Background to Modified Bond Model
CONCEPT v. 19-10-2012

Author:
Ir. E. Lantsoght
Background to Modified Bond Model
CONCEPT v. 19-10-2012

Author:
Ir. E. Lantsoght
Table of Contents

1. Introduction and Summary ................................................................. 4
2. Application of Hallgren’s model ......................................................... 7
   2.1 Kinnunen and Nylander model .......................................................... 7
   2.2 Hallgren’s model ........................................................................... 9
   2.3 Application of Hallgren’s model to experiments .................................. 20
   2.4 Application to experiments .............................................................. 22
3. Application of beam action and arch action ......................................... 30
   3.1 Kani’s tied arch ........................................................................... 30
   3.2 Arching action as described by Kim et al .......................................... 32
4. Development of the Modified Bond Model .......................................... 37
   4.1 Alexander’s Bond Model .............................................................. 37
   4.2 Application to concentrated loads close to the support ...................... 41
   4.3 Extension to slabs at continuous support ......................................... 43
   4.4 Extension to loads at and near the edge ......................................... 45
   4.5 Extension to slabs on bearings ...................................................... 47
   4.6 Extension to plain bars ................................................................. 52
   4.7 Size effect ................................................................................. 55
   4.8 Summary of Modified Bond Model ................................................. 56
5. Overview of results ............................................................................ 57
6. Discussion of the proposed model ...................................................... 63
7. Conclusions .................................................................................... 64
8. References .................................................................................... 65
1. Introduction and Summary

The aim of this report is to document the process that was followed to determine a suitable theoretical model for explaining the observations in the series of tests on slabs under a concentrated load close to the support. All reasoning behind the decisions is gathered in this document, while in the dissertation only the final version of the Modified Bond Model is described and compared to the experimental results.

The first approach that is studied is altering the Hallgren model (1996) by adding a compressive strut taking the direct load transfer from the load to the support into account. However, too many complications arose and it was found that the basic assumptions of the Hallgren model are not suitable for this extension. The main issues that in the basic assumptions are:

- The Hallgren model assumes axis-symmetrical conditions, such that polar coordinates can be used. For a rectangular slab under a concentrated load near the support, Cartesian coordinates need to be used, and the number of unknowns increases.

- The relation between the tangential strain and the distance to the column is assumed to be inversely proportional in the Hallgren model. For the region between the concentrated load and the support, a disturbed strain patterns occurs and the strains cannot be expressed as a function of the distance anymore.

- The failure criterion in the Hallgren model is valid for $a/d > 3.5$. For smaller $a/d$ ratios, the development of a failure criterion based on the principles of nonlinear fracture mechanics should rely on detailed experimental results. Such a criterion is not available for application yet.

- A direct implication of the Hallgren model for punching for the studied cases should result in a safe prediction, since direct load transfer is not taken into account yet. However, a large overprediction of the maximum load was obtained.

- For certain combinations of parameters, the Hallgren model results in numerical instability of the code and imaginary values are returned. This observation was also noted by Wei (2008).
In a second approach, the compressive strut and arching action are studied in more detail since this mechanism is assumed to contribute largely to the capacity of the slab under a concentrated load close to the support. Models for beams in shear, such as Kani’s model (1964) and Kim’s model (1999). It was found, however, that approaches for one-way shear cannot be directly implemented to the case of slabs under a concentrated load. Therefore, three-dimensional strut-and-tie models are studied.

A theory which is based on an improvement of such a three-dimensional strut-and-tie model is the Bond Model by Alexander and Simmonds (1992). As it was found that the compressive strut are not straight, but should be represented by arches, the strut-and-tie model was divided into radial strips and quadrants in which load is transferred in two directions, using one-way shear transfer through bond of the reinforcement in each direction. Therefore, this method was chosen as a starting point. While this method is suitable for concentric punching shear, the idea could be used for further development, which resulted in the Modified Bond Model. The advantages of the Modified Bond Model are:

- Since the Modified Bond Model is a lower bound approach, the principles from plasticity can be applied. The force flow in the quadrants can easily be altered for different situations in the geometry.
- The Modified Bond Model is most suitable for application to all experiments as it can be tailored and altered as necessary.
- Currently, the size effect is not taken into account in the Modified Bond Model, but a size effect factor could easily be applied onto the load factor $w$.
- By using two-way quadrants and radial strips, the Modified Bond Model connects one-way shear and two-way shear and is most suitable for the description of mechanisms which are combined beam shear and punching shear failures, such as slabs under a concentrated load close to the support.
- The Modified Bond Model uses elements from plasticity, critical shear stress approaches and strut-and-tie models, and shows that these different categories of shear approaches all have a common ground.
The benefit of the development of the Modified Bond Model is ultimately demonstrated by comparing the results from EN 1992-1-1:2005 versus the experimentally obtained shear capacities to the predictions according to the Modified Bond Model versus the maximum force at the concentrated load. The results are less conservative, yet still have a 5% lower bound of experimental to predicted value of larger than 1, and have a smaller coefficient of variation than the test-to-predicted results from EN 1992-1-1:2005.
2. Application of Hallgren’s model

2.1 Kinnunen and Nylander model

Kinnunen and Nylander developed a mechanical model for punching shear in 1960 (Kinnunen and Nylander, 1960). The model is based on the equilibrium of forces on a polar-symmetrical slab supported by a column, Fig. 2.1a. In experiments, it was observed that the portion outside the failure crack rotates as a rigid body. Therefore, forces are studied on a segmental part of the slab portion outside of the shear crack, Fig. 2.1b.

![Mechanical model by Kinnunen and Nylander (1960).](image)

The failure criterion is that the tangential concrete strain at the bottom surface of the slab reaches a critical value, which is also observed in experimental measurements. It is then assumed that the tangential concrete strain and the steel strain are inversely proportional to the radius from the slab center.

Based on a vertical projection, Fig. 2.1.c, the ultimate load can be described as:

\[ P_u = 1.1\pi \frac{By \left(1 + 2y/B \right)}{d^2 \left(1 + y/B \right)} f(\alpha)\sigma_d d^2 \]  \hspace{1cm} (2.1)
Background to Modified Bond Model
Application of Hallgren’s model - Kinnunen and Nylander model

with

\[ f(\alpha) \text{ a function of } \alpha \text{, the inclination of the conical shell} \]

\[ f(\alpha) = \sin \alpha \cos \alpha (1 - \tan \alpha) \]  \hspace{1cm} (2.2)

\[ \sigma \]  \text{ the stress in the conical shell,} \]

\[ c = 1.1 \text{ a magnification factor since dowel action is not considered in the model.} \]

The value of \( \alpha \) is determined by solving:

\[ (K_y \tan \alpha - 1) \frac{1 - \tan \alpha}{1 + \tan^2 \alpha} = \frac{1}{2 \times 2.35} \left( \frac{1 + \frac{y}{B}}{0.5B/d + \frac{y}{d}} \right)^{\ln \left( \frac{0.5c/d}{0.5B/d + \frac{y}{d}} \right)} \]  \hspace{1cm} (2.3)

with

\[ K_y = \frac{0.5c - B/2}{d - y/3} \]  \hspace{1cm} (2.4)

\( c \)  \text{ the diameter of the slab area with negative radial bending moment above the column.} \]

The stress in the conical shell is based on tests and can be written as:

\[ \sigma = 2.35 \sigma_{CT, r=B/2+y} = 2.35 E_c \varepsilon_{CT, r=B/2+y} \]  \hspace{1cm} (2.5)

with \( \sigma_{CT, r=B/2+y} \) and \( \varepsilon_{CT, r=B/2+y} \) the tangential stress and strain at \( r = \frac{B}{2} + y \) from the center of the column. In the Kinnunen and Nylander model (1960), the elastic modulus of the concrete is expressed as:

\[ E_c = 10^4 \left( 0.35 + 0.3 \frac{f_{c, cube}}{15} \right) \]  \hspace{1cm} (2.6)

The tangential strain at ultimate can be expressed as:

for \( \frac{B}{d} \leq 2 \) : \( \varepsilon_{CT, r=B/2+y} = 0.0035 \left( 1 - 0.22 \frac{B}{d} \right) \)  \hspace{1cm} (2.7)

for \( \frac{B}{d} > 2 \) : \( \varepsilon_{CT, r=B/2+y} = 0.0019 \)

The rotation of the slab portion outside the shear crack at ultimate, \( \psi \) is expressed as:

\[ \psi = \left( 1 + 0.5 \frac{B}{y} \right) \varepsilon_{CT, r=B/2+y} \]  \hspace{1cm} (2.8)

The radius of the slab area in which the reinforcement yields, \( r_s \), can be expressed as:

\[ r_s = \frac{E_c}{f_{fy} \psi \left( 1 - \frac{y}{d} \right) d} \]  \hspace{1cm} (2.9)
The distance from the column center to the concentric shear crack, $c_o$, is based on experimental observations:

$$c_o = 0.5B + 1.8d$$  \hspace{1cm} (2.10)

The other approach to describing the ultimate load $P_u$ is by taking moments around $Q_1$ in Fig. 2.1c:

$$P_u = 1.1 \times 4\pi \rho f_{y_d} dr \left[1 + \ln \left(\frac{0.5 c}{c_o}\right)\right] \frac{d - y/3}{c - B}$$

for $r < c_o$

$$P_u = 1.1 \times 4\pi \rho f_{y_d} dr \left[1 + \ln \left(\frac{0.5 c}{r}\right)\right] \frac{d - y/3}{c - B}$$

for $r > c_o$  \hspace{1cm} (2.11)

Iterations for the value of $y$ are then carried out until $P_u$ in Eq. (2.1) equals $P_u$ in Eq. (2.11). This procedure results in the ultimate punching load $P_u$ according to Kinnunen and Nylander.

### 2.2 Hallgren’s model

The goal of Hallgren’s model (Hallgren, 1996) is to derive the ultimate tangential concrete strain from a simple fracture mechanical model which reflects the brittleness of the concrete and the size effect. Based on the mechanical model of Kinnunen and Nylander (1960), the Hallgren model also takes into account that the slope of the shear crack varies with the geometry and the material properties. The result is a model for symmetric punching for slabs without shear reinforcement. The model is valid for $a/d > 3.5$ and it is pointed out that a different failure criterion is required for smaller $a/d$ ratios.

The geometry of the model is similar to the Kinnunen and Nylander model.
Background to Modified Bond Model
Application of Hallgren’s model - Hallgren’s model

Fig. 2.2: Geometry used in Hallgren’s model (1996).

The load is assumed to be carried from the slab portion outside the shear crack to the column via a truncated wedge through an inclined compressive force $T$. The compressive force is inclined at an angle $\alpha$. As a result, the length of the wedge, $y$, can be expressed as a function of the depth of the tangential compression zone $x$.

$$y = x(1 + \tan \alpha)$$  \hspace{1cm} (2.12)

The distance from the column center to the shear crack at the level of the flexural reinforcement is:

$$c_o = \frac{B}{2} + x + \frac{d - x}{\tan 1.5\alpha}$$  \hspace{1cm} (2.13)

The angle of the shear crack is assumed to be $1.5\alpha$. The value of 1.5 is determined from tests and nonlinear finite element analysis and is proportional to the ratio of the slope of the shear crack to the inclination of the largest principal compressive stress at the slab-column root.

An important point of consideration in Hallgren’s model is the expression for the tangential strains. It is assumed that the tangential strains for the concrete in compression and the steel in tension are inversely proportional to the radial distance from the column. This assumption is supported by experimental measurements and results from finite element analysis. Moreover, it is observed that the tangential steel strain remains constant within the shear crack and that the tangential concrete strain on the compression surface at $r \geq x$ is constant or might even decrease. The equilibrium of forces is calculated on a slab portion outside of the crack. Using the
assumption of the inverse proportionality of the tangential strains and the radial distance, the strain profiles can be expressed as:

\[ \varepsilon_{ct} = C_1 \frac{1}{r} \quad \text{for } r \geq \frac{B}{2} + y \]  

(2.14)

with \( C_1 \) a constant. The ultimate value of the strain \( \varepsilon_{cTu} \) is reached for \( r = \frac{B}{2} + y \):

\[ \varepsilon_{cTu} = C_1 \frac{1}{\frac{B}{2} + y} \]

such that the constant \( C_1 \) is found to be \( C_1 = \left( \frac{B}{2} + y \right) \varepsilon_{cTu} \).

