Characterization of a novel Optical Micro-machined Ultrasound Transducer

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Characterization of a novel Optical Micro-machined Ultrasound Transducer

Proefschrift

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door

Suzanne Margriet LEINDERS

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We design and demonstrate a prototype ultrasound sensor based on a photonic micro-ring resonator integrated on a silicon membrane, and show that it can detect very low pressure ultrasound waves. The use of integrated photonics in future array transducers has several benefits: for instance it provides a small spatial footprint, compatibility with MRI due to the lack of electrical wiring, easy interrogation of the array of elements and ease of mass production, which may result in cost-effective fabrication of array transducers. To understand the working principle of the sensor, we have modeled the basic sensor element, fabricated the sensor and measured the response of the sensor to ultrasound. We have studied the response of the optical resonator separately before we integrate the resonator on the membrane and measure the response of the entire sensor. Besides the characterization of the sensor, we have expanded the existing knowledge of acoustical noise to determine the noise mechanism of the sensor.

Although ultrasound sensors are widely used in both industrial and medical applications, this thesis mainly focuses on use in medical diagnostics. Currently, ultrasound transducers are amongst others used to image such diverse objects as the coronary arteries, heart valves, liver, kidneys, prostate, brain and thyroid, but the most familiar application is the echo imaging of the fetus. The basis of the conventional ultrasound transducer consists of piezoelectric material that converts an electrical signal into a pressure wave and vice versa. During the last decades the single element transducers are replaced by one-dimensional arrays of sensors and during the last few years we have seen the emergence of two-dimensional arrays of sensors. Hence the development is focused on miniaturization of the sensor elements and fabrication of large dense arrays.

In this thesis, we have designed the basic element of a possibly future array ultrasound sensor that is based on integrated photonics. We have used the resulting
Optical Micro-machined Ultrasound Sensor (OMUS) to investigate the working principle. The element consists of a silicon optical waveguide that is coupled to a photonic racetrack-shaped optical resonator with a spatial footprint of 50 µm by 10 µm. The optical resonator is positioned on a silicon-dioxide membrane with a thickness of 2.5 µm and a diameter of about 100 µm. When a pressure wave is incident on the element, the membrane deforms and thus deforms the optical resonator. The deformation of the optical resonator shifts the optical resonance frequencies. This shift is recorded by an external interrogation system.

We have fabricated the photonic circuit of the sensor in a semi-industrial CMOS line. The resulting wafer-piece (die) contains 220 nm high and 400 nm wide silicon waveguides on top of a 2 µm thick silicon-dioxide layer on top of a 250 µm thick silicon substrate. Next, we have deposited a 0.5 µm thick silicon dioxide cladding on top of the die to isolate the waveguide from the water. Finally, we have created the membrane by locally removing the entire silicon substrate with use of deep reactive ion etching from the back of the die.

The prototype sensor is used to give proof of concept. We have used a laser and photo-receiver to interrogate the optical resonator and a conventional ultrasound transducer to transmit the pressure waves that were received by the sensor. We have found that the sensor had a resonance frequency of 0.76 MHz with a -6 dB bandwidth of 19 %. Furthermore, we have demonstrated that this prototype is capable of detecting pressures as low as 0.4 Pa. The latter matches the performance of the state of the art piezo-electric transducers while the spatial footprint of our sensor is 65 times smaller.

The sensor elements were further investigated by characterizing the strain-induced shift of the resonances of the optical resonator. We have applied a well defined strain to the racetrack resonators and have identified three different influences on the shift of resonances: dispersion, change in effective index and change in track-length. We have found that the linear wavelength shift per applied strain varies between 0.5 and 0.75 pm/microstrain depending on the width of the waveguide and the orientation of the silicon crystal for infrared light around 1550 nm wavelength. The influence of the increasing ring circumference is about three times larger than the influence of the change in waveguide effective index, and the two effects oppose each other.

The influence of the membrane is investigated by applying increasing static loadings to the sensor. In this study, we have integrated a short optical resonator on the membrane, which experiences a radial deformation rather than a uniform elongation. The measurement results show a non-linear response with high sensitivity at the beginning of the curve where small loadings are applied and lower sensitivity to the larger loadings.

The dynamic behavior of the sensor is investigated with a laser vibrometer. Due to simultaneous read-out of the sensor and the vibrometer, the read-out of the sensor can be compared to the vibration of the membrane and vibration of the entire sensor. We were able to show that the sensitivity of the sensor to low pressure signals is indeed induced by the membrane of the sensor, as the back of the sensor shows no significant vibrations. Furthermore, we have shown that the resonance frequency of the sensor can be tuned using different membrane diameters.
The noise level of our sensor is different from conventional ultrasound transducers due to the lack of electrical circuits. The noise of the optical sensor is only caused by acoustical noise, which is not well described in literature. Therefore, we have expanded the existing knowledge of acoustical noise by analyzing two mechanisms of acoustical noise. We show that in thermodynamic equilibrium the noise power delivered by the medium to the sensor balances the noise power delivered by the sensor to the medium. Moreover, we show that for sensors with vanishing aperture area, the noise pressure due to the molecular motion in the medium will reach a well-defined finite limit.
In dit proefschrift ontwerpen en demonstreren we een prototype ultrageluidsensor die bestaat uit een optische micro-ring-resonator op een silicium membraan. We laten zien dat deze sensor zeer lage ultrageluid drukgolven kan detecteren. Het gebruik van geïntegreerde optica in toekomstige matrix transducenten heeft verschillende voordelen: de technologie past bijvoorbeeld op een klein oppervlak, is te combineren met MRI omdat er geen elektrische bedrading nodig is, maakt dat een matrix van elementen makkelijk uitleesbaar is en is via massa productie te produceren hetgeen mogelijk resulteert in kosteneffectieve fabricatie van matrix transducenten. Om de werking van de sensor te begrijpen hebben we een model gemaakt van het basis element, deze sensor gefabriceerd en de respons op ultrageluid gemeten. We hebben eerst apart de respons van de optische resonator bestudeerd voordat we de resonator in het membraan hebben geïntegreerd en de respons van de hele sensor hebben gemeten. Naast het karakteriseren van de sensor hebben we de bestaande kennis van akoestische ruis verruimd om het ruismechanisme van de sensor te kunnen bepalen. Hoewel ultrageluidsensoren worden gebruikt in zowel industriële als medische toepassingen, focust dit proefschrift zich op de medische diagnostiek. De meest bekende toepassing hierin is de echo van een foetus, maar de ultrageluidsensoren worden ook gebruikt om beelden te maken van allerlei objecten zoals de kraanslagaders, hartkleppen, lever, nieren, prostaat, hersenen en schildklier. De basis van de conventionele ultrageluidtransducenten bestaat uit piezo-elektrisch materiaal dat een elektrisch signaal in een drukgolf converteert en andersom. Al sinds tientallen jaren hebben enkel-elementstransducenten plaatsgemaakt voor een één-dimensionale array van transducentelementen. Als gevolg daarvan heeft de ontwikkeling zich met name gericht op het verkleinen van de transducentelementen en het fabriceren van matrices met hoge dichtheid van elementen.
In dit proefschrift hebben we het basis element ontworpen van een mogelijk toekomstige matrix ultrageluidsensor dat gebaseerd is op geïntegreerde optica. We hebben de resulterende optische micro-gefabriceerde ultrageluidsensor (OMUS) gebruikt om de werking daarvan te onderzoeken. Het element bestaat uit een optische golfgeleider van silicium die aan een optische ellips-vormige resonator wordt gekoppeld. Deze resonator heeft een spatieel oppervlak van 50 µm bij 10 µm. De optische resonator is gepositioneerd op een membraan van siliciumdioxide dat een dikte heeft van 2.5 µm en een diameter rond de 100 µm. Wanneer een drukgolf het element raakt vervormt het membraan en daarmee ook de optische resonator. De deformatie van de optische resonator zorgt voor een verschuiving van zijn optische resonantiefrequenties. Deze verschuiving wordt geregistreerd door een extern uitlees systeem.

We hebben het optische circuit van de sensor in een semi-industriële CMOS lijn gefabriceerd. De resulterende silicium schijf (wafer) bevat 220 nm hoge en 400 nm brede silicium golfgeleiders op een 2 µm dikke siliciumdioxide laag op een 250 µm dik silicium substraat. Hierna hebben we de golfgeleiders met een 0.5 µm dikke laag silicium bedekt om de golfgeleiders van het water te isoleren. Tenslotte hebben we het membraan gemaakt door lokaal via de achterkant met een diepe reactieve ion ets het volledige silicium substraat te verwijderen.

Het prototype sensor is gebruikt om te laten zien dat het concept werkt. We hebben gebruik gemaakt van een laser en fotodiode om de optische resonator uit te lezen en een conventionele ultrageluidstransduant om de drukgolf op te wekken. We hebben een resonantie frequentie van 0.76 MHz gevonden met een -6 dB bandbreedte van 19 %. Verder laten we zien dat dit prototype een minimale druk van 0.4 Pa kan detecteren. Deze druk komt overeen met die van de meest moderne piezo-elektrische transducenten terwijl het oppervlak van onze sensor 65 keer kleiner is.

De sensorelementen zijn verder onderzocht door het karakteriseren van de rekgeïnduceerde verschuiving van de resonanties van de optische resonatoren. We hebben een goed gedefinieerde rek aangebracht op de ellips-vormige resonatoren en hebben drie verschillende invloeden op de resonatieverschuiving kunnen onderscheiden; dispersie, verschil in effectieve index en verschil in baanlengte. We hebben voor infrarood licht met een golflengte rond 1550 nm een variatie tussen 0.5 en 0.75 pm/microrek van de lineaire golflengte-verschuiving per aangebrachte rek gevonden, die afhankelijk is van de breedte van de golfgeleider en de oriëntatie van het silicium kristal. De invloed van de toenemende baanomtrek is ongeveer drie keer zo groot als de invloed van het verschil in de effectieve index van de golfgeleider. Daarnaast werken de twee effecten tegengesteld.

De invloed van het membraan is onderzocht door een toenemende statische last op de sensor aan te brengen. In dit onderzoek is een korte optische resonator op het membraan geïntegreerd, die een radiële deformatie ondervindt in plaats van een uniforme verlenging. De meetresultaten laten een niet-lineaire respons zien met hoge gevoeligheid in het begin van de curve waar kleine lasten worden aangebracht en lagere gevoeligheid voor de hogere lasten.

Het dynamische gedrag van de sensor is met een laservibrometer onderzocht. Door het gelijktijdig uitlezen van de sensor en de vibrometer kan de meetwaarde van de
sensor vergeleken worden met de vibratie van het membraan en de vibratie van de gehele sensor. Hierdoor kunnen we laten zien dat de gevoeligheid van de sensor voor lage druk signalen ingerdaad door het membraan wordt geïnduceerd omdat de achterkant van de sensor geen significante trillingen laat zien. Verder laten we zien dat de resonantiefrequentie van de sensor kan worden beïnvloed door gebruik van membranen met verschillende diameter.

Het ruisniveau van onze sensor verschilt van conventionele ultrageluidsensoren door het ontbreken van elektrische circuits. De ruis van de optische sensor wordt alleen veroorzaakt door akoestische ruis, die niet goed is beschreven in de literatuur. Daarom hebben we de bestaande kennis van akoestische ruis uitgebreid door twee ruismechanismen van akoestische ruis te analyseren. We laten zien dat in thermodynamisch evenwicht het ruis vermogen dat door de omgeving aan de sensor wordt afgegeven in balans is met het ruis vermogen dat door de sensor aan zijn omgeving wordt afgegeven. Bovendien laten we zien dat voor sensoren met een oneindige klein oppervlak de ruis druk door moleculaire beweging in het medium een goed gedefinieerde eindige limiet bereikt.
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### 7 Conclusions and discussion

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Chapter 1

General introduction

This thesis describes the development and characterization of an ultrasound sensor based on integrated photonics. The use of light as information carrier makes this sensor different from the state-of-the-art piezo-electric ultrasound transducers. It provides a large data transfer capacity due to a broad bandwidth. Moreover the sensor is made using standard Complementary Metal Oxide Semiconductor (CMOS) fabrication processes, which allows for mass production. To design a high quality sensor that has comparable or even better image qualities than the state-of-the-art transducers, complete understanding of the sensor is necessary. Therefore we investigated the optical, mechanical and acoustical aspects of the sensor.

This chapter starts with an introduction of the main background of medical imaging and development of conventional ultrasound transducers (Sec. 1.1). Then we describe the aim of the research that was carried out in Sec. 1.2. In this part, we briefly describe the working principle of the sensor to get some understanding of the sensor. For a detailed description of the sensor as well as its operating principle we refer the reader to Chapter 4. In Sec. 1.3 we will discuss the development in the use of guided light in sensors and the semiconductor industry as fabrication platform. The outline of this thesis is presented in Sec. 1.4.

1.1 Medical imaging

Medical ultrasound is often used for diagnostic imaging of patients. Compared to other imaging techniques as CT and MRI, it has a lot of benefits; the images are real-time, it is safe (no radiation), inexpensive and the system is portable. The name ultrasound imaging refers to the use of high ($\geq 1$ MHz) frequency sound waves to image the inside of the body. A device, named transducer, is used to
emit soundwaves into the body. These waves partly reflect on the inhomogeneities inside the body. The transducer receives the reflected waves (echoes) and with use of the time of flight the axial depth of the tissue can be determined. A single transmission provides the location of the scatterers in the beam line of the transducer. To create an image, multiple lines are needed and hence the area of interest has to be scanned. Three typical ultrasound images are shown in Figure 1.1 with on the left an image of a fetus, in the middle an image of the 4 chambers and valves of the heart and on the right an ultrasound image of an artery.

The development of diagnostic ultrasound instrumentation as we know it today was initiated around the time of the end of the Second World War [1]. The conventional transducers used to make an image consist of piezoelectric material. Piezoelectricity is defined as an electrical polarization related to mechanical strain. The polarization is proportional to the strain and changes sign with it [2]. Thus piezoelectric materials accumulate an electric charge in response to an applied mechanical stress and vice versa. The first ultrasound transducers were made of Quartz. This material has a high mechanical strength and low internal friction, but needs large amplitude voltages to be driven. Later, piezoelectric ceramics were used which had an improved efficiency and could be processed into varying shapes and sizes. In 1954, lead titanate-zirconate compositions (PZT) were discovered as piezoelectric material which had the additional benefits of a larger operating temperature range [3]. Currently, PZT is still used as transduction material in the state-of-the-art transducers, while research for new materials like single crystals (e.g. Lead Magnesium Niobate-Lead Titanate (PMN-PT)) or composites continues [1].

The efficiency of a transducer in conversion of energy depends next to the material on the frequency of excitation. To improve the energy transfer between the piezoelectric material and the tissue, matching layers and backing layers are often used. A scheme of a conventional transducer is shown in Figure 1.2. This transducer contains a large backing block that prevents movement of the bottom part of the piezoelectric layer. The matching layer, which has an impedance in between the piezoelectric material and the tissue, is here indicated as plastic 'nose'.

Next to the development of new materials, other aspects of the transducers were also adapted. The transducers changed from single element transducers into arrays, which contain many small transducer elements in a line (linear array) or in a plane (matrix array). Arrays give the option to illuminate an entire area at once, or electrically scan the area of interest [1]. With these arrays higher resolution images could be obtained with a much faster acquisition time [4].

The transducers used for medical imaging are designed for specific applications. The materials, element dimensions, shape and resonance frequency are optimized to obtain the penetration depth and resolution needed for each procedure. The applications that require dense arrays with small outer dimensions of the transducer are the ones where our sensor can provide most added value. Such applications can be found in the field of intra-operative image guidance (e.g. Intravascular Cardiac Echography (ICE)), where the device should not interfere with the procedure [5], or in the field of internal medical diagnostics, for instance Transesophageal Echocardiography (TEE) or Intravascular Ultrasound (IVUS). In these last two diagnostic applications the transducer is positioned on a catheter which is posi-
tioned inside the body. With a TEE image the catheter containing the transducer is positioned via the mouth or nose in the esophagus to image the heart. In case of IVUS the catheter is positioned inside a coronary artery to image the artery wall. In both cases a small transducer is required that nevertheless consists of multiple elements. Because the transducer is mounted on a catheter, the number of coaxial cables that connects the catheter to the computer is also limited, which provides an additional design challenge.

Figure 1.1: Ultrasound images of a) fetus obtained with 3D ultrasound [6], b) apical 4-chamber view of the heart obtained with transthoracic echocardiography [7] and c) a cross-section of a coronary artery obtained with IVUS [8].

Figure 1.2: Scheme of ultrasound transducer [9].
1.2 An optical micro-machined ultrasound sensor for (medical) ultrasound

The aim of this research project was to design, fabricate and test an ultrasound sensor based on a new operation principle. This operation principle makes use of integrated photonics. Therefore our read-out system tracks the light spectrum over time. When an acoustical wave is incident on the sensor the transmission spectrum changes. We can explain this by describing the working principle of the sensor. The sensor consist of a photonic waveguide (bus waveguide) that guides the light from one side of the sensor towards the other side. We positioned a photonic ring resonator, which is a looped waveguide with distinct resonances, in the middle of the sensor. A part of the light spectrum is coupled from the photonic waveguide into this ring resonator. When we transmit a broad light spectrum through the bus waveguide and record the transmission spectrum at the end of the waveguide, instead of a flat spectrum we measure resonance dips consistent with the resonance frequencies of the ring resonator. We want to obtain a modulation of this spectrum due to ultrasound waves. This is done by positioning the ring resonator on an acoustical membrane. When a pressure wave is incident on the acoustical membrane, the membrane and thus the integrated ring resonator will deform. The deformation of the ring resonator influences the position of the resonances in the transmission spectrum. When one resonance is monitored over time using light with a very narrow spectrum, an incident pressure wave causes a time dependent shift of this resonance dip, which is observed as a modulation of the transmitted light intensity.

A main benefit of this new operating principle is the sensors in-susceptibility to electromagnetic interference. Therefore the sensor can be used in combination with MRI and other radiative environments. An other advantage is the possibility to stack data of several sensors in the spectrum. As a result only a single fiber is required as read-out of an entire array.

To get a good understanding of this sensor and be able to determine the feasibility as ultrasound sensor we investigated several aspects. We studied the optical components on the chip [10] as well as the influence of strain on the ring resonators (Chapter 3). We provided proof of concept with the first ultrasound measurements performed with a prototype (Chapter 4). We determined the influence of the membrane with a static pressure analysis and a study of the membrane motion (Chapter 5). Finally we derived a theory about the noise pressure levels that determines the minimal pressure level that can be measured with this type of sensor (Chapter 6).

1.3 Integrated photonic systems and micromachining fabrication technology

Our sensor uses light as information carrier to benefit from fiber optics. Fiber optics, first developed in the 1970s, revolutionized the telecommunications industry. Compared to copper wires, optical fibers have the benefit of low attenuation and
interference, high reliability over long distances, a long life span and a very high information capacity [11]. We choose silicon and silicon dioxide as materials for the index-guided based photonic waveguides. This allows us to use silicon-on-insulator technology for fabrication and hence profit from 50 years of development in semiconductor fabrication technology. This mainly means ease of fabrication and hence mass production. Chip production today is based on photolithography. In this process high energy UV-light is shone through a mask upon a photosensitive film covering the silicon wafer. The illuminated part of the photosensitive layer forms the pattern of the chip and is developed into a layer that protects the underlying silicon. With etching processes the silicon that is not protected by the mask is removed. The wafer is cleaned by removing the mask. Most designs require several of these fabrication sequences [12]. Lots of structures can be created with this fabrication technique, but there are limitations that have to be taken into account when a design is made. For instance, there are variations present in the height of the silicon light-guiding layer and the chip can contain structures that have a slightly different size than designed for. Furthermore the lithography can only be optimized for one feature size, meaning that only this feature size is according to specifications. A final consideration is the fact that the sides of the patterns are not perfectly straight, but have an angle of about 10 degrees [10].

1.4 Outline of this thesis

With this brief introduction to the different fields involved we have obtained a general idea why this new sensor can be of importance to the medical field. In the remaining part of this thesis, we discuss the work done to obtain this sensor as well as the investigation of the different aspects of the sensor. Chapter 2 presents a short overview of the theory that is important for the design of our sensor. It first describes the basic concepts that we used to model the sensor. The second part of this chapter describes the modeling results. With the models we obtained possible dimensions and corresponding resonance frequencies for our prototype device. We also calculated the first values of the pressure levels that could be received.

Chapter 3 studies the shift in the optical resonances due to an applied static mechanical strain to the ring resonator. The influence of different physical aspects on the ring resonator is investigated, such as: the elongation of the track, the change in cross-section of the waveguide due to the Poisson ratio, the change in the refractive index of the silicon and silicon dioxide due to the photo-elastic effect and the dispersion in the waveguide.

Chapter 4 presents the proof-of-concept of the sensor. We present the first measurement results of the response of our sensor to an acoustical pressure wave. We determined the transfer function and hence its resonance frequency. We found a noise equivalent pressure that indicated that we fabricated a very sensitive ultrasound sensor.
Chapter 5 describes the characterization of the sensor. We investigated what the behavior of the sensor is under static and dynamic loading as well as the initial shape. Due to the fabrication technique of the silicon wafer, we found that initial strain is present, which influences the behavior of the membrane. Instead of linear theory, large deflection theory is needed to understand the static pressure measurements. Furthermore it is shown that only the membrane itself is significantly responding to the ultrasound waves.

Chapter 6 presents a theory to determine the noise level of small sensors. The noise floor of our sensor is different from the state-of-the-art transducers due to the lack of piezoelectric material. Hence our sensor is only prone to the noise generated by thermal motion of the atoms in the membrane and the water, while the piezoelectric transducers also have to deal with the high electrical noise from the electrical impedance.

Chapter 7 concludes and summarizes the thesis.
Abstract – To be able to design a prototype sensor the basic concepts of the sensor should be known. This chapter describes the basis concepts of photonic waveguides, couplers and ring resonators as well as the linear theory for membrane deflection. With use of these concepts the membrane thickness and diameters are obtained. It is shown that a membrane thickness of 1.2 μm of silicon left underneath the 2.5 μm silicon dioxide results in the most induced strain in the layer of the photonic waveguide for a resonance frequency of 1 MHz. Membrane diameters from 60 to 100 μm result in resonance frequencies of 3.2 to 0.9 MHz in water. Calculations showed a minimum detectable pressure level in the order of 600 Pa, which is sufficient for ultrasound sensing.

This chapter is based on the following publication:
The general concept of the sensor is that the transmitted light traveling through the optical waveguide is modulated by the mechanical deformation of the optical ring resonator due to an incident ultrasound wave. To design such a sensor we need to understand the concepts of guided light and its mechanical response to pressure. An overview of the main concepts is given in the first part of this chapter, starting with the theory of photonic waveguides followed by the linear theory of membrane deflection. The second part shows the first modeling results for the dimensions and the corresponding resonance frequency of the sensor. These are used to obtain an estimate for the membrane diameter and thickness as starting values for the design of the prototype.

2.1 Photonic waveguides

This section describes the theory for high-index-contrast silicon waveguides. It only describes the theory that is of interest in our design. The reader is referred to [10] for a broader description of the theory and newly derived theory for these kind of waveguides. We start with the description of waves that can propagate in a waveguide (Section 2.1.1). Then the coupling of light from the straight waveguide towards the ring resonator via a directional coupler is described (Section 2.1.2) and we finalize with the description of a ring resonator (Section 2.1.3).

2.1.1 General description of a dielectric photonic waveguide

Photonic waveguides confine light and transport it over a given distance. The confinement is done by index guiding where the core of the waveguide has a higher refractive index than the surrounding. The photonic waveguides that are discussed are rectangular waveguides made of silicon. They have a SiO$_2$ buried oxide (BOX) substrate and an air or SiO$_2$ cladding. The waveguides have a typical height of 220 nm and width of 400 nm. The assumed free-space wavelength $\lambda$ of the light is 1.55 $\mu$m. A schematic picture of such a silicon-on-insulator (SOI) waveguide is shown in Figure 2.1a, with its fundamental transverse electric (TE) mode depicted in Figure 2.1b. The fundamental mode has most of its energy in the

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.1.png}
\caption{Cross-section of a SOI waveguide. a) Sketch of the waveguide b) Sketch of the $E_x$ component of the fundamental mode in color. Dark blue represents a large field, white represents zero field [10].}
\end{figure}
2.1. Photonic waveguides

The modes can be described with Maxwell’s equations. In our description we approximate silicon and silicon dioxide as linear dielectrics with permittivity $\epsilon$, which means that the material polarization is proportional to the electric field. We neglect magnetic behavior of the dielectrics by using the permeability of vacuum $\mu_0$. All optical descriptions given are valid for monochromatic light with angular frequency $\omega$ and vacuum wavelength $\lambda = \omega/2\pi c$ where $c$ is the speed of light in vacuum. The physical electromagnetic fields are described by the real components of the complex vector fields $\mathbf{E}$ and $\mathbf{H}$ of the electric and magnetic fields respectively. Maxwell’s complex equations for monochromatic light in an isotropic linear dielectric medium without charges are given by [13]

$$
\nabla \times \mathbf{E} = -i\omega\mu_0\mathbf{H}, \quad (2.1)
$$

$$
\nabla \times \mathbf{H} = i\omega\epsilon\mathbf{E}, \quad (2.2)
$$

The latter two equations (2.3) and (2.4) are not independent and follow directly from the first two equations (2.1) and (2.2). The refractive index $n$ depends on the permittivity as $n = \sqrt{\epsilon/\epsilon_0}$, with $\epsilon_0$ the permittivity of vacuum. The permittivity profile $\epsilon(x, y, z)$ describes the devices as is indicated in Figure 2.1a and hence describes how the electromagnetic field behaves.

