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First progressive report on
reliability analysis of drag dominated offshore platforms

On the application of probabilistic methods for structural elements under extreme sea levels

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1.1 Introduction

In practical design of the structures, engineers are generally aware from the determination of absolute reliability. The reason for this conscious action is that structures are practically neither feasible nor tenable due to the lackage of the avialable information. Since the lack of absolute reliability is a consequence of uncertainty, the evaluation of reliability naturally requires sufficient knowledge of the unknown uncertainties. Therefore, we may know the increasing gaining acceptance based on the belief that the reliability approach can be implemented without difficulties within the farmework of conventional structural analysis and design procedures (Fredenthal 1975 [18])[1]. Thus, the only requirements of this approach is to provide a logical systematic analysis of the reliability and safety of a design in the face of uncertainties that are specified with the probabilistic approximation of "load" and "resistance". However the distributions of the statistical variables are neither known nor obtainable by simple probability distributions. The main problem of a reliability analysis is thus the understanding of uncertainties and principles of probabilistic approaches. It is concluded that for assessment of reliability and structural safety, the concepts of probabilistic methods are playing a central role.

Before that we start with the definitions of reliability concepts, it is necessary to distinguish between the risk and uncertainty assessments. Because the risk is a measureable uncertainty of loss or of a damge, the faulty of a risk assessment is that its result is often related to the financile success of the design and the consequences of structural damge is often omitted. On the other hand, by application of the probabilistic theory to design circumstances, the result of analysis are constructed with a credible model which is consisted from physical reality. In other words, the limitation of measurable values for the subjective excitation and structural response are logically involved only in a reliability approach. A good definition of uncertainty is required to identify the sources of basic data information which is needed as a first step in reliability design procedure.

1.2 Uncertainty

Uncertainty is defined in *Webester's New Twentieth Century* Dictionary with six different meanings:

1. Not certainly known; questionable; problematical.
2. Vague; not definite or determined.
3. Doubtful; not having certain knowledge; not sure.
4. Ambiguous.
5. Not steady or constant; varying.
6. Liable to change or vary; not dependable or reliable.

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[1] Numbers in brackets designate References at end of paper.

If one concentrates on the difinations which seems are different in first instance, he will receive to the conclusion that all the definitions can be captured by two distintict meanings; Ambiguity and Vagueness [30]. In general, the *vagueness* is associated with the difficulties of making sharp or precise distinctions in the world, in other words, the reason of uncertainty is that the boundaries of a quantity is not known in a boundary domain. On contrary, the *ambiguty* is related to the several concepts as well as, one to many relation, variety, generality, diversity and divergence. In regarding to structural reliability theories, most of work has been concentrated to the analysis of ambiguty not to the vagueness. The question of how to measure vagueness or (*fuzziness*) has been one of the issues assosiated with the development of the theory of fuzzy sets. The application of fuzzy theory to the structural reliability of mooring pipelines has been considerd by Japan's researchors in order to find the collision damage. The evaluation of fuzziness for expert systems (computer networks managements and so on) is considered at the faculty of Informatics of Delft University of Technology.

1.3 Probability and probability of combined events

Bertrand Russell (1926) believed that the probability is the most important concept in modern science. In his intelligent statement, nobody has not the absolute definition for the probability and nobody knows what it means. In any case, in human living, there are a lot of problems that, the people encounter with the probability estimation. For example a decision may not be chosen as a "best" decision without consideration of a decision criteria. In most of deciisions, the final result is found under condition of uncertainty and a best decision can be found by a decision maker with a lot of experience. It can be concluded that in spite of probability, non of any decisions can not be accepted. However in most of cases the nature of belief, extremely affects the result of a decision. For this reason one decision which is pessimistic in the opinion of one person, may be considered as a wrong decision by a optimistic person. (A pessimistic person belives that nature always works against him while an optimistic person believes that luck is always on his side.)

Before that we introduce a mathematical definition for probability of an event, it seems necessary to define the event itself. Any problem involving the uncertainty, the "true situation" is, of course, not known for certain. In addition the existing condition will be turned and there will be different outcomes for a certain state. In probabilistic approach, possible outcomes are called *events*. Alfredo H.S. Ang and Wilson H. Tang [1] have defined the event by means of *sample space*. If all posibilities in a probabilistic approach is collected in a space which is called a sample sapce, then each of the invidual possibilities can be estimated as an event. Therefore the event is defined by a subset of sample space. An event may be represented by a *Venn diagram*. In a Venn digram, the sample space is represented by a rectangular; the event E itself is then represented by a closed region inside of the rectangular. The remainder part of the rectangular in the outside of this region is called the *complementary event* $\overline{E}$ (see Figure 1.1).

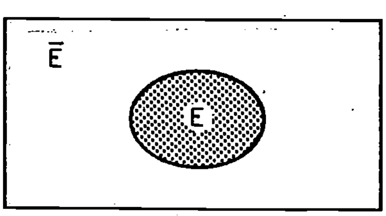


Figure 1.1 A Venn diagram

Knowing the meaning of an event, the probability has been defined in reference [39] as: "A mathematical concept which assigns a number between 0 and 1 to an event or a combination of events is called probability of the event or events. The likelihood of event is equal to 0 when it is impossible to occur and is equal to 1 when it is almost certain. Probabilities are dimensionless quantities."

In reality, true values for probabilities can be obtained only from an infinite events. Since infinite trial of any event is not existed in real world, thus the mathematical definition of probabilities are often found in a fixed number of trial events. Consider the result of a failure test of a component (take for example tensile strength of a bar). Suppose that the fixed number of components is given with N under test. In time t, the number surviving components is found by $N_s(t)$ while the number of failing components will be $N_f(t)$ (where $N_s(t) + N_f(t) = N$). The probability of survival (or *reliability*) and the probability of failure at time t are defined by the following equations

$$Pr\{E\} = Pr\{N_s \leq N\} = R(t) = \frac{N_s(t)}{N} \tag{1.1}$$

In which, $Pr\{E\}$ denotes the probability of occurrence of the event (usually called success). The probability of non-occurrence of the event or complement of E ($\overline{E}$, usually called failure) is defined by

$$Pr\{\overline{E}\} = Pr\{N_f \leq N\} = F(t) = \frac{N_f(t)}{N} \tag{1.2}$$

These are complementary, i.e. the survival and failure events can be added together and we have $R(t) + F(t) = 1$. By a certain event, the probability is equal to 1 and by an impossible event which does not occur in any time, the probability takes a zero account.

In most of cases, an event occurs in a combination of different events which are correlated with the formats of "AND", "OR", "NOT" and a combination of these three relations. For example, in describing the stae of supply of construction material, if E_1represents the shortage of concrete and E_2 represents the shortage of steel, then the shortage of material is given by the *union* $E_1 \cup E_2$ as shown in Venn diagram of events E_1 and E_2 [Figure (1.2)].

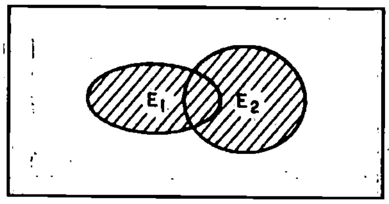


Figure 1.2 Union events

1.4 Fundamental laws of probability

Consider the probability of an event A, i.e. $Pr\{A\}$ or simply $P[A]$, is given by a number between *0* and *1*. Thus the probability that event A happens will be equal to $P[A]$. For example, $P[A] = 0.1$ in evaluation of flood (i.e. event flood $\equiv A$) in this year means that the probability of occurrence of a flood is equal to *0.1*.

In probability engineering, the relationships between sets are restricted by certain operation rules. Who has some problems with the probability concepts, he must be care from the initial definition of the used symbols. The operational symbols have been classified in accordance to the relations beteen an one dimensional category system. These symbols are given in Table 1.1 [1]. By definition, the *union probability* A and B, i.e. $P[A \cup B]$, is the sum of the individual probabilities of A and B minus the probability that both events (A and B) occurring simultaneously. Thus

Table 1.1 Operational symbols in probability

Symbol	Description
$\cup$	Union
$\cap$	Intersection
$\subset$	Belongs to, or is contained in
$\supset$	Contains

$$P[A \cup B] = P[A] + P[B] - P[A \cap B] \tag{1.3}$$

In special case when two events exclude each other, i.e. $P[A \cap B] = 0$, then the probability of two events can be added by the simple algebraic summation. This axiom is called the *addition law of probability* in the literature.

The *conditional probability* for A is the probability of occurrence of event A provided that event B has taken palce. By definition, the conditional probability is the ratio of the occurrence of both events A and B simultaneously $P[A \cap B]$ to the probability of occurrence ~~for event B. In fact the conditional probability of A assuming B has occurred, means that the~~ the occurrence of both events with the appropraite normalization (respect to the first occurrence or B), i.e.

$$P[A \mid B] = \frac{P[A \cap B]}{P[B]} \tag{1.4}$$

which can be rewritten as

$$P[A \cap B] = P[A \mid B].P[B] \tag{1.5}$$

If two events A and B are not correlated to each other (the occurrence of an event does not depend on the occurrence of another event), then

$$P[A \mid B] = P[A] \tag{1.6}$$

Or

$$P[A \cap B] = P[A].P[B] \tag{1.7}$$

For three events A, B and C the equation (1.7) can be written as follows

$$P[A \cap B \cap C] = P[A \mid B \cap C].P[B \mid C].P[C] \tag{1.8}$$

An advantage result can be obtained by implementation of this theory for n uncorrelated events B_i in which $i = 1,2,3,\ldots,n$ [see Figure (1.3)]. The probability of event A is given by

$$P[A] = P[A \cap B_1] + P[A \cap B_2] + \ldots + P[A \cap B_n] \tag{1.9}$$

$$P[A] = \sum_{i=1}^{n} P[A \cap B_i] = \sum_{i=1}^{n} P[A \mid B_i].P[B_i] \tag{1.10}$$

With the same definition of conditional probability for event A, the probability of occurrence of all events A, $B_{i,\ i=1,2,\ldots,n}$ can be written by using equation (1.5) as follows

$$P[A \cap B_i] = P[B_i \mid A].P[A] \tag{1.11}$$

Thus the conditional probability of $P[B_i|A]$ is calculated by equation (1.12). In practice, the lack of statistical data and model uncertainty are important features for the conditional probabilities.

$$P[B_i \mid A] = \frac{P[A \cap B_i]}{P[A]} = \frac{P[A \mid B_i].P[B_i]}{\sum_{j=1}^{n} P[A \mid B_j].P[B_j]} \tag{1.12}$$

The basic problem is that the statistical data can be described in term of subjective judgement.The traditional approach to statistical inference does not take into acount this past experience. The frequentistic philosophical is accepted as an objective degree of belief for the event with prior probability. By a *bayesian* philosophy, the *subjective* event (*posterior* probability) is obtained from the *objective* one (*prior* probability). Therefore the posterior probabaility distribution is updated from the prior probability distribution through a likelihood function. The fundamental of the bayesian philosophy is used in experimental practice for the analogy of quality of a material B_i by contribution of many laboratory test results defined by A. The extraction of subjective event (posterior probability) in term of the objective event (prior probability) requires the knowledge of the entire distribution of a random variable as defined by equation (1.12).

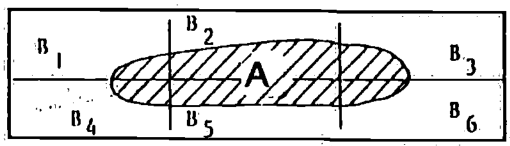


Figure 1.3 Conditional probability

1.5 Discrete and (or) Continuous Probability distribution functions

A real-valued function $F(x)$ is called a uni-variate cumulative distribution function (*c.d.f.*) or simply distribution function if

i) $F(x)$ is non-decreasing, i.e., $F(x_1) \leq F(x_2)$ for $x_1 \leq x_2$.
ii) $F(x)$ is everywhere continous from the right, i.e., $F(x) = \lim F(x+\epsilon)$; $\epsilon \to 0^+$.
iii) $F(-\infty)=0$, $F(\infty)=1$.

Thus the probability of the event "$X \leq x$" (where X is a random variable) defines by the cumulative distribution function, i.e., $Pr\{ X \leq x \} = F(x)$.

In the numerical analysis, the probabilistic distributions may not always be complete compatable with the continuous distributions and the discrete distributions which are characterized by the random variable X taking an enumerable number of values at discritized points (x_1, x_2, x_3, ..., x_n) with point probabilities are defined as follows

$$f_n = Pr\{X=x\} \geq 0 \tag{1.13}$$

The only requirement for the point probability f_n is that it should be satisfy the condition of

$$\sum_n f_n = 1 \tag{1.14}$$

And the corresponding distribution functions can be written as

$$F(x) = Pr\{X \leq x\} = \sum_{x_n \leq x} f_n \tag{1.15}$$

where the summation is over all values of x for which $x_n \leq x$. Examples of discrete distributions are *single point* or *degenerate* distribution, *binominal* distribution, *Hypergeometric* distribution, *Poisson* distribution, *Negative binominal* distribution and *geometric* distribution. For more information about the characteristics of the distributions the reader is reffered to *Handbook of mathematical functions* written by M. Abramowitz and I.A. Stegun (see reference [52]).

On the other hand, if the derivatives of a cumulative distribution function was absolutely continuous, then the distribution is called a continuous distribution. Examples of continuous distributions are *Normal* distribution, *Log-Normal* distribution, *Exponential* distribution, *Gumbel* or *Maxima Extreme-value Type I*, *Frechet* or *Maxima Extreme-value Type II*, *Fisher-Tippett* or *Minima Extreme-value Type I*, *Weibull* or *Minima Extreme-value Type III*, *Rayleigh* distribution, *Special Erlangian*, *Triangular* distribution, *Error function*, *Cauchy* distribution, *Laplace* distribution, *Pearson Type III* or *Gamma* distribution, *Beta* distribution and *rectangular* distribution [9,52,31].

The application of Gumbel distribution to model the environmental maximum of a quantity can be found in wide area of science (take for example the maximum wind speed). For the first time, E.J. Gumbel has introduced this distribution in his book "*Statistics of extremes*" published in 1958. The Gumbel distribution is in fact doubly exponential distribution. Therefore if Y is Type II Maxima distributed then Z = Ln Y is Type I maxima distributed which is often called the Gumbel distribution. On the other hand, Weibull distribution (3-parameter or 2-parameter) is also applied for the prediction of extreme conditions. In practice, it is common to plot the cumulative probability and environmental variable in a so-called "Weibull Scale" paper. A "good-fit" of environmental data will achieve by a straight line on this scale unless the Weibull distribution is not suitable for the given data. However based on the "Main study report of Hutton Area" [26], it has been found that when approxiamtely 30 years of data are used, the difference between Gumbel and Weibull distributions is fairly predictable. The extremes for 1 and 10-year are approximately evaluated with the same quantities, but on the other hand by the comparison of the 50 and 100-years data, the Gumbel results are always higher than those given by the Weibull distribution [26].

Note that the rectangular distribution with unit length is often called the *Uniform* distribution [31]. The distributions which have been introduced earlier, have found their applications in a variety of sciences. In offshore engineering as well as other hydraulic engineerings, for meteorological and Hydrological events, the most important distributions are: Normal, Log-normal (3-parameter or 2-parameter), Gumbel, Pearson, *Log Pearson* and Frechet (or Maxima Extreme-value Type II). In fractural mechanics and fatigue problems, the Weibull distribution (3-parameter or 2-parameter) and Log-normal distribution are implemented in majoraty of reliability problems. In each case, the characteristics of all important distributions can not be prepared in the context of this report and the reader should be study the advantage books about the mathematical discription of these distributions. However because of that in most physical problems the distribution of random variables are coinciding with the normal distribution, it will be useful to point out some characteristics of the normal probability distribution function in this context.

1.6 Normal or Guassian probability function

A random variable X is said to be normally distributed with mean $\bar{x}$ and variance σ_x^2 if the probability that X is less than or equal to x is given by:

$$Pr(X \le x) = \frac{1}{\sigma_x\sqrt{2\pi}} \int_{-\infty}^{x} \exp[-\frac{(t-\bar{x})^2}{2\sigma_x^2}]dt = F(\frac{x-\bar{x}}{\sigma_x}) \tag{1.16}$$

The corresponding probability density function (p.d.f.) is found by the differentiation of the above formula, i.e. equation (1.17).

$$\frac{\partial}{\partial x}F(\frac{x-\bar{x}}{\sigma_x}) = f(x) = \frac{1}{\sigma_x\sqrt{2\pi}}\exp(-\frac{(x-\bar{x})^2}{2\sigma_x^2}) \tag{1.17}$$

The normal distribution is symmetric around $\bar{x}$ and the inflexion points of the probability density function are at $\overline{x} \pm \sigma_x$. The direct integration of equation (1.16) would be cumbersome. It is possible to avoid this direct integration for manual calculations by providing a table in which a standardized normal function has been integrated. The normal density function may be placed in standardized form as follows:

$$f(z) = \frac{1}{2\pi} \exp[-\frac{z^2}{2}] \tag{1.18}$$

where $z = \dfrac{x - \overline{x}}{\sigma_x}$

In other words, if the characteristics of the normal distribution are taken equal to

$\overline{x} = 0$, $\sigma_x = 1$

In that case the integral of the normal distribution may be stored in a table for evaluation of integrals of a normal density function given a $\overline{x}$ and σ_x. The standard normal probability distribution function and its derivaties has been given by M. Abramowitz and I.A. Stegun in reference [52].

In general, the multivariate joint normal probability density function is considered in the n-dimensional *basic variables space* ω. For a given structure, the basic variables are defined by a realization of a random vector $X = (X_1, X_2, X_3, \ldots, X_n)$ where the number of basic variables n is assumed finite.

$$f([x]) = \frac{1}{(2\pi)^{\frac{n}{2}}} \cdot \frac{1}{C^{\frac{1}{2}}} \cdot \exp[-\frac{1}{2} \sum_{i,j=1}^{n} (x_i - \overline{x}_i) M_{ij} (x_j - \overline{x}_j)] \tag{1.19}$$

where $[x] = (x_1, x_2, x_3, \ldots, x_n)$, $[\overline{x}] = (\overline{x}_1, \overline{x}_2, \overline{x}_3, \ldots, \overline{x}_n)$ and $M = C^{-1}$ where C is the *covariance matrix* defined by equation (4.8).

$$C = \begin{Bmatrix} Var[x_1] & Cov[x_1, x_2] & Cov[x_1, x_n] \\ Cov[x_2, x_1] & Var[x_2] & Cov[x_2, x_n] \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ Cov[x_n, x_1] & Cov[x_n, x_2] & Var[x_n] \end{Bmatrix} \tag{1.20}$$

The covariance matrix of X can be written in a more convenient way by using vector and matrix notation as follows.

$$C = E[(X - \overline{X})(X - \overline{X})^T] \tag{1.21}$$

where the superscript '*T*' denotes the transpose of matrix. Since *C* is positive definite, there exists an orthogonal matrix, *T*, and that TCT^T is a diagonal matrix. (Its diagonal elements will be the eigenvalues of *C*.) Let $\underline{Y= TX}$. Then

$$E[(Y-\bar{Y})(Y-\bar{Y})^T] = T.C.T^T \tag{1.22}$$

will be a diagonal matrix, and the variables, *Y*, will be the required uncorrelated variables.

M. Abramowitz and I.A. Stegun have also considered the *standard bivariate normal* probability density function which is defined by pair of random variables (X_1,X_2). The formulation is similare to the formulation of multivarite joint normal probability function (Equ. (1.19)), but with introducing of the correlation factor ρ between two random varibles X_1 and X_2. (The correlation factor is defined by the ratio of covariance of variables to the product of their standard deviations). A simplified case for bivarite probability density function occures when the standard deviations of random variables are equal and the variables are independent , i.e., the correlation factor is zero ($\rho = 0$). In this case, the function is called the *circular normal probability density function* which can be written as follows:

$$f(\frac{x_1-\bar{x}_1}{\sigma},\frac{x_2-\bar{x}_2}{\sigma}) = f(\hat{x}_1,\hat{x}_2) = \frac{1}{2\pi\sigma^2}\exp\{-\frac{(x_1-\bar{x}_1)^2+(x_2-\bar{x}_2)^2}{2\sigma^2}\} \tag{1.23}$$

However, the multi-integral of the probability density function, equation (1.19), which is called the cumulative joint probability distribution function of the random variables, can be evaluted analytically only in some special cases. For the case of the *bivariate normal probability function*, the double integral of probability density function is called the joint probability distribution function of the random pairs. M.Abramowitz and I.A. Stegun have introduced some simplified graphs for the integration of the bivariate normal probability distribution function.

An alternative for the integration of the probability distribution functions has been found by use of the numerical methods. In fact, in the case of multivariates with several distributions, the combersome of the analytical methods makes it impossible to evalute the integrals without application of digital computers and it seems the numerical approach is the only one.It is necessary that all the uncertainties in a design have been involved in the joint probability density function *f*(*x*), and that *f*(*x*) is known. Owing the lack of data, these probability distribution functions are seldom known precisely, and the numerical evaluation of the joint probability distribution function is extremely difficult or even impossible. In structural reliability analysis, sometimes only the first and second order moments (the mean and variance) may be known. Different approaches of first-order second-moment (*FOSM*) methods simplifies the functional relationships and mitigates these difficulties outlined before.

On the other hand, generally speaking, the basic variables cannot be satisfactory modelled by a normal distribution. In order to simplify the development of the concept of reliability analysis method, an assumption is made that the basic variables are considered to contain just two *normal* or *log-normal* distributins.

1.7 Log-Normal probability function

The characteristics of log-normal distribution is very similare to the characteristics of normal distribution. Consider a random variable x is related to a second random variable x_1 by the relation

$$x_1 = Ln\ x \tag{1.24}$$

where *Ln* denotes the logarithms of the variable x in exponential base. If the random variable x is normally distributed, then x_1 is said to have a log-normal distribution. In terms of the mean value of x_1 , i.e. $\bar{x}_1$ the mean of the logaritms of variable x, and the standard deviation of x_1 , *i.e.* σ_{x1} the standard deviation of logaritms of x, the probability density function of x is given by:

$$f(x) = \frac{1}{x.\sigma_{x_1}.\sqrt{2\pi}}.\exp(-\frac{1}{2\sigma_{x_1}^{2}}((Ln\ x - \bar{x}_1)^2)\ ; \quad x>0 \tag{1.25}$$

The mean and the standard deviation of Log-Normal distribution, i.e. $\bar{x}_1$ and σ_{x1}, can be calculated in term of the mean and standard deviation of the random variable x. If the coefficient of variation of random variable x is defi ned in term of its mean and standard deviation as given by equation (1.26), then the mean and the standard deviation of new variables are easily calculated by formula's (1.27) and (1.28).

$$C_x = \frac{\sigma_x}{\bar{x}} \tag{1.26}$$

$$\bar{x}_1 = Ln\bar{x} - \frac{1}{2}Ln(1 + C_x^2) \tag{1.27}$$

$$\sigma_{x_1}^{2} = Ln(1 + C_x^2) \tag{1.28}$$

And the cumulative probability distribution function of x is written as equation (1.29) in which Φ denotes the cumulative distribution function for the standard normal distribution. Using Monte-Carlo approach, the cumbersome of integration for logaritms of many values would be facilitated by application of simple computer programes.

$$F(x) = \Phi[\frac{Lnx - \bar{x}_1}{\sigma_{x_1}}] \tag{1.29}$$

One of the characteristics of Log-normal distribution is that its *mode*, *median* and *mean* are different from each other. (Note that the mode of a density function would be peak of the distribution, the median of a data is the midpoint of the data if the data values are arranged in either ascending or descending order and finally the arithmatic mean is the measure of central tendency or average of the sample data).

1.8 Hazard

Although there are many definitions for the concept of hazard as well as risk, peril, ... but in the term of potential condition of a component, equipment or a system of components, the hazrd is implied threat or danger and often harm. Thus by occurrence of a hazard, the potential condition can be changed to condition of injury, death, damgae, loss of part or whole of the object.

In probability engineering, the *hazard rate*, is defined in term of probability of survival (or reliability). If we rewrite the probability of survival as follows

$$P_R(t) = \frac{N - N_f(t)}{N} = 1 - \frac{N_f(t)}{N} \qquad (1.30)$$

By differentiating we will get the following expression

$$\frac{dP_R(t)}{dt} = -\frac{1}{N}\frac{dN_f(t)}{dt} \qquad (1.31)$$

In which $\{dN_f(t) \,/\, N.dt\}$ is the instantaneous probability when dt approaches to zero. The instantaneous probability is often called the *probability density function* (*pdf*) for the case of continuous random variable and *probability function* or *point probability function* for the case of the discrete random variable. Thus the relation between probability of survival $P_R(t)$ and the probability density function $f(t)$ is obtained as follows

$$\frac{dP_R(t)}{dt} = -f(t) \qquad (1.32)$$

A rearrangement of equation equation (1.30) gives an expression for the *failure rate*.

$$\frac{dN_f(t)}{dt} = -N\frac{dP_R(t)}{dt} \qquad (1.33)$$

The failure rate can be interpreted as the number of components failing in time interval dt, between t and $t + dt$. If both sides of equation (1.33) are divided by $N_s(t)$, the *instantaneous failure rate* or the *hazard rate* will be equal to

$$\frac{1}{N_s(t)}\frac{dN_f(t)}{dt} = -\frac{N}{N_s(t)}\frac{dP_R(t)}{dt} \equiv \lambda(t) \qquad (1.34)$$

Substitution of equation (1.30) into the equation (1.34) results to the following expression

$$\lambda(t) = -\frac{1}{P_R(t)}\frac{dP_R(t)}{dt} \tag{1.35}$$

Hazard rate is always time dependendent but the case of constant hazard rate is a practical interest. An application of constant hazard rate will be shown later.

1.9 Probabilistic modelling

The accuracy of a reliability method is highly dependent to the probabilistic models. Normal distribution is the conventional distribution for the probabilistic modelling and often is used with the large number of variables. An important reason for the wide applicability of the normal distribution is in fact it fits with most quality controles and some reliability observations. The goodness of normal distribution for large number of variables is proved by central limit theorem. Because of extensive application of Normal distribution, the non-normal distributions are often converted to the equivalent normal distribution. Although these transformations are widely used in level *II* reliability analysis but the result is often lapsed by the applied transformations.

If the random variables are product of a large number of quanteties, then the log-normal distribution often gives a better approximation because of central limit theorem. Further, the extreme value distribution is used for the largest or smallest quantity of random variables. Although any distribution may be transformed to the normal distribution, but it should be considered that the results will not be accurate specificly in the tails. Although many types of distributions may be considered in the reliability analysis but usually it is a difficult task to find the best distribution for a random sample data.

The random variables that are often called *basic variables* mainly are collected by loading, strength (or resistance) and geometrical quantities. Concerning the loading, the basic variables will be treated in term of physical nature of loading and its statistical simulation. The main types of loadings are classified as Functional, Environmental and Accidental loads that may act individually or simultaneously. The important point in the reliability analysis is the effect of simultaneous loadings' particularry when the accidental loadings are included. If two or more loads are acting to the structure then the combined effects will be considered ultimately. However, the uncertainty of current action and wind turbulences or the damage to hatch covers caused by local loads will not be considered here. Concerning the uncertainties for the local loads, structural details of this type should always be designed with higher nominal safety margins than the substructures or single members. The collapse due to the usual over loading may be considered as the "fail safe" case (Lindemann et al. [34]). In the following chapter, the uncertainty in environmental loading especially the wave effect will be discussed with a sufficient attention. In consideration of the resistance parameteres, basic variables are often modelled as time independent variables (Thoft Christensen and Murotsu [50]). The distribution for the yield, buckling and collapse failures are often concides with the normal or log-normal distribution. For fatigue ultimate parameteres often the Weibull, expotential and Gumbel distributions are used in the response analysis.

1.10 Reliability method and its Level

The reliability method is a design tool for limit states. Essentially, it is based on probabilistic approach, which can be used to making decisions for important structures. Today the concepts of reliability analysis are applying for the design of nuclear plants, highway bridges, offshore structures and so on.

The probability of performance of a function is defined the reliability of the component, equipment or system for a stated period of time. On contrary, the probability of failure is defined in term of inability of the component, equipment or system for a required function.

An efficient reliability method treats the efficiency and the accuracy of requirements. Although in reliability engineering both efficiency and accuracy are important to making decisions, but in practice the models of reliability methods are divided into two major branches. Duddeck (1977) calls two kinds of models the *reseach models* and the *technical models*. The reasearch model compromises the reliability data to minimize the difference between the idealized reliability (P_R') and the true reliability (P_R). Here the emphesis is to develop an accurate reliability method. In contrary, a technical model improves the efficiency and the objective of the model aids to making decisions [37].

The advantage of a relibility method is positioned on its probabilistic nature. In the first place, by a probabilistic method it is possible to compare different failure modes. This method gives a uniform meaning in the safety of a structure. Furthermore, the relibility methods may be compared with the common sensivity analysis. Here, the advantage of a probability method is that it gives a coherent picture from the sensivity componant.

Different classical techniques are used to assess the reliability of engineering structures. *Level I* method comprises calculation based on characteristic values and (partial) safety factors or safety margins. Strictly speaking, a calculation at Level I does not involve failure probabilitis. It does, however, provide a method of checking whether a defined level of safety is satisfied. This type of calculation is more particulary suitable for every day practice. The interrelation of various Levels will be dealt latter on.

The transformation techniques are used to transfer all probability density functions to the probability density functions of normal distribution. This is the basis for a *Level II* (or *second moment*) method which comprises a number of approximate methods for linearization of limit state margin. Two important second-moment methods are the mean value first order second-moment (*MVFOSM*) and the advanced first order second-moment (*AFOSM*) methods that will be described in chapter *4*. The classification of two mentioned methods are as follows:

a) The mean value first order second-moment method is based on the variation of mean and standard deviation of random variables and is formulated on the basis of linear approximation of failure surface.

b) The advanced first order second-moment method is based on the recalculation of design points in terms of sensitivity values in an iterative form, and finally calculation of the reliability index (β) while the limit state equation is formulated by the linear approximation of failure criteria.

