Fatigue Crack Growth Predictions of Surface Cracks under Constant-Amplitude and Variable-Amplitude Loading

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ABSTRACT

Failures of thick components in aircraft structures are very often caused by fatigue crack growth of part-through cracks (surface or corner cracks). The focus of the present study is on fatigue crack growth of semi-elliptical surface cracks, both under constant-amplitude and variable-amplitude loading. The study includes several test series and an analysis of the results by predictions with models based on crack closure. The tests were carried out on plate specimens (thickness 9.6 mm) of 2024-T3 and 7075-T6, provided with a small semi-elliptical starter notch. Starter notches varied from shallow (a/c = 0.2) to semicircular (a/c = 1) in view of studying crack shape developments. Two types of variable-amplitude load sequences were used, i.e. a simplified flight-simulation and a realistic flight-simulation load history based on the wing load spectrum of the CN-235 aircraft. The crack opening stress ($S_{op}$) in constant-amplitude tests was measured along the crack front with a fractographic technique. This information is essential for the application and evaluation of prediction models. Fractography was also adopted to study crack shape developments, for which marker loads were applied. The analytical part of the study concentrated on predictions of crack shape developments, fatigue crack growth and crack growth lives until breakthrough. The numerical prediction technique includes crack increment predictions at several points of the curved crack front. The CORPUS model and the modified CORPUS model were used for the predictions of the variable-amplitude test results. Some adjustments of the CORPUS model were made to accommodate the prediction of semi-elliptical surface crack growth.

Keywords: surface crack, semi-elliptical surface crack, fatigue crack growth, crack growth prediction models, CORPUS model, modified CORPUS model, crack opening stress, fractography
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LIST OF SYMBOLS

a  crack depth
a/c  aspect ratio of an elliptical surface crack
a/t  relative crack depth
adpfx(J)  x coordinate of plastic zone border extending from a point J
adpy(J)  y coordinate of plastic zone border extending from a point J
ADP  plastic zone size measured from the center of specimen (for through cracks)
ADPH  memorized plastic zone size
c  crack length on the surface
C  crack growth rate constant in the Paris equation
C_A  crack growth rate constant in the Paris equation for the depth direction
C_C  crack growth rate constant in the Paris equation on the surface
CA  constant amplitude
da  crack growth increment in the thickness direction
dc  crack growth increment on the surface
da/dN  crack growth rate in the thickness direction
dc/dN  crack growth rate on the surface
dp(J)  primary plastic zone size calculated at a point J
dps(J)  secondary plastic zone size calculated at a point J
D  plastic zone size (CORPUS model)
D_sn  plane strain plastic zone size (CORPUS model)
D_ss  plane stress plastic zone size (CORPUS model)
eff  effective
f_w  finite width correction factor
f_v  angular function derived from embedded elliptical crack solution
F  stress intensity factor correction for semi elliptical surface cracks
F_A  stress intensity factor correction for the deepest point
F_C  stress intensity factor correction on the surface
K  stress intensity factor
K_A  stress intensity factor at the deepest point
K_C  stress intensity factor on the surface
ΔK  range of stress intensity factor (K_max - K_min)
$\Delta K_{\text{eff}}$ range of effective stress intensity factor ($K_{\text{max}} - K_{\text{op}}$)

$m$ crack growth exponent in the Paris relation

$m^c$ relaxation parameter (CORPUS model)

$n$ number of cycles

$op$ opening

$OL$ overload

$PPZ$ primary plastic zone

$Q$ shape factor for elliptical crack

$R$ stress ratio ($S_{\text{min}}/S_{\text{max}}$)

$r_p$ plastic zone size

$S_{gr}$ ground stress

$S_{\text{max}}$ maximum stress

$S_{\text{mf}}$ mean stress in flight

$S_{\text{min}}^*$ minimum stress in the $S_{\text{op}}$ measuring block

$\Delta S_{\text{min}}^*$ difference of subsequent minimum stress in the $S_{\text{op}}$ measurement

$S_{\text{op}}$ opening stress

$SPZ$ secondary plastic zone

$S_{0.2}$ yield stress

$t$ thickness

$U$ $\Delta S_{\text{eff}}/\Delta S = \Delta K_{\text{eff}}/\Delta K$

$UL$ underload

$ULZ$ underload effected zone (modified CORPUS model)

$VA$ variable amplitude loading

$W$ specimen width

$x(J)$ coordinate of a point $J$ in the X direction (parallel to specimen surface)

$y(J)$ coordinate of a point $J$ in the Y direction (in the thickness direction)

$\gamma$ $K_{\text{eff}}/K_{\text{max}}$

$\phi$ parametric angle

$\Phi$ complete elliptical integral of the second kind
Chapter 1

Introduction

Fatigue cracks in thick components usually start as surface cracks or corner cracks, which do not occur through the full thickness of the component. In the literature it is referred to as part-through cracks. A through crack is growing through the full thickness of the component. The stress analysis of part-through cracks is fairly complex due to the three-dimensional nature of the problem. The stress intensity factor K varies along the crack front. A first publication towards the K solution of a surface crack is the work of Green and Sneddon in 1952 [1.1], which treated an elliptical crack in an infinite solid. Based on the results of Green and Sneddon, Irwin [1.2] derived an expression of K for an infinite solid and an approximate solution for a surface crack in a finite plate. It may be noted that the approximate solution of Irwin is the first attempt to estimate K of a surface crack in a finite plate. Since then a number of approximate solutions were proposed. The state of the art of the K solution in the beginning of the 1970's can be found in: "The surface crack: Physical problems and computational solutions" [1.3]. It probably were the first proceedings devoted to surface crack problems. A wide variety of approximation methods is now available to estimate the value of K in a finite plate, including the finite element method, the boundary element method, the alternating method, the line spring method, and the body force method. In the past the solutions sometimes differed significantly, especially at the deepest point of deep flaws. However, recent solutions are in reasonable agreement, which shows improvements and convergence of the available solutions.

Currently a large amount of crack growth rate data of through cracks are available. Because surface crack problems require a three-dimensional analysis, while through cracks imply a two dimensional analysis, a relevant question to be raised is whether the through cracks data are applicable for part-through cracks. This topic was addressed in an ASTM symposium "Part-through crack fatigue life prediction" [1.4] held in 1977. The symposium proceedings were the result of an exercise in predicting the lives of surface cracked specimens, based on through crack data from compact tension specimens.
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Different $K$ solutions and crack growth accumulation schemes were used. Although it was said that sufficient accuracy was obtained in predicting the lives of surface cracks using compact tension specimen data, a closer look to the individual results in [1.4] shows a high variation of the predicted lives.

Complexities of prediction of surface cracks life arise from the fact that $K$ and the state of stress vary along the crack periphery. Close to the surface of the material plane stress conditions prevail, while far from the surface plane strain conditions predominate. The difference of the state of stress along the crack periphery implies a different size of plastic zone, which in turn leads to a different crack closure stress along the crack front. A quantitative description of the crack opening stress along the crack front is not available.

Experimental results have shown that fatigue cracks growing under variable amplitude (VA) loading will experience interaction effects. Crack increment in one cycle depends not only on the current load cycle, but also on the preceding cycles. A high overload will cause crack growth retardation in the following cycle. Several mechanisms have been proposed to explain the interaction effects. Among the mechanisms proposed, crack closure has been widely discussed. The crack closure mechanism can explain, at least qualitatively, the empirical trends.

The present prediction models for fatigue crack growth under VA loading can be classified into three main groups [1.5] i.e

- Yield zone models
- Crack closure models
- Strip yield models.

All models in these groups involve calculation of plastic zone sizes to account for interaction effects. However, the plastic zone is treated differently and the calculations are also performed differently. The yield zone models are based solely on the relationship of the current plastic zone size to the previous one. These models do not include a crack closure mechanism. Crack closure models and recent strip yield models take into account crack closure. In these models, the crack increment in one cycle is controlled by the effective stress intensity factor range, $\Delta K_{eff}$. The magnitude of $\Delta K_{eff}$ accounts for the range of the stress intensity when the crack is fully open. According to the crack closure
mechanism, cracks are closed and opened at positive stresses. Closure occurs because of the residual deformation left in the wake of the crack as it grows through plastically deformed material. Crack growth is controlled by the behaviour of the plastic zone (ahead or in the wake of the crack), therefore the calculation of the plastic zone size is important in these models. As mentioned previously the transition from plane stress condition on the surface to plane strain condition in the interior complicates the plastic zone size calculations.

Prediction models presented in the literature were developed for through cracks. The crack is assumed to have a straight crack front, so that $K$ is constant along the crack front. The crack closure level is also assumed to be constant. The assumption of a straight crack front is not always true. Experimental results show a curved crack front, especially for cracks growing under VA loading, thus $K$ will have different values along the crack front. Measurements of $S_{op}$ also show varying $S_{op}$ along the crack front. For through cracks variation of $K$ and $S_{op}$ can lead to levelling out of $\Delta K_{eff}$ variations [1.6], in agreement with a constant crack growth rate along the crack front. For surface cracks the situation is different, the crack growth rate is not constant along the crack front, which means a varying $\Delta K_{eff}$. In this case the variation of $S_{op}$ is needed for the prediction of the crack growth rate at different locations along the crack front. Measurements of the $S_{op}$-variation of surface cracks are the first aim of the present investigation. A second aim is to incorporate the measured $S_{op}$ variation in an existing prediction model, for which the CORPUS model and the modified CORPUS model have been chosen.

In chapter 2 some examples of the surface crack problems in structures will be presented. A typical example is the surface crack found at the F-111 wing pivot point made of D6AC steel. The flaw was introduced during the manufacturing of the material. The aircraft crashed after a small number of flight hours. This failure has led the US Air Force to issue the Damage Tolerance Requirements, which include the assumption that initial flaws exist in aircraft structures due to material defects or manufacturing processes.

Methods to estimate the stress intensity factor along the crack front of a semi-elliptical surface crack are discussed in chapter 3. Selected results will be presented.
The subject of chapter 4 is the growth of surface cracks under constant amplitude (CA) loading. It consists of two main parts. The first part is on the measurements of the crack opening stress of surface cracks. A fractographic technique, originally proposed by Sunder [1.7], was adopted. The technique is based on a load sequence that can produce distinct striation characteristics on the fracture surface. The crack opening stress $S_{op}$ is deduced from the striation pattern. With this technique the variation of $S_{op}$ along the crack periphery can be obtained.

The second part of chapter 4 covers the surface crack growth prediction under CA loading. A crack growth prediction of a surface crack must simulate a continuous process along the crack front. For practical reasons crack growth predictions are made for a limited number of points along the crack front. The first step made is to analyse the number of calculation points sufficient for an accurate simulation. CA fatigue crack growth data from another source are adopted to evaluate the crack growth prediction schemes. The results of crack closure measurements in the first part of chapter 4 are included in the prediction scheme to see the importance of crack closure mechanism in predicting surface crack growth under CA loading. Predicted lives and predicted crack growth rates are compared to test results. In addition predicted crack shape developments are also compared to experimental data.

In chapter 5, the CORPUS model and the modified CORPUS model will be used for predictions on the fatigue life of surface cracks under flight simulation loading. The CORPUS model and the modified CORPUS model are based on the crack closure concept. The CORPUS model was developed by de Koning [1.8]. Padmadinata [1.9] proposed the modified CORPUS model, which intended to remove certain shortcomings of the CORPUS model. Both models will be applied to surface crack growth under different flight simulation load histories, adopted in the present investigation, i.e., the Misawa load sequence and the CN-235 flight simulation. Results from tests with manoeuvre dominated load spectra, presented in the literature, are also used. The variation of measured $S_{op}$ along the crack front will be incorporated in the model. Some adjustments of the CORPUS model were made to accommodate the prediction of surface crack growth.
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Chapter 2

Surface Cracks in Structures and Components

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Chapter 2, Surface cracks in structures and components

2.1. Introduction

In this section examples of service failures involving surface cracks are presented. A typical example of a fatal accident due to a surface crack in aircraft structures was the crash of F-111 fighter-bomber aircraft. The crash has generated tremendous interest of the US Air Force in fracture mechanics design of aircraft structures, which finally lead to the Air Force damage tolerance requirements. In section 2.2 the failure of the wing pivot in F-111 will be presented followed by failures in some other aircraft components. Examples of failures in pressure vessels are presented in section 2.3 which include failures of rocket motor case.

2.2. Surface cracks in aircraft structures and components

2.2.1. Wing pivot fitting of F-111 aircraft [2.1].

The F-111 is a fighter bomber aircraft manufactured by the General Dynamics. An important feature of the F-111 is the variable sweep wing concept (see Fig.2.1) implemented for an optimum aerodynamic performance at low and high speed. The outer part of the wing can be moved during the flight to have a different sweep angle. The movable part of the wing was attached at the wing pivot fitting made of D6ac steel forging welded from a number of sub-components [Fig.2.2]. Inspection of the interior of the assembly was not possible.

The operational life of the aircraft was specified to be 4000 flight hours with 4000 landings. One of the aircraft crashed after accumulating 107 flight hours. The crash was caused by the separation of the outer left wing at the wing pivot fitting when the aircraft performed an in-flight manoeuvre which was less than half of the design limit load factor. Fig.2.3 shows the section of the wing where separation occurred. The failure investigation revealed that the fracture at the wing pivot fitting was caused by a surface flaw in the lower plate (see arrow in Fig.2.3). Fig.2.4 shows the flaw and the surrounding structure in more detail. The lower part of the plate represents the honeycomb fairing. A close up of the fracture area, shown in Fig.2.5, reveals the shape of the initial flaw with a length of approximately 25 mm and depth of 6 mm, which is a relatively large flaw. The dark part of the flaw was probably introduced during the forging process. The bright narrow band around the dark part was supposed to be the crack extension during 107 flights.
before the crash. Predictions were performed to estimate the crack growth and final failure of the component. The predictions correlated reasonably well the occurrence of the final fracture.

The failure of the wing pivot fitting of the F-111 had caused the US Air Force to issue the current Air Force damage tolerance requirements, which include the assumption that initial flaw exist in aircraft structures due to material defects or manufacturing processes. These initial flaws are assumed to exist in the most unfavourable location relative to the applied stress. Safety is achieved by the design concept (fail safe or slow crack growth structures) and the degree of inspectability of the structure. Inspection plays an important role in the new regulations to assure that fatigue cracks throughout the operational life of the aircraft can be found and repaired. Crack growth and residual strength analysis has to be performed and substantiated by tests. The objective of the crack growth and residual strength analysis is to provide an inspection program for the structural components. The damage tolerance design philosophy was later adopted for the civil aircraft. It may be noted that the fatigue analysis of the F-111 structures was based on the Miner rule. The structural design was based on the safe-life philosophy, in which analysis and test should demonstrate that there will be no fatigue failures within the operational life of the aircraft. The presence of initial flaws was not taken into account in the Miner analysis and the safe-life design philosophy.

Damage tolerance requirements also exist for nuclear pressure vessels, contained in the ASME boiler and pressure vessel code section XI. The ASME code has a different approach in realizing the damage tolerance in pressure vessels as compared to the US Air Force damage tolerance for aircraft structures. The ASME codes specify that if a crack larger than a prescribed size is detected in service then two options are available, i.e. to do weld repair or to perform crack growth analysis to assure safe operation until the next inspection.

2.2.2. Fatigue failure of landing gear components.

Landing gear components are usually machined from a thick forging. If fatigue cracks initiate, they will grow as surface cracks. Three cases of fatigue failures of landing gear components involving the growth of surface cracks are presented in this section.
Fatigue failure of a landing wheel [2.2].

The broken wheel of an aircraft main landing gear is shown in Fig.2.6a. The wheel was machined from a 2014-T6 aluminum forging, the wall thickness is approximately 12.7 mm. An enlarged view of the fracture surface is shown in Fig.2.6b and Fig.2.6c. The figures show a large corrosion pit at the initiation site. Beach marks are clearly visible in Fig.2.6c indicating that the crack was growing as a surface crack up to more than half the wall thickness.

Another example of cracked aircraft wheel is shown in Fig.2.7a [2.3]. The wheel was machined from 2014-T6 forging, it was removed from service because a crack was discovered during an inspection. To reveal the fracture surface the cracked part was broken open (Fig.2.7b and 2.7c). The fracture surface in Fig.2.7c showed five distinct semi-elliptical beach marks (arrow 1 to arrow 5). Arrow 0 marks the origin of the crack. Further observation of the crack initiation site revealed that the crack initiation was caused by material imperfection in the form of undissolved grain refiner.

Fatigue failure of landing gear hubs [2.4].

Fig.2.8a shows the hub of a forged 2014-T6 aluminum alloy aircraft main landing gear fractured due to fatigue crack. The fatigue crack initiated at the inner side surface of the hub (Fig.2.8b), which later grew as a surface crack as indicated by the beach marks. An enlarged view of the initiation site revealed a forging defect usually known as bright flakes (arrow in Fig.2.8c), which are small cracks due to the formation of gas filled voids during solidification.

The fracture surface of another main-landing-gear wheel hub [2.4] (different aircraft type from the previous example) is shown in Fig.2.9. The hub was forged from 2014-T6 aluminum. Investigation revealed that the fatigue crack was initiated at the corrosion pits found on the surface of the hub (see arrow). The crack was then growing as a surface crack as indicated by the beach marks.

2.2.3. Fatigue failure in jet and propeller engine components

Fatigue fracture of a jet engine turbine blade [2.3]

Fig.2.10 shows the fracture surface of a turbine blade of a jet engine made of nickel alloy. The arrow in the figure indicates the nucleation site on the convex side of the airfoil. The crack probably initiated by some high temperature corrosion damage. A few beach marks
can be seen on the fracture surface showing that the shape of the crack front is close to part elliptical. In components operating at high service temperature, such as the turbine blade, fracture can be affected by the creep phenomena. Creep is predominant if the mean stress is relatively high compared to the alternating stress. However, the fracture surface in Fig.2.10 shows a characteristic fatigue failure, which means that alternating stresses were responsible for this fracture.

*Fatigue failure of compressor blades of a jet engine [2.5].*
A number of compressor blades of a jet engine made of light alloy fractured in service due to fatigue cracking. The fracture surface of one of the blades is shown in Fig.2.11a. The fracture surface clearly shows characteristic features of a fatigue failure with a bright area of the fatigue part with beach marks, and a dark area of final failure. The enlarged view of the initiation site (Fig.2.11b) clearly shows the initial and subsequent growth as a surface crack. Most probably the crack initiation in this blade was due to a machining scratch.

*Fatigue failure of the propeller blade of a Harvard aircraft [2.6].*
Fig.2.12a shows the propeller blade of a Harvard aircraft which fractured during a flight. The fracture surface is shown in Fig.2.12b. Investigation revealed that a fatigue crack was initiated at the upper side of the profile. Traces of the beach marks on the fracture surface presented in Fig.2.12c clearly show that the crack propagates as a surface crack in a large area of the propeller cross section. The small area of the final fracture indicates that the fatigue stress level was low. Further investigation at the crack initiation site did not reveal the cause of the fatigue crack initiation.

*Fatigue cracking of a steel connecting rod for a reciprocating aircraft engine [2.3].*
Fig.2.13 shows part of a connecting rod of an aircraft engine with an elliptical surface crack found during an inspection. The rods were forged from 4337 steel. The crack started at as a circumferential surface crack, then propagated transversely into the bearing-bore wall. Examination of the fracture surface revealed three large inclusions, which were responsible for the initiation of the fatigue crack. Beach marks can be seen around the inclusions.
Fatigue fracture of a steel articulated rod of an aircraft engine [2.3]

Fig. 2.14 shows the fracture surface of a 4337 steel articulated rod of an aircraft engine fractured in service. Visual examination revealed that a fatigue crack was initiated at an electroetched identification mark on the flange surface. Concentric semi-elliptical beach marks are clearly visible extending almost the full width of the flange and about half of the web. Metallographic examination of a polished and etched section through the fracture origin revealed a notch that was caused by arc erosion during electroetching.

Fatigue fracture of the Sikorsky S-61L helicopter blade [2.7]

A Sikorsky helicopter crashed due to the separation of the main rotor spindle blade from the rotor hub. The spindle was made from 4340 steel. Fig. 2.15a and 2.15b show the location of the fracture and a surface crack originated from shallow pits. The pits were found in an area where the hardness was lower than specified. Residual stresses resulting from nickel plating of the bearing might have contributed to the initial growth of the fatigue crack. The crack probably had been present when the spindle was inspected with the fluorescent magnetic particle two month before the accident.

2.3. Fracture of pressure vessels

Pressure vessels often contain flaws or defects in the materials or introduced during fabrication processes, especially by welding. Fracture in pressure vessels occurred as early as 1919 when a large molasses tank failed resulting in 12 deaths, 40 injuries and massive properties damage. During the period 1960's a number of thin walled pressure vessels made of very high strength material in rocket and space vehicles failed during hydrostatic proof tests. These failures were attributed to small preexisting flaws that could not be detected by non-destructive inspection. These costly failures have lead to the use of linear elastic fracture mechanics to account for an initial defect by employing the fracture toughness of the material. Two examples of the failure of rocket motor case are presented below.

Fracture of a rocket motor case due to a delayed quenching crack [2.3]

Fig. 2.16a shows a D-6ac steel rocket motor case failed during proof testing. Visual examination revealed that brittle fracture started at the forward dome, and propagated circumferentially and then radially to the adjacent cylinder. The fracture origin was an
oxidised wide shallow surface crack (dark region at B in Fig.2.16b). Further examination revealed that the surface cracks were the result of delayed quenching.

**Fracture of the second-stage of A-1 Polaris rocket motor due to a welding crack [2.3].**

Fig.2.17 shows the fracture surface of a second stage A-1 Polaris rocket-motor case failed in a hydrostatic proof test. The case was gas tungsten arc welded from AMS 6434 steel. The fracture started on the inside surface of the motor case (shown by arrow in Fig.2.17) at a pre-existing small surface crack due to welding. Slow stable crack growth extended from the initial crack to the crack arrest line (between points A). The small bright area beyond the crack arrest line again indicates slow stable crack extension followed by unstable, rapid fracture in the region with a rough appearance. The crack arrest line and the following bright area shows that the surface crack has a somewhat irregular crack border.

2.4. **Concluding remarks**

**Assumption of a surface crack with a semi-elliptical shape.**

The occurrence of surface cracks is very common in thick components, where cracks initiate on the surface or close to the surface. Many examples of fatigue failures in service have shown that surface cracks grow with a shape close to a semi-ellipse. However, there are as many examples where the shape of the crack does not resemble a semi-ellipse, especially in components with a complex shape or when there are multiple crack initiation points. The linking up of the multiple cracks, such as due to corrosion pits, often lead to a non-elliptical crack. Analytical tools such as the finite element method can be used to obtain stress intensity factors for a non-elliptical crack. However, if fatigue crack growth predictions have to be carried out based on the stress intensity factors of the irregular crack front, several finite element calculations must be performed for different crack sizes and crack shapes. This procedure is too costly for practical fatigue and fracture analysis. Hence, surface cracks are usually assumed to be semi-elliptical. A number of stress intensity factor solutions of semi-elliptical cracks are available in the literature. Some solutions give the value of the stress intensity factor along the crack border as equations, which can be used for semi-elliptical cracks with different aspect ratios and different crack depths. A review of the available stress intensity solutions of semi-elliptical cracks is given in Chapter 3.
Crack initiation and fatigue crack propagation.
Examples of service fractures due to surface cracks presented in section 2.2 and 2.3 revealed different sources of initiation i.e.
1. material defects such as inclusions.
2. manufacturing defects due to machining, welding, forging, or identification marks.
3. corrosion pits.
Initial flaws due to material and manufacturing defects are present in a structure or a component before they are put into service. Very often fatigue cracks also initiate due to local stress concentrations or environmental conditions which lead to corrosion. The damage tolerance requirements for aircraft structures mentioned earlier, require that the cracks can not develop to a dangerous size throughout the service life of the structure. Fatigue crack growth prediction and residual strength analysis have to be performed for every component whose failure, if it remains undetected, can lead to the loss of the aircraft. For a surface crack the fatigue life may be covered mainly by the growth before the depth of the crack reaches the other surface (the condition that the crack depth reaches the other surface is often called breakthrough). Some examples presented in the previous sections, e.g. the F-111 wing pivot failure, shows that final fracture did occur before breakthrough. Therefore, crack growth prediction of surface cracks before breakthrough is very important for thick components. Crack propagation models have to be able to model different conditions along the crack front, e.g. different stress intensity factors resulting from crack front curvature. The fatigue crack propagation prediction of surface cracks under constant amplitude loading and variable amplitude loading is presented in Chapter 4 and Chapter 5 respectively. Available K-solutions for a semi elliptical surface cracks are presented first in Chapter 3.

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Chapter 2, Surface cracks in structures and components.

different sweep angle

Fig. 2.1. F-111 aircraft with the variable sweep wing concept.

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Fig.2.3. Section of the F-111 wing fractured during an in-flight manoeuvre [2.1].

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(a)

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Chapter 3

The Stress Intensity Factor of Surface Cracks

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3.1. Introduction
Experimental evidence indicates that a crack, initiated at the surface of a thick component, propagates with a near semi-elliptical crack front. Therefore, considerable analytical work has been done on deriving stress intensity factors for semi-elliptical surface cracks in a finite three-dimensional solid. Due to the curved boundary of a surface crack the value of K varies along the crack periphery. It is not constant as in the case of a through crack. Nevertheless, three-dimensional elasticity analysis has shown that the inverse root singularity and the angular variation of stresses close to the crack tip are the same as those of through cracks.
Due to the complex boundary conditions, an exact solution of K for a semi-elliptical surface crack in a finite plate is not available. The existing approximate solutions are usually based on assigning boundary correction factors, which modify the stress intensity factor of an elliptical crack embedded in an infinite solid.

3.2. Stress intensity factor of an elliptical crack in an infinite solid
The results of Green and Sneddon [3.1] for the stress distribution around an elliptical crack in an infinite solid were used by Irwin [3.2] to derive the stress intensity factor for such a crack. The result was

\[ K(\varphi) = \frac{\sigma \sqrt{\pi a}}{\Phi} \left( \frac{a^2 \cos^2 \varphi + \sin^2 \varphi}{c^2} \right)^{1/2} \]  

(3.1a)

with the parametric angle \( \varphi \) and the ellipse axes as indicated in the sketch. \( \Phi \) is the complete elliptical integral of the second kind:

\[ \Phi(k) = \int_0^{\pi/2} \sqrt{1 - k^2 \sin^2 \theta} \, d\theta \]

with \( k^2 = 1 - a^2/c^2 \)

It should be noted that Eq.(3.1a) is valid for \( a \leq c \). For \( a > c \) the equation should be replaced by:
\[ K(\varphi) = \frac{\sigma \sqrt{\pi c}}{\Phi} \left( \frac{c^2 \sin^2 \varphi + \cos^2 \varphi}{a^2} \right)^{\frac{1}{4}} \]  \hspace{1cm} (3.1b)

In the literature, \( \Phi \) is usually replaced by the shape factor \( Q \), which is equal to \( \Phi^2 \). The shape factor can be accurately approximated by

\[ Q = 1 + 1.464 \left( \frac{a}{c} \right)^{1.6} \quad \text{for} \ a/c \leq 1 \]  \hspace{1cm} (3.2a)

\[ Q = 1 + 1.464 \left( \frac{c}{a} \right)^{1.6} \quad \text{for} \ a/c > 1 \]  \hspace{1cm} (3.2b)

For a surface crack in a plate with a finite thickness and a finite width (see Fig.3.1.), \( K \) will also depend on the plate thickness (\( t \)) and the width (\( W \)), which is accounted for by the geometry ratio \( a/t \) and \( c/W \). The stress intensity factor at any point along the crack front can thus be written as:

\[ K(\varphi) = \sigma \sqrt{\pi a} \left[ \frac{\pi a}{Q} \right] F\left( \frac{a}{t}, \frac{a}{c}, \frac{c}{W}, \varphi \right) \]  \hspace{1cm} (3.3)

In the literature Eq.(3.3) is sometimes written as:

\[ K(\varphi) = M(\varphi) K(\varphi)_w \]  \hspace{1cm} (3.4)

with \( K(\varphi)_w \) as the solution for the infinite solid (Eq.3.1) and \( M(\varphi) \) as the magnification factor. It then follows from Eqs. (3.1), (3.3), and (3.4) that

\[ M(\varphi) = \frac{F\left( \frac{a}{t}, \frac{a}{c}, \frac{c}{W}, \varphi \right)}{\left( \frac{a^2 \cos^2 \varphi + \sin^2 \varphi}{c^2} \right)^{\frac{1}{4}}} \]  \hspace{1cm} (3.5)

In the past the effect of finite thickness for a surface crack was sometimes separately accounted for by a front face correction and a back face correction factor.
3.3. **Approximate methods and results of K of a semi-elliptical surface crack in a finite plate.**

Early solutions to the K value of a semi-elliptical crack in a finite plate (see Fig.3.1) were usually based on clever estimates of the influence of the back face and the front face on the K value of an elliptical crack in an infinite solid. Later, numerical methods were developed. Recently, many numerical results are becoming available. Different methods are employed including the finite element method, the boundary element method, the alternating method, the line spring method, and the body force method. The line spring method and the alternating method are reported to provide considerable savings in computer expense compared to the finite element method. For analyzing complex three-dimensional cracks, the boundary element method and the alternating method used in conjunction with the finite element method or the boundary element method offer a simple mesh idealization of the body. A three-dimensional body in the boundary element method, is meshed only on the surface, thus it reduces data preparation and leads to a smaller system of algebraic equations to be solved. In the alternating method the crack tip stress field is modelled analytically, hence it is not necessary to take into account the presence of the crack in the mesh idealization. In this section some approximate methods and selected results of K solutions of a semi-elliptical surface crack are briefly discussed.

**Irwin’s engineering estimate**

The first attempt to approximate the K value of a semi-elliptical surface crack was made by Irwin [3.2] in 1962. Irwin considered the effect of front face, back face corrections and plasticity. For the deepest point, the point of maximum depth(φ=90°), Irwin’s engineering estimate gave

\[
K = \frac{1.12\sigma\sqrt{a}}{\sqrt{\phi^2 - 0.212\left(\frac{\sigma}{\sigma_y}\right)^2}} \quad \text{(3.6)}
\]

In the range \(0 \leq a/t \leq 0.5\) and \(0 \leq a/c \leq 1\), the factor 1.12 is supposed to account for the effect of the front surface, while the back surface effect is supposed to be still insignificant. The term \(0.212(\sigma/\sigma_y)^2\) (\(\sigma_y =\) yield stress) is a plastic zone correction. It is applied [3.2]
because crack tip plasticity introduces a behaviour as if the crack is slightly longer. For fatigue stress cycles the correction is very small.