---

Fig. 2.3: Distribution of tangential steel \( \varepsilon_{st} \) and concrete strains \( \varepsilon_{ct} \) along the radius \( r \). (Hallgren, 1996).

The tangential steel strains can be expressed similarly:

\[ \varepsilon_{st} = C_2 \frac{1}{r} \]

(2.15)

with \( C_2 \) a constant. The relation between the steel and concrete tangential strains can be found as \( \varepsilon_{st} = \frac{d-x}{x} \varepsilon_{ct} \). Therefore, the constant \( C_2 \) is given as:

\[ C_2 = \left( \frac{d-x}{x} \right) C_1 = \left( \frac{d-x}{x} \right) \left( \frac{B}{2} + y \right) \varepsilon_{cTu} \]

(2.16)
The next step is the determination of the depth of the compression zone $x$. This depth can be expressed in terms of the tangential concrete and steel strains and the stress-strain states at $r = c_0$. Bond slip is neglected and the strain is assumed to be linear over the cross-section. The stress-strain curves from Fig. 2.4 are used for concrete and steel.

The yield strain of the concrete can be expressed as: $\varepsilon_{cy} = \frac{f_{cc}}{E_c}$ with $f_{cc}$ the concrete compressive strength. The yield strain of steel is expressed as $\varepsilon_{sy} = \frac{f_{sy}}{E_s}$ with $f_{sy}$ the yield stress of the concrete. For concrete in the elastic phase, a triangular stress block is used and after reaching yield, a bilinear stress block is used. The force in the concrete stress block is $F_c = \alpha_f f_{cc} x dr$ with $\alpha_f = 1 - \frac{\varepsilon_{cy}}{2\varepsilon_{ct}}$; Fig. 2.5. The value of $x$ can then be determined from the compatibility and stress-strain states at $r = c_0$.

Four cases are considered:
1. Steel and concrete are elastic

From horizontal force equilibrium, it is known that \( F_s = F_c \), which results in

\[
\sigma_s \rho d \, dr = \alpha_c f_c \, x \, dr
\]  

(2.17)

The value of \( \alpha_c \) is \( \frac{1}{2} \) when concrete is elastic. With the stress-strain relations, the following expression for the depth of the concrete compression zone is found:

\[
x = \frac{E_c}{k_E E_c} \rho d \left( \sqrt{1 + \frac{2E_c k_E}{\rho E_s}} - 1 \right)
\]  

(2.18)

In Eq. (2.18), \( k_E \) is a stiffness factor > 1 which takes into account the increased stiffness due to biaxial compression at \( r = c_o \). The stiffness factor can be determined from Hooke’s law for plane stress in polar coordinates:

\[
\begin{align*}
\varepsilon_c & = \frac{1}{E_c} \left( \sigma_c - \nu \sigma_{cr} \right) \\
\varepsilon_{cr} & = \frac{1}{E_c} \left( \sigma_{cr} - \nu \sigma_{ct} \right)
\end{align*}
\]  

(2.19)

In experiments, it is measured that \( \varepsilon_{cr} \approx 0.5 \varepsilon_{ct} \). The radial \( \sigma_{cr} \) and tangential stresses \( \sigma_{ct} \) can be expressed as:

\[
\begin{align*}
\varepsilon_{cr} & = 0.5 \varepsilon_{ct} \iff \frac{2}{E_c} \left( \sigma_{cr} - \nu \sigma_{ct} \right) = \frac{1}{E_c} \left( \sigma_{ct} - \nu \sigma_{cr} \right) \\
\iff \sigma_{cr} & = \sigma_{ct} \left( \frac{1 + 2\nu}{2 + \nu} \right)
\end{align*}
\]  

(2.20)

Combining Eq. (2.19) and Eq. (2.20) results in:

\[
\varepsilon_{ct} = \frac{1}{E_c} \sigma_{ct} \left( 1 - \frac{\nu + 2\nu^2}{2 + \nu} \right)
\]  

(2.21)

Therefore, the stiffness factor \( k_E \) can be expressed as:

\[
k_E = \frac{\sigma_{ct}}{\varepsilon_{ct}} = \left( 1 - \frac{\nu + 2\nu^2}{2 + \nu} \right)^{-1}
\]  

(2.22)

For a Poisson ratio of \( \nu = 0.2 \) a stiffness factor \( k_E = 1.15 \) is found.

2. Concrete has yielded at \( r = c_o \)

For the case in which steel is elastic and concrete has yielded, the concrete strain \( \varepsilon_{ct} > \varepsilon_{cy} \). The horizontal equilibrium \( F_s = F_c \) can be used to determine the depth of the compressive zone:

\[
\alpha_c k_E f_c x \, dr = \sigma_s \rho d \, dr
\]  

(2.23)
Background to Modified Bond Model
Application of Hallgren’s model - Hallgren’s model

with \( \alpha_{c_o} = 1 - \frac{\varepsilon_{cy}}{2\varepsilon_{cTo}} \).

The depth of the compressive zone can be expressed as :

\[
x = \frac{E_{cTo} \rho d}{\alpha_{c_o} f_{cc}} \left( \sqrt{1 + \frac{4\alpha_{c_o} f_{cc}}{\rho E_{cTo} \varepsilon}} - 1 \right)
\]  

(2.24)

3. Steel has yielded at \( r = c_o \)

The depth of the concrete compressive zone for the case in which the steel has yielded and the concrete is elastic is found again from the horizontal equilibrium: \( F_s = F_c \):

\[
f_{sy} \rho d = \frac{1}{2} f_{sy} x \leftrightarrow x = \frac{2 \rho df_{sy}}{k_e c_{To} E_c}
\]  

(2.25)

4. Both steel and concrete have yielded at \( r = c_o \)

Again, from the horizontal equilibrium, the depth of the compressive zone can be found, \( F_s = F_c \):

\[
f_{sy} \rho d = \alpha_{c_o} f_{cc} \leftrightarrow x = \frac{\rho df_{sy}}{\alpha_{c_o} f_{cc}}
\]  

(2.26)

The next step is to determine the **compressive force in the concrete and the tension in the steel**. The forces on a slab segment outside the shear crack are used to determine these values. The compressive force \( R_{cT} \) can be found from the tangential concrete stresses, by integrating the stresses from \( r = \frac{B}{2} + y \rightarrow r = \frac{c}{2} \), which means that the part between \( r = \frac{B}{2} + x \rightarrow r = \frac{B}{2} + y \) is neglected. For all tangential concrete in the elastic state \( \varepsilon_{cTu} \leq \varepsilon_{cy} \), such that the integration of stresses becomes:

\[
R_{cT} = \int_{\frac{B}{2} + y}^{\frac{c}{2}} \frac{1}{2} E_c x d \varepsilon_{cT} = \int_{\frac{B}{2} + y}^{\frac{c}{2}} \frac{1}{2} E_c x \varepsilon_{cTu} \left( \frac{B}{2} + y \right) \frac{1}{r}
\]

\[
R_{cT} = \frac{1}{2} E_c \varepsilon_{cTu} x \left( \frac{B}{2} + y \right) \ln \left( \frac{c}{B + 2y} \right)
\]  

(2.27)

If the concrete has partially yielded \( \left( \varepsilon_{cTu} > \varepsilon_{cy} \right. \) at \( r > \frac{B}{2} + y \)) an additional distance can be defined: \( r = \frac{C_1}{\varepsilon_{cy}} \). To meet convergence criteria, \( r \leq \frac{c}{2} \).
For \( r_c > \frac{B}{2} + y \) the compressive force in the concrete is expressed as:

\[
R_{cT} = \int_{\frac{B}{2} + y}^{r_c} \alpha_f f_{cT} x dr + \int_{r_c}^{r_c + B} \frac{1}{2} E_c x e_{cT} dr
\]

Similarly, the tension in the steel can be expressed by integrating the stresses from \( c_o \) to \( c/2 \). For the case in which all steel is in the elastic state, the tension in the steel is:

\[
R_{sT} = \int_{c_o}^{c/2} \sigma_s \rho d dr = C_e E_s \rho d \ln \left( \frac{c}{2c_o} \right)
\]

Fig. 2.6: Distribution of tangential concrete stresses \( \sigma_{cT} \) in the compression zone along the radius \( r \) of the slab (Hallgren, 1996).

Fig. 2.7: Tangential steel stresses \( \sigma_{sT} \) of the flexural reinforcement along the radius \( r \) of the slab (Hallgren, 1996).
For the case in which the steel yields at \( r > c_o \), an additional location is defined as \( r_s = \frac{C_2}{\varepsilon_y} \). To reach convergence, the yield radius needs to be limited by \( r_s \leq \frac{C}{2} \). For the case in which \( r_s > c_o \) and thus \( \varepsilon_{sR} > \varepsilon_y \) the tension in the steel is expressed as:

\[
R_{st} = \int_{c_o}^{c} f_{st} \rho d.d.r + \int_{r}^{c} \cdot \frac{C_2}{r} \rho d.d.r
\]

\[
R_{st} = \rho d \left( f_{st} (r_s - c_o) + C_2 \ln \left( \frac{c}{2r_s} \right) \right)
\]  

At \( r = c_o \) at the intersection of the shear crack, it is assumed that \( \varepsilon_{sR} = \varepsilon_{sR} \). With these assumptions, the radial steel tension \( R_{sR} \) at \( r = c_o \) is determined. The resulting radial force is:

\[
\begin{align*}
\text{if } r_s < c_o, & \quad \varepsilon_{sR} < \varepsilon_y : R_{sR} = E_c \varepsilon_{sR} \rho d 2\pi c_o \\
\text{if } r_s \geq c_o, & \quad \varepsilon_{sR} \geq \varepsilon_y : R_{sR} = f_{st} \rho d 2\pi c_o
\end{align*}
\]

The **dowel force** is determined in a following step. The dowel force is determined from the load at which the splitting crack opens along the reinforcement layer. Based on empirical observations, the dowel force is assumed as:

\[
D = 27.9 \varphi^{2/3} c_o \left( 1 - 1.6 \frac{\rho d}{\varphi} \right) f_{c2}^{1/3}
\]  

If the reinforcement bar yields at the shear crack \( \varepsilon_{sR} \geq \varepsilon_y \), then \( D = 0 \) for the sake of simplicity.

With all forces known, it is possible to write the **equilibrium equations**. The unknowns are the external load \( P \), the compression force \( T \) and the inclination of the compression force \( \alpha \). These unknowns are determined based on the equilibrium of forces on a slab segment outside the shear crack. A vertical projection results in:

\[
T \frac{\Delta \phi}{2\pi} \sin \alpha + D \frac{\Delta \phi}{2\pi} - P \frac{\Delta \phi}{2\pi} = 0
\]

\[
T \sin \alpha + D = P
\]  

A horizontal projection results in:
Background to Modified Bond Model
Application of Hallgren’s model - Hallgren’s model

\[
T \frac{\Delta \phi}{2\pi} \cos \alpha - R_{sfr} \frac{\Delta \phi}{2\pi} - R_{sfr} \Delta \phi + R_{cT} \Delta \phi = 0
\]

\[
T = \frac{R_{sfr} + R_{sfr} 2\pi - R_{cT} 2\pi}{\cos \alpha}
\]  
(2.34)

Expressing the moment around Q and assuming that \( R_{cT} \) is applied at \( \frac{2x}{3} \), which is valid for elastic concrete and is otherwise a simple approximation, it is found that:

\[
P \frac{\Delta \phi}{2\pi} \left( \frac{c - B}{2} - x \right) - R_{sfr} \Delta \phi (d - x) - R_{sfr} \frac{\Delta \phi}{2\pi} (d - x) - D \frac{\Delta \phi}{2\pi} \left( c - B \right) \frac{\Delta \phi}{2\pi} \left( c - B \right)
\]

\[
-T \frac{\Delta \phi}{2\pi} \frac{x}{2 \cos \alpha} - R_{cT} \Delta \phi \frac{2x}{3} = 0
\]

\[
P = \frac{\left( R_{sfr} 2\pi + R_{sfr} \right)(d - x) + R_{sfr} 2\pi \frac{2x}{3} + D \left( c - B \right) }{2 \cos \alpha} + T \frac{x}{2 \cos \alpha}
\]  
(2.35)

In a final step, the **failure criterion** is defined based on nonlinear fracture mechanics.

