The feasibility of the
TWO-DIMENSIONAL PARTICLE TRACKING VELOCIMETRY
for the study of
VERTICAL SEDIMENT TRANSPORT
IN A TURBULENT FREE SURFACE FLOW

Graduation Committee:
Prof. dr. ir. J.A. BATTJES
Drs. R. BOOIJ
dr. ir. W.S.J. UIJITTEWAAL
Prof. dr. ir. Y. ZECH

Graduation report presented by
Frédéric ADRIAENS et
Lionel VAN RILLAER, in order to
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ABSTRACT

The objective of the present work is to see whether the transport of sediment in a turbulent free surface flow can be studied with a new method of video image analysis, called the Two-Dimensional Particle Tracking Velocimetry.

A slice of a two-dimensional turbulent flow in a laboratory flume is illuminated and particles are injected. After the recording of the illuminated particles, the video images are digitised and a software package called DigImage is used to analyse them, in order to give for each image the number of particles and their two-dimensional positions and velocities.

Four parameters are varied in the experiments: the diameter of the particles, the depth-averaged flow velocity, the distance between the injection and the point of observation, and the height of the injection. An analysis is then carried out in order to see the influence of these parameters on the concentration profiles and on the particles velocities. This analysis compares the measurements and the model of suspension of particles by coherent structures of turbulence (bursting phenomena).

This work concludes on one hand that the Two-Dimensional Particle Tracking Velocimetry by DigImage is reliable for the study of sediment transport as long as the video recordings are of good quality. It confirms on the other hand that the concentration profiles are not described well by a diffusion model. Finally, it concludes that some measurements agree with the bursting phenomena, in particular the observation of larger downward velocities and that some other measurements are more difficult to interpret, in particular downward velocities larger than the settling velocity when the injection is near the surface and near the point of observation.

RESUME

L'objectif de ce travail est de voir s'il est possible d'étudier le transport de sédiments dans un écoulement turbulent à surface libre avec une nouvelle méthode informatique d'analyse d'images vidéo, appelée "Two-Dimensional Particle Tracking Velocimetry".

Une partie de l'écoulement bidimensionnel réalisé dans un canal de laboratoire est illuminée et des particules y sont injectées. Après avoir filmé les particules illuminées, les images vidéo sont digitalisées et un logiciel (appelé DigImage) les analyse, afin de donner pour chaque image le nombre de particules, leur position et leur vitesse (selon deux directions).

Quatre paramètres sont modifiés lors des expériences: diamètre des particules, vitesse de l'écoulement, distance entre l'injection et le point d'observation et enfin hauteur de l'injection. Une analyse est ensuite réalisée afin de voir l'influence de ces paramètres sur les profils de concentration et les vitesses des particules. Enfin, une comparaison est entreprise entre les mesures effectuées et le modèle de mise en suspension des particules par des structures cohérentes de turbulence (phénomène du "bursting").

Ce travail conclut d'une part que le logiciel utilisé est efficace pour l'étude du transport de sédiments, à condition que les enregistrements vidéo soient de bonne qualité. D'autre part, il confirme que les profils de concentration mesurés ne sont pas bien décrits par le modèle classique de diffusion.

Enfin, il conclut que certaines mesures correspondent au modèle du bursting, en particulier l'observation de vitesses verticales (vers le bas) de particules plus élevées près du fond. D'autres observations restent difficiles à interpréter, notamment des vitesses verticales (vers le bas) de particules supérieures à la vitesse de sédimentation lorsque l'injection est en surface et proche du point d'observation.
INTRODUCTION

The transport of sediments in rivers and coastal zones has a great importance in relation to several engineering topics like erosion of river banks and around structures. In Bangladesh for instance, people are obliged to desert their houses because the river bank is moving several hundred of meters each year. On the other hand, erosion around the foundations of a bridge may cause its ruin. Another big problem in relation to the transport of sediment is the sedimentation in estuaries, reservoirs and channels. It leads to high costs all over the world: in the case of navigable channels, the zones of deposition have to be dredged; in the case of a reservoir, the diminution of the effective volume can oblige to end the hydroelectric production or the irrigation activities.

So, all the studies on the transport of sediment particles may have practical applications if they contribute to a better comprehension of the fundamental mechanisms that make particles move in a flow. This transport is ruled by several phenomena, so that it is common to split it into three modes of transports: bed load transport, suspended load transport and wash load transport. In the first one, the sediment particles are more or less in continuous contact with the bed during the transport: the particles roll, slide or jump along the bed. The suspended load consists of sediment moving without contact with the bed over relatively long distances (compared to the size of the sediment). Finally, the wash load is made up of very fine particles which are not present in the bed and that will not settle. This theoretical distinction is in practice not easy to make, as some particles may occasionally be moving on the bed or in suspension [14], [15].

A lot of empirical formulae exist to predict the bed load transport, the suspended transport or the total sediment transport (Meyer-Peter & Müller, Bagnold, Einstein, Yalin…), but "A simple deterministic description of sediment transport is still not available." (Fredsoe & Deigaard, 1992). Some models based on physical processes were also developed before the years 1960's and more recently many authors try to link the sediment transport and the model of turbulent structures. Unfortunately, up to now "The general state of our fundamental knowledge of much of the sediment physics remains relatively primitive" (Nezu and Nakagawa, 1993).

In order to develop a model that describes this transport, it is very important to have experimental data on the sediment concentration profiles at one's disposal. But this gives only information on the average transport of sediment particles, and not on the way a single particle is moving. So the idea of tracking particles suspended in a water flow was suggested for the first time by Batchelor in 1965 [11], and many experimental techniques have been developed for this. As one can imagine, it is very difficult to carry out such measurements in situ, and there are more parameters in a river than in a laboratory flume. So it is more convenient for the understanding of the fundamental mechanisms of sediment transport to first study them in a laboratory flume. But even in that case the analyses were done up to now by hand, yielding a small amount of data for long and boring measurements. It was not possible in these conditions to measure ensemble averages. Fortunately, the up-to-date technology supplies new methods for this kind of analysis.
The feasibility of 2D-PTV for the study of sediment vertical transport

The objective of the present work is to see whether it is possible to use a new method of flow visualisation, the Two-Dimensional Particle Tracking Velocimetry (2D-PTV), to collect experimental data on the vertical transport of particles for a turbulent free surface flow, in a laboratory flume. This method has already been used for the study of flows: with very small suspended particles which are assumed to follow the movements of the fluid perfectly. The reconstruction of the particles paths gives then the paths of the fluid parcels. Here the idea is different, as the particles are tracked for the study of their own movements.

The data collected will be analysed in relation to coherent structures of turbulence, which are held responsible for most of the transport in the upward direction. More particularly we will measure some ensemble averaged concentration profiles over the depth ($\bar{c}(z)$) and two-dimensional velocities of suspended sediment particles ($u_p(z), w_p(z)$) (Lagrangian point of view). These two variables are very important as they permit to calculate the ensemble average suspended load transport $\overline{Q_{sus}}$ in the streamwise direction (per unit width for a steady situation):

$$\overline{Q_{sus}} = \int_a^h \bar{u}_p(z)\bar{c}(z)dz$$

where $h$ is the flow depth and $a$ is a reference height for the suspended load.

The stages of the experiments are the following:
- injection of particles in the flow
- creation of an illumination sheet in the flow
- recording on a video tape of an illuminated "slice" of this flow
- digitising the images
- analysis of the digitised images, in order to determine for each image the number of particles in the slice, their positions and their velocities
- analysis of the influence of some parameters on the concentration profiles and the two-dimensional particles velocities (diameter of the particles, depth-averaged flow velocity, distance between the injection and the point of observation, height of injection).
To begin, the first chapter gives a theoretical approach to the sediment transport in a turbulent free surface flow, including a brief description of the theoretical settling velocity of a single particle, the classical diffusion and stochastic models, and finally the new approach that considers the coherent structures of turbulence.

The second chapter gives the principles of the Two-Dimensional Particle Tracking Velocimetry (2D-PTV) by the software package DigImage, which is used for the determination of two-dimensional positions and velocities of the sediment particles.

After that, the third chapter describes the experimental set-up, and more particularly the particles, the injection system, the recording system and the apparatus used for the measurement of the characteristics of the flow.

The two following chapters study the flow in which the recordings were made, the parameters studied in the different experiments (diameter of the particles, depth-averaged flow velocity, distance between the injection and the point of observation, height of injection) and the practical procedure used to carry out the experiments.

Chapter six analyses the influence of the different parameters studied on the concentration profiles and on the two-dimensional particles velocities.

Finally conclusions are drawn concerning the feasibility of the Two-Dimensional Particle Tracking Velocimetry by DigImage and concerning the particle concentrations and velocities.
The feasibility of 2D-PTV for the study of vertical sediment transport
Chapter 1  Vertical transport of sediment in a turbulent free surface flow

Chapter 1

VERTICAL TRANSPORT OF SEDIMENT IN A TURBULENT FREE SURFACE FLOW

Introduction

In order to describe a "turbulent flow" it is essential to make the distinction between laminar flows, turbulent flows and, of course, flows that are between these two extremes. A definition of turbulence is not easy to give. It is more convenient to list some characteristics of it [3]: irregularity, large transports (mass, momentum, heat), three dimensional nature, large dissipation of energy and continuum (the smallest scales for turbulence are much larger than the molecular scales). An idea about the occurence of turbulence is given by the Reynolds number, which is dimensionless and defined as:

\[ \text{Re} = \frac{UL}{\nu} \]  

(1.1)

with  \( U = \) characteristic difference of velocity in the flow  
\( L = \) characteristic length for this difference of velocity  
\( \nu = \) kinematic viscosity of the fluid

For instance, in the case of a free surface flow in a rectilinear channel,

\[ \text{Re} = \frac{U_mh}{\nu} \]  

(1.2)

with  \( U_m = \) depth-averaged velocity in the streamwise direction  
\( h = \) water depth

And in the case of a pipe flow,

\[ \text{Re} = \frac{U_mD}{\nu} \]  

(1.3)

with  \( U_m = \) mean velocity in the cross section of the pipe  
\( D = \) diameter of the pipe

In actual fact the turbulent flows are characterised by a large Reynolds number (i.e. larger than 1000). In such flows the turbulent transports (mass, momentum, heat) are so large that the molecular transports can be neglected. In particular, the upward sediment transport in a free surface flow is due solely to turbulence.

The first paragraph of this chapter studies the settling velocity of a single particle and the following study some models of vertical transport of mass in turbulent free surface flows.
1.1. Settling velocity

The \textit{settling velocity} \(w_s\) (or fall velocity) of a particle is defined as the terminal velocity attained when the grain is settling in an extended fluid under action of gravity. It depends on several parameters of which the most important are: particle size, relative density (see 1.10), particle shape, dynamic viscosity of the fluid and strength of the gravity field.

Its determination may appear a very easy problem to solve. In fact, it gives an idea about the difficulties met when comparing measurements and theoretical models: a lot of formulae exist, and it is not always easy to see which one is theoretical, which one is empirical and what is the validity of each.

The theoretical settling velocity of a single particle is found by resolving the equilibrium equation (see figure 1.1):

\[ F_D + B = G \]  \hspace{1cm} (1.4)

with \(F\) the drag force, \(B\) the buoyancy and \(G\) the gravity force:

\[ F_D = \frac{1}{2} \rho U_r^2 C_D A \]  \hspace{1cm} (1.5)

\[ B = \text{Vol} \rho g \]  \hspace{1cm} (1.6)

\[ G = \text{Vol} \rho_p g \]  \hspace{1cm} (1.7)

\[
\begin{align*}
\text{Figure 1.1: Equilibrium of a settling particle}
\end{align*}
\]

with

\(\rho\) = mass density of the fluid
\(\rho_p\) = mass density of the particle
\(C_D\) = \text{drag coefficient}, a dimensionless parameter defined by equation (1.5)
\(U_r\) = relative velocity between the flow and the particle
\(A\) = area of the projection of the body upon a plane normal to the flow direction
\(\text{Vol}\) = volume of the particle
\(g\) = gravitational acceleration
In the case of a spherical particle of diameter \( d \) \( (A = \frac{\pi}{4} d^2; \ \text{Vol} = \frac{\pi}{6} d^3) \) falling in a still fluid \( (U_r = w_s) \), equation (1.4) becomes:

\[
\frac{1}{2} \rho w_s^2 C_D \frac{\pi}{4} d^2 + \frac{\pi}{6} d^3 \rho g = \frac{\pi}{6} d^3 \rho_p g
\]

which gives:

\[
w_s = \sqrt{\frac{4(s-1)gd}{3C_D}} \tag{1.9}
\]

with \( s \) the relative density (found in dimensional analysis) defined as:

\[
s = \frac{\rho_p}{\rho} \tag{1.10}
\]

The dimensional analysis shows also that the drag coefficient depends exclusively on the particle Reynolds number, another dimensionless parameter, defined as:

\[
\text{Re}_p \equiv \frac{w_s d}{\nu} \tag{1.11}
\]

1.1.1. Theoretical formulae

It is possible to predict for a simple case (i.e. when the particle is spherical and when the flow is laminar) the drag force as a function of the velocity \( U_r \). As it is easy to estimate the buoyancy and the gravity force, the solving of equation (1.4) will give the settling velocity (for \( U_r = w_s \)). Two classical formulae exist: the formula of Stokes (valid for \( \text{Re}_p < 0.1 \)), and the formula of Oseen and Goldstein. It is experimentally shown that the validity of these two formulae is \( \text{Re}_p < 0.1 \) and 1.0 respectively.

A) For a spherical particle with \( w_s \) and \( d \) such that \( \text{Re}_p \) is small \((< 0.1)\), the drag force is given in first approximation by the formula of Stokes:

\[
F_D = 3 \pi \mu U_r d \quad \text{with} \quad U_r = w_s \tag{1.12}
\]

with \( \mu \) the dynamic viscosity of the fluid \((\mu = \rho \nu, \ \text{with} \ \nu \ \text{the kinematic viscosity of the fluid})\). The drag coefficient is in this case:

\[
C_D = \frac{F_D}{1/2 \rho w_s^2 A} = \frac{3 \pi (\rho \nu) w_s d}{1/2 \rho w_s^2 \frac{\pi}{4} d^2} = \frac{24}{\text{Re}_p} = \frac{24 w_s d}{\nu} \tag{1.13}
\]
Putting equation (1.13) in equation (1.9) gives:

\[ w_s = \frac{(s - 1)g}{18v} d^2 \]  

(1.14)

This formula is valid for a single particle of diameter \(d\). But in practice there is always a large number of particles with different diameters. We consider now a set of particles from diameter \(d_{\text{min}}\) to \(d_{\text{max}}\) and we define the interval of diameter as \(\Delta d \equiv d_{\text{max}} - d_{\text{min}}\). Assuming that the mass density is independent of the diameter and that the distribution of the diameter is uniform (for natural sediment the distribution is lognormal, but if we consider a set with a small \(\Delta d\) this approximation is rational), we can calculate the mean settling velocity of this set (\(w_{s,\text{set}}\)) as following:

\[
w_{s,\text{set}} = \frac{1}{\Delta d} \int_{d_{\text{min}}}^{d_{\text{max}}} w_s(d) \, dd = \frac{1}{d_{\text{max}} - d_{\text{min}}} \int_{d_{\text{min}}}^{d_{\text{max}}} \frac{(s - 1)g}{18v} d^2 \, dd
\]

\[
= \frac{(s - 1)g}{18v} \left( \frac{d_{\text{max}}^2 + d_{\text{min}}^2 + d_{\text{max}}^2}{3} \right) = \frac{(s - 1)g}{18v} d_{\text{m}}^2
\]

(1.15)

Defining now the mean diameter of the set (\(d_{\text{m}}\))

\[
d_{\text{m}} = \frac{d_{\text{min}} + d_{\text{max}}}{2}
\]

(1.16)

we may write

\[
d_{\text{m}}^2 = \frac{d_{\text{min}}^2 + d_{\text{max}}^2 + d_{\text{max}}^2}{3} = d_{\text{min}}^2 + d_{\text{max}}^2 + \frac{(\Delta d)^2}{3}
\]

(1.17)

\[
d_{\text{m}}^2 = \left( \frac{d_{\text{min}} + d_{\text{max}}}{2} \right)^2 = d_{\text{min}}^2 + d_{\text{max}}^2 + \frac{(\Delta d)^2}{4}
\]

(1.18)

Formulae (1.17) and (1.18) show that the mean settling velocity of a set of particles (given by equation 1.15) is estimated well by the settling velocity of the mean diameter of this set when \(\Delta d\) is not too large.

B) Another theoretical formula for \(C_D\) in the case of a spherical particle and a small Reynolds particle number (\(Re_p < 1\)), which considers the inertial terms in the Navier-Stokes equations, is given by Oseen and Goldstein as

\[
C_D = \frac{24}{Re_p} \left( 1 + \frac{3}{16} \frac{Re_p}{1280} + \frac{19}{20480} Re_p^3 + \ldots \right)
\]

(1.19)
Chapter 1  *Vertical transport of sediment in a turbulent free surface flow*

One of the hypotheses used for finding this formula is $Re_p < 1$, so that we obtain in first approximation ($3/16 \approx 0.2 ; 19/1280 \approx 0.015$)

\[
C_D = \frac{24}{Re_p} + 4.5
\]  

(1.20)

1.1.2. Empirical formulae

In the case of a non spherical particle or a particle with $w$ and $d$ such that $Re_p$ is large ($> 1$), the relation between $C_D$ and $Re_p$ is determined empirically (by measuring the settling velocity of natural sand, spherical particles...), and has typically the form depicted in figures 1.2.a and 1.2.b.

![Figure 1.2.a: Variation in $C_D$ with $Re_p$ for spheres](image)

![Figure 1.2.b: Variation in $C_D$ with $Re_p$ for natural sand and gravel](image)

Note that for $Re_p > 10^4$ the drag coefficient increases and then fall down abruptly. This must fortunately not be considered here, as such high Reynolds numbers are never reached with natural sediment particles in suspension.
The related expression depicted in these figures is given by many authors in the form:

$$C_D = \frac{\alpha}{Re_p} + \beta$$  \hspace{1cm} (1.21)

where $\alpha$ et $\beta$ are two dimensionless coefficients given in the table 1.1.

<table>
<thead>
<tr>
<th>Author(s)</th>
<th>References</th>
<th>Remarks</th>
<th>Magnitude of $Re_p$</th>
<th>$\alpha$</th>
<th>$\beta$</th>
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<tr>
<td>Stokes</td>
<td>[6, 8, 16]</td>
<td>theoretical laminar</td>
<td>$Re_p &lt; 0.1$</td>
<td>24</td>
<td>0</td>
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<td>Oseen (1927)</td>
<td>[16]</td>
<td>theoretical laminar</td>
<td>$Re_p &lt; 1$</td>
<td>24</td>
<td>4.5</td>
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<td>Goldstein (1929)</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Brown &amp; Hutchinson (1979)</td>
<td>[8]</td>
<td>empirical</td>
<td>$0.1 &lt; Re_p$ and small</td>
<td>24</td>
<td>0.44</td>
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<td>Govan (1984)</td>
<td></td>
<td></td>
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<tr>
<td>Engelund &amp; Hansen (1967)</td>
<td>[8]</td>
<td>empirical</td>
<td>$Re_p &lt; 10^4$</td>
<td>24</td>
<td>1.5</td>
</tr>
<tr>
<td>Rubey (1933)</td>
<td>[8, 16]</td>
<td>empirical</td>
<td>wide range of $Re_p$</td>
<td>24</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 1.1: Formulae for the relation between $C_D$ and $Re_p$

The figure 1.3 on the following page compares all these formulae on the same graph.

Notice that for coarse particle ($Re_p > 1000$), $C_D = \beta = constant$. According to equation (1.9) we find in this case a settling velocity proportional to the square root of the diameter of the particle.

Putting equation (1.21) in (1.9) gives:

$$\beta w_s^2 + \frac{\alpha v}{d} w_s - \frac{4}{3} (s - 1) gd = 0$$  \hspace{1cm} (1.22)

For $\beta = 0$ (Stokes; $Re_p < 0.1$), the equation is linear in $w_s$ and the settling velocity is given by equation (1.14). If $\beta \neq 0$ (the inertial effects are not negligible), the solution is:

$$w_s = \frac{\alpha v}{2 \beta d} \left( \sqrt{1 + \frac{16 \beta}{3 \alpha^2} \frac{(s - 1) gd^3}{v^2}} - 1 \right)$$  \hspace{1cm} (1.23)

This formula gives the settling velocity for a single particle. If we want to know the mean settling velocity of a set of particles ($d_{min}, d_{max}$), we have to do the same integration as in (1.15). Assuming that $Ad = d_{max} - d_{min}$ is small in comparison with $d_{min}$ and $d_{max}$, we may suppose that the mean settling velocity of the set will be estimated well by the settling velocity of the mean diameter of this set.
Comparison of the different formulae for the relation \((\text{Re}_p, \text{C}_D)\)

\[
\text{C}_D = \alpha / \text{Re}_p + \beta
\]

- Stokes: \(\alpha = 24, \beta = 0\)
- Oseen: \(\alpha = 24, \beta = 4.5\)
- Brown & Hutchinson: \(\alpha = 24, \beta = 0.44\)
- Engelund & Hansen: \(\alpha = 24, \beta = 1.5\)
- Rubey: \(\alpha = 24, \beta = 2\)
- Fredsoe & Deigaard: \(\alpha = 36, \beta = 1.4\)

Figure 1.3: Comparison of the different formulae for the relation between the drag coefficient and the particle Reynolds number
Equation (1.23) may be written:

\[ \text{Re}_p = \frac{\alpha}{2\beta} \left( \sqrt{1 + \frac{16\beta (s-1)g}{3\alpha^2 \nu^2}} \right) - 1 \]  

(1.24)

In practice, the procedure for the computation of a settling velocity is the following:
- computation of the particle Reynolds number with (1.24) for the coefficients \( \alpha \) and \( \beta \) of the different formulae
- looking which values of \( \text{Re}_p \) are in the domain of validity of the corresponding formula
- compute \( w_s \) with (1.23) for all the valid formulae and compare the results.

Notice that defining a length \( \lambda \) as:

\[ \frac{1}{\lambda^2} = \frac{16\beta (s-1)g}{3\alpha^2 \nu^2} \]  

(1.25)

equation (1.23) becomes:

\[ w_s = \frac{\alpha}{2\beta} \frac{\nu}{d} \left( 1 + \left( \frac{d}{\lambda} \right)^3 - 1 \right) \]  

(1.26)

This shows that for
- very small particles \( (d/\lambda << 1) \), \( \sqrt{1 + \left( \frac{d}{\lambda} \right)^3} \equiv 1 + \left( \frac{d}{\lambda} \right)^3 / 2 \) (Taylor), so that \( w_s \sim d^2 \), which agree with the formula of Stokes (1.14)
- coarse particles \( (d/\lambda >> 1) \), \( \sqrt{1 + \left( \frac{d}{\lambda} \right)^3} - 1 \equiv \sqrt{\left( \frac{d}{\lambda} \right)^3} - 1 \equiv \left( \frac{d}{\lambda} \right)^3 \), so that \( w_s \sim \sqrt{d} \).

Notice also that defining the dimensionless particle diameter \( d_* \) (found in dimensional analysis) as

\[ d_* = \left( \frac{16\beta (s-1)g}{3\alpha^2 \nu^2} \right)^{\frac{1}{3}} d \]  

(1.27)

(1.23) may be written

\[ w_s = \frac{\alpha}{2\beta} \frac{\nu(s-1)g}{d_*} \left( 1 + \frac{16\beta}{3\alpha^2 d_*^2} \right) - 1 \]  

(1.28)
1.2. Diffusion models

1.2.1. Basic equation for the vertical transport of suspended particles

The *Ricci notations* are used: the subscript \( i \) denotes the component of a vector in direction \( i \); \( \nabla_i f \equiv \frac{\partial f}{\partial x_i} \) is the partial derivative of \( f \) with respect to the direction \( j \). Use is also made of the *Einstein summation convention*: if a subscript appears twice in a term, it denotes a sum of three terms in which the subscript takes the values 1, 2 and 3; in particular,

\[
\nabla_i^2 = \nabla_i \nabla_i = \nabla_i \nabla_i V_i + \nabla_i \nabla_i V_1 + \nabla_i \nabla_3 = \frac{\partial}{\partial x^i} + \frac{\partial}{\partial y^i} + \frac{\partial}{\partial z^i}.
\]

The source of the basic equations of hydrodynamics is the balance equation, also called "convection-diffusion equation", which expresses the variation of an extensive variable \( G \) (i.e. a variable proportional to the volume considered) in a control volume (Eulerian point of view):

\[
\frac{\partial c}{\partial t} + \nabla_i (V_i c) + \nabla_i T_i + D = 0
\]

with \( c = \frac{dG}{dVol} \), the concentration of \( G \) [dimensions of \( G/m^3 \)]

\( t = \) time

\( V_i = \) component of the fluid velocity in the direction \( i \)

\( T_i = \) transport of \( G \) per unit time and per unit surface through a surface perpendicular to the direction \( i \) [dimensions of \( G/(m^2s) \)]

\( D = \) loss of \( G \) per unit time en per unit volume [dimensions of \( G/(m^3s) \)].

The first term of this equation represents the variation in time of \( G \) per unit volume; the second represents the net effect of convective transport of \( G \); the third represents the diffusive transport and the last represents the inner variation of \( G \) (e.g. by chemical reaction).

The appendix 1.1 gives the basic equations of hydrodynamics, obtained with equation (1.29) applied to the mass and the momentum. It then explains the fundamental problem encountered in these equations when the flow is turbulent, and the splitting of an instantaneous value in a sum of an ensemble average value and an instantaneous deviation. Finally, it gives the basic equations of turbulence found with this splitting.

The hypothesis of a *gradient-type transport* is taken for the diffusive transport of mass for a substance mixed with water: \( T_i = -D_m \nabla_i c \) [kg/(m²s)]

with \( D_m = \) coefficient of molecular diffusion [m²/s]

\( c = \) concentration in weight of the substance mixed with water [kg/m³]

Putting this equation in the convection-diffusion equation (1.29) gives the *molecular diffusion equation*:

\[
\frac{\partial c}{\partial t} + \nabla_i (v_i c) - D_m \nabla_i^2 c + \varepsilon_0 = 0
\]

with \( v_i \) the local velocity of the substance mixed with water in the direction \( i \) (= \( V_i \) if this substance is soluble)

\( \varepsilon_0 \) the loss of concentration (e.g. by chemical reaction) per unit time [kg/(m³s)].
Notice that using the equation of continuity (C) (cf. appendix 1.1), this equation becomes:

\[ \frac{\partial \bar{c}}{\partial t} + \bar{v}_i \cdot \nabla_i \bar{c} - D_m \nabla^2 \bar{c} + \varepsilon_G = 0 \]  

(1.32)

with \( \cdot \) the scalar product between two vectors.

For the study of the transport in a turbulent flow, each variable is replaced by the sum of its ensemble average value and its instantaneous deviation value (see appendix 1.1). Doing after that an ensemble average of the equation, we obtain:

\[ \frac{\partial \bar{c}}{\partial t} + \nabla_i \left( \bar{v}_i \ \bar{c} \right) + \nabla_i \left( \bar{v}'_i \ c' \right) - D_m \nabla^2 \bar{c} + \varepsilon_G = 0 \]  

(1.33)

As in the case of the ensemble averaging of the Navier-Stokes equation (see appendix 1.1), a term due to the turbulent fluctuations appears; it represents the turbulent transport.

The following hypotheses are now made:

1) The turbulent transport is a gradient-type diffusion transport (i.e. the turbulent transport can be described as a diffusion process):

\[ (T)_{turb} = \bar{v}_i c' = -D_0 \nabla_i \bar{c} \]  

(1.34)

with \( D_0 \) a tensor (3 x 3) containing the turbulent diffusion coefficients.

2) The co-ordinates axis are defined such that they correspond to the principal axis of the turbulent movements, i.e. \( D_{ij} = 0 \) for \( i \neq j \).

3) The coefficient of molecular diffusion \( D_m \) is negligible compared to the coefficients of turbulent diffusion of the principal axis \( D_{ii} \).

With these three hypotheses and using the ensemble average equation of continuity (\( \bar{C} \)), equation (1.32) becomes the so called "semi-empiric diffusion equation":

\[ \frac{\partial \bar{c}}{\partial t} + \bar{v} \cdot \nabla_i (\bar{c}) - \nabla_i (D_n \nabla_i \bar{c}) + \varepsilon_G = 0 \]  

(1.35)

Should the turbulence been homogeneous and isotropic, then

\[ \nabla_i (D_n \nabla_i \bar{c}) = \frac{\partial}{\partial x} \left( D_{xx} \frac{\partial \bar{c}}{\partial x} \right) + \frac{\partial}{\partial y} \left( D_{xy} \frac{\partial \bar{c}}{\partial y} \right) + \frac{\partial}{\partial z} \left( D_{xz} \frac{\partial \bar{c}}{\partial z} \right) \]

\[ = D \left( \frac{\partial^2 \bar{c}}{\partial x^2} + \frac{\partial^2 \bar{c}}{\partial y^2} + \frac{\partial^2 \bar{c}}{\partial z^2} \right) = D \nabla^2 \bar{c} \]  

(1.36)

and equation (1.35) is analogous with equation (1.32).
In the case of a steady situation \( \frac{\partial \bar{c}}{\partial t} = 0 \) with a conservative substance \( \bar{c}_0 = 0 \), equation (1.34) becomes:

\[
\bar{v} \cdot \nabla f(\bar{c}) = \nabla f(D_\theta \nabla \bar{c})
\]  

(i.e. the convective transport and the turbulent diffusive transport are in equilibrium.

Considering now supplementary hypotheses:
* the flow is uniform in directions \( x \) and \( y \) \( \partial \bar{c} / \partial x = 0 \), \( \partial \bar{c} / \partial y = 0 \)
* all the particles have the same settling velocity \( w_s = \text{constant} \)
* the gravitation is the only external force \( (u, v, w) = (0, 0, -w_s) \)

then equation (1.35) simplifies in:

\[
w_s \frac{\partial \bar{c}}{\partial z} + \frac{\partial}{\partial z} \left( D_{zz} \frac{\partial \bar{c}}{\partial z} \right) = 0
\]  

(1.38)

with \( D_{zz} \) the turbulent diffusion coefficient in the vertical direction \([\text{m}^2/\text{s}]\).

The integration of this equation gives:

\[
\int w_s \frac{\partial \bar{c}}{\partial z} \, dz + \int \frac{\partial}{\partial z} \left( D_{zz} \frac{\partial \bar{c}}{\partial z} \right) \, dz = 0 \quad \Rightarrow \quad w_s \bar{c} + D_{zz} \frac{\partial \bar{c}}{\partial z} = \text{constant}
\]  

(1.39)

As there is no transport at the water surface, the constant of integration is zero:

\[
w_s \bar{c} = -D_{zz} \frac{\partial \bar{c}}{\partial z}
\]  

(1.40)

This equation represents the equilibrium of the suspended particles settling through a unit area parallel to the bed \( (w_s \bar{c}) \), and the turbulent diffusion of the suspended sediment \( D_{zz} \frac{\partial \bar{c}}{\partial z} \).

\[\text{Figure 1.4: Equilibrium of the sediment transport}\]

This equation shows that the equilibrium concentration profile \( \bar{c}(z) \) depends on the settling velocity of the particles and on the turbulent diffusion coefficient in the vertical direction. For the integration of this equation, \( D_{zz} \) must be expressed as a function of the height \( z \). This is done in different ways by many authors.
1.2.2. Rouse-Einstein distribution

In order to determine $D_{zz}$, Rouse and Einstein made use of the Reynolds analogy, which is the analogy between the turbulent transport of mass in the vertical direction (characterised by the turbulent diffusion coefficient $D_{zz}$) and the turbulent transport of momentum (characterised by the eddy viscosity $\nu_e$) (notice that the same analogy holds for the transport of heat).

In both cases the transport is described by the mixing length model of Prandtl (1925): "fluid parcels" move about a characteristic length $l_m$, called the mixing length, without exchanging any property (mass, momentum, heat) with the surrounded fluid parcels. After that they completely mix them with the surrounding fluid. As the transport of momentum and mass is assumed to occur in the same way, we may write:

$$D_{zz} \equiv \nu_e = \kappa u_* z \left(1 - \frac{z}{h}\right)$$  \hspace{1cm} (1.41)

The appendix 1.2 explains the different hypotheses made for finding this expression of the eddy viscosity $\nu_e$, and gives a physical interpretation of equation (1.40).

Equation (1.39) may thus be written:

$$w_z \bar{c} = -D_{zz} \frac{\partial \bar{c}}{\partial z} = -\kappa u_* z \left(1 - \frac{z}{h}\right) \frac{\partial \bar{c}}{\partial z}$$  \hspace{1cm} (1.42)

As the integration of this equation gives a logarithm for $z$, it is not possible to integrate it from $z = 0$; it is then done from a reference height $a$ where the concentration is $C_a$:

$$\int_{C_a}^{c(z)} \frac{dc}{c} = \int_a^z \frac{dz}{\kappa u_* z \left(1 - \frac{z}{h}\right)}$$  \hspace{1cm} (1.43)

$$\ln \left(\frac{c}{C_a}\right) = -\frac{w_z}{\kappa u_*} \left[ \ln \left(\frac{z}{1 - \frac{z}{h}}\right) - \ln \left(\frac{a}{1 - \frac{a}{h}}\right) \right] = -\frac{w_z}{\kappa u_*} \ln \left(\frac{z}{h-z} \frac{h-a}{a}\right)$$  \hspace{1cm} (1.44)

which gives the Rouse-Einstein distribution:

$$\bar{c}(z) = C_a \left(\frac{a}{h-a} \frac{h-z}{z}\right)^{\frac{1}{\kappa u_*}} \left(\frac{w_z}{\kappa u_*}\right)$$  \hspace{1cm} (1.45)

with $C_a$ = reference concentration of suspended sediment at a distance $a$ above the bed

$\frac{w_z}{\kappa u_*} \equiv Ro = \text{Rouse parameter}$ (dimensionless).

For more details about the Rouse-Einstein distribution, see appendix 1.3.
The distribution found with the classical diffusion model is an exponential curve, a property that agrees qualitatively with the experiments. Experimental curves which agree with this distribution can be found in [2,20]. In contradistinction to these results, some recent experiments [4, 6, 7, 8, 11, 14, 15] show that it do not give quantitative good results (in general the concentrations found are too small).

1.2.3. Variants of the Rouse-Einstein distribution

In fact the description of the Rouse-Einstein distribution in 1.2.2 is a simplified version of the original version. In this one the diffusion coefficient is not taken equal to the eddy viscosity, but proportional to it:

$$D_{zz} = \beta v_t$$  \hspace{1cm} (1.46)

with $\beta$ the coefficient of proportionality. Equation (1.78) becomes with this modification:

$$\bar{c}(z) = C_o \left( \frac{a}{h-a} \frac{h-z}{z} \right)^{\frac{1}{\beta \kappa u_\ast}}$$  \hspace{1cm} (1.47)

As shown in appendix 1.3 this will strongly modify the shape of the profile (replace the Rouse parameter Ro in figure 1.3 by its new value Ro/$\beta$).

$\beta$ may be found empirically by fitting the theoretical distribution to the measurements by a graphical method (draw some curves with different values for $\beta$ and decide with the eye which one fits the measurements best) or by a numerical method, for instance the method of least squares. As the previous equation may be written as:

$$\ln \left( \frac{\bar{c}(z)}{C_o} \right) = \frac{1}{\beta \kappa u_\ast} \ln \left( \frac{a}{h-a} \frac{h-z}{z} \right)$$  \hspace{1cm} (1.48)

and as that the method of least squares gives for a set of $N$ pairs $(x_i,y_i)$ the linear regression

$$y = \alpha x \text{ with: } \alpha = \frac{\sum_{i=1}^{N} x_i y_i}{\sum_{i=1}^{N} x_i^2},$$  \hspace{1cm} (1.49)

the coefficient $\beta$ is computed from the measured pairs $(z_i, \bar{c}_i = \bar{c}(z_i))$ with:

$$\beta = \frac{1}{\kappa u_\ast} \frac{\sum_{i=1}^{N} \left[ \ln \left( \frac{a}{h-a} \frac{h-z_i}{z_i} \right) \right]^2}{\sum_{i=1}^{N} \left[ \ln \left( \frac{a}{h-a} \frac{h-z_i}{z_i} \right) \ln \left( \frac{\bar{c}_i}{C_o} \right) \right]}$$  \hspace{1cm} (1.50)

Notice that $\beta$ is proportional to the Rouse parameter, which shows once again the importance of this parameter.
Some authors found a relation between the coefficient estimated and the physical properties of the flow and the sediment. For instance, van Rijn (1984) [6] proposes for small concentrations:

$$\beta = 1 + 2 \left( \frac{W_s}{u_*} \right)^2 \text{ if } 0.1 < \frac{W_s}{u_*} < 1 \quad (1.51)$$

and for large concentrations:

$$D_{zz} = \beta \sigma v_l \quad (1.52)$$

with

$$\sigma = 1 + \left( \frac{\bar{c}(z)}{c_0} \right)^{0.8} - 2 \left[ \frac{\bar{c}(z)}{c_0} \right]^{0.4} \quad (1.53)$$

in which $c_0 = 0.65$ is the maximum bed concentration.
1.2.4. Other diffusion models [20]

For the determination of the turbulent diffusion coefficient in the vertical direction \(D_{zz}\), some authors use the same process as the one of Rouse and Einstein, but with another formula for the gradient of the streamwise velocity in the vertical direction (see appendix 1.2).

For instance, **Hunt** (1954) [20] uses von Karman velocity distribution:

\[
\frac{\partial \left( \frac{\overline{U}}{u_*} \right)}{\partial \eta} = \frac{1}{2\kappa \left( B - \sqrt{1 - \eta} \right)} \tag{1.54}
\]

where \(\eta\) is the relative height defined as \(\eta \equiv z / h\) (cf. appendix 1.2). 
\(B\) is "very slightly smaller than unity".