**SESA *best-estimate* solution**

In order to improve the state-of-the-art of three-dimensional fracture analysis, a workshop was held at the Battelle Columbus Laboratories under the auspices of the Society of Experimental Stress Analysis (SESA). A series of benchmark crack configurations was selected to be studied in more detail including a semi-elliptical surface crack with a/c = 0.5. K solutions of the benchmark crack from different investigators were compared in [3.3]. It was concluded that the finite element results of Raju and Newman [3.4] and Atluri and Kathiresan [3.5], together with the boundary element method of Heliot et al. [3.6] gave acceptable results with ± 5% precision. Based on the above mentioned numerical results, a best estimate curve for a/c = 0.5 with a/t = 0.25 and 0.75 was proposed (see Fig. 3.2). The best estimate curve includes an error band of ± 3%. Compared to photoelastic data the proposed curve is within ± 10%. The agreement was considered reasonable in view of differences of crack geometry and Poisson's ratio between the photoelastic specimen and analysis. Moreover, photo-elastic measurements have a limited accuracy.

**The finite element method**

The finite element method has been used over the past two decades as one of the most powerful numerical tools for the solution of crack problems in fracture mechanics. Much efforts have been given on applying the finite element method to evaluate stress distributions around the crack tip, which are characterized by the square root singularity. The square root singularity of the stress field around the crack tip can be modelled with special crack tip elements. One type of crack tip elements induced a $\sqrt{r}$ displacement in the shape function representation, which results in $1/\sqrt{r}$ stress singularity. The square root stress singularity in two-dimensional problems can also be obtained by collapsing an eight noded quadrilateral element into a triangular element by coalescing nodes along one side and moving mid-side nodes to a quarter point position. Newman and Raju used a singular element developed by Tracey [3.7] to analyse part-through crack problems. Singular elements in the form of a pentahedron were used at the crack front (see Fig. 3.3). The singular elements were surrounded by isoparametric elements. The shape functions of displacement distributions of the singular elements were
obtained by modifying the shape functions of the isoparametric elements to give square root functions. The value of $K$ was extracted by using the nodal force method, where the nodal forces normal to the crack plane, ahead of the crack front were used to calculate local $K$-values. The value of $K$ at the crack front was then obtained by extrapolation. The results were given in graphical form [3.8]. Calculations were extended later to include bending load. Moreover, the results were also approximated by equations [3.9 and 3.10]. Additional corrections for $a/c \geq 1$ were included in [3.11]. The function $F$ in Eq.(3.3) was chosen to be:

$$F = [M_1 + M_2(a/t)^2 + M_3(a/t)^4] g f_q f_w$$  \hspace{1cm} (3.7)$$

where $M_1$, $M_2$, $M_3$ are functions of $a/c$, $f_q$ is the angular function from the embedded elliptical crack solution (the same function as in Eq.3.1), $g$ is a crack depth correction as a function of angle $\varphi$, and $f_w$ is the finite width correction. The expression for the terms in Eq.3.4 can be found in Appendix A.

Using 20 node collapsed quarter point singular elements Wu [3.12] calculated $K$ of semi-elliptical surface cracks. The value of $K$ was evaluated from nodal point displacements. $K$ at the crack tip was obtained by extrapolation. A favourable agreement with the Newman-Raju results was found (no more than 1-3% differences in most case).

The boundary element method

Methods to obtain the square root singularity in the boundary element method are similar to methods used in the finite element method, i.e. special crack tip elements or quarter-point elements are adopted. The latter elements have been extensively used in the application of the boundary element method to both two- and three-dimensional crack problems. Palusamy and Heliot [3.13] compared $K$ values for various part-through crack configurations calculated with the boundary element method and the finite element method. Quarter-point elements were used in the boundary element method to simulate the square root singularity at the crack tip. The value of $K$ was obtained by extrapolation of displacements close to the crack tip. For the case of a semi-elliptical surface crack under tension the difference between the boundary element method and the finite element method is less than 5%.
The alternating method

In the alternating method, solutions of two stress analysis problems are needed, namely the stresses in the uncracked body and the stresses in an infinite solid with cracks. The former problem can be solved with either the finite element method or the boundary element method, and the second problem with available analytical solutions. The boundary conditions of the cracked body are satisfied by alternating between the two solutions. The procedure of the computation is as follow:

1. Calculate the stress distribution in the uncracked body, including the region where the crack is present.

2. To fulfill the boundary condition at the crack surface, the normal stresses on the crack surface have to be eliminated. Use the opposite of the normal stress at the crack surface obtained in step 1, and calculate K for an infinite solid as the first step of the iteration procedure.

3. The normal stresses in 2 are also used to calculate stresses in the infinite solid to obtain the stresses on all the external surfaces of the finite solid.

4. The opposite of stresses on the external surfaces are then considered as the load on the uncracked solid. Iteration steps 2 and 3 are then repeated, which gives the first correction to K obtained in step 2.

The iteration process is continued until the normal stresses on the crack surface are negligibly small. For a semi-elliptical crack four to five iterations are needed to obtain a satisfactory K solution. It should be noted that the stresses in steps 2 and 4 have to be fitted to a polynomial function.

Raju et al. [3.14] applied the finite-element-alternating method and the finite element method to obtain K for small surface and corner cracks. The relative crack depth (a/t) range is from 0.05 to 0.2, and the crack aspect ratio (a/c) range from 0.2 to 1. The small crack geometry (a/t ≤ 0.2) was not analysed in the previous work of Newman and Raju [3.8-3.11]. The results obtained with the finite element alternating method agreed well (within 5%) with the finite element results. The finite element results were also compared to the empirical equations in [3.9-3.11]. The K values from the empirical equations are generally within 3% of the finite element results for small cracks. Thus the empirical equations are also accurate for small cracks.
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The line spring model
The basic idea of the line spring model is to replace a plate containing a semi-elliptical crack with a plate containing a through crack, see Fig.3.4. The presence of the semi-elliptical crack is represented by the introduction of line springs each being equivalent to plain strain single-edged crack plates. Each spring is carrying a normal load T and a bending moment M which are related to displacement δ and rotation θ. The quantities T, M, δ, and θ are assumed to be functions of the coordinate along the through crack. Once the relation between T, M and δ, θ at each spring is defined, K is evaluated from the two-dimensional elasticity problem of a plate with additional boundary conditions along the line spring.
Miyoshi et al. [3.15] used the line spring model together with the finite element method. Eight noded thick shell elements are used to represent the plate or shell, while the surface crack is modelled with quadratic isoparametric line spring elements. Compared to the finite element results of Newman and Raju, the line spring model of Miyoshi differs less than 3%.

The body force method
The body force method uses a stress field derived from point forces acting in an infinite solid. The boundary conditions are satisfied by applying body forces (continuously embedded point forces) along the boundary (e.g. crack face, back face). In [3.16] Nisitani et al. used the body force method to analyse a surface crack in a finite thickness plate. The relative crack depth (a/t) range is from 0.1 to 0.6, and the crack aspect ratio (a/c) range from 0.25 to 1. The finite thickness plate is replaced by a semi-infinite solid, which means that the boundary conditions of the front surface are automatically satisfied. The boundary conditions at the crack surface and the back surface are satisfied by applying body forces along these imaginary surfaces in the semi-infinite solid. The density of the body forces is adjusted to satisfy the boundary conditions. The results were given in tabulated form and also as equations fitted to the numerical results. A reasonable agreement with the Newman-Raju results was obtained, the maximum difference was 4.5%. Fett [3.17] extended the numerical results of Isida in [3.16] and provided fitted formulas for the extended numerical results. The extended results were claimed to have a maximum error of less than 2%.
3.4. Free surface effect on the stress intensity factor

Analytical and numerical results, particularly for a through crack, showed that the square root singularity is lost when a crack intersects a free boundary at right angles. Benthem [3.18] showed that for a quarter plane crack in a semi-infinite body the singularity varies between -0.5 and -0.332 for Poisson's ratio between 0.0 and 0.5. Shivakumar and Raju [3.19] used the finite element method to study the crack tip singularity of a through crack. Two singularities were found along the crack front. For a Poisson ratio of 0.3 the square root singularity dominates over 96% of the interior of the crack front, while the second singularity dominates at the region close to free surface. The power of the second singularity corresponds to Benthem's results.

Raju and Newman [3.8] studied the effect of the free surface on the value of K of a semi-circular crack by refining the mesh near the free surface. The results showed that the stress intensity factor drops off rapidly in the "boundary-layer" region near to the surface. However, the stress intensity distribution in the interior was unaffected by the refinement. Moreover, the boundary-layer effect is confined in a very thin layer close to the surface. Experimental results of Smith et al. [3.20] with frozen stress photoelasticity also showed a reduction of the stress intensity factor close to the free surface.

Because of the existence of the boundary-layer effect the value of the stress intensity factor on the surface is debatable. However, in view of the highly localized boundary-layer effect the value of K on the surface can be interpreted as an average stress intensity factor near the free surface [3.8]. In the crack growth analysis of a semi-elliptical surface crack, care should be taken when analyzing the crack growth on the surface using the value of stress intensity factor of the surface point.

3.5. Experimental verification of stress intensity factor solutions

In contrast to the numerous numerical results of K for semi-elliptical surface cracks, only limited experimental investigations were performed. The methods adopted were:

-photo-elasticity
-Kr, measurements in static tests
-K derived from crack growth data in CA tests.
Smith [e.g. 3.20 and 3.21] developed an experimental technique using the frozen-stress photelastic method to estimate the K distribution along the crack front. Experiments were done on photelastic specimens. The photelastic material have a mechanical behaviour which depends on temperature. At room temperature, the material exhibit time dependent mechanical responses, while above a certain temperature called critical temperature, it is essentially elastic. Above the critical temperature the material is also optically more sensitive to load. An initial starter notch was produced by striking a sharp blade on the specimen surface. The model was heated to a critical temperature and loaded monotonically until the crack begins to grow. After reaching the desired size, the load was reduced and the model was cooled. After removal of the load, the deformations and stress fringes produced at the critical temperature were retained. Slices for photelastic analysis were made at several locations along the crack front. The stress fringes on the slices were proportional to the maximum shear stresses, which can be converted to the stress intensity factor. The experimental results in [3.21] revealed a scatter of up to 10% above the Newman and Raju solution. It should be noted that the Poisson's ratio (ν) of the photelastic material above the critical temperature is approximately 0.5, while the Newman and Raju solution was obtained for ν = 0.3. In [3.21] and [3.22] Smith et al. quoted unpublished finite element results of Newman and Raju for ν = 0.45 which gave an average increase of K of 6% above the results for ν = 0.3. The numerical results with ν = 0.45 compares well with the photelastic results, with a difference of less than 5%. The difference was within the scatter band of experimental results (6%).

Using fracture data of surface cracked tension specimens made of brittle epoxy material Newman [3.22] examined fourteen K solutions of semi-elliptical surface crack available up to the year 1977. The data included results of 150 fracture tests on specimens with a/t ranging from 0.15 to 1. The average fracture toughness (denoted as $K_{\infty}$) was calculated for each solution using K at failure for data where a/t $\leq$ 0.5. The small value of a/t ratio was adopted to determine $K_{\infty}$, because at small a/t ratio the fourteen solutions analysed agree within 5%, thus the calculated fracture toughness should also be in fair agreement. The calculated K at failure for all data were then compared to the fracture toughness, $K_{\infty}$. The analysis showed that the Raju-Newman solution [3.8] gave the best results by correlating 95% of the data within ± 10% scatterband.
Aboutorabi and Cowling [3.23] obtained empirical K-values by comparing the crack growth rates at different points along the crack front of a surface crack to the crack growth rates of compact tension specimens. Empirical K-values were calculated by substituting the crack growth rates of the surface cracks into the Paris relation obtained from compact tension specimens. A good agreement was found except for \(a/t \geq 0.65\) where the largest difference was in the order of 10%.

### 3.6. Some illustrations of the Newman-Raju equations

Because the Newman-Raju expressions do not easily give an impression of the geometry effects on the K-values along the crack front, two illustrations are presented in Figs. 3.5 and 3.6. Figure 3.5 shows K as a function of \(a/c\) for three values of the crack depth, and for \(S = 100\) MPa, \(W = 100\) mm, \(t = 10\) mm. The curves apply to the deepest point A (\(\varphi = 90^\circ\)) and the surface point C (\(\varphi = 0^\circ\)). The graph shows:

1. Increasing K-values for deeper cracks (larger a).
2. Decreasing K(A)-values for a decreasing c and constant a (increasing a/c).
3. K(A) > K(C) for small a/c (slender cracks), but K(C) > K(A) for larger a/c (deep cracks).
4. A tendency for decreasing \(K_c\) for decreasing a/c (more slender cracks), except for a = 8 mm.

The trends agree with intuitively expected influences. The increase of \(K_c\) for a = 8 mm (a/t = 0.8) at low a/c is due to the width effect (for a/c = 0.2 and a = 8 mm, 2c/W = 0.80).

The second illustration is given in Figure 3.6 for a/t = 0.5. The distribution of K along the crack front is shown for five different crack shapes, varying from a slender crack (a/c = 0.2) to a relatively deep crack (a/c = 2). For the most slender crack (a/c = 0.2) \(K_{\text{max}}\) occurs at the deepest point (\(\varphi = 90^\circ\)). For a relatively deep crack (a/c = 2) \(K_{\text{max}}\) occurs at the surface (\(\varphi = 0^\circ\)). For a/c = 0.6, 0.8, and 1.0 the variation of K along the crack front is rather limited.

It should be pointed out that the Newman-Raju equations are different for a/c ≤ 1 and a/c > 1. For a/c ≤ 1 the equations are based on FEM calculations for a/c = 0.2, 0.4, 0.6 and 1 respectively, which seems to be a reasonable coverage. However, the equations for a/c > 1, FEM results of two a/c values could be used only, i.e. for a/c = 1 and a/c = 2. Since the curves in Figure 3.5 smoothly continue from the (a/c < 1) regime to the (a/c > 1)
regime, while the K-decrease until $a/c = 2$ is relatively small, it may be expected that the Newman-Raju equations give reasonable K-estimates, at least up to $a/c = 2$. It should be realized that using the equations outside $0.2 \leq a/c \leq 2$ is not substantiated by FEM calculations.

3.7. Concluding remarks

Different methods have been used to obtain $K$ solutions for semi-elliptical surface cracks in a finite solid. Recent solutions are in reasonable agreement. Selected results quoted in section 3.3 show that differences of less than 5\% can be achieved with different techniques. In fact, a large number of solutions have been reported with similar results. This agreement shows the convergence of the solutions to the "right answer".

In the literature the Newman-Raju solution is always used as a basis of comparison to other solutions. If the comparison is considered as a check to the Newman-Raju solution, an extensive check has been done, which confirm the usefulness of the Newman-Raju solution. The available experimental results also support this conclusion. The solution is widely used as the reference solution in the weight function method to obtain $K$ for a more complex loading, such as polynomial crack face loading. The closed form solution of the Newman-Raju facilitates the analysis of semi-elliptical surface cracks particularly the cycle-by-cycle prediction of fatigue life.

The lack of lateral constraint at the free surface complicates the stress analysis to evaluate $K$ at the surface. However, it was shown that the square root singularity still dominates 96\% of the interior of the crack front. This means that the boundary layer, where another singularity is dominant, is confined to a very thin layer close to the surface. Moreover, the stress intensity factor distribution in the interior is not affected by the presence of the thin boundary layer.

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Chapter 3. Stress intensity factors of semi-elliptical surface cracks


Chapter 3, Stress intensity factors of semi-elliptical surface cracks

Fig. 3.1. A surface crack in a finite plate subjected to tension load.

![Diagram of a surface crack in a finite plate](image)

Fig. 3.2. SESA "best-estimate" magnification factor $M(\phi)$ with 3% scatter band.

![Graph showing $M(\phi)$ vs. parametric angle $\phi$](image)
Fig. 3.3. A singular pentahedron element at the crack tip. The pentahedron is obtained from a hexahedron by coalescence of nodes 1, 5 and 2, 6.

Fig. 3.4. The basic idea of the line spring model. The plate with a semi-elliptical crack (Fig. 3.4a) is modelled as a plate with a through crack (Fig. 3.4b) and a row of plane strain single-edged crack plates (Fig. 3.4c and Fig. 3.4d), each being equivalent to a spring (Fig. 3.4e).
Fig. 3.5  Effects of crack depth (a) and crack shape (a/c) on K at points A and C.

Fig. 3.6  Effect of crack shape on K along crack front (a/t = 0.5).
Chapter 4

Growth of Surface Cracks under CA Loading

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4.1. Introduction

The growth of surface cracks under fatigue loading is a complex process due to the three-dimensional nature of the problem. Three-dimensional stress analysis using different methods have resulted in accurate K solutions of surface cracks. Another three-dimensional aspect of surface crack problem is the varying state of stress along the crack front. The crack front at the surface is in plane stress condition, while in the interior plane strain predominates. Different states of stress imply different plastic zone sizes along the crack front. Different plastic zone sizes left in the wake of the crack lead to a different crack closure stress ($S_{op}$) level along the crack front. Due to the lack of access, this variation of the $S_{op}$ can only be measured with a fractographic technique.

In section 4.2 characteristic features of surface crack growth under fatigue loading will be discussed, including crack shape development, crack closure behaviour, and crack edge trailing. The fractographic technique to measure $S_{op}$ mentioned in the literature will be addressed in section 4.3. Crack closure measurements of the present investigation are introduced in section 4.4. A prediction model for fatigue crack growth of surface cracks under CA loading, employing the $S_{op}$ distribution deduced from the measurements, is presented in section 4.5.

4.2. Common features of surface crack growth

4.2.1. Crack shape development in a semi-infinite body.

In an infinite body with an elliptical crack, or in a semi-infinite body with a semi-elliptical surface crack (see sketch), there are two dimensions only, viz. the elliptical axes $a$ and $c$. 

![Diagram of infinite and semi-infinite bodies with elliptical cracks](image-url)
Under constant amplitude loading crack growth will generally cause a change of the aspect ratio \( a/c \). A slender crack (low \( a/c \)) will have a larger \( K \)-value at point A and a lower \( K \)-value at point C. As a consequence it will grow to a more circular shape. It is easily understood that for an infinite body the crack will grow towards a circular shape. A semi-elliptical surface crack in a semi-infinite body will not grow to a semi-circle, because then \( K_C > K_A \), which implies that \( \frac{dc}{dN} > \frac{da}{dN} \). The crack will grow to a stable shape with an \( a/c \) ratio smaller than 1. If a stable ratio is reached then the requirement to maintain the same ratio is:

\[
\frac{da/dN}{dc/dN} = \frac{a}{c}
\]

(4.1)

A simple estimate of the stable \( a/c \) ratio can be obtained if it is assumed that the same Paris relation is applicable to crack growth in all directions. Equation (4.1) then requires:

\[
\frac{C \Delta K_A^n}{C \Delta K_C^m} = \frac{a}{c}
\]

(4.2)

Following the terminology of the previous chapter, \( K \) can be written as:

\[
K = S \sqrt{\frac{\pi a}{Q}} F
\]

(4.3)

with \( Q \) as the shape factor (Eqs.3.2a and 3.2b) and \( F \) as a correction factor to account for the interrelations between the free surface, the crack depth, the crack shape, and the parametric angle \( \phi \). Substitution in Eq.(4.2) leads to:

\[
\left( \frac{F_A(\phi = \pi/2)}{F_C(\phi = 0)} \right)^m = \frac{a}{c}
\]

(4.4)

Adopting the Newman-Raju equations (Appendix A) the final result for the stable \( a/c \) ratio is:

\[
\frac{a}{c} = \left( \frac{1}{1.1} \right)^{2m/(2+m)}
\]

(4.5)

Since \( m \) is usually larger than 2 (i.e. in the range of 3 to 4) the exponent \( 2m/(2+m) \) is
slightly larger than 1. As a consequence the stable a/c ratio will be lower than 1, but still fairly close to 1 (semi-circle). An illustrative calculation was made for a surface crack starting with a slender crack, a/c = 0.2, and adopting the Paris relation found in the present investigation for 7075-T6 (see section 4.5). The correction factors, $F_A$ and $F_C$, were calculated with the Newman-Raju equations, assuming that for the semi-infinite body $a/t$ and $c/w$ are zero. The result is given in Fig.4.1, which shows the successive crack front contours after each 10000 cycles in the upper graph and the increase of a/c in the lower graph. The stabilized a/c value according to Eq.(4.5) ($m = 2.88 \rightarrow a/c = 0.894$) is indicated by the dashed line. Initially the crack rapidly grows in the depth direction to a much less slender shape. The steep increase of a/c is then followed by a slow asymptotic increase towards the stabilized value.

In order to see the significance of the above trends for finite dimensions, it has to be realized that some simplifications were made:

(i) The crack shape was assumed to remain semi-elliptical, and crack growth was calculated for points A and C only.

(ii) The occurrence of crack closure was ignored, which is significant because more crack closure should be expected for point C (surface) than for point A (deepest point).

(iii) The dimensions were infinite. Since significant crack growth is required to approach the stabilized a/c, this may not occur in specimens with a finite thickness and width.

These aspects are discussed in sections 4.3 to 4.5.

4.2.2. Crack shape development in specimens

There is ample experimental evidence to show that the shape of a surface crack is changing as the crack grows under fatigue loading. The aspect ratio (a/c) of an initially semi-elliptical notch changes during fatigue. Fig.4.2 shows the changes of the crack aspect ratio during fatigue in plates under cyclic tension. The results were compiled in [4.1] from test data on different materials (various steels, aluminum alloys, and a titanium alloy). An observation from Fig.4.2 is the general trend that semi-elliptical surface cracks grow in such a way that the crack aspect ratios converge to a common value.

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The change of the crack shape during fatigue loading can be predicted by assuming again that the shape remains semi-elliptical and that the Paris equation is applicable to both directions, i.e.

\[
\frac{da}{dN} = C_a(\Delta K_a)^m \quad (4.6.a)
\]

\[
\frac{dc}{dN} = C_c(\Delta K_c)^m \quad (4.6.b)
\]

where \(\frac{da}{dN}\) and \(\frac{dc}{dN}\) are the crack growth rates at the deepest point and on the surface respectively. In Eqs. (4.6a) and (4.6b) the same slope factor \(m\) is assumed, but crack growth in the two directions may occur at different rates \((C_A \neq C_C)\). Equations (4.6.a), (4.6.b) and (4.3) give

\[
\frac{da}{dc} = \frac{C_a(F_s)^m}{C_c(F_c)^m} \quad (4.7)
\]

\(F_s\) and \(F_c\) are the \(K\) correction functions for the deepest point \((\varphi = 90^\circ)\) and on the surface \((\varphi = 0^\circ)\) respectively. Using a closed form solution of \(F\), e.g. the Newman-Raju solution discussed in the previous chapter, a relation between \(a/c\) and \(a/t\) can be derived and a crack-shape development curve \((a/c\ as\ a\ function\ of\ a/t)\) can be plotted for different initial semi-elliptical notches.

The crack shape development was used to assess the accuracy of \(K\) solutions of surface cracks. The predicted crack shape development in comparison to empirical results is an indirect measure of the validity of the solution. Hosseini and Mahmoud [4.1] made such a comparison for several \(K\) solutions. The predicted crack-shape developments were compared to data taken from 20 references [see Fig.4.2]. The ability to predict the crack shape development was evaluated by considering the standard deviation of the error of the predicted crack shape. It was found that the Newman-Raju solution gave the lowest standard deviation. Similar predictions of the crack shape development were made by Nair [4.2], Jolles and Tortoriello [4.3], Muller et al. [4.4], Gorner [4.5] and Xian [4.6]. In the crack shape development predictions based on Eq.(4.7), the crack growth rate on the surface is assumed to be slower than at the deepest point through a difference in the
constants in the Paris crack growth relation, \( C_a = (0.9)^n C_a \). This assumption was based on the experiments reported by Corn [4.7], showing that small semi-circular surface cracks propagate as a semi-circular. Because the value of \( K \) for a semi-circular crack on the surface (point C) is about 10% higher than at the deepest point (point A) then \( C_a \) must be lower than \( C_a \). Newman-Raju suggested that this observation is due to a different crack growth resistance at the surface and at the deepest point. Jolles and Tortoriello argued that the slower crack growth at the surface has to be associated with a difference between the crack closure level at the surface and at the deepest point. It should be pointed out here that the Newman-Raju argument implies a different crack growth resistance, whereas Jolles and Tortoriello start from a different crack driving force, implicitly assuming the same crack growth resistance. Jolles and Tortoriello did not measure \( S_y \) for surface cracks. Crack closure was measured on compact tension specimens, which was assumed to be representative for the crack closure level of surface cracks at the material surface. Crack closure at the deepest point was estimated from the crack growth analysis of shallow semi-circular surface cracks. Using \( U_c/U_a = 0.91 \) (following the definition of Elber \( U = \Delta S_{cr}/\Delta S = \Delta K_{cr}/\Delta K \), see Fig.4.3) shallow surface cracks propagates as a semi-circle, which agrees with the result of Corn [4.7] mentioned before.

The use of Eq.(4.7) in predicting the crack shape development implies that an influence of the cyclic stress level is not present. The experimental results of Hodulak [4.8],[4.9], [4.10] showed that this is not completely true, especially when the maximum stress of the cyclic stress is close to the yield point, \( S_{0.2} \). For such high stress levels a lower final crack aspect ratio was obtained (the final \( a/c \) for \( S_{max} = S_{0.2} \) is about 0.7 of \( a/c \) at a lower maximum cyclic load). The effect of the load level was explained by considering the plastic zone correction to crack size. Due to the plane stress condition near the surface, a larger plastic zone occurs. The occurrence of plasticity made the crack behave as if it were larger than its actual size. Because the plastic zone size and such a crack length correction depend on the load level, the effective crack size will depend also on the stress level. A higher load increases the effective length-to-depth ratio of a surface crack. However, a full explanation should also consider crack closure.

It may be noted here, that the recommended procedure in ASME Boiler and Pressure Vessel Code (section XI and Appendix A) for the analysis of surface cracks does not
consider the aspect ratio change. It is assumed that the crack grows with a constant aspect ratio.

4.2.3. Closure behaviour of surface cracks

*Plane strain and plane stress closure level of through cracks*

Experimental evidence has indicated that the level of crack closure is not constant along the crack front of a through crack. Ewalds and Furnee [4.11] measured the variation of the crack opening stress \( S_{op} \) of 2024-T3 aluminum alloy. Measurements were made along the crack front by thinning a specimen in a number of steps by a chemical milling process. The measurements showed that after the first thinning \( S_{op} \) decreased considerably, while further thinning steps did not affect \( S_{op} \) significantly. The large decrease of \( S_{op} \) after the first thinning step was related to the removal of the shear lips (which are usually associated with the plane stress condition), where contact between the crack surfaces mostly occurred. The role of shear lips on crack closure was also observed by Shaw and Le May [4.12]. Their fractographic observations show that rubbing of the crack surfaces occurred mainly on the shear lips, which means that closure contacts take place mainly on the shear lips. Using a fractographic technique (this technique is discussed in section 4.3), Pelloux et al. [4.13], Sunder and Dash [4.14], and Dawicke et al. [4.15] measured \( S_{op} \) of through cracks. They all found that more closure occurred at the surface compared to the center of the specimens. This observation is usually explained by considering the difference between plane strain and plane stress conditions at the crack tip. At the material surface plane stress conditions predominate, while in the interior plane strain conditions prevail. The difference of the state of stress along the crack front implies different sizes of the plastic zone. As a first approximation the size of the plastic zone can be estimated with:

\[
\tau_p = \frac{1}{\pi \alpha} \left( \frac{K}{S_{e2}} \right)^2 \tag{4.8}
\]

The factor \( \alpha \) is usually taken to be 3 for plane strain and 1 for plane stress. This difference in plastic zone size leads to a higher closure level near to the specimen surface, which is in the plane stress condition. Three-dimensional finite element calculations by Chermahini et al. [4.16] on through crack specimens confirm the higher \( S_{op} \) at the surface.
Chapter 4, Growth of surface cracks under CA loading

$S_{op}$ of surface cracks

Different states of stress along the crack front also occur for a surface crack, hence a similar variation of $S_{op}$ can be expected. Fleck et al. [4.17] measured $S_{op}$ of a surface crack in specimens made of BS 4360 50B structural steel. Crack closure on the surface (plane stress) was measured with a strain gage placed behind the crack tip. The plane strain crack closure was measured with a special push rod technique at the specimen mid-thickness, and by a crack mouth gage. Crack closure measurements on the specimen with CA loading ($R = 0.05$) showed that more closure occurred on the surface ($U_c = 0.75$ or $S_{op}/S_{max} = 0.29$), compared to the mid-thickness material ($U_a = 0.85$ or $S_{op}/S_{max} = 0.19$). Note that the $S_{op}$-levels are fairly low.

Ray and Grandt [4.18] performed experiments to measure $S_{op}$ of surface cracks in specimens made of a transparent polymer, polymethylmethacrylate (PMMA), subjected to a cyclic bending load ($R = 0.1$). An optical-interference method was used to determine the crack surface displacement field and to measure local crack closure levels along the crack front. The optical-interference fringe pattern showed that a significant portion of the crack plane, away from the specimen surface is still open at zero load. At increasing bending load, the crack surfaces started to open outwards, and finally reached the surface. A typical results of their crack opening stress measurements is shown in Fig.4.4, which shows that $S_{op}$ on the surface is higher than at other locations along the crack front. For $\varphi \geq 45^\circ$ the crack surface was fully open even at zero load. It should be noted that the higher $S_{op}$ on the surface was partly due to the applied bending fatigue load, which leads to a larger plastic zone on the surface.

4.2.4. Crack edge trailing

Experimental results of surface crack growth under fatigue loading always show crack edge trailing, where the crack front near the surface forms an angle not perpendicular to the specimen surface. An example is shown in Fig.4.5 [4.19]. In the previous section an estimation of the plastic zone in the plane stress and plane strain region has lead to the conclusion that the crack closure stress level near the surface will be higher than in the interior. For a growing fatigue crack it implies that the effective stress intensity factor near the surface is lower than at mid thickness. As a consequence the crack front on the surface will trail behind. The occurrence of trailing behind is usually obvious for part-
through cracks (surface cracks or corner cracks) under VA-loading. The high peak loads cause larger plastic zones and thus promote the trailing phenomenon as illustrated by Fig. 4.5.

