]
\[ p, q, r \quad \text{Rotational rates along the body axes, rad/s} \]
\[ Q, W \quad \text{Weighting matrices} \]
\[ S \quad \text{Wing surface, ft}^2 \]
\[ T \quad \text{Thrust force, lbf} \]
\[ u \quad \text{Input vector} \]
\[ V \quad \text{Airspeed, ft/s} \]
\[ X, Y, Z \quad \text{Forces along the body axes, lbf} \]
\[ x \quad \text{State vector} \]
\[ \alpha, \beta, \phi \quad \text{Angles of attack, sideslip, and roll, deg} \]
\[ \delta \quad \text{Deflection, deg} \]
\[ \rho \quad \text{Air density, slug/ft}^3 \]
\[ \tau \quad \text{Moment vector} \]

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I. Introduction

The future air combat environment demands low-observable super-maneuverable fighter aircraft with fault-tolerance capabilities. These features push research toward tailless configurations, as removal of the tail planes significantly reduces the radar cross-section and increases agility. The loss of control surfaces in the tail area, and the fault-tolerance requirement, necessitates the installation of unconventional control effectors that provide multi-axis moments and redundancy.

An interesting research object in this field is the tailless fighter concept developed under the Innovative Control Effector (ICE) program. The aircraft (figure 1) is characterized by its relaxed directional stability, nonlinear dynamics, and a total of 13 effectors with a high level of redundancy and aerodynamic interaction. The large number of effectors and their overlapping functionality however yield challenges in control law design. Since the number of inputs (13 effectors) exceeds the number of directions to control (3 rotations) the control allocation (CA) problem is under-determined and an infinite number of solutions exists. A convenient way to guide the selection of the ‘best’ solution is to introduce a secondary objective. Its role is twofold: it provides uniqueness to the solution, while it exploits the excess control power to increase the operational performance of the system. The topic of this paper is to find the lowest aerodynamic drag solution.

Older research propose generalized-inverse methods for CA such as direct allocation, daisy chaining, and the redistributed pseudo-inverse, because their solutions are feasible with respect to actuator limits and they are computationally cheap. However these methods are proven to produce sub-optimal solutions, and, more importantly, they lack the freedom to introduce specified secondary objectives.

More favorable solutions can be generated by constrained quadratic programming (QP) methods. The CA problem is then expressed in a sequential least squares (SLS) or weighted least squares (WLS) form and solved by one of the many available QP solvers. This branch of CA was long considered to be too computationally expensive, but the increase in onboard computational power in the past decade make optimization problems well solvable in real-time nowadays. In this paper the mixed objective form is used, minimizing the allocation error and a specific secondary objective in a single step. The solver is based on the active set algorithm from Ref. 7.

The main contribution of this paper is the assessment of two CA objectives to minimize drag for the ICE aircraft. The simplest one only uses a small part of the system model but requires tweaking of certain parameters, while the other is model-based and requires a full internal model of the aerodynamics. The performance of both objectives is analyzed in simulations of a mixed flight mission using two onboard system models with a different degree of accuracy. Because it is considered that future generation fighters feature the same characteristics as the ICE model, the outcome of this research is valuable in control system design for new aircraft with higher performance and lower operational costs.

The paper is structured as follows. In section II the aircraft model and simulation environment for this study are described. In section III the theory of incremental control allocation is discussed. In section IV two techniques to minimize drag in control allocation are presented. Both methods are evaluated and the results are shown and discussed in section V. Finally in section VI conclusions are drawn.

Figure 1: Sketch of the ICE aircraft.
II. Simulation framework

The Innovative Control Effector (ICE) study was a two-phased research started in 1993.\textsuperscript{1,2} The primary goal was to identify and quantify the aerodynamics and performance of different low-observable tailless aircraft configurations with innovative control effectors. This paper focuses on one of the two final designs, the land-based 65 degree leading edge swept delta wing shown in figure 1. The model features 13 control effectors: four leading edge flaps (LEFs), two elevons, two all-moving wing tips (AMTs), two spoiler slot deflectors (SSDs), pitch flaps, and thrust vectoring in pitch and yaw direction. An overview of the main modules of the simulation environment are given in figure 2. The specifications of the effectors are listed in table 1.

![Figure 2: Block diagram of the simulation setup.](image)