He derives with this distribution the following concentration profile:

\[
\frac{\overline{c}(\eta)}{C_o} \frac{1 - C_o}{1 - \overline{c}(\eta)} = \left[ \frac{1 - \eta}{1 - \eta_o} \frac{B - \sqrt{1 - \eta_o}}{B - \sqrt{1 - \eta}} \right]^{1/w_o} \tag{1.55}
\]

**Feng-Ming Chang, Simons and Richardson** (1967) [20] use the following distribution velocity, found with Prandtl's mixing length hypothesis:

\[
\frac{\overline{U}_m - \overline{U}}{u_*} = \frac{2}{\kappa} \left[ \ln \left( \frac{\sqrt{\eta}}{1 - \sqrt{1 - \eta}} \right) - \sqrt{1 - \eta} - \frac{1}{3} \right] \tag{1.56}
\]

and derive the following concentration profile:

\[
\overline{c} = C_o \frac{\eta}{\eta_o} \left( \frac{1 - \sqrt{1 - \eta_o}}{1 - \sqrt{1 - \eta}} \right)^2 \tag{1.57}
\]

which is very strange, as it does not depend on the characteristics of the particles \(w_o\) nor on the characteristics of the flow \(u_*\).
1.3. Other models

Other kinds of models have also been developed. Two of them are briefly described: the gravitational model of Velikanov (1955) and a stochastic model due to Yalin and Krishnappar (1973).

A. Gravitational model (Velikanov) [20]

This model is based on energetic considerations: the settling of the sediment is due to the gravity force, whereas the displacement of the particles in the positive vertical direction is due to the "non conservative hydrodynamic forces". As the flow brings particles up continuously, it loses a certain amount of its energy continuously. Velikanov expresses this loss of energy and writes an energetic equilibrium for the flow. He so derived the differential equation of the gravitational theory, which is completely analogous to the one of the diffusion theory:

\[ w_s \ddot{c} + \frac{\dddot{U} \tau_{xx}}{(\rho_p - \rho)g} \dddot{c} = 0 \]  \hspace{1cm} (1.58)

Velikanov took the classical linear expression for the shear stress and the following expression for the velocity

\[ \dddot{U} = \frac{u_*}{\kappa} \ln \left( \frac{\eta}{\psi} \right) \]  \hspace{1cm} (1.59)

with \( \psi = \frac{k}{30h} \) the equivalent sand roughness of Nikuradse (dimensionless).

The integration is unfortunately not possible analytically, and gives:

\[ \dddot{c} = C_n \exp \left[ -\frac{w_s}{u_*} \frac{\kappa}{S_0} (s-1) \int_{\eta_s}^{\eta} \frac{d\eta}{(1-\eta)\ln(\eta/\psi)} \right] \]  \hspace{1cm} (1.60)

with \( S_0 \) the slope of the channel.

Some experiments were carried out with a photoelectric method and with a cinematographic method and confirm the validity of this formula.
B. Stochastic model (Yalin & Krishnappar) [20]

The basic idea is the following: as the particles are transported by the turbulent fluctuations which are random (by the authors), the transport can be described as a random process. On the other hand in the case of homogeneous turbulence, turbulent diffusion takes place so as to yield a normal density curve for the dispersion of fluid particles in the vertical direction (for a two dimensional uniform flow). But the turbulence in a two-dimensional free surface flow is not homogeneous. So Yalin and Krishnappar consider very narrow stripes of height \( \Delta z \) and say that into these stripes the turbulence may be considered homogeneous.

After that they develop a "generating function" which expresses the probability that the particle on position \( z_n \) at time \( t_n \) will be at the position \( z_{n+1} \) at the time \( t_{n+1} = t_n + \Delta t \), and which is the normal density function:

\[
f_n(z_{n+1}, z_n) = \frac{1}{\sigma_{\Delta t} \sqrt{2\pi}} \exp \left[ -\frac{1}{2} \left( \frac{z_{n+1} - (z_n - w z \Delta t)}{\sigma_{\Delta t}} \right)^2 \right]
\]

(1.61)

with \( \sigma_{\Delta t} \) the variance of the normal function. This equation expresses that the probability that a particle at the end of the \( n+1 \)th step will be at the level \( z_{n+1} \) depends solely on its level \( z_n \) at the end of the \( n \)th step (Markov chain). The variance is then expressed as the product of the vertical flow velocity at the height \( z_n \) and the interval of time \( \Delta t \).

Yalin and Krishnappar give after that a condition on \( \Delta t \) such that \( \Delta z \) is small enough to consider that the turbulence is homogeneous. Finally, they demonstrate that the concentration profile can be computed with:

\[
C = C_0 \alpha_0 \sum_{n=0}^{\infty} f_n(z; \alpha)
\]

(1.62)

with \( \alpha_0 \) a constant of proportionality. This is done with a computer simulation (see figure 1.5) which displace the particle (initially in \( z = \alpha \)) in each step to a random level distributed according to the normal distribution.

Figure 1.5: Computer simulation based on the stochastic model of Yalin and Krishnappar
1.4. Vertical transport and coherent structures

The weakness of all the models seen above is that they do not incorporate modern knowledge of turbulence. They are not based on the mechanics of turbulence but use only the continuity equation for sediment transport or parametric probability functions. Furthermore they are kinematic (and not hydrodynamic) models.

The basic idea of a recent approach came after the discovery (with techniques of flow visualisation at the end of the 1960's) of "coherent structures of turbulence". This idea is that the sediment particles are almost only transported by these large turbulent structures. This concept yields to a revolution because all the models which use local descriptions (and thus also the famous classical diffusion model) cannot describe well the process as they do not consider these structures. This idea was confirmed by some Lagrangian measurements of particle displacements [14, 15], but "... sediment transport has not yet been so well described on the basis of relevant turbulent knowledge" (Nezu and Nakagawa, 1993).

The next paragraphs give a brief description of these large structures of turbulence and of a model using them. For a more detailed description of the mechanism of particle suspension, see the review of the experiments of Sumer & Oguz [14] and of Sumer & Deigaard [15], who discuss also the experiments of Corino & Brodley (1969), Grass (1971), Nychas & al (1973) and the models of this mechanism by Sutherland (1967) and by Offen & Kline (1973, 1975).


Coherent structures are identified with motion of fluids parcels that have a life cycle including birth, development, interactions and breakdown. In rivers they are classified in two categories: the bursting phenomena and the large-scale vortical motions. In a smooth laboratory flume, only the bursting phenomena are observed, as the vortical motions occur only with a rough bottom or with the presence of obstacles in the flow (bridge, bed protection work,...). The most important characteristics of the bursting phenomena are described briefly in this paragraph.

The bursting phenomena are composed of a complete sequence of lift-up, oscillation, ejection and sweep motions, which generates turbulent energy and Reynolds stresses. This begins in the smooth boundary layer, where two kinds of structures are found: the high-speed streaks (characterised by high velocities) and the low-speed streaks (characterised by low velocities). At a certain time, an individual low-speed streak lifts away from the wall, oscillates (in three dimensions) and then breaks down, while a portion of it is ejected into the outer flow ("ejection"). Subsequently, a high-speed parcel of fluid approaches the wall and sweeps away the retarded fluid that remains there from the ejection process ("sweep").
Chapter 1  *Vertical transport of sediment in a turbulent free surface flow*

The sequence of bursting motions is a **quasi-cyclic process**, i.e. it displays, on average, a nearly periodic motion in space and time, but it is not perfectly periodic in either time or space. The following relation was found for free surface flows and boundary layers by Rao et al (1971; adapted by Laufer & Narayanan in 1977), Lu & Willmarth (1973) and Nakagawa & Nezu (1978):

\[
\frac{T_B U_{\text{max}}}{h} = \frac{T_E U_{\text{max}}}{h} = \frac{T_S U_{\text{max}}}{h} = (1.5 - 3.0) \tag{1.63}
\]

with
- \( T_B \) = mean bursting period
- \( T_E \) = mean ejection period
- \( T_S \) = mean sweep period
- \( U_{\text{max}} \) = maximum velocity (assumed at the surface).

This relation shows that a sequence of bursting motions is composed of one ejection and one sweep on the average. It shows also that the mean periods are not dependent on the Reynolds number or the wall roughness bursting phenomena occur with both a rough and a smooth wall).

The most plausible model for the bursting phenomena is a "**horse-shoe vortex**" (also called by different authors "hairpin vortex", "Π-shaped eddy", "Ω-shaped eddy", "Λ-shaped vortex" or "loop vortex"), which is represented in figure 1.6:

![Figure 1.6: Conceptual model of cyclic bursting phenomena near a wall (Wallace, 1985) [11]](image-url)
It was shown that the instantaneous velocity profiles associated with the bursting phenomena show unstable inflections:

![Bursting phenomena and velocity profiles](image)

**Figure 1.7: Bursting phenomena and velocity profiles [3]**

A side view of the development of a burst is represented in the two following figures:

![Side view of the development of a burst](image)

**Figure 1.8: Side view of the development of a burst**

[14, adapted from Offen & Kline 1973]
1.4.2. Model of particle transport with the bursting phenomena (Wiltink, 1995) [19]

Wiltink made a model in order to find concentration profiles with a numerical simulation which considers the bursting phenomena. This model is based on the movement of individual particles which are dropped at the same point with a constant interval of time. The model simulates the individual paths of the particles and counts afterwards for a given time how many particles are in the different interval of height defined.

The model splits the particle paths in three parts: movement over the bottom before pick-up (1), movement of the particle in 20° upward the direction with a burst (2) and settling of the particle (3).

As shown on figure 1.10, this model considers that when a particle settles down it can be lifted up by another burst before it reaches the bottom.
The feasibility of 2D-PTV for the study of vertical sediment transport
Chapter 2

TWO-DIMENSIONAL
PARTICLE TRACKING VELOCIMETRY

Introduction

A lot of techniques have been developed over the years for obtaining qualitative and quantitative measurements of fluid flow. The first observations were qualitative and try to explain the structures present within some finite part of the flow. As for the quantitative measurements, they require a laborious work by hand. For example the Laser-Doppler flow meter gives measurements of the velocity at one or more isolated points within the flow with great accuracy but it does not offer a fast and accurate insight into the structures present.

To overcome these physical limitations (number of points remains relatively small) the measurement technique can be based on a multi-dimensional visualisation of the flow. One of these techniques is the Particle Tracking Velocimetry, in which the flow is seeded with small, (nearly) neutrally buoyant particles which are assumed to follow fluid elements without affecting the flow itself. The flow is illuminated in some manner such that the particles in some finite domain are visible. A series of images of the flow is captured, to record how the particles move in response to the fluid flow. These images are analysed to determine how the particles, and hence the fluid elements, move in time.

The Particle Tracking Velocimetry uses a time series of images, which offers a great volume of information on the particle positions as a function of time. Knowing the approximate location of a particle at a relatively large number of times enables a much more accurate estimation of the position of a particle at a given time, and of its velocity, provided the sampling frequency is much higher than the highest frequency in the particle motion. To use this information some sophisticated algorithms must be available for tracking particles from one image to the next.

The accuracy with which the velocities may be measured is limited by the accuracy with which the individual particle images may be located and the time period over which the velocity may reasonably be evaluated. The accuracy of location depends on the particle size and the method used to determine their positions. The spatial resolution is limited primarily by the number of particles in the flow: the more particles, the higher the resolution. Eulerian as well as Lagrangian descriptions may be obtained, utilising a suitable interpolation method, if the particle seeding density is sufficiently high.

This chapter outlines and describes the Two-dimensional Particle Tracking technique used by DigImage. DigImage is a software package created by Stuart Dalziel (Cambridge Environmental Research Consultants Ltd., 1993-1995).
2.1. Two-Dimensional Particle Tracking Velocimetry by DigImage

The projection of a lightsheet through the water will produce a two-dimensional image and illuminate some particles (see chapter 3). With the help of a camera, images of the lightsheet with particles will be made which are subsequently recorded with the video tape recorder (VTR) on a video-tape. After that, we can start the Particle Tracking Velocimetry. A computer controls the VTR and its monitor, grabs the images from the video-tape (see paragraph 2.1.1.) and determines the position of the particles in each image (see paragraph 2.1.2.). Once the particles have been located, their paths have to be determined. To recognise which particle in an image is the same particle in a further image, we will use the matching algorithm which determines the combinations between the particles in two following images. The probability of the particles path is given by a cost function (see paragraph 2.1.3.).

![Image of DigImage set-up](image)

Figure 2.1: DigImage set-up

2.1.1. Image capture

The Particle Tracking requires the physical dimension of the images. A link between this and the pixels of the screen will be established by the world co-ordinate system which has to be determined before the tracking proceeds. This can be done with a grid which is temporally placed in the experimental apparatus (in the flume). A number of locations on this grid can be recognised in the video-picture and so specify the world co-ordinates. Generally a simple linear mapping between pixels and world co-ordinate spaces will be chosen.

During the tracking phase, the video-tape is replayed and the images are captured by digitising the video signal. The Super VHS video tape recorder is fitted with an interface allowing computer control of all the video functions. The interface allows the computer to interrogate the VTR to determine what it is doing and where it is. To increase the accuracy
of these positions, each video tape will be pre-formatted with a time-code pulse on one of the audio-tracks. This short tone pulse is recorded every eight video fields (four frames). The relative position of these pulses is used as a check for the field count, during the subsequent tape operations.

It is always better to ensure clean experimental images than to throw away information by trying to remove unwanted noise. But sometimes when an image has been captured, it may be necessary to use some filters to increase the quality of the image.

Generally one problem occurs: the camera has an electronic shutter which is open on a cycle of 50 Hz, whereas the complete video frame is produced at only 25 Hz. Thus the information on one half of the interlace (the even lines) corresponds to an earlier time than the information on the other half of the interlace (odd lines) (see figure 2.2).

![Figure 2.2: Odd and even lines of a particle](image)

When the flow velocities are large (this is the case with a flow velocity in the flume of 10, 15 or 20 cm/s), the particles move a distance of one or more particle diameters between the two halves of the interlace, and the particles appear in two positions in one image. Under these circumstances it is necessary to reduce the vertical resolution from 512 lines to 256 lines, using only one half of the interlace to determine the particle positions. To reduce this phenomenon, the camera can produce an interlaced signal for which both video fields contain information at the same time.

2.1.2. Particle location

When an image with particles is taken, we have to locate these in this image. Generally, the particles have a higher intensity (brightness) than the background; if they have the same intensity as the background, we cannot detect them with a grey-level camera. DigImage uses a method of determining where a particle is located by fitting a Gaussian intensity distribution to the particle image and then determine the centroid of the Gaussian.

A particle in DigImage is defined as an area of an enhanced image satisfying a number of criteria, based on the intensity, size and shape of the particles. The most basic criterion is the intensity which is used to identify potential particles or blobs within an image.

Blobs can be defined as group of connected pixels in an image with an intensity higher than the background. The detection of blobs can be done with the help of a threshold. When a
region satisfying the threshold criterion has been found, it is marked (as found) and its properties determined to see if it is in fact a particle. Only blobs which fall within a given range of sizes (lower and upper size limit) and linear dimensions (minimum horizontal and vertical size), shapes (based on the x-y correlation of pixels satisfying the threshold) and which have an average intensity sufficiently above the threshold will be treated as particles.

![Intensity profile of an image](image)

Figure 2.3: *Intensity profile of an image*

This works very nicely if the background has an even intensity. A one-dimensional example is illustrated in figure 2.3. Using one threshold in this image, we might find only one or two blobs which are candidates for particles. Two or more (up to 8) thresholds can be used to find more blobs and thus more candidates for particles.

A first guess at the thresholds might be made either by looking at the intensity structure of the image (acquired from the video tape). The threshold and other controlling parameters may then be modified by an iterative process to determine which particles (and how many) are found. We will try to maximise the number of particles found, but simultaneously minimise the number of blobs rejected.
2.1.3. Particle matching

After finding the location of particles in several images, DigImage will try to reconstruct the paths of the particles. The matching procedure used for this is described below [5], [17], [18].

We will take a set of associations between two sets of entities, such that the set of associations is optimal in the sense that it minimises some linear function of the associations it includes. Suppose we have some or a lot of particles at $t = t_n$ in an image and also in an image at $t = t_{n+1}$:

\[
\begin{align*}
&t = t_n : & P &= \{p_1, \ldots, p_i, \ldots, p_M\} \\
&t = t_{n+1} : & Q &= \{q_1, \ldots, q_i, \ldots, q_N\}
\end{align*}
\]

with:

- $P$ : a set of $M$ particles ($p_i$) found at $t = t_n$
- $Q$ : a set of $N$ particles ($q_i$) found at $t = t_{n+1}$

We now define a set of association variable, between $p_i$ and $q_j$ as:

\[
\alpha_{ij} = \begin{cases} 0 & \text{if } p_i \text{ at } t = t_n \text{ is a different particle as } q_j \text{ at } t = t_{n+1} \\ 1 & \text{if } p_i \text{ at } t = t_n \text{ is the same particle as } q_j \text{ at } t = t_{n+1} \end{cases}
\]

For any given $p_i$ only one or none $\alpha_{ij}$ can be equal to one. Else a particle would have to be at two places at the same time. Identical arguments can be applied for each $q_j$.

![Figure 2.4: $\alpha_{ij}$ matrix](image)

Generally $M$ is different from $N$, which means that the image at the time $t = t_n$ has more or less particles than the image at the time $t = t_{n+1}$. There are many reasons to explain this; for examples: the particle may have moved outside (inside) the region of the flow being tracked, either by moving outside (inside) the bounds of the tracking window, or by moving
out of (into) the illuminated region or a blob can or cannot be recognised as a particle. Therefore we define $\alpha_{0j}$ and $\alpha_{ij}$ as:

$$\alpha_{0j} = 1 : \text{represents a particle (p)}_j \text{ that disappeared at } t = t_{n+1}$$
$$\alpha_{ij} = 1 : \text{represents a particle (q)}_j \text{ that appeared at } t = t_{n+1}$$

We have to determine the optimal set of nonzero $\alpha_{ij}$. We define a cost function to be optimised: $Z$ (hypothesis: the relation between $\alpha_{ij}$ and $c_{ij}$ is linear) as:

$$Z = \sum_{i=1}^{M} \sum_{j=1}^{N} \alpha_{ij} \cdot c_{ij}$$

with: $c_{ij}$ : cost of associating particle $p_i$ at $t = t_n$ with particle $q_j$ at $t = t_{n+1}$

The optimal solution will be chosen so as to minimise the objective function $Z$.

We suppose the basic cost of an association between the particle image $p_i$ and $q_j$:

$$B_{ij} = |\bar{x}_j + \bar{u}_i \cdot \Delta t - \bar{x}_j|$$

with: $\bar{x}_i$ : position of $p_i$ at $t_n$
$\bar{x}_j$ : position of $q_j$ at $t_{n+1}$
$\bar{u}_i$ : a measure of the velocity of $p_i$ at $t_n$
$\Delta t : t_{n+1} - t_n$

$B_{ij}$ is the distance between the place where particle $p_i$ is estimated to be at $t_{n+1}$ based on the velocity at $t_n$ and the place where the particle $q_j$ is at $t_{n+1}$.

Additional cost parameters can be added to the basic cost function. For instance particles whose criteria of ellipticity, size, shape or average intensity lie outside a certain range can get an additional cost value to make it more difficult to match them in comparison to more normal looking particles. A total cost function $c_{ij}$ can be defined per association as:

$$c_{ij} = B_{ij} \cdot \eta_i \cdot \varepsilon_j \cdot \varphi_j \cdot \tau_j \cdot I_{ij} + N_i + (1 - X_{ij}) \cdot Y_i$$

with: $\eta_i$ : new particle discount
$\varepsilon_j$ : ellipticity premium
$\varphi_j$ : size premium
$\tau_j$ : threshold premium
$I_{ij}$ : background intensity premium
$N_i$ : new particle joining fee
Chapter 2  Two-Dimensional Particle Tracking Velocimetry

\[ X_{ij} : \text{ cross-correlation function between subregions of the image centred on } p_i \text{ and } q_j \]
\[ Y_i : \text{ cross-correlation costing} \]
\[ c_{0j} : \text{ cost of a particle leaving the tracking region} \]
\[ c_{0j} : \text{ cost of a particle entering the tracking region} \]

The new particles (entering the image at \( t = t_0 \)) have no velocity history and Digimage is not able to predict accurately where it may be at \( t = t_{n+1} \). To overcome this problem, there are three solutions: the cost of these associations can be reduced, their matching distance can be increased or we can give them a predicted velocity (in our case the flow velocity).

To avoid mismatching, we only take during the subsequent analysis the particles appearing in three or more next samples during the tracking phase.

Digimage is able to follow a particle even if the particle concerned is not visible for one sample and so it maintains a velocity history for that particle so that its future position may be predicted (these particles are stored for a later use).

Notice that the matching will work correctly only if the mean distance between two particles is larger than the mean distance covered by a particle between two consecutive images. This is not a restriction due to DigImage: a matching done by hand in the same conditions (for instance on basis of photography) would not be possible.

With help of this cost-function we can determine the optimal set of \( \alpha_{ij} \)'s. In order to find a solution we could simply calculate all possible solutions and take the solution with the lowest sum of \( c_{ij} \)'s. This method resembles closely the transportation algorithm. The principle of the solution algorithm is outlined below.

Once the costs \( c_{ij} \) have been specified, any value of \( c_{ij} \) exceeding \( c_{0j} \) or \( c_{0j} \) (i.e. the cost of a particle leaving the region which is given by the lesser of the cost based on its distance to the boundary and that based on the maximum matching distance) and the cost is set to infinity: that matching will never occur so we need not consider it. An initial guess is made at \( \alpha_{ij} \) such that:

\[
\forall i, \quad \alpha_{ij} = 1 \quad \text{for } j = J \quad \text{with } c_{ij} = \min_{j=1,N} (c_{ij}) \\
\alpha_{ij} = 0 \quad \text{for } j \neq J
\]

Frequently this initial solution will be very close to optimal, but that will not always be the case. It is now necessary to iterate until the optimal solution is found.

For each iteration, we scan through the list of zero \( \alpha_{ij} \) which have finite costs, and evaluate the cost of bringing that association into the solution. Consider the zero association \( \alpha_{IJ} \) and suppose that \( L \) with \( \alpha_{IL} = 1 \) and it exists \( K \) with \( \alpha_{KJ} = 1 \). Now if \( \alpha_{IJ} \) were to be brought in (\( \alpha_{IJ} = 1 \)), then \( \alpha_{IL} \) and \( \alpha_{KJ} \) must leave the solution (\( \alpha_{IL} = 0 \), \( \alpha_{KJ} = 0 \)), otherwise this particle, at \( t = t_n \), will be two places at once, at \( t = t_{n+1} \). Further, as particle images \( i = K \) and \( j = L \) must be related to some physical particle, it will be necessary to let \( \alpha_{KL} \) enter the solution.

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Thus the reduced cost of bringing in $\alpha_{ij}$ is $c_{ij} + c_{KL} - c_{IL} - c_{KJ}$. If this reduced cost is negative, then bringing $\alpha_{ij}$ into the solution is favourable (will decrease the objective function). After scanning through all zero $\alpha_{ij}$ with finite costs, we bring the $\alpha_{ij}$ in which had the most negative reduced cost and start the next iteration. If all $\alpha_{ij}$ have reduced costs greater than or equal to zero, then the optimal solution has been found.

Example: 

$$i = 3$$
$$j = 4$$

$$\begin{align*}
(c_{ij}) &= \begin{pmatrix} 2 & 4 & 3 & 7 \\ 8 & 10 & 9 & 1 \\ 4 & 2 & 7 & 1 \end{pmatrix} & \Rightarrow & (\alpha_{ij}) &= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix}
\end{align*}$$

suppose that: $\alpha_{12} = 1$ $\Rightarrow$ $(\alpha_{ij}) = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$

we have thus:

$$\begin{align*}
I &= 1 \\
J &= 2 \\
L &= 1 \\
K &= 3
\end{align*}$$

$$c_{ij} + c_{KL} - c_{IL} - c_{KJ} = c_{12} + c_{31} - c_{11} - c_{32} = 4 + 4 - 2 - 2 = 4 > 0$$

$\Rightarrow$ bringing $\alpha_{12}$ in the solution is unfavourable
2.2. Subsequent analysis

Tracking the particles is only a small part of the experimental analysis. It will almost always be necessary to summarise the data obtained from the tracking procedure.

After the tracking process, Digimage has created four permanent files:

- (base_name.PAR) stores all the tracking parameters used to control the tracking,
- (base_name.WLD) records details on the co-ordinate system used;
- (base_name.IND) contains information relating to the status of the tracking at each time step;
- (base_name.PRT) contains information about a single particle at a single time step.

Once our experiment has been particle-tracked, we can use Trk2dVel (part of DigImage) to take a qualitative look at our data. Then the files can be exported in the form of *.PV files for further processing and do some quantitative analysis. If the tracking has not worked well then they will be exported in the form of *.ERR files. Although Trk2dVel has many functions (graphical or statistics of the flow) only a few of them are really necessary to investigate our data for the first impression.

2.3. Example

An example of our experiment is given here with all the steps made by the Two-Dimensional Particle Tracking Velocimetry. Digimage can digitise the image from the video tape and filter them when demanded (see figure 2.5), find the position of the particles (see figure 2.6), and match them on consecutive images (see figure 2.7 and 2.8).
The feasibility of the 2D-PTV for the study of vertical sediment transport

Figure 2.5.a: Image digitised by DigImage from the video tape
Figure 2.5.b: *Image filtered by DigImage*
The feasibility of the 2D-PTV for the study of vertical sediment transport

Figure 2.6: Position of the particles
**Principle of Particle Matching**

Basic cost function:

\[ B_{ij} = \frac{1}{|x_i - u_i \cdot \Delta t - x_j|} \]

- \( B_{ij} \) = cost for matching particle i and j
- \( x_i \) = position of particle i at frame 1
- \( x_j \) = position of particle j at frame 2
- \( \Delta t \) = time between frame 1 and 2
- \( u_i \) = velocity estimate at frame 1

Total cost of matching two frames:

Sum of the costs of a selected set of particle associations between frame 1 and frame 2.

Optimal solution is:

Best combination of particle associations with the lowest total cost.

---

**Figure 2.7: Matching algorithm**
Figure 2.8: Particles matched on consecutive images

- $z_p$ [cm]
- $x_p$ [cm]
The feasibility of 2D-PTV for the study of vertical sediment transport
Chapter 3

EXPERIMENTAL SET-UP

3.1. Overview

The principal object is the flume, around and on which all the equipment is put (the choice of this flume will be explained in 3.2.). Its proportions are given in figure 3.1.a. (see next page). In order to make the comprehension of the experimental set-up easier, the scale of the figures 3.1.b (top view) and 3.1.c (side view) is larger in the transversal and vertical directions than in the longitudinal direction, and the different apparatuses are also drawn on a larger scale.

From upstream to downstream, the following apparatus are found (the numbers put in parentheses indicate the distance between the apparatus and the beginning of the flume):

• a valve for the adjustment of the discharge
• a water tank which gives a constant discharge in the flume
• a flow depth meter (5 m)
• a Laser-Doppler Flow Meter (6.9 m), made up of the laser, the signal processor and two detectors
• two apparatus linked to the Laser-Doppler Flow Meter: an oscilloscope and a personal computer
• the device for the injection of particles, made up of a vessel, a pump, a "constant head-vessel" and an injection pipe. It is set at a variable distance ($d_{inj} = 0.5; 1.0; 1.5; 2.0$ and 5.0 m) from the centre of the camera (see below)
• a camera (10.2 m), linked to a video tape recorder and a monitor
• a diapositive projector (10.6 m) which illuminates a mirror put in the flow
• a vertical adjustable sluice gate (14.3 m) which is used for the regulation of the water depth
• a sieve, intended for the recovery of the injected particles.

All these apparatuses are described in more detail in the paragraphs below.
Figure 3.1: Overview of the experimental set-up
3.2. Flume

As said before, the aim of the experiments is to study the transport in the vertical direction in a uniform free surface flow. This necessitates a flume long enough (in order to avoid the perturbing effects upstream and downstream) and wide enough (in order to avoid the wall effects). It is also more convenient to work with a hydraulically smooth flow (in order to study a basic situation), which necessitates a smooth bottom. As particles will be injected into the flow and then recorded on a video tape, another criterion for the choice of the flume is the possibility of watching the flow laterally through windows.

These conditions are fulfilled by one of the flumes of the «Laboratorium voor Vloeistofmechanica» (Laboratory of Fluid Mechanics) of the «Technische Universiteit Delft» (Delft University of Technology, The Netherlands), in which the experiments were carried out (see figure 3.2).

![Figure 3.2: The flume](image)

Its dimensions are: length: 14.3 m; width: 0.4 m; depth: 0.4 m. It has an adjustable slope from +0.05% to -0.10%, but in practice we put it in a horizontal position.

The discharge is maintained constant by the upstream tank which has a constant water head and is adjusted by a valve. Unfortunately, it is not possible to have this discharge exactly equal on consecutive days by adjusting the valve, because it does not have a precise indication of how large the opening is. It is possible to adjust the water depth downstream by means of the vertical sluice gate.

The top of the flume is equipped with bars to put some apparatus (EMS flow meter, flow depth meter, device for the injection of particles) on a rolling frame, so that it is easy to move them along the flume.
A sieve is placed at the end of the flume in order to recover the particles. It is made of two different sheets of sieve. The first one has square holes of 0.1 mm. Just under this one we put another with square holes of 1.2 mm; its function was to maintain the first sieve in place and give it a certain rigidity and support.

The flow depth meter is a pointer gauge with an accuracy of 0.1 mm. As the determination of one flow depth necessitates two measurements (one for the bottom and one for the water surface), the accuracy on the flow depth is about 0.2 mm.
3.3. Particles

3.3.1. Choice of the particles

For the study of the influence of the coherent structures of turbulence on the vertical transport of particles, it is convenient to dispose of particles which will easily rise, i.e. which have a small settling velocity. As the settling velocity depends principally on the diameter of the particle and on its mass density, two kinds of particles are envisaged: very small ones with a mass density of natural sediment (\( \approx 2650 \text{ kg/m}^3 \)) and relatively large ones with a very small mass density.

Another criterion for the choice of the particles is that they must be located and matched by the Two-Dimensional Particle Tracking Velocimetry, i.e. they must not be too small. Finally, different diameters are required for the study of the influence of the diameter on the transport.

Two different sources of polystyrene particles of variable diameter (d) were available and fulfilled these conditions:

- source A: a few micrometers \( \leq d \leq 0.75 \text{ mm} \).
- source B: \( 0.8 \leq d \leq 1.5 \text{ mm} \).

Observation with a microscope showed that the shape of all these particles is almost spherical.

One can also observe with the naked eye that there is a very small proportion of expanded polystyrene (\( \approx 1 \) particle out of 500), and that there are two kinds of particles in both sources: opaque and translucent, in different proportions for each source (4 translucent particles out of 5 for the source A, and 1 translucent particle out of 5 for the source B).

For studying the influence of the diameter on the transport of sediment, particles with almost the same diameter are required. So the first operation was the sieving of the particles.

3.3.2. Sieving

Sieves with square holes were used. Unfortunately their dimensions (and thus their capacity) are relatively small (20 cm in diameter).

The chosen combination of sieves is put on a vibrating machine, which vibrates with a determined intensity (scale 0 to 10) during a period determined by a timer (1 to 60 minutes).

3.3.2.1. Choice of the sieves

The first idea was to work with three different diameters of particles. The criterion for the choice of these diameters was to have three different settling velocities, in the ratio 1 / 3 / 10:
The feasibility of 2D-PTV for the study of vertical sediment transport

\[ \frac{w_{s2}}{w_{s1}} = 0.33 \quad \frac{w_{s3}}{w_{s1}} = 0.1 \quad (3.1) \]

with
\[ w_{s1} \] the settling velocity of the largest particles (diameter \( d_1 \))
\[w_{s2} \] the settling velocity of the particles of intermediate diameter (diameter \( d_2 < d_1 \))
\[ w_{s3} \] the settling velocity of the smallest particles (diameter \( d_3 < d_2 < d_1 \))

As the mass density of these particles is very small, we may consider in first approximation that the settling velocity is proportional to the square of the diameter (see equation 1.14). Assuming that the mass density is the same for all the particles (whatever the diameter may be) we have:

\[ w_{si} = \frac{(s-1)g}{18\nu} \cdot d_i^2 \quad (i = 1,2,3) \quad (3.2) \]

Taking \( d_1 \) as a reference diameter, \( d_2 \) and \( d_3 \) may be expressed as a fraction of \( d_1 \):

\[ d_2 = \sqrt{0.33} \cdot d_1 \equiv 0.57 \cdot d_1 \quad (3.3a) \]
\[ d_3 = \sqrt{0.1} \cdot d_1 \equiv 0.32 \cdot d_1 \quad (3.3b) \]

For instance, 1.1 mm, 0.6 mm and 0.35 mm.

So the following sieves were chosen (size of the square holes in mm):
- Source A: 0 / 0.297 / 0.42 / 0.5 / 0.71
- Source B: 0 / 0.84 / 1 / 1.19

3.3.2.2. Parameters for the sieving

The three parameters for a sieving operation are the intensity of the vibrations (scale 0 to 10), the quantity of particles sieved (Q) and the vibration time (t = 1 to 60 minutes).

When sieving, one can observe that:
- with a large quantity of particles, all the holes of the sieves at the top of the stacked combination are filled in, and even when increasing the vibration time some particles can not pass through holes larger than them. The intensity of vibration does not have any influence on this phenomenon.
- with a small quantity of particles, they all pass directly through the sieves, even when the vibration time and the intensity of vibration are very small.

Some measurements of the distribution of the different diameters are done with a very long time of vibration and a small quantity of particles, in order to have a reference measurement (combination 1 below). The quantity of sieved particles for two different combinations of quantity Q and vibration time t are then measured. These measurements are carried out in order to increase the quantity sieved per unit time (Q/t), when having almost the same

50
proportions of sieved set of particles as the ones for the reference measurements (see table 3.1).

<table>
<thead>
<tr>
<th>Combination</th>
<th>Quantity sieved (Q) [g]</th>
<th>Vibration time (t) [min]</th>
<th>Q/t [g/min]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (reference)</td>
<td>17.5</td>
<td>6</td>
<td>2.9</td>
</tr>
<tr>
<td>2</td>
<td>35</td>
<td>6</td>
<td>5.8</td>
</tr>
<tr>
<td>3</td>
<td>17.5</td>
<td>3</td>
<td>5.8</td>
</tr>
</tbody>
</table>

Table 3.1: Combinations of the quantity sieved and the vibration time

The following graphics represent the cumulative (or grain-size) curves for an average of 4 measurements. The ordinate indicates how many percent (by weight) of the total sample is finer than the diameter of the sieve on the abscissa.

Figure 3.3.a: Grain-size curve for the source A of particles

Figure 3.3.b: Grain-size curve for the source B of particles
These curves let show different behaviours for the two sources:

- **source A**: it is better to work with (6 min; 35 g) than with (3 min; 17.5 g): no sieve is filled up, and the quality of the sieving increases with the duration of vibration.
- **source B**: it is better to work with (3 min; 17.5 g) than with (6 min; 17.5 g). One can easily explain this when watching trough the glass above the largest sieve: the holes of this one are rapidly obstructed (45% of the particles do not pass through the first sieve).

In order to spend time it is convenient to sieve at the same time the two kinds of particles, by superposing the sieves used for the two sources of particles. A compromise for doing this is to sieve with:

\[
Q = \frac{3}{4} \times 35 \text{ g} \\
t = 4 \text{ min}
\]

This combination has the extra advantage that it maximizes the quantity sieved per unit time: \( Q/t = 26.3 \text{ g} / 4 \text{ min} = 6.6 \text{ g/min} \)

3.3.2.3. Sieved quantities

Unfortunately, the sieving takes a lot of time for relatively small quantities of particles. After one day, a sufficient quantity of particles (i.e. 600 g; cf. 3.3.2.4) can be harvested for only two different diameters:

- \( 0.30 \text{ mm} \leq d \leq 0.42 \text{ mm} \), that we will call the **small particles**
- \( 1.00 \text{ mm} \leq d \leq 1.19 \text{ mm} \), that we will call the **large particles**

Fortunately the mean diameter of these two sets of particles (\( d_m \)) is 0.36 mm for the small particles and 1.095 mm for the large particles, and so they are representative of two "extreme" behaviours (for the velocities used in the experiments) for the transport: the large particles have a diameter three times larger than the small ones (which means a volume 30 times larger) and the ratio between the settling velocities is in first approximation:

\[
w_{s,\text{large}} / w_{s,\text{small}} = d_{m,\text{large}}^{2} / d_{m,\text{small}}^{2} = (1.095 / 0.36)^{2} \approx 9
\]

Notice that the difference between the maximum diameter and the minimum diameter for the small particles (\( \Delta d = d_{\text{max}} - d_{\text{min}} = 0.12 \text{ mm}; \Delta d/d_m = 33\% \)) is relatively more important than for the large particles (\( \Delta d = 0.19; \Delta d/d_m = 17\% \)). This fact is very clear when looking at the particles with a microscope.
3.3.2.4. Required quantities

To estimate the required quantity for all the experiments was not easy, because of the difficulty to predict the quantity required for one experiment, the number of experiments, the duration of one experiment, the lateral dispersion in the flume and some others parameters. Assuming that half the particles injected will be in the lightsheet (this is optimistic for large distances between the injection and the camera) and that the duration of one recording (including many combinations for the height of injection and the distance between the injection and the camera) is 30 minutes, the following quantities are found for one experiment (see appendix 3.1):

<table>
<thead>
<tr>
<th></th>
<th>Mean flow velocity [cm/s]</th>
<th>Number of particles in the lightsheet</th>
<th>Required mass of particles [g ]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small part.</td>
<td>10</td>
<td>50</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>50</td>
<td>150</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>100</td>
<td>300</td>
</tr>
<tr>
<td>Large part.</td>
<td>15</td>
<td>10</td>
<td>800</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>25</td>
<td>2000</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>25</td>
<td>3000</td>
</tr>
</tbody>
</table>

Table 3.2: Quantities of particles required for one experiment

These simple computations show that for the same number of particles in the lightsheet, a much larger weight of large particles is needed than for small particles. Unfortunately, the harvested quantities after sieving are almost the same for the small and the large particles (≤ 600 g per day). If we sieve during one day before each series of experiments, there will thus be approximately 20 particles per image for the large particles, and 75 for the small particles.