4.3. Fractographic technique for $S_{op}$ measurements

The fractographic method to determine $S_{op}$ is based on observations of the striation spacing. A striation is the trace of crack extension during an individual cycle, which is a function of $\Delta K_{eff}$. It thus can provide an indication of $S_{op}$. In order to produce striations that can give indications of $S_{op}$, a characteristic load sequence should be applied. For part-through cracks this technique is qualitatively superior to others ones because it can provide information on the variation of $S_{op}$ along the crack periphery.

Pelloux et al. [4.13] carried out constant-amplitude tests (stress ratio $R=0.05$) on 2124-T351 aluminium alloy specimens ($t = 12.7$ mm). They introduced a block of 10 smaller cycles with the same $S_{max}$ but a smaller $\Delta S$, i.e. a higher $S_{min}$ as for the basic fatigue load see Fig. 4.6. Different $S_{min}$ values were used for the 10 cycles. The magnitude of $S_{min}$ which produced equal striation spacings in the 10 inserted cycles and in the basic fatigue loads was supposed to be $S_{op}$ (same spacing $\rightarrow$ same $\Delta K_{eff} \rightarrow$ same $S_{op}$ because same $S_{max}$). The results indicated a fairly low $S_{op}$ ($= 0.2 S_{max}$, $U = 0.84$).

The method of Pelloux et al. is based on the assumption that $S_{op}$ remains constant during the whole load sequence. The same assumption was also made by Sunder and Dash [4.14], who improved the technique by introducing load sequences as shown in Fig. 4.7. The load sequences consist of cycles with a constant maximum stress ($S_{max}$) and a varying minimum stress ($S_{min}^\ast$). For each subsequent cycle the minimum stress is reduced (or increased) by $\Delta S_{min}$ until it reaches the minimum (or maximum) stress of the first cycle. The maximum and minimum stress in the tests were chosen in such a way that $S_{min} < S_{op} < S_{max}$. For cycles with $S_{min}^\ast < S_{op}$ the effective stress range, $\Delta S_{eff}$ is assumed to be constant, which should result in a constant striation width. Cycles with $S_{min}^\ast > S_{op}$ should leave striations with an increasing width because of the gradual increase of $\Delta S_{eff}$ (and also $\Delta K_{eff}$). By counting the number of striations with equal width, $S_{op}$ can be determined from:

$$S_{min}^\ast < S_{op} < S_{min} + \Delta S_{min}$$
where \( S_{\text{min}} \) is the minimum stress corresponding to the first striations with constant width. Tests of Sunder and Dash on 2024-T3 aluminium alloy specimens of 3 mm thickness resulted in \( S_{\text{op}}/S_{\text{max}} \) in the range between 0.35 to 0.44. For 5 mm thickness specimens, \( S_{\text{op}} \) was found to be:

\[
0.35 < S_{\text{op}} / S_{\text{max}} < 0.43 \quad \text{at the surface}
\]

\[
0.14 < S_{\text{op}} / S_{\text{max}} < 0.21 \quad \text{at mid thickness}
\]

In other words, also in these tests more crack closure occurred near the material surface. The method of Sunder and Dash was recently adopted by Dawicke et al. [4.15] in constant \( \Delta K \) tests (\( R=0.1 \)) on 2024-T351 specimens with a thickness of 9.6 mm. They also found a higher crack opening level near the material surface (\( S_{\text{op}}/S_{\text{max}}=0.3 \)) than at mid thickness (\( S_{\text{op}}/S_{\text{max}}=0.2 \)). They emphasize the 3-dimensional character of crack closure along the crack front.

4.4. **Constant-amplitude tests of the present investigation for \( S_{\text{op}} \) measurements of surface cracks.**

4.4.1. Experimental details

The experimental details and results have been presented in [4.20] and [4.21]. A brief description of the specimens and load sequence will be given here. Specimens were cut from 9.6 mm thick 7075-T6 aluminium alloy plate material, the length was 300 mm and the width was 100 mm. Five different aspect ratios were applied: \( a/c = 1 \) (semi circular), \( a/c = 0.8 \), \( a/c = 0.6 \), \( a/c = 0.4 \) and \( a/c = 0.2 \) (slender surface notch).

The load sequence developed by Sunder and Dash is adopted with the addition of marker load blocks to give an indication of the crack front development. The load sequence is applied as a periodical spectrum consisting of 3 blocks (see Fig.4.8). The first block is the nominal constant-amplitude fatigue load on the specimen. The stress ratio in this block was 0.1, \( S_{\text{op}} \) of this block has to be determined. The second block is the crack opening measurement block with decreasing \( S_{\text{min}} \). The last block is the marker load block which consists of constant-amplitude cycles with high \( R \)-value (\( R = 0.9 \)). Within the whole block \( S_{\text{max}} \) was kept constant. The \( R \) ratio adopted for the marker loads were based on the experience of Minderhoud [4.22] and Friedrich and Schijve [4.23] in testing of corner
cracks at open holes and in lug specimens respectively.

Table 4.1 gives a summary of the tests, which includes the number of cycles in each block, and the initial notch geometry.

4.4.2. Fractographic observations and results
Figure 4.9 shows a typical fracture surface with white bands of marker loads, which represent the crack fronts after a certain number of cycles. The width of the widest marker bands were found to be less than 0.2 mm. The shape of the crack fronts were recorded with an optical profile measuring machine and the scanning electron microscope (SEM). Semi-ellipses were fitted to the measured crack fronts using the least square method. Results show that the shape is very close to a semi-ellipse except near to the surface, where the crack trails behind (see Fig. 4.10).

A typical SEM micrograph is shown in Fig.4.11. It shows striations induced by constant-amplitude cycles with equal-spacing striations. The decreasing-S_{\text{min}} cycles left striations with gradually increasing width. Marker loads did not induce striations on the fracture surface. S_{op} was determined by counting the number of constant width striations of the decreasing-S_{\text{min}} block. As explained in section 4.3 the value of S_{op} should be between S_{\text{min}} of the first striation with equal width and the preceding S_{\text{min}}, i.e.

\[ S_{\text{min}}^* < S_{op} < S_{\text{min}}^* + \Delta S_{\text{min}} \]

where S_{\text{min}}^* is the minimum stress of the first striation with equal spacing. As a first approximation the mean value of these two stresses is taken as S_{op}.

Detailed results of the S_{op} measurements have been presented in [4.20 and 4.21]. The curves of S_{op} as a function of the normalized crack depth (a/t) are shown in Fig. 4.12. Figure 4.13 illustrates S_{op} as a function of the angle \( \varphi \) at different a/t. These figures show the development of S_{op} during crack extension along the crack front. Some tendencies can be observed in these figures i.e.:
1. In general S_{op} close to the specimen surface (\( \varphi = 15^\circ \)) is higher than at any other larger angle \( \varphi \) (Fig.4.13). Exceptions are found for the more slender notches (low a/c_p).
2. During crack extension the S_{op} value for a constant angle \( \varphi \) develops to a maximum
followed by a subsequent decrease near breakthrough (Fig. 4.12).

4.4.3. Discussion

Development of $S_{op}$ and the Ligament Effect

The development of $S_{op}$ for a constant $\varphi$ as shown in Fig. 4.12 can be explained as follows. At the beginning of the crack extension the lower $S_{op}$ is caused by the absence of a plastic zone in the wake of the crack. As the crack extends, plastic deformation left in the wake of the crack also develops resulting in an increasing $S_{op}$. After some growth in the thickness direction the ligament size (see sketch) becomes smaller with a corresponding decrease of restraint to plastic deformation in the ligament. When the plastic zone reaches the back surface net-section yield occurs in the ligament. The large plastic deformation induced by the net-section yielding, i.e. plastic deformation ahead of the crack, will keep the crack tip in an open condition. As a result the crack surface contact is reduced, resulting in a lower $S_{op}$. The crack depth at the occurrence of the net-section yielding can be roughly estimated by assuming that the ligament area carries the yield stress, and the cross sectional area away from the ligament carries the nominal stress:

$$S_{0.2}(t.2c - \pi ac /2) = S_{max}(t.2c) \quad \text{or} \quad a/t = 4/\pi \left(1 - S_{max}/S_{0.2}\right)$$

Substitution of $S_{max}$ and $S_{0.2}$ gives $a = 8.6$ mm or $a/t = 0.9$. Figure 4.12 shows that the $S_{op}$ started to decrease at lower values, i.e. at $a/t = 0.6$ for specimens PCA2, PCA14, and PCA13 and $a/t = 0.75$ for specimen PCA15. This discrepancy might be caused by the fact that the growth of the plastic deformation leading to the net-section yielding does not take place abruptly but progressively. The deviations also arise from the rough estimate in the above equation. However, it is believed that the similar order of magnitude supports the net section yield idea.

As mentioned above an exception to larger $S_{op}$ values near the surface ($\varphi = 15^\circ$) was found
for the more slender notch (Fig.4.13). The reason for this tendency is related to crack initiation and growth at the deepest point, which occurred much earlier than on the surface (see Fig.4.10). After the initiation the crack grew predominantly in the thickness direction. Crack growth along the surface became significant when the crack in thickness direction had grown to approximately half the thickness (see Fig.4.10). At that moment there is already plastic deformation in the wake of the crack at the deepest point, and $S_{op}$ has increased.

**Crack opening stress level**

Figures 4.12 and 4.13 illustrate that $S_{op}$ on the average appears to be fairly low. $S_{op}/S_{max}$ values vary from 0.1 to 0.29. It is easily derived (see Fig.4.3) that:

$$U = \frac{\Delta S_{eff}}{\Delta S} = \frac{1.5S_{op}/S_{max}}{1-R}$$

(4.9)

$S_{op}/S_{max} = 0.1$ to 0.29 implies for $R=0.1$ that $\Delta S_{eff}/\Delta S$ varies from 0.79 to 1.0. In other words the amount of crack closure is rather limited. The low crack closure level can be explained by recalling that:

1. The state of stress is predominantly plane strain due to the thick specimen used.
2. The yield stress of the material (7075-T6) is relatively high ($S_{0.2} = 509$ MPa).

Both effects will result in a small plastic zone around the crack tip. The plastic zone size can be estimated with Eq.(4.8). With $\alpha = 3$ and $S_{0.2} = 509$ MPa, a point at mid thickness with $a = 4.8$mm and $c = 5.5$mm gives $r_p = 0.03$ mm, which is indeed a small value compared to the crack size.

Preliminary crack growth tests and $S_{op}$ measurements using the same 9.6 mm thick Al 7075-T6 plate specimens with through cracks were performed (see Appendix B for detailed results). The stress ratio was again 0.1. $S_{op}$ was measured with an extensometer placed close to the crack tip. The results indicate $S_{op}/S_{max} = 0.28$ ($\Delta S_{eff}/\Delta S = 0.8$), which is still a low crack opening stress level. This value is approximately the same as the highest crack closure of part-through cracks in the present investigation ($S_{op}/S_{max} = 0.29$ for specimen PCA14 at $a/t = 0.6$, $\phi = 15^\circ$). It is interesting to recall the results of Pelloux et al. [4.13], Dawicke et al. [4.15] and Fleck et al. [4.17] discussed before. Although their tests were
done on different materials and different specimen thickness they all found $S_{op}/S_{max} \approx 0.20$ at the interior points. For locations near the surface Dawicke et al. found $S_{op}/S_{max} = 0.3$ which is in agreement with the results of Fleck et al. who used a near tip strain gage to measure plane stress closure ($S_{op}/S_{max} = 0.29$). From the results of Dawicke et al. and Fleck et al. it can be deduced that $S_{op}/S_{max}$ in the interior is about 66% of $S_{op}/S_{max}$ on the surface. Sunder and Dash [4.14] found a somewhat higher $S_{op}/S_{max}$ near the surface. The higher $S_{op}/S_{max}$ at the material surface is generally attributed to more plastic deformation (plane stress) at the surface. It implies that the large plastic zone will increase $S_{op}$ at the surface, and at the same time reduce $S_{op}$ away from the surface due to the propping effect.

$S_{op}$ variation along the crack front

To obtain an average trend of the variation of $S_{op}$ along the crack front, curves of $S_{op}/S_{max}$ vs $\varphi$ for $a/t = 0.6$ was plotted for each specimen (see Fig.4.14a). Specimen PCA6 (slender notch) was not included because of reasons mentioned above. The value $a/t = 0.6$ was chosen because at that thickness the plastic zone has developed and $S_{op}$ is not yet influenced by ligament yielding. Figure 4.14a shows that the variation of $S_{op}/S_{max}$ along the crack front is approximately linear and the slope of each specimen is similar. The curves $S_{op}/S_{max}$ were then normalised to the values of $S_{op}/S_{max}$ on the surface, which was obtained by extrapolation. Figure 4.14b shows that the ratio of $S_{op}/S_{max}$ at the deepest point to $S_{op}/S_{max}$ at the surface varies from 0.71 to 0.80. The high ratio indicates that the variation of $\Delta S_{eff}$ along the crack front is not large. The results of Dawicke and Fleck discussed before showed a ratio of 0.66.

Limitation of the fractographic method to determine $S_{op}$

The fractographic method to determine $S_{op}$ is based on the observation of striations. Not all materials show regular striations. Materials with limited deformation possibilities (limited slip systems), such as high strength steel, show poorly defined striations or striations may not develop at all. For these materials the fractographic method can not be used to determine $S_{op}$.

For materials with regular striations patterns, the method described in section 4.3 has a limited accuracy. The first source of the limited accuracy is the measurement of the striation width. In some cases the difference of the striation width is rather small, to judge which striation is the first one having equal width. The second source is the
approximation that \( S_{op} \) is taken as the mean value of \( S^*_{min} \) and \( S^*_{min} + \Delta S \), which implies that the accuracy is \( \Delta S / 2 \). Because all tests were done with \( \Delta S = 3 \text{MPa} \), then the maximum accuracy of the measured \( S_{op} \) is 1.5 MPa.

4.4.4. Conclusions

1. The crack closure level \( S_{op} \) varies along the crack front. In general the variation is relatively small (\( \Delta K_{eff} / \Delta K \) from 0.79 to 1.00). It is also remarkable that \( S_{op} \) is relatively low, which is associated with the plane strain conditions and the high yield stress. More closure occurred at the specimen surface (plane stress effect). The ratio of \( S_{op}/S_{max} \) at the deepest point to \( S_{op}/S_{max} \) on the surface indicates that \( S_{op} \) does not change significantly along the crack front. Ratios of 0.71 to 0.8 implies that \( S_{op} \) at the deepest point is only 20% to 29% lower than on the surface.

2. A remarkable observation is that \( S_{op} \) becomes smaller at the deepest point of the crack if the crack is relatively deep. This is attributed to an increasing size of the plastic zone ahead of the crack with a tendency to net section yield in the ligament material near the deepest point of the crack.

4.5. Fatigue life prediction under CA loading

4.5.1. Introduction

Fatigue life prediction of a cracked component is a very important objective of fracture mechanics analysis. For fatigue life predictions of part-through cracks (surface cracks or corner cracks) the variation of \( K \) along the crack front must be considered. As discussed before the \( K \) solutions, e.g. the Newman-Raju equations, are sufficiently accurate in describing the \( K \) variation along the crack front of a semi-elliptical surface crack. As long as the surface crack geometry does not deviate very much from a semi-elliptical shape, the use of the \( K \)-solutions is justified.

Crack growth of surface cracks, in fact occurs at all points along the crack front. However, in the past the crack growth process was usually modelled by crack growth at one point or two discrete points along the crack front in order to avoid complex calculations. Fatigue life predictions based on a single point of the crack front only, consider crack growth in the depth direction. The crack is assumed to grow with a constant aspect ratio (a/c constant). Empirical evidence and analytical considerations indicate this approach to be
invalid. Most predictions of surface cracks in the literature consider crack growth at two points. This includes the assumption of the semi-elliptical crack shape with two characteristic dimensions, i.e. crack length(c) and crack depth(a). Crack growth calculations are made separately for the crack depth(a) and crack length(c), thus the crack aspect ratio may change. To simulate a continuous crack growth process along the crack front, calculations at more than two points are desirable. Lof [4.24] performed crack growth predictions of corner cracks at open hole specimens and in pin-loaded lugs with 3 points. The coordinates of the points were defined by \( \varphi = 22.5^\circ, 45^\circ, \) and \( 67.5^\circ. \) The points on the surface \( (\varphi = 0^\circ) \) and on the bore-surface intersection \( (\varphi = 90^\circ) \) were not used because K-values at these points were considered to be unreliable.

Measurements of \( S_{op} \) of surface cracks discussed in the previous section have shown that \( S_{op} \) varies along the crack front. The results discussed in the previous section showed that, on the average the ratio between the closure level at the deepest point to the point near the surface is approximately 0.80. Incorporating different \( S_{op} \) along the crack front might lead to more accurate predictions.

Two different prediction schemes for crack growth prediction of surface cracks under CA loading are evaluated in this section. In both predictions, cycle by cycle crack growth is calculated. The first prediction scheme does not take into account crack closure. Fatigue life predictions are performed for an increasing number of points along the crack front (2, 4, 8, 16 and 32 \( \varphi \)-values). The purpose of performing predictions with an increasing number of calculation points is to find the smallest number of calculation points, which results in predicted fatigue lives close to predictions using many points. The second part is focused on the influence of incorporating \( S_{op} \) on the fatigue life prediction. The results of the prediction models will be compared to our own test results and to data from the literature. Only crack growth life until break-through will be predicted. At the moment of break-through the crack front moves rapidly to a full through crack with two crack fronts perpendicular to the material surface. This conversion will not be considered, but it should be pointed out that the remaining number of cycles after break-through is relatively small. Prediction models which can give the best prediction results will be used in the fatigue life prediction of surface cracks under variable-amplitude loading later in this investigation.
4.5.2. CA test data of surface cracks

Data from two sources could be used for the evaluation of the above prediction exercise, viz. our own data on 7075-T6 presented in section 4.3 and the results of Hall et al.[4.25] on 2219-T851 aluminum alloy, 6Al-4V titanium alloy and 9Ni-4Co-0.2C steel. The input data are recapitulated below.

**Fatigue test results of the present investigation.**

The fatigue life results of the 7075-T6 aluminum alloy specimens discussed in section 4.3 on crack closure measurements, are given in Table 4.1. It includes the initial semi-elliptical notch geometry, load level, and number of cycles to breakthrough. The $\gamma(R)$ relation was derived from crack closure measurements with through cracks in preliminary tests mentioned before (see Appendix B),

$$\gamma = \frac{S_{op}}{S_{max}} = 0.25 + 0.185R + 0.223R^2 + 0.12R^3 \quad \text{for} \quad R \leq 0.346 \quad (4.10a)$$

$$\gamma = R \quad \text{for} \quad R > 0.346 \quad (4.10b)$$

The crack increment in each cycle is calculated using the Paris equation

$$\frac{da}{dN} = 2.29 \times 10^{-10}(\Delta K)^{2.88} \quad \text{for} \quad R = 0.1 \quad (4.11)$$

With the $\gamma(R)$ equation and $U = (1 - \gamma)/(1 - R)$ it implies:

$$\frac{da}{dN} = 4.20 \times 10^{-10}(\Delta K_{eq})^{2.88} \quad (4.12)$$

In this equation the units are m/cycle for $\frac{da}{dN}$ and MPa\(\sqrt{m}\) for $K$.

**CA fatigue test results from the literature.**

Although much experimental work has been done on surface cracks under CA loading, only a few sources provide sufficient information for the present analysis, such as numerical data needed for prediction, CA crack growth equation for through cracks, number of cycles to breakthrough. For a prediction taking into account crack closure, a $U(R)$ or $\gamma(R)$ relation for the material is not always available.

Among the data available in the literature, the work of Hall et al.[4.25] provides the
necessary numerical data for prediction except crack closure data. Crack closure measurements were not performed in [4.25]. The materials used were 2219-T851 aluminum alloy, 6Al-4V titanium alloy and 9Ni-4Co-0.2C steel. The dimensions of the specimens, load levels, initial crack geometry, and test results are given in Tables 4.2 to 4.4. Crack growth rate data obtained for through cracks in double cantilever beam specimens were given in tabulated form. The Paris equation is fitted to the data, the constants of the Paris equation are given in Table 4.5. For the relation between ΔK_{eff} and ΔK, γ(R) relations are adopted from the literature. For the 2219-T851 aluminum alloy the γ(R) relation for 7075-T6 is adopted (Eqs.4.10a and 4.10b.). A γ(R) relation for 6Al-4V titanium alloy was found in [4.26]

\[
γ = \frac{S_{op}}{S_{max}} = 0.27 - 0.09R + 0.82R^2
\]  

(4.13)

The plot of this equation is shown in Fig.4.15 together with the plot of γ(R) relation of 7075-T6 Al. The γ(R) curve shows a minimum at R = 0.055 with again high γ values for negative R ratios. This appears to be fully unrealistic. Since a minimum of the γ(R) is unacceptable only predictions ignoring crack closure will be performed for the 6Al-4V titanium alloy. Also for 9Ni-4Co-0.2C steel only predictions ignoring crack closure will be made, because no S_{op} measurements can be found in the literature.

4.5.3. Flow diagram of the calculation

A description of the flow diagram of the prediction model is given in this section. Cycle-by-cycle calculation of crack growth is made at M points along the crack front.

1. The coordinates of the points i along the crack front are determined with

\[
x_i = c \cos φ
\]

(4.14a)

\[
y_i = a \sin φ
\]

(4.14b)

\[
φ = \frac{(i - 1)(\pi)}{(M - 1)2}
\]

(4.14c)

where φ is defined in Fig.3.1 and i = 1 to M.

2. ΔK_i is calculated using the Newman-Raju solution (Appendix A), where S is replaced
by \( \Delta S = S_{\text{max}} - S_{\text{min}} \).

3. Cycle-by-cycle crack increments at each point are calculated by using the Paris equation.

\[
\frac{dl}{dN} = C(\Delta K)^m
\]  

(4.15)

The crack increment \( dl \) is taken perpendicular to the semi-ellipse at point \( i \) (see Fig.4.16). The values of \( C \) and \( m \) of the Paris equation are assumed to be the same for all growth directions. The component of the crack increment in \( x \) and \( y \) directions are \( \Delta x_i \) and \( \Delta y_i \), respectively.

4. The new coordinates after crack increment are obtained by adding the components of the crack increment, \( \Delta x_i \) and \( \Delta y_i \), to the previous coordinates

\[
x_i = x_i + \Delta x_i
\]  

(4.16a)

\[
y_i = y_i + \Delta y_i
\]  

(4.16b)

5. An ellipse is fitted to the \( M \) pairs of the new coordinates \( (x_i, y_i) \) by using the least-square method. The fitted semi-ellipse is assumed to be the new crack front, with \( a_i \) and \( c_i \) of the fitted semi-ellipse as the new crack depth and half crack length respectively.

6. Using the new crack size \( (a_i, c_i) \), calculate again the coordinates of \( M \) points on the new crack front and repeat the same loop. The calculation is terminated when the crack depth \( (a) \) is equal to the specimen thickness \( (t) \). The flow diagram of the calculation is shown in Fig.4.17.

In predictions accounting for crack closure it is assumed that \( S_{op} \) varies linearly with \( \phi \), as discussed before. \( S_{min} \) is replaced by \( (S_{op}) \). The maximum \( S_{op} \) is assumed to occur on the surface and the minimum \( S_{op} \) at the deepest point, which is assumed to be 20% lower than \( S_{op} \) on the surface. The value of \( S_{op} \) at the deepest point is assumed to be equal to the measured \( S_{op} \) of the through crack reported in Appendix B. The variation of the crack closure stress as a function of \( \phi \) can then be written as,
\[ S_{op}(1) = S_{op}(\varphi = 0) - 0.2 S_{op}(\varphi = 0) \frac{\varphi}{\pi/2} \] (4.17)

It should be noted that the linear variation of \( S_{op} \) as a function of \( \varphi \) does not imply that \( S_{op} \) varies linearly along the crack front. The \( M \) points along the semi-elliptical crack are not distributed homogenously along the semi-elliptical crack front as in the case of a semi-circular crack. Fig.4.18 shows the position of 8 points along the crack front, for a more slender crack (\( a/c < 1 \)). The variation of \( S_{op} \) along the crack front is larger at the surface than at the deepest point. That appears to be logical.

4.5.4. Prediction Results.

Prediction without closure

Prediction without crack closure are performed with different numbers of calculation points along the crack front (2, 4, 8, 16, and 32 points). The continuous crack growth process is assumed to be simulated by crack growth at 32 points along the crack front. Predicted crack propagation lives for different materials, and the ratio of the predicted life to the prediction with 32 points are presented in Tables 4.6 to 4.9. The predicted life ratios are shown in Fig.4.19.

Figure 4.19 indicates that predictions with less calculation points give shorter lives compared to predictions based on more calculation points. Predictions with only two points result in the shortest life. The lowest average ratio of the predicted life calculated with 2 points is 0.85 for the Ti-alloy (see Table 4.8). A significant improvement for this material is already obtained by using 4 points, where the average ratio increased to 0.95. Smaller improvements are obtained for predictions with 8 and 16 points. The small differences obtained for calculations using 4, 8, 16 points show that the predictions rapidly converge to the results for a continuous crack growth process. Fig.4.19 also demonstrates a systematic effect of the initial crack aspect ratio (\( a_p/c_p \)) for predictions with two points. A lower life ratio is found for cracks with an initial shape close to a semi-circle (\( a_p/c_p \)), but this effect practically disappeared for predictions with eight points. Further evaluations will be based on predictions with 8 points. The choice is somewhat arbitrary, but the calculation time is still not excessive even with a PC (most calculations were carried out on a PC having a 80486 processor and a clock frequency speed of 50MHz).
Test lives and predicted crack propagation lives using 8 points are shown in Tables 4.10 to 4.13. Predicted crack growth curves for R = 0.1 are compared to test results in Fig.4.20a to 4.20d. The tables and figures show a reasonable agreement between the test results and predictions. The experimental crack growth rates are presented in Fig.4.21a to 4.21d. The solid line in these figures is the Paris equation used in the prediction model. For crack growth in the thickness direction the scatter band of the test results can be represented by the Paris equation. The results of Hall for Al-alloy, Ti-alloy, and Steel showed that the crack growth rate on the surface (dc/dN) is lower then predicted by the Paris equation.

The crack shape development curves (a/c vs a/t) are presented in Fig.4.22a to Fig.4.22d. In general the predicted curves follow the trend of the test results. Slender cracks (low a/c) grow to a less slender shape, i.e. they grow to a more semi-circular shape.

Prediction with crack closure
As mentioned before, predictions with a varying crack closure level along the crack front are performed with 8 calculations points. The predicted crack propagation lives compared to the test results and predictions without crack closure are shown in Table 4.10 to 4.12. For 2219-T851 Al alloy (result of Hall et al.) at R = 0.5, predictions with crack closure are not performed, because at this value of R the γ(R) relation resulted in $S_{op} / S_{min}$, which physically means that the crack remains fully open. The predictions with crack closure become equal to the predictions ignoring crack closure. Table 4.10 and Table 4.11 show that the difference between predictions ignoring crack closure and predictions taking into account crack closure for 7075-T6 Al and 2219-T851 Al differ only slightly. The predicted crack propagation life with crack closure is longer than without crack closure, the average difference is about 15%.

The crack propagation curves of predictions with crack closure are shown in Figs. 4.23a and 4.23b for R = 0.1 results. The crack growth rate curves (da/dN vs $\Delta K_{eff}$) presented in Figs.4.21a and 4.21b give a similar result to da/dN vs $\Delta K$ curves. The predicted crack shape development curves (with and without taking into account crack closure) compared to test results are shown in Figs.4.22a and Fig.4.22b. In general, predictions with crack closure resulted in a slightly higher crack aspect ratio during the crack growth.
4.5.5. Discussion

*The number of calculation points*

In the previous section, it was shown that predictions with 2 calculation points resulted in a shorter crack propagation life compared to predictions with more calculation points. However, predictions with more calculation points converge rapidly. Predictions using 4 points, which means an insertion of two calculation points between the point on the surface and the deepest point, increase the life compared to predicted life for 32 points to an average ratio of 0.97. For 8 points the average ratio is 0.99. A further increase of the number of calculation points has a rather small influence on the improvements of the predictions.

*Crack growth rates and crack propagation life*

Three-dimensional stress analyses of a surface crack have shown that \( K \) varies along the crack front (see Chapter 3). The difference in \( K \) will result in different crack growth rates along the crack front. The similitude concept for crack growth implies that for CA loading the same \( \Delta K \) and \( R \) in a load cycle will produce the same crack growth rate. As a consequence, the crack growth rate for the same \( R \) will be a function of \( \Delta K \) regardless of the crack geometry and specimen/structural geometry. If the similitude concept is applied to the growth of surface cracks, it is expected that the growth rate of a point on the crack front of cracks of different sizes, but having the same \( \Delta K \), will also be the same. Moreover, the crack growth rate will correspond to the growth rate of a through crack with the same \( \Delta K \). A plot of the crack growth rate as a function of \( \Delta K \) should result in same crack growth rate for surface cracks and through cracks. Fig.4.21a to 4.21d show the scatter of crack growth rates as a function of \( \Delta K \). The straight line in each figure is the Paris equation derived from through crack data. It should be noted that the \( R \) value of all specimens in Fig.4.21a to 4.21d is 0.1.

Different states of stress along the crack front of a surface crack will result in different \( S_{op} \). Due to the plane stress condition, \( S_{op} \) on the surface is higher than at deeper points. A linear variation of the \( S_{op} \) was adopted in the prediction with crack closure. Taking into account crack closure, the similitude concept implies that the same \( \Delta K_{eff} \) will produce the same crack growth rate. The plot of the crack growth rate as a function of \( \Delta K_{eff} \) in Figures 4.21a and 4.21b do not really lead to a collapse of all data points along the Paris equation.
Although for 7075-T6 specimens some alignment of the data points to the Paris equation can be observed, the data points from Hall et al. (2219-T851) show noticeable deviations from the Paris equation.