<table>
<thead>
<tr>
<th>Effector</th>
<th>Abbr.</th>
<th>Position limits, deg</th>
<th>Rate limit, deg/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Left inboard leading edge flap (LEF)</td>
<td>llefi</td>
<td>0/40</td>
<td>40</td>
</tr>
<tr>
<td>2 Left outboard leading edge flap (LEF)</td>
<td>llefo</td>
<td>-40/40</td>
<td>40</td>
</tr>
<tr>
<td>3 Left all-moving wing tip (AMT)</td>
<td>lamt</td>
<td>0/60</td>
<td>150</td>
</tr>
<tr>
<td>4 Left elevon</td>
<td>lele</td>
<td>-30/30</td>
<td>150</td>
</tr>
<tr>
<td>5 Left spoiler slot deflector (SSD)</td>
<td>lssd</td>
<td>0/60</td>
<td>150</td>
</tr>
<tr>
<td>6 Pitch flap</td>
<td>pf</td>
<td>-30/30</td>
<td>150</td>
</tr>
<tr>
<td>7 Right inboard leading edge flap (LEF)</td>
<td>rlefi</td>
<td>0/40</td>
<td>40</td>
</tr>
<tr>
<td>8 Right outboard leading edge flap (LEF)</td>
<td>rlefo</td>
<td>-40/40</td>
<td>40</td>
</tr>
<tr>
<td>9 Right all-moving wing tip (AMT)</td>
<td>ramt</td>
<td>0/60</td>
<td>150</td>
</tr>
<tr>
<td>10 Right elevon</td>
<td>rele</td>
<td>-30/30</td>
<td>150</td>
</tr>
<tr>
<td>11 Right spoiler slot deflector (SSD)</td>
<td>rssd</td>
<td>0/60</td>
<td>150</td>
</tr>
<tr>
<td>12 Pitch thrust vectoring</td>
<td>ptv</td>
<td>-15/15 \textsuperscript{a}</td>
<td>150</td>
</tr>
<tr>
<td>13 Yaw thrust vectoring</td>
<td>ytv</td>
<td>-15/15 \textsuperscript{a}</td>
<td>150</td>
</tr>
</tbody>
</table>

\textsuperscript{a} Thrust vectoring has circular limits. In control allocation pitch and yaw thrust vectoring limits are set to 10.6 deg, such that the combined deflection cannot exceed 15 deg.
II.A. Aircraft model

The aerodynamic model consists of 108 data-tables covering wind-tunnel measurements. The six force and moment coefficients $C_A, C_Y, C_N, C_l, C_m, C_n$ are each the sum of 17 to 19 sub-coefficients. These sub-coefficients are a nonlinear function of multiple states and inputs evaluated by linear or cubic interpolation of the wind-tunnel data. Though in some regions the data coverage is sparse, and linear interpolation is doubtful, the model is for now assumed to represent the real behavior of the aircraft. The implementation in MATLAB/Simulink is documented in Ref. 8.

It is noteworthy that the aircraft suffers from severe longitudinal and directional instability under certain circumstances. Furthermore the aerodynamics of the control surfaces are nonlinear and interacting, which means that control effectiveness is affected by the state of the effector, as well as that of other effectors. Examples of the aerodynamics of the model are given in figures 3a and 3b.

![Figure 3: Insight in the ICE aerodynamics at Mach 0.6.](image)

The aerodynamic model only covers the mainframe and control surfaces aerodynamics. Forces and moments caused by thrust and thrust vectoring are added later on:

\[
X = -C_A \cdot \frac{1}{2} \rho V^2 S + T \cdot \cos(\delta_{ptv}) \cos(\delta_{ytv})
\]

\[
Y = C_Y \cdot \frac{1}{2} \rho V^2 S + T \cdot \cos(\delta_{ptv}) \sin(\delta_{ytv})
\]

\[
Z = -C_N \cdot \frac{1}{2} \rho V^2 S - T \cdot \sin(\delta_{ptv}) \cos(\delta_{ytv})
\]

\[
l = C_l \cdot \frac{1}{2} \rho V^2 S b
\]

\[
m = C_m \cdot \frac{1}{2} \rho V^2 S \tilde{c} - T \cdot l_{tv} \sin(\delta_{ptv}) \cos(\delta_{ytv})
\]

\[
n = C_n \cdot \frac{1}{2} \rho V^2 S b - T \cdot l_{tv} \cos(\delta_{ptv}) \sin(\delta_{ytv})
\]

$X, Y, Z$ are the boxy forces and $l, m, n$ the body moments in a right-handed axis system with $X$ pointing forward and $Z$ pointing downward. $\rho$ is the air density, $V$ the total airspeed, $S$ the wing surface, and $b$ and $\tilde{c}$ the wing span and mean aerodynamic chord. $T$ is the thrust force and $l_{tv}$ the arm of thrust vectoring.

The original implementation discussed in Ref. 8 does not include actuator dynamics and limits. These are added between the controller and the aircraft model using transfer functions, rate limits and position limits in that order. Position and rate limits are given in table 1. For the leading edge flaps the dynamics are represented by

\[
\frac{\delta}{\delta_c}(s) = 18 \cdot \frac{400}{(s + 18)(s + 100)}
\]

and for all other effectors by

\[
\frac{\delta}{\delta_c}(s) = 40 \cdot \frac{400}{(s + 40)(s + 100)}
\]

with $\delta$ the real deflection and $\delta_c$ the commanded deflection. Because the gain of both transfer functions is 4, actuator control consists of a gain of 1/4. No more detailed actuator controller is applied for now.
II.B. Nonlinear Dynamic Inversion

This section briefly describes the control laws implemented in the motion control module from figure 2. Consider the equations of motion in the affine form:

\[
\dot{x} = f(x) + g(x)\tau
\]

\[
\tau = \Phi(x, u)
\]

in which \(x \in \mathbb{R}^n\) is the state vector, \(\tau \in \mathbb{R}^l\) is the vector of moment coefficients, and \(u \in \mathbb{R}^m\) is the input vector. Control of these systems is commonly separated in two parts: motion control to calculate the required moment \((\tau_c)\) to track a reference signal, and control allocation to spread the required moment over the available effectors.

