BEYOND A COMPLETE FAILURE: THE IMPACT OF PARTIAL CAPACITY REDUCTIONS ON PUBLIC TRANSPORT NETWORK VULNERABILITY

Oded Cats
o.cats@tudelft.nl
Department of Transport and Planning, Delft University of Technology

Erik Jenelius
erik.jenelius@abe.kth.se
Department of Transport Science, KTH Royal Institute of Technology

ABSTRACT

Disruptions often result with partial capacity reduction without resulting with a complete breakdown. This study aims to move beyond the analysis of complete failure by investigating the impacts of partial capacity reduction on public transport network performance. We analyse the relation between the extent of capacity reduction at the line level and its consequences on societal costs by performing a full network scan. This analysis framework is applied to planned temporary disruptions in the rapid public transport network in Stockholm, Sweden. Our results indicate that the network is highly vulnerable since it is characterized by greater negative impacts in a disproportional relation to the increase in the original capacity reduction. The non-linear properties of network effects and route choice result in non-trivial relation which carry implications on disruption management the deployment of mitigation measures.

Keywords: Network vulnerability; Disruption; Capacity; Public transport.

1. INTRODUCTION

Public transport systems are subject to degradations of its services which may result in degradation of system performance. Some disruptions are unplanned and unexpected (e.g., technical failures in vehicles or infrastructure and accidents), while others are planned (e.g., capacity reductions due to construction work or repairs) or at least known some time in advance (e.g., crew strikes).

Due to rigid constraints in terms of line operations, timetables, vehicle and personnel stock etc., service disruptions in public transport networks (PTN) are prone to have wide and sustained implications. In addition to the immediate effect on the lines directly concerned, the dynamic nature of public transport supply results in impacts on service availability and capacity further downstream and in some cases even upstream. The impact for travellers in terms of delays and inconvenience depends on the availability of alternative travel options, i.e., the amount of redundant capacity in the public transport network.

Most of the previous studies have analysed PTN vulnerability by investigating the impacts of network topology on link or node failure (e.g., Angeloudis and Fisk 2006, Colak 2010 and von Ferber et al. 2012). The impact of random and targeted attacks on system performance was assessed through their implications on network centrality measures and by analysing the process of breaking into disconnected sub-networks. Berche et al. (2009) considered two different graph representations of PTN, where nodes correspond to stations and node failure means that no traffic can pass or stop at the station, respectively. Rodríges-Núñes and García-Palomares (2014) generalize the purely topological studies by assuming that link travel times and an OD travel demand matrix are known and that travellers choose the fastest route in the network to reach their destinations. The closure of a link can have two distinctly different outcomes: (1) the network is separated into two non-connected components, or (2) some travellers have to make a detour to reach their destinations. Following Jenelius et al. (2006) they define the importance of a link in case (1) as the unsatisfied demand, i.e., the number of trips that cannot be carried out and in case (2) as the increase in average travel time assuming that affected travellers make the fastest possible detour.

However, the impact of a disruption on network performance depends on the propagation of service deterioration, cascading and spill-over effects as well as the dynamics of route choice adaptation. Cats and Jenelius (2014a) developed a dynamic approach to PTN vulnerability analysis. The approach was applied on an abrupt capacity reduction due to temporary link failure to identify particularly critical links.

In general, vulnerability has been analysed based on the impact of complete link failure (e.g., caused by infrastructure breakdown or signal system malfunction). However, most disruptions do not amount to complete failures but are caused by partial reductions in service capacity. Network resilience should hence not only consider system performance in case of extreme and rare failures but also investigate the impact of moderate and recurring disturbances. Such reductions may arise from maintenance and construction works, traffic incidents or cancelled services. Furthermore, many service disruptions in practice, such as vehicle breakdowns and cancelled trips, do not affect a specific network link but rather a public transport line operating on the network. Cats et al. (2011) demonstrated that disruptions at the link and line levels may lead to distinctively different flow distribution patterns.

The aim of this study is thus to fill an important gap in the knowledge about PTN vulnerability and analyse the relation between the extent of capacity reduction at the line level and its consequences on network performance. A full-scan analysis of partial capacity reduction is performed for the case study multimodal network of the rapid public transport system of Stockholm, Sweden. For every line in the network, a sequence of scenarios with varying degree of capacity reduction is simulated. The general relation between the level of capacity reduction and the total disruption impacts is then assessed.