For the linearization of limit state function either one or two expression from the Taylor series are often used in the vicinity of design points. Depending on the approximation on the expanding of Taylor series the methods are often called *FORM / SORM* methods in the literature. Which formulation is to be chosen depends on the nature of the problem. It means that if the limit state function only consists of linear relationships, then a FORM procedure will give the accurate results otherwise the SORM formulation is implemented in the frame work of reliability analysis. On the other hand, the approximate full joint distribution method is based on derivation of normal distribution values in term of non-normal informations based on the normal tail or weighted fracture approximation. These transformation methods have been discussed in detail by Shu-Ho Dai and Ming-O Wang [11].

In *level III* method the joint distribution of required uncertian parameteres are used to access probability of failure. In either case, the reliabilty of total joint distribution for the structure can be found by means of numerical integration procedures (take for example Monte Carlo method). The method comprises calculations in which the complete probability density functions of the stochastic variables are introduced and the possibly non-linear character of the reliability function is exactly taken into account.

Recently, a reliability method that compares the structural prospect with a reference prospect according to the principles of structural economic analysis has introduced in literature by a so-called *level IV* method. The interests in the analysis are the consideration of costs and benefits, maintenance, repair and consequences of failures. Such methods are appropriate for structures that are high economic importance such as nuclear power plants, highway bridges and so on. The full definition of these methods are given in chapter *4*.

1.11 Jackup risk versus fixed platforms

The risk has existed in human life since the dawn of human history. As time went by, the safety plan took on a social form. The advances usually had negative as well as positive effects on all eras and all peoples. Humanity soon became technical in the means to provide the objects and conditions necessary for sustenance and physical contentment. Nowadays humans must deal with those hazards that occur through carelessness and the unguarded or inadequately organized uses of their devices and substances. It was such a safety discipline that assisted in putting men on the moon. Such disciplines made air transportation so safe that it is the common way to travel over long distances. But again note to the paradox of technology versus hazard which has been explained in Table 1.2.

We now have learned some methods of avoiding the pitfalls of human weakness. A modern poet from P. Hein (1966) put it very clearly (see reference [44]).

The road to wisdom? Well it's plain and simple to express: Err and err, and err again, but less, and less, and less.

Scientists nowadays believe that the natural events are so complicated that we can not handle all of them in the framework of mathematical relations such as the differential equations. Indeed the type of a system is rather complicated in the order of it's characteristics behavior (The system is always defined by a set of variables). Thus one problem arises with the incompatibility of real behavior of the adopted system and its relative response observed by an unknown supplier (usually this also holds for the designer of the system). Informally stated, as well as the complexity of a system increases, our ability to make precise and yet significant statements about its behavior diminishes. Thus the precise analyses of the behavior of a complex system is not likely to have much real-worlds such as in the design of complex systems as well as aircrafts, nuclear plants, highway bridges and offshore platforms. The responsibility of the designer requests the implementation of fullified methods for insurance of operating systems and their passengers.

Table 1.2 Technology versus hazard

The Technological Advance	The Hazard
Fire	Burns, conflagrations
Knives	Inadvertent trauma
Fossil fuels	Atmospheric pollution
High speed transportation systems	Accident induced damage and injury
Pesticides	Food chain toxicity
Food preservatives	Carcinogens
Nuclear energy	Ionizing radiation

Before that we start to identify the basic concepts of reliability theory in general, it should be clearly noticed that which type of structures are studied in the skeleton of this thesis and what kind of hazards will be discussed as a source of the structural collapse?

In the concepts of offshore industry different kind of mobile platforms are usually used for the production of oil and gas resources. In spite of all type of mobile operating platforms, a jack-up platform is a mobile offshore structure consisting of a hull supported by 3 or more legs. The leg is towed to the location and then the hull is jacked above the water level.

For the fixed platforms 60339 is the total number of platform-years exposure while for the jack-ups the corresponding exposure is equal to 4097 rig-year in the period 1970-'87. Therefore a relative exposure ratio of approximately (60339/4097 $\simeq$ 15) is obtained for this period. The relative exposure ratio for the fixed platforms was fifteen times higher than that of jack-up structures in period of 1970-'87 and it may be interpretated with the vast experience of fixed platforms with comparison to the jack-up structures.

The philosophy for the integrity assesment of offshore structures can be found with consideration of different uncertainties in design parameteres and assumed models. Let us to start we the relative reliability of jack-ups compared with the fixed platforms. M. Efthymiou has extracted the exposure statistics of losses for the jackups and fixed platforms in the period 1970-'87 from the Worldwide Offshore Accident Databank WOAD [15].

Focussing on the most severe accidents, i.e., those found with the total loss of the unit, two important results can be drawn (see for instance reference [33]). Firstly the total loss of integrity for the jacket type structures is considerably less than for the jack-ups, i.e. when we consider only one of the periods of 1970-'79 and 1980-'87. Secondly, whereas the *frequency rate* of accidents for the jacket structures shows a significant improvement with time (from 7.3 to 1.1), such a trend cannot be observed for the jack-ups (162 vs. 97). Note that the frequency rate of accidents is determined by the total losses during the period of investigation (years) per 10,000 unit-years (for example the total losses of fixed platforms during 1970-'79 was 17 and the number of unit-years of exposure was 23,204 in the same period, thus the frequency rate of total losses is [(17/23,204) × 10,000 = 7.3]). Accidents in the WOAD databank are defined as those events or conditions which have caused damage to supporting structures or equipment and environment causing death or injury to individuals). On the other hand, if the same issue is used for the total losses due to the environmental loading or foundation failure, i.e. excluding total losses due to blowout, fire, collision or during transportation, then the frequancy rate of accidents for the fixed platforms will be (1.7 vs. 0.5) for the period of 1970-'79 and 1980-'87, while for jack-ups the frequency rate of accidents will be equal to (35 vs. 34) for the period of 1970-'79 and 1980-'87 respectively.

With the frequaency rate of accidents, we are able to find the (*instantaneous*) *failure rate* or hazard rate for the total losses of two kinds of platforms for two mentioned failures, i.e. either for the total loss or for losses resulting from environmental loading or foundation failure.

In the last section, it proved that if the probability that an item (platform) will survive for a stated interval is denoted by the *s-reliability*, and it is given by the reliability function $P_R(t)$, then the conditional probability of failure in the unit interval (take for example one year) is obtained by the equation (1.35). Rearrangement of equation (1.35) and integrattion from time *0* to time *t* gives general reliability function in term of failure rate $\lambda(t)$ as follows

$$P_R(t) = \exp[-\int_0^t \lambda(t).dt] \tag{1.36}$$

For the case of constant failure rate, i.e. $\lambda(t)$ independent of time, is a practical interest. In that case, the relation between reliability and failure rate can be written as:

$$P_R(t) = e^{-\lambda t} \tag{1.37}$$

where *e* denotes the exponential function.

Returning to our discussion over the losses of platforms, the failure rates for periods of 1970-'79 and 1980-'87 are shown in Table 1.3. It can be simply concluded that the relative failure rates for total losses are *22* and *89* in the period of 1970-'79 and 1980-'87 respectively. The relative failure rates for losses from environmental loading and foundation failure can be estimated from Table 1.3 which are equal to *21* and *68* in the period of 1970-'79 and 1980-'87 respectively.

Table 1.3 Failure rates for fixed platforms and jack-ups in the period 1970-'87
(Numbers in the table should be multiplied to 1/1,000,000)

Total losses consist of environmental loading, fire, transportation, ...				**Losses from environmental loading and foundation failure**			
1970 - 1979		1980 - 1987		1970 - 1979		1980 - 1987	
Fixed	Jackup	Fixed	Jackup	Fixed	Jackup	Fixed	Jackup
73.03	1633	11	947.7	17	350.4	5	340.6

As a matter of fact, the total loss of integrity would be assembeled by addition of individuale losses during the life time. In first instance, it is necesseray to focus on the subset of losses in the elevated condition excluding losses caused by blowout, fire, explosion and collision or loss during the transportation of the unit. This subset can be split up into two parts, namely those caused by the external circumstances or by the internal conditions. Thus the sources of the integrity losses for a jack-up structure can be investigated properly by two ways:

(a) Firstly, the designer can be interested to the variation of external circumstances with a set of design conditions. In this case, the most unfavourable external conditions are considered by the extreme waves and their probabilistic modelling. The stochastic response of the jack-up structure is obtained with the stochastic models for the wave loading. The response of the structure is aimed to find the worst probabilistic extreme wave conditions.

(b) Secondly, the structural conditions (physical and geometrical characteristics) can be changed during the design life. The method is based on the derivation of best design for a required environmental conditions. In this case, the sensitivity of jack-up in real environmental condition requires a more severe investigation into the added risks. The structural conditions can be changed during the design life including the effects of reformed structure after inspection. The stochastic models for material strengths can be considered to lead an effective cost design for the structure.

1.12 Developments in structural analysis of jack-ups

Traditionally the jack-up platforms have been designed for static deterministic (design wave) approach. The dynamic response has been studied for the self-operating uints when it was recognized that the reaction of jack-up units would be different when the units have moved to deeper waters and/or harsher environments. In 1982 Youicho Hattori et al. [22,23] have suggested that an equivalent single degree of freedom system (SDOF) would be suitable for the dynamic analysis of jack-up oil rigs. Their study was not only a way to find the natural vibration of jack-up units but also they have considered the virtual mass of a leg vibrating in water, and the supporting condition of the sea bed among others.) In the foregoing study by Y. Hattori et al., the three-dimensional idealized model of the unit was composed of beams and rigid plates. The beams formed the lattice structure of the legs and the rigid plates formed the box structure of the platform. An example of a three dimensional model of unit which contains 209 nodes, 450 beams and 76 plates is adopted from their study and is shown in Figure 1.4. The leg-hull interface in this model is represented by a weight-less rigid bar. The bar is tightened to the connection and the gap between leg and leg-guide is fulled by inserting a wedge into the gap.

A comprehensive study has been carried out by I.J. Bradshaw presented in the proceedings of Mobile Offshore Structures Conference held in City University London [5]. His study is consist of the comparison of analytical methods and software used for the jack-up analysis. Three analysis methods have been discussed including the design wave method, Two types of Frequency Domain (random dynamic) methods and Time Domain (random dynamic) method. At that time, only the software for the design wave method was capable to represent almost the all options of analysis (In Shell Internatinale Petroleum Maatschappij). The computer model used for the design wave method, i.e. static deterministic analysis, was a three dimensional space frame for the lattice legs, stiff elements for the hull and jack-housing and Pseudo members with specific properties and geometric planes for the leg-hull interface. This interface was able to model the additional loads resulting from the rack teeth in wave load of members. For the Frequancy Domain analysis, two different approaches have been used. These approaches were different in three lights; the wave linearisation procedure, the free surface effects and the statistical distribution of peak values. In issue of the linearisation method, the method (1) was implemented by a so called "constant wave steepness" approach while the method (2) was developed by Borgman approach. The shortcome of inclusion of non-linear effects due to wave loading and free surface effects have been overcome by implementation of limited, but still quite lengthy, time domain analysis. This provides the large amount of significantly lengthy time domain analysis for three dimensional model of unit which prohabits this approach. Thus an alternative idealized model of structure was used in the time domain analysis which has been called the "single stick representation" of the jack-up by Bradshaw [5]. Again this model introduces some errors with regarding to the spatial seperation effects (hydrodynamic cancellation phenomenon).

One of the recent studies on the up-to-date methods of jack-up analysis has been investigated by M.J.R. Hoyle [24]. In his case study, the jack-up was represented by an equaivalent model of hull grillage, collinear beam elements for lattice legs and a number of linear springs for the leg-hull interface. Two approaches were compared adopted from the *JIP* (Joint Industry jack-Up committee) and from *Noble Denton*. The main conclusion of the comparison was that there is no significant difference between the the response obtained from the JIP or Noble Denton. The only expection is that the JIP results are much larger than the results from the Noble Denton study. Indeed the application of Pierson-Moskowitcz spectrum has lead to the additional wave energy comparing to the results of the Noble Denton which have used a Jonswap spectrum with the peak enhancement factor $\gamma = 3.3$.

In viewing the ongoing research for the offshore structures, a new discussion of practical system reliability approach will be discovered in the further developments of the research. A combination of extreme environmental loading with fatigue loading will be considered. The probabilistic formulation for such a combined failures may occur due to the initial failure in fatigue and subsequent collapse under an extreme wave. The system reliability approach will be used in order to compare the most probable cause of failure for an individuale member and for the overall structure. The previous work by Jan Inge Dalane [12] has proved that for an individual element, the dominant cause of failure is fatigue, but on the other hand, for overall sturactural failure, overload and (or) a combination of fatigue and overload are more important. In fact, they have shown that because of redundancy effect, the failure of an individual section does not constitute structural collapse. However, the probability of failure for series and parallel system in conjunction with Boolean algebra will be used in the formulation of the system reliability.

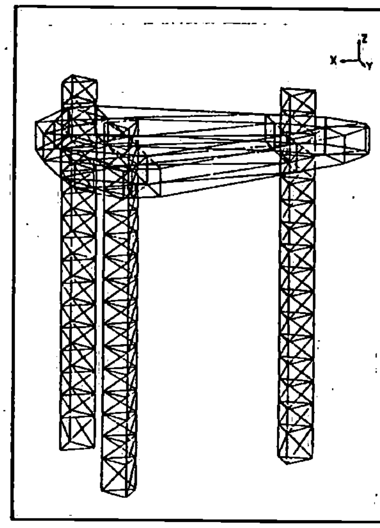


Figure 1.4 Idealized model for a jack-up rig

Application of system reliability within inspection and maintenance planning will be demonstrated in the context of this study. Using the *failure path approach* which often is referred as *"failure tree approach"*, the sequences of events up to failure will be analyized in the framework of fatigue failure. This method overcomes the problems arising by application of plasticity based approaches. It represent a more general approach which also can be used for the reliability assessment of truss or beam type elements. However in the domain of basic variables, using the correlation factors, increases the complexity of the formulation and the probability computation requires an extra dimension in integration. To reduce the problem of joint probability distributions, the important sequences of failure are included in the reliability evaluation and unimportant sequences are ignored. Based on the experience, it has been concluded that the joint probability integral may not be integrated with the basic variables more than *4* [50]. Since in most of structural systems, the failure function is often formulted in term of several basic variables, the implementation of numerical tools for evaluation of integrals is unavoidable.

In order to apply the standard methodologies of reliability problems for old offshore platforms, it seems a practical and general procedure should be developed for the assessment and requalification of existing platforms. In a recent report presented by Robert G. Bea [2] in OMAE 1993, four levels of *Reserve Strength Ratio (RSR)* has been introduced as an issue of qualification of offshore structures. The paper addresses how much the relaibility methods can be used to develop rational and reasonable criteria for the requalification of offshore platforms. The purpose of the paper is to quantify "false positives" with application of simpler methods, and for this purpose the *Fitness for Purpose (FFP)* criteria has been evaluted in the framework of reliability approaches. A graphical representation of this methodology is demonstrated in Figure 1.5 which has been adopted from the same paper [2] with some differences.

1.13 Exposing the problem

Continuous research in structural engineering has pointed a way for applying the reliability theory to the complex structures as well as offshore platforms. Generally, the reliability methods provide a common basis for comparative analysis. Figure 1.6 shows the steps involved in the calculation of structural reliability. At first, the hindcast data are assembeled together to provide the basic representitive combination of extreme environmental conditions. The failure of the structure for different direction of load set is evaluted to find the critical failure surface corresponding to the load set directions. The probability of failure is adopted in term of uncertainties in load set, strength characteristics and geometrical dimensions of structure in a frame work of quasi-static push-over analysis. It should be emphasized that the cumulative probability of failure P_f is normally obtained in term of cumulative directional distributions of long term extreme wave loads with the distribution of individual effects of current, wind, tides and etc. However, it seems that we are still many years away from being able to rely on reliability methods to give us an absolute sense of structural risk. In particular, taking into account the effects of inspection programs and the normal maintenance procedures into the reliability model have not been study very well. This presents that the only rational way to apply this theory, as we don't know explicitly that if a probability of failure of 10^{-7} is good or not (as an example). The primary use for reliabilty analysis is for design of new and different types of platforms, where we have no experience base. The

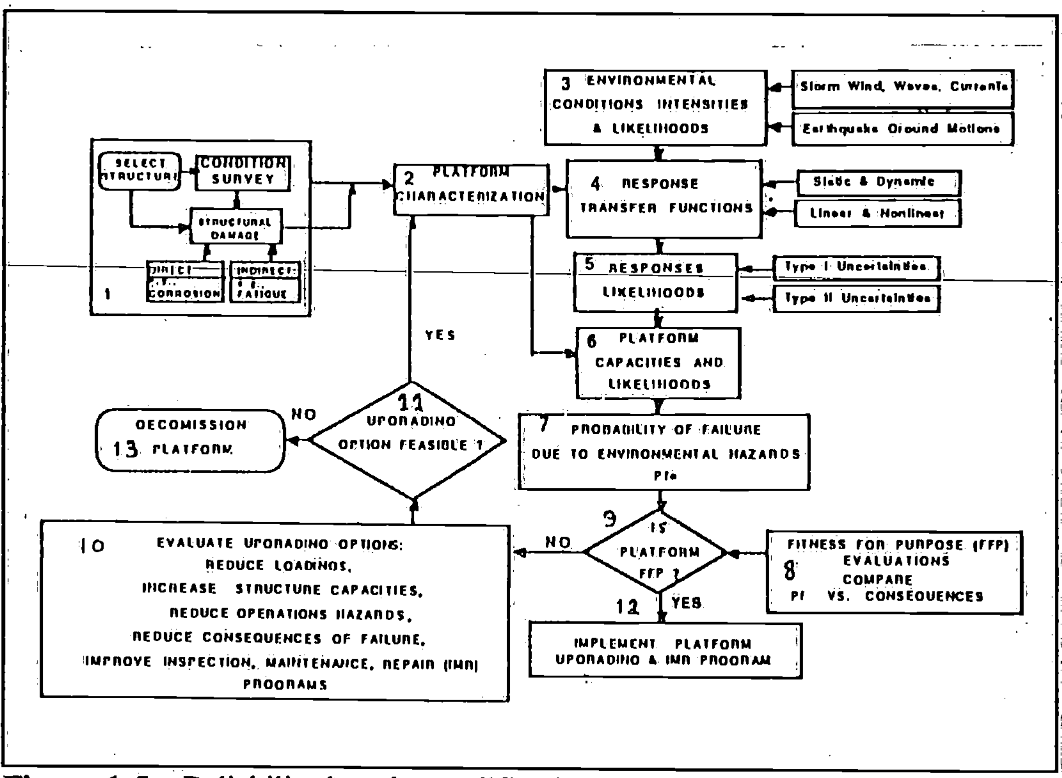


Figure 1.5 Reliability based requalification

configuration and consequence of failure of, for instance, a new jack-up for deep water is so difficult from conventional jack-ups that a different level of reliability would seem appropraite. Wouldn't the results be the same if instead of requiring a lower probability of failure, we simply reduced design stress when adopting new concepts?

With regard to the interpretation of the probability of failure within the context of the present report, we may arbitrary make use of the *frequency sense* or *cyclic interpretation* (see reference [38]). In a frequency sense, the probability of failure of, say, 0.001 indicates that, one in a thousand identical jack-ups operating under similar operating conditions is expected to fail during lifetime. The cyclic interpretation may be possible if the loading on the jack-up is constructed in terms of extreme value in N cycles, but in that case care must be taken from the nonstationary character of the waves over the lifetime of the jack-up.

The calibration of safety factors for associated structures may be specified in term of nominal loads (or stresses) and strength instead of mean values. In this case, the nominal values already contain safety factors that are not well defined. Therefore by application of nominal values, an ambiguity may arise as to the meaning of the resulting additional partial safety factors. However, in the definition of the mean values, there is no ambiguity and the partial safety factors may be find wihout any ambiguity.

An offshore platform can fail in a variety of hazards, for example, overturning, blowout, collision, storm condition, seabed collapse, fire, war and possibly fatigue. For jack-up platforms the highest risks occur by moving from one location to other place [45]. The risk of damage due to storm has been found less than 2.8×10^{-4} if a site assessment is carried out. However this context is not aimed to quantify the risk assessment of jack-ups but rather it contains a basic theory which is needed for comparison of structural damage due to different failure modes.

In fact, there are two appropriate aims from the issue of this report. Initial purpose is the collection of the foregoing results of reliability engineering researches and the secondary reason is the evaluation of difficulties on the reliability integrity of offshore structures.

The methodology is given hier can be used for the integrity assessment of offshore platforms relative to fatigue and extreme environmental loading. The rational program consist of either a design procedure for new platforms or the inspection and repair process for the existing rigs. In addition, the reliability analysis will be revealed for the structural model of jack-up platforms consist of three different parts. Although the fundamental issue of this report is to focus on the reliability of legs but in general the jack-up structure is represented by a *hull*, a *lattice leg* model (or may be a combination of lattice and *stick* model) and *leg-hull interface*. The hull structure may be represented by a suitable membrane element consists of main deck, sides, bottom, main structural bulkheads and Helicopter deck. The legs comprise either accurate models of three dimensional structure or stick modelling of legs with equivalent properties (Formulation of equivalent characteristics of legs has been given in classification notes e.g. Det Norske Vertias [13, 14] or Joint Industry Jack-up Committee [27, 28]). The leg-hull structure may be decided to model by the Hybrid structure including the following sub-structures: Leg-guide subsystem, Leg-pinion subsystem and combination of these subsystems into the total interface.

In addition, for the development of the program with practical objects, jack up structure is considered on the southeast coast of the Caspian Sea located at the north of the town Neka in Iran. The unit shall be capable of operating, all year around, in water depth ranging from *7.7 m* to *91.5 m*. Technical specification has been given by *Rauma-Repola* offshore company which is the rig-builder of semi-submesibles, jack-ups, drillships and pipe-laying vessels from Finland. The structure is now constructed in sections on the specialist production line in the workshop, and the deck is completed onshore. The legs are fed from beneach, so high lifts are avoided. The rigs are launched totally complete.

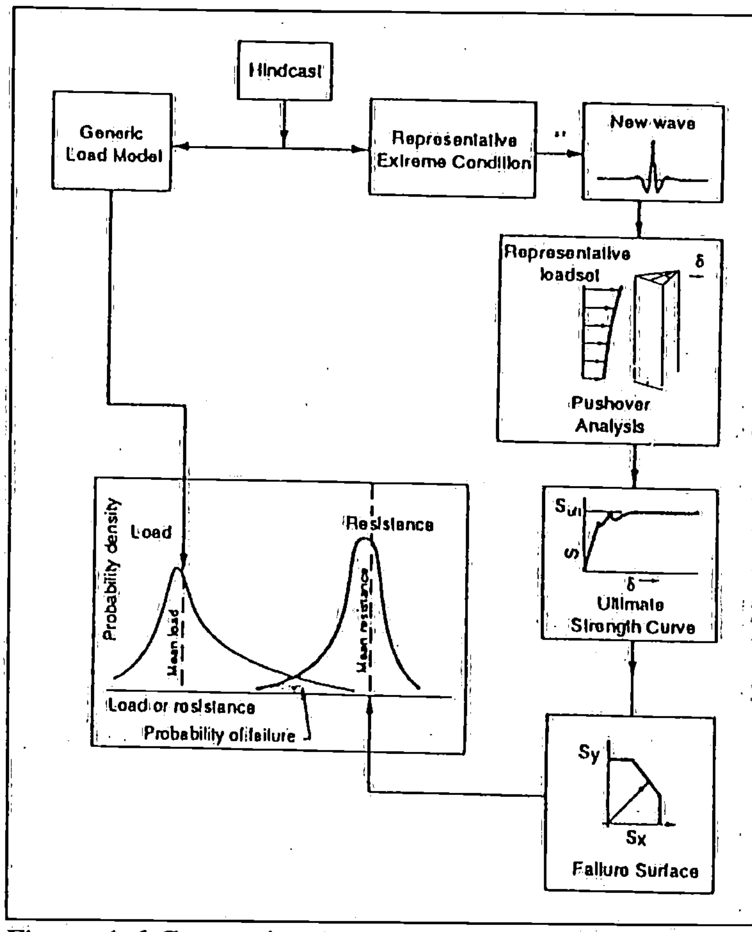


Figure 1.6 Conventional approach for reliability assessment of offshore structures (adopted with some differences from [3])

The legs are designed as a lattice type frame work with three chords and tubular bracings. The chords of the legs have a construction with two gear racks on each chord and are manufactured of a high tensile steel. The three chords of every leg are interconnected by *K*-type tubular bracings. Free ends of the *K*-type tubular bracing are flame-cut to shape and size prior to welding to the chords.

2- Wave statistics

2.1 Introduction

Our first purpose is to find a mathematical description of the waves, which apperantly seems impossible at first sight, since wind generated waves are irregular and vary non-periodic. The sea state in a storm or hurricane is measured by the variation of the water surface, described by the time and varies with the position. Instead of looking to the wave record at a fixed point, one can imagine that the water surface was measured at instant time in direction of horizontal wave propagation. This way of recording is, however, difficult and expensive and often the statistical characteristic of both methods are samilare for both types of signals. It is common practical to record at a fixed point.

Concerning the assumption that the wave record has been adopted in a suitable meteorological condition, it should be noticed that the characteristics of ergodicity and stationary are involved. A process is called an ergodic process whereby the characteristics of the process is derived by the time-averaging of the single sample record instead of the calculation of the essembled averages of the records at specific instants of time. In other words, a measurement of $\eta_1(t)$ is typical for all other measurements from an ergodic process $\eta(t)$. The record is called stationary if the statistical properties are independent of the origin of the time measurement. For a sample record the stationary process can be ensured if a sharp limitation of recording time is taken into account where there is usually quick variation in wave and wind conditions.

There are two basic approaches defined in choosing the design wave environment for an offshore structure. One of the methods consideres a single design wave with a given wave height and period (*deterministic approach*). One reason for this approach is the simplicity in design and the easy determination of response due to the given sea conditions. If this method for the calculation of response is considered, then it is recommended that single waves are considered to find the worst loads experienced for any of design wave conditions.

The other approach to selecting the design wave environment is based on the wave spectrum (*stochastic approach*). In this case, a suitable design wave spectrum is chosen representing an approparaite density distribution of the sea waves at the site under consideration. The most suitable spectrum is a measured design wave spectrum at the site, although such a spectrum is seldom available. As an alternative, one chooses one of the theoritical spectrum models available base on the fetch, wind and other meteorological conditions of site. The chosen wave spectrum, of course, describes a short term wave condition.

2.2 Wave energy spectrum

Although the wave propagation in general is three dimentional but in the following it is assumed that the waves are one dimensional and long crested in horizontal direction. First assume that the random wave with unknown amplitude and phase angle is decomposed of n sinusoidal waves that their constant parameteres can be found by Fourier analysis. The components of n harmonic waves with amplitudes a_n and phase angles α_n are superimposed to find the random wave surface.

$$\eta(x,t)=\sum_{n=1}^{N} a_n\cos(k_n x-\omega_n t+\alpha_n) \tag{2.1}$$

Where $k_n=2\pi/L_n$, L_n = wave length at the frequency ω_n and N = number of slices made in the wave spectrum. The frequency of a random wave is determined in the frequency corresponding to the n th $\Delta\omega$ slice. The frequncy ω_n and the wave number k_n are related by the transcendental equation (the linear dispersion equation).

$$\omega_n^2=gk_n\tanh(k_n d) \tag{2.2}$$

From linear theory of gravity waves, it is well known that the energy of a harmonic wave is proportional to the square of the amplitude a and its quantity per unit area is given by $\rho g a^2/2$ where ρ is the density of the water. With this approach, we assume that the energy spectrum of wave record is discritized in n interval whereby the frequency increments is given by df (or $d\omega$). The wave spectrum for a typical record is shown in Figure 2.1. For simplicity, we omit the constant factor ρg where the later results in the variance of wave spectrum by $a^2/2$.

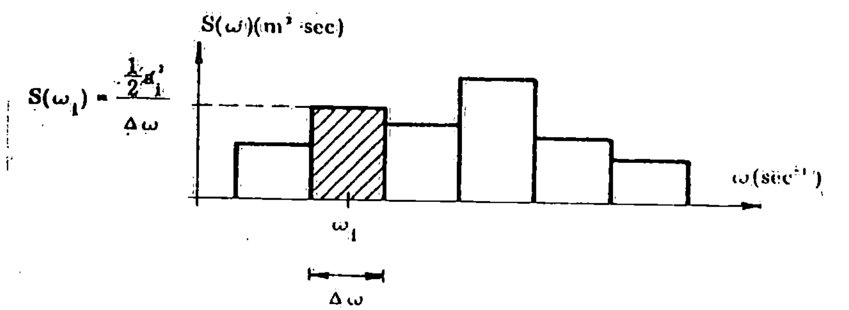


Figure 2.1 Stepped variance spectrum

When the frequancy interval $d\omega$ (or df) tends to zero, the stepped curve may be converted to the smooth continous curve. In most of cases our knowledge relies to the empirical data. In this situation, the characteristic values of wave spectrum is obtained by the information of wind records at the desired site. Having obtained the wave spectrum, its characteristics is normally defined by the moment of spectrum which is taken respect to the origin of ω or f in the wave spectrum diagram.

$$m_n=\int_0^{\infty} f^n S(f)df=\frac{1}{(2\pi)^n}\int_0^{\infty}\omega^n S(\omega)d\omega \tag{2.3}$$

The area of the wave spectrum is equal to the zeroth moment m_0 and the relationship between $S(f)$ and $S(\omega)$ for the zeroth moment is derived by

$$S(f)=2\pi S(\omega) \qquad (2.4)$$

2.3 Spectral width parameter

The spectral width parametere is a measure of higher frequency portion of the spectrum. For a stationary Guassian process it can be found either in the time domain or in the frequency domain. In time domain analysis, the width of the spectrum is measured by the mean zero upcrossing period T_z and the mean crest period T_c. The mean crest period is almost smaller than the mean zero upcrossing period because most ocean waves have local maxima (or also minima) that do not cross MWL. (i.e. in any case $N_c \geq N_z$, where N_c is the total number of crest points and N_z is the total number of zero upcrossing points.) If T_c is close to T_z, then wave spectrum is considered as a narrow band spectrum where in that case the energy is concentrated over a small frequency band. If the ratio α is defined by the division of T_c over T_z, then the spectral width parameter can be obtained as

$$\varepsilon_t=(1-\alpha^2)^{\frac{1}{2}}=(1-\frac{T_c^2}{T_z^2})^{\frac{1}{2}} \qquad (2.5)$$

On the other hand, the spectral width parameter may be obtained from the moments of wave spectrum. For a stationary Gaussian process, the expected rate of peaks f_p and the expected rate of zero upcrossing period f_z are used for the defination of spectral width parameter.

$$f_p=(\frac{m_4}{m_2})^{\frac{1}{2}} \qquad (2.6)$$

and

$$f_z=(\frac{m_2}{m_0})^{\frac{1}{2}} \qquad (2.7)$$

Thus the irregularity factor α is obtained by division of two frequncies that can be used as a measure for the broadness of the wave spetrum.

$$\alpha=\frac{f_z}{f_p} \qquad 0\leq\alpha\leq 1 \qquad (2.8)$$

Having obtained the irregularity factor α, the spectrum width parameter is often expressed by the spectral moments.

$$\varepsilon_s=(1-\alpha^2)^{\frac{1}{2}}=(1-\frac{m_2^{\,2}}{m_0 m_4})^{\frac{1}{2}} \tag{2.9}$$

It is very important to note that the presence of any noise in the estimation of energy spectral density will be amplified particularly in the calculation of higher moments. Errors in estimation of second and fourth moments are the main reasons of divergence from true spectral width parameter.

