“Modeling and design of origami mechanisms with compliant facets.”

Jelle Rommers

Report no : MSD 2015.004
Coach : Giuseppe Radaelli
Professor : Just Herder
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Modeling and design of origami mechanisms with compliant facets

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J. Rommers

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Board of examiners:
prof. dr. ir. J.L. Herder, PME, 3ME, TU Delft
i.r. G. Radaelli, PME, 3ME, TU Delft
dr. ir. J.J. van den Dobbelsteen, BMechE, TU Delft
dr. ir. I. Buijnsters, PME, 3ME, TU Delft

Faculty of Mechanical, Maritime and Materials Engineering (3mE) · Delft University of Technology
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The subject of this thesis project is an origami inspired mechanism with an interesting working principle, which was designed for a course in the Master curriculum. During the project, the exciting possibilities of this new principle have been explored and efforts have been made to make it more generally applicable.

In the back of this report you will find a prototype of the mechanism which is analyzed in the first paper. The prototype is meant to provide additional understanding. It was possible to include the prototype because of possibly the most interesting property of origami mechanisms: their inherent deployability.

I would like to thank my supervisors Just Herder and Giuseppe Radaelli for their support by always providing new perspectives and inspiration, which motivated me a lot.

Jelle Rommers, March 2015
Introduction

Origami is the Japanese art of paper folding, where ori means folding and kami means paper. Origami is encountered often in daily life. Think about a folded milk carton or a road map which can be unfolded by one single pulling motion on opposite corners. This last example shows that a few simple folding motions can result in a complex kinematic system.

In the past decades, the application of mathematics has led to increasingly complex origami designs. Efforts have been done to understand the underlying kinematic principles, and design tools like Treemaker© are developed which automatically produce the origami pattern for almost every user-defined 3D shape. Principles from origami mathematics have been used to solve engineering issues, for example in the case of airbag designs [1].

More recently, there has been an increased interest in origami from a mechanism design perspective [2, 3, 4, 5, 6]. In these origami mechanisms, the paper is replaced by more common engineering materials and the creases are replaced by joints. Advantages are the inherent deployability and the potentially low-cost planar fabrication method.

In the major part of literature, the facets (plate material between the joints) are designed as stiff elements, whilst flexibility of these facets can be used to the designers advantage.

The goal of this thesis project is to explore and generalize the principle of using flexible facets to incorporate spring behavior in origami mechanisms.

The report consists of three papers which contain the main work, supplemented by appendices that provide additional details. In the first paper, a model which describes the spring behavior of the bending facets for a common origami mechanism is proposed. In the second paper, this model is used to develop a design tool and example designs are presented. The design tool is used in the third paper to design a more complex mechanism. The introductions of the three papers are all written from a different perspective on origami mechanisms.
Chapter 1

Paper: A Pseudo-Rigid Body model of a Single Vertex Compliant Facet Origami Mechanism (SV-COFOM)
ABSTRACT

Recently, there has been an increased interest in origami art from a mechanism design perspective. The deployable nature and the planar fabrication method inherent to origami provide potential for space and cost efficient mechanisms. In this paper, a novel type of origami mechanisms is proposed in which the compliance of the facets is used to incorporate spring behavior: Compliant Facet Origami Mechanisms (COFOMs). A simple model that computes the moment characteristic of a Single Vertex COFOM has been proposed, using a semi-spatial version of the Pseudo-Rigid Body (PRB) theory to model bending of the facets. The performance of this PRB model has been evaluated numerically and experimentally, and showed performance comparable to a Finite Element model with 122 elements. The PRB model is a potential starting point for a design tool which would provide an intuitive way of designing this type of mechanisms including their spring behavior, with very low computational costs.

1 INTRODUCTION

Origami, the art of paper folding, has inspired the engineering community for decades. Applications range from solutions in packaging, to stiff sandwich panels with an origami inspired core. Recently there has been an increased interest in origami from a mechanism design perspective. In this view, the paper is replaced by plate material and the creases by hinges. These hinges are usually constructed by introducing a flexible material, categorizing the origami mechanisms as compliant mechanisms. We will refer to these surrogate creases as hinge lines. The plate material between those hinge lines will be referred to as facets. Mainly action origami patterns are of interest, which are designed to exhibit motion in the deployed state. Existing origami patterns have been converted to mechanical designs, like a solar array that can be stowed in a square satellite and deployed in space [1] or a stent that can be deployed in the desired place in an artery [2]. Advantages of origami mechanisms are the deployable nature and the 2D fabrication method which produces 3D mechanisms.

Existing literature tries to bridge the gap between origami art and engineering, mainly from a kinematic perspective. Greenberg et al. [3] used graph theory to display the different coupled mechanisms of which origami models consist. Bowen et al. [4] view each intersection of folding lines in an origami pattern as the origin of a spherical mechanism. The authors investigated approximately 130 action origami patterns and categorized those in different chains and networks of connected spherical mechanisms. Wilcox et al. [5] brought this categorization a step further towards mechanism design by proposing subcategories based on observed input/output motions and efficient placement of potential actuators.

Literature in which the facets in origami mechanisms are considered as compliant instead of rigid, is very scarce. Schenk and Guest [6] investigated the macro-scale deformation modes of Folded Textured Sheets, incorporating the bending of facets. Tachi [7] addressed the same topic with emphasis on design. Saito et al. [8] presented a theoretical design of a self-deployable rigid origami tesselated sheet which uses strain energy from one compliant facet.

In current research the flexibility of the facets is primarily seen as an unwanted side-effect, whilst this property can be used to the designers advantage. Compliant facets can function as springs, adding a fundamental attribute to origami mechanisms. The very scarce literature on origami with compliant facets only deals with small deformations and macro-scale behavior of lattices, instead of origami from a mechanism perspective.

Subject of this paper is the basic origami mechanism which
The mechanism in Fig. 2 is an example of a novel type of origami mechanisms where the flexibility of the facets is used to incorporate spring behavior. We will call these Compliant-Facet Origami Mechanisms, abbreviated as COFOMs. Analog to the categorization of Bowen et al. we will categorize the mechanism of interest as a Single Vertex COFOM, or SV-COFOM.

The goal of this paper is to propose and validate a simple yet accurate model that computes the moment curve of the mechanism in Fig. 2 as a function of the indicated $\theta_{\text{joint}}$. The main challenge is to describe the spring behavior of the bending facets. Such a simple model could be the starting point for a design tool which would provide an intuitive and insightful way of designing with low computational costs.

In the Methods section, the model will be proposed and the evaluation procedure will be explained. Subsequently, the results of this evaluation will be stated and discussed. Finally, conclusions will be presented.

2 METHODS

The approach to construct the model which computes the moment curve of the SV-COFOM will be analog to the Pseudo-Rigid Body (PRB) model theory from L.L. Howell [9]. Applying this theory will lead to a parametric formula with two unknowns, the model parameters. These will be used to fit the formula on data from a Finite Element (FE) model for a defined standard design of the mechanism. With the two obtained model parameters, the model will be tested by comparing it to a FE model for altered designs of the mechanism. The FE model will be validated experimentally.

2.1 PRB model

The Pseudo-Rigid Body model is normally used as a simple model to describe the large deflection of beams. A beam is approximated by two rigid beams connected by a torsion spring. The location and stiffness of this torsion spring are unknown and are used to fit the PRB model on existing force and deflection
FIGURE 3. Pseudo-Rigid Body (PRB) model of the clamped SV-COFOM. Bending of the bottom facets is modeled by introducing virtual hinge lines (dashed lines) with a torsion spring, dividing both compliant facets in two rigid ones. Point \( C_m \) is constrained to lie in the XZ-plane, creating a 1 DOF mechanism. By writing the angular rotation of the torsional springs \( \tau_B \) as a function of \( \theta_{\text{joint}} \), the moment curve of the mechanism can be constructed.

Data.

Figure 3 shows how PRB theory can be applied to the bending of the bottom facets of the SV-COFOM. The left figure shows the mechanism in flat folded state. Bending of the facets is approximated by introducing virtual hinge lines (dashed) with virtual torsion springs. The modeled mechanism now exists out of six rigid facets. The angular rotation of the virtual springs \( \tau_B \) is denoted by \( \tau_B \). This angle is defined to be zero when the bottom facets align, corresponding to the unbent facets in the real model. The clamped PRB model has two DOF. A constraint will be added by constraining point \( C_m \) to be in the XZ-plane. In the Discussion, we will reflect on this added constraint. Because of symmetry, only the right half of the SV-COFOM needs to be modeled. The resulting moment curve will be multiplied by two afterwards.

**Parametric formula.** The torsional stiffness in the real hinge lines is assumed to be zero and gravity is neglected, making the virtual torsion springs the only attribute capable of storing strain energy. To construct the parametric formula from the PRB representation in figure 3, we need to find the relation between the angular rotation of the virtual hinge lines \( \tau_B \) and the joint angle \( \theta_{\text{joint}} \). Once we have found this relation, the moment curve can be constructed by first computing the strain energy in one spring:

\[
V_{\text{spring}}(\theta_{\text{joint}}) = \frac{1}{2} \kappa_{\text{virtual}} \tau_B(\theta_{\text{joint}})^2
\]  

And secondly taking the derivative of the energy of the two springs with respect to the angle of the SV-COFOM, resulting in the moment curve:

\[
M_{\text{PRB}}(\theta_{\text{joint}}) = 2 \cdot \frac{\partial V_{\text{spring}}(\theta_{\text{joint}})}{\partial \theta_{\text{joint}}}
\]  

Note that the energy is multiplied by two because two virtual springs exist in the mechanism.

**Model parameters.** The torsion stiffness of the virtual hinge line is defined as a torsion constant times the length of this hinge line:

\[
\kappa_{\text{virtual}} = \xi_1 E^* t^3 \ast \frac{w}{2 \sin(\theta_B)}
\]  

Where \( \xi_1 \) is the first model parameter. The angle of the virtual hinge line \( \theta_B \) is defined to be half way between \( \theta_A \) and the bottom vertical hinge line in figure 3, plus the second model parameter \( \xi_2 \):

\[
\theta_B = \frac{\pi + \theta_A}{2} + \xi_2
\]

These model parameters are unknown and will be obtained by fitting the PRB model on data from a FE model for a defined standard design of the SV-COFOM.

**Kinematic relation between \( \tau_B \) and \( \theta_{\text{joint}} \).** In order to find the relation between the angular rotation of the virtual hinge line \( \tau_B \) and the angle \( \theta_{\text{joint}} \), the PRB model of figure 3 is viewed as a spherical mechanism. Figure 4 shows that the intersection of the real and virtual hinge lines forms the origin of a virtual sphere. In this sphere, a circular cutout of the right half of the mechanism is drawn, as indicated in the inserted figure in the top right. Points A, B, and C in the figure move on the surface of this sphere. Note that point C is constrained to be in the XZ plane, as defined in the PRB model. Point B is fixed, because the bottom of the SV-COFOM is fixed. Point B’ is meant to aid the calculation and is constructed by rotating point B at an angle of \( \theta_{\text{foot}}/2 \) around the z-axis, positioning it in the xz-plane.

