THE GAS/LIQUID FLOW IN A LINE-RISER SYSTEM

EXPERIMENTS AND MODELLING

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Master's thesis

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Preface

This thesis was written for the purpose of acquiring my master's degree in applied physics at the Delft University of Technology. The research was performed under supervision of professor van den Akker at the "Kramers Laboratorium voor Fysische Technology", which forms part of the Faculty of Applied Physics.

I wish to thank everyone who supported me during recent years and all those who have taken small or great efforts to help me out with the numerous difficulties that occurred during my graduation work. A special word of thanks to professor van den Akker, who gave me the opportunity to graduate on this challenging subject. Furthermore, I would like to thank Erik Legius for the pleasant and constructive supervision during the larger part of this research. Besides, I would like to express my gratitude towards Rob Mudde for taking over Erik's supervision in such a smooth way when he left the laboratory. In spite of all this supervision it would not have been possible to perform this research without the support of all the other personnel at the laboratory, whom I also wish to thank.

I especially want to express my sincere thanks to my parents. Without your endless support and understanding during my whole education, from the Apple tree until my graduation, I would never have made it this far. In addition to your encouragements there were the words from Geert Jan, who was always there to turn hardship into laughter. Thanks!

I also owe a great deal of thanks to my fellow students and friends. In particular I wish to thank my friends from "Tempest" and "Kap Nāh", who helped me turn my life as a student into a time which I will never forget. Furthermore, I would like to thank my colleagues from "kamer 1" for the many laughs and profound discussions in between our graduation work.

To all those, who are innocent enough not to make it any further than the introduction, I would like to express my appreciation for having tried. Drop by one day and I will explain it to you over a glass of Bourbon! Those who do make it through a large part of this thesis must take note that all typing errors are deliberate and designed to keep you awake!

Delft, 28 august 1997

Michiel Engelsman
Physics is the greatest adventure
the human mind has ever begun!

Richard P. Feynman
To my parents
and my brother
Summary

In industry, simultaneous transportation of gas and liquid through pipeline systems often occurs. The gas and the liquid can be distributed over the cross-sectional areas of the pipes in different ways. Strong pressure and gas-fraction fluctuations can occur and can lead to undesirable process conditions. It is not well understood how two-phase flows behave in complex pipeline structures. Especially the effects of geometry changes are unknown.

This research project focussed on the joining of a horizontal and a vertical pipeline by means of an upward turned bend. The aim of the project was to acquire a qualitative description of what happens in the facility. There has typically been looked at the characteristics of the flow upstream and downstream of the bend and at slugging frequencies.

A better understanding of the flow behaviour was acquired through visual observations and from images made by means of both a normal and a digital camera. Two main mechanisms were distinguished, one in the bend and one just before the bend, which turn a stratified flow in the line into a slug flow in the riser. This transition to slug flow was modelled and simulated by means of a code written in FORTRAN 77. The model is based on the calculation of the stable liquid level in a horizontal stratified flow, the instability level by means of a Kelvin-Helmholtz analysis, and the shedding behind a slug rising in a vertical pipe. The modelling results were in agreement with the visual observations and were further verified through measurements. These measurements were performed by means of pressure transducers and glass fibre probes. The emphasis was on the determination of the slushing frequencies.

The mechanism in the bend was found to have a frequency of about 1.5 to 2 Hz, whereas the mechanism just before the bend exhibits a frequency lower than approximately 0.5 Hz. Measurements showed that the flow in the riser rapidly changes into a developed slug flow of about 1 Hz. On the other hand, the flow far upstream in the line is not affected by the bend, as the waves created by the bend decay rapidly. The conclusion of this work is that the bend turns a stratified flow in the line into a slug flow in the riser, but that the flow far upstream and downstream of the bend is not affected.
Samenvatting

In de industrie vindt veelvuldig het gelijktijdig transport van gas en vloeistof door pijpleidingsystemen plaats. Het gas en de vloeistof kunnen zich op verschillende manieren over de doorsneden van de pijpleidingen verdelen. Sterke fluctuaties in de druk en de gasfractie kunnen daarbij optreden, die kunnen leiden tot ongewenste procescondities. Er is nog weinig begrip van het gedrag van twee-fasen stromingen in complexe pijpleidingstructuren. Met name het effect van geometrie veranderingen is onbekend.

Dit onderzoek was gericht op de verbinding van een horizontale met een verticale pijpleiding door middel van een opwaartse bocht. Het doel was het verkrijgen van een kwalitatieve beschrijving van wat er gebeurt in de opstelling. Er is gekeken naar de karakteristieken van de stroming stroomop- en stroomafwaarts van de bocht en naar "slugging"-frequenties. Begrip van het stromingsgedrag werd verkregen door middel van visuele waarnemingen en uit beelden gemaakt met zowel een gewone als een digitale camera. De twee voornaamste mechanismen werden onderscheiden. Deze mechanismen, één in de bocht en één net voor de bocht, zetten een gestratificeerde stroming in de horizontale buis om in een "slug" stroming in de verticale buis. Deze overgang naar "slug" stroming werd gemodelleerd en gesimuleerd door middel van een code geschreven in FORTRAN 77. Het model is gebaseerd op de berekening van het stabiele vloeistofniveau in een horizontale gestratificeerde stroming, het instabiele niveau door middel van een Kelvin-Helmholtz analyse en de "shedding" achter een slug die stijgt in een verticale pijp. De modelleer resultaten waren in overeenstemming met de visuele waarnemingen en werden verder geverifieerd door middel van metingen. Deze metingen werden verricht met behulp van druk sensors en glasvezelprobes. De nadruk lag op het bepalen van de "slugging"-frequenties.

Het mechanisme in de bocht bleek een frequentie van ongeveer 1.5 tot 2 Hz te hebben, terwijl het mechanisme net voor de bocht een frequentie heeft lager dan ongeveer 0.5 Hz. Metingen lieten zien dat de "slug" stroming in de verticale pijp snel overgaat in een ontwikkelde "slug" stroming van ongeveer 1 Hz. Daarnaast wordt de stroming ver stroomopwaarts in de horizontale buis niet beïnvloed door de bocht, aangezien de golven die door de bocht worden gecreeerd snel dempen. De conclusie van dit werk is dat de bocht een gestratificeerde stroming in de horizontale pijp omzet in een "slug" stroming in de verticale pijp, maar dat de stroming ver stroomop- en stroomafwaarts niet door de bocht wordt beïnvloed.
Nomenclature

This enumeration is not applicable to appendix D.

**Roman**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
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</thead>
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<tr>
<td>$a$</td>
<td>acceleration</td>
<td>$m/s^2$</td>
</tr>
<tr>
<td>$a'$</td>
<td>coefficient AR-model</td>
<td>$-$</td>
</tr>
<tr>
<td>$A$</td>
<td>area</td>
<td>$m^2$</td>
</tr>
<tr>
<td>$A'_i$, $A'_g$</td>
<td>derivative of area to height of phase</td>
<td>$m$</td>
</tr>
<tr>
<td>$b$</td>
<td>coefficient AR-model</td>
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</tr>
<tr>
<td>$c$</td>
<td>coefficient discrete Fourier transform</td>
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</tr>
<tr>
<td>$C$</td>
<td>wave velocity</td>
<td>$m/s$</td>
</tr>
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<td>$C_0$</td>
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<tr>
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<td>$C_{IV}$</td>
<td>critical wave velocity for the inviscid analysis</td>
<td>$m/s$</td>
</tr>
<tr>
<td>$C_V$</td>
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<td>$d$</td>
<td>distance between AR-models</td>
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<td>$dF$</td>
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<td>$kgm/s^2$</td>
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<td>$dR$</td>
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<td>$dZ$</td>
<td>gravitational force</td>
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<td>$D$</td>
<td>diameter</td>
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mass flux $kg/m^2s$
height $m$
height $m$
derivative of area to $A_i', A_g'$ $m$
imaginary number $-$
index $-$
drift flux $m/s$
wave number $1/m$
index $-$
order AR-model $-$
viscosity factor $-$
length $m$
index $-$
mass $kg$
index $-$
number of observations $-$
pressure $kg/m^2s$
perimeter $m$
volumetric flow rate $m^3/s$
radius $m$
autocovariance array $-$
autocovariance function $-$
time $s$
time-span $s$
velocity $m/s$
average velocity $m/s$
superficial velocity $m/s$
superficial velocity at $m/s$
a cross-section
mixture velocity $m/s$
relative velocity $m/s$
drift velocity $m/s$
rise velocity of a single bubble $m/s$
in a stagnant fluid
frictional velocity $m/s$
volume $m^3$
mass flow $kg/s$
space coordinate $m$
signal $-$
observation $-$
### Greek

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<td>α</td>
<td>hold-up</td>
<td>—</td>
</tr>
<tr>
<td>α</td>
<td>penalty factor</td>
<td>—</td>
</tr>
<tr>
<td>β</td>
<td>ratio of the Taylor bubble length to the slug unit length</td>
<td>—</td>
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<tr>
<td>α, β</td>
<td>dimensionless geometrical parameters (hold-up relation)</td>
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<td>γ</td>
<td>angle</td>
<td>rad</td>
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<td>δ</td>
<td>thickness</td>
<td>m</td>
</tr>
<tr>
<td>ε</td>
<td>interfacial roughness</td>
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<tr>
<td>ε</td>
<td>amplitude of perturbation</td>
<td>m</td>
</tr>
<tr>
<td>εt</td>
<td>random process</td>
<td>—</td>
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<tr>
<td>η</td>
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</tr>
<tr>
<td>θ</td>
<td>angle</td>
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<td>λ</td>
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<td>σε²</td>
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<td>τ</td>
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### Dimensionless numbers

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<td>-------------</td>
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<tr>
<td>b</td>
<td>bubble</td>
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<td>crit</td>
<td>instability level</td>
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<td>film</td>
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<td>gas, superficial</td>
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<td>h</td>
<td>homogeneous</td>
</tr>
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<td>i</td>
<td>interface</td>
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<td>time-step</td>
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<tr>
<td>I</td>
<td>imaginary</td>
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<tr>
<td>j</td>
<td>phase (liquid or gas)</td>
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<td>k</td>
<td>element of mass</td>
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<td>liquid</td>
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<tr>
<td>ls</td>
<td>liquid, superficial</td>
</tr>
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<td>mixture</td>
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<td>pressure</td>
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<td>r</td>
<td>relative</td>
</tr>
<tr>
<td>s</td>
<td>shear (stress)</td>
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<td>turbulent regime (friction factor)</td>
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<td>sh</td>
<td>shedding</td>
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<tr>
<td>SL</td>
<td>slug</td>
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<td>SU</td>
<td>slug unit</td>
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<td>t</td>
<td>translational</td>
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<tr>
<td>T</td>
<td>transposed</td>
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<td>Taylor bubble</td>
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<td>TD</td>
<td>Taitel-Dukler</td>
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<td>w</td>
<td>wall</td>
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<td>-</td>
<td>vector</td>
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<td>*</td>
<td>complex conjugate</td>
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<td>dimensionless</td>
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<td>-</td>
<td>perturbation</td>
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<td>average</td>
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<td>∞</td>
<td>equilibrium</td>
</tr>
<tr>
<td>0</td>
<td>constant</td>
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Abbreviations

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<thead>
<tr>
<th>Symbol</th>
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<tr>
<td>AR</td>
<td>Autoregressive</td>
</tr>
<tr>
<td>CCD</td>
<td>Charge Coupled Device</td>
</tr>
<tr>
<td>DACQ</td>
<td>Data-acquisition Unit</td>
</tr>
<tr>
<td>GIC</td>
<td>General Information Criterion for finite samples</td>
</tr>
<tr>
<td>HP-VEE</td>
<td>Hewlett Packard’s Visual Engineering Environment</td>
</tr>
<tr>
<td>ID</td>
<td>Internal Diameter</td>
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<td>IKH</td>
<td>Inviscid Kelvin-Helmholtz analysis</td>
</tr>
<tr>
<td>IReS</td>
<td>Image Registration System</td>
</tr>
<tr>
<td>MEM</td>
<td>Maximum Entropy Method</td>
</tr>
<tr>
<td>PSD</td>
<td>Power Spectral Density function</td>
</tr>
<tr>
<td>PVC</td>
<td>Polyvinyl Chloride</td>
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<tr>
<td>SOPHY-2</td>
<td>Solution Package for Hyperbolic functions</td>
</tr>
<tr>
<td>VAR</td>
<td>Variance</td>
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<tr>
<td>VKH</td>
<td>Viscous Kelvin-Helmholtz analysis</td>
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A Force balance on a slug

B Images of the two slugging mechanisms

C Specification of used equipment

D Code Slugmech
Chapter 1

Introduction

In industry simultaneous transportation of gas and liquid through pipeline systems often occurs at for example refineries and oil-platforms at sea. Due to the often rather complex structure of these systems the gas and the liquid can be distributed over the pipes in different ways creating different flow conditions. Often transitions between different conditions will take place somewhere along the way. As a result strong fluctuations in pressures and gas-fraction can occur. One can imagine that these fluctuations can lead to vibrations within the facility and eventually of the pipeline system itself. This is what is called "Fluid Structure Interaction". If this gets out of hand pipelines might even break. Furthermore, fluctuations in pressures and gas-fractions can lead to undesirable flow conditions within the system itself, like for example the creation of huge liquid slugs and gas bubbles.

An example is the typical situation when oil is being produced at an offshore site (see figure 1.1). The oil will be produced together with gas (and often water and sand). This mixture will then have to be transported from the well on the bottom of the ocean to the platform which may be miles away. The pipeline though will not lay exactly horizontal. As a result liquid may accumulate at a low point and eventually block the pipe. The gas can no longer pass, the liquid will keep accumulating and as a result a great liquid slug is being created. Especially when this occurs just before the pipe bends upward towards the platform the gravitational pressure build-up due to the growing liquid column will be larger than the pressure build-up in the gas until the column of oil reaches the top. After which there will no longer be a pressure build-up in the vertical pipeline, but only in the gas behind the slug. After some time the pressure in the gas behind the slug will become larger than that of the liquid column and all of a sudden the huge slug will be blown out of the system. As a result on the platform periods of only gas production and periods of huge oil production will alternate and "slug catchers" and storage capacity will have to be over-dimensioned. It goes without saying that one would like to predict and perhaps eventually prevent this behaviour of the two-phase flow.

As yet it is not very well understood how two-phase flows behave in complex pipeline structures. Especially the effect of geometry changes, as for example, T-junctions, sudden
contractions and bends is unknown. One can imagine that in industry one would very much like to predict the behaviour of the two-phase flows under these circumstances. Especially in the designing stage of pipeline systems the simulation of the flow is desired in order to construct these systems in such a way that the undesirable situations do not occur. For this reason a research project was carried out at the Kramers Laboratorium by Erik Legius on this subject of which the research as described in this thesis was a part. The goal of the larger project was to predict the dynamic behaviour of gas/liquid flows through complex pipeline systems by means of a two-phase flow code named SOPHY-2 (Solution Package for Hyperbolic functions). The emphasis of that project was on the strong pressure and gas-fraction fluctuations as these can lead to undesirable process conditions.

The focus of this research was on the joining of a horizontal and a vertical pipeline by means of an upward turned bend as can be seen in figure 1.2. We were interested in a qualitative description of what happens in such a line-riser system. We were not trying to find an exact solution or trying to obtain very accurate velocity profiles or exact frequencies. Given the heavy sloshing of the gas/liquid flow inside this system it is unlikely that more than a qualitative description can be obtained at all. Within the overall project two former graduate students have studied the effect of successively a T-junction and a sudden contraction.

The resulting flow in the system is described and the main mechanisms are modelled. We have typically looked at the characteristics of the flow upstream and downstream of the bend, the slugging frequencies and the influence of waves originating from the bend. The ultimate goal is to derive a (more or less) general model for the influence of the bend on the flow conditions in the straight pipe sections that can be used as boundary conditions.
in SOPHY-2. This code solves the mass and momentum balances for a one dimensional two-phase flow. Besides, the system is considered to be isothermal and thus the energy balance is not taken into account. The long term aim is to simulate the flow in a horizontal line with a boundary condition at it’s end and the flow in a vertical riser with a boundary condition at it’s begin. Together the result would be the flow as if the two were joined by means of an upward turned bend. The boundary conditions should not be seen as rigid conditions, but more as a kind of "coupling-conditions". In the code straight pipes are joined at points were something special happens, like a bend or a sudden contraction. This joining is done by means of a "coupling-condition" which accounts for the geometry change in a kind of black-box approach. The conditions take the dynamic behaviour of the two-phase flow into account.

The structure of this thesis is as follows. In the second chapter some basic theory on two-phase flow will be treated, although not very thorough. Only what is necessary to understand the remaining part of this thesis will be discussed. The third chapter will describe the facility used in the experiments. Furthermore, the experimental methods used will be discussed and some methods of data-analysis will be discussed. Next, the fourth chapter will give a description of what was observed in the system, what mechanisms play a role and what the work resulted in in general. In addition the development of the acquired model and it’s results will be treated. Then the fifth chapter will describe the experiments which were performed and their results. Finally, the sixth chapter will discuss the most important points which resulted from the research. In appendix A some details on a model which was developed during the first stages of the work will be treated. Appendix B will show two series of images illustrating the mechanisms on which the resulting model, developed in this project, is based.
As a general reference on two-phase flow theory Wallis (1969) was used. Priestley (1981) is a general reference in the area of signal analysis. For the model development which will be treated in this thesis, Landman (1991) was consulted as a reference concerning the calculation of the stable liquid height in a stationary stratified flow. Furthermore, the liquid level at which the flow becomes unstable was determined using the article by Barnea and Taitel (1994). Finally, two different references were consulted for the calculation of the shedding behind a liquid slug, namely Delfos (1996) and Kay and Nedderman (1985). Some of the work presented in this thesis has recently been published (Legius and van den Akker (1997)). This article can therefore also be consulted.
Chapter 2

General two-phase flow theory

As was discussed in the previous chapter, this thesis deals with the gas/liquid flow in a line-riser system. The simultaneous flow of gas and liquid through pipelines exhibits some rather complex phenomena. In order to deal with these difficulties in later stages of this thesis, some general theory on two-phase flow will be treated in this chapter. First the whole phenomenon of two-phase flow will be introduced in the first section. Then some basic theory will be the topic of the second section. The third section deals with the flow through a riser and a line respectively. Finally, the fourth section describes some different flow patterns in some more detail. Only the most important aspects will be treated. A more thorough treatment of the subject can be found in Wallis (1969), which was the most important source for this chapter. Furthermore Butterworth and Hewitt (1977), Oliemans (1995) and Barnea and Taitel (1985) were used.

2.1 Introduction to two-phase flow

Two-phase flow is the simplest of multi-phase flows. It is a term covering the interacting flow of two phases of matter (gas, liquid or solid) where the interface between the phases is influenced by their motion. The focus in this chapter will be on gas/liquid two-phase flow, although many aspects have parallels in for example solid/liquid flow. Familiar examples of two-phase flows are bubbles in lemonade, boiling of water, rain, clouds, beer poured from a bottle, blood, fire extinguisher etcetera. Furthermore over half of all chemical engineering is concerned with multi-phase flows, such as refrigerating, power generation, distillation, steel-making and so on.

A two-phase flow obeys all the basic laws of fluid mechanics. The equations are merely more complicated and numerous as in single-phase flow. Some very simple analytical models, which take no account of the details of the flow, can be very successful in describing two-phase flow. For example:
• Homogeneous model. The two components are treated as a pseudo-fluid with average properties without bothering with a difficult description of the flow pattern.

• Drift-flux model. In this model the attention is fully focussed on the relative motion between the two phases.

• Separated-flow model. The two phases are assumed to be flowing side by side. For each phase separate equations are being formed. The interaction between the two phases is also taken into account.

A first step in understanding two-phase flow can be taken by breaking it up into various regimes each of which is being governed by certain dominant geometrical or dynamic parameters. A part of the definition of flow regimes is the description of the arrangement of the components. This results in different flow patterns. The flow patterns are often already obvious from mere visual observations. An example of the flow patterns and of the complexity of two-phase flows can be given by observing the sequence of flow patterns in an evaporator as more and more liquid is converted to vapour (see figure 2.1). In different parts of the evaporator totally different flow regimes occur, each of which requires a different method of analysis.

Figure 2.1: Approximate sequence of flow patterns in a vertical tube evaporator (Wallis (1969)).
2.2. Basic relationships and models

2.2.1 Definition of variables

Some basic variables on two-phase flow will now be introduced. The two phases, gas and liquid, will be distinguished by means of the subscripts g and l.

A mass flow \( W_l \) (kg) of liquid of density \( \rho_l \) (kg/m\(^3\)) and a mass flow \( W_g \) of gas of density \( \rho_g \) will be flowing through a pipeline simultaneously. Of course the total mass flow \( W \) will be the sum of the two component flows. The volumetric liquid and gas flow rates are now defined as:

\[
Q_l = \frac{W_l}{\rho_l} \quad Q_g = \frac{W_g}{\rho_g} \quad \left(\frac{m^3}{s}\right)
\]  

We can then define the superficial velocities of the two phases as the velocity a phase would obtain when flowing alone in a pipe of cross-sectional area \( A \) (m\(^2\)):

\[
V_{ls} = \frac{Q_l}{A} = \frac{W_l}{\rho_l A} \quad V_{gs} = \frac{Q_g}{A} = \frac{W_g}{\rho_g A} \quad \left(\frac{m}{s}\right)
\]  

The sum of the two superficial velocities is usually called the mixture velocity \( V_m \) (m/s):

\[
V_m = V_{ls} + V_{gs}
\]  

The superficial velocity is also denoted as the volumetric flux or the volumetric flow rate per unit area. The mixture velocity is then the average volumetric flux. A correlation which is also of importance in two-phase flow calculations is the phase volume fraction:

\[
\lambda_l = \frac{Q_l}{Q_l + Q_g} = \frac{V_{ls}}{V_{ls} + V_{gs}} \quad \lambda_g = \frac{Q_g}{Q_l + Q_g} = \frac{V_{gs}}{V_{ls} + V_{gs}}
\]  

It is obvious that \( \lambda_l + \lambda_g = 1 \).

Every part of the pipe will be occupied by one of the two components flowing through it. When gas and liquid flow through a pipe together, the local liquid volume fraction will be greater than that under non-flowing conditions due to the effect of slip between
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The gas phase, which is the lighter of the two, will move faster, whereas the liquid will have the tendency to accumulate, especially at low points in the pipeline system. The liquid will occupy a segment $A_l$ of the total cross-section of the pipe and the gas will occupy a segment $A_g$ (see figure 2.2).

![Figure 2.2: Simplified cross-section of a pipe.](image)

Under two-phase flow conditions the volume fraction of a certain phase is known as the hold-up and is defined as:

$$\alpha_l = \frac{A_l \cdot dx}{A \cdot dx} = \frac{A_l}{A} \quad \alpha_g = \frac{A_g \cdot dx}{A \cdot dx} = \frac{A_g}{A}$$

(2.5)

Where once again $\alpha_l + \alpha_g = 1$. Due to the slip effect the liquid hold-up will be larger than the liquid volume fraction. Only for the no-slip condition, like for example in the homogeneous flow case, will the two be equal. In two-phase flow the gas hold-up $\alpha_g$ is usually called the void fraction. Once the phase hold-up is known it is possible to compute the average phase velocities $V_l$ and $V_g$ from the superficial velocities:

$$V_l = \frac{Q_l}{A_l} = \frac{Q_l}{A} \cdot \frac{A}{A_l} = \frac{V_{ls}}{\alpha_l} \quad V_g = \frac{Q_g}{A_g} = \frac{Q_g}{A} \cdot \frac{A}{A_g} = \frac{V_{gs}}{\alpha_g}$$

(2.6)

Note that these velocities are larger than the superficial velocities.

In case one considers the density to be uniform, the mass flux can be written as:

$$G_l = \rho_l V_{ls} \quad G_g = \rho_g V_{gs} \quad \left( \frac{kg}{m^2s} \right)$$

(2.7)

And thus: $G = G_l + G_g$. For the average mass flux it then follows that:

$$G_g = \frac{W_g}{A}$$

(2.8)

Usually the flow will be considered to be one-dimensional with averaged properties across the duct. Only when large variations in the properties occur, the theory will prove to be inadequate and a more detailed analysis will be necessary.