Recalling that the difference of $S_{op}$ between the surface and the deepest point is relatively small (approximately 20%), it is not surprising that $da/dN$ and $dc/dN$ vs $\Delta K_{ef}$ do not change significantly the position of the data points relative to the Paris equation. As a consequence predicted crack propagation lives will not differ significantly between predictions taking into account crack closure and predictions without crack closure.

If predictions were performed with the same $S_{op}$ level along the crack front, then the predicted crack propagation life would be the same as a prediction without closure. The assumed linear variation of $S_{op}$ with 20% lower value at the deepest point will result in slower crack growth rate at the surface point, hence a longer crack propagation life. Results in Tables 4.10 and 4.11 show that predictions with crack closure indeed result in longer crack propagation lives compared to predictions without crack closure. Nevertheless, the difference is not very significant, the average ratio of predicted lives with and without crack closure differ only about 15% for 7075-T6 Al and 2219-T851 Al. For fatigue life prediction of surface cracks under CA loading, incorporating $S_{op}$ in the prediction model is not really necessary, due to small differences of $S_{op}$ along the crack front. However, the situation may be different for VA loading where interaction effects occur.

**Crack shape development**

The crack shape development curves in Figs. 4.22a and 4.22b show that predictions without and with crack closure generally indicate the same trend as the test results. The higher aspect ratio predicted with crack closure is the result of the lower crack growth rate in the surface direction discussed before. It should be pointed out that a good agreement between the predicted crack shape and the experimental results does not automatically imply a good predicted crack growth life. Predicted crack shape development curves of 2219-T851 Al specimens presented in Figure 4.22b show a reasonable agreement with test results, while the ratio of predicted crack propagation life to test life of one specimen is 1.64, which is fairly high. The discrepancy is not too surprising, because the crack shape development curve only indicates the crack aspect ratio during crack
extension, eliminating the number of cycles needed to reach that aspect ratio. If the crack growth of surface cracks can be described with K and \( S_{op} \) only, the favourable agreement between predicted crack shape development and test results suggests that the K solution used in the prediction is accurate. The use of the crack shape development curve to assess the accuracy of K solutions has been discussed in section 4.2.2.

Predictions of the crack shape development performed in [4.1], and [4.6] adopted an assumption that the crack growth rate on the surface is slower than at the deepest point through the difference in the Paris equation constant, \( C_a = (0.9)^m C_a \). In other words, crack growth rate in the depth direction is faster than along the surface by a factor \((1.1)^m\). If crack shape development predictions are performed with the assumption that \( S_{op} \) at the deepest point is 20% lower than on the surface, it can be shown that a similar factor is obtained. Assuming \( C_a = C_c \), Equations (4.6.a) and (4.6.b) can be written as

\[
\frac{da}{dc} = \frac{[F_a(S_{max} - S_{op}(a))]^m}{[F_c(S_{max} - S_{op}(c))]^m}
\]

(4.18)

with \( S_{op} = \gamma S_{max} \) and \( S_{op}(A) = 0.8S_{op}(C) \) then

\[
\frac{da}{dc} = \frac{[F_a(1-0.8\gamma)]^m}{[F_c(1-\gamma)]^m}
\]

(4.19)

Using the \( \gamma(R) \) relation of 7075-T6 for \( R = 0.1 \) gives

\[
\frac{da}{dc} = \left[ \frac{F_a}{F_c} \right]^m (1.1)
\]

(4.20)

The factor \((1.1)^m\) is the same as the factor obtained with different Paris equation constants. For different \( R \) values the factor does not vary significantly.

The crack shape development was further analysed by predictions including crack closure for different initial crack shapes \((a/c)\) and different initial crack depths. The results are shown in Fig.4.24. It can be seen that a semi-circular crack grows as a semi-circle crack \((a/c = 1)\) up to \( a/t = 0.25 \), which is in agreement with the results of Corn [4.7] mentioned before. Very small cracks of different shapes \((a/c = 0.2\) to \( a/c = 5 \)) grow very rapidly to a
semi-circular shape, which is maintained up to \( a/t = 0.25 \). After further growth, \( a/c \) for several cracks converges to an common value of \( 0.85 \). For other materials the final \( a/c \) does not differ significantly as shown in the table below. The similar final aspect ratio for different materials explains the tendency found in Fig.4.2. It may be noted that slender cracks (\( a/c = 0.2 \) in Fig.4.24) do not always have the possibility to grow to the the stabilized value if the initial \( a/t \) is too large.

<table>
<thead>
<tr>
<th>Material</th>
<th>( m )</th>
<th>( (a/c)_f )</th>
<th>( (a/c)_f )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>no closure</td>
<td>closure included</td>
</tr>
<tr>
<td>9Ni-4Co-0.2C Steel</td>
<td>2.62</td>
<td>0.78</td>
<td>-</td>
</tr>
<tr>
<td>7075-T6</td>
<td>2.88</td>
<td>0.77</td>
<td>0.85</td>
</tr>
<tr>
<td>2024-T3</td>
<td>3.18</td>
<td>0.76</td>
<td>0.84</td>
</tr>
<tr>
<td>2219-T851</td>
<td>3.44</td>
<td>0.75</td>
<td>0.82</td>
</tr>
<tr>
<td>6Al-4V Ti</td>
<td>3.65</td>
<td>0.74</td>
<td>-</td>
</tr>
</tbody>
</table>

For comparison, predicted crack shape development curves ignoring crack closure are shown in Fig.4.25. The figure shows that small cracks of different shapes rapidly grow to semi-elliptical crack having \( a/c = 0.9 \). The final crack aspect ratio is slightly lower than predictions taking into account crack closure, as can be seen in the table above.

The previous simple calculation and predictions show that the assumed linear variation of \( S_{op} \) is compatible with the usual assumption of different crack growth rates at the deepest point and on the surface. The assumption of the linear variation of \( S_{op} \) will be used in prediction of surface cracks growth under VA loading, because it has a more sound physical basis.

All test results presented in this chapter were performed with an initial notch having \( a/c \) \( < 1 \). It may be recalled that an example of a service failure mentioned in Chapter 2 was initiated at corrosion pits, which can result in a deep notch especially when the corrosion attack is parallel to the rolling direction. Scheerder [4.27] performed fatigue tests on specimens with a single corrosion pit created with the anodic polarization method. In some specimens the corrosion pit were created parallel to the rolling direction of the plate specimen resulting in a very deep pit having \( a/c \) \( > 3 \). Fig.4.26
shows the crack shape development curves of cracks initiated at deep corrosion pits. It can be seen that the crack aspect ratio rapidly drops to approximately 1 (a semi-circular crack) and then slowly becomes slightly more slender (a/c \( \approx \) 1). The rapid drop of the aspect ratio to a/c = 1 indicates that the initial crack growth rate on the surface was very high leading to a rapid increase of the crack length c. The rapid drop of a/c is in agreement with the prediction in Fig.4.24. For a very deep crack with a/c \( \geq \) 1 the stress intensity factor on the surface is very high. As an example, for a crack penetrating half of the thickness of the plate specimen used in section 4.5.2, and having an aspect ratio of 4, the Newman-Raju solution results in \( K_a/K_c = 2.24 \). The Paris equation with the power m = 2.88 then predicts in crack growth on the surface to be approximately 8 times faster than at the deepest point. The tests of Scheerder emphasize the danger of deep corrosion pits in cyclically loaded structures. The small size of the corrosion attack on the surface grows rapidly to a wide crack with a deep penetration, i.e. the depth of the pits.

4.5.6. Concluding remarks

1. Fatigue crack growth predictions of surface cracks have to reflect the continuous growth process along the crack front. It is assumed here that the continuous crack growth process can be represented by predictions of crack growth increments at 32 points along the crack front. Predictions with lower numbers of points are then evaluated in comparison to the predictions with 32 points. Predictions using 2 points, i.e. on the surface and at the deepest point, resulted in shorter predicted lives compared to predictions with more points. Moreover, the underprediction is systematically affected by the initial crack shape (a_0/c_0). If 8 points are used all predictions are above 97% of the predictions with 32 points, the average being 99%. A further increase of the number of calculation points has a rather small influence. Further applications in this study will employ 8 calculation points, because computing time is still not yet excessive.

2. Predictions of crack growth lives, adopting a linear variation of S_{op} along the crack front, as observed in the fractographic S_{op} measurements, results in a slightly more unconservative predictions compared to prediction without crack closure. The small difference is caused by the small difference of S_{op} between the surface and the deepest point. For the prediction of the crack growth life of surface cracks under CA loading, the incorporation of S_{op} in the prediction model is not really necessary.
3. The plot of $\frac{dc}{dN}$ or $\frac{da}{dN}$ of test data of different specimens and materials as a function of $\Delta K$ for the same $R$ ratio does not result in an accurate alignment of the data points to the Paris equation as suggested by the similitude concept. Plotting the data points as a function of $\Delta K_{eff}$ does only marginally improve the position of the data points relative to the Paris equation.

4. Test results show that under fatigue loading slender cracks become less slender while semi-circular cracks become more slender. The tendency can be well predicted either without or with taking into account crack closure. The favourable agreement of the crack shape development does not guarantee a well predicted crack growth life. However, the favourable agreement does imply that the $K$ solution used in the prediction is satisfactory.

References, Chapter 4


### Chapter 4, Growth of surface cracks under CA loading

#### Table 4.1
Summary of test parameters of CA loading tests of surface flawed 7075-T6 Aluminum alloy specimens, $W = 100\text{mm}$, $t = 9.6\text{mm}$, $S_{\text{max}} = 150\ \text{MPa}$, $R = 0.1$. The number of cycles in the marker load block and in the crack opening measurement block are 30000 and 45 respectively.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Initial Crack Geometry (dimension in mm)</th>
<th>Final Crack Geometry</th>
<th>Test lives (cycles)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$a_0$</td>
<td>$c_0$</td>
<td>$a_0/t$</td>
</tr>
<tr>
<td>PCA6</td>
<td>1.92</td>
<td>9.60</td>
<td>0.20</td>
</tr>
<tr>
<td>PCA15</td>
<td>2.88</td>
<td>7.20</td>
<td>0.30</td>
</tr>
<tr>
<td>PCA2</td>
<td>1.92</td>
<td>3.20</td>
<td>0.20</td>
</tr>
<tr>
<td>PCA14</td>
<td>1.92</td>
<td>2.40</td>
<td>0.20</td>
</tr>
<tr>
<td>PCA13</td>
<td>1.92</td>
<td>1.92</td>
<td>0.20</td>
</tr>
</tbody>
</table>

#### Table 4.2
Summary of test parameters of CA loading tests of surface flawed 2219-T851 Aluminum alloy (Hall et al.[4.25]).

<table>
<thead>
<tr>
<th>Specimen geometry (mm)</th>
<th>Load (MPa)</th>
<th>Initial Crack Geometry (mm)</th>
<th>Final Crack Geometry</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$a_0$</td>
<td>$c_0$</td>
</tr>
<tr>
<td>$t = 11.39, W = 228.6$</td>
<td>$S_{\text{max}} = 19.46$</td>
<td>3.25</td>
<td>8.51</td>
</tr>
<tr>
<td></td>
<td>$R = 0.1$</td>
<td>3.61</td>
<td>6.12</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5.49</td>
<td>6.40</td>
</tr>
<tr>
<td>$t = 11.40, W = 228.6$</td>
<td>$S_{\text{max}} = 10.81$</td>
<td>6.25</td>
<td>19.18</td>
</tr>
<tr>
<td></td>
<td>$R = 0.1$</td>
<td>7.54</td>
<td>12.88</td>
</tr>
<tr>
<td></td>
<td></td>
<td>9.27</td>
<td>10.97</td>
</tr>
<tr>
<td>$t = 11.40, W = 228.6$</td>
<td>$S_{\text{max}} = 32.43$</td>
<td>2.90</td>
<td>8.46</td>
</tr>
<tr>
<td></td>
<td>$R = 0.5$</td>
<td>3.76</td>
<td>6.17</td>
</tr>
<tr>
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<td></td>
<td>5.33</td>
<td>6.10</td>
</tr>
<tr>
<td>$t = 11.35, W = 228.6$</td>
<td>$S_{\text{max}} = 19.46$</td>
<td>5.41</td>
<td>19.05</td>
</tr>
<tr>
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<td>7.21</td>
<td>12.88</td>
</tr>
<tr>
<td></td>
<td></td>
<td>9.14</td>
<td>10.34</td>
</tr>
<tr>
<td>Specimen geometry (mm)</td>
<td>Load (MPa)</td>
<td>Initial Crack Geometry (mm)</td>
<td>Final Crack Geometry</td>
</tr>
<tr>
<td>------------------------</td>
<td>------------</td>
<td>-----------------------------</td>
<td>----------------------</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$a_0$ $c_0$ $a_0/t$ $a_0/c_0$</td>
<td>$a_0/t$ $a_0/c_r$</td>
</tr>
<tr>
<td>$t = 9.55$</td>
<td>$S_{max} = 48.64$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$W = 152.50$</td>
<td>$R=0.1$</td>
<td>1.82 6.30 0.191 0.290</td>
<td>1.000 0.836</td>
</tr>
<tr>
<td>$t = 9.40$</td>
<td>$S_{max} = 29.19$</td>
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</tr>
<tr>
<td>$W = 152.40$</td>
<td>$R=0.1$</td>
<td>2.56 4.50 0.268 0.570</td>
<td>1.000 0.896</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3.86 4.27 0.404 0.902</td>
<td>1.000 0.886</td>
</tr>
<tr>
<td>$t = 9.45$</td>
<td>$S_{max} = 75.67$</td>
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<td></td>
</tr>
<tr>
<td>$W = 152.50$</td>
<td>$R=0.5$</td>
<td>2.69 4.52 0.285 0.594</td>
<td>0.780 0.770</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3.76 4.29 0.398 0.878</td>
<td>0.815 0.858</td>
</tr>
<tr>
<td>$t = 9.70$</td>
<td>$S_{max} = 45.40$</td>
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</tr>
<tr>
<td>$W = 152.73$</td>
<td>$R=0.5$</td>
<td>5.03 16.03 0.519 0.314</td>
<td>1.000 0.522</td>
</tr>
<tr>
<td></td>
<td></td>
<td>7.85 8.91 0.810 0.882</td>
<td>1.000 0.680</td>
</tr>
</tbody>
</table>

Table 4.3. Summary of test parameters of CA loading tests of surface flawed 6Al-4V βA Titanium alloy (Hall et al.[4.25]).

<table>
<thead>
<tr>
<th>Specimen geometry (mm)</th>
<th>Load (MPa)</th>
<th>Initial Crack Geometry (mm)</th>
<th>Final Crack Geometry</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$a_0$ $c_0$ $a_0/t$ $a_0/c_0$</td>
<td>$a_0/t$ $a_0/c_r$</td>
</tr>
<tr>
<td>$t = 7.77$</td>
<td>$S_{max} = 47.56$</td>
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<td></td>
</tr>
<tr>
<td>$W = 152.58$</td>
<td>$R=0.1$</td>
<td>1.35 4.11 0.173 0.326</td>
<td>0.964 0.812</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.90 2.95 0.245 0.644</td>
<td>1.000 0.88</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.67 2.97 0.343 0.894</td>
<td>1.000 0.854</td>
</tr>
<tr>
<td>$t = 7.77$</td>
<td>$S_{max} = 32.43$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$W = 152.60$</td>
<td>$R=0.1$</td>
<td>3.76 12.62 0.484 0.298</td>
<td>1.000 0.536</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4.98 8.46 0.642 0.588</td>
<td>1.000 0.746</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6.22 7.06 0.802 0.882</td>
<td>1.000 0.876</td>
</tr>
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<td>$t = 7.62$</td>
<td>$S_{max} = 64.86$</td>
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<td></td>
</tr>
<tr>
<td>$W =152.42$</td>
<td>$R=0.5$</td>
<td>1.40 4.24 0.183 0.330</td>
<td>1.000 0.792</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.96 3.07 0.257 0.636</td>
<td>0.984 0.836</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.82 3.20 0.370 0.880</td>
<td>1.000 0.852</td>
</tr>
<tr>
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<td>$S_{max} = 48.64$</td>
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<td></td>
</tr>
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<td>$R=0.5$</td>
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<td>1.000 0.522</td>
</tr>
<tr>
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<td></td>
<td>4.95 8.43 0.670 0.588</td>
<td>1.000 0.680</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6.17 7.21 0.835 0.856</td>
<td>1.000 0.804</td>
</tr>
</tbody>
</table>

Table 4.4. Summary of test parameters of CA loading tests of surface flawed 9Ni-4Co-0.2C Steel alloy (Hall et al.[4.25]).
### Table 4.5.
The constants of the Paris equation for 2219-T851 Al alloy, 6Al-4V Ti alloy and 9Ni-4Co-0.2C Steel[4.25].

<table>
<thead>
<tr>
<th>Material</th>
<th>da/dN vs $\Delta K^*$</th>
<th>da/dN vs $\Delta K_{\text{eff}}^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$R$</td>
<td>$C$</td>
</tr>
<tr>
<td>2219-T851 Al-alloy</td>
<td>0.1</td>
<td>$3.90\times10^{-11}$</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>$5.12\times10^{-11}$</td>
</tr>
<tr>
<td>6Al-4V Ti-alloy</td>
<td>0.1</td>
<td>$1.74\times10^{-12}$</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>$1.31\times10^{-12}$</td>
</tr>
<tr>
<td>9Ni-4Co-0.2C Steel</td>
<td>0.1</td>
<td>$2.57\times10^{-11}$</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>$2.93\times10^{-11}$</td>
</tr>
</tbody>
</table>

* da/dN in $\mu$m/cycle and $\Delta K$ or $\Delta K_{\text{eff}}$ in MPa$\sqrt{\text{m}}$

### Table 4.6.
Test lives and predicted lives of 7075-T6 Aluminum alloy. Predictions were performed with different number of calculation points along the crack front. The ratios in the table are defined as the ratio relative to the predicted life with 32 calculation points.
<table>
<thead>
<tr>
<th>$a/c_0$ of specimens</th>
<th>2 points</th>
<th>4 points</th>
<th>8 points</th>
<th>16 points</th>
<th>32 points</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>lives</td>
<td>ratio</td>
<td>lives</td>
<td>ratio</td>
<td>lives</td>
</tr>
<tr>
<td>0.382</td>
<td>16656</td>
<td>0.92</td>
<td>17666</td>
<td>0.98</td>
<td>17899</td>
</tr>
<tr>
<td>0.588</td>
<td>19054</td>
<td>0.92</td>
<td>20321</td>
<td>0.98</td>
<td>20631</td>
</tr>
<tr>
<td>0.856</td>
<td>15688</td>
<td>0.84</td>
<td>17657</td>
<td>0.94</td>
<td>18299</td>
</tr>
<tr>
<td>0.326</td>
<td>21921</td>
<td>0.93</td>
<td>23118</td>
<td>0.98</td>
<td>23345</td>
</tr>
<tr>
<td>0.586</td>
<td>29736</td>
<td>0.88</td>
<td>32732</td>
<td>0.97</td>
<td>33395</td>
</tr>
<tr>
<td>0.844</td>
<td>23453</td>
<td>0.81</td>
<td>27719</td>
<td>0.96</td>
<td>18500</td>
</tr>
<tr>
<td>0.342</td>
<td>11146</td>
<td>0.93</td>
<td>11782</td>
<td>0.98</td>
<td>11914</td>
</tr>
<tr>
<td>0.610</td>
<td>11021</td>
<td>0.91</td>
<td>11872</td>
<td>0.98</td>
<td>12048</td>
</tr>
<tr>
<td>0.874</td>
<td>9015</td>
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<td>0.96</td>
<td>10347</td>
</tr>
<tr>
<td>0.284</td>
<td>14101</td>
<td>0.93</td>
<td>14939</td>
<td>0.98</td>
<td>15103</td>
</tr>
<tr>
<td>0.560</td>
<td>16351</td>
<td>0.88</td>
<td>18006</td>
<td>0.97</td>
<td>18395</td>
</tr>
<tr>
<td>0.884</td>
<td>12448</td>
<td>0.79</td>
<td>14949</td>
<td>0.95</td>
<td>15408</td>
</tr>
<tr>
<td></td>
<td>0.88</td>
<td>0.97</td>
<td>0.99</td>
<td>1.00</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.7. Test lives and predicted lives of 2219-T851 Aluminum alloy (test data of Hall et al.[4.25]).
### Table 4.8

Test lives and predicted lives of 6Al-4V βA Ti-alloy (test data of Hall et al.[4.25]).

<table>
<thead>
<tr>
<th>$a_p/c_0$ of specimen</th>
<th>2 points</th>
<th>4 points</th>
<th>8 points</th>
<th>16 points</th>
<th>32 points</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>lives</td>
<td>ratio</td>
<td>lives</td>
<td>ratio</td>
<td>lives</td>
</tr>
<tr>
<td>0.290</td>
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<td>0.97</td>
<td>17547</td>
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<td>18411</td>
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<td>18915</td>
</tr>
<tr>
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<td>0.84</td>
<td>13970</td>
<td>0.94</td>
<td>14507</td>
</tr>
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<td>0.92</td>
<td>12039</td>
<td>0.98</td>
<td>12175</td>
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<tr>
<td>0.854</td>
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<td>0.79</td>
<td>12335</td>
<td>0.95</td>
<td>12749</td>
</tr>
<tr>
<td>0.290</td>
<td>16983</td>
<td>0.88</td>
<td>18668</td>
<td>0.96</td>
<td>19078</td>
</tr>
<tr>
<td>0.594</td>
<td>17729</td>
<td>0.86</td>
<td>19548</td>
<td>0.95</td>
<td>20153</td>
</tr>
<tr>
<td>0.878</td>
<td>13634</td>
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<td>15598</td>
<td>0.92</td>
<td>16363</td>
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<td>0.97</td>
<td>11648</td>
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<td>16805</td>
<td>0.91</td>
<td>17812</td>
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<tr>
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<td>0.95</td>
<td></td>
<td>0.98</td>
<td></td>
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</table>

Chapter 4, Growth of surface cracks under CA loading
### Table 4.9. Test lives and predicted lives of 9Ni-4Co-0.2C Steel (test data of Hall et al.[4.25]).

<table>
<thead>
<tr>
<th>$(a/c_0)$ of specimen</th>
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<th>4 points</th>
<th>8 points</th>
<th>16 points</th>
<th>32 points</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>lives</td>
<td>ratio</td>
<td>lives</td>
<td>ratio</td>
<td>lives</td>
</tr>
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<td>0.96</td>
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<td>0.91</td>
<td>23531</td>
<td>0.98</td>
<td>23876</td>
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<td>0.98</td>
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</tr>
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<td>0.98</td>
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### Table 4.10. Comparison of predicted life with and without crack closure to test results for 7075-T6 Al alloy.

<table>
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<th>$a/c_0$ of specimens</th>
<th>Test results</th>
<th>Prediction results</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>without crack closure</td>
</tr>
<tr>
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<td></td>
<td>lives</td>
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<td>25063</td>
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<td>24000</td>
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<td></td>
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</tr>
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</table>

76
<table>
<thead>
<tr>
<th>$a_0/c_0$ of specimens</th>
<th>Test results</th>
<th>Prediction results</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>without crack closure</td>
<td>with crack closure</td>
</tr>
<tr>
<td></td>
<td></td>
<td>lives</td>
<td>ratio</td>
</tr>
<tr>
<td>0.382</td>
<td>19500</td>
<td>17899</td>
<td>0.92</td>
</tr>
<tr>
<td>0.588</td>
<td>18800</td>
<td>20631</td>
<td>1.10</td>
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<tr>
<td>0.856</td>
<td>16000</td>
<td>18299</td>
<td>1.14</td>
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<td>0.88</td>
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<td></td>
<td>1.10</td>
<td>1.22</td>
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<tr>
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<td>11914</td>
<td>0.76</td>
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<tr>
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<td>16200</td>
<td>12048</td>
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<td>10347</td>
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<tr>
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<td>28200</td>
<td>15103</td>
<td>0.54</td>
</tr>
<tr>
<td>0.560</td>
<td>30200</td>
<td>18395</td>
<td>0.61</td>
</tr>
<tr>
<td>0.884</td>
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<td>15408</td>
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</tr>
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<td>0.90</td>
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</tr>
</tbody>
</table>

Table 4.11. Comparison of predicted lives with and without crack closure for 2219-T851 Al alloy (data of Hall et al.[4.25]).
<table>
<thead>
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<th>$a_{\infty}/c_0$ of specimens</th>
<th>Test results</th>
<th>Prediction results without crack closure</th>
<th></th>
</tr>
</thead>
<tbody>
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<td></td>
<td>lives</td>
<td>ratio</td>
</tr>
<tr>
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<tr>
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</tr>
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<td>0.314</td>
<td>16100</td>
<td>11648</td>
<td>0.72</td>
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<tr>
<td>0.882</td>
<td>11500</td>
<td>17812</td>
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</tr>
<tr>
<td></td>
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<td>0.96</td>
</tr>
</tbody>
</table>

Table 4.12. Comparison of predicted lives with and without crack closure for 6Al-4V Ti alloy (data of Hall et al.[4.25]).
| $\frac{a}{c_0}$ | Test results | Prediction results without crack closure |  |
|----------------|--------------|----------------------------------------|--|---|
|                |              | lives | ratio |  |
| 0.326          | 57000        | 50049 | 0.88  |  |
| 0.644          | 57000        | 53510 | 0.94  |  |
| 0.894          | 43500        | 44681 | 1.03  |  |
| 0.298          | 29200        | 21483 | 0.74  |  |
| 0.588          | 27300        | 23876 | 0.87  |  |
| 0.882          | 21900        | 19614 | 0.90  |  |
| 0.330          | 22800        | 21379 | 0.94  |  |
| 0.636          | 22300        | 22235 | 1.00  |  |
| 0.880          | 16400        | 17839 | 1.09  |  |
| 0.298          | 23700        | 22002 | 0.93  |  |
| 0.588          | 22500        | 24809 | 1.10  |  |
| 0.856          | 15000        | 18305 | 1.22  |  |
|                |              | 0.97  |       |  |

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Chapter 5

Prediction of Surface Crack Growth under Flight Simulation Loading with the CORPUS and the Modified CORPUS Models

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5.1. Introduction
Fatigue test with VA-loading has shown the occurrence of interaction effects. Crack extension in one cycle, Δa does not only depend on the load level of the cycle but also on preceding cycles. Crack growth retardation following an overload is a typical example of an interaction effect. Crack growth prediction models must be able to predict the occurrence of such interaction effect. In this chapter two prediction models based on the crack closure concept, i.e. the CORPUS model and the modified CORPUS model, will be applied to surface crack growth under VA-loading. A brief review of the available prediction models is given in section 5.2 with an emphasize on crack closure models. A description of the CORPUS model and the modified CORPUS model is covered in section 5.3. A summary of prediction results of both models for through cracks from the literature is also presented. Fatigue test results of surface cracks under flight simulation loading of the present test series and from the literature are presented in section 5.4. The modelling of surface cracks with CORPUS and the modified CORPUS models is given in section 5.5. Results of predictions for surface cracks are presented in section 5.6. Prediction results are discussed in section 5.7. Some concluding remarks are presented in section 5.8.

5.2. Prediction models for VA-loading
As previously mentioned in Chapter 1, the crack growth prediction models for VA-loading can be classified into three main groups i.e.,

1. Yield zone models
2. Crack closure models
3. Strip yield models

A short description of the models is given below.

Yield zone models
The yield zone models are represented by the Wheeler model [5.1] and the Willenborg model [5.2]. Both models consider plastic zone sizes for prediction of retardation effects (see Fig.5.1). The large plastic zone, r_{p,OL} was caused by a high previous overload, while r_{p,i} is the plastic zone of the current cycle. Wheeler introduced a retardation factor C_p, defined by
\[
\frac{\text{da}}{\text{dN}}_{VA} = C_p \frac{\text{da}}{\text{dN}}_{CA}
\]

(5.1)

\[
C_p = \left( \frac{r_{pi}}{\lambda} \right)^m
\]

(5.2)

The retardation factor \( C_p \) is assumed to depend on the current plastic zone size \( r_{pi} \) and the remaining distance of the crack tip to the plastic zone boundary of the overload, \( \lambda \). An empirical factor \( m \) was introduced in Eq.5.2. The factor is not a material constant, it depends on the type of the load spectra. For fatigue life prediction of a certain load spectra, the factor \( m \) has to be determined empirically with the same spectra.

In the Willenborg model retardation was assumed to be caused by a reduction of \( K_{\text{max}} \) and \( K_{\text{min}} \) to effective values \( K_{\text{max,eff}} \) and \( K_{\text{min,eff}} \) of cycles following a high overload. The reduction depends on \( \lambda \), \( r_{pi} \), \( a_p \), and \( a_{oli} \) (see again Fig.5.1). If \( K_{\text{min,eff}} \) is negative, it is assumed that \( K_{\text{min,eff}} \) is equal to zero. As long as \( K_{\text{min,eff}} \) is positive \( \Delta K \) does not change, because the reduction is applied to both \( K_{\text{max}} \) and \( K_{\text{max}} \), but the reduction implies a decrease of the stress ratio, \( R \). The reduction of \( K_{\text{max}} \) and \( K_{\text{min}} \) to the effective values \( K_{\text{max,eff}} \) and \( K_{\text{min,eff}} \) is not based on plausible physical arguments, moreover it does not agree with the present understanding of the crack closure mechanism.

In the formulation of both models only retardation was considered, therefore other interaction effects, such as acceleration and delayed retardation can not be accounted for. Moreover, the models do not include crack closure. Since the occurrence of crack closure at a positive stress is a physical reality, it should be an essential element of a crack growth prediction method. Some modifications were proposed for both models, but since the modified models are still based on the same basic assumptions as the original models, they still can not include other interaction effects than retardation.