Electromagnetic fields in a homogeneous isotropic medium without charges obey the wave equations

$$
(\nabla^2 + n^2k^2)\mathbf{E} = 0, \quad (2.5)
$$

$$
(\nabla^2 + n^2k^2)\mathbf{H} = 0, \quad (2.6)
$$

with $k = \omega/c$ the free-space propagation constant [13, Ch. 9]. A dielectric waveguide is fully described by its permittivity profile $\epsilon(x, y)$ which is invariant in the $z$-direction, i.e. the direction in which the light propagates. We may assume that the propagating wave solutions have the form

$$
\mathbf{E}(x, y, z) = \mathbf{E}(x, y)e^{-i\beta z}, \quad \mathbf{H}(x, y, z) = \mathbf{H}(x, y)e^{-i\beta z}, \quad (2.7)
$$

with $\beta$ the propagation constant.

To find a solution in the form of Eq. 2.7 for the light propagation through rectangular dielectric waveguides, we can use the extended description of Marcatilli’s approximate analytical approach [14] or numerical mode solvers. The field on the chip has to deal with losses. These losses are introduced by imperfections of the silicon and by waveguide bends. In case of the waveguide bends the mode in the bend changes with respect to the mode of the straight waveguide. This happens because the wave fronts at the outside of the bend have to propagate faster than the wave fronts at the inside. As a result the power of the mode moves towards the outside of the bend [15]. This effect is stronger for sharper bends. When we include losses the electrical field of the TE mode is described by

$$
\mathbf{E}(x, y, z) = \mathbf{E}(x, y)e^{-i\beta z - \alpha_p z}, \quad (2.8)
$$
with $\alpha_p$ the propagation loss.

The effective index is often expressed in terms of the propagation constant as

$$n_e \equiv \frac{\beta}{k}. \quad (2.9)$$

The effective group index $n_g$ is defined as

$$n_g \equiv \frac{\partial \beta}{\partial k}. \quad (2.10)$$

We can also find an explicit expression for the group index in terms of the first-order dispersion in the effective index $n_e$ as

$$n_g = n_e - \lambda \frac{\partial n_e}{\partial \lambda}. \quad (2.11)$$

In our case, we only use a small wavelength span around a center wavelength $\lambda_c$. Therefore we approximate the wavelength-dependence of the effective index $n_e(\lambda)$ as linear so that

$$\beta(\lambda) \approx 2\pi \left[ \frac{n_e(\lambda_c) - n_g(\lambda_c)}{\lambda_c} + \frac{n_g(\lambda_c)}{\lambda} \right]. \quad (2.12)$$

### 2.1.2 Directional couplers

It is possible to couple light from one waveguide into another by means of a directional coupler. Such a directional coupler consists of two parallel single-mode waveguides positioned close together as is shown in Figure 2.2a-b. We use a directional coupler to couple a fraction of the light from the straight waveguide into the ring resonator. The electric field in the coupler $E^c$ can be approximated as a superposition of the two modes of the isolated waveguides $a$ and $b$. The amplitudes of the two modes vary along the length of the coupler as is shown in Figure 2.2c. The electromagnetic field is approximated as

$$E^c(x, y, z) \approx E^a(x, y)u_a(z) + E^b(x, y)u_b(z), \quad (2.13)$$

with $u_a$ and $u_b$ the complex modal amplitudes of waveguides $a$ and $b$ respectively and $E^a$ and $E^b$ the modal electric fields of the waveguides. When light propagates through waveguide $b$ at the start of the coupler, i.e. $z = 0$, all energy is in this waveguide and therefore the amplitude of this mode is maximal and the mode in waveguide $a$ is zero ($u_a(0) = 0$). At the effective coupling length of the coupling, i.e. $z = \tilde{L}$, the amplitudes of both modes in the coupler is given by

$$u_b(\tilde{L}) = \tau u_b(0), \quad u_a(\tilde{L}) = \kappa u_b(0). \quad (2.14)$$

The effective coupling length $\tilde{L} = L + \Delta L$ consist of the length of the coupler $L$ and an contribution $\Delta L$ from a part of the bends of the waveguide. The complex amplitudes $\tau$ and $\kappa$ are calculated using coupled mode theory. The derivation is
given in [10], but the final formula for the coupling amplitudes for two waveguides is given by

$$\tau = \left( \cos s\tilde{L} - \frac{i\delta}{s} \sin s\tilde{L} \right) e^{i(\beta_b + \kappa_{bb} - \delta)\tilde{L}}, \quad (2.15)$$

$$\kappa = -\left( \frac{i\kappa_{ab}}{s} \sin s\tilde{l} \right) e^{i(\beta_b + \kappa_{bb} - \delta)\tilde{L}}, \quad (2.16)$$

where $\beta_b$ is the propagation constant of mode $b$, $\kappa_{bb}$ is the correction to this propagation constant originating from the other waveguide, $\delta \equiv 1/2(\beta_b + \kappa_{bb} - \beta_a - \kappa_{aa})$ is the difference between the corrected propagating constants of the guides and $s = \sqrt{\kappa_{ba}\kappa_{ab} + \delta^2}$ is the coupling coefficient. Coupling coefficient $\kappa_{ab}$ represents the coupling from the mode of waveguide $b$ to the mode of waveguide $a$ and $\kappa_{ba}$ represents the coupling from the mode of waveguide $a$ to the mode of waveguide $b$. These coefficients dominate $s$. The guides in the coupler that we study are designed to be identical, but we experimentally observed non-zero $\delta$ in our couplers.

![Directional coupler and its behavior](image)

**Figure 2.2:** Directional coupler and its behavior. a) Top view of a directional coupler with bus waveguide $b$ and second waveguide $a$. b) Optical microscope photo of a directional coupler in SOI. The very narrow pinkish lines are the waveguides. c) The normalized power in each mode at a certain coupling length $z$. The coupling coefficient $s$ is 0.1 for identical waveguides ($\delta = 0$) [10].

### 2.1.3 Ring resonators

Ring resonators consist of a looped optical waveguide and a coupling mechanism to couple light into the loop. The shape of the ring is arbitrary. We use racetrack resonators which are elongated rings with a straight part between the bends (Fig. 2.3). As coupling mechanism we use two directional couplers (Sec 2.1.2). A losless coupler without reflections is generally described by [16]

$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} \tau^* & \kappa \\ -\kappa^* & \tau \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, \quad (2.17)$$

where $a_1$, $a_2$, $b_1$ and $b_2$ are the waveguide terminals and $\kappa$ and $\tau$ are the coupling coefficients from the bus waveguide to the ring resonator and vice versa as indicated in Figure 2.3. The relation between the coupling coefficients is given by $|\tau|^2 + |\kappa|^2 = 1$ and the * is the complex conjugate.
When the wave travels through the racetrack resonator it experiences after one round-trip a phase delay $\phi_\rho$ and a decay by a factor $\alpha$, so that
\[
a_2 = \alpha e^{i\phi_\rho} b_2.
\]  
(2.18)
We can obtain the power in the output waveguide $|b_1|^2$ by substituting Eq. (2.18) in Eq. (5.1) to find
\[
b_2 = \frac{-\kappa^*}{1 - \tau \alpha e^{i\phi_\rho}} a_1,
\]  
(2.19)
and
\[
b_1 = \left( \tau^* - \frac{\kappa \kappa^* \alpha e^{i\phi_\rho}}{1 - \tau \alpha e^{i\phi_\rho}} \right) a_1,
\]  
(2.20)
with $\tau = |\tau| e^{i\phi_\tau}$ and $\phi_\tau$ the phase delay due to the coupler. We can rewrite Eq. (2.20) into
\[
b_1 = -\alpha + |\tau| e^{i(\phi_\rho + \phi_\tau)} \frac{e^{-i\phi_\rho} - \alpha |\tau| e^{i\phi_\tau}}{e^{-i\phi_\rho}} a_1.
\]  
(2.21)
We compute $|b_1|^2 = b_1^* b_1$ and use $2 \cos \theta = e^{i\theta} + e^{-i\theta}$ to get
\[
|b_1|^2 = \frac{\alpha^2 + |\tau|^2 - 2\alpha |\tau| \cos \theta}{1 + \alpha^2 |\tau|^2 - 2\alpha |\tau| \cos \theta} |a_1|^2,
\]  
(2.22)
where $\theta = \phi_\rho + \phi_\tau$ is the net phase delay of traveling through the ring and coupler. In case of two bus waveguides with identical couplers we include the transmission through the second coupler in the track round-trip by replacing $\alpha$ with $\alpha |\tau|$. Eq. (2.23) changes then into
\[
|b_1|^2 = \frac{(\alpha^2 + 1 - 2\alpha \cos \theta) |\tau|^2}{1 + \alpha^2 |\tau|^2 - 2\alpha |\tau| \cos \theta} |a_1|^2,
\]  
(2.23)
with in this case $\theta = \phi_\rho + 2\phi_\tau$. When the racetrack including the couplers has length $l$, the transmission through the directional couplers with effective length $\tilde{L}$ is given by Eq. (2.15). The phase delay due to propagation through a waveguide with length $l - 2\tilde{L}$ is equal to $\phi_\rho = \beta (l - 2\tilde{L})$. The total phase delay of the ring is therefore given by
\[
\theta = -\beta l + 2\delta \tilde{L} - 2\kappa_{bb} \tilde{L} + 2 \arg \left\{ \cos s \tilde{L} - \frac{i \delta}{s} \sin s l \right\}.
\]  
(2.24)
For a coupler with two identical waveguides, neglecting the $\kappa_{bb}$ and with linear dispersion of the effective index, the phase delay reduces to
\[
\theta = -\beta l = -2\pi \left[ \frac{n_e - n_g}{\lambda_c} + \frac{n_g}{\lambda} \right] l,
\]  
(2.25)
with $n_e \equiv n_e(\lambda_c)$ and $n_g \equiv n_g(\lambda_c)$.
When we measure the transmission spectrum of a waveguide coupled with a single directional coupler to a ring resonator, given by $|b_1|^2$ as a function of wavelength,
we observe resonance dips for $\theta = 2\pi m$ with $m$ an integer number. The resonance wavelengths $\lambda_m$ for two identical waveguides in the coupler are given by

$$m\lambda_m = n_e(\lambda_m)l.$$  \hspace{1cm} (2.26)

A typical transmission spectrum for our sensor is shown in Figure 2.4. The difference between two successive resonance dips is indicated by the free spectral range (FSR). We may approximate the FSR by linearizing the relation between $m$ and $\lambda(m)$ in Eq.(2.26) and then computing $|\Delta \lambda|$ for $\Delta m = 1$ to obtain,

$$\Delta \lambda_{FSR} = \left| \frac{\partial \lambda}{\partial m} \right| \Delta m = \frac{\lambda^2}{(n_e - \lambda \frac{\partial n_e}{\partial \lambda})l} = \frac{\lambda^2}{n_gl}.$$  \hspace{1cm} (2.27)

At resonance we have $\cos \theta = 0$ and thus Eq.(2.23) becomes

$$|b_1|^2 = \frac{(\alpha - |\tau|)^2}{(1 - \alpha |\tau|)^2} |a_1|^2.$$  \hspace{1cm} (2.28)

This relation shows that there is no transmission at the resonance wavelengths when $|\tau| = \alpha$, hence when the round-trip loss of the racetrack is equal to the power coupled to the racetrack. This condition is called critical coupling. The minimum transmitted power $|b_{1,\text{min}}|^2$ occurs at resonance while the maximum transmitted power $|b_{1,\text{max}}|^2$ occurs in between the resonances. The extinction ratio, defined as $r \equiv |b_{1,\text{min}}|^2/|b_{1,\text{max}}|^2$, and the full-width at half-max (FWHM) of the transmission spectrum show the shape of the resonances as a function of the waveguide and coupler properties and are given by

$$r = \frac{(\alpha - |\tau|)^2(1 + \alpha |\tau|)^2}{(\alpha + |\tau|)^2(1 - \alpha |\tau|)^2},$$  \hspace{1cm} (2.29)

and

$$\Delta \lambda_{FSR} = \frac{\lambda^2}{\pi n_g \cos^{-1} \left[ \frac{2\alpha |\tau|}{1 + \alpha^2 |\tau|^2} \right]}. $$  \hspace{1cm} (2.30)

The FWHM depends on the losses in the resonator and scales with the FSR, while for the extinction ratio the critical coupling is most important. When $\alpha$ is replaced
by \( \alpha |\tau| \), Eq. (2.26)-(2.30) are also valid for two couplers, as the second coupler acts as an additional source of loss.

\[ D \nabla^4 w(r, \theta, t) + \rho h \frac{\partial^2 w(r, \theta, t)}{\partial t^2} = q(r, \theta, t), \quad (2.31) \]

where \( \nabla^4 = \nabla^2 \nabla^2 \) with \( \nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} \) the Laplacian in polar coordinates and \( \rho \) is the density of mass. The flexural rigidity \( D \) of the plate is given by

\[ D = \frac{E}{1 - \nu^2} \int_{-h/2}^{h/2} z^2 dz = \frac{E h^3}{12(1 - \nu^2)}, \quad (2.32) \]
2.2. Membrane deflection

Figure 2.5: Differential element of a thin plate with neutral plane $n$ at half the thickness $h$ of the plate and lamina at $z$ below the middle plane.

with $E$ the Young’s modulus and $\nu$ the Poisson ratio. When different materials are used, we integrate over the separate layers with their specific material properties to obtain the flexural rigidity of the entire plate [17]. The displacement $w(r, \theta, t)$ measures the deflection of the middle plane of the plate. When we look at a lamina with thickness $dz$ located at a distance $z$ below the middle plane $n$ of the plate indicated in Figure 2.5, the normal strains defined as the relative elongation of the lamina ($\epsilon_l \equiv \Delta l/l$) are given by

$$\epsilon_x = \frac{z}{r_x},$$  \hspace{1cm} (2.33)
$$\epsilon_y = \frac{z}{r_y},$$  \hspace{1cm} (2.34)

where $r_x$ and $r_y$ are the radii of curvature in the $x$, $z$-plane and $y$, $z$-plane respectively. We assume small deflections and slopes and therefore the curvatures may be approximated with use of the second order derivatives of the displacement (e.g. $r_x \approx -1/\partial^2 w/\partial x^2$) such that

$$\epsilon_x = -z \frac{\partial^2 w}{\partial x^2},$$  \hspace{1cm} (2.35)
$$\epsilon_y = -z \frac{\partial^2 w}{\partial y^2}.$$  \hspace{1cm} (2.36)

The shear strain is given by

$$\gamma_{xy} = -2z \frac{\partial^2 w}{\partial x \partial y}.$$  \hspace{1cm} (2.37)

From Hooke’s law the strains relate to the stresses as

$$\epsilon_x = \frac{1}{E}(\sigma_x - \nu \sigma_y),$$  \hspace{1cm} (2.38)
$$\epsilon_y = \frac{1}{E}(\sigma_y - \nu \sigma_x).$$  \hspace{1cm} (2.39)
with \( \nu \) the Poisson ratio. Combining Eqs. (2.35)-(2.39) results in a radial and transverse stress along the thickness of the membrane in polar coordinates these stresses are

\[ \sigma_r = \frac{Ez}{1 - \nu^2} \left( \frac{\partial^2 w}{\partial r^2} + \nu \frac{\partial^2 w}{\partial \theta^2} \right), \]  
(2.40)

\[ \sigma_\theta = \frac{Ez}{1 - \nu^2} \left( \frac{\partial^2 w}{\partial \theta^2} + \nu \frac{\partial^2 w}{\partial r^2} \right), \]  
(2.41)

\[ \tau_{r\theta} = G\gamma_{r\theta} = -\frac{2Gz}{r} \frac{\partial^2 w}{\partial r \partial \theta}. \]  
(2.42)

In a static situation, Eq. (2.31) can be simplified to:

\[ D\nabla^4 \omega(r, \theta) = q(r, \theta). \]  
(2.43)

With a symmetrical load the deflection of a circular membrane is independent of \( \theta \) hence \( w(r, \theta) = w(r) \). Assuming that the membrane has clamped boundaries, hence \( w(a) = 0 \), and \( \frac{\partial w(a)}{\partial r} = 0 \), and the deflection has a maximum at the center of the membrane, i.e. \( \frac{\partial w(0)}{\partial r} = 0 \), this equation can be solved resulting in [17]

\[ w(r) = \frac{q}{64D} \left( a^2 - r^2 \right)^2. \]  
(2.44)

We can determine the maximum or minimum stress at the lower or upper face of the plate. These maximum or minimum stress is the same in the lower and upper face but has an opposite sign. In the radial direction of a face, the absolute maximum is found at the boundary of the plate where \( \sigma_r = -\frac{3qa^2}{4h^2} \) and a slightly lower value is found at the center of the plate with opposite sign where \( \sigma_r = \frac{3(1 + \nu)qa^2}{8h^2} \) [17]. We do not know the exact boundary conditions of our sensor as the membrane is part of the chip itself. When we therefore model the displacement of the chip under uniform loading we can get the displacement and radial curves for a 100 \( \mu \)m diameter membrane (Fig. 2.6). A typical curve of the radial strain in a layer has positive values at the edge of the membrane where it is extended, while towards the center negative values are present due to a compression of the surface. Between the center and edge is a circular area without radial strain which is around 1/3 of the radius from the edge of the membrane.

### 2.2.2 Resonance frequency of membranes

To obtain the fundamental mode of the membrane we solve Eq. (2.31) without loading. We assume a solution \( w(r, \theta, t) = W(r, \theta)e^{-i\omega t} \) with \( \omega \) the angular frequency and \( t \) the time. If we go through the mathematics, we find the solution for full circular plates as

\[ W(r, \theta) = \{AJ_n(\beta r) + CI_n(\beta r)\} \begin{bmatrix} \sin n\theta \\ \cos n\theta \end{bmatrix}, \]  
(2.45)

with \( J_n \) the Bessel function of the first kind, \( I_n \) the modified Bessel function of the first kind and \( \beta^4 = \omega^2 \rho h / D \) [18,19]. When we apply clamped boundary conditions
2.2 Membrane deflection

![Diagram of membrane deflection and radial strain curve](image)

**Figure 2.6:** Typical displacement curve of a circular membrane with a radius of 50 µm (blue line) and corresponding radial strain curve in a lamina of the plate (red line).

we get

\[ I_n(\beta a) J'_n(\beta a) - J_n(\beta a) I'_n(\beta a) = 0. \] (2.46)

Each value of \( n \) in this equation will have an infinite number of roots, which give the resonance frequencies. We define \( \lambda_{nm} = \beta_{nm}a \) where \( n \) is the integer arising in Eq. (2.46) and \( m \) is corresponding to the order of the root for a given \( n \). The fundamental mode has for instance a value of \( \lambda_{01}^2 = 10.216 \). The normal modes are given by

\[
W_{nm}(r, \theta) = \left\{ J_n(\beta_{nm}r) - \frac{J_n(\beta_{nm}a)}{I_n(\beta_{nm}a)} I_n(\beta_{nm}r) \right\} \left[ \begin{array}{c} \sin n\theta \\ \cos n\theta \end{array} \right].
\] (2.47)

Because \( \lambda_{nm}^2 = \omega a^2 \sqrt{\rho h/D} \), we are able to calculate the resonance frequency \( f = \omega/2\pi \) in air which is given by [19]

\[
f = \frac{1}{2\pi} \frac{\lambda_{nm}^2}{a^2} \sqrt{\frac{D}{\rho h}}.
\] (2.48)

One side of the sensor will be submerged in water instead of air, which will give an extra load on the membrane. The resonance frequency will shift downwards by a factor that could be approximated by

\[
f_w = \frac{f_a}{\sqrt{1 + \gamma \Gamma}},
\] (2.49)

where \( f_w \) is the resonance frequency for water, \( f_a \) the resonance frequency for air, \( \Gamma \) a non-dimensional added virtual mass factor, which is a function of mode shapes and boundary conditions and \( \gamma = \frac{\rho_w a}{\rho_p h} \) with \( \rho_w \) the density of mass of the water and \( \rho_p \) the density of mass of the plate [20].
2.3 Membrane modeling

The theory in the previous section demonstrated that the resonance frequency of the membrane can only be exactly determined for air and for known boundary conditions. To deal with water loading and unknown boundary conditions we have to solve the problem numerically. We like to obtain values for the ideal thickness of the membrane, given a certain resonance frequency and find the corresponding resonance frequency in water of certain membrane diameters. We will first describe the modeling parameters followed by the results of the static and dynamic simulations.

2.3.1 Specifications and modeling parameters

In the numerical simulation we use a 2D axis-symmetric domain and modeled the device as a 250 $\mu$m silicon layer below the 2.5 $\mu$m SiO$_2$ layer (including the top layer). The 220 nm thick silicon waveguides are neglected. We assume for simplicity that the strain induced at 0.5 $\mu$m from the top, i.e. the waveguide layer, will represent the strain induced in the waveguide. The acoustical membrane is created by removing a part of the substrate. In the static analyses, described in Section 2.3.2, the thickness of silicon left under the SiO$_2$ layer is varied each time. In the dynamic analyses, described in Section 2.3.2, we model three different membranes with diameters of 60 $\mu$m, 80 $\mu$m and 100 $\mu$m. In these dynamic models, the membranes consist only of SiO$_2$, and the area underneath the membrane is filled with air. The remaining part of the modeling domain is filled with water. The discretization of the receiver design contained at least 12 points per wavelength. The water domain is large enough to avoid interference from reflecting waves from the boundaries of the domain. In both studies we used a density of mass of 2329 kg/m$^3$ for the silicon and an isotropic Young’s modulus of 170 GPa and Poisson ratio of 0.28. The silica has a density of 2200 kg/m$^3$, Young’s modulus of 70 GPa and a Poisson ratio of 0.17.

2.3.2 Modeling results

To analyze the strain profiles and determine the response of the receiver, we perform two different analyses which are described in this section. The first part is a static analysis by which we compute the dimensions of the membrane. The second part is a dynamic study in which the response and sensitivity of the receiver are obtained.

Static analysis

The acoustical membrane can be developed for all kinds of applications. We designed a membrane that has an acoustical resonance at 1 MHz when it is submerged in water. To find the optimal dimensions, i.e. inducing the highest amount of strain in the optical resonator, we investigate whether an optimum is present when the thickness of the membrane is varied. The thickness of the membrane consists of at least the 2.5 $\mu$m SiO$_2$ layer, so only the thickness of the silicon was varied from...
Table 2.1: Response of three prototype receivers due to an incident plane wave

<table>
<thead>
<tr>
<th>Membrane diameter [$\mu$m]</th>
<th>Resonance frequency [MHz]</th>
<th>Sensitivity [microstrain/kPa]</th>
<th>Minimum detection level [Pa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>3.2</td>
<td>1.3</td>
<td>1540</td>
</tr>
<tr>
<td>80</td>
<td>1.6</td>
<td>3.4</td>
<td>590</td>
</tr>
<tr>
<td>100</td>
<td>0.9</td>
<td>3.2</td>
<td>625</td>
</tr>
</tbody>
</table>

0 $\mu$m to 10 $\mu$m. For each thickness of the membrane the corresponding radius is calculated by Eqs. (2.48) and (2.49) to obtain a resonance frequency of 1 MHz. For every thickness and corresponding radius, we determine with a finite element model the maximum absolute strain that can be induced in the membrane. We used a static pressure of 1 kPa. The results show that the maximum strain varies from 0.17 to 0.45 microstrain. The maximum value is attained when 1.2 $\mu$m of silicon is left underneath the 2.5 $\mu$m SiO$_2$ layer. Although an optimum exists, the difference in strain in the range of 0 $\mu$m to 4 $\mu$m of silicon is only a factor of 1.5. In the fabrication process of the membrane, the flatness and thickness of the silicon layer is difficult to control. A well-controlled fabrication can be performed when all the silicon up to the SiO$_2$ layer is removed, using a chemically selective etch process. Therefore we have finally choosen to remove all the silicon in the design of the prototypes and in the remaining study.