Note that the arcs in Fig. 4 correspond with the angles in
FIGURE 4. Spherical representation of the PRB model. Only the right half of the facets are drawn, as indicated in the inserted figure in the top right. Goal is to calculate the angular rotation of the virtual hinge line \( \tau_B \) as a function of \( \theta_{\text{joint}} \).

Fig. 3 as:

\[
\begin{align*}
\widehat{AC} &= \theta_A \\
\widehat{AB} &= \theta_B - \theta_A \\
\widehat{BC'} &= \widehat{B'C'} = \theta_B
\end{align*}
\]

First we calculate the arc on the unit sphere between point \( B \) and point \( B' \):

\[
\sin(\widehat{BB'}) = 2 \sin(\theta_{\text{foot}}/4) \sin(\theta_B)
\]

Note that \( \widehat{BB'} \neq \theta_{\text{foot}}/2 \) because that arc would not be on the virtual sphere.

Arc \( BC \) (not drawn) will first be calculated. Using the cosine rule for spherical systems we can write:

\[
cos(\angle B'BC) = \frac{\cos(\theta_B) - \cos(\theta_B) \cos(\widehat{BB'})}{\sin(\theta_B) \sin(\widehat{BB'})}
\]

By substituting (5) in (6) and rearranging, an explicit expression for \( \widehat{BC} \) can be obtained in a straightforward manner. Again using the cosine rule for spherical systems:

\[
cos(\angle ABC) = \frac{\cos(\theta_A) - \cos(\widehat{BC}) \cos(\theta_B - \theta_A)}{\sin(\widehat{BC}) \sin(\theta_B - \theta_A)}
\]

\[
cos(\angle CBC') = \frac{\cos(\theta_{\text{joint}}) - \cos(\widehat{BC}) \cos(\theta_B)}{\sin(\widehat{BC}) \sin(\theta_B)}
\]

Note that \( \tau_B \) is the angle between the arcs \( \widehat{AB} \) and \( \widehat{B'C'} \). The rotation of the virtual hinge line can now be calculated as:

\[
\tau_B(\theta_{\text{joint}}) = \angle ABC - \angle CBC'
\]

Note that this is one of the two possible solutions for the angle \( \tau_B \). In the second solution, the angles in (9) would be added instead of subtracted. This would mean that the facets would be bending inwards instead of outwards. The formula is checked on errors by drawing the PRB model with a Computer Aided Design package and measuring the angles between the facets for different values of \( \theta_{\text{joint}} \).

2.2 Fitting and evaluation of the PRB model

The two model parameters \( \xi_1 \) and \( \xi_2 \) will be obtained by fitting the moment curve of the PRB model on data from a FE model for a defined standard design of the SV-COFOM. Fitting will be done by minimizing the Root Mean Squared Error (RMSE) between the two models:

\[
RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (M_{\text{PRB},i} - M_{\text{FEM},i})^2}
\]
The standard design is defined by the following standard design variable set, also depicted in figures 1 and 2:

\[
\begin{bmatrix}
\theta_A
\
\theta_{foot}
\
w
\xi
\end{bmatrix} = \begin{bmatrix}
50 \text{ [deg]}
\
60 \text{ [deg]}
\
150 \text{ [mm]}
\
0.3 \text{ [mm]}
\
193 \text{ [GPa]}
\end{bmatrix}
\]

Note that the parametric formula is not dependent on the height \( h \) of the SV-COFOM and the Poisson ratio of the material. In the standard design, the height is chosen as 300 mm and the Poisson ratio as 0.24. The effect of varying the height and Poisson ratio on the moment curve will be investigated with the FE model.

Next, the prediction performance of the PRB model with the obtained model parameters will be evaluated. This is done by comparing the PRB model to the FE model for deviating designs of the SV-COFOM. The deviating designs are constructed by varying one of the standard design variables at a time minus and plus a third of the original value (the values of the angles \( \theta_A \) are rounded to 30 deg and 70 deg). Changing the E-modulus will not be tested because this only affects the moment curve by multiplication, in both the PRB and FE model. The performance of the PRB model is quantified by the RMSE between the PRB and the FE model data.

To show the extend to which the PRB model could be improved by making the model parameters dependent on the design variables, the PRB model will be refitted on the FE data of each separate design variable set. This will result in new model parameters and RMSE values for each set.

To avoid stress concentrations in the region where the hinge lines of the SV-COFOM intersect, we will assume that there is a circular cutout around this point with radius 15 mm. This cutout is expected to have a non-negligible influence on the moment curve. However, it is not included in the PRB model. The reason for this is that the purpose of the model is to aid in a design process. It is anticipated that the radius will only have an effect on the stiffness of the SV-COFOM, i.e. it will serve as a multiplication factor to the moment curve. This stiffness can already be manipulated by changing the width, E-modulus or thickness. The designer will most probably minimize the radius, depending on the material properties. To validate the assumption that the cutout radius mainly has effect on the stiffness of the SV-COFOM, moment curves for cutout radii of 10 mm and 20 mm will be computed using the FE model.

2.3 Finite Element Model

The SV-COFOM is modeled in the Finite Element software package Ansys. The facets of the mechanism are modeled as slender plates composed of Shell63 elements with the nonlinear geometry option enabled. The elements are based on Kirchhoff-Love theory and have four nodes with 6 DOF.

Hinge lines are introduced by connecting nodes at the edges of the facets. Two connected nodes are constrained to have the same displacements, but are free in all relative rotations. Only the right half of the mechanism is modeled because of symmetry, to save computational power. The vertical hinge lines are constrained to be in the XZ plane of Fig. 2. A rotation is applied to the end of the mechanism in steps to construct the reaction moment curve. The simulation starts at the maximum joint angle (the last drawing in Fig. 2) to avoid the buckling point at \( \theta_{joint} = 0 \). In the last steps approaching this point, the FE model will not be able to find a solution. This data will be omitted. Gravity forces are not incorporated in the FE model.

The FE model will be run for the standard design using 121, 288, 1006 and 4020 elements. The model with 1006 elements is assumed to be ‘valid’ and this number of elements will be used throughout the paper to fit and test the PRB model. This validity assumption is tested by comparing to empirical data. The models with the other numbers of elements are used to test if the model with 1006 elements can be considered to be converged. Also, they serve as benchmarks to compare the performance of the PRB model.

2.4 Experimental setup

The results of the FE model for the standard design of the SV-COFOM are validated experimentally. The mechanism is fabricated using spring steel AISI 304 plates as facets. Hinge lines are introduced by applying Mylar© tape with pressure sensitive adhesive in an alternating pattern, as in Fig. 5.

The experimental setup consists of a load cell attached to a rotating arm as in Fig. 6. The mechanism is connected to the load cell via a rod that can freely move parallel to the arm, such that mainly forces transverse to the arm are measured. The rotation of the arm is measured by a potentiometer. The arm is moved up and downwards manually ten times to record the moment curve.

The empirical data will be compared to the FE model by computing the RMSE. The empirical data will be averaged over the ten consecutive up and down motions before the RMSE is computed. This way, the average of the hysteresis loop is obtained. The raw data will be plotted to show the spread of the measurements and the magnitude of the hysteresis loop.

3 RESULTS

Figure 7 shows the moment curve of the SV-COFOM with the standard design variables set. The PRB model (blue solid line) is fitted on the FE model by varying the model parameters. The optimized model parameters are \( \xi_1 = 0.994 \text{ m}^2 \) and \( \xi_2 = 16.9 \text{ deg} \). This gives a RMSE of 1.0 \( \text{e-2 Nm} \) between the two models. Positive values of the reaction moment curve indicate that the mech-
FIGURE 5. Fabrication of the SV-COFOM. Hinge lines are created by applying Mylar® tape between two spring steel plates in an alternating pattern.

FIGURE 6. Experimental setup. The moment curve of the clamped SV-COFOM is recorded using a load cell and a potentiometer.

The mechanism tends to snap to the state $\theta_{\text{joint}} = 0$. Note that the curve does not start at zero because the FE model was not capable of handling the buckling behavior at this point.

The prediction performance of the PRB model with the obtained model parameters $\xi_1 = 0.994 \, m^{-2}$ and $\xi_2 = 16.9 \, \text{deg}$ is shown in Fig. 8. The PRB model is compared to the FE model for designs of the SV-COFOM that deviate from the standard design. The RMSE values between the two models are shown in the second column of Table 1. In this table, also the RMSE values for the refitted case are given, with the corresponding optimized model parameters.

Figure 9 shows the effect of varying the height of the mechanism. Changing this design variable from 300 mm to 200 mm and 400 mm, results in RMSE values of 2.3 $\times$ 10^{-2} Nm and 1.7 $\times$ 10^{-2} Nm, respectively. Varying the Poisson ratio from 24 to 0.4 results in a RMSE value of 1.5 $\times$ 10^{-2} Nm. Figure 10 shows FE data of the effect of a changed cutout radius.

The convergence of the FEM solution with respect to the number of shell elements is shown in Table 3. The FEM data is constructed for the standard design of the SV-COFOM. The RMSE values of the different moment curves are calculated with

<table>
<thead>
<tr>
<th>Varied design variable</th>
<th>RMSE $\times 10^{-2}$</th>
<th>RMSE $\times 10^{-2}$</th>
<th>$\xi_1$</th>
<th>$\xi_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard design</td>
<td>1.0</td>
<td>1.0</td>
<td>0.994</td>
<td>16.9</td>
</tr>
<tr>
<td>$\theta_A = 30 , \text{deg}$</td>
<td>0.9</td>
<td>0.3</td>
<td>0.861</td>
<td>20.5</td>
</tr>
<tr>
<td>$\theta_A = 70 , \text{deg}$</td>
<td>5.8</td>
<td>4.0</td>
<td>1.13</td>
<td>14.3</td>
</tr>
<tr>
<td>$\theta_{\text{foot}} = 40 , \text{deg}$</td>
<td>1.7</td>
<td>1.5</td>
<td>0.985</td>
<td>19.4</td>
</tr>
<tr>
<td>$\theta_{\text{foot}} = 80 , \text{deg}$</td>
<td>1.0</td>
<td>0.8</td>
<td>1.01</td>
<td>14.2</td>
</tr>
<tr>
<td>$w = 100 , \text{mm}$</td>
<td>1.2</td>
<td>0.9</td>
<td>0.901</td>
<td>23.1</td>
</tr>
<tr>
<td>$w = 200 , \text{mm}$</td>
<td>1.8</td>
<td>1.1</td>
<td>0.992</td>
<td>13.3</td>
</tr>
<tr>
<td>$t = 0.2 , \text{mm}$</td>
<td>0.4</td>
<td>0.3</td>
<td>1.05</td>
<td>16.4</td>
</tr>
<tr>
<td>$t = 0.4 , \text{mm}$</td>
<td>2.9</td>
<td>2.5</td>
<td>0.955</td>
<td>17.6</td>
</tr>
<tr>
<td>Average</td>
<td>1.8</td>
<td>1.4</td>
<td>0.985</td>
<td>17.3</td>
</tr>
</tbody>
</table>

TABLE 1. Effect of refitting the PRB model on the test cases. The values in the second column correspond to the graphs in Fig. 8.
(a) Varying $\theta_A$ from the standard value of 50 deg

(b) Varying $\theta_{foot}$ from the standard value of 60 deg.