The analysis consists of simulating disruption scenarios using the same dynamic public transport operations and assignment tool as used by Cats and Jenelius (2014a), which represents individual vehicles and passengers. The model enforces strict on-board capacity constraints. It hence facilitates the analysis of upstream, downstream and horizontal cascading effects. The disruption of a link or line causes a redistribution of passenger flows in the network using a probabilistic and en-route choice model.

The paper is organized as follows. Section discusses the features of partial line capacity reductions and hypothesizes the relation between capacity reduction and the impact on network performance. Section 3 presents the case study application and the simulation model used to carry out the experiments. Section 4 presents and discusses the results, and Section 5 concludes the paper.

2. ON PARTIAL CAPACITY REDUCTION

A partial reduction in service capacity on an important line may lead to a substantial deterioration in system functionality and deviate from the normal state of operations. Vulnerable systems are characterized by greater negative impacts in a disproportional relation to the increase in the original capacity reduction (Taleb 2014). Figure 1 illustrates linear, convex and concave functions between the capacity reduction for line i, Δc_i, and the change in total passenger welfare in the network when compare with the undisrupted scenario, Δw_i. For example, a convex relation suggests that a single extreme failure will result with greater damage than two half-sized failures.

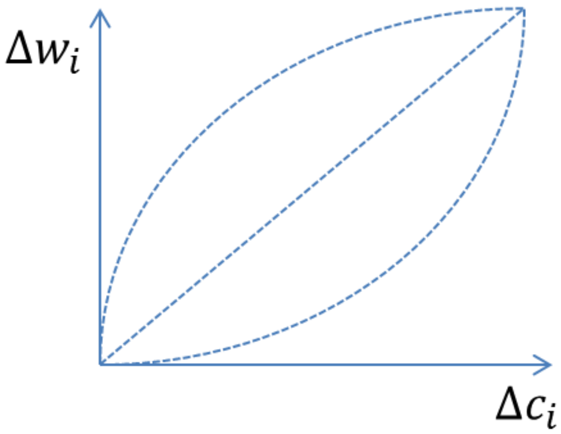

Figure 1. An illustration of possible relations between capacity reduction and the impact on network performance

The non-linear properties of network effects, traffic dynamics and route choice may result in a non-trivial relation between the magnitude of the failure and its consequences on network performance. This is particularly true for systems that operate close to capacity, as many urban public transport systems do, since the impact of capacity reduction depends on the availability of redundant on-board capacity and alternative paths.

A direct consequence of reduced line frequency is an increase in waiting times on the line itself. As an example, moving from twelve departures per hour to nine departures per hour implies a 25% reduction in frequency; however, based on the average headway this reduction leads to a 33% increase in expected waiting time assuming travellers arrive randomly to the stops. Moving from nine departures to six departures per hour is another decrease of 25% of the base level but leads to a 100% increase in the expected waiting time compared with the base case. However, part of this difference might be counteracted by obtaining better service regularity on lower-frequency services.

Further, reduced line frequency may have several effects due to increased passenger loads. Larger number of boarding and alighting passengers lead to delays, which induce uneven headways between vehicles and further delays; if vehicles are full there will also be denied boarding. Reduced line frequency may also induce passengers to switch to less direct or otherwise normally less attractive lines, causing increased congestion and the associated negative effects on those lines. It is expected that the influence of such secondary effects will increase with increasing levels of capacity reduction.

3. APPLICATION

3.1 Simulation Model

The evaluation of service disruptions requires a dynamic tool that can represent service supply dynamics and passengers response to such events. BusMezzo, a dynamic public transport operations and assignment tool was used in this study as the evaluation tool. The model fulfils the desired requirements as it represents individual vehicles and passengers including service uncertainties and passengers en-route decisions. A description of the supply representation is available in Toledo et al. (2010) and the relevant demand representation are presented in Cats et al. (2011). Cats and Jenelius (2014a) detail the modelling of service disruptions and related spill-over effects in BusMezzo. A brief presentation of the most relevant model features is therefore given hereunder.