**Crack closure models**

A number of prediction models based on crack closure were proposed. They were based on the crack closure concept proposed by Elber. The plastic zone left in the wake of a growing crack will cause contact of the crack faces at positive stresses, which means that the crack
tip singularity does not exist during the part of the load cycle where the crack is closed. Elber suggested that only that part of the load cycle, where the crack is fully open until the crack tip, will contribute to crack extension, which leads to the definition of an effective stress range,

\[ \Delta S_{\text{eff}} = S_{\text{max}} - S_{\text{op}} \]  

(5.3)

In terms of the stress intensity factor, Eq.5.3 can be written as,

\[ \Delta K_{\text{eff}} = K_{\text{max}} - K_{\text{op}} \]  

(5.4)

Elber measured \( S_{\text{op}} \) with a compliance technique on 2024-T3 Al-alloy specimens. He defined a ratio

\[ U = \frac{\Delta K_{\text{eff}}}{\Delta K} = \frac{\Delta S_{\text{eff}}}{\Delta S} \]  

(5.5)

and obtained following empirical relation from test results

\[ U = 0.5 + 0.4R \]  

(5.6)

Elber found this relation to be independent of the crack length \( a \). Instead of \( U \) another ratio \( \gamma \), to be used later, can also be defined

\[ \gamma = \frac{K_{\text{op}}}{K_{\text{max}}} = \frac{S_{\text{op}}}{S_{\text{max}}} = 1 - (1 - R)U \]  

(5.7)

In terms of \( \Delta K_{\text{eff}} \) and \( U \), the Paris crack growth relation can be written as

\[ \frac{da}{dN} = C(\Delta K_{\text{eff}})^m = C(U \Delta K)^m \]  

(5.8)

Crack closure models for VA-loading require cycle-by-cycle calculation of the crack opening stress, \( S_{\text{op}} \) and the corresponding \( K_{\text{op}} \). A \( U(R) \) relation, such as Eq.5.6, is used to calculate \( S_{\text{op}} \). \( \Delta K_{\text{eff}} \) is then obtained from Eq.5.4. The crack extension is determined from crack growth data obtained under CA loading such as the Paris relation (Eq.5.8).

Crack closure models are represented by PREFFAS [5.3], ONERA [5.4], and CORPUS [5.5].
Crack closure was introduced in the models using intuitive arguments based on empirical observations. The main concern was how the preceding load history influences $S_{op}$ of the current cycle. Test results have shown that high positive peak loads will increase $S_{op}$, while high negative peak loads will reduce $S_{op}$ of subsequent cycles. However, not all cycles will change $S_{op}$. A limited number of high historical peak values ($SH_{max}$ and $SH_{min}$, H for historical) of the preceding load history will influence the current $S_{op}$ level. The models are different in the assumptions made to determine the historical stress levels.

Padmadinata [5.6] performed a thorough investigation of crack closure models for predictions of fatigue crack growth of through cracks under flight simulation loading. The aim of the investigation was to improve the existing crack growth prediction models. Three crack closure models mentioned previously, i.e. PREFFAS, ONERA and CORPUS were considered to be potential candidates for detailed evaluations. Critical analysis of the models and comparisons of prediction results with flight simulation test data collected from different sources were made. A better understanding of how the models are working was obtained by visualising the value of $S_{op}$ in every cycle in a flight. Characteristics and limitations of each model were pointed out. In his concluding remarks on the CORPUS model Padmadinata commented "The CORPUS model is an interesting model because of its relation to the physical world of crack growth under variable amplitude loading". The hump mechanism, to be discussed in section 5.3, used to visualise the crack opening behaviour, is in agreement with the physical understanding of how $S_{op}$ behaves under overloads and underloads. The modified CORPUS model introduced by Padmadinata was postulated mainly to overcome a certain limitation of the CORPUS model to be discussed in section 5.3.

**Strip yield models**

The strip yield models (Newman [5.7] and de Koning [5.8]) are based on the Dugdale model for calculating the plastic zone size. Dugdale [5.9] adopted the assumption used by Irwin, that the presence of the plastic zone at the crack tip, causes the crack to behave as if it were longer that its actual size. Dugdale further assumed that in the strip between the physical crack tip and the fictitious crack tip, the stress $\sigma$ is equal to the yield stress. He then applies a superposition for the case of the fictitious crack under (i) the external load and (ii) the yield stress applied as a crack closing edge stress in the "strip yield" zone. The size of
the plastic zone (i.e. the length of the strip) is obtained by requiring that the singularity vanishes at the fictitious crack tip. The plastic elongation in the strip is assumed to be equal to the elastic crack opening in the strip yield zone. After crack growth it leads to plastic deformation in the wake of the crack. In the strip yield models the plastic zone is idealized as vertical bar elements having rigid-perfectly-plastic material behaviour. Crack opening occurs when bar elements in the wake of the crack loose contact. The model thus predicts the $S_{op}$ level, contrary to the older crack closure models, where the value of $S_{op}$ is obtained from empirical results. Compared to crack closure models discussed previously, the strip yield models are more complex due to the non-linear material behaviour considered and the changing crack geometry during opening and closing of the crack surface. As pointed out by Schijve [5.10] the approach of the strip yield model may be debatable because it applies two elastic solutions to obtain the plastic zone size, and secondly, because the plastic deformation in the strip yield zone is again the result of superposition of two elastic calculations. Extensive computer capacity is required due to the iterative procedures used in the calculation. Several predictions were reported both for simple load histories and flight simulation tests. In general good prediction results were obtained. Delayed retardation can be predicted with these models.

5.3 Description of the CORPUS model and the modified CORPUS model

5.3.1 The CORPUS model

The CORPUS model was proposed by De Koning [5.5] for crack growth prediction under flight simulation load sequences. Crack closure is visualized by the formation of humps on the crack faces. Higher loads will create larger humps, which can be flattened by later negative loads. Each cycle creates its own hump with associated $S_{op}$ level, thus the hump mechanism requires a cycle-by-cycle evaluation of $S_{op}$. Intuitively it can be deduced that a larger hump corresponds to a higher $S_{op}$ level and a smaller hump to a lower $S_{op}$. The evaluation of the $S_{op}$ level to be considered for the crack extension is the essential part of the CORPUS model. The CORPUS model has some characteristic features i.e.

- Recognition of primary and secondary plastic zones
- Considerations of plane stress and plane strain conditions for calculating the plastic zone size.
- Multiple overload effects to account for interactions between two or more overloads.
Description of the CORPUS model

Although the CORPUS model was announced by de Koning as a simple model, it is in fact a fairly complex model. It is not always easy to understand how it works. The following description largely follows the analysis of the CORPUS model as presented in the thesis of Padmadinata [5.6].

The hump mechanism and crack closure

According to the CORPUS model, plastic zones left in the wake of the crack are visible as humps on the fracture surface. Fig. 5.2 shows a hump created by an overload. The height of the hump depends on the magnitude of the overload, the higher the hump, the larger the stress to open the crack and thus the larger $S_{op}$. A subsequent underload flattens the hump, therefore it will decrease $S_{op}$. To illustrate how the hump mechanism works, Figures 5.3 to 5.7 are taken from [5.5] and [5.6]. Finite element calculations were performed to quantify the effect of the hump formation on the crack closure level. Results shown schematically in Fig. 5.3 indicate that $S_{op}$ decreases as the crack grows into the plastic zone. However, for simplicity the CORPUS model assumed $S_{op}^n$, i.e. $S_{op}$ due to cycle number $n$, to be constant. The extent where the value of $S_{op}$ is assumed to be constant is taken as the width of the plastic zone size. If the crack has grown beyond the plastic zone border, it is assumed that the effect of this hump has vanished and $S_{op}$ of this hump is zero. This condition can be written as follow

$$S_{op}^n = g(S_{max}^n, S_{min}^n) \quad \text{if} \quad a^n \leq a \leq a^n + D^n \quad (5.9a)$$

$$S_{op}^n = 0 \quad \text{if} \quad a > a^n + D^n \quad (5.9b)$$

where $g(S_{max}^n, S_{min}^n)$ is the hump opening function and $D^n$ is the plastic zone size.

De Koning used the term "delay switch" to describe this behaviour; a delay switch is turned on upon the application of an overload and it is turned off if the crack has grown through the plastic zone.

To determine the hump opening function De Koning performed experiments with different overload and underload combinations. The hump opening stress is defined as the maximum stress of subsequent constant amplitude cycles which give crack arrest. With this procedure De Koning determined the hump opening function for 2024-T3 and 7075-T6 materials as

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polynomials given by:

\[ g(S_{\text{max}}^n, S_{\text{min}}^n) = S_{\text{max}}^n(-0.4 R^4 + 0.9 R^3 - 0.15 R^2 + 0.2 R + 0.45) \quad \text{for } R > 0 (5.10a) \]

\[ g(S_{\text{max}}^n, S_{\text{min}}^n) = S_{\text{max}}^n(0.1 R^2 + 0.2 R + 0.45) \quad \text{for } -0.5 \leq R \leq 0 (5.10b) \]

Equations 5.10a and 5.10b are valid for the determination of the crack closure level following a single overload and underload combination. For a more complex loading sequence with different overload levels, each overload level creates its own hump having different \( S_{\text{op}} \). The crack is assumed to be closed as long as there is a hump still in contact. Thus, the largest hump will determine the \( S_{\text{op}} \). Fig.5.4 shows the plastic wake of the crack covered with three humps. The last hump that will lose contact is hump number 3 which implies that \( S_{\text{op}} \) is equal to \( S_{3} \).

Factors affecting the crack closure level

a. The effect of a high load level

Using the finite element method, Newman [5.7] demonstrated that \( S_{\text{op}} \) does not only depend on \( S_{\text{max}}^n \) and \( S_{\text{min}}^n \), but also on the level of \( \sigma_{\text{max}} \) in comparison to a yield stress for which Newman adopted the average \( (\sigma_{0.2} + \sigma_y)/2 \). To account for the influence of high load levels De Koning defined a correction factor, \( h \), to be applied to the \( S_{\text{op}} \) value. The correction function was obtained by a curve fitting procedure to Newman's results. The correction function \( h \) is equal to

\[ h = 1 - 0.2(1 - R^n)^3 \left( \frac{S_{\text{max}}}{1.15\sigma_y} \right)^3 \] (5.11)

Using this correction function Eq.5.9a can be written as

\[ S_{\text{op}}^n = g(S_{\text{max}}^n, S_{\text{min}}^n) h \] (5.12)

b. The effect of severe underloads

As mentioned previously, severe underloads will reduce the hump, which means that the hump opening stress will also be reduced. Consider a hump that was formed at cycle \( n \) with maximum stress, \( S_{\text{max}}^n \), and minimum stress, \( S_{\text{min}}^n \), a more negative underload occurring at
a later cycle i with $S_{\min}^n$ will reduce the hump opening stress according to

$$S_{op,i}^n = g(S_{\max}^n, S_{\min,i}^n) h^i$$  \hspace{1cm} (5.13)

A still lower underload in cycle j will further reduce the hump opening stress to

$$S_{op,j}^n = g(S_{\max}^n, S_{\min,j}^n) h^j$$  \hspace{1cm} (5.14)

Fig.5.5 illustrates how a lower underload decrease the $S_{op}$ level of previous cycles. The figure is a plot of load versus crack length, the insert shows the load sequence. A hump is created at $S_{\max}^1$, the subsequent $S_{\min}^1$ will flattened the hump. The opening stress of this hump is

$$S_{op}^1 = g(S_{\max}^1, S_{\min}^1) h^1$$  \hspace{1cm} (5.15)

The effective stress range in the next cycle is

$$\Delta \sigma_{eff}^1 = S_{\max}^2 - S_{op}^1$$  \hspace{1cm} (5.16)

A second hump is created at $S_{\max}^2$ and flattened by $S_{\min}^2$, the hump opening stress of the second hump is

$$S_{op}^2 = g(S_{\max}^2, S_{\min}^2) h^2$$  \hspace{1cm} (5.17)

Since $S_{\max}^1$ is higher than $S_{\max}^2$, $S_{op}^1$ will be higher than $S_{op}^2$. The magnitude of $S_{op}^1$ is not reduced by $S_{\min}^2$ because $S_{\min}^2$ is equal to $S_{\min}^1$. $S_{op}^1$ determines the crack opening stress of the next cycle. The effective stress is given by

$$\Delta \sigma_{eff}^2 = S_{\max}^3 - S_{op}^1$$  \hspace{1cm} (5.18)

The third hump created by $S_{\max}^3$ will be flattened by $S_{\min}^3$. Since $S_{\min}^3$ is lower than $S_{\min}^1$ and $S_{\min}^2$ the first and the second hump will also be flattened leading to lower hump opening stresses. The new hump opening stresses are

$$S_{op}^1 = g(S_{\max}^1, S_{\min}^3) h^1$$  \hspace{1cm} (5.19a)

$$S_{op}^2 = g(S_{\max}^2, S_{\min}^3) h^2$$  \hspace{1cm} (5.19b)
\[ S_{op}^3 = g(S_{max}^3, S_{min}^3) h^3 \]  \hfill (5.19c)

The new value of \( S_{op}^1 \) is still the highest hump opening stress, which means that it still governs the crack opening stress. The maximum stress of the next cycle \( S_{max}^4 \) is larger than all previous \( S_{max} \). The hump created in this cycle will be the largest, which means that it will determine the hump opening stress. In this case, all previous humps become insignificant for future considerations.

**Plastic zone size, delay switch and material memory**

The calculation of the plastic zone size is an important part of the CORPUS model, because it determines the delay switch mentioned before and it influences the overload interaction to be discussed later. The model distinguishes plastic zones developing into virgin (elastic) material, and plastic zones extending in plastically deformed material, i.e. in existing crack tip plastic zones. The former ones are called primary plastic zones and the latter ones are called secondary plastic zones.

A primary plastic zone size \( D \) is calculated based on Irwin's approach, but it was modified by de Koning to account for large zones if \( S_{max} \) approaches the net section yield stress. Using the Dixon width correction factor for a central cracked specimen, the equation for the primary plastic zone size for 2024-T3 Al-alloy becomes:

\[
D = \frac{1 - \gamma \left( \frac{S_{max}}{\sigma_y} \right)^2 \left( \frac{a}{W} \right)^2}{\sqrt{1 - \gamma \left( \frac{S_{max}}{\sigma_y} \right)^2 \left( \frac{a}{W} \right)^2 - 4 \left( \frac{a}{W} \right)^2}} - 1 \quad (5.20)
\]

where \( W \) is the width of the specimen and \( \gamma = 1/1.32 \) for plane stress conditions and \( \gamma = 1/9 \) for plane strain conditions.

For 7075-T6 Al-alloy a simpler equation was obtained by de Koning:
\[ D = a\gamma \left( \frac{S_{\text{max}}}{\sigma_y} \right)^2 \left[ 1 + \left( \frac{a}{W} \right)^2 + \gamma \left( \frac{S_{\text{max}}}{\sigma_y} \right)^2 \right] \] (5.21)

Prediction results of Padmadinata have shown that applying the less complex equation (Eq.5.21) to 2024-T3 material does not have a significant influence on the predicted crack propagation lives. As mentioned before the value of \( \gamma \) depends on the state of stress, for plane strain \( \gamma = 1/1.32 \) and for plane stress \( \gamma = 1/9 \). The state of stress is related to the size of the plastic zone relative to the thickness. Plane stress condition is assumed if the plastic zone size calculated under plane stress condition (\( D_{ss} \)) is equal or larger than half of the sheet thickness (\( D_{ss} \geq 0.5t \)). If \( D_{ss} \leq 0.35t \), plane strain conditions are assumed. In the transition from plane strain condition to plane stress condition (\( 0.35t \leq D_{ss} \leq 0.5t \)), the plastic zone size is calculated with

\[ D = D_{ss} + 2D_{ss} \left[ \frac{D_{ss} - 0.35}{t} \right]^4 \frac{(D_{ss} - D_{ss})}{t} \] (5.22)

where \( D_{ss} \) the size of the plane strain plastic zone.

Secondary plastic zones are assumed to develop under plane strain conditions. The size is calculated also with the Irwin approach

\[ D = \frac{1}{9\pi} \left( \frac{\Delta K_{\text{eff}}}{2\sigma_y} \right)^2 \] (5.23)

The term \( 2\sigma_y \) is used because the secondary plastic deformation occurs in the primary plastic zone after reversed plastic deformation in the primary zone.

A schematic picture of primary and secondary plastic zones is shown in Fig.5.6 [5.6]. The first plastic zone was formed at \( a = a_1 \), and a second one at \( a_2 \). The second plastic zone is considered as a primary plastic zone (PPZ) because it penetrated the elastic material. Plastic zones created at \( a_3 \) and \( a_4 \) are secondary plastic zones (SPZ) because they are confined to the already plastically deformed material.
The size of the plastic zone has an important role in the delay switch and the material memory considerations. A delay switch is turned on upon application of a load that creates a PPZ and turned off if the crack has grown through that plastic zone. In Fig. 5.6, the delay switch of the first PPZ is turned off at a crack size a + dp. At this moment, the first PPZ can be removed from the memory and the second PPZ becomes the dominant one. If $S^2_{\text{max}} < S^1_{\text{max}}$, two PPZ's are stored in the material memory, i.e. the first PPZ and the second PPZ. The second PPZ is stored because the plastic zone penetrates the elastic material. The second PPZ will become the dominant plastic zone if the delay switch of the first PPZ is turned off. If $S^2_{\text{max}}$ is higher than $S^1_{\text{max}}$, a larger hump is created, the second hump will become dominant immediately after the application of $S^2_{\text{max}}$, and any effect of the first PPZ is no longer present. The first PPZ can be erased. The removal of PPZ's from the material memory means that only a limited number of plastic zones must be stored in the material memory. $S_{\text{max}}$ values to be stored (labelled as $S^i_{\text{max}}$, $i$ for historical) must form a series of decreasing values because larger $S_{\text{max}}$ will erase all previous $S_{\text{max}}$. Similarly, historical $S_{\text{min}}$ forms a series of increasing, or at least equal $S_{\text{min}}$ levels. An $S_{\text{op}}$ associated with a pair of $S^i_{\text{max}}$ and $S^i_{\text{min}}$ is labelled as $S^i_{\text{op}}$. Historical $S_{\text{op}}$ values do not necessarily form a series of increasing or decreasing values.

If a load cycle does not create a PPZ because the plastic zone does not penetrate into the elastic material an SPZ is created. Historical values of SPZ follow the same rules as the PPZ, i.e. $S^i_{\text{max}}$ form a decreasing series while $S^i_{\text{min}}$ form an increasing series, and no specific sequence applies to $S^i_{\text{op}}$. Fig. 5.7 shows a schematic picture of the series of historic values of PPZ and SPZ. With respect to $S^i_{\text{op}}$, there is another condition that reduces the number of historic SPZ's, i.e. $S^i_{\text{op}}$ of the SPZ must be higher than the lowest $S^i_{\text{op}}$ of PPZ. This condition stems from the fact that the hump of the SPZ remains less significant if its $S^i_{\text{op}}$ is lower than the $S^i_{\text{op}}$ of the PPZ. Padmadinata investigated the significance of the SPZ by performing predictions in which the SPZ was ignored. The results showed that for the F-27 wing load spectrum the difference between predictions taking into account the SPZ and predictions ignoring SPZ was very small (a few percent). However, some effect of ignoring the SPZ appeared for the TWIST spectrum, where predictions were in the order of a 25% shorter crack growth life. The SPZ can be significant for load spectrum with many small cycles with a high mean stress. These small cycles could have an $S_{\text{op}}$ higher than $S_{\text{op}}$. 

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of the PPZ.

Overload interaction

Experimental results have shown that the crack growth retardation following overloads depends on the number of overloads and the number of cycles between overloads. More retardation occurs for multiple overloads due to an increasing crack closure level. In the CORPUS model this effect is considered. The hump opening stress, $S_{op}^n = g(S_{max}^n, S_{min}^n)$, given by equations 5.10a and 5.10b is valid for a single overload $S_{max}^n$ combined with underload $S_{min}^n$. If a series of overloads is applied, de Koning assumes that $S_{op}^n$ will reach an upper bound stationary level defined by

$$g(S_{max}^n, S_{min}^n) + m_{st} [S_{max}^n - g(S_{max}^n, S_{min}^n)]$$  \hspace{1cm} (5.24)

where $m_{st}$ is a parameter to define the stationary level of $S_{op}^n$. He observed from experiments that $m_{st}$ depends on the crack growth increment between the overloads ($\Delta a$) and the plastic zone size of the overloads ($D^n$). For the case of plane stress condition the value of $m_{st}$ was supposed to be:

$$m_{st} = 1.1 + 0.2 \frac{\Delta a}{D^n} \quad \text{if} \quad 0 < \frac{\Delta a}{D^n} \leq 0.25$$  \hspace{1cm} (5.25a)

$$m_{st} = 0.15 \quad \text{if} \quad 0.25 < \frac{\Delta a}{D^n} \leq 1$$  \hspace{1cm} (5.25b)

$$m_{st} = 0 \quad \text{if} \quad \frac{\Delta a}{D^n} > 1$$  \hspace{1cm} (5.25c)

Eq.5.25 implies that for small distances between the overloads ($0 < \Delta a/D^n < 0.25$) $m_{st}$ increases with $\Delta a$. The limiting case is the CA loading where $\Delta a$ is small compared to $D^n$, which means that $\Delta a/D^n$ goes to zero. In this case $m_{st} = 0.1$. If the crack has grown through the overload plastic zone ($\Delta a/D^n > 1$), the overload interaction is ignored (Eq.5.25c).

If a series of overloads is applied, the hump opening stress will reach the stationary value given by Eq.5.24 after a large number of overload cycles. After the application of each
overload the value of $S_{op}^n$ is increased step by step according to the following equation

$$S_{op}^n = g(S_{max}^n, S_{min}^n) + m^n[S_{max}^n - g(S_{max}^n, S_{min}^n)]$$  \(5.26\)

$m^n$ in Eq.5.26 is called "the relaxation parameter". The value of $m^n$ is not constant, it is updated when an overload of level $n$ is applied. The value of $m^n$ is calculated with the following equation

$$m_{new}^n = m_{old}^n + \delta [m_{old}^n - m_{old}^n \frac{\Delta a}{D^d}]$$  \(5.27\)

In the above equation, $\delta$ is the relaxation factor, $\Delta a$ is the crack extension between the overloads and $D^d$ is the plastic zone size of the dominant hump. The relaxation factor was taken to be 0.28 for 2024-T3 Al-alloy. The value is valid for interaction effects of overloads with the same level in plane stress condition. For a more general case, where overloads of different levels interact at different states of stress, two correction factors were introduced. The corrected relaxation factor is

$$\delta = 0.28 \delta_1 \delta_2$$  \(5.28\)

$\delta_1$ accounts for interaction of different overload levels and $\delta_2$ accounts for the effect of reduced interaction in plane strain condition. The values of $\delta_1$ and $\delta_2$ can be calculated with

$$\delta_1 = \frac{4D^d D^n}{(D^d + D^n)^2} \quad \text{if} \quad D^n \leq D^d$$  \(5.29a\)

$$\delta_1 = \frac{D^d}{D^n} \quad \text{if} \quad D^n > D^d$$  \(5.29b\)

$$\delta_2 = \frac{D^n}{D_{\text{m}}^n}$$  \(5.30\)

where $D^d$ is plastic zone size of the dominant hump, $D^n$ and $D_{\text{m}}^n$ are the current plastic zone size and its value for the hypothetical case of pure plane stress.

The $m_{old}^n$ in Eq.5.27 is replaced by $m_{old}^d \delta_1$, if the latter one is larger than $m_{old}^n$.

It should be noted that only interaction between the current overload and the overload
associated with the dominant hump is considered. The overload interaction is not applied to 7075 Al-alloy because empirical evidences showed that the interaction of overloads was not significant for this material. However, for the more ductile 2024-T3 alloy the overload interaction was observed experimentally as said before. De Koning's modelling of the multiple overload effect seems to be plausible, but it cannot be denied that the mathematical formulation is rather heuristic.

Summary of prediction results for through cracks
A summary of results of CORPUS predictions for through cracks will be presented in this section. Predictions were performed by Padmadinata [5.6] in an evaluation of crack closure models. The CORPUS model was used to predict crack propagation life of specimens made of 2024-T3 Alclad, 2024-T3 bare material, and 7075-T6 clad material. The thickness of all specimens were 2 mm. Flight simulation test data were collected from the literatures[5.11-5.17]. Flight simulation sequence applied in the tests were:
- Misawa sequence (simplified flight simulation)
- F4 (simplified flight simulation)
- F-27
- CN-235
- TWIST and MiniTWIST
- FALSTAFF and MiniFALSTAFF

Characteristic features of the flight simulation load histories are given in Appendix C. The Misawa sequence and the CN-235 flight simulation load history were applied in the present investigation. In the simplified Misawa flight-simulation tests all flights applied in one test are similar. Different combinations of the overload stress level ($S_{OL}$) and the ground stress level ($S_{gr}$) were used, see Fig.5.8. Misawa and Schijve [5.17] applied two overload stress levels ($S_{OL} = 160$ MPa and $200$ MPa) and three ground stress levels ($S_{gr} = -0$ MPa, -40 MPa, and -80 MPa) in tests on specimens with a central through crack. In the CN-235 ten different types of flight (Fig.5.9) were applied in blocks of 1000 flights with a random sequence of the flights in each block (see Fig.5.10).

Padmadinata made comparisons between test results and predictions by (i) comparing crack growth lives, (ii) crack growth rates, and (iii) in some cases the crack extension in one flight. The variation of $S_{pp}$ was sometimes presented to clarify trends found in predictions.
In general, predicted crack growth lives were in good agreement with test data. Ratio of predicted lives to test lives were in the range 0.5 to 2.0 (see Fig.5.11). Predicted crack growth rates are also considered to be good. On the average the predicted crack growth lives were 11% lower than the test results or in other words, the predicted results on the average were slightly conservative. The predicted crack growth rates slightly overestimated the test results. However, crack extension in rarely occurring very severe flights was underpredicted, a difference which increased for longer crack length.

In general, the effect of changing load parameters in flight simulation tests was correctly predicted by the CORPUS model. Such effects include the effect of gust load severity, the effect of ground stress level, the effect of design stress level, the effect of load sequence, the effect of truncation, and the effect of omitting small loads. The general conclusion was that the CORPUS model was quite satisfactory.

Two shortcomings of the CORPUS model mentioned in [5.6] were:
- Conservative predictions were obtained if rarely occurring downward gust loads, which were more negative than the ground load, occurred. This effect was not very well predicted.
- A load sequence effect was not always predicted correctly, especially for simple load sequences.

Some comments on these shortcomings are given below

a. Effect of rarely occurring severe negative gust loads.

The most severe flight in a flight simulation sequence usually occurs only once in one large block of flights (1000, 2500 and 4000 flights in one block of the CN-235, F-27, and TWIST spectrum respectively). That flight has the most severe positive and the most severe negative gust load. In some load histories the most severe negative load was more compressive than the ground loads. According to the hump mechanism mentioned previously this most severe negative load will reduce all previously created humps. The opening stress of the dominant hump is then calculated with this severe negative load. There is no mechanism in the model to eliminate or to relax the influence of this rarely occurring compressive load. Consequently, it leads to a relatively low crack opening stress for a long period, which causes a higher crack growth rate, and apparently an
underestimation of the crack growth life. Fig. 5.12 shows the crack growth rate under the F-27 spectrum. In the three cases shown, the most severe downward load is a gust load. The ratio between predicted lives and test results are 0.33, 0.42, and 0.62 for the cases SL100, NL100, and LL100 respectively. The predominant effect of the low gust load was found especially in 2024-T3 Al-alloy. For 7075-T6 Al-alloy such a low crack opening stress does not occur very long, due to small plastic zone sizes (delay switches turned off).

b.Load sequence effect for simple load cases
The simplified flight simulation sequences of Misawa are shown in Fig. 5.8. In load sequence II the overload occurs at the beginning of the flight, whereas in sequence III it occurs at the end of the flight. Test results did not show a significant difference between the two sequences, while the CORPUS prediction indicates a significant effect, especially for \( m = 100 \) (cycles in one flight). The average ratio of predictions to test results is 1.52 and 0.74 for sequence II and III respectively. This ratio indicates that CORPUS overpredicts sequence II, while it underpredicts sequence III. The difference is associated with the difference in the predicted \( S_{op} \) in the two sequences, as indicated in Fig. 5.13. In sequence II \( S_{op} \)-level is determined by the maximum stress of the overload and the minimum stress of the flight load, which leads to a higher \( S_{op} \) than in sequence III, where \( S_{op} \) is determined by the maximum stress of the overload and the ground stress. The lower \( S_{op} \)-level in sequence III causes shorter predicted lives. In a more complex load sequence, i.e. miniTWIST, modification of the load sequence did not result in significant changes of predicted lives, which is in agreement with the test results.

5.3.2 The modified CORPUS model
Based on the study of the three crack closure models i.e. PREFFAS, ONERA, and CORPUS, Padmadinata [5.6] proposed a modification of the CORPUS model. The modification is intended primarily to remove the influence of the most severe gust which is more negative than the ground load. As mentioned in section 5.3.1, the severe underload flattened the dominant hump lowering the \( S_{op} \)-level of the dominant hump. Since this lower \( S_{op} \)-level is associated with the maximum overload and the most negative underload, it will remain for along time. There is no mechanism in the CORPUS model to remove this influence, and thus it can last for the entire flight simulation block depending on the plastic zone size of the dominant hump. In the modified CORPUS model, Padmadinata introduced the
underload affected zone to uncouple the effect of the underload from the maximum overload. The procedure for selecting \( S_{op} \) is also simplified by introducing the local \( S_{op} \) instead of \( S_{op} \) of the SPZ. The modified model applied the multiple overload interaction also to 7075 material, where in the original CORPUS model it is only applied to 2024 material.

**Description of the modified CORPUS model**

**Underload affected zone (ULZ)**

To confine the influence of the severe underload the ULZ is introduced in the modified CORPUS model. The ULZ will determine when the effect of the severe underload has to be removed from the material memory. The extent of the ULZ is associated with the reversed plastic deformation induced by the underload. The size of the underload zone is approximated by assuming that it develops under plane strain condition:

\[
D_u = \frac{1}{9\pi} \left( \frac{K_{op} - K_{min}}{2\sigma_y} \right)^2
\]

(5.31)

The size of ULZ is governed by the range of \( K_{op} - K_{min} \) because at the moment that \( K \) is equal to \( K_{op} \) the crack is closed. A more downward load will create additional reversed plasticity in the zone \( D_u \). In general the behaviour of the ULZ is very similar to the PPZ, i.e.

- a ULZ can overlap with another ULZ.
- a more severe underload overrules the previous lighter underloads.
- the series of ULZ is characterized by an increasing series of \( SH_{min} \) values.
- from the series of ULZ, the one caused by the most severe underload is considered to be the dominant ULZ.
- if the crack is growing beyond a ULZ the \( SH_{min} \) is reset.

Historical values of \( S_{op} \) are determined from \( SH_{max} \) and \( SH_{min} \) of the ULZ.