Dynamic analysis

We designed three prototypes with different membrane diameters to determine the response and influence of the membrane on the performance of the sensor. In a time domain analysis, we used an incident acoustic plane wave with Gaussian pulse shape with a maximum amplitude of 50 kPa and pulse width of 0.6 $\mu$s. The time and frequency response of the 80 $\mu$m diameter membrane are shown in Figure 2.7. The results of all the receivers are listed in Table 2.1. For every receiver, we numerically obtained the resonance frequency of the membrane. The maximum strain values, present in the time simulations, were used to calculate the sensitivity by dividing the strain by the maximum of the incoming pressure field. To determine the minimum pressure detection level, we assumed to have an optical detector with a resolution of 1 pm and we used 0.5 pm/microstrain for the shift in the light spectrum due to strain from [21].

The results show that our device can be well used for ultrasound sensing. The three times less sensitivity of the smallest membrane suggest that there is a minimum footprint of the element. Furthermore, we conclude that the membrane with a 80 $\mu$m diameter is the most sensitive receiver.
2.4 Conclusion

With use of the basic concepts of waveguides and membrane deflection we have been able to design a prototype sensor. When we look at the static strain profiles we conclude that the optical resonator needs to be positioned at maximum distance from the neutral plane, i.e. at the top or at the bottom, and either in the center or at the edge of the membrane. When the sensor is realized, the waveguide needs to be protected from the water and will therefore be positioned below a 0.5 µm cladding. Therefore the maximum distance from the neutral plane is a little bit less in the final design. Because of the current racetrack shape of the ring resonators we will position it in the center of the membrane where the largest area with similar strain values is present. Because this area with similar strain is limited, the maximum length of the ring resonator is limited to 2/3 of the membrane diameter.

The thickness of the membrane should in optimal situation contain a layer of 1.2 µm silicon underneath the silicon dioxide for a membrane with resonance around 1 MHz. Although this optimum exist we prefer to build a prototype that could easily be compared to models and hence contains a flat and evenly thick membrane. Therefore we will remove all the silicon with a well-controlled chemically selective etch process.

The dynamic simulations show that the obtained sensitivity of the sensor is sufficient for ultrasound sensing. Furthermore it is possible to fabricate the membrane diameters that correspond to resonance frequencies in the MHz range.
Chapter 3

Optical characterization of strain sensors based on silicon waveguides

Abstract – Microscale strain gauges are widely used in micro electro-mechanical systems (MEMS) to measure strains such as those induced by force, acceleration, pressure or sound. We propose all-optical strain sensors based on micro-ring resonators to be integrated with MEMS. We characterized the strain-induced shift of the resonances of such devices. Depending on the width of the waveguide and the orientation of the silicon crystal, the linear wavelength shift per applied unit of strain varies between 0.5 and 0.75 pm/microstrain for infrared light around 1550 nm wavelength. The influence of changing ring circumference is about three times larger than the influence of the change in waveguide effective index, and the two effects oppose each other. The strong dispersion in 220 nm high silicon sub-wavelength waveguides accounts for a decrease in sensitivity of a factor 2.2 to 1.4 for waveguide widths of 310 nm to 860 nm. These figures and insights are necessary for the design of strain sensors based on silicon waveguides.

This chapter is based on the following publication:

3.1 Introduction

Microscale strain gauges are widely used in micro electro-mechanical systems (MEMS) to measure strains such as those induced by force, acceleration, pressure or (ultra)sound [22, 23]. These sensors are traditionally based on a piezoresistive or piezoelectric material which transduces the strain to an electrical signal. Alternatively, optical resonators can be used as sensing element, providing particular benefits: high-speed readout, small sensor size, small multiplexer size (1 mm²), insensitivity to electromagnetic interference, and no danger of igniting gas explosions with electric sparks.

Integrated optics technology allows the optical strain sensors, as well as their multiplexing circuit, to be integrated with MEMS. The sensing elements and their multiplexers can often be fabricated in a single processing step. Silicon-on-insulator (SOI) has emerged as one of the focus platforms for integrated optics, and is relatively straightforward to integrate with MEMS, as MEMS are most commonly made of silicon. Micro-electronic research institutes have tailored CMOS fabrication processes to the demands of SOI optical circuits, and now offer cheap and reproducible wafer-scale fabrication [24, 25]. The high contrast of the refractive index of SOI ridge waveguides allows for a small device footprint, and single-mode guides have a cross-section of only 400 × 220 nm².

We employ ring resonators as sensing element. Such a resonator consists of a waveguide which is looped, forming a closed cavity which has specific optical resonance wavelengths. Any change in the size or in the refractive index of this waveguide shifts its resonances, and this shift can be accurately recorded.

Several groups have reported on sensor micro opto-electro-mechanical systems (MOEMS) that are based on silicon integrated optical ring resonators, such as strain gauges [26, 27], or pressure sensors [28–30]. An application of particular interest is as ultrasound sensor for medical intravascular ultrasonography (IVUS). IVUS has been recommended for the diagnostics of atherosclerosis [31, 32]. IVUS is an invasive technique for blood vessel imaging where the sensor is attached to a catheter and brought inside the artery. Using an array of sensors improves the image quality but wiring many piezoelectric sensors with coaxial cables requires too much space for this application. As solution, we proposed a micro-machined ultrasound transducer (MUT) with optical readout [33]. This sensor consists of a silicon ring resonator integrated in a membrane that deforms due to ultrasonic waves. Integrated optical multiplexers allow high-speed read-out of many sensors via one optical fiber and, moreover, insensitivity to electromagnetic interference allows usage inside MRI scanners.

The relation between strain and silicon waveguides is of broader interest than sensing. Electro-mechanical modulation of silicon optical resonators may be employed to modulate optical signals, for application in the field of telecommunication [34]. As alternative to silicon waveguide-based ring resonators, it is also possible to use photonic crystals cavities, which have their own dispersion relations [35]. Strain has also been used to modify the birefringence of larger SOI rib waveguides [36]. Strain is inevitable when using silicon photonic circuits on a flexible substrate [37]. Another interesting field of research is the strain-induced change in the electronic
band-gap and the optical refractive index of silicon, with the possibility to intro-
duce second-order nonlinearity [38–40]. However, details of the relation between
an applied strain and the shift in optical resonance of ring resonators have not
been studied.
We characterized the shift of the resonance wavelengths which is caused by a
well-defined strain. We quantified the contribution of three physical effects: the
strain-induced change in circumference of the resonator, the strain-induced change
in effective index of the waveguide, and the dispersion which is strong in sub-
wave-length silicon waveguides. This knowledge is required for the design of me-
chanical sensors based on silicon integrated optics, such as ring resonators or Mach-
Zehnder interferometers.
This chapter is organized as follows: first we present the devices which are used to
study the effect of strain on silicon optical waveguides (Sec. 3.2), then we derive
opto-mechanical theory describing these devices (Sec. 3.3), after which we detail
the experimental setup and methodology (Sec. 3.4). The characterization and the
analysis of the devices are presented in Sec. 3.5 and Sec. 3.6, respectively, and we
draw some conclusions in Sec. 3.7.

3.2 Devices

In this section, we first describe the SOI-technology in which the devices are fab-
ricated, secondly detail the ring resonators and then describe the sets of devices
that we studied.
The integrated optical devices are in silicon-on-insulator technology, with 220 nm
thick waveguides of mono-crystalline silicon. The guides are on top of a 2 \( \mu \text{m} \) thick
buried oxide (BOX) layer which is on top of a 675 \( \mu \text{m} \) thick silicon substrate. We
deposited a 2 \( \mu \text{m} \) thick SiO\(_2\) cladding layer using plasma-enhanced chemical vapor
deposition (PEVCD), so that the silicon waveguides are embedded in silica. The
devices were fabricated via the EU-funded ePIXfab consortium at IMEC (Leuven,
Belgium) [24,25]. IMEC fabricated the devices in their CMOS line with 193 nm
deep-UV lithography. The waveguides are not exactly rectangular but have a
side-wall-angle of 10 degrees. We measured the average widths of the fabricated
waveguides with a helium ion microscope (Carl Zeiss SMT), providing an accuracy
of 15 nm.
We designed long racetrack-shaped ring resonators in an “add-drop” configuration
(Fig. 3.1), and excite the “input” waveguide with infrared light with wavelengths
\( \lambda \) around a center wavelength \( \lambda_c \) of 1550 nm. A directional coupler couples light
from the “input/output” waveguide to the resonator, and an identical coupler is
used half-way the racetrack to couple light to a “drop” waveguide. The transmit-
ted spectrum \( T(\lambda) \) in the “output” port has dips at the resonance wavelengths
of the resonator. We characterized the couplers and 59% of the power is coupled
from the waveguide to the track, such that the power which goes straight-through
the coupler \( |t|^2 = 41\% \). Having a strong coupling in a symmetric add-drop config-
uration gives resonance dips with good extinction ratio even for high losses in the
racetrack or for variations in coupling (e.g. due to fabrication) [16].
Chapter 3. Optical characterization of strain sensors based on waveguides

Figure 3.1: Sketch of racetrack resonator with in/output ports (not to scale). Long straight waveguide has length $l_s = 1000 \, \mu m$ and width $w$. Taper section has length $l_t = 64.35 \, \mu m$ (4.35 $\mu m$ long waveguide of width 400 nm, taper with a length varying from 0 to 60 $\mu m$, followed by a waveguide of width $w$ to close the space.) Coupler section has length $l_c$ and consists of two 10 $\mu m$ long parallel guides (width $\sim$400 nm, gap 220 nm), and bends with a radius of 5 $\mu m$.

Silicon is anisotropic, so its deformation depends on the direction in which a force is applied. Therefore two sets of devices were fabricated and characterized; one with the long side of the racetrack parallel to the $\langle 110 \rangle$ direction of the silicon crystal and one with the long side parallel to the $\langle 100 \rangle$ direction of the silicon crystal. Reference [41] explains the crystal planes in a “(100) wafer” as we used. We characterized the influence of the width of the waveguide on the shift in resonance, therefore each set of devices consists of resonators with waveguide widths varying from 310 nm up to 860 nm.

3.3 Theory

This section presents the theory of ring resonators such as presented in the previous section, i.e. a looped waveguide with a varying width. First, Sec. 3.3.1 presents the optical theory of ring resonators, then Sec. 3.3.2 derives the opto-mechanical theory. Sec. 3.3.3 applies the theory to the long racetrack resonators under study. The relations derived in this section are used as fitting function of the measured spectra, and as basis for the analysis of the measurements.

3.3.1 Ring and racetrack resonators

The transmitted optical power $T$ of a micro-ring resonator with two lossless couplers in an add-drop configuration such as shown in Figure 3.1 is [16]

$$T = \frac{\alpha^2|t|^2 + |t|^2 - 2\alpha|t|^2 \cos(\delta)}{1 + \alpha^2|t|^4 - 2\alpha|t|^2 \cos(\delta)}, \quad (3.1)$$

where $|t|^2$ is the straight-through power of the coupler and $\alpha^2$ is the power transmission due to one round-trip through the ring ($\alpha = 1$ means zero loss). $T$ thus
describes the optical power transmitted from the input to the output of the connecting waveguide, and is wavelength dependent because $\delta$ is wavelength-dependent. The phase delay $\delta$ of one round-trip through the ring (including passing the couplers) is

$$\delta = \int n_e(\rho, \lambda) \frac{2\pi}{\lambda} d\rho = \langle n_e(\lambda) \rangle \frac{2\pi}{\lambda} l,$$  

(3.2)

where the waveguide effective index $n_e(\rho, \lambda)$ is averaged over the position $\rho$ in the track with circumference $l$ as $\langle n_e(\lambda) \rangle \equiv \frac{1}{l} \int n_e(\rho, \lambda) d\rho$. The effective index in the coupler is approximated equal to the effective index of a single isolated waveguide. The strong modal dispersion in sub-wavelength silicon waveguides is approximated to be linear around the center wavelength $\lambda_c$, and is expressed in terms of the effective group index $n_g \equiv n_e - \lambda \frac{\partial n_e}{\partial \lambda}$, so

$$n_e(\rho, \lambda) = n_e(\rho) + (n_e(\rho) - n_g(\rho))(\frac{\lambda}{\lambda_c} - 1),$$  

(3.3)

where $n_e$ and $n_g$ at the right-hand-side, denoted without $\lambda$ dependence, are evaluated at $\lambda_c$. As $\lambda$ and $\rho$ are independent,

$$\langle n_e(\lambda) \rangle = \langle n_e \rangle + (\langle n_e \rangle - \langle n_g \rangle)(\frac{\lambda}{\lambda_c} - 1),$$  

(3.4)

from which it is observed that the transmission spectrum of a racetrack resonator with varying width is described by the same relation as a resonator with a single waveguide, but with averaged effective index $\langle n_e \rangle$ and group index $\langle n_g \rangle$.

Equation (3.1) with Eqs. (5.11) and (3.4) will be fitted to the measured resonance spectra to accurately obtain $\langle n_g \rangle$ and $\langle n_e \rangle$, from which the resonance wavelengths are calculated.

### 3.3.2 Strain-induced resonance shift of ring resonators

This section details the shift in the resonances of a ring resonator due to an applied mechanical strain. Four physical effects play a role when elongating a ring- or racetrack resonator. First, the circumference of the track $l$ increases. Second, the cross section of the waveguide shrinks due to the Poisson effect. Third, the refractive indices of the silicon and SiO$_2$ change due to the photo-elastic effect. The latter two effects together influence the effective index $n_e$ of the waveguide. Fourth, the shift in resonance is affected by the dispersion in the waveguide. In our case, a homogeneous strain $S_z$ is applied parallel to the long sides of the racetrack resonator (the z-direction). The transmitted spectrum of the connecting waveguide shows dips at the resonance wavelengths $\lambda_m$ when $\delta = m2\pi$, or

$$m\lambda_m = \int n_e(\rho, \lambda_m, S_z)(1 + S_\rho(\rho, S_z)) d\rho.$$

(3.5)

The effective index of the waveguide depends on a mechanical deformation. The local strain in the direction of the track $S_\rho$ is taken into account by stretching each element $d\rho$ to $(1 + S_\rho)d\rho$. For the straight waveguide of the racetracks as in
Figure 3.1, the $z$- and $\rho$-directions coincide, whereas they do not for the coupler section. We found that the relation between an applied strain $S_z$ and the shift in resonance wavelength is linear, which is explained by the fact that the applied strains are small. A description of this linear influence can be found by taking the first derivative of Eq. (5.12) with respect to $S_z$,

$$m \frac{\partial \lambda_m}{\partial S_z} = \int \left\{ \left( \frac{\partial n_e}{\partial S_z} + \frac{\partial n_e}{\partial \lambda_m} \frac{\partial \lambda_m}{\partial S_z} \right)(1 + S_\rho) + n_e \frac{\partial S_\rho}{\partial S_z} \right\} d\rho,$$

which we evaluate at zero strain (i.e. $S_z = S_\rho = 0$). Solving this equation for $\partial \lambda_m / \partial S_z$, substituting $m$ from Eq. (5.12), and dividing by track circumference $l$ gives

$$\frac{\partial \lambda_m}{\partial S_z} = \frac{\lambda_c}{\langle n_g \rangle} \int \left( \frac{\partial n_e}{\partial S_z} + n_e \frac{\partial S_\rho}{\partial S_z} \right) d\rho,$$

(3.6)

with $\lambda_c$ the resonance wavelength $\lambda_m$ without deformation. This equation is easiest understood when considering a resonator with a single waveguide shape (i.e. $\langle n_e \rangle = n_e$ and $\langle \lambda_g \rangle = \lambda_g$). In that case,

$$\frac{\partial \lambda_m}{\partial S_z} = \frac{n_e}{n_g} \langle \frac{\lambda_c \partial n_e}{\partial S_z} \rangle + \frac{\lambda_c}{\langle n_g \rangle} \int \frac{\partial S_\rho}{\partial S_z} d\rho,$$

(3.7)

where the influence of the different physical effects are indicated. Without dispersion, $n_e / n_g = 1$. For the part of the track which is in the direction of the applied strain $S_\rho = S_z$, so $\partial S_\rho / \partial S_z = 1$, hence the contribution of the track-length change is simply $\lambda_c$.

### 3.3.3 Strain-induced resonance shift of long racetracks

We measured very long racetracks because this will allow for neglecting the influence of the tapers and the couplers. In the long racetrack resonators, Eq. (5.12) reads

$$m \lambda_m = 2l_sn_s(1 + S_z) + \int_{\text{tapers}} n_e (1 + S_\rho) d\rho + \int_{\text{couplers}} n_e (1 + S_\rho) d\rho,$$

(3.8)

where the contributions of the different sections of the track are separated (see Fig. 3.1, with $l_s$, $l_t$, and $l_c$ indicating the straight, taper and coupler sections, respectively) and $n_s$ is the effective index of the long straight waveguide. We calculate the first-order influence of strain on this racetrack similarly to Eq. (3.6), and rewrite the equation such that the influence of the tapers and the couplers is
written as a correction to the shift caused by the long straight guides,

\[
\langle n_g \rangle \frac{\partial \lambda_m}{\partial S_z} = \lambda_c \left( \frac{\partial n_s}{\partial S_z} + n_s \right) + \frac{\lambda_c}{l} \int_{\text{tapers}} \left( \frac{\partial n_e}{\partial S_z} + n_e \frac{\partial S_p}{\partial S_z} - \frac{\partial n_s}{\partial S_z} - n_s \right) \, d\rho \\
+ \frac{\lambda_c}{l} \int_{\text{couplers}} \left( \frac{\partial n_e}{\partial S_z} + n_e \frac{\partial S_p}{\partial S_z} - \frac{\partial n_s}{\partial S_z} - n_s \right) \, d\rho.
\]

(3.9)

We will justify later that the second and third term of the right-hand-side of this equation are small compared to the first one, and hence can be neglected, resulting in

\[
\langle n_g \rangle \frac{\partial \lambda_m}{\partial S_z} \approx \lambda_c \left( \frac{\partial n_s}{\partial S_z} + n_s \right).
\]

(3.10)

The taper is a waveguide in the z-direction with a width varying from 400 nm up to the width \( w \) of the long section waveguide. The second term at the right-hand-side of Eq. (3.9) is the relative contribution of the taper to the resonance shift, with respect to the contribution of a waveguide with width \( w \) of the same length. The relative contribution of the taper is smaller than the relative contribution of a 400 nm wide waveguide of the same length. Using Eq. (3.10), it is thus found that the second term of the right-hand-side of Eq. (3.9) is smaller than

\[
\left| \frac{4l_t}{l} \right| \frac{\langle n_g \rangle \frac{\partial \lambda_m}{\partial S_z}}{\text{width under study}} - \frac{\langle n_g \rangle \frac{\partial \lambda_m}{\partial S_z}}{\text{width 400 nm}}.
\]

(3.11)

The third term of Eq. (3.9) comes from the effect of the couplers including the bend waveguides. This contribution can be either positive (as for the long waveguides) or negative (as the path-length might shrink due to the Poisson effect). We expect the magnitude to be smaller than twice the effect of a straight waveguide of equal length that is strained in its long direction. Thus the 3rd term in Eq. (3.9) is smaller in magnitude than

\[
2 \frac{2l_c}{l} \left( \langle n_g \rangle \frac{\partial \lambda_m}{\partial S_z} \right).
\]

(3.12)

As will be shown in Sec 3.5, the maximum measured difference in \( \langle n_g \rangle (\partial \lambda_m/\partial S_z) \) for the devices under study with different waveguide widths is 10%. For these long racetracks, \( 4l_t/l = 11\% \), so the second term in Eq. (3.9) is smaller than 1.1%. The third term is smaller than \( 4l_c/l = 5\% \). Equation (3.10) is used in the characterization of the measurements. We characterized both \( \langle n_g \rangle \) and \( \partial \lambda_m/\partial S_z \). The effective index of the straight waveguide \( n_s \) is computed with a numerical mode solver, which allows us to extract the strain-induced change in effective index.
Chapter 3. Optical characterization of strain sensors based on waveguides

Figure 3.2: Example of resonance shift due to an applied strain. (a) Small span of 7 measured spectra for increasing values of applied strain. Resonance dips shift to the red. Racetrack in the <100> crystalline direction, with waveguide width 400 nm. (b) The wavelengths of the resonance dips $\lambda_m$ in (a) are plotted versus the applied strain $S_z$. The wavelengths of the resonance dips for decreasing values of strain are also plotted. Resonance shift $\frac{\partial \lambda_m}{\partial S_z}$ is obtained from a linear fit.

Similar to Eq. (5.13) in the more general Sec. 3.3.2, we indicate the effects of the different phenomena in Eq. (3.10)

$$\frac{\partial \lambda_m}{\partial S_z} = \frac{n_s}{\langle n_g \rangle} \left( \frac{\lambda_c}{n_s} \frac{\partial n_s}{\partial S_z} + \lambda_c \right).$$  

This result is used in the interpretation of the measurements in Sec. 3.6. In fact, track-averaged group index $\langle n_g \rangle$ can be approximated as the group index $n_g$ of the straight waveguide. We have used the numerical mode solver to show that this approximation is valid within 1%. In our analysis, we use the track-averaged group index which was accurately measured.

3.4 Methodology

We characterized the photonic chips in an automated setup in which they are bent such that the top layer with the racetrack resonators is strained. Transmission spectra of the resonators were recorded for elongations varying from 0 to 275 microstrain. As example, Figure 3.2a shows a resonance dip of the measured spectra for increasing strain. The resonance wavelengths, and the group index $n_g$, were extracted from fitting a relation for ring resonator transmission. Figure 3.2b shows the resonance wavelength $\lambda_m$ plotted versus the applied strain.
3.4. Methodology

3.4.1 Mechanical setup: four point bending

We designed and fabricated a mechanical setup in which the chips are bent such that the top layer with the photonic circuit is uniformly strained (Fig. 3.3). The setup is equipped with elastic elements to provide an accurate bending moment to the chip, without hysteresis or other non-linearities. Between the two inner supports, the chip experiences a constant bending moment $M$ (known as pure bending) [42]

$$M = \frac{a_1(a_4 - a_3)}{4a_2}L,$$

with $L$ the load applied at the lever. The bending of the chip is described by plate bending theory for thin plates with small deflections [43], as its thickness $H$ is small compared to its width $W$ and length. An assumption in this theory is that the normal stresses in the x-direction can be neglected, so that there is no strain $S_x$ in the x-direction and the width $W$ of the chip does not change due to the applied load. In the setup, the deflection of the chip at considerable distance from its ends can be assumed to be cylindrical. In this mechanical analysis the influence of upper layers of the chip (BOX layer, waveguide layer, and cladding layer) is neglected as their total thickness of 4.220 $\mu$m is much smaller than the
chip thickness of 675 µm. Hooke’s law and plate bending theory give the relation between the stress $\sigma_z$ and strain $S_z$ in the chip [41,43]:

$$\sigma_z = \frac{E_z}{1 - \nu_{xz}\nu_{zx}} S_z,$$

(3.15)

with Young’s modulus$^1$ $E_z$ and anisotropic Poisson’s ratios$^2$ $\nu_{xz}$ and $\nu_{zx}$. Combining Eqs. (3.14) and (3.15) with the cylindrical deflection of the chip gives the strain $S_z$ on the top surface of the chip in the mechanical setup:

$$S_z = \frac{3a_1(a_4 - a_3)(1 - \nu_{xz}\nu_{zx})}{2a_2W^2H^2E_z} L. $$

(3.16)

A precise linear stage (Newport MFA-CC) applies a force to the lever, while a load cell (Omega LECB5) measures the actual applied load $L$. It was observed that the relation between the displacement of the linear stage $Y$ and the applied load $L$ is linear in the regime of our measurements, and also that the repeatability of the linear stage position $Y$ was higher than the repeatability of the load cell. Therefore, we extracted a single number for the resistance of the chip to bending, $\partial Y/\partial L$, from all the measurements performed on a chip.

### 3.4.2 Optical setup

The transmission spectra of the racetracks were measured with near infrared light around $\lambda_c = 1550$ nm. An amplified spontaneous emission light-source (OptoLink C-band ASE) was used to emit this light, and a 5 nm span of the spectra were recorded with an optical spectrum analyzer (Yokogawa AQ6370B). The input and output waveguides of the racetrack resonators are routed to out-of-plane grating couplers at convenient locations on the chip, and coupled to cleaved optical fibers via free-space [44,45]. These fibers were mounted on stages with piezo positioning, and automatically actively aligned in the horizontal $(x,z)$-plane before recording a spectrum. All transmission spectra are normalized to the transmission spectrum of a reference waveguide, which was smoothened by convolution with a 1 nm wide Gaussian window to remove Fabry-Pérot resonances originating from reflections of the out-of-plane grating couplers.