The relative velocity can now be defined as:

$$V_{gl} = (V_g - V_l) = -V_{lg}$$

(2.9)
2.2. BASIC RELATIONSHIPS AND MODELS

Drift velocities are defined as the difference between the component velocities and the mixture velocity as follows:

\[ V_{lm} = V_l - V_m \quad V_{gm} = V_g - V_m \]  \hspace{1cm} (2.10)

The drift flux now represents the volumetric flux of a component relative to a surface moving at the mixture velocity, i.e.:

\[ j_{gl} = \alpha_g(V_g - V_m) \quad j_{lg} = (1 - \alpha_g)(V_l - V_m) \]  \hspace{1cm} (2.11)

Considering that \( V_m = V_{ls} + V_{gs} \) and that \( V_{gs} = \alpha_g V_g \) we can now write:

\[ j_{gl} = \alpha_g V_g - \alpha_g V_m = V_{gs} - \alpha_g(V_{ls} + V_{gs}) = V_{gs}(1 - \alpha_g) - \alpha_g V_{ls} \]  \hspace{1cm} (2.12)

And similarly:

\[ j_{lg} = V_{ls} \alpha_g - (1 - \alpha_g) V_{gs} \]  \hspace{1cm} (2.13)

Therefore \( j_{gl} = -j_{lg} \). This symmetry is an important and useful property of the drift flux. By means of equation 2.6 we can now find:

\[ j_{lg} = \alpha_g(1 - \alpha_g)(V_l - V_g) = \alpha_g(1 - \alpha_g)V_{lg} \]  \hspace{1cm} (2.14)

The drift flux is thus proportional to the relative velocity.

2.2.2 Two-phase flow models

As was discussed in the introduction two-phase flows can be described by the basic laws of fluid mechanics. Three simple models were mentioned which can be successful in describing two-phase flows without taking the details of the flow into account. Each one will now be discussed shortly.

Homogeneous model

This model provides the simplest technique for analysing two-phase flows. The assumption is that the two phases flow as a homogeneous mixture, so complete interaction between the two phases is assumed. The mixture is then treated as a pseudo-fluid, with average physical properties, which obeys the equations of single-phase flow. All standard methods of fluid mechanics than apply.

As pseudo properties of the fluid the weighted averages of the properties of the two separate phases are taken. Differences in velocities, temperature and chemical potential induce momentum, heat and mass transfer between the two phases. Often these processes proceed very rapidly, especially in the case of one phase being finely dispersed in another phase.
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The resulting properties of the mixture in equilibrium can than be considered to be equal to the average properties of the individual components.

The two phases are treated as a single fluid travelling at the mixture velocity \( V_m \). The phases are well mixed and no slip occurs, so \( \alpha_l = \lambda_l \). As discussed the fluid is considered to have a homogeneous density:

\[
\rho_h = \alpha_g \rho_g + \alpha_l \rho_l
\]  
(2.15)

Furthermore, the fluid will be considered to have a homogeneous viscosity. Various suggestions have been made by different researchers. Straight forward is to consider the analogy to the homogeneous density:

\[
\mu_h = \alpha_g \mu_g + \alpha_l \mu_l
\]  
(2.16)

An alternative is to take into account that most of the liquid will be flowing close to the wall. One might then expect that it is appropriate to take:

\[
\mu_h = \mu_l
\]  
(2.17)

The homogeneous flow model only works well in a limited number of cases. For some applications it is even totally inappropriate, like for example, for countercurrent vertical flows. These flows are driven by gravity acting on the different densities of the phases and can therefore not be described by a suitable average velocity.

Drift-flux model

The drift-flux model is a two-phase model with slip in which the two phases are considered together as a single-fluid. The attention is focussed on the relative motion rather than on the motion of the individual phases. The theory can be developed in a very general way, but the model is particularly useful when the relative motion is determined by a few key parameters and is independent of the flow rate of each phase. When, for example, the situation of a dispersed bubble flow in a vertical pipeline is considered, the relative motion between the bubbles and the liquid is governed by a balance between buoyancy and drag forces. The relative motion then becomes a function of the void fraction and not of the flow rate. The drift-flux model can be successfully used in for example the bubble and slug flow regimes.

As we have seen \( V_m = V_{ls} + V_{gs} \) and so equation 2.12 can be expressed in the alternative forms:

\[
V_{ls} = (1 - \alpha_g)V_m - j_{gl}
\]  
(2.18)

\[
V_{gs} = \alpha_g V_m + j_{gl}
\]  
(2.19)
These equations show that the volumetric flux of a component is equal to the sum of the volumetric concentration times the average volumetric flux and a flux $-j_{pl} = j_{lg}$ due to the relative motion. Therefore the drift flux provides a convenient way of modifying the homogeneous theory to account for the relative motion. All further properties of the flow, such as void fraction, mean density, and momentum flux can be expressed as the homogeneous flow value together with a correction factor or an additional term which is a function of the ratios of $j_{pl}$ to the component fluxes.

As an example of a typical situation in which the drift-flux model is appropriate a dispersed bubble flow in a vertical riser was mentioned. In this situation the actual gas velocity is supposed to respond to the centre-line velocity in the pipe. This velocity tends to be approximately 25% greater than the mixture velocity under turbulent flow conditions. Due to their buoyancy the gas bubbles will experience a bubble rise velocity $V_b$. As a result the actual gas velocity, in the case of vertical flow, becomes:

$$V_g = C_0 V_m + V_b$$

(2.20)

The parameter $C_0$, known as the distribution parameter, reflects the effect of the centre-line velocity and has the typical value of 1.25.

**Separated-flow model**

A much more complicated model is the separated-flow model. This model takes into account that the two phases can have differing properties and different velocities. It is a model which can be developed with various degrees of complexity. The simplest is when only one parameter, such as the velocity, is allowed to differ. The conservation equations are then written for the combined flow. The most sophisticated way of developing this model is by writing down separate equations for continuity, momentum and energy for each phase. The acquired six equations are then solved simultaneously together with the closure equations which describe how the phases interact with each other and with the walls of the duct. Although the accuracy is increased in this way, the disadvantage is the extra complexity that is introduced. In case the number of variables to be determined exceeds the available number of equations, simplifying assumptions or empirical correlations are introduced. The latter ones are often only valid within certain ranges of some parameters.

As the separated model considers the two components of the flow separately it is also called the two-fluid model. It is, among others, useful for the description of stratified and annular flows in ducts.
2.3 Two-phase flow in a line and a riser

One of the most striking features of two-phase flows is the tendency to adopt a wide variety of geometrical configurations, known as flow patterns or flow regimes. The different patterns are determined by the shape the interface between the two phases adopts. There is an almost infinite range of possibilities, although in general the surface tension tends to create curved interfaces leading to spherical shapes (at least what the smaller bubbles and droplets are concerned). In the case of non-vertical flow there is the additional non-symmetrical effect of gravity, which tends to pull the liquid to the bottom of the pipe.

By classifying the different interfacial distributions in the already mentioned flow regimes, the description of two-phase flow can be simplified. One should note, however, that this classification, although extremely useful, is highly qualitative and often very subjective. Many different regimes have been defined and a lot of different names have been used. The definitions used here are chosen for their relative generality of acceptance.

As was mentioned in the introduction on two-phase flow a method often used to describe when the various flow patterns occur is by drawing a flow-regime map. Just as with the description of the regimes itself, a lot of different maps have been introduced by many authors. As the areas the regimes occupy in the maps can change drastically when certain parameters are just slightly changed, it is hardly possible to come up with one generally valid map.

In the remaining part of this section the regimes which can be distinguished for the gas/liquid flow through respectively a riser and a line respectively will be discussed. Furthermore, two general flow maps will be presented. The different regimes will then be discussed in some more detail in the next section, along with the transitions between the regimes.

2.3.1 Vertical upward flow

When a gas/liquid flow is flowing upwards through a vertical pipe the different regimes which can be distinguished are illustrated in figure 2.3.

As the amount of gas flowing through the riser is gradually increased the following regimes can be distinguished:

- **Bubble flow.** Here, there is a dispersion of bubbles in a continuum of liquid. The bubbles are of approximately uniform size. This regime is also called dispersed bubble flow.
- **Slug flow.** As the concentration of bubbles becomes high, bubble coalescence takes place and, progressively, the bubble diameter approaches that of the tube. Large
bubble-shaped bubbles then alternate with liquid slugs (in which small bubbles can still be present). The bullet-shaped bubbles are normally referred to as Taylor bubbles. This is known as slug flow, intermittent flow and in the case of vertical flow also plug flow. Furthermore, the situation when the slugs are free from small bubbles is sometimes referred to as elongated bubble flow.

- **Churn flow.** Due to the further increased gas flow, the velocity of the slug flow bubbles is increased and, ultimately, these bubbles will break down leading to a highly unstable regime of an oscillatory nature in which liquid is moving upwards and downwards. This regime is also called froth flow.

- **Annular flow.** The liquid now flows as a film on the wall of the tube and partly as small droplets distributed in the gas flowing through the centre. To point out that there are small droplets entrained in the gas core this regime is sometimes called annular dispersed flow.

In order to get an idea of when these different patterns occur a flow regime map for an air/water system is presented in figure 2.4. The map gives the situation for a tube of 5.1 cm internal diameter, the solid lines are the transition points according to the theory, whereas the symbols represent measurements.
2.3.2 Horizontal flow

As was already mentioned the main complicating feature for horizontal flows is formed by the gravitational forces acting on the liquid and pulling it downwards to the bottom of the channel. The different flow regimes occurring in a horizontal line are illustrated in figure 2.5.

Again the various regimes will be described as the gas flow rate through the line is increased:

- **Bubble flow.** For low gas flow rates the gas travels as bubbles at the top of the tube. As for vertical flow this is also called dispersed bubble flow.

- **Plug flow.** The bullet-shaped bubbles once again occur but they are now moving along the top of the channel.

- **Stratified flow.** In this situation the gravitational separation is complete, the gas flows through the upper part of the tube and the liquid along the bottom. This regime is also called stratified smooth flow.

- **Wavy flow.** As the gas velocity is increased, waves are formed on the gas/liquid interface. The waves will become higher for further increased gas flow rates, whereby liquid drops may be entrained in the gas stream. Often this regime is called stratified wavy flow.

- **Slug flow.** As the waves grow large enough they may reach the upper surface of the tube. Large slugs of liquid will then be interspersed with regions where there is a
2.3. TWO-PHASE FLOW IN A LINE AND A RISER

Flow patterns in a line.

- **Plug flow.**

- **Slug flow.** Again this regime is also referred to as elongated bubble flow in the case the slugs are free of small bubbles.

- **Annular flow.** As in vertical tubes, for high gas flow rates, the liquid will flow as a film on the wall and the gas will flow through the centre of the tube. Liquid droplets may get entrained in the gas core (often called annular dispersed flow) and a further complication is that the film will be thicker at the bottom of the channel than at the top due to the effect of gravity giving drainage around the periphery. For very high gas flow rates the liquid film may get wavy. This is then called annular wavy flow.

Just as for the riser a flow regime map of an air/water system is presented in order to get an idea of when the various regimes occur. Figure 2.6 gives the situation for a 5.1 cm internal diameter tube, again the solid lines represent the theoretical transitions and the symbols represent measurements.
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2.4 Flow patterns

In the previous section the different flow patterns that occur in gas/liquid flow through a line and a riser have been introduced. Not all of these patterns are of interest in the remaining part of this thesis. The bubbly flows for example never occurred within the range in which the experimental set-up which was used operates. The same can be said of the annular flows. The churn flow regime did occur in the riser, but was not the focus of this study. The main patterns of flow that were dealt with were the slug flows in the line and the riser and the stratified flow in the line. These regimes will therefore be further discussed in the subsequent part of this section. This chapter will then be concluded with a short treatment of the phenomenon of flow transitions.

2.4.1 Stratified flow

Stratified flow is characterised by the complete separation of the two phases due to the effect of gravity. The heavier liquid flows along the bottom of the channel, while the gas phase flows on top of it. As the phases are completely separated it will be no surprise that in this case it will be appropriate to use the separated-flow model.

It was previously discussed that in its most general form this model starts with six equations for the conservation of mass, momentum and energy for the different phases. The model can be drastically simplified by considering the flow to be steady and isothermal without mass transfer between the phases. The two momentum equations will then lead to

FIGURE 2.6: Flow regime map for a 5.1 cm diameter horizontal line under normal atmospheric conditions (Barnea, D. and Taitel, Y. (1985)).
two equations for the unknown phase hold-up and the two-phase pressure gradient. This, as the superficial velocities are considered to be known. By means of equation 2.6 the average phase velocities can be calculated once the phase hold-up is determined. To be able to solve these equation several empirical correlations will still be necessary.

The following momentum equations can now be written down:

\[
-\alpha_g \frac{dp}{dx} - \tau_{wg} \frac{P_g}{A} - \tau_l \frac{P_l}{A} - \alpha_g \rho_g g \sin \theta + \alpha_g \rho_g g \cos \theta \frac{dh_g}{dx} - \frac{d}{dx} (G_g V_g) = 0
\] (2.21)

\[
-\alpha_l \frac{dp}{dx} - \tau_{wl} \frac{P_l}{A} + \tau_l \frac{P_l}{A} - \alpha_l \rho_l g \sin \theta - \alpha_l \rho_l g \cos \theta \frac{dh_l}{dx} - \frac{d}{dx} (G_l V_l) = 0
\] (2.22)

Here \( p \) denotes the pressure, \( \tau_{wg} \) and \( \tau_{wl} \) the frictional stresses of the wall on the gas and the wall on the liquid, \( \tau_l \) the shear stress at the interface, \( P_l, P_g \) and \( P_l \) the perimeters of the interface and the gas and the liquid along the wall, \( g \) the gravitational acceleration, \( \theta \) the angle of the line relative to the horizontal, \( h \) the height of a phase in the duct and \( G \) the momentum of a phase per unit of volume. The situation is illustrated in figure 2.7.

In formulas 2.21 and 2.22 the terms with the \( \cos \theta \) factors take into account a possible change in the liquid height in the channel. Mostly a stationary situation with a constant liquid level will be considered. The terms with the \( \cos \theta \) factors then disappear from the equations. The terms with the momentum per unit volume \( G \) are the acceleration terms. The shear stresses can be written in the following familiar forms, in which \( f \) denotes the Fanning friction factor:

\[
\tau_{wg} = f_g \rho_g \frac{V_g^2}{2}
\] (2.23)

\[
\tau_{wl} = f_l \rho_l \frac{V_l^2}{2}
\] (2.24)
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\[
\tau_i = \frac{f_i \rho_g (V_g - V_i)^2}{2}
\]  
(2.25)

In the calculations of the friction factors the actual average velocities and the hydraulic diameters are used. Hereby the gas is considered to flow through a closed duct and the liquid through an open channel, thus:

\[
D_l = 4 \frac{A_l}{P_l} \quad D_g = 4 \frac{A_g}{P_g + P_i}
\]  
(2.26)

In the case of the interfacial friction factor an additional difficulty is the determination of the interfacial velocity \(V_i\) and the interfacial roughness \(\varepsilon_i\).

From equations 2.21 and 2.22 the pressure gradients can be eliminated and when the acceleration terms are ignored a relation can be found by which the liquid hold-up \(\alpha_l\) can be calculated:

\[
\tau_{wg} \frac{P_g}{A_g} - \tau_{ul} \frac{P_l}{A_l} + \tau_i P_i \left( \frac{1}{A_l} + \frac{1}{A_g} \right) - (\rho_l - \rho_g) g \sin \theta - \Delta \rho g D \cos \theta \frac{\partial h_i}{D} \frac{\partial \alpha_l}{\partial x} = 0
\]  
(2.27)

The relations between the top angle \(\gamma\) and the liquid hold-up and the perimeters are as follows (see figure 2.7):

\[
\alpha_l = \frac{\gamma - \sin \gamma}{2\pi}
\]  
(2.28)

\[
h_i = \frac{1}{2} \left[ 1 - \cos \left( \frac{\gamma}{2} \right) \right] D
\]  
(2.29)

\[
P_l = \gamma \frac{D}{2}
\]  
(2.30)

\[
P_i = \sin \left( \frac{\gamma}{2} \right) D
\]  
(2.31)

\[
P_g = \pi D - P_l
\]  
(2.32)

It is convenient to write equation 2.27 in dimensionless form, whereby it is assumed that \(\partial \alpha_l/\partial x = 0\) (corresponding to a stationary situation with a constant void fraction):

\[
X^2 f_i \left[ \frac{\bar{P}_l}{A_l} \tilde{V}_i^2 - \left[ \frac{\bar{P}_g}{A_g} \tilde{V}_g^2 + f_i \left( \frac{\bar{P}_i}{A_g} + \frac{\bar{P}_l}{A_l} \right) \tilde{V}_r^2 \right] \right] - 4Y = 0
\]  
(2.33)

Where \(V_r = V_g - V_i\) is the relative velocity, \(\bar{f}_g\) and \(\bar{f}_i\) are the ratios of the actual to superficial friction factors and \(\bar{f}_i\) is the ratio of the interfacial friction factor to the superficial...
2.4. FLOW PATTERNS

gas friction factor. Furthermore, $X$ denotes the Lockhart-Martinelli parameter, which is defined as the square root of the ratio of the single-phase frictional pressure gradients of the liquid and the gas flow. These are the pressure gradients the phases would have in case they were flowing alone through the tube. $Y$ takes the angle of the line into account and is called the Taitel-Duckler inclination parameter. In equation form $X$ and $Y$ are written as:

$$X^2 = \frac{\left(\frac{dp}{dx}\right)_{ls}}{\left(\frac{dp}{dx}\right)_{gs}} = \frac{2f_{ls} \frac{\rho_l V_{ls}^2}{D}}{2f_{gs} \frac{\rho_g V_{gs}^2}{D}}$$

(2.34)

$$Y = \frac{(\rho_l - \rho_g)g \sin \theta}{\left(\frac{dp}{dx}\right)_{gs}} = \frac{-(\rho_l - \rho_g)g \sin \theta}{2f_{gs} \frac{\rho_l V_{gs}^2}{D}}$$

(2.35)

In order to solve the acquired set of stratified flow equations, correlations are still required for the calculation of the interfacial friction factor and for the liquid interface velocity. For the friction factor numerous empirical correlations exist. For the interface velocity the average liquid velocity can be taken in the turbulent case, while in the laminar case twice that value is appropriate.

2.4.2 Slug flow

Slug flow was previously described as a series of large bullet-shaped bubbles (Taylor bubbles) which alternate with liquid slugs. These Taylor bubbles almost fill the whole pipe cross-section. For vertical upward flow they rise centrally through the riser with a liquid film around them, while in a horizontal line they cling to the top of the channel, thereby creating an area of stratified flow between the liquid slugs.

When one is capable of describing the dynamics of a single individual bubble, some of the overall properties of the slug flow regime can be easily predicted. The bubble dynamics are determined by the mixture velocity, the corresponding velocity profile in the liquid slug, the pipe geometry and the fluid properties. The velocity profile in the liquid slug is a function of the pipe roughness and the Reynolds number based on the mixture velocity and the pipe diameter. Therefore the bubble dynamics are dependent on the mixture velocity and not on the individual superficial velocities of the phases. In almost every case the bubble length is not an important variable, since the dynamics of the bubble are entirely governed by its nose and tail. Furthermore, in case each unit cell (this is a cell consisting of one bubble and one slug) is independent, the bubble dynamics are independent of the void fraction.

The pressure drop in slug flow can be divided into three parts, namely the drop over a liquid slug, the drop over a Taylor bubble and the drop around the ends of the bubble.
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The viscosity and density of the gas are normally much lower than that of the liquid and as a result the gas in the bubble will generally have an approximately constant pressure. As the bubbles have a cylindrical shape and the gas/liquid interface has a constant curvature the pressure drop around the ends of the bubble will also be negligible. All that results is the pressure drop in the liquid slug, which can be calculated using single-component techniques. As the slugs and Taylor bubbles alternate in slug flow, the pressure will oscillate in a saw-tooth like manner and this may result in oscillations in the system and of the pipeline structure itself.

Due to the effect of gravity there is some difference between a slug flow in a riser and a slug flow in a line. The flow patterns were already described and as could be seen they were not the same. As a consequence both situations will also have to be treated separately. First the situation in a riser will be described, then the situation in a line.

Slug flow in a riser

When describing the vertical upward slug flow we will consider the flow to be stationary and fully developed. Slug flow has been characterised by the alternation of liquid slugs and Taylor bubbles. As a result it can be described by viewing one unit cell consisting of one slug and one bubble, as is illustrated in figure 2.8.

![Figure 2.8: A unit cell in vertical upward slug flow.](image-url)
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The Taylor bubble occupies just about the entire cross-section and moves upward with a velocity $V_{TB}$. It has a length $L_{TB}$ and can be considered as a volume of constant pressure. The friction with the liquid film will be neglected. The liquid film of thickness $\delta_l$ will be a point of study in a later stage in this thesis. The average velocity of the gas in the Taylor bubble section is denoted by $V_{g,TB}$. The liquid film has a velocity of $V_{l,TB}$. The average void fraction in the Taylor bubble section is written as $\alpha_{TB}$. For the slug section analogous definitions apply.

The velocity of the Taylor bubble can be described by the Nicklin relation (Nicklin et al. (1962)):

$$V_{TB} = C_0 V_m + V_{\infty,TB}$$

(2.36)

Here $C_0$ is denoted as the distribution coefficient, which is equal to the ratio of the velocity of the liquid at the centre of the slug to the average liquid velocity. It has typically the value 1.2. $V_{\infty,TB}$ is the rising velocity of a single Taylor bubble in a stagnant fluid. It has been determined to equal: $0.35\sqrt{gD}$.

The Taylor bubbles are separated by liquid slugs, which will normally contain lots of small bubbles. The movement of these bubbles can be described by the formula of the bubbly flow regime. The velocity of the bubbles was discussed with the drift-flux model which resulted in equation 2.20. Due to entrainment of the gas from the tail of the Taylor bubbles into the slugs, the void fraction in the slugs will be higher just behind the bubbles then in the rest of the slugs. The length of the slugs remains constant.

For the time in which a Taylor bubble passes a fixed point we can write (see figure 2.8):

$$\Delta t_{TB} = \frac{L_{TB}}{V_{TB}}$$

(2.37)

And thus the volume of the gas which passes is equal to:

$$V_{g,TB} = V_{g,TB} A_{TB} \Delta t_{TB} = L_{TB} A_{TB} \alpha_{TB} \frac{V_{g,TB}}{V_{TB}}$$

(2.38)

Note that $V$ denotes a velocity and $V$ a volume. Analogous we can write for the time in which a slug passes and the volume of gas passing with it:

$$\Delta t_{SL} = \frac{L_{SL}}{V_{TB}}$$

(2.39)

$$V_{g,SL} = V_{g,SL} A_{SL} \Delta t_{SL} = L_{SL} A_{SL} \alpha_{SL} \frac{V_{g,SL}}{V_{TB}}$$

(2.40)
CHAPTER 2. GENERAL TWO-PHASE FLOW THEORY

The total volume of gas passing with one slug unit now equals (with 2.38 and 2.40):

\[ Q_g (\Delta t_{TB} + \Delta t_{SL}) = V_{gs} A \left[ \frac{L_{TB} + L_{SL}}{V_{TB}} \right] \]  

(2.41)

Here \( Q_g \) denotes the volumetric flow rate. In case we consider the gas to be incompressible (so that the mass and volume conservations are equal) conservation of mass gives (with 2.41):

\[ V_{gs} A \frac{L_{SL}}{V_{TB}} = L_{TB} A \alpha_{TB} \frac{V_{g_{TB}}}{V_{TB}} + L_{SL} A \alpha_{SL} \frac{V_{g_{SL}}}{V_{TB}} \]  

(2.42)

With \( \beta = \frac{L_{TB}}{L_{g_{SU}}} \) this can also be written as:

\[ V_{gs} = \beta \alpha_{TB} V_{g_{TB}} + (1 - \beta) \alpha_{SL} V_{g_{SL}} \]  

(2.43)

The conservation of mass for the liquid can be treated similarly. The only difference is that the negative liquid flow rate of the liquid film around the Taylor bubble has to be taken into account. The result is:

\[ V_{ls} = (1 - \beta)(1 - \alpha_{SL}) V_{l_{SL}} - \beta(1 - \alpha_{TB}) V_{l_{TB}} \]  

(2.44)

By means of the resulting equations one can calculate, for example, the hold-up in the Taylor bubble or the slug unit.

Slug flow in a line

An ideal slug flow in a horizontal line consists of liquid slugs alternated by huge bubbles which travel centrally through the channel with a liquid film around them of uniform thickness. Actually in this way the influence of the gravity, which gives the horizontal slug flow its distinguished look, is being neglected. This situation is instructive though to show that the bubbles do not move with the same average velocity as the liquid. If the film has a mean thickness \( \delta \), the available area for gas in the bubble is:

\[ A_b = \pi \left( \frac{D}{2} - \delta \right)^2 \]  

(2.45)

Since there is no pressure drop along the length of the bubble, the liquid film on the wall will be stationary. Continuity of the volumetric flux at this cross-section now demands that:

\[ \frac{V_b}{V_m} = \frac{A}{A_b} \]  

(2.46)
2.4. FLOW PATTERNS

This then gives:

\[ V_b = \frac{1}{1 - \frac{4\rho}{D}} V_m \]  \hspace{1cm} (2.47)

Hence, the velocity of the bubble \( V_b \) is larger than the mixture velocity.