3.3.3. Mass density

As mentioned in chapter 1, the settling velocity \( w_s \) of a particle is a very important parameter of the models of sediment transport. The formulas that allow to calculate it use the factor \((s-1)\), where \( s = \rho_p / \rho \) is the relative density, ratio between the mass density of particles \( (\rho_p) \) and the one of the fluid \( (\rho) \).

The problem in our case is that the mass density of the polystyrene particles is nearly the same as the one of water. So the relative density \( s \) will be nearly 1. As in the computation of the theoretical settling velocity the factor \((s-1)\) is multiplied, the theoretical value will be very inaccurate if the relative inaccuracy on \( \rho_p \) is large. Three methods were used in order to have the best possible measurement of \( \rho_p \): the "classical method", the "picnometer method" and the "salt method".

Appendix 3.2 explains these three methods and gives the measurements of the mass density of the polystyrene particles. The values obtained are 1031 kg/m³ for the large particles and 1038 kg/m³ for the small ones.
3.3.4. Settling velocity

3.3.4.1. Theoretical settling velocity

The choice of a formula adapted for the calculation of the settling velocity ($w_s$) will depend on the magnitude of the particle Reynolds number ($Re_p$) (see chap 1), depending itself on the settling velocity. This means that we have to take a formula, doing implicitly the hypothesis that $Re_p$ is adapted for it, and verify afterwards whether it was true. Let us try with the simplest formula, i.e. the Stokes formula (1.14). Considering the inaccuracy in (s-1), the theoretical settling velocities are (see appendix 3.3):

| Small particles: $w_{s,sm}$ | 2.8 mm/s ± 0.4 mm/s |
| Large particles: $w_{s,lm}$ | 21.6 mm/s ± 4.0 mm/s |

The corresponding particle Reynolds numbers are:

$$Re_{p,small} = 1.0 > 0.1$$
$$Re_{p,large} = 24 >> 0.1$$

This means that the Stokes formula is not valid and that $w_s$ must be calculated by the formula (1.23). However, because the particle Reynolds number for the small particles is not very much higher than 0.1, the settling value found for the small particles is nearly correct (see figure 1.3: for values of $Re_p \leq 1$ the Stokes formula does not differ much from the other formulas, except the one of Fredsoe, not valid for such small $Re_p$).

The formula (1.23) uses the dimensionless particle diameter $d_*$, which is in our case:

<table>
<thead>
<tr>
<th>$d_{*,p} = 6725 , d$</th>
<th>$d_m$</th>
<th>$d_{*,m}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small particles</td>
<td>0.36</td>
<td>2421</td>
</tr>
<tr>
<td>Large particles</td>
<td>1.095</td>
<td>7364</td>
</tr>
</tbody>
</table>

Table 3.3: Dimensionless particle diameter of the polystyrene particles

As the dimensionless particle diameter is a characteristic number for the particle, it is possible to see which kinds of quartz sediment ($\rho_{quartz} = 2650 \, \text{kg/m}^3$) correspond to our particles, by equalising the dimensionless particle diameter of the quartz sediment ($d_{*,quartz}$) and the one of the polystyrene particles ($d_{*,p}$):

$$d_{*,quartz} = d_{*,p} \Rightarrow \sqrt[3]{\frac{(s_{quartz} - 1)g}{\nu^2}} \, d_{quartz} = \sqrt[3]{\frac{(s - 1)g}{\nu^2}} \, d_p$$

$$\Rightarrow d_{quartz} = \sqrt[3]{\frac{s - 1}{s_{quartz} - 1}} \, d_p \quad (3.4)$$

54
with

\[ s_{\text{quartz}} = \text{relative density of quartz sediment} \]
\[ s_p = \text{relative density of polystyrene} \]

Considering the mean diameter of each set, we find

for the small particles \((s_p = 1.038)\), \(d_{\text{quartz}} = 0.28\) \(d_p = 0.10\) mm

for the large particles \((s_p = 1.031)\), \(d_{\text{quartz}} = 0.27\) \(d_p = 0.29\) mm.

which gives:

<table>
<thead>
<tr>
<th></th>
<th>(d_p) [mm]</th>
<th>(d_{\text{quartz}}) [mm]</th>
<th>corresponding type of sediment [8]</th>
</tr>
</thead>
<tbody>
<tr>
<td>small particles</td>
<td>0.36</td>
<td>0.10</td>
<td>very fine sand</td>
</tr>
<tr>
<td>large particles</td>
<td>1.095</td>
<td>0.29</td>
<td>medium sand</td>
</tr>
</tbody>
</table>

Table 3.4: Similitude of the polystyrene particles and natural sediment

The settling velocities calculated with equation (1.14) or (1.23), for the different numerical coefficients given in chapter 1 are summarised below:

<table>
<thead>
<tr>
<th></th>
<th>Stokes</th>
<th>Oseen</th>
<th>Brown</th>
<th>Engelund</th>
<th>Rubey</th>
<th>Fredsoe</th>
</tr>
</thead>
<tbody>
<tr>
<td>(w_s) [mm/s]</td>
<td>2.8</td>
<td>2.4</td>
<td>2.8</td>
<td>2.7</td>
<td>2.6</td>
<td>1.8</td>
</tr>
<tr>
<td>(\Delta w_s) [mm/s]</td>
<td>0.4</td>
<td>0.3</td>
<td>0.4</td>
<td>0.4</td>
<td>0.4</td>
<td>0.3</td>
</tr>
<tr>
<td>(Re_p) [ ]</td>
<td>1.0</td>
<td>0.9</td>
<td>1.0</td>
<td>1.0</td>
<td>0.9</td>
<td>0.7</td>
</tr>
</tbody>
</table>

Table 3.5: Theoretical settling velocities of the small particles computed with formula (1.18)

Table 3.5 shows that the settling velocities computed for the small particles with all the different formulas are nearly the same \((\equiv 2.65\) mm/s), except with the one of Fredsoe, which is not valid for such small values of the particle Reynolds number. This can be easily seen when looking at the figure 1.3: all the curves are nearly superposed (except the one of Fredsoe) for particle Reynolds number close to 1.

<table>
<thead>
<tr>
<th></th>
<th>Stokes</th>
<th>Oseen</th>
<th>Brown</th>
<th>Engelund</th>
<th>Rubey</th>
<th>Fredsoe</th>
</tr>
</thead>
<tbody>
<tr>
<td>(w_s) [mm/s]</td>
<td>26.2</td>
<td>8.1</td>
<td>16.3</td>
<td>11.9</td>
<td>10.9</td>
<td>10.1</td>
</tr>
<tr>
<td>(\Delta w_s) [mm/s]</td>
<td>4.0</td>
<td>1.0</td>
<td>2.5</td>
<td>1.6</td>
<td>1.4</td>
<td>1.5</td>
</tr>
<tr>
<td>(Re_p) [ ]</td>
<td>29</td>
<td>9</td>
<td>18</td>
<td>13</td>
<td>12</td>
<td>11</td>
</tr>
</tbody>
</table>

Table 3.6: Theoretical settling velocity of the large particles computed with formula (1.18)

Table 3.6 shows that the settling velocities computed for the large particles are nearly the same \((\equiv 11\) mm/s) for all the formulas valid for the \(Re_p\) found (Engelund, Rubey, Fredsoe). This can also be seen on the figure 1.3: for a particle Reynolds number close to 12, the curves of Engelund, Rubey and Fredsoe are close to each other (notice that the scale on figure 1.3 is logarithmic, so that small differences in ordinate are in fact large differences).
3.3.4.2. Measurement of the settling velocity

Two methods were carried out: one with a classical method and the other with the Two-Dimensional Particle Tracking Velocimetry.

3.3.4.2.1. Classical method

When passing through the water surface, the particle decelerates, then accelerates over a small distance and finally moves with a constant velocity \( w_s \) if no wall effects. It is easy to measure the settling velocity of a single particle in a large glass cylinder, by dropping it above the water level and by measuring the time \( t \) required to cover a height \( H \) when the particle has its constant velocity: \( w_s = H/t \).

The measurement is done between two reference heights, at a distance from:

- the **water surface** such that the velocity of the particle is already constant. One can see that it is the case after only a few millimetres.
- the **bottom** such that the settling is not influenced by wall effects. A quantitative idea of these effects is given by the formula of Faxen [16]. It expresses, for a particle moving perpendicular to a plane wall, the drag coefficient as a function of the diameter of a sphere and the distance \( l \) between the particle and the wall (for low \( \text{Re}_p \)):

\[
C_{D,\text{wall}} = C_D \left[ 1 + \frac{9}{8} \left( \frac{d}{2l} \right) + \left( \frac{9}{16} \frac{d}{2l} \right)^2 \right]
\]

(3.5)

with

\[
C_{D,\text{wall}} = \text{drag coefficient modified by the wall effects} \\
C_D = \text{drag coefficient without wall effects}
\]

The particle must also be at a certain distance \((l/d \gg 1)\) from the glass cylinder, in order to avoid wall effects. Another formula of Faxen gives the drag coefficient for a particle moving parallel to a plane wall:

\[
C_{D,\text{wall}} = C_D \left[ 1 - \frac{9}{16} \left( \frac{d}{2l} \right) + \frac{1}{8} \left( \frac{d}{2l} \right)^3 - \frac{45}{256} \left( \frac{d}{2l} \right)^4 - \frac{1}{16} \left( \frac{d}{2l} \right)^5 \right]^{-1}
\]

(3.6)

Figure 3.4 shows that the influence of the wall effect is minute for \( l = 5d \), i.e. 2 mm for the small particles and 6 mm for the large ones. So it is very easy to avoid the wall effects with a cylinder of a sufficient diameter.
Influence of the wall effects on $C_D$

**Figure 3.4: Influence of the wall effects on the drag coefficient for a particle moving perpendicular or parallel to a plane wall**

In practice the measurements start at a distance of 8.8 cm from the water surface and end at a distance of 4.2 cm from the bottom.

The large particles are dropped from 10 cm above the water surface. The small particles in the same conditions do not pass through the water surface, so that it is necessary to push them into the water for the first millimetres. The measurements are done separately for the opaque and the translucent particles, in order to see whether there is a difference in mass density between these kinds of particles.

**Figure 3.5: Experimental set-up for the classical measuring of the settling velocity**

The settling velocities measured with this method are:

<table>
<thead>
<tr>
<th>Particles</th>
<th>Number of measurements</th>
<th>$W_{s, \text{mean}}$ [mm/s]</th>
<th>standard deviation [mm/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small</td>
<td>10</td>
<td>3.6</td>
<td>0.2</td>
</tr>
<tr>
<td>Large opaque</td>
<td>34</td>
<td>11.8</td>
<td>0.8</td>
</tr>
<tr>
<td>Large translucent</td>
<td>15</td>
<td>12.8</td>
<td>0.7</td>
</tr>
</tbody>
</table>

Table 3.7: Settling velocities measured with the classical method
This shows a small difference between the large opaque and translucent particles. The mean settling velocity computed for all the large particles is **12.1 mm/s** (standard deviation: 0.9 mm/s).

The values given in table 3.10 are nearly the same as the theoretical values for the large particles (11.0 mm/s), and somewhat different for the small ones (2.8 mm/s).

Notice that the distribution of the settling velocity for the opaque large particles seems to present two different statistical populations (but because of the small number of measurements it is nevertheless imprudent to infer this).

![Distribution of the settling velocity for the opaque large particles](image)

**Figure 3.6: Distribution of the settling velocity for opaque particles with the classical measurement done for 34 particles**

For the small particles $Re_p \approx 1$, so that the settling velocity can be estimated by the Stokes formula (1.10). Transforming this equation, this measurement permits to calculate the mass density of the small particles:

$$w_s = \frac{(s-1)g}{18\nu} d^2 \Rightarrow \rho_p = \left(\frac{18w_s}{gd^2} + 1\right) \rho_w = 1.043 \rho_w = 1042 \text{ kg/m}^3 \quad (20^\circ\text{C}) \quad (3.7)$$

which is nearly the value obtained from the combination of the picnometer method and the salt method (1038 kg/m$^3$).
3.3.4.2.2. Measurement with Two-Dimensional Particle Tracking Velocimetry

The aim of this measurement was to see whether the used particles could be detected and tracked by the Two Dimensional Particle Tracking Velocimetry by DigImage, and to get used to this new method of measurement.

The method used is the following: put some water in a rectangular glass tank (height: 30 cm; length: 24 cm; width: 10 cm), where there is a mirror (height: 15 cm; width: 2 cm) in a vertical position. It is illuminated by a diapositive projector, in which the diapositive is replaced by a slot in order to obtain a sheet of light with a width of approximately 1.5 cm. A camera records the settling of the particles in a window located at half the height of the tank (height of the window: 8 cm). The images are then analysed with the DigImage Two-Dimensional Particle Tracking Velocimetry analysis system.

This measurement was done for the small and the large particles, and we noted in both cases the presence of some circulating flow in the tank, due to the difference in temperature between the water and the panes.

![Diagram of experimental set-up](image)

Figure 3.7: Experimental set-up for the measurement of the settling velocity with the Two-Dimensional Particle Tracking Velocimetry method

Small particles

We had some problem to make the small particles settle, because they stayed on the water surface when they were dropped from 10 cm above it. We had to help them to pass through the surface by hitting them with a finger. We so created extra turbulence which perturbed the measurement of the settling velocity.

Fifteen images were analysed, on which there were 95 particles on average. The mean settling velocity calculated is 6.1 mm/s, which differs considerably from the theoretical values (2.8 mm/s) and the value measured with the previous method (3.6 mm/s).
The mean settling velocity for each image is reported in the graph here below, which shows that it is constant for the short interval of time studied (15 x 25 ms = 375 ms).

![Mean settling velocity for small particles with 2D-PTV (15 images)](image_url)

*Figure 3.8: Mean settling velocity for small particles with the Two-Dimensional Particle Tracking Velocimetry method*

Nevertheless this measurement is interesting for the distribution of the settling velocity, were it appears clearly that there are two statistical populations:

![Distribution of w_s for the small particles measurement with 2D-PTV for 15 images](image_url)

*Figure 3.9: Distribution of the settling velocity for the small particles with the Two-Dimensional Particles Tracking Velocimetry method*

Two explanations could be given: the existence of two populations of diameters or the disturbing effects due to the presence of the circulating flow, which is the most likely.

In the following graph, all the settling velocities of the different images are put one behind the other, in order to see if the dispersion of the measured settling velocity varies in time.
The time is written between inverted commas because all the velocities of one image are in reality measured on the same time, but are represented on this graphic at times slightly different:

**Figure 3.10: Settling velocity as a function of time for small particles with the Two-Dimensional Particle Tracking Velocimetry method**

This figure shows that the dispersion of the velocity is constant with time, i.e. the situation is stationary.

**Large particles**

The particles were dropped in a group from 10 cm above the water surface just before the beginning of the recording. Forty images were analysed. The figure 3.11 is the same graph as the one of figure 3.10, but for the large particles:
One may observe that the dispersion of the velocity becomes more and more large with the time. This is because too many particles were dropped at the same time, which created drag effects. The mean settling value measured is 16.8 mm/s, which is larger than the value measured with the classical method (12.1 mm/s). As the settling velocity of the set varies with time, this mean value has no real interesting physical significance and we cannot have more information on the settling velocity with this measurement (e.g. on the distribution of the settling velocity as done for the small particles on figure 3.9).
3.4. Injection of the particles

An injection of particles in the flow with a constant velocity and with a constant concentration in particles is required. So the particles will be distributed homogeneously in the flow and can give us information about the local flow velocity. The procedure described below will be used to satisfy these conditions.

The sieved particles are mixed in a pail with water coming from the flume (in order to have the same temperature for the injection water) and a little bit of soap (to avoid that the particles stick to each other). When they are mixed, we put them in a vessel (a) (the letters between the parentheses refer to the figure 3.12).

The dimension of this vessel are: height: 48 cm
diameter: 49.5 cm
volume: 93 l

It is equipped with a mixer (c) so the mixture (b) (particles, water, soap) stays homogeneous and the particles do not go to the bottom of the vessel. When mentioning the mixture, in the further paragraphs, we include in this term the particles, the water and the soap.

![Injection system](image)

*Figure 3.12: Injection system*

In the bottom of the vessel, there is a valve which is connected to the pump (p) with a rubber tubing (d). At the outside of the pump there are two rubber tubings: the first one (e)
goes to the ‘constant-head vessel’ (f) (see next paragraph) and the second one (g) goes back to the vessel. The last one is equipped with a tap so we can control the flow rate of the mixture which goes back to the vessel and thus the discharge which goes to the constant-head vessel.

In order to have a constant velocity for the injection of the mixture, a constant-head vessel is used to maintain a constant water height and to induce the movement of water from the pressure of the injection pipe.

The constant-head vessel consists of two tubes (in PVC): one small (h) (diameter: 90 mm) and another bigger (i) (diameter: 230 mm). The bottom of the small one is closed and there are two holes in it. One for the inflow of the mixture from the pump in one small rubber tubing (diameter: 11.5 mm) and the second for the transfer to the injection pipe (k) via another rubber tubing (same diameter: 11.5 mm) (see further paragraph).

The quantity of mixture arriving in the first PVC tube is greater than the quantity which is injected in the flume so this tube is always full of mixture and the excess of mixture overflows and falls in the second tube. The bottom of the biggest tube is also closed and there is one hole connected to a rubber tubing (l) (diameter: 38 mm) which brings back the excess mixture to the vessel. The big tube is attached to a post (m).

We can vary the height of this constant-head vessel in order to change the difference of the water level in it (top of the small PVC tube) and the water level in the flume. It is simple to change the injection velocity of the mixture in the flow.

The injection of the mixture in the flow is realised with a metallic L-shaped tube (diameter: 7.8 mm), which is connected to the rubber tubing coming out of the constant-head vessel. The end of this tube (the horizontal part which is beneath the free surface) has a length of 60 cm in order to suppress the influence of the elbow-pipe on the mixture injection. The airtightness of the various joints must be perfect otherwise the air will come inside the tubings (there is an underpressure in the highest part of the tubing) and will collect in the higher part of these tubes, causing a decrease in discharge (the effective tube cross-section decreases). This injection pipe is fastened to a rolling frame over the flume so it is easy to change the place where we inject particles in the flow.
3.5. Recording of the particles

The study of the local turbulent structures and velocities are of interest to us. For this purpose, only a small part of the flow will be recorded and then analysed: the centre of the flume. Otherwise the local coherent structures cannot be analysed and the influence of the wall effects from the side of the flume will be too important. To analyse the recordings with the Particle Tracking Velocimetry, the particles in this zone must be illuminated with a sufficient light intensity and the quality of these images must also be good. It is generally better to start with high quality images of an experiment, than to try to extract data from low quality by throwing away some of the information. To obtain thus the necessary informations from the flow, the particles in a longitudinal slice in the centre of the flow will be illuminated with a lightsheet and then recorded.

A mirror, placed on a metal frame, is put in the centre of the flow (see figure 3.1), with an angle of 45 degrees with regard to the flow direction and it is illuminated by a diapositive projector, equipped with a special diapositive in metal with a variable slit. The beam of light goes through the slit from the side of the flume, comes into the flume, is reflected from the mirror and illuminated the flow and the particles in the desired zone with a lightsheet.

The width of the beam of light can be adjusted with the variable diapositive slit (the width is generally included between 1.5 and 2.0 cm). The intensity of the light has to be maximum in this lightsheet; it can be increased if the lens of the diapositive projector is focused or defocused. The light is always very intensive in the centre and a little bit less intensive along the edges of the lightsheet.

Figure 3.13: Darkened flume
To avoid the recording of particles out of the lightsheet but illuminated by the daylight, the flume around the camera is darkened by some dark clothing sheets put over all the glass faces of the flume (see figure 3.13). Two small openings are left: the first one allows the light to enter the flow from the diapositive projector and the second one allows the recording of the particles in the flow. The top of the flume is covered with boards and two small boards are placed at each extremity of this zone to prevent the light from entering the flume. During the recording, the lights in the laboratory are switched off. With all these precautions, we obtain a very dark flume around the camera and so only the particles in the illuminated zone are recorded.

A black and white camera is used for the recording these particles. This camera is on a frame and inside a dark texture to shield it from the disturbing daylight. The pictures taken with the camera are made at 35 cm upstream from the mirror; so the mirror does affect the flow in the measuring section.

A video tape recorder (JVC Super VHS HR-55000 E) is connected to the camera and to the monitor (Mitsubishi). This monitor is used to adjust the lens of the camera to obtain a clearness image and to control instantaneous the quality of the images and the functioning of the video tape recorder which enables us to stop the injection of the particles if the quality of the images decreases or if a problem occurs.
3.6. Measurement of the flow velocity

There are various instruments to measure the water velocity. Each instrument has advantages and disadvantages. The accuracy of the measurements depends on the size of the measurement volume; a large one does not allow to have very accurate measurements and thus to measure the local turbulent structures contrary to a small one. We first used an electro magnetic flow meter (EMS) for measurement of the average velocity profile and then a Laser-Doppler Flow Meter (LDFM) for detailed measurements.

3.6.1. EMS probe

It is an electromagnetic probe which is based on the induction law of Faraday. The probe is put in the flow, fastened to a rolling frame, at a certain height and the apparatus gives us two electric signals (A and B) which represent the variation of the magnetic field. We can calculate the flow velocity in the X (U) and Y (V) direction (two directions in a plane parallel to the bottom) from these two outputs with the calibration formulas given from the manufacturer. The characteristics of this probe and the manner to calculate its offset and to have a mean flow velocity are explained in appendix 3.4.

Its advantages are:
- it does not need transparent water;
- it is easy to move it along the flume (it is attached to a rolling frame over the flume);
- it gives a first idea of the water velocity rapidly (for small A and B: \( U \approx 0.1 |A| \) and \( V \approx 0.1 |B| \)).

Its disadvantages or limitations are:
- the probe is in the flow thus it disturbs the flow;
- the maximum frequency is 1 Hz and it has a large measurement volume; thus it is not possible to measure the turbulent velocity (\( U' \) and \( V' \)) accurately;
- the probe cannot measure near the bottom of the flume and just beneath the surface of the flow (it needs always to be under 1.5 cm of water);
- it can be influence by some metal pieces (for example: the frame of the flume);
- its offset varies in the time and with the temperature;
- the relation between the outputs and the velocities is not linear;
- it does not give the velocity in the Z direction, which is of great interest to us for the transport of sediment particles.
3.6.2. Laser-Doppler Flow Meter (LDFM)

Introduction

Normally a fluid contains very small particles, such as dust, etc., with a diameter of the order of one micrometer (1 \( \mu \text{m} \)). If a beam of light is projected into the fluid, part of the light incident on these particles is scattered in all directions, most of it in a forward direction. The scattered light can be observed with a detector.

![Laser-Doppler Flow Meter](image)

*Figure 3.14: Laser-Doppler Flow Meter*

If the particles are moving relatively to the light source and the detector, the frequency of the light \( (f_s) \) scattered in a certain direction will undergo a small shift relative to the frequency of the incident light \( (f_i) \). This frequency shift is called the Doppler shift (or Doppler frequency) \( (f_d) \). The value of this Doppler frequency is proportional to the velocity of the particle and depends on the direction of the light beams. The relationship between the Doppler frequency and the velocity depends only on the velocity, frequency of light and some geometrical data.

The velocity of the fluid is determined from the measurement of \( f_d \) with which it has a linear relationship. The presence of small particles is essential for the application of the method. Fortunately the normal water supply contains enough of these particles. The magnitude of the Doppler shift is relatively small. The only way to measure the Doppler frequency \( (f_d) \) is
by comparing the frequency of the scattered light with that of incident beam and not to measure the light frequency directly. One way of doing this is by mixing, on the surface of a photo detector, the scattered light from a certain direction with light from the same direction that has not undergone this shift, referred to as the reference beam. The reference beam can be much weaker than that of the incident main beam. The output signal of the detector contains, amongst other things, the Doppler shift ($f_d$) between the two frequencies.

For this kind of measurement it is essential that both beams, the scattered light beam and the reference beam, are optically coherent. This means that the two beams must be obtained from one and the same light source, a laser, using a beamsplitter.

![Figure 3.15: Laser beams trajectory](image)

**Apparatus**

The apparatus must be allowed to warm up for at least a quarter of an hour before it is used to avoid the influence of varying temperature on the measurement. The receivers have to be lined up in the direction of the reference beams. When the detectors are positioned correctly and when the received Doppler signals A and B are sufficient, the indicators signals of the LDFM will be satisfactory (the green LED’s light up) which means that these signals can be analysed and so can give us the velocity of the flow.

Output signals 1 and 2 can be represented by velocity vectors. The relation between the output signals 1 and 2 and the velocity vectors A and B (respectively) is linear:

$$1 \text{ Volt} = 20 \text{ cm/s}.$$  

Looking from the optical receivers to the laser, the velocity vectors can be defined as follows, assuming that the transmitter is set horizontally:
It is then easy to obtain the velocity in the direction X (U) and Z (W):

\[
U = 20 \left( \frac{A - B}{\sqrt{2}} \right) \quad [\text{cm/s}] \\
W = 20 \left( \frac{A + B}{\sqrt{2}} \right) \quad [\text{cm/s}]
\]

(3.7) \quad (3.8)

where A and B are in Volts.

The offset and the manner to measure it is explained in appendix 3.5.

Its advantages are:
- the laser beam does not disturb the flow;
- it has a very small measurement volume with a high frequency thus it has a big accuracy: it can measure turbulent velocity;
- the relation between the outputs and the velocities is linear;
- its offset is nearly constant in the time;
- it gives rapidly a first idea of the water velocity (\(U \approx 15(A-B)\) and \(W \approx 15(A+B)\)).

Its disadvantages and limitations are:
- it needs transparent water and two transparent and clean glass faces;
- it is not easy to move it along the flume;
- in our flume, it cannot measure near the bottom (down to 7 mm from bottom) of the flume and just under the surface of the flow (up to 5 mm from water surface);
- the laser may be dangerous for the eyes.
The feasibility of 2D-PTV for the study of vertical sediment transport
Chapter 4

STUDY OF THE FLOW

In order to know the characteristics of the flow where the particles are recorded, measurements were done for the mean velocity profile, the characteristic parameters of turbulence and the influence of the injection and the injection pipe.

As mentioned before, the Reynolds number is a characteristic number for turbulence in a flow. Two Reynolds numbers are computed here: one for the flow in the flume and one for the flow in the injection pipe (cf. equations 1.2 and 1.3). The height of the constant-head vessel is assumed to be such that \( \bar{U}_{m,\text{injection}} = \bar{U}_{m,\text{flow}} = \bar{U}_{m} \).

The water depth is always taken as \( h \equiv 8 \text{ cm} \) and the diameter of the injection pipe is \( D = 7.8 \text{ mm} \). As \( h/D \equiv 10 \), \( Re_{\text{flow}} \equiv 10 \cdot Re_{\text{injection}} \). The table 4.1 gives the different values computed for the three depth-averaged flow velocities used in the experiments:

\[
\begin{array}{|c|c|c|c|}
\hline
\bar{U}_{m} & [\text{cm/s}] & 10 & 15 & 20 \\
\hline
Re_{\text{flow}} & & 8000 & 12000 & 16000 \\
Re_{\text{injection}} & & 800 & 1200 & 1600 \\
\hline
\end{array}
\]

*Table 4.1: Reynolds numbers of the flow and the injection*

4.1. Mean velocity profile

4.1.1. Theoretical mean velocity profile

The flume and the recording zone are chosen (cf. 3.2) in order to have a two-dimensional steady flow in this place. Many authors [3, 8, 20] use the following definitions of dimensionless variables for the description of the mean velocity profile:

\[
u^+ = \frac{\bar{U}}{u_*} \quad z^+ = \frac{zu_*}{\nu} \quad (4.1) \& (4.2)
\]

The computation of these variables requires the value of the shear velocity \( u_* \). As the flume is perfectly horizontal, \( u_* \) cannot be computed with \( u_* = \sqrt{\tau_b / \rho} = \sqrt{ghS_b} \) (i.e. the regime is not perfectly uniform). It can be "directly" measured (with the measurement of a velocity profile and its fitting to a theoretical formula or with the measurement of turbulent variables - see 4.2) or estimated with empirical formulae. As a "direct" measurement requires a lot of velocities measurements and computations, it is more handy to use in practice an empirical formula.
We used the Darcy-Weisbach equation [14,15], which gives in the case of a smooth bottom a relation between the shear velocity \( u_* \), the depth-averaged velocity of the flow \( \bar{U}_m \) and the characteristic parameters of the flow (kinematic viscosity \( \nu \) and hydraulic radius \( R \)). For more details about this equation, see appendix 4.1.

Many authors divide a turbulent free surface flow nearby a wall in three zones:

- \( z^+ < 5 \) **laminar zone** or viscous sublayer, where the Reynolds stresses are negligible with respect to the viscous stresses
- \( 5 < z^+ < 30 \) **buffer zone**, where both the viscous and the turbulent stresses are important
- \( 30 < z^+ \) **turbulent zone**, where the viscous stresses are negligible with regard to the Reynolds stresses.

Notice that some authors do not use the concept of buffer zone and prefer to consider that the viscous sublayer goes to \( z^+ = 11.6 \) (where the velocity is equal to 11.6 \( u_* \)) [20, 23].

According to the theory and to many experiments, the velocity profile has typically the shape depicted in figure 4.1:

![Velocity profile in a two-dimensional turbulent free surface flow with a smooth boundary plane](image)

**Figure 4.1:** Velocity profile in a two-dimensional turbulent free surface flow with a smooth boundary plane [8]
• In the **viscous sublayer**, the relation between $u^+$ and $z^+$ is linear:

$$u^+ = z^+ \quad (4.3)$$

• In the **buffer zone**, which is a transition zone, there is no handy expression for the velocity profile.

• In the **turbulent zone**, the integration of equation (A1.2/18) derived in the appendix 1.2 with Prandtl's mixing length model (valid for a smooth bottom):

$$\frac{\partial \overline{U}}{\partial z} = \frac{u_*}{\kappa} \frac{1}{z}$$

(4.4)

This gives

$$\overline{U} = \frac{1}{u_*} \kappa \ln \left( \frac{z}{z_0} \right)$$

(4.5)

with

$\overline{U} =$ ensemble average velocity at the height $z$

$u_*$ = shear velocity

$\kappa =$ constant of von Karman

$z =$ height in the water ($z = 0$ at the bottom and $z = h$ at the water surface)

$z_0 =$ *equivalent "roughness" of the bottom*, such that $\overline{U}(z_0) = 0$.

The depth-averaged flow velocity may be estimated by:

$$\overline{U}_m = \frac{1}{h} \int_0^h \overline{U} \, dz \approx \frac{1}{h} \int_{z_0}^h \overline{U} \, dz = \frac{u_*}{\kappa} \left( \ln \left( \frac{h}{z_0} \right) - 1 \right)$$

(4.6)

This shows that the velocity measured at the height $z = h/e$ (with $e \approx 2.72$ the basis of the Napierian logarithm) estimates the depth-averaged flow velocity. This was the method used in practice to determinate the depth-averaged flow velocity of the experiments. On the other hand, the maximum velocity ($\overline{U}_{\text{max}}$) can be estimated with:

$$\overline{U}_{\text{max}} = \overline{U}(z = h) = \frac{u_*}{\kappa} \ln \left( \frac{h}{z_0} \right) = \overline{U}_m + \frac{u_*}{\kappa}$$

(4.7)
Defining now the shear Reynolds number:

\[ \text{Re}_s = \frac{u_s z_0}{v} \quad (4.8) \]

and using the dimensionless values defined above, equation (4.5) becomes:

\[ u^+ = \frac{1}{\kappa} \ln \left( \frac{z u_s}{v} \right) - \frac{1}{\kappa} \ln \left( \frac{z_0 u_s}{v} \right) = \frac{1}{\kappa} \ln (z^+) + \frac{1}{\kappa} \ln \left( \frac{1}{\text{Re}_s} \right) \quad (4.9) \]

In the case of a free surface flow with a hydraulic smooth bottom, many authors found that the shear Reynolds number is equal to 0.11 \pm 0.02 in the turbulent zone. Thus this equation may be written (assuming that \( \kappa = 0.4 \)):

\[ u^+ = 2.5 \ln(z^+) + (5.5 \pm 0.4) = 5.76 \log(z^+) + (5.5 \pm 0.4) \quad (4.10) \]

As mentioned before, the depth-averaged flow velocities \( \overline{U}_m \) used in our experiments are 10, 15 and 20 cm/s. This gives the following limits for the different zones (assuming in first approximation that \( u_s = 1/13.2 \overline{U}_m \); see appendix 4.1):

<table>
<thead>
<tr>
<th>Viscous sublayer</th>
<th>Buffer zone</th>
<th>( \overline{U}_m ) [cm/s]</th>
<th>10</th>
<th>15</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>( z^+ ) [ ]</td>
<td></td>
<td>5</td>
<td>0.7</td>
<td>0.44</td>
<td>0.33</td>
</tr>
<tr>
<td>( z ) [mm]</td>
<td></td>
<td>30</td>
<td>4.0</td>
<td>2.6</td>
<td>2.0</td>
</tr>
<tr>
<td>( z ) [mm]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( z ) [mm]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4.2: Limits of the different zones for the flows used in our experiments

These computations show that the height of the viscous sublayer is not negligible with regard to the dimensions of the particles used in our experiments (the mean diameter of the set "small particles" is 0.36 mm and the one of the set "large particles" is 1.095 mm).
4.1.2. Velocity profiles in practice

Experiments show that the velocity profile of equation (4.10) describes well the measurements, with some modifications to the two numerical coefficients. Equation (4.10) is from [3] and the following expressions can be found in the literature:

\[
\begin{align*}
    u^+ &= 2.5 \ln(z') + 5.5 = 5.75 \log(z') + 5.5 \quad [8] \tag{4.11} \\
    u^+ &= 2.78 \ln(z') + 5.5 = 6.40 \log(z') + 5.5 \quad [9] \tag{4.12} \\
    u^+ &= 2.54 \ln(z') + 5.56 = 5.85 \log(z') + 5.56 \quad [13] \tag{4.13} \\
    u^+ &= 2.40 \ln(z') + 5.8 = 5.53 \log(z') + 5.8 \quad [14, 15, 19] \tag{4.14}
\end{align*}
\]

In practice, the method of the least squares gives the following dimensional equation for the measurements:

\[
\bar{U} = \alpha \ln(z) + \beta \quad \tag{4.15}
\]

with for instance \( \bar{U} \) in cm/s and \( z \) in mm.

The theoretical formula (cf. equation 4.5) can be written:

\[
\bar{U} = \frac{u_*}{\kappa} \ln(z) - \frac{u_*}{\kappa} \ln(z_0)
\]

thus

\[
\begin{align*}
    \alpha &= \frac{u_*}{\kappa} \\
    \beta &= -\frac{u_*}{\kappa} \ln(z_0) = -\alpha \ln(z_0) \Rightarrow z_0 = e^{-\beta/\alpha}
\end{align*}
\]

(4.17)

The depth-averaged velocity is then computed with:

\[
\bar{U}_m = \frac{1}{h_0} \int_0^h \bar{U}(z) \, dz = \frac{1}{h} [\alpha h (\ln(h) - 1) + \beta h] = \alpha (\ln(h) - 1) + \beta
\]

(4.18)

The shear velocity can be computed with the Darcy-Weisbach equation. One can then compute the ratio between the depth-average velocity and the shear stress, the shear Reynolds number (cf. equation 4.8) and the constant of von Karman \( \kappa = u_*/\alpha \); cf. equation 4.17).

After that, the dimensionless variables \( z' \) and \( u' \) are computed and the relation \( (\ln(z'), u') \) can be plotted on a graph. Normally this relation is linear (for a smooth bottom) and is like the equations (4.10) to (4.14).
4.1.3. Measurements of velocity profiles

Four measurements of a mean velocity profile were carried out in the middle of the flume: the first with an EMS probe (cf. 3.6.1) and three, more accurate, with a Laser-Doppler Flow Meter (cf. 3.6.2).

4.1.3.1. Velocity profile measured with EMS probe

The measurements are done in the recording zone with a frequency of 1000 Hz and with an interval of time for the computation of the average $\Delta t = 30$ s. The profile measured for a water depth of 8 cm and for a mean velocity about 20 cm/s is given in figure 4.2:

![Velocity profile measured with EMS probe](image)

Figure 4.2: Mean velocity profile measured with the EMS probe

The velocities measured for the extreme heights 69 and 79 mm are not reliable because the probe is partly out of the water.

As explained in 3.6.1, such a measurement gives only velocities for large measurement volumes and is not very accurate. In order to increase the accuracy, other velocity profiles were measured with a Laser-Doppler Flow Meter, which has two great advantages: the measurement volume is very small ($\approx 0.1 \text{ mm}^3$) and it does not perturb the flow (see 3.6.2).

4.1.3.2. Velocity profile measured with Laser-Doppler flow meter (LDFM)

A first velocity profile was measured in order to see the influence of the offset and of the fact that the LDFM is not perfectly horizontal (contrary to the flume; see 3.6.2). After that, two other velocity profiles were measured on different days, in order to see if nearly the same profile is always found.
4.1.3.2.1. Influence of the offset and of the fact that the LDFM is not perfectly horizontal

In order to see the influence of the offset and of the fact that the LDFM is not perfectly horizontal, a velocity profile is measured and the of the LDFM outputs (A, B; see appendix 3.5) are computed in different ways (water depth = 9.76 cm; depth-averaged velocity about 20 cm/s). The measurements are done with the oscilloscope, for an interval of time of 40 s. The figure 4.3 gives the results of these measurements, where the value of $R^2$ is by definition:

$$R^2 = 1 - \frac{\sum (y_i - \hat{y}_i)^2}{(\sum y_i^2) - \left(\frac{\sum y_i}{n}\right)^2}$$  \hspace{1cm} (4.19)

with

- $y_i =$ value measured
- $\hat{y}_i =$ value estimated with the equation of regression
- $n =$ number of pairs (x,y) on the graph.