**The selection of \( S_{op} \)-level**

The second difference in the modified CORPUS is the procedure to select the \( S_{op} \)-level. In the original CORPUS model, the \( S_{op} \) is selected from the maximum \( SH_{op} \) of the PPZ and the SPZ. The modified CORPUS model does not consider the SPZ, instead the local \( S_{op} \) related to the current stress level \( (S_{max,i}, S_{min,i}) \) is introduced. The local \( S_{op} \) is applied to the next cycle only if it exceed \( SH_{op} \). With this procedure the selection of \( S_{op} \) is simplified because no
historical SPZ has to be stored.

Introduction of multiple overload interaction to 7075-T6

In the modified CORPUS model, Padmadinata applied the multiple overload interaction also to 7075-T6 Al-alloy. The material constant in the relaxation parameter $\delta$ (Eq. 5.28) for 7075-T6 Al-alloy was chosen to be 0.15. This value was obtained by comparing prediction results for different values of $\delta$.

Summary of prediction results for through cracks

In most cases prediction results with the modified CORPUS model do not differ very much from the original CORPUS model. The average ratio of predicted lives to test lives ($N_p/N_t$) for 83 test series is 0.89 for the CORPUS model and 0.87 for the modified CORPUS model. Fig. 5.14 shows the comparison of the average ratios for different test series. However, the standard deviation is significantly lower for the modified CORPUS model (standard deviation 0.275 for the CORPUS model and 0.182 for the modified CORPUS model).

Better prediction results were obtained with the Modified CORPUS model for cases, where the most severe gust is more compressive than the ground stress level. For the three cases of the F-27 spectrum mentioned in section 5.3.1, where the CORPUS model gave $N_p/N_t = 0.62$, 0.42, and 0.33, the modified CORPUS model results are 1.02, 1.02, and 1.05. The effect of different ground stress levels is also predicted correctly because the influence of the most negative gust load, which masks this effect, is eliminated. Better predictions were also obtained for different sequences applied in the Misawa tests. The modified CORPUS predict a negligible difference between sequence II and sequence III, which is in agreement with the test results.

In addition to a correct prediction of the effect of different ground stress levels and different load sequences, the modified CORPUS model also correctly predicts the effects of (1) design stress level, (2) truncation of high loads, and (3) omission of small cycles. The last three trends were also predicted correctly with the CORPUS model. However, the crack increments in the rarely occurring most severe flights are still poorly predicted as in the CORPUS model.
5.4. **Flight Simulation test data and results**

5.4.1. Flight simulation tests of the present investigation

Two types of flight simulation sequences were applied in the present investigation i.e.
- Misawa load sequence
- CN-235

A survey of the test program with the flight simulation sequences is presented in Table 5.1. The information includes the type of load sequence and the load levels.

As mentioned previously, the Misawa load sequence is a simplified flight-simulation sequence, where all flights applied in one test are similar. Three different flights were defined, and different combinations of the overload stress level \((S_{OL})\) and the ground stress level \((S_{gr})\) were applied (see again Fig.5.8). For the present investigation the stress level of the CA cycles in flight was taken to be \(S_{max} = 150\) MPa and \(S_{min} = 50\) MPa, the overload stress levels were \(S_{OL} = 200\) MPa and \(250\) MPa, the same ground stress levels as in the Misawa tests were adopted, i.e. \(S_{gr} = 0\) MPa, \(-40\) Mpa, and \(-80\) MPa. The combinations of \(S_{OL}\) and \(S_{gr}\) applied in the present investigation are given in Table 5.1a.

As described previously, the CN-235 flight simulation sequence has ten different flight types in a block of 1000 flights (see again Figs.5.9 and 5.10 for the 10 different flight profiles and flight sequence in one block respectively). The stress levels adopted in the present investigation are given in Table 5.1b.

A further description of the flight simulation sequence is given in Appendix C.

Tests were done on surface cracked specimens of 7075-T6 and 2024-T3 Al-alloy. The specimens have the same dimensions as used for CA tests (total length 300 mm, width = 100 mm, and thickness 9.6 mm). The initial notch in all specimen has the same dimension i.e. \(a_0 = 1.92\) mm, \(c_0 = 3.2\) mm \((a_0/t = 0.2\) and \(a_0/c_0 = 0.6\).

Marker load cycles were applied in tests with the Misawa sequence to reveal the crack front position after a certain number of flight cycles. In the CN-235 tests no marker load cycles were applied because flight A and B of the spectrum create a regular pattern on the fracture surface which can be easily recognised.
5.4.2. Flight-simulation tests reported by Hall et al. [5.18]
In a series of tests with surface cracked specimens Hall et al. [5.18] applied load spectrum representative for bomber aircraft and fighter aircraft which are manoeuvre dominated spectra. A short description of the two flight simulation sequences is given in Appendix C. The materials used were 2219-T851, Ti-6Al-4V, 9Ni-4Co-0.2C steel. Results of the titanium specimens and the steel specimens will not be discussed, because of an inappropriate $\gamma R$ relation for titanium (see section 4.5.2 of Chapter 4), while $\gamma R$ for this type of steel is not available. Test data, including specimen dimensions and the size of initial notches, are given in Table 5.2. Two specimens were tested under one type of load spectrum. Three initial flaws of different size were introduced along the length of each specimen. The distance between the initial flaws was 88.9 mm.

5.4.3. Fatigue crack growth lives and crack growth rates
5.4.3.1 Results of the present test series

Results for 7075-T6 Al-alloy with the Misawa sequence

The fracture surface of 7075-T6 material shows easily recognised marker bands, the position of the marker bands was measured with a microscope equipped with a coordinate measuring table. Fig.5.15 is an example of the fracture surface of specimens loaded with flight simulation sequences of the present investigation. Figs.5.16 show examples of the records of the marker bands on the fracture surface of different specimens. The discrete symbols represent the measured positions, and the semi ellipses fitted to the discrete points are also shown. Crack edge trailing is found in all specimens especially for the higher overload of $S_{OL} = 250$ MPa. The occurrence is more pronounced for a longer crack length. Despite the occurrence of the crack edge trailing, the distance ($\Delta c$) of two successive real crack fronts on the surface is similar to the distance of the two fitted semi-ellipses (see Fig.5.17). The figure shows the true crack increment as a function of the crack increment obtained from the fitted semi-ellipses. Almost all points are in the range of 0.75 to 1.25. The agreement implies that the crack growth rate on the surface is similar to the crack growth rate of the assumed semi-elliptical crack.

Crack growth lives until breakthrough are presented in Table 5.3. The lives until breakthrough are obtained by an extrapolation of the crack propagation curve ($a$ vs $n$) until $a = t$. Fig.5.18 shows examples of crack propagation curves of load sequences II and III. It
should be noted that the half crack length (c) in Fig.5.18 refers to the fitted semi-ellipse.

Microfractographs of three specimens are shown in Fig.5.19. Load sequence type I was applied on the specimen shown in Fig.5.19a. The figure shows four striations created by the gust cycles and one larger striation created by the GAG cycle. Fig.5.19b shows the fracture surface of a specimen with load sequence type II, \( m = 5 \) and \( S_{OL} = 250 \) MPa. This figure shows that the crack growth in one flight is dominated by the crack extension of the overload. A different situation is found for the larger number of cycles in a flight (\( m = 100 \)) and the lower overload (\( S_{OL} = 200 \) MPa, sequence III), see Fig.5.19c. Crack growth in one flight occurred more in the smaller gust cycles, the contribution of the overload cycle is not dominant.

**Result for 2024-T3 with the Misawa sequence**

Fig.5.20 shows examples of records of marker bands on the fracture surface of different specimens. The records do not show essential differences with those of the 7075-T6 specimens, a similar crack edge trailing is found for the more ductile 2024 specimens. Crack growth lives until breakthrough are given in Table 5.4 and examples of crack propagation curves are shown in Fig.5.21.

Tests with the Misawa sequences for through cracks in 2 mm 2024-T3 sheet material quoted in [5.6] showed that load sequences II and III with the same number cycles resulted in a very similar crack growth lives (Table 9.7a and Table 9.7b) of [5.6]. However, the present tests with surface cracks show more scatter for load sequence II and load sequence III for the same number of smaller cycles.

**Results for 7075-T6 Al-alloy with the CN-235 flight simulation sequence.**

The CN-235 flight simulation sequence presented in Fig.5.10 shows that in block of 1000 flights there is one flight type A (flight 591) and two flights type B (flight 239 and flight 921). The occurrence of flight type A and flight type B can be recognised on the fracture surface of 7075-T6 specimens as a repeated sequence of a dark band (flight type A) followed by two lighter bands (flight type B), see Fig.5.15. The visibility of the sequential bands is more obvious in the measuring microscope. The trace of flight type A and B, obtained with the measuring microscope, on the fracture surface before breakthrough for 7075-T6
specimens are shown in Fig.5.22. The total crack propagation lives and the crack propagation lives to breakthrough of the 7075-T6 specimens are presented in Table 5.5. The number of flights at the occurrence of crack breakthrough in Table 5.5 was obtained by extrapolating the crack growth curve (a vs number of flights) in Fig.5.23. For specimens tested with the lower truncation level (maximum stress of flight C) more bands are visible on the fracture surface, which are supposed to be flight type C. Flight type A in these specimens can be distinguished from flight type B and C because it left darker bands compared to other bands. It was not possible to distinguish flight B from flight C, because they have a similar appearance. Therefore, only traces of flight A were drawn in Fig.5.22 for this specimen.

Detailed SEM observations were performed on one 7075-T6 specimen, i.e. a specimen tested with the untruncated load spectrum and S_{max} = 162 MPa. The observations were intended to reveal the retardation following the more severe flights, i.e. flights A and B. SEM observations were done mainly close to the specimen surface. In the depth direction, it was difficult to distinguished different flights because the striations width are too small. Fig.5.24 is an example of the fractograph close to the specimen surface. This fractograph showed that there are a group of striations which form a repeated unit. Later it was recognised that the group of the striations represent an individual flight. Flight H appears as a group of striations with one wider striation of the GAG cycle followed by one small striation, a wider striation of the highest stress in flight H and four smaller striations (see flight profile of flight type H in Fig.5.9 for comparison). By comparing the flight profile and the appearance of the striations other flight types can be identified. The reconstitution of the striations in Fig.5.24 reveals that the observed area represents flights 195 - 229. Observations were made at several locations on the fracture surface of the specimen. The features of all flight types can be distinguished, except for flight C which was not found in the observed area. From microfractographs such as Fig.5.24 the average crack extension of a particular flight can be measured. For flights A and B it was not possible to determine the end of the flight due to the absence of striations of the following flights. Crack growth rates of flight F to H at different positions close to the specimen surface are shown in Fig.5.25. The position of flights A and B are also indicated in Fig.5.25. The decrease of crack growth rates following flights B are quite obvious. At small crack length, crack growth rates following flights A do not show a decrease, only the last observed flight A shows a decrease of the crack growth rate. The absence of retardation in the observed area following flight

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A was due the failure to observe striations close to flights A, striations of other flights were visible after some distance, where the retardation effect might already have vanished. For flights B, shortly after those flights, striations of other flights were visible, in this region retardation effects caused by flight B were still present.

Results for 2024-T3 Al-alloy with the CN-235 flight simulation sequence
For the 2024-T3 specimens the sequence of traces of flight type A followed by two flights type B before crack break-through could not be identified with the measuring microscope. An example of the fracture surface of a 2024-T3 specimen can be seen in Fig.5.15. Traces became visible on the fracture surface after crack break-through, especially at larger crack length. To estimate the occurrence of crack break-through the crack length after break-through was plotted as a function of the number of flights. The plot was extrapolated to the point of break-through, i.e. at the middle of the specimen, to obtain the flight number at the occurrence of break-through. The estimated number of flights to breakthrough and the total lives of 2024-T3 specimens are given in Table 5.5. In the specimen with spectrum truncation to the maximum load of flight C, it was not possible to distinguish different flight types (A, B, and probably C), because the traces of these flights have a similar appearance. SEM observations failed to characterise different flights, because most flights appeared as one striation with a similar width. It confirmed that 2024-T3 is not as easy a material for fractographic studies as 7075-T6.

5.4.3.2 Result of bomber and fighter spectrum of Hall et al.[5.18]
Crack growth lives for the bomber and the fighter spectrum of 2219-T851 specimens are given in Table 5.6. Crack growth curves for the bomber and the fighter spectra are shown in Fig. 5.26. No fractographic observation were reported in [5.18] for these spectrum tests.

5.5 The CORPUS and the modified CORPUS models for surface cracks.
5.5.1 Modelling crack growth at discrete points
In applying the CORPUS model and the modified CORPUS model to surface cracks, some adjustments must be made because of the difference in geometry of surface cracks and through cracks. A through crack is usually assumed to have a straight crack front, therefore the value of K is the same along the crack front. As a consequence the same crack increment and the same plastic zone size can be assumed along the crack front. This
assumption can not be applied for surface cracks because $K$ varies along the crack front. For this reason crack growth calculations have to be made at a number of discrete points along the crack front. Based on the results of prediction of surface cracks under CA loading in Chapter 3, the number of calculation points was chosen to be 8. In this section some adjustments of the CORPUS model to accommodate calculation at discrete points are presented.

*Calculation of the crack increment*

During crack growth, the shape of the crack is assumed to be semi-elliptical with crack depth $a$ and crack length $c$. Because of symmetry only half of the semi-elliptical crack is analysed. Fig.4.16 shows the geometry of the crack as already used for the CA prediction. It will also be used here. The angle $\varphi$ and the coordinates of a point $J$ are calculated with Eq.(4.14a), Eq.(4.14b), Eq.(4.14c):

$$\varphi = \frac{(J-1)(\pi/2)}{(M-1)}$$

$$x(J) = c \cos \varphi$$

$$y(J) = a \sin \varphi$$

where $M$ is the number of calculation points, in this case $M = 8$. The Paris equation (Eq.5.8) is used to calculate the crack increment ($\Delta l_J$) in the direction perpendicular to the tangent of the crack front at point $J$. The component of $\Delta l_J$ in x and y directions are added to the previous coordinates to obtain a new coordinate of the crack front. For $M = 8$, a discrete set of eight coordinates of the new crack front is obtained. The new crack front is obtained by fitting a semi-ellipse to the discrete points.

*Plastic zone size calculation*

The extent of the primary plastic zone for each calculation point is calculated assuming plane strain conditions along the crack front. A simple equation is adopted for the calculation of PPZ:
\[
d p(J) = \frac{1}{3\pi} \left( \frac{K_{\text{max}}}{\sigma_{0.2}} \right)^2
\]  

(5.32a)

Secondary plastic zones are calculated by replacing \(\sigma_{0.2}\) in Eq. 5.31 with \(2\sigma_{0.2}\):

\[
d p_s(J) = \frac{1}{3\pi} \left( \frac{\Delta K}{2 \sigma_{0.2}} \right)^2
\]  

(5.32b)

A plastic zone enclave is fitted to the discrete extent of the plastic zones calculated for each point. The shape of the plastic zone enclave is also assumed to be semi-elliptical. For further calculations the extent of the plastic zone of a point is taken as the distance to the fitted plastic zone enclave, and not from Eq. (5.32a) and (5.32b). The derivation of the magnitude of the distance to the plastic zone enclave is given in Appendix D.

**Check for delay switch and the formation of a new PPZ**

A delay switch is turned off if the coordinate of a point \(J\) after crack extension lies outside the previous (historical) plastic zone enclave. For a historical plastic zone enclave having axes \(\text{adpaH}(1, J)\) and \(\text{adpcH}(1, J)\) the condition can be written as

\[
\frac{x(J)^2}{\text{adpcH}(1, J)^2} + \frac{y(J)^2}{\text{adpaH}(1, J)^2} > 1
\]  

(5.33)

\(H\) stands for historical and \(1\) for PPZ.

A new plastic zone is formed if the fitted plastic zone extent of a point occurs outside the historical plastic zone enclave (see figure in Appendix D for a schematic picture of the current crack front with the fitted plastic zone enclave):

\[
\frac{\text{adpfx}^2}{\text{adpcH}(1, J)^2} + \frac{\text{adpfx}^2}{\text{adpaH}(1, J)^2} > 1
\]  

(5.34)

where \(\text{adpfx}\) and \(\text{adpfy}\) are the coordinates of the plastic zone extent from a point \(J\) (see Appendix D for the calculation of \(\text{adpfx}\) and \(\text{adpfy}\)). It should be noted that the condition in Eq. (5.33) and Eq. (5.34) can be satisfied for some points but not for others. For Eq. (5.33), if a point \(J\) satisfies the condition, the delay switch of that point is turned off, while other points still hold the delay switch.
Overload interaction factor

In the calculation of the overload interaction parameter $\delta_1$, the size of the dominant plastic zone $D^d$ and the current plastic zone $D^n$ are involved, Eq. (5.29a) and (5.29b):

$$
\delta_1 = \left[ \frac{4D^d D^n}{(D^d + D^n)^2} \right]^4 \quad \text{if } D^n \leq D^d
$$

$$
\delta_1 = \frac{D^d}{D^n} \quad \text{if } D^n > D^d
$$

For the surface crack growth calculation, where calculations are performed at points defined by angle $\phi$, $D^d$ and $D^n$ do not lie on the same line (see Fig. E in Appendix E). For this reason $D^n$ is not taken as the plastic zone extent at the point, but on the line of $D^d$ instead. The derivation of $D^n$ is given in Appendix E.

For the modified CORPUS model the underload affected zone (ULZ) from each calculation point is calculated with Eq. (5.32b):

$$
D_1(J) = \frac{1}{3\pi} \left( \frac{K_{op}(J) - K_{min}(J)}{2\sigma_{0.2}} \right)^2
$$

An ULZ enclave is fitted to discrete $D_1(J)$ assuming the shape to be semi-elliptical. Similar to PPZ an ULZ switch is turned off if a point J after crack extension lies outside the previous (historical) ULZ enclave. The criterion for a new ULZ to be formed is also similar to the criterion for PPZ i.e. it is formed if the fitted ULZ of a point lies outside the historical ULZ enclave.

5.5.2 Flow diagram of the calculation

Fig. 5.27 shows the flow diagram of the CORPUS model. There are three loops in the flow diagram, in each loop repeating calculations are made for discrete points along the crack front defined by angle $\phi$.

Several steps in the flow diagram can be recognized:

1. Read $S_{max,j}$, $S_{min,j}$ (each cycle starts with a maximum, followed by a minimum stress).
2. Calculate $S_{op,j}$, assuming CA-conditions.
3. If $S_{\text{max}} < S_{\text{op,\,max}}$ the crack is not opened then $\Delta a = 0$.

4. Calculate the stress intensity factor correction, $\text{Corr}(J)$ with the Newman-Raju solution and the effective stress intensity range, $\Delta K_{\text{eff}}(J) = \text{Corr}(J)(S_{\text{max}} - S_{\text{op,\,j}})$. Calculate $\Delta a$ with the Paris relation. Repeat the calculations for all points. Discrete points of a new crack front are obtained, with coordinates $x(J), y(J)$.

5. Fit a semi-ellipse to the discrete points $x(J), y(J)$ with the least square method. The axis of the semi-ellipse in $y$ direction "a" is the depth of the new crack front, and the axis in $x$ direction "c", is the half length of the new crack at the surface.

6. Calculate coordinates of discrete points defined by $\varphi$ on the new crack front where calculations will be performed. At each point, calculate the extent of the plastic zone $dp(J)$ with Eq.5.31. The components of $dp(J)$ in $x$ and $y$ directions are $dpx$ and $dpy$ respectively. The coordinates of the extent of the plastic zone are $(x(J) + dpx(J)), (y(J) + dpy(J))$.

7. A semi-ellipse is fitted to the discrete points $(x(J) + dpx(J)), (y(J) + dpy(J))$ to obtain the plastic zone enclave, having axes $adpa$ and $adpc$.

**Aspects of PPZ**

8. Check whether a point $x(J), y(J)$ lies outside the plastic zone enclave $adpaH(1,J), adpcH(1,J)$. If this condition (Eq.(5.33)) is satisfied, then a delay switch of a PPZ must be turned off. Memory values of remaining PPZ must be renumbered.

9. If $S_{\text{max}}$ exceeds an $SH_{\text{max}}(1,J)$ value a new PPZ will be created and an old one will be overruled. All SPZ’s are erased.

10. If the coordinate of the border of the plastic zone from a point $J$ is outside the last created PPZ, a new PPZ is created. The values of the overload interaction parameter $m(L,J)$ and $S_{\text{op}}(J)$ are calculated, then select the highest $S_{\text{op}}(J)$.

11. If $S_{\text{min}}$ is lower than $SH_{\text{min}}(1,J)$ then the values of $SH_{\text{min}}(1,J)$ and $SH_{\text{op}}(1,J)$ will be reset. All SPZ’s are deleted.

12. If a new PPZ is created there are no SPZ’s to be considered. Go to D to perform calculations for other points.

**Aspects of SPZ**

13. If there is no previous SPZ ($N2 = 0$), a new SPZ will be created.

14. Check whether a point $x(J), y(J)$ is outside the plastic zone enclave $adpa(2,J), adpc(2,J)$. 136
If the point is outside the enclave, a delay switch of SPZ must be turned off.

15. If \( S_{\text{max},i} > SH_{\text{max}}(2,J) \) one or more SPZ's will be overruled and can be erased from the memory.

16. If \( S_{\text{min},i} < SH_{\text{min}}(2,J) \) then \( SH_{\text{min}}(2,J) \) and \( SH_{\text{op}}(2,J) \) will be reset.

17. Check whether \( S_{\text{op},i}(J) \) is larger than the maximum \( S_{\text{op}}(J) \) of the PPZ. New SPZ is created if \( S_{\text{op},i} > \max SH_{\text{op}}(1,J) \).

18. Choose the valid value of \( S_{\text{op}}(J) \) for the next cycle, which is the highest \( S_{\text{op}} \) of PPZ's and SPZ's if present.

19. Repeat calculation for other points.

20. Go to the next load cycle.

The flow diagram of the modified CORPUS model is presented in Fig.5.28. Compared to the CORPUS model the flow diagram is simpler because secondary plastic zones are not considered.

5.6. Prediction Results
Crack growth lives predicted with the CORPUS and the modified CORPUS models are presented in Tables 5.3 to 5.6 together with the test results. Non-interaction prediction results are also included to see the extent of interaction effects predicted by the CORPUS and the modified CORPUS models. The ratio of prediction results to test results are shown in Fig.5.29. Examples of predicted crack propagation curves compared to test results are shown in Fig. 5.18 (7075-T6 with the Misawa load sequence), Fig.5.21 (2024-T3 with the Misawa load sequence), Fig.5.23 (7075-T6 with CN-235 flight simulation load), Fig.5.26 (2219-T851 with bomber spectrum and fighter spectrum). Fig.5.30 shows the comparison of predicted crack shape development curves to test results of some specimens of different load spectrum. Comparisons of predicted crack growth rate to test results are shown in Fig.5.31.

**Prediction results for the Misawa sequence**

**Non-interaction predictions**
The non-interaction predictions were performed by adding the crack extension in each cycle, calculated with the Paris equation, \( \Delta a = f(\Delta K_{\text{eff}}) \). The \( S_{\text{op}} \) level in each cycle is obtained from \( S_{\text{max}} \) of the current cycle and \( S_{\text{min}} \) of the preceding cycle. Table 5.3 shows that the
average ratio of $N_p/N_i$ for non-interaction predictions of 7075-T6 specimens is 0.98, whereas for 2024-T3 (Table 5.4) the ratio is 0.75. These results imply that interaction effects on the average are not large, but apparently more significant for 2024-T3 material with this particular load sequence. Predictions results for $m = 100$ are more conservative compared to predictions for $m = 5$, both for 7075-T6 and 2024-T3. Fractographic observations of a specimen with $m = 5$ mentioned in section 5.4.3 (see Fig.5.19b) have shown that crack extension in one flight ($m = 5$) occurs primarily in the overload cycle. Therefore, retarded crack growth in the smaller cycles becomes less important in the overall crack extension. Unconservative predictions for $m = 5$ and $S_{os} = 250$ MPa should be due the underprediction of crack growth in the overload cycles. For $m = 100$, Fig.5.19c showed that the retarded crack growth of the smaller cycles in one flight is more predominant compared to the crack extension of the overload cycle. Non-interaction prediction for load sequence III indicates a similar retardation compared to load sequence II, as shown by the small difference of predicted lives for the same $m$ and the same $S_{OL}$.

Padmadinata obtained similar results for non-interaction predictions of the Misawa sequence. For load sequence II with $m = 5$ his predictions for different ground stress resulted in average $N_p/N_i = 1$, while for $m = 100$, the average $N_p/N_i = 0.53$. The explanation of the occurrence of retardation of the present results of surface cracks should also apply to the results of through cracks predicted by Padmadinata.

**CORPUS predictions**
In the CORPUS model and the modified CORPUS model the $S_{op}$ level of the current cycle is determined by considering of $S_{op}$-levels of previous cycles (see section 5.3.1 and 5.3.2), left from humps in the wake of the crack. Interactions between different load levels are thus taken into account in the calculation and determination of $S_{op}$ for the next cycle. If interaction effects can be modelled by considering the variation of $S_{op}$ during variable-amplitude loading, it might be expected that $N_p/N_i$ will be equal to one for the CORPUS and the modified CORPUS predictions.

Table 5.3 and 5.4 show that the average ratio of $N_p/N_i$ for the CORPUS model is 1.11 and 0.99 for 7075-T6 and 2024-T3 respectively. For load sequence II with $m = 100$ the CORPUS prediction for 7075-T6 gives accurate results with $N_p/N_i$ is equal to 1.06 and 1.03, while for
2024-T3 the predictions are slightly unconservative with \( N_p/N_t \) is equal to 1.16 and 1.25. Padmadinata [5.6] found more unconservative predictions for this particular load sequence i.e., an average \( N_p/N_t = 1.52 \). Padmadinata showed that the \( S_{op} \) level of the smaller cycles is excessively high (see Fig.5.13) because it is obtained from \( S_{OL} \) and \( S_{min} \). The history of \( S_{op} \) of a surface crack on the surface and at the deepest point of load sequence II with \( m = 100 \) for 7075-T6 is shown in Fig.5.32a. Compared to the result of Padmadinata shown in Fig.5.13, the \( S_{op} \) in Fig.5.32a is fairly low, which results in a faster crack growth than in through-crack specimens and thus a less conservative predictions. The same value of \( S_{op} \) applies for the load sequence with \( m = 5 \), however, predictions with the CORPUS model now result in non-conservative predictions, particularly for \( S_{OL} = 250 \) MPa, see Table 5.3, for load sequence II, \( m = 5 \) and \( N_p/N_t = 1.62 \) for 7075-T6, while \( N_p/N_t = 1.32 \) for 2024-T3. Fig.5.19b shows that the crack extension in one flight of the load sequence with \( m = 5 \) is dominated by the crack extension during the overload cycle, the unconservative predictions should be due to underprediction of the crack extension of the overload cycle.

The CORPUS model predicts a lower \( S_{op} \) of the smaller cycles for load sequence III (see Fig.5.32b) compared to load sequence II, because the \( S_{op} \) in this case is obtained from \( S_{OL} \) and \( S_{gr} \). As a consequence predicted lives are less than predictions for load sequence II, especially for \( m = 100 \).

**Modified CORPUS prediction**

In general good prediction results for the Misawa sequence are obtained with the modified CORPUS model for 7075-T6 and 2024-T3. The average ratios are of \( N_p/N_t = 1.26 \) for 7075-T6, and a slightly conservative result \( N_p/N_t = 0.84 \) for 2024-T3. The latter conservative result is caused by the predictions for load sequences with \( m = 100 \) (sequence II, \( N_p/N_t = 0.60 \) and 0.57, sequence III, \( N_p/N_t = 0.59, 0.62, 0.64, 0.53 \)). Conservative predictions were also found by Padmadinata for load sequence III.

For both load sequences II and III, \( S_{op} \) of the smaller cycles is determined by \( S_{OL} \) and \( S_{gr} \). Therefore, the predicted lives of the two sequences are very similar. The history of \( S_{op} \) for load sequence II and load sequence III at the deepest point in a flight with \( S_{OL} = 250 \) MPa, \( S_{gr} = -40 \) MPa, and \( m = 100 \) is shown in Fig.5.33. The value of \( S_{op} \) following the overload is determined by the local \( S_{op} \) calculated from \( S_{OL} \) and \( S_{min} \) of the small cycles,
because this $S_{op}$ is higher than $S_{op}$ of $S_{OL}$ and $S_{gr}$. $S_{op}$ of the following cycles is determined from $S_{OL}$ and $S_{gr}$. An increase of $S_{op}$ at the deepest point occurs after a number cycles (see Fig.5.33). $S_{op}$ increases to the value calculated from $S_{OL}$ and $S_{min}$ because the crack has grown through the underload affected zone. A reset occurs to the previous value of $SH_{min}$, i.e. $S_{gr}$ is replaced by $S_{min}$. It may be pointed out here, that in reality such discontinuous changes will not occur. However, this is a consequence of the model, which simplifies the effect of plastic zones as discussed previously in relation to Fig.5.3.

**Trend prediction for the Misawa tests**

Test results for the Misawa sequence of through cracks quoted by Padmadinata and surface cracks of the present tests, especially for 2024-T3, did not show appreciable differences between load sequence II and load sequence III. The average life ratio of test results of sequence II/III for through cracks in [5.6] was 1.03. The table below shows the life ratio of sequence II/III of the present investigation.

<table>
<thead>
<tr>
<th>$S_{gr}$</th>
<th>$S_{OL}$</th>
<th>$m$</th>
<th>7075-T6</th>
<th>2024-T3</th>
</tr>
</thead>
<tbody>
<tr>
<td>-40</td>
<td>200</td>
<td>5</td>
<td>1.29</td>
<td>1.13</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td></td>
<td>1.32</td>
<td>0.98</td>
</tr>
<tr>
<td></td>
<td>250</td>
<td>5</td>
<td>1.06</td>
<td>1.11</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td></td>
<td>1.29</td>
<td>1.09</td>
</tr>
<tr>
<td>average</td>
<td></td>
<td></td>
<td>1.24</td>
<td>1.08</td>
</tr>
</tbody>
</table>

The life ratio for 2024-T3 in this table confirm the trend for through cracks mentioned above, where the life ratio was 1.03. To see how the models indicate this trend a similar table is presented for the three predictions models (non-interaction, the CORPUS, and the modified CORPUS).
The average life ratios of the two sequence (II/III) in the above table shows that the modified CORPUS model predicts the trend very well, while the non-interaction prediction is slightly conservative and the CORPUS model fails to predict the trend.