Under normal circumstances gravity will pull on the liquid film and as a result the film will be thicker at the bottom of the channel. Furthermore, buoyancy will make the bubble cling to the top of the pipe. What results is a situation as is illustrated in figure 2.9. In effect this geometry consists of two parts. The bubble has the stratified flow configuration, whereas the liquid cylinder, which will normally contain bubbles, has the bubble flow configuration. The stratified part can be described by the earlier discussed separated flow model, the bubble flow part by means of the drift-flux model. For the translational velocity of the slug one can write:

\[ V_t = C_0 V_m \]  \hspace{1cm} (2.48)

Here \( C_0 \) is the distribution coefficient already discussed with equation 2.36. As is obvious the rise velocity of a bubble is nil in this horizontal situation.

\[ \text{FIGURE 2.9: Slug flow in a line.} \]

2.4.3 Flow transitions

A different aspect of two-phase flow which has not yet been mentioned is the study of the transitions between different flow regimes. One of the main things one would like to know is when certain flow patterns occur and especially under which circumstances the flow shifts to a different pattern. A tremendous amount of effort has been taken by numerous researchers to describe the transitions and most of all to figure out when they occur.

In this thesis we will run into two transitions. The most important one is the transition from stratified flow to slug flow in the line. This transition will be described in detail in later chapters. The second transition is the one between slug flow and churn flow in the riser. This transition is not studied in detail. Instead only the positioning of this transition in the flow map has been determined. In this section the treatment of the transitions between various regimes will be limited to a short description of the dominant mechanisms.
Transitions in a horizontal line

**Bubble to plug to stratified transition**
At very low gas flow rates tiny bubbles are being finely dispersed in the continuum of liquid. The bubbles move along at the top of the tube. The amount of bubbles is too small for them to collide and to coalesce. As the gas flow rate becomes higher this will happen and at first some larger randomly shaped bubbles will be formed. As these bubbles grow they approach a rather bullet like shape. The flow has now come in the plug flow regime. For even higher flow rates the bubbles become so large that one could say that "one single huge bubble" has developed. The gas is now flowing on top of the liquid and a stratified flow has arisen.

**Stratified smooth to wavy transition**
As the gas flow rate is further increased waves will be generated on the liquid surface. Pebbled waves are caused by wind action, i.e. the imbalance between the wind energy fed to the waves and the viscous dissipation. Roll waves, on the other hand, occur when the destabilising effect of the liquid inertia and the pressure variation over long waves overcomes gravity. Atomisation takes place as a result of the pressure variation overcoming the surface tension. Stratified wavy flow exists under conditions where the velocity of the gas is high enough to cause waves to form, but slower than that needed for rapid wave growth, which, as will be seen, is the major cause of slug flow.

**Stratified to slug transition**
Consider a smooth stratified flow in the channel. As the liquid flow rate is increased, the liquid level will rise and at some point a wave may be formed. As the gas flow rate over the crest of the wave has to remain constant (conservation of mass), the gas will have to accelerate. The Bernoulli effect (which states that the sum of the kinetic energy per unit mass, the potential energy per unit mass and the pressure divided by the density has to remain constant) then causes the pressure over the crest of the wave to drop. As a result of this pressure drop the wave will grow. The only opposing force is the gravity. Under certain conditions, described by the Kelvin-Helmholtz theory, the gravity will be overtaken and the waves will rapidly grow. These growing waves can block the pipe and form a competent bridge and as a result slug flow ensues. The Kelvin-Helmholtz analysis will be treated in detail at a later stage in this thesis.

**Slug to annular transition**
In the slug flow regime growing waves block the pipe and create slugs. At higher gas flow rates, the amount of liquid flowing through the system will become insufficient to form slugs. Instead the liquid will be swept up and around the pipe to form an annular flow. This transition does not have a sharp, well-defined change, but rather a gradual one, since it is difficult to distinguish between a highly aerated slug and a wavy annular flow.
2.4. FLOW PATTERNS

Transitions in a vertical riser

Bubble to slug transition
As gas is being introduced at low flow rates it is distributed into small, discrete bubbles. As the flow rate becomes higher the bubbles will grow larger and above a critical size the bubbles will begin to deform. The bubbles will now rise in a kind of a zig-zag path. The result is that the bubbles will collide and coalesce, forming larger bubbles with a spherical cap shape. The bubbles are not yet large enough to occupy the whole cross-section of the pipe and create slug flow. When the gas flow rate is increased further, the bubble density will increase causing many collisions and thus larger bubbles. This ultimately results into slug flow.

Slug to churn transition
For higher gas flow rates the slug flow will change to churn flow. There exist different explanations for this transition. The transition can be said to occur when the gas velocity relative to the falling liquid film around a Taylor bubble approaches the condition for flooding. Flooding is the point where the velocity of the film shifts from downwards to upwards. The transition can also be attributed to the Helmholtz instability of the liquid film surrounding a Taylor bubble. Further the transition can be attributed to the growing of the Taylor bubbles. Added to that the churn flow region is sometimes considered to be just an entry region phenomenon associated with the existence of slug flow further along the pipe. Whenever one observes slug flow in a riser the condition near the entry seems to be churning. The distance over which this churn flow can be witnessed depends on the rate at which the slug flow develops into a developed slug flow. This is dependent on the flow rates and the pipe-size.

Transition to annular flow
For high gas flow rates a film will be flowing upwards along the wall of the riser and the gas will flow in the centre carrying entrained liquid droplets. The upward flow of the film against gravity results from the forces exerted by the fast moving gas core. The film will normally have a wavy interface and the waves will shatter and enter the gas core as entrained droplets. Annular flow can only exist when the gas flow is high enough to lift the entrained droplets. When the flow rate is insufficient, the droplets will fall back, accumulate, form a bridge and create slug or churn flow.
Chapter 3

Experimental set-up, methods and data-analysis

Within this project experiments were performed by means of a test facility. This facility will be described in the first section of this chapter. Next, the second section will deal with the experimental methods used and the data-analysis performed. Some details on the used equipment can be found in appendix C.

3.1 Set-up

The two-phase flow, concurrent, air/water, line-riser system (as illustrated in figure 3.1) consists of a horizontal line, which is 9 meters long, and a vertical riser of 4 meters in length. All the pipes have an internal diameter of 100 mm and are made of transparent PVC. The line and the riser are connected by means of an upward turned bend with a radius of 630 mm.

A buffer tank of 1.5 m³ in volume contains an amount of tap water, which is being circulated through the system by a centrifugal pump. The pump delivers a constant power (and not a constant flow rate) and can give a maximum flow rate of approximately 500 l/min. This is equal to a superficial liquid velocity of about 1 m/s. Once the air/water flow has gone through the entire system, the mixture will plunge back into the storage vessel, which then functions as a separator.

The liquid mass flow rate is measured by an electromagnetic flow meter just behind the liquid pump. The water passes through a honeycomb structure located at the beginning of the line. Immediately after the honeycomb air at near atmospheric pressure comes into the line parallel to the water flow. It is let into the system through a pipe with an internal diameter of 38 mm. The flow rate is measured by two parallel rotameters. The maximum superficial air velocity is approximately 11 m/s.
CHAPTER 3. EXPERIMENTAL SET-UP, METHODS AND DATA-ANALYSIS

The line contains nine measuring points for pressure transducers. They are all placed one meter apart. Furthermore, the bend contains five such points placed symmetrically between the connection points to the line and the riser. In addition, the riser has three points designed for pressure measurements, again placed one meter apart. Between each of these measuring points in the riser and between the highest one and the top of the riser, an additional pair of two glass fibre measuring points are located. These three pairs are all located symmetrically between two pressure measuring points. A pair is placed exactly 300 mm apart.

Ultimately, the mixture will flow back into the storage vessel through the return line. As can be seen in the figure there are two flexible hoses present in the facility. One just before the line and one between the bend and the riser. Furthermore, there is a lifting tackle located half-way through the line. These are constructed in order to be able to place the line under different angles with the horizontal. The study of the air/water flow in such a system is planned for future research, but was not a part of the present project. To make sure the line was placed exactly horizontal a laser beam was used for levelling.
3.2 Methods and data-analysis

3.2.1 Pressure transducers

As can be seen in figure 3.1 there are nine points in the line and three points in the riser where pressure measurements can be performed. Furthermore, there are five such points present in the bend. A pressure signal gives information on the fluctuation of the pressure at a certain spot. This fluctuation is a superposition of all pressure effects within the system. As a result only limited information on the local situation can be acquired through pressure measurements. By means of pressure measurements the frequency of an aspect of the flow which makes the pressure in the entire system fluctuate, like the creation of slugs, can be determined. Besides, the change in an overall property of the flow, like the flow regime, can be detected. An example of a raw pressure signal in case there is slug flow in the line and the riser is given in figure 3.2.

In order to measure the pressure at a certain point in the system, piezo-electrical pressure transducers are used. These sensors measure the pressure difference with regard to the external pressure outside the system. The end of a sensor will be placed flush with the wall inside the pipe, just in the two-phase mixture. The difference between the pressure inside the tube and the surrounding pressure is turned into a voltage. The sensors can measure pressure differences varying between nil and 150 kPa with an accuracy of 99.5 % of it’s range.
The pressure transducers are connected to a power source which amplifies the analog signals. The analog voltages are then turned into a discreet signal through a high-speed multiplexer and a data-acquisition unit (DACQ). The maximum sampling frequency is 100 kHz. The DACQ is controlled by a personal computer by means of a program designed in Hewlett Packard's Visual Engineering Environment (HP VEE), which is a graphical programming environment for engineering purposes. Within this program input values like the sampling frequency and the number of scans can be given.

The sampled signals from the pressure measurements can be analysed by means of the code Analyze, written in FORTRAN 77 by Erik Legius and Bas Meeuwissen. This code performs a statistical analysis of the signals. It can perform an analysis on several signals at once, with a maximum of $2^{17}$ scans per signal, both in the time and the frequency domain. Figures of the functions determined can be viewed by means of the program Gnuplot. The algorithms for the signal analysis in the code Analyze came for a large part from Press et al. (1992).

In this research pressure measurements were used to determine the slugging frequencies. The frequencies in the signal can be determined directly by letting Analyze calculate the power spectral density function of the signal. An other method used is to first determine an autoregressive model of the signal and then estimate the spectrum from this model. This method, called Maximum Entropy Method, is further discussed in section 3.2.3.

The pressure measurements were also used to locate flow regime transitions in the flow map. The transitions from stratified flow to slug flow in the line and from slug flow to churn flow in the riser were located. This is further discussed in section 3.2.4.

### 3.2.2 Glass fibre probes

By means of glass fibre probes measurements can be performed in the riser in case a slug flow is present. The frequency of the slug flow, the lengths of the slugs and the Taylor bubbles and their velocities can be determined.

Each probe consists of a glass fibre. Light is sent through the fibre from one end, while the other end is placed in the two-phase mixture. When the light reaches the end of the fibre within the flow, most of it will leave the fibre in case the end is surrounded by water, as the refraction indices of water and glass are just about the same. On the other hand the light will be reflected in case the end of the fibre is surrounded by air, due to the difference of the refraction indices of air and glass. The light beam will then return through the fibre. At the entrance of the fibre the entering and returning beams are being split and so the signal can be detected. The signal is turned into a voltage which will be maximal when air is detected and minimal when water is detected.
3.2. METHODS AND DATA-ANALYSIS

Just as was the case with the pressure signals, the signals from the glass fibres are amplified by a power source. The analog signal is then turned into a digital one by a high-speed multiplexer and a data acquisition unit (DACQ). Once again the HP-VEE program on the personal computer controls the DACQ. The sampled signals can then be processed by the code Fiber, written in FORTRAN 77 by Bas Meeuwissen. Some changes were made to the code to make it applicable for this research. The code Fiber first turns the raw signal into a neat block-signal representing the alternating slugs and Taylor bubbles, then it calculates the lengths and velocities of the slugs and the Taylor bubbles. The code Analyze can once again be used to determine the dominant frequencies in the block-signal.

At first a straight tip aimed perpendicular to the flow direction was used (see figure 3.3). This presented the problem that every single small bubble or droplet was being detected. In essence the fibres worked too well. As the flow in this facility is rather rough, there are a lot of small bubbles present in the slugs and numerous small droplets in the Taylor bubbles. As a result it was hardly possible to distinguish the slugs and the Taylor bubbles in the signal. A typical signal is illustrated in figure 3.4.

![Figure 3.3: Glass fibre configurations of a straight and a bent tip.](image)

In order to get rid of a great deal of the small bubbles and droplets in the signal, the straight tip was bend upwards (see figure 3.3). The objective was to make the small upward flowing droplets and bubbles bounce off the bend. The slugs and Taylor bubbles rise centrally through the riser and will be detected as they are way larger than the bubbles and droplets. The result was rather well as can be seen in figure 3.4. As the probes are placed in the centre of the pipe, there is no problem with the falling liquid film at the wall, though other liquid falling down can effect the measurements.

The signal we now have, as illustrated in figure 3.4, still has to be turned into a neat block-signal. Some criteria are necessary to achieve that. The first thing which can be used is that when a bubble hits the probe under an angle larger than 60 degrees with a line perpendicular to the surface of the bubble, the signal will not be maximal. As
a Taylor bubble rises centrally in the riser and as the probes are placed in the centre, a Taylor bubble will always be detected under a smaller angle. So when a peak in the signal is not maximal it is almost for sure that it represents a small bubble. So a threshold of 80% has been used to disregard all smaller peaks. Of course the opposite is valid for a droplet in a Taylor bubble, so between two maxima a threshold of 20% has been used.

A problem which still has to be tackled is that it is possible that a single droplet within a Taylor bubble is detected and splits up the Taylor bubble. As a result the Taylor bubbles will be too small and can even be missed altogether. Therefore the criteria has been used that if 5 points before and/or after a minimum, which lies between two maxima, are all maxima, the minimum will be turned into a maximum. Finally, the criterion is used that a Taylor bubble has a minimal length of one diameter of the pipe. Smaller bubbles will be disregarded.

Figure 3.4: Raw signals of glass fibres with a straight tip and a bent tip.
3.2. METHODS AND DATA-ANALYSIS

In case two droplets are being detected at the edge of a Taylor bubble it is hard to tell exactly where the Taylor bubble begins or ends. With the criteria mentioned a kind of average is taken as the droplet closest to the Taylor bubble will be disregarded as it has a lot of maxima next to it. On the other hand the second droplet will not meet this condition and will remain. Only when three droplets are being detected quickly after each other or when a droplet is being detected twice, will a Taylor bubble be split up.

The code Fiber turns the raw signal into a block-signal according to the mentioned criteria. Then it estimates the lengths of the slugs and Taylor bubbles. The velocities can also be estimated as two fibres are always placed exactly 30 cm apart. Comparison of the two signals which should be the same except for a time shift gives an estimation of the velocities. An example of two block-signals is given in figure 3.5. The first fibre is located 30 cm below the second one. It can be seen that they reasonably follow each other up, although some Taylor bubbles are split or combined differently in the two cases.

![Figure 3.5: Example of two block-signals representing a slug flow. Fibre 2 is placed 30 cm above fibre 1.](image)
3.2.3 Maximum Entropy Method

Two different ways to determine the dominant frequencies in a signal have been used. The first one is the ordinary determination of the power spectral density function. This function determines to what extent a frequency is present in the signal: $PSD(f)df$ represents to what extent the frequencies between $f$ and $f + df$ are present. An example is given in figure 3.6. As can be seen in the figure this function tends to fluctuate heavily.

Through this first method, a signal is being split into spectral components by determining its Fourier transform. The Fourier transform of a signal $x$ with a time-span equal to $T$ is given by:

$$\mathcal{F}(x) = \int_0^T x(t)e^{-2\pi ift}dt$$ (3.1)

The power spectral density function of a signal is equal to the convolution of the Fourier transform of the signal and its complex conjugate (denoted by $\mathcal{F}^*(x)$):

$$PSD(f) = \lim_{T \to \infty} \frac{1}{T} \mathcal{F}(x)\mathcal{F}^*(x)$$ (3.2)

The $PSD$ can be estimated from $N$ observations by making use of the coefficients $c_k$. These coefficients are determined by the discrete Fourier transform from $N$ observations:

$$c_k = \sum_{j=0}^{N-1} x_j e^{2\pi jik/N}$$ (3.3)

The contribution of the frequency $k/N$ to the "energy" of the overall frequency spectrum is given by $c_k^2$. An estimation of the PSD is acquired by plotting the values $c_k^2$, normalised with $\sum c_k^2$, against the frequency.

A different method to determine the dominant frequencies in a signal is called the Maximum Entropy Method (MEM). This method produces the averaged course of the frequencies. The advantage is that the maximum is more clearly visible. As an example the result for the same signal as for which the PSD was illustrated in figure 3.6 is shown in figure 3.7. The MEM method determines the spectrum using the autoregressive model of the signal.

In case a number of observations have been made of a process varying in time, it is possible to determine a model of this process. This model will have to take the typical properties of the process into account. A method to estimate this model is by writing an observation as a linear combination of previous observations. This is called an autoregressive model of the signal (AR-model). An autoregressive model of the order $k$ is defined as:

$$x_t + a_1x_{t-1} + \cdots + a_kx_{t-k} = \epsilon_t$$ (3.4)
3.2. METHODS AND DATA-ANALYSIS

**FIGURE 3.6**: Power Spectrum by PSD, $V_{ts} = 0.25$ m/s, $V_{gs} = 1.30$ m/s.

**FIGURE 3.7**: Power Spectrum by MEM, $V_{ts} = 0.25$ m/s, $V_{gs} = 1.30$ m/s.
here \( x_t \) denotes an observation at time \( t \) and \( \{a_1, \ldots, a_k\} \) are the coefficients of the AR-process. The function \( \epsilon_t \) is a random process with average nil and variance \( \sigma^2 \) and is a measure for the deviation of the predicted value from the real value. The coefficients can be estimated using the Burg method (see Childers (1978)). This is a procedure which heightens the order of the AR-model with one step at a time and recalculates each coefficient at every step.

When an AR-model is being fit to a process, two estimation errors can occur: overfitting and underfitting. Overfitting means that superfluous information is being used and underfitting means that relevant information is not being taken into account. A good order selection criterion results in an order which gives the best balance between the two. One of these is the so called General Information Criterion for finite samples (GIC), which states for \( N \) observations (see Broersen and Wensink (1993)):

\[
GIC(k, \alpha) = \ln[\sigma^2] + \alpha \frac{k}{N}
\]  

(3.5)

Where \( \alpha \) denotes the penalty factor. The optimal coefficient is \( \alpha = 3 \) (see Broersen and Wensink (1996)). The best order with which to estimate an AR-model from \( N \) observations is that value \( k \) for which \( GIC(k, 3) \) is minimal.

Once the AR-model of a signal has been determined, the spectrum of the signal can be estimated by (see Priestley (1981)):

\[
MEM(f) = \frac{\sigma^2}{2\pi |1 + a_1 \exp(-if) + \cdots + a_k \exp(-ikf)|^2}
\]  

(3.6)

This is what is called the Maximum Entropy Method. The dominant frequency is given by that frequency for which \( MEM(f) \) is maximal. With Maximum Entropy it is denoted that the spectrum chosen is the spectrum corresponding to the most random or most unpredictable time series whose autocorrelation function agrees with the one for which we tried to estimate the spectrum.

The variance of the PSD(f) is given by:

\[
Var\{PSD(f)\} = E^2\{f\}
\]  

(3.7)

Here \( E(g) \) denotes the expected value of \( g \). The variance of this equation is thus equal to 100 percent. In order to reduce this variance the signal is split up into a number of parts. The PSD is then averaged over these parts. It is very difficult to exactly determine the variance of this estimator. The variance of the estimation of the spectrum by means of the Maximum Entropy Method is given by:

\[
Var\{MEM(f)\} \approx \frac{k}{N} MEM^2(f)
\]  

(3.8)
3.2. METHODS AND DATA-ANALYSIS

A 65% confidence interval is given by the standard deviation (i.e. the root of the variance), whereas a 95% confidence interval is given by twice the standard deviation.

Both methods were incorporated in the FORTRAN 77 code Analyze and used in this project.

3.2.4 Autoregressive modelling

A pressure signal measured will contain information on the two-phase flow in the facility. So a pressure measurement performed in a certain flow regime will have characteristic properties corresponding to that regime. The flow regime can thus be determined by comparing pressure signals. The pressure signal is a superposition of all the pressure effects within the facility and will thus only contain the information on the overall flow in the entire system.

By comparing pressure signals it is possible to determine the location of a transition from one regime to another. Consider certain flow regimes to be present in the line and the riser. A pressure measurement can be performed to represent this situation. Now either the gas or the liquid flow rate can be slightly altered and again a measurement can be performed. By doing this over and over again until the flow has clearly changed into a different regime in either the line or the riser, a whole series of pressure measurements results. In case all these measurements are compared a sudden change will become visible in the properties of the signal at the location of the flow transition.

The properties of the pressure signals are compared by looking at the autoregressive models describing the signals (see equation 3.4). One signal, clearly in a certain regime, is taken as the reference signal with which the other signals are compared. The variance between two AR-models represents the "distance" between the signals and can be seen as a measure for the difference between the two.

Consider \( \{a_1, \cdots, a_k\} \) and \( \{b_1, \cdots, b_k\} \) to represent the coefficients of two AR-models A and B. Model A will be considered to be the reference model. The difference vector is defined as:

\[
d = \begin{pmatrix}
1 \\
a_1 - b_1 \\
\vdots \\
a_k - b_k \\
\end{pmatrix}
= \begin{pmatrix}
1 \\
d_1 \\
\vdots \\
d_k \\
\end{pmatrix}
\]  \hspace{1cm} (3.9)

The distance between the models A and B can then be defined as (see Broersen and Wensink (1995)):

\[
\Delta AR_{AB} = d^T R d 
\]  \hspace{1cm} (3.10)
Where $R$ denotes the autocovariance array of the AR-model $A$, which is defined as:

$$R = \begin{pmatrix}
R(0) & R(1) & \cdots & R(k) \\
R(1) & R(0) & \cdots & R(k-1) \\
\vdots & \vdots & \ddots & \vdots \\
R(k) & R(k-1) & \cdots & R(0)
\end{pmatrix} \quad (3.11)$$

Here $R(r)$ denotes the autocovariance function defined as (see Priestley (1981)):

$$R(r) = \frac{1}{N} \sum_{i=1}^{N-|r|} (x_i - \bar{x})(x_{i+|r|} - \bar{x}) \quad (3.12)$$

In this equation $\bar{x}$ denotes the average of the observations $x_i$ in the signal.

Once the distance of every signal of a series with regard to the reference signal is determined, these distances can be drawn in a graph with the varying superficial velocity as the independent variable. The distances are normalised by means of $R(0)$ (the value of the distance of the reference signal with itself). An example is given in figure 3.8.

The locations of the flow regime transitions, from stratified flow to slug flow in the line and from slug flow to churn flow in the riser, have been determined using this method of autoregressive modelling. The coefficients of the autoregressive models were determined with the code Analyze. Next, the distances were calculated by means of the code Distance, also written in FORTRAN 77 by Bas Meeuwissen. Finally, the graphs were made using the program Gnuplot. As a general reference on this method of autoregressive modelling Meeuwissen (1996) can be used.
Figure 3.8: Distance by AR-modelling (stratified to slug transition in the line).
Chapter 4

Model development

In the previous chapter the configuration of the experimental set-up has been presented, we continue by describing what has been observed within the facility. In the first section of this chapter the overall view will be presented and the occurring flow regimes mentioned. Two mechanisms will be distinguished which are dominant in the system. Following this discussion a model will be developed in the second section, based on the observed mechanisms. Within this model a few key parameters need to be calculated. The theory on which they are based will be treated in the third section. Next, the fourth section will present the modelling results. Two other models which were developed in the early stages of this work are shortly discussed in the fifth section.

4.1 Flow behaviour and mechanisms

4.1.1 Overall view and flow patterns

One of the first things which can be observed when watching the system operate at different flow rates is that the bubbly flow regime is totally absent in the riser. For the given flow rate ranges the riser only operates in the slug flow and the churn flow regime. The absence of the bubbly flow rate is rather surprising as it would normally, for a single vertical pipe, always occur for low gas flow rates. It is therefore immediately obvious that the bend has quite some impact on the system.

What the line is concerned, there are also two flow regimes which can be distinguished within the given ranges of the flow rates. These are the stratified flow regime and the slug flow regime. The stratified flow always seems to be wavy to some extend. Even when the flow would normally be smooth. Waves which are induced by the bend make the surface wavy. Besides, the air inlet also has the unfortunate property of creating waves on the air/water interface.
CHAPTER 4. MODEL DEVELOPMENT

The line operates in the stratified flow regime for liquid flow rates below typically a superficial velocity of about 0.4 m/s. For higher liquid flow rates the flow shifts to the slug flow regime. The transition in the riser from slug flow to churn flow takes place at relatively high gas flow rates. For superficial gas velocities typically above about 4 m/s, the riser operates in the churn flow regime.