![Velocity profile measured with LDFM](image)

Figure 4.3: Mean velocity profile measured with a Laser-Doppler Flow Meter

One can see that the profile found is described well by a logarithmic law. The equation of regression is

$$\bar{U} \text{ [cm/s]} = 3.074 \ln(z \text{ [mm]}) + 11.22$$

and the computed characteristic parameters of this flow are (cf. 4.1.2):

$$z_0 = 26 \mu m$$

$$\bar{U}_m = 22.2 \text{ cm/s} \quad u_* = 1.12 \text{ cm/s} \quad \frac{\bar{U}_m}{u_*} = 19.8$$

$$Re_* = \frac{u_*z_0}{\nu} = 0.29 \quad \kappa = 0.36$$

The value of the shear Reynolds number differs strongly with the value 0.11 ± 0.02 (assuming that the value of the shear velocity computed with the Darcy-Weisbach equation is reliable).
Knowing the value of the shear velocity, the dimensionless variables can be computed and plotted in a graph:

![Velocity profile with the dimensionless variables](image)

\[ u^+ = 2.74 \ln(z^+) + 3.38 \]
\[ R^2 = 0.99 \]

**Figure 4.4: Mean velocity profile measured with a Laser-Doppler Flow Meter as a function of the dimensionless variables \( u^+ \) and \( z^+ \)**

In order to see the influence of the offset and the fact that the LDFM is not totally horizontal (contrary to the flume), the electric outputs are computed another time. The velocities computed in three situations are compared in table 4.3: assuming that the offset is equal to zero and that the LDFM is horizontal; assuming that the offset is different from zero and that the LDFM is horizontal; assuming that the offset differs from zero and that the LDFM is not horizontal (see 3.6.2 for the formulae used, the significance and the determination of \( \zeta \); \( z \) = height of the measurement volume):

<table>
<thead>
<tr>
<th>( z ) [mm]</th>
<th>without offset ( \zeta = 45^\circ )</th>
<th>with offset ( \zeta = 45^\circ )</th>
<th>with offset ( \zeta = 46.16^\circ )</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.5</td>
<td>16.82, 1.99</td>
<td>16.67, 1.02</td>
<td>16.69, 0.68</td>
</tr>
<tr>
<td>10</td>
<td>18.74, 1.36</td>
<td>18.59, 0.4</td>
<td>18.59, 0.02</td>
</tr>
<tr>
<td>15</td>
<td>19.24, 1.24</td>
<td>19.09, 0.38</td>
<td>19.09, -0.01</td>
</tr>
<tr>
<td>20</td>
<td>20.27, 1.33</td>
<td>20.12, 0.36</td>
<td>20.12, -0.05</td>
</tr>
<tr>
<td>25</td>
<td>20.89, 1.36</td>
<td>20.74, 0.39</td>
<td>20.74, -0.03</td>
</tr>
<tr>
<td>30</td>
<td>21.80, 1.36</td>
<td>21.64, 0.39</td>
<td>21.65, -0.05</td>
</tr>
<tr>
<td>35</td>
<td>22.00, 1.40</td>
<td>21.85, 0.44</td>
<td>21.85, -0.01</td>
</tr>
<tr>
<td>40</td>
<td>22.44, 1.46</td>
<td>22.29, 0.49</td>
<td>22.29, 0.04</td>
</tr>
<tr>
<td>45</td>
<td>23.17, 1.40</td>
<td>23.02, 0.44</td>
<td>23.02, -0.03</td>
</tr>
<tr>
<td>50</td>
<td>23.07, 1.43</td>
<td>22.92, 0.46</td>
<td>22.92, 0.00</td>
</tr>
<tr>
<td>55</td>
<td>23.49, 1.46</td>
<td>23.34, 0.50</td>
<td>23.34, 0.02</td>
</tr>
<tr>
<td>60</td>
<td>23.86, 1.45</td>
<td>23.71, 0.48</td>
<td>23.71, 0.00</td>
</tr>
<tr>
<td>65</td>
<td>24.29, 1.47</td>
<td>24.14, 0.51</td>
<td>24.14, 0.02</td>
</tr>
<tr>
<td>75</td>
<td>24.81, 1.44</td>
<td>24.66, 0.48</td>
<td>24.66, -0.03</td>
</tr>
<tr>
<td>85</td>
<td>24.64, 1.48</td>
<td>24.49, 0.51</td>
<td>24.49, 0.02</td>
</tr>
<tr>
<td>90</td>
<td>25.03, 1.58</td>
<td>24.88, 0.61</td>
<td>24.88, 0.11</td>
</tr>
</tbody>
</table>

**Table 4.3: Influence of the offset and the angle between the LDFM and the horizontal on the time average velocities**

80
The conclusion of this comparison is that the influence of the offset and of an angle $\zeta \neq 45^\circ$ is minute on the time average streamwise velocity $\overline{U}$ (see figure 4.5). On the opposite it has a great influence on the values of $\overline{V}$, which became nil on average in place of values about 1.5 cm/s. This means that for the determination of a velocity profile it is not necessary to consider these two parameters, but that for the measurement of turbulence we have to consider them.

![Influence of the offset and of the inclination of the LDFM on the velocity profile](image)

*Figure 4.5: Influence of the offset and the angle between the LDFM and the horizontal on the mean velocity in the longitudinal direction*

Table 4.4 shows that this influence is minute on the computed characteristics of the flow (about 1%):

<table>
<thead>
<tr>
<th></th>
<th>Without offset $\zeta = 45^\circ$</th>
<th>With offset $\zeta = 45^\circ$</th>
<th>With offset $\zeta = 46.16^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equ. Reg.</td>
<td>$U=3.0737 \ln(z)+11.22$</td>
<td>$U=3.0747 \ln(z)+11.07$</td>
<td>$U=3.0742 \ln(z)+11.07$</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td>$\overline{U}_m$ [cm/s]</td>
<td>22.23</td>
<td>22.08</td>
<td>22.08</td>
</tr>
<tr>
<td>$u_*$ [cm/s]</td>
<td>1.121</td>
<td>1.114</td>
<td>1.114</td>
</tr>
<tr>
<td>$z_0$ [\mu m]</td>
<td>26.0</td>
<td>27.3</td>
<td>27.3</td>
</tr>
<tr>
<td>$\overline{U}<em>m / u</em>*$</td>
<td>19.83</td>
<td>19.82</td>
<td>19.82</td>
</tr>
<tr>
<td>Re$_*$</td>
<td>0.29</td>
<td>0.30</td>
<td>0.30</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>0.365</td>
<td>0.362</td>
<td>0.362</td>
</tr>
<tr>
<td>Equ. Reg.</td>
<td>$u^* = 2.74 \ln(z^*) + 3.38$</td>
<td>$u^* = 2.76 \ln(z^*) + 3.28$</td>
<td>$u^* = 2.76 \ln(z^*) + 3.28$</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
</tr>
</tbody>
</table>

*Table 4.4: Influence of the offset and the angle between the LDFM and the horizontal on some characteristics of the flow*
4.1.3.2.2. Measurements of some velocity profiles

Two other velocity profiles were measured, for a flow depth of 7.87 cm and 8.01 cm respectively (each profile is determined by the measurement of 7 velocities). The results of these measurements and the previous are summarised in table 4.5:

<table>
<thead>
<tr>
<th>Equ. Reg.</th>
<th>1 h = 97.6 mm</th>
<th>2 h = 78.7 mm</th>
<th>3 h = 80.1 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R^2$</td>
<td>0.99</td>
<td>0.97</td>
<td>0.99</td>
</tr>
<tr>
<td>$\bar{U}_m$ [cm/s]</td>
<td>22.08</td>
<td>18.71</td>
<td>19.21</td>
</tr>
<tr>
<td>$u_*$ [cm/s]</td>
<td>1.114</td>
<td>0.982</td>
<td>1.003</td>
</tr>
<tr>
<td>$z_0$ [µm]</td>
<td>27.3</td>
<td>10.5</td>
<td>13.9</td>
</tr>
<tr>
<td>$\bar{U}<em>m / u</em>*$ [ ]</td>
<td>19.82</td>
<td>19.05</td>
<td>19.14</td>
</tr>
<tr>
<td>$Re_*$ [ ]</td>
<td>0.30</td>
<td>0.103</td>
<td>0.140</td>
</tr>
<tr>
<td>$\kappa$ [ ]</td>
<td>0.36</td>
<td>0.42</td>
<td>0.40</td>
</tr>
<tr>
<td>Equ. Reg</td>
<td>$u' = 2.76 \ln(z') + 3.28$</td>
<td>$u' = 2.41 \ln(z') + 5.46$</td>
<td>$u' = 2.50 \ln(z') + 4.92$</td>
</tr>
<tr>
<td>$R^2$ [ ]</td>
<td>0.99</td>
<td>0.97</td>
<td>0.99</td>
</tr>
</tbody>
</table>

Table 4.5: Characteristic equations and values of some velocity profiles measured

The two last velocity profiles measured correspond much better to the velocity profiles found in the literature. Moreover they are relative to flow depths of the same magnitude of the one used during the experiments. Thus they will be taken as reference velocity profiles for the experiments. It would be better for the data analysis (see chapter 6) to dispose of the measured velocity profiles of the flows we used in the experiments. Unfortunately, problems of timing and technical problems made that we could not do it. However, the depth-averaged velocity was for each experiment estimated, by measuring the velocity at the height $h/2.71$ with a Laser-Doppler Flow Meter (cf. equation 4.6). Using the Darcy-Weisbach equation for the estimation of the shear velocity, the velocity profile can be estimated with:

$$u' = 2.45 \ln(z') + 5.2$$  \hspace{1cm} (4.20)

It is important to notice that the coefficients of the equations of regression ($\bar{U}$; $z$) and ($u'$; $z'$) vary rapidly with the presence of one wrong pair (because the number of pairs is only 7). Another important remark is that these two coefficients do not change much the shape and the position of the curve (see figure 4.6.a et 4.6.b, drawn for $u_* = 1$ cm/s).
Influence on the concentration profile of the variation of $A$ in $u^+ = A \ln(z^+) + B$; $u_\tau = 1 \text{ cm/s}$

![Graph showing influence of $A$.](image)

Figure 4.6.a: Influence on the concentration profile of the variation of $A$ in $u^+ = A \ln(z^+) + B$ for a shear velocity of 1 cm/s

Influence on the concentration profile of the variation of $B$ in $u^+ = A \ln(z^+) + B$; $u_\tau = 1 \text{ cm/s}$

![Graph showing influence of $B$.](image)

Figure 4.6.b: Influence on the concentration profile of the variation of $B$ in $u^+ = A \ln(z^+) + B$ for a shear velocity of 1 cm/s
4.2. Measurement of the important parameters of turbulence

The most important parameters of turbulence are the correlation of the turbulent fluctuations of $U$ and $W$ ($U'W'$), giving the Reynolds shear stresses (see appendix 1.1), and the intensity of the turbulent velocities (instantaneous deviations of the velocity from to the ensemble average velocity; see appendix 1.1), defined as:

$$
\text{Intensity of } U' = \sqrt{U'^2} \quad \text{Intensity of } W' = \sqrt{W'^2} \tag{4.21}
$$

A measurement of these values was done with another Laser-Doppler Flow Meter, which permits to measure the instantaneous velocities nearby the bottom (until 1 mm). The height of water is about 7.00 cm, the mean velocity about 15 cm/s, the frequency of measurement 100 Hz and $\Delta t = 100$ s (thus 10 000 measurements for one height). The computing method is explained in appendix 4.2.

The mean velocity profile measured is once again logarithmic:

![Velocity profile diagram](image)

Figure 4.7: Velocity profile of the flow used for the measurement of turbulence

and has the following characteristic equations and values:

$$
\overline{U} = 2.15 \ln (z) + 8.75
$$

$$
\overline{U}_m = 15.7 \text{ cm/s} \quad u_* = 0.85 \text{ cm/s} \quad z_0 = 17 \mu\text{m}
$$

$$
\frac{\overline{U}_m}{u_*} = 18.4 \quad \text{Re}_* = \frac{u_* z_0}{\nu} = 0.15 \quad \kappa = 0.40
$$

$$
u^+ = 2.52 \ln (z^+) + 4.87
$$
The correlation of the turbulent fluctuations of the streamwise and the vertical velocity are depicted in figure 4.8:

![Correlation of U' and W']

Figure 4.8: Measurement of the correlation of the turbulent fluctuations of U and W

As one can see, the absolute value of the correlation between $U'$ and $W'$ (and thus the Reynolds stresses) is decreasing when going up to the surface, which corresponds with the measurements given in [11]. The variation is nearly linear, except near the bottom. The extrapolation of the line intersects the abscissa at a value which is useful for the estimation of the shear stress at the bed ($\tau_b$) and the shear velocity ($u_\tau$):

$$u_\tau \equiv \sqrt{\frac{\tau_b}{\rho}} = \sqrt{\frac{U'W'}{\text{extrapolated}_{z=0}}}$$

(4.22)

which gives:

$\frac{U'W'}{\text{extrapolated}_{z=0}} \equiv 0.75$ cm$^2$/s$^2$

$\tau_b \equiv 0.075$ N/m$^2$

$u_\tau \equiv 0.87$ cm/s.

Notice that the value of the shear velocity given by the Darcy-Weisbach equation is $u_\tau = 0.84$ cm/s. This justifies the use of this equation in all the previous computations.

According to [11], this correlation should decrease for small values of $z$. This was not measured here; notice that the lowest measure ($z = 1$ mm) is not reliable (see appendix 4.2). For a correlation of for instance -0.5 cm$^2$/s$^2$, the corresponding Reynolds stress is 0.05 N/m$^2$, which corresponds to a normal pressure of a column of 0.05 mm of water.
The intensities of $U'$ and $W'$ are depicted in figure 4.9:

![Intensity of $U'$ and $W'$](image)

**Figure 4.9: Measurement of the intensity of $U'$ and $W'$**

The intensity of $U'$ is larger than that of $W'$, and they both decrease when going up to the surface. This agrees with the description found in Nezu & Nakagawa [11]. In order to compare quantitatively these measurements with the curves found in [11]:

\[
\frac{\sqrt{U'^2}}{u_*} = 2.30 \ e^{-z/h} \quad \frac{\sqrt{W'^2}}{u_*} = 1.63 \ e^{-z/h}
\]

(4.23)

the dimensionless turbulent intensities are computed as a function of the relative height:

![Dimensionless turbulent intensities as a function of the relative height](image)

**Figure 4.10: Dimensionless turbulent intensities as a function of the relative height**

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4.3. Influence of the injection

The injection perturbs the flow for two reasons: firstly by the presence of the injection pipe in the flow and secondly because the velocity of the injected mixing (fluid + particles) could be different from the local flow velocity. The momentum per unit length of the flow and that of the injection are:

\[ \bar{I}_{\text{flow}} = \bar{U}_{m,\text{flow}} \rho hb \]  \hspace{1cm} (4.24)

with \( b = \text{width of the flume} = 40 \text{ cm} \)

\[ \bar{I}_{\text{injection}} = \bar{U}_{m,\text{injection}} \rho \frac{\pi}{4} D^2 \]  \hspace{1cm} (4.25)

Assuming that the height of the constant-head vessel is such that the mean velocity of the injection is equal to the mean velocity of the flow, the ratio of the two momenta is:

\[ \frac{\bar{I}_{\text{injection}}}{\bar{I}_{\text{flow}}} = \frac{\frac{\pi}{4} D^2}{hb} = 1.5 \times 10^{-3} \]  \hspace{1cm} (4.26)

Thus the influence of the injection on the total momentum is small. This is really negligible and will stay negligible even if there is a small difference between the two velocities. But in that case it influences the dispersion of the particles when they come out of the injection pipe. If the injection is very far from the camera this is not important, because the situation has the time to stabilise. But for small distances (the smallest used in these experiments is 50 cm) a visual observation shows that it can be significant. So the height of the constant-head vessel will always be such that the local flow velocity and the injection velocity are equal (cf. appendix 5.1).

The measurement of the influence of the injection and of the injection pipe on the velocity profile requires the measurement of small differences in velocity on small measurement volumes. This is possible with the use of a Laser-Doppler Flow Meter. A first reference velocity profile is measured without the injection pipe. Afterwards the injection pipe is placed in the flow (at a height of 40 mm for a water depth of 80 mm) and velocities profiles are measured at different distances from the injection. The frequency of the measurement is 100 Hz and \( \Delta t = 30 \text{ s} \).

Figure 4.11 on the next page gives the results of these measurements.

It is not easy to compare the influence of the injection on the velocity profile at the different distances from the injection (\( \Delta x \)). It is in fact very difficult to split the total influence in the influence of the pipe itself and the one of the injection of the mixing. We can just say that it has a small influence on the velocity profile (about 2%), except at the height of the injection (about 5%). This can however be minimised if the height of the constant-head vessel is well adjusted.
Figure 4.11: Influence of the injection and the injection pipe on the velocity profile
Chapter 5

EXPERIMENTS

Now that all the environments for the experiments have been studied (the Two-Dimensional Particle Tracking Velocimetry, the experimental set-up and the flow), we will describe the parameters changed for these experiments, the procedure used and the analysis of the recordings.

5.1. Parameters

In order to see the influence of several parameters on the possibilities of the Two-Dimensional Particle Tracking Velocimetry and on the concentration profiles, we made a set of experiments which differ from each other with only one parameter.

The different parameters studied are:

- the **diameter of particle**: large ones or small ones;
- the **distance between the end of the injection pipe and the centre of the camera** ($d_{inj}$, see figure 5.1); 5 different distances were taken: 0.5, 1, 1.5, 2, 5 m;
- the **height of the injection** ($h_{inj}$, see figure 5.2: defined as the distance between the centre of the injection pipe and the bottom of the flume); three different heights were chosen: just beneath the water surface, in the middle of the flow and near the bottom of the flume;
- the **mean flow velocity** ($\overline{U}_m$); three different velocities were chosen: 10, 15 and 20 cm/s.

![Figure 5.1: $d_{inj}$](image1)

![Figure 5.2: $h_{inj}$](image2)

We worked with a mean flow velocity of 15 and 20 cm/s for the large particles and 10 and 15 cm/s for the small particles. An experiment with the large particles carried out with 10 cm/s showed that these particles did not move at all after they reached the bottom. The
mean flow velocity of 15 cm/s was used for the two types of particles and permits to compare the influence of the diameter.

We made four series (A to D) of 12 (13 for the two lasts) experiments (0 to 13) with the following parameters:

<table>
<thead>
<tr>
<th>Name of the experiment</th>
<th>Type of the particles</th>
<th>Mean flow velocity $\bar{U}_m$ [cm/s]</th>
<th>Distance between point of observation and injection $D_{nj}$ [cm]</th>
<th>Height of the injection $H_{nj}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A01-A12</td>
<td>Large</td>
<td>20</td>
<td>50 100 150 200</td>
<td>bottom beneath surface</td>
</tr>
<tr>
<td>B01-B12</td>
<td>Large</td>
<td>15</td>
<td>50 100 150 200</td>
<td>bottom beneath surface</td>
</tr>
<tr>
<td>C01-C13</td>
<td>Small</td>
<td>15</td>
<td>50 100 150 200</td>
<td>bottom beneath surface</td>
</tr>
<tr>
<td>D00-D12</td>
<td>Small</td>
<td>10</td>
<td>50 100 150 200</td>
<td>bottom beneath surface</td>
</tr>
</tbody>
</table>

Table 5.1: Parameters used for the different series of experiments

In order to dispose of information on the influence of the diameter, we made the series of experiments B and C the same day in the same flow.
5.2. Choice of the duration of one experiment

In order to compute ensemble averaged values of the variables studied (concentration profile and velocities of the particles) it is important to dispose of a representative number of independent images. Two characteristic times are important in these experiments: the time taken by a particle to go through the recording zone (which will give the number on different, and thus independent, particles recorded) and the mean bursting period (which will give the number of independent periods recorded). We take arbitrarily a duration of recording of one minute and we check afterwards whether it is enough for having a sufficient number of independent data.

We can consider in first approximation that the width of the window of recording is 15 cm and that the streamwise velocity of a particle \( u_p \) is equal to the mean velocity of the flow \( \bar{U}_m \). With these hypotheses a particle will go through this window in 1.5, 1 and 0.75 seconds respectively for the different velocities used in the experiments (10, 15 and 20 cm/s). This means that with 60 seconds recording we dispose of 40, 60 and 80 independent sets of particles respectively.

On the other hand, the mean bursting period \( \bar{T}_b \) in a free surface flow can be estimated with (cf. 1.4):

\[
\bar{T}_b = (1.5 \text{ to } 3.0) \frac{h}{\bar{U}_{max}}
\]  

(5.1)

with: 
- \( h \) = depth flow
- \( \bar{U}_{max} \) = maximum value of the streamwise flow velocity

\( \bar{U}_{max} \) can be estimated with (cf. chapter 4):

\[
\bar{U}_{max} = \bar{U}(z = h) = \bar{U}_m + \frac{u_*}{\kappa}
\]

(5.2)

where \( u_* \) can be estimated with the Darcy-Weisbach equation.

This gives for the different velocities used in the experiments:

<table>
<thead>
<tr>
<th>( \bar{U}_m )</th>
<th>[cm/s]</th>
<th>10</th>
<th>15</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>( u_* )</td>
<td>[cm/s]</td>
<td>0.57</td>
<td>0.81</td>
<td>1.04</td>
</tr>
<tr>
<td>( \bar{U}_{max} )</td>
<td>[cm/s]</td>
<td>11.4</td>
<td>17.0</td>
<td>22.6</td>
</tr>
<tr>
<td>( \bar{T}_b )</td>
<td>s</td>
<td>1.1 to 2.2</td>
<td>0.7 to 1.4</td>
<td>0.5 to 1.0</td>
</tr>
</tbody>
</table>

Table 5.2: Mean bursting period for the flows used in the experiments

So with 60 seconds recording we dispose for the three velocities used of 38, 57 and 76 times the mean bursting period respectively. This shows also that the computed averages will be better for a mean flow velocity of 20 cm/s than for a mean flow velocity of 10 cm/s.
These considerations show that with one minute of recording, the number of independent measurements is sufficient to state that the averages computed estimate the ensemble averages well.

5.3. Procedure

In order to compare the different series of experiments, it is very important to follow at each time the same procedure. The procedure described below will be taken as reference for all the series of experiments.

The sieve (to recover the particles) is put at the end of the flume and the flume is placed horizontally. The diapositive-projector is placed and correctly focused to obtain a great light intensity in recording zone. The width of the lightsheet is measured (generally between 1.5 and 2.0 cm) at the two extremities of the recording window. Then an offset of the Laser-Doppler Flow Meter is recorded in a glass box filled with still water. When this offset is registered, the valve can be opened and the water comes into the flume. The water depth and flow velocity are adjusted with the valve and the vertical sluice gate at the end of the flume to obtain a water depth of more or less 8 cm and the mean flow velocity desired (10, 15 or 20 cm/s). The mean flow velocity is measured with the Laser-Doppler Flow Meter and estimated by the measurement of the velocity at $z = 82.7$ cm (cf. chapter 4). The injection velocity depends on the height of the constant-head vessel (see appendix 5.1.) and it will be placed at the right height to have the same injection velocity as the flow velocity at this height (so it does not disturb it).

The video-tape is placed in the video tape recorder and it is unwound and rewound to ensure that the tension of the video-tape is constant. It is then placed on the right place for the recording of the new experiments. The distinctness of the camera is adjusted. The piece of plastic with the world co-ordinates and reference points is put in the water in the lightsheet just in front of the camera and is recorded (about 60 seconds to be sure to have a good image). The top and the two extremities of the flume around the recorded zone are now covered with boards.

Now all the environmental set-ups are prepared, the mixture can be prepared. The amount of particles is placed in a pail with a little bit of soap and some water coming from the flume (so it has the same water temperature). All these components are mixed in the pail and then put in the vessel. In this, the mixer will keep the mixture homogeneous, but we remark that during a series of experiments the particle concentration decreases.

Between two different experiments of one series, a data sheet will be recorded. This data sheet contains all the data concerning this experiment:

- the date of the experiment;
- the letter of the series (from A to D) and the number of the experiment (from 0 to 13);
- the **information about the injection**: its height, the distance between the injection point and the camera and the height of the constant-head vessel;
- the **information about the flow**: its mean velocity, its depth and the height of the measurement volume;
- the **width of the lightsheet** (at the two extremities of the recorded zone);
- the **information about the particles**: their type and the concentration of the mixture in the vessel.

<table>
<thead>
<tr>
<th>Series n°</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Experiment n°</td>
<td></td>
</tr>
<tr>
<td>Date</td>
<td></td>
</tr>
<tr>
<td>Injection</td>
<td></td>
</tr>
<tr>
<td>$h_{\text{inj}}$ [mm]</td>
<td></td>
</tr>
<tr>
<td>$d_{\text{inj}}$ [cm]</td>
<td></td>
</tr>
<tr>
<td>constant-head vessel height [cm]</td>
<td></td>
</tr>
<tr>
<td>Flow</td>
<td></td>
</tr>
<tr>
<td>velocity [cm/s]</td>
<td></td>
</tr>
<tr>
<td>depth [mm]</td>
<td></td>
</tr>
<tr>
<td>height of the measurement volume [mm]</td>
<td></td>
</tr>
<tr>
<td>Lightsheet</td>
<td></td>
</tr>
<tr>
<td>width [mm]</td>
<td>and</td>
</tr>
<tr>
<td>Particles</td>
<td></td>
</tr>
<tr>
<td>diameter [mm]</td>
<td></td>
</tr>
<tr>
<td>sieved (yes/no)</td>
<td></td>
</tr>
<tr>
<td>concentration [ml/l]</td>
<td></td>
</tr>
<tr>
<td>Remarks</td>
<td></td>
</tr>
</tbody>
</table>

*Figure 5.3: Data sheet*

An experiment may start when all these preparations are done. The lights in the laboratory are switched off and the pump for the injection is started. The head of the small PVC tube of the constant-head vessel is then closed with a hand (always the same hand) in order to increase the pressure until the injection of the mixture in the flow starts. The series of experiments begins. After each experiment, the camera is turned (its legs do not move so the camera records always the same part of the flow) to record the data sheet (10 seconds), then it is turned again to its initial position (marked with some tape placed on the glass of the flume), the dark texture is put on it and the recording of the particles in the flow starts again (about 60 seconds). The injection of the mixture is not stopped between the experiments of one series, thus all the changes of parameters have to be made very quickly, because the number of particles is limited (cf. chapter 3.3). Before a new recording, the pressure in the constant-head vessel is again increased in order to get rid of eventual air
bubbles accumulating in the injection pipe. In order to take the least time between two changes, the sequence of the recordings is the following:

- with a constant height of injection, we vary the distance between the injection and the camera (4 recordings);
- the height of the injection is changed and also the height of the constant-head vessel in order to have the same velocity.

The first series of experiments is now made and the same procedure will be used for the three other series of 12 experiments. Each series is about 15 minutes of recording but it takes nearly one day to prepare and to record it (not counting the time to sieve the particles).

5.4. Analysis with the Two-Dimensional Particle Tracking Velocimetry

When we have a video-tape with a recording with the world co-ordinates and with the experimental data, an analysis can be done with the Two-Dimensional Particle Tracking Velocimetry. All these analyses have been made in the Technical University of Eindhoven.

This analysis can be divided in a few steps:

1. recording the audio signals
2. setting the world and reference co-ordinates
3. setting the particle location parameters
4. setting the tracking parameters
5. checking the particle tracking data.

1. Recording the audio signal

The video-tape is placed in the VTR and it is unwound and rewound to ensure that the tension of the video-tape is constant. We first need to record an audio-signal on the audio channel of the video-tape to enable the computer to find the right image during the tracking (see chapter 2).

The audio-signal is firstly recorded on a small part of the video-tape so it can be checked. It must be recorded at equal time intervals; this can be checked with the help of a headphone: a monotonous sound like the sound of a metronome must be heard. A second more extensive check can be made by computer. When these two checks are right, the audio-signal can be recorded during the whole duration of the experiment.
2. Setting the world and reference co-ordinates

An image containing the reference points and world co-ordinates is grabbed in a buffer (place in the memory of the computer where the image is stored). With the help of the cursor, these points are localised on the screen and their co-ordinates are specified with the keyboard. The more points are entered, the more accurate will be the determination of the position of the particles and thus the accuracy of all the steps which follow. Generally about ten points are entered. Among the proposed choices, a linear mapping between the world and the pixel co-ordinate system is chosen.

3. Setting the particle location parameters

The parameters for the particle location will first be checked on a static image representative of the experiment. This image will be grabbed and placed in a buffer. The parameters will be chosen with the help of this grabbed image: as the velocities of the particles are large owing to the width of one image, this grabbed image has to be filtered to avoid having double particles (when the odd lines and the even lines are shifted of 1/50 second, cf. chapter 2). The most important location parameters are:

- **ellipticity**: the value is 1.0 if the form of the blob has not an importance to catch the particle. Whereas if the blob has to be nearly circular to be considered as a particle, this value will be 0.9;
- **lower size limit**: the minimum area-pixels that a blob must have to be considered as a particle;
- **upper size limit**: the maximum area-pixels that a blob must have to be considered as a particle;
- **lower threshold**: the lower threshold value for blob detection to be considered as a particle;
- **upper threshold**: the upper threshold value for blob detection to be considered as a particle;
- **number of threshold**: the number of threshold used (from 2 up to 8);
- **min X-size**: the minimal horizontal size for blob detection considered as a particle;
- **min Y-size**: the minimal vertical size for blob detection considered as a particle.

All these parameters can be determined easily on a grabbed image in the buffer with the help of the cursor. A zoom can be made on a blob (or on a particle) and so its dimension and its intensity can be found. There is a function to check if all the particle location parameters are correctly entered. This function locates all the particles present in the buffer according to the parameters set and displays the results in another buffer by putting a cross where blobs are identified as a particle. All these parameters are verified on a representative image and are checked afterwards on 3 or 4 other representative images of the flow: it is thus very important to chose these images correctly.

The accuracy of the 2D-PTV for the location of a blob is at least 1 pixel. As the resolution of the screen is 512 pixels and the width of the recording zone is 15 cm, this gives in our case an accuracy about \( \frac{15}{512} = 0.3 \text{ mm} \). In practice, turning the camera, locating the reference points and the world co-ordinates... make that such an accuracy is not reached.
Thus the accuracy of this analysis technique is excellent but is not reached because of experimental manipulations.

4. Setting the tracking parameters

Now that DigImage can recognise the particles, the parameters for the tracking must be introduced. The important parameters are:

- **tracking window**: the window in which the Particle Tracking occurs (allows to suppress a mirror effect beneath the surface of water and the bottom of the flume where there are particles which do not move);
- **interlace filter**: to avoid that the particles are double, it will work with 256 lines instead of 512 (DigImage only considers the odd or the even lines);
- **timing**: the total track time;
- **new particle behaviour**: it allows to introduce here a constant velocity for the new particles which have no history; generally the value of this velocity will be taken equal to the mean velocity of the flow. These velocities can also be determined if two consecutive images are taken in two buffers and we measure the distance covered by a particle between these two images. These velocities can be given either in pixels per seconds or in real co-ordinates per seconds. It is not possible to enter a velocity varying with the height (logarithm profile);
- **maximum matching distance** \((x,y)\): when a particle was already observed in a previous image \((t = t_n)\), its velocity is known and thus DigImage can predict were it will be in this image \((t = t_{n+1})\). This distance represents a circle (of an ellipse if \(x \neq y\)) around the predicted position in which the Particle Tracking will search this particle.
- **maximum new path error** \((x,y)\): it is the same as the maximum matching distance but for the new particle with the constant velocity (see new particle behaviour). The matching area is greater in this case than for the particle with a history.

When all these parameters are introduced, we can check if the tracking is made correctly by starting it and viewing on the screen the reconstitution of the particle paths made by DigImage. The parameters for each experiment in a series are the same because the mean flow velocity and the type of particles are the same, so the whole series can be tracked in one run. Once all of them are correct, we can start the particle tracking. If these parameters are introduced by two different persons, the results may vary a little bit. The tracking for a series of 12 experiments takes about 17 hours (with a Pentium 115 MHz)! These trackings were made during the night.
5. Checking the particle tracking data

Once the computer has finished the particle tracking, four permanent files are created with all the results. The *.IND files were printed to control the tracking (it puts indications when a problem occurs). The number of particles that are matched for each grabbed image is written in it. When it did not match any particle for a long time (about 50 images = 2 seconds) it means that it was the paper which introduces a new experiment. After that Trk2dVel permits us to transform the file with all the result in *.PV files (particle velocity files) so they can be exported. For this option we have to enter the first and last sample of each experiment, the number of samples during the tracking phase which must be passed through to accept this path and the name of the current experiment. Thus we have one large series of files for each experiment.

The 2D-PTV permits to analyse a lot of images in a short time. Once the parameters of location and of tracking are well set, the computer can work during hours without any help. It is very convenient to start the analysis at the end of the day and to obtain the results the next morning.

Post-processing

All these files coming out of the Particle Tracking were transferred from Eindhoven to Delft via the network. These series of files were converted into one file for each experiment, with the help of a program written in DOS by W.S.J. Uijttewaal. Now these files can be analysed. Each line of these files represents a particle and is composed of four columns: the first two for the position of this particle and the other two for its velocity in the X- and Y-direction. To analyse this information and to make some graphs, we have worked with the Macros of Microsoft Excel.
The feasibility of 2D-PTV for the study of vertical sediment transport
Chapter 6

DATA ANALYSIS

This analysis follows the two main objectives of this work: firstly to investigate the feasibility of Two-Dimensional Particle Tracking Velocimetry by DigImage works well for the study of turbulent sediment transport. Secondly to compare some models of particle suspension in a turbulent flow (cf. 1.4) and the measurements.

After some preliminary remarks, the different variables computed (two-dimensional positions and velocities) will be analysed for a snapshot. Then these variables will be studied for the averages computed for all the images of one experiment, i.e. for ensemble averages when the number of particles tracked is satisfactory. This analysis shows the influence of the different parameters modified (diameter of the particles, depth-averaged flow velocity, distance between the injection and the camera and height of the injection) on the variables studied.

6.1. Preliminary remarks

The positions (and the velocities respectively) of the particles in the streamwise and vertical directions are written \( x_p \) and \( z_p \) (\( u_p \) and \( w_p \) respectively). A negative \( w_p \) denotes a particle going down in the flow.

The Particle Tracking Velocimetry with any automated analysis system must be realised very carefully: if for instance the world co-ordinates are not given well, all the positions and velocities of the particles will be completely erroneous. On the other hand, if the audio pulse is not recorded well, the images will not be identified well and the tracking between consecutive images will be erroneous. Finally, all kinds of problems of particle location or tracking can occur. That means that one has to be very careful and critical when analysing the data.

Once the analysis with DigImage is done, *.PV files (cf. chapter 5) must be made for the data post-processing. For doing this, the number of consecutive images on which a particle must be tracked must be chosen. We took it as 3, in order to eliminate the tracking errors which can occur on two consecutive images. That means that the particle paths will not be studied (this has already been done in the experiments carried out by Sumer & Oguz [14] and Sumer & Deigaard [15]). But this kind of analysis would also be possible with DigImage with the use of a special lens on the camera and by making the *.PV files with particles which were tracked on a large number of consecutive images.

As mentioned in chapter 5, we made 4 series of 12 (or 13) experiments. All the video recordings were not of top quality and the number of particles in the images was sometimes too small to get files with particles which were tracked on three consecutive images. Many
experiments carried out with the large particles were not a success, especially when working with $\bar{U}_w = 15 \text{ cm/s}$ (this velocity was too small to lift up the particles).

The appendix 6.1 gives the particle concentrations in the vessel used in the four series of experiments and the total number of particles injected. As the volume of one large particle is about 30 times the one of a small particle, we find for the large particles larger volumetric concentrations for a smaller number of particles injected. As one can see, the total number of particles injected is huge: about 700 000 for each series carried out with large particles and about 12 000 000 for each series carried out with small particles.

In order to see the efficiency of the method of tracking used in these experiments, the appendix 6.2 gives among other things these particle concentrations and the number of particles tracked (notice that a particle may be tracked many times in a series of three consecutive images, and thus this number does not represent the real number of distinct particles tracked). It shows that the number of particles tracked in a short time (four nights of work for a computer) is huge with regard to other techniques of particle location or tracking: more than 320 000 particles were tracked (with their two-dimensional positions and velocities) on three consecutive images. This number can be compared to other experiments of particle location or particle tracking: Sumer & Oguz [14] analysed with a photographic method the paths of 1141 particles. Later, Sumer & Deigaard [15] used another photographic technique and analysed the paths of about 2000 particles (but their tracking was most complicated as they worked with three-dimensional positions and velocities and as they tracked particles over a distance about 2.5 m). Finally, Wiltink [19] analysed with a video recording method (followed by an analysis by hand of the images) about 10 images with an average of 40 particles on each.

The appendix 6.2 shows also that the average number of particles tracked for one experiment with the large particles (2158 for the series A and 4124 for the series B) is smaller than the one for the small particles (11 030 for the series C and 11 774 for the series D). This means that the ensemble averages will better estimate the true values for the experiments carried out with the small particles than the ensemble averages computed for the experiments carried out with the large particles.

In order to carry out the data analysis, it is important to dispose of the streamwise velocity profile of the flow in which the recordings were done, for two reasons: firstly to compare it with the streamwise velocities of the particles, and secondly for the determination of the shear velocity, which is very important to compare the concentration profiles measured with the models (cf. chapter 1) and to have an idea about the turbulence intensities (cf. appendix 1.2: $U' \equiv W' \equiv u_\ast$). The way this velocity profile is measured or estimated was explained in chapter 4.