(c) Varying width $w$ from the standard value of 150 mm.

(d) Varying the thickness $t$ from the standard value of 0.3 mm.

**FIGURE 8.** Prediction performance of the PRB model with the earlier obtained model parameters. The PRB model has not been refitted.

### TABLE 2. Convergence of the FEM solution for the standard design of the SV-COFOM.

The model with 1006 elements is considered to be converged and is used throughout this paper. The model with 121 elements shows a performance comparable to the PRB model.

<table>
<thead>
<tr>
<th>Number of elements</th>
<th>RMSE $\times 10^2$ Nm w.r.t. the case of 4020 elements</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; 121</td>
<td>(no solution)</td>
</tr>
<tr>
<td>121</td>
<td>2.9</td>
</tr>
<tr>
<td>288</td>
<td>1.0</td>
</tr>
<tr>
<td>1006</td>
<td>0.2</td>
</tr>
<tr>
<td>4020</td>
<td>0</td>
</tr>
</tbody>
</table>

The prediction performance of the PRB model in the executed tests is comparable to a FE model with 121 elements. Using 121 elements will give a RMSE of 2.9 $\times 10^{-2}$ Nm with respect to the FE model which is considered valid. The average RMSE for the curve constructed with 4020 elements is 4.5 $\times 10^{-2}$ Nm.

### 4 DISCUSSION

Figure 7 shows that the PRB model can be fitted well on the standard design of the SV-COFOM. The prediction tests of the PRB model in Fig. 8 also show good results. Note the large scaling differences in the y-axes of the graphs, showing that the test cases cover a broad range. The test cases do not include a simultaneous change of parameters, but the results are considered as a good indication that the model produces useful output.

Figure 11 shows the empirical validation of the FE model. The RMSE is 4.5 $\times 10^{-2}$ Nm.
of the PRB model is 1.8 $\times$ 2 Nm. The performance of the PRB model when $\theta_0$ is changed to 70 deg is an exception with an RMSE of 5.8 $\times$ 2 Nm. However, when the RMSE values of the FE model with 121 elements and PRB model would be normalized by dividing by the minimum reaction moment value, the PRB model would score equal to better.

The PRB model could be used in the early design stages of origami mechanisms with compliant facets providing an easy, insightful way of designing with very low computational costs. The low computational costs also make it more feasible to use optimization algorithms to search for the right design variables to approach a desired moment curve.

The PRB model could be improved by making the model parameters dependent on the design variables. Table 3 shows that the average RMSE in the test cases from Fig. 8 decreases from 1.8 $\times$ 2 Nm to 1.4 $\times$ 2 Nm when the PRB model would be refitted on the FE data of each of the deviating designs. Note that the model should be tested on new designs to assess the new prediction performance.

The SV-COFOM is only considered with point $C_m$ constrained in the y-direction as in Fig. 3, also in the empirical and numeric tests. However the experience in the empirical tests was that in the dominant motion of the SV-COFOM point $C_m$ does not tend to move in the y-direction, this has not been proven.

Changing the height or Poisson ratio of the mechanism results in RMSE values comparable to those in the prediction tests of Fig. 8. However, it is anticipated that if the height would be decreased more, the model errors will exceed that of the prediction tests. The height was changed from 300 mm to 200 and 400 mm, showing that there is still some design space in this variable.

The cutout radius mainly has effect on the stiffness of the mechanism, as shown in Fig. 10. This means that fixing the radius will not limit the design space, because the same results can be obtained by changing the thickness, width or E-modulus of the SV-COFOM.

Figure 11 shows good correspondence of the FE model with the measurements. The empirical data around the buckling point $\theta_{\text{joint}} = 0$ shows lower values than the FE data. This is probably due to imperfections, as normally the case when buckling is measured empirically. The empirical data curve shows some hysteresis. This is probably due to energy dissipation due to strain in the Mylar® tape, or play in the hinge lines or attachment to the load cell.
The SV-COFOM exhibits a large range of negative stiffness, meaning that the moment values decrease at an increasing angle $\theta_{\text{joint}}$. This property could potentially be used to statically balance compliant mechanisms [10].

5 CONCLUSIONS

A new type of origami mechanisms has been proposed in which compliance of the facets is used to incorporate spring behavior: Compliant Facet Origami Mechanisms (COFOMs).

A simple and accurate 1 DOF model which computes the moment curve of a Single Vertex COFOM was proposed, using a semi-spatial version of Pseudo-Rigid Body theory to model bending of the facets. The performance of the model has been evaluated numerically and experimentally. The model showed an average RMSE of 1.8 $\times$ 10^{-2} Nm (on a magnitude in the order of 0.45 Nm) which is comparable to a Finite Element model with 122 elements. The PRB model is a potential starting point for a design tool which would provide an intuitive way of designing this type of mechanisms including their spring behavior, with very low computational costs.

The FE data shows good correspondence with an experimental test, with a RMSE of 4.5 $\times$ 10^{-2} Nm on an order of magnitude of 0.45 Nm.

REFERENCES


Chapter 2

Paper: A design tool for a Single Vertex Compliant Facet Origami Mechanism (SV-COFOM) including torsional hinge lines
A DESIGN TOOL FOR A SINGLE VERTEX COMPLIANT-FACET ORIGAMI MECHANISM (SV-COFOM) INCLUDING TORSIONAL HINGE LINES

Jelle Rommers, Giuseppe Radaelli, Just Herder
Dep. of Precision and Microsystems Engineering, 3ME
Delft University of Technology
Delft, 2628 CD, The Netherlands
jellerommers@gmail.com, g.radaelli@tudelft.nl, j.l.herder@tudelft.nl

ABSTRACT
Principles from origami art are applied in the design of mechanisms and robotics increasingly frequent. A large part of the application-driven research focuses on devices where the creases (‘hinge lines’) are actuated and the facets are constructed as stiff elements. In this paper, a design tool is proposed in which hinge lines with torsional stiffness and flexible facets are used to design passive, instead of active mechanisms. The design tool is an extension of a model of a Single Vertex Compliant Facet Origami Mechanism (SV-COFOM) and is used to approximate a desired moment curve by optimizing the design variables of the SV-COFOM. Three example designs are presented: a Constant Moment Joint, a Gravity Compensating Joint and a Zero Moment Joint. The Constant Moment Joint design has been evaluated experimentally, resulting in a RMSE of 0.064 Nm on a constant moment value of 0.39 Nm. This indicates that the design tool is suitable for a coarse estimation of the moment curve of the SV-COFOM in early stages of a design process.

1 INTRODUCTION
Principles from origami art are applied in the design of mechanisms and robotics increasingly frequent. Examples are a robot with an origami wheel which is adjustable in size [1] or a surgical forceps which is produced from a single sheet of material [2]. The paper of the origami models on which the mechanisms are based is replaced by more common engineering materials like plastics. Creases are introduced by locally reducing the stiffness of the plate or by adding a second, more flexible material. We will refer to these mimicked creases as hinge lines and to the plate material between the creases as facets. Because motion is made possible through deflection of the material, origami mechanisms are commonly categorized as Compliant Mechanisms. Advantages of fabricating a mechanism from a flat sheet are cost reduction and deployability.

A large part of the recent application-driven origami mechanisms research focuses on self-folding robots and structures. In this field, the hinge lines are actuated so that the device can fold itself, starting from a printed flat sheet [4–6]. The vast majority of this research is based on the premise of rigid facets. In a previous work by the authors (Rommers et al. [3]) Compliant Facet Origami Mechanisms (COFOMs) are introduced, in which flexible facets act like springs. Subject is the common origami mechanism of figure 1. This mechanism is categorized as a Single Vertex COFOM (SV-COFOM), named after the single inter-
FIGURE 2. The SV-COFOM acting as a joint with non-linear torsional stiffness. The bottom of the mechanism is clamped, forcing the bottom facets to bend during joint movement. Goal is to approach a desired moment curve by changing the design variables in figure 1. Figure from Rommers et al. [3].

section of hinge lines. When this mechanism is clamped at the bottom as in figure 2, the bottom facets are forced to bend during the indicated motion. This bending has a similar effect as including springs in the mechanism. The mechanism acts globally as a joint with a non-linear spring. In the previous work, a simple model is proposed which computes the moment curve that results from bending of the facets.

The model in [3] is developed on the assumption of zero torsional stiffness in the hinge lines. Aside from the fact that this assumption is never fully valid in reality, including torsional stiffness could lead to interesting moment curves of the mechanism. The SV-COFOM could be used as a building block of which the stiffness curve can be manipulated.

The goal of this paper is to propose a design tool for the SV-COFOM of figure 2, by extending the model in the previous work to include torsional stiffness $\kappa_A$ and $\kappa_C$ in the hinge lines as in figure 1. The design tool will output the moment curve for a certain input of design variables. An optimization algorithm will be used to find the design variables that give the closest approximation of a desired moment curve. Instead of actuating the creases as common in recent literature, this approach will result in passive mechanisms.

In the Methods section, the model of the previous work in [3] will be extended to a design tool. The optimization procedure used to design three example mechanisms will be stated. Furthermore, the fabrication and experimental evaluation method of one of the designs is explained. The resulting example designs will be presented in the Results section, along with the experimental results of one of the examples. The design examples and experimental results will be discussed. Finally, conclusions will be presented.

2 METHODS

In this section, the design tool will be developed. Subsequently, the optimization procedure will be explained and the experimental tests will be described.
2.1 Design tool

In Rommers et al. [3], the deflection of the bottom facets of the SV-COFOM is modeled by applying Pseudo-Rigid Body (PRB) model theory from L.L. Howell [7]. Figure 3 shows the PRB model of the mechanism. Each bottom facet is divided into two rigid ones by introducing a virtual hinge line (dashed) with a certain equivalent torsional stiffness. The position angle of the virtual hinge line \( \theta_B \) and the equivalent torsional stiffness are determined numerically in the previous work and validated within certain design boundaries. The angular rotation of the virtual hinge lines are denoted by \( \tau_B \). The rotation is defined to be zero when the adjacent facets align.

The model will be extended by introducing the torsional stiffness in the real hinge lines \( \tau_A \) and \( \tau_C \). First, the global procedure to construct the moment curve from the PRB model is outlined. Second, the necessary kinematic formulas will be derived.