The motion controller in this research is based on the nonlinear dynamic inversion (NDI) theory. The control laws are derived in Ref. 9. For the control of the body angular rates Eq. 4 reads as follows:

\[
\begin{bmatrix}
\dot{p} \\
\dot{q} \\
\dot{r}
\end{bmatrix} = -I^{-1}\left(\begin{bmatrix} p \\ q \\ r \end{bmatrix} \times I \begin{bmatrix} p \\ q \\ r \end{bmatrix} + \frac{1}{2}\rho V^2 SI^{-1} \begin{bmatrix} b & 0 & 0 \\ 0 & c & 0 \\ 0 & 0 & b \end{bmatrix} \begin{bmatrix} C_l \\ C_m \\ C_n \end{bmatrix}\right)
\]

in which \(I\) is the moment of inertia. Inversion of this system results in the following function for the moment command \(\tau_c\):

\[
\begin{bmatrix} C_l \\ C_m \\ C_n \end{bmatrix}_c = \frac{I}{\rho V^2 S} \begin{bmatrix} b & 0 & 0 \\ 0 & c & 0 \\ 0 & 0 & b \end{bmatrix}^{-1} \left(\begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix}_c + I^{-1}\left(\begin{bmatrix} p \\ q \\ r \end{bmatrix} \times I \begin{bmatrix} p \\ q \\ r \end{bmatrix}\right)\right)
\]

II.C. System identification

Model-based control requires an estimation of the system parameters. As will be made clear in the remainder of this paper, for control allocation the current forces and moments acting on the aircraft, and their derivatives to every input are essential information. A system identification module is implemented in the simulation framework, based on the following.

**Exact model**

An exact estimation of the system parameters can be obtained by copying the ‘real’ table-based aerodynamic model from section II.A into the system identification module. Feeding in the current states \((x_0)\) and inputs \((u_0)\) gives the current force- and moment coefficients \(C_{X_0}, C_{Y_0}, C_{Z_0}, C_{l_0}, C_{m_0}\), and \(C_{n_0}\). Because the functions are not analytic, derivatives with respect to the inputs cannot mathematically be derived. They can however be obtained by the taking the central difference of the parameter values with one input slightly increased and decreased. For this study the central difference approximation with \(u_i^+ = u_{i,0} + 0.1\) deg and \(u_i^- = u_{i,0} - 0.1\) deg is applied, such that

\[
\frac{\partial C}{\partial u_i} = \frac{C(x_0, u_0, u_i^+) - C(x_0, u_0, u_i^-)}{u_i^+ - u_i^-}
\]

Because thrust vectoring is not a part of the aerodynamic model, its contribution to the force- and moment coefficients and their derivatives are established according to the equations in table 2.

**Spline model**

The central difference method to calculate force- and moment derivatives requires 22 extra evaluations of the six main coefficients, namely twice for every control surface. With an analytic model of the aircraft,
The control derivatives can be calculated mathematically at a lower computational cost. In Ref. 10 a multivariate simplex B-spline model of the ICE aerodynamics is developed. Splines are piecewise polynomial approximations of a function defined in barycentric coordinates. They are known for their flexibility and remarkable competence to model nonlinear systems with local irregularities. The spline model of the ICE aircraft from Ref. 10 has an accuracy unable to achieve with an ordinary polynomial model, though it is not perfect as can be seen in figure 3. The use of this model for system identification reveals the sensitivity of the to be developed controller to model inaccuracies. Calculating the derivatives of the force- and moment approximations of a function defined in barycentric space) with respect to physical inputs (in Cartesian space) is already researched but not perfect as can be seen in figure 3. The use of this model for system identification reveals the sensitivity of the to be developed controller to model inaccuracies. Calculating the derivatives of the force- and moment functions (in barycentric space) with respect to physical inputs (in Cartesian space) is already researched and documented in Ref. 9. Note that the spline model only covers the aerodynamics, so it is also expanded with the thrust vectoring model from table 2.

### III. Incremental control allocation

The motion controller from section II.B sets the commanded moment coefficients \( \tau_c \) such that the reference signal \( \bar{x}_c \) is tracked. It is the task of control allocation (CA) to spread this control demand over the available inputs \( u \). Most existing CA methods consider a linear effector model, such that Eq. 5 can be written in the affine form

\[
\tau = B(x)u
\]

with \( B \) called the control effectiveness matrix. The linear system from Eq. 9 is efficient for solvers, but it is also limited in three ways. At first it assumes that \( \tau \) is caused purely by inputs, leaving the aircraft mainframe aerodynamics out of the equation. Secondly it assumes that the control effectiveness is constant over the entire range of the effector. Finally it does not account for aerodynamic interactions between effectors. All three assumptions are not valid for the ICE aircraft.