At the location \( r = \frac{B}{2} + y \) the concrete is in a biaxial state of compression. If the shear deformation is neglected, \( \varepsilon_{cT} = \varepsilon_{cR} \) can be assumed.

---

**Fig. 2.8: States of stress (Hallgren, 1996).**

Based on biaxial compression test results (Kupfer et al., 1969), it is found that the compressive strain in the x- and y-direction are more or less equal to the transverse tensile strain in the z-direction, such that before failure \( \varepsilon_{cT} = -\varepsilon_{cZ} \) at \( r = \frac{B}{2} + y \).
After reaching the ultimate stress, softening behavior can occur at $r = \frac{B}{2} + y$ and $\sigma_{\text{ult}}$ at the column root decreases. When the three-axial stress state becomes unstable, the shear crack can break through the compression zone and a punching failure can occur. To describe the failure mechanism, the opening of the macro-crack is based on the fictitious crack model by Hillerborg et al. (1976) which is a nonlinear fracture mechanics approach. The fictitious crack is the part of the crack where the tensile strength is exceeded and the crack opens but some tensile stress can still be transferred as a result of post-peak concrete softening. The critical crack opening $w_c$ is the crack opening for which tension can no longer be transferred and the macro-crack opens.

*Fig. 2.9: Fictitious crack model by Hillerborg et al. (1976).*

The failure criterion for $\varepsilon_{c,Tu}$ is based on this simplified model, in which it is assumed that a horizontal crack opens in the tangential compression zone at $r = \frac{B}{2} + y$. The average strain across the compression zone $\varepsilon_{c,Zu}$ when the critical opening $w_c$ is reached can be written as:

$$\varepsilon_{c,Zu} = \frac{w_c}{x}$$

$\Rightarrow |\varepsilon_{c,Tu}| = |\varepsilon_{c,Zu}| = \left| \frac{w_c}{x} \right|$ \hspace{1cm} (2.36)

And the failure criterion can thus be expressed as:

$$\varepsilon_{c,Tu} = \frac{w_c}{x}$$

(2.37)

The value of the critical crack opening $w_c$ can be determined from the bilinear tension-softening curve by Petersson and Gustavsson (1980).
Fig. 2.10: Bilinear tension-softening curve by Petersson and Gustavsson (1980).

The area underneath the curve is the fracture energy $G_F$, the energy necessary to create one unit area of crack. Based on the bilinear tension-softening curve, the relation between the fracture energy $G_F$ and the critical crack opening $w_c$ is found to be:

$$w_c = 3.6 \frac{G_F}{f_a} \quad (2.38)$$

Moreover, the fracture energy $G_F$ is size-dependent. The structural size can be expressed in terms of $x$, the depth of the concrete compression zone, which is approximately equal to the radial length of the zone with tensile strains at about $r = \frac{B}{2} + y$. Using the multifractal scaling law, the fracture energy is:

$$G_F = G_F^\infty \left( 1 + \frac{\alpha_F d_a}{x} \right)^{-\frac{1}{2}} \quad \text{with} \quad G_F^\infty = G_F^R \left( 1 + \frac{13d_a}{d^R} \right)^{\frac{1}{2}} \quad (2.39)$$

with:

- $G_F^\infty$ the fracture energy for an infinitely large structural size,
- $d_a$ the maximum aggregate size,
- $\alpha_F$ an empirical factor $\approx 13$,
- $d^R$ the depth of the RILEM test beam.

Combining Eq. (2.38) and Eq. (2.39) results in the failure criterion:

$$\epsilon_c = \frac{3.6 G_F^\infty}{f_a f_T} \left( 1 + \frac{13d_a}{x} \right)^{\frac{1}{2}} \quad (2.40)$$
2.3 Application of Hallgren’s model to experiments

The described procedure is applied to S1T1 from the experiments (Lantsoght, 2011). This codified procedure is first verified with the results from Hallgren’s experiments and his calculated values. The Matlab code for the algorithm is given here:
Background to Modified Bond Model
Application of Hallgren’s model - Application of Hallgren’s model to experiments
For S1T1, a punching load of 1912kN is found, while in the experiment a value of 954kN was obtained.

### 2.4 Application to experiments

The direct application of Hallgren’s model to the experimental results of slabs under a concentrated load close to the support (Lantsoght, 2011) does not lead to good results, as expected. It is necessary to take the direct load transfer between the load and the support into account. Moreover, all the assumptions made by Hallgren (1996) need to be critically revisited before the method can be tailored for the case of loads near to the support.

The first assumption which needs to be verified is the axis-symmetrical conditions. It is obvious that for a concentrated load on a rectangular plate near to the support, axis-symmetrical conditions are not met. Therefore, the force equilibrium needs to be expressed in terms of Cartesian coordinates, Fig. 2.11.
Fig. 2.11: Modified Hallgren model including compression strut (Figures from Wei, 2008).
To gain a better understanding of the strain profiles, nonlinear finite element models are used. As shown in Fig. 2.12, the principal strains have a disturbed pattern around the load and they cannot be expressed inversely proportional to the distance to the load.

![Strain profile around load](image)

*Fig. 2.12: Strain profile around load*

In the finite element models (modified from Prochazkova, 2012), the compressive strut/arch can clearly be noticed from the stress profiles.
Background to Modified Bond Model
Application of Hallgren’s model - Application to experiments

Fig. 2.13: Cut along the width at the location of the load showing the stresses.

Fig. 2.14: Cut along the width halfway between the load and the support showing the stresses.
Background to Modified Bond Model
Application of Hallgren’s model - Application to experiments

Fig. 2.15: Cut along the width halfway at the location of the support showing the stresses.

Fig. 2.16: Cut along the span showing the compressive strut and cracks between the load and the support.

An expression for the compression strut can be sought such that the fan of struts (or arches) can be replaced by a single force. The fan of struts can be sketched as in Fig. 2.17; and this can be projected onto the $xz$ plane (Fig. 2.18) or the $xy$ plane (Fig. 2.19). The projection on the $xy$ plane is used to determine an equivalent force that can be applied onto the $xz$ plane, from which the problem might be treated two-dimensionally, or the equivalent compressive force can be added to Hallgren’s model (Fig. 2.11).
The equivalent force can be expressed as a sum of compressive forces at different angles:

\[
ΣC_{\text{strut},1} = C_{\text{strut},1} + 2(C_{\text{strut},2} + ... + C_{\text{strut},n})
\]  
(2.41)

All compressive forces under an angle can be expressed in terms of \(C_{\text{strut},1}\):
Background to Modified Bond Model
Application of Hallgren’s model - Application to experiments

\[ C_{\text{strat},i} = \frac{C_{\text{strat},1}}{\cos \alpha_i} \]  \hspace{1cm} (2.42)

Combining Eq. (2.41) and Eq. (2.42) results in:

\[ \sum C_{\text{strat},i} = C_{\text{strat},1} + 2 \left( \frac{C_{\text{strat},1}}{\cos \alpha_2} + \ldots + \frac{C_{\text{strat},1}}{\cos \phi} \right) \]

\[ \sum C_{\text{strat},i} = C_{\text{strat},1} + 2C_{\text{strat},1} \left( \frac{1}{\cos \alpha_2} + \ldots + \frac{1}{\cos \phi} \right) \]  \hspace{1cm} (2.43)

\[ \sum C_{\text{strat},i} = C_{\text{strat},1} + 2C_{\text{strat},1} \sum_{i=2}^{n} \frac{1}{\cos \alpha_i} \]

The sum can be expressed as an integral, in which the bound from \( \alpha_2 \) to \( \alpha_n \) are replaced by bounds from 0 to \( \phi \):

\[ \int_{0}^{\phi} \frac{1}{\cos x} \, dx = \int_{0}^{\phi} \sec x \, dx \]

\[ = \ln |\sec x + \tan x| \bigg|_{0}^{\phi} \]

\[ = \ln |\sec \phi + \tan \phi| - \ln |1| \]

\[ = \ln |\sec \phi + \tan \phi| \]  \hspace{1cm} (2.44)

The equivalent compressive force is then:

\[ C_{\text{eq}} = C_{\text{strat},1} + 2C_{\text{strat},1} \ln |\sec \phi + \tan \phi| \]

\[ C_{\text{eq}} = C_{\text{strat},1} (1 + 2\ln |\sec \phi + \tan \phi|) \]  \hspace{1cm} (2.45)

The value of the angle of dispersion of the fan \( \phi \) is initially not known. The compressive force \( C_c \) in Fig. 2.11 is partially carried directly to the support by this fan of struts.

When applying the strut force into the model, the fact that the failure criterion is not just a function of \( f_c \) resulted in numerical instability. The resulting values of \( \phi \) were not consistent.

A few changes were made in the modified Hallgren model to study the sensitivity and the stability of the algorithm: the value of \( R_{c1} \) was neglected, the angle over which the fan of strut acts was fixed and the failure criterion was replaced by a maximum concrete strain of \( \varepsilon_{cu} = 0.0035 \). However, serious complications in the modified Hallgren model occurred, and showed that the modified model is not stable for parameter variations, often resulting in imaginary numbers, as also observed by Wei (2008).
As Hallgren (1996) indicated that the failure criterion is not valid for deep beams, a brief study of the literature was carried out to see if a failure criterion for deep beams based on nonlinear fracture mechanics is available. Another idea which was explored was studying the failure criterion for the compressive strut in compression. However, the tri-axial stress state in the slab might alter the failure criterion. None of these tracks led to fruitful results.

To conclude it can be stated that, when altering the Hallgren model for the application to slabs under concentrated loads near to the support, the boundaries of the applicability of the assumptions are transgressed.
3. Application of beam action and arch action

A second approach is to study arching action and the compressive strut in more detail, to gain a better insight of the direct transfer of load from its point of application to the support.

3.1 Kani’s tied arch

In a first approach, Kani’s tied arch (Kani, 1964) is used to study the capacity of the arch, Fig. 3.1. From the lines between C and the support, it can be derived that:

\[
\frac{y}{y_o} = \frac{d}{d-s+y_o}
\]

with

- \( y \) the depth of the reduced compression zone,
- \( y_o \) the depth of the compression zone,
- \( s \) the crack spacing.

The following relation is assumed:

\[
\frac{M_{cr}}{M_{fl}} = \frac{y}{y_o}
\]

with

- \( M_{cr} \) the moment which results at the maximum value for the concentrated load \( P \),
- \( M_{fl} \) the theoretical flexural moment at failure.

Now it is assumed that \( s = y_o \). Ultimately the following relation is obtained:
Background to Modified Bond Model
Application of beam action and arch action - Kani’s tied arch

\[ M_{cr} = \frac{M_{fl} d}{0.9 a} \]  

(3.3)

The factor 0.9 is used for the influence of the biaxial stress conditions.

To compare this approach to the experiments on slabs under a concentrated load, the flexural capacity \( M_{fl} \) is determined based on the ACI rectangular stress block.

\[ \text{Fig. 3.2: Rectangular stress block diagram as used in ACI.} \]

An overview of the results as compared to a reference subset from the experiments is given in Table 3.1. The large scatter on the results indicates that this method is not suitable for determining the capacity of the compressive strut or arch and hence using this as an indication for the moment at failure is not an adequate method.

\[ \text{Table 3.1: Comparison between Kani’s method for quantifying the capacity of the arch and the experimental results.} \]

<table>
<thead>
<tr>
<th>Test</th>
<th>( a ) (mm)</th>
<th>( f_{ck} ) (MPa)</th>
<th>( A_s ) (mm²)</th>
<th>( f_y ) (MPa)</th>
<th>( a ) (mm)</th>
<th>( \beta )</th>
<th>( c ) (mm)</th>
<th>( M_{fl} ) (kNm)</th>
<th>( M_{exp} ) (kNm)</th>
<th>( M_{cr} ) (kNm)</th>
<th>( M_{exp}/M_{cr} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1T1</td>
<td>600</td>
<td>29</td>
<td>6597</td>
<td>601</td>
<td>64</td>
<td>0.84</td>
<td>76</td>
<td>925</td>
<td>475</td>
<td>454</td>
<td>1.05</td>
</tr>
<tr>
<td>S2T1</td>
<td>600</td>
<td>28</td>
<td>6597</td>
<td>601</td>
<td>66</td>
<td>0.84</td>
<td>78</td>
<td>920</td>
<td>673</td>
<td>451</td>
<td>1.49</td>
</tr>
<tr>
<td>S3T1</td>
<td>600</td>
<td>42</td>
<td>6597</td>
<td>601</td>
<td>44</td>
<td>0.74</td>
<td>59</td>
<td>963</td>
<td>675</td>
<td>473</td>
<td>1.43</td>
</tr>
<tr>
<td>S5T4</td>
<td>400</td>
<td>40</td>
<td>6597</td>
<td>601</td>
<td>47</td>
<td>0.76</td>
<td>62</td>
<td>957</td>
<td>615</td>
<td>705</td>
<td>0.87</td>
</tr>
<tr>
<td>S8T1</td>
<td>600</td>
<td>63</td>
<td>6597</td>
<td>601</td>
<td>30</td>
<td>0.65</td>
<td>45</td>
<td>992</td>
<td>731</td>
<td>487</td>
<td>1.50</td>
</tr>
<tr>
<td>S9T1</td>
<td>400</td>
<td>67</td>
<td>6597</td>
<td>601</td>
<td>28</td>
<td>0.65</td>
<td>43</td>
<td>996</td>
<td>733</td>
<td>733</td>
<td>0.74</td>
</tr>
</tbody>
</table>

| AVG  | 1.18  |
| STD  | 0.34  |
| COV  | 0.29  |
3.2 Arching action as described by Kim et al

Kim et al. (1999) analytically described the beam action and arch action for beams in shear. The main assumption in the theory by Kim et al. (1999) is that the following ratio can be used:

\[
\frac{z_0}{z_m} = \frac{T_m}{T_0}
\]  

(3.4)

with

- \(z_0\) the calculated inner lever arm length,
- \(z_m\) the actual inner lever arm length,
- \(T_m\) the measured tension in the steel,
- \(T_0\) the steel tension as calculated from beam theory.