**Crack shape development and crack growth rate**

Crack shape development curves of test results of 2024-T3 Al-alloy specimens with the Misawa load are compared to the CORPUS and the modified-CORPUS predictions in Fig.5.30a. The agreement between the test results and the predicted crack shape development is good. This conclusion implies that the predicted crack shape is in agreement with the crack shape in the tests. However, the agreement of the crack shape development curves does not guarantee good predicted lives, which is also true for the constant-amplitude predictions. The same crack shape development in the tests and the predictions does suggest that the calculated K variation along the crack front represents the true geometry effect of the semi-elliptical crack.

Predicted and test crack growth rates for 2024-T3 Al-alloy with the Misawa load sequence are shown in Fig.5.31a. Test results show that at the beginning the crack growth rate at the deepest point is faster than on the surface, after some growth the crack growth rate on the surface is higher. This tendency is followed by both the CORPUS predictions and
the modified-CORPUS predictions. For three cases shown in Fig.5.31a the CORPUS predictions and the modified-CORPUS predictions gives a similar crack growth curve. Only in one case i.e., load sequence II, $S_{OL} = 250$ MPa, $m = 100$, the predicted crack growth curves of the CORPUS model and the modified-CORPUS model differ significantly. The reason of the significant disagreement is the difference in the $S_{op}$ calculated with the two models as already mentioned previously.

Prediction results for the bomber and fighter spectra of Hall [5.18].

Non-interaction predictions

Tables 5.6a and 5.6b show that the average $N_p/N_i$ for the bomber spectrum ($N_p/N_i = 0.71$) is higher than the fighter spectrum ($N_p/N_i = 0.44$). It implies that more retardation occurred in the fighter spectrum. The load spectra shown in FIG.C-2 and Fig.C-3 (Appendix C) illustrate that the fighter spectrum contains more smaller cycles than the bomber spectrum. It can then be expected that retarded crack growth during the smaller cycles contribute to the lower ratio of $N_p/N_i$ in the fighter spectrum.

CORPUS predictions

Table 5.6a and Table 5.6b also show that the CORPUS predictions for 2219-T851 gives satisfactory results for both spectra, i.e. an average ratio of $N_p/N_i$ of 1.15 and 0.97 respectively. The $S_{op}$ histories at the surface predicted by the CORPUS model for the bomber spectrum and the fighter spectrum are shown in Fig.5.34a and 5.34b respectively. For the bomber spectrum $S_{op}$ of most smaller cycles is equal to $S_{min}$. The use of Eq.4.10 to obtain $S_{op}$ predicts that the crack is fully open ($S_{op} = S_{min}$) for $R \geq 0.346$. However, the CORPUS model still predicts the occurrence of some interaction effects ($N_p/N_i$ of the CORPUS model is higher than the non-interaction prediction), due to interaction effects of the high overloads on cycles with $R < 0.346$. The $S_{op}$ history of the fighter spectrum shown in Fig.5.34b indicates a relatively constant $S_{op}$ of the dominant hump. A local increase of $S_{op}$ due to high loads is brought back to the $S_{op}$ of the dominant hump due to the negative underloads.

Modified-CORPUS prediction

In Tables 5.6a and 5.6b the modified-CORPUS predictions result in small differences compared to the CORPUS model. The average $N_p/N_i$ for predictions of the bomber
spectrum decrease from 1.15 (the CORPUS model) to 1.06 (the modified CORPUS), while for the fighter spectrum the ratio increases from 0.97 to 1.09. The small differences between the CORPUS predictions and the modified-CORPUS predictions should be due to small difference of the $S_{op}$ in the two models. Fig.5.34a and Fig.5.34b show the $S_{op}$ history for the bomber and the fighter spectrum calculated with the modified CORPUS model. The $S_{op}$ of the dominant hump calculated with the modified-CORPUS model does not differ from the $S_{op}$ obtained with the CORPUS model.

Crack shape development and crack growth rate

Fig.5.30b shows examples of experimental and predicted crack shape development curves of 2219-T851 specimens with the bomber and the fighter spectrum. The CORPUS prediction and the modified-CORPUS prediction give a very similar crack shape development curves, both predictions follow the tendency found in the experiment.

Two examples of experimental and predicted crack growth rates for the bomber spectrum and the fighter spectrum are shown in Fig.5.31b. It may be noted that the crack growth rates in Hall's experiments were quite high due to: (1) large initial flaw in a number of specimens, and (2) high overloads are inserted quite frequently in the spectra (every 6 and 18 missions in fighter spectrum and every 10 and 100 missions in bomber spectrum). Only predictions with the modified-CORPUS model are presented for clarity. Prediction results for the bomber spectrum show a regular increase of the crack growth rate in missions with high overloads. A very small degree of retardation is predicted following missions with high overloads, which is not too surprising because the stress ratio $R$ of most smaller cycles is than larger than 0.346, and no retardation is predicted for those cycles. Only after mission 100, where a higher overload is introduced, a more discernable retardation is predicted. The predicted crack growth rate agrees reasonably good with the measured average crack growth rate of the first 99 missions. In mission 100, where the highest overload is introduced, the crack growth rate is underpredicted. After mission 100 the agreement between experimental and predicted crack growth rate on the surface is still reasonably good, while crack growth rate at the deepest point is underpredicted. The high crack growth rate at the deepest point should be due to ligament yield which is caused by the combination of the high overload during mission 100 and the small ligament (mission 100 occurred at $a/t = 0.95$). The occurrence of ligament yield was also
found in the $S_{op}$ measurements presented in Chapter 4, which causes the decrease of $S_{op}$ close to break-through. In the present model the occurrence of a decreasing $S_{op}$ for large $a/t$ is not included, because it takes place close to break-through where the remaining life is small.

Predictions for the fighter spectrum in Fig.5.31b show that the crack growth rate in missions with high overloads (missions 1, 7, 13, and 19) are also underpredicted as it was found for the bomber spectrum. The crack growth curve in the thickness direction ($da/dN +$ ) shows an overprediction for missions where high overloads were not inserted. Compared to the bomber spectrum, more retardation can be observed in missions following the missions with overloads.

**Prediction results for the CN-235 load sequence.**

*Non-interaction prediction*

The following table shows the results of non-interaction predictions for the CN-235 load spectrum.

<table>
<thead>
<tr>
<th>Material</th>
<th>$S_{max}$</th>
<th>Trunc.</th>
<th>Test</th>
<th>Prediction</th>
<th>$N_p/N_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>7075-T6</td>
<td>162</td>
<td>A</td>
<td>18591</td>
<td>19591</td>
<td>1.05</td>
</tr>
<tr>
<td></td>
<td>200</td>
<td>A</td>
<td>5616</td>
<td>10682</td>
<td>1.90</td>
</tr>
<tr>
<td></td>
<td>200</td>
<td>B</td>
<td>3879</td>
<td>10684</td>
<td>2.75</td>
</tr>
<tr>
<td>2024-T3</td>
<td>162</td>
<td>A</td>
<td>50630</td>
<td>37584</td>
<td>0.74</td>
</tr>
<tr>
<td></td>
<td>200</td>
<td>A</td>
<td>28113</td>
<td>19296</td>
<td>0.67</td>
</tr>
<tr>
<td></td>
<td>200</td>
<td>B</td>
<td>14642</td>
<td>19310</td>
<td>1.32</td>
</tr>
</tbody>
</table>

For 7075-T6 the non-interaction prediction lives are higher than the test lives. In terms of interaction effects, it implies that some acceleration of crack growth must have occurred. In the literature, crack growth acceleration was reported in a simple variable amplitude loading, where an overload or a number of overload cycles is inserted between CA cycles. The crack extension of the overload cycles was found to be higher than
predicted from CA crack growth data (accelerated crack growth). However, the occurrence of an overall crack growth acceleration for the CN-235 spectrum should be disregarded. The CN-235 spectrum contains a few overloads (in the severe flights) and numerous small cycles (in less severe flights), where retardation of crack growth should prevail. If crack growth acceleration contributes to the overall crack growth, the contribution should be limited to the small number of the overload cycles. The most probable cause of the high predicted lives is an inappropriate crack growth rate equation for the numerous small cycles. The crack growth rate equation used in the prediction, Eq.4.12, was derived from tests with through crack specimens of 7075-T6 (9.6 mm thick) with R = 0.1. Accounting for crack closure, \( da/dN \) as a function of \( \Delta K_{\text{eff}} \) was obtained

\[
da/dN = 4.20 \times 10^{-10} (\Delta K_{\text{eff}})^{2.88}
\]

According to the similarity approach Eq.4.12 should be valid for different \( R \) values including \( R > 0.346 \) where the crack is fully open. Fig.5.35 shows that Eq.4.12 agrees very well with test results for \( R = 0.30 \), where the crack is fully open. To check whether the agreement is still valid for higher \( R \) values, additional tests with \( R = 0.67 \) and \( R = 0.80 \) were performed. The crack growth rate results for \( R = 0.67 \) and \( R = 0.80 \) are also shown in Fig.5.35. It obvious that the crack growth rate curves for the higher \( R \) values do not coincide with the crack growth curves for lower \( R \) values. For further predictions the following equations, derived from for \( R = 0.67 \) and 0.80 will be used for cycles with \( R > 0.30 \)

\[
da/dN = 1.17 \times 10^{-10} (\Delta K)^{3.94} \quad \text{for } 0.30 < R \leq 0.67 \quad (5.36a)
\]

\[
da/dN = 3.95 \times 10^{-10} (\Delta K)^{3.85} \quad \text{for } 0.67 < R < 1.00 \quad (5.36b)
\]

Using Eq.5.36 predicted crack growth lives for the CN-235 spectrum are shown in Table 5.5 For tests with the untruncated spectrum (A) non-interaction predictions result in conservative predictions indicating the occurrence of crack growth retardation. Non-interaction predictions can not predict the effect of truncation, because predicted lives for truncation to the maximum stress in flight B are practically the same as for the untruncated spectrum.
CORPUS prediction

The CORPUS prediction for 7075-T6 Al and 2024-T3 Al gave an average ratio \( N_p/N_e = 1.20 \) (see Table 5.5). Although this average ratio is good, Table 5.5 reveals that the predicted lives for the truncated spectrum (B) and the untruncated spectrum are very similar, whereas the test results showed that the truncated spectrum gives much shorter lives as usually observed in flight-simulation tests. In other words, the effect of truncation is not predicted. For the CN-235 load spectrum the stress ratio of all cycles of the unsevere flights (flight types F,G,H,I, and J) is larger than 0.346, except for the air-to-ground cycles. \( S_{op} \) of these cycles, according to Eq.4.10b, is equal to \( S_{min} \) (crack fully open) of that cycles. Since \( S_{op} \) of these cycles are most probably higher than \( S_{op} \) of the dominant hump (truncated or un-truncated), then the crack growth of these cycles is not influenced by truncation. \( S_{op} \) in the CN-235 load spectrum predicted with the CORPUS model is shown in Fig.5.36. It can be seen that \( S_{op} \) of the dominant hump is low and it influences only a small number of cycles mainly in flights A,B,and C.

Modified-CORPUS prediction

Table 5.5. shows that the modified-CORPUS prediction results are similar to the CORPUS predictions. The truncation level effect is also not predicted. The same explanation as for the CORPUS prediction is also valid for the modified-CORPUS prediction. Fig.5.37 shows the \( S_{op} \) history predicted by the modified-CORPUS model. The similar \( S_{op} \) development in the modified-CORPUS and in the CORPUS model explains the similar predicted lives.

Crack shape development and crack growth rate.

Fig.5.30c shows the comparison between experimental and predicted crack shape development curve for the CN-235 spectrum with \( S_{max} = 200 \) MPa. It can be seen that the predicted crack shape development follows the tendency of the test results. The CORPUS prediction and the modified-CORPUS prediction give practically the same curves, which was also found for the prediction of the Misawa sequence, and the bomber and the fighter spectrum of Hall et.al. [5.18].

The comparison of predicted and test crack growth rates for the CN-235 spectrum is shown in Fig.5.31c. It should be noted that test crack growth rate in this figure is the average crack growth rate between two severe flights which leave visible marks on the
fracture surface. Up to \( a/t = 0.4 \) no marker of the severe flights are visible, therefore crack growth data for \( a/t < 0.4 \) are not available. Fig 5.31c shows that for \( a/t > 0.4 \) the predicted crack growth rate do not differ substantially from the test results. The crack growth rate then is already quite high. Differences between test and predicted lives for the truncated spectrum of 7075-T6 specimen (see Table 5.5) should thus result from differences in crack growth rates at small \( a/t \).

5.7. Discussion

1. The \( \gamma(R) \) relation and the thickness effect (\( \gamma = S_{op}/S_{max} \))

Predictions of crack growth lives of surface cracks were performed by assuming plane strain conditions along the crack front. The assumption was made because the plate specimens are sufficiently thick to expect that plane strain conditions will prevail along the crack front. The \( \gamma(R) \) relation for crack opening was obtained by measurements on 9.6 mm thick 7075-T6 plate specimens with through cracks. The relation (Eq.4.10) was assumed to be representative for plane strain. It is shown in Fig.5.38 in comparison to the \( \gamma(R) \) relation adopted by De Koning in CORPUS for crack growth in thin sheet material (Al-alloy sheet specimens of 2 mm), see Eq.(5.10). Apparently, De Koning’s \( S_{op}/S_{max} \) levels are significantly higher than the levels used in the present investigation. However, in thin sheet material the state of stress can not be as close to plane strain as in thick material. Higher \( S_{op} \)-values should be expected for thin sheets, and lower values for plate material. The tendency of relatively low \( S_{op} \)-levels for thick plate specimens is confirmed by the fractographic \( S_{op} \) measurements along the crack fronts of the surface cracks reported in section 4.4.2.

Another indication for a thickness effect on crack closure under flight-simulation load histories can be obtained from several literature sources. In an extensive survey on flight-simulation test results by Schijve [5.19] test series on the thickness issue were also summarized. A systematic thickness effect was generally found in tests with both gust spectra and manoeuvre spectra. An illustration reproduced from [5.19] gives results from Schra and ‘t Hart [5.20] (Fig.5.39). For two aluminium alloys it shows a much faster crack growth in the 10 mm thickness specimens than in the 2 mm sheet specimens. In other words, there is less retardation in the plate specimens. That should be associated with a lower crack closure stress level, due to smaller plastic zones under plane strain conditions. There is more retardation in the thin sheet specimens, where crack closure is more
significant, due to the larger plastic zones obtained under predominantly plane stress conditions.

2. Aspect of the variation of $S_{op}$ along the crack front.

Based on the results of $S_{op}$ measurements along the crack front presented in Chapter 4 a linear variation of $S_{op}$ is assumed (see Eq.4.17). $S_{op}$ decreases linearly from the surface to the deepest point which is assumed to have $S_{op}$ 20% lower than on the surface. For the constant amplitude predictions the incorporations of the variation of $S_{op}$ results in a slightly more conservative predictions. It is also interesting to compare prediction results assuming the same $S_{op}$ along the crack front to predictions assuming a linear variation of $S_{op}$. The constant $S_{op}$ to be used in this prediction is calculated from Eq.4.10, which in the previous predictions was assumed to occur at the deepest point. It is thus assumed that $S_{op}$ along the crack front is the same as $S_{op}$ at the deepest point. Tables 5.7a to 5.7f show the comparison of prediction results for the modified-CORPUS model for different spectra. As should be expected the predictions assuming constant $S_{op}$ leads to lower $N_p/N_t$ values. The average $N_p/N_t$ values are in the range of 9% to 17% lower than the prediction incorporating the $S_{op}$ variation. The differences are not very significant from an engineering point of view. However, the occurrence of a decreasing $S_{op}$ from the surface to the inner part of the material is physically true. Although the assumption of a linear variation of $S_{op}$ is a simplification, it is still physically better than assuming a constant $S_{op}$ along the crack front.

3. Crack edge trailing.

In the present prediction scheme a new crack front is obtained by fitting an ellipse to the discrete crack increments. The shape has been assumed to be and to remain semi-elliptical. The occurrence of crack edge trailing can thus not be predicted. If crack edge trailing should be predicted a much more complex function has to be fitted to the discrete crack increments. Unfortunately $K$ solutions for other than semi-elliptical surface crack are not available. A three-dimensional finite element model can be used to obtain $K$, but cycle-by-cycle calculations of $K$ in a prediction model is prohibitive. The relevant question is, whether such a sophisticated approach is necessary for an engineering prediction technique. Test results in Fig.5.17 have shown that the distance of successive ellipses fitted to the crack front is very close to the crack increment on the surface for cracks with
trailing edges. This encouraging result implies that the crack growth rate on the surface is hardly influenced by the occurrence of crack edge trailing, provided that the deviation from the semi-elliptical shape occurs only close to the surface. Some examples of service failures presented in Chapter 2 shows that surface cracks grow fairly close to the semi-elliptical shape, even for a rather complex geometry.

4. The relaxation factor in overload interactions
The relaxation factor was introduced by de Koning for the calculation of the overload interaction parameter in Eq.5.28. De Koning applied a constant to the relaxation factor, which for 2024-T3 was taken to be 0.28. For 7075-T6 overload interaction was excluded and the relaxation constant was assumed to be zero. Based on predictions with several relaxation constants, Padmadinata chose 0.15 for 7075-T6. In fact the constant in the relaxation factor should be related to the yield stress of the material. Materials with a lower yield stress have a larger plastic zone compared to materials with a high yield stress. As a consequence overlapping of PPZ's will occur more frequently and hence a larger degree of overload interactions can be expected. Therefore a higher relaxation factor should be valid for these materials. For materials with a high yield stress the plastic zone sizes should be very small and overlapping PPZ's will seldom occur, so that the overload interaction can be neglected. The determination of the relaxation factor until now was a trial and error procedure to obtain a value that fits the test results. For the prediction of the 2219-T851 results the constant was taken to be 0.28 both for CORPUS and modified-CORPUS. The same value as for 2024-T3 was taken, because the yield stress of 2219-T851 (362 MPa) is very close to the yield stress of 2024-T3 (360 MPa).

5. Measurements of crack size and shape
In the present investigation the crack size and shape were measured on the fracture surface from either the trace of marker bands or marks of severe flights. Observations of the markers were carried out in an optical microscope equipped with a coordinate measuring table. For constant amplitude tests and the tests with the Misawa sequence, marker loads were inserted periodically, which induced marker bands clearly visible in the optical microscope. In general the visibility of marker bands in 7075-T6 is better than in 2024-T3. For 7075-T6 marker bands close to the initial notch were usually still visible, which for 2024-T3 were not visible. In the CN-235 sequence, flights A and flights B
induced visible markers in 7075-T6 after the crack had grown to approximately half the plate thickness, whereas in 2024-T3 no markers were visible before crack break-through. Therefore crack growth data at small crack depth are not available for 7075-T6, and for 2024-T3 no crack growth data became available before crack break-through. Since crack growth data close to the initial notch is also important, it is necessary to adopt other methods for surface crack size and shape observations. The use of the electrical potential technique seems to be promising in reconstruction of the size of the crack (Kubo [5.21] and Lai [5.22]). It should be noted that in this method the crack is assumed to be semi-elliptical, thus crack edge trailing can not be described.

5.8. Concluding remarks.
In summary, the CORPUS model and the modified-CORPUS model were applied to predict crack growth lives of surface cracks. Test data were obtained with a very simple flight simulation (the Misawa sequence), a more complex flight simulation (the bomber and the fighter sequences of Hall et.al.[5.18]) and a realistic flight simulation (CN-235 sequence). Crack growth calculations were performed at eight discrete points along the crack front. It is assumed that the crack is growing as a semi-ellipse, therefore the discrete crack extension is fitted to a semi-ellipse to obtain a new crack front. With this assumption the occurrence of crack edge trailing can not be predicted. PPZ's and SPZ's in the CORPUS model and ULZ's in the modified-CORPUS model are also assumed to have a semi-elliptical shape.

Some remarks on prediction of surface cracks are given below.
1. In general similar results were obtained for the CORPUS and the Modified-CORPUS prediction.
2. On the average the CORPUS prediction and the modified-CORPUS predictions are fairly accurate. For all specimens the ratios of \( \frac{N_p}{N_t} \) are in the range 0.5 to 2.0. Crack shape developments are accurately predicted. The influence of truncation in the CN-235 spectrum is predicted poorly.
3. Surface cracks usually occur in thick components, which are supposed to be in a plane strain state of stress. In this state of stress the plastic zone sizes are small, and as a consequence the crack closure level is lower than in thin sheets (plane stress condition). Therefore, crack closure measurements on thick specimens to obtain \( \gamma(R) \) relation is
necessary for crack growth prediction of thick components. The use of a \( \chi(R) \) relation for thin sheets will lead to highly unconservative predictions.

4. Although the crack closure level in plane strain conditions is low, non-interaction predictions, in most cases give conservative result, suggesting the occurrence of interaction effects.

5. The use of a varying \( S_{op} \) along the crack front in the modified-CORPUS model results in slightly longer predicted lives. Since the incorporation of the varying \( S_{op} \) in the model is rather simple, it is recommended to include it in the prediction model.

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under constant-amplitude and simplified flight simulation loading," Faculty of

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### Load Parameters

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Table 5.1a Load parameters of tests with the Misawa sequence, material 2024-T3 and 7075-T6. Sequences I, II and III are shown in Fig.5.8.

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Table 5.1b Load parameters of the tests with the CN235 load spectrum, materials 2024-T3 and 7075-T6.

Table 5.1 Survey of the load parameters of the present investigation of tests with flight simulation loading on specimens with surface cracks ($a_s/c_0 = 0.6$, $a_s/t = 0.2$).
<table>
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<td>(a₀/c₀)</td>
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Table 5.2. Specimen data of Hall et al. [5.17], material 2219-T851 Al-alloy. Specimen dimensions are t = 11.43 mm, W = 228.60 mm, L = 685.80 mm. Three surface cracks were introduced along the length of each specimens.
### Table 5.3

Test results and predicted crack growth of 7075-T6 specimens lives loaded by the Misawa sequence. Specimens with surface cracks, $a_0/c_0 = 0.6$, $a_0/t = 0.2$, $t = 9.6$ mm. Three prediction models are used, i.e. non-interaction (linear model), CORPUS, and modified CORPUS model.
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Table 5.4 Test results and predicted crack growth lives of 2024-T3 specimens loaded by the Misawa sequence. Specimens with surface cracks, $a_i/c_0 = 0.6$, $a_i/t = 0.2$, $t = 9.6$ mm. Three prediction models are used, i.e. non-interaction (linear model), CORPUS, and modified CORPUS model.
Table 5.5. Test results and predicted crack growth lives of 7075-T6 and 2024-T3 specimens loaded by the CN235 load spectrum. Specimens with surface cracks, $a_0/c_0 = 0.6$, $a_0/t = 0.2$, $t = 9.6$ mm. Three predictions models are used, i.e., non-interaction (linear model), CORPUS, and modified CORPUS models.
### Chapter 5, Prediction of surface crack growth under flight simulation...

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Table 5.6a Results for bomber spectrum

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Table 5.6b Results for fighter spectrum

Table 5.6. Initial crack geometry, test results and predicted crack growth lives of 2219-T851 specimens loaded by bomber and fighter spectra. Specimens dimensions $W = 9228.6$ mm and $t = 11.43$ mm. Three prediction models are used, i.e. non-interaction (linear model), CORPUS, and modified CORPUS.
### Table 5.7a.

Comparison of prediction results of the MISAWA load (material 7075-T6 Al) with the modified-CORPUS model assuming a linear variation of $S_{op}$ and a constant $S_{op}$ along the crack front.

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Table 5.7b  Comparison of prediction results of the MISAWA load (material 2024-T3) with the modified CORPUS model assuming a linear variation of $S_{op}$ and a constant $S_{op}$ along the crack front.
### Table 5.7c.
Comparison of prediction results of the bomber spectrum (material 2219-T851) with the modified CORPUS model assuming a linear variation of $S_{op}$ and a constant $S_{op}$ along the crack front.

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### Table 5.7d.
Comparison of prediction results of the fighter spectrum (material 2219-T851) with the modified-CORPUS model assuming a linear variation of $S_{op}$ and a constant $S_{op}$ along the crack front.

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Table 5.7e. Comparison of prediction results of the CN-235 spectrum (materials 7075-T6 Al and 2024-T3 Al) with the modified CORPUS model assuming a linear variation of S<sub>op</sub> and a constant S<sub>op</sub> along the crack front.
Fig. 5.1 Plastic zone sizes in the Wheeler and the Willenborg models.

Fig. 5.2 A hump created by an overload, and flattened by an underload.
Fig. 5.3 The hump opening behaviour assumed in the CORPUS model compared to experimental result and finite element analysis [5.5].

Fig. 5.4 The opening behaviour of a crack tip with 3 significant humps on the crack surface [5.5].
Fig. 5.5 Example of crack opening behaviour of four humps [5.5].

Fig. 5.6 Overlapping plastic zones in the CORPUS model [5.6].
Fig. 5.7 Series of decreasing $S_{H_{\text{max}}}$ values and increasing $S_{H_{\text{min}}}$ values in the CORPUS model. Consequences of some $S_{\text{max}}$ and $S_{\text{min}}$ values if they occur [5.6].

$S_{\text{ol}} = 160$ or 200 MPa

$m = 5$ cycles or 100 cycles

$S_{\text{gr}} = 0$, -40, or -80 MPa

Fig. 5.8 Three different sequences in the Misawa sequence
Fig. 5.9 Flight profiles for different flights of CN-235 spectrum [5.6]
CN-235, flight sequence in one block of 1000 flights.

Numbers of most severe flights:
- type A: 591
- type B: 239, 921
- type C: 186, 168, 412, 581, 684, 831

Fig. 5.10 The random sequence of flights in one block of 1000 flights of the CN-235 load spectrum [5.6].

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Fig. 5.14 The average ratio between predictions and test results, predicted by the CORPUS and the modified CORPUS models [5.6]
Fig. 5.15 The fracture surface of specimens loaded by the flight simulation sequences (Misawa and CN-235).
Fig. 5.16 The position of marker bands on the fracture surface of 7075-T6 specimens for the Misawa load sequence. Discrete points are the recorded positions of the actual crack fronts, the full lines are fitted semi-ellipses.
Fig. 5.17 Comparison between true crack extension on the surface and crack extension calculated from fitted semi-ellipses. Crack extension data obtained from 7075-T6 and 2024-T3 specimens for the Misawa sequence.
Fig. 5.18 Comparison between experimental and predicted crack growth curves of 7075-T6 Al-alloy specimens for the Misawa load sequence.
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Fig. 5.19a Test result for load sequence type I.

Fig. 5.19b Test result for load sequence type II with $m = 5$ and $S_{0L} = 250\text{MPa}$.

Fig. 5.19 Samples of microfractograph of 7075-T6 Al-alloy specimens for the Misawa load sequence.

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Fig. 5.19c Test result for load sequence type III with $m = 100$ and $S_{OL} = 200$ MPa.

Fig. 5.19 (cont'd) Samples of microfractograph of 7075-T6 Al-alloy specimens for the Misawa load sequence.
Fig. 5.20 The position of marker bands on the fracture surface of 2024-T3 specimens for the Misawa load sequence. Discrete points are the recorded positions of the actual crack fronts, the full lines are fitted semi-ellipses.
Fig. 5.21 Comparison between experimental and predicted crack growth curves of 2024-T3 Al-alloy specimens for the Misawa load sequence.
Fig. 5.22 The traces of flight A and flight B on the fracture surface of 7075-T6 specimens for the CN-235 spectrum at different load levels and different truncation levels.
Fig. 5.22 (cont'd). The traces of flight A and flight B on the fracture surface of 7075-T6 specimens for the CN-235 spectrum at different load levels and different truncation levels.
Fig. 5.23. Comparison between experimental and predicted crack growth curves of 7075-T6 specimens for the CN-235 spectrum.
Fig. 5.24. An example of microfractograph of 7075-T6 specimen for the CN-235 load sequence ($S_{\text{max}} = 162$ MPa).
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Fig. 5.25. Crack growth rates in a 7075-T6 specimen with the CN-235 spectrum derived from fractographic observations. Crack growth rates of flights F, G, H were measured before and after flights A and B.
Fig. 5.26. Comparison between experimental and predicted crack growth curves of 2214-T851 Al-alloy specimens with the bomber sequence (results of Hall et al. [5.17]).
Fig. 5.27 Flow diagram of the CORPUS model.
Fig. 5.27 (cont'd) Flow diagram of the CORPUS model.
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Fig. 5.27 (cont'd) Flow diagram of the CORPUS model.
Fig.5.28 Flow diagram of the modified CORPUS model.
Fig. 5.28  (cont'd) Flow diagram of the modified CORPUS model.
Fig. 5.28 (cont'd) Flow diagram of the modified CORPUS model.
Fig. 5.29a. Results for 7075-T6 Al alloy with the Misawa load sequence.

Fig. 5.29b. Results for 2024-T3 Al alloy with the Misawa load sequence.

Fig. 5.29. Ratio of prediction to test lives of three predictions model.
Fig. 5.29c. Results for 2219-T851 Al alloy with bomber and fighter spectrum.

Fig. 5.29d. Results for the CN-235 spectrum (7075-T6 Al and 2024-T3 Al).

Fig. 5.29. (cont'd) Ratio of prediction to test lives of three predictions model.
Fig. 5.30a  Results for the Misawa load sequence type II and III (2024-T3 specimens).

Fig. 5.30. Comparison between experimental and predicted crack shape development curve (a/c vs a/t).
Fig. 5.30b Results for the bomber spectrum and the fighter spectrum (2219-T851 specimens, test results of Hall et al. [5.17]).

Fig. 5.30c Results for CN-235 spectrum (7075-T6 Al specimens).

Fig. 5.30. (cont'd) Comparison between experimental and predicted crack shape development curve (a/c vs a/t).
Fig. 5.31a. Results for the Misawa load sequence type II and type III (2024-T3 specimens).

Fig. 5.31. Comparison between experimental and predicted crack growth rate as a function of a/t.
Fig.5.31b. Results for the bomber spectrum and the fighter spectrum (2219-T851 specimens, test results of Hall et.al. [5.18]).