A brighter approach is to locally linearize the system about the current state and inputs, as is described in Ref. 11 and for the spline-based variant in Ref. 9. The idea is then that input increments are calculated based on the control demand increment. Consider the form

\[
\tau = \tau_0(x, u_0) + B(x, u_0)\Delta u
\]

\[
u = u_0 + \Delta u
\]

such that \( \tau_0 \) represents the current moment vector and \( B \) is the incremental control effectiveness matrix containing local control derivatives. In fact \( B \) is the Jacobian of the control directions w.r.t. the inputs, evaluated at the current state and inputs:

\[
B(x, u_0) = \begin{bmatrix}
\frac{\partial C_1}{\partial u_1}|_{x, u_0} & \cdots & \frac{\partial C_1}{\partial u_m}|_{x, u_0} \\
\frac{\partial C_m}{\partial u_1}|_{x, u_0} & \cdots & \frac{\partial C_m}{\partial u_m}|_{x, u_0} \\
\frac{\partial C_n}{\partial u_1}|_{x, u_0} & \cdots & \frac{\partial C_n}{\partial u_m}|_{x, u_0}
\end{bmatrix}
\]

See figure 4 for a graphic explanation of the difference between global and local control effectiveness estimation. As long as input increments are small, so the update frequency of the controller is high, incremental CA is a powerful tool to deal with nonlinear input dynamics.
One important feature of the ICE model not taken into account for now are non-monotonic nonlinearities in the input dynamics. Parabolic functions such as in figure 3b can be misleading if control effectiveness is evaluated beyond the slope reversal. Although for now this phenomenon is not encountered in practice, it is worthwhile a more detailed analysis in future research.

Figure 4: Explanation of linear CA and incremental CA. Local linearization about $u_0$ and calculating the increment $\Delta u$ gives a better estimation of the real control effectiveness at $u = u_0 + \Delta u$.

Incremental control allocation can be applied to both generalized inverse methods and optimization-based CA. The unconstrained pseudo-inverse\textsuperscript{12} for example can just as well be used to calculate the input increment:

$$\Delta u = B^+(\tau_c - \tau_0) + (I - B^+B)(u_p - u_0)$$

$$u = u_0 + \Delta u$$

but keep in mind to subtract $u_0$ from the preferred input. For the ICE model however a weighted least squares (WLS) optimization is suggested. If the secondary objective is to force the inputs to a preferred state ($u_p$), the CA problem becomes

$$\min_{\Delta u} \left\| Q(B\Delta u + \tau_0 - \tau_c) \right\|_2 + \left\| W(\Delta u + u_0 - u_p) \right\|_2$$

subj. to $u \leq \Delta u \leq \bar{u}$

with

$$\bar{u} = \max(-\dot{u}, u_{\text{min}} - u_0)$$

$$u = \min(\dot{u}, u_{\text{max}} - u_0)$$

$Q$ and $W$ are weighting matrices to prioritize allocation in certain directions or minimizing certain inputs. $Q \gg W$ should be used to emphasize the importance of allocation error minimization over control minimization. $\|\cdot\|_2$ represents the $l_2$-norm and is defined as

$$\|a\|_2 = \left( \sum_{i=1}^{\dim(a)} |a_i|^2 \right)^{1/2} \quad \text{(Ref. 11)}$$

Equation 13 can be solved by one of the many available quadratic programming methods. In this research active set programming is used, based on Ref. 11, solving the primary and secondary objective in one step, e.g.

$$\min_{u \leq \Delta u \leq \bar{u}} \left\| \begin{bmatrix} QB & Q(\tau_0 - \tau_c) \\ W & W(u_0 - u_p) \end{bmatrix} \Delta u \right\|_2$$

American Institute of Aeronautics and Astronautics
Two important adjustments are made to the original algorithm. The initial solution is set to zero rather than to the optimal solution from the previous time step, because the increment is calculated and not the entire input. Accordingly the initial working set is chosen empty instead of equal to the previous optimal working set. Secondly the pseudo-inverse is chosen as the internal unconstrained solver instead of the original QR-decomposition method. QR-decomposition is computationally cheaper, but can cause difficulties if the rank of the first matrix of the combined objective is not full. This is the case with one of the objectives from the following section.

IV. Minimum drag objective

Previous research focused on optimization-based CA algorithms to solve Eq. 13 with fixed secondary objectives. The most common choice is

\[ W = \text{diag}(1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1) \] (17)

\[ u_p = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]^T \]

which minimizes the total control effort. In general this strategy comes up with a convenient set of inputs, but the focus here is on the uniqueness of the solution rather than on its physical impact.

Given the excessive number and diverse nature of the controls on the ICE aircraft, it is considered that more though-out secondary objectives can yield a significant improvement of the operational performance. This section proposes two control allocation strategies to reduce the aerodynamic drag of the aircraft.

IV.A. Effector prioritization

James M. Buffington first experimented with exploiting the redundancy of the ICE aircraft using mission-specific \( W \)-matrices. The idea is that a higher \( W \)-value penalizes the deflection of an effector, such that the use of the others is prioritized. To minimize the wing loading the AMTs were penalized, whereas for the minimum radar cross-section mode the SSDs were given the highest \( W \)-value.