It is now obvious that for relatively low liquid flow rates in combination with high gas flow rates there will be a stratified flow present in the line and a churn flow in the riser. As the gas drags the water into the bend, the water will get entrained in the gas stream due to the roughness of the air/water interface and the high gas velocity. Within the range of the gas flow rate in which the facility can operate the velocity of the gas cannot become high enough to create an annular flow in the riser. The water will fall back at regular intervals, although the time averaged water flow is still upwards. The resulting alternating flow was denoted as churn flow. Although a study of this configuration can be most interesting, this was not done in detail in the present work as the aim was to identify and describe the effect of the bend on the flow conditions. In the circumstance of a stratified flow in the line and a churn flow in the riser this effect is minimal. The situation is reasonably steady and nothing peculiar seems to happen.

On the other hand a slug flow in the line and in the riser will be present for high liquid flow rates and not too high gas flow rates. This is a regime where we quickly run into the limits of the experimental set-up. What happens is that for high liquid flow rates the level of the water in the line becomes so high that the air inlet gets under the water level. As a result a stationary wave pattern is being induced by the air inlet and at some point, when the flow rate is high enough, one of these waves will reach the top of the channel and thereby block the passage for the gas stream. A slug has then been created and due to the gas pressure this slug will rapidly start moving downstream. While it is travelling through the line the slug will grow and can easily reach a length of a couple of meters.

After passing through the bend the slug will climb upwards into the riser. This has as a consequence that the slug will induce a high hydrostatic pressure in the line and thus behind the liquid pump. It was mentioned in section 3.1 that this pump delivers a constant power and not a constant liquid flow rate. The result of the high hydrostatic pressure is then that the liquid flow rate and thus the water level will drop and consequently new slugs will not be formed at the air inlet. The slug in the riser will keep moving onwards and will ultimately go through the return line and plunge back into the storage vessel. As the slug moves into the return line it will no longer induce a hydrostatic pressure upstream. The pressure in the line will drop back to it's normal value. This will once again increase the water flow rate, heighten the liquid level in the line, increase the height of the waves at the air inlet and in this way induce a new slug.
4.1. FLOW BEHAVIOUR AND MECHANISMS

The consequence is that, for flow rates where we run into a slug flow in the line and the riser, the properties of the flow become geometry induced. We can measure the frequency of the slugging and determine where the transition to this regime is located in the flow regime map, but for modelling purposes it is not suitable. Besides, there is almost no influence of the bend as the slugs are being created in the line and move through the bend into the riser seemingly unaltered. This configuration is therefore of less concern in this project. An additional problem, when performing experiments within this flow regime, is that the liquid flow rate fluctuates heavily as was described (typically ±50 l/min) and can thus not be kept within a reasonable margin of the desired value.

As now only one situation of great interest remains. It is the situation where we have a stratified flow in the line and a slug flow in the riser. This regime occurs in the largest part of the flow regime map within the ranges of the flow rates in this facility. This situation is extremely interesting for this research project as there is a clear effect of the bend on the flow. What we have is a steady stratified flow in the line, which is being transformed in the bend into a slug flow in the riser. While these slugs are being created in the bend, a side effect is that waves are formed which travel upstream into the line. These waves induce a second slugging mechanism in the line, just before the bend. In the riser a slug flow results which will quickly develop into a developed, vertical upward, slug flow.

The main emphasis in this thesis will thus be on the situation where we have a stratified flow in the line and a slug flow in the riser. The focus will be on the two slugging mechanisms induced by the bend, which turn the horizontal stratified flow into a vertical upward slug flow. In the next subsection these mechanisms will be treated in some more detail. Then in the following section a model will be developed based on the hereafter described mechanisms.

4.1.2 Slugging mechanisms

As will be clear a major part of this research was to figure out what was actually happening within the facility. This was done in the main part by mere visual observations. In order to analyse the mechanisms by which the stratified flow in the line was turned into a slug flow in the riser, video images made with a normal video camera and pictures taken by a digital camera were used. The two slugging mechanisms will now each be separately treated.

Slugging mechanism in the bend
The mechanism by which the stratified flow is turned into a slug flow in the bend is illustrated in figure 4.1.

Consider the situation when a slug is moving upwards through the riser. This is being illustrated by the first picture in the figure. While this slug moves upwards it will shed liquid as a film at it's back. This liquid film will flow downwards along the walls of the
CHAPTER 4. MODEL DEVELOPMENT

FIGURE 4.1: Slugging mechanism in the bend.

pipe. When it flows into the bend the liquid will accumulate at the lower edge of the bend as the gas flow from the line will tend to flow through the upper section and in addition the gravity will also pull the liquid down.

It is obvious that there will be an incoming liquid flow rate into the bend originating from the line. This incoming liquid flow and the down coming liquid flow from the riser will bump into each other at the beginning of the bend. This is shown in the second picture of figure 4.1. The liquid level in the bend will rise where the two flows meet and a bump in the liquid level will result. Once this level reaches the critical height due to the Kelvin-Helmholtz instability based on the Bernoulli effect, the liquid will be "sucked up" and the bend will be slammed shut. The result is a tiny slug and as can be seen in the third picture a Taylor bubble has been caught in between the previous slug and this one. The new slug will move up into the riser and once again the situation with which we started will result.

By means of visual observation the frequency of this slugging mechanism can be estimated to be somewhere between 1.5 and 2 Hz. A whole series of images taken with the digital camera at 15 Hz is presented in Appendix B.

This slugging mechanism in the bend we just discussed can also be simply seen as the shifting of a stratified flow in a pipeline towards a slug flow as the line is being held under
4.1. FLOW BEHAVIOUR AND MECHANISMS

**Figure 4.2:** Flow regime map for a 5.1 cm diameter horizontal line under normal atmospheric conditions (Barnea, D. and Taitel, Y. (1985)).

**Figure 4.3:** Flow regime maps for several angles of inclination; identical pipe and conditions as fig. 4.2 (Barnea, D. and Taitel, Y. (1985)).
an angle with the horizontal, with upward flow as a result. This can be illustrated by five flow maps of a line under different positive angles. As we can see in figures 4.2 and 4.3, the area in which there is a stratified flow under horizontal conditions rapidly diminishes when the line is held under a positive angle to the horizon. The area is instead being occupied by the slug flow regime.

Slugging mechanism just before the bend
The slugging mechanism in the bend has a side effect. Due to the rough way in which the bend is being slammed shut when a slug is being created, waves on the air/water interface are created. These waves will move upstream into the line and will then slowly decay.

At first the water level will be relatively low in the line and nothing peculiar will occur (see figure 4.4). As water is coming into the line from the bend and of course due to the normal liquid flow from the pump, the water level will gradually rise. After some time the accumulated water will be so great that the crest of a wave reaches the critical level as described by the Kelvin-Helmholtz instability. This wave is then once again being "sucked up" towards the top of the duct and as a result the channel will be blocked and a slug created (see pictures two and three of figure 4.4). This slug will then start moving out of the line and will take the additional water which had accumulated with it and the water level will thus drop considerably, resulting in the situation at which we started off.

Through visual observations it can once again be determined that the frequency of this mechanism lays somewhere between nil and 0.5 Hz. What the liquid levels are concerned visual observations show that the instability level lays somewhere between 80 and 90 mm, whereas the level drops considerably after a slug has moved out of the line. The level then seems to be something like 40 to 50 mm. The stable liquid level of the stratified flow can
4.2 MODELLING THE MAIN MECHANISM

be seen at the beginning of the line and appears to be about 60 to 70 mm. Again a series of pictures made with the digital camera can be found in Appendix B.

The stable liquid level can be seen at the beginning of the line as observations show that this mechanism only occurs in approximately the last two meters of the line, just before the bend. The waves probably decay faster than that the water level rises and as a result the crests of the waves will not reach the critical level anymore if they have not already done so in the first two meters in which they travelled from the bend.

An overall picture has now been presented of what is happening in the facility. We have especially found two slugging mechanisms for the regime where we have a stratified flow in the line and a slug flow in the riser. In the following section a model, based on the observations made in this section, will be developed.

4.2 Modelling the main mechanism

Now that the behaviour of the facility has been presented, the next step is to turn it into a model by which the flow can be described. Ideal would be a model which exactly describes the flow the way it really is. Unfortunately though, this is generally not possible. The situation on hand in this project is way too complex to be exactly modelled. Therefore simplifications have to be made. The model must give a description of the most important features of the flow, without becoming too complex to be dealt with. The model which was developed is illustrated in figure 4.5.

The model consists of several different, already existing theories which were joined together and which will be discussed in detail in the next section. We start of with a stationary stratified smooth flow in the line with a stable liquid level. For a single horizontal channel the stable liquid level of a stationary stratified smooth flow can be calculated using the theory presented by Landman (1991).

When performing experiments the flow rates of the water and the air will always be exactly known and anyway they can under most circumstances be measured with relative ease. Therefore it is fair to consider the flow rates and thus the superficial velocities of the liquid and the gas to be known. As a result the incoming liquid flow rate can be easily calculated in the model when the superficial liquid and gas velocities are taken as input parameters which have to be set in advance.

We will take the view, that in the situation with which we start, a slug will be rising in the riser. So we will have a stratified smooth flow in the line and a slug flow in the riser. When a slug is rising in a vertical pipe, it will leak water at it’s back. This is what is called shedding. The shedding takes the form of a liquid film flowing downwards by the walls of
the pipe (that is, when the gas flow rate is not so high that the flooding point is reached). The amount of liquid that is shed by the slug and comes down into the bend and the line can be determined in two different ways. One is based on Delfos (1996) and is rather straightforward. The second method, based on Kay and Nedderman (1985), considers a turbulent film.

The calculated shedding from the slug and the known incoming liquid flow rate will heighten the level of the stratified flow in the line. The theory of the Kelvin-Helmholtz instability, based on the Bernoulli effect and treated in detail by Barnea and Taitel (1994), suggests that above a certain liquid level a stratified flow will become unstable and slug flow will result. Using this theory we can determine at which liquid level slug flow will arise in the system.

The model now basically consists of the following. We start with a stratified flow in the line and a slug flow in the riser. We can determine the stable liquid level in the line. Next, we can calculate the shedding from the slug and we already know the incoming liquid flow rate. Besides, we can use the Kelvin-Helmholtz analysis to find the liquid level at which the flow becomes unstable. Combining all this we can calculate the time necessary for the liquid level to rise from the stable level to the unstable level. Once this point is reached a slug is assumed to be instantly created. This slug will take all the additional, accumulated water with it out of the line and the situation with which we started results. The time it takes for the liquid to rise from the stable level to the unstable level can be seen as representative for the slugging frequency.
4.3 Theory on the calculation of key parameters

In the presented model the whole line is taken as the playing ground. This while it was pointed out in the previous section that this slugging mechanism only takes place in approximately the last two meters of the line just in front of the riser. As a result the frequency will be underestimated, but we cannot take only a certain part of the line into account as in doing so we would impose a frequency. On the other hand the model starts with a liquid level corresponding to a stable stratified flow. It was also pointed out in the previous section that this level lays above the level the liquid drops to just after a slug has passed. This results in an overestimation of the frequency, but again adjusting the model to start with a certain lower liquid level would impose a frequency. The theories used are independent and the model which has been developed gives a result only depending on the flow rates and the geometries of the entire facility. All that we can hope for is that the two mentioned estimation errors will reasonably cancel each other out and that the frequency found corresponds to the one measured experimentally for what the order of magnitude is concerned.

4.3.1 Stable liquid level

Taitel and Dukler (1976) derived a model for the prediction of the liquid level for horizontal and near horizontal gas/liquid flows through pipelines. Landman (1991) validated the model and gave a more thorough explanation of the results. This model is developed analogous to the general case as described in section 2.4.1.

The model starts by performing a one-dimensional momentum balance on each of the liquid and gas phases in equilibrium, so that:

\[ \frac{dp}{dx} + F_l = 0 \quad \frac{dp}{dx} + F_g = 0 \]  

(4.1)

In these relations the pressure gradient is balanced by the forces due to shear and interfacial stresses and, in case of an inclined flow, by the gravity force, i.e.:

\[ F_l = \frac{1}{A_l} \left( \tau_{w} P_l - \tau_{l} P_l + \rho_l A_l g \sin \theta \right) \]  

(4.2)
In figure 2.7 the geometrical parameters are defined and the set-up illustrated. The flow is assumed to be uniform along the line, in other words the stratified flow is assumed to be in equilibrium. As a result the interfacial level gradients are neglected. The wall shear stresses are given by equations 2.23 and 2.24. Where the friction factors can be written in the form:

\[ f_l = C_l Re_l^{-n} \quad \text{and} \quad f_g = C_g Re_g^{-m} \]  

(4.4)

Here \( Re_j (j = l, g) \) denotes the Reynolds number. This dimensionless number represents the ratio between inertia forces and friction forces and is equal to \( Re_j = \rho_j V_j D_j / \mu_j \). The Reynolds number used is based on the hydraulic diameters \( D_j \). These were introduced by equation 2.26. In this way it is as if the liquid flows through an open channel and the gas flows through a closed duct. In order to determine the friction factors, the constants \( C_j \) are defined as (see Taitel and Dukler (1976)):

\[
\begin{align*}
C_l &= C_g = 16, \quad n = m = 1 \\
C_l &= C_g = 0.046, \quad n = m = 0.2
\end{align*}
\]

for laminar flow

for turbulent flow

The assumption now made by Taitel and Dukler, which reduces the formula to a rather simple form, is that the interfacial stress is the same as that for the gas phase at the wall. In other words \( \tau_l = \tau_{wg} \). This is based on the assumption that \( V_l \ll V_g \).

The relations can be nondimensionalised by means of the diameter of the pipe and the superficial gas and liquid velocities. By eliminating the pressure drop from equation 4.1 and substituting equations 4.2 and 4.3, the Taitel Dukler hold-up relation can be found:

\[ \alpha X^2 - \beta - 4Y = 0 \]

(4.5)

This relation gives the solutions for the liquid level in a stratified flow. \( X \) is the Lockhart-Martinelli parameter introduced by equation 2.34 and \( Y \) is the Taitel Dukler inclination parameter given by equation 2.35. Furthermore, \( \alpha \) and \( \beta \) are dimensionless geometrical parameters:

\[
\alpha = (\bar{V}_l \bar{D}_l)^{-n} \bar{V}_l^2 \frac{\bar{P}_l}{A_l}
\]

(4.6)

\[
\beta = (\bar{V}_g \bar{D}_g)^{-m} \bar{V}_g^2 \left[ \frac{\bar{P}_g}{A_g} + \frac{\bar{P}_l}{A_l} + \frac{\bar{P}_g}{A_g} \right]
\]

(4.7)
4.3. THEORY ON THE CALCULATION OF KEY PARAMETERS

These geometrical parameters are explicit functions of the liquid height, which in turn is a function of the hold-up.

The solutions of the Taitel-Dukler hold-up relation are displayed in figure 4.6. The liquid height is shown as a function of the logarithm of the Lockhart-Martinelli parameter for various values of the Taitel-Dukler inclination parameter. For the horizontal case this inclination parameter equals nil. The solution is especially interesting in the case of upward flow corresponding to negative values of the inclination parameter. For certain inclinations up to three solutions for the liquid level are possible. Landman suggests that under certain circumstances hysteresis between the solutions might occur. The operating point then depends on the history of the gas/liquid flow rates. Barnea and Taitel (1994) suggested that the thinnest solution is structurally stable, the middle solution is linearly unstable and the thickest solution is unstable to finite disturbances. Here we will only consider the thinnest solution to be stable. This is the solution with which we will work in the rest of the model. This is also the solution most in agreement to what was observed during the experiments, namely that the liquid level drops below the stable liquid level after a slug has passed. In the present work the line has not been placed under an angle. For the horizontal case that was studied, the inclination parameter is nil and as can be seen in the figure there is only one solution in this case.
4.3.2 Instability level of the liquid

An overview on the stability of separated flow is given by Barnea and Taitel (1994). The stability of separated flow is rather complex and can be approached in several ways: linear vs non-linear, inviscid vs viscous and one-dimensional vs two and multi-dimensional analysis.

Due to interfacial instability waves will be created on the interface of the stratified flow. These waves can either lead to a wavy interface or to conditions where the waves reach the top of the channel and cause transition from stratified flow. Usually, it is conceded that instability leading to the growth of short waves will result in pebbly flow, since short waves tend to saturate quickly. The only effect will be the roughening of the interface. Long-wave instability, on the other hand, is associated with the transition from stratified flow to slug or annular flow.

The instability responsible for the transition from stratified flow is usually attributed to the Kelvin-Helmholtz instability which in turn is attributed to the Bernoulli effect. In other words to the decrease in the pressure over the wave crest due to the velocity acceleration. This effect acts against the stabilising effect of gravity. An exact three-dimensional analysis of the Kelvin-Helmholtz instability is extremely difficult. Furthermore, we are interested in the long waves in a straight pipe. A long wave, one-dimensional analysis is well-suited for describing the instability.

Barnea and Taitel have shown by means of simulations that the non-linear analysis usually confirms the results obtained by the linear-analysis. Only for downward flow a slight deviation between the two was observed. For the purpose of the present work the use of the linear analysis therefore seems to be justified. In the remaining part of this section we will develop the one-dimensional, long wavelength, linear Kelvin-Helmholtz analysis. There will still be two alternative approaches, namely the inviscid (IKH) and the viscous Kelvin-Helmholtz (VKH) analysis. In the inviscid case the shear stresses are neglected. In both cases the steady equilibrium liquid level is obtained by the theory treated in the previous section which does take the shear stresses into account.

The basic theory will be developed using the viscous analysis. The continuity equations are:

$$\frac{\partial}{\partial t}(\rho_l A_l) + \frac{\partial}{\partial x}(\rho_l A_l V_l) = 0 \quad (4.8)$$

and,

$$\frac{\partial}{\partial t}(\rho_g A_g) + \frac{\partial}{\partial x}(\rho_g A_g V_g) = 0 \quad (4.9)$$
4.3. THEORY ON THE CALCULATION OF KEY PARAMETERS

The momentum equations for each phase are:

\[ \frac{\partial}{\partial t}(\rho_l A_l V_l) + \frac{\partial}{\partial x}(\rho_l A_l V_l^2) = -\tau_{wl} P_l + \tau_{l} P_l - A_l \frac{\partial p_l}{\partial x} + \rho_l A_l g \cos \theta \frac{\partial h_l}{\partial x} - \rho_l A_l g \sin \theta \]  \tag{4.10}

and,

\[ \frac{\partial}{\partial t}(\rho_g A_g V_g) + \frac{\partial}{\partial x}(\rho_g A_g V_g^2) = -\tau_{wg} P_g - \tau_{l} P_l - A_g \frac{\partial p_g}{\partial x} + \rho_g A_g g \cos \theta \frac{\partial h_g}{\partial x} - \rho_g A_g g \sin \theta \]  \tag{4.11}

In these equations \( p_g \) and \( p_l \) are the pressures in each phase. Considering the flow to be incompressible the two momentum equations can be combined. The pressure terms can be eliminated using the approximate relation:

\[ p_g - p_l = \sigma \frac{\partial^2 h_l}{\partial x^2} \]  \tag{4.12}

Where \( \sigma \) denotes the surface tension. This yields the following equations:

\[ \frac{\partial h_l}{\partial t} + \frac{A_l}{A_l'} \frac{\partial V_l}{\partial x} + V_l \frac{\partial h_l}{\partial x} = 0 \]  \tag{4.13}

\[ \frac{\partial h_g}{\partial t} - \frac{A_g}{A_g'} \frac{\partial V_g}{\partial x} + V_g \frac{\partial h_l}{\partial x} = 0 \]  \tag{4.14}

and,

\[ \rho_l \frac{\partial V_l}{\partial t} - \rho_g \frac{\partial V_g}{\partial t} + \rho_l V_l \frac{\partial V_l}{\partial x} - \rho_g V_g \frac{\partial V_g}{\partial x} + (\rho_l - \rho_g) g \cos \theta \frac{\partial h_l}{\partial x} - \sigma \frac{\partial^3 h_l}{\partial x^3} = F \]  \tag{4.15}

where,

\[ F = -\frac{\tau_{wl} P_l}{A_l} + \frac{\tau_{wg} P_g}{A_g} + \tau_{l} P_l \left( \frac{1}{A_l} + \frac{1}{A_g} \right) - (\rho_l - \rho_g) g \sin \theta \]  \tag{4.16}

and \( A'_l \) is equal to \( dA_l/dh_l \).

The steady state solutions follow by setting \( F = 0 \). Equations 4.13 until 4.15 can now be linearised about the steady state solution. A perturbed liquid level can then be substituted into the linearised equations:

\[ \dot{h}_l = \epsilon \exp[i(\omega t - kx)] \]  \tag{4.17}
Where $e$ is the amplitude and $k$ the wavenumber of the perturbation. This yields the following dispersion relation for the angular frequency $\omega$:

$$
\omega^2 - 2(ak - bi)\omega + ch^2 - dk^4 - eki = 0
$$

(4.18)

where,

$$
a = \frac{1}{\rho} \left( \frac{\rho_l V_l}{\alpha_l} + \frac{\rho_g V_g}{\alpha_g} \right) 
$$

(4.19)

$$
b = \frac{1}{2\rho} \left[ \left( \frac{\partial F}{\partial V_{ls}} \right)_{v_{ls,\alpha_l}} - \left( \frac{\partial F}{\partial V_{gs}} \right)_{v_{ls,\alpha_l}} \right]
$$

(4.20)

Here the subscripts denote that the terms in the subscript have to be kept constant. In other words, the term with the subscript has to be determined for the steady state. It is not seen as a locally varying property. Furthermore,

$$
c = \frac{1}{\rho} \left[ \frac{\rho_l V_l^2}{\alpha_l} + \frac{\rho_g V_g^2}{\alpha_g} - (\rho_l - \rho_g)g \cos \theta \frac{A}{A_l} \right]
$$

(4.21)

$$
d = \frac{\sigma}{\rho} \frac{A}{A_l}
$$

(4.22)

$$
e = -\frac{1}{\rho} \left( \frac{\partial F}{\partial \alpha_i} \right)_{v_{ls},V_{gs}}
$$

(4.23)

$$
\rho = \frac{\rho_l}{\alpha_l} + \frac{\rho_g}{\alpha_g}
$$

(4.24)

All the variables in these equations are evaluated at steady-state conditions.

The solution for $\omega$ is:

$$
\omega = (ak - bi) \pm \sqrt{(a^2 - c)k^2 - b^2 + d^2 + (ek - 2abk)i}
$$

(4.25)

This is the solution for the viscous case. The steady-state solution will become unstable whenever the imaginary part of the complex number $\omega$ becomes negative. This will lead to exponential growth of the perturbed liquid level. The imaginary part $-\omega_i$ is called the amplification factor. The solution for the inviscid case is ($b=e=0$):

$$
C = \frac{\omega}{k} = \frac{\frac{\rho_l V_l}{H_l} + \frac{\rho_g V_g}{H_g}}{\frac{\rho_l}{H_l} + \frac{\rho_g}{H_g}} \pm \sqrt{\frac{(\rho_l - \rho_g)g \cos \theta}{\frac{\rho_l}{H_l} + \frac{\rho_g}{H_g}} - \frac{\rho_l \rho_g (V_g - V_l)^2}{\left(\frac{\rho_l}{H_l} + \frac{\rho_g}{H_g}\right)^2} + \frac{\sigma k^2}{\frac{\rho_l}{H_l} + \frac{\rho_g}{H_g}}}
$$

(4.26)
Here $H_l = A_l/A'_l$, $H_g = A_g/A'_g$ ($A'_g = dA_g/dH_g$) and $C$ denotes the wave velocity. The amplification factor will in this case remain nil as long as the term under the square root is positive. When it becomes negative two conjugate solutions for the imaginary part will exist. The one with the negative sign then contributes to the instability.