A relation for ring resonator transmittance, Eqs. (3.1)-(3.4), was fitted to the recorded spectrum. The ring length $l$ and straight-through power of the coupler $|t|^2 = 41\%$ were fixed, while the effective index $\langle n_e \rangle$, group index $\langle n_g \rangle$, resonator waveguide loss $\alpha^2$ and fiber-coupling loss were fitted. The resolution bandwidth of the optical spectrum analyzer (OSA) was incorporated in this fitting by convoluting the calculated spectrum with a 20 pm wide Gaussian curve. For the zero-strain measurement, the mode number $m$ of the resonance closest to $\lambda_c$ was estimated from Eq. (5.12) where the effective index $n_e(\rho, \lambda_c, 0)$ was calculated using a mode solver (film mode matching method in FimmWave by PhotonDesign [46]). This dip was followed over consecutive measurements. An accurate initial guess of $\langle n_e \rangle$ and

---

$^1$ $E_i$ is the Young’s modulus along axis $i$.

$^2$ $\nu_{ij}$ is the Poisson’s ratio that corresponds to a contraction in direction $j$ when an extension is applied in direction $i$.
⟨ng⟩ (thus the wavelengths of the resonance dips) is necessary for the Levenberg-Marquardt fitting algorithm [47]. Therefore, the resonance dips were first found using findpeaks [48] and from this ⟨ne⟩ and ⟨ng⟩ were estimated via Eq. (5.12). This initial estimate allows for automated fitting of the spectra. With this fitting, the free parameters in Eqs. (3.1)-(3.4) could be obtained, and λ_m was calculated from Eq. (5.12) with an accuracy much higher than the resolution bandwidth of the OSA.

### 3.4.3 Measurements

We characterized chips with the racetracks in the <110> crystalline direction and with the racetracks in the <100> direction. The measurements were repeated several times. First, the chip was manually placed in the setup. Then resonators with different widths of the straight waveguide were automatically measured. The strain of the racetrack was increased and decreased from 0 to approximately 275 micro-strain, with 6 steps in each direction (see Fig. 3.2a). The transmittance spectrum was recorded for each applied strain, and the resonance position λ_m that started closest to λ_c was extracted. The effective group index ⟨ng⟩ was also extracted from this spectrum. For each value of applied strain, the measured load L and the position of the linear stage Y were recorded. Per measurement set of increasing and decreasing strain, the resonance shift per displacement of the load cell, ∂λ_m/∂Y, was obtained from a linear fit, and so was the relation between the displacement and the applied load, ∂Y/∂L. We observed that both relations were indeed linear in this regime. The strain-induced resonance shift is then

\[
\frac{\partial \lambda_m}{\partial S_z} = \frac{\partial \lambda_m}{\partial Y} \cdot \frac{\partial Y}{\partial L} \cdot \frac{\partial L}{\partial S_z},
\]

(3.17)

in which the first two terms on the right-hand-side are measured and the last term is calculated from Eq. (3.16).

The relation between the displacement of the load cell and the measured load can be interpreted as the resistance of the chip and setup to bending. The average value for the chip with the racetracks in the <110> direction is ∂Y/∂L = 0.128 µm/mN and the average value for the chip with the racetracks in the <100> direction ∂Y/∂L = 0.135 µm/mN.

### 3.4.4 Numerical mode solver

For the analysis of the measurements, we calculated the effective index at zero strain, ne(ρ, λ_c, 0) using the film mode matching method in FinmmWave [46]. Also the effective group index ng(ρ) at λ_c was calculated using this mode solver. The track-averaged effective index ⟨ne⟩ and group index ⟨ng⟩ are then straightforward to calculate.

### 3.4.5 Measurement uncertainty analysis

The uncertainty in the measurements was estimated following the guidelines of Ref. [49]. The relative errors of the three terms on the right-hand-side of Eq. (3.17)
Table 3.1: Material properties, dimensions, and estimated uncertainties of mechanical setup

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<th>Quantity</th>
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<tbody>
<tr>
<td>$E_z/(1-\nu_{zx}\nu_{xx})$, &lt;110&gt;</td>
<td>170 GPa$^a$</td>
<td>4 GPa</td>
</tr>
<tr>
<td>$E_z/(1-\nu_{zx}\nu_{xx})$, &lt;100&gt;</td>
<td>141 GPa$^a$</td>
<td>4 GPa</td>
</tr>
<tr>
<td>$a_1$</td>
<td>156 mm</td>
<td>0.3 mm</td>
</tr>
<tr>
<td>$a_2$</td>
<td>24 mm</td>
<td>0.03 mm</td>
</tr>
<tr>
<td>$a_3$</td>
<td>5 mm</td>
<td>0.03 mm</td>
</tr>
<tr>
<td>$a_4$</td>
<td>20 mm</td>
<td>0.03 mm</td>
</tr>
<tr>
<td>Chip $W$</td>
<td>24 mm</td>
<td>0.3 mm</td>
</tr>
<tr>
<td>Chip $H$</td>
<td>0.675 mm</td>
<td>0.01 mm</td>
</tr>
</tbody>
</table>

$\partial L/\partial S_z$, <110> 12.7 mN/µstrain 0.6 mN/µstrain 4.7%
$\partial L/\partial S_z$, <100> 10.5 mN/µstrain 0.5 mN/µstrain 4.7%

$^a$ From Ref. [41]
$^b$ Estimated.
$^c$ Measured with digital electronic calipers, maximum error 0.5 mm.
$^d$ Estimated fabrication uncertainty, maximum error 0.05 mm.
$^e$ Measured with digital electronic calipers, maximum error 0.02 mm.

are added quadratically, as they are independent. The chips with the racetrack resonators in the <110> and <100> directions were placed in the mechanical setup and measured 6 and 5 times, respectively.

\[ \partial\lambda_m/\partial Y \]

The value for $\partial\lambda_m/\partial Y$ is averaged over the repetitive measurements, and the uncertainty is estimated as the standard deviation. The relative uncertainty did not significantly depend on the width of the waveguide, and the maximum relative uncertainty (of all widths) is used. The uncertainty for the chips with the waveguides in the <110> and <100> directions are 3.1% and 1.1%, respectively. The measurement-to-measurement difference mainly originated from repositioning the chip in the setup, which was done before each measurement. Repeating a measurement without repositioning the chip in the setup gives a measurement-to-measurement difference which is negligible. We could not attribute this difference to a slight tilt of the chip with respect to the setup (around the y-direction). We do not fully understand why the uncertainty in the <110> direction is higher, but the strong angle dependency of Poisson’s ratio around the <110> direction may play a role. Also, we had to reassemble the setup between various <110> measurements, while the measurements of the <100> chip were performed consecutively in a mainly empty laboratory.
\( \partial Y/\partial L \)

The value of \( \partial Y/\partial L \) did not significantly depend on the position of the chip in the setup. All measurements (for different widths of the waveguide, and repetitions of the measurements) are averaged to obtain \( \partial Y/\partial L \). The statistical uncertainty (arising from random fluctuations) is estimated as the standard deviation, and the systematic uncertainty of the system (load cell, load cell voltage source, and A/D converter) is estimated as 3%. The standard deviation of the 30 measurements in the \(<110>\) direction is 2.8%, and the standard deviation of the 45 measurements in the \(<100>\) direction is 0.7%. This difference can be explained by the fact that we increased the integration time of the read-out of the load cell from 50 samples at 1 kHz for the \(<100>\) direction measurements to 1000 samples at 1 kHz for the \(<110>\) direction measurements. The output voltage of the load cell is a few mV, which required this longer integration time of our A/D converter (National Instruments USB-6251 DAQ). The uncertainties of \( \partial Y/\partial L \) are thus 5.8% and 3.7% for the chips with the racetracks in the \(<110>\) and \(<100>\) directions, respectively.

\( \partial L/\partial S_z \)

The mechanics of the setup is described by Eq. (3.16). The material properties, dimensions, and uncertainties that are used in this equation are listed in Table 3.1. The uncertainty \( \sigma \) of a quantity whose uncertainty is estimated as a maximum deviation \( u \) is given by \( \sigma = u/\sqrt{3} \) [49]. In the computation of the uncertainty of \( \partial L/\partial S_z \), we have treated all uncertainties as independent and approximated the influence of all the uncertainties as linear.

**Group index \( \langle n_g \rangle \)**

We found that the track-averaged effective group index does not depend on the applied strain. Therefore all measurements of a device are averaged, and the uncertainty is estimated as the standard deviation. These were 78 and 65 measurements for the racetracks in the \(<110>\) direction and \(<100>\) directions, respectively. The relative uncertainty did not depend much on the width of the straight waveguide in the racetrack nor on the crystalline orientation, so that we have used the maximum of 0.03%.

**Effective index \( \langle n_e \rangle \)**

In the analysis of the measurements, we calculated the effective index with a numerical mode solver. We do not know the uncertainty, as it is mostly related to the difference between the simulated waveguide and the fabricated waveguide. Therefore, we estimated the uncertainty in the effective index as the difference between the measured effective group index \( \langle n_g \rangle \) and the track-averaged group index as calculated with the same mode solver (see Fig. 3.4b).
Figure 3.4: (a) Measured resonance shift per applied strain $\partial \lambda_m / \partial S_z$, with resonance wavelength $\lambda_m$ and strain $S_z$. (b) Measured and calculated track-averaged effective group indices $\langle n_g \rangle$. Measured for racetracks in the $<100>$ and in the $<110>$ silicon crystalline directions. (c) Change in effective index of a straight waveguide, $n_s$, due to a strain, $S_z$, applied in the direction of the guide.
3.5 Characterization

We characterized two chips with long racetrack resonators; one chip with the resonators in the $<110>$ silicon crystalline direction and one with the resonators in the $<100>$ direction. A strain $S_z$ was applied to the top surface of the chips, where the resonators are placed. The strain was increased to approximately 275 microstrain, and then decreased to zero strain in steps of approximately 45 microstrain. Per applied strain, the transmission spectrum of the resonator was recorded, as is shown in Figure 3.2a. The wavelength of the resonance dip which started closest to 1550 nm is extracted from each of the spectra, and plotted versus the applied strain in Figure 3.2b. The shift per applied strain $\partial \lambda_m / \partial S_z$ is obtained from a linear fit. This is done for racetracks with different widths, and the resonance shifts per strain are presented in Figure 3.4a. It can be seen that the racetracks in the $<110>$ direction are slightly more sensitive than the tracks in the $<100>$ direction, and that the resonators with wider waveguides are more sensitive to strain than the ones with narrower waveguides widths. The latter can be attributed to the dispersion in the waveguide, as shown in Sec. 3.6. The estimated uncertainties are with respect to the absolute value of $\lambda_m / S_z$, and a large part of the uncertainty is a systematic bias and equal for all measurements. Considering only the statistical (or random) uncertainties, we found that the racetracks in the $<110>$ and $<100>$ directions have a significantly different shift in resonance.

The track-averaged effective group indices $\langle n_g \rangle$ were also extracted from the spectra and are presented in Figure 3.4b. We also calculated the effective group index with the numerical mode solver. The calculated and measured track-averaged effective group indices agree within 3%.

The change in effective index due to strain, $\partial n_e / \partial S_z$, is calculated using Eq. (3.10). We measured the resonance shift $\partial \lambda_m / \partial S_z$ and the effective group index $\langle n_g \rangle$, and we calculated the effective index $n_s$ of the straight waveguide with the numerical mode solver. The resulting $\partial n_s / \partial S_z$ is shown in Figure 3.4c.

3.6 Analysis

In this section, we interpret the measured shift and indicate the contributions of different physical effects: the elongation of the track, the change in effective index and the dispersion of the waveguide. Equation (3.13) shows how these effects shift the resonance wavelength. The effect of the elongation of the track ($\lambda_c$) and the effect of the change in effective index are added. The change in effective index $\partial n_s / \partial S_z$ is negative, so the two effects oppose each other. The dispersion of the waveguide, $n_s / n_g$, is smaller than unity, and thus damps the shift. Figure 3.5a presents $n_s / \langle n_g \rangle$, in which it can be seen that this damping is stronger for small waveguides. Figure 3.5b presents the resonance wavelength shift with dispersion excluded. The shift due to the change in the effective index increases (in magnitude) with increasing width of the guide. The higher resonance shift for wider waveguides $\partial \lambda_m / \partial S_z$ is thus due to the dispersion, and not due to the change in effective index of the waveguide.
3.7 Conclusion

We measured the strain-induced shift of the resonances of optical racetrack resonators in silicon-on-insulator technology. For waveguides with a width of 400 nm, the resonance wavelength shift per applied strain is 0.57 pm/microstrain when the racetrack is parallel to the \(<100>-\)direction of the silicon crystal, and 0.66 pm/microstrain when the racetrack is parallel to the \(<110>-\)direction. We observed largest sensitivity for wider waveguides; a racetrack with 860 nm wide waveguides oriented in the \(<110>-\)direction has a resonance shift of 0.75 pm/microstrain. We have studied elongations up to 275 microstrain, and observed a linear relation between the resonance wavelength and the applied strain.

The effect of the strain-induced increase in track circumference and the effect of the strain-induced change in waveguide effective index oppose each other. The effect of the strain-induced increase in circumference is about three times larger than the effect of the change in effective index. The strong dispersion in the sub-wavelength silicon waveguides lowers the change in wavelength shift approximately by a factor two. In fact, the lower dispersion of the wider waveguides is the reason that these devices are more sensitive.
This chapter addressed how micro-ring resonators transduce an applied strain into a shift of their optical resonance wavelengths. Sensor MEMS or MOEMS consist, next to the transducer, also of a mechanical structure that acts on the transducer and an interrogation system that reads the transducer. A variety of interrogation concepts exist for monitoring the shift in the resonances of optical resonators \cite{50-56}. The detection limit of MOEMS sensors depends on the designs of the mechanical structure and the interrogation system. These designs are application specific as they depend, for example, on the bandwidth of the measured signal (e.g., Ref. \cite{33} presents the design of an ultrasound sensor).

In this work, we have characterized a novel type of optical strain sensors which can be integrated in micro-electro-mechanical systems (MEMS). We believe these sensors open opportunities in different fields of applications such as in the medical, petrochemical, or oil&gas markets, by offering specific advantages such as high-speed readout over kilometer distances, integrated optical multiplexing, and small device size. Moreover, by removing the need for galvanic connections, susceptibility to electromagnetic disturbance is eliminated.
Abstract – With the increasing use of ultrasonography, especially in medical imaging, novel fabrication techniques together with novel sensor designs are needed to meet the requirements for future applications like three-dimensional intercardiac and intravascular imaging. These applications require arrays of many small elements to selectively record the sound waves coming from a certain direction. Here we present proof of concept of an optical micro-machined ultrasound sensor (OMUS) fabricated with a semi-industrial CMOS fabrication line. The sensor is based on integrated photonics, which allows for elements with small spatial footprint. We demonstrate that the first prototype is already capable of detecting pressures of 0.4 Pa, which matches the performance of the state of the art piezoelectric transducers while having a 65 times smaller spatial footprint. The sensor is compatible with MRI due to the lack of electronical wiring. Another important benefit of the use of integrated photonics is the easy interrogation of an array of elements. Hence, in future designs only two optical fibers are needed to interrogate an entire array, which minimizes the amount of connections of smart catheters. The demonstrated OMUS has potential applications in medical ultrasound imaging, non destructive testing as well as in flow sensing.

This chapter is based on the following publication:
4.1 Introduction

The wide use of ultrasonography, especially in medical imaging, has resulted in the development of different types of ultrasound transducers such as piezo-electric transducers, micro-machined ultrasound transducers (MUTs) and optical ultrasound sensors. Conventional transducers are made of piezo-electric material and nowadays contain multiple piezo elements to form a one or two dimensional array. These arrays enable beam focusing and steering, and hence are able to scan the area or volume of interest and create real-time 2D or 3D images [1, 2]. The frequencies of the transducers are ranging from 1-20 MHz and typical bandwidths are 70-80% [57]. Recently, Xia et al. [58] presented a piezo-electric array of 5 mm × 5 mm containing 25 elements of 0.9 mm × 0.9 mm. This ultrasound array transducer is used in photoacoustic breast tomography and has a center frequency of 0.9 MHz and a bandwidth of 80%. The noise equivalent pressure is 0.5 Pa per single element. We shall use this element as reference to our sensor.

Because small diagnostic transducers are paramount for extending the possibilities of intravascular and transesophageal ultrasound, one of the goals in the development of piezo-electric transducers is miniaturization [1]. There are two main challenges in the fabrication of small arrays. The first challenge is to obtain the small elements. The conventional fabrication technique uses a diamond saw to divide a single piezo-electric slab into multiple elements [1]. The minimal distance between two neighboring elements is determined by the thickness of the dicing blade (minimal ∼10-15 µm), and advanced fabrication techniques like laser cutting are required when this distance should be decreased.

The second issue is related to the individual wiring of the elements, especially when a large number (order of thousands) of elements is needed [59–61]. Bonding techniques, in which wires are attached to each individual element, are too labor-intensive for arrays containing large amounts of elements. Hence, the use of flexible circuits or other complex techniques are required for the electrical interconnections. Moreover, dense electrical wiring generally brings the disadvantage of intrinsic susceptibility to electromagnetic interference and cross-talk [59, 61].

The mentioned production difficulties make that increasing miniaturization of conventional piezo-electric transducer arrays is intricate. To circumvent these difficulties, capacitive and piezo-electric micro-machined ultrasound sensors (CMUTs and PMUTs) are developed [62–64]. A MUT consists of a flexible membrane that is deformable by ultrasonic pressure waves. The micro-machining technology used for the fabrication of MUTs benefits from the developments in the integrated circuit (IC) technology, which allows for a relatively easy fabrication of complex transducer patterns, as well as the integration of accompanying electronic circuitry [65, 66]. However, these MUTs still have a high electrical impedance resulting in a low signal-to-noise ratio and, consequently, a low imaging depth. To obtain the imaging depth that is currently attained with piezo-electric arrays, the signal-to-noise ratio of MUTs should be improved [57, 63].

As far as the receive function is concerned, optical micro-sensors are another alternative for conventional piezo-electric transducers [67, 68]. These optical devices share the fabrication benefits of MUTs. An additional benefit of these
4.2 Concept, design and fabrication

The OMUS consists of a waveguide and a photonic ring resonator that are integrated onto a membrane (Fig. 4.1a). When a broad spectrum of light is trans-
mitted through the waveguide, a part of the spectrum is coupled into the ring resonator by means of a directional coupler [77]. The transmitted spectrum at the output port of the waveguide shows dips at the optical resonance wavelengths $\lambda_m$ of the ring resonator, given by

$$m\lambda_m = n_e l,$$

where $m$ is an integer number, $l$ the circumference of the ring and $n_e$ its effective index of refraction. Incident acoustical pressure waves strain the membrane and hence the resonator. The induced strain in the resonator causes a shift in the optical resonance curves with respect to the undeformed state. The magnitude of this shift due to induced strain depends on three aspects: the amount of elongation of the optical resonator track, the change in effective index of refraction, and the change in dispersion of the material of the track [21, 78]. The shift in resonance is what we want to observe. To do so without using a dedicated detector, we transmit light, and hence measure the transmitted intensity, at one optical wavelength. This wavelength is chosen on one flank of the optical resonance curve of the undeformed resonator, such that a shift of the resonance curve directly translates into a modulation of the transmitted optical intensity, as is shown in Figure 4.1b. The intensity is measured using a photo diode.

![Figure 4.1: a) Sketch of the OMUS, showing the photonic micro-ring resonator on top of the membrane. b) Sketch of three intensity curves at the end of the waveguide, representing the different transmittances for different strain values of the photonic micro-ring resonator.](image)

The design of the membrane is chosen such that it has a resonance frequency around 1 MHz. To determine the dimensions we use plate bending theory, because the deflection of the membrane is expected to be small compared to its height [79]. With this theory the resonance frequency of the freely moving membrane can be calculated analytically for specific boundary conditions. However, for medical diagnostic purposes the sensor will be submerged in water-like substances, and the predicted resonance frequency in air will shift downwards [20]. Therefore, we use a finite element method (COMSOL Multiphysics) to numerically determine the diameter and thickness of the membrane [80] that will give the desired resonance frequency. The membrane of the OMUS was designed as a 2.7 $\mu$m thick silicon
4.3 Results

We have optically characterized the OMUS by measuring the transmittance of light. Moreover we performed acoustical measurements to measure the time responses to transmitted acoustical pulses with different frequencies. Finally we determined the bandwidth, sensitivity and noise equivalent pressure of the sensor. The optical characterization starts by determining the shape of the optical resonance dips. The shape of the ring resonator and its position on the membrane determine the shift in optical wavelength due to the deformation of the membrane. The steeper the flank of the optical resonances, the more sensitive the sensor. An optimization problem now arises, in that a long racetrack has a narrower resonance dip and therefore a higher sensitivity. However, the strain patterns of a statically deformed membrane showed that a short ring resonator located in the middle of the membrane results in the most strain per pressure. Based on the mechanical FEM analysis and the model in Ref [77], we chose a racetrack-shaped optical ring-resonator with a bending radius of 5 µm and 40 µm straight track. We use two directional couplers on opposite sides of the ring resonator with a 6 % coupling efficiency of optical power. For details on the design, we refer the reader to our previous work [10,81]. We use an out-of-plane grating coupler at each end of the waveguide to direct the light upwards out of the waveguides and into optical fibers. We realized the sensor using the following procedure. First, the fine optical circuitry was fabricated using a semi-industrial CMOS fabrication line at IMEC (Leuven, Belgium) via the ePIXfab platform [82]. The resulting wafer-piece (die) contains 220 nm high and 400 nm wide silicon waveguides on top of a 2 µm thick silicon-dioxide layer on a 250 µm thick silicon substrate. A 0.5 µm thick silicon-dioxide cladding was deposited to isolate the waveguide from the water. Second, we etched the membrane (124 µm diameter) from the back of the die using deep reactive ion etching with sulfur hexafluoride (SF₆) as etchant and we used the silicon-dioxide layer as a well-defined etch-stop. A photograph of the membrane with photonic circuitry is shown in Fig 4.2. Third, we glued the chips on two 1 mm thick glass plates. The first plate contained a hole of 4 mm diameter, which was positioned behind the membrane. The second glass plate sealed the chip from the backside, ensuring that only air is present behind the membrane. Finally, we connected the optical fibers to the silicon photonic circuit, referred to as packaging. We developed a packaging method that is suitable for under-water usage. This involves attaching to the chip a Pyrex block in which the connecting optical fiber ended. The block had a polished angle facet with an evaporated aluminum coating to reflect light from the fiber into the grating coupler at the end of the waveguide [83].
Chapter 4. Proof of concept of an OMUS

Figure 4.2: Microscopic image of an OMUS, showing a membrane with an optical resonator and two directional couplers. This OMUS has a circular membrane with a diameter of 78 µm and a thickness of 2.7 µm. The membrane is visible because it is illuminated from the back. The OMUS reported in this chapter has a larger membrane diameter of 124 µm.

Figure 4.2: Microscopic image of an OMUS, showing a membrane with an optical resonator and two directional couplers. This OMUS has a circular membrane with a diameter of 78 µm and a thickness of 2.7 µm. The membrane is visible because it is illuminated from the back. The OMUS reported in this chapter has a larger membrane diameter of 124 µm.

nance curve of the ring resonator in a static situation, hence without deformation of the membrane. We measured the transmission $T(\lambda)$ from the input to the output of the OMUS by stepping through successive optical wavelengths. The intensity $I$ at the output of the chip for a static situation is given by $I = T I_0$ with $I_0$ the maximum output intensity far away from the resonance dip. This is directly related to the maximum output power $P_0$, which currently is 10 µW (Fig. 4.3a). The transmittance of the OMUS shows a dip at resonance of the ring resonator which has a FWHM of 100 pm. The transmittance of a membrane at rest is denoted by $T_0$ and is the initial transmittance of every acoustical measurement. When the membrane and hence the optical resonator is deformed, the transmittance of the ring resonator is shifted. We write the deformed transmittance $T(\lambda)$ in first order as $T(\lambda) \approx T_0(\lambda + \Delta \lambda)$, where $\Delta \lambda$ is the wavelength shift due to the deformation of the ring (see Fig. 5.1b).

For the acoustical measurements we observe the time modulation of the output intensity at one particular optical wavelength $\lambda_l$ set by the laser (indicated as measurement wavelength in Fig. 5.1b). When an incident acoustical wave deforms the membrane and hence the resonator over time, the time dependent intensity at the output of the chip can be described as

$$I(\lambda_l, t) = T(\lambda_l) I_0 \approx T_0 (\lambda_l + \Delta \lambda(t)) I_0,$$

(4.2)

with $\Delta \lambda(t)$ the ultrasound-induced optical wavelength shift. When we interrogate by measuring the intensity at one particular optical wavelength over time, the sensitivity of interrogation is determined by the gradient of the initial transmittance curve and hence is the largest when the measurement wavelength $\lambda_l$ is at the point of largest gradient of the transmittance. To show this dependence, we applied an acoustical wave (described in section Materials and Methods) and measured the maximal output intensity swing versus the chosen measurement wavelength. The result is shown in Figure 4.3b (blue line). The curve shows equal sensitivity be-
tween the left and right side of the optical resonance dip (dashed black line) as is expected by the symmetry of the resonance dip. The maximal sensitivity is reached at around 1/3 from the bottom of the dip. We also determined the noise versus the chosen optical wavelength (red line) by measuring the intensity fluctuations in the absence of an incident acoustical wave. As is shown in the figure, the noise depends only slightly on the output intensity (dashed black line). Therefore, the signal-to-noise-ratio will mainly depend on the sensitivity curve. For every measurement we will hence use the optical wavelength that gives a maximal output intensity swing; this will be chosen on the left flank.