The relation between the contributions of arching and beam theory is expressed as:

\[
T_A = \alpha T_B
\]

(3.5)

with

- \(T_A\) the contribution of the arching action,
- \(T_B\) the contribution of the beam action.

Therefore;

\[
T_m = (1 + \alpha)T_B = (1 + \alpha)T_0
\]

(3.6)

Combining Eq. (3.4) and Eq. (3.6) results in:

\[
\frac{z_0}{z_m} = 1 + \alpha \Rightarrow z_m = \frac{1}{1 + \alpha} z_0
\]

(3.7)

For simplicity, the profile of the arch can be described as (Fig. 3.3):

\[
z(x) = \left(\frac{x}{a}\right)^\gamma z_0
\]

(3.8)

Fig. 3.3: Assumed internal moment arm length variation (Kim et al., 1999).
An empirical relation is determined for the power $r$:

$$r = k \left( \frac{d}{a} \right)^{n_1} \rho^{n_2} \leq 1$$  \hspace{1cm} (3.9)

with $k$, $n_1$ and $n_2$ as empirical constants. The value of $z_0$ can be determined with linear theory as well as with limit theory. Here, it is assumed to be:

$$z_0 = \left(1 - \rho^{1/2}\right)d$$  \hspace{1cm} (3.10)

The contribution of the arching action to the shear capacity is determined as $T \frac{dz}{dx}$.

Taking the tension in the steel $T$ as $T = f_s \rho bd$ results in the following expression for the contribution of the arching action to the shear capacity:

$$V_2 = f_s \rho \ast bd \ast r \left( \frac{x}{a} \right)^{r-1} \frac{1}{a} z_0 = \rho \left(1 - \rho^{1/2}\right) r \left( \frac{x}{a} \right)^{r-1} f_s bd^2$$  \hspace{1cm} (3.11)

The value of $r$ is a constant $\leq 1$ and the shear is studied at $x = d$.

A comparison between the predicted values for the arching contribution and the experimental results is given in Table 3.2. The value of $r$ is determined with the recommended values of $n_1 = 0.6$ and $n_2 = -0.1$. It is found that the calculated value of the shear force due to arching is too small as compared to the shear force occurring in the experiments. A positive observation is that the expression for the arching action in Eq. (3.11) does not depend on the concrete compressive strength, a parameter which traditionally is important in the determination of the shear capacity, but did not have a major influence on the outcome of the maximum shear capacity in the experiments.

<table>
<thead>
<tr>
<th>Test</th>
<th>$a$ (mm)</th>
<th>$b_{load}$ (mm)</th>
<th>$b_{sup}$ (mm)</th>
<th>$P_a$ (kN)</th>
<th>$\rho$ (%)</th>
<th>$R_2$ (kN)</th>
<th>$V_{max}$ (kN)</th>
<th>$d_1$ (mm)</th>
<th>$r$</th>
<th>$V_2$ (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1T1</td>
<td>600</td>
<td>200</td>
<td>100</td>
<td>954</td>
<td>0.996</td>
<td>804</td>
<td>799</td>
<td>265</td>
<td>0.6127</td>
<td>295</td>
</tr>
<tr>
<td>S2T1</td>
<td>600</td>
<td>200</td>
<td>100</td>
<td>1374</td>
<td>0.996</td>
<td>1135</td>
<td>1129</td>
<td>265</td>
<td>0.6127</td>
<td>295</td>
</tr>
<tr>
<td>S3T1</td>
<td>600</td>
<td>300</td>
<td>100</td>
<td>1371</td>
<td>0.996</td>
<td>1137</td>
<td>1131</td>
<td>265</td>
<td>0.6127</td>
<td>295</td>
</tr>
<tr>
<td>S5T4</td>
<td>400</td>
<td>300</td>
<td>100</td>
<td>1755</td>
<td>0.996</td>
<td>1550</td>
<td>1544</td>
<td>265</td>
<td>0.7814</td>
<td>450</td>
</tr>
<tr>
<td>S8T1</td>
<td>600</td>
<td>300</td>
<td>100</td>
<td>1481</td>
<td>0.996</td>
<td>1232</td>
<td>1226</td>
<td>265</td>
<td>0.6127</td>
<td>295</td>
</tr>
<tr>
<td>S9T1</td>
<td>400</td>
<td>200</td>
<td>100</td>
<td>1523</td>
<td>0.996</td>
<td>1361</td>
<td>1355</td>
<td>265</td>
<td>0.7814</td>
<td>450</td>
</tr>
</tbody>
</table>
However, in this approach it is assumed that the shear capacity is mostly determined
by the contribution of the arching action, as the load is placed close to the support. To
determine the ratio $\alpha$ which indicates how much of the shear capacity comes from
beam action and how much from arching action, Kim and Jeong (2011) created an
algorithm. The results from this algorithm show however that it cannot be directly
applied to slabs as the results which are found indicate that barely any force transfer
relies upon arching action. A disadvantage of the reported method lies as well in the
fact that there are many issues in the theory which are not discussed in depth by the
authors, resulting in an unclear procedure, in which it is by times not even possible to
determine the units in the given formulas. The signs of the stresses and strains are
obscure in the publication. An earlier publication by Kim and Kim (2008) contains a
different flowchart as well as different expressions. This method was applied as well,
still resulting in very low values for the contribution of the percentage of the
contribution of the arching action. It can therefore be concluded that applying
methods for shear in beams does not give satisfactory results for shear in slabs and
another approach is necessary, taking into account the three-dimensional behavior in
slabs.

The algorithm for the determination of the contribution of the arching action to the
shear capacity is given here:
Background to Modified Bond Model
Application of beam action and arch action - Arching action as described by Kim et al
Background to Modified Bond Model
Application of beam action and arch action - Arching action as described by Kim et al
4. Development of the Modified Bond Model

4.1 Alexander’s Bond Model

After the development of a three-dimensional strut-and-tie model to describe punching in slab-column connections with an external moment acting on the connection (Alexander and Simmonds, 1986, 1987), further experimental studies were carried out. These experiments indicated that the radial compression struts are curved and not straight and parallel to the reinforcement in plan, requiring fundamental changes to the mechanics of the strut-and-tie model. The result of these modifications is the bond model for concentric punching shear (Alexander, 1990; Alexander and Simmonds, 1992) in which radial arching action and the concept of a critical shear stress on a critical section are combined. The model gives a simple lower bound estimate of the ultimate slab shear strength based on a combination of the truss model with the concept of a limiting one-way shear stress. The basis of the method is the following expression of the shear force:

\[ V = \frac{d(Tjd)}{dx} = \frac{d(T)}{dx} jd + \frac{d(jd)}{dx} T, \quad (4.1) \]

in which the first part is carried by beam action (requiring strong bond forces) and the second part by arching action (requiring only remote anchorage of the reinforcement).

In Eq. (4.1) the following parameters are used:

- \( T \): the steel tension force;
- \( jd \): the effective moment arm.

The geometry of the curved arch (which replaced the compression strut) is not governed by conditions at the intersection of the arch and the reinforcement tying the arch, but rather by the interaction between the arch and the adjacent quadrants of the slab. The radial strips (Fig. 4.1) extend from the loaded zone, up to a “remote end”, which is a position of zero shear. The shear carried in the radial compression arch varies from a maximum near the loaded zone where the slope of the arch is large, to a minimum at the intersection of the arch and the reinforcing bar, where the slope is small. The shear carried by a radial strip needs to be dissipated some distance away from the loaded area, depending on the curvature of the arch. Fig. 4.2 describes the radial strip as a cantilever beam. The length \( l \) is called the loaded area, and \( w \) the
uniformly distributed load. For four radial strips extending from the loaded area, a lower bound of the shear capacity is expressed as:

\[ P = 8\sqrt{M_s w} \]  \hspace{1cm} (4.2)

Fig. 4.1: Layout of radial strips, Alexander and Simmonds (1992).

Fig. 4.2: Equilibrium of radial strip, Alexander and Simmonds (1992).

The flexural capacity of the strip \( M_s \) and the loading term \( w \) are consequently defined to meet two conditions:
- the equilibrium of the strip has to be satisfied, and
- both the flexural capacity and shear capacity of the strip may not be exceeded at any point in the strip.

The flexural capacity depends upon the amount of reinforcement that effectively acts within the strip and is composed of the negative and positive moment capacity.

\[ M_{neg} = \rho_{neg} f_y j d^2 c \] \hspace{1cm} (4.3)
\[ M_{pos} = k_r \rho_{neg} f_y j d^2 c \] \hspace{1cm} (4.4)

In these equations, the following symbols are used:
Background to Modified Bond Model

Development of the Modified Bond Model - Alexander’s Bond Model

\[ \rho_{neg} = \frac{A_{sT}}{bd} \]
the negative effective reinforcing ratio;

\[ \rho_{pos} = \frac{A_{sB}}{bd} \]
the positive effective reinforcing ratio;

\( A_{sT} \) the total cross-sectional area of top steel within the radial strip plus half the area of the first top bar on either side of the strip;

\( A_{sB} \) the total cross-sectional area of bottom steel within the radial strip plus half the area of the first top bar on either side of the strip;

\( b \) the total distance between the first reinforcing bars on either side of the radial strip;

\( d \) the effective depth;

\( jd \) the internal moment arm;

\( c \) the width of the radial strip;

\( f_y \) the yield stress of the reinforcement;

\( k_r \) a factor which accounts for the proportion of the bottom steel that can be developed by the rotational restraint at the remote end of the strip.

This value is zero if the remote end is simply supported.

The loading term \( w \) represents a lower bound estimate of the maximum shear load that may be delivered to one side of a radial strip by the adjacent quadrant of the slab. Fig. 4.3 shows any load applied directly to the strip \( (q) \) and the internal shears and moments. The near side face of the half-strip is loaded in shear \( (v) \), torsion \( (m_t) \) and bending \( (m_b) \). Two approximations are made: the direct load \( (q) \) and the torsional shear are neglected. The maximum value of the loading term \( w \) can be based on the maximum value of beam action shear or on a direct estimation of the bond strength. In Alexander’s bond model, the loading term is described based on the one-way shear capacity from ACI 318.

\[ w_{ACI} = 0.166d \sqrt{f_c} \quad (4.5) \]

The bond model also explains how load may be carried in the presence of diagonal cracking. Test results have shown that diagonal cracking occurs at 50 to 70% of the ultimate load.
Fig. 4.3: Free-body diagram of one-half radial strip, Alexander and Simmonds (1992).

Fig. 4.4 shows the predicted failure load compared to 115 test results reported in the literature. The mean value is 1.29. This value is reasonably close to unity, which suggests that the mechanics of the bond model are not unrealistic. The effect of the reinforcement is taken into account by estimating the flexural capacity $M_s$ of the radial strip and, thus, can be combined with an estimate of the one-way shear strength which does not take the reinforcement ratio into account. The bond model is limited to plates with a value $c/d$ larger than 0.66; and can thus be applied to the slab shear experiments.