The situation when the imaginary part of $\omega$ is nil is said to be the condition of marginal stability. For the viscous case this gives the following stability criterion:

$$\left( \frac{e}{2b} - a \right)^2 - (a^2 - c) - dk^2 < 0$$  \hspace{1cm} (4.27)

Substituting equations 4.19 until 4.24 gives:

$$\left( C_V - C_{IV} \right)^2 + \frac{\rho_l \rho_g}{\rho^2 \alpha_l \alpha_g} (V_g - V_l)^2 - \frac{\rho_l - \rho_g}{\rho} g \cos \theta \frac{A}{A'_l} - \frac{\sigma}{\rho} \frac{A}{A'_l} k^2 < 0$$  \hspace{1cm} (4.28)

For the inviscid case the stability criterion is given by the last three terms, as the first term becomes equal to nil. The first term is the additional effect of the shear stresses. The fourth term is the contribution of the surface tension and is the only term depending on the wavelength. As was discussed previously we are looking for the stability criterion for long wavelengths. In this case the last term will be neglected as it does not affect the neutral stability criterion for long wavelengths. $C_V$ is the critical wave velocity on the inception of instability:

$$C_V = \frac{\rho}{2b} \frac{\left( \frac{\partial F}{\partial \alpha_l} \right)_{V_l, V_{*g}}}{\left[ \left( \frac{\partial F}{\partial V_{*g}} \right)_{V_{*l}, \alpha_l} - \left( \frac{\partial F}{\partial V_{*l}} \right)_{V_{*g}, \alpha_l} \right]}$$  \hspace{1cm} (4.29)

And $C_{IV}$ is the critical wave velocity for the inviscid analysis:

$$C_{IV} = \frac{\rho_l V_l \alpha_g + \rho_g V_g \alpha_l}{\rho_l \alpha_g + \rho_g \alpha_l}$$  \hspace{1cm} (4.30)

The shear stresses from the wall on the gas and the wall on the liquid can be evaluated using equations 2.23, 2.24 and 4.4. The hydraulic diameters were given in equation 2.26. For the constants in the terms for the friction factors the same is valid as was mentioned in relation to equation 4.4. For the frictional stress at the interface the following relation can be used:

$$\tau_i = f_i \rho_g (V_g - V_l)^2$$  \hspace{1cm} (4.31)

The interfacial friction factor can be assumed to have a constant value of 0.014. Only when $f_g > 0.014$ it is better to take $f_i = f_g$. 
The Kelvin-Helmholtz neutral stability criterion can be rearranged to give:

\[ (V_g - V_l) < K \sqrt{\left( \rho_l \alpha_g + \rho_g \alpha_l \right) \frac{\rho_l - \rho_g}{\rho_l \rho_g} g \cos \theta \frac{A}{(dA_I/dh_I)}} \]  \hspace{1cm} (4.32)

Using equations 2.28 and 2.29 it is not difficult to find that:

\[ \frac{dA_I}{dh_I} = \frac{D}{2 \sin(\gamma/2)} (1 - \cos \gamma) \]  \hspace{1cm} (4.33)

This relation is only valid for \( h_I / D < 0.5 \), otherwise the same relation can be used with \( D - h_I \) as liquid height. For the top angle \( \gamma \) we can write:

\[ \gamma = 2 \arccos \left( \frac{2h_I}{D} \right) \]  \hspace{1cm} (4.34)

For the inviscid case \( K = 1 \) and for the viscous case \( K = K_v \):

\[ K_v = \sqrt{1 - \frac{(C_V - C_{IV})^2}{\rho_l - \rho_g g \cos \theta \frac{A}{(dA_I/dh_I)}}} \]  \hspace{1cm} (4.35)

Taitel and Dukler (1976) used a simplified inviscid analysis and suggested to use a speculative factor:

\[ K = K_{TD} = 1 - \frac{h_I}{D} \]  \hspace{1cm} (4.36)

Especially at low viscosities this factor successfully accounts for the viscous effects. Figure 4.7 compares \( K_{TD} \) with \( K_V \) for several viscosities. In the case of water (\( \mu = 1 \text{ cP} \)) the Taitel and Dukler factor is almost identical to \( K_V \). Therefore the speculative factor is used in the model.

As can be seen in figure 4.7 the viscous factor \( K_v \) approaches unity for high viscosities. The consequence is that the viscous case approaches the inviscid case as the viscosity becomes higher. This is rather surprising as one would expect the opposite. The same can be seen in the speculative factor. This factor is obviously smaller then unity. As a result the neutral stability is reached for lower liquid and gas velocities in the viscous case than in the inviscid case. Again this is opposite to what one would expect. In order to understand this, two effects have to be taken into consideration, first the surface tension and second the shear stresses introduced into the system by the viscosity. The surface tension tends to stabilise the flow by reducing disturbances on the interface. As was previously pointed out we are only interested in the long wavelength disturbances. For long wavelengths the influence of the surface tension is greatly reduced. As the factor containing the surface tension was inversely proportional to the wavelength squared, it was neglected. The second
4.3. THEORY ON THE CALCULATION OF KEY PARAMETERS

FIGURE 4.7: Taitel and Dukler speculative factor.

...effect, which is introduction into the system by the viscosity, is the introduction of shear stresses. These shear stresses tend to amplify any disturbance on the interface. This is the reason for the seemingly abnormal observations.

We have seen that the instability is dictated by the complex part of $\omega$. Once this amplification factor becomes negative, the interface will become unstable and a transition to another flow regime might occur. For a superficial gas velocity of 5 m/s the amplification factor has been plotted against the superficial liquid velocity in figure 4.8. It can be seen that at point a the amplification factor becomes negative for the viscous analysis, while it becomes negative according to the inviscid analysis in point b. Between these two points the factor of amplification is rather small, only as point b is reached does the amplification factor rise rapidly.

Figure 4.9 presents the results for the neutral stability criterion for both the viscous and the inviscid analysis. Below the stability lines it is obvious that a stratified smooth flow will be present. The interface will remain stable. On the other hand above the stability lines the flow will be unstable and a transition to another flow regime will occur. As the waves grow rapidly they will reach to top of the channel and create slugs. The result is a slug flow. Only in case of a too low liquid level this will not be possible as there will not be enough liquid present to create waves high enough to reach the top of the channel. In this case an annular flow will result from the transition. Barnea and Taitel assumed the critical liquid level below which slug flow cannot be created to be half the pipe diameter, although they do not present any justification for this value.
CHAPTER 4. MODEL DEVELOPMENT

**Figure 4.8:** Amplification factor as a function of the superficial liquid velocity (here $\omega_{12}$ denotes the amplification factor).

**Figure 4.9:** Flow pattern prediction by VKH and IKH analysis.
4.3. THEORY ON THE CALCULATION OF KEY PARAMETERS

The most difficult part to interpret is the area between the two stability curves. As we have seen the amplification factor is very low in this area. Waves will thus not grow extremely fast. It is now assumed that for low liquid levels (again without any justification said to be below half a diameter) transition will not occur. The waves will not grow high enough to create slug flow and roll waves will exist. In case the liquid flow rate is high enough the waves will be able to reach the top of the duct and create slug flow.

4.3.3 Shedding behind a slug

The calculation of the shedding behind a slug rising in the riser has been done in two different ways. First a rather straightforward approach will be treated. This is based on some considerations made by Delfos (1996). Next, an approach by Dukler and Bergelin, which can be found in Kay and Nedderman (1985), will be discussed.

Delfos approach

A relatively thin film in a round tube can be transformed into a two-dimensional flow by opening the tube and flattening it. The result is a liquid film flowing down a wall with width $\pi D$. The spanwise direction can be further ignored as it can be regarded to be free of dynamics. The flow around a Taylor bubble is illustrated in figure 4.10.

As we have seen in chapter two, the mixture velocity is equal to the sum of the superficial gas and liquid velocities. At the cross-section $yy'$ the mixture velocity is equal to $V_m =$

![Flow around a Taylor bubble](figure4.10.png)

**Figure 4.10:** Flow around a Taylor bubble.
For the rising velocity of the Taylor bubble we can use the Nicklin relation as given by equation 2.36. The amount of gas flowing through the cross-section has to remain constant, thus:

\[
\frac{\pi D^2}{4} V'_{gs} = \frac{\pi (D - 2\delta_l)^2}{4} V_{TB}
\]  

(4.37)

Where \(\delta_l\) is the film thickness. Using the mentioned equations it follows that:

\[
V'_{is} = \left(\frac{D - 2\delta_l}{D}\right)^2 V_{TB} - V_{m}
\]

(4.38)

The down coming liquid flow equals:

\[
Q_l = \pi D^2 \frac{V'_{is}}{4}
\]

(4.39)

Instationary waves on the film surface are neglected. The liquid flow rate will then remain constant through every cross-section, so per unit width \(Q_f = V_f \delta_l\). And so:

\[
\delta_l(z) = \frac{Q_f}{V_f(z)} = \frac{Q_l / \pi D}{V_f(z)}
\]

(4.40)

This results in:

\[
\delta_l(z) = \frac{V'_{is} D}{4V_f(z)}
\]

(4.41)

After some time a balance will appear between the gravity and the friction and so an equilibrium film velocity \(V_{f\infty}\) and film thickness \(\delta_{l\infty}\) will arise. A relation will now have to be found for the equilibrium film thickness.

A smooth film flowing down the walls of a riser will soon become unstable and ripples will arise. A stable wave pattern can be formed on the film surface. For high Reynolds numbers the waves will no longer grow linearly and can acquire an amplitude a few times the film thickness. The surface tension will then start playing a role. The velocity of the larger waves will become higher than that of the film due to the effect of gravity. A wavy surface generates a higher liquid flow rate than a smooth film. As we will be dealing with a turbulent film, the parabolic velocity profile of a laminar film will not be applicable.

For the average velocity of a turbulent film we will have to make use of an empirical correlation. Delfos compared several correlations and found that a correlation by Brötz (1954) was most satisfactory. It fitted well with experimental data, is the simplest and is probably sufficiently accurate. The Brötz correlation is valid for Reynolds numbers for the film (based on the equilibrium film velocity and thickness) between 100 and 4300 and is written in the form:

\[
V_{f\infty} = \frac{3}{\sqrt[3]{590}} \frac{g Q_f}{3}
\]

(4.42)
4.3. THEORY ON THE CALCULATION OF KEY PARAMETERS

As \( Q_f = V_{f\infty} \delta_{f\infty} \), this can be written as:

\[
V_{f\infty} = \sqrt{\frac{590}{3} g \delta_{f\infty}} \tag{4.43}
\]

We now have a closed set of equations from which the equilibrium film thickness can be determined.

The shedding behind a slug will be equal to:

\[
Q_{sh} = \left( \frac{\pi D^2}{4} - \frac{\pi (D - 2\delta_{f\infty})^2}{4} \right) V_{f\infty} \tag{4.44}
\]

This can be reduced to:

\[
Q_{sh} = \pi \delta_{f\infty} (D - \delta_{f\infty}) V_{f\infty} \tag{4.45}
\]

Dukler and Bergelin approach

An alternative to the previous method is the approach first proposed by Dukler and Bergelin. This method immediately starts by regarding the film to be turbulent. In the absence of a comprehensive theory of turbulence which would permit the calculation of the Reynolds stresses and the corresponding heat and mass transfer quantities, one of the analogies based on the mixing length theory has to be used. The Von Karman extension of the Prandtl-Taylor analogy is one of those and will be used here. It is illustrated in figure 4.11.

This theory states that a turbulent film consists of three different layers. The first layer is the one adjacent to the wall and is called the viscous layer. Here kinetic energy is transferred by means of molecular transfer processes. In this layer the flow is almost laminar,

\[
\begin{align*}
\text{core} & \quad \bar{V} = 5.5 + 2.5 \ln \bar{y} & \text{eddy transfer} \\
\bar{y} = 30 & \\
\text{buffer} & \quad \bar{V} = -3.05 + 5.0 \ln \bar{y} & \text{molecular & eddy transfer} \\
\bar{y} = 5 & \\
\text{viscous} & \quad \bar{V} = \bar{y} & \text{molecular transfer} \\
\bar{y} = 0
\end{align*}
\]

Figure 4.11: Von Karman’s extension of the Prandtl-Taylor analogy.
as viscous effects dominate the flow. The second layer, called the buffer layer, is in effect a mixture of the first and third layer. In this layer molecular and eddy transfer processes both occur simultaneously. The third layer consists of the really turbulent flow. It is called the core layer and here only eddy transfer of kinetic energy is found.

Each layer in the film will have a different velocity profile as can be seen in figure 4.11. Here $\tilde{V}$ is the dimensionless velocity and $\tilde{y}$ the dimensionless height. With $\tau_w$ as the wall shear stress and $V_\ast = \sqrt{\tau_w/\rho}$ as the frictional velocity, they have the form:

$$\tilde{u} = \frac{V}{V_\ast} \quad \tilde{y} = y\frac{V_\ast}{\nu}$$

(4.46)

Where, $\nu$ denotes the kinematic viscosity (equal to the dynamic viscosity divided by the density).

The dimensionless liquid flow rate per unit width can now be calculated by integrating the velocity profiles over the film thickness as follows:

$$\tilde{Q} = \int_0^\eta \tilde{V} d\tilde{y} \quad \eta = \frac{\delta \sqrt{g \delta_t}}{\nu}$$

(4.47)

In this relation $\eta$ denotes the dimensionless film thickness. It follows that:

$$\tilde{Q} = \int_0^5 \tilde{y} d\tilde{y} + \int_5^{30} (5 \ln \tilde{y} - 3.05)d\tilde{y} + \int_{30}^\eta (2.5 \ln \tilde{y} + 5.5)d\tilde{y}$$

(4.48)

The Reynolds number based on the film thickness and the superficial liquid velocity at a cross-section through the Taylor bubble can be expressed as a function of the dimensionless liquid flow rate:

$$\frac{Re}{4} = \frac{\tilde{Q}}{\nu}$$

(4.49)

Making use of this relation, the solution to equation 4.48 can be written as:

$$\frac{Re}{4} = 2.5\eta \ln \eta + 3\eta - 64$$

(4.50)

From this equation the film thickness can now be determined. Now that the film thickness is known the relations 4.43 and 4.45, used in the Delfos approach, can further be used to calculate the shedding.
4.4 MODELLING RESULTS

4.4 Modelling results

A model describing the flow in the facility has been developed and some theories concerning the calculation of key parameters were treated. The model has been simulated by means of a code written in FORTRAN 77. It deals with an air/water flow in a facility with the geometry of the experimental set-up. The code requires a superficial gas and a superficial liquid velocity as input. Furthermore, an angle of the line to the horizontal has to be given. In this work only the horizontal situation has been considered, but the angle has been incorporated in the code in view of future research. For a combination of input parameters the code returns the slugging frequency, the stable liquid level, the instability level of the liquid, the entering liquid flow rate, the shedding rate, the equilibrium film thickness and the equilibrium film velocity. The code is displayed in appendix D.

The code was written for an air/water system as that is what was also experimentally studied. In essence the code can also be used for other fluids by simply changing the fluid properties in the code. The only problem which will then have to be overcome is that the code makes use of the Taitel-Dukler speculative factor for the calculation of the instable liquid level. This factor is only valid for water (see figure 4.7). In order to make the code valid for other liquids the normal factor given in equation 4.29 has to be entered into the code. A problem will be the calculation of the critical wave velocity on the inception of instability.

As was discussed in the previous section it is possible that there are two or even three solutions for the stable liquid level in a stratified flow. It was also pointed out that only the thinnest solution will be stable. The code gives all the solutions, but continues the calculations with the lowest one. For the horizontal case it was shown that there was only one solution to the equations. Some solutions for the stable liquid level can be found in tables 4.1 until 4.4. As can be seen the solutions vary between 40 mm for low liquid flow rates to 80 mm for high liquid flow rates. Most solutions lie in the neighbourhood of 60 to 70 mm, fully in agreement with the visually observed values.

It was discussed in the previous section that there were two ways to come to a solution for the instability level of the liquid. These are the viscous and the inviscid analysis. The stability line for the viscous case was found to lie at lower flow rates than the stability line for the inviscid case. Beneath these two lines a stratified smooth flow will be present and above the lines a slug flow (or annular flow in case the instability level was found for liquid levels below half a diameter). Between these two lines Barnea and Taitel considered a slug flow to be present for liquid levels above half a diameter and stratified wavy flow for lower liquid levels. This means that in the case under study here the instability level would have to be given by the viscous solution, as the stable liquid levels are already rather high.

Some values for the instability found by the inviscid analysis can be found in tables 4.1 until 4.4. On the other hand some levels found by means of the viscid analysis are displayed in table 4.5. It is striking to see that the levels found through the inviscid analysis give
CHAPTER 4. MODEL DEVELOPMENT

**Table 4.1:** Modelling results obtained by means of IKH stability analysis and the Dukler and Bergelin film thickness calculation \((h_{crit}\) denotes the instability level).  

<table>
<thead>
<tr>
<th>(V_{is}) (m/s)</th>
<th>(h_{l}) ((10^{-3} \text{ m}))</th>
<th>(h_{crit}) ((10^{-3} \text{ m}))</th>
<th>(Q_{\text{inlet}}) ((10^{-3} \text{ m}^3/\text{s}))</th>
<th>(Q_{\text{sh}}) ((10^{-3} \text{ m}^3/\text{s}))</th>
<th>(\delta_{l}) ((10^{-3} \text{ m}))</th>
<th>(V_{\infty}) (m/s)</th>
<th>(Re_f) (-)</th>
<th>(f) (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>50</td>
<td>85</td>
<td>0.4</td>
<td>0.1</td>
<td>0.4</td>
<td>0.9</td>
<td>360</td>
<td>0.02</td>
</tr>
<tr>
<td>0.10</td>
<td>60</td>
<td>85</td>
<td>0.8</td>
<td>0.1</td>
<td>0.4</td>
<td>0.9</td>
<td>360</td>
<td>0.05</td>
</tr>
<tr>
<td>0.15</td>
<td>65</td>
<td>85</td>
<td>1.2</td>
<td>0.1</td>
<td>0.4</td>
<td>0.9</td>
<td>360</td>
<td>0.10</td>
</tr>
<tr>
<td>0.20</td>
<td>70</td>
<td>85</td>
<td>1.6</td>
<td>0.1</td>
<td>0.4</td>
<td>0.9</td>
<td>360</td>
<td>0.15</td>
</tr>
<tr>
<td>0.25</td>
<td>70</td>
<td>85</td>
<td>2.0</td>
<td>0.1</td>
<td>0.4</td>
<td>0.9</td>
<td>360</td>
<td>0.20</td>
</tr>
<tr>
<td>0.30</td>
<td>70</td>
<td>85</td>
<td>2.4</td>
<td>0.1</td>
<td>0.4</td>
<td>0.9</td>
<td>360</td>
<td>0.30</td>
</tr>
<tr>
<td>0.35</td>
<td>75</td>
<td>85</td>
<td>2.7</td>
<td>0.1</td>
<td>0.4</td>
<td>0.9</td>
<td>360</td>
<td>0.45</td>
</tr>
</tbody>
</table>

**Table 4.2:** Modelling results obtained by means of IKH stability analysis and the Dukler and Bergelin film thickness calculation \((h_{crit}\) denotes the instability level).  

<table>
<thead>
<tr>
<th>(V_{is}) (m/s)</th>
<th>(h_{l}) ((10^{-3} \text{ m}))</th>
<th>(h_{crit}) ((10^{-3} \text{ m}))</th>
<th>(Q_{\text{inlet}}) ((10^{-3} \text{ m}^3/\text{s}))</th>
<th>(Q_{\text{sh}}) ((10^{-3} \text{ m}^3/\text{s}))</th>
<th>(\delta_{l}) ((10^{-3} \text{ m}))</th>
<th>(V_{\infty}) (m/s)</th>
<th>(Re_f) (-)</th>
<th>(f) (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>40</td>
<td>75</td>
<td>0.4</td>
<td>0.2</td>
<td>0.5</td>
<td>1.0</td>
<td>500</td>
<td>0.02</td>
</tr>
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<td>0.10</td>
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<td>1.0</td>
<td>500</td>
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</tr>
<tr>
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<td>0.2</td>
<td>0.5</td>
<td>1.0</td>
<td>500</td>
<td>0.10</td>
</tr>
<tr>
<td>0.25</td>
<td>60</td>
<td>75</td>
<td>2.0</td>
<td>0.2</td>
<td>0.5</td>
<td>1.0</td>
<td>500</td>
<td>0.20</td>
</tr>
<tr>
<td>0.30</td>
<td>65</td>
<td>75</td>
<td>2.4</td>
<td>0.2</td>
<td>0.5</td>
<td>1.0</td>
<td>500</td>
<td>0.25</td>
</tr>
<tr>
<td>0.35</td>
<td>65</td>
<td>75</td>
<td>2.7</td>
<td>0.2</td>
<td>0.5</td>
<td>1.0</td>
<td>500</td>
<td>0.35</td>
</tr>
</tbody>
</table>
### 4.4. MODELLING RESULTS

**Table 4.3:** Modelling results obtained by means of IKH stability analysis and the Dukler and Bergelin film thickness calculation ($h_{crit}$ denotes the instability level).

<table>
<thead>
<tr>
<th>$V_{gs}$ (m/s)</th>
<th>$h_{t}$ (10^-3 m)</th>
<th>$h_{crit}$ (10^-3 m)</th>
<th>$Q_{inlet}$ (10^-3 m^3/s)</th>
<th>$Q_{sh}$ (10^-3 m^3/s)</th>
<th>$\delta_t$ (10^-3 m)</th>
<th>$V_{f\infty}$ (m/s)</th>
<th>$Re_f$ (\text{-})</th>
<th>$f$ (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>75</td>
<td>90</td>
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<td>0.4</td>
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<tr>
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<td>0.4</td>
<td>0.9</td>
<td>360</td>
<td>0.09</td>
</tr>
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<td>0.2</td>
<td>0.6</td>
<td>1.1</td>
<td>660</td>
<td>0.07</td>
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<tr>
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<td>45</td>
<td>70</td>
<td>1.2</td>
<td>0.2</td>
<td>0.7</td>
<td>1.1</td>
<td>660</td>
<td>0.07</td>
</tr>
</tbody>
</table>

**Table 4.4:** Modelling results obtained by means of IKH stability analysis and the Dukler and Bergelin film thickness calculation ($h_{crit}$ denotes the instability level).

<table>
<thead>
<tr>
<th>$V_{gs}$ (m/s)</th>
<th>$h_{t}$ (10^-3 m)</th>
<th>$h_{crit}$ (10^-3 m)</th>
<th>$Q_{inlet}$ (10^-3 m^3/s)</th>
<th>$Q_{sh}$ (10^-3 m^3/s)</th>
<th>$\delta_t$ (10^-3 m)</th>
<th>$V_{f\infty}$ (m/s)</th>
<th>$Re_f$ (\text{-})</th>
<th>$f$ (Hz)</th>
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</thead>
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<tr>
<td>0.5</td>
<td>80</td>
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<td>2.4</td>
<td>0.1</td>
<td>0.4</td>
<td>0.9</td>
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</tr>
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<td>85</td>
<td>2.4</td>
<td>0.1</td>
<td>0.4</td>
<td>0.9</td>
<td>360</td>
<td>0.30</td>
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<tr>
<td>1.5</td>
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<td>80</td>
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<td>0.1</td>
<td>0.5</td>
<td>1.0</td>
<td>500</td>
<td>0.30</td>
</tr>
<tr>
<td>2.0</td>
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<td>75</td>
<td>2.4</td>
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<td>0.5</td>
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<td>0.6</td>
<td>1.1</td>
<td>660</td>
<td>0.25</td>
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<tr>
<td>3.0</td>
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<td>70</td>
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<td>0.2</td>
<td>0.6</td>
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<td>0.7</td>
<td>1.2</td>
<td>840</td>
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</table>
rather good results, whereas the levels given by the viscous analysis are not all that good. In the viscous case the solutions for high liquid flow rates are lower than those found for the stable liquid level. Of course this is erroneous. For slightly lower liquid flow rates the instability level lies so close to the stable level that this results in high frequencies. These high slugging frequencies have never been observed and seem rather unlikely to occur. For low liquid flow rates the viscous method gives normal results, but the theory suggest that especially for low flow rates (and thus low liquid levels) it is more likely that the inviscid case gives the correct solutions. The solutions to the instability level of the liquid found by means of the inviscid analysis give very reasonable results. The values lie between 70 and 90 mm which seems reasonable compared to the visually observed 80 to 90 mm.

<table>
<thead>
<tr>
<th>$V_{ls}$ (m/s)</th>
<th>$V_{gs} = 2.0$ m/s, $\theta = 0^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$h_{l}$</td>
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</tr>
<tr>
<td>0.30</td>
<td>65.0</td>
</tr>
<tr>
<td>0.35</td>
<td>67.5</td>
</tr>
</tbody>
</table>

In the area between the viscous and the inviscid stability lines the amplification factor was shown to be very low. For this reason waves created with a low liquid level were considered not to grow fast enough to reach the top of the channel and create slugs. The perturbation will have moved out of the system before the instability level is reached. On the other hand slugs would be formed if the liquid level was sufficiently high. Barnea and Taitel assumed that the intersection between these to situations lies at a liquid level of half a pipe diameter, although they did not give any justification for this. As transitions to slug flow have not been visually seen to occur at the low levels the viscous analysis suggests, the results given by the inviscid analysis have further been considered to be the correct ones. As mentioned these are shown in tables 4.1 until 4.4.