Figure 4.3: a) The measured transmittance versus optical wavelength, for a maximum output power $P_0$ of 10 µW. b) The measured output intensity swing of the OMUS (blue line), the RMS value of the noise in the output intensity (red line), and the transmittance (dashed black line; normalized for visibility), all versus the chosen optical wavelength of the laser.

After we determined a suitable optical wavelength for our measurements, we investigated the response of the OMUS to an acoustical Gaussian modulated sine wave (see Section Materials and Methods), being transmitted by an external ultrasound source. Two measured time traces of the output signal due to an incident acoustical wave are shown in Fig 4.4. The presented signals are an average of 500 individual signals. The upper time trace shows the response to a transmitted acoustical wave with a 0.42 MHz center frequency, which is below the acoustical resonance peak of the membrane. The second time trace shows the response to a transmitted acoustical wave around the resonance frequency of the membrane ($f_0 = 0.77$ MHz). The signal below the resonance frequency of the membrane resembles the transmitted acoustical pulse quite well, while there is a large acoustical ring-down time present in the signal around the acoustical resonance frequency of the OMUS.
Chapter 4. Proof of concept of an OMUS

Figure 4.4: Time responses of the OMUS (right, red) for two transmitted acoustical pulses (left, black) with different center frequencies. The presented signals are an average of 500 individual signals.

The normalized transfer function of the OMUS is shown in Fig 4.5 and is obtained by sweeping the ultrasound frequency from 0.4 MHz to 1.4 MHz using narrowband pulses. The -6 dB bandwidth of the OMUS is 19% and the center frequency is 0.76 MHz. The used maximum output power $P_0$ for these and subsequent measurements was 70 $\mu$W.

We determined the sensitivity and noise equivalent pressure of the OMUS for incident acoustic waves with a frequency that corresponds to the maximum of the transfer function ($f_0 = 0.76$ MHz). The measurement results are shown in Fig 4.6 which shows the amplitude of the detected signals versus the amplitude of the
exciting acoustical signal. For small deformations of the membrane, and hence for small optical wavelength shifts of the optical resonance, the sensor response is linear. The OMUS has a sensitivity of 2.1 mV/Pa for pressures up till approximately 150 Pa (determined with a linear fit). For higher pressures, the sensitivity curve starts to deviate from a linear behavior. This nonlinear response of the OMUS occurs because of the used detection system. With this system the dynamic range of the OMUS is determined by the width of the optical resonance curve. If the deformation of the membrane, and hence the shift in optical resonance is too large, the response of the OMUS is distorted. We determined a noise equivalent pressure (NEP) of 0.4 Pa by the intersection of the sensitivity curve and the RMS value of the measured output noise (0.8 mV). To transform the NEP value to an equivalent minimum detectable wavelength shift we use the derivative of equation (4.2) with respect to the wavelength. The minimum detectable wavelength shift $\Delta \lambda_{\text{noise}}$ is thus calculated from the RMS noise $\Delta I_{\text{noise}}$ as

$$
\Delta \lambda_{\text{noise}} = \frac{\partial \lambda}{\partial I} \Delta I_{\text{noise}} = \frac{\partial \lambda}{\partial T} \frac{\Delta I_{\text{noise}}}{I_0}.
$$

(4.3)

To determine $\partial \lambda/\partial T$, we use the derivative of the normalized transmittance (Fig 4.3a), resulting in $\partial T/\partial \lambda = 10.5 \text{ nm}^{-1}$. With measured values $I_0 = 69 \mu W$ and $\Delta I_{\text{noise}} = 21 \text{ nW}$, this results in $\Delta \lambda_{\text{noise}} = 29 \text{ fm}$.

**Figure 4.6:** The measured amplitude of the OMUS output signal (blue points) versus the amplitude of the incident pressures, calibrated with a hydrophone. The sensitivity was obtained with a linear fit through the linear region of the system (dashed black line) and has a tangent of 2.1 mV/Pa. The red line is the RMS value of the measured output noise (0.8 mV). The crossing of the lines gives the NEP at 0.4 Pa. The results apply to the center frequency of the OMUS ($f_0 = 0.76 \text{ MHz}$).
4.4 Discussion

The results demonstrate that we fabricated a very sensitive and small ultrasound sensor compared to the polymer ring resonators [67] and even state of the art piezo-electrical devices. This sensor can easily be repeated into an array of sensors by using waveguide gratings or echelle gratings [75]. These passive optical (de)multiplexers convert a broad spectrum into many wavelength channels and vice versa. When we use grating coupled ring resonators [76] only one of the many resonance peaks per ring resonator will be present in a wavelength channel, hence allowing to successively stack as many sensors as possible into the available spectrum. Underneath these parallel photonic waveguides with grating coupled ring resonators multiple membranes can be created using dry etching or other techniques [84]. A dense array of sensors is particularly beneficial in the field of intravascular ultrasound, where the outer dimensions of the catheter are restricted to 1 mm and every catheter can only be used once due to hygiene codes. This application will require integration with a separate source (e.g., a piezo-electric element) to form a hybrid transducer. Moreover, an increase of the acoustical center frequency to 15-20 MHz will be necessary, which in principle should be feasible through redimensioning the sensor. Another interesting field of application is photoacoustics where high SNR and a small bandwidth are needed [85].

In view of its small spatial footprint of 124 µm diameter, the question arises whether sensor is more prone to noise. This topic will be further addressed in Chapter 6. Nevertheless, the obtained detection limit, or NEP, of 0.4 Pa is better than the NEP of 0.5 Pa offered by the state of the art piezo-electrical device with a 65 times larger surface area.

To optimize our sensor as medical imaging device, there are a few aspects that can be improved. The most important one is addition of absorbing layers or other adjustments to the membrane to increase the bandwidth, which is now 4 times less than for conventional transducers. The resonance frequency of the current OMUS is comparable with the photo-acoustic breast tomography sensor, but needs to be increased for almost all other medical applications. This can be achieved by further reduction of the membrane diameter or by designing a different membrane shape.

Improvements can also be obtained in the dynamic range. With the current detection system, the dynamic range is limited. This could be drastically increased when we use a similar interrogator system as the one used by Rosenthal et al [68]. In their paper, coherence-restored pulse interferometry (CRPI) is introduced which enables them to track the location of an optical resonance peak or dip over time. In that case, the dynamic range will not be limited by the optical interrogation system.

Although further development is needed, with this first prototype we gave proof of the OMUS concept and demonstrated a very sensitive sensor that may form the basis of future ultrasound transducers.
4.5 Method

The optical path of the measurement set-up starts with a tunable laser (Agilent, 81940A) to generate the light. The light passes through an in-fiber isolator (Opto-link Cooration Ltd., OLISO-I-S155), in-fiber attenuator (Opto-link Corporation Ltd., OLVAO-MN-155-2TA) and an in-fiber polarization controller (Thorlabs, FPC560) and is then coupled into the chip. The polarization controller is required because the optical waveguide on the OMUS only accepts one polarization state. The other fiber of the chip is connected to a photo-receiver (Newport, Newfocus 1811-FC-AC).

The acoustical part of the set-up contains a 1 MHz ultrasound transducer (Olympus, Panametrics V314) as source, which is connected to an arbitrary waveform generator (Agilent, 33521A). All the equipment is connected to a computer and we use 16-bit AD-cards (Spectrum, M314142-exp) with a sampling rate up to 250 MSa/s to acquire all the data. The ultrasound source and OMUS are placed in a water basin on a fixed distance of 23.2 cm from each other. The temperature of the water basin is regulated within 0.1°C by a thermostat (Grant Instruments, GD120) and we use a needle hydrophone (1 mm, Precision Acoustics) to measure the pressure generated at the position of the OMUS.

Every series of measurements starts with a short scan of the optical transmission to determine the optical resonance wavelength. Then we step through the optical wavelengths of the laser, starting from a position halfway the flank, to find the optical wavelength with the highest sensitivity. At this optical wavelength, the successive measurements were performed, by transmitting different sound waves towards the chip. The transmitted acoustical pressure has the shape

\[ p(t) = A \exp \left[ -\left( \frac{t - \tau_d}{N \frac{2\pi}{f_0}} \right)^2 \right] \sin(2\pi f_0 [t - \tau_d]), \quad (4.4) \]

where \( p \) is the acoustical pressure as a function of time \( t \), \( A \) is the amplitude of the acoustic wave, \( \tau_d \) is a fixed time delay, \( N \) is half the number of sine periods that are roughly visible below the gaussian envelope, and \( f_0 \) is the center frequency of the sine. We determined the transfer function of the OMUS by transmitting narrow-band pulses (\( N=20 \)) at subsequent acoustical frequencies, and adjusted the amplitude in such way that we compensated for the transfer function of the source, as to keep the emitted pressure the same for every acoustical frequency. We measured the sensitivity by altering the amplitude at a fixed frequency of \( f_0 = 0.76 \) MHz. The noise measurements were performed without a transmitted acoustical signal. We performed every measurement series twice, one with the OMUS and one with the hydrophone, in which the last series provided a reference for the pressure values at the surface of the OMUS.
Characterization of the optical micro-machined ultrasound sensor

Abstract – The development of ultrasound sensors for real time 3D ultrasound imaging has focused on miniaturization and extension from single elements towards large dense arrays. We proposed an optical micro-machined ultrasound sensor as basis for such future arrays. To optimize the basis element for a specific application complete understanding of the element is needed. This chapter describes the characterization of the basis element. We show that the measured response to changes in temperature correspond to literature with sensitivities around 80 pm/K. We measured a stronger response of the sensor to static pressure than predicted by a finite element model. We already showed that the sensor was very sensitive to ultrasound with a noise equivalent pressure of 0.4 Pa. In this chapter we are able to show that this sensitivity is indeed induced by the membrane of the sensor. Furthermore we show that the resonance frequency of the sensor can be tuned with the diameter of the membrane and we show that similar membrane diameters result in similar responses, i.e. the sensor is reproducible.

This chapter is based on the following publication:
5.1 Introduction

Real-time 3D ultrasound imaging techniques require transducers that consist of dense arrays [4]. Applications like intravascular ultrasound (IVUS), endoscopic ultrasound (EUS) or image guidance based intra-operative ultrasound restrict the outer dimensions of the transducer [86–90]. Hence in the last decade the development of ultrasound sensors is focused on miniaturization and extension from single elements towards large dense arrays [1]. The conventional transducers are made of piezo-electric material. Miniaturization and fabrication of small dense arrays is challenging for these conventional transducers especially in the wiring of the individual elements [59, 61]. Therefore alternative sensors and fabrication methods are widely investigated [91–95]. We proposed an alternative ultrasound sensor fabricated with micro-machining techniques comparable to CMUTs and PMUTs but based on integrated photonics instead of piezo-electric material [21, 78, 96]. The applied integrated circuit fabrication technology (CMOS) allows for ease of production. Furthermore, the advantage of the use of integrated photonic circuitry is the possibility to design small spatial footprints of the sensor elements (∼10 µm [74]). Other advantages are its insensitiveness to electromagnetic interference, high speed communication and a broad bandwidth for data transmission. We fabricated this optical micro-machined ultrasound sensor and provided proof of concept [96]. This sensor can be used as a basis for building large dense arrays but further development of this sensor towards any application first requires complete understanding of its behavior.

This chapter presents a full characterization of the sensor. We performed temperature measurements, static measurements and dynamic measurements. The temperature measurements and static measurements are compared to literature [97, 98]. Next to the literature, the static measurements also compared to a finite element model as well. The dynamic measurements show to what extend the membrane read-out of the sensor corresponds to the motion of the membrane.

This chapter is organized as follows: first we describe the working principle of the sensor and the devices used for the study, then we present the theory to determine the influence on the optical wavelength shift of the sensor due to temperature and due to a static load. The influence of a static load can be extended to an expected wavelength shift due to an ultrasound pressure wave. Next, we describe the method and measurement set-up, followed by the results. We end with the conclusions.

5.2 Working principle and device description

The working principle of the sensor as well as the fabrication process and design considerations are already described in Ref [96]. For convenience we sum up the most important aspects in this section.

The optical micro-machined ultrasound sensor consists of a waveguide and a photonic ring resonator that are integrated onto a membrane (Fig. 5.1a). When a broad light spectrum is transmitted through the waveguide, a part of the spectrum is coupled into the ring resonator by means of a directional coupler [77]. The trans-
mitted spectrum at the output port of the waveguide shows dips at the optical resonance wavelengths $\lambda_m$ of the ring resonator, given by

$$m\lambda_m = n_e l,$$

(5.1)

where $m$ is an integer number, $l$ the circumference of the ring and $n_e$ its effective index of refraction. Incident acoustical pressure waves strain the membrane and hence the resonator. The induced strain in the resonator causes a shift in the optical resonance curves with respect to the undeformed state. The shift in resonance is what we want to observe.

When we load the sensor in several steps, we can obtain a translation of the resonance dip from the wavelength shift in successive scans of the entire spectrum. This is indicated in Figure 5.1b by the three different spectra in blue, red and green, which correspond to different added loads. This scanning procedure can be used for static measurements, but is too slow for detection of the ultrasound waves without use of a dedicated detector. To determine fast optical wavelength shifts, we transmit light, and hence measure the transmitted intensity, at one optical wavelength. This wavelength is chosen on one flank of the optical resonance curve of the undeformed resonator (in this case the red curve), such that a shift of the resonance curve directly translates into a modulation of the transmitted optical intensity, as is shown in Figure 5.1b. The intensity is measured using a photo diode. We realized the sensor using the following procedure. First, the miniature optical circuitry was fabricated using a semi-industrial CMOS fabrication line at IMEC (Leuven, Belgium) via the ePIXfab platform [82]. The resulting wafer-piece (die) contains 220 nm high and 400 nm wide silicon waveguides on top of a 2 $\mu$m thick silicon-dioxide layer on a 250 $\mu$m thick silicon substrate. A 0.5 $\mu$m thick silicon-dioxide cladding was deposited to isolate the waveguide from the water. Second, we etched the membrane (124 $\mu$m diameter) from the back of the die using deep reactive ion etching with sulfur hexafluoride (SF$_6$) as etchant and we used the silicon-dioxide layer as a well-defined etch-stop.

**Figure 5.1:** a) Sketch of the OMUS, showing the photonic micro-ring resonator on top of the membrane. b) Sketch of three intensity curves at the end of the waveguide, representing the different transmittances for different strain values of the photonic micro-ring resonator. Adapted from [96]
The static and dynamic measurement set-ups require different sensors. Therefore, the sensors for the dynamic measurements were further processed. We glued those chips on a 1 mm thick glass plate. The plate contained a hole of 4 mm diameter, which was positioned behind the membrane. Finally, we connected optical fibers to the silicon photonic circuit of these sensors, which is referred to as packaging [83]. An overview of the used sensors is given in Table 5.1. The difference between the sensors is found in the post-processing of the device, diameter of the membrane and length of the optical ring resonator.

**Table 5.1:** Sensor description and use in different measurements. The first column list the sensor number, followed by the diameter of the membrane and race track (i.e. resonator) length. The glass layer indicates whether a glass layer is present or not. The last column indicates the measurements in which the device is used, where $T$ is the temperature measurements, $V$ is the vibrometer measurement and $P$ is the pressure measurement.

<table>
<thead>
<tr>
<th>Sensor</th>
<th>membrane diameter [$\mu$m]</th>
<th>racetrack length [$\mu$m]</th>
<th>glass layers</th>
<th>measurements</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chip 4</td>
<td>60</td>
<td>30</td>
<td>yes</td>
<td>T</td>
</tr>
<tr>
<td>Chip 11</td>
<td>124</td>
<td>40</td>
<td>yes</td>
<td>V,T</td>
</tr>
<tr>
<td>Chip 13</td>
<td>101</td>
<td>40</td>
<td>yes</td>
<td>V,T</td>
</tr>
<tr>
<td>Chip 16</td>
<td>104</td>
<td>40</td>
<td>no</td>
<td>P</td>
</tr>
<tr>
<td>Chip 17</td>
<td>63</td>
<td>30</td>
<td>no</td>
<td>P</td>
</tr>
</tbody>
</table>

5.3 Theory

This section describes the influence of temperature on the optical resonance and the physical relations between the deformation of the membrane and the expected optical resonance shift. We start with the dependence on temperature as this is independent of the mechanical deformation (Sec. 5.3.1). Then we describe the deformation of the membrane. The membrane has an initial hollow position. When a pressure load is applied to the sensor, the membrane will deform further. This deformation induces (more) strain in the optical ring resonator, which causes a shift in the optical resonances. These different aspects are discussed below. First we discuss the initial condition of the membrane (Sec. 5.3.2). Second we discuss the relation between the pressure and deformation of the membrane (Sec. 5.3.3) and third we describe the expected optical wavelength shift due to this deformation (Sec 5.3.4).

5.3.1 Influence of temperature

Temperature influences the propagation of light through the silicon waveguide. The refractive index is dependent on temperature according to:

$$
\beta(T) = \frac{1}{n} \frac{dn}{dT},
$$

(5.2)
with $dn/dT$ being the thermo-optic coefficient [97, 99]. The change of refractive index is significant for silicon ($\beta = 1.79 \times 10^{-4}/$K) and is an order of magnitude lower for SiO$_2$. The thermal expansion coefficient is given by

$$\alpha(T) = \frac{1}{h} \frac{dh}{dT},$$

(5.3)

with $h$ being the sample thickness. The thermal expansion of Si measured around room temperature at 1530 nm wavelength is low ($\alpha = 2.58 \times 10^{-6}$/K) compared to the change of refractive index [99]. This means that the effect of the expansion of the cross-section of the waveguide is negligible compared to other effects.

The change in the direction of propagation can give rise to noticeable spectrum changes. To determine the total effect on the silicon waveguide due to changes in temperature, the wavelength dispersion of the effective index has to be taken into account as well. A derivation of this formula is beyond the scope of this thesis, but is given by [100]

$$\lambda = \frac{L}{m} \left(1 - \left(\frac{L}{m}\right) \left(\frac{\partial n_{\text{eff}}}{\partial \lambda}\right)\right)^{-1} \left(\frac{\partial n_{\text{eff}}}{\partial \epsilon_c}\right) 2n_{\text{si}} \left(\frac{\partial n_{\text{si}}}{\partial T}\right) (T - T_0) + \frac{L}{m} n_{\text{eff}}(\lambda_0),$$

(5.4)

with $\epsilon_c$ the relative dielectric constant for the material that is changing with temperature, in this case the silicon waveguide.

### 5.3.2 Initial condition

It is known from literature that the fabrication process of thin films can cause residual stress [101–107]. In general it is difficult to determine the state of residual stress in a thin film because it depends on the specifics of the fabrication process and varies through the film thickness [103]. There are different processes that can contribute to the residual stress, but the main cause in our case is due to the difference in thermal expansion coefficients of the materials [105]. Although this difference is not important when the device is used, it is of importance when the CMOS structures are made at elevated temperatures. The silicon dioxide is deposited onto a substrate of single-crystal silicon at these elevated temperatures. The wafer of our sensor is made with the smart cut technique [108]. Due to the different thermal expansion coefficients of silicon and silicon dioxide thermal stresses are induced in these materials during manufacture and remain there during the subsequent use of the devices. For thermal mismatch problems, the elastic strain needed to fit the film to the substrate is given by

$$\epsilon = -(\alpha_f - \alpha_s)(T - T_0) = -\Delta \alpha \Delta T,$$

(5.5)

where $\alpha_f$ and $\alpha_s$ are the linear thermal expansion coefficients of the film and substrate, respectively, $T$ is the current temperature and $T_0$ is the initial temperature at which the film and substrate were in a stress-free state [101]. The varying values for the silicon dioxide expansion coefficient in literature and the fact that thermal expansion coefficients in a thin film can differ from the bulk values make it difficult to predict the strain values present in the membrane [102, 107]. The stress in a thin
film deposited on a substrate can be approximated in second order by a biaxial homogeneous stress superimposed on a linear biaxial stress gradient. The stress in the film is in this approximation biaxial at all points, but varies linearly from the top surface to the bottom surface [106]. Both compressive and tensile stresses can be present, but it is known that silicon dioxide resulting from a thermal oxidisation of silicon is in a state of compression on the silicon surface [105]. When the membrane is created by locally removing the substrate, the internal stress in the membrane will relax and the device will deform [102, 106].

5.3.3 Deflection

The deflection of uniformly loaded plates is described depending on the size of the displacement. For small displacements, linear theory can be used. When the displacement is in the order of the thickness of the plate, large deflection theory is used. This theory assumes that the vertical displacement $w$ is much larger than the lateral displacements $u$ and $v$ and that the normals to the undeformed middle surface remain the normals to the deformed middle surface [17, 109–112]. The governing differential equations or Kármán equations for the bending of a circular plate due to a load $P$ are [111]

$$P = \frac{Eh^3}{12(1-\nu^2)} \nabla^4 w - \frac{h}{r} \frac{\partial \phi}{\partial r} \frac{\partial^2 w}{\partial r^2},$$

$$\nabla^4 \phi = -\frac{E}{r} \frac{\partial w}{\partial r} \frac{\partial^2 w}{\partial r^2},$$

where $E$ is the Young’s modulus, $\nu$ is the Poisson ratio and $\phi$ is the Airy stress function. The biharmonic operator $\nabla^4$ is given by

$$\nabla^4 = \left( \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} \right)^2.$$  \hspace{1cm} (5.8)

In case of very large deflection theory the strain relations contain extra terms compared to the linear theory, with second order dependence on both the $u$ and $w$ direction resulting in [17]

$$\epsilon_r = \frac{du}{dr} + \frac{1}{2} \left[ \left( \frac{du}{dr} \right)^2 + \left( \frac{dw}{dr} \right)^2 \right] \quad \text{and} \quad \epsilon_\theta = \frac{u}{r},$$

where $\epsilon_r$ is strain in the radial direction, $\epsilon_\theta$ is the strain in the tangential direction and $u$ is the displacement in the radial direction. These strain relations can be extended with intrinsic strain [110, 111].

Next to the intrinsic strain our sensor has an initial displacement in the order of the thickness $h$ of the membrane. In such case superposition of the initial condition and the deflection due to a load can not be applied [113]. Therefore we will use a finite element model to determine the amount of strain that is induced by the initial deformation and the pressure load together.

For dynamic measurements we assume that the deformation of the membrane at
its fundamental mode has a similar shape as the static loading. In that case the
displacement in time is given by

\[ w(t) = w_{stat} + \Delta w(t), \]
\[ = w_{stat} + \frac{\partial w}{\partial \Delta P} \Delta P(t). \]  
\[ (5.10) \]

So at the position of the static curve where the load matches the load of the water,
the slope should match the displacement per load of the dynamic measurements.

5.3.4 Optical wavelength shift

We now determine the optical wavelength shift due to a certain strain at the posi-
tion of the ring resonator. Equation 5.1 gives the dip position of the transmitted
spectrum of the undeformed ring resonator. When strain \( S \) is applied these dips
at the resonance wavelengths \( \lambda_m \) appear when the phase delay \( \delta \) of one round trip
through the ring is

\[ \delta = \int n_e(\rho, \lambda) \frac{2\pi}{\lambda} \, d\rho = m2\pi, \]  
\[ (5.11) \]
or

\[ m\lambda_m = \int n_e(\rho, \lambda_m, S_z) (1 + S_\rho(\rho, S_z)) \, d\rho, \]  
\[ (5.12) \]
where \( z \) indicates the specific direction of the long sides of the racetrack resonator
and \( \rho \) indicates the direction along the racetrack in general. The effective index of
the waveguide depends on its mechanical deformation. Moreover, the local strain
in the direction of the track \( S_\rho \) is taken into account by stretching each element
\( d\rho \) to \( (1 + S_\rho) d\rho \). From previous strain measurements, we know that the optical
wavelength shift is linear with respect to strain. We derived a formula of this shift
in such way that it shows dependence on three different physical aspects \([78]\)

\[ \frac{\partial \lambda_m}{\partial S_z} = \left\{ \begin{array}{c} n_e \frac{\lambda_e}{n_g} \frac{\partial n_e}{\partial S_z} \langle \text{dispersion} \rangle \\
\frac{\lambda_e}{n_g} \frac{\partial n_e}{\partial S_z} \langle \text{eff. index} \rangle + \frac{\lambda c}{n_g} \frac{\partial S_\rho}{\partial S_z} \langle \text{track-length} \rangle \end{array} \right\}, \]  
\[ (5.13) \]

where \( n_g \) is the group index and \( \lambda_c \) is the resonance wavelength without deformation.