The same can be applied to decrease aerodynamic drag. The weighing matrix has to penalize the most resistant controls, such that their use is avoided if the control power is sufficient. It is found that for the ICE aircraft the SSDs generally cause the most drag and the leading edge flaps and thrust vectoring the least. Hence the following weighing is chosen:

\[ W = \text{diag}(1, 5, 5, 10, 5, 1, 1, 5, 5, 10, 1, 1) \] (18)

\[ u_p = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]^T \]

Note that the order of the effectors is given in table 1.

Although the values in Eq. 18 are based upon an analysis of the system and iterative experiments, they might not be the most suitable in every single situation. This is inherent to fixed-value control allocation objectives.

IV.B. Model-based

It is also possible to deviate from the regular secondary objective formulation with \( W \) and \( u_p \), and in fact introduce an analytic expression for drag. The optimization objective becomes

\[
\min_{u \leq \Delta u \leq u_0} ||Q(B\Delta u + \tau_0 - \tau_c)||_2 + ||(C_D(x, u))||_2
\]

In order for the minimum drag objective to fit in the WLS incremental form, drag first has to be expressed as an affine function of \( \Delta u \). That is achieved as follows. The drag coefficient is defined as the negative force coefficient in \( X \)-direction in the aerodynamic frame. The force coefficients in the aerodynamic frame are calculated by the following transformation:

\[
\begin{bmatrix}
  C_X' \\
  C_Y' \\
  C_Z'
\end{bmatrix} =
\begin{bmatrix}
  \cos \alpha & 0 & \sin \alpha \\
  0 & 1 & 0 \\
  -\sin \alpha & 0 & \cos \alpha
\end{bmatrix}
\begin{bmatrix}
  \cos \beta & \sin \beta & 0 \\
  -\sin \beta & \cos \beta & 0 \\
  0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
  C_X \\
  C_Y \\
  C_Z
\end{bmatrix} - \begin{bmatrix}
  C_T \\
  0 \\
  0
\end{bmatrix}
\] (20)
in which \( C_T = \frac{T}{\frac{1}{2} \rho V^2 S} \) is the thrust coefficient. This term has to be subtracted to make sure drag only consists of control surface resistance and the loss of thrust in forward direction due to thrust vectoring. This loss is considered to be \( T \cdot (1 - \cos \delta_{ptv} \delta_{gtv}) \). Given Eq. 20

\[
C_D = -C'_X = -C_X \cos(\alpha) \cos(\beta) - C_Y \cos(\alpha) \sin(\beta) - C_Z \sin(\alpha) + C_T \cos(\alpha) \cos(\beta)
\]

(21)

with \( C_X(x, u), C_Y(x, u), \) and \( C_Z(x, u) \) in the body frame as recorded in the model. The drag coefficient is a linear combination of the force coefficients in the body frame, so the drag derivative w.r.t an input is

\[
\frac{\partial C_D}{\partial u_i} = -\frac{\partial C_X}{\partial u_i} \cos(\alpha) \cos(\beta) - \frac{\partial C_Y}{\partial u_i} \cos(\alpha) \sin(\beta) - \frac{\partial C_Z}{\partial u_i} \sin(\alpha)
\]

(22)

The derivatives toward all inputs are combined in the drag effectiveness matrix

\[
E(x, u_0) = \left[ \frac{\partial C_D}{\partial u_1} \bigg|_{x, u_0}, \cdots, \frac{\partial C_D}{\partial u_n} \bigg|_{x, u_0} \right]
\]

(23)

and so the incremental drag formula reads

\[
C_D = C_{D_0}(x, u_0) + E(x, u_0) \Delta u
\]

(24)

The model-based minimum drag CA problem is then as follows:

\[
\min_{\Delta u \leq u \leq \Delta u} \|Q(B \Delta u + \tau_0 - \tau_c)\|_2 + \|E \Delta u + C_{D_0}\|_2
\]

(25)

The secondary objective in Eq. 25 does not 'know' the exact minimum drag position of the effectors. Rather, the incremental nature implies a gradient decent search toward this position. The magnitude of each \( E \)-value represents the amount of resistance per unit increment of the particular effector, while the sign of the \( E \)-value indicates if the minimum drag position is higher or lower than the current position. If \( Q \) is chosen too low, drag is considered more important than the allocation error, and the effectors are indeed driven toward their minimum drag position in a couple of time steps, but the aircraft destabilizes. Appropriate \( Q \)-values are found to be in the range of 5 to 10.

At first instance the method described looks to be related to a control minimization objective (Eq. 13) with \( W = \text{diag}[E] \) and \( u_p \), the minimum drag position. Though \( W \) is in that scenario treated as the global drag effectiveness from \( u_p \) to \( u_0 \), while \( |E| \) actually represents the local resistance about \( u_0 \). Moreover finding the minimum drag position each time step for each effector requires a lot of computations, making the approach from Eq. 25 far more useful.

Please also note that the objective from Eq. 25 has only 3 parameters (3 \( Q \)-values), whereas Eq. 13 has 29 (3 \( Q \)-values, 13 \( W \)-values, and 13 \( u_p \)-values). Hence the model-based objective requires a lot less tuning, for it gets most information through system identification. This makes the method less sensitive to preliminary decisions, but more to modeling errors.

V. Results

Three control allocation modes with different secondary objectives are evaluated in a mixed flight scenario. All modes are in incremental form and solved by the active set algorithm from section III. The names, abbreviations, optimization objectives, and other settings are as follows.