Vehicle travel times are composed of running, queuing, dwelling and recovery times. The former are determined by a speed-density relationship in a joint car and public transport mesoscopic traffic simulation model. Queuing times at intersections are obtained from stochastic turning movement servers. Dwell times are based on flow-dependent functions, while recovery times depend on vehicle scheduling and dispatching. Different public transport modes have different vehicle types, capacities, operating speeds and control strategies. Furthermore, they exercise a varying level of interaction with other vehicles (e.g., busses in mixed traffic, bus lanes, underground), which results in different characteristics in terms of traffic regimes and travel time variability.

Passenger travel times are composed of access, wait, on-board, transfer and egress times. First, a non-compensatory rule-based choice-set generation model produces a set of alternative paths for each OD pair. Each element in the path alternative is a set, or hyper-path, created by grouping those public transport lines that provide an equivalent connection between a given pair of stops or several public transport stops which are connected by the same public transport lines. Second, each passenger undertakes a series of dynamic path decisions based on the expected travel attributes associated with alternative travel decisions. Travel expectations depend on the information available to each passenger. All travel decisions are modelled within the framework of discrete random utility models. Travellers' decisions are triggered and influenced by how the public transport service evolves and their ability to carry out their decisions depends on service availability and vehicle occupancy.

3.2 Case Study Description

The Stockholm public transport network represented in BusMezzo in this study includes the seven metro lines, four inner-city buses and the cross-radial light rail train. Figure 2 presents the network graph where nodes correspond to either stops or transfer hubs and links to line segments. The metro is characterized by a radial structure and constitute the backbone of the network. The metro lines are clustered into three trunks identified by their color: blue (10-11), red (13-14) and green (17-19). Inner-city trunk lines provide high coverage in the inner city while the light rail line functions as an orbital service connecting major interchange stations strategically located along the southern and western edges of the inner city.

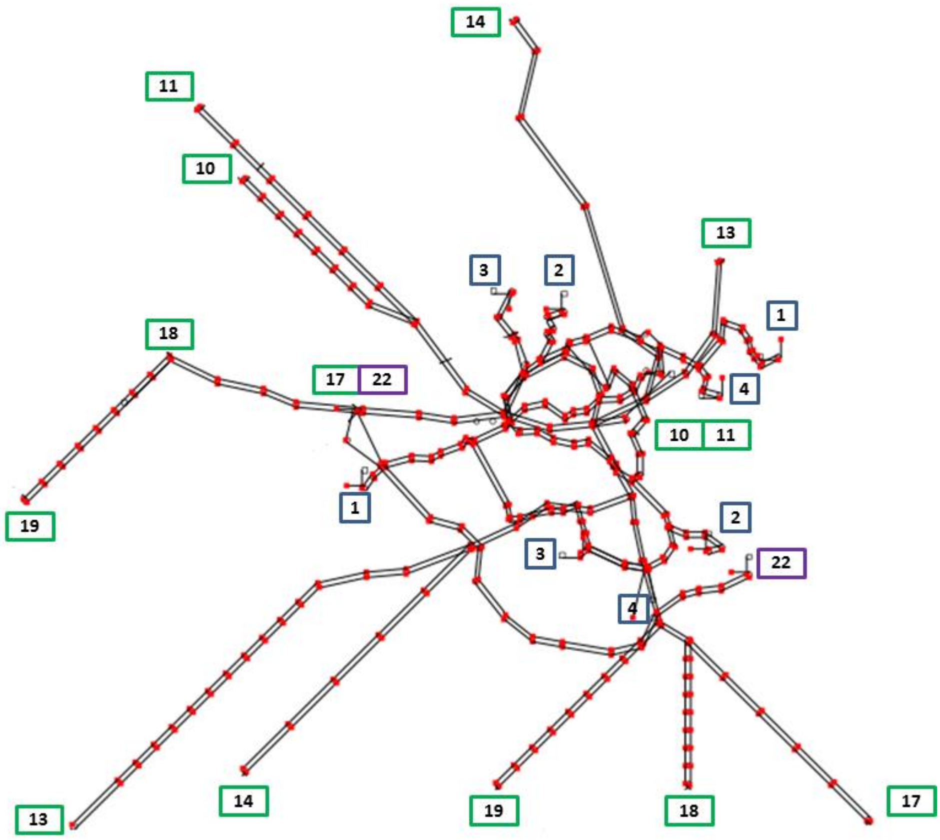

Figure 2. The Stockholm rapid public transport network as shown in BusMezzo graphical editor. The seven metro lines (10-11, 13-14, 17-19), the orbital light rail train (22) and the four trunk bus lines (1-4).