5.4 Description and numerical modeling parameters of the measurement set-up

The characterization of the sensor is performed with three different measurements;
one set-up to determine the dependence on temperature (Sec. 5.4.3), one in which
we measured the response due to a static pressure (Sec. 5.4.1) and one in which
we measured the deflection of the membrane due to an incident acoustic wave with
a simultaneous optical read-out of the sensor (Sec. 5.4.2). This section describes
all measurement set-ups as well as the parameters that were used in the numerical
FEM model (5.4.4).
5.4.1 Static measurement set-up description

As described in Section 5.2, we use unpackaged sensors for the pressure measurements. Therefore, we have to align two single optical fibers above the grating couplers to couple the light into the sensor. The optical path of this setup contains a tunable laser (Agilent, 81960A) to generate the light followed by an in-fiber polarization controller (Thorlabs, FPC560). This fiber ends at an translational stage which can be positioned above the sensor. The fiber at the other end of the sensor is connected to an optical power meter (Agilent, N7744A). The sensor is positioned on a chuck. Using a membrane vacuum pump we are able to decrease the pressure under the membrane. We measure the pressure with a calibrated pressure sensor (Vacuubrand, CVC 3000). The set-up is shown in Figure 5.2a. For every measurement series, we sweep the wavelength of the laser through 400 pm around the dip position and measure the mean dip position for a given pressure during 75 seconds.

5.4.2 Dynamic measurement set-up description

The basic equipment, used for the dynamic vibrometer measurements was similar to the setup used in Ref [96] (Fig. 5.4b). In the optical path of the measurement set-up we use a tunable laser (Agilent, 81940A) to generate the light. The light passes through an in-fiber isolator (Opto-link Cooration Ltd., OLISO-I-S155), an in-fiber attenuator (Opto-link Corporation Ltd., OLVAO-MN-155-2TA) and an in-fiber polarization controller (Thorlabs, FPC560), and is then coupled into the chip. The other fiber of the chip is connected to a photo-receiver (Newport, Newfocus 1811-FC-AC). The acoustical part of the set-up contains a 1 MHz ultrasound transducer (Olympus, Panametrics V314) as a source, which is connected to an arbitrary waveform generator (Agilent, 33521A). The ultrasound source is placed in a water basin on a fixed vertical distance of 10 cm below the sensor. The OMUS is attached to a hollow tube and positioned in such way that the sensor itself is submerged in water while the back of the tube is still above the water surface so only air is present behind the membrane. The ultrasound source and OMUS are positioned in line with a laser vibrometer (Polytec, OFV5000) that can measure the displacement of the OMUS. The vibrometer has a displacement card (Polytec, dd-300) that can measure displacements up to 150 nm with a resolution of 0.1 pm/√Hz at a maximum sample frequency of 24 MHz. We use a 20x objective (f=200, M Plan Apo NIR) to focus on the center of the back of the membrane or on the back of the chip. All the equipment is connected to a computer and we use 16-bit AD-cards (Spectrum, M314142-exp) with a sampling rate up to 250 MSa/s to acquire all the data.

Every series of measurements starts with a short scan of the optical transmission spectrum to determine the optical resonance wavelength. Then we step through the optical wavelengths of the laser, starting from a position halfway the flank, to find the optical wavelength with the highest sensitivity. At this optical wavelength, the successive measurements were performed, by transmitting different
sound waves towards the chip. The transmitted acoustical pressure has the shape

\[ p(t) = A \exp \left[ -\left( \frac{t - \tau_d}{N f_0} \right)^2 \right] \sin \left( 2\pi f_0 [t - \tau_d] \right), \quad (5.14) \]

where \( p \) is the acoustical pressure as a function of time \( t \), \( A \) is the amplitude of the acoustical wave, \( \tau_d \) is a fixed time delay, \( N \) is the number of sine periods that are roughly visible below the gaussian envelope, and \( f_0 \) is the center frequency of the sine. We determined the transfer function of the OMUS by transmitting narrow-band pulses (\( N=30 \)) at subsequent acoustical frequencies. We measured the sensitivity by altering the amplitude at a fixed frequency of \( f_0 = 0.8 \) MHz for Chip 11 and at \( f_0 = 1.05 \) MHz for Chip 13.

5.4.3 Temperature measurement set-up description

The optical equipment used for these measurements is exactly the same as the one used for the dynamic vibrometer measurements. Because the sensors are packaged, we can submerge them in the water basin. We position them in the middle of the basin and measure the temperature of the water with a calibrated analog thermometer that has a precision of 0.005°C. The water temperature is regulated with a thermostat (Grant Instruments, GD120) within 0.1°C. We vary the temperature with steps of 1°C and perform measurements when the temperature is stable.

5.4.4 Numerical modeling parameters and methodology

We numerically model our sensor as a membrane with clamped boundary conditions. We only model a quarter of the membrane thus employing the axisymmetric conditions. The membrane consist of three layers; first the 2 \( \mu \text{m} \) main SiO\(_2\) layer, second the 0.22 \( \mu \text{m} \) optical Si layer, third the 0.5 \( \mu \text{m} \) SiO\(_2\) cover layer. The Young’s moduli are 70 GPa and 170 GPa for the SiO\(_2\) layer and Si layer respectively. The Poisson ratio is 0.19 for SiO\(_2\) and 0.27 for Si. The FEM model uses curved triangular finite rotation shell elements that can handle the expected deformations [114]. The pressure is increased from 0 to 7 MPa.

To determine the wavelength shift per strain (Eq. 5.13), we need to have estimates for the effective index, \( n_e \), group index \( n_g \), resonance wavelength \( \lambda_c \) and change in effective index due to strain \( \frac{\partial n_e}{\partial S} \). We obtained these values for several long ring resonators and waveguides with different widths [78]. We use these results as estimate for the ring resonators. With an estimated effective index of 2.2, group index of 4.32, resonance wavelength of 1550 nm and a change in effective index \( \frac{\partial n_e}{\partial S} = -0.72 \) we get an optical wavelength shift due to strain of 0.55 pm/microstrain.

5.5 Results

We started the characterization of the OMUS by determining its dependence on temperature. Next we measured the response to a static load and compared this to the FEM model. We continued by measuring the membrane displacement as
well as the chip displacement due to an incident acoustical pressure wave. All measurement results are described below.

5.5.1 Temperature measurement results

The influence of temperature was first visualized in the transmission curve of the sensor. The local temperature is influenced by the transmit intensity of the laser. To show this, we determined the shape of the optical resonance curve by measuring the transmittance $T(\lambda)$ from the input to the output of the OMUT. The intensity $I$ at the output of the chip for a static situation is given by $I = T I_0$ with $I_0$ the maximum output intensity far away from the resonance dip. This maximum output intensity $I_0$ is related to the maximum output power $P_0$ that we measure with the photo-receiver. We measured the transmittance by stepping through successive wavelengths with the laser for two different output powers. Figure 5.3 shows the transmittance for output powers $P_0$ of 10 $\mu$W and 60 $\mu$W. The low intensity curve (blue line) shows the expected symmetric resonance curve with a FWHM of 100 pm. The curve for higher power (red line) shows a different transmittance. The asymmetry of the resonance dip can be explained by the heating of the ring resonator due to the dissipation of light. Silicon has a strong thermo-optical effect and the resulting shift in resonance wavelength is about 80 pm/°C [97]. During the measurement we stepped through the successive wavelengths and therefore changed the amount of light in the ring, and thus the dissipation. On the left flank of the resonance dip, the amount of light in the ring resonator increases for increasing laser wavelength, resulting in a shift to the right of the resonance dip during the measurement sequence. On the right flank of the resonance dip, the amount of light for increasing laser wavelength decreases and the resonance dip thus shifts to the left. Therefore, with this measurement method, at relatively high power values the transmission curve is altered due to the heat generated by the laser source.
5.5. Results

To determine the thermo-optical effect more thoroughly we measured the optical wavelength shift of three sensors. The straight part of the racetrack of two resonators is 40 µm and one ring has a straight part of 30 µm. The temperature was varied from 23°C to 27°C in steps of 1°. At every temperature several measurements were performed. The measurement results of Chip 13 are similar to the other measurements and clearly show that all the measured temperature shifts are on a straight line (Fig. 5.4). The measured sensitivity of the optical wavelength shift per degree centigrade is 80.0°C, 82.5°C and 80.9°C for chip 4, chip 11 and chip 13 respectively. The coefficient of determination $R^2$ is 0.99 for all measurements and shows that the results are easily fitted to the linear regression [115]. The small difference between the three devices may have to do with small fabrication variations. The measurements correspond well with calculations and other measurements in literature where the optical wavelength shift varies between 70 pm/K to 90 pm/K for different waveguide widths [97].

5.5.2 Static measurement results

When we look at the initial condition of the membrane, it can be seen through a microscope that the membrane is in a hollow position. This initial displacement is confirmed with a white light interferometer (Bruker, ContourGT-K). The result of the latter is shown in Figure 5.5. Due to the rough surface of the device the interferometer measurement can only be used to get an estimate for this initial displacement, which is in the order of 4 µm.

We measured the optical wavelength shift of the membrane due to a static loading. The results are shown in Figure 5.6 and show a nonlinear response. The response is sensitive for small pressures and less sensitive for the higher pressures. The maximum wavelength shift is in the order of 200 pm. These results are similar to the measurement results of Hallynck [98]. They measured a similar curve in the response to the same pressure load with a maximal optical wavelength shift of 208 pm for a similar device with a rectangular membrane and a ring resonator.
We modeled the device to see whether we could fit the measurements and obtain the membrane displacement. Because we have no exact measurements of the initial deflection, we vary the initial strain in the model to obtain different initial deflections of the membrane. We modeled two cases, one in which the membrane has no initial strain or displacement and one in which the initial displacement of the membrane is obtained with an initial strain of 1%. The results are shown in Figure 5.7 with on the left hand side the displacement of the membrane and on the right hand side the change in strain in the layer of the waveguide due to loading. We use the change in strain to determine the optical resonance shift due to loading, using the initial resonance wavelength as starting position. It is shown that an initial strain of 1% results in an initial displacement of 4.4 µm. Furthermore, it can be seen that the general trend for the displacement and strain becomes nonlinear, but that the pressure load needed for this non-linearity is much larger than the one used for the measurements. In the regime of interest both the displacement and strain are linear. The initial strain causes a change in sign of the strain in the top layer. Although the membrane is buckled due to the initial strain, there is still strain present in the entire membrane when there is no loading. We obtain -108 µstrain at 70 kPa with no initial strain and 48 µstrain at 70 kPa with an initial strain of 1%. When we use the conversion of 0.55 pm/µstrain we obtain an expected wavelength shift of -59 pm and 26 pm. These values are much lower than the shifts that we have measured. The sign of the measurements correspond to the model with no initial strain.

Figure 5.4: Measured optical wavelength shift change versus temperature change. The linear fit has a $R^2$ of 0.99.
Figure 5.5: Interferometer measurement of the sensor. The color indicates the measured height of the sensor where a red color represents the flat surface and a blue color the maximum displacement. There are three different (isolated) ring resonators visible with straight lengths from 20 to 60 μm. The hollow membrane is clearly visible around the 40 μm ring resonator. The measurement contains too much noise to obtain accurate quantitative values of the displacement of the membrane.

Figure 5.6: The optical induced wavelength shift due to pressure for two sensors, Chip 16 and Chip 17.

5.5.3 Dynamic measurement results

We first investigated whether the read-out of the ring resonator is only influenced by the deformation of the membrane. We obtained two time-traces simultaneously,
The modeling results for a membrane with no initial strain (blue) and an initial strain of 1% (red). The displacement in both cases is shown in a) and the corresponding strain difference (the initial strain with zero pressure subtracted) in b).

Figure 5.7: Two simultaneous recorded timesignals are shown of Chip 11 transmitted with a center frequency of 0.8 MHz. One of the optical read-out of the OMUS (read) and one of the displacement of the membrane (blue). A zoom of both time traces is shown in the lower right corner.

Figure 5.8: Two simultaneous recorded timesignals are shown of Chip 11 transmitted with a center frequency of 0.8 MHz. One of the optical read-out of the OMUS (read) and one of the displacement of the membrane (blue). A zoom of both time traces is shown in the lower right corner.
displacement as well as the OMUS signal are shown in Figure 5.9 for two sensors. It is shown that Chip 11 has its resonance frequency at 0.8 MHz. This is comparable to the OMUS of Ref [96], which is a sensor with a similar membrane diameter (124 µm). Chip 13 has a higher resonance frequency of 1.05 MHz which is expected in view of the smaller membrane diameter of 100 µm. Clearly, the membrane displacements and the OMUS signal have almost the same transfer functions.

![Graph](image)

**Figure 5.9:** The transfer functions of the membrane and the sensor signal. a) the results for Chip 11 and b) the results for Chip 13. The signals are normalized with respect to the maximum of each function. These maxima correspond for Chip 11 to 270 mV (vibrometer signal) and 199 mV (OMUS signal) and for Chip 13 to 111 mV (vibrometer signal) and 172 mV (OMUS signal).

Next we determined the sensitivity of the OMUS, which is related to the membrane displacement for a given amplitude of the incident acoustical pressure wave. This is determined by transmitting acoustical pressure waves with increasing amplitude and center frequency $f_0$ of the sensor. The results are shown in Figure 5.10. For small deformations of the membrane, and hence for small optical wavelength shifts, the sensor response is linear. For higher pressures, the sensitivity curve starts to deviate from the linear curve. This nonlinear response of the OMUS occurs because of the used interrogator system. With this system the linear dynamic range of the OMUS is determined by the width and shape of the optical resonance curve. If the deformation of the membrane, and hence the shift in optical resonance is too large, the response of the OMUS is distorted. From the response of the vibrometer it can be seen that the displacement of the membrane is still linear for higher pressures.

The difference between the chip displacement (black line) and membrane displacement (red line) confirms that for both sensors only the membrane is significantly excited by the incident pressure wave. The optical resonance shift $\Delta \lambda$ (blue line)
is obtained from the intensity modulation $\Delta I$ of the photodiode according to

$$\Delta \lambda = \frac{\partial \lambda}{\partial T} \frac{\Delta I}{I_0},$$

(5.15)

where $\partial \lambda / \partial T$ is determined from the derivative of the normalized transmittance $T$ (Figure 5.1b). We obtained a derivative of $\partial T / \partial \lambda = 13.2 \text{ nm}^{-1}$ for Chip 11 and $\partial T / \partial \lambda = 8.3 \text{ nm}^{-1}$ for Chip 13 from the measured spectra. By dividing the optical resonance shift with the measured displacement we obtain a sensitivity of $2.1 \text{ pm nm}^{-1}$ (resonance shift over membrane displacement) for Chip 11 at 0.8 MHz and $1.6 \text{ pm nm}^{-1}$ for Chip 13 at 1.05 MHz.

![Figure 5.10](image)

**Figure 5.10**: Measured optical wavelength shift (blue line), membrane displacement (red line) and chip displacement (black line). a) the results for Chip 11 and b) the results for Chip 13. We obtained the sensitivity of the OMUS signal by making a linear fit through the first measurement points (dashed blue line). The sensitivity of the displacement was determined by making a linear fit through the vibrometer data (dashed red line).

### 5.6 Discussion

We characterized the sensor by determining its response to temperature, a static loading and a pressure wave. We did this with three different measurements applied to several sensors. We showed that the OMUS is sensitive to changes in temperature. The response is linear and corresponds to calculated values and measured values in literature. However, we do not expect that temperature effects will influence our acoustical measurements, because the influence of temperature is a relatively slow effect (order of seconds) compared to the ultrasound measurements that take place in a short amount of time (order of microseconds). Furthermore, the displacements of the membrane are small and hence the differences in wavelength shifts are small. Therefore we will write the influence of temperature only
as an extra translation $\Delta \lambda_h$ of the low-power curve in Figure 5.3, and consider this translation to be constant during the ultrasound measurements. We measured the initial position of the membrane and found that the membrane is buckled. The main reason of this buckling is strain due to the fabrication process. There are several methods known from literature that can avoid this internal strain, for instance by deposition of another film [116, 117] or by ion implantation [118]. Another solution is reported by Iwase et al. who adjusted the structure of the sensor in such way that it corrects for the buckling by design [119]. All these methods have consequences for the design of the membrane, which means that they influence the resonance frequency and possible sensitivity. This is important to take into account when the design needs to be optimized for an application as ultrasound sensor.

Optical ring-resonators have already been used to measure static loading with a linear response between the load and optical wavelength shift. The sensitivity of the sensor depends on the radius of the ring resonator as well as the thickness of the membrane [120]. The buckling of the membrane due to internal strain in the sensor seems to change the linear static behavior since we measured a non-linear response to a static pressure. We were not able to explain the measurement results of the static measurements with a finite element model: the non-linearity occurs at much higher loadings in the model as in the measurements. There are several aspects that need to be investigated further to find a possible explanation. First, the exact values of the initial displacement and material constants are unknown. This makes it difficult to build an exact model to fit our measurements. Second, the status of initial strain is unknown. We showed with the model that the amount of strain as well as the sign is completely different for a situation with no initial strain and one with initial strain. Third, the linear response of the ring resonator is only investigated for cases of pure bending, where in the situation of static loading, membrane strain is an important factor as well.

The stronger response of the device due to static loading may also be an explanation for its sensitivity to ultrasound. Due to the missing information of the static displacement of the membrane it is not yet possible to obtain a full description of the displacement of the sensor as described by Eq. 5.10. Instead, we are able to match the optical wavelength shift of the sensor due to pressure in a similar manner. The highest derivative of the static measurements is found at the beginning of the curve and has values in the order of 10 pm optical wavelength shift/kPa. These values are in the same order of magnitude as the linear fit of the vibrometer measurements. Because the sensors are barely submerged under water when the vibrometer measurements are performed, the static loading due to the water column is in the order of several pascals. Therefore the steep derivative of the strain measurements seems to match the sensitive response to ultrasound.

With the vibrometer measurements we are able to show that the response of the sensor is a result of the oscillating membrane. We conclude from the measurements that only the membrane is significantly oscillating due to the incident pressure wave and the back of the sensor is not. Hence the response of the ring resonator matches the response of the membrane. With the two devices that were fabricated for the vibrometer measurements we showed that the sensor is reproducible.
Abstract — In medical diagnostics, ultrasound devices are often used to image organs or tissue from positions inside the human body. Therefore it is paramount to design small and sensitive sensors. In search for miniature ultrasound sensors we developed an opto-mechanical micro-machined ultrasound sensor based on integrated photonics instead of piezo-electric material. To predict the lowest detectable pressure we wanted to determine the noise level of this sensor. Due to the lack of electrical circuits the noise of the sensor is only caused by two noise mechanisms; one is the molecular agitation in the medium hitting the surface of the sensor and the other the mechanical thermal noise induced in the sensor itself. The resulting acoustical noise is not well described in literature. Therefore this paper expands the existing knowledge of acoustical noise by analyzing both noise mechanisms. We show that in thermodynamic equilibrium the noise power delivered by the medium to the sensor indeed balances the noise power delivered by the sensor to the medium. Moreover we show that for sensors with vanishing aperture area, the noise pressure due to the molecular motion in the medium will reach a well-defined finite limit.


6.1 Introduction

The need of 3D real time images in medical ultrasound requires ultrasound transducers that consist of dense arrays of elements [4]. Next to the increasing amount of elements, developments of transducer design also involve further miniaturization by the introduction of micro-machining processes [1]. In search for miniature acoustic sensors we developed an opto-acoustical ultrasound sensor [96]. We showed that this sensor is very sensitive and can measure pressure levels down to 0.4 Pa. We wanted to determine the absolute noise floor and found that the noise floor was much smaller than predicted from calculations found in literature. The pressure noise in our sensor originates from intrinsic thermo-mechanical fluctuations in the sensor itself [121] and thermo-acoustical noise due to molecular agitation in the medium [122]. Gabrielson [123] reviews several techniques to calculate the mechanical thermal noise induced in micro-machined sensors. However, some aspects are missing in the literature. First of all, the noise due to molecular agitation in the medium is not explicitly derived. Second, the current description of the acoustical noise received by a transducer uses the specific acoustic impedance and is defined as the acoustic impedance of the medium \( \rho c \) times the surface area of the transducer. When we use this specific acoustic impedance to determine the noise floor of our sensor, the noise blows up for sensors with a small aperture. Third, with noise calculations of conventional piezo-electric transducers this acoustical noise is only implicitly taken into account as a partial contribution to the total noise at the electrical output [1, 2]. Hence there is a need to derive a theory that describes the expected acoustical noise of these small non-electric sensors.

This paper presents a theory to determine the acoustical noise of a piston transducer. The theory is based on full thermodynamic equilibrium conditions, which means that the noise power produced by the sensor itself must equal the power produced by the medium. In this way are we able to cross-check the power delivered by both mechanisms. The theory shows that the noise level is not only dependent on the surface area of the transducer, but that a factor is needed to correct for the piston behavior of the transducer when the radius \( a \) of the transducer’s surface is small compared to the wavelength \( \lambda \) (i.e., \( a/\lambda < 0.3 \)). Furthermore is it shown that due to this factor the acoustical noise induced by the medium converges to a finite value for transducers with a vanishing surface area.

This paper is organized as follows: We derive the theory in Section 6.2, where we start the analysis with a short overview of the derivation that is needed to determine the noise pressure at the transducer surface. Next we derive the noise power and the noise pressure due to the sensor and the medium, and we conclude that section with a description of the noise pressure levels. We show simulation results that describe the dependence of the acoustical noise on the ratio \( a/\lambda \) of the piston surface for our sensor. In Section 6.3 we will compare these noise pressure to the noise levels that can be obtained with conventional piezo-electric transducers based on a finite element model. We finalize this paper with conclusions in Section 6.4.
6.2 Theory

When a system is in full thermodynamic equilibrium with its surroundings at temperature $T$, no net power can flow between the system and the surroundings (Fig. 6.1a) [124]. If the system contains a damping element, it means that there is a mechanism for mechanical dissipation in that system. In that case the fluctuation-dissipation theorem [125] states that there will be a component of mechanical fluctuation in that system that is directly related to the dissipation. As a result of this theorem any mechanical system, no matter how complex, that is in full thermodynamic equilibrium, can be analyzed for mechanical-thermal noise by adding a noise force generator alongside each damping element [123]. In the equation of motion of our sensor the oscillation of the membrane is approximated by piston motion. Therefore only one damping mechanism is present, which represents the internal mechanical resistance against the oscillation of the membrane. In an equivalent circuit this mechanical sensor can be described as a damper with mechanical resistance $R_M$ and a parallel force generator both connected to the piston (Fig. 6.1b). On the other hand, if the system is in contact with a surrounding medium via a membrane, molecular agitation against the membrane will be a second cause of noise (Fig. 6.1c). This kind of noise can be analyzed in a similar manner as the electrical noise received by an antenna. Because the sensor is in thermal equilibrium with the medium, the total spectral power received by the sensor from the medium must equal the spectral power delivered by its internal noise force generator to the medium [126]. Based on this principle we are able to check the noise power generated by both mechanisms. Next, we determine the noise pressure at the surface of the transducer. We start the analysis by determining the amount of noise power produced by the force generator in the noisy mechanical resistor (Sec 6.2.1), and the noise power density present in the medium (Sec 6.2.2). We proceed by calculating the mechanical radiation impedance of the piston and its effective area in reception (Sec 6.2.3). Then we determine the maximum amount of noise power that is collected by the sensor (Sec 6.2.4) and finalize by deriving the noise pressure present at the surface of the sensor (Sec 6.2.5).

6.2.1 Energy generated by the sensor

When the sensor is in thermal equilibrium with its surroundings at an absolute temperature $T$, there are intrinsic thermodynamic fluctuations present, which are described by the force generator alongside the damper. The magnitude of the mean square displacement fluctuations can be solved from the equipartition theorem [127, Ch. 7.5] which states that each independent quadratic term in the total energy of the system corresponds to a degree of freedom with a mean fluctuation energy equal to $k_B T/2$ where $k_B = 1.38 \times 10^{-23}[\text{J/K}]$ is the Boltzmann constant. To determine the thermal agitation of the sensor we follow the Nyquist reasoning [126] for thermal agitation of electric charge in electrical resistors, but in our case we take two mechanical dampers.