Fig. 4.4: Bond model results using ACI one-way shear, Alexander and Simmonds (1992).
4.2 Application to concentrated loads close to the support

In the bond model by Alexander and Simmonds (1992), the shear is carried by the arch from a maximum at the face of a column where the slope of the arch is large, to a minimum, or perhaps zero, at the intersection of the arch and the reinforcing steel, where the slope is small. Translating this approach to the case of a slab under a concentrated load near to the support requires taking direct load transfer between the load and the support into account, Fig. 4.6. For the additional three strips towards unrestrained edges, the original approach is used, Fig. 4.5.

Fig. 4.5: Sketch of application of bond model to slabs under a concentrated load close to the support.

Fig. 4.6: Direct load transfer in the strip between the load and the support.

The load \( w \) has to be the lower bound estimate of the maximum shear that can be delivered by the adjacent plate to one side-face of the strip, Fig. 4.3. As a result, every strip is loaded with \( 2w \). The flexural capacity of the strip results from the sum of the
negative and positive moment capacities. Using both moment capacities is further investigated in §4.3. The loading $2w$ is placed over a length $l$ such that the total load is maximized and the equilibrium is met, Fig. 4.2.

For the unrestrained sides, the maximum load can be expressed as:

$$P_{\text{load}, y} = 2\sqrt{M_{s,y}w_y}$$

$$P_{\text{load}, x} = 2\sqrt{M_{s,x}w_x}$$

(4.6)

Note that in Eq. (4.6) a distinction is made between the $x$- and $y$-direction. In Alexander’s bond model (1990), concentric punching shear was studied and the reinforcement ratios in both directions were identical. For the case of a one-way slab under a concentrated load in shear, typically the reinforcement in the $y$-direction is only 20% of the reinforcement in the main longitudinal $x$-direction. Therefore, the capacity of the strips in the $x$- and $y$-directions are studied separately.

For the restrained side, direct load transfer between the load and the support needs to be taken into account. The increase in capacity can be taken into account by using the factor $\frac{a_v}{2d}$ as proposed by Regan (1982) for slabs under a concentrated load near to the support to take the increase in capacity of the part of the punching perimeter at the support into account. The factor $\frac{a_v}{2.5d}$ as proposed for the combination with EN 1992-1-1:2005 for slabs under concentrated loads (RBK, 2012) is also tried for taking direct load transfer into account for the strip between the load and the support. Depending on the distance $a_v$ where the start of increasing capacities is expected (Kani, 1964) the factors $\frac{a_v}{2d}$ and $\frac{a_v}{2.5d}$ can both be obtained from the geometry, Fig. 4.7:

$$k = \frac{V_{\text{strut}}}{V_{EC,2}} = \frac{dV_{EC,2}}{a_v} = 2d$$

(4.7)
Background to Modified Bond Model
Development of the Modified Bond Model - Extension to slabs at continuous support

Fig. 4.7: Geometry for magnification factor.

As the width of the strip depends on the size of the loading plate, this width also needs to be taken into account when determining $A_{sT}$ and $A_{sB}$. As in a strut-and-tie model, the layout of the reinforcement is important: $A_{sT}$ and $A_{sB}$ are determined as the top and bottom steel within a strip, plus one-half the area of the first top or bottom bar on either side of the strip. In S1T1, the size of the loading plate is 200mm x 200mm. The main longitudinal reinforcement is $\varphi 20 \,–\, 125$mm. Thus, in a strip 2 bars can be used, and then half of the bottom bar on each side of the strip is added. In total, 3 bars are used, such that $A_{sB} = 942\text{mm}^2$. The distance between the bars is $b_x = 375$mm. With $d_x = 265$mm, a reinforcement ratio $\rho_x = 0.95\%$ is used for the $x$-direction strips. The first results of the bond model, taking the stronger strip towards the support into account are very promising, Table 4.1.

Table 4.1: Comparison between experimental results and bond model with fortified strip towards the support.

<table>
<thead>
<tr>
<th>Test</th>
<th>$f_{cc}$ (MPa)</th>
<th>$M_{negx}$ (kNm)</th>
<th>$M_{negy}$ (kNm)</th>
<th>$w_{ACTx}$ (kN/m)</th>
<th>$w_{ACTy}$ (kN/m)</th>
<th>$P_x$ (kN)</th>
<th>$P_y$ (kN)</th>
<th>$a_v$ (mm)</th>
<th>$P_{load}$ (kN)</th>
<th>$P_{tot}$ (kN)</th>
<th>$P_{meas}$ (kN)</th>
<th>$P_{meas}/P_{calc}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1T1</td>
<td>29</td>
<td>65</td>
<td>8</td>
<td>240</td>
<td>226</td>
<td>249</td>
<td>87</td>
<td>450</td>
<td>293</td>
<td>716</td>
<td>954</td>
<td>1.33</td>
</tr>
<tr>
<td>S2T1</td>
<td>28</td>
<td>97</td>
<td>12</td>
<td>235</td>
<td>222</td>
<td>302</td>
<td>105</td>
<td>400</td>
<td>400</td>
<td>912</td>
<td>1374</td>
<td>1.51</td>
</tr>
<tr>
<td>S3T1</td>
<td>42</td>
<td>100</td>
<td>25</td>
<td>288</td>
<td>272</td>
<td>340</td>
<td>164</td>
<td>400</td>
<td>451</td>
<td>1119</td>
<td>1371</td>
<td>1.23</td>
</tr>
<tr>
<td>S5T4</td>
<td>40</td>
<td>100</td>
<td>25</td>
<td>278</td>
<td>262</td>
<td>333</td>
<td>161</td>
<td>200</td>
<td>883</td>
<td>1539</td>
<td>1755</td>
<td>1.14</td>
</tr>
<tr>
<td>S8T1</td>
<td>63</td>
<td>103</td>
<td>25</td>
<td>352</td>
<td>332</td>
<td>381</td>
<td>182</td>
<td>400</td>
<td>504</td>
<td>1249</td>
<td>1481</td>
<td>1.19</td>
</tr>
<tr>
<td>S9T1</td>
<td>67</td>
<td>69</td>
<td>17</td>
<td>362</td>
<td>342</td>
<td>316</td>
<td>151</td>
<td>250</td>
<td>669</td>
<td>1287</td>
<td>1523</td>
<td>1.18</td>
</tr>
</tbody>
</table>

|                | AVG 1.26    | STD 0.14     | COV 0.11 |

4.3 Extension to slabs at continuous support

To extend the bond model for application to all experiments, further development of the method is required, which will result in the Modified Bond Model. Alexander
(1990) proposes using a factor \( k_r \) on the positive moment capacity. The factor \( k_r \) ranges from 0 (for simply supported edges) to 1 (for fully restrained cases). For the application to continuous slabs under a concentrated load, it is necessary to better define the parameter \( k_r \) and study on which strips it can be applied. It is chosen to define \( k_r \) as \( k_r = \frac{M_{\text{sup}}}{M_{\text{span}}} \) with \( M_{\text{sup}} \) the moment at the support and \( M_{\text{span}} \) the moment at the location of the concentrated load. The contribution of the top reinforcement can be taken into account on 3 strips (all strips facing the support) or on all 4 strips.

\[
\begin{align*}
\text{Fig. 4.8: Application of the bond model for loads near the continuous support.}
\end{align*}
\]

Another possibility is to use the enhancement factor \( \alpha_{\text{Regan}} \) on the calculated capacity \( P_{\text{tot}} \), as described by Regan (1982). The value of \( \alpha_{\text{Regan}} \) equals \( \alpha_{\text{Regan}} = \frac{M_1 + M_2}{M_1} \) with \( M_1 \) the absolute value of the largest moment in the shear span and \( M_2 \) the absolute value of the smallest moment. Using \( \alpha_{\text{Regan}} \) led to good results when applied to the shear provisions from EN 1992-1-1:2005 (Lantsoght, 2012). Combined with the bond model, this approach did not lead to satisfying results.
Background to Modified Bond Model
Development of the Modified Bond Model - Extension to loads at and near the edge

The best results, Table 4.2, are obtained when the influence of the top reinforcement is taken into account with the factor $k_r$ on 3 strips facing the support, Fig. 4.8. This result could be expected, as the moment over the support influences the two quadrants between the support and the load, and thus 3 strips.

Table 4.2: Comparison between Modified Bond Model and experimental results close to the continuous support.

<table>
<thead>
<tr>
<th>Test</th>
<th>$f_{ck}$ (MPa)</th>
<th>$M_{negx}$ kNm</th>
<th>$M_{negy}$ kNm</th>
<th>$w_{ACx}$ kN/m</th>
<th>$w_{ACy}$ kN/m</th>
<th>$P_x$ kN</th>
<th>$P_y$ kN</th>
<th>$k_r$</th>
<th>$M_{posx}$ kNm</th>
<th>$M_{posy}$ kNm</th>
<th>$P_{sup}$ kN</th>
<th>$P_{tot}$ kN</th>
<th>$P_{test}/P_{calc}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1T2</td>
<td>29</td>
<td>65</td>
<td>8</td>
<td>238</td>
<td>225</td>
<td>248</td>
<td>95</td>
<td>0.21</td>
<td>64</td>
<td>8</td>
<td>321</td>
<td>760</td>
<td>1.35</td>
</tr>
<tr>
<td>S2T4</td>
<td>28</td>
<td>97</td>
<td>12</td>
<td>234</td>
<td>221</td>
<td>301</td>
<td>123</td>
<td>0.38</td>
<td>96</td>
<td>12</td>
<td>467</td>
<td>1014</td>
<td>1.40</td>
</tr>
<tr>
<td>S3T4</td>
<td>42</td>
<td>100</td>
<td>25</td>
<td>286</td>
<td>270</td>
<td>339</td>
<td>190</td>
<td>0.34</td>
<td>100</td>
<td>25</td>
<td>520</td>
<td>1239</td>
<td>1.08</td>
</tr>
<tr>
<td>S5T1</td>
<td>40</td>
<td>100</td>
<td>25</td>
<td>277</td>
<td>261</td>
<td>332</td>
<td>183</td>
<td>0.29</td>
<td>99</td>
<td>25</td>
<td>1001</td>
<td>1700</td>
<td>1.06</td>
</tr>
<tr>
<td>S8T2</td>
<td>63</td>
<td>103</td>
<td>25</td>
<td>350</td>
<td>330</td>
<td>379</td>
<td>209</td>
<td>0.33</td>
<td>102</td>
<td>25</td>
<td>579</td>
<td>1376</td>
<td>0.99</td>
</tr>
<tr>
<td>S9T4</td>
<td>67</td>
<td>69</td>
<td>17</td>
<td>360</td>
<td>340</td>
<td>315</td>
<td>173</td>
<td>0.32</td>
<td>68</td>
<td>17</td>
<td>765</td>
<td>1425</td>
<td>1.29</td>
</tr>
</tbody>
</table>

| AVG  | 1.19 |
| STD  | 0.17 |
| COV  | 0.15 |

4.4 Extension to loads at and near the edge

To extend the bond model for application on rectangular slabs under concentrated loads, the method needs to be suitable for loads near the edge. For the situation of a load placed directly at the edge, it is clear that only 3 strips can be used, 2 of which are loaded by $w$ and one by $2w$, Fig. 4.9.

Fig. 4.9: Application of bond model to load near the edge.
Background to Modified Bond Model
Development of the Modified Bond Model - Extension to loads at and near the edge

For the case in which the load is placed in the vicinity of the edge, some load transfer can occur in the slab quadrants between the $x$-direction strips and the free edge, Fig. 4.10. If it is assumed that in an undisturbed quadrant, $w$ is transferred in each direction to the radial strips, as used in Alexander’s bond model, then the theory can be extended by assuming that a reduced amount of one-way shear stress can be transferred via bond to the radial strips near the edge. This reduced amount can be expressed as $\alpha w$, with $\alpha < 1$. Therefore, the amount of load on the radial strips can be expressed in terms of $\alpha$. For the strips in the $x$-direction, the occurring load is $x_1w = (1+\alpha)w$ and $x_1 > 1$. For the strip between the load and the free edge, the considered loading is $x_2w = 2\alpha w$. To compare this approach to the experimental results, a subset of experiments at the simple support and with the load near to the edge is selected. Since it is observed in the experiments that geometrical considerations have a large influence on the resulting shear capacity, the factor $\alpha$ is determined as $\alpha = \frac{b_{r,side}}{b_{r,mid}} = \frac{438mm}{1250mm} = 0.35$.