The twelve lines included in the case study serve 437 stops with approximately 700 vehicle runs performed by more than 200 vehicles during the morning peak period. Table 1 provides summary information on each line. The information was extracted from the annual statistical report of the regional transport administration and passenger counts (SL 2009). With the exception of the light rail train, line 22, all other lines operate with a frequency of 12 departures per hour (planned headway of 5 min). As could be expected, the metro lines, and in particular lines 13-14 and 17-19, known as the red and green lines, respectively, carry the largest passenger volumes. Among the trunk bus lines, line 4 is the busiest line in Stockholm, surpassing the number of passengers served by the light rail train. The trunk bus lines have slight variations in route length and number of stops for the two service directions depending of route layout and service alignment.

Table 1. Key characteristics of case study lines

Line	Mode	Frequency [dep/hr]	Length [km]	Number of stops	Peak hour ridership [pass]
1	Trunk bus	12	10.7-10.9	32-33	7000
2	Trunk bus	12	7.8-7.9	22-23	4500
3	Trunk bus	12	9.4-9.7	25-26	6000
4	Trunk bus	12	12.1-12.6	29-31	11,000
10	Metro	12	14.3	14	20,500
11	Metro	12	15.0	12	
13	Metro	12	26.7	25	49,000
14	Metro	12	19.1	19	
17	Metro	12	19.0	24	60,000
18	Metro	12	26.2	23	
19	Metro	12	28.4	35	
22	Light rail train	8	11.5	17	8000

The case study network was simulated for the morning peak period (6:00-9:00). Each public transport mode is simulated with distinct vehicle types, vehicle capacities, operating speeds, traffic regimes (mixed traffic, dedicated lane, separate right-of-way), dwell time functions and control strategies. These sets of operational attributes yield different levels of reliability and capacity depending on service design and right-of-way.

Approximately 125,000 passenger trips are generated during the morning peak hour, travelling between more than 4,500 different origin-destination pairs. The overall travel demand is considered fixed under all scenarios, assuming that the temporary disruption does not lead to substantial changes in travel patterns. The demand matrix was generated based on the regional travel demand model. The master choice set includes more than 615,000 alternative paths. Given that all lines in the case study network operate at high frequencies, travellers are assumed to depart randomly from the origins without consulting timetables. Passengers are thus generated in the simulation following a Poisson arrival process.

3.3 Scenario Design

A full-scan approach was taken in simulation disruption scenarios. A partial capacity reduction was therefore simulated independently for each line in the network. Capacity reduction was conceived in terms of a bi-directional reduction in service frequency in this study. Such a disruption could be caused by for example construction or maintenance works or a limited strike. For each line, service frequency was incrementally reduced by 25%. Frequency reduction was considered uniform over the simulation period. Hence, each line was simulated with 75%, 50% and 25% of the current frequency level. This implies frequencies of 9, 6 and 3 departures per hour or headways of 6.67, 10 and 20 minutes, respectively, for all lines except for line 22 (Table 1). The corresponding values for the latter are 6, 4 and 2 departures per hour or headways of 10, 15 and 30 minutes. By offering services on all lines in all scenarios, network integrity was sustained avoiding the issue of disconnected travellers that are unable to execute their trips.

Passengers are assumed to have perfect knowledge of the planned service in terms of lines and planned frequencies. Moreover, they are fully aware of the planned disruption and its implications on planned service frequency. However, uncertainty will result in discrepancies between the planned service and the actual provisioned service. In addition, the simulation reflects the availability of real-time information displays at all stops in the case study network. This information is generated in BusMezzo based on the prediction scheme that is used in practice in Stockholm (Cats et al., 2011). Consequently, the availability of real-time information at stops does not correspond to perfect information as service conditions as well as real-time information are subject to uncertainty (Cats and Gkioulou, 2015).

The aforementioned scenario design results in 37 scenarios: the base case scenario (100% capacity on all lines) and three capacity reduction scenarios (25%, 50% and 75%) for each of the 12 lines included in this case study network. Each of the disruption scenarios is denoted by the disrupted line and the percentage of capacity reduction (e.g., L1-25%). Each scenario was evaluated based on 10 simulation replications. This number of replications yielded a maximum allowable error of less than 1% for the average passenger travel time. The simulation generates a series of output files including the paths that were taken by each traveller and the corresponding travel time components. Passenger travel times are thus calculated based on the disaggregate demand representation and the time difference between simulation events.