Consider two dampers, each with a mechanical resistance $R_M$ and the same temperature $T$, connected as indicated in Figure 6.2. Due to thermal agitation of
Chapter 6. Derivation of the acoustical noise pressure

\[ A_0 = \rho_0 c_0 \]

Medium
\[ R_M \]
Noiseless
\[ F_{\text{noise}} \]
\[ p_{\text{noise}} \]
\[ R_M \]
\[ p_{\text{noise}} \]

**Figure 6.1:** a) Sensor with a piston \( A \) and matched mechanical resistance \( R_M \), in contact with a medium with impedance \( Z_0 \). The sensor and medium are contained in (large) cavities. The whole configuration is in full thermodynamic equilibrium at temperature \( T \). The only thermodynamic connection between both sides is via the surface area \( A \) of the sensor. b) The system described for the two approaches; on the left (1) we consider the noise power produced by the intrinsic force generator and delivered to a noiseless medium and on the right (2) the noise power produced by the medium and received by the transducer with a noiseless mechanical resistance.

**Figure 6.2:** Scheme of two connected dampers \( I \) and \( II \) each with a mechanical resistance \( R_M \). a) The mechanical network diagram of the two dampers fixed on one side and connected together on the other side. b) The equivalent network model.

A damper 1 a force \( F \) generates a velocity \( v \) in damper 2. This velocity, obtained by dividing the force by 2 \( R_M \), causes absorption of power in damper 2. This absorbed power is equal to the product of the mechanical resistance and the square of the velocity. So power is transferred from damper 1 to damper 2. In the same manner power is transferred from damper 2 to damper 1. Since the two dampers are at the same temperature it follows from the second law of thermodynamics that the power flowing in one direction is exactly equal to the power flowing in the other direction. This equilibrium condition holds for the power exchanged within any frequency band [126]. Hence the force is a universal function of frequency, mechanical resistance and temperature and of these variables only. To determine the form of this function we consider again the two dampers each of resistance \( R_M \), but now connected via a long non-dissipative thin rod (Fig. 6.3). The length
of the rod is \( l \), it has a density \( \rho \), cross-section \( A \) and the velocity of propagation along the rod is defined as \( c \). This rod has the same characteristic impedance \( Z = R_M \) as the mechanical resistance so there is no reflection at either end of the rod. Let the absolute temperature of the system be \( T \) after thermal equilibrium has been established. We can describe the delivered power to the rod as two trains of energy traversing the rod. One train is traveling from left to right, being the power delivered by damper 1 and absorbed by damper 2. The other train is traveling in the reverse direction. At a certain instant after equilibrium has been established, we isolate the rod from the two dampers. Under these conditions there is complete reflection at the two ends of the rod and the energy which was on the rod at the time of isolation remains trapped. Now instead of describing the waves on the rod as two trains traveling in opposite directions we describe the rod as vibrating at its natural frequencies. The lowest frequency has a force node at each end and no intermediate nodes. The frequency corresponding to this mode of vibration is \( c/2l \). The next higher natural frequency has a node at each end of the rod and one in the middle and is described by \( 2c/2l \). In this way there are natural frequencies at \( 3c/2l, 4c/2l \), etc. Consider a frequency range extending from \( f \) cycles per second to \( f + \Delta f \) cycles per second, i.e. a frequency range of width \( \Delta f \). The number of modes of vibration, or degrees of freedom, lying within this range may be taken to be \( 2l\Delta f/c \), provided \( l \) is taken sufficiently large to make this expression a great number. Under this condition it is permissible to speak of the average energy per degree of freedom as a definite quantity. To each degree of freedom there corresponds an energy equal to \( k_B T \) on the average, on the basis of the equipartition law: \( 1/2k_B T \) for the open-ended modes and \( 1/2k_B T \) for the fixed-ended modes. The total energy of the vibrations within the frequency interval \( \Delta f \) is then seen to be \( 2lk_B T\Delta f/c \). But since there is no reflection this is the energy within that frequency interval which was transferred from the two dampers to the rod during the time of transit \( l/c \). The average power, transferred from each damper to the rod within the frequency interval \( \Delta f \) during the time interval \( l/c \) is therefore \( k_B T\Delta f \). Therefore the maximal power that the sensor can deliver to the medium is

\[ P_{sens} = k_B T\Delta f. \tag{6.1} \]

### 6.2.2 Energy present in the medium

In this analysis we want to determine the noise power density present in the medium. In the present case, the cavity is assumed to contain liquid with impedance \( Z_0 \) as indicated in Figure 6.1. The energy present in this cavity is determined by the thermal motion of the particles of the fluid. There are two types of atomic motion present in a liquid: phonon motion and diffusional motion [128]. The phonon motion consist of one longitudinal mode and may contain two transverse modes. Whether these transverse modes are present or not depend on the solid-like ability of liquids to support shear waves. This is indicated by the Frenkel frequency \( \omega_F \) that is minimally needed to support those waves. The second atomic motion is the diffusional motion which happens when an atom is jumping between two equilibrium positions. Taking all possible motions into account, the
total energy present in the liquid can be described as

\[ E = K_l + P_l + K_s(\omega > \omega_F) + P_s(\omega > \omega_F) + K_d + P_d, \]  

where \( K \) is the kinetic energy and \( P \) the potential energy [128]. The subscripts define the three different motions with \( l \) the longitudinal mode, \( s \) the shear modes and \( d \) the diffusing atoms. This energy equation can be simplified because the energy of the diffusional motion is so low that it can be omitted. Furthermore, the frequency range of ultrasound sensors is much lower than the Frenkel frequency of water (~10^{11} Hz at 150 MPa) [128], which allows us to omit the shear phonons as well. Hence the only relevant energy left in the liquid is the longitudinal mode of phonon motion. The energy of the longitudinal mode in a liquid contains the same energy as a longitudinal mode in solid state physics [128–130]. Because the energy of longitudinal modes in solid state physics have been extensively described [131–133], we will assume that we can use this description to determine the energy in the medium. A practical justification for this assumption may be found in the abundant use of finite element and finite difference models for computing acoustic wave fields. In these models we use regular discrete grids to describe the wave propagation through a fluid. These models are thoroughly validated for frequencies in the MHz range. We see no reason to doubt that for computing noise in the same frequency range it is equally allowed to replace wave propagation through the continuous fluid medium by wave propagation through a regular grid. The latter is exactly what is considered in solid state physics.

The energy in the medium can be obtained via the vibrational modes of the corresponding system of particles. Similar to the description of the energy trapped in the rod, we will use standing waves as sums of traveling waves to describe the vibrational modes. The simplest system grid contains cubic cells with sides \( L \). Whereas the traveling waves have positive and negative wave-vectors to describe their behavior, the standing waves consist of a superposition of two opposite traveling waves and can thus be described by only the positive wave-vectors [131]. Therefore the mode spectrum is represented in an octant of the three-dimensional \( \mathbf{k} \)-space as a simple cubic lattice with sides \( \pi / L \). The wave-vector \( \mathbf{k} \) is given by \( \mathbf{k} = \hat{i}k_x + \hat{j}k_y + \hat{l}k_z \). The distance from each point to the origin is proportional to
the resonant frequency of the mode. The number of standing wave states inside a volume element \(d^3\vec{k}\) in this octant of \(k\)-space is

\[
\rho_s(\vec{k})d^3\vec{k} = \left(\frac{L}{\pi}\right)^3 d^3\vec{k} = \frac{V}{\pi^3} d^3\vec{k},\tag{6.3}
\]

where \(V = L^3\) is the volume of the cubic cell and \(\rho_s(\vec{k})\) is the density of states for standing waves in a three-dimensional crystal [131]. If we introduce spherical coordinates \((\theta, \phi, k)\) and the number of standing wave states in a spherical shell between \(k\) and \(k + dk\) we obtain

\[
\rho_s(\theta, \phi, k)k^2 \sin \theta dk d\theta d\phi = \frac{V}{8\pi^3} k^2 \sin \theta dk d\theta d\phi = \frac{V}{8\pi^3} k^2 dk d\Omega,\tag{6.4}
\]

where the factor 1/8 is present to correct for the fact that the density is given for the entire space while in Eq.6.3 only positive \(k\) values were allowed. Moreover, \(d\Omega = \sin \theta d\theta d\phi\) is the solid angle enclosed by \(\theta\) and \(\theta + d\theta\), and \(\phi\) and \(\phi + d\phi\).

The elementary vibrational disturbance of a lattice of atoms that uniformly oscillates at a single frequency is called a phonon in the quantum mechanical description. The quantum of energy of a phonon of frequency \(f\) is \(hf\), where 

\[h = 6.626 \times 10^{-34} \text{[Js]}\]

is Planck’s constant. The total energy of a particular vibrational mode is a multiplication of the average number of phonons per mode and the energy per phonon. The average number of phonons \(\langle n \rangle\) per mode at temperature \(T\) is described by the Bose-Einstein factor as [131]

\[
\langle n \rangle = \frac{1}{e^{(hf/k_BT)} - 1}.\tag{6.5}
\]

Therefore, the average energy per mode is given by

\[
\langle E \rangle = \langle n \rangle hf = \frac{hf}{e^{(hf/k_BT)} - 1}.\tag{6.6}
\]

With the known mode density of the system and the average energy per mode we are able to determine the total energy density of the system. The energy density (energy per unit volume) \(dU\) per solid angle in a band between \(k\) and \(k + dk\) is given by the average energy per mode times the number of modes per solid angle and per unit volume as

\[
dU(\theta, \phi, k) = \frac{hf}{e^{(hf/k_BT)} - 1} \frac{k^2}{8\pi^3} dk.\tag{6.7}
\]

We assume that \(hf/k_BT \ll 1\) which is valid for frequencies below the GHz range. With this assumption the exponential \(e^x = 1 + x/1! + x^2/2! + ...\) can be approximated as \(e^{hf/k_BT} \approx 1 + hf/k_BT\). So below 1 GHz the energy density flowing within the cavity between \(f\) and \(f + df\) is

\[
dU(\theta, \phi, k) \approx k_BT \frac{k^2}{8\pi^3} dk.\tag{6.8}
\]
The intensity per solid angle of the modes in a band $dk$ is

$$d\bar{I}(\theta, \phi, k) = c_0 dU(\theta, \phi)\bar{e}_r = c_0 k_B T \frac{k^2}{8\pi^3} dk\bar{e}_r,$$

(6.9)

where $\bar{e}_r = \frac{k}{|k|}$ is the direction of the wave and $c_0$ is the speed of sound in the medium. We like to express $dP$ with dependence on frequency $f$ instead of $k$. Because $k = 2\pi f/c_0$, we obtain for the intensity

$$d\bar{I}(\theta, \phi, f) = k_B T f^2 df\bar{e}_r = \frac{k_B T}{\lambda^2} df\bar{e}_r.$$

(6.10)

To describe the sound intensity that is produced by the medium we want to introduce the term loudness $L$. This loudness (in $\text{Wm}^{-2}\text{sr}^{-1}\text{Hz}^{-1}$) is the spectral power density produced per solid angle and is thus given by

$$L(\theta, \phi, f) = \frac{k_B T}{\lambda^2}.$$

(6.11)

### 6.2.3 Radiated power from a piston

The connection between the mechanical resistor and the medium is made by the surface area of the sensor in the rigid baffle that separates both cavities in Figure 6.2. This connection has influence on the amount of energy that can travel from one side of the cavity towards the other side. From the mechanical resistor’s side towards the medium this is indicated by the mechanical radiation impedance and from the medium towards the sensor this is indicated by an effective area of the surface. The formulation of both the mechanical radiation impedance and the effective area of the surface will be derived in this section.

When we have a piston surface $S$ in a rigid baffle at $z = 0$, the energy that radiates into the medium can be determined by the Rayleigh integral. The pressure at constant frequency at any position $(x, y, z)$ in the medium is given by

$$\hat{p} = \frac{-i\omega \rho}{2\pi} \int \int_S \hat{v}_n(x_s, y_s) e^{ikR} dR,$$

(6.12)

where the hat indicates the complex amplitude of the quantity, $R = \sqrt{(x - x_S)^2 + (y - y_S)^2 + z^2}$, with $x$, $y$ and $z$ the coordinates in cartesian space with $z$ directed perpendicular to the baffle. The subscript $s$ denotes the source coordinates. The uniform normal velocity of the surface is denoted by $v_n$. We use the acoustic energy flux or acoustic intensity $\bar{I} = p\bar{v}$ to determine the time average power in the far field radiated from the source as [134]

$$P_{av} = \int \int_S \bar{I}_{av} \cdot \bar{n}_{out} dS,$$

(6.13)

where $\bar{n}_{out}$ indicates the normal vector of the surface $S$. When we write the pressure $p$ as $p = \text{Re} \{\hat{p}e^{-i\omega t}\}$ and the velocity as $\bar{v} = \text{Re} \{\hat{v}\bar{e}^{-i\omega t}\}$, the average product is given by

$$(p\bar{v})_{av} = \frac{1}{2} \text{Re} \left\{ \hat{p}\bar{v}^* \right\}.$$

(6.14)
Hence, the time average of the acoustic energy flux $\vec{I}_{av}$ is given by

$$\vec{I}_{av} = \frac{1}{2} \text{Re} \left\{ \hat{p} \hat{v}^* \right\}. \quad (6.15)$$

We can now calculate the average power in the far field as

$$P_{av} = \frac{1}{2} \text{Re} \left\{ \int_S \hat{v}^* \hat{p} dS \right\},$$

$$= \frac{|\hat{v}_n|^2}{2} \text{Re} \left\{ -\frac{i \omega \rho}{2 \pi} \int_S \int_S e^{ikR} dx_s dy_s dxdy \right\},$$

$$= \frac{|\hat{v}_n|^2}{2} \text{Re} \left\{ Z_{m,rad} \right\}, \quad (6.16)$$

where $Z_{m,rad}$ is the mechanical radiation impedance [134]. The radiation impedance is defined as the force $\hat{F}_z$ divided by the normal velocity $\hat{v}_n$. For a piston this is similar to the area integral of the specific radiation impedance $\hat{p}/\hat{v}_n$. The quadruple integral can be solved for a baffled circular piston resulting in a mechanical radiation impedance of

$$Z_{m,rad} = \frac{-i \omega \rho}{2 \pi} \int_S \int_S \int_S e^{ikz} (R dx_s dy_s dxdy),$$

$$= \pi a^2 Z_0 \left[ R_1(2ka) - iX_1(2ka) \right], \quad (6.17)$$

with

$$R_1(2ka) = 1 - \frac{2J_1(2ka)}{2ka}, \quad X_1(2ka) = \frac{2H_1(2ka)}{2ka}. \quad (6.18)$$

The $J_1(\eta)$ and $H_1(\eta)$ are the Bessel function and the Struve function of first order [134] and $a$ is the radius of the piston. So the radiated power from a circular piston with area $A = \pi a^2$ in a rigid baffle results in

$$P_{av} = \frac{|\hat{v}_n|^2}{2} A Z_0 R_1(2ka). \quad (6.19)$$

The radiated power can also be obtained from the maximum intensity $I_{max}$ and the normalized radiation pattern $P_n(\theta, \phi)$ of the circular piston as

$$P_{av} = \int_0^{2\pi} \int_0^{\pi/2} I_{max} P_n(\theta, \phi) r^2 \sin \theta d\theta d\phi. \quad (6.20)$$

The maximum intensity in the far field is given by

$$I_{max} = \frac{1}{2} \text{Re} \left\{ \hat{p}_{FF} \hat{v}_{FF}^* \right\} = \frac{|\hat{p}_{FF}|^2}{2Z_0}. \quad (6.21)$$

For the pressure in the far field we can state that $z \gg x_S$ and $z \gg y_S$. Therefore Eq.(6.12) can be approximated as

$$\hat{p}_{FF} = \frac{-i \omega \rho}{2 \pi} \hat{v}_n \int_S \int_S \frac{e^{ikz}}{z} dx_s dy_s,$$

$$= \frac{-i \omega \rho A e^{ikz}}{2\pi z} \hat{v}_n. \quad (6.22)$$
We are only interested in the modulus of the field, which results in
\[ |\hat{p}_{FF}| = \frac{Z_0 A}{\lambda z} |\hat{v}_n|. \]  
(6.23)

When we substitute Eq.(6.23) and Eq.(6.21) into Eq.(6.20), we obtain for the radiated power
\[ P_{av} = \frac{Z_0 A^2}{2\lambda^2} |\hat{v}_n|^2 \Omega_A, \]  
(6.24)

where
\[ \Omega_A = \int_0^{2\pi} \int_0^{\pi/2} P_n(\theta, \phi) \sin \theta d\theta d\phi, \]  
(6.25)
is known as the beam solid angle of the piston and is the integral over the normalized radiation pattern. Both calculated radiated powers in Eq.(6.19) and Eq.(6.24) should be the same and by equating we are able to find a relationship between the effective area of the piston, the wavelength and the radiation pattern as
\[ \lambda^2 = A_{\text{eff}} \Omega_A, \]  
(6.26)

where
\[ A_{\text{eff}} = \frac{A}{R_1}. \]  
(6.27)
The validity of this relationship can also be shown mathematically by solving the integral of the beam solid angle. With use of the far field approximation the pressure can be expressed in the form of an outgoing spherical wave and nonuniform directivity \( D(\theta, \phi) \) according to
\[ \hat{p} \approx D(\theta, \phi) \frac{e^{ikr}}{r}, \]  
(6.28)

where
\[ D(\theta, \phi) = \frac{i \omega \rho}{2\pi} \int_S \hat{v}_n(x_s, y_s) e^{-ik\vec{x}_s \cdot \vec{e}_r} dx_s dy_s, \]  
(6.29)
and contains the two-dimensional Fourier transform of \( \hat{v}_n(x_s, y_s) \). For a circular piston, \( D(\theta, \phi) \) is only dependent on \( \theta \) as \[ D(\theta) = -\frac{i \rho c \hat{v}_n k a^2}{2} \frac{2J_1(ka \sin \theta)}{ka \sin \theta}. \]  
(6.30)
Hence the normalized radiation pattern is given by
\[ P_n(\theta, \phi) = \frac{|D(\theta, \phi)|^2}{\max \{|D(\theta, \phi)|^2\}} = \frac{2J_1(ka \sin \theta)}{ka \sin \theta}. \]  
(6.31)
Therefore the effective area can be calculated with Eq.(6.26) using this relationship for the directivity
\[ A_{\text{eff}} = \frac{\lambda^2}{2\pi \int_0^{\pi/2} \left[ \frac{2J_1(ka \sin \theta)}{ka \sin \theta} \right]^2 \sin \theta d\theta}. \]  
(6.32)
This integral can be solved

\[
\int_0^{\pi/2} \left[ \frac{2 J_1(ka \sin \theta)}{ka \sin \theta} \right]^2 \sin \theta d\theta = \frac{2ka - 2J_1(2ka)}{(ka)^3},
\]  

which, in view of Eq.(6.18), results in

\[
A_{\text{eff}} = \frac{\lambda^2 (ka)^3}{2 \pi [2ka - 2J_1(2ka)]} = \frac{\pi a^2}{2} \frac{2ka}{2ka - 2J_1(2ka)} = \frac{A}{R_1(2ka)}.
\]

This confirms the validity of Eq.(6.27) for the specific case of a circular piston.

### 6.2.4 Energy collected by the sensor

In this section we want to determine the power that is maximally delivered by the medium to the sensor. The maximum delivered power to the surface area of the sensor is given by [135, 136]

\[
P_{\text{med}} = \int_{f}^{f+\Delta f} A_{\text{eff}} \int_{0}^{2\pi} \int_{0}^{\pi/2} L(\theta, \phi, f) P_n(\theta, \phi) \sin \theta d\theta d\phi df.
\]

Using Eqs.(6.11),(6.26) and (6.25) we obtain for the maximum delivered power

\[
P_{\text{med}} = \int_{f}^{f+\Delta f} \frac{\lambda^2}{\Omega_A} k_B T \int_{0}^{2\pi} \int_{0}^{\pi/2} P_n(\theta, \phi) \sin \theta d\theta d\phi df,
\]

We already mentioned that when the sensor is in full thermodynamic equilibrium with the medium the powers produced by the sensor and by the medium should be the same. We just determined the maximum power produced by the medium and collected by the sensor as \( k_B T \Delta f \). We already determined the maximum power produced by mechanical resistor of the sensor as \( k_B T \Delta f \) (Eq. (6.1)). So these two powers are indeed equal.

### 6.2.5 Noise induced pressure at the sensor

We use the derived maximum noise power generated by the sensor and by the medium to determine the noise pressure generated at the transducer surface. We draw two equivalent circuits corresponding to the two different noise mechanisms. The circuit containing the Nyquist spectral power produced by the mechanical resistor is shown in Figure 6.4a. It contains the mechanical impedance of the noiseless damper, the noise pressure source due to the force generator and the mechanical radiation impedance of the piston. The maximum noise power is delivered
when there is ideal coupling between the sensor and the medium, i.e. \( Z_{m,rad} = Z_m \). Based on this coupling we are able to determine the noise pressure source with use of the known power delivered to \( Z_{m,rad} \). This dissipated power is \( k_B T \Delta f \) (Sec 6.2) and is equal to \( P_{II} \). From this equation we can determine the noise source pressure \( \bar{p}_{sens} \) due to the sensor itself as

\[
P_{sens} = k_B T \Delta f = \frac{A^2 \bar{p}_{mem}^2}{\text{Re} \{ Z_m \}} = \frac{A^2 \bar{p}_{sens}^2}{4\text{Re} \{ Z_m \}} \tag{6.37}
\]

\[
\bar{p}_{sens} = \frac{\sqrt{4k_B T \Delta f \text{Re} \{ Z_m \}}}{A}. \tag{6.38}
\]

Here a bar is used to indicate the RMS value of a quantity.

The second equivalent circuit is shown in Figure 6.4b and contains the maximum radiation noise power delivered by the medium. Similar to the derivation above, the noise source pressure \( \bar{p}_{med} \) due to the medium is found as

\[
P_{med} = k_B T \Delta f = \frac{A^2 \bar{p}_{mem}^2}{\text{Re} \{ Z_{m,rad} \}} = \frac{A^2 \bar{p}_{med}^2}{4\text{Re} \{ Z_{m,rad} \}} \tag{6.39}
\]

\[
\bar{p}_{med} = \frac{\sqrt{4k_B T \Delta f \text{Re} \{ Z_{m,rad} \}}}{A}, \tag{6.40}
\]

The factor \( R_1 \) for piston motion is the main difference between our relation for the noise pressure of the medium and the method in the literature that just uses the specific acoustic impedance \( Z_0 \) of the medium. The influence of this factor is shown in Figure 6.5 where the noise pressure \( \bar{p}_{med}/\sqrt{\Delta f} \) is plotted against \( ka \) of our sensor. It is shown that both methods converge to the same value for elements with large \( ka \) values. This means that for transducer elements with \( ka > 2 \) the specific impedance can be used to determine the noise floor. When we have transducer elements with \( ka < 1.9 \) or \( a/\lambda < 0.3 \), both methods differ significantly. The method without factor \( R_1 \) goes to infinity while the method with the factor \( R_1 \)
converges. The limiting value can be calculated with the expansion series of the Bessel function as

\[ J_1(\eta) = \frac{\eta/2}{(1!)^2} - \frac{2(\eta/2)^3}{(2!)^2} + \frac{3(\eta/2)^5}{(3!)^2} - \cdots \]  

(6.41)

When we substitute Eq. (6.41) into Eq. (6.18) we obtain

\[ R_1(2ka) = \frac{(2ka)^2}{4 \cdot 2} - \frac{(2ka)^4}{6 \cdot 4^2 \cdot 2} + \frac{(2ka)^6}{8 \cdot 6^2 \cdot 4^2 \cdot 2} - \cdots \]  

(6.42)

For a circular piston with \( ka \to 0 \) the noise source pressure \( \bar{p}_{\text{med}} \) converges to 0.3 mPa/√Hz.

\[ \bar{p}_{\text{med}}/\sqrt{\Delta f} \text{ using } Z_m = AR_1Z_0 \]

\[ \bar{p}_{\text{med}}/\sqrt{\Delta f} \text{ using } Z_m = AZ_0 \]

Figure 6.5: The noise source pressure \( \bar{p}_{\text{med}}/\sqrt{\Delta f} \) versus \( ka \) for circular sensors with \( k \) the wavenumber and \( a \) the radius of the sensor. The black curve is the noise pressure based on the impedance of water and the surface area of the sensor while the red curve is calculated with use of an extra factor \( R_1 \) that corrects for the radiation behavior of the piston.