**Moment curve.** In order to construct the moment curve from the PRB model in figure 3, the relation between the hinge lines and \( \theta_{\text{joint}} \) must be found. Once this is done, the potential energy in the SV-COFOM can be calculated as (gravity is neglected):

\[
V(\theta_{\text{joint}}) = \frac{1}{2} \left( 2\kappa_A \tau_A^2 + 2\kappa_{\text{virtual}} \tau_B^2 + \kappa_C \tau_C^2 \right)
\]  
(1)

Note that \( \kappa_A \) and \( \kappa_{\text{virtual}} \) are multiplied by two because these hinge lines appear twice in the SV-COFOM. The derivative of this energy with respect to \( \theta_{\text{joint}} \) gives the moment curve of the mechanism:

\[
M(\theta_{\text{joint}}) = \frac{\partial V(\theta_{\text{joint}})}{\partial \theta_{\text{joint}}}
\]  
(2)

In [3], the torsional stiffness of the virtual hinge line is defined as:

\[
\kappa_{\text{virtual}} = \xi_1 E t^3 \frac{w}{2 \sin(\theta_B)}
\]  
(3)

with:

\[
\theta_B = \frac{\pi}{2} + \theta_A + \xi_2
\]  
(4)

Where it was determined numerically that \( \xi_1 = 0.994 \text{ m}^{-2} \) and \( \xi_2 = 16.9 \text{ deg.} \)

![FIGURE 4](image-url) Adjusted figure of Rommers et al [3]: including the torsional stiffness of the hinge lines in the spherical representation of the PRB model. Goal is to calculate the angular rotation of the hinge lines \( \tau_A, \tau_B \) and \( \tau_C \) as a function of the joint angle \( \theta_{\text{joint}} \) of the SV-COFOM.

**Kinematic relation between hinge lines and \( \theta_{\text{joint}} \).** In order to write the angular rotations of the virtual hinge lines as a function of \( \theta_{\text{joint}} \), the PRB model of Fig. 3 is viewed as a spherical mechanism in [3]. Please refer to this work for further explanation. Figure 4 shows this spherical representation extended to include the torsional stiffness of the real hinge lines \( \tau_A \) and \( \tau_C \). Following will be the calculation of the angular rotation of these hinge lines as a function of \( \theta_{\text{joint}} \). Forming the formulas that are added to the original model to form the complete design tool.

Note that the arcs in fig. 4 correspond to the angles in fig. 3 as:

\[
\widehat{AC} = \theta_A \\
\widehat{AB} = \theta_B - \theta_A \\
\widehat{BC'} = \theta_B
\]

First we calculate the arc on the unit sphere between point \( B \) and point \( B' \):
\[
\sin(\overline{BB'}) = 2\sin(\theta_{foot}/4)\sin(\theta_B) \tag{5}
\]

Note that \(\overline{BB'} \neq \theta_{foot}/2\) because that arc would not be on the virtual sphere.

Arc BC (not drawn) will first be calculated. Using the cosine rule for spherical systems we can write:

\[
cos(\angle BB'C) = \frac{\cos(\theta_B) - \cos(\theta_B)\cos(\overline{BB'})}{\sin(\theta_B)\sin(\overline{BB'})} = \frac{\cos(\overline{BC}) - \cos(\theta_B - \theta_{joint})\cos(\overline{BB'})}{\sin(\theta_B - \theta_{joint})\sin(\overline{BB'})} \tag{6}
\]

By substituting (5) in (6) and rearranging, an explicit expression for \(\overline{BC}\) can be obtained in a straight forward manner.

Computation of \(\tau_A\):

\[
cos(\angle BAC) = \frac{\cos(\overline{BC}) - \cos(\theta_B - \theta_A)\cos(\theta_A)}{\sin(\theta_B - \theta_A)\sin(\theta_A)} \tag{7}
\]

\[
\tau_A = \pi - \angle BAC \tag{8}
\]

Computation of \(\tau_C\):

\[
cos(\angle BCB') = \frac{\cos(\overline{BB'}) - \cos(\overline{BC})\cos(\theta_B - \theta_{joint})}{\sin(\overline{BC})\sin(\theta_B - \theta_{joint})} \tag{9}
\]

\[
cos(\angle ACB) = \frac{\cos(\theta_B - \theta_A) - \cos(\theta_A)\cos(\overline{BC})}{\sin(\theta_A)\sin(\overline{BC})} \tag{10}
\]

\[
\tau_C = 2(\angle BCB' + \angle ACB) - \pi \tag{11}
\]

For completeness, we will also include the relation between the virtual hinge line and \(\theta_{joint}\) from [3]:

\[
cos(\angle ABC) = \frac{\cos(\theta_A) - \cos(\overline{BC})\cos(\theta_B - \theta_A)}{\sin(\overline{BC})\sin(\theta_B - \theta_A)} \tag{12}
\]

\[
cos(\angle CBC') = \frac{\cos(\theta_{joint}) - \cos(\overline{BC})\cos(\theta_B)}{\sin(\overline{BC})\sin(\theta_B)} \tag{13}
\]

\[
\tau_B = \angle ABC - \angle CBC' \tag{14}
\]

All equations are checked by drawing the PRB model in a Computer Aided Design package.

Resulting is a closed-form formula which outputs the moment curve of the SV-COFOM as a function of its design variables.

### 2.2 Optimization

This section will explain the optimization procedure used to approach a certain desired moment curve of the SV-COFOM by changing the design variables.

#### Optimization procedure

The design variables will be optimized by constructing an objective function which quantifies in a single score how close the moment curve of a certain design approaches the desired curve. A grid search is performed, in which all design variables are discretized between boundaries with a certain sample rate. All possible combinations of the resulting design variable values are used as input in the design tool, which computes the corresponding moment curves. All moment curves are scored using the objective function and the design with the highest score is chosen. The sample rate is increased until this highest score converges. The grid search approach strongly reduces the possibility of finding a local optimum, as in other optimization tools. The low computational power used by the design tool makes the use of the grid search approach possible.

#### Design variables and boundaries

The design variables of the SV-COFOM are depicted in Figs. 1 and 2: the torsional stiffness of the hinge lines \(\kappa_A\) and \(\kappa_C\), the position angle of the hinge lines \(\theta_A\), the foot angle \(\theta_{foot}\) and the width, thickness and E-modulus. The last three variables all have the same effect on the moment curve (a multiplication), so only the width will be
varied. The E-modulus and thickness are fixed at 193 Gpa (AISI 304 spring steel) and 0.3 mm respectively.

The same assumptions and boundaries as in Rommers et al. [3] will be used: The height of the mechanism is fixed at 300 mm, a circular cutout of radius 15 mm is introduced at the intersection of the hinge lines and $\theta_A$ is limited at 70 deg.

The torsion constants for the hinge lines are not bounded, but the resulting values will be evaluated on feasibility during the experimental validation.

**Example designs.** Three example designs are worked out. In each example, the design of the SV-COFOM is optimized to approach a certain desired moment curve.

The first example design is a Constant Moment Joint (CMJ). The objective is to construct a moment curve which has a constant value for a range as large as possible. A constant range is defined as the range for which the moment values are within boundaries of 3% of the average of these values.

The second example is a Gravity Compensating Joint. In this design, a virtual mass of 0.4 Kg is attached at the top of the SV-COFOM (point $C_m$ in Fig. 3). The goal is to counteract the gravity forces on the mass, i.e. to get a reaction moment curve of zero of the mechanism including the mass. The objective function determines the range in which the reaction moment curve is between $\pm$ 3% of the maximum exerted moment of the mass. One extra design variable is added: a tilt of the mechanism, measured from the vertical in the positive y-direction.

The third example is a Zero Moment Joint. The design is optimized to obtain a moment curve that stays within boundaries around zero for a range as long as possible. The boundaries are defined as $\pm$ 3% of the absolute value of the minimum moment of the contribution from the virtual hinge lines.

**2.3 Experimental setup.**

The design of the Constant Moment Joint is evaluated experimentally. Figure 5 shows the fabricated CMJ. AISI 304 spring steel plates of 0.3 mm thickness form the facets of the mechanism. Hinge lines are introduced by applying Mylar® tape in an alternating pattern as shown schematically in the upper left of Fig. 5. Torsional stiffness is added by clamping spring steel wire AISI 301 between adjacent facets. When the facets rotate, the steel wire is subjected to torsion. We will refer to the steel wire pieces as *torsion bars*.

The dimensions of the torsion bars for a desired torsion stiffness can be calculated, but during the experiments this approach appeared not consistent enough. This could be due to a varying shear modulus for different diameter of the bars, or fabrication errors in the diameters. Instead, the desired torsion values are approached in an iterative way by experimentally testing torsion bars with different diameters as in Fig. 6. The torsion bars are clamped to two plates with hinge lines constructed the same way as in the CMJ, the only difference being the larger thickness of the plates to mitigate the effect of bending of the plates. The plates are rotated by manually pulling a load cell attached to one of the plates by a cord. This is done two times for three angles of each torsion bar to test for linearity. This way, each torsion bar is tested six times.

The moment curve of the CMJ is recorded by a load cell.
FIGURE 7. Experimental setup used to record the moment curve of the Constant Moment Joint design.

connected to a rotating arm as in Fig. 7. The arm is moved up and downwards manually ten times to record the moment curve. The load cell is connected to the CMJ by a rod that can move freely parallel to the indicated arm, such that mainly forces orthogonal to the arm are measured. The rotation of the arm is measured by a potentiometer.

The empirical data will be compared to the model output by computing the Root Mean Squared Error (RMSE):

\[ RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (\text{Empirical}_i - \text{Model}_i)^2} \]

Where \( i = 1...n \) denotes the constant range of the CMJ. The empirical data will be averaged over the ten consecutive up and down motions before the RMSE is computed. This way, the average of the hysteresis loop is obtained. The raw data will be plotted to show the spread of the measurements and the magnitude of the hysteresis loop.

3 RESULTS

Figure 8 shows the moment curve of example design 1: the SV-COFOM is optimized to exhibit a constant moment. The grey dashed curves show the separate contributions of the hinge lines, summing up to the constant moment curve in blue solid. The moment is constant for a range of 77 deg, where the curve is within boundaries of ±3% of the mean value of 0.45 Nm. If these boundaries are taken as 5%, the same curve will have a range of 85 deg. Negative values of the reaction moment curve indicate that the SV-COFOM has a tendency to move in the positive direction of \( \theta_{\text{joint}} \) (downwards in Fig. 2).

Example design 2, the Gravity Compensating Joint, is shown in Fig. 9. The separate contributions of the hinge lines and the mass to the total moment curve are shown in dashed grey. Note that no curve for \( \kappa_C \) is shown, because this torsion constant is optimized to zero. The SV-COFOM compensates the attached mass of 0.4 Kg for a range of 80 deg, where the maximum absolute deviation from zero is 3% of the maximum moment of the
FIGURE 10. Design 3: Zero Moment Joint. Range of 66° within 3% error of the minimum value of the $\kappa_{\text{virtual}}$ curve.