- **Standard mode (ST)**

\[
\min_{\Delta u \leq u \leq \Delta u} \left\| \begin{pmatrix} Q \tau & \tau_c \end{pmatrix} \right\|_2
\]

\[
Q = \text{diag}(10000, 10000, 10000)
\]

\[
W = \text{diag}(1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)
\]

The model-based minimum drag CA problem is then as follows:
• Prioritization mode (PR)

\[
\min_{\Delta u \in \mathbb{R}} \left\| \begin{pmatrix} QB \\ W \end{pmatrix} \Delta u + \begin{pmatrix} Q(\tau_0 - \tau_c) \\ W \end{pmatrix} \right\|_2
\]

\[
Q = \text{diag}(20000, 20000, 20000)
\]
\[
W = \text{diag}(1, 1, 5, 5, 10, 5, 1, 5, 5, 1, 10, 5, 1, 1)
\]

• Model-based mode (MB)

\[
\min_{\Delta u \in \mathbb{R}} \left\| \begin{pmatrix} QB \\ E \end{pmatrix} \Delta u + \begin{pmatrix} Q(\tau_0 - \tau_c) \\ C_D \end{pmatrix} \right\|_2
\]

\[
Q = \text{diag}(5, 5, 5)
\]

The Q-matrices are chosen such that the relative importance of all secondary objectives is about equal.

V.A. Flight with exact model

The flight test consists of moderate maneuvering with roll and sideslip (aiming) commands at Mach 0.6 at 600 ft altitude. After 5 seconds of stabilization a 5 second left turn is taken in a bank angle of -30 degrees. Between 15 and 20 seconds a -20 degrees angle of sideslip command is given. The mission ends with a steep right turn of half a minute in a bank angle of 60 degrees. It is known that especially the latter maneuver is important in air combat, but often limited by the available power. Drag reduction in this scenario increases the odds over the competitor.

For assessment altitude is kept constant through an outer loop controlling the angle of attack. Constant thrust is set to 6500 lbf. No climbing maneuvers are included in the simulation, because it is found in a climb situation most drag, both from the mainframe and from the effectors, is lift-induced drag. For a given angle of attack it is possible to reduce drag by placing control surfaces parallel to the airflow, but this will be accompanied by a decrease of lift. To compensate for that, a higher angle of attack has to be flown and the drag reduction in the end is not significant.

The attitude response, airspeed, altitude, and aerodynamic drag are shown in figure 5. The flight path and plots of the effector positions can be found in the appendix. The values of average drag are given in table 3. For clarity of the graphs the turbulence model is turned off, but drag values of simulations with turbulence are also given in table 3.

<table>
<thead>
<tr>
<th>Mode ( \text{turbulence on} )</th>
<th>Average drag [lbf]</th>
</tr>
</thead>
<tbody>
<tr>
<td>ST ( \text{turbulence off} )</td>
<td>6462.6</td>
</tr>
<tr>
<td>PR ( \text{turbulence off} )</td>
<td>6041.2 ((-6.52%))</td>
</tr>
<tr>
<td>MB ( \text{turbulence off} )</td>
<td>6046.1 ((-6.44%))</td>
</tr>
<tr>
<td>ST ( \text{turbulence on} )</td>
<td>6503.1</td>
</tr>
<tr>
<td>PR ( \text{turbulence on} )</td>
<td>6098.6 ((-6.22%))</td>
</tr>
<tr>
<td>MB ( \text{turbulence on} )</td>
<td>6080.1 ((-6.51%))</td>
</tr>
</tbody>
</table>

It is clear from figure 5 that under constant bank angles (5s – 10s and 25s – 55s) the PR-mode and MB-mode reduce drag with respect to the ST-mode. Also sideslip commands (15s – 20s) are well tracked by these modes, while reducing drag with about 8% relative to the standard mode. The difference in average drag values between the PR-mode and MB-mode is from table 3 not significant, but from figure 5d it may be clear that the PR-mode is beneficial during the dynamic part of the maneuver, whereas the MB-mode reduces more drag in the steady parts. Overall the PR-mode performs slightly better, though in turbulence it is the other way around. Notice that in cruise flight drag reduction is not significant, which is the reason that climb maneuvers are not investigated for now.
It is interesting to look how the minimum drag modes accomplish the drag reduction. Figures 6a and 6b show the effector positions in the steady part of the sideslip maneuver (at 19s) and right turn (at 35s) respectively. Some specific choices are clear. Whereas the ST-mode uses primarily the left SSD (effector 5) in a sideslip, the PR-mode selects yaw thrust vectoring (13) and the left outboard LEF (2), and the MB-mode chooses yaw thrust vectoring (13) and the right inboard LEF (7). To suppress sideslip in the right turn the ST-mode again uses the yaw power of the left SSD, and the other modes choose thrust vectoring. Effector usage in the dynamic parts of the mission can be read from the figures in the appendix.