Finally, it is possible to draw up a list of "equivalent experiments", i.e. experiments which differ with respect to only one parameter. The appendix 6.3 gives the multiple practical possibilities of comparison, considering only the experiments with a sufficient number of tracked particles (more than approximately 150 particles). As one can see, there are many possibilities to study the influence of a parameter: 3 for the diameter, 15 for the depth-averaged velocity, 9 for the distance between the injection and the point of observation and 11 for the height of injection. As we are interested in three different variables (concentration
profiles, streamwise velocity and vertical velocity), a complete analysis would compare 114 situations. This is of course not very practical. So we will describe as best as possible the characteristic influences of the parameters on the variables studied with a reduced number of situations which are representative.
6.2. Analysis of snapshots

Now that DigImage has written for each experiment hundreds files containing the two-dimensional positions and velocities of the particles for snapshots, it is convenient to draw some graphs. They will help to visualise the different trends of the particle motions as a function of the different parameters used.

A first kind of graph containing all the information can be drawn: it represents an image with all the particles and their two dimensional velocity vectors (the length of the vector is proportional to the velocity):

Figure 6.0.a: Positions and velocities vectors for a snapshot of experiment A03
Large particles; $\bar{U}_m = 20$ cm/s; $d_{ej} = 1$ m; $h_{ej} =$ surface
Figure 6.0.b: Positions and velocities vectors for a snapshot of experiment B04
Large particles: $\bar{U}_m = 15$ cm/s; $d_{inf} = 0.5$ m; $h_{inf} = $ surface

These graphs give a good idea of the general trends of the particle motion, in these cases a movement towards the bottom. One can also see that one or two particles, located in the same area, are going up, contrary to the general trend. In the figure 6.0.a. there are some particles leaving the bottom. However, these graphs do not give any information about the concentration profile or about the values and the distribution of the velocities.

In order to have more information, six representative graphs are drawn for a snapshot:
- **position** of the particles ($x_p, z_p$)
- **concentration profile**, i.e. histogram of the vertical position ($z_p$)
- **streamwise** (longitudinal) **velocity** of the particles as a function of the height ($u_p, z_p$) on a linear scale
- streamwise (longitudinal) **velocity** of the particles as a function of the height ($u_p, z_p$) on a logarithmic scale
- **histogram** of the **vertical velocity** ($w_p$)
- vertical velocity as a function of the height ($w_p, z_p$).

The figures 6.1 and 6.2 give these graphs for small particles injected in a flow of 20 cm/s, at 1 m from the camera and beneath the surface, for two snapshots taken with a time interval of $3*(1/25$ s).

Notice that in the case of a non fully developed situation (i.e. a situation which evolves in the streamwise direction) all these values can vary from one part of the image to the other. It is nevertheless assumed that the situation observed is uniform for the whole image (i.e. the dimensions of the image are assumed to be small with regard to the characteristic lengths for the variation of the variable studied). This hypothesis is the better fulfilled as the distance between the injection and the point of observation is increased.
Figure 6.1: Characteristic graphs for a snapshot of experiment C13

Small particles; \( U_m = 15 \text{ cm/s} \); \( d_{\text{inj}} = 0.5 \text{ m} \); \( h_{\text{inj}} = \text{surface} \)
Figure 6.2.a: Position

Figure 6.2.b: Concentration profile

Figure 6.2.c: Longitudinal velocities (linear scale)

Figure 6.2.d: Longitudinal velocities (logarithmic scale)

Figure 6.2.e: Distribution of vertical velocities

Figure 6.2.f: Vertical velocities (linear scale)

Figure 6.2: Characteristic graphs for a snapshot of experiment C13
Small particles; \( U_m = 15 \text{ cm/s} \); \( d_{inj} = 0.5 \text{ m} \); \( h_{inj} = \text{surface} \)
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An important observation from these figures is that the streamwise velocities of the particles follow well a logarithmic law (which will be compared further with the one of the fluid) and that the vertical velocity seems to be nearly independent of the height. One may also observe that the distribution of the vertical velocities follows a gaussian-like profile.

The comparison between the two snapshots taken with an interval of time of 3*(1/25 s) shows large variations in the concentration profile and small ones in the histogram of w_p. These comparisons can be made for lot of images, and the conclusion is in general always the same: all these graphs show already important characteristics of the sediment transport, but one snapshot is only representative of one instantaneous situation, and two snapshots taken with some tenths of second of interval can be highly different. This means that a subjective analysis could take some “representative” snapshots over the thousands available and any law wanted could be concluded.

The same analysis and conclusions can be done for snapshots which refer to small particles and for others distances or heights of injection.

A more reliable analysis can therefore be done with a large number of images and the computation of averages, which will estimate well the ensemble averages (in our steady case, time averages) when the number of particles tracked is sufficient. This is also the only way to compare the experimental measurements and existing models of concentration profile (cf. chapter 1), which always give them for ensemble averages and never for instantaneous situations.
6.3. Analysis of ensemble averages

6.3.1. Comparison between a snapshot and the corresponding ensemble average

The figures 6.3 and 6.4 permit to compare the characteristic graphs for a snapshot and its corresponding ensemble average in the case of large particles. It shows that a snapshot can neither estimate the concentration profile nor the streamwise velocity profile nor the histogram of the vertical velocity. The same analysis and conclusion can be drawn for small particles and for others distances or heights of injection.

6.3.2. Analysis of the characteristics graphs of an ensemble average

Figure 6.4 shows that:
- the position of the particles is random in almost the whole window of recording, i.e. there are no problems of scratches with the camera lens or with the glass of the flume.
- the concentration profile follows a bell-shaped profile (but is not symmetrical).
- the streamwise velocities of the particles \( w_p \) follow a logarithmic profile.
- the distribution of the vertical velocities of the particles \( w_p \) has a bell-shaped profile (but is not symmetrical).
- the vertical velocity of the particles \( w_p \) does not depend on the height.

An interesting observation is that there are many particles with a small \( z_p \) which appear to have a streamwise velocity much larger than the mean flow velocity; some of them reach nearly 40 cm/s whilst \( \bar{U}_m = 20 \text{ cm/s} \). This observation occurs for most of the experiments with the large particles and for several experiments with the small particles. This is without any doubt due to a tracking problem (so there are wrong particles data even when we take only the ones which are on three consecutive images): near the bottom, the average distance between two particles is less than the distance travelled by a particle between to consecutive images and thus the tracking does not work well (cf. chapter 2). Notice that this is not a problem due to the DigImage system: an analysis done by hand would under the same conditions not be possible.

If experiments were done with very low concentrations (such that the average distance between two particles near the bottom is smaller than the distance travelled by a particle between two images), it would be possible to study the bed load transport with the 2D-PTV analysis system. But in such a case the suspended transport would be non-existent, because the bed load transport is in practice characterised by much larger concentrations than the suspended load (except for particles with a very small Rouse parameter). This means that it is not possible to study at the same time (unless with very small Rouse parameters) the bed load transport and the suspended load transport.

As the tracking does not work well near the bottom, a threshold of height must be chosen beneath which we do not consider the results of the 2D-PTV analysis as reliable. It is taken as 1.0 cm, which is quite large as it represents 12.5 % of the flow depth. That means that in all the following graphs we do not consider as reliable all the variables related to particles with \( z_p < 1.0 \text{ cm} \), i.e. the bed load transport cannot be studied.
Figure 6.3: Characteristic graphs for a snapshot of experiment A03

Large particles; $U_m = 20 \text{ cm/s}$; $d_{\text{inj}} = 1 \text{ m}$; $h_{\text{inj}} = \text{surface}$
Figure 6.4.a: Position

Figure 6.4.b: Concentration profile

Figure 6.4.c: Longitudinal velocities (linear scale)

Figure 6.4.d: Longitudinal velocities (logarithmic scale)

Figure 6.4.e: Distribution of vertical velocities

Figure 6.4.f: Vertical velocities (linear scale)

Figure 6.4: Characteristic graphs for a snapshot of experiment A03
Large particles; $U_m = 20 \text{ cm/s}$; $d_{inj} = 1 \text{ m}$; $h_{inj} = \text{ surface}$
6.3.3. Problem met during the analysis

The six characteristic graphs for the averages were drawn for each experiment and the properties mentioned above are common to all of them. There are only variations in the concentration profiles, which follows a bell-shaped profile when the situation is not fully developed (e.g. when small particles are injected at the surface and near the camera) and an exponential profile when the situation is fully developed (e.g. when large particles are injected near the bottom far away from the camera). However, peculiar distributions of streamwise velocities appeared for the experiments C09 to C12 (corresponding to small particles in a flow with $\bar{U}_m = 15$ cm/s):

![Streamwise velocity of the particles for the experiment C10](image1)

Figure 6.5: Streamwise velocity of the particles for the experiment C10
Small particles; $\bar{U}_m = 15$ cm/s; $d_{inf} = 1$ m; $h_{inf} = surface$

One may observe the presence of two statistical populations which follow a logarithmic profile. The first one has a depth-averaged velocity of about 7.5 cm/s and the second one about 15 cm/s. When comparing this graph and the one of other analogous experiments (see figure 6.6) it is clear that the first population has no physical significance.

![Streamwise velocity of the particles for the experiment C07](image2)

Figure 6.6: Streamwise velocity of the particles for the experiment C07
Small particles; $\bar{U}_m = 15$ cm/s; $d_{inf} = 1$ m; $h_{inf} = middle$
In order to find the origin of this problem, all the particles were then sorted with regard to their streamwise velocity and the mean vertical velocity of these two statistical populations were computed. The ratio between these two values was about 2. As the velocities are computed with

\[ u_p = \frac{\Delta x_p}{\Delta t} \quad \quad v_p = \frac{\Delta z_p}{\Delta t} \]  

(6.5)

and as the tracking gave velocities two times smaller than they had to be, either the positions are twice as small or the interval of time is twice as large. Wrong positions could be due to a problem with the even and the odd lines on the video tape or to a problem of image filtering; wrong times could be due to a problem of audio code. The reading of the index file of these experiments (cf. chapter 2) shows that the audio code was correct. It shows also that in many places the tracking from one image to the other was wrong each time that the buffer 15 was analysed: there was for instance an average of 40 particles tracked for the 14 first buffers and then only 5 particles for the buffer 15. At many other places the same phenomenon occurred but more randomly with regard to the buffer number.

The most likely explanation is either a problem with the odd and even lines on the video tape or a filtering problem. This could be due to the bad quality due to the video tape, which was not new when we made the recordings of our experiments. Moreover, because of a problem with the hard disk of the DigImage computer, the analysis of these experiments had to be done a second time. And for each analysis the video tape is read about 5 times by DigImage. Thus it was already non optimal when the final analysis was carried out.

This problem shows that one has to be very careful with the data given by any automated system of particle tracking, and that some procedure must be applied (comparison with equivalent experiments, comparison with theoretical estimations...) to see if the results are reliable.

The problematic experiments were then modified, by throwing away all the particles that had a streamwise velocity \( u_p < 10.0 \text{ cm/s} \). This arbitrary threshold was chosen after observation of the two statistical populations, and remembering that the particles with \( z_p < 1.00 \text{ cm} \) are not considered as reliable.
The streamwise velocity profile of the particles becomes then:

\[ u_p \ [\text{cm/s}] \]

\[ z_p \ [\text{cm}] \]

![Figure 6.7: Modified streamwise profile of the particles for the experiment C10](image)

Small particles; \( \bar{U}_m = 15 \ \text{cm/s}; d_{\text{inj}} = 1 \ \text{m}; h_{\text{inj}} = \text{surface} \)

The clear distinction for the value \( z_p = 6.5 \ \text{cm} \) shows the tracking window; there are more particles near the bottom compared with figure 6.5 because these graphs represent only the 4000 first pairs given (this is a limitation of Microsoft Excel 95 which with these graphs were drawn). The modified velocity profiles for the particles are similar to the ones of equivalent experiments; that means that the problematic data are not lost but can be used further in the analysis.

The analysis of the data could be done in two different ways: take each variable separately and study the influence of each parameter on it; or take each parameter and study its simultaneous influences on the different variables. Both ways have advantages and disadvantages, and could be preferred by any according to his feeling of interest: while the river engineer will be overall interested in the concentration profiles and the streamwise velocities (these two variables permit to compute the suspended load), the physicist will be more interested in the interactions between the concentration profiles and the particle velocities. We chose the first method, i.e. the "river engineer" point of view.

The analysis that follows will study three variables (concentration profiles, streamwise velocities of the particles, vertical velocities of the particles) and the influence of each parameter (diameter of the particles, depth-averaged flow velocity, distance between the injection and the point of observation, height of injection) on them.
6.4. Concentration profiles

As said before, the concentration profiles follow an exponential or a bell-shaped curve (but not symmetrical), depending whether the situation is fully developed or not.

6.4.1. Influence of the diameter

The influence of the diameter on any variable analysed can be studied with the comparison of the series of experiments B and C, both carried out the same day in the same flow, with a velocity of 15 cm/s. Unfortunately, this velocity was too small to lift up the large particles in a significant way, so that these experiments gave satisfactory results only when the injection was near the camera and not on the bottom (i.e., with a non-fully developed situation). So there are only three experiments totally comparable (see appendix 6.3). The comparison can for instance be done when the distance between the injection and the point of observation is 1 m and when the particles are injected just beneath the surface.

![Figure 6.8.a: Experiment B03; Large particles](image)

![Figure 6.8.b: Experiment C10; Small particles](image)

Figure 6.8: Influence of the diameter, $U_m = 15$ cm/s; $d_{inj} = 1$ m; $h_{inj} = surface$

The influence is very clear: when the large particles are already near the bottom (notice that the average vertical velocity for all the particles located above 1 cm $\tilde{w}_p$ is negative: $\tilde{w}_p = -10.0$ mm/s, i.e. the particles are settling and the profile is not fully developed; cf. 6.5), the small ones are always in the upper part of the flow ($\tilde{w}_p = -3.4$ mm/s, i.e. the profile is not fully developed).

Another influence of the diameter on the concentration profiles can be seen when the fully developed situations are reached (equilibrium). One easy way to determine if the equilibrium is reached is to see if the average vertical velocity of the particles is nearly equal to zero. This is the case for instance for the large particles when $U_m = 20$ cm/s with the injection at 0.5 m from the camera and on the bottom ($\tilde{w}_p = -0.9$ mm/s), and for the small particles when $U_m = 15$ cm/s; $d_{inj} = 2$ m; $h_{inj} = bottom$ ($\tilde{w}_p = +0.5$ mm/s) or $U_m = 10$ cm/s; $d_{inj} = 5$ m; $h_{inj} = bottom$ ($\tilde{w}_p = -0.2$ mm/s). The figures 6.9.a to c give these equilibrium concentration profiles.
One can see that it tends more to a uniform distribution for the small particles than for the large ones. This tendency is well described by the classical diffusion model (cf. figure A1.3/2). A strange result is that the concentration profile for the small particles with $\bar{U}_m = 15$ cm/s tends less to an uniform profile than in the case $\bar{U}_m = 10$ cm/s. This is in contradistinction to the classical diffusion model and to an intuitive view (the average suspended load transport, dependant to $u(z)\bar{c}(z)$, should be larger in the second case when it has a smaller velocity). One explanation could be that the profiles are not yet fully developed (i.e. they are not equilibrium profiles), but this seems not be the case when looking at the average vertical velocity of the particles (see 6.6); another explanation could be a variation of the influence of the mirror (or of the injection pipe) when the flow velocity is different.
6.4.2. Influence of $U_m$

The influence of the depth-averaged flow velocity can be seen for the large particles for instance for the experiments carried out with the injection at 1 m and just beneath the surface:

![Figure 6.10.a: Experiment A03](image1)

$U_m = 20$ cm/s

![Figure 6.10.b: Experiment B03](image2)

$U_m = 15$ cm/s

Figures 6.10: Influence of $U_m$; Large particles; $d_{nj} = 1$ m; $h_{nj} = $ surface

A very important influence of the flow velocity concerns the maximum heights that are reached by the suspended particles: while lots of particles go up to 6 cm when $U_m = 20$ cm/s (and some up to 7 cm), the particles in the flow with $U_m = 15$ cm/s reach hardly 4 cm. As the suspended load transport is dependent of the product of the local particle velocity (which can be assumed to be equal to the local flow velocity) and the concentration (cf. introduction), one see that the relation between the suspended load transport and the averaged-depth velocity is not linear (in this case).

One can also observe that the concentration profile has still a bell-shaped profile (with a maximal concentration of 2.05 cm) for the flow with $U_m = 20$ cm/s while it has already a exponential profile for the flow with $U_m = 15$ m/s (i.e. the initial bell shape has already disappeared).
For the small particles, the experiment carried out with the injection a 2 m from the point of observation and near the bottom; the following concentration profiles are obtained:

Figures 6.11: Influence of $\bar{U}_m$: Small particles; $d_{inj} = 2$ m; $h_{inj} = $ bottom

As mentioned before, the concentration profile for the smaller velocity ($\bar{U}_m = 10$ m/s) is steeper than the one for the larger velocity ($\bar{U}_m = 15$ cm/s), in contradiction with the classical diffusion model.

Another situation may also be compared for the small particles: when the injection is at 1 m from the point of observation and near the surface.

Figures 6.12: Influence of $\bar{U}_m$: Small particles; $d_{inj} = 1$ m; $h_{inj} = $ surface
6.4.3. Influence of $d_{ij}$

The comparison of the concentration profile measured for different distances between the injection and the point of observation shows how it evolves from a not fully developed distribution to its equilibrium distribution. This comparison may be done for instance for small particles injected in a flow with $U_m = 10$ cm/s near the bottom. $N$ represents the number of particles located above 1 cm of height ($z_p > 1$ cm):

Figure 6.13.a: Experiment D10
$d_{ij} = 0.5$ m; $N = 7723$; $\bar{W}_p = + 4.3$ mm/s

Figure 6.13.b: Experiment D09
$d_{ij} = 1$ m; $N = 5028$; $\bar{W}_p = + 2.7$ mm/s

Figure 6.13.c: Experiment D04
$d_{ij} = 1.5$ m; $N = 4262$; $\bar{W}_p = + 0.9$ mm/s

Figure 6.13.d: Experiment D03
$d_{ij} = 2$ m; $N = 4314$; $\bar{W}_p = + 0.7$ mm/s

Figure 6.13.e: Experiment D00
$d_{ij} = 5$ m; $N = 4379$; $\bar{W}_p = -0.2$ mm/s
One can clearly see that for a small \( d_{inj} \) the profile is not yet an equilibrium profile as it becomes more and more steeper when \( d_{inj} \) increases. The shape of the profile is nearly the same between \( d_{inj} = 1.5 \) m and \( d_{inj} = 2.0 \) m, showing that the equilibrium nearly reached. This is confirmed by the ensemble average vertical velocities of the particles (with \( z_p > 1 \)), which are \( \bar{w}_p = +0.9 \) mm/s and \( \bar{w}_p = +0.7 \) mm/s respectively: the equilibrium is not reached and the situation evolves very slowly. The same comparison can be done for all the experiments and it always shows a not fully developed profile tending to the equilibrium.

6.4.4. Influence of \( h_{inj} \)

The influence of \( h_{inj} \) can be clearly seen when comparing for instance the experiments carried out for large particles when \( \bar{U}_m = 20 \) cm/s and with \( d_{inj} = 0.5 \) m:

Figures 6.14: Influence of \( d_{inj} \); Large particles; \( \bar{U}_m = 20 \) cm/s; \( d_{inj} = 0.5 \) m

One can see that the height of injection influences strongly the distance necessary to have an equilibrium profile. One can also see that when the equilibrium profile is reached, a lot of particles are on the bottom and the tracking works not so good (the percentage of *PV files is of 89, 65 and 11% respectively).
6.4.5. Comparison of the measurements and the classical diffusion model

The comparison may be done between the experimental results and the classical diffusion model of Rouse and Einstein. An important remark is that this distribution is only valid for equilibrium situations (see appendix 1.3). The distances between the injection and the point of observation being relatively small in our experiments, the equilibrium is not often reached at the point of observation. To determine when the situation is at the equilibrium, the following criterion is used:

\[ |\bar{w}_p| < 0.15 \, w_s \quad (6.1) \]

with \( \bar{w}_p \) the mean vertical velocity for all the particles with \( z_p > 1 \) cm. This criterion applied for the two sets of particles gives:

\[ |\bar{w}_p| < 1.8 \, \text{mm/s for the large particles} \quad (6.2.a) \]
\[ |\bar{w}_p| < 0.54 \, \text{mm/s for the small particles} \quad (6.2.b) \]

Appendix 6.2 gives all the values of \( \bar{w}_p \) and let see that only five situations can be considered at the equilibrium with this criterion (when having a sufficient number of particles tracked, i.e. the experiments A01 and A09 are not considered):

<table>
<thead>
<tr>
<th>Nr Exp.</th>
<th>Particles</th>
<th>( \bar{U}_m ) [cm/s]</th>
<th>( d_{inj} ) [m]</th>
<th>( h_{inj} )</th>
<th>( \bar{w}_p ) [mm/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A06</td>
<td>Large</td>
<td>20</td>
<td>1</td>
<td>middle</td>
<td>-1.0</td>
</tr>
<tr>
<td>A12</td>
<td>Large</td>
<td>20</td>
<td>0.5</td>
<td>bottom</td>
<td>-0.9</td>
</tr>
<tr>
<td>C03</td>
<td>Small</td>
<td>15</td>
<td>1.5</td>
<td>bottom</td>
<td>+0.2</td>
</tr>
<tr>
<td>C04</td>
<td>Small</td>
<td>15</td>
<td>2</td>
<td>bottom</td>
<td>+0.5</td>
</tr>
<tr>
<td>D00</td>
<td>Small</td>
<td>10</td>
<td>5</td>
<td>bottom</td>
<td>-0.2</td>
</tr>
</tbody>
</table>

Table 6.1: Equilibrium situations

The ratio \( \beta \) between the turbulent diffusion coefficient in the vertical direction (\( D_m \)) and the eddy viscosity (\( v_e \)) can be computed by fitting the theoretical curve on the measured curves. As explained in appendix 1.3, it can be done by an numerical method or by a graphical method.

In both cases, one must chose a reference height \( \alpha \). When the ensemble average estimates well the true values, the largest concentration measured is at the lowest point and \( \alpha \) is taken as this height (= 1.35 cm in our case). This is the case for experiments C03 and C04. In the case of experiments A06 and A12, the number of particles tracked (146 and 333 respectively) is to small to consider that the ensemble average concentration profile estimates well the true concentration profile. The choice of \( \alpha \) as the lowest height where a concentration is measured is then not necessary the best choice. Finally, in the case of experiment D00, the concentration profile is very steep and the concentration at the lowest point of measurement is not the largest (cf. 5.2: the ensemble averages estimate better the true values when the depth-averaged velocity is large; this recording was done during two minutes, which is the double of the duration of the other recordings, but it seems that it was still not sufficient).
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The reference height \( a \) must then be chosen such that it permits at the curve to pass near most of the points. This choice is made after a visual observation, and remembering that:

- the theoretical curve pass always through the reference pair \((a, C_a)\)
- the theoretical curve pass always through the point \((z = h, \bar{c} = 0)\).

For the fitting of the theoretical curve on the measurements, one can work with the relative heights \( h' = z/h \) and with the relative concentrations \( \bar{c}(z)/C_a \):

\[
\bar{c}(z) = \frac{\text{mass of particles at height } z}{\text{Volume of the lightsheet}} / \frac{\text{mass of particles at height } a}{\text{Volume of the lightsheet}} = \left\{ \frac{(\text{number of particles located at height } z \text{ during } \Delta t)}{\Delta t} / (\text{number of particles located at height } a \text{ during } \Delta t) / \Delta t \right\} \times \text{mass of one particle}
\]

\[
= \frac{\text{number of particles located at height } z \text{ during } \Delta t}{\text{number of particles located at height } a \text{ during } \Delta t}
\]

(6.3)

For the computation of \( \beta \), it is here made use of a numerical method well known: the least square method (see appendix 1.3). It gives the following expression for \( \beta \):

\[
\beta = Ro \frac{\sum_{i=1}^{N} \left[ \ln \left( \frac{a/h}{1-a/h} \cdot \frac{1-z_i/h}{z_i/h} \right) \right]^2}{\sum_{i=1}^{N} \left[ \ln \left( \frac{a/h}{1-a/h} \cdot \frac{1-z_i/h}{z_i/h} \right) \cdot \ln \left( \frac{\bar{c}_i}{C_a} \right) \right]}
\]

(6.4)

with \( Ro \) the Rouse parameter \( Ro = \frac{w_s}{\kappa u_s} \)  

(6.5)

The value found for \( \beta \) is dependent of the value of the reference height and its concentration, and strongly dependent of the Rouse parameter, which is only estimated:

- \( w_s \) is estimated with the measurements of the "classical method" (see 3.3; the accuracy on the average values found is assumed to be \( \Delta w_s = 0.2 \text{ mm/s, which is optimistic)\)}
  - \( w_s = 12.1 \text{ mm/s for the large particles and } w_s = 3.6 \text{ mm/s for the small particles.} \)
- \( u_s \) estimated with the Darcy-Weisbach equation (the relative accuracy of this estimation is assumed to be \( \Delta u_s / u_s = 5\% \))
- \( \kappa \) is assumed to be equal to 0.4.
Once the value of $\beta$ is computed, the fitted theoretical concentration profile can be drawn with:

$$
\frac{c(z)}{C_a} = \left[ \frac{1}{\eta} \right]^{1/\beta} \left[ \frac{1}{\eta_a - 1} \right]^{1/\eta_a} \left[ \frac{1 - 1/\eta}{\eta - 1} \right]^{1/\beta \eta} w_z
$$

(6.6)

On the following graphs, three curves are drawn, corresponding to three values of $\beta$:

- one with the **minimal Rouse parameter** $Ro_{\text{min}} = \frac{w_z - \Delta w_z}{\kappa (u_* + \Delta u_*)} = \frac{Ro}{1.05} - \frac{\Delta w_z}{1.05 \kappa u_*}$

  (6.7)

- one with the **Rouse parameter** $Ro = \frac{w_z}{\kappa u_*}$

  (6.8)

- one with the **maximal Rouse parameter** $Ro_{\text{max}} = \frac{w_z + \Delta w_z}{\kappa (u_* - \Delta u_*)} = \frac{Ro}{0.95} + \frac{\Delta w_z}{0.95 \kappa u_*}$

  (6.9)

**Equilibrium profile of experiment A06**

As mentioned before, the number of particles tracked is only of 144, i.e. the ensemble average does not approximate very well the true values of the concentration profile.

The reference height is chosen at $z = 1.35$ cm, and the fitting of the theoretical curve is given in figure 6.15:

![Figure 6.15: Fitting the Rouse-Einstein distribution on the measurements with the method of least squares for the experiment A06 (equilibrium situation)](image)

One can see that the different values of $\beta$ found when considering the inaccuracy on the Rouse parameter change hardly the shape and the position of the curve. This observation can be done for all the equilibrium situations studied here.
Equilibrium profile of experiment A12

The reference height is here chosen as $z = 2.05$ cm; the number of particles tracked is 333.

Figure 6.16: Fitting the Rouse-Einstein distribution on the measurements with the method of least squares for the experiment A12 (equilibrium situation)

Equilibrium profile of experiment C03

As mentioned before, the number of particles is here huge (11 695) and the shape of the concentration profile measured is exponential, so that the reference height is chosen as the lowest height, i.e. $z = 1.35$ cm:

Figure 6.17: Fitting the Rouse-Einstein distribution on the measurements with the method of least squares for the experiment C03 (equilibrium situation)
Equilibrium profile of experiment C04

The number of particles is once again huge (15 703), and the reference height is chosen as the lowest height, i.e. $z = 1.35$ cm:

![Graph](image)

Figure 6.18: Fitting the Rouse-Einstein distribution on the measurements with the method of least squares for the experiment C04 (equilibrium situation)

One may observe that the curve drawn with the value computed with the method of the least squares does not pass near the pairs measured. In fact, this is due to this numerical method itself, which is put on the wrong track by the presence of two particles detected in $z = 7.65$ cm. In fact, these two particles would not be there so alone if the interval of height had been chosen in a different way. Moreover, two particles on 15 703 do not really represent something consistent.

If the same computation of $\beta$ is done without this pair, the following graph is found:

![Graph](image)

Figure 6.19: Fitting the Rouse-Einstein distribution on the measurements with the method of least squares for the experiment C04 (equilibrium situation), without the "problematic pair"
Table 6.2 gives the different values of $\beta_{\text{min}}$, $\beta$ and $\beta_{\text{max}}$ for different values of located particles in $z = 7.65$ cm:

<table>
<thead>
<tr>
<th>$\bar{c}(z = 7.65 \text{ cm})$</th>
<th>0</th>
<th>2</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_{\text{min}}$</td>
<td>2.10</td>
<td>1.11</td>
<td>1.22</td>
<td>1.31</td>
<td>1.37</td>
<td>1.41</td>
<td>1.48</td>
<td>1.54</td>
<td>1.58</td>
</tr>
<tr>
<td>$\beta$</td>
<td>2.33</td>
<td>1.24</td>
<td>1.35</td>
<td>1.45</td>
<td>1.52</td>
<td>1.57</td>
<td>1.65</td>
<td>1.71</td>
<td>1.76</td>
</tr>
<tr>
<td>$\beta_{\text{max}}$</td>
<td>2.59</td>
<td>1.37</td>
<td>1.50</td>
<td>1.61</td>
<td>1.69</td>
<td>1.75</td>
<td>1.83</td>
<td>1.90</td>
<td>1.96</td>
</tr>
</tbody>
</table>

Table 6.2: Influence of an "extreme pair" on the value of $b$ computed with the method of the least squares.

**Equilibrium profile of experiment D00**

The number of particles is once huge (16 374), but the concentration profile found has not an exponential shape. That is the reason why the reference height is chosen at $z = 2.75$ cm ($\eta = 0.34$):

![Equilibrium profile](image)

*Figure 6.20: Fitting the Rouse-Einstein distribution on the measurements with the method of least squares for the experiment D00 (equilibrium situation)*

The problem met in experiment C04 and the problem of the choice of a reference height lead to the conclusion that the least square method applied in a systematic way (i.e. with always the same reference height and without have a look at the curve plotted on the same graph as the measurements) is not a good way to fit well the Rouse-Einstein distribution on the experimental results.
Table 6.3 summarises the different values of b computed in these equilibrium situations. The last line of each square represents the relative inaccuracy on $\beta$, computed with

$$\Delta \beta_r = \frac{\max(\beta - \beta_{\text{min}}, \beta_{\text{max}} - \beta)}{\beta}$$

(6.10)

<table>
<thead>
<tr>
<th></th>
<th>A06</th>
<th>A12</th>
<th>C03</th>
<th>C04</th>
<th>D00</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>1.6/1.7/1.8</td>
<td>2.6/2.8/3.0</td>
<td>1.6/1.8/2.0</td>
<td>2.1/2.3/2.6</td>
<td>9.4/10.4/11.6</td>
</tr>
<tr>
<td>%</td>
<td>7%</td>
<td>7%</td>
<td>11%</td>
<td>11%</td>
<td>11%</td>
</tr>
</tbody>
</table>

Table 6.3: Ratio between the turbulent diffusion coefficient in the vertical direction and the eddy viscosity, computed with the least square method

Table 6.3 shows that the relative inaccuracy on $\beta$ is not too large. Notice that the same computations with a larger $\Delta w_s$ or a larger $\Delta u_e$ give values of $\beta_{\text{min}}$ and $\beta_{\text{max}}$ which differ more of the value of $\beta$. Thus the curves are more different and the inaccuracy on $\beta$ is larger.

Comparing the equilibrium situation between the series A (large particles) and the series C (small particles), one can see that $\beta$ is smaller for the small particles (average = 2.05) than for the large particles (average = 2.25), which agrees with the diffusion model (the small particles move nearly as the water parcels, while the big ones have more inertia and are more influenced by drag effects). However, the comparison with the series D let me see another tendency. But in fact these series are not really comparable as the mean depth-averaged velocity is different.

One can however see that all the values computed for $\beta$ are always larger than one, i.e. the turbulent diffusion coefficient taken in the diffusion model is too small. This agrees with other experiments [4].
6.5. Streamwise velocities of the particles ($u_p$)

Figure 6.21 give the streamwise particle velocities as a function of the height ($u_p, z_p$), for small particles with $\bar{U}_m = 15$ cm/s, $d_{\text{inj}} = 2$ m and $h_{\text{inj}} = \text{middle}$:

![Figure 6.21: Typical distribution of the streamwise particle velocities
Experiment C05
Small particles; $\bar{U}_m = 15$ cm/s; $d_{\text{inj}} = 2$ m; $h_{\text{inj}} = \text{middle}]

It shows that its distribution is in first approximation logarithmic, and that some particles, although they are above the height of 1 cm (that was chosen as a critical threshold under which the particles given by the tracking are not reliable, cf. 6.3.2), have a streamwise velocity of the flow much larger than the depth-averaged velocity. This point will be studied later, together with the vertical velocities of the particles.

A question one can ask is if the ensemble average streamwise velocity of the particles ($\bar{u}_p$) has to be equal to the mean streamwise flow velocity ($\bar{U}_m$). The first is computed with the particles, which are on different heights and which are not uniformly distributed over the height (cf. 6.4), whilst the second is computed by giving the same weight to each height. This means that $\bar{u}_p$ will be strongly influenced by the mean velocity of the height where the concentration is the highest. In the case of a fully developed distribution, the highest concentration are near the bottom, thus $\bar{u}_p < \bar{U}_m$ should be found, unless the concentration profile tends to an uniform concentration (i.e. when the Rouse parameter is very small), in which case $\bar{u}_p \approx \bar{U}_m$ should be found.

These considerations are only valid if the velocities are computed over the whole depth. Unfortunately, the particles with $z_p < 1$ cm are not considered as reliable in this analysis, so that the mean value is computed for the interval (1 cm; h). This means that the lowest velocities are not considered for the computation of $\bar{u}_p$, yielding higher values for it.
A coarse estimation of the difference between the depth-averaged velocity and the average computed between 1 cm and h can be done with the hypotheses that the distribution is uniform and that the velocity profile of the particles follows on average the one of the fluid:

\[
\overline{u}_p(z) \equiv \overline{U}(z) = \frac{u_*}{\kappa} \ln \left( \frac{z}{z_0} \right)
\]  

This yields

\[
\overline{u}_{p,m} = \frac{1}{N} \sum_{i=1}^{N} u_{p,i} = \frac{1}{h} \int_{0}^{h} \overline{u}_p(z) \, dz = \frac{u_*}{\kappa} \ln \left( \frac{h}{z_0} \right) - 1
\]  

whilst the average computed for the interval \([h/8; h]\) is:

\[
\overline{u}_p = \frac{1}{N} \sum_{i=1}^{N} u_{p,i} \quad \text{when } z_p > \frac{h}{8} \, \text{cm} = \frac{1}{h - h/8} \int_{h/8}^{h} \overline{u}_p(z) \, dz = \frac{u_*}{\kappa} \ln \left( \frac{h}{z_0} \right) - \frac{1}{7} \ln \left( \frac{1}{8} \right)
\]  

The difference is thus

\[
\overline{u}_p - \overline{u}_{p,m} = \frac{u_*}{\kappa} \frac{1}{7} \ln \left( \frac{1}{8} \right) = 0.74 \quad u_*
\]

which gives for the different series of experiments:  
A: 0.78 cm/s;  
B and C: 0.61 cm/s;  
D: 0.43 cm/s.

But as the concentrations are non uniform in most cases, it is difficult in these conditions to give the physical significance of the value of \(\overline{u}_p\). The appendix 6.2 gives for the different experiments these values, which are always smaller than \(\overline{U}_m\), and close to \(\overline{U}_m\) in the case of a not fully developed situation with high concentrations near the surface, i.e. when the injection is at 0.5 m and just beneath the surface. The values of \(\overline{u}_p\) are in this case:

<table>
<thead>
<tr>
<th>Experiment</th>
<th>(\overline{U}_m) [cm/s]</th>
<th>(\overline{u}_p) [cm/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A04</td>
<td>20.5</td>
<td>19.1</td>
</tr>
<tr>
<td>B04</td>
<td>15.4</td>
<td>14.3</td>
</tr>
<tr>
<td>C09</td>
<td>15.4</td>
<td>15.2</td>
</tr>
<tr>
<td>D12</td>
<td>10.2</td>
<td>10.7</td>
</tr>
</tbody>
</table>

Table 6.4: Average streamwise particles velocities when \(d_{inf} = 0.5\) m and \(h_{inf} = \) surface
A more interesting analysis concerning the streamwise velocities of the particles is the comparison with the velocity profiles of the flows used in the experiments. As mentioned in chapter 4, technical problems encountered during the measurements made that we do not dispose of these profiles. And as it is not possible to create later exactly the same flow (cf. 3.2), it was not possible to measure these profiles after the experiments. This is the weak point of this analysis.

The velocity profiles on the graphs of this paragraph are estimated with the following equation (cf. 4.1):

\[
\overline{u} = 2.45 \ln(z^*) + 5.2
\]

\[
\Leftrightarrow \quad \overline{U} = u_* \left[ 2.45 \ln \left( \frac{z}{u_*} \right) + 5.2 \right]
\]

where \( u_* \) is estimated with the Darcy-Weisbach equation (with the depth-averaged velocity, measured at the height \( z = h/2.72 \) with a Laser-Doppler Flow Meter).

![Graph showing streamwise particles velocities and streamwise flow velocity](image)

**Figure 6.22:** Streamwise particles velocities and streamwise flow velocity

*Experiment C05*

Small particles; \( \overline{U}_m = 15 \text{ cm/s} \); \( d_{inf} = 2 \text{ m} \); \( h_{inf} = \text{middle} \)

One can see that the particles have nearly the same velocity as the surrounding fluid, but that the majority of the particles have an velocity smaller than the local velocity of the flow. This is quite peculiar, especially for the small particles which should follow well the movements of the fluid. This difference could be due either to a scaling problem of the Two-Dimensional Particle Tracking Velocimetry or a bad estimation of the velocity profile, i.e. a bad estimation of the shear velocity with the Darcy-Weisbach equation, or a bad estimation of the numerical coefficients in equation 6.15 (cf. chapter 4), or a bad measurement of the velocity at the height \( z = h/2.72 \).
Notice that in the series D of experiments, the flow velocity profile goes really better through the particles velocities (for situations which tends to the equilibrium):

![Graph showing streamwise particles velocities and streamwise flow velocity](image)

**Figure 6.23: Streamwise particles velocities and streamwise flow velocity**

*Experiment D08*

*Small particles; $\overline{U}_m = 10 \text{ cm/s}; d_{inf} = 1 \text{ m and } h_{inf} = \text{middle}*

This figure shows also that even if the velocity profile of the flow was better surrounded by the particles velocities, the concordance would not be perfect. This means that the flow can not be studied well with the particles which are suspended in it, contrarily to experiments that use the 2D-PTV with minute particles which are assumed to follow the flow perfectly (the aim of these experiments being the study of the flow itself).