<table>
<thead>
<tr>
<th>Design variable</th>
<th>Constant Moment Joint</th>
<th>Gravity Compensator</th>
<th>Zero moment joint</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa_A$ [Nm/rad]</td>
<td>0.1430</td>
<td>0.1871</td>
<td>0</td>
</tr>
<tr>
<td>$\kappa_C$ [Nm/rad]</td>
<td>0.2513</td>
<td>0</td>
<td>0.4346</td>
</tr>
<tr>
<td>$\theta_A$ [deg]</td>
<td>70</td>
<td>70</td>
<td>70</td>
</tr>
<tr>
<td>$\theta_{\text{foot}}$ [deg]</td>
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<td>63</td>
<td>63</td>
</tr>
<tr>
<td>Width [mm]</td>
<td>150</td>
<td>146</td>
<td>113</td>
</tr>
<tr>
<td>Thickness [mm]</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>E-modulus [Gpa]</td>
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<td>193</td>
<td>193</td>
</tr>
<tr>
<td>Tilt [deg]</td>
<td>0</td>
<td>27</td>
<td>0</td>
</tr>
</tbody>
</table>

TABLE 1. Optimized design variables for the three example designs.

Experimental results Iteratively approaching the desired stiffness of the torsion bars for the Constant Moment Joint design $\tau_A = 0.1430$ Nm/rad and $\tau_C = 0.251$ Nm/rad, resulted in values of $\tau_A = 0.1250$ Nm/rad (desired was 0.1430 Nm/rad) and $\tau_C = 0.3204$ Nm/rad (desired was 0.2513 Nm/rad). The average constant value in the original constant region changed from 0.45 Nm to 0.39 Nm.

4 DISCUSSION

The design examples show that the moment curve of the SV-COFOM can be manipulated to a large extend. This is mainly due to the negative stiffness contribution of the virtual hinge line, meaning that the slope of this moment curve contribution is negative. For example, in the Constant Moment Joint, the positive stiffness of the hinge lines are compensated by the negative stiffness of the virtual hinge line, adding up to a straight moment curve. The fact that the SV-COFOM acts globally as a joint pro-
FIGURE 12. Empirical validation of the Constant Moment Joint design. (1) is the mechanism with $\kappa_A = \kappa_C = 0$. (2) is the mechanism with added torsional stiffness $\kappa_A = 0.1250\text{Nm/rad}$ and $\kappa_C = 0.3204\text{Nm/rad}$. The RMSE between model and empirical data for the original constant range of the CMJ is 0.064 Nm.

provides the possibility to include it in origami mechanisms as a joint with configurable stiffness. The negative stiffness of the SV-COFOM could also be used to compensate positive stiffness of other components of an origami mechanism, e.g. the bending of a compliant flexural hinge. This provides the possibility of \textit{statically balancing} the total mechanism [8].

The empirical validation of the Constant Moment Joint design in Fig. 12 indicates that the design tool gives a good rough estimation of the moment curve. The tool could be used in early stages of a design process, where the simplicity and the high computational efficiency of the model can be useful to quickly evaluate designs or perform optimizations. In later stages, the design could be fine-tuned, for example by using a Finite Element Model.

The empirical data shows higher reaction moment values than the model in the region around $\theta_{\text{joint}} = 20\text{deg}$. The rest of the measured moment curve shows lower values. This could be due to deformations of the top facets, which were observed during the experiments. The top facets could have acted as extra springs in series, resulting in lower effective torsional stiffness values of the hinge lines. The effect of lower torsional stiffness values can be best understood by looking at the separate moment contributions in Fig. 8. Lowering the torsional stiffness $\kappa_A$ will have the effect of a multiplication factor between zero and one to the graph of $\kappa_A$. This will result in a declining moment curve of the total CMJ. The higher moment values around $\theta_{\text{joint}} = 20\text{deg}$ can be explained by a lower stiffness value of $\kappa_C$, which could also be due to bending of the top facets. The moment values of the curve of $\kappa_C$ will become less negative around this point, resulting in a local increase in the total moment curve. The decline of the empirical curve could also be partly due to gravity forces, which are not incorporated in the model.

A different fabrication method of including the torsional stiffness could decrease the error between the empirical data and the model. For example, clamping the torsion bars at the inside of the mechanism instead of the outside could decrease the bending of the facets. Also, decreasing the height of the top facets could improve the results by mitigating the bending deformations of these facets. The model could also be extended by including bending of the top facets.

Iteratively finding the right dimensions for the torsion bars showed to be tedious, because in practice the diameter and length of these bars is varied in steps. The measured stiffness values of the torsion bars showed a small standard deviation. This means (together with a sufficient number of measurements) that the assumption of linear stiffness of the hinge lines can be considered valid. The stiffness values of the hinge lines have not been bounded in the three optimizations. The experimental results show that the resulting stiffness values are of a realistic magnitude.

5 CONCLUSIONS

A design tool which computes the moment curve of a clamped Single Vertex Compliant Facet Origami Mechanism (SV-COFOM) was proposed by extending the PRB model in Rommers et al. [3] to include torsional stiffness in the hinge lines.

The design tool was used to approximate a desired moment curve by optimizing the design variables of the SV-COFOM. Three example designs have been presented: a Constant Moment Joint, a Gravity Compensating Joint and a Zero Moment Joint. The design examples showed that the moment curve of the SV-COFOM can be manipulated to a large extend.

The Constant Moment Joint design has been evaluated experimentally. The RMSE between the model output and the experimental data for the constant range showed to be 0.064 Nm, on constant moment value of 0.39 Nm. This indicates that the design tool is suitable for a course estimation of the moment curve of the SV-COFOM in early stages of a design process.

REFERENCES


Chapter 3

Paper: Design of a gravity compensator consisting of two Single Vertex Compliant Facet Origami Mechanisms (SV-COFOMs)
ABSTRACT
The past years show an increase of a special type of Lamina Emergent Mechanisms which are inspired by origami art: Origami mechanisms. Advantages of these compliant mechanisms which are fabricated from a sheet material are low production costs and an inherent deployability. In this paper, a gravity compensator is designed by connecting two Single Vertex Compliant Facet Origami Mechanisms (SV-COFOMs) in series. These SV-COFOMs globally act as joints with a nonlinear stiffness curve which can be manipulated by changing the design variables. An energy approach is used in the design process. Resulting is a gravity compensator with a range of 230 mm on an effective mechanism length of 350 mm. A practical design example of the gravity compensator in Lamina Emergent Mechanism-form is presented.

INTRODUCTION
Lamina Emergent Mechanisms (LEMs) are "mechanical devices fabricated from planar materials (laminae) with motion that emerges out of the fabrication plane" [1]. Motion is possible due to deflection of the material, categorizing LEMs as compliant mechanisms. The past years show an increase of a special type of LEMs: origami mechanisms. These mechanisms do not have a formal definition, but are designed using the vast knowledge available in origami patterns. Also, the mechanisms are often designed without a distinct base part situated on the ground, as is the case in most LEMs. Possible advantages of fabricating a mechanism from a sheet material are lower costs and deployability. The planar fabrication methods used to fabricate Microelectromechanical systems (MEMS) make it possible to construct 3D origami mechanisms at the micro-scale [2, 3].

In earlier work by the authors [4, 5] Compliant Facet Origami Mechanisms (COFOMs) are introduced, in which bending of the facets and hinge lines (surrogate creases) are used to store strain energy. The authors proposed a design tool for a common origami mechanism that globally acts similar to a joint with a nonlinear stiffness spring. Figure 1 shows this mechanism, which is constructed by clamping a Single Vertex (SV) COFOM at the bottom. Single Vertex refers to the single intersection of hinge lines. The mechanism acts as a planar joint in the XZ-plane. The design tool is used to compute the strain energy and reaction moment in the range of $\theta_{\text{joint}}$. Outside of the LEM and origami field, Radaelli et al. [6] proposed a method in which planar mechanisms consisting of links and joints with linear torsion springs are designed, using a graphical energy approach. These mechanisms can be replaced by their equivalent compliant versions, as in [7]. If the stiffness characteristics of these torsion springs could be manipulated instead of being linear, this could lead to interesting results.

The goal of this paper is to design a serial chain of two SV-COFOMs that behaves as a gravity compensator, using the energy method in [6]. This means that if a weight would be attached to the end point of the mechanism, the reaction force at this point should remain zero in the vertical direction throughout a predefined range of motion.

In the Methods section, it will be explained how the energy method will be used to design the theoretical mechanism. The Results section will include the outcome of this design. In the Discussion, this design will be evaluated and finally, conclusions will be presented.

METHODS
The mechanism concept of the two COFOMs connected by links is shown in Fig. 2. Link 1 is attached to the ground. Links 2 and...
The hinge lines have torsional stiffness $\kappa_A$ and $\kappa_C$. When clamped at the bottom, the mechanism can be viewed as a planar joint in the XZ-plane with nonlinear stiffness. The links are assumed to only move in the XZ-plane. The joints which connect the links are replaced by the SV-COFOMs, essentially adding torsional springs with configurable stiffness. Refer to the axes in Figs. 1 and 2 to understand how the SV-COFOMs are included. The workspace is limited due to the finite rotations of the SV-COFOMs. In the two extreme positions, the bottom facets prevent the top facets from rotating further. Goal is to change the design variables of the SV-COFOMs and the link lengths such that end point $e(x,z)$ exerts a constant upward directed reaction force throughout a spatial range as large as possible. This mechanism will be able to compensate the gravity forces of a mass, attached at end point $e(x,z)$. The links are assumed to have zero mass.

Energy method. The basic idea of the energy method of [6] is to plot for each possible end point position the energy which is stored in the mechanism in that configuration. The energy graphs of different components can be added to design a desired energy field, i.e. a desired reaction force behavior of the end point. Note that the slope of the energy graph in a certain direction is equal to the magnitude of the reaction force on the end point in that direction. Computing the energy field is in our case simple because the configuration of the whole mechanism is determined by the end point position. The stored energy is the strain energy in the SV-COFOMs, added to the potential gravitational energy from the mass attached at the end point $V_I$:

$$V_I(x_e,z_e) = V_1(\theta_1, \phi_1) + V_2(\theta_2, \phi_2) + mgz_e$$

Where $V_1$ and $V_2$ are the strain energy in joints 1 and 2 respectively, $\phi_1$ and $\phi_2$ are the design variables of the joints. Furthermore:

$$\theta_1(x_e, z_e, L_1, L_2)$$
$$\theta_2(x_e, z_e, L_1, L_2)$$

Computing the joint angles requires standard inverse kinematics. $m$ denotes the mass attached at the end point and $g$ is the gravitational constant. The workspace will be discretized and for every end position, the total energy will be calculated. Resulting will be a plot in three dimensions, in which the third dimension will be shown in color. The x and z axes will show the position of end point $e(x,z)$ and the third dimension shows the total energy of
the mechanism including the mass, for the corresponding mechanism configuration. This graph will give an intuitive representation of the behavior of the mechanism. A change in the design variables will yield a change in the energy plot.