4. ANALYSIS AND RESULTS

The results for each disruption scenario are summarized in Table 2. The average nominal travel time in the case study network is 25.6 minutes under nominal operations. This value increases by 6.33% to 27.2 minutes in the worst-case scenario which is caused by a 75% capacity reduction on metro line 14. The number of transfers remains almost unchanged in most scenarios with the exception of disruptions on trunk line 3. This line provides direct connections between key locations in the city, and many passengers prefer switching to indirect alternatives when faced with limited availability.

The scenarios were evaluated based on their societal costs as measured by their impact on total passenger welfare. Based on the Swedish value of time, the total passenger welfare in the peak morning hour amounts to a loss of 7.65 million SEK under normal operations, which are 61 SEK per passenger (10 SEK worth approximately 1 € as of December 2014). The welfare change is presented in the two last columns of Table 2 in absolute monetary terms and as a percentage compared with the baseline scenario without disruption. The general trend is that increasing capacity reductions yield increasing societal costs. The magnitude of the disruption impact varies from negligible to 484,900 SEK, depending on the capacity reduction and the disrupted line. The societal costs for the entire network increase by 3-6% in the case of a 75% decrease in the frequency of a single metro line. Since all the metro lines in Stockholm are grouped into trunks (Figure 2), a shortage of capacity on one line can be substituted by one or several other metro lines on the high-demand stations along the common corridor.

The results suggest that the network is relatively robust to partial planned line disruptions compared with unplanned link closures. A previous study by the authors (Cats and Jenelius 2014b) found that an unplanned 30 minutes long failure of the most critical segment in this network results in a 11.2% increase in the nominal travel times and yielded a societal cost of 807,000 SEK. There are several important differences between the disruption scenarios considered in this paper versus the disruptions considered in Cats and Jenelius (2014b). First, the disruption is at the line-level rather than the segment-level. The latter implies that multiple lines may be affected by the segment closure, while the disruptions investigated in this paper are constrained to a single line. Second, the disruption results in a partial capacity reduction rather than a complete failure. Third, the disruption is planned and hence allows travellers to adapt their path upon departure from their origins, whereas an unplanned disruption induces delays to travellers that are constrained on-board or downstream and rerouting decisions are taken under uncertainty. For all these reasons, unplanned disruptions, even if the initial disruption is limited to a segment rather than an entire line, can be more disruptive than planned line-level capacity reductions.

Table 2. Passenger travel time and relative welfare change for each disruption scenario

Scenario	**Average travel time** $t^{walk} + t^{wait} + t^{ivt}$ **[sec]**	**Transfers** $trans$	**Total passenger welfare** $[10^5$ **SEK]**	**Total welfare change due to disruption** $[10^3$ **SEK]**	**Relative change in welfare [%]**
no disruption	1 535	1.17	-76.50		
L1-25%	1 535	1.17	-76.50	0.00	-0.00
L1-50%	1 545	1.17	-77.02	-51.78	-0.68
L1-75%	1 557	1.17	-77.57	-107.29	-1.40
L2-25%	1 539	1.17	-76.70	-19.93	-0.26
L2-50%	1 538	1.17	-76.65	-14.95	-0.20
L2-75%	1 542	1.16	-76.84	-33.56	-0.44
L3-25%	1 536	1.25	-76.54	-4.06	-0.05
L3-50%	1 539	1.32	-76.71	-21.40	-0.28
L3-75%	1 544	1.39	-76.95	-45.01	-0.59
L4-25%	1 535	1.17	-76.50	0.00	-0.00
L4-50%	1 543	1.16	-76.90	-39.85	-0.52
L4-75%	1 545	1.14	-77.01	-50.73	-0.66
L10-25%	1 542	1.14	-76.85	-34.94	-0.46
L10-50%	1 558	1.16	-77.64	-114.10	-1.49
L10-75%	1 595	1.16	-79.49	-298.63	-3.90
L11-25%	1 546	1.14	-77.03	-52.95	-0.69
L11-50%	1 560	1.15	-77.72	-122.29	-1.60
L11-75%	1 589	1.17	-79.17	-267.02	-3.49
L13-25%	1 547	1.16	-77.08	-57.58	-0.75
L13-50%	1 567	1.15	-78.08	-157.97	-2.06
L13-75%	1 619	1.18	-80.68	-417.65	-5.46
L14-25%	1 550	1.17	-77.23	-73.24	-0.96
L14-50%	1 564	1.16	-77.94	-144.19	-1.88
L14-75%	1 632	1.17	-81.35	-484.90	-6.34
L17-25%	1 537	1.19	-76.60	-10.12	-0.13
L17-50%	1 539	1.12	-76.70	-20.18	-0.26
L17-75%	1 610	1.17	-80.26	-376.05	-4.92
L18-25%	1 543	1.17	-76.89	-38.93	-0.51
L18-50%	1 546	1.17	-77.04	-54.47	-0.71
L18-75%	1 561	1.16	-77.82	-131.91	-1.72
L19-25%	1 535	1.16	-76.50	0.00	-0.00
L19-50%	1 556	1.17	-77.56	-105.57	-1.38
L19-75%	1 594	1.18	-79.44	-294.16	-3.85
L22-25%	1 541	1.17	-76.78	-27.62	-0.36
L22-50%	1 545	1.16	-76.99	-48.67	-0.64
L22-75%	1 550	1.15	-77.27	-77.21	-1.01