2.4 Probability distribution of water surface η

The most promising distribution for the water surface is considered with the assumption of normal distribution. The normal density function supposes equal probability for positive and negative water surface which persumes that the mean value of distribution is expected to zero. The density function for water surface elevation is given by:

$$p(\eta)=\frac{1}{\sqrt{2}\pi\sigma_\eta}\exp[-\frac{\eta^2}{2\sigma_\eta^2}] \tag{2.10}$$

Where σ_η is the standard deviation of the water surface elevation and is formulated in term of spectral moments as

$$\sigma_\eta=\sqrt{m_0} \tag{2.11}$$

The normal distribution for the wave elevation is fitted with deep water condition where the low steepness of waves are predominant. In shallow water condition, the nonlinearity of sea waves produces some deviation from the normal distribution which is discussed in the following.

2.5 Nonlinearity of sea waves

The extent of normal distribution for waves requires the measure of characteristics of the skewness λ_3 and kurtosis χ_4. These quantities are derived in terms of mean water elevation $\bar{\eta}$ and root mean square value of the profile σ_η.

$$\lambda_3=\frac{1}{\sigma_\eta^3}.\frac{1}{N}.\sum_{n=1}^{N}(\eta_i-\bar{\eta})^3 \tag{2.12}$$

$$\lambda_4'=\frac{1}{\sigma_\eta^4}.\frac{1}{N}.\sum_{n=1}^{N}(\eta_i-\bar{\eta})^4 \tag{2.13}$$

Where N is total number of points in the record. In case of normal distribution, $\lambda_3=0$ and

$X_4 = 3.0$. The new quantity which is called the coefficient of excess $\lambda_4 = X_4 - 3.0$ is often introduced to measure the deviation from Guassian process. The characteristics of skewness λ_3 is that it demonstrates the symmetry of the distribution about the mean value. For the positive values of skewness, the distribution has a longer tail within the range of values greater than mean.

For a narrow - band wave, the skewness is proportional with the wave steepness. Skewness is also related to the band width of the spectrum. For broad spectrums, the Skewness is inversely proportional with the spectrum width parameter. The same law is valid for the kurtosis. Kurtosis indicates the peakedness of the mode of statistical distribution. The long tails of the distribution for higher values of kurtosis is observed on the possitive side of the spectrum. The probablity density function of nonlinear sea is expanded by use of skewness and kurtosis factors in a series form as

$$p(\hat{\eta})=\phi(\hat{\eta})\ \{\ 1+\frac{\lambda_3}{3!}H_3(\hat{\eta})+\frac{\lambda_4}{4!}H_4(\hat{\eta})+\ ...\ \} \tag{2.14}$$

Where $\phi(\hat{\eta})$ and H_n are the standard normal distribution and nth order of Hermite polynomial of the variable $\hat{\eta}$ respectively.

$$\hat{\eta}=\frac{\eta-\bar{\eta}}{\sigma_\eta} \tag{2.15}$$

and

$$H_3(\hat{\eta})=\hat{\eta}^3-3\hat{\eta} \tag{2.16}$$

$$H_4(\hat{\eta})=\hat{\eta}^4-6\hat{\eta}^2+3 \tag{2.17}$$

Concerning the quantities of skewness and kurtosis, the Hermitian moment b_n is used as defined below.

$$\lambda_3=\frac{b_3}{\sigma_\eta^3} \tag{2.18}$$

$$\lambda_4=\frac{b_4}{\sigma_\eta^4} \tag{2.19}$$

Where b_n is formulated in term of Hermitian moments.

$$b_n=\sigma_\eta^n.H_n(\hat{\eta}) \tag{2.20}$$

Figure 2.2 shows the normal and non-normal distributions for a wave profile in shallow water waves. (λ_3= 1.5, λ_4= 2.814)

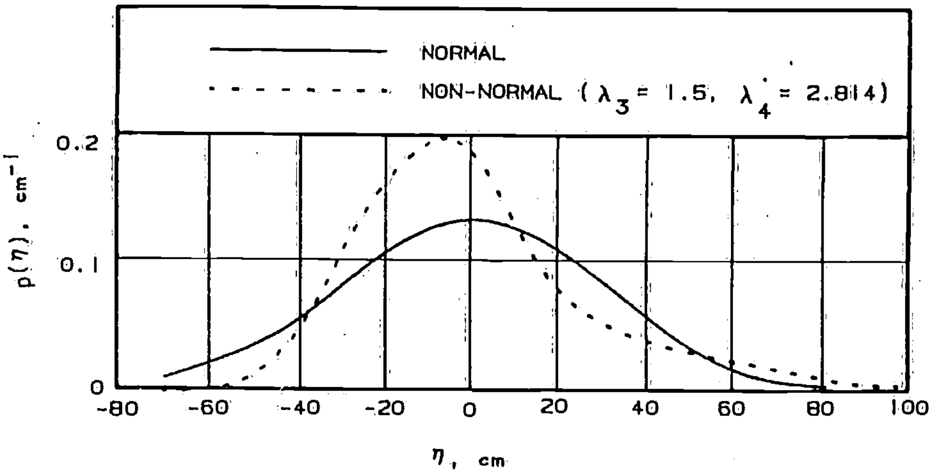


Figure 2.2 Probability density functions of sea surface Bitner (1980) adopted from reference [9]

2.6 Probability distribution of positive maxima extremes

The probability distribution of wave surface in a linear or non-linear wave condition was discussed earlier. The emphasis of this section is to find the probability distribution of positive maxima. For a random process, the wave surface at a given reference location is formulated in term of wave frequency and phase angle.

$$\eta=\sum_{n=1}^{N} a_n \cos(\omega_n t-\alpha_n) \tag{2.21}$$

The local positive maxima for a given wave is defined by two boundary conditions $\dot{\eta}=0$ and $\ddot{\eta}<0$, where $\dot{\eta}$ and $\ddot{\eta}$ are the first and second derivatives of random wave equation. The joint probability distribution of η, $\dot{\eta}$, $\ddot{\eta}$ and $p(\eta,\dot{\eta},\ddot{\eta})$ are evaluated in term of moments of wave spectrum according to Cartwright and Longuet-Higgins (see reference [9]):

$$p(\eta,\dot{\eta},\ddot{\eta})=\frac{1}{(2\pi)^{\frac{3}{2}}(m_2\Delta)^{\frac{1}{2}}}\exp\{-\frac{1}{2}[\frac{\dot{\eta}^2}{m_2}+\frac{1}{\Delta}(m_4\eta^2+2m_2\eta\ddot{\eta}+m_0\ddot{\eta}^2)]\} \tag{2.22}$$

where

$$\Delta=m_0m_4-m_2^2 \tag{2.23}$$

The expected number of occurances (or the mean crossing frequency) during which a maximum η takes a value between η_0 and $\eta_0+d\eta_0$ is obtained from the integral of the joint probability density function.

$$\mu(\eta_0)=\int_{-\infty}^{0} p(\eta_0,0,\ddot{\eta})\,|\ddot{\eta}|\,d\ddot{\eta} \tag{2.24}$$

Substitution of $p(\eta_0,\ 0,\ \ddot{\eta})$ gives the expected number of occurrence for $\eta_0 \leq \eta \leq \eta_0+d\eta$ as follows :

$$\mu(\eta_0)=\frac{\sqrt{\Delta}}{(2\pi)^{\frac{3}{2}}m_0\sqrt{m_2}}\exp\{-\frac{1}{2}(\frac{\eta_0}{\sqrt{m_0}})^2\}[\exp\{-\frac{1}{2}\frac{\alpha^2}{\varepsilon^2}(\frac{\eta_0}{\sqrt{m_0}})^2\}$$

$$+\frac{\alpha}{\varepsilon}(\frac{\eta_0}{\sqrt{m_0}})\int_{-\frac{\alpha}{\varepsilon}(\frac{\eta_0}{\sqrt{m_0}})}^{\infty}\exp\{-\frac{x^2}{2}\}dx] \tag{2.25}$$

Or

$$\mu(\eta_0)=\frac{1}{2\pi\sqrt{m_0}}\sqrt{\frac{m_4}{m_2}}[\frac{\varepsilon}{\sqrt{2\pi}}\exp\{-\frac{1}{2\varepsilon^2}(\frac{\eta_0}{\sqrt{m_0}})^2\}$$

$$+\alpha(\frac{\eta_0}{\sqrt{m_0}})\exp\{-\frac{1}{2}(\frac{\eta_0}{\sqrt{m_0}})^2\}\{1-\Phi(-\frac{\alpha}{\varepsilon}\frac{\eta_0}{\sqrt{m_0}})\}] \tag{2.26}$$

In which Φ is the standard normal distribution and $\alpha=\sqrt{1-\epsilon^2}$.

The expected number of positive values of maxima (or the mean crossing frequency for positive maxima) is given by the integral of $p(\eta,\ 0,\ \ddot{\eta})$ with two variants elevation η and acceleration $\ddot{\eta}$.

$$\nu_m^+=\int_0^{\infty}\int_{-\infty}^{0} p(\eta,0,\ddot{\eta})\,|\ddot{\eta}|\,d\ddot{\eta}\,d\eta \tag{2.27}$$

After simplification the integral is reduced to

$$\nu_m^+ = \frac{1}{4\pi}\left(\sqrt{\frac{m_2}{m_0}} + \sqrt{\frac{m_4}{m_2}}\right) = \frac{1}{4\pi}\left(\frac{1+\alpha}{\alpha}\right)\sqrt{\frac{m_2}{m_0}} \tag{2.28}$$

Having obtained the values of $\mu(\eta_0)$ and ν_m^+ in equations (2.26) and (2.28), the probability density function of positive maxima is derived by:

$$p(\eta_0) = \frac{\mu(\eta_0)}{\nu_m^+} \tag{2.29}$$

$$p(\eta_0) = \frac{2/\sqrt{m_0}}{1+\alpha}\left[\frac{\varepsilon}{\sqrt{2\pi}}\exp\left(-\frac{1}{2\varepsilon^2}\left(\frac{\eta_0}{\sqrt{m_0}}\right)^2\right)\right.$$

$$\left. + \alpha\left(\frac{\eta_0}{\sqrt{m_0}}\right)\exp\left(-\frac{1}{2}\left(\frac{\eta_0}{\sqrt{m_0}}\right)^2\right)\left(1-\Phi\left(-\frac{\alpha}{\varepsilon}\frac{\eta_0}{\sqrt{m_0}}\right)\right)\right] \qquad 0 \leq \eta_0 < \infty \tag{2.30}$$

If η_0 is nondimensionalized by dividing by $\sqrt{m_0}$ $(\hat{\eta} = \eta_0/\sqrt{m_0})$, the probability density function becomes:

$$p(\hat{\eta}) = \frac{2}{1+\alpha}\left[\frac{\varepsilon}{\sqrt{2\pi}}\exp\left(-\frac{\hat{\eta}^2}{2\varepsilon^2}\right) + \alpha\hat{\eta}\exp\left(-\frac{\hat{\eta}^2}{2}\right)\left(1-\Phi\left(-\frac{\alpha}{\varepsilon}\hat{\eta}\right)\right)\right] \qquad 0 \leq \hat{\eta} < \infty \tag{2.31}$$

Hier it is interesting to notice a physical phenomenonIn case of long crested waves , the distribution of individual crest of ripples follows the probability distribution of positive maxima while the height of swell of complies the normal probability density function.

However the probability distribution of a narrow band spectrum is obtained by substitution of $\epsilon = 0$ in equation (2.31).

$$p(\hat{\eta}) = \hat{\eta}\exp\left(-\frac{\hat{\eta}^2}{2}\right) \qquad 0 \leq \hat{\eta} < \infty \tag{2.32}$$

For a wide band spectrum ($\epsilon = 1$), the probability density function is found in the form of a truncated ($\hat{\eta} = 0$) normal distribution as follows

$$p(\hat{\eta}) = \sqrt{\frac{2}{\pi}} \exp(-\frac{\hat{\eta}^2}{2}) \qquad 0 \le \hat{\eta} < \infty \qquad (2.33)$$

Thus if bandwidth is finite, the probability distribution falls between a normal and a Raleigh distribution functions [equations (2.33) and (2.32)]. The probability density function of $\hat{\eta}$ for various values of ϵ is shown in Figure 2.3.

The cumulative distribution of $\hat{\eta}$ is derived by integration of $p(\hat{\eta})$ in equation (2.31):

$$P(\hat{\eta}) = \frac{2}{1+\alpha}[-\frac{1}{2}(1-\alpha) + \Phi(\frac{\hat{\eta}}{\varepsilon}) - \alpha \exp(-\frac{\hat{\eta}^2}{2})(1-\Phi(-\frac{\alpha}{\varepsilon}\hat{\eta}))] \qquad (2.34)$$

For narrow band spectrum ($\epsilon=0$), we will find

$$P(\hat{\eta}) = 1 - \exp(-\frac{\hat{\eta}^2}{2}) \qquad (2.35)$$

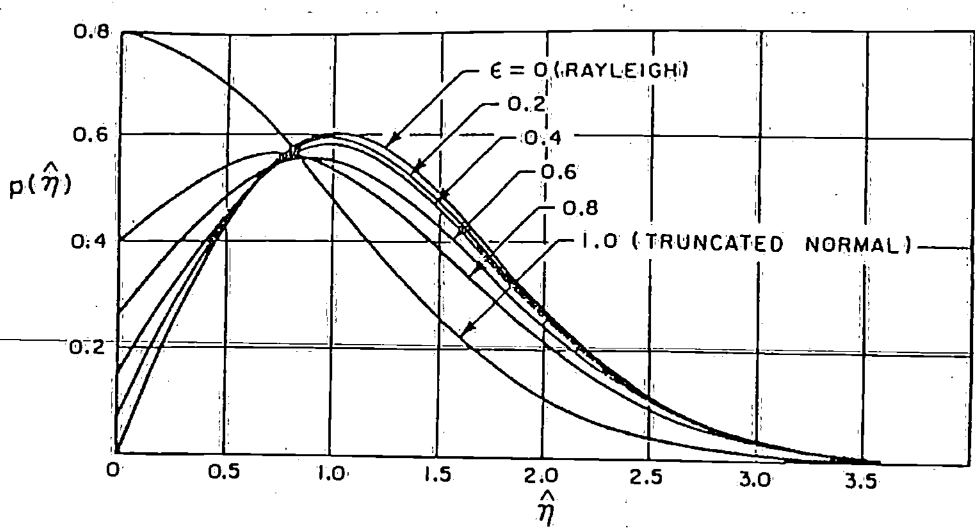


Figure 2.3 Probability density functions of non-dimensional maxima watheter surface as a function of band width parameter ϵ

The application of joint successive level crossing technique has investigated by Naess and Tayfun [16]. Naess deduced that the cumulative probability for the non-dimensionalized amplitude can be replaced with the following approximation:

$$P(\hat{\eta}) = 1 - \exp(-\frac{\hat{\eta}^2}{1-r}) \qquad (2.36)$$

where

$r' = R(T/2)$ & $R(\tau) = \int_0^\infty S(\omega).\cos\omega\tau.d\omega$

T= dominant wave priod with condition $R(T) = 0$.

2.7 Generalized probability distribution of H

Let us start with the definition of statistical height parameteres which are commonly used in random waves. If all heights in a record are evaluted from the difference between the crest and trough then the following equations can be used to describe the height of the wave H by three different symbols as follows:

1- The mean height $\overline{H}$

The mean of the heights in the record is called the mean height $\overline{H}$:

$$\overline{H} = \frac{1}{N}\sum_{i=1}^{N} H_i \tag{2.37}$$

2- The significant wave height H_s

The mean of the highest one-third of all heights is defined by the significant wave height H_s or $H_{1/3}$.

$$H_s = H_{\frac{1}{3}} = \frac{3}{N}\sum_{i=1}^{N/3} H_i \tag{2.38}$$

3- The root mean square height H_{rms}

The root mean square (rms) height is calculated in term of square of wave heights as

$$H_{rms} = [\frac{1}{N}\sum_{i=1}^{N} H_i^2]^{1/2} \tag{2.39}$$

It is clear that the probability density of troughs (the expected numCmaxilyma with negative values) is similar to the probability density of crests (the expected number of maxima with positive values). Thus the probablity density of amplitudes is obtained by the probability of crests or troughs because the wave surface elevation is proportional with wave amplitude [equation (2.1)]. Substitution of amplitude notation a instead of water surface notation η_0 in equation (2.30), the probability density of wave amplitudes becomes

$$p(a)=\frac{2/\sqrt{m_0}}{1+\alpha}[\frac{\varepsilon}{\sqrt{2\pi}}\exp\{-\frac{1}{2\varepsilon^2}(\frac{a}{\sqrt{m_0}})^2\}+\alpha(\frac{a}{\sqrt{m_0}})\exp\{-\frac{1}{2}(\frac{a}{\sqrt{m_0}})^2\}\{1-\Phi(-\frac{\alpha}{\varepsilon}\frac{a}{\sqrt{m_0}})\}] \quad (2.40)$$

The probability density of amplitudes a for a random wave follows a Rayleigh or normal distribution depending on the spectrum width parameter (ϵ= 0 Rayleigh, and ϵ= 1 normal distribution).

Assuming the Airy wave theory is valid, the relation $H= 2a$ can be introduced into the equation (2.40). Thus the probability density of H is given by the half of probability density of amplitude a as follows

$$p(H)=\frac{1/\sqrt{m_0}}{1+\alpha}[\frac{\varepsilon}{\sqrt{2\pi}}\exp\{-\frac{1}{2\varepsilon^2}(\frac{H}{2\sqrt{m_0}})^2\}$$

$$+\alpha(\frac{H}{2\sqrt{m_0}})\exp\{-\frac{1}{2}(\frac{H}{2\sqrt{m_0}})^2\}\{1-\Phi(-\frac{\alpha}{2\varepsilon}\frac{H}{\sqrt{m_0}})\}] \quad (2.41)$$

The cumulative distribution function of H is derived by integration of equation (2.41)

$$P(H)=\frac{2}{1+\alpha}[-\frac{1}{2}(1-\alpha)+\Phi(\frac{H}{2\varepsilon\sqrt{m_0}})-\alpha\exp\{-\frac{1}{2}(\frac{H}{2\sqrt{m_0}})^2\}\{1-\Phi(-\frac{\alpha}{2\varepsilon}\frac{H}{\sqrt{m_0}})\}] \quad (2.42)$$

2.8 Narrow band Guassian wave

For a narrow band Guassian wave, the probability density function of H is obtaned by substitution of ϵ= 0 (or α=1) in equation (2.41)

$$p(H)=\frac{1}{2\sqrt{m_0}}\frac{H}{2\sqrt{m_0}}\exp\{-\frac{1}{2}(\frac{H}{2\sqrt{m_0}})^2\}=\frac{H}{4m_0}\exp(-\frac{H^2}{8m_0}) \quad (2.43)$$

The nth moment of the function $p(H)$ about the origin (H= 0) is given by the following equation. It should be noticed that the moment is equal to the expected value of H in order n.

$$E[H^n]=\int_0^\infty H^n p(H)dH \quad (2.44)$$

The integral is evaluted for odd and even numbers of n separately as

$$E[H^{2n}] = 2^{3n}(n!)\,m_0^n \tag{2.45}$$

and

$$E[H^{(2n+1)}] = \frac{2^{n+1}(2n+1)!}{n!}\sqrt{\frac{\pi}{2}}\,m_0^{(2n+1)/2} \tag{2.46}$$

It is clear that the zeroth moment gives the value of cumulative probability which corresponds to a unit area under probability density function that $E[\,H^0] = 1$). The mean height $\bar{H}$ is evaluted by the first moment as follows

$$\bar{H} = E[H] = \sqrt{2\pi m_0} \tag{2.47}$$

The root mean square height H_{rms} may be calculated by the square of second moment

$$H_{rms} = \sqrt{E[H^2]} = 2\sqrt{2m_0} \tag{2.48}$$

The variance of heights is formulated in term of the first and second moments. For this purpose, we use the defination of variance as follows

$$\sigma_H^2 = E[(H - E[H])^2] = E[(H - \bar{H})^2] \tag{2.49}$$

where σ_H^2 is the variance (σ_H is the standard deviation) and E denotes the expected value. After expansion of equation (2.49) noting that $E[\bar{H}] = \bar{H}$, the variance of wave height is evaluated as

$$\sigma_H^2 = E[H^2] - 2\,E[H]\,\bar{H} + \bar{H}^2 \tag{2.50}$$

Because $\bar{H} = E[H]$, thus equation (2.50) is reduced to

$$\sigma_H^2 = E[H^2] - (E[H])^2 = E[H^2] - \bar{H}^2 \tag{2.51}$$

With substitution of the first and second moment in equation (2.51), we find the variance as

$$\sigma_H^2 = [8 - 2\pi]\,m_0 \tag{2.52}$$

Thus replacing m_0 by H_{rms} in equation (2.43), the probability density function of H for narrow band spectrum becomes

$$p(H) = \frac{2H}{H_{rms}^2}\exp\left(-\frac{H^2}{H_{rms}^2}\right) \tag{2.53}$$

and

$$P(H) = 1 - \exp(-\frac{H^2}{H_{rms}^2}) \tag{2.54}$$

On the other hand, the significant wave height H_s is often represented in term of H_{rms}. By definition, H_s is the mean of the highest of one third waves. First we find the height H_0 which corresponds with the highest of one third of wave heights.

$$P(H_0) = \frac{2}{3} = 1 - \exp(-\frac{H_0^2}{H_{rms}^2}) \tag{2.55}$$

and H_0, the corresponding height for the highest one third of wave heights is equal to

$$H_0 = 1.048 H_{rms} \tag{2.56}$$

The centriod of the area for $H \geq H_0$ under the probability density function is used to find the H_s as follows

$$H_s = \frac{\int_{H_0}^{\infty} H p(H)\, dH}{\int_{H_0}^{\infty} p(H)\, dH} \tag{2.57}$$

where $p(H)$ is given by equation (2.53). We then integrate equation (2.57) to find H_s in term of wave spectrum parameters

$$H_s = 1.416 H_{rms} \approx 4\sqrt{m_0} \tag{2.58}$$

2.9 Modified Rayleigh distribution and joint probability distribution of (H, T)

The probability distributtion of computed wave heights (in dimensionless form) has been compared with the measured data by Naess [see reference 16]. He found that the modified Rayleigh distribution with the parameter $r' = -0.71$ is a better approximation for the wave height distribution.

Theoritical distributions for wave heights are not limited to the disTheoreticalwhich have been discussed by Cartwright, Longuet-Higgins [9] and Naess [16]. Cavanié et al [16] have given a distribution which is resonably in agreement with wave data for a narrow band spectrum. Lindgren and Rychlic [16] have formulated a distribution which is not in a closed form and needs more complex numerical computations. The distribution predicts the difference between maxima and minima as well as their time interval.

The joint probability distribution of wave height and period has been considered to provide useful informations for the statistics of waves. However we will not going to discuss the joint probability distribution of wave heights and period and only it might be useful that in application of different probability distributions for wave height and wave period two points should be considered carefully. Firstly the definition of wave height and periods are not the same in most of publications and they must be used with care. Secondly the joint probability distribution of wave height and period for a broad spectrum have not been adequately described with the update available distributions.

2.10 Long term probability distribution of wave heights

The long term probability distribution of wave heights can be obtained either in term of extreme individual wave heights H or the significant wave heights H_s. The first choice is often used in association with the ultimate limit state analysis while the second is usually applied for the ultimate fatigue analysis. The importance of a long term distribution is established by two types of storms during the design life of the structure. One is the locally generated storm condition and other is the distant storm appearing as swells. The locally generated storms often are random in nature while the swells are generally low- frequency waves and regular in nature.

In evaluation of long term wave distribution, first a design life is considered for the structure. If the wave data for the long term criterion is lacking such as many cases in offshore sites, then the probability distribution of long-term wave data is calculated by the available distribution functions of short term waves. The long term distribution of wave heights is found by ascension of wave heights during design life. The individual wave height distributions are assumed be independent and therefore it is possible to apply one of the extreme value distributions for the long term cumulative function.

Assume that N is the number of waves in the planned service life ($N= T_L / T_s$ where T_L is the return period and T_s is the short term period.) . The extreme value distribution of type I is based on the maximum N individually independent random waves. The wave heights are numbered from 1 to N in ascension order, thus the cumulitive probability of the largest H_{max} is defined in term of H_i in which $i=$ 1, 2, 3, ...,N

$$P(H_{\max} \leq H) = P(H_1 \leq H, H_2 \leq H, H_3 \leq H, \dots, H_N \leq H) \tag{2.59}$$

Since H_i are independent, $P(\, H_{max} \leq \, H)$ may be written by the product of cumulative distributions as

$$P(H_{\max} \leq H) = \prod_{i=1}^{N} P(H_i \leq H) = P(H_1 \leq H)\, P(H_2 \leq H) P(H_3 \leq H) \dots P(H_N \leq H) \tag{2.60}$$

We assume that the cumulative probability functions are equal for any short term wave height H_i. This simplifies equation (2.60) to:

$$P(H_{\max} \leq H) = P(H).P(H).P(H).... = [P(H)]^N \tag{2.61}$$

Or

$$p(H_{max} \leq H) = N\, p(H)\, [P(H)]^{N-1} \tag{2.62}$$

Substitution of equations (2.41) and (2.42) in (2.62), the long term probability density function of wave heights is derived as

$$p(H_{max} \leq H) = \frac{N/\sqrt{m_0}}{(1+\alpha)^N}\left[\frac{\varepsilon}{\sqrt{2\pi}}\exp\{-\frac{1}{2\varepsilon^2}(\frac{H}{2\sqrt{m_0}})^2\} + \alpha(\frac{H}{\sqrt{2m_0}}\exp\{-\frac{1}{2}(\frac{H}{2\sqrt{m_0}})^2\}\{1-\Phi(-\frac{\alpha}{2\varepsilon}\frac{H}{\sqrt{m_0}})\right.$$

$$\times[-\frac{1}{2}(1-\alpha)+\Phi(\frac{H}{2\varepsilon\sqrt{m_0}}) - \alpha\exp\{-\frac{1}{2}(\frac{H}{2\sqrt{m_0}})^2\}\{1-\Phi(-\frac{\alpha}{2\varepsilon}\frac{H}{\sqrt{m_0}})\}]^{N-1} \tag{2.63}$$

The cumulative distribution is given by the integral of equation (2.63) as follows

$$P(H_{max} \leq H) = \frac{1}{(1+\alpha)^N}[-\frac{1}{2}(1-\alpha)+\Phi(\frac{H}{2\varepsilon\sqrt{m_0}})-\alpha\exp\{-\frac{1}{2}(\frac{H}{\sqrt{2m_0}})^2\}\{1-\Phi(-\frac{\alpha}{2\varepsilon}\frac{H}{\sqrt{m_0}})\}]^N \tag{2.64}$$

Instead of using the generalized probability function, sometimes the Weibull cumulative distribution is employed to predict the long term wave heights. In that case, the extreme value distribution of type *II* may be applied in the Weibull cumulative distribution. The probability of exceedence for $P(H_{max} > H)$ is determined in term of N independent probability of exceedence as follows

$$P(H_{max} > H) = P(H_1 > H, H_2 > H, \ldots, H_N > H) \tag{2.65}$$

Since H_i are independent

$$P(H_{max} > H) = Q(H_1 > H)\; Q(H_2 > H)\; \ldots\; Q(H_N > H) \tag{2.66}$$

Substitution of probability of exceedence for H_1, H_2, H_3, …, H_N, equation (2.64) is reduced to

$$P(H_{max} > H) = [1 - P(H)]^N \tag{2.67}$$

and the cumulative probability of H_{max} is written as

$$P(H_{max} \leq H) = 1 - [1 - P(H)]^N \approx 1 - N[1 - P(H)] \tag{2.68}$$

This approximation needs to curtail for small values of wave height, otherwise the negative values are found for the cumulative distribution function. The absolute error is almost nil for large values of H which are interested in the design condition. However a three parameter Weibull distribution is assumed for the short term wave height in the following

$$P(H) = 1 - \exp[-(\frac{H-A}{B})^C] \tag{2.69}$$

The long term distribution is found by the substitution of equation (2.69) in (2.68)

$$P(H_{max} \leq H) = 1 - N\exp[-(\frac{H-A}{B})^C] \tag{2.70}$$

Because the number of waves N in the working life is known, so that the cumulative probability distribution may be formulated in form of Weibull distribution

$$P(H_{max} \leq H) = 1 - \exp[-(\frac{H-\acute{A}}{B})^C] \tag{2.71}$$

Before application of Weibull distribution, a method for prediction of extreme wave heights was developed by Gumbel [20]. The long term probability distribution of wave heights is sometimes obtained by log-normal probability distribution. Ochi (1978) [reference 9] has compared the cumulative distribution of significant wave heights from ocean weather station India in the North Atlantic. It was concluded that log-normal distribution is fitted with the data except at the upper extremes because it overestimates the data for large values of H_s. On contrary, the Weibull distribution compares well for large values of H_s while it fails to satisfy the data in other range than near the upper tail. Thus the generalized form of Weibull distribution was found a better represent instead of two parameter distribution, since it encompasses two parameter distribution and other alternative distributions as well. On the other hand, Mathisen and Bitner-Gregersen (1990) [reference 9] have suggested a "Lonowe" (Log normal) distribution. They have shown that Lonowe distribution provides a good result in many sets of wave data. The only problem is the position of transition point which can be selected arbitrary.