In reality, noise is caused by both the medium and the sensor itself. The equivalent circuit of a real case is shown in Figure 6.4c. Because the noise sources are uncorrelated we can not add the pressures but need to add the corresponding powers in the circuit. Hence the pressure at the surface \( \bar{p}_{\text{mem}} \) of the transducer is given by

\[
\bar{p}_{\text{mem}} = \sqrt{\left( \frac{\bar{p}_{\text{sens}}}{|Z_I + Z_{II}|} \right)^2 + \left( \frac{\bar{p}_{\text{med}}}{|Z_I + Z_{II}|} \right)^2},
\]

(6.43)

\[
= \sqrt{4k_BT\Delta f/A} \left( \frac{R_I|Z_{II}|^2}{|Z_I + Z_{II}|^2} + \frac{R_{II}|Z_I|^2}{|Z_I + Z_{II}|^2} \right).
\]

(6.44)
The real and imaginary parts of $Z_I$ follow from the construction of the sensor, and the real and imaginary parts of $Z_{II}$ follow from Eqs. 6.17 and 6.18. Depending on the relative magnitude of $Z_I$, we can distinguish the following cases:

- If $|Z_I| \gg |Z_{II}|$ we have a very stiff transducer, and this will result in
  \[ \bar{p}_{\text{mem}} = \bar{p}_{\text{med}}. \] (6.45)
  In this case the noise from the medium is dominant.

- If $|Z_I| \ll |Z_{II}|$ we have a very compliant transducer, and this will give
  \[ \bar{p}_{\text{mem}} = \bar{p}_{\text{sens}}. \] (6.46)
  In this case the noise from the sensor is dominant. If $ka < 1$, then
  $|Z_I| \ll |Z_{II}|$ implies that $R_I \ll R_{II}$, and we will have
  \[ \bar{p}_{\text{sens}} \ll \bar{p}_{\text{med}}. \] (6.47)

To summarize, $\bar{p}_{\text{med}}$ can be regarded as the upper limit of the total acoustical noise of a sensor.

### 6.3 Piezo-electric model

With the derived theory we are able to investigate the contribution of the acoustical noise due to the medium to the total noise of a piezo-electric ultrasound transducer. We made a finite element model that can be used to calculate the expected noise floor and mechanical radiation impedance. This section first describes the model (Sec. 6.3.1) and then presents the results (Sec.6.3.2).

#### 6.3.1 Model description

We made a 2D and 3D finite element model in PZFlex (Weidlinger Associates, UK). The 2D model is made to determine the electrical impedance of the transducer and to determine the sensitivity as there is no significant difference compared to the 3D model while having a much faster calculation time. The 3D model is used to calculate the mechanical radiation impedance. The geometry of the transducer is similar in both models and will be described first. The piezo-electric element is optimized for a resonance frequency around 0.8 MHz, which is comparable to one of our opto-acoustical sensors. The transducer has a square surface with a similar area as the circular sensor. The width $x_{\text{width}}$ of the square element is each time calculated as $x_{\text{width}} = \sqrt{\frac{a^2}{\pi}}$, with $a$ the radius of the circular sensor. In the 2D model there is an option to scale the electrode in such way that a square element is obtained. The design of the transducer is similar to the one used in Ref [92] except for the backing layer which is changed into an electrical conducting glue. The stack of the transducer starts with a 120 $\mu$m backing layer followed by a 1.6 mm piezo-electric layer and a 350 $\mu$m matching layer. The width of the entire stack $x_{\text{width}}$ is each time varied and runs from 85 $\mu$m to 2.54 mm corresponding
6.3. Piezo-electric model

to a $ka$ of 0.2 to 6 respectively. The transducer is loaded with water or void. For the electrical impedance we first determine the resonance frequency of the transducer by excitation of the piezo-electric element with a 90 ns pulse. With this short pulse we are able to excite all frequencies of interest. At resonance we determine the electrical impedance. We need to determine the sensitivity of the transducer to be able to translate the electrical impedance to a noise pressure. We transmit a 3-cycle sinusoid acoustical plane wave with center frequency of 1 MHz through the water and measure the induced voltage on the electrode of the sensor. We divide the peak to peak value of the voltage by the peak to peak value of the pressure at the surface of the transducer to find the sensitivity.

The mechanical radiation impedance is obtained with the 3D model. We used the method of Ref. [137] and excite the transducer with a 3-cycle sinusoid with 1 MHz center frequency. We compute the average pressure and the average velocity over the surface of the transducer. The modulus of the maximum of the average pressure divided by the maximum of the average velocity is computed. We calculate the relative radiation impedance by dividing this modulus by the specific acoustic impedance of water, which is 1.5 MRayl.

6.3.2 Modeling results

First we determined the electrical impedance of the transducer for several $ka$ values by changing the width of the transducer. We modeled two cases; an unloaded transducer and a water loaded transducer. The difference in the electrical impedance between the two models is the addition of a contribution due to the acoustic radiation impedance. The results are shown in Figure 6.6a. It can be seen that the curves are quite similar. At higher $ka$ values ($ka > 2$) the electrical impedance has the same order of magnitude and for lower $ka$ values ($ka \leq 2$) the electrical impedance with air loading is a little bit lower than the electrical impedance of the water loaded transducer. It can also be seen that the electrical impedance goes to infinity for small $ka$ numbers.

We also determined the relative radiation impedance in a 3D model. Figure 6.6b shows the modeling results compared to the analytical relative radiation impedance where we plotted $|Z_\text{m,rad}/\pi a^2 Z_0|$ for the analytical radiation impedance and performed the same corrections for the modeling results. The curves are rather similar. One difference between the two curves may be explained by the fact that the analytical radiation impedance is obtained for a circular piston while the surface element of the model is rectangular. Another explanation for the higher value is the fact that the element is not positioned in a rigid baffle, as is the case in the analytical solution, but has a void next to it. The last comment that can be made about the higher values for larger $ka$ is the fact that ideal piston motion is no longer occurring. In the regime where $ka > 1.6$ piston motion is no longer guaranteed as can be seen in the motion pattern of the model. For the larger $ka$ values second and even third order vibrations of the surface are visible.

We are able to determine the noise pressure of the transducer based on the electrical impedance when we know its sensitivity. We can use the Johnson-Nyquist noise
of a resistor which will result in noise voltages of

\[ V_{\text{noise}} = \sqrt{4k_B T R} \]

where \( R = \text{Re} \{ Z_{\text{elec}} \} \). With the sensitivity of the transducer in volts per pascal we can determine the equivalent noise pressure. We determined the sensitivity and the results for \( ka \leq 2 \) are listed in Table 6.1. It can be seen that the sensitivity has a maximum when \( ka = 1.4 - 1.6 \). The resulting noise pressures \( \bar{p}_{\text{mem}}/\sqrt{\Delta f} \) for a piezoelectric transducer are shown in Figure 6.7. The curve shows higher equivalent pressure values than the sensor noise pressure which is estimated at \( \bar{p}_{\text{med}} \) because we assume that a PZT transducer is relatively stiff. The differences in Figure 6.7 are expected because of the extra electrical noise in the PZT transducer. Furthermore, the trend seems similar to the curve where the specific acoustic impedance \( Z_0 \) is used without \( R_1 \) which hence goes to infinity.

6.4 Discussion

In this paper we derived the acoustical noise pressure at the surface of a transducer. We showed that for smaller elements an extra factor is needed to take radiation effects into account. This factor ensures that for sensors with element areas with a \( ka < 2 \) the medium noise converges. For a sensor with a circular surface the limit of the noise pressure due to molecular agitation in the fluid is 0.3 mPa/\( \sqrt{\text{Hz}} \). This means that from the total noise level of our sensor with \( ka < 1 \) and a bandwidth of 1 MHz, a maximum of 0.3 Pa is noise that can be assigned to noise contributions
Table 6.1: Modeling results of a piezo-electric transducer. The first column lists the \( ka \) value of the transducer, the second and third column list the electrical impedance of the loaded and unloaded transducer respectively and the last column contains the sensitivity values.

<table>
<thead>
<tr>
<th>( ka )</th>
<th>Impedance with water</th>
<th>Impedance with air</th>
<th>Sensitivity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( k \Omega )</td>
<td>( k \Omega )</td>
<td>( \mu V/Pa )</td>
</tr>
<tr>
<td>0.2</td>
<td>6980</td>
<td>2496</td>
<td>27</td>
</tr>
<tr>
<td>0.4</td>
<td>1550</td>
<td>825</td>
<td>40</td>
</tr>
<tr>
<td>0.6</td>
<td>550</td>
<td>408</td>
<td>51</td>
</tr>
<tr>
<td>0.8</td>
<td>272</td>
<td>248</td>
<td>58</td>
</tr>
<tr>
<td>1.0</td>
<td>162</td>
<td>189</td>
<td>64</td>
</tr>
<tr>
<td>1.2</td>
<td>107</td>
<td>149</td>
<td>68</td>
</tr>
<tr>
<td>1.4</td>
<td>78</td>
<td>85</td>
<td>72</td>
</tr>
<tr>
<td>1.6</td>
<td>51</td>
<td>60</td>
<td>72</td>
</tr>
<tr>
<td>1.8</td>
<td>30</td>
<td>36</td>
<td>71</td>
</tr>
<tr>
<td>2.0</td>
<td>27</td>
<td>22</td>
<td>66</td>
</tr>
</tbody>
</table>

from the medium and from the mechanical system of the sensor. Although this theory is different from the acoustical literature its derivation is partially similar to the one used in radio astronomy. In radio astronomy antennas are used to detect signals from outside the atmosphere of the earth to study celestial objects at the radio frequencies. Noise is an important factor in this field and we followed the same methodology to derive the theory. Based on thermodynamic equilibrium noise contributions of the antenna and environment are determined and matched. Because antennas often have no clear physical area they use the effective area as measure for the effectiveness of the antenna. We used the effective area in the same manner to take piston motion into account which influences the effectiveness of the surface element.

It is clear that the derived noise level is of importance for mechanical sensors without electrical noise. To relate the noise pressure of mechanical sensors to noise pressures of sensors with an electrical system we modeled a PZT-transducer with thickness mode vibrations. The results suggest that the contribution from electrical noise in small piezo-electric transducers is dominant. It is also shown that when the corresponding noise pressures are obtained, the electrical impedance yields a trend similar to the acoustical impedance without the \( R_1 \) factor, but with higher values.

We may conclude from the noise pressure curve of piezo-electric transducers that there is no use in building piezo-electric sensors with \( ka < 1 \) as the noise pressure level goes rapidly up due to the increase of the dominant electrical impedance. An advantage of opto-mechanical transducers is that these devices are still able to detect low noise pressures while having surface elements with very low \( ka \) values.
Figure 6.7: The noise pressure levels versus $ka$ of the example PZT transducer. The black curve shows the acoustical noise pressure from the medium, as based on the impedance of the water and surface area of the sensor; the red curve is the acoustical noise pressure calculated with use of an extra factor $R_1$ that corrects for piston motion and the magenta curve is the total noise pressure obtained with the finite element model. This noise pressure is based on the electrical impedance of the transducer and hence contains the electrical noise and the acoustical noise. The sensitivity of the transducer is used to determine the corresponding noise pressure.
We demonstrated that silicon micro-ring resonators are sensitive to strain and when they are integrated on an acoustical membrane can be used to sense ultrasound. In this thesis we mainly focused on the use of this sensor in medical applications, but the sensor can also be optimized for other applications such as the oil and gas markets. In medical procedures it is an advantage that the sensor is not susceptible to electromagnetic disturbance and hence can be used in combination with MRI. An other important benefit is the safety during medical procedures. Because the sensor is not electrically driven, safety concerns like high voltages or leakage currents are not present. A main benefit in the fabrication process of the sensor is the use of CMOS technologies that allow for ease of production and do not pose design limitations on the device.

In this section we will discuss the feasibility (Sec. 7.1) and prospects (Sec. 7.2, Sec. 7.3) of this ultrasound sensor in medical diagnostics. We will also discuss the use as temperature sensor (Sec. 7.4) and static pressure sensor (Sec. 7.5). We finalize this Chapter with a discussion about the noise pressure levels of small sensors (Sec. 7.6).

### 7.1 Feasibility of the sensor

We designed, fabricated and characterized an ultrasound sensor that is based on integrated photonics. The prototype consist of a 220 nm high silicon photonic ring-resonator (spatial footprint: 10 µm by 50 µm) integrated on an acoustically tuned silicon dioxide circular membrane (124 µm diameter, 2.5 µm thick). The ultrasound sensor has a resonance frequency of 0.76 MHz with a -6 dB bandwidth of 20 %. The noise equivalent pressure of 0.4 Pa is slightly better than the state-of-the-art ultrasound transducers with the same resonance frequency, while these
have a 65 times larger spatial footprint (Sec. 4.3). The vibrometer measurements (Sec. 5.5.3) show that the optical intensity modulation corresponds with the oscillation of the membrane and that there is no significant oscillation present in the silicon chip. Furthermore it is shown that the fabrication process is reproducible, hence a second sensor with similar membrane diameter will have a similar resonance frequency.

The sensor is interrogated by use of a laser and a photoreceiver. The wavelength of the laser is chosen such that it is positioned on the flank of a resonance dip of the ring resonator. In this way, a shift in optical resonance translates into a modulation of the light intensity at that particular wavelength. The noise equivalent pressure of 0.4 Pa corresponds to a minimal detectable wavelength shift of 29 fm. As this is the first prototype there is a lot of room for improvement of the sensor for each specific application. The most important adjustment, needed for almost every application, is addition of absorbing layers or other adjustments to the membrane to increase the bandwidth. Improvement of the bandwidth directly means that the Q-factor of the sensor has to be decreased, i.e. the increase in bandwidth is at the cost of its sensitivity. But there is also room to increase the sensitivity of the sensor. As a first step, the ring resonator can be optimized to the strain distribution of the membrane. One can think of racetracks positioned in a smaller area in the center with on average higher strain values or on the opposite by using larger rings exactly positioned on the edge of the membrane. Furthermore it is possible to enhance the induced strain in the membrane by optimizing the design of the membrane itself. By locally adding or removing layers of the membrane it may be possible to design a shape that contains areas that act as a mechanical strain amplifier.

When the application allows a lower sensitivity than the current 0.4 Pa, there is also the option of increasing the bandwidth by using the frequencies below the resonance frequency of the sensor. The curve of the transfer function (Fig. 4.5) is much flatter below the resonance frequency, resulting in a broader bandwidth. With a value of -17 dB this corresponds to a noise level of 2.8 Pa.

The resonance frequency of the current OMUS is comparable with that of the photo-acoustic breast tomography sensor used as comparison in Chapter 4, but needs to be increased for almost all other medical applications. This can be achieved by further reduction of the membrane diameter. Reducing the radius of the membrane results in an increase of the resonance frequency as is shown in Sec. 2.2.2 for a membrane diameter of 100 µm with the higher resonance frequency of 1.05 MHz. Other options to increase the resonance frequency are an stiffer membrane or different membrane shape.

The dynamic range of the OMUS is limited to low pressures as is shown in Figure 4.6 by the deviation from the linear curve. The vibrometer measurements show that this limitation is due to the interrogator and not due to a nonlinear response of the membrane (Sec. 5.5.3). Hence the dynamic range of the sensor could be drastically increased when we use a similar interrogator system as the one used by Rosenthal et al [68], where the resonance dip is tracked instead of the intensity modulations at a single wavelength.

We can conclude that the OMUS is applicable as ultrasound sensor. Even while
this sensor is not optimized for a specific application, but designed to be comparable to models and to assess robustness with fabrication, it is already very sensitive with a noise equivalent pressure of 0.4 Pa. The sensor can easily be improved for a specific application and has enough options as described above to implement these improvements.

7.2 Matrix probe

There are two medical imaging applications where the OMUS has most added value in view of the design requirements of the probe. The first application is TransEsophageal Echocardiography (TEE) in which the heart is imaged with a probe located inside the esophagus. In ideal situations the probe is inserted via the nose, which limits the volume of the head of the probe to around 1 cm$^3$ [138]. The second application is Intravascular Ultrasound (IVUS) where an ultrasound probe mounted on a catheter is inserted into the coronary artery to image the artery wall and determine the amount of atheromatous plaque. In this case the outer diameter of the probe is limited to 1 mm [139].

In both cases the spatial footprint of the probe is limited, while a dense matrix array is required to obtain real-time 3D images [4]. There are many different ways to design such an array, but Raghunathan et al. [138] show that a matrix design with a good radiation pattern and fast acquisition options consist of a few transmit elements, located in the middle of the array, and many receiver elements around. By repeating one sensor of the OMUS we should be able to build a very sensitive matrix sensor with high potential for imaging applications in IVUS and TEE. With TEE we have the option to build this array of sensors as TEE probe and transmit the acoustical pulse from the outside with a conventional ultrasound transducer. The other option, which is also needed for IVUS, is to integrate a few piezoelectric elements on the chip to transmit the beam.

The array should contain an incoming bus waveguide with passive optical demultiplexers that convert the wide spectrum of each bus waveguide into many small spectrum channels. We can obtain the spectrum for each ring resonator with grat ing couplers [76]. These grating coupled ring resonators allow coupling of only one particular resonance peak per ring. The spectrum of the outgoing bus waveguide is created with a multiplexer that successively stacks the modulated spectra of as many sensors as possible back into the available wide spectrum. The membranes positioned underneath the ring resonators can be created using dry etching or other techniques [84]. The current interrogator system should be changed into one that is capable of tracking the different resonant dips of all the successive ring resonators. Hence the ideal interrogator is capable to interrogate one or more channels where each channel contains several peaks. Technobis (Alkmaar, The Netherlands) already fabricated an interrogator named the LadyGator that is capable of simultaneously interrogating 6 channels with a resolution of 1 fm wavelength shift and maximum interrogation speed of 2 MHz.

When we have an array of sensors we are able to solve the limited bandwidth of the current sensor by designing membranes with successive resonance frequencies
as is done in some pMUT designs [62, 140, 141]. Although the integrated photonics and membranes are small enough to meet the design requirements of a possible matrix array, the current packaging is not. It is possible to package the chip with use of a fiber facet connected directly to the waveguide that fits the outer dimensions for IVUS, as is shown by Rosenthal et al. [68].

### 7.3 Photoacoustics

Another interesting field of application is photoacoustics. Not only can photoacoustics be used to image, but it can also provide functional information in the form of blood oxygenation, blood flow and temperature [142, 143]. It combines the high-contrast and spectroscopic-based specificity of optical imaging with the high spatial resolution of ultrasound imaging, and has a greater penetration depth than purely optical imaging modalities [143]. With this technique tissue is irradiated with short light pulses. Due to the transient thermo-elastic expansion of the irradiated tissue, wideband ultrasonic waves are generated. The frequency range of photoacoustic waves is inversely proportional to the dimensions of the absorbing volume. The local optical energy deposition is wavelength dependent due to the wavelength dependent absorption coefficient of tissue and determines the magnitude of the wave. The photoacoustic waves hence contain physiologically specific information which is not present in normal ultrasound images. Due to the ANSI standards [144] only a limited amount of energy can be deposited in the human body, which results in a maximum inducible pressure. This is different from ultrasound images where the pressure of the acoustic wave is set by the transducer. Normal pressure with photoacoustics are in the order of kPa due to an increase in temperature in the range of mK [5, 145]. This pressure can be easily detected with our sensor.

Only the addition of a light source is required to use our OMUS for photoacoustic imaging. Polymer ring resonators as well as a Fabry-Perot optical interferometers have already been tested for future use in photoacoustics [146, 147]. They both use external sources and illuminate the object from a different position than the sensor. In some imaging systems this light source is separated from the sensing device [143], but for other applications like intravascular photoacoustics we need to integrate the light source with the sensor. The absorption spectrum of the tissue of interest has its wavelength between 400 nm and 1300 nm for lipid, collagen and elastin [145]. Therefore the silicon waveguide can not be used as light source because it only supports propagation of light waves with wavelengths around 1.55 µm [15]. Hence an additional fiber is needed to radiate the light. From the integration of an optical fiber and a state-of-the-art IVUS transducer [142, 148, 149] it can be concluded that it is possible to integrate our sensor with a light source in the same manner.
7.4 Temperature sensor

We studied the response of the sensor to temperature. This behavior is well described in literature and our measurements show similar results. The optical waveguide is very sensitive to temperature changes with an optical wavelength shift around 80 pm/°C. With its linear temperature response, the sensor can be used for accurate measurements of changes in temperature.

The sensitivity to temperature changes may be of interest in the field of hyperthermia therapy where specific parts of the body are heated to kill or damage tumor tissue or to make this more susceptible to ionizing radiation or chemotherapy [150, 151]. When ultrasound is used to monitor the temperature, our sensor can be used, next to the imaging, for extra local measurement of the temperature at the position of the sensor. Because the temperature during treatment changes slowly compared to the fast ultrasound waves, the slow translation of the resonance dip can be used to determine the change in temperature and the fast modulation on this translation can be applied to image ultrasound pressure waves.

7.5 Strain sensor and pressure sensor

We studied the effect of pure bending on the optical racetrack resonators. We found that three different physical aspects have influence on the final shift in resonance wavelength: elongation of the waveguide, change in effective index of the waveguide, and dispersion. The effect of the strain-induced increase in track circumference and the effect of the strain-induced change in waveguide effective index oppose each other, where the first effect is three times larger than the latter. The strong dispersion in silicon sub-wavelength waveguides (400 nm by 220 nm) accounts for a decrease in sensitivity of about a factor two. The shift in resonance wavelength per applied strain is dependent on the width of the waveguide and the orientation of the silicon crystal and varies between 0.5 and 0.75 pm/microstrain for infrared light around 1550 nm wavelength. We have studied elongations up to 275 microstrain, and observed a linear relation between the resonance wavelength and the applied strain, which makes the silicon racetrack resonators suitable as strain sensors.

With the knowledge of this linear behavior we investigated the response of the sensor to a static loading. Therefore we created a membrane around one of the racetrack resonators. This membrane changed the circumstances of the measurements completely. First, the case of pure bending has changed into an applied strain due to deformation of the membrane under static loading. This means that the strain is present in the radial direction of the membrane and hence is no longer aligned with the direction of the racetrack. Second, we found that the chip has internal strain. A part of this strain is released by the initial deformation of the membrane. With the static measurements, we showed that these two effects have a major influence on the measurement results as they describe a non-linear response to the applied pressure. Hence our sensor is at this moment less suited as pressure sensor. We know from literature that linear responses can be obtained when thicker membranes are used [120, 152].
7.6 Noise pressure level of sensors with a small spatial footprint

We wanted to determine the absolute noise floor of our newly developed optical micro-machined ultrasound sensor. We already measured a noise pressure of 0.4 Pa and found that this is a much lower value than the theoretical noise pressure levels obtained from literature, which go to infinity when the radius of the membrane goes to zero. Therefore we derived a theory to determine the acoustical noise for sensors with a small spatial footprint. With this theory we showed that the acoustical noise pressure levels of all sensors converge for a vanishing aperture. The difference between our theory and the one described in literature is an extra factor that accounts for the radiation effects of the surface. When we calculate the acoustical noise floor for our sensor using a bandwidth of 20%, we obtain a maximum pressure level of 0.13 Pa that can be assigned to noise contributions from the medium and from the sensor itself. This means that with our first sensor design we are already able to measure a pressure level that is very close to the pressure level of internal noise contribution values from the sensor and surroundings. With further optimization of the interrogating system it may be possible to reach this acoustical noise contribution level. We also showed that the acoustical noise pressure level can never be obtained with a piezo-electric sensor with similar $ka$ values as the electrical impedance goes to infinity when the aperture becomes zero, i.e. $ka \rightarrow 0$. As a result these optical micro-machined sensors can uniquely be used to detect pressure waves using apertures with a small diameter compared to the wavelength of the pressure wave.


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Suzanne M. Leinders was born in The Hague, the Netherlands on November 15, 1983. She followed secondary school (Gymnasium) at Christelijk Gymnasium Sorghvliet (The Hague, the Netherlands) and graduated in 2002. She went to Delft University of Technology to study Applied Physics. She interrupted her studies for a year in 2005 to take a full-time position as orchestra commissioner in the board of Krashna Musika. After this year she continued her studies in combination with a position in the board of Delft Challenge as skipper of a sailing team. She sailed several sailing regattas including the Tour the France à la voile in 2007 and 2008. After this experience she focused on her studies and started the master programme. During her master she went for an internship at Philips (Eindhoven, the Netherlands) to measure on the skin and skin structures using 3D Polarization sensitive optical coherence tomography. Her M.Sc. graduation project was a collaboration between the Daniel den Hoed kliniek and the department of radiation science & technology. The project focused on the possible gain in adaptive radiotherapy for liver stereotactic body radiation therapy (SBRT) compared with the clinical protocol. She received her Master degree in 2010. After her graduation she was employed as a scientific researcher at the Daniel den Hoed kliniek to publish on her graduation work. In March 2011, she started her Ph.D. research on the design, fabrication and validation of an ultrasound sensor based on integrated photonics. This research led to this Thesis.
Publications of the author

Papers in preparation


Journal publications


**Proceedings**


**Conference contributions**


• S. M. Leinders, W. J. Westerveld (*presenting*), “Sensitive optical micro-machined ultrasound sensors based on a silicon photonic ring resonator on an acoustical membrane,” in *EMN Meeting on Photonics*, Barcelona, Spain, September 2016 (*invited*)