![Fig. 4.10: Modified Bond Model for load in the vicinity of the edge.](image)

A comparison between the experimental results and the proposed method is given in Table 4.3. Again, an excellent agreement between the experimental results and the proposed Modified Bond Model is found.
Table 4.3: Comparison between proposed Modified Bond Model for loads near the edge and the experimental results.

<table>
<thead>
<tr>
<th>Test</th>
<th>$f_{ck}$ (MPa)</th>
<th>$a_v$ (m)</th>
<th>$P_{test}$ (kN)</th>
<th>$M_{augs}$ kNm</th>
<th>$w_{ACI x}$ kN/m</th>
<th>$w_{ACI y}$ kN/m</th>
<th>$P_x$ kN</th>
<th>$P_y$ kN</th>
<th>$P_{sup}$ kN</th>
<th>$P_{tot}$ kN</th>
<th>$P_{exp}/P_{calc}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>S4T1</td>
<td>42</td>
<td>0.4</td>
<td>1160 100</td>
<td>25</td>
<td>286</td>
<td>270</td>
<td>229</td>
<td>221</td>
<td>410</td>
<td>860</td>
<td>1.35</td>
</tr>
<tr>
<td>S4T2</td>
<td>42</td>
<td>0.4</td>
<td>1110 100</td>
<td>25</td>
<td>286</td>
<td>270</td>
<td>229</td>
<td>221</td>
<td>410</td>
<td>860</td>
<td>1.29</td>
</tr>
<tr>
<td>S6T4</td>
<td>41</td>
<td>0.2</td>
<td>1366 100</td>
<td>25</td>
<td>283</td>
<td>267</td>
<td>228</td>
<td>220</td>
<td>814</td>
<td>1261</td>
<td>1.08</td>
</tr>
<tr>
<td>S6T5</td>
<td>41</td>
<td>0.2</td>
<td>1347 100</td>
<td>25</td>
<td>283</td>
<td>267</td>
<td>228</td>
<td>220</td>
<td>814</td>
<td>1261</td>
<td>1.07</td>
</tr>
<tr>
<td>S7T1</td>
<td>67</td>
<td>0.4</td>
<td>1121 103</td>
<td>25</td>
<td>361</td>
<td>341</td>
<td>261</td>
<td>249</td>
<td>466</td>
<td>976</td>
<td>1.15</td>
</tr>
<tr>
<td>S7T5</td>
<td>67</td>
<td>0.4</td>
<td>1063 103</td>
<td>25</td>
<td>361</td>
<td>341</td>
<td>261</td>
<td>249</td>
<td>466</td>
<td>976</td>
<td>1.09</td>
</tr>
<tr>
<td>S10T1</td>
<td>68</td>
<td>0.25</td>
<td>1320 69</td>
<td>17</td>
<td>362</td>
<td>341</td>
<td>213</td>
<td>204</td>
<td>609</td>
<td>1026</td>
<td>1.29</td>
</tr>
<tr>
<td>S10T2</td>
<td>68</td>
<td>0.25</td>
<td>1116 69</td>
<td>17</td>
<td>362</td>
<td>341</td>
<td>213</td>
<td>204</td>
<td>609</td>
<td>1026</td>
<td>1.09</td>
</tr>
</tbody>
</table>

| AVG    | 1.18        |
| STD    | 0.11        |
| COV    | 0.10        |

4.5 Extension to slabs on bearings

For slabs on bearings, two approaches can be followed: the punching capacity of the bearing can be studied, and the capacity of the slab under a concentrated load can be studied.

First, the punching capacity at the bearings is studied. The load at punching can be directly determined from the measured force in the load cell of the individual bearing. From the experimental results, the most heavily loaded bearing is determined and compared to the punching capacity from the Modified Bond Model. The size of the bearing is 350mm x 280mm, and therefore the x-direction strips are 350mm and the y-direction strips 280mm. Initially the situation at the support is considered using 3 strips, similar to the situation at the edge, Fig. 4.11. The considerations from the Modified Bond Model near the edge and at the continuous support are also applied to this method. For application at the continuous support, only the x-direction strip is affected by the moments in the shear span. Comparing these results to the experimental results, however, does not result in clear answers. There is a lot of scatter on the results, and the measurements do not show a significant difference between loading near the simple or the continuous support. Unlike for concentrated loads on the slab, in which the continuous support results in a higher capacity, for the bearing forces, slightly higher capacities are found at the simple support. Sometimes, higher reaction forces are observed on bearings near the edge for loading near the edge than on the middle bearing for loading in the middle of the width. Therefore,
direct relation between the concentrated load and the bearing forces does not always occur. It should also be noted that, in the end, it is not surprising that this method does not lead to satisfactory results. It was noted in the experiments that punching of the bearings was a secondary failure mechanism, occurring shortly after shear failure at the concentrated load.

---

Fig. 4.11: Application of bond model to punching of bearings at the support line.

Therefore, the approach in which the slab supported by bearings is studied in the same way as the other slab specimens under a concentrated load near the support is followed. However, since the support length is reduced and smaller maximum concentrated loads occurred in the experiments, further modification of the bond model is required. Similar to previous approaches where the load $w$ is reduced under certain conditions when the quadrants are disturbed such as placing the load in the vicinity of the free edge, a reduction of the load can also be proposed for this case. The reduction of the one-way shear transferred through beam action and bond is
For loads near the edge, the initial approached is a combination of the method sketched in Fig. 4.10 and the method for slabs on bearings from Fig. 4.12. Combining the reduction factor of the bearings at the support and loading near the edge could be expressed by the product of the reduction factor for the slab supported by bearings (0.42w) and the reduction factor for loading near the edge (0.35w): 0.35*0.42 = 0.15. This approach is used in Fig. 4.13. However, the stiff strip between the load and the support is assumed to be loaded by more than 0.57w as shown in Fig. 4.13. Therefore, the quadrants near the edge are considered from a different point of view: n the strong direction (x-direction), only the influence of the edge is taken into account, and 0.35w is used for the loading term in the x-direction, while the combined effect of the edge and the bearings (0.15w) is used in the y-direction.
Fig. 4.13: Application of the Modified Bond Model to slabs on bearings with the load near the edge.

Fig. 4.14: Proposed application of the Modified Bond Model to slabs on bearings with the load near the edge, taking redistribution into account.

All results of experiments on undamaged slabs S15 to S18 are used for comparison to the Modified Bond Model: slabs at the simple and continuous support, and slabs
Background to Modified Bond Model
Development of the Modified Bond Model - Extension to slabs on bearings

...loaded near the edge and in the middle of the width. The properties of S15 to S18 and the experiments are repeated in Table 4.4 and the results of the comparison between the Modified Bond Model and the experiments are given in Table 4.5. For experiments with the concentrated load near the continuous support, the influence of the top reinforcement is taken into account by applying the factor $k_r$ on 3 strips. It is shown in Table 4.5 that a good agreement between the experimental results and the predictions from the Modified Bond Model are achieved. This result might be surprising, as a few generalizing assumptions are made. However, the method seems to be very flexible for adaptation, as most plasticity approaches, and good results can still be obtained. In the original bond model, the loading on the strips is assumed to be purely from the internal loading, and torsion is neglected, Fig. 4.2. Alexander (1990) calculated that the influence of torsion would allow for about 20% more capacity, and therefore omitting the torsion leads to safe results.

### Table 4.4: Overview of results from S15 to S18

<table>
<thead>
<tr>
<th>Test</th>
<th>$f_{ck}$</th>
<th>$b_r$</th>
<th>$a_r$</th>
<th>$P_{max}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>S15T1</td>
<td>42.8</td>
<td>1.25</td>
<td>0.36</td>
<td>1040</td>
</tr>
<tr>
<td>S15T4</td>
<td>42.8</td>
<td>1.25</td>
<td>0.36</td>
<td>1127</td>
</tr>
<tr>
<td>S16T1</td>
<td>43.9</td>
<td>0.438</td>
<td>0.36</td>
<td>932</td>
</tr>
<tr>
<td>S16T2</td>
<td>43.9</td>
<td>0.438</td>
<td>0.36</td>
<td>815</td>
</tr>
<tr>
<td>S16T4</td>
<td>43.9</td>
<td>0.438</td>
<td>0.36</td>
<td>776</td>
</tr>
<tr>
<td>S16T5</td>
<td>43.9</td>
<td>0.438</td>
<td>0.36</td>
<td>700</td>
</tr>
<tr>
<td>S17T1</td>
<td>43.1</td>
<td>1.25</td>
<td>0.16</td>
<td>1365</td>
</tr>
<tr>
<td>S17T4</td>
<td>43.1</td>
<td>1.25</td>
<td>0.16</td>
<td>1235</td>
</tr>
<tr>
<td>S18T1</td>
<td>42.7</td>
<td>0.438</td>
<td>0.16</td>
<td>1157</td>
</tr>
<tr>
<td>S18T2</td>
<td>42.7</td>
<td>0.438</td>
<td>0.16</td>
<td>1079</td>
</tr>
<tr>
<td>S18T4</td>
<td>42.7</td>
<td>0.438</td>
<td>0.16</td>
<td>1122</td>
</tr>
<tr>
<td>S18T5</td>
<td>42.7</td>
<td>0.438</td>
<td>0.16</td>
<td>1104</td>
</tr>
</tbody>
</table>

### Table 4.5: Comparison between Modified Bond Model and experimental results of slabs on bearings.

<table>
<thead>
<tr>
<th>Test</th>
<th>$M_{max}$</th>
<th>$M_{maxx}$</th>
<th>$w_{ACT}$</th>
<th>$w_{ACTx}$</th>
<th>$M_{pmax}$</th>
<th>$M_{pmaxx}$</th>
<th>$P_x$</th>
<th>$P_y$</th>
<th>$P_{strong}$</th>
<th>$P_{side}$</th>
<th>$P_{exist}$</th>
<th>$P_{tot}$</th>
<th>$P_{test}/P_{calc}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>S15T1</td>
<td>64</td>
<td>104</td>
<td>277</td>
<td>253</td>
<td>0.39</td>
<td>64</td>
<td>16</td>
<td>267</td>
<td>237</td>
<td>187</td>
<td>180</td>
<td>172</td>
<td>928</td>
</tr>
<tr>
<td>S15T4</td>
<td>64</td>
<td>104</td>
<td>277</td>
<td>253</td>
<td>0.19</td>
<td>9</td>
<td>17</td>
<td>267</td>
<td>230</td>
<td>159</td>
<td>180</td>
<td>151</td>
<td>886</td>
</tr>
<tr>
<td>S16T1</td>
<td>64</td>
<td>105</td>
<td>280</td>
<td>256</td>
<td>0.27</td>
<td>9</td>
<td>17</td>
<td>269</td>
<td>232</td>
<td>160</td>
<td>181</td>
<td>152</td>
<td>615</td>
</tr>
<tr>
<td>S16T2</td>
<td>64</td>
<td>105</td>
<td>280</td>
<td>256</td>
<td>0.34</td>
<td>9</td>
<td>17</td>
<td>269</td>
<td>232</td>
<td>160</td>
<td>181</td>
<td>152</td>
<td>615</td>
</tr>
<tr>
<td>S16T4</td>
<td>64</td>
<td>105</td>
<td>280</td>
<td>256</td>
<td>0.55</td>
<td>64</td>
<td>16</td>
<td>269</td>
<td>242</td>
<td>199</td>
<td>181</td>
<td>189</td>
<td>663</td>
</tr>
</tbody>
</table>
### 4.6 Extension to plain bars

The next step in the development of the Modified Bond Model is the application to the slabs with plain bars S11 to S14. As the bond properties of plain bars are different, the impact on the Modified Bond Model is assumed to be large. However, all the influence of the bond properties are reflected by the expression of the one-way shear transferred by beam action and thus bond through the loading term \( w \).