2.11 Hydrodynamic forces

The statistics of wave amplitudes was discussed earlier in both short term and long term conditions. For strength analysis of an offshore structure thethatabilistic models of wave characteristics are introduced in the wave force models. The square of *Response amplitude operator* $(\overline{RAO^2})$ for each response is multiplied to the wave energy spectrum in order to find the structure response spectrum. This method of analysis for any excitition is called the *spectral analysis method* which has been discussed elsewhere with more details (see for instance [9]). The spectral method was initially developed for the linear systems. The system is defined linear when the excitation of the wave is prothatonal to the amplitude of wave such as for inertia dominated wave force. In that case, the principle of superpostion may be applied for random wave distribution. It is highly desirable that in any case the ideal linearization may not be achieved even for the fixed structures.

The nonlinearity of the response may be arised due to the selected wave theory, the drag term in Morison equation, the interaction of wave and current, the nonlinearity of strength parameters and physical conditions of sea-bed. Nevertheless in reality nonlinear systems can not be treated without some approximations with regarding to the linearization. Because in each case an unic *RAO* can only be obtained for a linear system, thus the application of common spectral analysis is restricted to the linear modelling of external wave and current forces.

With respect to the reliability analysis, it is important to seperate two classes of uncertainties in modelling of loads [42]. The class *I* uncertainties are naturally inherent variables which are *random* through time and space. The class *II* uncertainties are variables due to the modelling, measurement and observation errors which are *systematic* through time and space. The class *I* variables are always independent with respect to data while the class *II* variables are sensetive by data information, improvement of physical or numerical modelling and accuracy of meseurements. The distinction between two variables in statistics of loads is highly valuable for two reasons. First the extension of uncertainties to other environmental conditions (sea-state, annual periods) may not be achieved without gathering the nature of characteristics. Second, the contribution of risks due to two types of uncertainties is important in order to making decisions or evluting the reliability of existing structures.

Although the old formula for wave forces, Morison equation, originally has been derived for oscillatory flow applied to the cylinderical members but the wave forces in inclined cylinders, the effect of structural velocity, the interaction of current and wave forces are obtained with the improvement of the Morison equation. The primary information with respect to the Morison equation becomes available with the dimensional analysis. For dimensional description of the wave force *f(t)*, a group of independent parameteres has been considered by different researches (see for instance [9]). The relationship between force on the unit length with space-time variables is given by following expression

$$f(t) = \Phi(t,T,a,D,D_r,\rho,\nu) \tag{2.72}$$

where a is the amplitude, D is the characteristic dimension (such as diameter), D_r is the roughness mean size, ρ is the density and ν is the kinematic viscosity of water.

Assume that the linear wave theory is valid in description of wave loads. The Morison equation for a vertical cylinder gives the wave forces per unit length of the member with diamater D as

$$f(t) = \rho\frac{\pi}{4}D^2C_M\dot{u}_x + \frac{1}{2}\rho D C_D |u_x| u_x \tag{2.73}$$

where
f = wave force per unit length of the vertical cylinder
u = horizontal component of water particle velocity
C_M = inertia
C_D = drag

If the non-dimensionalized form of equation (2.72) in term of water particle velocity is compared with the Morison equation, the infulence parameters for the inertia and drag coefficients are obtained. The relationship between hydrodynamic coefficients and nondimensionalized parameters is estabilished by using the foregoing expressions.

$$C_m = C_m(\omega t, \frac{2a}{D}, \frac{u_0 D}{\nu}, \frac{D_r}{D}) \tag{2.74}$$

$$C_d = C_d(\omega t, \frac{2a}{D}, \frac{u_0 D}{\nu}, \frac{D_r}{D}) \tag{2.75}$$

where $u_0 = (2\pi a)/T$.

Note that the most common parameteres which are affecting the wave forces are Reynolds number Re and Keulegan-Carpenter number KC (if the roughness is assumed constant). Both Re and KC numbers are defined in terms of the amplitude of velocity u_0 and the period T as follows.

$$KC = \frac{u_0 T}{D} \qquad Re = \frac{u_0 D}{\nu} \tag{2.76}$$

Returning back to the Morison equation, formula (2.73), one of the wave theories is applied in the estimation of hydrodynamic forces. Even with the application of linear Airy wave theory, the drag term in Morison equation causes the deviation of the distribution of wave force from distribution of water surface. The result of foregoing discussion is thus the distribution of hydrodynamic force deviates from the water surface distribution except by the linearization of drag term in the spectral analysis, the wave force follows the same distribution as well as the wave surface (Guassian distribution).

2.12 Invarient uncertainties of hydrodynamic forces

For practical application of the reliability format, the traditional methods has been developed by the approximation of quasi-staic loading instead of dynamic one.
Even for the deterministic methods, two types of loadings are distinguished from each other which are called static and dynamic loadings. The first issues of reliability engineering in 1970 proposed the substitution of dynamic loadings with equivalent quasi-static loadings. By this way, the time invariant of basic varibles are formulated in general format of reliability analysis. The push-over analysis of fixed and floating platforms is the first product of this approach and uncertainties of this time - invarient reliability method is discussed in the following.

The first issue of class *II* uncertainty of wave forces is found in determination of wave heights. Generally speaking, the uncertainty of wave forces comes from three origins which can be defined as *model uncertainty*, *statistical uncertainty* and *measurement uncertainty*. The model uncertainty indicates the fitness of one proposed distribution with the observed data. An average value thatefficient of variation $V = 0.11$ has been given by Guedes Soares and Moan [19] for the model uncertainty. The statistical uncertainty arises from the method of parameter estimation (likelihood method, method of moments or graphical approach) and the limited amount of data. An approximate value of uncertainty $V = 0.$ has been given in reference [19]. The measurement uncertainty represents the errors in the source of measurements which differs by method of data collection. For observed data the bound for the coefficient of variation is taken in the interval V= 0.05-0.10 while for the instrumental measurements and handcasting data, Pianc [43] suggests the values of V= 0.10-0.20 and V= 0.20 respectively. One reasonable value of measurement uncertainty V= 0.09 has been given for the instrumental data which is valid for the North sea (see reference [19]). In fact the method of data collection influences the measurement uncertainty as well as the statistical uncertainty. The large number of data for the visual observation causes a smaller statistical uncertainty but a larger measurement uncertainty than instrumental data.

With all these values, the systematic uncertainty of wave heights may be found by the multiplicative law. As described in the previous section, the systematic uncertainty is a measure of actual variable to predicted variable and often in reliability engineering is called the *bias factor* or *the ratio of mean to nominal value* $(\mu/X_{nom.})$. The coefficient of variation for bias factor is found as

$$V_{B_H}=\sqrt{0.11^2+0.08^2+0.09^2}=0.16 \tag{2.77}$$

where B_H is the bias factor for extreme wave height and V_{BH} is the coefficient of variation.

Olufsen and Bea [42] has been discussed the bias factors for the extreme wave heights and significant wave heights. The distribution of bias in hindcast wave heights has been described by log-normal distribution with a mean value and coefficient of variation equal to 0.95 and 0.15 for North Sea (*NS*). The corresponding values for Gulf of Mexico (*GoM*) has been found as 0.85 and 0.20 for the mean value and standard deviation respectively. The bias is not only the class *II* uncertainty but also a small amount of class *I* uncertainty. The class *I* variability is considered very small with respect to the class *II* variability and because the contribution of each one can not be seperated precisely thus the bias for hindcast data is estimated as class *II* uncertainty. However the wave height in the relibility analysis is described by the predicted wave height multiplied to the bias factor.

The coefficient of variation for sytematic uncertainty of wave periods may be found seperately or by the joint distribution of wave heights and periods [19]. The typical values for the model uncertainty, the statistical uncertainty and measurement uncertainty for the NS are respectively 0.1, 0.05 and 0.1. Thus the coefficient of variation for bias factor in NS is evaluted as

$$V_{B_T}=\sqrt{0.10^2+0.05^2+0.10^2}=0.15 \tag{2.78}$$

where the B_T is the bias for wave period and V_{BT} is the coefficient of variation.

2.13 Distribution of hydrodynamic coefficients and marine growth

The source of uncertainty for hydrodynamic coefficients are substantially located on class *II* uncertainties. Due to the complex variation of hydrodynamic coefficients with wave amplitude and phase angle, even with large amount of data which are available for the smooth circular cylinders, the limited data on hydrodynamic coefficients for non-circular shapes with surface roughness and *Re* & *KC* numbers dependency can not be found in the literature. It is noticeable that for high Reynolds numbers (say above 10^5), the hydrodynamic coefficients show a negligible dependency to the Reynolds number. Therefore the dependency of Reynolds number for the hydrodynamic coefficients in jack-up legs can be neglected in extreme wave conditions [40].

For example, experiments with full scale measurements in Christchurch Buy Tower have illustrated that the hydrodynamic coefficients diminish with increasing the depth of water but for design purposes the condition of near surface seems to be safer [47]. For near surface Soding et al [47] suppose the following values:

$C_M = 1.8$ & $C_D = 0.66$	$KC > 30$;
C_M and C_D both are increased when KC decreases	$5 < KC < 30$;
$C_M = 2.0$ (for $KC < 5$ C_D is not important)	$KC < 5$.

In reference [50], the mean value and coefficient of variation for hydrodynamic coeffficients in a jacket structure are taken with normal distribution where the mean values and coefficient of variation are given as:

Drag coefficient	$\overline{C_D} = 0.75$	$V_{CD} = 0.30$
Inertia coefficient	$\overline{C_M} = 1.80$	$V_{CM} = 0.30$
Correlation factor between C_D and C_M		$\rho = -0.90$

Goedes Soares and T. Moan [19] assume that the hydrodynamic coefficients are often complied by normal distribution with coeffficient of variation equal $t^o0.1$ (both inertia and drag coefficient). One reprort by R. Loseth and L. Hauge [36] has been applied the log-normal distribution for hydrodynamic coefficients with taken into account the influence of non-circular sections of lattice legs with racks and marine growth. The effect of marine growth must be discussed for two reasons. Firstly, the increase of diameter leads to the increased projected area and hence increased hydrodynamic loading. Secondly, the increase of roughness which influences the estimate of drag force and therefore increased wave force.

$C_M = 2.0$	$\mu = 1.75$, $\sigma = 0.125$;
$C_D = 0.65$ for 0 *mm.* marine growth thickness	$\mu = 1.00$, $\sigma = 0.200$;
$C_D = 0.80$ for 80 *mm.* marine growth thickness.	

The prediction of wave forces especially the drag coeffiecients for lattice legs has been investigated from 1975 in engineering science data unit [46]. A great number of wind tunnel tests have been calibrated to find the mathematical model for darg coefficient in different configuartions of legs. Unfortunately, the tunnel wind tests are the only experimental way for the prediction of drag coefficients because in most of cases, the wave basins are not large enough to test full legs and the evalution of results for wide range of *Re* or *KC* numbers are impractical. The comparison of tunnel wind tests with analytical method for both square and triangular legs are discussed by N.P. Smith and C.A. Wendenburg [46]. It has been concluded that when the effect of roughness on the drag coefficient is neglected, the drag coefficient is underpredicted by 23% for all-cylinderical legs and by 9% for the standard 116 leg with triangular chords. The difference between the test results and the calculated values is less than 4%. Therefore even with the application of high order wave theories, the effect of roughness on the marine growth should be evaluted properly in the prediction of drag term coefficient. Sometimes by applying the anti-fouling coating, the effect of marine growth is restricted for a few years but the experience have learned that the coating is not a proper way to prevent the increasing marine growth. In Recommended practice [27] it is assumed that the severe marine growth is not allowed for the jack-ups by the regulary cleaning of legs. Also the painting of legs influences the roughness and therefore the drag coefficient (effect of salt water during long period).

3- Strength statistics

3.1 Introduction

Until recently the design of fixed offshore structures has been customarily based on the basic allowable stresses. The *AISC* building code (1978) and *API-RP-2A* (1981) allow to use the method of working stress in design of fixed offshore structures. On contratry, in Europe, the limit state design philosophy has been used for the design of fixed platforms. The method has been adopted by regulatory authorities worldwide (Chen 1985, reference [10]). The application of limit state design method for both fixed and floating platforms requires sufficient and accurate information about the behaviour of the structure throughout the entire range of loading up to the collapse process.

In this chapter, the uncertainty due to the strength parameters is discussed for steel offshore structures. Given a statistical distribution for the loads (or the stresses), the most probable largest load (or stress) amplitude during a certain time period can be determined. The probability of exceeding a certain load (or stress) level can also be found.

For the designer of a offshore platform it would be more useful to get information about the probability of failure. It is therefore necessary to know the strength in terms of a stress which will cause failure. This has to be the real failure stress and not a design stress including safety margins.

The uncertainties of strength of materials are often proposed time independent unless the chemical compisition of the steels will be altered during the structure life. The chemical composition gives the reduction in the plate thickness of members by corrosion and this increases the uncertainty which is then time dependent.

The emphasis of this chapter is to investigate the uncertainty of steel in marine structures for the extreme conditions. Although an overview on the strength characteristics of marine steels as well as chemical composition, mechanical properties, weldability, fatigue strength, hardebility, corrosion is needed but in the following sections the uncertainties due to the predictions of strength parameteres with importance in mechanical properties of marine steels are considered.

3.2 Application of steel in marine structures

According to the unified requirements of classification societies, the marine structural steels are divided into three principle groups. The distinction among three groups are classified by the minimum yield stress (strength) according to the unified requirements of classification societies as shown in Table 3.1 (see reference [25]). Muesgen et al [41] have divided the high strength steels in offshore structures to three groups *StE355*, *StE460* and *StE690*. They have reported the application of each type of steels in large offshore projects. Based on the recent reports, while the majority of ships are built using the mild steels, in construction of the offshore platforms and especially the jack-up structures, the high strength steels play an important role.

To obtain considerable higher strength, additional elements such as vanadium, and higher nickel and copper contents are required. It has been shown that the probabilistic modelling of each type of steels requires sufficient data based on the tensile strength of steels. Lindemann et al [34] have proved a negative correlation between tensile strength σ_y with the plate thickness t. It has been found that the yield strength is decreased with increasing the thickness but no data on the correlation factor has not be reported. However, the sources of uncertainties in probabilistic modells of steels are mainly influenced by many factors as well as [25]:

- steel grade,
- chemical composition,
- manufacturing process,
- plate of profile and test specman thickness,
- testing procedure,
- parameter estimation method.

Table 3.1 Characteristics of marine structural steels

No.	Type	Abbreviation	Yield strength (N/mm^2)
(1)	Normal strength steels	MS	235 < σ_y minimum
(2)	Higher strength steels	HTS	315 < σ_y minimum > 390
(3)	Very high strength steels	VHTS	420 < σ_y minimum

3.3 Steel in jack-up platforms

The characteristics of steels for jack-up platforms are usually verified by the same methods in other types of offshore structures. The specifications of steels are generally given from the existing standards in the world like *ASTM, API, BSI, AWS, IIW, ASM* and other International Codes that are used in the classification of structural steels. Sometimes the operators of offshore platforms are using their own specifications that are the modified versions of national standards (see reference [29]).

The application of *high strength steels* is attributed more interest in recent offshore structures. It is well known that in deep water the applications of high strength steels have saved the weight and the cost of oil and gas platforms. Since last four years, many studies have been made for the application of materials which can be used in deep waters. High strength steel is only one of the them. Others may be found in the literature as the application of composite materials of steel and concrete or the high strength concrete.

3.4 Importance of probabilistic modelling for strength of steel members

If we consider the common reliability methods in the literature, it may be concluded that the statistic of strength parameters has a minor effect on the overall reliability analysis. To illustrate the importance of statistical characteristics for strength parameters, consider the diagrams of Fig. 3.1.

The influence of material model on the Hasofer - Lind reliability index has been calculated for the pressure in the semisubmersible hull with $D = 2000$ *mm* and $h = 600$ *m* (see reference [25]). The mean and coefficient of variation have been assumed equal to 420 *MPa* and 0.06 respectively. The numbers on the figures indicate the state of limit condition. While the case No. (1) is valid for the membrane yielding, the other two cases with No. (2) and (3) represent the ultimate state of elastic instability and elastic/plastic instability respectively. By comparison of diagrams, it is explicitly resulted that the yield stress modifies the reliability index more than the Young's Modulus.

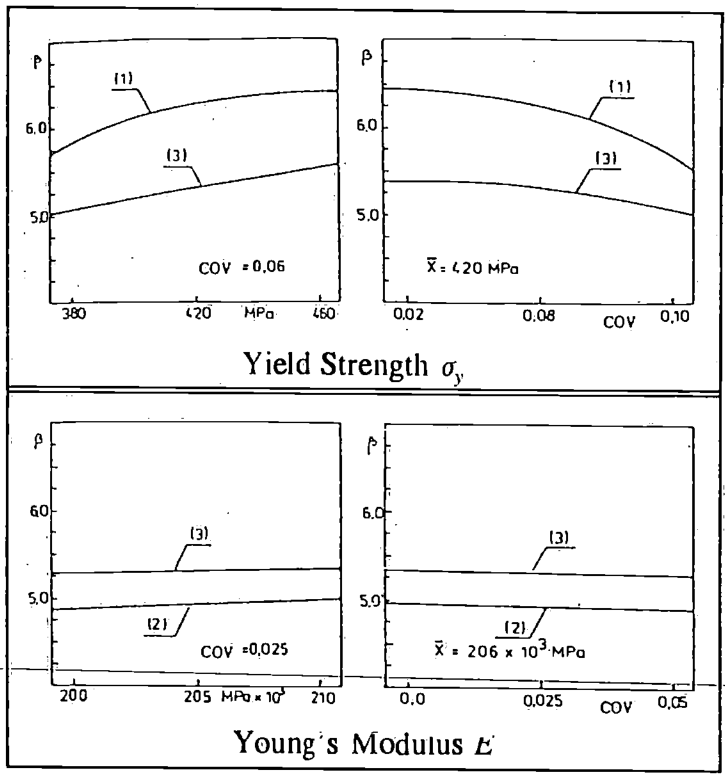
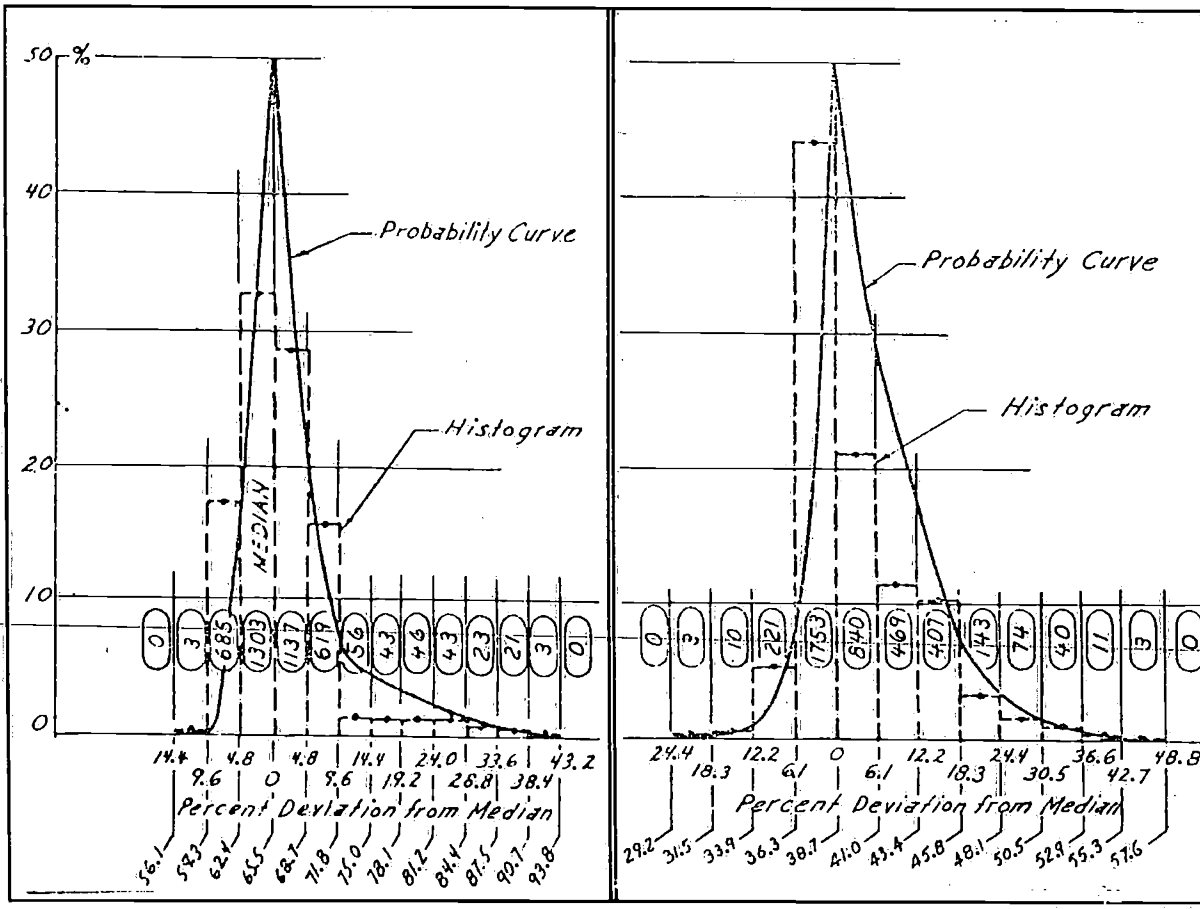


Fig. 3.1 The influence of material model on the the *Hasofer-Lind* reliability index of a spherical semisubmersible hull

3.5 Probabilistic modelling of mill steels in tension and compression

Julian (1957) has considered the uncertainty of test results for *ASTM A-7* structural mill steels (see reference [29]). According to his investigation, the statistical distribution of ultimate strength and yield strength pertains the normal distribution. The data has been collected from 3982 tests for the ultimate strength and 3974 tests for the yield strength representing 33000 tons of steels used on nine projects between 1938 and 1951. Figs. 3.1 (a) and (b) are given by Julian using the results of ultimate strength and the yield strength of structural steels. The ordinates of the histrograms shown as cells drawn in dash lines represent the relative frequency of the test results occuring with the cell widths indicated by the abscissas at their bases. To achieve a standard for comparison, the width of the cells in graphs has been arbitrary chosen as one probable deviation according to the normal distribution.

The plots are also representing the percentages for the deviation from medium. The figures in circles indicate the number of test results pertainting to each cell of the histograms. The solid lines represent the cumulative frequency or probability of occurance or $P_r(X \leq x)$. The ordinates should be interpreted by two separate occurance. For the left of the median, the percentage of total number of samples testing below the value indicated by the corresponding abscissa. For the right of the medium, the percentage of total number of samples testing above the value indicated by the corresponding abscissa.



(a) Ultimate strength (*k.s.i*) (b) Yield strength (*k.s.i.*)

Fig. 3.2 Histogram and probability curve of ultimate and yield strength of *ASTM A-7* structural steels (From Oliver G. Julian 1957)

The histogram and probability curve of ultimate strength and yield strength have been transformed into a histogram for the member compression strength by P.S. Tromans and J.W. Van de Graaf [51]. They have performed the risk assessment for a jacket structure located in the Gulf of Mexico (GOM). They assumed that the member compression strength will be reduced according to the Toma and Chen equation (see reference [10]). In that case, the reduced slenderness was equal to $\lambda = 0.55$ and it was assumed that it has a coefficient of variation of *10 %* according to the investigation of J.W. Cox. The reduced slenderness λ is defined in term of the yield strength and the slenderness parameteres of member as follows:

$$\lambda = \frac{1}{\pi}\sqrt{\frac{\sigma_y}{E}}\frac{KL}{r} \tag{3.1}$$

Since this uncertainty is dominated by the uncertainty of yield strength given equal to *8%*, thus it has to be reasonable that the distribution of compression strength to be similar to that of the yield strength of structural steels which has been discussed earlier. To obtain the histogram of compression member strength, it is necessary to calculate the reduced member compression strength. Toma and Chen [10] have compared different formulations for the member compression strength P_u. For fabricated steel tubular columns as used in offshore structures, they recommended two formulas for evalution of the compression strength in term of the yield strength. The first formula has been used to correlate the member yield strength P_u with the reduced slenderness parameter λ in the risk assessment of a platform prepared by P.S. Tromans and J.W. Van de Graaf [51] :

$$P_u = (1.0 - 0.091\lambda - 0.22\lambda^2)P_y \qquad 0<\lambda<1.41 \tag{3.2}$$

In the same report, they discritized the member strength histogram into six values with associated probabilities. The failure mode is found by a progressive collapse method developed by G. Stewart and J.W. Van de Graaf (see reference [48]) which corresponds with the collapse of six comprssion members. First they assume that the compression strength of the six members are uncorrelated. For six strength values and six critical members, the required collapse analysis has been performed $6^6 = 46656$ times with the push over analysis based on an efficient method described by Stewart and Van de Graaf (1991). However if the member strengths are statistically independent, the coefficient of variation of the collapse strength will reduce [51], because the simultaneous failing of the several compression members should be occured. The coefficient of variation for the fully uncorrelated strength of compression members is estimated to be *4%*. The ratio of environmental load to mean collapse strength (*1.39*) has been compared with the almost same deterministic value of *1.4* calculated by using the mean member compressive strength. In one recent report by M. Si Boon-Ing et al (see reference [3]), the coefficient of variation for the broadside collapse is evaluated equal to *3.5%*. The risk analysis has been performed for determinitic strength charcteristics and the results have been compared with the results for the fully correlated member strength *10%* and fully uncorrelated member strength *3.5%*.

3.6 Comments on the common distribution functions for strength parameters

The uncertainty of strength parameteres for steels are substantially originated due to the following reasons:

1- The unstable upper yield strength is considered rather than the stable lower yield strength which causes to a difference about *5-10%* in the results.

2- The strain rate for tests are considerably higher than the practical experience which makes approximately *10%* deviation from the mean test results.

3- At cross sections consisting of webs and flanges, the result indicates the yield strength of the web rather than the strength of thicker metal in the flanges. To account the effect of thicker flanges a difference about *5-10%* is proposed.

The values for three uncertainties are added to find the uncertainty of strength parameteres for mill steels. The combined uncertainty is evaluted equal to *20-30%* for the yield strength. By an mean value of *25%* the reduced strength for an average yield strength of *40* ksi is found to be *30* ksi. It should be noted that these values are given as examples and they may not be typical (Julian 1957).

Lindemann et al 1977 (see reference [34]) has proved a negative correlation of the yield stress σ_y with the plate thickness t based on the material tests by Augestad. They showed that the yield stress will decrease with increasing thickness. They poved that the normal distribution is not a good representation for the histograms and they proposed the double exponential distribution. They pointed out that the distribution tails in most of the cases are largely contributing to the computed of failure. This means that the truncation of the probability density functions should be considered very carefully because the probability of failure depends particularly on the shape of the tails of the probability density functions.

Mansour et al 1984 (see reference [38]) discussed the steel properties and their uncertainties on the base of more than *60000* samples of various steel types and test methods. The calculated weighted average of coefficient of variation for the yield strength was *0.089*. They also remarked that Galambos in reviewing much the same data suggests that any numerical analysis is probably worthless since the measurements are so varied. They conclude that for rolled shapes the mean yield stress be taken as $1.05\ \sigma_y$ in flanges and $1.10\ \sigma_y$ in webs with coefficient of variations of 0.10 and 0.11 respectively. The results for ultimate strength (*4200* samples) were more significant when compared, since this measurement is not so affected by strain rate. The weighted average of the coefficient of variation representing several different types of steel was *0.068*. In the same paper they presented the results for the Young's modulus (*300* samples) by the mean and the weighted average of the coefficient of variation as follows

$$\mu = 207.2 \times 10^3\ MPa \text{ and } COV = 0.031$$

Huther et al 1992 (see reference [25]) recalculated the same data (given in [38]) and they proved that the results for ultimate strength (*4200* samples) were more significant when compared, since the measurement is not so affected by strain rate. They also considered the effect of steel grade on the results of mean and coefficient of variation for three parameters (the yield strength, the ultimate strength and the Young's Modulus) and by this investigation, they proved that the mean of the yield strength strongly depends on steel quality. In the same paper, they summarized statistical results given by Stiansen et al (1980). The analysis of available data from Staugaitis for the tensile yield and ultimate strength in *ABS* grade *B* and *C* steel plates of *19* and *32 mm* have been calculated. The normal distribution was acceptable at any resonable significance level for all grades investigated except for the tensile strength of the *C* steel. The calculated coefficient of variation were *4.4* to *6.9%* against *6-8%* proposed by *ISSC*.

3.7 Proposed model using the new statistical data

In chapter one, the difficulties encounters with the choice of a distribution have been discussed briefly. We already established that the normal distribution is compateble with the data when a large number of data are available. Hutler et al (1992) have compared four common probabilistic models for the strength parameters. The probabilistic models were:

Normal law, Log Normal law, Gumbel and Weibull law.

They considered the statistical models of the yield strength, ultimate strength and Young's Modulus for the various steel grades. The probabilistic models were checked with the χ^2 and Kolmogorov-Smirnov tests. By fitting of data, they have concluded that the Normal and Gumbel laws do appear acceptable as they can allow negative values for very low probabilities.

The new statistical data of the marine steels showed that the coefficient of variation and bias of σ_y decreased when nominal characteristics increased, while the data for the tensile (or rupture) strength results that the standard deviation is particulary independent of the nominal characteristics. About *1000* test samples were considered in the recent investigation by Huther et al. The following are the results obtained from their calculations:

Table 3.2 Probabilistic models and basic variables for marine steels [25]

Type of Distribution	Yield Strength (N/mm^2)			Ultimate Strength(N/mm^2)		
Parameters	a	b	c	a	b	c
Normal	386	16	-	511	10	-
Log-normal	254	4.0[2]	0.015	468	3.74	0.058
Gumbel	378	15	-	507	9.4	-
Weibull	335	56	3.5	474	43	4.4

In Table 3.2, the parameteres *a*, *b* and *c* are defined by the type of distribution. For normal distribution *a* and *b* represent the mean μ and standard deviation σ of distribution type while for log-normal distribution *a*, *b* and *c* are the Origin parameter, the mean and the standard deviation of log-normal distribution, i.e. $\overline{x}_1$ and σ_{x1} as defined in section 1.7. For Gumbel distribution *a* and *b* represent the Mode parameter and the Scale parameter which are

[2] In original paper the parameter has been given by $b = 40\ N / mm^2$!

calculated in term of mean and standard deviation while for Weibull distribution *a*, *b* and *c* are thę Origin parameter, the Scale parameter and Exponent parameter respectively. The probability density function (pdf) and the cumulative distribution function (cdf) of *4* distributions are plotted in Figures 3.3 - 3.10 with the calculated parameteres obtained from Huther et al [25].