It looks like the most drag reduction can be achieved by just keeping the SSDs closed. A simulation with only this constraint already shows a drag reduction of almost 5%. At low angles of attack the SSDs are indeed a poor choice for yaw control because they generate a lot of drag and degrade the effectiveness of the elevons. Though in extreme situations extra yaw power can be necessary, and additionally the SSDs are very useful at high angles of attack to recover airflow over the trailing edge flaps.
Effector use

(a) Effector states during sideslip, measured at 19s.  
(b) Effector states during sideslip, measured at 35s.

Figure 6: Effector use during steady parts of the mission.
ST = standard mode, PR = prioritization mode, MB = model-based mode.

V.B. Flight with spline model

In the previous simulations the onboard model of the system is assumed perfect, i.e. \(\tau_0, B, C_{D_0},\) and \(E\) used in control allocation are the exact values. This however is an utopia, since the internal model always has some degree of inaccuracy. The same mission if therefore flown with the spline model from section II.C delivering information to the control allocation module. The aerodynamic angles, airspeed, altitude, and drag plots are given in figure 7, with the remainder of the plots in the appendix. Average drag values are listed in table 4.

Table 4: Results of flight with spline model.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Average drag [lbf]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>turbulence off</td>
</tr>
<tr>
<td>ST</td>
<td>6523.2</td>
</tr>
<tr>
<td>PR</td>
<td>6174.4 (-5.35%)</td>
</tr>
<tr>
<td>MB</td>
<td>6307.6 (-3.30%)</td>
</tr>
</tbody>
</table>

From figures 7a and 7b it can be concluded that the tracking performance is similar to when an exact model is used (figures 5a and 5b). Hence the spline model is accurate enough for decent operation, at least for the kinds of maneuvers tested. Also the drag values of the ST-mode and PR-mode are comparable.

However from table 4 and figure 7d it can be deduced that the performance of the MB-mode is notably degraded by the use of the spline model. The reason for this is that an imperfect model sometimes provides misleading (drag-)information to the controller. See for example figure 8. According to the spline model the deflection of the left AMT can be a couple of degrees without producing much drag. In reality however the drag is significantly higher, up to 8% at \(\delta_{lamt} = 4\) deg. The aerodynamic model consists of 108 sub-models, each of them with this kind of inaccuracies, adding up to the loss in performance of the MB-mode. The PR-mode does not use this information, so its drag reduction capability is less affected by the quality of the model. Table 5 shows the results if the real aircraft dynamics were based on the spline model. Then the original model is inaccurate and the spline model is ‘exact’, i.e. the roles are reversed. It is clear that again the MB-mode is favorable if the onboard model is precise and the PR-mode if it is not.
Figure 7: Aerodynamic angles, airspeed, altitude, and drag during flight with spline model. ST = standard mode, PR = prioritization mode, MB = model-based mode.

Figure 8: Drag coefficient vs. left AMT deflection at Mach 0.6 and $\alpha = 3$ deg.
Table 5: Results of flight with the aircraft dynamics based on the spline model. ST = standard mode, PR = prioritization mode, MB = model-based mode.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Average drag [lbf]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>original model</td>
</tr>
<tr>
<td>ST</td>
<td>6300.1</td>
</tr>
<tr>
<td>PR</td>
<td>5875.3 (-6.74%)</td>
</tr>
<tr>
<td>MB</td>
<td>6018.1 (-4.48%)</td>
</tr>
</tbody>
</table>

V.C. Discussion

From tables 3 and 4 it can be concluded that mission specific control allocation can indeed optimize operational performance, in this case drag reduction. The advantages are most apparent in the steady states of a turn or aiming maneuver, and in the roll-in and roll-out phase. In cruise flight the drag reduction is marginal. The validity of the comparison of drag values is proven by figures 5 and 7. Under all three modes the same maneuver is flown: altitude is kept constant and the aerodynamic angle tracking exactly matches. Each test the thrust setting is the same, but the final airspeeds of the PR-mode and MB-mode are higher than that of the ST-mode. This indicates that the aircraft encountered less resistance along the way. When thrust for the PR-mode and MB-mode is set to 6100 lbf the final airspeeds are comparable (with the exact model), underlining these modes make the aircraft more full efficient.

The simulations described are performed at Mach 0.6 at an altitude of 600 ft. Simulations of the same maneuvers in other conditions prove that the performance is maintained, see e.g. figure 9a. The PR-mode turns out to be more sensitive to Mach number than the MB-mode. The influence of altitude is negligible for both modes. The maneuvers are not very suitable for supersonic flight or high altitudes, so the ability of drag reduction in this part of the flight envelope is a topic for future research. Figure 9b shows the drag reduction in a right turn with various bank angles. The sharper the turn, the more drag is reduced. It is also clear that the MB-mode outperforms the PR-mode, primarily in the steady part as is argued earlier. Steeper bank angles than 60 degrees can be flown, but the aircraft easily destabilizes. Better tuning of the outer loops and flight envelope protection is required for further testing.

![Drag reduction at various Mach numbers](image1.png)

(a) Drag reduction at various Mach numbers.
Maneuver is the same as previously.
Altitude is 10000 ft.

![Drag reduction at various bank angles](image2.png)

(b) Drag reduction at various bank angles.
Maneuver consists of 5 seconds cruise, 20 seconds turn, and 5 seconds cruise. Mach 0.6 at 600 ft.