**Path of e(x,z) and force in z direction.** Before the force in z direction can be calculated, the vertical path of e(x,z) must be determined as if the end point has a forced displacement in the z direction. This is done by taking the x value with the lowest energy for every z value from the workspace of e(x,z), resulting in \( V_{\text{path}}(z_e) \). Generally, this will be the path that the end point will follow if it would have a forced displacement in the z direction, because a mechanism will always tend to go in the direction of most decreasing or least increasing energy. Note that this approach is not always valid and the eventual results need to be checked by inspecting the energy plot. The force in z direction can now easily be calculated by:

\[
F_{\text{path}}(z_e) = -\frac{\partial V_{\text{path}}(z_e)}{\partial z} \tag{1}
\]

**Optimization procedure.** At this point, a graph is obtained which describes the vertical force as a function of the z position of the end point e(x,y). The corresponding x values describe the path of the end point. The vertical force graph will be dependent on the sets of design variables of the two SV-COFOMs \( \phi_1 \) and \( \phi_2 \), the length of the links and the added weight. The design variables of the joints are depicted in Fig. 1. The design tool in [5] will be used to compute the moment curve that results from a certain set of design variables, with low computational costs. In [5], the height parameter is set at 300 mm and a circular cutout of radius 15 mm around the intersection point of the hinge lines is assumed. Furthermore, \( \theta_A \) is bounded to a maximum value of 70 deg. We will do this accordingly. The thickness and E-modulus parameter are not varied because this will have the same result in the model as changing the width. The widths of the two SV-COFOMs will be varied simultaneously to decrease computational costs.

The goal of the optimization is to obtain a reaction force graph around zero for a range as long as possible. The force is considered to be zero if it is within 1% of the weight of the mass. The optimization is performed by computing the range of zero force for a certain set of design variables, using a genetic algorithm. The best scoring design will be shown in the Results section, along with the moment curves of the separate joints.

**RESULTS**

Figure 3 shows the energy of the total optimized mechanism including the mass. Every point in the colored graph represents a possible end position of the mechanism. The energy which the total mechanism has when the end point is at this position, is plotted at this position in the corresponding color. The calculated path of the end point for a forced displacement in the z direction is plotted in red dashed.

Figure 4 shows the separate energies of the mechanism when following the path from Fig. 3. The derivative of the total mechanism energy with respect to the z direction is shown in Fig. 5. This is the reaction force of the mechanism with added mass in the vertical direction. The mechanism compensates the weight for a range of 230 mm measured along the vertical, within an error of 1% of the weight of the mass. Figures 6 and 7 show the moment curves belonging the SV-COFOMs of the resulting design. Table 1 shows the optimized design variables.

**DISCUSSION**

The resulting design of the gravity compensator has a range of 230 mm. Compared to the length of 350 mm of effective links 2 and 3, this is a reasonably large range.

When inspecting the energy plot in Fig. 3, the approach shows to be valid in this case. The end point always goes in the direction of the least increasing or most decreasing energy. The
The Z-coordinate of end point e(x,z) following path [m]

**FIGURE 4.** The separate energies of the mechanism and mass when following the path from figure 3.

The Z-coordinate of end point e(x,z) following path [m]

**FIGURE 5.** Reaction force of the mechanism with added mass in the vertical direction, when following the path from figure 3. The mechanism compensates the weight for a range of 230 mm, within an error of 1% of the weight of the mass. The distorted part of the graph is due to the discrete end point positions.

end point of the gravity compensator will follow the indicated path in Fig. 3 automatically when subjected to a forced vertical displacement. This means that there is no need for additional kinematic constraints (e.g. a rail, linkage or cam system).

The gravity compensator is assumed to only move in the xz-plane. In a more practical design, the stiffness in the y-direction will need to be taken into account.

The path of the end point is not a straight vertical line. A straight path would in some applications be more desirable, for example when the mechanism would be used as a vibration isola-

**FIGURE 6.** Reaction moment curve of joint 1 of the resulting design.

**FIGURE 7.** Reaction moment curve of joint 2 of the resulting design.

**TABLE 1.** Optimized design variables of the resulting design.

<table>
<thead>
<tr>
<th>Design variable</th>
<th>Joint 1</th>
<th>Joint 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa_A$ [Nm/rad]</td>
<td>0.1822</td>
<td>0.0439</td>
</tr>
<tr>
<td>$\kappa_C$ [Nm/rad]</td>
<td>0.0402</td>
<td>0.2327</td>
</tr>
<tr>
<td>$\theta_A$ [deg]</td>
<td>70</td>
<td>70</td>
</tr>
<tr>
<td>$\theta_{foot}$ [deg]</td>
<td>61.72</td>
<td>60</td>
</tr>
<tr>
<td>Width [mm]</td>
<td>182</td>
<td>182</td>
</tr>
<tr>
<td>Thickness [mm]</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>E-modulus [Gpa]</td>
<td>193</td>
<td>193</td>
</tr>
<tr>
<td>Link lengths 2 and 3 [m]</td>
<td>0.2</td>
<td>0.1515</td>
</tr>
<tr>
<td>Mass [Kg]</td>
<td>0.250</td>
<td></td>
</tr>
</tbody>
</table>
FIGURE 8. Practical LEM design example of the gravity compensator (top view of a flat plate). The black dashed lines represent hinge lines with torsional stiffness. The red dashed lines represent hinge lines which are designed to deform permanently to erect and form the mechanism from its flat initial state.

The design in this paper is mainly conceptual. Figure 8 shows a global idea of what a more practical design of the two SV-COFOMs connected in series could look like. The figure shows a piece of sheet material, which includes the gravity compensator in unfolded state. The black dashed lines show the hinge lines, with the torsional stiffness $\kappa_A$ and $\kappa_C$ from Fig. 1. The red dashed lines are hinge lines which are designed to deform permanently when folded, for example by plasticly deforming the material. The SV-COFOMs from Fig. 1 are implemented by adding an extra piece of material between two, instead of one vertical hinge line. The mechanism is supposed to be erected by folding the bottom red dashed hinge line. The second step is to fold the other red dashed hinge lines to form the angle $\theta_{foot}$ similar to Fig. 1. Link 1 has a reinforcement rib which must be unfolded to stiffen the link. The next step in the design would be to find a suitable fabrication method to construct the hinge lines with torsional stiffness. Refer to Rommers et al. [5] for an example implementation of these torsional hinge lines.

CONCLUSIONS

A conceptual design for a gravity compensator based on two Single Vertex Compliant Facet Origami Mechanisms connected in series was proposed using the energy approach method from Radaelli et al. [6] and the design tool of Rommers et al. [5]. The design compensates a mass attached at the end point in a range of 230 mm, on an effective mechanism length of 350 mm (within boundaries of 1% of the weight).

A practical design example of the gravity compensator in Lamina Emergent Mechanism-form was presented.

REFERENCES


A new type of origami mechanisms has been proposed in which compliance of the facets is used to incorporate spring behavior: Compliant Facet Origami Mechanisms (COFOMs).

A simple and accurate 1 DOF model which computes the moment curve of a Single Vertex COFOM was proposed, using a semi-spatial version of Pseudo-Rigid Body (PRB) theory to model bending of the facets. The performance of this PRB model has been evaluated numerically and experimentally. The model showed an average RMSE of $1.8 \times 10^{-2}$ Nm (on a magnitude in the order of 0.45 Nm) which is comparable to a Finite Element model with 122 elements.

A design tool which computes the moment curve of a clamped SV-COFOM was proposed by extending this PRB model to include torsional stiffness in the hinge lines.

The design tool was used to approximate a desired moment curve by optimizing the design variables of the SV-COFOM. Three example designs have been presented: a Constant Moment Joint, a Gravity Compensating Joint and a Zero Moment Joint. The design examples showed that the moment curve of the SV-COFOM can be manipulated to a large extend.

The Constant Moment Joint design has been evaluated experimentally. The RMSE between the model output and the experimental data for the constant range showed to be 0.064 Nm, on constant moment value of 0.39 Nm. This indicates that the design tool is suitable for a course estimation of the moment curve of the SV-COFOM in early stages of a design process.

A conceptual design for a gravity compensator based on two SV-COFOMs connected in series was proposed using the energy method from [7] and the design tool. The design compensates a mass attached at the end point in a range of 230 mm, on an effective mechanism length of 350 mm (within boundaries of 1% of the weight).

A practical design example of the gravity compensator in Lamina Emergent Mechanism-form was presented.
Appendix A

Fabrication of hinge lines in a plate material

In this appendix, fabrication methods to introduce hinge lines in a plate material are investigated in order to find a possible production method for the SV-COFOM. In literature, an ideal hinge line is commonly defined as having no on-axis stiffness and infinite off-axis stiffness. However, paper 2 and 3 of this report show that an on-axis stiffness can also be desirable. Mechanical hinges are excluded from the results because their properties are well known. The appendix will start with a short literature review, after which the different test results are described.

A-1 Literature

Delimont et al. [6] present a very comprehensive overview on hinge lines. However, the hinge lines all have an off-axis stiffness which is too low to produce the SV-COFOM. All hinge lines are 2D designs which work on the principle of extending the hinge line path, for example in Figs. A-5 and A-6. Wilding et al. [8] do a similar analysis from the starting point of Lamina Emergent Mechanisms. Francis et al. [9] also analyse the possibility of creating hinge lines in different plate materials, but the authors do this by virtue of folding. A metric which quantifies how well a material is suitable for folding is used. All mentioned literature is used as an inspiration for the following paragraphs.

A-2 Experimental tests

Sewing principle. The working principle behind the alternating tape pattern in papers 1 and 2 is that the tape is not subjected to peeling forces. A drawback is the fact that a hinge is only created at the place where two tape pieces touch. It would be best if the tape is very narrow, to create a dense line of hinge points along the line. This would reduce ‘opening’ of
Fabrication of hinge lines in a plate material

the hinge lines, especially at the ends. This would be the case when a cord is 'sewed' through
the plates. Figures A-1 and A-2 show the fabrication results. Small holes are made using a
laser cutter. Drawbacks of this method are the residual stresses in the plate material from
the heat of the laser, the fact that this method is not directly suitable for mass production
and the difficulties of keeping sufficient tension in the cord.

Living hinges. The most commonly encountered hinge lines in everyday products are living
hinges. Think about plastic packaging boxes which hinge when they are opened, or a plastic
binder. In these examples, the stiffness of the plate material is locally reduced by thinning.
This is done by pressing, partly cutting the material or injection molding the plate in a
shape with a thinned line. Tests are performed by pressing a blunt edge into a 1 mm, thick
Polypropylene sheet as in Figs. A-3 and A-4. The results are a well functioning hinge. A
major drawback is the visco-elastic effect that occurs when the facets bend.

Extending the hinge line path. This concept is most encountered in literature. See Figs. A-
5 and A-6 for clarity. Effectively, the hinge line exists of bars subjected to torsion. The length
of these torsion bars can be increased by an alternating motion. Drawback is a relatively poor
off-axis stiffness.