In the previous section it was shown that two different methods to calculate the film thickness and thus the shedding behind a slug were possible. The results found by means of the
Dukler and Bergelin approach are shown in tables 4.1 until 4.4. The results found through the Delfos approach on the other hand are shown in tables 4.6 until 4.9. Both methods give fine results for as far as the slugging frequencies are concerned. With the Delfos approach the frequencies found are slightly higher than with the Dukler and Bergelin approach. This is due to the higher shedding rate in the Delfos results. This is a result of the thicker film and the higher superficial film velocity found. To be able to compare the resulting film thicknesses from the two models we can make use of some experimental data obtained by Karapantsios (1989) and (1995) and Takahama et al. (1980). These are displayed in figure 4.12.

Comparing the results found for the film thicknesses with their Reynolds numbers it is easily seen that the results found by the means of the Dukler and Bergelin approach reasonably correspond to the experimental data shown. On the other hand the film thicknesses given by the Delfos approach are too large compared to their Reynolds numbers. Furthermore, a problem arises with the Reynolds numbers of the Delfos approach. For the equilibrium film velocity the empirical relation found by Brötz was used. As was mentioned this correlation is valid for Reynolds numbers within the range of 100 to 4300. The numbers of the Delfos case amply exceed these values. The use of this relation is therefore not allowed in this case. As a result the solutions given by the Dukler and Bergelin approach should be seen as the better ones.
### CHAPTER 4. MODEL DEVELOPMENT

#### Table 4.6: Modelling results obtained by means of IKH stability analysis and the Delfos film thickness calculation ($h_{crit}$ denotes the instability level).

<table>
<thead>
<tr>
<th>$V_{ls}$ $(m/s)$</th>
<th>$Q_{sh}$ $(10^{-3} m^3/s)$</th>
<th>$\delta_l$ $(10^{-3} m)$</th>
<th>$V_{f\infty}$ $(m/s)$</th>
<th>$Re_f$</th>
<th>$f$ $(Hz)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>2.6</td>
<td>3.4</td>
<td>2.6</td>
<td>8840</td>
<td>0.10</td>
</tr>
<tr>
<td>0.10</td>
<td>2.6</td>
<td>3.4</td>
<td>2.6</td>
<td>8840</td>
<td>0.15</td>
</tr>
<tr>
<td>0.15</td>
<td>2.6</td>
<td>3.4</td>
<td>2.6</td>
<td>8840</td>
<td>0.25</td>
</tr>
<tr>
<td>0.20</td>
<td>2.7</td>
<td>3.4</td>
<td>2.6</td>
<td>8840</td>
<td>0.35</td>
</tr>
<tr>
<td>0.25</td>
<td>2.7</td>
<td>3.4</td>
<td>2.6</td>
<td>8840</td>
<td>0.50</td>
</tr>
<tr>
<td>0.30</td>
<td>2.7</td>
<td>3.4</td>
<td>2.6</td>
<td>8840</td>
<td>0.65</td>
</tr>
<tr>
<td>0.35</td>
<td>2.7</td>
<td>3.4</td>
<td>2.6</td>
<td>8840</td>
<td>0.85</td>
</tr>
</tbody>
</table>

#### Table 4.7: Modelling results obtained by means of IKH stability analysis and the Delfos film thickness calculation ($h_{crit}$ denotes the instability level).

<table>
<thead>
<tr>
<th>$V_{ls}$ $(m/s)$</th>
<th>$Q_{sh}$ $(10^{-3} m^3/s)$</th>
<th>$\delta_l$ $(10^{-3} m)$</th>
<th>$V_{f\infty}$ $(m/s)$</th>
<th>$Re_f$</th>
<th>$f$ $(Hz)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>2.8</td>
<td>3.6</td>
<td>2.6</td>
<td>9360</td>
<td>0.10</td>
</tr>
<tr>
<td>0.10</td>
<td>2.8</td>
<td>3.6</td>
<td>2.6</td>
<td>9360</td>
<td>0.15</td>
</tr>
<tr>
<td>0.15</td>
<td>2.8</td>
<td>3.6</td>
<td>2.6</td>
<td>9360</td>
<td>0.20</td>
</tr>
<tr>
<td>0.20</td>
<td>2.8</td>
<td>3.6</td>
<td>2.6</td>
<td>9360</td>
<td>0.30</td>
</tr>
<tr>
<td>0.25</td>
<td>2.8</td>
<td>3.6</td>
<td>2.6</td>
<td>9360</td>
<td>0.40</td>
</tr>
<tr>
<td>0.30</td>
<td>2.9</td>
<td>3.6</td>
<td>2.6</td>
<td>9360</td>
<td>0.50</td>
</tr>
<tr>
<td>0.35</td>
<td>2.9</td>
<td>3.6</td>
<td>2.6</td>
<td>9360</td>
<td>0.70</td>
</tr>
</tbody>
</table>
4.4. MODELLING RESULTS

**Table 4.8:** Modelling results obtained by means of IKH stability analysis and the Delfos film thickness calculation ($h_{\text{crit}}$ denotes the instability level).

<table>
<thead>
<tr>
<th>$V_{gs}$ (m/s)</th>
<th>$Q_{sh}$ ($10^{-3} \text{ m}^3/\text{s}$)</th>
<th>$\delta_l$ ($10^{-3} \text{ m}$)</th>
<th>$V_{\infty}$ (m/s)</th>
<th>$Re_f$ (-)</th>
<th>$f$ (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
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<td>2.5</td>
<td>8250</td>
<td>0.35</td>
</tr>
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<td>2.5</td>
<td>8500</td>
<td>0.25</td>
</tr>
<tr>
<td>1.5</td>
<td>2.7</td>
<td>3.5</td>
<td>2.6</td>
<td>9100</td>
<td>0.25</td>
</tr>
<tr>
<td>2.0</td>
<td>2.8</td>
<td>3.6</td>
<td>2.6</td>
<td>9360</td>
<td>0.20</td>
</tr>
<tr>
<td>2.5</td>
<td>2.9</td>
<td>3.6</td>
<td>2.6</td>
<td>9360</td>
<td>0.20</td>
</tr>
<tr>
<td>3.0</td>
<td>3.0</td>
<td>3.7</td>
<td>2.7</td>
<td>9990</td>
<td>0.20</td>
</tr>
<tr>
<td>3.5</td>
<td>3.0</td>
<td>3.7</td>
<td>2.7</td>
<td>9990</td>
<td>0.20</td>
</tr>
</tbody>
</table>

**Table 4.9:** Modelling results obtained by means of IKH stability analysis and the Delfos film thickness calculation ($h_{\text{crit}}$ denotes the instability level).

<table>
<thead>
<tr>
<th>$V_{gs}$ (m/s)</th>
<th>$Q_{sh}$ ($10^{-3} \text{ m}^3/\text{s}$)</th>
<th>$\delta_l$ ($10^{-3} \text{ m}$)</th>
<th>$V_{\infty}$ (m/s)</th>
<th>$Re_f$ (-)</th>
<th>$f$ (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>2.6</td>
<td>3.3</td>
<td>2.5</td>
<td>8250</td>
<td>0.90</td>
</tr>
<tr>
<td>1.0</td>
<td>2.7</td>
<td>3.4</td>
<td>2.6</td>
<td>8840</td>
<td>0.65</td>
</tr>
<tr>
<td>1.5</td>
<td>2.8</td>
<td>3.5</td>
<td>2.6</td>
<td>9100</td>
<td>0.60</td>
</tr>
<tr>
<td>2.0</td>
<td>2.9</td>
<td>3.6</td>
<td>2.6</td>
<td>9360</td>
<td>0.50</td>
</tr>
<tr>
<td>2.5</td>
<td>2.9</td>
<td>3.6</td>
<td>2.7</td>
<td>9720</td>
<td>0.50</td>
</tr>
<tr>
<td>3.0</td>
<td>3.0</td>
<td>3.7</td>
<td>2.7</td>
<td>9990</td>
<td>0.50</td>
</tr>
<tr>
<td>3.5</td>
<td>3.0</td>
<td>3.7</td>
<td>2.7</td>
<td>9990</td>
<td>0.50</td>
</tr>
</tbody>
</table>
Summing it all up it can be said that the results given by tables 4.1 until 4.4 represent the final modelling results. The frequency of slugging found thus lies between nil and 0.5 Hz., fully in agreement with the observed visual observations. In the next chapter the experimental results will be treated and will be compared to this modelling result.

4.5 Other developed models

In the earlier stages of this work two other attempts were made to model the flow, before the idea of the finally resulting model, which was presented in the previous sections, was born. The first model was based on an idea of my predecessor, Luc Prieels (see Prieels 1996). This model will first be shortly discussed. Hereafter the second model will be shortly presented. A more thorough treatment of the first model can be found in appendix A.

Force balance on a slug
The first model which was developed considered the forces acting on a slug as it moves through the bend. Luc Prieels had attempted to evaluate the strong mechanical stresses induced in the bend as a long slug, moving at a velocity close to the air velocity, penetrated the bend (see Prieels 1996). An error was found in the force balance he had written down, so the first step taken was to correct the model.

The whole idea behind the further development of the model was, that in case a slug moved through the system it would have to acquire a certain minimal initial velocity in the line in order to be able to make it all the way up the riser. The walls of the duct invoke frictional stresses on the slug. These stresses, together with the gravity, will slow the slug down as it rises in the bend and the riser. The slug was considered to have a constant velocity as it moved through the line. When going up into the bend and the riser it’s kinetic energy would be turned into potential energy and in case the amount of kinetic energy was not sufficient for the slug to make it to the top of the riser and into the return line, it would at some point in the riser come to a stand still. The idea was that at this point the slug would collapse, fall down into the bend, create waves on the surface of the stratified flow in the line and perhaps initiate the formation of a new slug.

A code was written in FORTRAN 77 by which this model was simulated. The forces acting on the slug and the velocity of the slug as it travelled through the bend could be determined. In essence the results of this model were fine. The problem though was that we never got any further. It was not clear how the model could be extended to include series of slugs, slugs falling back down or the vibration that would be induced on the facility. It was especially obscure how there would ever come a frequency of slugging out of this model. Besides, the falling down of slugs on which the model was based has never been
4.5. OTHER DEVELOPED MODELS

observed visually. Therefore the development of this model was ceased. The details about
the model can be found in appendix A.

Model of a collapsing front

A second attempt to model the flow in the facility consisted of the following. Take as a
starting point the situation where there is a stratified smooth flow present in the line and
a column of water in the bend and the riser (see figure 4.13). Between these two regimes
a rigid front is considered to be present. At first the system will be in equilibrium and
the front will be at rest. As gas is coming into the line at the air inlet, this will induce a
pressure build up in the line. Due to the higher pressure in the line the water column will
be pushed upwards and out of the bend and the riser. The front between the stratified
flow and the water column will thus move into the bend. As the water column is reduced
in size, so will it's weight and as a result the front will be pushed even further into the
bend. On the other hand the pressure in the stratified part will drop due to expansion,
additional frictional losses (due to an additional length travelled by the stratified flow) and
the gravity as the front moves upwards in the bend.

![Diagram of fluid flow](image)

**Figure 4.13: Situation at the start.**

The whole idea behind this model was that it might be possible for the velocity of the
front to drop back to nil at some point or even become negative. At that instant the liquid
column would be considered to collapse and fall back into the bend enclosing a Taylor
bubble within the liquid column. Then the whole calculation can start all over again with
a bubble in the riser. This might then lead to slugging frequencies.

Unfortunately this did not work out this way. It turned out that the pressure build up
in the line due to the incoming gas flow dominates the entire process. The pressure keeps
rising at a high rate and so the front keeps accelerating. Actually, this can be compared to
the slugging situation at an offshore sight as was described in the first chapter. Here huge slugs are being formed and once they start moving they flow out of the pipeline at once. In literature this situation is called "severe slugging".

A second attempt to model the flow in this way was done by placing a Taylor bubble at the bottom of the riser at the start. The idea was that once this bubble reaches the top of the riser, there will be a period of time in which there will be no water flowing out at the top of the riser. The hydrostatic pressure before the front will then remain constant for some time and perhaps in this way the front could decelerate. Pitiful enough it did not work out that way. The build up of the pressure in the stratified flow part due to the incoming gas flow rate keeps dominating the process. At this point the development of the second model was also ceased.
Chapter 5

Experimental results

Pressure transducers and glass fibre probes were used to determine the frequencies of slugging in the facility. In the first section some parameters of the experiments, like scanning frequencies and numbers of scans, will be discussed. The obtained results will be discussed in the second section of this chapter. The third section will discuss the results obtained through autoregressive modelling of pressure signals.

5.1 Measuring parameters

Before any measurements can be performed some parameters will have to be set. This concerns the scanning frequencies and the numbers of scans. Furthermore, the optimal orders for which to calculate the autoregressive models for the maximum entropy method and the autoregressive modelling will have to be determined.

In order to avoid aliasing the frequency of measuring should be at least twice the maximal frequency one wants to detect. On the other hand it is best to measure as long as possible in order to be able to find the lowest frequencies in the signal. For this reason the frequency should not be chosen too high as the number of scans that can be made is limited. For the pressure measurements a frequency of 10 Hz was chosen. The amount of scans used was $2^{13}$ ($2^{15}$ was also used but gave similar results).

For what the glass fibre measurements are concerned some other considerations will have to be taken into account. In case we consider a Taylor bubble to have a minimal length of 100 mm (one pipe diameter) and a velocity of 2 m/s, it will pass a probe in 5/100 of a second. With a scanning frequency of 100 Hz this bubble will be scanned 5 times by a probe. At lower frequencies we will not be able to distinguish this bubble. At higher frequencies the risk of splitting up the bubble by scanning a droplet inside the Taylor bubble multiple times becomes higher. As a result a frequency of 100 Hz was used for the glass fibre measurements. The amount of scans used was $2^{14}$ (again $2^{15}$ was also used but gave similar results).
Both for the pressure and the glass fibre measurements the signals were divided into blocks of $2^{10}$ scans. The spectra resulting from each block were then averaged in order to heighten the accuracy. The blocks were chosen in such a way that they overlapped for 50%.

In order to use the maximum entropy method for the determination of the dominant frequencies in a signal and to determine the locations of the flow regime transitions by means of autoregressive modelling, the autoregressive model of the signal will first have to be calculated. In order to do so the order of the AR-model will first have to be chosen.

In the case of the autoregressive modelling the optimal order, giving the best balance between overfitting and underfitting, has to be determined for the reference signal. In case of the transition from stratified flow to slug flow in the line, a slug flow was chosen as reference flow. While for the transition from slug flow to churn flow in the riser a churn flow was chosen as reference flow. These transitions were determined for different flow rates, but for each type of transition only one reference signal was used.

For the autoregressive modelling pressure signals were used. The reference signal was determined by performing 10 measurements in the reference flow, with a scanning frequency of 10 Hz and $2^{14}$ scans. They were performed at the last measuring point in the line, just before the bend. For each signal equation 3.5 was used to determine the GIC-value for all orders between 1 and 1000. That order for which this value is minimal is the optimal order for that signal. The median of the ten optimal orders of the signals was considered to be the optimal order of the reference flow. The signal with the optimal order closest to the median value was chosen as the reference signal. Figure 5.1 illustrates the resulting GIC-values as a function of the order of the autoregressive model. The signal used in the illustration is the reference signal in the case of transition from stratified flow to slug flow in the line.

The optimal order of the slug flow signal, used as reference for the transition from stratified flow to slug flow in the line, appeared to be 85. For the churn flow signal, used as reference for the transition from slug flow to churn flow in the riser, the optimal order was found to be 25. The reference slug flow had superficial velocities of $V_{ls} = 0.55$ m/s and $V_{gs} = 1.55$ m/s, whereas the reference churn flow had superficial velocities of $V_{ls} = 0.20$ m/s and $V_{gs} = 10$ m/s.

When an autoregressive model of order 85 (or 25) has to be determined of a signal, only as much scans are necessary. When the amount of scans is lower, the AR-model of that order can not be determined. For a higher amount of scans, the AR-model is determined with higher precision. So in essence the pressure measurements for the rest of the modelling can be rather short. In this work a number of $2^{10}$ (=1024) scans was chosen, again with a scanning frequency of 10 Hz.
5.2. MEASURING RESULTS

In the case of the normal pressure and glass fibre measurements the spectra were determined making use of the maximum entropy method. This is based on the autoregressive model of a signal. So once again an order has to be chosen for which to calculate the autoregressive models. The problem is that for all the different signals in the various regimes, the optimal order varies between approximately 5 and 180. As can be seen in figure 5.1, the balance between overfitting and underfitting is not affected very much between orders of 50 and 150. This is valid for all the signals. For this reason an order of 100 was used for all signals. As the results of the maximum entropy method were always in agreement with the results obtained by means of the power spectral density function, the chosen order seems to do fine.

5.2 Measuring results

The aim of the pressure and glass fibre measurements is to verify the modelling results discussed in the previous chapter. We are therefore looking for two slugging frequencies, namely the one corresponding to the slugging just before the bend and the one corresponding to the slugging in the bend. The first one was found (visually and through modelling) to have a frequency between nil and 0.5 Hz and the second one a frequency between 1.5 and 2 Hz.
**CHAPTER 5. EXPERIMENTAL RESULTS**

**Figure 5.2:** Raw pressure signal, $V_{ls} = 0.25$ m/s, $V_{gs} = 1.30$ m/s (stratified flow in the line and slug flow in the riser; sixth position in the line).

**Figure 5.3:** Power spectrum, $V_{ls} = 0.25$ m/s, $V_{gs} = 1.30$ m/s (stratified flow in the line and slug flow in the riser; ninth position in the line; 95% confidence interval: 0.00018).
5.2. MEASURING RESULTS

**Figure 5.4:** Power spectrum, $V_{ls} = 0.25 \text{ m/s}$, $V_{gs} = 1.30 \text{ m/s}$ (stratified flow in the line and slug flow in the riser; second position in the riser; 95% confidence interval: 0.000060).

**Figure 5.5:** Power spectrum, $V_{ls} = 0.15 \text{ m/s}$, $V_{gs} = 1.30 \text{ m/s}$ (stratified flow in the line and slug flow in the riser; second position in the riser; 95% confidence interval: 0.000044).
CHAPTER 5. EXPERIMENTAL RESULTS

5.2.1 Pressure measurements

We can start by performing pressure measurements at different points in the system. As the pressure fluctuations are a superposition of all pressure effects in the facility it is to be expected that the results of the pressure measurements are the same at all measuring points. A typical raw pressure signal, in the regime where a stratified flow is present in the line and a slug flow in the riser, is shown in figure 5.2. It concerns a pressure signal from the sixth measuring point from the entrance (located approximately 3 meters before the bend). The signal measured here is about the same as the signal measured just before the bend at the ninth point. The frequency of slugging just before the bend is clearly present in the alternating peaks, but the higher frequency on top of these peaks will obviously be very hard to distinguish.

The pressure signals in the line only have low frequencies present in their spectra, corresponding to the slugging just before the bend. A typical result is displayed in figure 5.3. The slugs formed just before the bend can reach lengths of 1 to 2 meters and generally have a relatively low void fraction. They induce large pressure variations and can be easily found in the pressure signals. On the other hand the slugs created in the bend are rather small and have a high void fraction. The pressure variations are relatively small and therefore the frequency is hard to distinguish in this heavily sloshing facility.

The spectrum found through pressure measurements halfway up the riser is illustrated in figure 5.4. Again, just as was the case in the line only the low frequency of slugging is
5.2. MEASURING RESULTS

**Figure 5.7:** Power spectrum, $V_{ls} = 0.65$ m/s, $V_{gs} = 0.25$ m/s (slug flow in the line and the riser; sixth position in the line; 95% confidence interval: 0.0035).

**Figure 5.8:** Power spectrum, $V_{ls} = 0.65$ m/s, $V_{gs} = 0.25$ m/s (slug flow in the line and the riser; second position in the riser; 95% confidence interval: 0.00035).
CHAPTER 5. EXPERIMENTAL RESULTS

**Figure 5.9:** Probability density function of the Taylor bubble length, $V_{ls} = 0.25$ m/s, $V_{gs} = 1.35$ m/s (stratified flow in the line and slug flow in the riser; lowest measuring point in the riser).

**Figure 5.10:** Probability density function of the slug length, $V_{ls} = 0.25$ m/s, $V_{gs} = 1.35$ m/s (stratified flow in the line and slug flow in the riser; lowest measuring point in the riser).
5.2. MEASURING RESULTS

present. The higher frequency is hardly there. Only very occasionally this frequency does arise clearly, as can be seen in figure 5.5. Although different flow rates were used for these two figures, the appearance of the high frequency peak did not appear to be flow rate dependent. It was tried to make the higher peak appear in all the signals by changing the number of scans and the block sizes. The spectrum was determined by the normal power spectral density function and the maximum entropy method. In both cases the results stayed the same and the high peak did not arise. In order to determine the high peak, glass fibre measurements in the riser will be necessary.

In case of the different flow regime with a slug flow present in the line and the riser, the slugs are formed in the beginning of the line and grow rapidly until they reach the bend. These slugs have lengths of 2 to 3 meters and are hardly aerated. As a result large pressure fluctuations are induced and thus the slugging frequency can be easily found in the signal. The raw pressure signal displayed in figure 5.6 clearly shows the large slugs. The spectra found in the line at the sixth position and halfway up the riser are shown in figures 5.7 and 5.8. One clear frequency peak at ±0.1 Hz is visible, this is the geometry induced frequency of the slugs initiated by the air-inlet.

5.2.2 Glass fibre measurements

By means of the glass fibre measurements the frequencies of the slug flow in the riser can be determined. Furthermore, the lengths and the velocities of the slugs and the Taylor bubbles can be found.

First the regime of a stratified flow in the line and a slug flow in the riser will once again be considered. The probability density functions of the Taylor bubble lengths and the slug lengths can be found in figures 5.9 and 5.10. As a slug flow is developing, Taylor bubbles are split up. We can indeed see that the smaller the length, the more Taylor bubbles there are. This is in agreement with the splitting up process. The slugs seem to have an average length if approximately 1 meter. Some larger slugs are also present. The large amount of small slugs represent the little slugs formed in the bend. The larger ones then represent the slugs formed in the line.

The velocities of the slugs and Taylor bubbles are illustrated by means of the probability density functions shown in figures 5.11 and 5.12. It can be seen that these velocities can vary quite a bit. On average they seem to be somewhere between 1 and 3 m/s.

For what the frequencies are concerned, we again expect to find the frequencies corresponding to the two mechanisms. We should keep in mind though that a slug flow rising in a vertical pipe always rapidly changes into a developed slug flow of 1 Hz. That this happens quite quickly is illustrated by figure 5.13. This figure represents a measurement at the highest glass fibre measuring point (as was mentioned, in the regime of a stratified flow in
CHAPTER 5. EXPERIMENTAL RESULTS

**Figure 5.11:** Probability density function of the Taylor bubble velocity, $V_t = 0.25$ m/s, $V_{gs} = 1.35$ m/s (stratified flow in the line and slug flow in the riser; lowest measuring point in the riser).

**Figure 5.12:** Probability density function of the slug velocity, $V_t = 0.25$ m/s, $V_{gs} = 1.35$ m/s (stratified flow in the line and slug flow in the riser; lowest measuring point in the riser).
5.2. MEASURING RESULTS

**Figure 5.13:** Power spectrum through a glass fibre measurement, $V_s = 0.30$ m/s, $V_{gs} = 0.50$ m/s (stratified flow in the line and slug flow in the riser; highest measuring point in the riser; 95% confidence interval: 0.0023).

**Figure 5.14:** Power spectrum through a glass fibre measurement, $V_s = 0.30$ m/s, $V_{gs} = 0.50$ m/s (stratified flow in the line and slug flow in the riser; lowest measuring point in the riser; 95% confidence interval: 0.00029).
the line and a slug flow in the riser). It is clear that a developed slug flow has arisen. This accounts for all the measurements performed at the highest point in the riser, although sometimes the frequency is still slightly higher than 1 Hz (the flow is still developing).

At the lowest measuring point we do expect to find two frequencies. As the lowest point is located more than a meter upwards into the riser, the frequencies will already have moved somewhat towards 1 Hz. It turned out to be rather difficult to find two peaks. The higher peak of around 1.5 Hz is always there, but the lower one is just a saddle in the plot. Only when \( 2^{17} \) scans were used (two measurements of \( 2^{16} \) added, as the data acquisition unit can not handle more points) the second frequency clearly appeared (see figure 5.14).

In the regime where there is a slug flow in the line and the riser, we expect to find the frequency of about 0.1 Hz with glass fibre measurements, which was already found through pressure measurements and visual observations. In this case the normal power spectral density function gives more clear results. For the lowest measuring point the result is shown in figure 5.15. We can clearly observe the frequency peak we are looking for and besides it is visible that the flow is starting to develop as a higher frequency peak is beginning to arise. Figure 5.16 shows that the higher frequency peak of the developed slug flow has become dominant at the top of the riser. So once again it can be observed that the slug flow rapidly develops in the riser.