The relation between a reduction in line capacity and the increase in societal cost follows a convex function. Figure 3 plots for each line the societal costs inflicted by a certain reduction in its capacity, corresponding to the fifth column in Table 2. The undisrupted case serves as the benchmark for all disruption scenarios. Disruptions on the metro lines have particularly adverse effects, with the exception of line 18 that results in moderate societal costs due to the relatively low demand levels on the south branch which it serves exclusively. Capacity reductions on metro lines 13 and 14, which jointly form the north-east to south-west trunk, result in the most adverse effect on network performance. As for trunk bus lines, network performance is most susceptible to disruptions on line 1 when high capacity reductions occur.

There is a pronounced increase in the slope of the disruption costs for increasing capacity reductions, albeit the extent of this trend varies considerably for different lines. Reductions of 25%, 50% and 75% of metro line 14 frequency result in societal costs of 73, 144 and 485 thousand SEK during the morning peak hour, respectively. A marginal capacity decrease of 25% thus results in approximately the same reduction in total welfare for the first two increments (25% and 50%). In contrast, a further reduction of 25% induces marginal losses that are 4.85 times greater than the previous increments. This disproportional effect is also apparent for other metro lines. On average, the increase in societal costs is more than 2.5 higher for the 50% metro frequency reduction than for the 25% reduction, and almost 8.5 higher in the case of a 75% reduction (a 3.15 times higher marginal increase than the previous increment). A considerably more modest increase in the marginal effect of capacity reductions was found for the trunk bus lines and the light rail line.

Network topology and travel patterns result in a non-trivial relation between capacity reduction and its impact for various lines. Line ridership (Table 1) is not necessarily a good predictor of the consequences of disruptions. For example, trunk line 4 is the busiest bus line but reducing its frequency results in less adverse implications than corresponding disruptions on trunk line 1. This is presumably due to the considerable overlap between line 4 and other bus lines and the availability of attractive metro alternatives from major stations. Similarly, network performance is more robust to a disruption on one of the green metro lines (17-19) than to a corresponding disruption on one the red lines (13-14), even though the former serve more passengers. The availability of two additional lines rather than one additional line on the trunk sections make the system less vulnerable to a disruption on one of the green lines. Furthermore, the trunk segments are more extensive in the case of the green lines than for the red lines.