The concerned parameters for the evaluation of parameters were σ_y, σ_r, the strain at rupture, the steel thickness and the steel strength. According to their investigations, the log-normal distribution can be used for the probabilistic modelling of the yield strength of (MS) and (HTS) steels classified in table 3.1. It is extremely important to note that the data for the yield strength have been collected from the test of (MS) and (HTS) steels and due to the lackage of data for the (VHTS) steels, Hutler et al proposed that the (VHTS) steels can be better modelled with the normal distribution. The coefficient of variation for the yield strength of mild steels and higher tensile steels are calculated to be *0.100* and *0.080*. The bias factor are assumed to be *1.125* and *1.100* for the mild steels and the high tensile steels respectively.

For the tensile (or rupture) strength, they concluded that the hypothesis of the log normal distribution can be rejected, then it has been recommended to use the normal distribution instead of log normal distribution. The coefficient of variation for the tensile strength of the MS and HTS steels are evaluted to be *0.050* and *0.040*. The bias factor are also computed equal to *1.100* and *1.050* for the mild steels and the high tensile steels respectively. However for the very high strength steels (VHTS), there is not sufficient statistical data available at least in the present time and their suitable probalistic models are the subject of future investigation.

The Young's Modulus for three types of marine steels are assumed to coincide with the normal distribution. Mansour et al [38] have been given the mean and coeffficient of variation of *300* sample tests resulted from the tension and compression tests. They recalculated the weighted average of the mean value is 200.67×10^3 *MPa* and the weighted average of the coefficient of variation is *0.031*. In many cases the Young's Modulus may be treated as deterministic, without making large mistakes, except for slender columns.

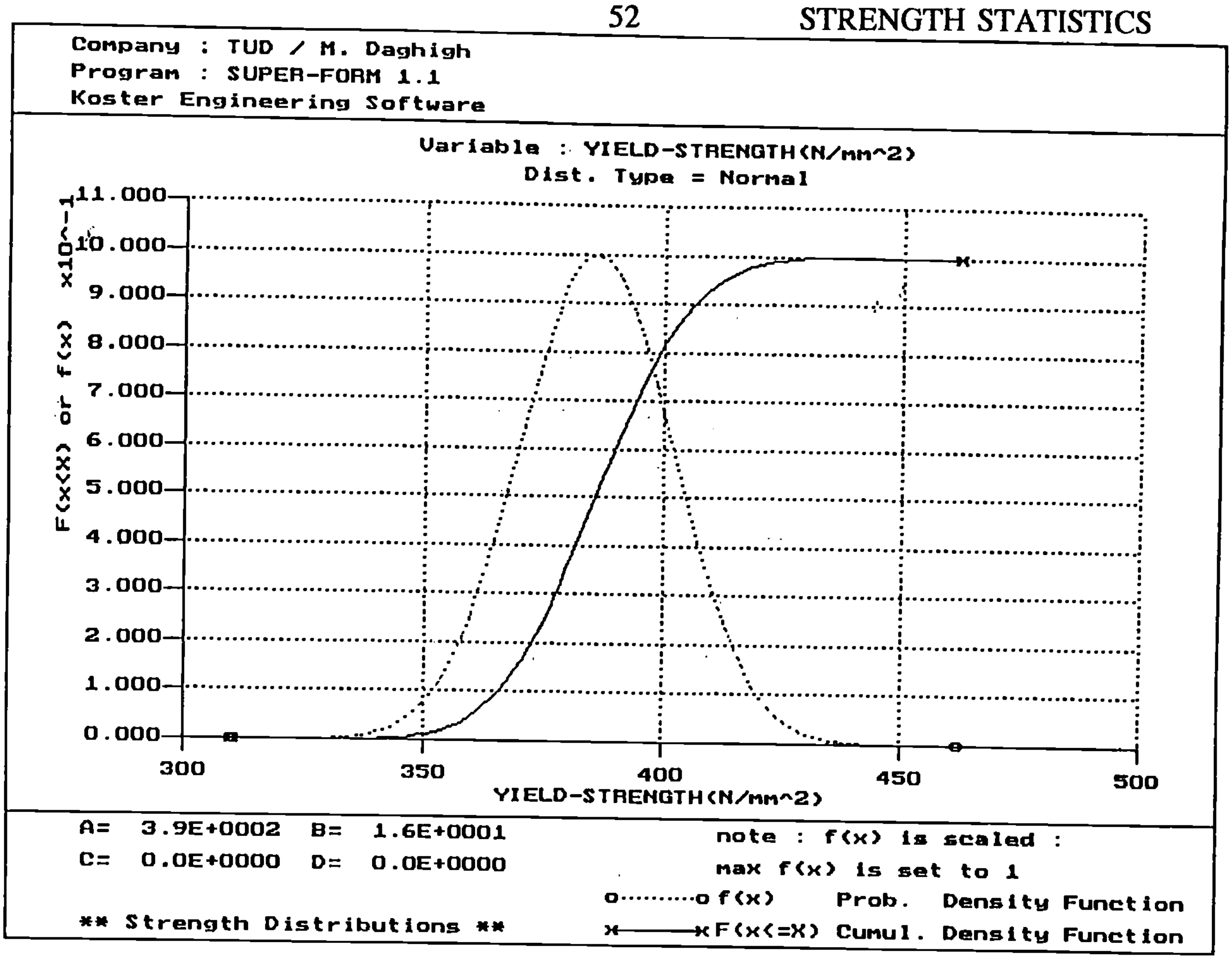


Figure 3.3 Implementation of normal distribution for Yield Strength

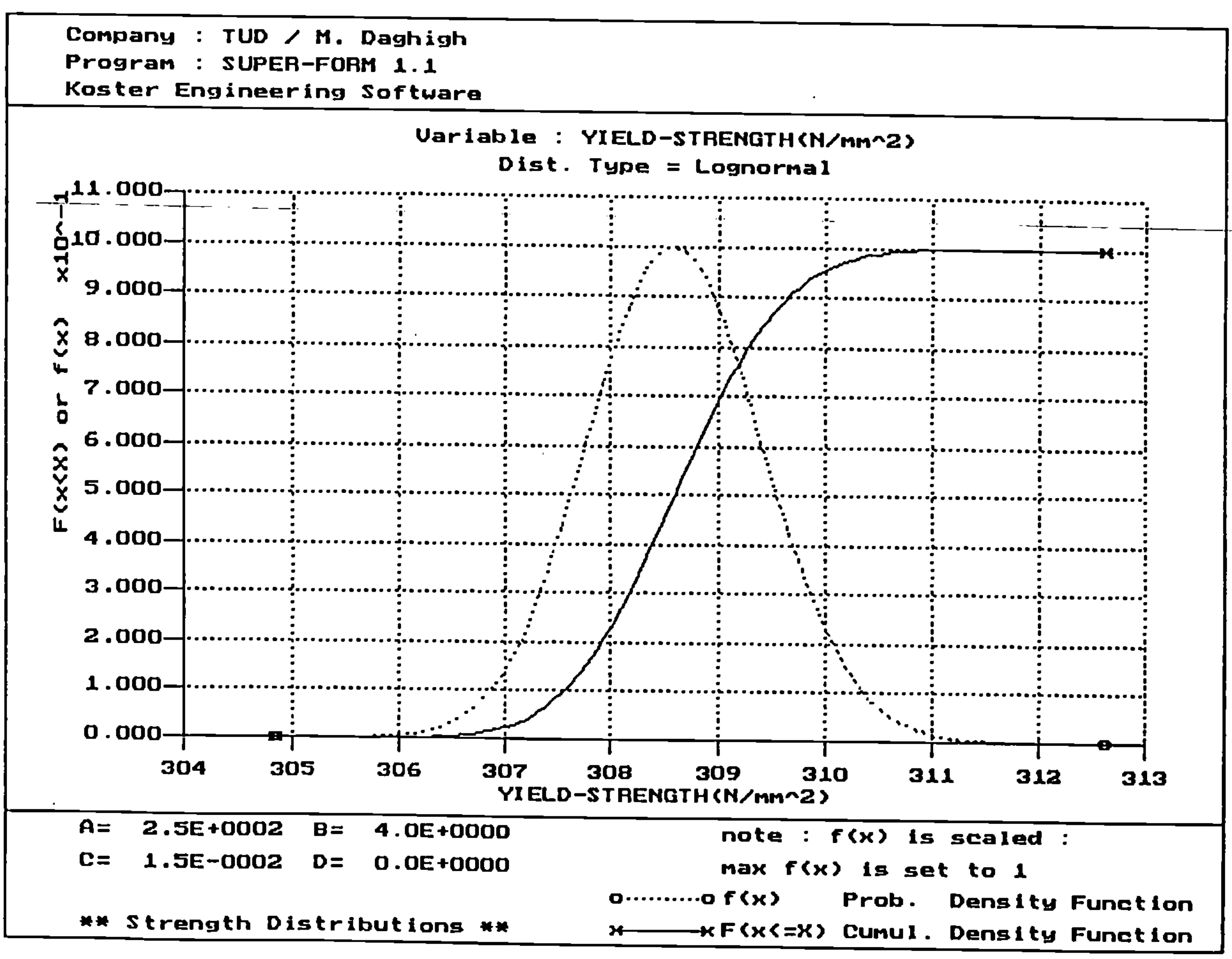


Figure 3.4 Implementation of log-normal distribution for Yield Strength

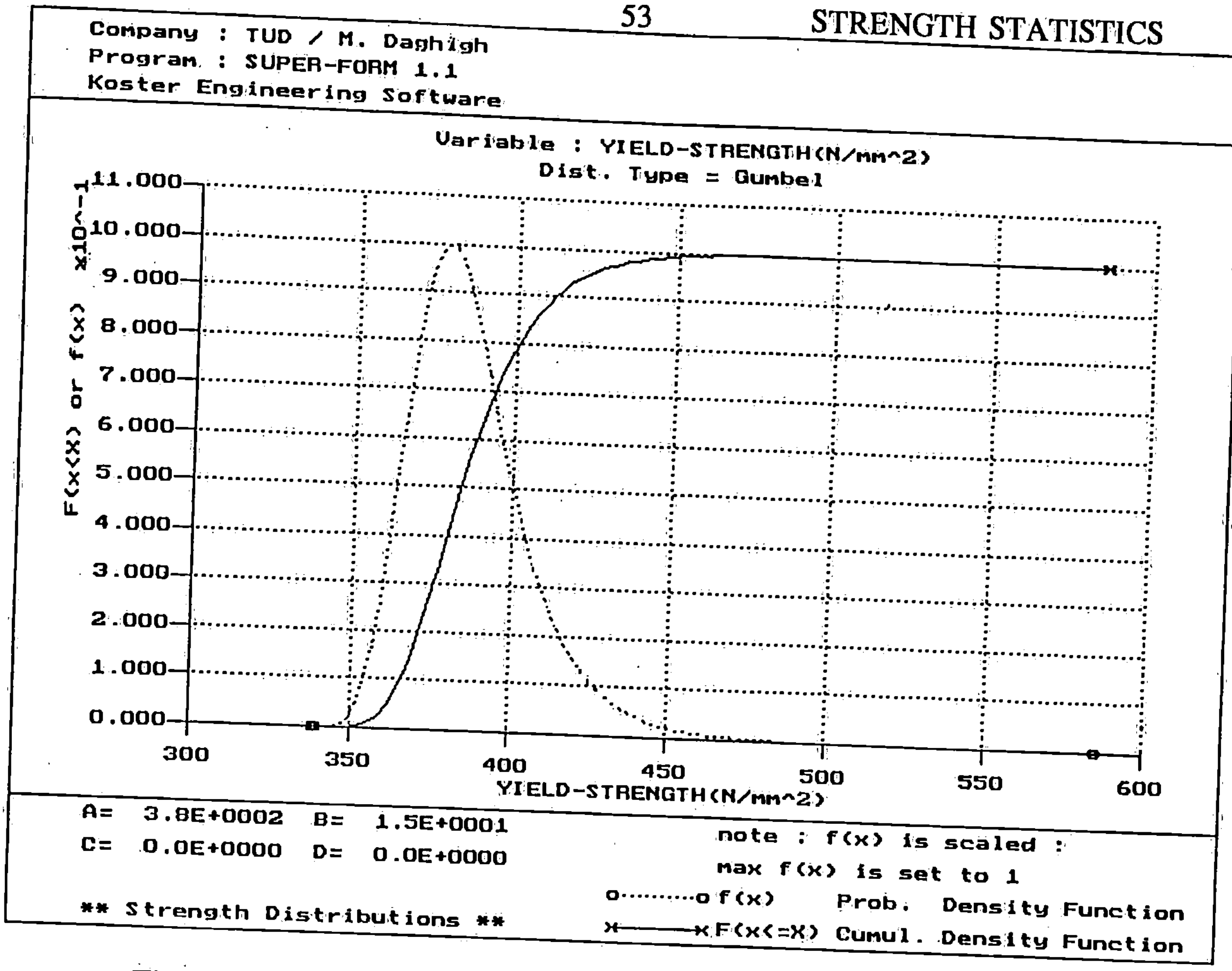


Figure 3.5 Implementation of Gumbel distribution for Yield Strength

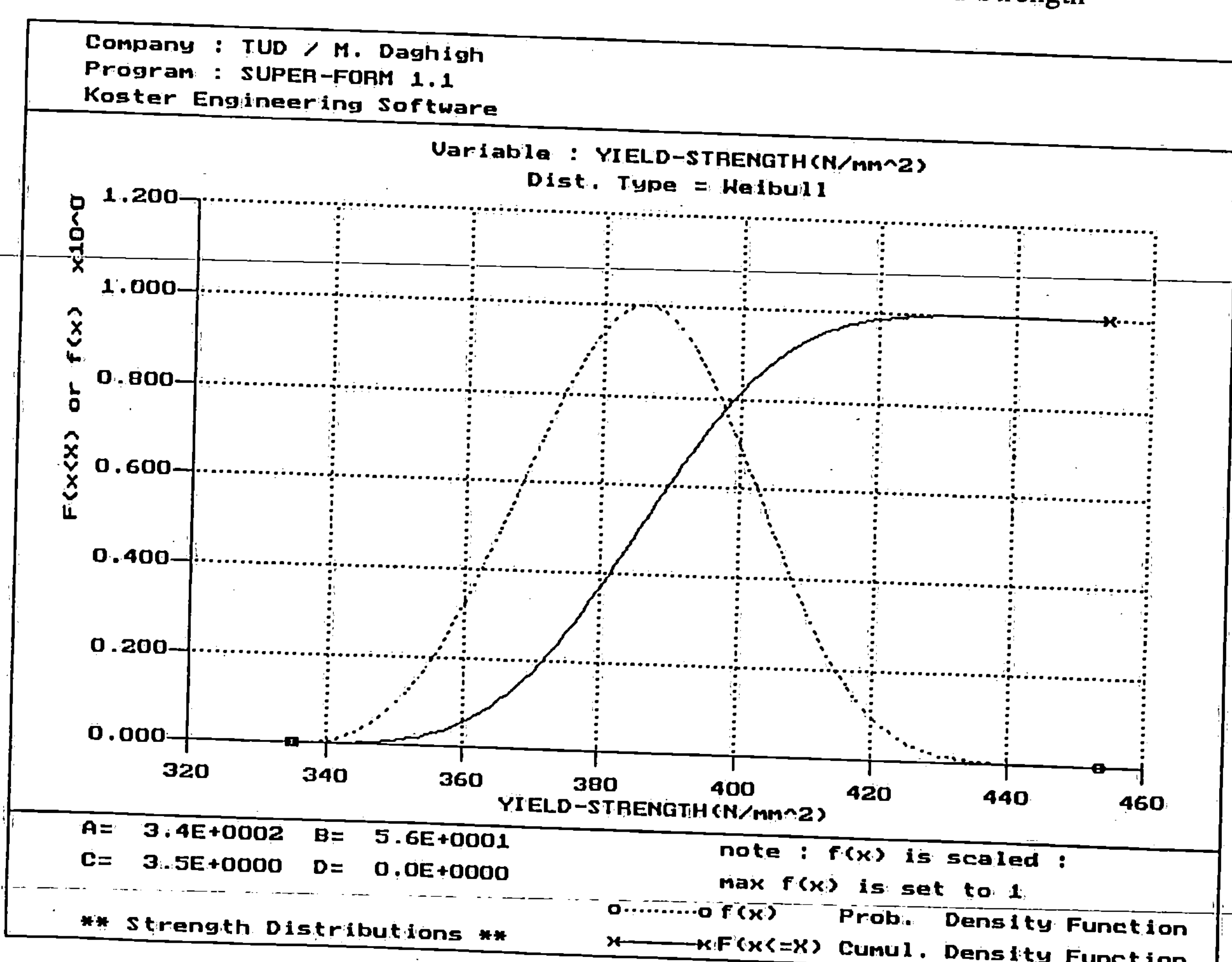


Figure 3.6 Implementation of Weibull distribution for Yield Strength

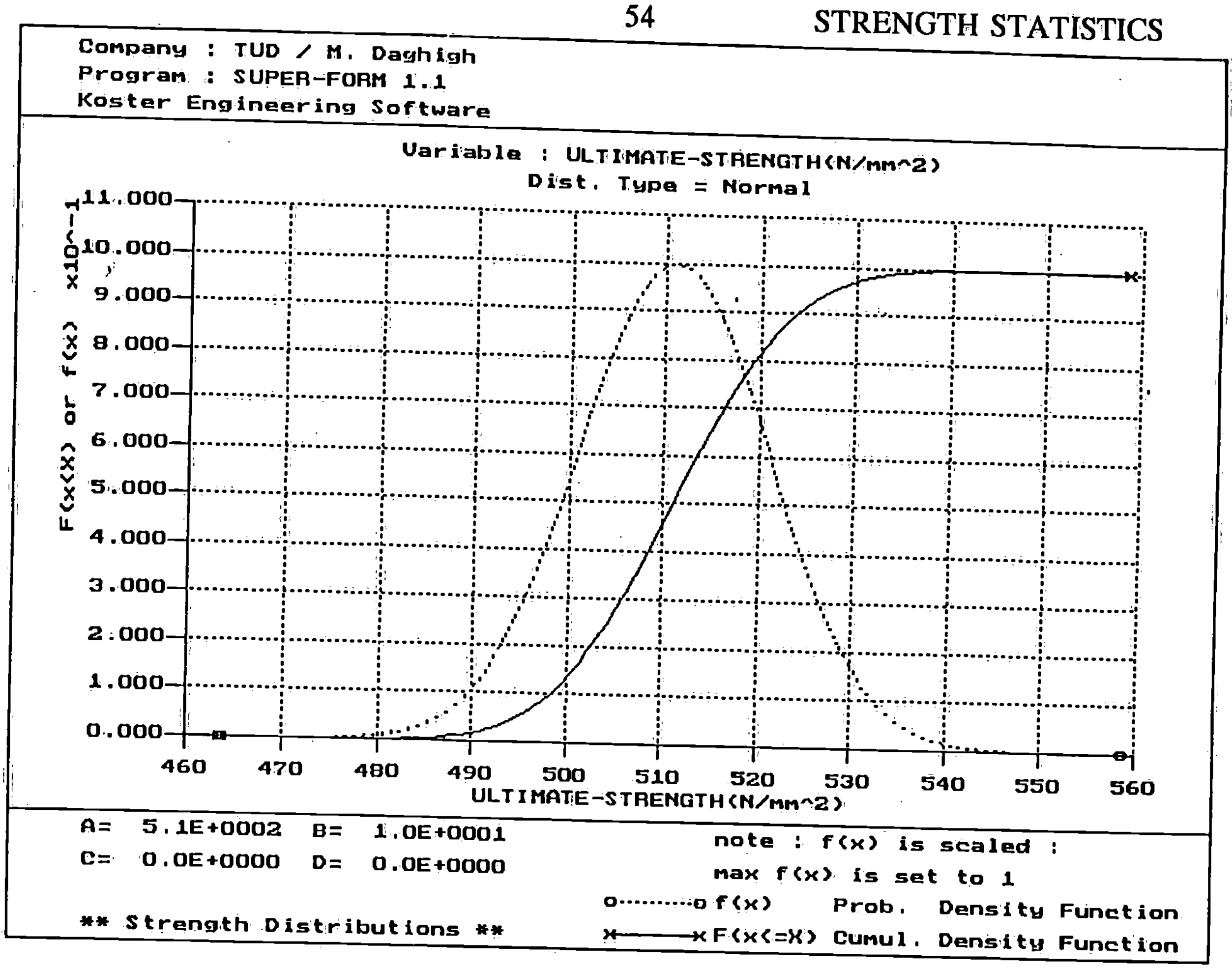


Figure 3.7 Implementation of normal distribution for Ultimate Strength

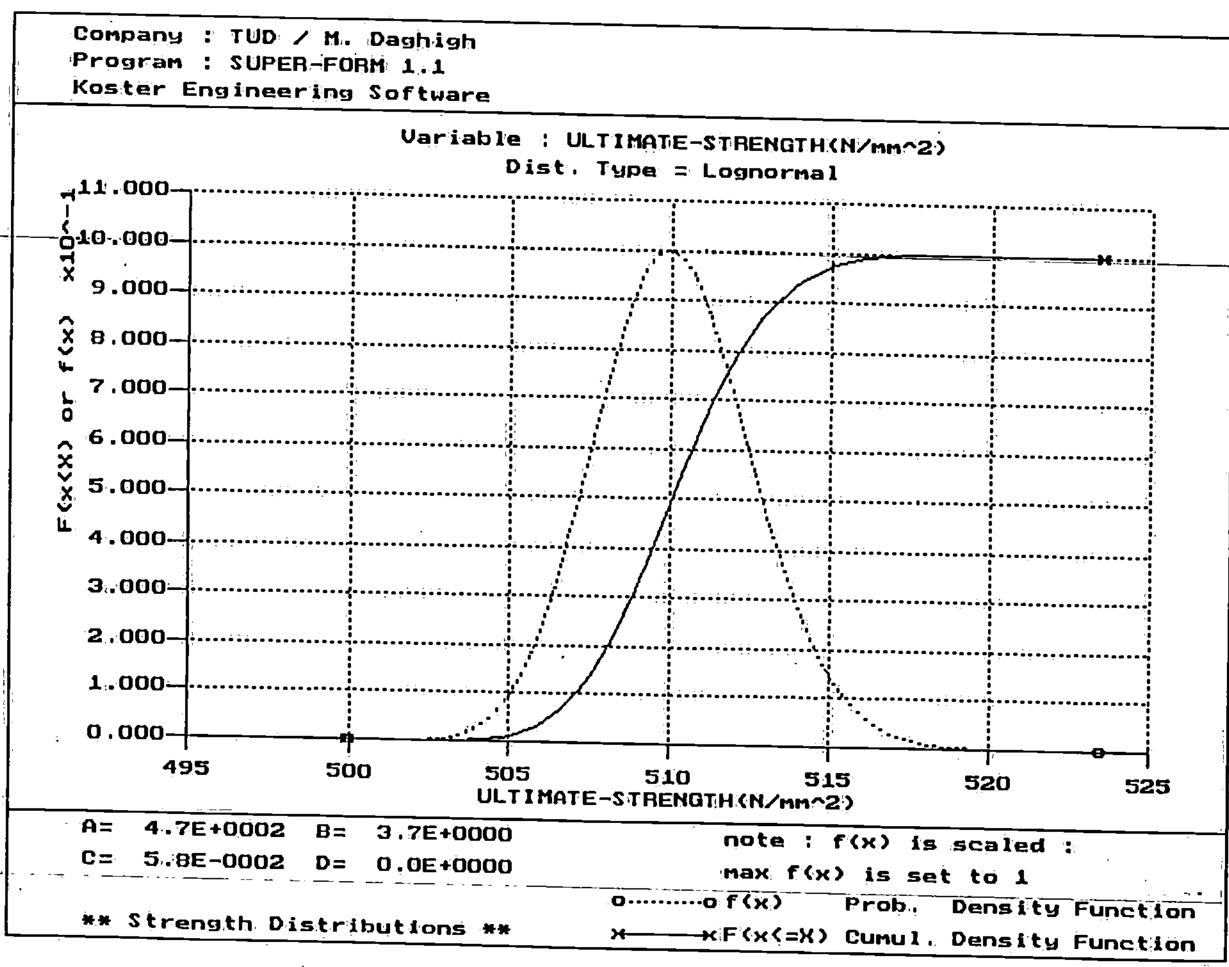


Figure 3.8 Implementation of log-normal distribution for Ultimate Strength

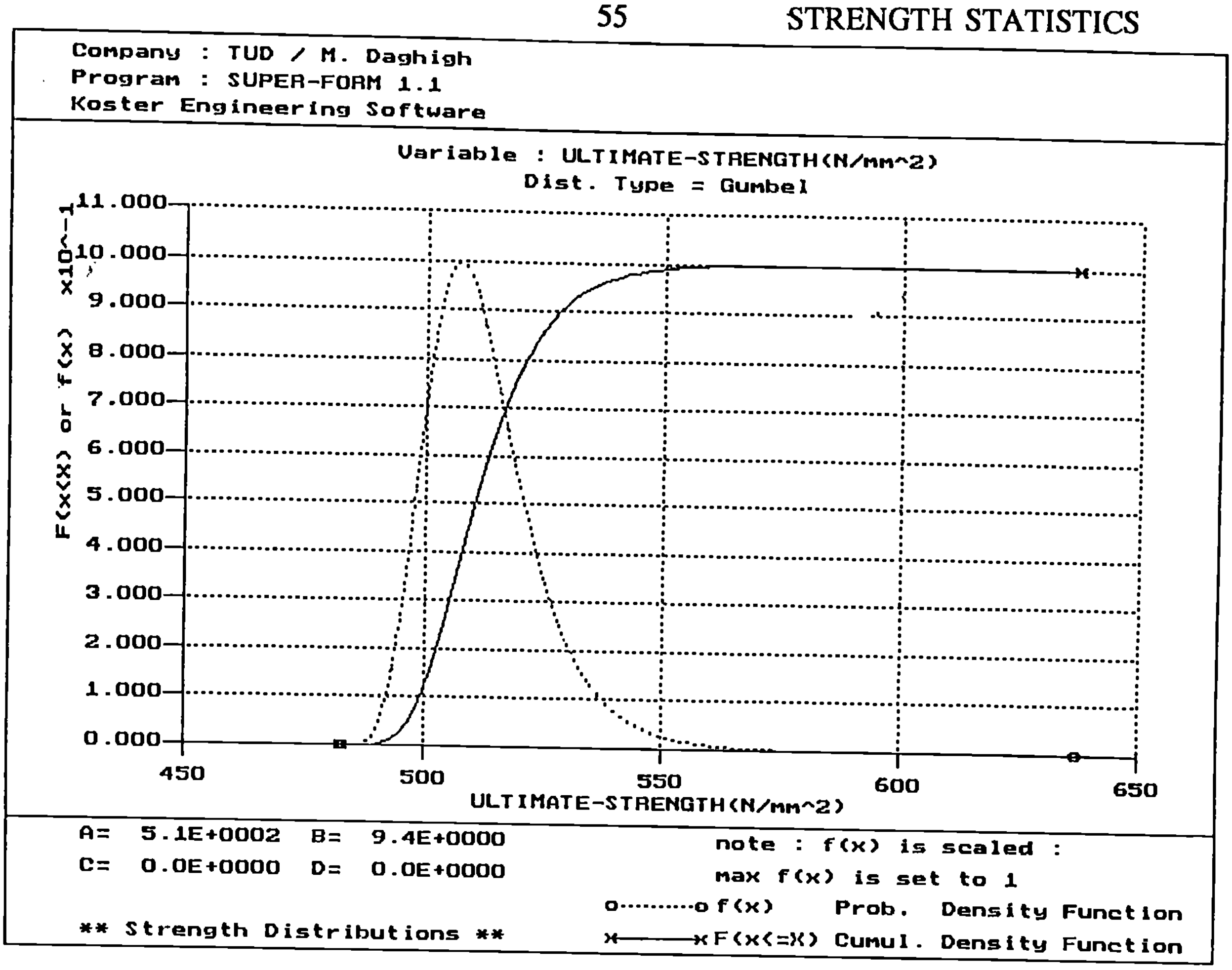


Figure 3.9 Implementation of Gumbel distribution for Ultimate Strength

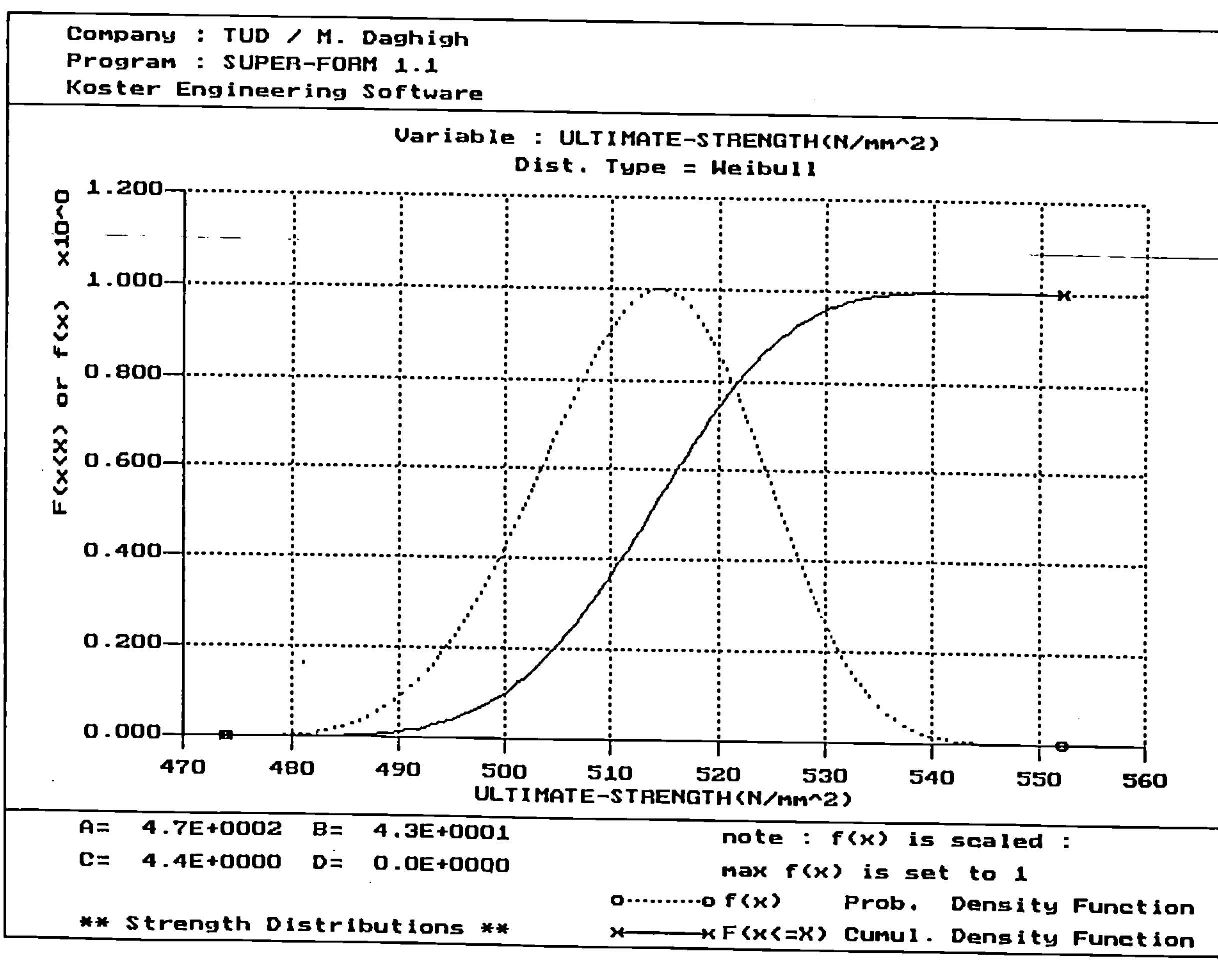


Figure 3.10 Implementation of Weibull distribution for Ultimate Strength

4- Limit states design - First order methods

4.1 Introduction

The accuracy of a stochastic analysis for an offshore structure depends on the reliable data for determinig the design loads, the fatigue loads and the operational aspects. J.R. Lloyd in the second offshore symposium has been related the probability of occurance of the design sea-state to the projected life of the structure (see reference [35]). He has mentioned the so-called '5 times rule' which states that the design condition should have a probability of occurance of 5 times the projected life of the structure. For example a structure with a 30 year projected life should be designed to withstand a sea-condition with a probability of occurance of once in 150 years.