Finally, figures 5.17 and 5.18 show the lengths and the velocities of the slugs in the regime of a slug flow in the line and the riser. We expect to find larger slugs travelling at a greater velocity than in the case of a stratified flow in the line and a slug flow in the riser, as was visually observed. Indeed some peaks at higher values can be seen, although the smaller and slower slugs are still predominantly present. Between the slugs a stratified flow is present in the line. This is the case most of the time. In this period the slugging mechanism in the bend with a frequency of 1.5 to 2 Hz is still present. As a result there are by far more small slugs with a low velocity. As every little slug counts for one, they dominate in the probability density function.

5.2.3 Comparison with the modelling results

The lengths of the slug given by figure 5.10 can be compared with the length found by means of the model. For \( V_{ls} = 0.25 \, m/s \) and \( V_{gs} = 1.35 \, m/s \), the model calculates a frequency of 0.20 Hz, a shedding of \( 1.32 \times 10^{-4} \, m^3/s \) and an entering flow rate from the entrance of \( 1.96 \times 10^{-3} \, m^3/s \). This corresponds to an amount of water accumulated in the line in every cycle of \( 1.046 \times 10^{-2} \, m^3 \). If we consider the slug created to take all this water with it out of the line, the slug would have a length of 1.33 m. This is in reasonable accordance to the result found experimentally and illustrated in figure 5.10.
5.2. MEASURING RESULTS

**Figure 5.15:** Power spectrum by glass fibre measurement, $V_{ls} = 0.55$ m/s, $V_{gs} = 1.55$ m/s (slug flow in the line and the riser; lowest measuring point in the riser).

**Figure 5.16:** Power spectrum by glass fibre measurement, $V_{ls} = 0.55$ m/s, $V_{gs} = 1.55$ m/s (slug flow in the line and the riser; highest measuring point in the riser).
CHAPTER 5. EXPERIMENTAL RESULTS

Figure 5.17: Probability density function of the slug length through a glass fibre measurement, \( V_{ls} = 0.55 \text{ m/s}, V_{gs} = 1.55 \text{ m/s} \) (slug flow in the line and the riser; lowest measuring point in the riser).

Figure 5.18: Probability density function of the slug velocity through a glass fibre measurement, \( V_{ls} = 0.55 \text{ m/s}, V_{gs} = 1.55 \text{ m/s} \) (slug flow in the line and the riser; lowest measuring point in the riser).
5.3. FLOW REGIME TRANSITIONS

The experimental results for the velocities of the Taylor bubbles, as shown in figure 5.11, can be compared to the theoretical value given by the Nicklin relation (see equation 2.36). For $V_{lb} = 0.25$ m/s and $V_{gb} = 1.35$ m/s, this relation gives a Taylor bubble velocity of 2.27 m/s. This result is fully in accordance to the experimental results.

By making combined use of the pressure measurements and the glass fibre measurements, we did indeed find the two frequencies found with the model. The modelling results were already in agreement with the visual observations. It can therefore be concluded that the model gives a reasonable description of the slugging mechanisms. The mechanism in the bend has a frequency of about 1.5 to 2 Hz, whereas the mechanism just before the bend has a frequency lower than approximately 0.5 Hz.

5.3 Flow regime transitions

The first one, at the "Kramers laboratorium", to use the autoregressive modelling in order to determine flow regime transitions was Bas Meeuwissen (see Meeuwissen 1996). The technique was used in this project primarily to see if it is also applicable in this facility, as it fluctuates much more heavily than the set-up Bas Meeuwissen used. As a result of these fluctuations, the distances found by means of the autoregressive modelling also fluctuated rather heavily. This gave results which were not all too well. Therefore three measurements were performed at each point, resulting in three distances. These distances were then averaged and finally a smooth curve was drawn through the resulting distances. This gave rather good results. As a smooth curve is drawn, values lower than 1 can occur in the plots.

One should note that for the transition from stratified flow to slug flow in the line one single reference signal was used for all the plots at various flow rates. Analogous, one reference signal was used for all the plots for the transition from slug flow to churn flow in the line. Furthermore, it is important to realize that it is not possible to exactly determine the location of a transition. The locations given are estimations based on the course of the plot together with visual observations.

In the case of the transition from stratified flow to slug flow in the line the results of the autoregressive modelling are displayed in figures 5.19 until 5.22. Every figure represents a different superficial gas velocity. Each consists of 25 measuring points. As can be seen the point of transition drops to lower values for the superficial liquid velocity as the superficial gas velocity is increased. This is in agreement with the Kelvin-Helmholtz instability theory, as that is based on the Bernoulli effect which in fact is in it's turn based on a velocity difference. The velocity at which instability occurs will be reached for lower liquid flow rates as the gas flow rate is increased.
CHAPTER 5. EXPERIMENTAL RESULTS

The flow regime transitions found are in accordance to the visual observations. They can also be compared to the flow regime map of figure 5.26. The transitions found by means of the autoregressive modelling are illustrated in figure 5.28. In this figure the lowest transition points are shown (in practise one wants to avoid slugs, so the lowest possible points should be used). As can be seen the transitions are found for somewhat higher superficial liquid velocities in this project. This is rather surprising as the slug flow is induced by the air-inlet. One would assume slug flow to occur for lower superficial liquid velocities than without the effect of the geometry. Apparently there is still some sort of stabilising effect present.

For the case of the transition between slug flow and churn flow in the riser, the results are shown in figures 5.23 until 5.25. They were acquired analogously to the previous ones. In this case it is rather difficult to give an exact transition range as the slug flow is developing towards a churn flow from the start onwards. For the two lowest liquid flow rates churn flow can be considered to be present above a superficial gas velocity of approximately 4 m/s. For higher liquid flow rates the problem arises that for a certain range of gas flow rates a slug flow is present in the line. This can be seen in the last figure. This slug flow changes the point at which a churn flow in the riser arises, as relatively large slugs, formed in the line, will enter the riser. Besides, it is hardly possible to keep the liquid flow rate reasonably constant within this range where slug flow arises. Therefore, no measurements were performed for higher superficial liquid velocities.

If we compare the location of this transition to the one in the flow map of a riser in figure 5.27, we can see that the transition point is found for higher superficial gas velocities in this project. This is probably due to the fact that the bend prefers to create slugs. Again the results found by means of autoregressive modelling are shown in figure 5.28.

Finally, a test was done to see how dependent the results are on the choice of the optimal order with which to determine the autoregressive models. For the transition in the line the optimal order was found to be 85. Around this value there is a whole range of orders for which the GIC-value is just about minimal. If a value within this range is chosen the result should be approximately the same, as the balance between overfitting and underfitting is about equal. To see if this is correct the location of the transition was determined for order 50. The result is displayed in figure 5.29. When we compare this to figure 5.20, it can be seen that the difference indeed is minimal.

The main question was if this method of autoregressive modelling was applicable in this project. It seems to be reasonable to say it is.
5.3. FLOW REGIME TRANSITIONS

**Figure 5.19:** Transition in the line.

**Figure 5.20:** Transition in the line.
CHAPTER 5. EXPERIMENTAL RESULTS

**Figure 5.21:** Transition in the line.

**Figure 5.22:** Transition in the line.
5.3. Flow Regime Transitions

Figure 5.23: Transition in the riser.

Figure 5.24: Transition in the riser.
CHAPTER 5. EXPERIMENTAL RESULTS

FIGURE 5.25: Transition in the riser.

FIGURE 5.26: Flow regime map for a 5.1 cm diameter horizontal line under normal atmospheric conditions (Barnea, D. and Taitel, Y. (1985)).
5.3. FLOW REGIME TRANSITIONS

Figure 5.27: Flow regime map for a 5.1 cm diameter riser under normal atmospheric conditions (Barnea, D. and Taitel, Y. (1985)).

Figure 5.28: Flow map found through AR-modelling of pressure signals (the lowest transition point was used).
CHAPTER 5. EXPERIMENTAL RESULTS

FIGURE 5.29: Transition in the line; determined with AR-models of order 50 instead of the optimal 85.
Chapter 6

Concluding remarks

6.1 Conclusions

The flow in the facility falls into one of three various categories, namely a stratified flow in the line with a churn flow in the riser, a slug flow in both the line and the riser, and a stratified flow in the line with a slug flow in the riser. The first regime occurs at high gas flow rates (superficial gas velocity above typically 0.4 m/s). It is a rather steady regime in which nothing peculiar seems to happen. The second regime with slug flow in the line and the riser is geometry induced and has a low frequency (about 0.1 to 0.2 Hz). It occurs for somewhat higher liquid flow rates (superficial liquid velocity typically above 0.3 m/s for higher gas flow rates to 0.4 m/s for lower gas flow rates). In both cases the effect of the bend is minimal. For this reason they were not studied in detail.

The third regime is the most interesting regime as the bend has a clear effect on the flow. Furthermore, it occurs in the larger part of the flow regime map within the ranges in which the facility can operate. The bend is responsible for the transition of the steady stratified flow in the line into a slug flow in the riser. Two slugging mechanisms, one in the bend and one just before the bend, can be distinguished.

The mechanism of slugging in the bend comprises of two liquid flows bumping into each other. These are the incoming liquid flow from the entrance and a flow coming from the riser as a result of the shedding behind a slug. These flows meet at the beginning of the bend and create a bump in the liquid level. Once this bump reaches the instability level, at which the flow becomes unstable due to the Bernoulli effect, a slug will be formed. This slug will take the accumulated water with it into the riser. A side effect is that the slugging mechanism in the bend sends waves upstream into the line.

The slugging mechanism in the line occurs when the liquid level in the line is high enough so that the crest of a wave reaches the instability level and creates a slug. This mechanism occurs in the last few meters just before the bend, as the waves created in the bend rapidly
decay in upstream direction. At first, there will be a low liquid level in the line. The incoming liquid flow from the entrance and the liquid flow coming back from the bend will increase the liquid level until the slug can be formed. The slug will then take all the accumulated water with it and the situation at the start will once again occur.

A model describing the transition from stratified flow in the line to slug flow in the riser has been successfully developed. It starts by calculating the stable liquid level of a steady stratified flow in the line. The level of instability is determined by means of a Kelvin-Helmholtz analysis based on the Bernoulli effect. Furthermore, the shedding behind a slug rising in the riser is determined. The time necessary for the incoming liquid flow rate together with the shedding to increase the liquid height in the line, from the stable level to the instability level, is taken as an estimate of the slugging frequency. In this way, the model especially describes the slugging mechanism in the line. The mechanism in the bend can be seen as the shifting of a stratified flow into a slug flow as the line is being held under an ever increasing angle.

The model gives a good description of the slugging mechanism in the line. The modelling results, the experimental results and the visual observations all point out that the frequency of slugging in the line is lower than about 0.5 Hz and that the frequency of slugging in the bend lies between approximately 1.5 and 2 Hz.

The bend has no effect on the flow conditions far up- and downstream of the bend. The waves in the line rapidly decay and thus the flow in the line far upstream from the bend is just a regular flow. Furthermore, the experiments pointed out that the resulting flow in the riser quickly develops into a developed slug flow of about 1 Hz as it rises in the riser.

The method of locating the positions of the flow regime transitions, due to changing flow rates, by making use of autoregressive modelling of pressure signals, gives reasonable results. The transitions from a slug flow to a churn flow in the riser and from a stratified flow to a slug flow in the line have been located. The critical flow rates have already been mentioned in the first paragraph. They were in accordance to the visual observations. The transition in the riser was found to occur at higher flow rates than found in the literature. This is due to the fact that the bend facilitates slug formation. On the other hand, the transition in the line was also found to occur at higher flow rates. This is surprising as one would assume the flow to be less stable due to the influence of the bend and also because the transition was found to be geometry induced. Apparently, there is still some sort of stabilising effect present.
6.2 Recommendations

In view of future research, a few points might be taken into consideration:

- The model developed particularly describes the slugging mechanism just before the bend. Perhaps, a second model can be developed to describe the mechanism of slugging in the bend more accurately. Once this is done, the coupling of the two might give an even better description of the flow in the facility.

- An adjustment can be made to the developed model in order to make it possible to incorporate other fluids in the simulations. The Taitel-Dukler speculative factor then must be replaced by the real factor for the viscous case. The determination of the critical wave velocity on the inception of instability will provide difficulty.

- Installing a pressure transducer at the bottom of the line might make it possible to measure the frequency of slugging by detecting the pressure variations due to the fluctuating water level in the line. In case the line is placed under a positive angle with the horizon, a possible hysteresis in the water level may also be detected in this way.

- Perhaps the air inlet can be changed in order to investigate the effect of the air inlet on the creation of slug flow in the line.

- By using a larger buffer tank, it might be possible to reduce the fluctuations in the liquid flow rate. A more stable liquid flow rate might increase the accuracy of the experiments, reduce the effect of the air inlet and thereby make measurements in the slug flow regime in the line more useful.

- In addition to the last two points, it might also be possible to use a pump which delivers a constant flow rate rather than a constant power. In addition to the advantage of a more stable liquid flow rate, this might also increase the range in which the facility can operate. This might make the study of some other flow regimes possible.

- It is definitely very interesting to see how the facility operates when the line is placed under an angle with the horizon. One of the foremost questions will be if the mechanisms distinguished in the present work are also present then and of course if the model is still applicable. One should keep in mind though that some new phenomena may arise, like severe slugging for negative angles, roll waves for positive angles and hysteresis of the liquid level for upward flow.
Bibliography


Appendix A

Force balance on a slug

In chapter four it was mentioned that a model based on the force balance on a slug had been developed in the early stages of this work. In this appendix the details of the model will be presented.

The situation at the start is illustrated in figure A.1. A slug with length $L$ and mass $M$, moving with a constant velocity $V_0$ has just arrived at the bend at time $t_0$.

A few assumptions are made:

- The water is considered to be incompressible.
- The reaction forces of the bend on the slug are assumed to be in the radial direction.
- The additional pressure drop over the slug, due to the compression of the gas behind the slug as the slug velocity decreases once it moves up into the bend, will not be taken into account.
As the slug moves through the line it is considered to have a constant velocity. As a result there will be a balance between the pressure drop over the slug and the friction from the walls of the channel on the slug. From this balance the pressure drop over the slug can be determined. This pressure drop is considered to remain constant as the slug moves into the bend. Furthermore, the pressure is considered to drop linearly over the slug. As the gravity will start to work against the slug once it moves into the bend, the slug velocity will decrease and so will the friction from the walls on the slug. For the pressure drop over the slug we can write:

$$\Delta p = \frac{4f_s}{D} \frac{1}{2} \rho_l V_0^2 L$$  \hspace{1cm} (A.1)

Here $f_s$ is the friction factor computed in the turbulent regime by:

$$f_s = 0.046 Re_0^{-0.2}, \quad Re_0 = \frac{\rho_l V_0 D}{\mu_l}$$  \hspace{1cm} (A.2)

The process of the slug moving through the bend can be divided into small time-steps of constant length $\Delta t$. After one period $\Delta t$ the slug will have moved into the bend over an angle:

$$\Delta \theta_1 = \frac{V_0 \Delta t}{R}$$  \hspace{1cm} (A.3)

The mass present in the bend at that instant can be written as:

$$\Delta m_1 = \rho A R \Delta \theta_1$$  \hspace{1cm} (A.4)

The slug velocity has dropped to $V_1 < V_0$. In the next time step a new element of mass will move into the bend. As the time-step remains constant and the velocity of the slug drops, this second element of mass will be smaller than the first one. Besides, it will move into the bend over a smaller angle and the velocity of the slug will drop even further. Continuing in this manner, the slug will be decomposed into small elements $\Delta m_i$. The position of each element at time $t_i$ is:

$$\theta_i(\Delta m_k) = \theta_{i-1}(\Delta m_k) + \Delta \theta_i$$  \hspace{1cm} (A.5)

for $k = 1, i$ and $\theta_{i-1}(\Delta m_i) = 0$. The situation at time $t_i$ is illustrated in figure A.2.

As each element of mass has a constant size over time, the momentum balance will be time-independent and thus reduce to a force balance describing the movement of an element $\Delta m_i$ penetrating the bend at time $t_i$. The forces acting on an element of mass in the bend are shown in figure A.3. Here $dZ$ denotes the gravity force, $dF$ the frictional force, $dR$ the reaction force from the bend and $F_a$ and $F_b$ forces performed on the element of mass by the parts of the slug in front of and behind $\Delta m_i$ respectively. The radial and tangential components of the acceleration of the slug are written as $a_r$ and $a_t$. 

A2
The following force balance can now be written down (the subscript $i$ is omitted for the acceleration and for the forces):

$$\Delta m_i a = dZ + dF + F_a + F_b + dR$$  \hspace{1cm} (A.6)

This equation can be decomposed along the radial and tangential axis ($F_a$ and $F_b$ are assumed to be tangential as $\Delta \theta_i$ will be small for small time-steps):

$$A_m i \Omega = dZ \cos \theta_i - dR_i$$  \hspace{1cm} (A.7)

$$A_m j \Omega = -dZ \sin \theta_i - dF_i - F_a + F_b$$  \hspace{1cm} (A.8)

Where $a_r = -V_i^2/R$ is the radial acceleration and $a_t$ is the tangential acceleration. The gravitational force is equal to $dZ = \Delta m_i g$. The frictional term can be determined using:

$$dF_i = \frac{4f_s 1}{D} \rho V_i^2 R \Delta \theta_i A = \frac{2f_s V_i^2 \Delta m_i}{D}$$  \hspace{1cm} (A.9)

The force $F_b$ which represents the resulting force of the part of the slug behind $\Delta m_i$ can be determined by means of a force balance on the part of the slug still present in the line. This can be written as:

$$(M - \sum_{k=1}^{i} \Delta m_k) a_t = -F_b - F + F_{pb}$$  \hspace{1cm} (A.10)

\hspace{3cm} A3
FIGURE A.3: Forces on an element of mass $\Delta m_i$ in the bend.

Here $F$ represents the friction force on the part of the slug still in the line and $F_{pb}$ denotes the pressure force behind the slug. The pressure behind the slug is equal to the sum of the atmospheric pressure and the pressure drop over the slug. Similarly the force $F_a$, representing the resulting force of the part of the slug in front of $\Delta m_i$, can be calculated using the force balance on that part of the slug (note that the relation is written for time step $i$ and that the summations are over mass elements $k$):

$$
\sum_{k=1}^{i-1} \Delta m_k \ddot{a}(i) = \sum_{k=1}^{i-1} d\ddot{Z}(k) + \sum_{k=1}^{i-1} d\ddot{R}(k) + \sum_{k=1}^{i-1} d\dot{R}(k) + \ddot{F}_a - \ddot{F}_{pa}
$$  \hspace{1cm} (A.11)

In this case $F_{pa}$ is the pressure force working against the front of the slug. This force equals the atmospheric pressure times the area. This last equation can be decomposed along the $x$ and $z$ axis respectively, as follows:

$$
\sum_{k=1}^{i-1} \Delta m_k (a_t(i) \cos \theta_t(k) - \frac{V_i^2}{R} \sin \theta_t(k)) = 
$$  \hspace{1cm} (A.12)

$$
- \sum_{k=1}^{i-1} dF(k) \cos \theta_t(k) - \sum_{k=1}^{i-1} dR(k) \sin \theta_t(k) + F_a \cos \theta_t(i) - F_{pa} \cos \theta_t(i)
$$
\[
\sum_{k=1}^{i-1} \Delta m_k (a_i(i) \sin \theta_i(k) + \frac{v_i^2}{R} \cos \theta_i(k)) = (A.13)
\]

\[- \sum_{k=1}^{i-1} dZ(k) - \sum_{k=1}^{i-1} dF_k \sin \theta_i(k) - \sum_{k=1}^{i-1} dR(k) \cos \theta_i(k) + F_a \sin \theta_i(i) - F_{pa} \sin \theta_i(i) \]

We can also write for the gravitational term:

\[
\sum_{k=1}^{i-1} dZ(k) = \sum_{k=1}^{i-1} dZ(k) (\sin^2 \theta_i(k) + \cos^2 \theta_i(k)) \quad (A.14)
\]

When we multiply equation A.7 with \(\cos \theta_i(k)\) and take the sum over all elements of mass in the bend, we can find:

\[
- \sum_{k=1}^{i-1} \Delta m_k \frac{V_k^2}{R} \cos \theta_i(k) = \sum_{k=1}^{i-1} dZ(k) \cos \theta_i(k) - \sum_{k=1}^{i-1} dR(k) \cos \theta_i(k) \quad (A.15)
\]

Equations A.13 and A.15 can now be summed to give:

\[
\sum_{k=1}^{i-1} \Delta m_k a_i(i) \sin \theta_i(k) = (A.16)
\]

\[- \sum_{k=1}^{i-1} dZ(k) \sin^2 \theta_i(k) - \sum_{k=1}^{i-1} dF_k \sin \theta_i(k) + F_a \sin \theta_i(i) - F_{pa} \sin \theta_i(i) \]

From this equation the force \(F_a\) can now be computed. A closed system of equations has now been found and thus the problem can be solved.

The model has been simulated by means of a code written in FORTRAN 77 for an air/water system. An initial velocity of the slug and the length of the slug have to be given in advance. The code then computes all the forces for every time-step and determines the velocity changes of the slug over time. The code starts the calculations when the slug just enters the bend and stops when the end of the slug has left the bend or when the velocity of the slug becomes nil or negative before that point has been reached. Of course the geometry chosen is the one of the experimental facility used in this project.

An estimation can be given for the total reaction force performed by the bend on the entire slug. The maximal force will be about the value reached for the total dissipation of all the kinetic energy, as the bend turns the horizontal momentum into vertical momentum. So \(F_{\text{max}} = \rho V_0^2 A \approx 1000\) Newton, in case the initial velocity was 10 m/s.

The simulations show that a slug of 1 meter in length needs to have an initial velocity of at least 6 m/s, whereas a slug of 3 meters in length at least needs an initial velocity of 9

A5
The larger slug needs a higher initial velocity due to the frictional losses. The results for the total reaction force on the entire slug are shown in figure A.4.

The graph shows that initially the total force rises quickly as more and more liquid from the slug is entering the bend. The force does not rise linearly as the slug velocity is already decreasing. After the whole bend is occupied by water from the slug the total force drops due to the decreasing velocity of the slug. Once the end of the slug has reached the bend it can be seen that the total force starts to drop more rapidly as less and less water is present in the bend. Finally, as the slug has left the bend the force becomes nil. It can be seen that the maximum indeed approaches the estimated value of 1000 Newton.

During the period that the bend is totally occupied by the slug, there is a saw-tooth like pattern visible in the graph. The reason for this is that in time the elements of mass entering the bend per time step become smaller. Once the front of an element of mass reaches the end of the bend this whole element is considered to have moved out of the bend. The element entering in that period of time will be smaller than the one that has left the bend. As a result the bend will no longer be fully filled. In the next steps the bend will fill again, until the second element leaves the bend. And so on, resulting in the saw-tooth pattern.

Figure A.5 shows the velocity of the slug over time and figure A.6 shows the reaction force on the first element of mass as it travels through the bend.
The results of this model found thus far are fine, but it was not clear how to continue from this point onwards. It was not clear how to include series of slugs, slugs falling back down or the vibration induced on the facility. Besides, it was not obvious how a frequency of slugging would result from this model. On top of this the falling back down of the slugs was never observed visually. Therefore the development of this model was ceased at this point.

**Figure A.5:** Velocity of the slug \((L=3 \text{ m}, V_0=9 \text{ m/s and } \Delta t=0.003 \text{ s})\).
**Figure A.6:** Reaction force on the first element of mass ($L=3$ m, $V_0=9$ m/s and $\Delta t=0.003$ s).
Appendix B

Images of the two slugging mechanisms

Two series of images made by means of a digital camera at a frequency of 15 Hz are presented in this appendix. First a series of the slugging mechanism in the bend will be shown, then a series of the slugging mechanism just before the bend is presented.

Slugging mechanism in the bend

In this first series the mechanism by which slugging is being created in the bend is illustrated. At first we can see a smooth liquid level in the bend. Then in the next images it is visible that incoming water from the line and from the riser bump into each other. A bump in the liquid level is being created. Once this level becomes high enough a slug is formed which quickly moves out of the bend. Moments later the situation returns to the one we started off with.
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\[ t = \frac{2}{15}\text{ sec.} \]

\[ t = \frac{3}{15}\text{ sec.} \]
$t = \frac{7}{15} \text{ sec.}$

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\[ t = \frac{10}{15} \text{ sec.} \]

\[ t = \frac{11}{15} \text{ sec.} \]

\[ t = \frac{12}{15} \text{ sec.} \]
\[ t = \frac{13}{15} \text{ sec.} \]

\[ t = \frac{14}{15} \text{ sec.} \]
Slugging mechanism just before the bend

In the second series the slugging mechanism which takes place in the line just before the bend is illustrated. The series starts with a situation of a high liquid level in the line. The crest of a wave reaches the instability level according to the Kelvin-Helmholtz analysis and creates a slug. Once formed this slug moves out of the line very rapidly and takes all the additional water that had accumulated in the line with it. In the following images it is visible that the liquid level stabilises at a much lower level. After some time the incoming liquid flow rate from the pump and due to the shedding in the riser will heighten the liquid level and the situation with which we started will once again result.

\[ t = 0 \text{ sec.} \]

\[ t = \frac{1}{15} \text{ sec.} \]

\[ t = \frac{2}{15} \text{ sec.} \]
$t = \frac{3}{15}$ sec.