Using a sandwich material. Hinges can be made from the sandwich material Hylite©. The
material consists of two thin aluminum plates with a Polypropylene (PP) core. The PP is
surprisingly well bonded to the metal: the bond is stronger than the PP itself. This is usually
a very difficult material combination to bond. The material is produced by a roll-to-roll
process, where the materials are bonded under high temperature and pressure. The material
is 1.2 mm thick. Figure A-8 shows hinge lines which are milled into the aluminum plates,
leaving the PP to function as a hinge. The concept works well en was not encountered in
literature. However, the facets are not capable of handling the relatively large deflections
needed for the SV-COFOM. A single sheet of spring steel, attached to one side of the PP
might be a good fabrication solution.

A-3 Conclusion

The hinge line designs proposed in literature have a off-axis stiffness which is too low to
fabricate the SV-COFOM. Conventional living hinges provide a good alternative, but visco-
elastic effects experienced during bending of the facets make the process unsuitable. An
appropriate method could be the use of a sandwich material like Hylite©, which combines
the large elongation capabilities of PP with low visco-elastic effects of aluminum.
Figure A-1: The prototype fabricated using the sewing principle. The holes are laser cutted. The principle can be seen as equivalent to the alternating tape method with very narrow tape pieces. A drawback is among others the problem of keeping tension on the cord.

Figure A-2: Close up of the sewing principle.
38 Fabrication of hinge lines in a plate material

(a) Locally thinned PolyPropylene

(b) Tools used to press the material

Figure A-3: Locally thinning a material by pressing.

Figure A-4: A SV-COFOM produced by pressing hinge lines in 1 mm Polypropylene. Additionally, the hinge lines are intermittently cut to reduce torsional stiffness. A drawback of this method is the visco-elastic effect that arises during bending of the facets.
Figure A-5: Laser cutted hinge lines in AISI 304 spring steel. Effectively, the hinge line exists of bars subjected to torsion. The length of these torsion bars can be increased by an alternating motion. Drawback is a relatively poor off-axis stiffness.

Figure A-6: The laser cutted hinge lines subjected to a rotation.
Figure A-7: Adding a flexible layer (rubber glue) at both sides of two steel plates. The thought behind this experiment was that the rubber in between the plates could provide resistance to the peeling forces. Result was the rubber shearing along the steel edges.

Figure A-8: Hinges made from the sandwich material Hylite®. Hylite consists of two thin aluminum plates with a polypropylene core. Hinge lines are created by milling the aluminum at both sides. This fabrication principle has not been encountered in literature.
Appendix B

Experimental validation

In this section, details of the prototypes and measurement setup are presented.

Figure B-1 shows the different prototypes constructed during the project. All prototypes are made from AISI 304 spring steel plates. The prototypes differ in thickness, design of the hinge line angles and tape. The two prototypes on the left use ripstop nylon tape. The ones on the right use mylar® tape. Both are stiff tapes. The prototype in the middle is constructed using the 'sewing' method described in the fabrication appendix.

Figure B-3 shows the prototype from paper 2, with the torsion bars disassembled.

Figure B-2 shows the attachment of the prototype to the load cell. A small rod is pushed through two hinges, which are bolted onto the mechanism. The rod can move freely in the longitudinal direction, as also showed in Fig. B-5 where the load cell is attached to the arm of the measurement setup.

The mechanism is clamped on the measurement setup at the bottom, as shown in Fig. B-4.
Figure B-1: Different prototypes.

Figure B-2: Attachment of the prototype to the load cell.
Figure B-3: The prototype of paper 2, with torsion bars disassembled.

Figure B-4: Enlarged picture of the measurement setup from papers 1 and 2.
Figure B-5: Attachment of the SV-COFOM to the load cell, once installed in the measurement setup. The pin-hole connection is meant to mainly measure forces orthogonal to the arm of the measurement setup.
Appendix C

Other COFOM creations

Fig. C-1 on the next page shows mock-up models of other COFOMs which were developed during the project.
Figure C-1: Other COFOM creations developed during the project.
The Finite Element Model is developed in Ansys© as described in paper 1. The next pages include the code which comprises the model, and a plot of the relative stresses (Fig. D-1).
!* This file computes the reaction moment curve of the SV-COFOM and writes this data in a .txt file.
!* Important:
!* - Watch the node numbers which connect the plates, they need to be redefined when the design changes.
!* - First timestep produces no sensible results
!* - Dataset only gets appended, not replaced!
!* - The slot joint should have enough steps for the coordinate projection

/CWD, 'C:\Users\Jelle\Dropbox\Afstuderen\FEM\CWD'

finish
/clear, start
!*Abbr, Input, /input,HUIDIG.txt
*Abbr, Deformed,PLDISP,1
!/SHOW,JPEG,,0
/eshape,1 ! Displays elements with shapes determined from the real constants or section definition. 
pi = acos(-1) ! Pi does otherwise not exist in Ansys.

! ---------------- INPUTS --------------------------
File_name = '10dec_A50' ! Watch out: Dataset only gets appended!
p_joint_max = 1.602375870036185 ! [rad] From matlab 
kp7x = 0.169199970874292 ! [m] From matlab. 
kp7y = 0.0417500000000000 ! [m] From matlab. 
kp7z = 0.217004159245069 ! [m] From matlab.

steps = 50 
p_min = 0.001*pi/180 
p_max = p_joint_max 
vari_min = -0.013 
vari_max = p_min-p_max

ei_edge = 0.005 ! watch out: nodes change. 
p_vouwlijn = 50*pi/180 ! watch out: nodes change. Measured from vertical 
Phi_foot = 60*pi/180 ! Angle between the standing facets 
sphere_radius = 0.015

! Material parameters. 
Poisson = 0.313 
Elastmod = 193E9

! Geometric parameters (flat folded) 
hoogte = 0.317 
width = 0.15 
h_vouw_begin = 0.150 
dikte = 0.0003 
h_vouw_eind = h_vouw_begin +(width/2)/tan(p_vouwlijn)
toplink = hoogte - h_vouw_begin
x_foot = cos(Phi_foot/2)*width/2
y_foot = sin(Phi_foot/2)*width/2

/prep7

! Define keypoints
k,1,0,0,0
k,2,x_foot,y_foot,0
k,3,x_foot,y_foot,h_vouw_eind
k,4,0,0,h_vouw_begin
k,5,0,0,h_vouw_begin
k,6,x_foot,y_foot,h_vouw_eind
k,7,kp7x,kp7y,kp7z
k,8,sin(p_joint_max)*(hoogte - h_vouw_begin),0,cos(p_joint_max)*(hoogte - h_vouw_begin)+h_vouw_begin
k,9,sin(p_joint_max)*(hoogte - h_vouw_begin)/2,0,cos(p_joint_max)*(hoogte - h_vouw_begin)/2+h_vouw_begin
A,1,2,3,4
A,5,6,7,8,9

lplot
wplane,1,0,0,h_vouw_begin,1,0,h_vouw_begin,0,1,h_vouw_begin
SPH4, 0, 0, 0, sphere_radius
ASBV,all,all,SEPO,DELETE,DELETE
!Delete first two areas
ADELE,1,2,1,0
LDELE,1,9,1,1
! KP's are also deleted
!KDELE
k,9,sin(p_joint_max)*(hoogte - h_vouw_begin)/2,0,cos(p_joint_max)*(hoogte - h_vouw_begin)/2+h_vouw_begin
ADELE,6
A,14,15,16,17,18
A,20,21,22,9,23,19

! From now on, only two areas are present: bottom is area 5, top is area 1

! Define local coordinate system
local,11,CART,0,0,h_vouw_begin,0,0,p_joint_max*(180/pi)-90 ! Ansys wants this in DEGREES
CSYS,0 !Go back to default

! element type 1: shell element for facets
ET, 1,shell63 ! Ansys help: 63 fine mesh, or 93 coarse (ansys help says to prefer 63 types for non-linear analysis)
! Assign material properties to label 1
mp, ex, 1, Elastmod
mp, nuxy, 1, Poisson
R,1,dikte !Real constant: area, moment of inertia, etc.

! element type 2: Element to make the hinge lines (spherical joint)
ET,2,MPC184 !Assigns element type to reference number 2.
Keyopt, 2, 1, 15

! Spherical joint. <<Sets element key options: Element type number as defined on the ET command, Number of the KEYOPT to be defined (KEYOPT(KNUM)), Value of this KEYOPT.!
sectype, 2, Joint, spherical

! element type 3
ET, 3, MPC184
Keyopt, 3, 1, 1
! Rigid beam
Keyopt, 3, 2, 1
! Use Lagrange multiplier method

! element type 4: Beam with negligible flexibility (very stiff)
ET, 4, BEAM188
! Assign material properties, etc. to label 4
mp, ex, 4, Elasticmod*10
mp, nu, 4, Poisson
SECTYPE, 4, BEAM, RECT
SECDATA, 0.02, 0.02

! element type 5: slider joint on a spherical joint (=slot joint)
ET, 5, MPC184
keyopt, 5, 1, 8
sectype, 5, JOIN, SLOT
secjoint, , 11
! secjoint, [blank], identifier of coordinate system at node I
CSYS, 0

! MESH (COPIED FROM LOG using Size Cntrls and mesh areas)
AESIZE, ALL, el_edge,
MSHKEY, 0
FLST, 5, 2, 5, ORDE, 2
FITEM, 5, 1
FITEM, 5, 5
CM, Y, AREA
ASEL, , , , P51X
CM, Y1, AREA
CHKMSH, 'AREA'
CMSEL, S, Y
!*
AMESH, Y1
!*
CMDELE, Y
CMDELE, Y1
CMDELE, Y2
!*

! Apply hinge element(s) between nodes (picked from GUI)
Type, 2
SECNUM, 2
    e , 71 , 535
    e , 74 , 551
    e , 75 , 550
    e , 76 , 549
    e , 77 , 548
! Create very stiff beam
CSYS,11
n,4001,0,0,0
n,4002,toplink,0,0
n,4003,toplink+0.0001,0,0
n,4004,toplink+0.0001,0,0
CSYS,0
node_kp22=node(kx(22),ky(22),kz(22)) ! Get node at kp 22
TYPE, 4  ! Extremely stiff beam
MAT, 4
SECNUM, 4
e,4001,4002
e,4002,4003
e,4004,node_kp22
! Attach beam to point of rotation
type,2
SECNUM, 2
e,4001         ! Other pair is now fixed to world
                ! Attach beam to end point of mechanism with ball-socket joint
type,5
secnum, 5
sectype,5,JOIN,SLOT
secjoint, ,11   ! secjoint,[blank],identifier of coordinate system at node I
e,4003,4004     ! Coordinate system is attached to first node
CSYS,0          ! Go back to default

!-----------------Stiffen line(s)
!type,3  ! Not necessary when using the extremely stiff beam
!!mesh,1