The design loads are often specified as the '*most probable maximum* value or (*mpm*) value' being the load value coincides with the peak of the distribution of the extreme loads. Regardless of analysis method (either in frequency domain or in time domain), the common way for extreme dynamic analysis of the structure has been explained in all of the Classification Notes (take for example DnV Classification Notes, Joint Industry Jack-up Committee, etc.). There are three different dynamic amplification factors for determining of the inertial loadsets. The first dynamic amplification factor (DAF_1) is expressed by the ratio of the standard deviation of dynamic response σ_{Rd} to the standard deviation of static response σ_{Rs}. The dynamic amplification factor 2, (DAF_2), is defined by the ratio of the most probable maximum value of the dynamic response to the most probable maximum value of the static one. The third dynamic amplification factor , (DAF_3), is calulated in term of the *extreme most probable maximum* value or (*mpme*) value of the dynamic and static responses. At last by taking into account the third dynamic amplification factor, the inertial loadsets for base shear, overturning moment are evaluated by the procedure described in the classification notes.

On the other hand, the non-linearities in the restoring force characteristics are a major of the deviation of the distribution function from predictions based on the assumption of a narrow band spectrum (i.e. *Rayleigh* distribution). In principle, it is concluded that analytical methods which are aimed at obtaining statistical data directly from knowledge of the equation of motion and the statistics of the forces, while giving insight in the main factors determinig the behaviour and loads, are as yet inadequate for application to many practical cases.

4.2 Probabilistic basis of structural reliability

A physical model for reliability expression is an abstract, simplified, practical one for studying the mechanism of failures. This model implies an assumption that the probability density functions for stress and strength are known. Let the density function for the stress be denoted by f_s and for the strength by f_r. In principle, R is a variable representing the variations in resistance between nominally identical structures, whereas S represents the maximum load effects within a period of time, say successive T years. Let, for the time being, assume that the distribution of R and S are both independent of time. Then the reliability is defined as the probability that the strength will exceed the stress during any reference period of duration T years. The mathematical expression is determined by equation (4.1) or (4.2) depending to the model whether it is the true reliability model (indicated by the small letters) or the idealized reliability model (indicated by the large letters).

$$P_R = Pr(r > s) = Pr(r - s > 0) \tag{4.1}$$

$$P'_R = Pr(R > S) = Pr(R - S > 0) \tag{4.2}$$

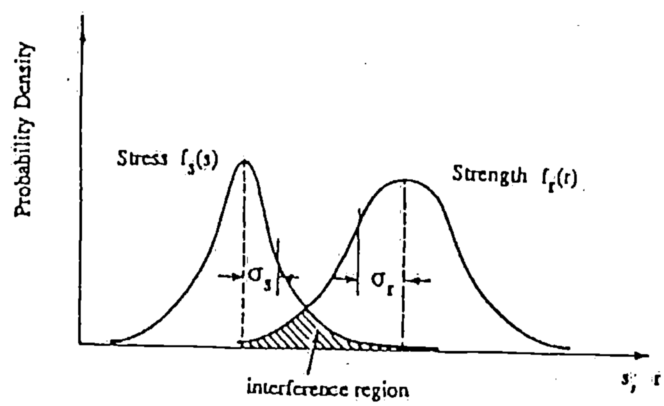


Fig. 4.1 Stress $f_s(s)$, and strength, $f_r(r)$, ditributions with interference region [34]

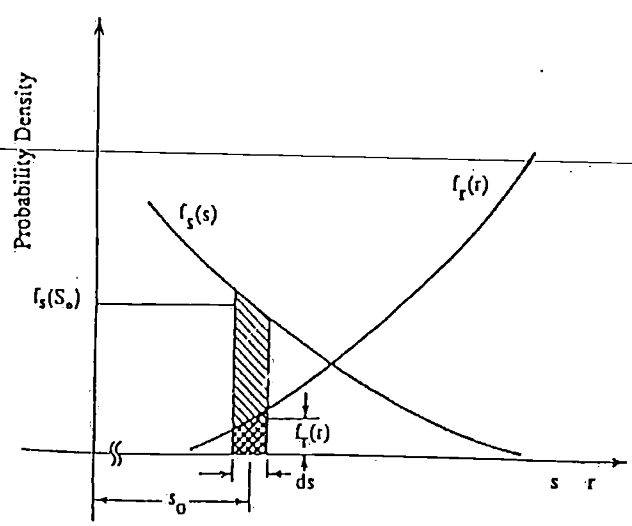


Fig. 4.2 Enlarged portion of interface region for deriving integral form of reliability

where $Pr(.)$ is the probability and S, R are the random variables for the stress s and the strength r respectively. For the simplicity we assume that there is no difference between the true reliability and idealized reliability and both are represented by P_R in the ongoing context. Thus the shaded region in Figure 4.2 can be used to find the probablity of strength r being greater than s_0 by integration of $f_r(r)$ in the interval $s_0 \leq r < \infty$:

$$Pr(r > s_0) = \int_{s_0}^{\infty} f_r(r)\, dr \tag{4.3}$$

On the other hand, the probability of the given stress s_0 is equal to the small area with width ds; that is,

$$Pr(s_0 - \frac{ds}{2} \leq s \leq s_0 + \frac{ds}{2}) = f_s(s_0)\, ds \tag{4.4}$$

The probability that the stress in the small interval ds and the strength r exceeding the stress can be calculated by the product of two probabilities calculated earlier as follows

$$f_s(s_0)\, ds . \int_{s_0}^{\infty} f_r(r)\, dr \tag{4.5}$$

The reliability is the probability that the strength r is greater than the stress s for all values of stress s (i.e. $r \geq s$) and is calculated by double integral of equation (4.5) as follows

$$P_R = \int_{-\infty}^{\infty} f_s(x)\, P_r(x)\, dx \tag{4.6}$$

where in equation (4.6) the parameter x represents the strength or stress variable and P_r indicates the chance of exceedance (or the probability of exceedance) for strength r. To find the probability of failure, equation (4.6) is used as follows

$$P_R = \int_{-\infty}^{\infty} f_s(s) [\int_{s}^{\infty} f_r(r)\, dr]\, ds \tag{4.7}$$

$$P_F = F = Pr(r \leq s) = 1 - \int_{-\infty}^{\infty} f_s(s) . [\int_{s}^{\infty} f_r(r)\, dr]\, ds \tag{4.8}$$

By substitution of new bounds for the integral (4.8) we will have

$$F = \int_{-\infty}^{\infty} f_s(s) [\int_{-\infty}^{s} f_r(r)\, dr]\, ds \tag{4.9}$$

Or

$$F = \int_{-\infty}^{\infty} f_s(x) . F_r(x)\, dx \tag{4.10}$$

where $F_r(.)$ denotes the cumulative distribution function of strength parameter.

The probability of failure F in equ. (4.10) is a measure of justification for the acceptance of a design. If this probability is less than a small value, ϵ, the "socially acceptable" probability of failure, the design is acceptable (Hasofer and Lind [21]). In second moment or *Level II* methods, the criterion $Pr\ (R \leq S) < \epsilon$ is replaced by a criterion involving the mean and standard deviation of two random variables i.e. the resistance (or strength) R and the stress (or load) S.

In some reliability engineering books (see for instance reference [11), the reliability expression is formulated with the probability that the stress will not exceed by the strength. The mathematical expression is determined by an equation similar to equation (4.1) or (4.2) and in this case the probability of $Pr\ (s \leq r)$ is evaluted for an interface region of $r_0 - dr/2 \leq r \leq r_0 + dr/2$. By the same way, the reliability expression is found as

$$P_R = \int_{-\infty}^{\infty} f_r(r) . [\int_{-\infty}^{r} f_s(s)\, ds]\, dr \tag{4.11}$$

Or

$$P_R = \int_{-\infty}^{\infty} f_r(x) . F_s(x)\, dx \tag{4.12}$$

where P_R is the probability that the strees will not exceed by the strength (or reliability) and $F_s(.)$ denotes the cumulative distribution function of stress parameter. The probability of failure is then found by a similar function in term of reliability expression given in equation (4.11) and we have

$$F = Pr(r_0 \leq s) = 1 - \int_{-\infty}^{\infty} f_r(r) [\int_{-\infty}^{r} f_s(s)\, ds]\, dr \tag{4.13}$$

By using new boundary conditions for the integral (4.13), the probability of failure is formulated as

$$F = \int_{-\infty}^{\infty} f_r(r) . [\int_{r}^{\infty} f_s(s)\, ds]\, dr \tag{4.14}$$

And finally by substitution of chance of exceedance for stress distribution in equation (4.14), the probability of failure is determined by equation (4.15):

$$F = \int_{-\infty}^{\infty} f_r(x).P_s(x).dx \tag{4.15}$$

where the parameter x represents the strength or stress variable and $P_S(.)$ indicates the chance of exceedance for stress S. Which formulation is to be chosen depends on the nature of the problem.

4.3 Generalization of the reliability expression

The probability of failure and the probability of survival (or the reliability) has been expressed in terms of the joint probability distribution of strength and stress. On the basis of limit state model, a so-called convolution integral explained with reference to R and S representing the nominal strength and stress. Usually R and S are functions of many random variables, i.e.

$$\bar{X}_R = R(X_1^{resis}, X_2^{resis}, X_3^{resis}, \ldots, X_m^{resis}) \tag{4.16}$$

$$\bar{X}_S = S(X_{m+1}^{stress}, X_{m+2}^{stress}, X_{m+3}^{stress}, \ldots, X_n^{stress}) \tag{4.17}$$

Thus the probability of failure for the general case is similar to the equations given in the last section . The generalized reliability expression will depend on the all basic variables which is represented by a multi-dimensional integral (instead of double-integral with basic variables of strength and stress) ; that is

$$P_F = \int_{\omega_f} \prod_{i=1}^{i=n} f_{x_i}(x_i) dx_i \tag{4.18}$$

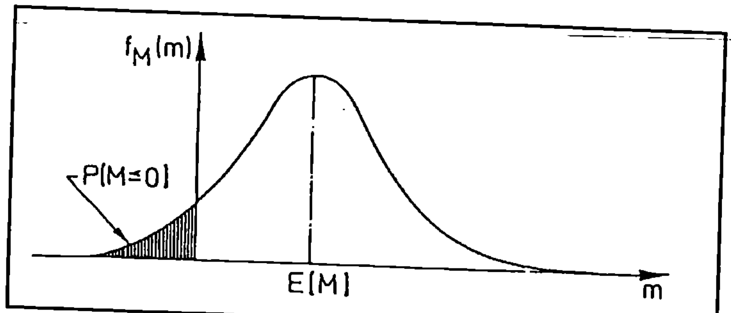


Figure 4.3 Illustration of probability of failure

where Π indicates the multiplication of probability distribution function of basic variables and ω_f denotes the failure region which is the corresponding margin represented by $M \leq 0$ in Figure 4.3. The variables x_i (or in random space X_i) in the formulation of probability of failure are called the basic variables and it was beforehand assumed that the probability density function of random variables are always known. In general the basic variables may have density functions which are consists of correlation parameteres and thus in the formulation of probability of failure it is usual to integrate the joint density function of x_1, x_2, ...,x_n in the coordinate of basic variables as follows

$$P_F = \int \int, \ldots, \int f_x(x_1, x_2, \ldots, x_n) dx_1 dx_2 \ldots dx_n \tag{4.19}$$

By this equation, the determination of the probability of failure comes down to calculating an n-fold integral, n being the number of basic variables. Integration of the equation (4.18) or (4.19) requires the probability knowledge for the resistance and load parameteres and also by application of modern computers, it is found that the integration of this diffrential equation consumes a lot of *CPU* time even for the simple engineering problems. However there are two convenient approximate methods for the evaluation of this integral which are called the *transformation methods* and the *simulation methods*. These methods have been classified as follows (see M.J. Marley [53]).

In the transformation method, the original integral is transformed to the integral in boundary of independent standard normally distributed variables. The first transformation is used for the transformation of correlated basic variables to uncorrelated variables that often requires the determination of *Jacobian* matrix. The second transformation is usually carried out by the *Rosenblatt* transformation that is used to find the independent standard normal varieties from the initial probability density functions (note that they are not necessarily normally distributed). By this formulation, the safety index can be evaluated by the so-called *Hohenbichler* algoritm.

In the simulation method, the integral is approximated by random *Monte Carlo* integration. For this purpose, a repeating process involves in the equation of integral which is simualted by a set of random variables generated in accordance with the corresponding probability distribution functions. It has been concluded that Monte Carlo method from finite samples in analysis are not 'exact' enough and the results of Monte Carlo simulation is questionable when we are dealing with low probability of failures (see Ang and Tang [54]).

The transformation methods are often splited to *FORM* and *SORM* methods. In the next section, we will discuss the linear transformation methods (or a so-called FORM procedure). It should be emphasised that the use of FORM/SORM or MCS (Monte Carlo Simulation) methods does require some caution, insight, and experience in reliability computation. The methods may be compared with the structural analysis methods whether in one problem the common sttiffness matrix theory or the finite element approach will lead to accurate results. Like structural engineering problems, it is useful to start with the classical and usual methods (i.e. the transformation methods in the relaibility analysis) for better understanding of the advanced methods (advanced MCS methods).

4.4 The mean-value first order second-moment (*MVFOSM*) method (Level II)

The closed form solutions for the integrals in equations (4.10) and (4.15) only exists for special cases [50]. Basically the rationale of a reliability method is a justification in term of a higher level. The level of a reliability method refers to the extent of information about the structural problem. The most common in the practical engineering is the traditional safety factor method that in fact can be estimated as *level I* or first moment reliability method. The second moment or *level II* method, often is used in which two different parameteres are contributed to describe the relevant random variables. Among the Level II reliability methods, in the MVFOSM method the linearization of the limit state function takes place at the mean value (*MV*). The method is formulated on the basis of linear approximation of failure surface and only the first-order (*FO*) terms are retained in Taylor series expansion. The mean-value first order second-moment (MVFOSM) method is based on the variation of mean and standard deviation of random variables and up to second moment (*SM*) of the random variables (means and covariances) are used in the reliabity measure. For application of *MVFOSM* method, the non-normal distributions for the stress and strength variables should be converted to the equivalent normal distributions. If S is a random variable representing the stress and R is a random variable representing the strength of the structure, then the *safety margin* is defined as

$$M = g(R,S) = R - S \qquad (4.20)$$

Failure occurs when the total applied stress S exceeds the ultimate capacity R, i.e., when the margin M is negative. Considering the safety margin in Figure 4.3, if R and S are normally distributed then the probability of failure P_f or F is bounded in the safety margin $M \leq 0$. The mean and the standard deviation of the margin ($\bar{M}$, σ_M) [or (μ_M, σ_M) ; ($E[M]$, σ_M)] are calculated by equations (4.21) and (4.22):

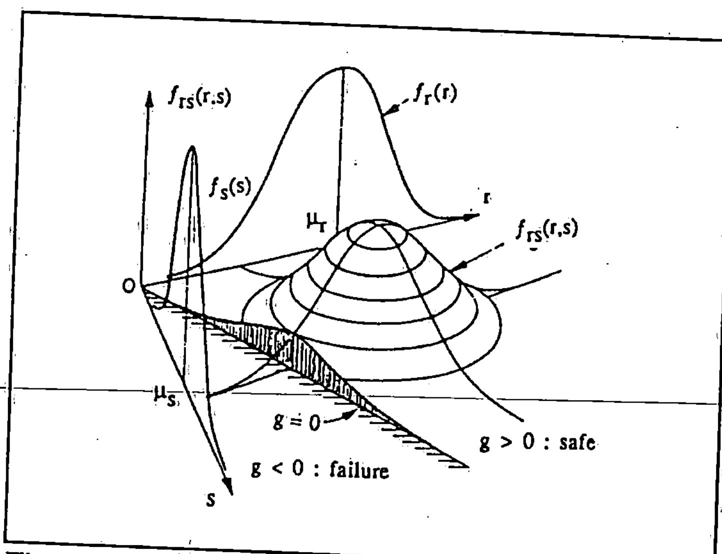


Figure 4.4 The R - S model, the marginal distributions, the joint distribution and Domain of the limit state surface $G(r,s) = 0$

$$\bar{M} = \bar{R} - \bar{S} \qquad (4.21)$$

$$\sigma_M^2 = \sigma_R^2 + \sigma_S^2 - 2\rho_{R,S}\,\sigma_R\,\sigma_S \qquad (4.22)$$

where $\bar{R}$, $\bar{S}$, σ_R and σ_S are the mean and the standard deviation of strength and stress variables and ρ is the *correlation coefficient* of two random variables. The Limit state surface $g(r, s) = 0$ is shown in Figure 4.4 together with the marginal and joint distribution of (R, *S*).

The correlation coefficient is defined by the ratio of *covariance* of *R* and *S* to the product of the standard deviation of strength and stress.

$$\rho_{R,S} = \frac{COV\{R,S\}}{\sigma_R \cdot \sigma_S} \tag{4.23}$$

where *COV{R, S}* is the covariance of *R* and *S* which is defined by

$$COV\{R,S\} = E[R-E(R)]E[S-E(S)] \tag{4.24}$$

Using equations (4.21) and (4.22), the standardized margin $\hat{M}$ which has a zero mean and a unit standard deviation can be written as (see reference [38]);

$$\hat{M} = \frac{M-\bar{M}}{\sigma_M} \tag{4.25}$$

The probability of failure is calculated in term of the safety margin M by equ. (4.26).

$$P_F = F = Pr\{\ M \leq 0\ \} \tag{4.26}$$

With substitution of $M = 0$ in equ. (4.25), the standardized margin becomes $\hat{M} = -\bar{M} / \sigma_M$ and thus the limit state for failure condition is determined in term of the standardized safety margin and we have

$$P_F = Pr\{\ \hat{M} \leq -\frac{\bar{M}}{\sigma_M}\ \} = \Phi_N(-\frac{\bar{M}}{\sigma_M}) \tag{4.27}$$

where Φ_N, which is often denoted by Φ in the literature, is the standard normal distribution function. The ratio of $\bar{M} / \sigma_M$ is the so-called the *safety index* β which is the inverse of the coefficient of variation of the safety margin ($C_M = \sigma_M / \bar{M}$). The function $\Phi(-\beta)$ must be looked up in Table 4.1.

Using equations (4.21) and (4.22), the safety index is formulated in term of four characteristics values i.e. the mean and covariance of the strength and stress variables.

$$\beta = \frac{\bar{M}}{\sigma_M} = \frac{\bar{R}-\bar{S}}{(\sigma_R^2 + \sigma_S^2 - 2\rho_{R,S}\sigma_R\sigma_S)^{1/2}} \tag{4.28}$$

In a comprehensive literature survay presented by Faulkner et al (reference [17]), a *central safety factor* θ is defined by the ratio of the mean value of the stress to the the mean value of the strength $\theta = \bar{R} / \bar{S}$. This model is used in many situations for the comparison of strength with respect to the stress. The θ-parameter is often defined as the safety factor and may be used by other convenient symbols as it has been used in literature (for example K parameter). Whether the component or system bahaves safely ($\theta > 1$), the safety factor must therefore be treated probabilistically, i.e. what is the *probability* that the event ($\theta > 1$) occurs. Introducing the θ parameter into the equation (4.28), an alternative definition of the safety index may be expressed by equation (4.29) written below:

$$\beta = \frac{\bar{M}}{\sigma_M} = \frac{\theta - 1}{(\theta^2\sigma_R^2 + \sigma_S^2 - 2\rho_{R,S}\theta\sigma_R\sigma_S)^{\frac{1}{2}}} \qquad (4.29)$$

Table 4.1 Cumulative distribution function for standard normal distribution

β	$\Phi(-\beta)$	β	$\Phi(-\beta)$	β	$\Phi(-\beta)$
0.0	5.0000×10^{-1}				
0.1	4.6018×10^{-1}	2.1	1.7864×10^{-2}	4.1	2.0658×10^{-5}
0.2	4.2075×10^{-1}	2.2	1.3903×10^{-2}	4.2	1.3346×10^{-5}
0.3	3.8209×10^{-1}	2.3	1.0724×10^{-2}	4.3	8.5399×10^{-6}
0.4	3.4458×10^{-1}	2.4	8.1975×10^{-3}	4.4	5.4125×10^{-6}
0.5	3.0854×10^{-1}	2.5	6.2097×10^{-3}	4.5	3.3977×10^{-6}
0.6	2.7426×10^{-1}	2.6	4.6612×10^{-3}	4.6	2.1125×10^{-6}
0.7	2.4197×10^{-1}	2.7	3.4670×10^{-3}	4.7	1.3008×10^{-6}
0.8	2.1186×10^{-1}	2.8	2.5551×10^{-3}	4.8	7.9330×10^{-7}
0.9	1.8407×10^{-1}	2.9	1.8658×10^{-3}	4.9	4.7920×10^{-7}
1.0	1.5866×10^{-1}	3.0	1.3500×10^{-3}	5.0	2.8665×10^{-7}
1.1	1.3567×10^{-1}	3.1	9.6760×10^{-4}	5.1	$\approx 1.7000 \times 10^{-7}$
1.2	1.1507×10^{-1}	3.2	6.8714×10^{-4}	5.2	$\approx 1.0000 \times 10^{-7}$
1.3	9.6800×10^{-2}	3.3	4.8342×10^{-4}	5.3	$\approx 5.8000 \times 10^{-8}$
1.4	8.0757×10^{-2}	3.4	3.3693×10^{-4}	5.4	$\approx 3.3000 \times 10^{-8}$
1.5	6.6807×10^{-2}	3.5	2.3263×10^{-4}	5.5	$\approx 1.9000 \times 10^{-8}$
1.6	5.4799×10^{-2}	3.6	1.5911×10^{-4}	5.6	$\approx 1.1000 \times 10^{-8}$
1.7	4.4565×10^{-2}	3.7	1.0780×10^{-4}	5.7	$\approx 6.0000 \times 10^{-9}$
1.8	3.5930×10^{-2}	3.8	7.2348×10^{-5}	5.8	$\approx 3.3000 \times 10^{-9}$
1.9	2.8717×10^{-2}	3.9	4.8097×10^{-5}	5.9	$\approx 1.8000 \times 10^{-9}$
2.0	2.2750×10^{-2}	4.0	3.1672×10^{-5}	6.0	9.8660×10^{-10}

4.5 The advanced first-order second moment method (AFOSM) method (Level-II)

The issue of *MVFOSM* method was the first step on the mathematical formulation of the Level two reliability methods. Historically the probabilistic calculation of failure by use of a computational model has been considered at the mean value of basic variables. Thus the *basic varible space* ω was divided into two sets called the *failure region* ω_f and the *safe region* ω_s. In reliability analysis, the seperation of the two regions ω_f and ω_s is called the *failure surface* and is defined by a *failure function*.

It should be emphasized that the failure function concept is deterministic and is usually determined by the traditional structural analysis methods. On the other hand, one important issue to bear in mind is that the same failure surface can be described by a number of different failure functions and thus the results obtained from the structural reliability assessments are more likely to be nominal rather than actual. Now the question is arising that what can we do in order to find the best approximation (or at least one better approximation) for the failure surface? In other words, because the margin is not unique for a given failure surface, what is the effect of different approximation in the evaluation of failure surface? This and many other questions have been discussed in the rest of this section.

Consider the definition of safety margin in term of basic variables (strength and stress) as has been given by equation (4.20). This is the one formulation for the evaluation of the reliability of a componant, or a collection of componants (or a *system*). Since in general we do not have enough information on the tail of the distributions of R and (or) S, thus we replace the criterion of "socially acceptable probability of failure, i.e., $P(M \leq 0) < \epsilon$" (where ϵ is a small value) by a criterion involving the mean and standard deviation of R and S (In genral all characteristics of probabilistic distribution of random variables R, S as well as skewness, kurtosis and etc.).

Using a linear safety margin M defined by equation (4.20) and assuming that the strength R and the stress S or in general all basic variables X_i, $i = 1,2,3,\ldots,n$, are normally distributed, the probability of failure is then given by

$$P_f = Pr\{\ M \leq 0\ \} = \Phi\left(\frac{0-\mu_M}{\sigma_M}\right) = \Phi(-\beta) \tag{4.30}$$

where μ_M and σ_M are the mean and standard deviation of linear safety margin of n basic variables and β is the *safety* (or *reliability) index*. The mean value first order second moment (MVFOSM) reliability approach is implemented by expanding the safety margin M in a Taylor series about the linearization point. This elementary reliability index can be given a simple geometrical interpration. First assume that the basic variables X_i, $i = 1,2,3,\ldots,n$ are trnasformed to the stochastically independent normally distributed variables Z_i while $i = 1,2,3,\ldots,n$. The standardized variables are calculated as follows

$$(U_1, U_2, \ldots, U_n) = \left(\frac{Z_1-\mu_{Z_1}}{\sigma_{Z_1}}, \frac{Z_2-\mu_{Z_2}}{\sigma_{Z_2}}, \ldots, \frac{Z_n-\mu_{Z_n}}{\sigma_{Z_n}}\right) \tag{4.31}$$

where U_1, U_2, ..., U_n are n independent standard normal variables. Inserting the corresponding expressions for Z_1, Z_2, ..., Z_n we have

$$(Z_1, Z_2, \ldots, Z_n) = (\mu_{Z_1} + U_1 . \sigma_{Z_1}\ ,\ \mu_{Z_2} + U_2 . \sigma_{Z_2}\ ,\ \ldots\ ,\ \mu_{Z_n} + U_n . \sigma_{Z_n}) \tag{4.32}$$

The principle of limit state design requires that the basic variables would be the functions of the resistance R and the stress S as given by equations (4.16) and (4.17). Thus the basic variables are mainly two types and the safety margin should be evaluated in term of both of the stength and the stress variables as follows.

$$M = a_0 + \bar{a}^T \bar{Z}_R - \bar{b}^T \bar{Z}_S = a_0 + (a_1 Z_1 + a_2 Z_2 + \ldots + a_m Z_m) - (b_1 Z_{m+1} + b_2 Z_{m+2} + \ldots + b_{n-m} Z_n) \quad (4.33)$$

where $\bar{a}^T = [a_1\ a_2\ \ldots\ a_m]$, $\bar{b}^T = [b_1\ b_2\ \ldots\ b_{n-m}]$ and $\bar{Z} = (Z_1, Z_2, \ldots, Z_n)$ are the constants and normally distributed basic variables respectively. The vectors $\bar{a}$ and $\bar{b}$ are assumed to be positive real values and the vectors $\bar{Z}_R$ and $\bar{Z}_S$ are represented the basic variables corresponding to the strength and stress parameters respectively.

Substitution of $M = 0$ in equation (4.33) leads to the failure surface equation, which is now indicates a straight line in the coordinates of R and S. By the vector symbols introduced in equation (4.33), the mean and the variance of safety margin are obtained as

$$E\{M\} = a_0 + \bar{a}^T . E\{\bar{Z}_R\} - \bar{b}^T . E\{\bar{Z}_S\} \quad (4.34)$$

$$Var\{M\} = \sum_{i=1}^{i=m} a_i^2 . \sigma_{Z_i}^2 + \sum_{i=m+1}^{i=n} b_i^2 . \sigma_{Z_i}^2 \quad (4.35)$$

where the mean value and the variance (the square of standard deviation) of the safety margin are represented by $E\{ M \}$ (is also called the expected value) and $Var\{ M \}$.