$\frac{4}{15}$ sec.

$\frac{5}{15}$ sec.

$\frac{6}{15}$ sec.
$t = \frac{7}{16}$ sec.

$\quad$

$t = \frac{8}{15}$ sec.

$\quad$

$t = \frac{9}{15}$ sec.
Appendix C

Specification of used equipment

In this appendix some of the equipment used in this project will be specified.

Pressure transducers

The pressure transducer used in this project is illustrated in figure C.1. It is a Druck LTD sensor of the type PDCR820/IS with a range of 1.5 bar and 10 Volts. The inaccuracy due to non-linearity and hysteresis is maximally ±0.1 %. The error due to temperature differences is maximally ±0.5 %. The measuring method is based on the deformation of a silicon window in the sensor due to a pressure difference over the window. The deformation results in a change in the resistance of the window and this is turned into a voltage.

![Pressure Transducer](image)

**Figure C.1:** The pressure transducer.

Glass fibre probes

The glass fibre used is illustrated in figure C.2. The fibre consists of a core of glass with a diameter of 200±8 μm, inside a silicon cladding and a teflon hull. The core is the part through which the light travels and thus the part of the glass fibre with which the two-phase flow is detected.
Data acquisition equipment

The pressure transducers and the glass fibres are connected to a 24 channel high-speed FET multiplexer of the type HP44711A, which is connected to a 13 bit high-speed voltmeter of 100kHz of the type HP44702B. This voltmeter is controlled by a data acquisition and control unit (DACQ) of the type HP3852A, which can be given orders directly or through a HP-IB interface in the programming language BASIC.

Flowmeter

Between the liquid pump and the entrance of the line, a flowmeter was present which was used to measure the liquid flow rate. It was a Krohne Altometer of the type IFS4000. Calibration measuring range(=100%): 84.823 m³/h

<table>
<thead>
<tr>
<th>Range in %</th>
<th>Deviation in %</th>
</tr>
</thead>
<tbody>
<tr>
<td>99</td>
<td>0.024</td>
</tr>
<tr>
<td>50</td>
<td>-0.030</td>
</tr>
<tr>
<td>22</td>
<td>0.008</td>
</tr>
</tbody>
</table>

Rotameters

Two rotameters were used to determine the air flow rate. They were both Brooks Full-view rotameters. Each one had a different range, namely 58.98 m³/h and 278 m³/h. The one with the smaller range was of the type 1110A12K1D2E0A0, the one with the larger range of the type 12-1110-10.
Digital camera

The digital camera used was of the type Dalsa CA-D1-0128. It is called a ccd-camera, where ccd stands for Charge Coupled Device. This is the name of the chip replacing the normal film. The images consist of 256×256 pixels. The maximal recording frequency is 750 Hz. A film can maximally consist of 1097 images. The camera is connected to a computer with a controlling program. The entire system is called IRsS (Image Registration System).
Appendix D

Code Slugmech

Code SLUGMECH to simulate the mechanism by which slugs are being created just in front of a bend in a two-phase flow line-riser system with an upward turned bend.

Written by: Michiel Engelsman
Copyright: Kramers Laboratorium voor Fysische Technologie, 1997.

input: Uls = the inlet superficial liquid velocity (m/s) (00.00-99.99,real)
Ugs = the inlet superficial gas velocity (m/s) (00.00-99.99,real)
theta = the angle of inclination of the line from the horizontal (0-90 for upward flow) (degrees) (000-090 or 270-360,integer)

PROGRAM slugmech (cUls,cUgs,comega)
IMPLICIT NONE
INTEGER k, error
CHARACTER*3 comega
CHARACTER*4 cUls, cUgs
REAL Uls, Ugs
REAL hmin, hcrit
DOUBLE PRECISION theta, omega
DOUBLE PRECISION pi
c------------------------------------------------------------------------
 c          MAIN PROGRAM
 c------------------------------------------------------------------------
 c

 IF ((cUls .eq. '') .or. (cUgs .eq. '') .or. (comega .eq. '')) THEN
   write(6,*)
   write(6,*) 'Please specify input parameters, e.g.'
   write(6,*) 'slugmech [Uls] [Ugs] [theta]'
   write(6,*)
   write(6,*) 'where,'
   write(6,*) 'cUls = superficial liquid velocity'
   write(6,*) 'Uls (m/s) (01.00-99.99, real)'
   write(6,*) 'cUgs = superficial gas velocity'
   write(6,*) 'Ugs (m/s) (01.00-99.99, real)'
   write(6,*) 'ctheta = angle of the line (degrees)'
   write(6,*) 'positive angle: 001-090 or'
   write(6,*) 'negative angle: 270-360, integer'
   STOP
 ENDIF

 c

 c***********************************************************************
 c          Convert input characters (cUls, cUgs, comega)
 c          to real numbers (Uls, Ugs, omega)
 c***********************************************************************
 c
 error=0

 Uls=0
 Ugs=0

 k=1
 Uls = Uls + (ichar(cUls(k:k)) - 48) * 1.0e1
 Ugs = Ugs + (ichar(cUgs(k:k)) - 48) * 1.0e1

 k=2
 Uls = Uls + (ichar(cUls(k:k)) - 48)
 Ugs = Ugs + (ichar(cUgs(k:k)) - 48)

 k=3
 Uls = Uls + (ichar(cUls(k:k)) - 48) * 1.0e-1
 Ugs = Ugs + (ichar(cUgs(k:k)) - 48) * 1.0e-1

 k=4
 Uls = Uls + (ichar(cUls(k:k)) - 48) * 1.0e-2
 Ugs = Ugs + (ichar(cUgs(k:k)) - 48) * 1.0e-2

 D2
omega = 0
k = 1
omega = omega + (ICHAR(omega(k:k)) - 48) * 1.0d2
k = 2
omega = omega + (ICHAR(omega(k:k)) - 48) * 1.0d2
k = 3
omega = omega + (ICHAR(omega(k:k)) - 48)

IF (omega .gt. 90) THEN
  IF (omega .le. 270) THEN
    WRITE(6,*) 'The angle of the bend has to be chosen '
    WRITE(6,*) 'between 0 and 90 degrees for positive '
    WRITE(6,*) 'angles and between 270 and 360 degrees '
    WRITE(6,*) 'for negative angles (of course 360 '
    WRITE(6,*) 'equals 0, a horizontal line). '
    WRITE(6,*) 'Please chose a new angle and try again! '
    error = 1
  ENDIF
ENDIF

IF (error .eq. 1) THEN
  GOTO 1000
ENDIF

--  change omega (degrees) to theta (radians)
pi = 4*DATAN(1.0d0)
theta = (pi/180)*omega

***************************************************************************
Call subroutine for the calculation of the liquid height
***************************************************************************

CALL liqlevel(Uls, Ugs, theta, hmin)

***************************************************************************
Call subroutine for the calculation of the critical liquid level
***************************************************************************

CALL instab(Uls, Ugs, theta, hcrit)

IF (hcrit .le. hmin) THEN
  WRITE(6,*) 'An erroneous situation has occurred! '
  WRITE(6,*) 'The calculated stable liquid level '
  WRITE(6,*) 'is higher than the calculated '
ENDIF
write(6,*) 'liquid level at which the interface becomes unstable. This implies that the interface will always be unstable! Unfortunately the code cannot cope with this situation.'
error=l
ENDIF

IF (hcrit .ge. 0.1) THEN
write(6,*) 'An erroneous situation has occurred! The critical liquid level at which the interface becomes unstable has been determined to be greater than the internal diameter of the line. Unfortunately the code cannot cope with this situation.'
error=1
ENDIF

IF (error .eq. 1) THEN
GOTO 1000
ENDIF

CALL calcslug(Uls,Ugs,theta,hmin,hcrit)

write(6,*) 'Mission accomplished '

1000 END

SUBROUTINE liqlevel(Uls,Ugs,theta,hmin)
IMPLICIT NONE

INTEGER i, k, b
INTEGER def, q, qmax

PARAMETER(def=10000)

REAL h, h(4), x
REAL Uls, Ugs, g, D
REAL rhol, nul, rhog, nug
REAL n, m, Cl, Cg, hmin

DOUBLE PRECISION pi, theta, gamma
DOUBLE PRECISION Plt, Plg, Pin
DOUBLE PRECISION alphal, alphag
DOUBLE PRECISION A, Al, Ag
DOUBLE PRECISION Dl, Dg, ul, ug
DOUBLE PRECISION Rel, Rela, Reg, Regs
DOUBLE PRECISION nomx, denomy, X2
DOUBLE PRECISION nomy, denomy, Y
DOUBLE PRECISION Dlt, Dgt, Alt, Agt
DOUBLE PRECISION ult, ugt, som
DOUBLE PRECISION Plit, Pint, Pgat
DOUBLE PRECISION alpha, beta
DOUBLE PRECISION var(def), dev

MAIN ROUTINE

write(6,*), 'Calculating liquid height...'

rholl=1000
nul=1.002e-3
rholl=1.185
nulg=18.3e-6
g=9.81
D=0.1
pi=4*datan(1.0d0)

h=0
i=0
q=1

start sequence for determining liquid level
DO 100 k=1,10000
i=i+1
h=k*1.0e-5
x=h/D
gamma=2*acos(1-2*x)

c--- calculate perimeters
Pli=gamma*D/2
Pga=(pi*D)-Pli
Pin=D*dsin(gamma/2)
c--- calculate holdup
alphal=(gamma-dsin(gamma))/(2*pi)
alphag=1-alphal
c--- calculate area's
A=(pi*(D**2))/4
Al=(gamma-dsin(gamma))*(D**2)/8
Ag=A-Al
c--- calculate hydraulic diameters
Dl=4*Al/Pli
Dg=4*Ag/(Pga+Pin)
c--- calculate velocities
ul=Uls/alphal
ug=Ugs/alphag
c--- calculate Reynolds numbers and superficial Reynolds numbers
Rel=(rhoi*Dl*ul)/nul
Rel=(rhoi*D*Uls)/nul
Reg=(rho*Dg*ug)/nug
Regs=(rho*D*Ugs)/nug
c--- determine constants of friction factor
IF (Rel .gt. 2.0d3) THEN
  Cl=46e-3
  n=0.2
ELSE
  Cl=16.0
  n=1
ENDIF

IF (Reg .gt. 2.0d3) THEN
  Cg=46e-3
  m=0.2
ELSE
  Cg=16.0
  m=1
ENDIF
c calculate Lockhart-Martinelli parameter
nomx=Cl*(Reis**(n*-l))*rhol*(Uls**2)
denomx=Cg*(Regs***(m*-l))*rhog*(Ugs**2)
X2=nomx/denomx
c calculate Taitel-Duckler inclination parameter
nomy=-l.O*(rhol-rhog)*g*sin(theta)
denomy=2.0*Cg*(Regs***(m*-l))*rhog*(Ugs**2)/D
Y=nomy/denomy
c determine dimensionless numbers (t added to normal parameter)
Dlt=Dl/D
Dgt=Dg/D
ult=ul/Uls
ught=ug/Ugs
Plit=Pli/D
Pgt=PGA/D
Pint=Pin/D
Alt=A1/(D**2)
Agt=Ag/(D**2)
c calculate constants of main formula
alpha=((Dlt*ult)**(n*-l))*(ult**2)*Plit/Alt
som=(Pgt/Agt)+(Pint/Alt)+(Pint/Agt)
beta=((Dgt*ught)**(m*-l))*(ught**2)*som
c calculate deviation of main formula from nil
dev=(alpha*X2)-beta-(4*Y)
var(i)=dabs(dev)
c determine possible heights of liquid
IF (i .gt. 3) THEN
IF (var(i-i) .lt. var(i-2)) THEN
IF (var(i-i) .lt. var(i)) THEN
hq=(k-1)*1.0e-5
q=q+1
ENDIF
ENDIF
ENDIF
100 CONTINUE
qmax=q-1

IF (qmax .gt. 1) THEN
write(6,*), '-> The possible liquid heights have just been',
write(6,*), 'determined to be:
ELSE
write(6,*), '-> The possible stable liquid height has just',
write(6,*), 'been determined to be:
ENDIF
DO 200 b=1,qmax
   write(6,*) ' hl(',b,')=',hl(b),'(m)'
   IF (b .eq. 1) THEN
      hmin=hl(b)
   ENDIF
   IF (b .ge. 2) THEN
      IF (hl(b) .lt. hmin) THEN
         hmin=hl(b)
      ENDIF
   ENDIF
200 CONTINUE
IF (qmax .gt. 1) THEN
   write(6,*) 'The lowest liquid level will be considered '
   write(6,*) 'to be the stable one and will therefore be '
   write(6,*) 'used in the rest of the program.'
   write(6,*) ' hmin=',hmin,'(m)'
ENDIF

SUBROUTINE instab(Uls,Ugs,theta,hcrit)

IMPLICIT NONE

CHARACTER*11 infile1
CHARACTER*12 infile2

INTEGER i, m, j, ghi
INTEGER t, q, def

PARAMETER(def=10000)
PARAMETER(ghi=10)

REAL Uls, Ugs
REAL rhol, rhog, g
REAL D, n
REAL hcrit
DOUBLE PRECISION theta, pi, A
DOUBLE PRECISION x, gamma
DOUBLE PRECISION R1, Rg, U1, Ug
DOUBLE PRECISION xx, xy, xz
DOUBLE PRECISION yx, yy, yz
DOUBLE PRECISION zx, zy, zz
DOUBLE PRECISION y, z, dAldhl
DOUBLE PRECISION u, v, w, ux, uy
DOUBLE PRECISION K, afw, var(def)
DOUBLE PRECISION dev(ghi), op1(ghi)

MAIN ROUTINE

write(6,*) 'Calculating critical liquid level...'

--- initiation
rhol=1.0e3
rhog=1.185e0
g=9.81e0
D=0.1e0
pi=4.0d0*datan(1.0d0)
A=(pi*(D**2.0d0))/4.0d0
m=0
j=0

infile1='./variantie'
infile2='./derivative'
open (unit=10,file=infile1,status='unknown')
open (unit=11,file=infile2,status='unknown')

--- start loop to determine liquid height at which interface becomes unstable
DO 100 i=1,10000
  m=m+1
  n=m*1.0e-5

--- calculation of geometries
--- dimensionless liquid height and top-angle
  x=n/D
  IF (x .gt. 0.5) THEN
    zz=1-x
  ELSE
    zz=x
  ENDIF
  gamma=2.0d0*dacos(1.0d0-(2.0d0*zz))
c liquid and gas fraction
Rl=(gamma-dsin(gamma))/(2.0d0*pi)
IF (x .gt. 0.5) THEN
  Rl=1.0d0-Rl
ENDIF
Rg=1.0d0-Rl
c average axial velocities
Ul=Uls/Rl
Ug=Ugs/Rg
c derivative of liquid area to liquid height
xy=1.0d0-dcos(gamma)
y=dsin(gamma/2.0d0)
z=xy/y
dAldhl=(D/2.0d0)*z
c write(11,9000) n, Rl
c calculation of parts of the stability criterion
u=(rhol*Rg)+(rhog*Rl)
v=(rhol-rhog)/(rhol*rhog)
w=A*g*dcos(theta)
ux=(u*v*w)/dAldhl
uy=ux**0.5d0
c determine if viscous or inviscid factor has to be used
IF (theta .le. (pi/2.0d0)) THEN
  IF (n .ge. 0.05d0) THEN
    c viscous
    K=1.0d0-x
  c===== overrule -> always inviscid
    K=1.0d0
  ELSE
    c inviscid
    K=1.0d0
  ENDIF
ELSE
  c inviscid
  K=1.0d0
ENDIF
c determine deviation of equation from nil
afw=Ug-Ul-(K*uy)
var(i)=dabs(afw)
c write(10,9000) n, var(i)

D10
C---    find the possible solutions
   IF (i .gt. 3) THEN
      IF (var(i-1) .lt. var(i)) THEN
         IF (var(i-1) .lt. var(i-2)) THEN
            j=j+1
            dev(j)=var(i-1)
            opl(j)=(m-1)*1.0e-5
            write(6,*) 'dev(',j,')=',dev(j)
            write(6,*) 'opl(',j,')=',opl(j)
            write(6,*) 'm=',m-1
         ENDIF
      ENDIF
   ENDIF
   CONTINUE

C---    determine ultimate solution
   IF (j .gt. 1) THEN
      q=1
      DO 200 t=2,j
         IF (dev(t) .lt. dev(q)) THEN
            q=t
         ENDIF
      CONTINUE
   ELSE
      q=1
   ENDIF

   hcrit=opl(q)
   write(6,*) '—> The critical liquid height at which the '
   write(6,*) 'interface becomes unstable has just been '
   write(6,*) 'determined to be: '
   write(6,*) 'hcrit=',hcrit,'(m)'

   close(unit=10)
   close(unit=11)
C==========================================================================================================

9000 format(2f14.6)
1000 END
Subroutine of the code Slugmech to do the main
calculation on the mechanism of slug creation.

SUBROUTINE calcslug(Uls, Ugs, theta, hmin, hcrit)

IMPLICIT NONE

CHARACTER*11 infile1

INTEGER i, j, k, p
INTEGER def, m, q, ghi

PARAMETER(def=10000)
PARAMETER(ghi=10)

REAL D, L, x, n
REAL Uls, Ugs, Umix
REAL hmin, nul
REAL rhol, g, hcrit

DOUBLE PRECISION theta, pi
DOUBLE PRECISION gamma, Pli
DOUBLE PRECISION A, AI, Vlent
DOUBLE PRECISION V10, Vlcrit
DOUBLE PRECISION liqent, shed
DOUBLE PRECISION totent, crittim
DOUBLE PRECISION freq, deltal
DOUBLE PRECISION u, v, w, yy
DOUBLE PRECISION xy, y, z
DOUBLE PRECISION var(def), zy
DOUBLE PRECISION opl(ghi), Vb
DOUBLE PRECISION ux, uy, UO
DOUBLE PRECISION dev(ghi), Ul
DOUBLE PRECISION afw, Uinf

MAIN ROUTINE

write(6,*) 'Main calculation in progress...'

--- initiation
D=0.1
pi=4*datan(1.0d0)
L=9
A=(pi*(D**2))/4
rhol=1000.0
g=9.81
null=1.002e-3

c
\[ V_10 = A_1 \cdot L \]

c--- calculation of volume of line occupied by liquid at start
x=hmin/D
\[ \gamma = 2 \cdot \arccos(1 - 2 \cdot x) \]
\[ P_{li} = \gamma \cdot D/2 \]
\[ A_l = (\gamma - \sin(\gamma)) \cdot (D^2)/8 \]
\[ V_{10} = A_l \cdot L \]

c--- calculation of volume of line occupied by liquid when
the critical liquid height has been reached
x=hcrit/D
\[ \gamma = 2 \cdot \arccos(1 - 2 \cdot x) \]
\[ P_{li} = \gamma \cdot D/2 \]
\[ A_l = (\gamma - \sin(\gamma)) \cdot (D^2)/8 \]
\[ V_{lcrit} = A_l \cdot L \]

c--- amount of liquid that has to enter the line to reach
the critical liquid level
\[ V_{lent} = V_{lcrit} - V_{10} \]

c--- amount of liquid entering the line at inlet per second
\[ \text{lqent} = U_{ls} \cdot A \]

c== calculation of the thickness of the film around
a Taylor bubble for a turbulent regime based on Von Karman's
extension of the Prandtl-Taylor analogy

c== initiation
m=0
i=0
j=0

c--- mixture velocity
\[ U_{mix} = U_{ls} + U_{gs} \]

c--- velocity of a Taylor bubble
\[ Vb = (1.2 \cdot U_{mix}) + 0.35 \cdot ((g \cdot D)^{0.5}) \]

c
\[ \text{infile1=}'./variantie' \]
\[ \text{open (unit=10,file=\text{infile1},status='unknown')} \]

c--- start loop to determine solution of equation
\[ \text{DO 100 } i=1,10000 \]
\[ m=m+1 \]
\[ n=m \cdot 1.0e-2 \]
u=(nul*n/rhol)**2.0
v=(u/g)**(1/3.0)
w=((D-2*v)/D)**2.0
xy=(w*Vb)-Umix
yy=(rhol/(g*nul))*(n**2.0)
zy=yy**(1/3.0)
y=2.5*n*log(n)
z=(3.0*n)-64.0
var(i)=dabs((xy*zy)-y-z)
c
cc write(10,9000) n, var(m)
c
IF (i .gt. 3) THEN
  IF (var(i-l) .lt. var(i)) THEN
    IF (var(i-1) .lt. var(i-2)) THEN
      j=j+1
      dev(j)=var(i-1)
      opl(j)=(m-l)*1.0e-2
      write(6,*) 'dev(',j,')=',dev(j)
      write(6,*) 'opl(',j,')=',opl(j)
      write(6,*) 'm=',m-l
    ENDIF
  ENDIF
ENDIF
ENDF
ENDF
cc close(unit=10)
c--- seek the ultimate solution
IF (j .gt. 1) THEN
  q=1
  DO 200 k=2,j
    IF (dev(k) .lt. dev(q)) THEN
      q=k
    ENDIF
  200 CONTINUE
ELSE
  q=1
ENDIF
cc--- calculate the film thickness
ux=((nul*opl(q))/rhol)**2.0
deltal=(ux/g)**(1.0/3)
c
cc--- calculate amount of liquid entering the line due to shedding
cc--- in the turbulent regime
cc--- superficial liquid velocity in film
uy=((D-2.0*deltal)/D)**2.0
Ul=(uy*Vb)-Umix
cc
D14
calculation of equilibrium film velocity according to Brotz

\[ U_{\text{finf}} = \sqrt{\frac{590}{3} g \Delta l} \]

shedding

\[ \text{shed} = \pi \Delta l U_{\text{finf}} (D - \Delta l) \]

calculation based on Delfos

initiation

\[ i = 0 \]
\[ j = 0 \]
\[ m = 0 \]

mixture velocity

\[ U_{\text{mix}} = U_{\text{ls}} + U_{\text{gs}} \]

velocity of a Taylor bubble

\[ V_b = (1.2 U_{\text{mix}}) + 0.35 \sqrt{g D} \]

\[ x = D^2 \left( V_b - U_{\text{mix}} \right) / (4 V_b) \]
\[ y = \left( \frac{(590 g D^2)}{3 (V_b^2)} \right)^{0.5} \]

infile1='./variantie'

open (unit=10, file=infile1, status='unknown')

start loop to determine solution of equation

\[ \text{DO } 300 1=1,10000 \]

\[ m = m + 1 \]
\[ n = m \times 1.0 \times 10^{-5} \]

\[ z = y \times (n^{1.5}) \]

\[ afw = n^2 - D^n - z + x \]

\[ \text{var}(m) = \text{dabs}(afw) \]

write(10,9000) n, var(m)

IF \( m > 3 \) THEN

IF \( \text{var}(m-1) \lt \text{var}(m) \) THEN

IF \( \text{var}(m-1) \lt \text{var}(m-2) \) THEN

\[ j = j + 1 \]

\[ \text{dev}(j) = \text{var}(m-1) \]

\[ \text{op1}(j) = (m-1) \times 1.0 \times 10^{-5} \]

ENDIF

ENDIF

ENDIF

300 CONTINUE

close(unit=10)

seek the ultimate solution

IF \( j > 1 \) THEN
q=1
DO 400 k=2,j
IF (dev(k) .lt. dev(q)) THEN
  q=k
ENDIF
400 CONTINUE
ELSE
  q=1
ENDIF

film thickness
deltal=opl(q)

superficial velocity of the liquid at the Taylor bubble
u=((D-2.0*deltal)/D)**2.0
Ul=(u*Vb)-Umix

=======
equilibrium film velocity
Uinf=((590.0/3)*g*deltal)**0.5

shedding
shed=pi*deltal*Uinf*(D-deltal)

total amount of liquid entering the line per second
totent=liqent+shed

time necessary to let the liquid reach the critical liquid level (interface becomes unstable => slug)
crittim=Vlent/totent

corresponding slugging frequency
freq=1/crittim

Result
write(6,*)'crittim=',crittim
write(6,*)'V10=',V10
write(6,*)'Vlcrit=',Vlcrit
write(6,*)'liqent=',liqent
write(6,*)'shed=',shed
write(6,*)'deltal=',deltal
write(6,*)'Uinf=',Uinf
write(6,*)'Vb=',Vb
write(6,*)'Ul=',Ul
write(6,*)'Umix=',Umix
write(6,*)'The calculated slugging frequency is:'
write(6,*)' => frequency=',freq,'(Hz)'

D16
900  write(6,*) 'The calculation has just been performed!'  
      
9000  format(2f14.6)  
1000  END  
      