Figure 9: Drag reduction at various Mach numbers and at various bank angles with exact model.
ST = standard mode, PR = prioritization mode, MB = model-based mode.

All in all, both minimum drag modes are able to reduce the operation drag of the ICE aircraft, and it turns out that the PR-mode produces better solutions if the onboard model lacks accuracy, as was expected. But does this make the PR-mode the preferred technique in practical applications? As mentioned earlier the MB-mode has a lot less parameters to be manually tuned than the PR-mode. Moreover the performance of the MB-mode turns out to be rather robust to the choice of $Q$-values, as long as the importance of the primary objective is clear ($Q \geq 5$). The reason for this is that in most situations, especially in the steady parts of the mission, the allocation error can easily be minimized to zero, so the relative importance of the
secondary objective does not play a role. Practically speaking control allocation does not have to make a concession between primary and secondary objective, because the primary objective can be achieved to the fullest, leaving enough excess power for the secondary objective whatever $Q$-value.

On the other hand, the PR-mode is very sensitive to the choice of its parameters. In fact the entire power of PR-mode relies on the iterative establishment of $W$-values. For example a simulation with a slightly different $W = \text{diag}(1, 1, 5, 5, 5, 5, 1, 5, 5, 5, 2, 2)$, shows that the drag reduction capability is more than halved: $-3.10\%$ against $-6.52\%$ originally. The $W$-matrix in this study is specifically optimized for the maneuvers performed, but in an other mission it can behave differently. This vulnerability of the PR-mode is inherent to its design and makes the MB-mode a preferable option in systems of which the dynamics are accurately mapped.

A final point of attention is that the model now assumed to be 'real' is in some parts very illogical. Linear interpolation of sparse data results in implausible input dynamics, see e.g. figure 3b. Multivariate simplex splines are not able to capture such sharp functions, which explains most of the inaccuracy of the spline model. In reality though, the aircraft is expected to feature smoother input dynamics, and the spline model will not be that imprecise. The drop is performance of the MB-mode by the use of the spline model is thus exaggerated in this study.

VI. Conclusions

The next generation fighter aircraft is likely to feature a tailless design with control surfaces for pitch and yaw relocated on the main wing. The large number of effectors and their overlapping functionality requires a smart control allocation approach that comes up with a unique and operationally attractive solution. This research focused on decreasing drag of the ICE aircraft through two specifically designed secondary control allocation objectives.

The first drag minimization mode (prioritization, PR) penalizes the deflection of the most drag-causing effectors, such that the use of low-resistant effectors is preferred. In the mission flown, the PR-mode reduces drag with about 6% relative to the standard control allocation mode. The reduction is primary the result of using thrust vectoring instead of the SSDs for yaw control. The performance of the PR-mode is relatively insensitive to the accuracy of the onboard model, yet it is vulnerable to the choice of the initial parameters. Slight changes in the parameters or a different flight scenario may largely affect the drag reduction capabilities of the PR-mode.

The second drag minimization mode (model-based, MB) incorporates the drag function in the control allocation problem and uses the excess control power to force the effectors to their minimum drag position. If the onboard model is precise the performance is comparable to that of the PR-mode, as it reduces drag with about 6.5%. With an inaccurate onboard model however the drag reduction is significantly less. In return the number of initial parameters is much less and the performance is rather insensitive to these parameters.

In conclusion, effector prioritization can be an effective way to reduce operational drag of an aircraft, but its efficiency depends on initial tuning. For practical application the technique is questionable because acceptable performance cannot be guaranteed over the entire flight envelope, especially not with one fixed set of parameters. On the other hand model-based drag reduction in control allocation design is theoretically substantiated and expected to maintain its performance in a much wider range of circumstances. The prerequisite is that the onboard model accurately estimates the actual behavior of the aircraft.

The tests in this study involved turning and aiming commands at subsonic speeds. A topic for future research is to investigate the performance of the modes in other parts of the flight envelope (e.g. supersonic speeds) and with other maneuvers. The most important recommendation is to improve the ICE aerodynamic model, such that physically implausible phenomena are corrected. A high fidelity model can expose the real potential of spline-based modeling and the opportunities for spline-based mission-specific control allocation for next generation fighter aircraft.

Appendix

This appendix includes additional plots of the simulation results. Figure 10 shows the effector states during the flight tests with the exact onboard model. Effector states during the flight tests with the onboard spline model are shown in figure 11. The appendix concludes with two plots of the flight paths in figure 12.
Figure 10: Effector states during flight with exact model.
ST = standard mode, PR = prioritization mode, MB = model-based mode.
Figure 11: Effector states during flight with spline model.
ST = standard mode, PR = prioritization mode, MB = model-based mode.
(a) Flight paths with exact model.

(b) Flight paths with spline model.

Figure 12: Flight paths. With the PR-mode and MB-mode the airspeeds increase relative to the ST-mode, explaining the different turn radii with the exact model. With a lower thrust setting for the PR-mode and MB-mode the paths are more alike. The difference in airspeeds with the spline model is less, and the PR-mode already drifts a little to the right in the aiming maneuver. The difference in turn radii is therefore less obvious than with the exact model.

References