FINISH

/SOLU
antype, static             !static analysis
nlgeom, on                  !nonlinear geometry
autots, on                  !auto time-stepping
!arclen, on                  !Cannot be used with MPC184 and lagrange multiplier method
! Number of substeps
NSUBST, 10 ! Defines in this case the substeps of first load step

! Define constraints on lines
dl, 14, 5, all
ndl, 17, 5, UY ! Bottom vertical line
ndl, 23, 1, UY ! Top full vertical line
dl, 1, 1, UY ! Top half vertical line
dl, 2, 1, UY

D, 4002, UY
D, 4002, ROTX
D, 4002, ROTZ

! Apply rotation or displacement in steps
vari_step = (vari_max - vari_min)/(steps - 1)
*do, vari, vari_min, vari_max, vari_step ! Vari goes from vari_min to vari_max in steps
  D, 4002, ROTY, vari
  solve
  *endo
FINISH

*status, all

/POST26
ksel, all
NSOL, 2, 4002, ROT, Y, Jintangle
RFORCE, 3, 4002, M, Y, Rmoment
IESOL, 3, 979, 4003, Fy, , Ftangent
XVAR, 2
PLVAR, 3

! WRITING SOLUTION TO .TXT FILE
*CREATE, scratch, gui ! Create new .gui called 'scratch'
*DIM, Table, TABLE, steps, 2 ! Create Empty table length, width
VGET, Table(1, 0), 2 ! Fill column 1 with variable 2
VGET, Table(1, 1), 3 ! Fill column 2 with variable 3
/OUTPUT, File_name, 'txt', 'C: \ Users \ Jelle \ Dropbox \ Afstuderen \ FEM \ Comparison with empirical test'
! Specify file name + extension + path
*VWRITE, 'Jintangle', 'Rmoment' ! Write caption for columns
%C, %C

*VWRITE, Table(1, 0), Table(1, 1) ! Write Columns
%G, %G
/OUTPUT, TERM ! Redirects text output to the screen (in txt file)
*END ! End Creation
/INPUT, scratch, gui

FINISH
Figure D-1: Plot showing the relative stress distribution in the mechanism at the indicated joint angle.
Appendix E

Matlab code for objective function in paper 3

This appendix includes the matlab code for the objective function used in paper 3.

The code is written so that is also possible to use 3 joints in series. This brings some extra difficulties, because now multiple mechanism configurations are possible per possible end position of the mechanism. The configuration with the lowest energy is chosen. Note that this approach is again not always valid and needs to be checked afterwards, just like in paper 3.

The code can be constructed more computationally efficient.

1 function [ Range ] = Multiple_joints_objective(design_vars)
2 % This objective function calculates the energy fields for 2 or 3
3 % SV-COFOMS in series, calculates the vertical path and outputs the range
4 % for which the path is zero.
5 d2r = pi/180;
6 r2d = 1/d2r;
7
8 % INPUT FROM OPTIMIZATIONALGORITHM
9 cC_v = [0; design_vars(1); design_vars(2)];
10 cD_v = [0; design_vars(3); design_vars(4)];
11 Width_v = design_vars(6)*[1;1;1];
12 Phi_v = d2r*[60; r2d*design_vars(7); 60];
13 pj1min_deg = d2r*0.0001; % Offset angle of first joint
14
15 % REST OF INPUT PARAMETERS
16 thickness = 0.3e-3;
17 E_modulus = 193e9;
18 CB_Nrad = 0.9900*E_modulus*thickness^3;
19 offset = d2r*16.9;
20 Alpha_v = d2r*[70;70;70];
Matlab code for objective function in paper 3

```matlab
jDir_v = [1; 1; 1];
cB_on_off_v = [0; 1; 1]*1;
Rw = 1/100;
Gravity_on_off = 1;

% Simulation parameters
plotData = 0;
N = 1; % # of angular positions of link 2
ngridX = 300*2; % # of grid divisions for end point positions
ngridY = 300*2;
pj1_min = d2r*pj1min_deg;
pj2_min = d2r*1e-15;
pj3_min = d2r*1e-15;
jmax1 = 1; % Part of maximum joint angle
jmax2 = 1;
jmax3 = 1;

% Plot parameters
PlotSingleLayer = 1;
PlotDataPoints = 0;
PlotPath = 1;
x_val = 0.14;
scaling = 1;

% CALCULATED INPUT PARAMETERS
% Compute virtual hinge line angle
Beta_v = Alpha_v + 0.5*(pi-Alpha_v)+ offset;

% Compute virtual and real hinge line constants
cB_v = cB_Nrad*Width_v./(2*sin(Beta_v));
cB_v = cB_on_off_v.*cB_v;

% Calculate maximum joint angles
pj1_max = jmax1*Maxjoint(Phi_v(1), Alpha_v(1));
pj2_max = jmax2*Maxjoint(Phi_v(2), Alpha_v(2));
pj3_max = jmax3*Maxjoint(Phi_v(3), Alpha_v(3));

% Calculate angles for joint 1
pj1_v = pj1_min:(pj1_max-pj1_min)/(N-1):pj1_max;

% Calculate vectors of joint 1 hinge line angles
pB1_v = 0*pj1_v;
pC1_v = 0*pj1_v;
pD1_v = 0*pj1_v;
for ij1 = 1: length(pj1_v)
    [pB1_v(ij1),~] = pB_calculation(pj1_v(ij1),Beta_v(1),Phi_v(1),Alpha_v(1));
    [pC1_v(ij1)] = pC_calculation(pj1_v(ij1),Beta_v(1),Phi_v(1),Alpha_v(1));
    [pD1_v(ij1)] = pD_calculation(pj1_v(ij1),Beta_v(1),Phi_v(1),Alpha_v(1));
end

% Calculate position points
posX_v = linspace(-L_v(4), sum(L_v)-L_v(1),ngridX);
posY_v = linspace(-L_v(4), sum(L_v),ngridY);
```
% Calculate energies per grid point
V = zeros(ngridY, ngridX, N*2) % Matrix of energies per position
V_layers = zeros(ngridY, ngridX, N*2,4);
Vf_layers = zeros(ngridY, ngridX, 4);

% Calculate energies per grid point
ctr = 1;
for ix = 1:ngridX
    for iy = 1:ngridY
        for iV = 1:(N*2)
            iV2 = ceil(iV/2); % Do everything twice (elbow up and down)
            elbow = 2-mod(iV,2); % Odd and even
            pj1 = pj1_v(iV2);
            pj1_g = 2*pi - pj1;
pB1 = pB1_v(iV2);
pC1 = pC1_v(iV2);
pD1 = pD1_v(iV2);

            % Calculate pj2_g and pj3_g
            % 2 solutions each.
            % If the point cannot be reached, output is NaN.
            [pj2_g_v,pj3_g_v] = angles3geo(pj1, L_v, posX_v(ix), posY_v(iy));
            pj2_g = pj2_g_v(elbow);
pj3_g = pj3_g_v(elbow);

            % Convert joint angles to local
            [pj2] = Global2Local(pj1_g,pj2_g);
pj3 = Global2Local(pj2_g,pj3_g);

            % Change sign according to the nature of the joint (left or right bend)
pj2 = pj2*jDir_v(2);
pj3 = pj3*jDir_v(3);
        end
        if (pj2<pj2_max)*(pj2>0)*(pj3<pj3_max)*(pj3>0)*(isnan(pj2) ==0)*(isnan(pj3)==0)

            % Calculate angles of hinge lines
            [pB2,~] = pB_calculation(pj2, Beta_v(2), Phi_v(2), Alpha_v(2));
pC2 = pC_calculation(pj2, Beta_v(2), Phi_v(2), Alpha_v(2));
pD2 = pD_calculation(pj2, Beta_v(2), Phi_v(2), Alpha_v(2));
            [pB3,~] = pB_calculation(pj3, Beta_v(3), Phi_v(3), Alpha_v(3));
Matlab code for objective function in paper 3

\[
\begin{align*}
[pC3] &= pC_{\text{calculation}}(pj3, \beta_v(3), \phi_v(3), \alpha_v(3)) ; \\
[pD3] &= pD_{\text{calculation}}(pj3, \beta_v(3), \phi_v(3), \alpha_v(3)) ; \\
\end{align*}
\]

% Calculate and store energies
\[
\begin{align*}
V_B &= 0.5 \times c_{\text{B}_v}(1) \times pB_1^2 + 0.5 \times c_{\text{B}_v}(2) \times pB_2^2 + 0.5 \times c_{\text{B}_v}(3) \times pB_3^2; \\
V_C &= 2 \times 0.5 \times c_{\text{C}_v}' \times [pC_1^2 + pC_2^2 + pC_3^2]; \\
V_D &= 0.5 \times c_{\text{D}_v}' \times [pD_1^2 + pD_2^2 + pD_3^2]; \\
\end{align*}
\]

% Find lowest energy per grid point
% Only necessary in the case of three joints.
[Vf] = Efilter(V, ngridX, ngridY);

%% ADD GRAVITY
\[
\begin{align*}
V_{\text{ones}} &= Vf ./ Vf; \\
Gz0 &= 0.5198; \\
dG &= 2.4493; \% weight in N \\
Gz_v &= \text{linspace}(-Gz0, dG * (posY_v(\text{end}) - \text{posY}_v(1)) - Gz0, \text{ngridY}); \\
Gz_v &= Gz_v'; \\
V_G &= \text{repmat}(Gz_v, 1, \text{ngridX}) \times V_{\text{ones}}; \\
Vf_{\text{no\_gravity}} &= Vf; \\
Vf &= Vf + \text{Gravity\_on\_off} \times V_G; \\
\end{align*}
\]

%% FIND VERTICAL PATH
\[
\begin{align*}
\text{for } \text{ivp} &= 1 : \text{ngridY}; \\
\text{VPx\_ind} &= \text{find}(Vf(\text{ivp},:) == \text{min}(Vf(\text{ivp},:))); \\
\text{if } \text{isempty}(\text{VPx\_ind}) == 0; \\
\text{VPx\_v}(\text{ivp}) &= \text{posX}_v(\text{VPx\_ind}); \\
\text{VPy\_v}(\text{ivp}) &= \text{posY}_v(\text{ivp}); \\
\text{VP\_E}(\text{ivp}) &= Vf(\text{ivp}, \text{VPx\_ind}); \\
\end{align*}
\]
end
end
% Take away energy point at coordinate 0,0,0
[~, VPzero_ind] = find(VPy_v==0);
VPmin = max(VPzero_ind);
VPx_v = VPx_v(VPmin+1:length(VPx_v));
VPy_v = VPy_v(VPmin+1:length(VPy_v));
VP_E = VP_E(VPmin+1:length(VP_E));

%% DETERMINE AMOUNT OF POINTS GAINED
threshold = 4;
Range = -1*y_mech(end)*(length(find(abs(VP_dF)<threshold))/ length(VP_dF)) ;
end
Bibliography