To study the influence of bond on the loading term \( w \), several approaches are followed. The recommendations for bond from ModelCode 2010 (fib, 2012) are studied. For loading through bond, the loading term needs to be expressed as a stress around the bars which is distributed to the section:

\[
w = jd \left( \frac{\pi \phi}{s} \tau_{bond} \right)
\]

with

- \( jd \): the internal lever arm based on a rectangular stress block diagram:
  
  \[
  jd = d - \frac{\rho bd f_y}{2 \times 0.85 f_y b},
  \]

- \( \phi \): the bar diameter,

- \( s \): the bar spacing,

- \( \tau_{bond} \): the bond capacity:
  
  \[
  \tau_{bond} = \min \left( 1.25 \sqrt{f_{cm}}, 5 \left( \frac{f_{cm}}{25} \right)^{0.25} \right)
  \]

The bond capacity can also be based on the design bond stress:

\[
f_{bd} = \eta_2 \eta_b f_{ad}
\]
Background to Modified Bond Model
Development of the Modified Bond Model - Extension to plain bars

with

\[ f_{cm} = 0.30 f_{ck}^{2/3} \]

for concrete classes \( \leq C50/60 \)

For ribbed bars \( \eta_1 = 2.25 \). Poor bond is assumed since the concrete cover is small, \( \eta_2 = 0.7 \). However, when bond models are used for the expression of \( w \), poor results are obtained. Therefore it is better to work with a maximum value of the beam action shear which is limited by a nominal one-way shear stress.

The first expression for the shear capacity which is studied is from ModelCode 2010:

\[ v_{Rd,c} = k_c \sqrt{f_{ck} z} \]  

(4.10)

with a maximum for \( \sqrt{f_{ck}} \leq 8 \text{ MPa} \) and \( k_c = \frac{180}{1000 + 1.25 z} \). Applying this method on the experimental results, leads to larger scatter than using \( w_{ACI} \).

Next, the expressions for shear from EN 1992-1-1:2005 are used:

\[ v_{Rd,c} = C_{Rd,c} k \left(100 \rho f_{ck}^{1/3}\right)^{1/3} \]

\[ v_{min} = 0.035 k^{1/2} f_{ck}^{1/2} \]  

(4.11)

with:

\[ k = 1 + \frac{\sqrt{200}}{d_i} \leq 2 \]  

(4.12)

Using \( v_{min} \) and \( v_{Rd,c} \) both lead to extremely conservative results when applied into the Modified Bond Model. It should also be kept in mind that the expression for \( v_{Rd,c} \) requires slightly more computational work than the expression for \( w_{ACI} \).

In a next step, the recommendations from the French National Annex to EN 1992-1-1:2005 are considered:

\[ v_{min} = 0.34 \sqrt{f_{ck}} \]  

(4.13)

This approach, however, gives unconservative results, just like when this approach is used to determine the shear capacity for slabs under a concentrated load (Lantsoght et al., 2012). Secondly, the minimum one-way shear capacity for beams from the French National Annex is studied:

\[ v_{min} = 0.053 k^{1/2} f_{ck}^{1/2} \]  

(4.14)

This method gives very conservative results.

Therefore, it is seen that \( w_{ACI} \) gives the best results from all considered methods. The choice for \( w_{ACI} \) is thus an element of empiricism in the Modified Bond Model.
Having made the final choice for using $w_{ACI}$ in the Modified Bond Model, a reduction factor can now be sought that can be applied for slabs with plain bars. A few ideas are explored. The first idea is to use the ratio between the factors form the recommended bond models from Model Code 2010: $\frac{1}{2.55} = 0.44$. In Dutch practice, a factor of 0.5 is typically used for plain bars. However, both these approaches result in very conservative results, not in line with previous calculations. Therefore, an additional element of empiricism is built into the model: the reduction factor to be used on $w$ for the case of plain bars is determined to be 0.8.

The choice for a larger factor can however be explained physically as well, based on the fact that the Modified Bond Model uses a combination of one-way shear in the quadrants and arching action in the strips. It is known that the mechanism of arching action is not penalized by using plain bars, and therefore it is assumed that the contribution of arching action is becoming larger when applying plain bars. In the Modified Bond Model, the distinction is clear between both mechanisms by using strips and quadrants, while these are interwoven in reality.

The properties and experimental results of S11 to S14 are repeated in Table 4.6. The comparison between the experimental results and the Modified Bond Model are given in Table 4.7. Good agreement is found between the experimental results and the Modified Bond Model.

**Table 4.6: Overview of the properties and experimental results of slabs with plain bars**

<table>
<thead>
<tr>
<th>Test</th>
<th>$f_{ck}$ (MPa)</th>
<th>$b_r$ (m)</th>
<th>$a_v$ (m)</th>
<th>$P_{max}$ (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S11T1</td>
<td>45</td>
<td>1.25</td>
<td>0.45</td>
<td>1194</td>
</tr>
<tr>
<td>S11T4</td>
<td>45</td>
<td>1.25</td>
<td>0.45</td>
<td>958</td>
</tr>
<tr>
<td>S12T1</td>
<td>45</td>
<td>0.438</td>
<td>0.45</td>
<td>931</td>
</tr>
<tr>
<td>S12T2</td>
<td>45</td>
<td>0.438</td>
<td>0.45</td>
<td>1004</td>
</tr>
<tr>
<td>S12T4</td>
<td>45</td>
<td>0.438</td>
<td>0.45</td>
<td>773</td>
</tr>
<tr>
<td>S12T5</td>
<td>45</td>
<td>0.438</td>
<td>0.45</td>
<td>806</td>
</tr>
<tr>
<td>S13T1</td>
<td>43</td>
<td>1.25</td>
<td>0.25</td>
<td>1404</td>
</tr>
<tr>
<td>S13T4</td>
<td>43</td>
<td>1.25</td>
<td>0.25</td>
<td>1501</td>
</tr>
<tr>
<td>S14T1</td>
<td>42</td>
<td>0.438</td>
<td>0.25</td>
<td>1214</td>
</tr>
<tr>
<td>S14T2</td>
<td>42</td>
<td>0.438</td>
<td>0.25</td>
<td>1093</td>
</tr>
<tr>
<td>S14T4</td>
<td>42</td>
<td>0.438</td>
<td>0.25</td>
<td>1282</td>
</tr>
<tr>
<td>S14T5</td>
<td>42</td>
<td>0.438</td>
<td>0.25</td>
<td>1234</td>
</tr>
</tbody>
</table>
Table 4.7: Overview of the comparison between the Modified Bond Model and the experimental results from S11 to S14 with plain bars.

<table>
<thead>
<tr>
<th>Test</th>
<th>M_{negx}</th>
<th>M_{negy}</th>
<th>w_{ACIx}</th>
<th>w_{ACIy}</th>
<th>k_{r}</th>
<th>M_{posx}</th>
<th>M_{posy}</th>
<th>P_{x}</th>
<th>P_{y}</th>
<th>P_{side}</th>
<th>P_{slab}</th>
<th>P_{tot}</th>
<th>P_{test}/P_{calc}</th>
</tr>
</thead>
<tbody>
<tr>
<td>S11T1</td>
<td>101</td>
<td>27</td>
<td>236</td>
<td>223</td>
<td>0,19</td>
<td>15</td>
<td>28</td>
<td>308</td>
<td>156</td>
<td>363</td>
<td>208</td>
<td>245</td>
<td>983</td>
</tr>
<tr>
<td>S11T4</td>
<td>101</td>
<td>27</td>
<td>236</td>
<td>223</td>
<td>0,59</td>
<td>101</td>
<td>27</td>
<td>308</td>
<td>156</td>
<td>458</td>
<td>208</td>
<td>309</td>
<td>1077</td>
</tr>
<tr>
<td>S12T1</td>
<td>101</td>
<td>27</td>
<td>236</td>
<td>223</td>
<td>0,23</td>
<td>15</td>
<td>28</td>
<td>308</td>
<td>156</td>
<td>363</td>
<td>208</td>
<td>245</td>
<td>663</td>
</tr>
<tr>
<td>S12T2</td>
<td>101</td>
<td>27</td>
<td>236</td>
<td>223</td>
<td>0,23</td>
<td>15</td>
<td>28</td>
<td>308</td>
<td>156</td>
<td>363</td>
<td>208</td>
<td>245</td>
<td>663</td>
</tr>
<tr>
<td>S12T4</td>
<td>101</td>
<td>27</td>
<td>236</td>
<td>223</td>
<td>0,31</td>
<td>101</td>
<td>27</td>
<td>308</td>
<td>156</td>
<td>416</td>
<td>208</td>
<td>280</td>
<td>699</td>
</tr>
<tr>
<td>S12T5</td>
<td>101</td>
<td>27</td>
<td>236</td>
<td>223</td>
<td>0,32</td>
<td>101</td>
<td>27</td>
<td>308</td>
<td>156</td>
<td>417</td>
<td>208</td>
<td>282</td>
<td>700</td>
</tr>
<tr>
<td>S13T1</td>
<td>100</td>
<td>27</td>
<td>230</td>
<td>217</td>
<td>0,21</td>
<td>15</td>
<td>28</td>
<td>303</td>
<td>153</td>
<td>643</td>
<td>205</td>
<td>434</td>
<td>1252</td>
</tr>
<tr>
<td>S13T4</td>
<td>100</td>
<td>27</td>
<td>230</td>
<td>217</td>
<td>0,38</td>
<td>100</td>
<td>27</td>
<td>303</td>
<td>153</td>
<td>755</td>
<td>205</td>
<td>510</td>
<td>1365</td>
</tr>
<tr>
<td>S14T1</td>
<td>100</td>
<td>27</td>
<td>228</td>
<td>215</td>
<td>0,21</td>
<td>15</td>
<td>28</td>
<td>302</td>
<td>153</td>
<td>640</td>
<td>204</td>
<td>432</td>
<td>842</td>
</tr>
<tr>
<td>S14T2</td>
<td>100</td>
<td>27</td>
<td>228</td>
<td>215</td>
<td>0,28</td>
<td>15</td>
<td>28</td>
<td>302</td>
<td>153</td>
<td>640</td>
<td>204</td>
<td>432</td>
<td>842</td>
</tr>
<tr>
<td>S14T4</td>
<td>100</td>
<td>27</td>
<td>228</td>
<td>215</td>
<td>0,34</td>
<td>100</td>
<td>27</td>
<td>302</td>
<td>153</td>
<td>742</td>
<td>204</td>
<td>501</td>
<td>911</td>
</tr>
<tr>
<td>S14T5</td>
<td>100</td>
<td>27</td>
<td>228</td>
<td>215</td>
<td>0,26</td>
<td>100</td>
<td>27</td>
<td>302</td>
<td>153</td>
<td>720</td>
<td>204</td>
<td>486</td>
<td>896</td>
</tr>
</tbody>
</table>

AVG 1,25
STD 0,18
COV 0,15

4.7 Size effect

All slabs in the series of shear experiments have a depth of 0.3m. No experimental results on the size effect for slabs in shear are thus available. However, as the Modified Bond Model is easily adapted to different situations, it is also possible to add a factor on the expression for $w$ which takes the size effect into account. A possible approach which can be applied to a three-dimensional strut-and-tie model is given by Rizk et al. (2012). These authors express the size effect as:

$$\xi = \left( \frac{l_{ch}}{d} \right)^{0.33}$$

(4.15)

with

$d$ the effective depth to the main reinforcement, and

$l_{ch}$ the characteristic length $l_{ch} = \frac{E_{c}G_{f}}{f_{ct}^{2}}$, with

$E_{c}$ the modulus of elasticity of the concrete,

$G_{f}$ the fracture energy,

$f_{ct}$ the direct tensile strength of the concrete.
Based on curve fitting, simplified empirical expressions for the characteristic length \( l_{ch} \) are available as well, for example:

\[
    l_{ch} = -3.84 f'_c + 580 \text{ in [mm]}
\]  

(4.16)

However, this method is currently not applied in the method, as no experimental results are available to quantify the influence of the size effect.

### 4.8 Summary of Modified Bond Model

The Modified Bond Model is based on the Bond Model by Alexander and Simmonds (1992) for concentric punching shear. The following alterations are made:

- For one-way slabs, the \( x \)- and \( y \)-direction strips are considered separately to take into account the different reinforcement in both directions.

- The capacity of the \( x \)-direction strip between the load and the support is increased by the factor \( 2d/a \), which takes direct load transfer into account.

- To take into account the support moment at the continuous support, the moment capacity is considered as a combination of the bottom reinforcement capacity and \( k_r \) times the top reinforcement capacity. The factor \( k_r \) is determined to be the ratio between the support moment and the span moment. The factor \( k_r \) is applied on the \( y \)-direction strips, and on the \( x \)-direction strip between the load and the support, as these strips are affected by the quadrants in which the sign of the moment changes.