Notice that as the mean velocity profile of the particles is nearly the same as the one of the flow, the suspended load is estimated well with:

\[
\overline{Q_{ms}} = \int_a^h \overline{\bar{u}_p(z)c(z)dz} = \int_a^h \overline{U(z)c(z)dz} \quad (6.17)
\]

This was assumed from a long time in some formulae that estimate the suspended load (Bagnold, 1966; Chang, Simons and Richardson, 1967 [20]).

Finally, as the flow velocity increases with the height, one can assume that a particle rising in the flow will have a smaller velocity than a particle located at the same height but going down. This gives rise to perform the computation of the conditionally averaged velocities, defined for each height as:

\[
< u_p(w_p > 0) > = \frac{1}{N^+} \sum_{i=1}^{N^+} u_{p,i} \text{ for the } N^+ \text{ particles with } w_p > 0 \quad (6.18.a)
\]

\[
< u_p(w_p < 0) > = \frac{1}{N^-} \sum_{i=1}^{N^-} u_{p,i} \text{ for the } N^- \text{ particles with } w_p < 0 \quad (6.18.b)
\]

This will be studied later, together with the conditionally averaged vertical velocities.
6.5.1. Influence of the diameter

Figures 6.24: Influence of the diameter on $u_p(z)$; $\overline{U}_m = 15$ cm/s; $h_{inj} = \text{surface}$

These graphs are not convenient to compare, as the large particles have already reached the bottom while the small ones are distributed over the whole depth. As said before, the averaged value of the streamwise velocity will be influenced by the heights of high particles concentrations, unless the profile of concentration is uniform. It is thus possible to compare the velocities of the large and the small particles on small interval of height where the concentration is nearly uniform. The following table give these average values and the number of particles in the considered interval of height for the figures 6.24.a and b (the velocity of the flow is estimated with equation 6.16):

<table>
<thead>
<tr>
<th>Interval</th>
<th>Exp. B04 $u_p(z)$ [cm/s]</th>
<th>Number of particles</th>
<th>Exp. C09 $u_p(z)$ [cm/s]</th>
<th>Number of particles</th>
<th>$\overline{U}$ [cm/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 cm &lt; $z_p$ &lt; 3.5 cm</td>
<td>14.56</td>
<td>1435</td>
<td>14.57</td>
<td>32</td>
<td>15.70</td>
</tr>
<tr>
<td>3.5 cm &lt; $z_p$ &lt; 4.0 cm</td>
<td>14.67</td>
<td>1271</td>
<td>14.89</td>
<td>73</td>
<td>15.99</td>
</tr>
<tr>
<td>4 cm &lt; $z_p$ &lt; 4.5 cm</td>
<td>14.84</td>
<td>931</td>
<td>15.15</td>
<td>139</td>
<td>16.24</td>
</tr>
<tr>
<td>4.5 cm &lt; $z_p$ &lt; 5.0 cm</td>
<td>14.93</td>
<td>468</td>
<td>15.13</td>
<td>133</td>
<td>16.47</td>
</tr>
<tr>
<td>5 cm &lt; $z_p$ &lt; 5.5 cm</td>
<td>14.98</td>
<td>221</td>
<td>15.28</td>
<td>159</td>
<td>16.67</td>
</tr>
</tbody>
</table>

Table 6.5: Influence of the diameter on the streamwise velocity profile
As one can see at 0.5 m of the injection, the velocities of the small particles are larger than the ones of the large particles. This is normal, as the small particles have less inertia.

6.5.2. Influence of \( \bar{U}_m \)

The streamwise flow velocity is of course the parameter which will influence the most the streamwise particle velocities. This is very clear when looking for instance the large particles in a flow with \( \bar{U}_m = 20 \) cm/s and in a flow with \( \bar{U}_m = 15 \) cm/s:

![Figure 6.25.a: Experiment A04](image) \( \bar{U}_m = 20 \) cm/s

![Figure 6.25.b: Experiment B04](image) \( \bar{U}_m = 15 \) cm/s

Figures 6.25: Influence of \( \bar{U}_m \) on \( u_p \); Large particles; \( d_{\text{inj}} = 0.5 \) m; \( h_{\text{inj}} = \text{surface} \)

This influence can be clearly seen with the averages computed on small intervals of height where the concentration is assumed to be uniform:

<table>
<thead>
<tr>
<th>Height Interval</th>
<th>( u_p(z) ) [cm/s]</th>
<th>Number of Particles</th>
<th>Height Interval</th>
<th>( u_p(z) ) [cm/s]</th>
<th>Number of Particles</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 cm &lt; ( z_p ) &lt; 3.5 cm</td>
<td>18.53</td>
<td>727</td>
<td>3.5 cm &lt; ( z_p ) &lt; 4 cm</td>
<td>18.95</td>
<td>938</td>
</tr>
<tr>
<td>4 cm &lt; ( z_p ) &lt; 4.5 cm</td>
<td>19.09</td>
<td>1148</td>
<td>4.5 cm &lt; ( z_p ) &lt; 5 cm</td>
<td>19.26</td>
<td>1237</td>
</tr>
<tr>
<td>5 cm &lt; ( z_p ) &lt; 5.5 cm</td>
<td>19.38</td>
<td>1196</td>
<td>5 cm &lt; ( z_p ) &lt; 6 cm</td>
<td>19.63</td>
<td>1285</td>
</tr>
</tbody>
</table>

Table 6.6: Influence of the diameter on the streamwise velocity profile
6.5.3. Influence of $d_{\text{inj}}$

The influence of the distance between the injection and the point of observation can for instance be studied with the comparison of the experiments carried out with small particles injected in a flow with $\overline{U}_m = 15 \text{ cm/s}$ and $h_{\text{inj}} = \text{middle}$.

![Graphs showing Influence of $d_{\text{inj}}$ on $u_p$](image)

Figure 6.26.a: Experiment C08  
$d_{\text{inj}} = 0.5 \text{ m}$

Figure 6.26.b: Experiment C07  
$d_{\text{inj}} = 1 \text{ m}$

Figure 6.26.c: Experiment C06  
$d_{\text{inj}} = 1.5 \text{ m}$

Figure 6.26.d: Experiment C05  
$d_{\text{inj}} = 2 \text{ m}$

Figures 6.26: Influence of $d_{\text{inj}}$ on $u_p$; Small particles; $\overline{U}_m = 15 \text{ cm/s}$; $h_{\text{inj}} = \text{middle}$

As one can see, these graphs are nearly the same, and it is very difficult to see whether there is a variation of $u_p(z)$ in relation with $\overline{U}(z)$. This means that after hardly 0.5 m (and without any doubt well before) the small particles have reached an equilibrium for their streamwise velocities.
6.5.4. Influence of $h_{inj}$

The influence of the height of the injection can be studied for instance in the case of small particles injected in a flow with $\bar{U}_m = 10$ cm/s, and $d_{inj} = 1$ m:

![Graph](image)

**Figure 6.27.a: Experiment D07**

$h_{inj} = \text{surface}$

![Graph](image)

**Figure 6.27.b: Experiment D08**

$h_{inj} = \text{middle}$

![Graph](image)

**Figure 6.27.c: Experiment D09**

$h_{inj} = \text{bottom}$

Figures 6.27: Influence of $h_{inj}$ on $u_p$: Small particles; $\bar{U}_m = 10$ cm/s; $d_{inj} = 1$ m

In contradistinction with the two first parameters studied, one can observe here a trend in the particles velocities: when injected near the surface, the particle velocities are larger than when they are injected near the bottom. This is really not surprising, as the particles going down in the flow go to a region of lower velocity.
6.6. Vertical velocities of the particles ($w_p$)

When plotting the vertical velocity of all the particles tracked (or of 4000 of them if there are more than 4000), we can observe two important things. Firstly, the vertical velocity seems not to depend very much on the height, with nevertheless a small tendency for larger vertical velocities near the bottom. Secondly, the vertical velocities of the particles are well distributed around a mean value, i.e. the distribution of $w_p$ follows a gaussian-like profile. This second observation is confirmed when drawing the histograms:

![Histograms for experiments A05, B04, and C04](image)

**Figure 6.28.a:** Vertical velocity of the particles for experiment A05
Large particles; $\bar{U}_m = 20$ cm/s; $d_{inj} = 0.5$ m; $h_{inj} = \text{middle}$

**Figure 6.28.b:** Vertical velocity of the particles for experiment B04
Large particles; $\bar{U}_m = 15$ cm/s; $d_{inj} = 0.5$ m; $h_{inj} = \text{surface}$

**Figure 6.28.c:** Vertical velocity of the particles for experiment C04
Small particles; $\bar{U}_m = 15$ cm/s; $d_{inj} = 2$ m; $h_{inj} = \text{bottom}$
We may also observe that some particles, although they are above the height of 1 cm (under which we do not consider the particles given by the tracking as reliable) have a very large vertical velocity (the same phenomenon was observed for the streamwise velocity). This is observed for particles going up \( (w_p > 0) \) and for particles going down \( (w_p < 0) \). For instance, in the experiment carried out with small particles; \( \bar{U}_m = 10 \text{ cm/s}; d_{\text{inj}} = 5 \text{ m}; h_{\text{inj}} = \text{bottom} \) (see figure 6.28.d), some particles have a vertical velocity about 20 mm/s and some others about -20 mm/s. These values are very large comparing to the settling velocity of these particles (3.6 mm/s). When reading the *.PV files we can just see that the particles which have an abnormal vertical velocity have almost always a normal streamwise velocity. Similarly, the particles which have an abnormal large streamwise velocity have almost always a normal vertical velocity. Unfortunately, it is difficult to say whether these large values are due to physical phenomena or to tracking errors.

In quantitative terms, we can for instance compute that for this experiment (which corresponds to an equilibrium situation), the number of particles which have \( w_p < -1.0 \text{ cm/s} \) or \( w_p > +1.0 \text{ cm/s} \) represent only 1.7% of all the tracked particles.

As mentioned above, the computation of the ensemble average vertical velocity of the particles shows an important characteristic of the experiment: if \( \bar{w}_p \equiv 0 \) (compared to the magnitude of the settling velocity), the situation is fully developed (equilibrium situation) and if \( \bar{w}_p \) is strongly different from zero, the situation is not fully developed. Nevertheless, this distinction is in practice difficult to make for the small particles, which give always small values for \( \bar{w}_p \), sometimes even if the concentration profile let clearly see that the situation is not fully developed. This is for instance the case of experiments D06 (not fully developed situation with \( \bar{w}_p = -0.6 \text{ mm/s} \)).
Figure 6.29: Concentration profile for experiment D06
Small particles; $\overline{U_m} = 10 \text{ cm/s}; d_{\text{inj}} = 1.5 \text{ m}; h_{\text{inj}} = \text{surface}$

Such a small value of $\overline{w_p}$ means that the situation tends very slowly to the equilibrium.

6.6.1. Influence of the diameter

The three equivalent experiments for the influence of the diameter (see appendix 6.3) show clearly that the average vertical velocity of the particles located above $z_p = 1 \text{ cm} \ (\overline{w_p})$ is larger (in absolute value) for the larger particles. This is normal as we saw (cf. 3.3.4) that the settling velocity of the large particles is three times larger than the one of the small particles. This ratio is always the same for the average downward velocity of the particles in the flow:

<table>
<thead>
<tr>
<th>$\overline{U_m}$</th>
<th>$d_{\text{inj}}$</th>
<th>$h_{\text{inj}}$</th>
<th>Experiment</th>
<th>$\overline{w_p}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>1</td>
<td>surface</td>
<td>B03 - Large</td>
<td>-10.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>C10 - Small</td>
<td>-3.4</td>
</tr>
<tr>
<td>15</td>
<td>0.5</td>
<td>surface</td>
<td>B04 - Large</td>
<td>-14.3</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>C09 - Small</td>
<td>-5.0</td>
</tr>
<tr>
<td>15</td>
<td>0.5</td>
<td>surface</td>
<td>B05 - Large</td>
<td>-7.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>C08 - Small</td>
<td>-1.1</td>
</tr>
</tbody>
</table>

Table 6.7: Influence of the diameter on $\overline{w_p}$

It is interesting to notice that when the particles are injected near the surface and that the point of observation is near the injection, both large and small particles have a larger (in absolute value) average vertical velocity than the settling velocity.
6.6.2. Influence of $\bar{U}_m$

In order to see the influence of the depth-averaged velocity on C the tables 6.8.a and b compare the equivalent experiments for the depth-averaged velocity of the flow:

<table>
<thead>
<tr>
<th>Series A</th>
<th></th>
<th>A03</th>
<th>A04</th>
<th>A05</th>
</tr>
</thead>
<tbody>
<tr>
<td>20 cm/s</td>
<td>[mm/s]</td>
<td>-7.1</td>
<td>-9.7</td>
<td>-6.9</td>
</tr>
<tr>
<td>Series B</td>
<td></td>
<td>B03</td>
<td>B04</td>
<td>B05</td>
</tr>
<tr>
<td>15 cm/s</td>
<td>[mm/s]</td>
<td>-10.0</td>
<td>-14.3</td>
<td>-7.5</td>
</tr>
</tbody>
</table>

Table 6.8.a: Influence of $\bar{U}_m$ on $\bar{w}_p$ for the large particles

<table>
<thead>
<tr>
<th>Series C</th>
<th></th>
<th>C01</th>
<th>C02</th>
<th>C03</th>
<th>C04</th>
<th>C05</th>
<th>C06</th>
</tr>
</thead>
<tbody>
<tr>
<td>15 cm/s</td>
<td>[mm/s]</td>
<td>+1.8</td>
<td>+1.8</td>
<td>+0.2</td>
<td>+0.5</td>
<td>-1.1</td>
<td>-1.3</td>
</tr>
<tr>
<td>Series D</td>
<td></td>
<td>D10</td>
<td>D09</td>
<td>D04</td>
<td>D03</td>
<td>D02</td>
<td>D05</td>
</tr>
<tr>
<td>10 cm/s</td>
<td>[mm/s]</td>
<td>+4.3</td>
<td>+2.7</td>
<td>+0.9</td>
<td>+0.7</td>
<td>-0.7</td>
<td>+0.6</td>
</tr>
<tr>
<td>Series C</td>
<td></td>
<td>C07</td>
<td>C08</td>
<td>C09/C13</td>
<td>C10</td>
<td>C11</td>
<td>C12</td>
</tr>
<tr>
<td>15 cm/s</td>
<td>[mm/s]</td>
<td>-2.2</td>
<td>-1.1</td>
<td>-5.0/-4.2</td>
<td>-3.4</td>
<td>-2.9</td>
<td>-3.6</td>
</tr>
<tr>
<td>Series D</td>
<td></td>
<td>D08</td>
<td>D11</td>
<td>D12</td>
<td>D07</td>
<td>D06</td>
<td>D1</td>
</tr>
<tr>
<td>10 cm/s</td>
<td>[mm/s]</td>
<td>+0.7</td>
<td>+1.9</td>
<td>-1.7</td>
<td>-1.0</td>
<td>-0.6</td>
<td>-1.2</td>
</tr>
</tbody>
</table>

Table 6.8.b: Influence of $\bar{U}_m$ on $\bar{w}_p$ for the small particles

One can observe that in the case of the large particles (series A and B), the mean downward velocity of the particles is larger (in absolute value) for a smaller depth-averaged velocity of the flow, but that in the case of the small particles (series C and D) the situation is opposite. This is an observation quite difficult to interpret.
6.6.3. Influence of $d_{\text{inj}}$

Let us now look at the variation of the average vertical velocity of the particles as a function of the distance of injection (for equivalent experiments), which gives a good idea of the distance necessary to reach the equilibrium situation. For instance, for the large particles injected in a flow with $\overline{U}_m = 20$ cm/s and just beneath the surface:

![Graph showing $w_p$ as a function of $d_{\text{inj}}$ for experiments A01 to A04](image)

*Figure 6.30: Mean vertical velocity of the large particles as a function of $d_{\text{inj}}$ for experiments A01 to 04: $\overline{U}_m = 20$ cm/s; $h_{\text{inj}} = \text{surface}*$

The particles injected just beneath the surface will settle until the equilibrium is reached. One can see that it is the case after about 2 m.

The same graph can be drawn for the small particles, for instance when they are injected near the bottom in a flow with $\overline{U}_m = 10$ cm/s:

![Graph showing $w_p$ as a function of $d_{\text{inj}}$ for experiments D00-D03-D04-D09-D10](image)

*Figure 6.31: Mean vertical velocity of the small particles as a function of $d_{\text{inj}}$ for experiments D00, D03, D04, D09 and D10: $\overline{U}_m = 10$ cm/s; $h_{\text{inj}} = \text{bottom}*$

One can see that the particles are lifted up until an equilibrium situation is reached, after about 2 m.
Other equivalent experiments give more peculiar curves;

![Graph showing \( w_p \) as a function of \( d_{\text{inj}} \) for experiments C05-C08]

**Figure 6.32:** Mean vertical velocity of the small particles as a function of \( d_{\text{inj}} \) for experiments C05 to C08: \( \overline{U}_m = 15 \text{ cm/s}; h_{\text{inj}} = \text{middle} \)

The fact that \( \bar{w}_p \) is larger (in absolute value) in 1.0 than in 0.5 m is quite difficult to interpret.

### 6.6.4. Influence of \( h_{\text{inj}} \)

The influence of the height of injection can be intuitively guessed: if the particles are injected
- near the **bottom**, they will globally move up to the surface, i.e. \( \bar{w}_p > 0 \)
- in the **middle** of the flow, they will globally stay in this part of the flow, dispersing them to the bottom and to the surface (in a way that depends on their diameter and of the flow velocity), i.e. \( \bar{w}_p \approx 0 \)
- just beneath the **surface**, they can only settle, i.e. \( \bar{w}_p < 0 \).

All this considerations are of course only valid if the point of observation is not too far from the injection, i.e. in a not fully developed situation.

Table 6.9 summarises some values of \( \bar{w}_p \) for equivalent experiments regarding to \( h_{\text{inj}} \), with \( d_{\text{inj}} = 0.5 \text{ m} \):

<table>
<thead>
<tr>
<th>Bottom [mm/s]</th>
<th>A12</th>
<th>C01</th>
<th>D10</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-0.9</td>
<td>+1.8</td>
<td>+4.3</td>
</tr>
<tr>
<td>Middle [mm/s]</td>
<td>A05</td>
<td>B05</td>
<td>C08</td>
</tr>
<tr>
<td></td>
<td>-6.9</td>
<td>-7.5</td>
<td>-1.1</td>
</tr>
<tr>
<td>Surface [mm/s]</td>
<td>A04</td>
<td>B04</td>
<td>C09/C13</td>
</tr>
<tr>
<td></td>
<td>-9.7</td>
<td>-14.3</td>
<td>-5.0/-4.2</td>
</tr>
</tbody>
</table>

**Table 6.9:** Influence of \( h_{\text{inj}} \) on \( \bar{w}_p \) for \( d_{\text{inj}} = 0.5 \text{ m} \)
We can consider in first approximation that the equilibrium situation will be reached when all the particles have had the time to reach the bottom once. The estimation of the distance between the injection and the point of observation necessary to have this can be done as follows:

![Diagram showing the estimation process](image)

**Figure 6.33: Estimation of the distance required to have an equilibrium situation**

The vertical velocity of the particles is assumed to be constant with the height and equal to the settling velocity \( w_s \). Assuming that the only movement possible for the particle is the settling, the time required for arriving at the bottom \( (t_s) \) is:

\[
 t_s = \sum_{h_w}^0 \frac{dz}{w_s - h_w} = \frac{h_w}{w_s} \tag{6.19}
\]

A **rough estimation** is to consider that the streamwise velocity of the particles is constant over the depth and equal to the depth-averaged velocity of water \( (u_p = \text{constant} = \overline{U}_w) \). The streamwise distance travelled by a particle before reaching the bottom \( (L_s) \) is then:

\[
 L_s = \int_0^{t_s} u_p(t) \, dt = \int_0^{t_s} \overline{U}_w \, dt = \frac{\overline{U}_w}{w_s} h_w \tag{6.20}
\]

A **better estimation** is to consider that the particles have a streamwise velocity equal to the local flow velocity \( (u_p(z) = \overline{U}(z)); \text{ cf. } 6.3.4)\):

\[
 L_s = \int_0^{t_s} u_p(t) \, dt = \int_0^{t_s} \overline{U}(t = t) \, dt = \int_0^{t_s} \overline{U}(z = h_w - w_s t) \, dt = \int_0^{t_s} \frac{u_p}{\kappa} \ln \left( \frac{h_w - w_s t}{z_0} \right) \, dt \tag{6.21}
\]
which yields:

\[
L_s = \frac{u_*}{\kappa} \left[ \ln \left( \frac{h_{inj}}{z_0} \right) - 1 \right] \quad h_{inj} = \frac{u_{inj} - u_*}{w_s} \frac{1}{h_{inj}}
\]  

(6.22)

The following computations are done with:

- \( u_{inj} = \bar{U}(z = h_{inj}) \) estimated by equation (6.16)
- \( u_* \) estimated with the Darcy-Weisbach equation (cf. appendix 4.1)
- \( w_s \) measured with the classical method (cf. 3.3.4.2.1)

and remembering that the injection pipe has a diameter of 7.8 mm. These computations are summarised in table 6.10:

<table>
<thead>
<tr>
<th></th>
<th>Large particles</th>
<th></th>
<th>Small particles</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>( w_s = 12.1 \text{ mm/s} )</td>
<td>( w_s = 12.1 \text{ mm/s} )</td>
<td>( w_s = 3.6 \text{ mm/s} )</td>
</tr>
<tr>
<td>( \bar{U}_m ) [cm/s]</td>
<td>20</td>
<td>15</td>
<td>15</td>
<td>10</td>
</tr>
<tr>
<td>( u_* ) [cm/s]</td>
<td>1.04</td>
<td>0.81</td>
<td>0.81</td>
<td>0.57</td>
</tr>
<tr>
<td>( h_{inj} ) [cm]</td>
<td>( L_s ) [m]</td>
<td>( L_s ) [m]</td>
<td>( L_s ) [m]</td>
<td>( L_s ) [m]</td>
</tr>
<tr>
<td>0.78 (bottom)</td>
<td>0.09</td>
<td>0.04</td>
<td>0.23</td>
<td>0.48</td>
</tr>
<tr>
<td>4.0 (middle)</td>
<td>0.59</td>
<td>0.46</td>
<td>1.54</td>
<td>1.08</td>
</tr>
<tr>
<td>7.22 (surface)</td>
<td>1.15</td>
<td>0.90</td>
<td>3.01</td>
<td>2.11</td>
</tr>
</tbody>
</table>

Table 6.10: Estimation of the distance required for the equilibrium situation as a function of the depth-averaged velocity and the height of injection

This theoretical view does not agrees well with the experimental results (see appendix 6.4): the prediction is confirmed only in 25 experiments out of 40. In particular, it is never confirmed when working with small particles injected near the bottom (which will globally go up in the flow).
6.7. Conditionally averaged velocities

The basic idea that motivates the computation of conditionally averaged velocities is that the mechanisms that make the particles rise is different from the one that makes them settle (cf. the bursting phenomena and the model of "ejections" and "sweeps" in 1.5).

The conditionally averaged velocities are defined:

- for the **streamwise** velocity as:
  
  \[
  \langle u_p(w_p > 0) \rangle = \frac{1}{N^+} \sum_{i=1}^{N^+} u_{p,i} \text{ for the } N^+ \text{ particles with } w_p > 0 \quad (6.23 \text{ a})
  \]
  
  \[
  \langle u_p(w_p < 0) \rangle = \frac{1}{N^-} \sum_{i=1}^{N^-} u_{p,i} \text{ for the } N^- \text{ particles with } w_p < 0 \quad (6.23 \text{ b})
  \]

- for the **vertical** velocity as:
  
  \[
  \langle w_p(w_p > 0) \rangle = \frac{1}{N^+} \sum_{i=1}^{N^+} w_{p,i} \text{ for the } N^+ \text{ particles with } w_p > 0 \quad (6.24 \text{ a})
  \]
  
  \[
  \langle w_p(w_p < 0) \rangle = \frac{1}{N^-} \sum_{i=1}^{N^-} w_{p,i} \text{ for the } N^- \text{ particles with } w_p < 0 \quad (6.24 \text{ b})
  \]

These velocities are here computed over the depth for intervals on height of 7 mm. As mentioned above, the variables in relation with the particle that have a height \( z_p < 1.0 \) cm are not considered. For some information on the conditionally averaged velocities near the bottom (up to 2 mm), see the experiments of Sumer & Oguz [14] and Sumer and Deigaard [15]. A very important remark is that for heights near the surface the number of particles is smaller (unless the injection is near the surface and near the point of observation) and thus the dispersion in the values computed at these heights is larger. This means **that the values corresponding to large heights are less reliable** (unless in the case mentioned).

6.7.1. Typical profile of conditionally averaged velocities

Figure 6.34 shows typical shapes for the conditionally averaged velocities. This figure refers to small particles with \( \bar{U}_m = 15 \) cm/s, \( d_{ij} = 0.5 \) and \( h_{ij} = \text{bottom} \). The numbers written near each measurement denote the number of particles that were available for the computation of the conditionally averages.

Figure 6.34 illustrates well the preliminary remark mentioned above: the dots in relation with the largest heights are less reliable as they are computed with a smaller number of particles.
Conditionally averaged streamwise velocity profiles

- **Flow velocity**
- ▲ <up(wp>0)>
- × <up(wp<0)>

Vertical velocity data corresponding to upward and downward paths

- ▲ <up(wp>0)>
- × <up(wp<0)>

\[ N^* = 723; N = 2989 \]

Figure 6.34: Velocity data for experiment C01

Small particles; \[ \bar{U}_m = 15 \text{ cm/s} \]; \[ d_{\text{inj}} = 0.5 \text{ m} \]; \[ h_{\text{inj}} = \text{Bottom} \]
The streamwise conditionally averaged velocities follows the velocity profile of the flow, which is estimated on this graph with $u^* = 2.45 \ln(y^*) + 5.2$, where $u^*$ is estimated from the depth-averaged velocity with the Darcy-Weisbach equation (cf. appendix 4.1). For the rising particles ($w_p > 0$) it is always smaller than for the settling particles. This is normal as a rising particle comes from a region where the fluid velocity is smaller, while the settling particles comes from a region where the fluid velocity is larger (remember the mixing length model of Prandtl, cf. appendix 1.2).

The vertical conditionally averaged velocities for the settling particles ($w_p < 0$) has globally a linear variation with regard to the height, with the largest vertical velocities (on absolute value) near the bottom. Notice that this agrees with the "sweep" concept, and that these conditional averaged vertical velocities near the bottom are always larger than the settling velocity. For rising particles ($w_p > 0$), the conditionally averaged velocity seems to be constant.

Notice that Sumer and Deigaard [15] arrives at the same conclusions for $<u_p(w_p < 0)>$ and $<u_p(w_p > 0)>$, but find another variation of $<w_p(w_p < 0)>$ and $<w_p(w_p < 0)>$ (but they only studied the lower part of the flow, and there is perhaps no contradistinction between their measurements and the ones of this work):
6.7.2. Influence of the different parameters

The comparison of the conditionally averaged velocities for equivalent experiments shows peculiar variations that are difficult to interpret.

For instance, the influence of the flow velocity can be studied for small particles injected at 0.5 m from the point of observation and in the middle of the flow (see figures 6.37.a and b). The streamwise conditionally averaged velocity follows well the velocity profile of the flow. As mentioned before, the velocity profile of the flow pass better through the particle velocities in the series D; this is seen also here. The vertical conditionally averaged velocity for rising particles is steeper in the case of the low flow velocity. Finally, the vertical conditionally averaged velocity for settling particles is nearly constant (remember that the measurements for large heights are less reliable) and its magnitude is about 8 mm/s for $\bar{U}_m = 10$ cm/s and about 4 mm/s for $\bar{U}_m = 15$ cm/s. This is once again difficult to interpret.

As the influence of the other parameters is also difficult to interpret and does not give extra information, this analyse is not pursued.
Conditionally averaged streamwise velocity profiles

Vertical velocity data corresponding to upward and downward paths

N' = 4679; N = 6272

Figure 6.37.a: Velocity data for experiment C08
Small particles; \( \overline{U} \_m = 15 \text{ cm/s}; d_{inf} = 0.5 \text{ m}; h_{inf} = \text{Middle} \)
Chapter 6  Data analysis

Conditionally averaged streamwise velocity profiles

- Flow velocity
- △ <up(wp>0)>
- × <up(wp<0)>

Vertical velocity data corresponding to upward and downward paths

N⁺ = 8103; N⁻ = 6095

Figure 6.37.b: Velocity data for experiment D11
Small particles; \( \overline{U_m} = 10 \text{ cm/s} \); \( d_{uij} = 0.5 \text{ m} \); \( h_{uij} = \text{Middle} \)
The feasibility of 2D-PTV for the study of vertical sediment transport
Conclusions

Two sets of conclusions come out of this work: one about the Two-Dimensional Particle Tracking Velocimetry by DigImage, and one about the characteristics of the sediment transport in a turbulent free surface flow.

The first set of conclusions is that the Two-Dimensional Particle Tracking (2D-PTV) by DigImage is a reliable method to locate and to match sediment particles when two conditions are respected: for a good location, the contrast between the background and the illuminated particles must be large; for a good matching, the average distance between two particles on an image must be larger than the average distance travelled by a particle between two consecutive images.

The accuracy of the measurement is very high when reference points of the experimental set-up are known well. Moreover, the number of particles which can be tracked in a short time is huge compared to classical methods, although the computer analysis requires quite a long time (more or less one hour for the analysis of 25 images with an average of 40 particles on each). Finally, the analysis must include a procedure to see whether there were no problems of particle location and/or tracking.

The second set of conclusions concerns the suspended load transport in a turbulent free surface flow. Firstly, the classical diffusion model (the so-called Rouse-Einstein distribution) does not describe this transport well quantitatively: in all the fully developed concentration profiles encountered, it predicted concentrations that were too low, i.e. the turbulent diffusion coefficient used in this model is too small.

Secondly, some measurements agree with the model of bursting process, in particular larger downward velocities near the bottom. Other observations are difficult to interpret, in particular downward velocities when the injection is near the surface and near the point of observation, and upward particle velocities which are constant with regard to the height (but this is not so manifest).
The feasibility of 2D-PTV for the study of vertical sediment transport
Research suggestions

The analysis we made in chapter six could be strongly improved if the flow velocity profiles were measured with a Laser-Doppler Flow Meter during the experiments, as close as possible to the point of observation. Another improvement would be a higher intensity of light, in order to be able to position the mirror far from the point of observation. It is in fact assumed in the analysis that the mirror has no effect on the flow where the recordings are made, but this point was not really checked (thus its influence as a function of the depth-averaged velocity is not known).

Similar experiments could be carried out with very small particles (for instance with a diameter about 0.1 mm) in order to see if the concentration profile tends to the one predicted by the classical diffusion model and if the streamwise velocity of the particles tends to follow the local streamwise velocity of the fluid more closely. The same could be done with particles with an intermediate diameter (we studied in fact two extreme situations with the "large particles" and "small particles").

On the other hand, the influence of other parameters could be studied, as the water depth, the presence of waves, the presence of a rough bottom or the Reynolds number.

At last, a convenient practical point for these experiments would be to dispose easily of a sufficient quantity of particles (i.e. without having to sieve two days before each 30 minutes of experiments). This would guarantee the persistence of the injection during a whole series of experimentation.

May 1996

ADRIAENS Frédéric
VAN RILLAER Lionel
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Appendix 1.1 Basic equations of turbulence

This appendix is based on the course of R. Booij (TUD) "Turbulentie in de waterloopkunde" (Turbulence in hydrodynamics) [3]. It gives the basic equations of hydrodynamics, obtained with the balance equation (1.28) applied to the mass and the momentum. It then explains the fundamental problem encountered in these equations when the flow is turbulent, and the splitting of an instantaneous value in a sum of an ensemble average value and an instantaneous deviation. Finally, it gives the basic equations of turbulence found with this splitting.

The balance equation (1.28) can be applied for two conservative (i.e. $D = 0$) physical variables well known, the mass and the momentum:

- for the **mass**: considering an incompressible fluid ($Dp / Dt = 0$), equation (1.28) becomes the so-called **equation of continuity**:

  \[
  \nabla_i v_i = 0 \quad \text{(A1.1/1)}
  \]

- for the **momentum**: considering a fluid with a dynamic viscosity ($\eta$) independent of the position, equation (1.28) becomes the so-called **Navier-Stokes equations** (one equation for each component of the momentum):

  \[
  \text{(NS)} \quad \frac{\partial}{\partial t} \left( \rho v_j \right) + \nabla_i \left( \rho v_i v_j \right) + \nabla_j p - \eta \nabla^2 v_j = k_j \quad (j = 1, 2, 3) \quad \text{(A1.1/2)}
  \]

  with $p =$ pressure

  $k_j =$ component $j$ of the external forces per unit mass

The continuity equation and the Navier-Stokes equations form a system of 4 equations (nonlinear because of the convection terms in NS) with 4 unknown quantities ($V_1$, $V_2$, $V_3$ and $p$). For a **small Reynolds number** (i.e. a laminar flow), the convective momentum flux is negligible compared to the molecular diffusion of momentum. The NS equations are in first approximation linear and the system is stable (i.e. small modifications of the boundary conditions induce small modifications in the solution).

For a **large Reynolds number** (i.e. a turbulent flow), the diffusion terms in the Navier-Stokes equations can be neglected with respect to the nonlinear convection terms. As a consequence, the system of equations becomes unstable, i.e. some small modifications in the boundary conditions induce large modifications in the solution. This means that it is not possible to know exactly the instantaneous characteristics of the flow unless the boundary conditions are perfectly known, which is not possible in practice. But the equations can still be used to determine the average characteristics of the flow.
For doing this, the ensemble average of a variable is defined as the average of the values of this variable measured at the same place and at the same time after the start of identical experiments carried out. In the case of a steady flow (i.e. a flow characterised by variables that have constant ensemble averages with respect to time), the ensemble average is estimated by the time average:

$$\bar{W} = \frac{1}{\Delta t} \int_{t-\Delta t/2}^{t+\Delta t/2} W(t) dt$$  \hspace{1cm} (A.1.1/3)

with $\Delta t$ an interval of time larger than a characteristic time for the flow studied.

The instantaneous characteristics are then split in the sum of an ensemble average value and an instantaneous deviation value. This gives, for instance for the instantaneous vertical velocity ($W$) and the instantaneous concentration ($c$):

$$W = \bar{W} + W'$$
$$c = \bar{c} + c'$$  \hspace{1cm} (A.1.1/4a) \hspace{1cm} (A.1.1/4b)

with $\bar{W}$ = ensemble average vertical velocity
$W'$ = instantaneous deviation of the vertical velocity
$\bar{c}$ = ensemble average concentration
$c'$ = instantaneous deviation of the concentration

By definition of the ensemble average values, it is obvious that the ensemble average values of the instantaneous deviations are equal to zero:

$$\bar{c}' = 0 \hspace{1cm} \bar{W}' = 0$$  \hspace{1cm} (A.1.1/5)

In order to study the ensemble averages of the different variables of a turbulent flow, this splitting is made for all the terms of the continuity equation and of the Navier-Stokes equations. Afterwards an ensemble average of the equation is done. This gives the ensemble average continuity equation ($\bar{C}$) and the ensemble average Navier-Stokes equation ($\bar{NS}$), known as the Reynolds equations (Re):

$$\nabla_i \bar{V}_i = 0$$  \hspace{1cm} (A.1.1/6)

$$\text{(Re) } \frac{\partial}{\partial t} (\rho \bar{V}_j) + \nabla_i (\rho \bar{V}_i \bar{V}_j) + \nabla_i (\rho \bar{V}_i' \bar{V}_j') + \nabla_j p - \eta \nabla_i^2 \bar{V}_j = \bar{k}_j \hspace{1cm} (j = 1,2,3)$$  \hspace{1cm} (A.1.1/7)
New terms appear in this equation, $\nabla_i (\rho \bar{V}'_i \bar{V}'_j)$, which have the dimensions of a stress [N/m²] and correspond to the correlation of the simultaneous fluctuations of two components of the velocity vector at the same point. These terms are called the Reynolds stresses and are written:

$$q_{ij} = \rho \bar{V}'_i \bar{V}'_j \tag{A1.1/8}$$

where $q_{ij}$ is the turbulent stress in a direction $i$ on a surface perpendicular to the direction $j$. The different $q_{ij}$ are often written in a tensor (3 x 3) containing the different velocity correlations.

The advent of these nine extra unknowns makes that the system composed by $\bar{C}_i$ and $\text{(Re)}$ is no longer solvable. In fact, as $q_{ij} = q_{ji}$, there are only six extra unknowns. As the gradient of the normal pressure ($\nabla_j \bar{p}$) is very much larger than the one of the normal Reynolds stresses $\nabla_i q_{ii}$, these one can be neglected. Unfortunately there are still three extra unknowns left. This means that in a general case (i.e. when no extra simplifications can be done) it is inevitable to find extra relations for the solution of the system. That is the reason for the models of turbulence. The next page indicates the position of the models for the study of turbulence and the most popular models.