Fig.5.31. (cont'd) Comparison between experimental and predicted crack growth rate as a function of a/t.
Fig. 5.31c. Results for the CN-235 spectrum (7075-T6 specimens).
Fig. 5.31. (cont'd) Comparison between experimental and predicted crack growth rate as a function of a/t.
Fig. 5.32a. Different $S_{op}$ levels at the surface and at the deepest point in sequence II of the Misawa load, calculated with the CORPUS model.
Fig. 5.32b. Different $S_{op}$ levels at the surface and at the deepest point in sequence III of the Misawa load, calculated with the CORPUS model.
Fig. 5.33. Crack opening stress history of load sequence II and III of Misawa load calculated with the modified-CORPUS model.
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Fig. 5.34a. Crack opening stress histories for the bomber spectrum calculated with the CORPUS model and the modified CORPUS model.
Fig. 5.34b. Crack opening stress histories for the fighter spectrum calculated with the CORPUS model and the modified-CORPUS model.
Fig. 5.35. da/dN as a function of ΔK or ΔK_{eff} for R values where the crack is fully open. The Paris equation is fitted for each R. For R = 0.33 the Paris equation is very close to da/dN vs. ΔK_{eff} derived from test with R = 0.1.
Fig 5.36. The $S_{op}$ history for the CN-235 load spectrum predicted by the CORPUS model, the maximum stress is 200MPa. Result for flights A to D.
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Fig 5.36 (cont'd). The $S_n$ history for the CN-235 load spectrum predicted by the CORPUS model, the maximum stress is 200MPa. Result for flights B to J.
Fig. 5.37. The $S_{op}$ history for the CN-235 load spectrum predicted by the modified-CORPUS model, the maximum stress is 200MPa. Result for flights A to D.
Fig. 5.37 (cont') The $S_{op}$ history for the CN-235 load spectrum predicted by the modified-CORPUS model, the maximum stress is 200Mpa. Result for flights E to J.
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Fig. 5.39. Thickness effect on crack growth curves. Results of Schra and 't Hart [5.20].
Chapter 6

Epilogue, summary and conclusion

In the present investigation fatigue crack growth of surface cracks was studied in two Al-alloy plate materials, viz. 2024-T3 and 7075-T6, with a thickness of 9.6 mm. Fatigue tests were carried out under Constant-Amplitude (CA) loading and Variable-Amplitude (VA) loading, including a simple flight simulation load sequence and a realistic flight-simulation load history. The investigation can be characterized in different ways.

(i) A considerable amount of experimental effort had to be made to obtain relevant information on the growth of surface cracks in plate material. Because the semi-elliptical crack fronts can not be seen by eye, marker loads or severe-flight marker traces had to be used. Subsequently, the crack fronts had to be recorded in numerical format by a coordinate microscope. An elliptical crack front has one dimension more (2 dimensions, a and c) than a through crack (1 dimension, a). It highly amplifies the efforts to be spent on recording crack growth.

Secondly, crack closure information was considered to be essential for the evaluation of crack growth models. Again, the 2-dimensional crack shape is a problem, because it prevents compliance measurements to determine the variation of $S_{op}$ along the crack front. Indirect fractographic measurements had to be made, and the experimental work was quite laborious. However, since information in the literature was extremely scarce, it was unavoidable to do this type of exploratory observations. In other words, a lot of fieldwork had to be done in order to obtain an empirical documentation of the topic to be analyzed.

(ii) A second approach to the present investigation is the comparison of the 2-dimensional crack growth behaviour to the behaviour of the 1-dimensional through cracks. The information on the behaviour of through cracks is extensive indeed, and it is believed that a qualitative understanding has been obtained. This understanding is essential for devising crack growth models in terms of fracture mechanics, and even more for recognizing limitations of such models. The question then is whether our conceptions are also applicable to the 2-dimensional surface
cracks, or whether new problems will arise.

(iii) Finally, a practical approach is the question, whether we can predict the growth of semi-elliptical surface crack. Such cracks are important in damage tolerance evaluations of heavy structural components. According to the new airworthiness requirements, recently proposed by the Federal Aviation Administration, fatigue crack growth from initial surface damage has to be considered. Service experience has shown that it is not an unreasonable requirement.

With respect to the above problem settings, the present investigation has led to a number of observations and conclusions. Several conclusions are directly bearing on the results and the analysis of the present investigation. Other conclusions have a more general meaning. The conclusions are presented below under different headings.

6.1 Conclusions, directly related to results and analysis of the present investigation

Observations on crack growth and crack closure

1. The fractographic observations indicate that crack growth of surface cracks under cyclic tension can be described in a reasonably accurate way as a progressive movement of a semi-elliptical crack front. However, the shape of the semi-ellipse (the aspect ratio $a/c$, $a =$ depth, $c =$ semi length at the material surface) will generally not be constant. A shallow crack (low $a/c$) will grow faster at the deepest point and become less shallow. A deep crack (high $a/c$) will grow faster along the material surface and become more semi-circular. This observation has been shown to be valid for CA loading and for VA loading.

2. The crack closure measurements have indicated that $S_{op}$ varied as the crack was growing. In the beginning, it was low, but it increased as the surface crack became larger, which is associated with building up plastic deformation in the wake of the crack. Later, when the crack depth exceeded about $2/3$ of the plate thickness, $S_{op}$ decreased again due to plastic yielding in the remaining ligament area. This occurred shortly before crack break through ($a = t$) in a relatively short period with relatively high crack growth rates.

3. The crack closure level also varied along the crack front. $S_{op}$ is lower at the deepest point and larger at the material surface. The variation is systematic, but it is still
fairly small (some 20% as a maximum). The variation is related to a transition of the state of stress, i.e. plane strain at the deepest point, and plane stress at the surface.

4. The crack front at the surface is trailing behind, which is associated with the higher $S_{op}$ near the material surface.

5. The crack closure levels in the present plate materials were found to be significantly lower than in thin sheet material of the same Al-alloys. This observation should also be related to the more predominantly plane strain conditions in thick plates.

**Crack growth predictions for CA loading**

6. In the past predictions for a surface crack were usually restricted to a prediction of crack growth at two points of the crack front only, viz. to $\varphi = 0$ (surface point) and $\varphi = 90^\circ$ (deepest point). It has been shown here, that this number is too small, because it leads to unnecessarily conservative predictions. Prediction at 8 points is a feasible compromise with an insignificant loss of accuracy.

7. Cycle-by-cycle crack growth calculations at 8 points of the crack front, taking into account a variation of $S_{op}$ along the crack front, can be done with a simple algorithm.

8. Predictions of the crack growth lives until break through, ignoring crack closure, were very satisfactory for tests of the present program and some test results from the literature. If crack closure was taken into account, the predicted crack growth lives were about 10% longer and slightly unconservative. It appears that for CA load predictions the occurrence of crack closure can be ignored, which can not be done for VA loading.

9. Predictions on the crack shape development ($a/c$ ratio), covering an initial crack shape ($a_i/c_o$) varying from 0.2 to 1.0, were very satisfactory. The agreement should not be considered to be a proof of good crack growth rate predictions, but it indicates that the K-solution (Newman-Raju) for semi-elliptical surface cracks is satisfactory.
Crack growth predictions for VA loading

10. Predictions of crack growth of a surface crack under VA loading with the CORPUS model have not yet been reported in the literature. For the present results certain adjustments of the CORPUS model (and also for the modified CORPUS model) had to be made in view of the 2-dimensional character of the crack shape. Calculations were made again for 8 points along the crack front. An algorithm was prepared for this purpose.

11. Predictions were made for a simple flight-simulation sequence and a realistic flight-simulation load history, based on the CN-235 aircraft wing structure. Predictions could also be made for some data from the literature (Hall et al., military spectra, 2219-T851 Al alloy), but there is hardly any sufficiently documented information available. In addition to predictions with the CORPUS model and the modified CORPUS model, non-interaction predictions have also been made, which gives a kind of a reference to see whether interaction effects have occurred.

In general similar results were obtained with the CORPUS model and the modified CORPUS model. On the average, predictions of both models are fairly accurate, but the scatter of \( N_f/N_p \) (ratio of test life to predicted life) showed a significant scatter (value were in the range of 0.5 to 2.0). The effect of truncation in the CN-235 tests was poorly predicted. The modified CORPUS model gave better predictions on the effect of ground-to-air cycles than the CORPUS model.

12. The analysis of the results have made it clear that the predictions heavily depend on the basic \( da/dN \) data adopted, and on the thickness effect on crack closure. In thin sheets more crack closure will occur. In plate material less crack closure will be present, as confirmed indirectly by results of flight-simulation tests on specimens with different thicknesses.

13. The non-interaction predictions were generally conservative, indicating that crack growth retardation was involved. However, it is remarkable that the non-interaction results were not excessively conservative. This is due to the limited amount of crack closure.

14. Crack edge trailing at the material surface was evident, especially in the flight-simulation tests with a high maximum load. The shape of the crack was still
approximated by a semi-ellipse, which did not upset the prediction accuracy. The prediction of the crack shape development was again satisfactory.

15. Fractographic observations with the electron microscope (SEM) indicated that crack increments during high load cycles were underpredicted (acceleration!). Since such high loads are very rare, it does not disturb the average crack growth prediction.

6.2 Conclusions with a more general meaning

16. The application of special load histories to determine \( S_{op} \) from striation spacings can be used. It is a rather time consuming procedure, while the accuracy can not be expected to be very high. Unfortunately, there is no obvious alternative method available.

17. The application of marker loads is a useful and reliable method to make records of successive curved crack fronts. During flight-simulation tests the traces of severe flights can be used for the same purpose, but this requires more experience. It turned out that the 7075-T6 alloy is a more easy material for fractographic observations than the 2024-T3 alloy, which is a pity, because the latter alloy must be preferred for fatigue critical components. Anyhow, without fractography the present investigation could not have been made. It should be emphasized that fractography is an indispensable means of fatigue studies on crack growth models.

18. The analysis of the results with the CORPUS model has indicated that it is not really a simple model, which can easily be understood with respect to what the model is doing during a complex flight-simulation load history. It helps significantly to draw flight profiles with indications of \( S_{op} \) in every load cycle. It also helps to have the non-interaction predictions. It might be suggested that the CORPUS model can be improved. However, it is doubtful of a further optimization is justified in view of several uncertainties, which can not easily be removed in an accurate way. The description of crack closure, the applicability of the basic \( da/dN \) data, as affected by the stress ratio, and the transition of the state of stress will remain uncertain factors.

19. The test results and the analysis suggest that there is no fundamental difference between the behaviour of a 1-dimensional through crack and a 2-dimensional
surface crack. However, the extra dimension is complicating the picture significantly, especially due to crack closure, while making observations becomes just another issue.

20. Since predictions on crack growth of semi-elliptical cracks can give reasonable results, it appears that fracture mechanics is a most useful tool to proof the damage tolerance of heavy aircraft components, which according to the official requirements are in the category of the slow crack growth option. It would be most worthwhile to carry out a similar study as the present one on quarter elliptical corner cracks at holes in plate material, for both unloaded and loaded hole configurations.
APPENDIX A THE NEWMAN-RAJU EQUATIONS FOR SEMI-ELLIPTICAL SURFACE CRACKS

The boundary-correction factors for a semi-elliptical surface crack in the Newman-Raju solution were chosen in [3.11] to be:

\[
F = [M_1 + M_2(a/t)^2 + M_3(a/t)^4] f_v f_w
\]

For \(a/c \leq 1\)

\[
M_1 = 1.13 - 0.09(a/c)
\]

\[
M_2 = -0.54 + \frac{0.89}{0.2 + (a/c)}
\]

\[
M_3 = 0.5 - \frac{1}{0.65 + (a/c)} + 14(1 - a/c)^{24}
\]

\[
g = 1 + [0.1 + 0.35(a/t)^2](1 - \sin \varphi)^2
\]

\[
f_v = [(a/c)^2 \cos^2 \varphi + \sin^2 \varphi]^{\frac{1}{4}}
\]

\[
f_w = \left[\text{sec} \frac{\pi c}{W} \sqrt{a/t}\right]^{\frac{1}{2}}
\]

For \(a/c > 1\)

\[
M_1 = \sqrt{c/a}(1 + 0.04c/a)
\]

\[
M_2 = 0.2(c/a)^4
\]

\[
M_3 = -0.11(c/a)^4
\]

\[
g = 1 + [0.1 + 0.35(c/a)(a/t)^2](1 - \sin \varphi)^2
\]

\[
f_v = [(c/a)^2 \sin^2 \varphi + \cos^2 \varphi]^{\frac{1}{4}}
\]

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The same width correction, $f_w$, is used for both $a/c \leq 1$ and $a/c > 1$.

It should be noted that the Newman-Raju equations and the $K$ equations for an elliptical crack in an infinite solid (Eqs. 3.1a and 3.1b) apply different angular functions, $f_\varphi$, for $a/c \leq 1$ and $a/c > 1$. The function for $a/c > 1$ was obtained by a rotation of the axes by $90^\circ$ (replacing a, c, $\varphi$ with c, a, [$\pi/2 - \varphi$]). It was shown by Schijve [A-1] that this rotation procedure is unnecessary, and he proposed to use the following accurate equation for the complete elliptical integral $\Phi$:

$$\Phi = \frac{\pi}{2} \frac{1}{(1 + m)[1 + \frac{m^2}{4} + \frac{m^4}{64}]} \quad \text{(A-1)}$$

where,

$$m = \frac{1 - a/c}{1 + a/c}$$

If this equation is used, Eq. 3.19 can also be used for $a/c > 1$, i.e., Eq. 3.1b can be dropped, and it is no longer necessary to know to equations for the shape factor (Eqs. 3.2a and 3.2b).

Reference, Appendix A

APPENDIX B CRACK OPENING STRESS MEASUREMENTS IN 7075-T6 AL-
ALLOY WITH THROUGH CRACKS

The dimensions of specimens used for measuring \( S_{op} \) of through cracked specimens were the same as specimens with part-through crack i.e. thickness 9.6 mm, width 100 mm, and length 300 mm. A starter notch was introduced by drilling a hole with a diameter of 1 mm and two saw cuts with a total nominal length of 3 mm.

CA loading was applied to the specimens. Crack opening displacement (COD) was measured with a clip gage placed approximately 1 mm behind the crack tip. From the plot of load as a function of COD, the value of \( S_{op} \) was deduced. Measurements were performed at crack length interval of approximately 5 mm. Averages value of 5 measurements were taken as \( S_{op} \) values.

The maximum stress of all tests was 120 MPa. Different values of the minimum stress were applied resulting in different stress ratios (R) for each specimen. The value of R in each test and the value of \( S_{op} \) deduced from the COD measurements is given in the following table.

<table>
<thead>
<tr>
<th>( S_{min} ) (MPa)</th>
<th>R</th>
<th>( S_{op} ) (MPa)</th>
<th>( S_{op}/S_{max} )</th>
</tr>
</thead>
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<tr>
<td>-120</td>
<td>-1.0</td>
<td>19.80</td>
<td>0.165</td>
</tr>
<tr>
<td>-90</td>
<td>-0.75</td>
<td>22.92</td>
<td>0.191</td>
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<td>-48</td>
<td>-0.40</td>
<td>24.70</td>
<td>0.205</td>
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<tr>
<td>-30</td>
<td>-0.25</td>
<td>24.90</td>
<td>0.208</td>
</tr>
<tr>
<td>1</td>
<td>0.01</td>
<td>29.88</td>
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<td>0.10</td>
<td>33.90</td>
<td>0.282</td>
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<td>0.20</td>
<td>33.80</td>
<td>0.323</td>
</tr>
<tr>
<td>36</td>
<td>0.30</td>
<td>36.00</td>
<td>0.300</td>
</tr>
</tbody>
</table>

Figure B-1 shows the plot of \( \gamma = S_{op}/S_{max} \) as a function of R. The value of \( \gamma \) at \( R = 0.3 \) is lower than \( R = 0.2 \). This result should not be expected, because it may well be assumed that \( \gamma(R) \) is an increasing function. If this discrepancy is attributed to scatter of the test.
results, the statistical rearrangement method [4.28] can be applied, which affect the position of $\gamma(R)$ of the last two points only ($R = 0.2$ and $R = 0.3$). Figure B-2 shows the rearrangement results with a curve obtained by a least squares regression method (regression to $\gamma$, subjected to scatter, on $R$, not subjected to scatter). The result is:

$$\gamma = 0.25 + 0.185R + 0.233R^2 + 0.12R^3$$

The curve is crossing the line $\gamma = R$ at $R = 0.346$, which implies that for $R > 0.346$, crack closure during cyclic loading does not occur anymore.
Fig.B-1. The plot of $\gamma$ ($S_{op}/S_{max}$) as a function of $R$ for through cracks of 7075-T6 Al specimens. Note that the value of $\gamma$ at $R = 0.3$ is lower than at $R = 0.2$.

Fig.B-2. $\gamma(R)$ curve of test results shown in Fig.B-1 after statistical rearrangement of the last two points.
APPENDIX C FLIGHT SIMULATION LOAD HISTORIES

Fatigue loads in several aircraft components are VA-amplitude loading, which quite often is a superposition of deterministic loads and stochastic loads. The mixture of these two types of loads can result in a complex load history on the aircraft structure. The load history of one aircraft will be different from that for another one, partly because of different utilization of the aircraft. During the design phase predictions have to be made on loads which will occur in service. The load spectrum is needed for damage tolerance analysis purposes and for full-scale fatigue testing of the aircraft. For these purposes realistic load sequences are required. A flight-by-flight simulation load sequence is supposed to represent a realistic simulation of the load history in service. Flight simulation sequences for general purposes were also developed, e.g. TWIST and FALSTAFF, they are usually used for comparative fatigue investigations, or to study the effect of certain variables of flight simulation tests.

In the past simplified flight simulation load sequences were applied in full-scale flight simulation tests (e.g. on the Comet aircraft). This simplified flight sequence consist of one flight type only, with CA cycles in flight (type I of Fig.5.8). The present flight simulation sequences are more realistic. They consist of different flight types arranged in a random sequence. Within each flight the sequence of the load is also random. Flight simulation load spectra are given for a fixed (large) number of flights, defined as one block, which is repeated until the end of the test. The load-time history for a flight simulation test is defined by a number of variables, which includes:

- truncation level
- omission of small cycles
- design stress level
- sequence of flights
- sequence of loads in each flight.

Various flight simulation sequences used in the present investigation and in [5.6] will be briefly described below.

TWIST [C-1]

TWIST is a standardized flight simulation sequence representative for loads on the lower wing skin of transport aircraft (TWIST = Transport WIng STandard). The spectrum is a
gust dominated one. Statistical information of the spectrum is given in Table C-1. One block of TWIST consists of 4000 flights of 10 different flight types (A to J in Table C-1) in a random sequence. The average number of cycles in one flight is 100. Flight A represents the most severe flight with severe storm, whereas flight J is a flight in nice weather. The characteristic stress level in TWIST is the mean stress in flight ($S_{mf}$). The stress level of the gust load and the ground stress ($S_{gr}$) are related to $S_{mf}$. $S_{gr}$ is defined as -0.50 $S_{mf}$.

Mini-TWIST [C-2]
Mini-TWIST was derived from TWIST by omitting most cycles of the lowest amplitude i.e. level X in Table C-1. The average number of cycles in one flight was reduced to about 15. The sequence of flights in mini-TWIST is the same as in TWIST.

FALSTAFF [C-3]
FALSTAFF is another standardized flight sequence. It represents loads at the wing root area of a fighter aircraft, thus it is base on a manoeuvre dominated spectrum. The number of flights in one block is 200, where each flight differs from the other ones. The average number of cycles per flight is 90. The characteristic stress level in FALSTAFF is the maximum level of the spectrum.

Short-FALSTAFF
Short-FALSTAFF was obtained from FALSTAFF by omitting small ranges in FALSTAFF. The average number of cycles per flight is about 50% of the number for FALSTAFF.

Misawa sequence [C-4]
The Misawa sequence is a simplified flight simulation load with constant gust cycles, and with the addition of single overload at either at the beginning or at the end of each flight. Three different flight sequences were defined (see Fig.5.8):
- sequence I: all gust cycles are equal
- sequence II: similar to sequence I with an overload at the beginning of the flight
- sequence III: similar to sequence I with an overload at the end of the flight.
During one test the same sequence is applied in all flights. Two different numbers of gust cycles (m) were applied, i.e. m = 5 or m = 100. The other variable is the ground stress level ($S_{gr}$).
The purpose of performing tests with this simplified sequence is to study the interaction effects especially retardation and acceleration that might turn up in such a sequence.

F-4 sequence [C-5]
The F-4 sequence is similar to the sequence I of the MISAWA load. The number of gust cycles in one flight is 10.5 (see Fig.C-1). The gust amplitude and the ground stress are defined as $0.32 S_{mr}$ and $-0.052S_{mf}$ respectively.

F-27 sequence [C-5]
The F-27 flight simulation load history is representative for the gust spectrum of the F-27 aircraft. Similar to the TWIST spectrum 10 different flights were defined. The stress levels were also related to $S_{mr}$ as in TWIST.

CN-235 sequence [C-6]
The load spectrum for the lower wing skin of the CN-235 aircraft is shown in Table C-2. The load levels were divided into 128 levels. Level 128 corresponds to the maximum stress in the most severe flight and level 1 corresponds to the lowest level of the same flight. Similar to FALSTAFF, the characteristic stress level in the CN-235 spectrum is the maximum stress. One block of spectrum consists of 1000 flights of 10 different flight types. The flight profile of all 10 flight types is shown in Fig.5.9.

Bomber spectrum and fighter spectrum of Hall[C-7]
In a series of research and development programs of the US Air Force to develop methods and to collect data for designing against fatigue fracture in military aircraft several tests were performed with surface cracked specimens. Spectrum loading were applied in some tests. The load spectrums were representative for bomber aircraft and fighter aircraft. Three different missions were defined for both spectra. The missions were basically the same (see Tables C-3 and C-4 for the load sequences in the bomber spectrum and the fighter spectrum respectively). Figs C-2 and Fig C-3 show flight profiles for the bomber and the fighter spectra respectively. The load level of each cycle is defined as a percentage of the limit load. In the bomber spectrum, two high overloads of different levels were introduced every 100 missions and three overloads of lower levels (also different magnitude) were introduced every 10 missions. Spectra for fighter missions differ in the addition of a high overload every 18 missions and a lower overload every 6 mission.
References, Appendix C.


C-5. Van der Linden,H.H., "NLR test results as data base to be used in a check of crack propagation prediction model. A Garteur activity," National Aerospace Laboratory, The Netherlands, 1979, NLR TR 79121.


<table>
<thead>
<tr>
<th>Flight Type</th>
<th>Number of flights in one block</th>
<th>level and magnitude (Sa/Smf) of amplitudes</th>
<th>Total number of cycles per flight</th>
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<tr>
<td></td>
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<td>II 1.50</td>
<td>III 1.30</td>
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</tr>
<tr>
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</tr>
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<td>E</td>
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</tr>
<tr>
<td>J</td>
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</tbody>
</table>

\( S_{\text{op}}/S_{\text{ref}} = 0.50 \)

Average number of cycles per flight = 100 (15)

Number of cycles for miniTWIST are given in brackets

Table C.1. TWIST and miniTWIST load spectrum.
<table>
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<tr>
<th>level (a)</th>
<th>number of maxima</th>
<th>level (a)</th>
<th>number of maxima</th>
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<th>number of minima</th>
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</table>

(a) Level 128 corresponds to $S_{\text{max}}$ of spectrum
Level 1 corresponds to -0.1728 $S_{\text{max}}$
Level x corresponds to $[0.009235(x-1)-0.1728]S_{\text{max}}$
$S = 0$ corresponds to level 19.7, $S_{\text{me}}$ corresponds to level 63.76

Table C.2. CN-235 load spectrum for one block of 1000 flights.
<table>
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<th>Cycle</th>
<th>Load(% limit)(^1)</th>
<th>cycles/mission</th>
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<th>Load(% limit)(^1)</th>
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\(^1\) The limit stress for 2219-T851 Al-alloy is 231.05 MPa, while for Ti-6Al-4V is 482.63 MPa.

Table C.3. The load sequence of the bomber spectrum of Hall [C.7].
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1 The limit load for 2219 Al-alloy is 213.05 MPa, while for Ti-6Al-4V is 426.10 MPa.
2 1 cycle every 6 missions starting with the first mission.
3 1 cycle every 18 missions starting with the eighteenth mission.

Table C.4. The load sequence of the fighter sequence of Hall [C-7].

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Fig C.1. The F-4 simplified flight simulation sequence.
Fig.C.2. Load sequence of the bomber spectrum of Hall et al. [C-7]. The load level is defined as percentage of the limit load (see Table C.3). Each flight is basically the same, two high overloads (A) were introduced every 100 missions and three overloads (B) of lower levels were introduced every 10 missions.
Fig. C.3. Load sequence of the fighter spectrum of Hall et al. [C-7]. The number of cycles of each load level can be seen in Table C-4. Each mission is basically the same, two high overloads were introduced, one every 18 missions and the other one every 6 missions.
The above figure shows the current crack front and the fitted current plastic zone enclave. For further calculations the extent of the plastic zone from a point A with a coordinate \(x(J), y(J)\) is taken as the distance from the point to the fitted plastic zone enclave. The distance is calculated, since \(dp(J)\) calculated with Eq.5.31 will be slightly different due to the fitting procedure.

The distance from the point to the fitted plastic zone enclave is \(dp\) (the dp stands for the fitted enclave). Line 1 is a tangent to the current crack front at point A. The equation of line 1 is,

\[
y = \frac{a^2}{y(J)} - \frac{a^2}{c^2} \frac{x(J)}{y(J)} x. \tag{D-1}
\]

The equation of line m, which is perpendicular to line 1 is

\[
y = y(J) + \frac{c^2}{a^2} \frac{y(J)}{x(J)} (x - x(J)) \tag{D-2}
\]

Point B is the intersection of line m with the plastic zone enclave. The coordinate of point
The coordinate of point B can be obtained by substituting Eq.(D-2) to the equation of the fitted plastic zone enclave, which is:

\[
\frac{x^2}{adpc^2} + \frac{y^2}{adpa^2} = 1
\]  

(D-3)

It is easily derived that the coordinate of point B is

\[
x(B) = \frac{-BB + \sqrt{BB^2 - 4 AA CC}}{2 AA}
\]  

(D-4)

\[
y(B) = y(J) + \frac{c^2 y(J)}{a^2 x(J)} (x(B) - x(j))
\]  

(D-5)

where

\[
AA = \frac{1}{adpc^2} + \frac{D^2}{adpa^2}
\]  

(D-6)

\[
BB = \frac{2 y(J) D - 2 x(J) D^2}{adpa^2}
\]  

(D-7)

\[
CC = \frac{y(J)^2 + D^2 x(J)^2 - 2 y(J) D x(J)}{adpa^2} - 1
\]  

(D-8)

\[
D = \frac{c^2 y(J)}{a^2 x(J)}
\]  

(D-9)
The above figure shows a point P, with parametric angle \( \varphi \) on the crack front of the dominant hump and the dominant plastic zone extent from that point, \( D^d \). The coordinate of P is xDOM(J), yDOM(J). After some growth, the crack front extents to a new crack front, indicated as the current crack in the figure. On the new crack front, S is the point with parametric angle \( \varphi \). The current plastic zone size from point S is \( D^r(\varphi) \). In the original CORPUS model, for through cracks \( D^u \) and \( D^d \) are on the same line i.e. the crack line. For surface cracks, \( D^d \) and \( D^r(\varphi) \) do not lie on the same line. To have both plastic zone sizes in the same line, the plastic zone \( D^u \), i.e the distance from point Q to point R is used as the current plastic zone size. To determine the distance the procedure in Appendix D is followed.
With
\[ D = \frac{c^2 y(Q)}{a^2 x(Q)} \]
\[ AA = \left( \frac{1}{adpc^2} + \frac{D^2}{adpa^2} \right) \]
\[ BB = \frac{2y(Q)D - 2x(Q)D^2}{adpa^2} 5-3 \]
\[ CC = \frac{y(Q)^2 + D^2 x(Q)^2 - 2y(Q)x(Q)D}{adpa^2} -1 \]

The coordinate of point R is then
\[ x(R) = \frac{-BB + \sqrt{BB^2 - 4*AA*CC}}{2*AA} \]
\[ y(R) = y(Q) + D(x(R) - x(Q)) \]

The distance between point Q and point R is:
\[ D^2 = \sqrt{(x(R) - x(Q))^2 - (y(R) - y(Q))^2} \]
Stellingen behorende bij het proefschrift

"Fatigue Crack Growth Predictions of Surface cracks under Constant-Amplitude and Variable-Amplitude Loading"

van

Ichsan Setya Putra
1. In comparison to through cracks in thin sheet material of aluminium alloys, the crack closure stress level \( S_{op} \) of surface cracks in plate material of the same alloys is relatively low.

2. The \( S_{op} \) variation along the crack front of a surface crack is rather limited. Predictions of crack growth life ignoring the \( S_{op} \) will result in a small difference compared to predictions which incorporate the \( S_{op} \) variation.

3. Crack shape development of surface cracks is a useful means to verify \( K \)-solutions of these cracks, but it cannot be used to check the accuracy of a prediction model.

4. Crack edge trailing causes real surface cracks to deviate from a semi-elliptical shape. Crack growth analysis can still be based on the semi-elliptical shape without a significant loss of accuracy.

5. Fractography can provide very useful information for fatigue analysis and predictions. It requires a most careful examination of fractographic pictures, which is highly time consuming. However, it is rewarding by giving detailed information, which can not be obtained otherwise.

6. In many publications on empirical fatigue investigations a missing link is the macroscopic analysis of the fracture surfaces, which is inexcusable from a scientific point of view, and in view of cost-effectivity considerations.

7. The proof of the validity of a new fatigue prediction model is incomplete if the author compares predictions with his own test results only. It is also incomplete if test results are not compared to non-interaction predictions.

8. The present editing software and laser printers have stimulated the creation of many different individual styles of lay out of reports, papers and drawings. They are usually obtained after throwing away thousands of pages and spending a lot of time. It is undesirable and inefficient.

9. Lecturers in engineering subjects should be engaged in real industrial projects.

10. The new forms of information carriers such as microfiches, diskettes and CDROM will not be able to replace printed paper because of the flexibility we have when reading from paper and since in our culture we are taught to read from a piece of paper.

11. According to an instructive statement of the University of California "research is a gamble". In order to be able to win the game you have to work hard and have a high motivation.