- For loads at the edge, three strips are used.

- For loads in the vicinity of the edge or away from the center of the width, the loading term \( w \) is reduced with \( 2b_r/b_{slab} \).

- When not the full width of the slab is supported, the loading term \( w \) is reduced by using a factor \( l_{bearings}/b_{slab} \).

- As shown for the case of a slab supported by bearings under a concentrated load in the vicinity of the edge, redistribution of the loads can be applied, and the different stiffness in the \( x \)- and \( y \)-direction strips can be taken into account.

- For slabs with plain bars, a reduction factor on the loading term \( w \) of 0.8 is used.
5. Overview of results

To assess the improvement based on the Modified Bond Model, the results from the comparison between the experimental results and EN 1992-1-1:2005 are repeated (Lantsoght, 2012). For S1 to S18, the results are shown in Fig. 5.1. The comparison shows the results using the effective width based on load spreading from the center of the load to the support $V_{Rd,c}$ and using the effective width based on load spreading form the far side of the loading plate $V_{Rd,\text{eff}}$. The latter is what ultimately is determined to be the best approach. As is shown in Fig. 5.1 the results tend to deviate from the $45^\circ$ line for larger experimental values. The statistical properties for $V_{\text{exp}}/V_{Rd,\text{eff}}$ are the following: the average value is 1,917; the standard deviation is 0,264 and the coefficient of variation is 14%.

![Fig. 5.1: Comparison with EN 1992-1-1:2005: experiments on an undamaged slab S1 to S18.](image)

For comparison, all experimental results of S1 to S18 and all predictions using the Modified Bond Model for the undamaged slabs are brought together in Table 5.1. The average value is still on the conservative side, but not as excessively underpredicting the shear capacity as EN 1992-1-1:2005. The scatter is much smaller, and the coefficient of variation is improved as well.
Table 5.1: Overview of all results as predicted by the Modified Bond Model.

<table>
<thead>
<tr>
<th>Test</th>
<th>$P_u$ (kN)</th>
<th>$P_{MBM}$ (kN)</th>
<th>$P_u/P_{MBM}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1T1</td>
<td>954</td>
<td>716</td>
<td>1.33</td>
</tr>
<tr>
<td>S1T2</td>
<td>1023</td>
<td>760</td>
<td>1.35</td>
</tr>
<tr>
<td>S2T1</td>
<td>1374</td>
<td>912</td>
<td>1.51</td>
</tr>
<tr>
<td>S2T4</td>
<td>1421</td>
<td>1014</td>
<td>1.40</td>
</tr>
<tr>
<td>S3T1</td>
<td>1371</td>
<td>1119</td>
<td>1.23</td>
</tr>
<tr>
<td>S3T4</td>
<td>1337</td>
<td>1239</td>
<td>1.08</td>
</tr>
<tr>
<td>S4T1</td>
<td>1160</td>
<td>860</td>
<td>1.35</td>
</tr>
<tr>
<td>S4T2</td>
<td>1110</td>
<td>860</td>
<td>1.29</td>
</tr>
<tr>
<td>S5T1</td>
<td>1804</td>
<td>1700</td>
<td>1.06</td>
</tr>
<tr>
<td>S5T4</td>
<td>1755</td>
<td>1539</td>
<td>1.14</td>
</tr>
<tr>
<td>S6T1</td>
<td>1446</td>
<td>1162</td>
<td>1.24</td>
</tr>
<tr>
<td>S6T2</td>
<td>1423</td>
<td>1185</td>
<td>1.20</td>
</tr>
<tr>
<td>S6T4</td>
<td>1366</td>
<td>1261</td>
<td>1.08</td>
</tr>
<tr>
<td>S6T5</td>
<td>1347</td>
<td>1261</td>
<td>1.07</td>
</tr>
<tr>
<td>S7T1</td>
<td>1121</td>
<td>976</td>
<td>1.15</td>
</tr>
<tr>
<td>S7T2</td>
<td>1172</td>
<td>928</td>
<td>1.26</td>
</tr>
<tr>
<td>S7T3</td>
<td>1136</td>
<td>943</td>
<td>1.20</td>
</tr>
<tr>
<td>S7T5</td>
<td>1063</td>
<td>976</td>
<td>1.09</td>
</tr>
<tr>
<td>S8T1</td>
<td>1481</td>
<td>1249</td>
<td>1.19</td>
</tr>
<tr>
<td>S8T2</td>
<td>1356</td>
<td>1376</td>
<td>0.99</td>
</tr>
<tr>
<td>S9T1</td>
<td>1523</td>
<td>1287</td>
<td>1.18</td>
</tr>
<tr>
<td>S9T4</td>
<td>1842</td>
<td>1425</td>
<td>1.29</td>
</tr>
<tr>
<td>S10T1</td>
<td>1320</td>
<td>1026</td>
<td>1.29</td>
</tr>
<tr>
<td>S10T2</td>
<td>1116</td>
<td>1026</td>
<td>1.09</td>
</tr>
<tr>
<td>S10T4</td>
<td>1511</td>
<td>989</td>
<td>1.53</td>
</tr>
<tr>
<td>S10T5</td>
<td>1454</td>
<td>985</td>
<td>1.48</td>
</tr>
<tr>
<td>S11T1</td>
<td>1194</td>
<td>983</td>
<td>1.21</td>
</tr>
<tr>
<td>S11T4</td>
<td>958</td>
<td>1077</td>
<td>0.89</td>
</tr>
<tr>
<td>S12T1</td>
<td>931</td>
<td>663</td>
<td>1.40</td>
</tr>
<tr>
<td>S12T2</td>
<td>1004</td>
<td>663</td>
<td>1.51</td>
</tr>
<tr>
<td>S12T4</td>
<td>773</td>
<td>699</td>
<td>1.11</td>
</tr>
<tr>
<td>S12T5</td>
<td>806</td>
<td>700</td>
<td>1.15</td>
</tr>
<tr>
<td>S13T1</td>
<td>1404</td>
<td>1252</td>
<td>1.12</td>
</tr>
<tr>
<td>S13T4</td>
<td>1501</td>
<td>1365</td>
<td>1.10</td>
</tr>
<tr>
<td>S14T1</td>
<td>1214</td>
<td>842</td>
<td>1.44</td>
</tr>
<tr>
<td>S14T2</td>
<td>1093</td>
<td>842</td>
<td>1.30</td>
</tr>
<tr>
<td>S14T4</td>
<td>1282</td>
<td>911</td>
<td>1.41</td>
</tr>
<tr>
<td>S14T5</td>
<td>1234</td>
<td>896</td>
<td>1.38</td>
</tr>
<tr>
<td>S15T1</td>
<td>1040</td>
<td>928</td>
<td>1.12</td>
</tr>
<tr>
<td>S15T4</td>
<td>1127</td>
<td>886</td>
<td>1.27</td>
</tr>
<tr>
<td>S16T1</td>
<td>932</td>
<td>615</td>
<td>1.52</td>
</tr>
<tr>
<td>S16T2</td>
<td>815</td>
<td>615</td>
<td>1.33</td>
</tr>
<tr>
<td>S16T4</td>
<td>776</td>
<td>663</td>
<td>1.17</td>
</tr>
</tbody>
</table>
Background to Modified Bond Model
Overview of results - Summary of Modified Bond Model

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S16T5</td>
<td>700</td>
<td>660</td>
<td>1.06</td>
</tr>
<tr>
<td>S17T1</td>
<td>1365</td>
<td>1159</td>
<td>1.18</td>
</tr>
<tr>
<td>S17T4</td>
<td>1235</td>
<td>1086</td>
<td>1.14</td>
</tr>
<tr>
<td>S18T1</td>
<td>1157</td>
<td>799</td>
<td>1.45</td>
</tr>
<tr>
<td>S18T2</td>
<td>1079</td>
<td>799</td>
<td>1.35</td>
</tr>
<tr>
<td>S18T4</td>
<td>1122</td>
<td>862</td>
<td>1.30</td>
</tr>
<tr>
<td>S18T5</td>
<td>1104</td>
<td>874</td>
<td>1.26</td>
</tr>
</tbody>
</table>

AVG 1.24
STD 0.15
COV 0.12

These results are also shown graphically in Fig. 5.2. Comparing this figure to the results in Fig. 5.1 show the clear improvement of the Modified Bond Model over the shear provisions from EN 1992-1-1:2005 for slabs under concentrated loads close to the support.

Fig. 5.2: Comparison between experimental results and Modified Bond Model

The comparison between the experimental results and the Modified Bond Model is also shown as a histogram in Fig. 5.3. The histogram shows that the distribution is not a normal distribution, but more a lognormal distribution. It also shows that the 5% lower bound is about 1.06. This result means that the Modified Bond Model has sufficient inherent safety.
To study the sensitivity of the Modified Bond Model to important parameters, the results of $P_u/P_{MBM}$ are studied as a function of the concrete compressive strength (Fig. 5.4), the distance between the load and the support (Fig. 5.5, Fig. 5.6) and the size of the loading plate (Fig. 5.7). These results show a consistent performance of the Modified Bond Model over the range of the studied parameters. It should be noted as well that the sensitivity to the concrete compressive strength is smaller when using the Modified Bond Model.
Fig. 5.4: Results of comparison between experiments and Modified Bond Model as a function of the concrete compressive strength.

Fig. 5.5: Results of comparison between experiments and Modified Bond Model as a function of the distance between the load and the support expressed as a/d.
The following parameters are determined from the analysis (Lantsoght, 2012) to have a large influence on the shear capacity of slabs under a concentrated load near to the support: the size of the loading plate, the moment distribution in the shear span and the distance between the load and the support. These parameters also influence the maximum load as determined in the Modified Bond Model, and thus it can be concluded that the behavior of the model is in line with the experimental observations.
6. Discussion of the proposed model

One of the advantages of the proposed model is that it shows the essential link between one-way and two-way shear for the case of a slab under a concentrated load in shear.

Using a relatively simple model with strips does not require much computational time. The Modified Bond Model uses strips over which the load is carried, and is easily adapted to different situations, where it still leads to good results. Just like with a plasticity approach, this lower bound method can easily be adapted and is easily used for the determination of the maximum load.

As the width of the strips depends on the size of the loading plate, the influence of this important parameter is taken into account. Moreover, the Modified Bond Model does not depend so strongly on concrete compressive strength as for example the expression for $V_{Rd,c}$ in EN 1992-1-1:2005, which leads to a better correspondence between the calculated values according to the Modified Bond Model as compared to the experiments.

Currently, empiricism is built into the model for two aspects: the choice of $w_{ACI}$ could be seen as a more empirical decision, and the reduction factor of 0.8 to be used on $w$ for plain bars is empirical as well.

The size effect is currently not taken into account, but the expression for $w$ could be expanded with a size effect factor. However, experimental results to quantify the size effect on slabs in shear under a concentrated load close to the support are not available.

A disadvantage of the proposed method is that it results in a maximum force at the concentrated load (as is typically done for punching shear) instead of a maximum shear force at the support, which allows for the comparison to combinations of loads such as the action of dead loads and live loads on a solid slab bridge.
7. Conclusions

This report gives the background to the development of the Modified Bond Model.

Initially, an extension of the Hallgren model is sought to apply this model to the case of slabs under a concentrated load close to the support. Since this approach implied that the bounds within which the assumptions of the Hallgren model are valid, it was discovered that the possibilities for extension of the model were limited.

To better understand the direct transfer of the load from its point of application to the support, efforts were geared towards the description of the arch and compressive strut. By studying this mechanism, it was found that the three-dimensional behavior of slabs under a concentrated load led to the fact that the arching capacity cannot be determined by using methods for beams in shear.

With this understanding, the Alexander and Simmonds bond model is studied. As this model is applicable to concentric punching shear, it has been extended for the application to slabs under a concentrated load in shear; the resulting model is called the Modified Bond Model. Further extension of the model to take into account the non-axis-symmetrical conditions in one-way slabs, the moment distribution in the shear span, loading near the edge, slabs on bearings and slabs with plain bars are developed. It is then found that the proposed model gives significantly better results than the previously used EN 1992-1-1:2005 for one-way shear.
8. References


Background to Modified Bond Model

References - Summary of Modified Bond Model