By substitution of reduced variables, i.e. equation (4.32), in the equation of safety margin (4.33), the following expression may be expressed for the safety margin

$$M = [a_0 + \bar{a}^T . E\{\bar{Z}_R\} - \bar{b}^T . E\{\bar{Z}_S\}] + [(a.\sigma_{Z_R})^T . \bar{U}_R - (b.\sigma_{Z_S})^T . \bar{U}_S] \quad (4.36)$$

To implement an invariable measure, both sides of equation (4.36) are divided by the standard deviation of the safety margin σ_M. By substitution of equation (4.34) into (4.36), the expresion for the normalized safety margin is obtained by

$$\tilde{M} = \frac{\mu_M}{\sigma_M} + \frac{(a.\sigma_{Z_R})^T}{\sigma_M} \bar{U}_R - \frac{(b.\sigma_{Z_S})^T}{\sigma_M} \bar{U}_S \quad (4.37)$$

In use of equation (4.37), the following notations are mainly substituted and a common equation is derived for the normalized safety margin as given in literature [55]. Consider the following expressions

$$\beta = \frac{\mu_M}{\sigma_M} \quad (4.38)$$

$$\bar{\alpha}_R = \cos(\theta_R) = -\frac{(a.\sigma_{Z_R})^T}{\sigma_M} \quad (4.39)$$

$$\bar{\alpha}_S = \cos(\theta_S) = \frac{(b.\sigma_{Z_S})^T}{\sigma_M} \tag{4.40}$$

The normalized safety margin is rewritten by

$$\tilde{M} = \beta - (\bar{\alpha}_R\ \bar{U}_R + \bar{\alpha}_S\ \bar{U}_S) = \beta - \bar{\alpha}^T.\bar{U} \tag{4.41}$$

where $\overline{\alpha}^T = [\overline{\alpha}_R \quad \overline{\alpha}_S]$, $\overline{U}^T = [\overline{U}_R \quad \overline{U}_S]$ are the vectors of sensivity measures and the uncorrelated basic variables respectively. In the (R,S) coordinate system space, $cos(\theta_R)$ and $cos(\theta_S)$ are the direction cosines of the unit outward normal vector for R and S respectively. Also $\overline{\alpha}$ is a unit outward normal vector to the limit state surface in U-space. Thus in dimension of n reduced basic variables U_i , the shortest distance from the origin to this linear failure surface (i.e. $\tilde{M} = 0$) is equal to β because $\overline{\alpha}^T.\alpha = 1$.

In a FORM approach (First Order Reliability Method), the linear safety margin M is obtained in terms of basic variables in a general expression as given by equation (4.41) and the elementary reliability index can be interpreted as the distance from the origin (mean value point) to the limit state surface in the standardized U-space. In MVFOSM method, the point closest to the origin in U coordinates, the most likely failure point, which is also known as the *design point*, is found by the zero safety margin or

$$\bar{U}^{\star} = \beta\,\bar{\alpha} \tag{4.42}$$

Clearly, the β-value obtained for a non-linear safety margin will depend on the choice of linearization point. The resulting β-value will also depend on the choice of failure function. (for example, compare the results for $M = R^2 - S^2$, $M = (R - S)^3$,... with the safety margin given in equation (4.20)). In other words, the choice of the mean value point for expanding Taylor series causes the invariance problem. Since the distance from the linearization points increases, thus some errors will be introduced at increasing distance by neglecting higher order terms in expanding Taylor series. In addition, even for certain linear forms of the safety margin, such as when the loads counteract one another, significant errors may be introduced in reliability problem. This shortcomings are avoided by using a procedure usually attributed to Hasofer and Lind [21].

Hasofer and Lind in 1974 have suggested a new definition for the safety index instead of that given by equation (4.38). They supposed that if the basic variables of a system $X = (X_1, X_2, X_3, \ldots, X_n)$ are uncorrelated, the first step is to transform the non-normal variables to the equivalent normal distributions. This can be down by both the *normal tail approximation* prodecure or by the *weighted fractile approximation* [11]. The normal tail approximation is often called the *Rosenblatt* transformation and has been originally introduced by M. Rosenblatt in 1952 [54]. The Rosenblatt transformation includes transformation of a set of multivariate random variables onto a set of independent random variables, each normally distributed. The weighted fractile approximation was introduced by Paloheimo and Hanus

(1974) [11] for an equivalent normal distribution in accordance to the probability of failure P_F at the design point. It can be interpreted as follows. The design point is equal to the weighted average of the mean value and the probability of failure P_F, or $(1 - P_F)$, fractile of each variable X_i, using sensivity factor α_i as the weighting factor. After the transformation was performed to the equivalent normal distributions, the second step would be the linear transformation of a normalized set of random variables $Z = (Z_1, Z_2, Z_3, \ldots, Z_n)$ to the standardized normally distributed random variables $U = (U_1, U_2, U_3, \ldots, U_n)$ by the following equation

$$U_i = \frac{Z_i - \mu_{Z_i}}{\sigma_{Z_i}} \quad ,i = 1,2,3,\ldots,n \tag{4.43}$$

When the basic variables $X = (X_1, X_2, X_3, \ldots, X_n)$ are correlated it is necessary as a first step to obtain a set of uncorrelated variables $Y = (Y_1, Y_2, Y_3, \ldots, Y_n)$ and then normalize this set of variables. It means that the non-normal stochastic quanteties $(Y_1, Y_2, Y_3, \ldots, Y_n)$ which are characterised by a distribution type and a number of parameters (eg. the mean and the standard deviation and etc.) can be always transformed to the normally distributed variables $(Z_1, Z_2, Z_3, \ldots, Z_n)$ by the Rosenblatt transformation or similar transformation methods [50]. By the linear transformation as given in equation (4.31), the standardized normally distributed variables $(U_1, U_2, U_3, \ldots, U_n)$ are calculated where in equation (4.31) it has been already assumed that the basic variables $(Z_1, Z_2, Z_3, \ldots, Z_n)$ are normally distributed (i.e. the set of basic variables which are used as the input data for linear transformation). Finally, the reliability index β has been defined by Hasofer and Lind in the u-coordinate system, namely as the smallest distance from the origin to the failure surface (see Figure 4.5).

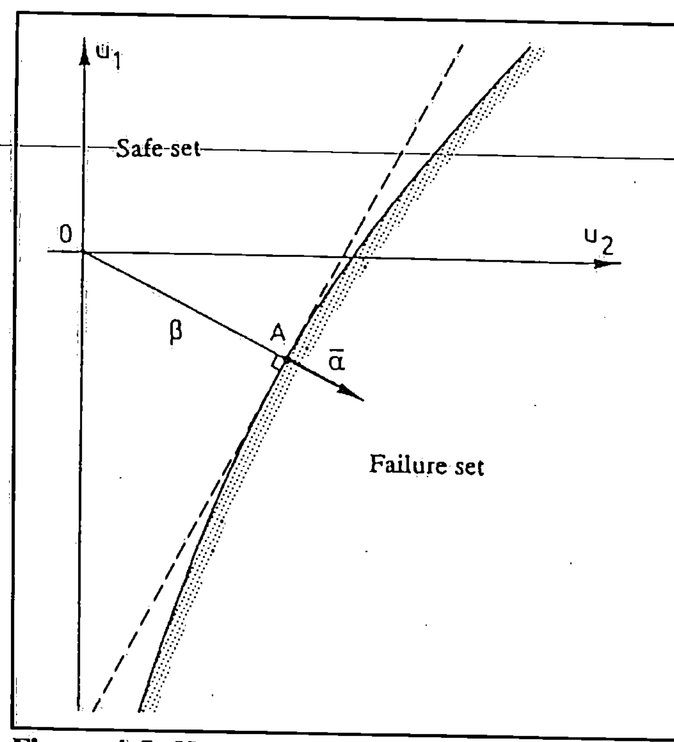


Figure 4.5 Hasofer and Lind reliability index

It should be emphasized that the reliability index in this method can be calculated independently of the distribution types of the basic variables, while the one-to-one relation expressed by equation (4.30), is invalid when the failure surface is non-linear. Experience shows that a good approximation to the probability of failure, P_f, can usually be obtained by using equation (4.30) and the Hasofer and Lind reliability index even for non-linear safety surfaces. In each case, the reliability index for a single structural element with respect to a single failure mode can be estimated by implementation of uncorrelated normalized variables in failure surface function and calculation of the distance, Δ, from the origin of the failure surface. Thus the second-moment reliability criterion will now be:

$$\Delta > \beta \tag{4.44}$$

In which, β is given by the ratio of mean and covariance of the strength and stress variables as indicated in equation (4.30). If the mean value of X (both R and S) are adopted, a so called mean value approximation is obtained. By this procedure, a more accurate approximation, however, is obtained by setting the design point as the point on the failure boundary where the probability density is greatest. Such as the MVFOSM method, the point which corresponds to the shortest distance from linear failure surface is reffered to as the *checking point* or *design point* or *β-point*. In the AFOSM method, the distance β (or the corresponding design point) can be determined by solving by iteration the $n+1$ equations in term of standardized variables

$$\hat{g}\,(U_1^*, U_2^*, \ldots, U_n^*) = \hat{g}\,(\beta\alpha_1, \beta\alpha_2, \ldots, \beta\alpha_n) = 0 \tag{4.45}$$

and

$$U_i^* = \alpha_i^* . \beta = -\frac{\left(\frac{\partial \hat{g}}{\partial U_i}\right)_{U_i^*}}{\left[\sum_i \left(\frac{\partial \hat{g}}{\partial U_i}\right)^2_{U_i^*}\right]^{\frac{1}{2}}} . \beta \qquad ; \; i = 1,2,3,\ldots,n \tag{4.46}$$

where α_i represents the relative effect of the corresponding random variable on the total variation and $\hat{g}$ indicates the safety margin in standardized coordinates. Equations (4.45) and (4.46) can be directly used for the estimation of safety index. For this purpose some additional parameters are often used to simplify the expressions of design point. For example the following notation is introduced in equation (4.46) to obtain a unit vector in term of the sensivity factors.

$$\alpha_i^* = -\frac{[\nabla \hat{g}(U_i^*)]_{U_i^*}}{|\nabla \hat{g}(U^*)|} = -\frac{\left(\frac{\partial \hat{g}}{\partial U_i}\right)_{U_i^*}}{\left[\sum_{i=1}^{i=n} \left(\frac{\partial \hat{g}}{\partial U_i}\right)^2_{U_i^*}\right]^{\frac{1}{2}}} = -\frac{\left(\frac{1}{\beta}\frac{\partial \hat{g}}{\partial \alpha_i}\right)_{(\beta\alpha_i^*)}}{\left[\sum_{i=1}^{i=n} \left(\frac{1}{\beta}\frac{\partial \hat{g}}{\partial \alpha_i}\right)^2_{(\beta\alpha_i^*)}\right]^{\frac{1}{2}}} \tag{4.47}$$

where $\nabla\hat{g}(U)$ is the gradient vector for the reduced safety margin $\hat{g}(U)$. The expression (4.45) expresses that the point A is on the failure surface as indicated on Figure 4.5. Thus by substitution of equation (4.47) in equation (4.46), the set of equations are simplified to the safety margin equation (4.45) plus the following n equations

$$U_i^{\star} = -\beta \frac{[\nabla \hat{g}(U_i^{\star})]_{U_i^{\star}}}{|\nabla \hat{g}(U^{\star})|} \quad ; \; i=1,2,3,...,n \tag{4.48}$$

As a practical approach, the set of $n+1$ nonlinear equations (in general case) are often solved by an alternative procedure. Let the uncorrelated normalized basic random variables in the design point may be found by

$$U_i^{\star} = \frac{Z_i^{\star} - \mu_{Z_i}}{\sigma_{Z_i}} = \beta^{ni} . \cos(\theta_{U_i}^{ni}) = \beta^{ni} . \alpha_i^{ni} \tag{4.49}$$

where β^{ni} , $cos(\theta_{Ui}^{ni})$ and α_i^{ni} are the reliability index, cosine direction and the sensivity factors in the n-th iteration (ni means the number of iterations). The so-called sensivity factors α_i^{ni} and reliability index β^{ni} are used to calculate the design point in the system coordinates of uncorrelated standardized basic variable U as follows

$$Z_i^{\star ni} = \mu_{Z_i} + \beta^{ni} . \cos(\theta_{U_i}^{ni}) . \sigma_{Z_i} \tag{4.50}$$

$$Z_i^{\star ni} = \mu_{Z_i} + \beta^{ni} . \alpha_i^{ni} . \sigma_{Z_i} \tag{4.51}$$

$$Z_i^{\star ni} = \mu_{Z_i} + U_i^{\star ni} . \sigma_{Z_i} \tag{4.52}$$

At last with substitution of equation (4.48) into equation (4.52), the set of n equations are obtained as

$$Z_i^{\star (ni+1)} = \mu_{Z_i^{\star ni}} - \beta^{ni} \frac{[\nabla \hat{g}(U_i^{\star ni})]_{U_i^{\star ni}}}{|\nabla \hat{g}(U^{\star ni})|} \sigma_{Z_i^{\star ni}} \tag{4.53}$$

The equation (4.53) together with equations (4.45) and (4.49) form a set of $n+1$ equations which can be solved for the parameteres β and Z_i (or U_i).

In general, for a linear limit state function, the Hasofer/Lind (or AFOSM) method will yield the same result for β as the MVFOSM method. For nonlinear state functions, this method yields a safety index β which is invariant to the formulation of the limit state function. The full procedure of the AFOSM method is summarized as follows:

1. Define an appropraite limit state function in term of basic random variables (X_i).
2. Implement a transformation for the basic variables in case of correlated variables (Y_i).

3. Implement the second transformation for the basic variables in case of non-normal distributions (Z_i).
4. Transform the approximaite limit state function (step 1) by use of new basic random variables.
3. Compute the mean and the standard deviation of the basic variables, i.e. μ_{Zi} and σ_{Zi} .
4. Compute the mean and the standard deviation of the transformed limit state function μ_M and σ_M by equations (4.34) and (4.35).
5. Compute the initial reliability index by the ratio of mean and standard deviation of M.
6. Define a set of standardized variables (U_i) in term of basic variables obtained in step 3 by using equation (4.31).
7. Identify the limit state function in the coordinates of reduced basic variables U_i.
8. Compute the sensivity factors α_i by equation (4.46) or the partial derivatives $\partial\hat{g}/\partial U_i$ from equation (4.45) .
9. With the initial variables at design point Z_i^{*ni} [in the first step $Z_i^{*0} = \mu(Z_i)$], the safety index β, the sensivity factors α_i or the partial derivatives from step 8, determine Z_i^{*ni+1} [in first step Z_i^{*1}] values from equation (4.53).
10. Check the calculated safety index β by equation (4.53) until a satisfactory result is calculated on the basis of successive iteration.
11. Repeat steps 8 to 10 until convergence has been attained.
12. Determine the probability of failure P_F by the results of last computation in step 10 and the cumulative standard normal distribution from equation (4.30).

Example 4.1 (Reliability analysis of a beam ; Excerpted from [11])

In this example we consider a beam which is only calculated for pure bending moment. The beam is designed in accordance with the *Euler-Cauchy* equilibrium condition. Let the critical stress is denoted by σ_c, the maximum moment and the inverse of the section modulus by M and Z^{-1} respectively. With application of these notations, the equilibrium condition can be stated by

$$\sigma_c = M Z^{-1} \tag{4.54}$$

Assume that all variables of σ_c, M and Z^{-1} are normally distributed. The mean and the standard deviation of basic variables are given in Table E4.1.1. Find the reliability index and the probability of failure P_F by the MVFOSM and AFOSM methods.

Table E4.1.1 Probabilistic characteristics of the design parameteres

	Mean	Standard deviation
Critical stress	$\mu_{\sigma c} = 700\ N/m^2$	$\sigma_{\sigma c} = 140\ N/m^2$
Maximum moment	$\mu_M = 18{,}750\ N.m$	$\sigma_M = 2812.5\ N.m$

Inverse of the section modulus	$\mu_Z^{-1} = 0.024\ m^{-3}$	$\sigma_Z^{-1} = 0.00144\ m^{-3}$

Solution

The limit state function is formulated as follows

$$g(\sigma_c, M, Z) = \sigma_c - M.Z^{-1} \tag{4.55}$$

The example has been solved by an iteration procedure in reference [11] with an initial reliability index $\beta = 2.5$, and the final solution was $\beta = 1.58$, with the probability of failure $P_F = 5.7026 \times 10^{-2}$. With the Super-Form program written by Koster Engineering, the relaibility index and the probability of failure have been calculated by the same procedure discussed earlier. The results are demonstrated in Table E4.1.2 which are comparable with the solutions of Dai and Wang [11] (pp. 84-85).

Table E4.1.2 The reliability indices and the probabilities of failure with FORM methods

Used method	β	P_F
MVFOSM	1.585	5.651×10^{-2}
AFOSM	1.580	5.707×10^{-2}

The AFOSM method gives a better estimate on the reliability approach. Using the characteristics of the iteration procedure for this method, a relatively accurate solution is obtained by 2 itertions. The final result for the design point and the square of the sensivity factors are indicated in Table E4.1.3.

Table E4.1.3 Design point and square of the sensivity factors for design point

	Design point value	α_i^2
σ_c	$5.049 \times 10^2\ N/m^2$	0.78
M	$2.067 \times 10^4\ N.m$	0.19
Z	$2.443 \times 10^{-2}\ m^{-3}$	0.04

By comparison of the sensivity factors for different designs, it can be concluded that the major importance for probabilistic design are attributed for the critical stress σ_c and the maximum moment M respectively. The sensivity of the design with respect to the section modulus can be ignored in case of complicated limit state design.

Example 4.2 (Reliability analysis of a fillet weld in compression and bending ; Excerpted from [11])

The reliability of a joint with fillet weld is considered in the combined effect of the bending moment and the axial loading. The structure is a rectification column and the fillet weld attachs the skirt of tank to the rectification column. The allowance stress of fillet S_a is a characteristic value which is assumed to have the following values expressed in *MPa*.

60 - 62 - 64 - 66 - 68 - 70 - 72 - 74 - 76 - 78 - 80

The throat area of the fillet weld is $A = 267\ cm^2$ and its section modulus is $Z = 13{,}800\ cm^3$. The column is operating under combined effect of axial force and flextural bending as shown in Figure E4.2.1.

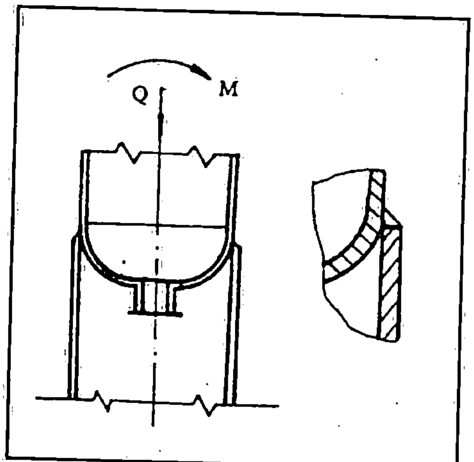


Fig. E4.2.1 A fillet weld attaching skirt to column

The axial load Q includes the total weight of the column and reacting medium under operation and is assumed to consistent with the normal distribution. The maximum bending moment M is obtained from the environmental effects and it is assumed to be log-noramlly distributed.The geometrical and physical characteristics of the joint together with the external load effects are given in Table E4.2.1.

Table E4.2.1 Probabilistic characteristics of the axial load and bending moment

	Mean	Standard deviation
Axial load Q	$\mu_Q = 3.3 \times 10^5\ N$	$\sigma_Q = 3.3 \times 10^4\ N$
Bending moment M	$\mu_M = 5.91 \times 10^5\ N.m$	$\sigma_M = 5.91 \times 10^4\ N.m$

Solution

A limit state function is defined as

$$g(X) = S_a - \left(\frac{Q}{A} + \frac{M}{Z}\right) \tag{4.56}$$

It has been stated that the bending moment obeys from a log-normal distribution. In order to implement the characteristics parameteres of both distributions in the Super - Form program, it is necessary to find the mean and the standard deviation of log-normal distribution. In section 1.7, it has been explained that the mean and the standard deviation of random variable x, (hier bending moment M), is related to the probabilistic characteristics of random variable x_1, (hier M_1) which obeys from log-normal distribution. The coefficient of variation of random variable M is defined by the ratio of its standard deviation to its mean [equation (4.57)]

$$C_M = \frac{\sigma_M}{\mu_M} \tag{4.57}$$

Thus the mean and the standard deviation of log-normal distribution are obtained from

$$\sigma_{M_1} = \sqrt{Ln(1 + C_M^2)} = 0.09975 \tag{4.58}$$

$$\mu_{M_1} = Ln\,\mu_M - \frac{1}{2}\sigma_{M_1}^{\;2} = Ln(5.91 \times 10^5) - \frac{1}{2}(0.09975)^2 = 13.2846 \tag{4.59}$$

With substitution of the characteristics of the fillet area and the section modulus, the limit state function can be rewritten as

$$g(X) = S_a - 3.74532 \times 10^{-5} Q - 7.2464 \times 10^{-5} M = 0 \tag{4.60}$$

The characteristics of both distributions of Q and M are used in Super-Form program. The results of calculation together with the diagram of the probability of failure for MVFOSM and AFOSM methods are shown in Figures (E4.2.2) - (E4.2.5). In Super - Form program for the case of MVFOSM method, the mean value and the standard deviation of the limit state function is found for each critical stress S_a. Moreover the reliability index and the probability of failure are calculated in the analysis way. For the case of AFOSM method, in addition to the above mentioned parameteres, the number of iterations and the value of reliability function in the calculated design point and finally the difference between the last two iterations for probability of failure are calculated. In addition the sensivity factors for each variable are established in both methods while for the AFOSM method, the sensivity factors indicate the solution for the last iteration.

For our example it has been found that the AFOSM method needs to evaluate the safety margin for three times. Similar to the Example 4.1, the results of calculation for relaibility index (or the probability of failure) are conservative by using the AFOSM method than the MVFOSM method. As much as the allowance stress for the fillet weld increases, the probability of failure comes down and thus by estimating a criteria for the probability of failure (or for the reliability index), the appropraite allowance stress of the fillet weld can be obtained. AISC specifications for a fillet weld can be used here. Depending on the particular load combination used, the criteria for the target reliability index will be varied for the given fillet weld. With the notations of D for dead load, L for live load and W for wind load (or alternatively earthquake load) it can be assumed that the following ratios are valid in this example as

$L / D = 2$ and $W / D = 5$.

Thus the target reliability should be greater or equal to $\beta = 3.3$ (see Table C.7.7, pp. 164 in reference [56]). Searching the results of Super-Form program it can be easily found that for the allowance stress of $S_a = 72$ *MPa* the desired reliability index is revealed. Therefore the design point is obtained at the point that the probability of failure is smaller than $P_F = 4.856 \times 10^{-4}$ where the difference between the last two iterations for probability of failure is equal to $\Delta P_F = 7.024 \times 10^{-7}$. Other characteristics of the desired design are summarized in Table E4.2.2.

Table E4.2.2 Design point and square of the sensivity factors for design point

	Design point value	α_i^2
Q	3.525×10^5 *N*	0.04
M	8.114×10^5 *N.m*	0.96

It can be concluded that the marginal effect of the bending moment M on the reliability design has a greater influence than from the axial load Q. However the results for the allowance stress of $S_a = 60$ *MPa* are identical with the results of Dai and Wang [11] when the solution is performed untill 3 iterations.

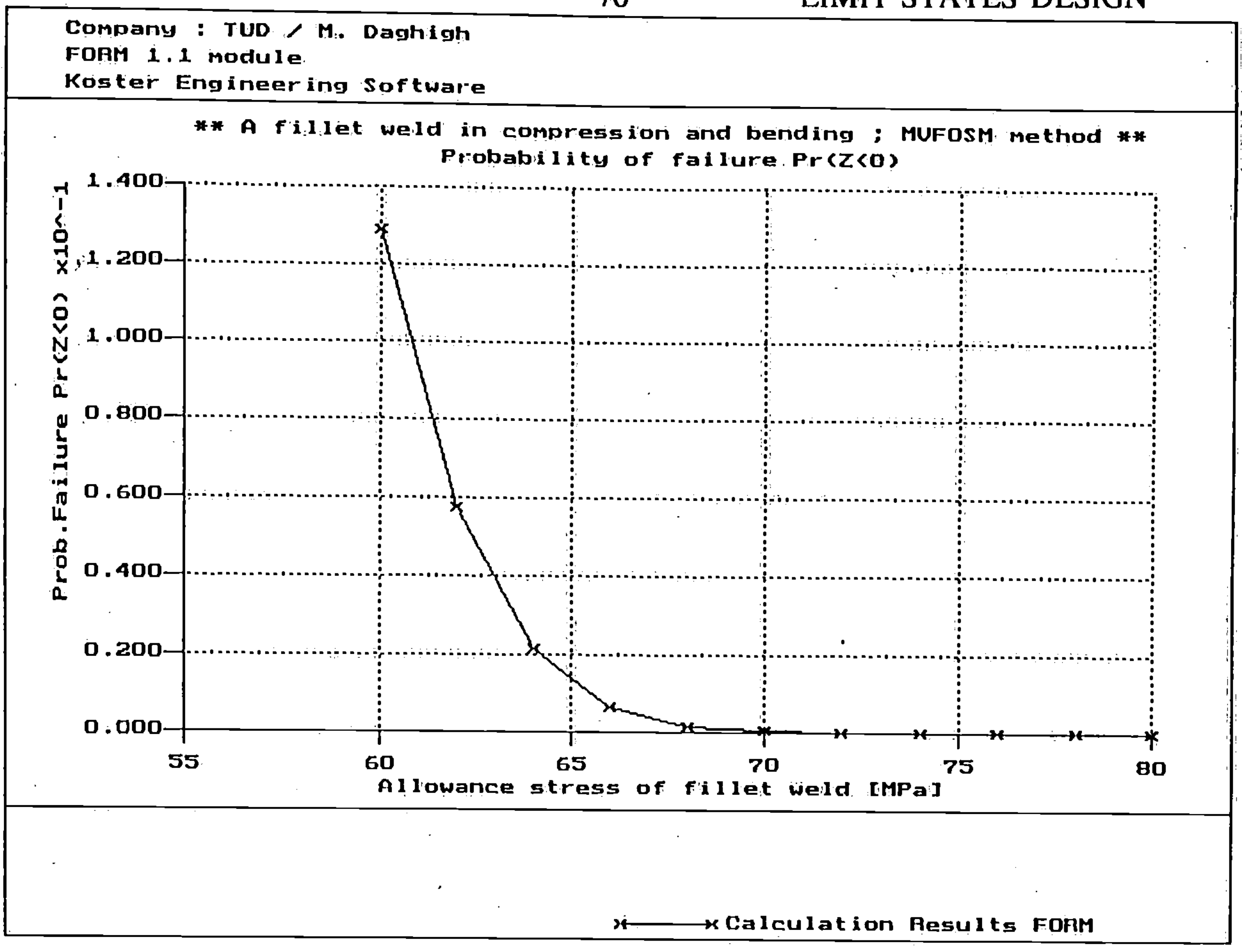


Figure E4.2.2 Probability of failure P_F for fillet weld conforming to MVFOSM

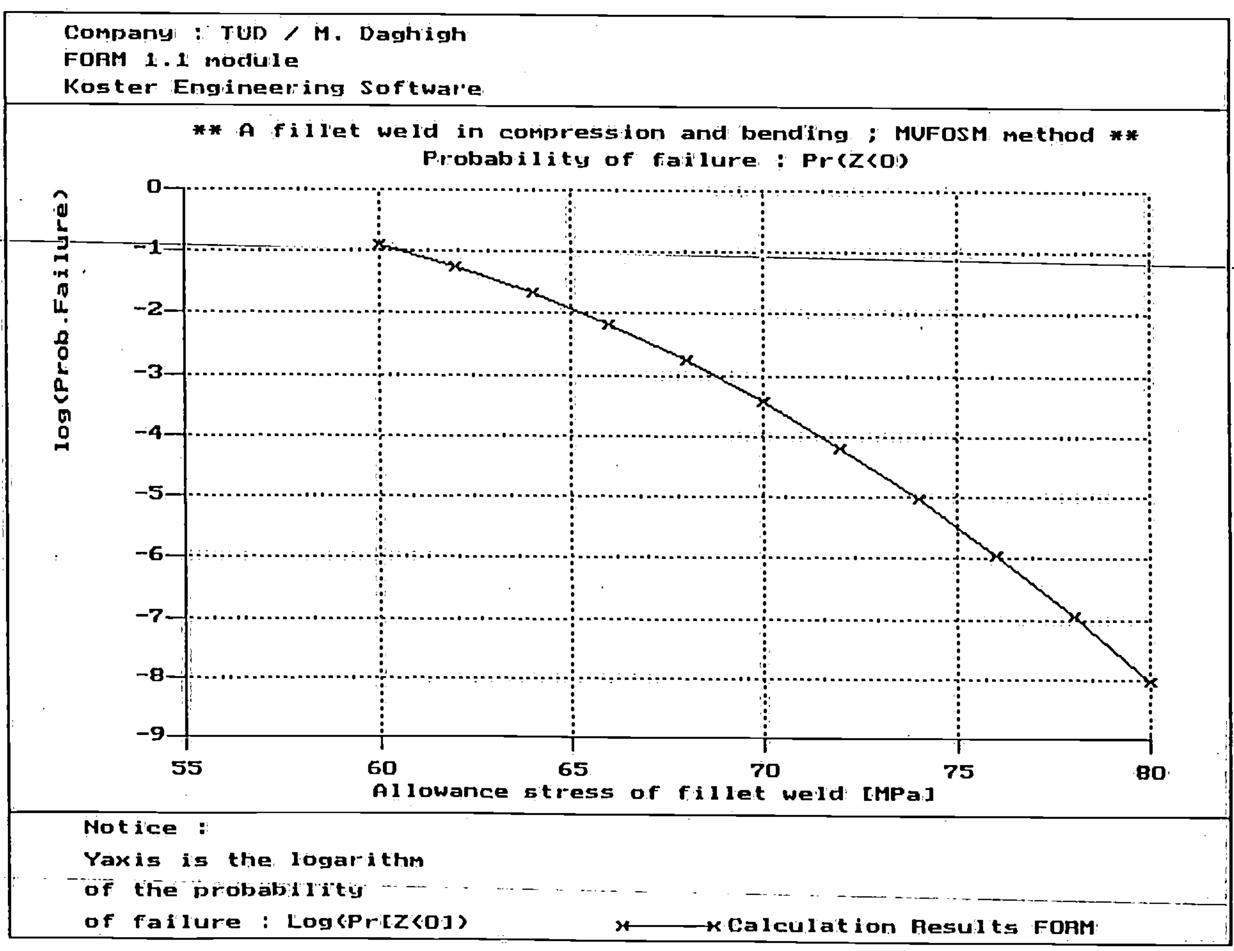


Figure E4.2.3 Logaritm of probability of failure P_F for fillet weld with the same