Amongst these models, the gradient-type transport models are often used. Their basic hypothesis is that the diffusive transport in the direction $i$ ($T_i$) of an (extensive or intensive) variable $G$ is proportional to the local gradient of this variable:

$$T_i \sim \frac{\partial G}{\partial x_i} \tag{A1.1/9}$$

In the case of the transport of momentum in the direction $i$ ($G = \rho V_i$) in a laminar flow, the transport is the viscous stress and the coefficient of proportionality is the kinematic viscosity $v$. For the description of a turbulent flow, the eddy viscosity ($v_i$) is defined by analogy with the viscous stresses:

$$\tau_{ij} = \rho v \left( \frac{\partial V_i}{\partial x_j} + \frac{\partial V_j}{\partial x_i} \right) \quad \text{--- analogy} \quad \rightarrow \quad q_{ij} = \rho v_i \left( \frac{\partial \bar{V}_i}{\partial x_j} + \frac{\partial \bar{V}_j}{\partial x_i} \right) \tag{A1.1/10}$$

with $\tau_{ij}$ the viscous stress in a direction $i$ on a surface perpendicular to the direction $j$. 
Figure A1.1/1: Position of the gradient-type transport models in the field of the study of turbulence
The feasibility of 2D-PTV for the study of vertical sediment transport
Appendix 1.2

Expression of the eddy viscosity

This appendix explains how the eddy viscosity is expressed as a function of the height in the classical model of vertical transport of sediment in a turbulent surface flow, which yields to the so-called Rouse-Einstein concentration profile studied in the appendix 1.3.

Assuming that the viscous shear stresses are negligible compared to the turbulent shear stresses, the total stresses may be expressed by:

\[ \tau_{xz} = \tau'_{xz} + q_{xz} = \rho \overline{U \dot{W}} = \rho v_i \frac{\partial \bar{U}}{\partial z} \Rightarrow v_i = \frac{\tau_{xz}}{\rho} \quad (A1.2/1) \]

Thus \( \tau_{xz} \) and \( \frac{\partial \bar{U}}{\partial z} \) must be expressed as a function of the height \( z \). It is derived like for a fluid without sediment, because the lack of information of the influence of the sediment concentration on the shear stresses and on the streamwise velocity gradient. This approximation is nevertheless rational for small concentrations.

An expression of \( \tau_{xz} \) is found by expressing the equilibrium of the forces acting on the control volume in the uniform situation drawn here below:

![Figure A1.2/1: Shear stress for an uniform free surface flow](image)

\[ \tau_{xz} \, dx \, dz = \rho \, g \, (h-z) \, dx \, dy \, S_0 \quad \Rightarrow \quad \tau_{xz} = \rho \, g \, (h-z) \, S_0 \quad (A1.2/2) \]

with \( S_0 \) the slope of the channel.

And more particularly at the bottom:

\[ \tau_b = \tau_{xz}(z = 0) = \rho \, g \, h \, S_0 \quad (A1.2/3) \]
Putting equation (A1.2/3) in equation (A1.2/2) gives:

$$\tau_{xz} = \tau_b \frac{h - z}{h}$$  \hspace{1cm} (A1.2/4)

Notice that defining the relative height

$$\eta = \frac{z}{h}$$

equation (A1.2/5) may be written:

$$\tau_{xz} = \tau_b (1 - \eta)$$  \hspace{1cm} (A1.2/6)

For finding an expression of $\frac{\partial \bar{U}}{\partial z}$, Rouse and Einstein used the law of universal distribution of velocity, which is derived here under. The following hypotheses are done:

1) In the region near to the bottom, the total shear stresses are constant and equal to $\tau_b$, the shear stress at the bed:

$$\tau_{xz} = \text{constant} = \tau_b$$  \hspace{1cm} (A1.2/7)

Notice that this hypothesis is confirmed by the experience [13].

2) The instantaneous fluctuation of the streamwise velocity ($U'$) is proportional to the local gradient of $\bar{U}$ in the vertical direction: if at a certain time $W' > 0$ (the packet of fluid is going up), this causes the following fluctuations of $U'$:

\begin{figure}[h]
\centering
\begin{subfigure}{0.5\textwidth}
\centering
\includegraphics[width=\textwidth]{figureA1.2.2A}
\caption{A. $\frac{\partial \bar{U}}{\partial z} > 0$}
\end{subfigure}
\hspace{0.5cm}
\begin{subfigure}{0.5\textwidth}
\centering
\includegraphics[width=\textwidth]{figureA1.2.2B}
\caption{B. $\frac{\partial \bar{U}}{\partial z} < 0$}
\end{subfigure}
\caption{Instantaneous fluctuations of the longitudinal velocity}
\end{figure}
Appendix 1.2 Expression of the eddy viscosity

- in the case of a positive gradient \( \frac{\partial \bar{U}}{\partial z} > 0 \):

\[
\uparrow W' > 0 \quad \Rightarrow \quad U' = \bar{U}(z - \ell_m) - \bar{U}(z) \equiv -\ell_m \frac{\partial \bar{U}}{\partial z} < 0 \quad (A1.2/8a)
\]
\[
\downarrow W' < 0 \quad \Rightarrow \quad U' = \bar{U}(z + \ell_m) - \bar{U}(z) \equiv +\ell_m \frac{\partial \bar{U}}{\partial z} > 0 \quad (A1.2/8b)
\]

thus for an ensemble average this must give \( \overline{U'W'} < 0 \).

- in the case of a negative gradient \( \frac{\partial \bar{U}}{\partial z} < 0 \):

\[
\uparrow W' > 0 \quad \Rightarrow \quad U' = \bar{U}(z - \ell_m) - \bar{U}(z) \equiv +\ell_m \frac{\partial \bar{U}}{\partial z} > 0 \quad (A1.2/9a)
\]
\[
\downarrow W' < 0 \quad \Rightarrow \quad U' = \bar{U}(z + \ell_m) - \bar{U}(z) \equiv -\ell_m \frac{\partial \bar{U}}{\partial z} < 0 \quad (A1.2/9b)
\]

thus for an ensemble average this must give \( \overline{U'W'} > 0 \).

Notice that this hypothesis is verified well for the molecular transports which occurs on distances so small that the local streamwise velocity gradient is nearly constant for a local transport. But with turbulence the transports (mass, momentum, heat) occur on larger distances, sometimes of the magnitude of the flow depth. Thus the turbulent transport can not be well described by a local gradient transport.

3) \( W' \) has the same magnitude as \( U' \); this can be justify when subtracting equation (C) and equation (\( \bar{C} \)) (see appendix 1.1), which gives for the studied two-dimensional case:

\[
\begin{aligned}
\left\{ \begin{array}{l}
\frac{\partial U}{\partial x} + \frac{\partial W}{\partial z} = 0 \\
\frac{\partial U}{\partial x} + \frac{\partial W}{\partial z} = 0
\end{array} \right. \Rightarrow \quad \frac{\partial U'}{\partial x} + \frac{\partial W'}{\partial z} = 0 \quad \Rightarrow \quad |U'| \equiv |W'|
\end{aligned} \quad (A1.2/10)
\]

(if the turbulence is isotropic)

Using the second hypothesis, one can find:

- in the case of a positive gradient \( \frac{\partial \bar{U}}{\partial z} > 0 \):

\[
\overline{U'W'} < 0 \quad \text{can be written as} \quad \overline{U'W'} = -\ell_m^2 \left( \frac{\partial \bar{U}}{\partial z} \right)^2 \quad (A1.2/11)
\]

- in the case of a negative gradient \( \frac{\partial \bar{U}}{\partial z} < 0 \):

\[
\overline{U'W'} > 0 \quad \text{can be written as} \quad \overline{U'W'} = +\ell_m^2 \left( \frac{\partial \bar{U}}{\partial z} \right)^2 \quad (A1.2/12)
\]
In order to have one equation which express this two situations, the following notation is used:

\[ \frac{U' W'}{\ell_m^2} \frac{\partial U}{\partial z} \frac{\partial U}{\partial z} \]  
\[ \text{(A1.2/13)} \]

4) The mixing length is not constant with the height, but vary linearly with it (close to the bottom):

\[ \ell_m = \kappa z \]  
\[ \text{(A1.2/14)} \]

with \( \kappa \) = dimensionless constant of von Karman. This constant was studied in many experiments and seems to be independent of the characteristics of the flow (flow depth, mean velocity...) and equal to 0.4 (with maximum 10 \% of variation for different flows). Nevertheless, this value decreases if the sediment concentration is high.

Using all these hypotheses, one can write (for the region near the wall):

\[ \tau_b = q_{xz} = \rho \frac{U' W'}{\ell_m^2} \frac{\partial U}{\partial z} \frac{\partial U}{\partial z} = \rho \kappa^2 z^2 \frac{\partial U}{\partial z} \frac{\partial U}{\partial z} \]  
\[ \text{(A1.2/15)} \]

So that:

\[ \frac{\partial U}{\partial z} = \sqrt{\frac{\tau_b}{\rho}} \frac{1}{\kappa z} \]  
\[ \text{(A1.2/16)} \]

Defining the shear velocity \( u_* \) [m/s] as follows:

\[ u_* = \sqrt{\frac{\tau_b}{\rho}} \]  
\[ \text{(A1.2/17)} \]

equation (A1.2/16) can be written in the classical form of the so-called law of universal distribution of velocity:

\[ \frac{\partial U}{\partial z} = \frac{u_*}{\kappa} \frac{1}{z} \]  
\[ \text{(A1.2/18)} \]

This equation was derived for a region near the wall (cf. hypothesis 1 and 4), but experiments showed that it suits nearly for the whole depth.

Notice that putting equations (A1.2/14) and (A1.2/18) in equations (A1.2/8 & 9) gives:

\[ \begin{cases} 
U' \equiv u_* \\
W' \equiv u_* 
\end{cases} \]  
\[ \text{(A1.2/19)} \]

which give a physical interpretation to the shear velocity.
Putting the expression of the shear stress (A1.2/4) and the expression of gradient of the streamwise velocity (A1.2/18) in the expression of the eddy viscosity (A1.2/1) gives:

\[
\nu_t = \frac{\tau_{xz}}{\frac{\partial U}{\partial z}} = \frac{\tau_b}{\frac{h}{h}} \frac{h - z}{h} = \nu_t = \kappa u_z z \left(1 - \frac{z}{h}\right) \tag{A1.2/20}
\]

Notice that using the relative height \( \eta \), equation (1.68) may be written:

\[
\nu_t = \kappa u_z h \eta(1 - \eta) \tag{A1.2/21}
\]

Notice also that the turbulent diffusion coefficient in the vertical direction \( D_{zz} \), which is assumed to be equal to the eddy viscosity in the classical diffusion model has the dimensions \([m^2/s]\) and can thus be expressed as the product of a length and a velocity:

\[
D_{zz} \equiv \nu_t \sim \ell_t u_t \tag{A1.2/22}
\]

with \( \ell_t \), a characteristic length and \( u_t \), a characteristic velocity for the structures of turbulence the most responsible for the turbulent transport, i.e. the large eddies.

As \( \nu_t = \kappa u_z z \left(1 - \frac{z}{h}\right) \) was found, one can write:

- \( \ell_t - z \left(1 - \frac{z}{h}\right) \) \tag{A1.2/23}

is the characteristic length for the structures of turbulence; it is not constant with the height, but tends to zero nearby the bottom and the water surface, where it varies proportionally to \( \pm z \):

\[
\frac{\partial \ell_t}{\partial z} (z = 0) = 1 = \frac{\partial z}{\partial z} \tag{A1.2/24}
\]

\[
\frac{\partial \ell_t}{\partial z} (z = h) = -1 = \frac{\partial (-z)}{\partial z} \tag{A1.2/25}
\]

- \( u_t \sim u_- \) \tag{A1.2/26}

is the characteristic velocity for the structures of turbulence.

- \( \kappa \) is the coefficient of proportionality between \( D_{zz} \) and \( \ell_t, u_t \).
The feasibility of 2D-PTV for the study of vertical sediment transport
Appendix 1.3

Study of the Rouse-Einstein distribution

The Rouse-Einstein distribution expresses the ensemble average (see appendix 1.1) concentration profile of suspended sediment (i.e. from the reference height $a$ to the water surface) in a two-dimensional steady flow as:

$$
\bar{c}(z) = C_a \left( \frac{a}{h-a} \frac{h-z}{z} \right)^{\frac{1}{\kappa u_*}}
$$

(A1.3/1)

with $z =$ height in the flow; $z = 0$ at the bottom and $z = h$ at the water surface
$h =$ flow depth
$\bar{c}(z) =$ ensemble average concentration at height $z$
$a =$ reference height
$C_a =$ reference concentration of suspended sediment at a distance $a$ above the bed
$u_* =$ settling velocity of the sediment
$\kappa =$ constant of von Karman $\approx 0.4$
$u_* =$ shear velocity

The coefficient $w_* / \kappa u_*$ is called the Rouse parameter and is dimensionless.

Notice that some values of the concentration are independent of the Rouse parameter:

$$
\bar{c}(z = a) = C_a \left( \frac{a}{h-a} \frac{h-a}{a} \right)^{\frac{1}{\kappa u_*}} = C_a
$$

(A1.3/2)

$$
\bar{c}(z = h) = C_a \left( \frac{a}{h-a} \frac{h-h}{h} \right)^{\frac{1}{\kappa u_*}} = 0
$$

(A1.3/3)

Using the reference height $\eta$, and defining $\eta_a \equiv \frac{a}{h}$,

(A1.3/4)

the Rouse-Einstein distribution may also be written:

$$
\bar{c}(z) = C_a \left( \begin{array}{c} \frac{1}{\eta} - 1 \\
\frac{1}{\eta} - 1 \\
\eta_a
\end{array} \right)^{\frac{1}{\kappa u_*}}
$$

(A1.3/5)

This equation shows that the relative concentration depends not on the height but on the relative height.
The distribution is determined if one concentration C_a at one reference height \( a \) is known. The following equation shows the influence of the choice of another reference height:\n\[
\bar{c}(z) = C_a \left( \frac{a}{h-a} \right)^{\frac{1}{\kappa u}} \left( \frac{h-z}{z} \right)^{\frac{1}{\kappa u}} = C_a \left( \frac{h-z}{z} \right)^{\frac{1}{\kappa u}} \tag{A1.3/6}
\]
if we chose another reference height \( a \) and its concentration \( C_a \), the concentration profile is multiplied by another constant \( C_0' \). This is depicted on the following graph, where the dots represent concentration measurements:

![Influence of the choice of a reference height for the Vanoni distribution](image)

Figure A1.3/1: *Influence of the choice of a reference height on the Rouse-Einstein distribution*

One can see that the choice of another reference height does not changes fundamentally the shape of the concentration profile. Opposite to this, the Rouse parameter influences strongly the shape of the distribution:

if \[ \frac{W_z}{\kappa u_*} \gg 1 \] i.e. relative big sediment for a weak flow,
\[
\bar{c}(z) \text{ decreases rapidly with increasing } z.
\]

if \[ \frac{W_z}{\kappa u_*} \ll 1 \] i.e. relative small sediment for a strong flow,
\[
\bar{c}(z) \text{ decreases slowly with increasing } z, \text{ i.e. the distribution tends to an uniform distribution.}
\]

This is shown on the figure A1.3/2, which represents the relative concentration \( \bar{c} / C_a \) as a function of the relative height \( \eta \) for different Rouse parameters and for \( \eta_* = 0.05 \).
Influence of the Rouse parameter on the Rouse-Einstein distribution

Figure A1.3/2: Influence of the Rouse parameter on the Rouse-Einstein distribution
Finally, if the distribution describes the concentration profile correctly, then the constant $C_0$ (cf. A1.3/6) should be the same, whatever the reference height chosen, i.e.

$$C_0 \left( \frac{a}{h-a} \right)^{ \frac{1}{2} } \propto \text{constant} \tag{A1.3/7}$$

This is a very convenient way to see whether the distribution fits well the experimental profile.

Vanoni made measurements of concentration profiles and concluded that the Rouse-Einstein distribution describes well them when $\alpha = 0.05 \ h \ (\leftrightarrow \eta_\alpha = 0.05)$ and when the Rouse parameter is small.
Appendix 1.3 Study of the Rouse-Einstein distribution
Appendix 3.1

Required quantity of particles for one experiment

The most important parameters for the determination of the required quantity of particles for one experiment are the number of particles in the light sheet, the volume of the light sheet, the lateral dispersion (which will vary with the distance between the injection and the camera) and the duration of one recording. Here will be estimate the quantity of particles needed according to the dimensions of the set-up and the choice of experimental parameters.

A. SMALL PARTICLES

<table>
<thead>
<tr>
<th>Particles</th>
<th>( \text{d}_m ) [mm]</th>
<th>( V_p = \pi \text{d}_m^2 / 6 ) [mm³/part.]</th>
<th>( n_p ) [part.]</th>
<th>( \rho_p ) [kg/m³]</th>
<th>( m_p = V_p \rho_p ) [mg/part.]</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean diameter</td>
<td>0.36</td>
<td>0.36</td>
<td>0.36</td>
<td></td>
<td></td>
</tr>
<tr>
<td>volume (1 particle)</td>
<td>0.024</td>
<td>0.024</td>
<td>0.024</td>
<td></td>
<td></td>
</tr>
<tr>
<td>number of part. in light sheet</td>
<td>50</td>
<td>50</td>
<td>100</td>
<td></td>
<td></td>
</tr>
<tr>
<td>mass density</td>
<td>1050</td>
<td>1050</td>
<td>1050</td>
<td></td>
<td></td>
</tr>
<tr>
<td>mass of one particle</td>
<td>0.026</td>
<td>0.026</td>
<td>0.026</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Injection pipe</th>
<th>( u_m ) [cm/s]</th>
<th>( d_i ) [mm]</th>
<th>( A_i = \pi d_i^2 / 4 ) [mm²]</th>
<th>( Q_{li} = A_i u_{mean} ) [l/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean velocity</td>
<td>10</td>
<td>15</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>diameter</td>
<td>7.8</td>
<td>7.8</td>
<td>7.8</td>
<td></td>
</tr>
<tr>
<td>area</td>
<td>47.8</td>
<td>47.8</td>
<td>47.8</td>
<td></td>
</tr>
<tr>
<td>discharge</td>
<td>0.005</td>
<td>0.007</td>
<td>0.007</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Light sheet</th>
<th>( h_L ) [mm]</th>
<th>( l_L ) [mm]</th>
<th>( w_L ) [mm]</th>
<th>( V_L = h_L l_L w_L ) [l]</th>
<th>( C_L = n_p / V_L ) [part./l]</th>
</tr>
</thead>
<tbody>
<tr>
<td>height</td>
<td>80</td>
<td>80</td>
<td>80</td>
<td></td>
<td></td>
</tr>
<tr>
<td>length</td>
<td>145</td>
<td>145</td>
<td>145</td>
<td></td>
<td></td>
</tr>
<tr>
<td>average width</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>volume</td>
<td>0.174</td>
<td>0.174</td>
<td>0.174</td>
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<td></td>
</tr>
<tr>
<td>concentration (# of part.)</td>
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<td>287</td>
<td>575</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Flow</th>
<th>( h ) [mm]</th>
<th>( L ) [mm]</th>
<th>( Q_L = u_m h L ) [l/s]</th>
<th>( Q_{ex} = C_L Q_L ) [part./s]</th>
<th>( a )</th>
<th>( Q_{sp} = a Q_{ex} ) [part./s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>height</td>
<td>80</td>
<td>80</td>
<td>80</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>width of the flume</td>
<td>400</td>
<td>400</td>
<td>400</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>discharge water</td>
<td>3.2</td>
<td>4.8</td>
<td>4.8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>discharge particles</td>
<td>920</td>
<td>1379</td>
<td>2759</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>lateral dispersion</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>discharge part. in light sheet</td>
<td>460</td>
<td>690</td>
<td>1379</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Tank</th>
<th>( C_i = Q_i / Q_L ), ( C_L m_p / a ) [g/l]</th>
<th>( T_{rec} ) [min]</th>
<th>( V_{req} = Q_L T_{rec} ) [l]</th>
<th>( V_o ) [l]</th>
<th>( M = V_o C_i ) [g]</th>
<th>( C_v = (M / \rho_p) / V_o ) [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>concentration (weight)</td>
<td>9.9</td>
<td>9.9</td>
<td>19.7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>duration of one recording</td>
<td>30</td>
<td>30</td>
<td>30</td>
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<td></td>
</tr>
<tr>
<td>required quantity of water</td>
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<td>13</td>
<td>13</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>used quantity of water</td>
<td>10</td>
<td>15</td>
<td>15</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>required mass of particles</td>
<td>99</td>
<td>148</td>
<td>296</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>concentration (volume)</td>
<td>0.94</td>
<td>0.94</td>
<td>1.88</td>
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<td></td>
</tr>
</tbody>
</table>

A 18
**Appendix 3.1  Required quantity of particles for one experiment**

### B. LARGE PARTICLES

<table>
<thead>
<tr>
<th>Particles</th>
<th>( d_m ) [mm]</th>
<th>( V_p = \pi d_m^3 / 6 ) ([\text{mm}^3/\text{part.}])</th>
<th>( n_p ) [part.]</th>
<th>( \rho_p ) [kg/m(^3)]</th>
<th>( m_p = V_p \rho_p ) [mg/part.]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( 1.095 )</td>
<td>( 0.687 )</td>
<td>( 10 )</td>
<td>( 1050 )</td>
<td>( 0.722 )</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Injection pipe</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean velocity</td>
</tr>
<tr>
<td>diameter</td>
</tr>
<tr>
<td>area</td>
</tr>
<tr>
<td>discharge</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Light sheet</th>
</tr>
</thead>
<tbody>
<tr>
<td>height</td>
</tr>
<tr>
<td>length</td>
</tr>
<tr>
<td>average width</td>
</tr>
<tr>
<td>volume</td>
</tr>
<tr>
<td>concentration (# of part.)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>height</td>
</tr>
<tr>
<td>width of the flume</td>
</tr>
<tr>
<td>discharge water</td>
</tr>
<tr>
<td>discharge particles</td>
</tr>
<tr>
<td>lateral dispersion</td>
</tr>
<tr>
<td>discharge part. in light sheet</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Tank</th>
</tr>
</thead>
<tbody>
<tr>
<td>concentration (weight)</td>
</tr>
<tr>
<td>duration of one recording</td>
</tr>
<tr>
<td>required quantity of water</td>
</tr>
<tr>
<td>used quantity of water</td>
</tr>
<tr>
<td>required mass of particles</td>
</tr>
<tr>
<td>concentration (volume)</td>
</tr>
</tbody>
</table>

The comparison of the computations for the small and the large particles for \( u_m = 15 \text{ cm/s} \) shows that a much larger quantity of large particles is needed for still less particles in the light sheet.
The feasibility of 2D-PTV for the study of vertical sediment transport
Appendix 3.2

Measurement of the particle mass density

As mentioned in chapter 1, the settling velocity $w_s$ of a particle is a very important parameter of the models of sediment transport. The formulae that allows to calculate it use the term $(s-1)$, where $s = \rho_p / \rho_w$ is the relative density, ratio between the mass density of particles ($\rho_p$) and the one of water ($\rho_w$).

The problem in our case is that the mass density of the polystyrene particles is nearly the same as the one of water. So the relative density $s$ will be nearly 1. As in the computation of the theoretical settling velocity the factor $(s-1)$ is multiplied, the theoretical value will be very inaccurate if the relative inaccuracy on $\rho_p$ is large. Three methods were used in order to have the as best as possible measurement of $\rho_p$: the "classical method", the "picnometer method" and the "salt method".

A. Classical method

The classical method of measurement is the following:

1) Place a graduated tube on a weighing machine, and put the meter on $M = 0$.

2) Fill the tube with water, and measure
   the weight: $M_w$
   the volume: $Vol_w$.

   The calculation of $\rho_w \equiv M_w / Vol_w$ and the comparison of this value with the theoretical value (function of the temperature) give an idea about the accuracy of the measurement.

3) Add particles and mix well, so that all the air goes out; then measure
   the weight: $M_{wp}$
   the volume: $Vol_{wp}$.

4) Calculate
   $M_p = M_{wp} - M_w$
   $Vol_p = Vol_{wp} - Vol_w$
   $\rho_p \equiv M_p / Vol_p$. 
The feasibility of 2D-PTV for the study of vertical sediment transport

The measurements with this method give, for two different graduated cylinders:

<table>
<thead>
<tr>
<th></th>
<th>$\rho_p$ [kg/m$^3$]</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
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<td>Cylinder B</td>
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<td>1018</td>
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<tr>
<td>$\Delta \rho_p$ [kg/m$^3$]</td>
<td></td>
<td>95</td>
<td>62</td>
<td>58</td>
<td>50</td>
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</tbody>
</table>

Table A3.1/1: Measurement of the mass density with the classical method

where $\Delta \rho_p$ is the inaccuracy on $\rho_p$, computed with the extreme values of $\rho_p$, found when considering the inaccuracy of the measurement on the volume ($\Delta \text{Vol} = 0.1 \text{ ml}$ for the cylinder A and 0.25 ml for the B) and on the mass ($\Delta M = 0.01 \text{ g}$).

The better measurement with this method is $\rho_p = 1018 \text{ kg/m}^3 \pm 50 \text{ kg/m}^3$. The computing of the relative inaccuracy on the parameter $(\rho_p/\rho_w - 1)$ with this value, even if assuming that the value of $\rho_w$ is exactly known ($\rho_w = 998 \text{ kg/m}^3 \pm 0 \text{ kg/m}^3$), gives:

$$\rho_p - \rho_w = 20 \text{ kg/m}^3 \pm 50 \text{ kg/m}^3$$

yielding a relative inaccuracy of 250 %. Thus, this measurement is not satisfactory.

B. Picnometer method

In order to improve the precision of the measurement of the mass density, the inaccuracy on both the weight and the volume have to be reduced. This is done by using a weighing machine with $\Delta M = 0.0005 \text{ g}$ and a picnometer with $\Delta \text{Vol} \equiv 0.05 \text{ ml}$.

The method is the following:

1) Put the picnometer when dry on the weighing machine, and measure $M_{\text{pic,dry}}$.

2) Calculate the volume of the picnometer:
   * fill the picnometer completely with water and measure $M_{\text{pic+w}}$
   * calculate $M_w = M_{\text{pic+w}} - M_{\text{pic,dry}}$
   * measure the temperature
   * read in tables the value of $\rho_w$ for this temperature
   * calculate $\text{Vol}_{\text{pic}} = M_w / \rho_w$. 
3) Measure the mass of the particles put into the picnometer:
   * empty the water out of the picnometer and weigh it (\(M_{\text{pic, wet}}\))
   * fill the picnometer with particles, and measure \(M_{\text{pic, wet + part}}\)
   * calculate \(M_{\text{part}} = M_{\text{pic, wet + part}} - M_{\text{pic, wet}}\)

4) Measure the volume of particles put into the picnometer:
   * fill in the picnometer with water, and mix the particles and the water, in order to
     evacuate all the air. This operation makes that the theoretical inaccuracy on mass (0.0005 g)
     is in practice much greater (we take the value 0.005 g for the further calculations). Then
     replace the thermometer on the picnometer. This was not possible for the small particles,
     because to many of them were floating. So none measurement of the mass density for the
     small particles could be done with this method.
   * measure \(M_{\text{pic + water + part}}\)
   * calculate
     \[M_w = M_{\text{pic + water + part}} - M_{\text{pic}} - M_{\text{part}}\]
     \[\text{Vol}_w = M_w / \rho_w\]
     \[\text{Vol}_p = \text{Vol}_{\text{pic}} - \text{Vol}_w\]

5) Calculate \(\rho_p = M_p / \text{Vol}_p\).

The mass densities computed with this method are given in table 3.4:

<table>
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<tr>
<th>(\rho_p) [kg/m(^3)]</th>
<th>1</th>
<th>2</th>
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<th>4</th>
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</thead>
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<tr>
<td>1026</td>
<td>1030</td>
<td>1033</td>
<td>1035</td>
<td></td>
</tr>
</tbody>
</table>

| \(\Delta \rho_p\) [kg/m\(^3\)] | 4.5 | 3.9 | 4.3 | 4.2 |

Table A3.1/2: Measurement of the mass density with the picnometer method
The feasibility of 2D-PTV for the study of vertical sediment transport

The following mean value is taken for the further computations:

$$\rho_p = 1031 \text{ kg/m}^3 \pm 5 \text{ kg/m}^3$$

Assuming that the value of $\rho_w$ is not exactly known ($\rho_w = 998 \text{ kg/m}^3 \pm 1 \text{ kg/m}^3$; inaccuracy due to temperature), the computing of the parameter ($\rho_p - \rho_w$) gives:

$$\rho_p - \rho_w = 31 \text{ kg/m}^3 \pm 6 \text{ kg/m}^3$$

yielding a relative inaccuracy of 20%. This method is better, but still not totally satisfactory. We have to remember this problem when calculating the theoretical settling velocities.

C. Salt method

As the mass density of the particles is almost the same as the one of water, it is possible to measure it by adding a soluble substance heavier than water, e.g. salt. The used salt is pure NaCl. Adding more and more of it in the water, an equilibrium situation will appear after a certain time (the particles are in suspension in the solution). Knowing the mass density of distilled water (i.e. knowing the temperature) and of NaCl (2164 kg/m$^3$), the mass density of the particles is determined at the equilibrium by:

$$\rho_p = \rho_{\text{mixing}} = \frac{dM}{dV} = \frac{M_w + M_{\text{salt}}}{V_w + V_{\text{salt}}} = \frac{M_w}{\rho_w} + \frac{M_{\text{salt}}}{\rho_{\text{salt}}}$$  \hspace{1cm} (A3.1/1)

This is very convenient, because only two measurements are needed ($M_w$ and $M_{\text{salt}}$), which means that the accuracy will be improved greatly. Another advantage of this method is that both small and large particles may be in the salt water. Thus it is possible to see whether there is a difference in mass density between these two sets of particles.

However, this method is only possible for the measure of a small mass density, because there is a limit from the solubility of salt. For NaCl at 20°C, the solubility is of 35.8 parts (in weight) for 100 parts of water. This means that we are able to measure a mass density of maximum:

$$\rho_{p,\text{max}} = \frac{100 + 35.8}{100 + 35.8} \text{ kg/m}^3 = 1163 \text{ kg/m}^3$$  \hspace{1cm} (A3.1/2)

As the picnometer method showed that the mass density of the polystyrene particles is about 1030 kg/m$^3$, this method is of use.
The following procedure is used:

1) Measurement of the mass of water ($M_w$) and of the mass density of water ($\rho_w$):
   * pour the sieved particles in a glass pot
   * place the pot on the weighing machine and put its meter on $M = 0$
   * pour more or less 100 ml of distilled water and measure $M_w$
   * measure the temperature (T), and read in tables the value of $\rho_w$.

2) Measurement of the quantity of salt required to bring the particles in suspension ($M_{salt}$):
   * place a small dish on the weighing machine and put its meter on $M = 0$
   * pour some dry NaCl on the dish and weigh it ($M_{salt,init}$)
   * add a small quantity of salt in the water and mix well
   * repeat this until the particles are in suspension
   * weigh the remaining salt ($M_{salt,end}$)
   * calculate $M_{salt} = M_{salt,init} - M_{salt,end}$

This procedure, though extremely accurate in theory, has unfortunately hidden sources of error and inaccuracy. Firstly, some water is lost during the experiment because it evaporates and because the mixing operation ejects some little drops out of the pot. As the mass of a two millimetre diameter drop is 0.004 g, the accuracy on the mass will rapidly decrease. Secondly, it is very difficult to say when the particles are in suspension: for a given quantity of salt, some particles are floating, some are in suspension and some others are on the bottom (i.e. the mass density of particles of the same diameter is slightly different).

So it is very difficult to estimate theoretically the inaccuracy of the measure. The analysis of the dispersion in the results is the only way to see the quality of this procedure.

The observation of the particles in suspension in the solution shows that there is statically no difference of mass density between opaque and translucent particles (for a given diameter). Another important observation is that the small particles have a larger mass density than the large particles.

The results of the measurements are given in table A3.1/3:

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<tr>
<th>$\rho_p$,small [kg/m$^3$]</th>
<th>1</th>
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<th>3</th>
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<th>Mean</th>
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<td>1026</td>
<td>1021</td>
<td>1021</td>
<td>1020</td>
<td>1023</td>
<td></td>
</tr>
<tr>
<td>$\rho_p$,large [kg/m$^3$]</td>
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<td>1020</td>
<td>1016</td>
<td>1015</td>
<td>1013</td>
<td>1016</td>
</tr>
</tbody>
</table>

Table A3.1/3: Measurement of the mass density with the salt method

This is different from the value 1031 kg/m$^3$ found for the large particles with the picnometer method. In fact, the picnometer method gives a mean value for a set of particles, while the salt method gives a threshold upon which we "decide" to say that the particles are in suspension (so it is always subjective). We will for this reason use the value of 1031 kg/m$^3$ (measured only for the large particles) for the further calculations.

Assuming that the subjective threshold was taken the same way for the small and the large particles, the mass density of the small particles can be taken as

$$\rho_{p,small} = 1031 + (1023 - 1016) = 1038 \text{ kg/m}^3.$$
The feasibility of 2D-PTV for the study of vertical sediment transport
Appendix 3.3  

Theoretical settling velocity

In order to see the influence of the diameter, 3 settling velocities are computed for each set of particles (small, large):

* one for the minimum diameter of the set (\(d_{\text{min}}\))
* one for the mean diameter of the set (\(d_{\text{m}}\))
* one for the maximum diameter of the set (\(d_{\text{max}}\))

In order to see the influence of the inaccuracy of the measurement of \(\rho_p\), 3 settling velocities are computed and for each diameter:

* \(w_{s,\text{min}}\) computed with \(s_{\text{min}} = (\rho_p/\rho_w)_{\text{min}} = (\rho_p - \Delta\rho_p)/(\rho_w + \Delta\rho_w)\)
* \(w_s\) computed with \(s = \rho_p/\rho_w\)
* \(w_{s,\text{max}}\) computed with \(s_{\text{max}} = (\rho_p/\rho_w)_{\text{max}} = (\rho_p + \Delta\rho_p)/(\rho_w - \Delta\rho_p)\)

The inaccuracy on the value of the settling velocity (\(\Delta w_s\)) is computed by:

\[
\Delta w_s = \max (w_s - w_{s,\text{min}}; w_{s,\text{max}} - w_s)
\]

The Reynolds particle number (\(Re_p\)) is computed for each diameter with the value of \(w_s\).

The values of \(\rho_p\) and \(\Delta\rho_p\) are taken from the measurement with the picnometer method (see 3.3.3.B), which are adapted for the small particles with the difference seen with the salt method (1031 kg/m\(^3\) + 7 kg/m\(^3\)) (see 3.3.3.C).

\[
\begin{align*}
\rho_p,\text{small} & \quad 1038 \quad [\text{kg/m}^3] \\
\rho_p,\text{large} & \quad 1031 \quad [\text{kg/m}^3] \\
\rho_w (20^\circ \text{C}) & \quad 998 \quad [\text{kg/m}^3] \\
\Delta\rho_p,\text{small} & \quad 5 \quad [\text{kg/m}^3] \\
\Delta\rho_p,\text{large} & \quad 5 \quad [\text{kg/m}^3] \\
\Delta\rho_w & \quad 1 \quad [\text{kg/m}^3] \\
v (20^\circ \text{C}) & \quad 1,00E-06 \quad [\text{m}^2/\text{s}] \\
g & \quad 9,81 \quad [\text{m/s}^2]
\end{align*}
\]

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<tr>
<th>diameter</th>
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<th>Large</th>
<th>relative density</th>
<th>Small</th>
<th>Large</th>
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</table>

1. Stokes

\[
Re_p < 0.1
\]

\[
C_D = \frac{24}{Re_p}
\]

\[
w_s = \frac{(s - 1) g}{18 \nu} \cdot d^2
\]

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<th>(w_s)</th>
<th>(w_{s,\text{max}})</th>
<th>(\Delta w_s)</th>
<th>(Re_p)</th>
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<tr>
<td>(d_{\text{m}})</td>
<td>2.4</td>
<td>2.8</td>
<td>3.3</td>
<td>0.4</td>
</tr>
<tr>
<td>(d_{\text{max}})</td>
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<td>3.9</td>
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<tr>
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<td>(d_{\text{min}})</td>
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<td>18.0</td>
<td>21.3</td>
</tr>
<tr>
<td>(d_{\text{m}})</td>
<td>17.7</td>
<td>\textbf{21.6}</td>
<td>25.6</td>
<td>\textbf{4.0}</td>
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<tr>
<td>(d_{\text{max}})</td>
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<td>4.7</td>
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</tbody>
</table>
2. Other formulas

\[ C_D = \frac{\alpha}{Re_p} + \beta \]

\[ w_s = \frac{\alpha}{2\beta d} \left( \frac{16\beta}{5\alpha^2} \frac{s-1}{v^2} d^2 - 1 \right) \]

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<th>( w_s )</th>
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<th>( w_s )</th>
<th>( w_{s,\text{max}} )</th>
<th>( \Delta w_s )</th>
<th>( Re_p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small</td>
<td>( d_{\text{min}} )</td>
<td>1.1</td>
<td>1.3</td>
<td>1.5</td>
<td>0.2</td>
<td>0.4</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( d_m )</td>
<td>1.6</td>
<td>1.8</td>
<td>2.1</td>
<td>0.3</td>
<td>0.7</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( d_{\text{max}} )</td>
<td>2.1</td>
<td>2.5</td>
<td>2.8</td>
<td>0.4</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>Large</td>
<td>( d_{\text{min}} )</td>
<td>7.6</td>
<td>8.9</td>
<td>10.2</td>
<td>1.3</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( d_m )</td>
<td>8.6</td>
<td>10.1</td>
<td>11.5</td>
<td>1.5</td>
<td>11</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( d_{\text{max}} )</td>
<td>9.6</td>
<td>11.2</td>
<td>12.7</td>
<td>1.6</td>
<td>13</td>
<td></td>
</tr>
</tbody>
</table>
This computations show that

* the relative inaccuracy on \( w_s \) due to the inaccuracy on \( \rho_p \) is about 15 %
* the relative difference for the settling velocity between \( d_{\text{min}} \) and \( d_{\text{max}} \) is about 90 % for the small particles and 20 % for the big particles.

The tables below resume the settling velocity we found.