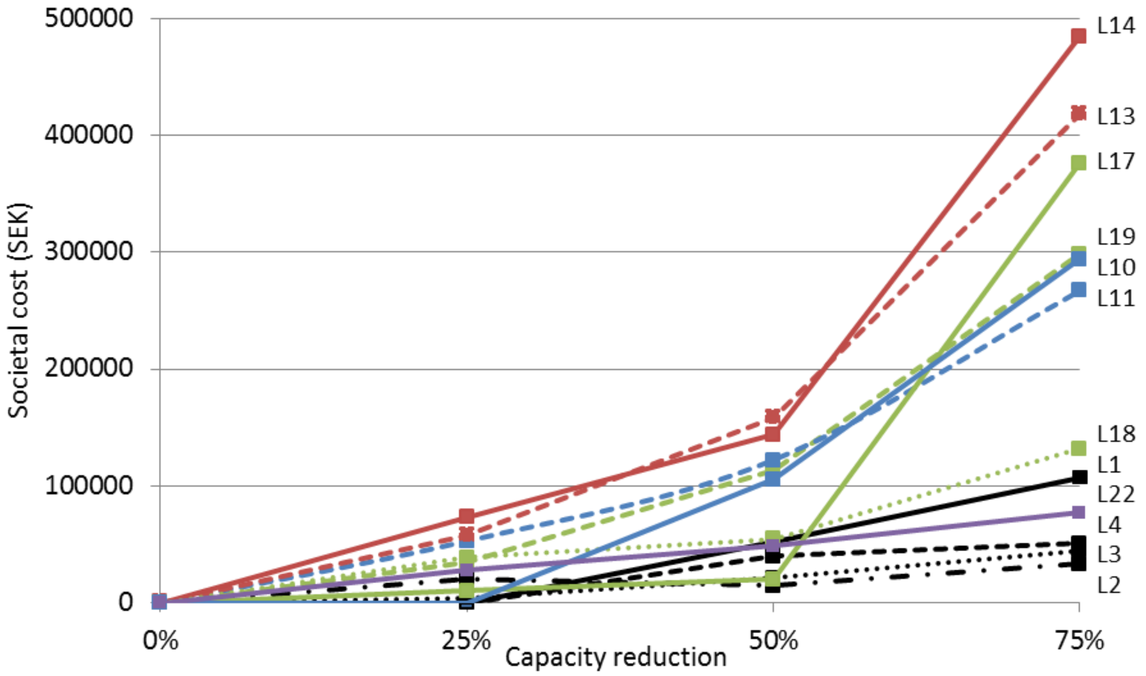

Figure 3. Relation between capacity reduction and change in total passenger welfare

The relation between capacity reduction and the societal costs it induces can support system operators when undertaking disruption management strategies. In particular, it could support the planning of temporary disruptions. The convex relation indicates that it is advisable to plan long and small rather than short and large capacity reductions. For example, a two weeks capacity reduction of 25% on metro line 10 will result in a societal cost of 349,400 SEK during morning peak hours on working days. The corresponding cost for a week-long reduction of 50% is 570,500 SEK, which, in turn, is less than the societal cost caused by a two-day reduction of 75%. Furthermore, the results of this study can help evaluate and prioritize alternative mitigation measures. Resources will be better allocated to relieve a high capacity reduction then restoring a smaller capacity reduction on an equivalent service. Increasing line capacity from 25% to 50% of the original levels will obtain substantially greater benefits than using the same resources (e.g., rolling stock, crew) to recover capacity from 50% to 75%.

5. CONCLUSION

The impacts of partial capacity reductions at the line-level were investigated and analysed for the Stockholm rapid public transport network. The results of this study suggest that policy makers and service operators should devote disproportional attention to major capacity reductions because such disruptions lead to disproportional consequences and societal costs. A twice as large capacity reduction is likely to result in more than twice as much delays. Arguably, the convex relation between capacity reduction and change in total passenger welfare is underestimated in this study because the disutility associated with in-vehicle time was considered independent of on-board crowding. Accounting for the impact of on-board crowding on the perceived in-vehicle time is expected to further contribute to this relation because a reduction in service provision results with higher saturation on the remaining services.

Whilst capacity reduction and change in total passenger welfare followed a convex function in the case study network, the generality of these results will be investigated in future research. It is postulated that this pattern will prevail also for other networks in other cities but the extent of these effects might vary considerably. In fact, significant differences were already evident in this case study network for different modes and lines.

The vulnerability analysis in this study considers a planned temporary reduction in service frequency. It was therefore assumed that passengers are informed on the disruption and can reroute accordingly. However, even in case of planned disruptions not all passengers may be well-informed on the disruption and its implication. Moreover, the total demand for public transport services is considered inelastic, whereas significant service reduction may have implications on other trip decisions such as mode and destination choices.

The results of this study could be used by public transport agencies and operators for assessing the need for replacement services to substitute reduced capacity on a disrupted service. The benefits from increasing the capacity of replacement services can be quantified for individual lines based on the extent of service frequency reduction.

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