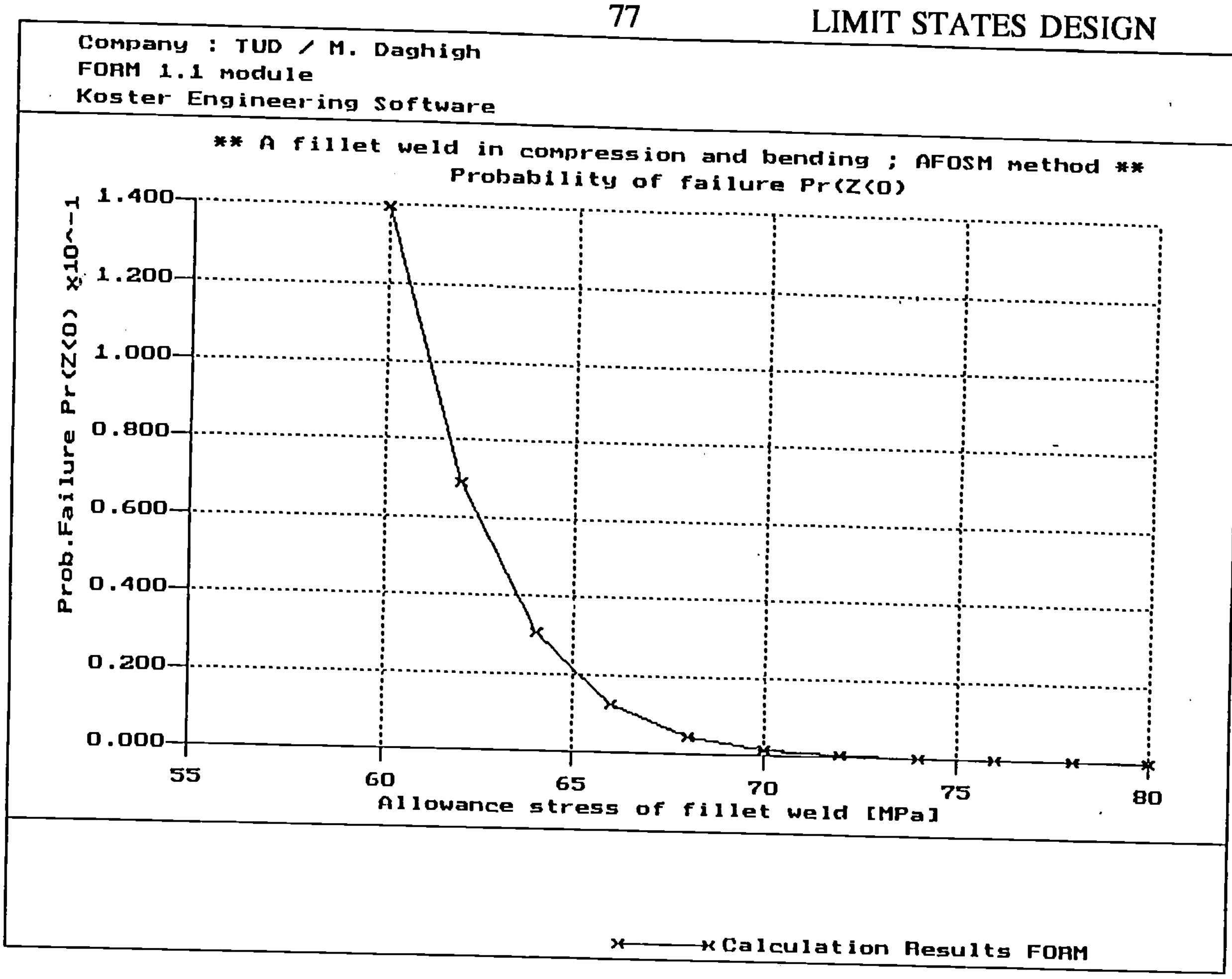


Figure E4.2.4 Probability of failure P_F for fillet weld conforming to AFOSM method

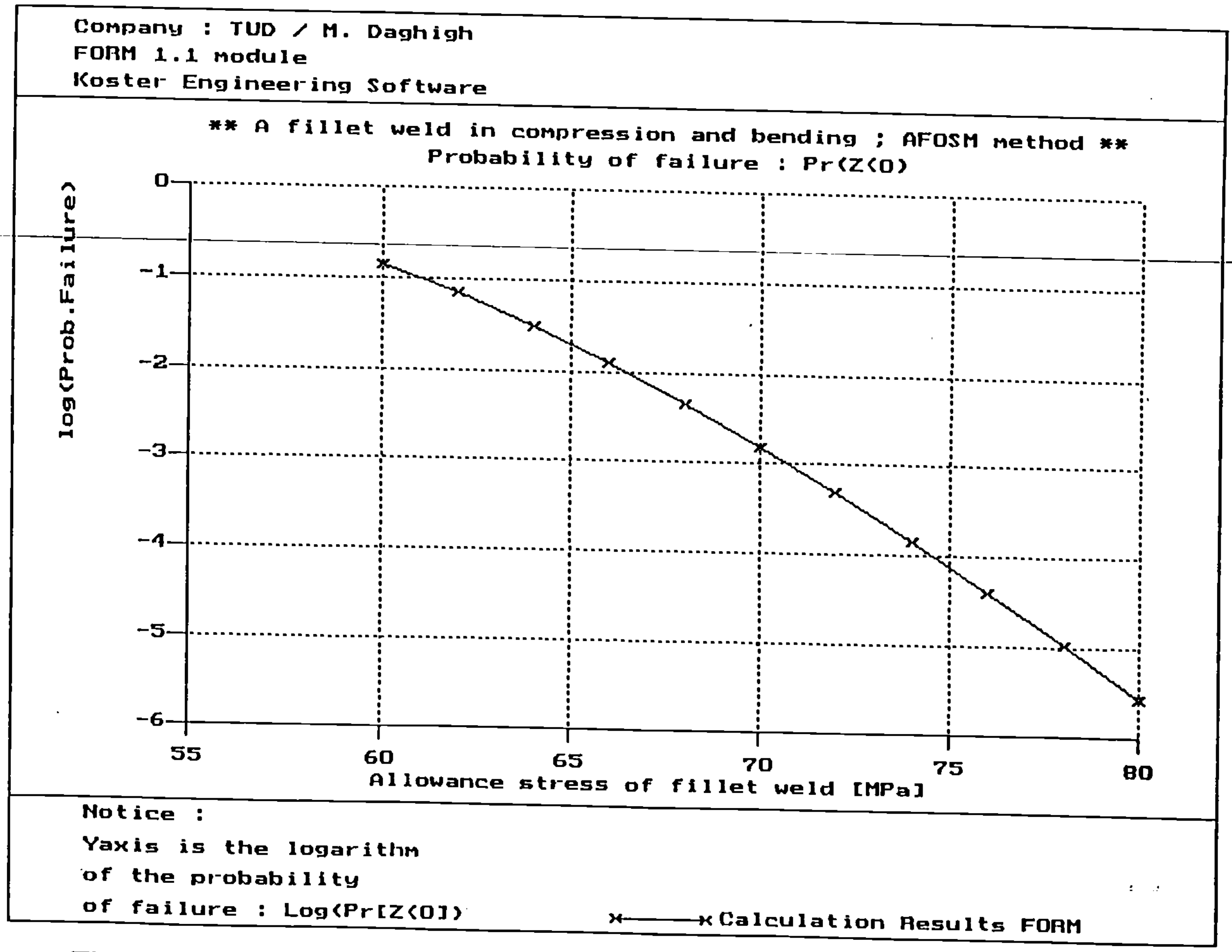


Figure E4.2.5 Logaritm of probability of failure P_F for fillet weld with the same

4.6 Relaibility analysis for simplified systems

The procedure of reliability analysis for single elements by FORM method has been discussed in sub-sections 4.4 & 4.5. Moreover Examples 4.1 & 4.2 have been proposed to illustrate the main concepts for the computational method of reliability analysis of single elements. In general , for a structural element, several different failure modes are usually of importance. As an example for truss and plane frame structures, the most important failure modes are failure in tension/compression due to axial loads and failure in bending and large deflections. It has be proved that a sequence of member failures can be represented by a *parallel system*. On the other hand, the total structural collapse occurs if any such failure sequence occur, i.e., the structure can be modelled as a *series system* of parallel sub-systems. In reliability analysis the statically determinate (*Isostatic*) structures will fail as a series system while for a statically indeterminate (*hyperstatic* or *redundant*) structure the system will fail by a mechanism of failure in a parrallel system. The failure modes are then combined in a series system to obtain the total probability of failure. The experience shows that in order to reduce the number of variables and calculation time it is satisfactory to find the probability of failure for the most dominant mechanisms. It has been shown that how the *β-unzipping* method and the *branch and bound* method are used to identify the significant mechanisms of failure [50].

Consider a structural element with two potential failure modes defined by margins $M_1 = g_1(U_1, U_2)$ and $M_2 = g_2(U_1, U_2)$, where U_1 and U_2 are standardized basic variables. The corresponding failure surface and reliability indices β_1 and β_2 are shown in Figure (4.6). The probability of failure is found by integration of the joint bivariate normal density function for the random variables in the failure region ω_f (if the random variables are normally distributed).

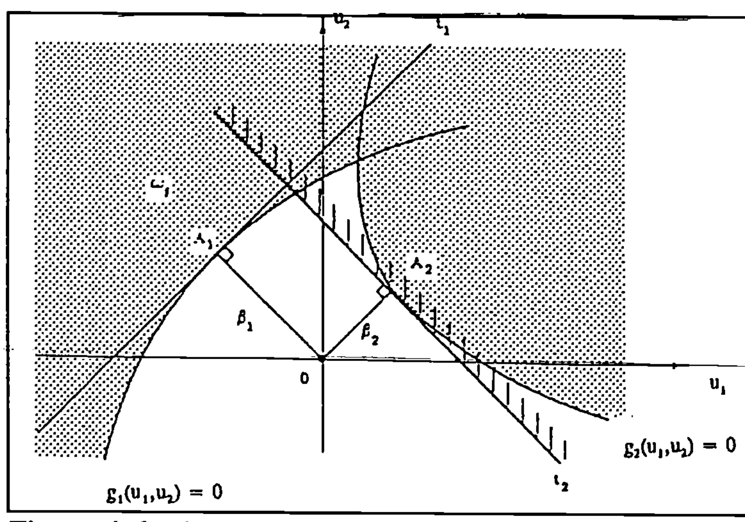


Figure 4.6 The probabilsitic of failure for a series system

$$P_F = \int_{\omega_f} f(u_1, u_2, 0) . d\bar{u} \qquad (4.61)$$

where f is the bivariate normal density function for the random vector $\bar{U} = (U_1, U_2)$. The shortest distance from the origin O to the failure surface is defined by the reliability index $\beta = \beta_2$ as shown in Figure (4.6). Thus the probability of failure is estimated by integration of the distribution function over the hatched area (the "right" of the tangent t_2). Such a modelling of the system reliability has been called *systems modelling at level* 0 by P. Thoft-Christensen and Y. Murotsu [50], which is in fact an element reliability modelling and gives an upper bound for the reliability index. They have proved that a better choice for the probability of failure can be found by modelling the reliability problem by a series system. In a series system, the probability of failure P_F of the element is

$$P_F = P_{F_1} \cup P_{F_2} \tag{4.62}$$

In fact, in this case the probability of two failure elements (or two modes of failure for one element) and the correlation factor between them are taken into account which is called *system modelling at level* 1. However the probability of failure for the series system of Figure 4.6 can be obtained by the probabilities of survival (or reliabilities) of two (elements or modes) P_{R1} and P_{R2} as follows (see Hohenbichler and Rackwitz [57])

$$P_F = 1 - [P_{R_1} \cap P_{R_2}] \tag{4.63}$$

Because U_1 and U_2 are the independent standardized normal variables and the safety margins M_1 and M_2 are approximated by the linearized margins in their design points A_1 and A_2, thus the safety margins can be formulated as follows

$$M_1 = a_1 U_1 + a_2 U_2 + \beta_1 \tag{4.64}$$

$$M_2 = b_1 U_1 + b_2 U_2 + \beta_2 \tag{4.65}$$

where a_1, a_2, b_1 and b_2 are the cosine direction of the unit vectors and β_1 and β_2 are the corresponding reliability indices to the safety margins. The correlation coefficient ρ can be obtained by calculation of the $cos(v)$ where v is the angle between the unit vectors $\overline{a}$ and $\overline{b}$. The safety margins M_1 and M_2 are also standardized normally distributed and the correlation factor between two safety margins M_1 and M_2 are simply obtained by the following expression

$$\rho = \sum_{i=1}^{2} a_i . b_i \tag{4.66}$$

In section 1, it was shown that the reliability measure (or the probability of survival) and the probability of failure are complementary, thus if the reliability index for an element is given by β_i then in case of normal distribution, the reliability (or the probability of survival) can be found by the cumulative distribution function as given by

$$P_{R_i} = \Phi(\beta_i) \tag{4.67}$$

The bivariate normal distribution function, defined in section 1, can be used in order to obtain the probability of failure for a series system with two elements as shown in Figure

4.6. Thus the probability of failure for the series system is obtained by the cumulative joint probability distribution of two elements as follows

$$P_F = 1 - F(\beta_1, \beta_2, \rho) = 1 - \Phi_2(\beta_1, \beta_2; \rho) \tag{4.68}$$

where F and Φ_2 are represented for the cumulative bivariate probability of uncorrelated reliability indices β_1 and β_2 while the correlation cofficient of safety margins are given by ρ. When the probability of failure determined by a level 1 approxiamtion of P_f, the reliability index may be calculated by

$$\beta = -\Phi^{-1}(P_f) \tag{4.69}$$

where Φ denotes the cumulative distribution function for the uni-variate standard normal distribution.

Example 4.3 (Reliability analysis of a simple structural system ; Excerpted from [50])

Assume that the simple structural system shown in Figure E4.3.1 loaded by a single concentrated load p. The collapse of system may be occurred by structural failure of elements 1 and (or) 2. The load-carrying capacity in the elements 1 and 2 are represented by $1.5 \times n_P$ and n_P respectively. Both of the basic variables are realizations of independent normally distributed random variables P and N_P with the following characteristics

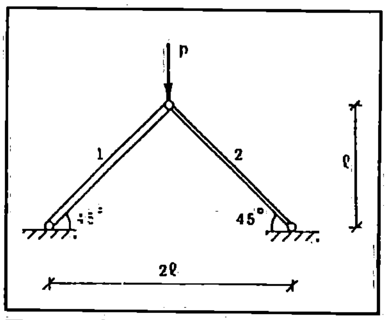


Fig. E4.3.1 Plane truss structure

Table E4.3.1 Probabilistic characteristics of P and N_P

	Mean	Standard deviation
Concentrated load P	$\mu_P = 4\ KN$	$\sigma_P = 0.8\ KN$
Measure of capacity N_P	$\mu_{NP} = 4\ KN$	$\sigma_{NP} = 0.4\ KN$

Find the probability of failure and the reliability index for the structural system.

Solution

The *performance function* (limit state function) can be defined for elements 1 and 2 as follows

$$M_1 = \frac{3}{2} N_F - \frac{\sqrt{2}}{2} P \tag{4.70}$$

$$M_2 = N_F - \frac{\sqrt{2}}{2} P \tag{4.71}$$

Suppose that the characteristics of basic variables are used in order to find the safety margins in term of the standardized random variables $U_1 = (N_F - 4) / 0.4$ and $U_2 = (P - 4) / 0.8$

$$M_1 = 0.728\, U_1 - 0.686\, U_2 + 3.486 \tag{4.72}$$

$$M_2 = 0.577\, U_1 - 0.816\, U_2 + 1.691 \tag{4.73}$$

By comparison with equations (4.64) and (4.65), the reliability indices are $\beta_1 = 3.846$ and $\beta_2 = 1.691$. The correlation coefficient between two safety margins is determined by equation (4.66) as follows

$$\rho = 0.728 \times 0.577 + 0.686 \times 0.816 = 0.98 \tag{4.74}$$

Thus the probability of failure of the system is determined by equation (4.68)

$$P_F = 1 - F(\beta_1, \beta_2, \rho) = 1 - \Phi_2(3.846, 1.691; 0.98) \tag{4.75}$$

where Φ_2 is represented for the standard bivariate normal distribution function (i.e zero means and unit standard deviations for both random values). In section 1, it has been discussed that the calculation of the uni-variate normal distribution have been performed numerically without any difficulties and a number of of tables exist for this purpose in the appendix of any probabilistics handbook [1,9,11,44,50,52]. M. Zelen and N.C. Severo [52] have been given a number of plots for approximate calculations of standard bivariate normal function (see Figs. 26.2-26.4 pp. 937-939 from referenec [52]). Although the solution has been performed by numerical integration methods but the Figures are not accurate especially for boundary values of the correlation coefficient ρ eg. for $\rho \approx -1$, 0, +1. Moreover the Hermite polynomial expansion method is also used for calculating the bivariate normal distribution function [50]. In author's opinion, it will be more appreciated if the numerical method is developed for the standard bivariate normal distribution. For this reason the cumulative function is expanded by the Taylor series with respect to the correlation coefficient near the origin (i.e. $\rho = 0$). By this method the integral is reduced from the double integral (for the random variables) to the single integral [50]. By the same notations used in section 1, the standard bivariate normal density function is rewritten as

$$f(t_1,t_2,\rho)=\{2\pi\sqrt{1-\rho^2}\}^{-1}\exp[-\frac{1}{2}(\frac{t_1^2+t_2^2-2\rho t_1 t_2}{1-\rho^2})] \quad (4.76)$$

The function f can be integrated for the variables t_1 and t_2 in order to find the joint cumulative distribution function as given by

$$F(u_1,u_2,\rho)=\int_{-\infty}^{u_1}\int_{-\infty}^{u_2} f(t_1,t_2,\rho)\,dt_1\,dt_2 \quad (4.77)$$

Let we assume that both variables t_1 and t_2 are represented by constant parameteres and the function f be the first order derivative of the cumulative function F with respect to ρ. By differentiation of equation (4.77) to the variable ρ, the standard bivariate normal density function can be written as follows

$$\frac{\partial^2 F(u_1,u_2,\rho)}{\partial u_1\,\partial u_2}=\frac{\partial F(u_1,u_2,\rho)}{\partial\rho} \quad (4.78)$$

Thus for the uni-variate function F, the Taylor series can be derived at the point by bivariate function with $\rho = 0$.

$$F(u_1,u_2,\rho)=F(u_1,u_2,0)+\int_0^{\rho}\frac{\partial F(u_1,u_2,\lambda)}{\partial\lambda}\Big|_{\lambda=\rho}\,d\rho \quad (4.79)$$

Obviously the first part on the right side of equation (4.79) can be represented by the product of the two uni-variate normal distribution functions and we have

$$F(u_1,u_2,\rho)=\Phi(u_1)\,\Phi(u_2)+\int_0^{\rho}\frac{\partial F(u_1,u_2,\lambda)}{\partial\lambda}\Big|_{\lambda=\rho}\,d\rho \quad (4.80)$$

Returning back to our example, we only need to determine the single integral or second expression on the right hand side of equation (4.80). For this purpose a lot of numerical methods have been used in literature (for example Hermite polynomial expansion method) but hier we consider the *PFD* (Polynomial Finite Difference) method which has been reported by the author on earlier report [58].

In the PFD method, we could eliminate the errors which are arising from the simplifications of the conventional finite difference method. Brifely, in the traditional (or conventional) finite difference method, the derivatives of a function are obtained from the minimum required degree of function and for example if the differential equation involves the second derivatives of function then a parabolic function is passed through each three assumed points. In the traditionary method, using the direct substraction methods or exapansion methods on series approach, the real function is replaced by a few of discontinuous functions (parabola's for the second derivative). In the PFD method, we have used polynomial functions in derivatives and thus for approxiamation of second order derivatives, the diffrentiation is taken place on the higher order function rather than the parabolic equation. By this way, the derivatives of small degrees (first and second) are obtained with very small errors compared to the traditionary finite differenec method. It has been shown that for the higher orders derivatives the PFD method gives a reasonable approximation and the shape of function is changed when it is compared with the conventional finite differenec method.

In order to integrate equation (4.80), it is necessary to divide the variable domain into some finite distances. For our example $\rho = 0.98$ and it has been assumed that the variable domain is divided by 7 equivalent distances each equal to $\Delta\rho = 0.14$. The first expression on the right side of equation (4.80) forms one the boundary conditions for the differential equation. Thus the product of two uni-variate normal functions is detremined as follows

$$F(\beta_1,\beta_2,0) = \Phi(3.846)\Phi(1.691) = (9.999409411D{-1}) \times (9.544224341333685D{-1}) \qquad (4.81)$$

where D denotes the double precision real data type. Note that the cumulative uni-variate normal function is excerpted from reference [52]. The product in equation (4.81) is calcualted by a simple program in FORTRAN with 15 accuracy decimal digits which is represented by the double precision variable as follows

$$F(\beta_1,\beta_2,0) = 9.543660669942732D{-1} \qquad (4.82)$$

The boundary conditions for the differential equation (4.80) are defined as follows

$$F(3.846, 1.691, 0) = 9.543660669942732D{-1} \qquad (4.83)$$

and

$$\frac{\partial F(3.846,1.691,\rho)}{\partial\rho} = \{2\pi\sqrt{1-\rho^2}\}^{-1}\exp[-\frac{1}{2}\frac{(3.846)^2+(1.691)^2-2\rho(3.846)(1.691)}{1-\rho^2}] \qquad (4.84)$$

Equation (4.84) holds for ρ = [0.14 , 0.28 , 0.42 , 0.56 , 0.70 , 0.84 , 0.98] . Thus equation (4.83) together with the 7equations obtained from equation (4.84) form a set of 8 linear systems of equations. The output will be the cumulative bivariate normal distribution for defferent parameters of ρ and the answer for $\rho = 0.98$ is obtained by

$$F(\beta_1,\beta_2,\rho) = \Phi_2(3.846, 1.691; 0.98) = 9.54417356D{-1} \qquad (4.85)$$

And the probability of failure P_F is determined by equation (4.75) as follows

$$P_F = 1 - \Phi_2(3.846, 1.691; 0.98) = 0.045582644 \qquad (4.86)$$

The reliability index for the series system in Figure (4.6) is calculated by the FORM method by

$$\beta = -\Phi^{-1}(P_F) \simeq 1.6893 \tag{4.87}$$

In reference [50], the *Ditlevsen* bounds for this example have been calculated and the probability of failure is expected to be close the the lower bound of Ditlevsen. The lower bound is the maximum probability of failure of any element and in this example the lower bound and the corresponding reliability index have been given by $P_{F1} = 0.04557757$ and $\beta_1 = 1.691$.

5- Conclusions and further recommendations

The following conclusions can be drawn from the reliability analysis methods:

1. The main objective of the research has been discussed in the first section of this report. A risk analysis has been reported for the jack-up failures vs. jacket failures by M. Efthymiou [15] and S.F. Leijten & M. Efthymiou [33]. The detailed investigation of the platform failures in two parts including the total loss and the environmental loading and foundation failure has been studied. The hazard rate for both type of platforms has been compared and it has been concluded that regardless of the selection of type of loss, the failure rate does not differ from the failure rate of previous decade for jack-up platforms. It included the loss of jack-up platform in case of total loss or loss due to the environmental loading and foundation failure.

2. Developments on the dynamic analysis of jack-up platforms has been outlined in the first section. This is due to large dynamic amplification for moderate and deep waters combined with the structural flexibility for jack-up structures. The method is implemented on the extreme load effect that will need the contribution of joint probability distribution of significant wave height and spectral peak period as discussed in the second section. The nonlinearities in the structural response due to the flexibility of the structure (so-called p-δ effect) may introduce highly dynamic behavior. Even with the advances for the reliability analysis methods it seems that most of the reliability approaches are not consistent with the state-of-the-art structural analysis techniques. For example in the estimation of reliability measure for dynamically sensitive offshore platforms, it has been assumed that the dynamic amplification factor (DAF) can be assumed as a deterministic parameter in the reliability analysis.

3. One of the promising applications of probabilistic methods is for reliability updating based on improved or new information. This can be implemented for the strength characteristics including the reliability updating based on improved or new information with respect to fatigue crack growth in weldings of platform. The reliability model may be improved by improved information about the environmental characteristics, the dynamic properties of the structure, the improved model for probabilistics characteristics of strength of materials and all geometrical restrictions.

4. A well known technique from classical methods of reliability analysis is the first order second moment (FOSM) methods in which a complex system reliability model is approximated by linear functional relationship between the output safety margins and the input basic variables. An efficient repeated computation is adopted which allows the determination of design point, probability of failure and reliability index. As the structural reliability is governed by the tail behavior of the normal probability distribution, it is apparent that the FORM methods based on second moment characteristics may not be sufficient in reliability studies. Adoptive techniques that combine FORM (or even SORM) calculations on the Monte Carlo Simulation (MCS) with point evaluations of the true failure function has been suggested e.g. in [59].

5. As indicated by P. Thoft Christensen [60], the safety margin for most failure elements is in general dependent on the stress effect history. There are some techniques for the formulation of reliability under combined random load sequences, but the stochastic model is comprising from the transformation of discrete time variant problem into a time invariant problem involving only the extreme of the time-variant variables. The tail behavior is again occurred when the basic variables are transformed to the uncorrelated standardized random variables. Superposition of the environmental loadings can be simply applied for highly redundant platforms as well as the fixed platforms but the robustness of the initial design will not be differ too much. This effect can be taken into account by introducing one or more model uncertainty variables. The problem is arising which distributions must then be estimated from information of unknown data. An additional problem is arising in the evaluation of the residual strength of a damaged element. For reliability approaches optimal inspection and maintenance strategies can not be obtained without and real information about the residual strengths.

6. Documented experience show that offshore structures are occasionally exposed to accidental loads and damages. The consequences of damage should require more attention and the reliability of the remaining damaged system may be useful than the estimated probability of failure for the initial undamaged structure. The motivation in the new regulations is to ensure that the local damage do not lead to structural collapse rather than the progressive collapse analysis of first design. Although there is not an union method for the evaluation of the reliability of damaged state but the structural redundancy concept should be accounted in the reliability measures in conjunction with the nonlinear effects of slender system.

7. The above mentioned factors need to be balanced to the type of platform, its side and the mode of operation, in order to optimize its overall performance and ensure the integrity of the platform during its service life. Some optimal design procedures regarding to the weight of platform have been studied by P. Thoft Christensen [60] but also it is not a fully optimal design procedure. It is possible to modify the position of joints in the set of optimization variables (shape optimization). On the other hand, in case of deep water fixed platforms, the inspection and maintenance of the deeply submerged parts of the structure may be difficult if not impossible. In this case, it may be necessary to build more redundancy into the structure in order to ensure its long term integrity. However the structure has to be able to continue its design operation or to provide sufficient time for safe evacuation and repairs. Sometimes it is much better to design the platform in such a way that sufficient safety is ensured during the total installation time of the structure than to rely on repair when needed.

8. The equivalent linear safety margin concept can be used in order to approximate the system reliability of complex structure. The method is often referred as the Hohenbichler & Rackwitz or Gollwitzer & Rackwitz algorithms [50]. An alternative approach has been developed in simulation methods which is called the Response Surface Methodology (RSM). In this method the complex system is directly approximated by a simple functional relationship between the reliability quantities (for example reliability indices) and the basic variables. As well as the transformation methods, the linear and quadratic functions are applied with an efficient repeated computation. Adaptive techniques that combine FORM/SORM calculation on the response surface function with point evaluations of the true

failure function has been reported by P. Bjerager [55].

6- Nomenclature

$\eta(t)$ = an ergodic record representing the water surface
a = wave amplitude
α = phase angle
k = wave number
L = wave length
ω = $2\pi f$ = wave frequency
d = water depth
m_n = $E[f^n]$ = nth moment of spectrum
T_z = mean zero upcrossing period
T_c = mean crest period
N_z = number of zero upcrossing points
N_c = number of crest points
ϵ_t = spectral width parameter
ϵ_s = spectrum width parameter
$p(.)$ = probability density function (p.d.f.)
μ_x = $\bar{x}$ = mean value
σ_x = standard deviation of variable x
λ_3 = skewness
λ_4 = kurtosis
$\Phi(x)$ = standard normal distribution of x
H_n = nth order of Hermite polynomial
b_n = Hermitian moments
H_s = $H_{1/3}$ = significant wave height
H_{rms} = root mean square height
N = number of waves in planned service
T_S = short term period
T_L = long term return period
RAO = response amplitude operator
D = characteristic dimension (diameter)
D_r = roughness mean size
ρ = density of water
ν = viscosity of water
$f(t)$ = wave force per unit length of vertical cylinder
u = horizontal component of water particle velocity
C_M = inertia
C_D = drag
Re = Reynolds number

KC = Keulegan-Carpenter number
B_H = bias factor for extreme wave height
V_{BH} = coffiecient of variation for B_H
σ_y = yield strength (N/mm^2)
σ_r = rupture strength (N/mm^2)
λ = slenderness parameter of an element
E = Young's Modulus
DAF = dynamic amplification factor
$Pr\{X \leq x\}$ = probability of the event "$X \leq x$"
$F(x)$ = uni-variate cumulative distribution function (c.d.f.)
f_n = point probability at ponit n
$exp()$ = exponential of .....
∂ = partial differentiation
ω_s = safe space region
ω_f = failure space region
X = $(X_1, X_2, X_3, \ldots, X_n)$ random vector
C = covariance matrix of random vector X
A^T = transpose of matrix A
ρ = correlation factor
Ln = logaritms of x in exponential base
C_x = coefficient of variation
z = standardized variable
R = resistance (or) strength
S = stress (or) load
$P_{R'}$ = idealized reliability
P_R = true reliability
M = safety margin
$\hat{M}$ = standardized margin
P_f = F = probability of failure
β = safety (or) reliability index
θ = central safety factor

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