<table>
<thead>
<tr>
<th>Small particles</th>
<th>Stokes</th>
<th>Oseen</th>
<th>Brown</th>
<th>Engelund</th>
<th>Rubey</th>
<th>Fredsoe</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w_s ) [mm/s]</td>
<td>2,8</td>
<td>2,4</td>
<td>2,8</td>
<td>2,7</td>
<td>2,6</td>
<td>1,8</td>
</tr>
<tr>
<td>( \Delta w_s ) [mm/s]</td>
<td>0,4</td>
<td>0,3</td>
<td>0,4</td>
<td>0,4</td>
<td>0,4</td>
<td>0,3</td>
</tr>
<tr>
<td>( \text{Re}_p ) [ ]</td>
<td>1,0</td>
<td>0,9</td>
<td>1,0</td>
<td>1,0</td>
<td>0,9</td>
<td>0,7</td>
</tr>
</tbody>
</table>

Table A3.1: Theoretical settling velocity for the small particles

This table shows that the settling velocity computed for the small particles is nearly the same with the different formulae, except with the one of Fredsoe, not valid for such small values of the particle Reynolds number. This can be easily seen when looking at the figure 1.3, which gives all the different formulae on the same graphic: all the curves are nearly superposed (behalf the one of Fredsoe) for particles Reynolds number close to 1.

<table>
<thead>
<tr>
<th>Large particles</th>
<th>Stokes</th>
<th>Oseen</th>
<th>Brown</th>
<th>Engelund</th>
<th>Rubey</th>
<th>Fredsoe</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w_s ) [mm/s]</td>
<td>26,2</td>
<td>8,1</td>
<td>16,3</td>
<td>11,9</td>
<td>10,9</td>
<td>10,1</td>
</tr>
<tr>
<td>( \Delta w_s ) [mm/s]</td>
<td>4</td>
<td>1</td>
<td>2,5</td>
<td>1,6</td>
<td>1,4</td>
<td>1,5</td>
</tr>
<tr>
<td>( \text{Re}_p ) [ ]</td>
<td>29</td>
<td>9</td>
<td>18</td>
<td>13</td>
<td>12</td>
<td>11</td>
</tr>
</tbody>
</table>

Table A3.2: Theoretical settling velocity for the large particles

This table shows that the settling velocity computed for the large particles is nearly the same when \( \text{Re}_p \) is such that the formula is valid (Engelund, Rubey, Fredsoe). This can also be seen on the figure 1.3: for a particle Reynolds number close to 12, the curves of Engelund, Rubey and Fredsoe are close to each other.
Appendix 3.4

Characteristics of the EMS probe

The flow velocity in the X (U) and Y (V) direction (two directions in a plane parallel to the bottom) can be calculated from the two outputs with the calibration formulas given from the manufacturer for our probe:

For the X-channel: \[ |U| = -5.274 \cdot 10^{-4} \cdot A^2 + 0.1049 \cdot |A| + 0.0042 \quad [\text{m/s}] \]

For the Y-channel: \[ |V| = -4.511 \cdot 10^{-4} \cdot B^2 + 0.1038 \cdot |B| + 0.0059 \quad [\text{m/s}] \]

where A and B are measured in mV.

We measured the offset of the probe every day before starting the experiment (\( A_{\text{offset}(th)} \) and \( B_{\text{offset}(th)} \) at a time \( t_b \)) and at the end (\( A_{\text{offset}(te)} \) and \( B_{\text{offset}(te)} \) at a time \( t_e \)) by putting the probe in a pail full with still water. This pail is put at the same place as the probe during the day, in order to avoid the magnetic influence of the metallic part of the flume.

For the calculation of the velocity measured in the day, we suppose that this offset is linear with the time. This means that the ‘real’ velocity is only known at the end of the day.

For an experiment at a time \( t \) (between \( t_b \) and \( t_e \)), the offset in A will be:

\[
A_{\text{offset}(t)} = \left( \frac{t - t_b}{t_e - t_b} \right) \cdot (A_{\text{offset}(te)} - A_{\text{offset}(th)}) + A_{\text{offset}(th)}
\]

and likewise for B. The two outputs are now:

\[
A = A_{\text{measured}} - A_{\text{offsettime}(t)}
\]
\[
B = B_{\text{measured}} - B_{\text{offsettime}(t)}
\]

This instrument gives us only instantaneous values for A and B and thus for U and V. We can connect this apparatus to an integrator to obtain a mean values. This integrator gives us for a chosen interval of time (\( \Delta t \)):

\[
\sum_{t=0}^{\Delta t} A(t) \quad \text{and} \quad \sum_{t=0}^{\Delta t} B(t)
\]
So we have for an experiment at a time $t$:

$$A_m = \frac{1}{f} \frac{1}{\Delta t} \left( \sum_{t=0}^{\Delta t} A(t) \right) - A_{\text{offsettime}(t)}$$

$$B_m = \frac{1}{f} \frac{1}{\Delta t} \left( \sum_{t=0}^{\Delta t} B(t) \right) - B_{\text{offsettime}(t)}$$

with: $f$ : frequency of the measurement  
$A_m$ : mean over the output A  
$B_m$ : mean over the output B

It gives us in first approximation:

For the X-channel: $|U_m| \equiv -5.274 \cdot 10^{-4} \cdot A_m^2 + 0.1049 \cdot |A_m| + 0.0042 \quad [\text{m/s}]$

For the Y-channel: $|V_m| \equiv -4.511 \cdot 10^{-4} \cdot B_m^2 + 0.1038 \cdot |B_m| + 0.0059 \quad [\text{m/s}]$

with: $U_m$ : mean flow velocity in the X direction for $\Delta t$  
$V_m$ : mean flow velocity in the Y direction for $\Delta t$

It is only a first approximation because the relations between the outputs and the velocities are second order functions. Nevertheless, it is a good approximation because the second order term is very small ($4\ldots5 \cdot 10^{-4} \cdot A$ or $B$) relative to the first order term ($0.1$) and in addition we worked with only small outputs ($A, B \ll 1$ Volt).
The feasibility of 2D-PTV for the study of vertical sediment transport
Appendix 3.5

Offsets of the Laser-Doppler Flow Meter

We have to measure an offset of this instrument in still water, the same way we did with the EMS probe, but with a glass box filled with water. So we find a mean value for A and B: \( A_{\text{offset}} \) and \( B_{\text{offset}} \) (over a chosen \( \Delta t \)). We have to include these values in the calculations of the \( U \) and \( W \) velocities (with the hypothesis: \( \zeta = 45^\circ \)):

\[
U = 20 \left( \frac{(A - A_{\text{offset}}) - (B - B_{\text{offset}})}{\sqrt{2}} \right) \quad \text{[cm/s]}
\]

\[
W = 20 \left( \frac{(A - A_{\text{offset}}) + (B - B_{\text{offset}})}{\sqrt{2}} \right) \quad \text{[cm/s]}
\]

Horizontal alignment of the Laser Doppler Flow Meter (\( \zeta \) not equal to 45 degrees)

How to find the value of \( \zeta \) from the experiments where we know that \( \bar{W} = 0 \) (\( \bar{W} \) is the ensemble average over \( W \), which can be calculated with a time average with \( \Delta t \) long enough; if \( \Delta t \) is long enough: \( W_m = \bar{W} \))?

The velocities are:

\[
U = 20[(A - A_{\text{offset}}) \cdot \cos(\zeta) - (B - B_{\text{offset}}) \cdot \sin(\zeta)] \quad \text{[cm/s]}
\]

\[
W = 20[(A - A_{\text{offset}}) \cdot \sin(\zeta) + (B - B_{\text{offset}}) \cdot \cos(\zeta)] \quad \text{[cm/s]}
\]

Thus:

\[
\bar{W} = 20 \left( (A - A_{\text{offset}}) \cdot \sin(\zeta) + (B - B_{\text{offset}}) \cdot \cos(\zeta) \right) = 0
\]

\[
\Rightarrow (A - A_{\text{offset}}) \cdot \sin(\zeta) + (B - B_{\text{offset}}) \cdot \cos(\zeta) = 0
\]

\[
\Rightarrow \zeta = \arctg \left( -\frac{(B - B_{\text{offset}})}{(A - A_{\text{offset}})} \right)
\]

We note that if \( (A - A_{\text{offset}}) = -(B - B_{\text{offset}}) \), we have \( \zeta = \arctg (1) = 45^\circ \)
Conclusion:

\[
U = 20[(A-A_{\text{offset}}) \cdot \cos(\zeta) - (B-B_{\text{offset}}) \cdot \sin(\zeta)] \quad \text{[cm/s]}
\]

\[
W = 20[(A-A_{\text{offset}}) \cdot \sin(\zeta) + (B-B_{\text{offset}}) \cdot \cos(\zeta)] \quad \text{[cm/s]}
\]

An other manner to estimate the inclination of the Laser Doppler with regard to the horizontal is with the help of a water-gauge:

Putting a sheet of paper parallel to the flow and drawing on it the impact of the 2 beams going to the detectors, show that they are not horizontal whereas the use of a water gauge shows that the flume is perfectly horizontal. It means that the angle \( \zeta \) (angle between the horizontal and a reference velocity vector of the Lazer-Doppler Flow Meter) is not equal to 45° and thus that the Laser-Doppler Flow Meter is not horizontal. In order to measure this angle the following measurements are done.

The water-gauge (length: 300 mm) is put on the detectors floor and we can see that it is not totally horizontal, its edge has to be lifted up (1 mm) to be totally horizontal; \( \varepsilon \) is the angle between the water-gauge and the detectors floor:

A sheet of paper is placed perpendicular to the detectors floor and the positions of the three beams of light are noted on it. The points represent the impact of the beams.

- the angle in the centre is a right angle (as we can see with the measure of the three sides):

\[
\sqrt{(7.55)^2 + (7.55)^2} = 10.68 \text{ cm}
\]
• estimation of the angle between the two beams and the detectors floor:

\[ \delta \equiv \arcsin \frac{18.9 - 18.65}{10.65} = 1.35^\circ \]

• estimation of the angle between the straight line (which pass through the two beams) and the horizontal (bottom of the flume):

\[ \gamma \equiv \delta - \varepsilon = 1.35^\circ - 0.19^\circ = 1.16^\circ \]

\[ \zeta \equiv 45^\circ + \gamma = 46.16^\circ \]

Conclusion: the water gauge confirms that \( \zeta \) is well not equal to \( 45^\circ \) and gives a value of \( 1.2^\circ (\gamma) \) with regard to the horizontal for the inclination of the Laser-Doppler Flow Meter.
The feasibility of 2D-PTV for the study of vertical sediment transport
Darcy-Weisbach equation

The Darcy-Weisbach equation [14,15] gives in the case of a smooth bottom a relation between the shear velocity \((u_*)\), the depth-averaged velocity of the flow \((\overline{U}_m)\) and the characteristic parameters of the flow (kinematic viscosity \(v\) and hydraulic radius \(R\)).

The **Darcy-Weisbach friction factor** \((f')\) [8] is defined by:

\[
\tau_b = \frac{f'}{8} \rho \overline{U}_m^2
\]  
**(A4.1/1)**

Remembering the definition of the shear velocity:

\[
u_* = \sqrt{\frac{\tau_b}{\rho}}
\]  
**(A4.1/2)**

this equation may be written:

\[
u_* = \sqrt{\frac{f'}{8} \overline{U}_m}
\]  
**(A4.1/3)**

An empirical expression of the friction factor valid for \(Re < 10^5\) is given by Henderson:

\[
f' = \frac{0.316}{Re^{1/4}}
\]  
**(A4.1/4)**

where the Reynolds number is based on the mean flow velocity and the hydraulic radius \(R\):

\[Re = \frac{4R\overline{U}_m}{v}
\]  
**(A4.1/5)**

where

\[R = \frac{A}{L}
\]  
**(A4.1/6)**

with \(A\) the area of the cross section of the flow and \(L\) its perimeter. In the case of a rectangular cross section of width \(b\), the hydraulic radius is given by:

\[R = \frac{bh}{b + 2h}
\]  
**(A4.1/7)**
Thus the variation of $u_*$ as a function of $\bar{U}_m$ is known if $b$ and $h$ are known. Putting equations (A4.1/4), (A4.1/5) and (A4.1/7) in equation (A4.1/3) gives:

$$u_* = \sqrt{0.0279 \left( \frac{b + 2h}{bh} \right)^{1/4} \bar{U}_m^{7/8}} \quad (A4.1/8)$$

i.e. the relation between the shear velocity and the depth-average velocity is nearly linear.

In our experiments $b$ is equal to 40 cm and $h$ is about 8 cm (thus $R = 5.71$ cm). Under these conditions the relation between the shear velocity and the depth-averaged velocity of the flow is given by the dimensional equation:

$$u_* \left[ \frac{cm}{s} \right] = \frac{1}{13.2} \bar{U}_m^{7/8} \left[ \frac{cm}{s} \right] \quad (A4.1/9)$$

which is plotted on figure A4.1/1:

![Darcy-Weisbach equation](image)

Figure A4.1/1: Shear velocity for a rectangular cross section flume (width = 40 cm) and for a flow depth of $h = 8$ cm, computed with the Darcy-Weisbach equation.

For the depth-averaged velocities used in these experiments (10, 15 and 20 cm/s), it gives:

<table>
<thead>
<tr>
<th>$\bar{U}_m$ [cm/s]</th>
<th>Re</th>
<th>$f^*$</th>
<th>$\bar{U}<em>m / u</em>*$</th>
<th>$u_*$ [cm/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>23000</td>
<td>$2.57 \times 10^{-2}$</td>
<td>17.6</td>
<td>0.57</td>
</tr>
<tr>
<td>15</td>
<td>34000</td>
<td>$2.32 \times 10^{-2}$</td>
<td>18.6</td>
<td>0.81</td>
</tr>
<tr>
<td>20</td>
<td>46000</td>
<td>$2.16 \times 10^{-2}$</td>
<td>19.2</td>
<td>1.04</td>
</tr>
</tbody>
</table>

Table A4.1/1: Estimation of the shear velocity in our experiments with the Darcy-Weisbach equation.
The feasibility of 2D-PTV for the study of vertical sediment transport
Measurement of the important parameters of turbulence

The Laser-Doppler Flow Meter used has other reference co-ordinates for the measurement of the velocity than the one used for the measure of the mean velocity profile (when watching from the laser to the detectors):

\[
\begin{align*}
U &= \lambda [(A - A_{off}) \cos(\zeta) + (B - B_{off}) \sin(\zeta)] \\
W &= \lambda [(A - A_{off}) \sin(\zeta) - (B - B_{off}) \cos(\zeta)]
\end{align*}
\]

(A4.2/1)

The velocities in the longitudinal and vertical directions are computed with:

\[
\begin{align*}
U &= \lambda [A \cos(\zeta) + B \sin(\zeta)] \\
W &= \lambda [A \sin(\zeta) - B \cos(\zeta)]
\end{align*}
\]

(A4.2/2)

with

- $U$ instantaneous streamwise velocity
- $W$ instantaneous vertical velocity
- $\lambda$ scaling factor between the voltage and the velocity [(m/s)/V]
- $A, B$ instantaneous electric outputs
- $A_{off}, B_{off}$ offsets of the two electric output
- $\zeta$ angle between the horizontal and the A-reference vector

(cf. figure A4.2/1).

The offset measured is of magnitude 2 or 3 mV, for outputs from 40 to 100 mV given with an accuracy of 5 mV. As the offset is smaller than the accuracy of the measurement, it may be neglected, and the velocities are computed with:
As the flow is steady, the flume horizontal and the variation of the flow depth in the streamwise direction minute \((dh/dx = 3 \times 10^3)\), the time average velocity in the vertical direction must be nearly equal to zero at each height:

\[
\overline{W} = \lambda \left[ A \sin(\zeta) - B \cos(\zeta) \right] = \lambda \left[ \overline{A} \sin(\zeta) - \overline{B} \cos(\zeta) \right] = 0 \quad \text{(A4.2/3)}
\]

If \(\zeta = 45^\circ\), then \(\overline{A} = \overline{B}\). But in practice this is not the case, and \(\zeta\) is computed for each height with:

\[
\zeta = \arctan \left( \frac{\overline{B}}{\overline{A}} \right) \quad \text{(A4.2/4)}
\]

The values found for this angle at the different heights is nearly constant and has a mean value of 46.33°, except for the lowest measurement \((z = 1\text{mm})\), which gives a value of 42.7°. This means that this first measurement is not reliable.

Once \(\zeta\) is computed, the instantaneous velocities \(U\) and \(W\) can be computed with equation (A4.2/2). The computing of the intensities of \(U\) and \(W\) and of the correlation \(\overline{U'W'}\) can be done as following:

\[
(U,W) \rightarrow (\overline{U},\overline{W}) \rightarrow (U',W') \rightarrow \begin{cases} 
U'^2 & \rightarrow \overline{U'^2} \rightarrow \sqrt{U'^2} \\
W'^2 & \rightarrow \overline{W'^2} \rightarrow \sqrt{W'^2} \\
U'W' & \rightarrow \overline{U'W'}
\end{cases} \quad \text{(A4.2/5)}
\]

If there are \(N\) measurements, the number of computations is:

- computing of \(\overline{U}\) and \(\overline{W}\): \(2N\) additions
- computing of \(U'\) and \(W'\): \(2N\) additions
- computing of \(U'^2\), \(W'^2\) and \(U'W'\): \(3N\) multiplication's
- computing of \(\overline{U'^2}\), \(\overline{W'^2}\) and \(\overline{U'W'}\): \(3N\) additions

Thus in total \(3N\) multiplication's and \(7N\) additions.
If we now use the following properties of the ensemble averages and the fluctuations:

\[
U'^2 = (U - \bar{U})^2 = U^2 - 2U\bar{U} + (\bar{U})^2 = U^2 - 2\bar{U}U + (\bar{U})^2 = U^2 - (\bar{U})^2 \quad (A4.2/6)
\]

\[
U'W' = (U - \bar{U})(W - \bar{W}) = UW - U\bar{W} - \bar{U}W + \bar{U}\bar{W} = UW - \bar{U}W - \bar{W}U + \bar{U}\bar{W} \quad (A4.2/7)
\]

we can do the computations as follows:

\[
(U, W) \rightarrow (UW \rightarrow U\bar{W}) \rightarrow \sqrt{U'^2}, \sqrt{W'^2}, U'W' \quad (A4.2/8)
\]

With this method, 3N multiplications and only 5N additions must be done.

The measurements were done for 19 heights; 15 from the bottom to surface (P1 to P15) and then 4 back down (P16 to P19). Four of them were thrown aside because they gave completely senseless results.
The feasibility of 2D-PTV for the study of vertical sediment transport
Appendix 5.1

Calculation of the injection velocity

As explained in the chapter 4, the injection velocity ($U_{inj}$) has to be the same as the flow velocity ($\bar{U}$). Three different injection heights and three different flow velocities will be taken; to avoid at each time to calculate the velocity profile, we derive a convenient relation between the injection height ($h_{inj}$) and the mean flow velocity ($\bar{U}_m$) to find the injection velocity:

$$U_{inj} = \bar{U}(h_{inj})$$

As seen in the chapter 4, the velocity profile is well estimated by:

$$\bar{U}(z) = \frac{u_*}{\kappa} \ln \left( \frac{z}{z_0} \right)$$

and the mean flow velocity can be measured at the height $z = \frac{h}{2.71}$

with: $h$ : water depth ($\approx 80$ mm)

The idea is to be able to determine $U_{inj}$ as a function of $h_{inj}$ with only the measure of the mean flow velocity:

$$\bar{U}_m = \bar{U}\left(\frac{h}{2.71}\right) = \frac{u_*}{\kappa} \left( \ln \left( \frac{h}{z_0} \right) - 1 \right) = \frac{u_*}{\kappa} \ln \left( \frac{h}{z_0} \right) - \frac{u_*}{\kappa}$$

which gives:

$$\frac{u_*}{\kappa} \ln \left( \frac{h}{z_0} \right) = \bar{U}_m \cdot \frac{u_*}{\kappa}$$

Unfortunately, $u_*$ and $y_o$ can not be determined with only one equation. But if we suppose that $u_* = \frac{\bar{U}_m}{17.5}$ (see chapter 4) is available for a flow velocity which varies between 10 and 20 cm/s (remark: this relation was only measured with 20 cm/s), we find:
The feasibility of the 2D-PTV for the study of vertical sediment transport

\[ \frac{u_\ast}{\kappa} \ln \left( \frac{h}{z_0} \right) = \bar{U} \frac{u_\ast}{\kappa} = \bar{U} \frac{1}{17.5\kappa} = \bar{U} \left( \frac{1}{17.5\kappa} \right) \equiv 1.134 \bar{U} \]

1. When \( h_{\text{inj}} \) = just beneath the water surface = \( h - \frac{D}{2} \)
   
   with: \( D \) : diameter of the injection pipe (\( \equiv 7.8 \text{ mm} \))

   \[ \bar{U}_{\text{inj}} = \bar{U} \left( h - \frac{D}{2} \right) = \frac{u_\ast}{\kappa} \ln \left( \frac{h - D/2}{z_0} \right) \equiv \frac{u_\ast}{\kappa} \ln \left( \frac{h}{z_0} \right) \text{ (because } h \gg D/2) \]
   
   \[ = 1.143 \bar{U} \]

2. When \( h_{\text{inj}} = \) in the middle of the flow = \( \frac{h}{2} \)

   \[ \bar{U}_{\text{inj}} = \bar{U} \left( \frac{h}{2} \right) = \frac{u_\ast}{\kappa} \ln \left( \frac{h/2}{z_0} \right) = \frac{u_\ast}{\kappa} \left[ \ln \left( \frac{h}{z_0} \right) - \ln 2 \right] \]
   
   \[ = \frac{u_\ast}{\kappa} \ln \left( \frac{h}{z_0} \right) - \frac{u_\ast}{\kappa} \ln 2 = 1.143 \bar{U} - \frac{u_\ast}{\kappa} \ln 2 \]
   
   \[ = \bar{U} \left( 1.143 - \frac{\ln 2}{17.5\kappa} \right) = 1.044 \bar{U} \]

3. When \( h_{\text{inj}} = \) just near the bottom of the flume = \( \frac{D}{2} \)

   \[ \bar{U}_{\text{inj}} = \bar{U} \left( \frac{D}{2} \right) = \frac{u_\ast}{\kappa} \ln \left( \frac{D}{2z_0} \right) = \frac{u_\ast}{\kappa} \ln \left( \frac{Dh}{2z_0} \right) \]
   
   \[ = \frac{u_\ast}{\kappa} \ln \left( \frac{h}{z_0} \right) - \frac{u_\ast}{\kappa} \ln \left( \frac{2h}{D} \right) = 1.143 \bar{U} - \frac{u_\ast}{\kappa} \ln \left( \frac{2h}{D} \right) \]
   
   \[ = \bar{U} \left[ 1.143 - \frac{1}{17.5\kappa} \ln \left( \frac{2h}{D} \right) \right] = 0.711 \bar{U} \]
Conclusion: in a first approximation $U_{\text{inj}}$ can be calculated with: $U_{\text{inj}} = \beta \overline{U}_m$ where $\beta$ is given by:

<table>
<thead>
<tr>
<th>$h_{\text{inj}}$</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>beneath surface</td>
<td>1.14</td>
</tr>
<tr>
<td>middle</td>
<td>1.04</td>
</tr>
<tr>
<td>bottom</td>
<td>0.71</td>
</tr>
</tbody>
</table>

The constant-head vessel height can be found from the graph below. To measure the injection velocity, the measurement volume of the Laser was placed just at the outside of the injection pipe. These points have thus been found experimentally for the injection in the middle and beneath the surface and for the bottom this is an extrapolation of the first two (the Laser-Doppler Flow Meter can not measure the flow velocities beneath 7 mm from the bottom).
The feasibility of 2D-PTV for the study of vertical sediment transport
Appendix 6.1

Particle concentrations

This appendix gives the concentration and the number of particles present in the mixture injected for the different series of experiments carried out. The different parameters are:

- \( p \) \( \text{porosity} = 45\% \) (assuming that the particles are spherical and have the same diameter)
- \( V_p \) volume of one particle
- \( V_{\text{app},p} \) \textit{apparent volume} of the particles put in the vessel (for one series of experiments)
  - i.e. real volume + volume of vacuum between the particles
- \( V_w \) volume of water put in the vessel
- \( V_{\text{real},p} \) \textit{real volume} of the particles put in the vessel (for one series of experiments)
  - i.e. apparent volume \(* (1 - \text{porosity})\)
- \( c_{r,\text{real}} \) \textit{real concentration} (in volume)
  - i.e. real volume / volume of water
- \( c_r \) particle concentration for one series of experiments
  - i.e. real concentration / volume of one particle
- \( \pi \) \text{particles} number of particles injected for one series of experiments
  - i.e. particle concentration * volume of water

<table>
<thead>
<tr>
<th>Series</th>
<th>( A )</th>
<th>( B )</th>
<th>( C )</th>
<th>( D )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_p ) [mm(^3)]</td>
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<td>0.687</td>
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<td>20</td>
<td>32</td>
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<tr>
<td>( V_{\text{real},p} ) [l]</td>
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<td>( c_{r,\text{real}} ) [l]</td>
<td>0.0275</td>
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<td>( c_r ) [# particles/l]</td>
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<td>38 000</td>
<td>456 000</td>
<td>463 000</td>
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<tr>
<td>( \pi ) particles</td>
<td>800 000</td>
<td>575 000</td>
<td>9 120 000</td>
<td>14 820 000</td>
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Table A6.1/1: Particle concentration for the 4 series of experiments

As the volume is proportional to \( d^3 \), it is obvious that for the same volume of particles injected, the number of particles is much larger for the small ones than for the large ones.
Appendix 6.2

Experiments data

This appendix summarises for the four series (A to D) of experiments (0 to 13) all their experimental and theoretical parameters and some of the information given by DigImage: the number of *PV and *ERR files and the longitudinal and vertical mean velocities of the particles during an experiment calculated by DigImage.

**series A**

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<th>Number of ERR-files</th>
<th>Percentage of PV-files</th>
<th>Number of tracked particles</th>
<th>Number of particles per image</th>
<th>Average streamwise velocity [cm/s]</th>
<th>Average vertical velocity [mm/s]</th>
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<td>1010</td>
<td>9</td>
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<td>-4,8</td>
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<td>Surface</td>
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<td>7919</td>
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<td>17,6</td>
<td>-7,1</td>
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<td>89,0</td>
<td>8111</td>
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<td>-9,7</td>
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<tr>
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<td>17,1</td>
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<th>Percentage of PV-files</th>
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<td>201</td>
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<td>2005</td>
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<td>13,5</td>
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A 50
### Appendix 6.2  Experiment Data

**series C**

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* Notice that the initial name of the series C was Z.

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* Notice that the initial name of the series D was Y.
The feasibility of the 2D-PTV for the study of vertical sediment transport

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<th>C</th>
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<td>576 000</td>
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<td>14 819 000</td>
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<td># particles tracked</td>
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<td>percentage of the tracked particles</td>
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<td>2.1</td>
<td>1.6</td>
<td>1.0</td>
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We can observe that the percentage of the tracked particles is very small and that it is about two times greater for the large particles than for the small ones.
Appendix 6.3

Equivalent experiments

This appendix summarises the “equivalent experiments” defined as experiments which differ only in one parameter.

The parameters are the following:

- **d**: diameter of the particles (large or small, see chapter 3.3)
- **d_{inj}**: distance between the point of observation and the injection (0.5, 1, 1.5, 2 m)
- **h_{inj}**: height of the injection (near the bottom, in the middle or beneath the surface)
- **\( \bar{U}_m \)**: depth-averaged velocity (10, 15 or 20 cm/s)

### 1. Influence of the diameter

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<th>h_{inj}</th>
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<tr>
<td>B04</td>
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<td>D09</td>
<td>Small</td>
<td>1</td>
</tr>
<tr>
<td>C03</td>
<td>D04</td>
<td>Small</td>
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</tr>
<tr>
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<td>D03</td>
<td>Small</td>
<td>2</td>
</tr>
<tr>
<td>C05</td>
<td>D02</td>
<td>Small</td>
<td>2</td>
</tr>
<tr>
<td>C06</td>
<td>D05</td>
<td>Small</td>
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</tr>
<tr>
<td>C07</td>
<td>D08</td>
<td>Small</td>
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</tr>
<tr>
<td>C08</td>
<td>D11</td>
<td>Small</td>
<td>0.5</td>
</tr>
<tr>
<td>C13</td>
<td>D12</td>
<td>Small</td>
<td>0.5</td>
</tr>
<tr>
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<td>D07</td>
<td>Small</td>
<td>1</td>
</tr>
<tr>
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<td>D06</td>
<td>Small</td>
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<tr>
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3. **Influence of $d_{\text{inj}}$**

<table>
<thead>
<tr>
<th>d</th>
<th>$\bar{U}_m$ [cm/s]</th>
<th>$h_{\text{inj}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A01</td>
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<td>A03</td>
</tr>
<tr>
<td>A05</td>
<td>A06</td>
<td>B04</td>
</tr>
<tr>
<td>B03</td>
<td>C05</td>
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<td>C10</td>
<td>C11</td>
</tr>
<tr>
<td>D02</td>
<td>D05</td>
<td>D08</td>
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4. **Influence of $h_{\text{inj}}$**

<table>
<thead>
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<th>d</th>
<th>$\bar{U}_m$ [cm/s]</th>
<th>$h_{\text{inj}}$ [m]</th>
</tr>
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<tr>
<td>A03</td>
<td>A06</td>
<td>Large</td>
</tr>
<tr>
<td>A04</td>
<td>A05</td>
<td>A12</td>
</tr>
<tr>
<td>B04</td>
<td>B05</td>
<td>Large</td>
</tr>
<tr>
<td>C01</td>
<td>C08</td>
<td>C13</td>
</tr>
<tr>
<td>C02</td>
<td>C07</td>
<td>C10</td>
</tr>
<tr>
<td>C03</td>
<td>C06</td>
<td>C11</td>
</tr>
<tr>
<td>C04</td>
<td>C05</td>
<td>C12</td>
</tr>
<tr>
<td>D01</td>
<td>D02</td>
<td>D03</td>
</tr>
<tr>
<td>D04</td>
<td>D05</td>
<td>D06</td>
</tr>
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<td>D07</td>
<td>D08</td>
<td>D09</td>
</tr>
<tr>
<td>D10</td>
<td>D11</td>
<td>D12</td>
</tr>
</tbody>
</table>

One can see that there are many possibilities to study the influence of one parameter: 3 for the diameter, 15 for the depth-average velocity, 9 for the distance between the injection and the point of observation and 11 for the height of injection. As we are interested in three different dependent variables (concentration profiles, streamwise velocity and vertical velocity), a complete analysis would compare 114 situations.
Distances required to reach an equilibrium situation

The objective of this appendix is to see whether the theoretical distance necessary to reach an equilibrium situation (\(L_e\)) corresponds to the measurements. The theoretical criterion for deciding whether an equilibrium situation is reached, is \(d_{nj} > L_e\) where \(L_e\) is the required distance for a particle settling from the injection to the bottom (assuming that the settling is the only possible movement). The practical criterion for deciding whether an equilibrium is reached is \(w_p < 0.2 w_c\).

**series A**

<table>
<thead>
<tr>
<th>Name</th>
<th>Time of recording [s]</th>
<th>(d_{nj}) [m]</th>
<th>(b_{nj})</th>
<th>Vertical mean velocity [mm/s]</th>
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<th>Situation in practice</th>
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</thead>
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<td>yes</td>
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<td></td>
<td></td>
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<td>no</td>
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<tr>
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<td>0.5</td>
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</table>

**series B**

<table>
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<th>(d_{nj}) [m]</th>
<th>(b_{nj})</th>
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<th>Situation in practice</th>
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</table>
### Appendix 6.4 Distances required to reach an equilibrium situation

#### series C

**Type of particles** Small  
**Flow depth [mm]** 80,7  
**Mean flow velocity [cm/s]** 15,4  
**Date** 22 april 96  

<table>
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<tr>
<th>Name</th>
<th>Time of recording [s]</th>
<th>$d_{ij}$ [m]</th>
<th>$h_{ij}$</th>
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<th>Situation in practice</th>
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**Series D**

**Type of particles** Small  
**Flow depth [mm]** 80,2  
**Mean flow velocity [cm/s]** 10,2  
**Date** 24 april 96  

<table>
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<th>Steady Theoretically</th>
<th>Situation in practice</th>
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<tr>
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<td>no</td>
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</tbody>
</table>

The prediction is verified in only 25 experiments out of 40. Thus the theoretical point of view does not agree well with the experimental results.
REFERENCES


The feasibility of 2D-PTV for the study of vertical sediment transport
Notations

Symbols

\(\bar{x}\)  
ensemble average value of the variable \(x\)

\(x'\)  
instantaneous fluctuation of the value of \(x\) relatively to \(\bar{x}\)

\(x_m\)  
mean value of the variable \(x\)

\(\Delta x\)  
difference in the variable \(x\)

\(|x|\)  
absolute value of \(x\)

\(\frac{du}{dx}\)  
derivative of \(u\) with respect to \(x\)

\(\frac{\partial u}{\partial x}\)  
partial derivative of \(u\) with respect to \(x\)

\(Du\)  
total derivative of \(u\) with respect to \(x\)

\(Dx\)  

\(\nabla, u\)  
partial derivative of \(u\) with respect to the direction \(i \equiv \frac{\partial u}{\partial x_i}\)

\(\nabla u\)  
gradient of \(u = \left(\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial u}{\partial z}\right)\)

\(\bar{u}\)  
vector \((u_x, u_y, u_z)\)

\(\bar{u} \cdot \bar{v}\)  
scalar product of the vectors \(\bar{u}\) and \(\bar{v}\)

\(x \approx y\)  
x proportional to \(y\)

\(x = y\)  
x defined as \(y\)

\(x \equiv y\)  
x nearly equal to \(y\)

[i]  
refers to the bibliography

Letters

\(a\)  
reference elevation for the concentration distributions

\(A\)  
surface area

\(A, B\)  
electric outputs

\(A, B\)  
umerical coefficients

\(\bar{A}, \bar{B}\)  
Laser-Doppler Flow Meter reference vectors

\(A_{\text{measured}}, B_{\text{measured}}\)  
electric outputs measured

\(A_{\text{offset}(t)}, B_{\text{offset}(t)}\)  
offset of the electric outputs at time \(t\)

\(b\)  
width of the flume

\(B\)  
buoyancy

\(B_{ij}\)  
distance between the place where particle \(p_j\) is estimated to be at \(t_{m+1}\) 
and the place where the particle \(q_j\) is at \(t_{m-1}\).
The feasibility of 2D-PTV for the study of vertical sediment transport

c  sediment concentration

c_{ij}  cost of associating particle p_i at t = t_a with particle q_j at t = t_{a+1}

C_s  reference concentration of suspended sediment (in z = 0)

C_D  drag coefficient

C_D,wall  drag coefficient modified by the wall effects

d  diameter of a particle

d_{ijn}  distance between the injection and the point of observation

D_m  mean diameter of a set of particles

D_{max}  maximal diameter of a set of particles

D_{min}  minimal diameter of a set of particles

d'_p  dimensionless particle diameter

D  diameter of the injection pipe; loss per unit time en per unit volume

D_{m}  coefficient of molecular diffusion

D_{ij}  turbulent diffusion coefficient

D_{zz}  turbulent diffusion coefficient in the vertical direction

f  frequency of the measurement

f'  Darcy-Weisbach friction factor

f_d  Doppler shift (or Doppler frequency)

f_i  frequency of the incident light

f_s  frequency of the light

F_D  drag force

g  gravitational acceleration

G  gravity force

h  flow depth

h_{ijn}  height of injection

I  momentum

I_{ij}  background intensity premium

I_{ij},K  indices for the particles of the set Q

k_j  component j of the external forces per unit mass

l  distance

l_m  mixing length

l_t  characteristic length for turbulence

I_{ij},L  indices for the particles of the set P

L  length

L_s  streamwise length travelled by a settling particle

M  mass

M, N  number of particles on an image

N_i  new particle joining fee

N  number of settling particles

N^+  number of raising particles

p  porosity; pressure

p_i  particle i at time t_a

P  set of M particles found at (t = t_a) = \{p_i, ..., p_n, ..., p_M\}

q_j  particle j at time t_{a+1}

q_{ij}  Reynolds stress in the direction i on a surface perpendicular to the direction j

Q  set of N particles found at (t = t_{a+1}) = \{q_1, ..., q_{N}, ..., q_N\}; quantity sieved
<table>
<thead>
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<td>$\overline{x}_j$</td>
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</table>
The feasibility of 2D-PTV for the study of vertical sediment transport

\( X_{ij} \)  cross-correlation function between subregions of the image centred on \( p_i \) and \( q_j \)

\( y \)  cross direction

\( Y_i \)  cross-correlation costing

\( z \)  vertical direction; height above the bottom in a vertical direction

\( z_0 \)  equivalent roughness of the bottom

\( z_p \)  vertical position of a particle

\( z' \)  dimensionless height

\( Z \)  cost function

Greek letters

\( \alpha_{ij} \)  association variable between \( p_i \) and \( q_j \)

\( \alpha, \beta \)  numerical coefficients

\( \beta \)  ratio between \( D_{zz} \) and \( v_t \)

\( \varepsilon_0 \)  loss of concentration per unit time

\( \varepsilon_j \)  ellipticity premium

\( \gamma, \delta, \varepsilon \)  angles

\( \phi_i \)  size premium

\( \eta \)  relative height

\( \kappa \)  von Karman constant

\( \lambda \)  scaling factor, length

\( \mu \)  dynamic viscosity of a fluid

\( \nu \)  kinematic viscosity of a fluid

\( \nu_t \)  eddy viscosity

\( \psi \)  equivalent sand roughness of Nikuradse

\( \rho \)  mass density of a fluid

\( \rho_p \)  mass density of the polystyrene particles

\( \rho_{quartz} \)  mass density of quartz sediment

\( \sigma \)  numerical coefficient

\( \tau \)  shear stress

\( \tau_b \)  shear stress at the bed

\( \tau^v \)  viscous stress

\( \tau_{ij} \)  shear stress in the direction \( i \) on a surface perpendicular to the direction \( j \)

\( \tau_j \)  threshold premium

\( \zeta \)  angle between the horizontal and the Laser-Doppler Flow Meter reference vectors

\( \eta \)  relative height

\( \eta_i \)  new particle discount

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