# LOGARITHMIC VARIANCE PROFILES AND THE CORRESPONDING $f^{-1}$ SPECTRA OF TEMPERATURE FLUCTUATIONS IN TURBULENT RAYLEIGH-BÉNARD CONVECTION

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*Abstract* We report experimental results for the temperature variance $\sigma^2(z)$ and the corresponding frequency spectra $P(f)$ in turbulent Rayleigh-Bénard convection (RBC) in a cylindrical sample of aspect ratio $\Gamma \equiv D/L = 1.00$ ($D = 1.12$ m is the diameter and $L = 1.12$ m the height). The measurements were conducted in the Rayleigh-number range $10^{11} \lesssim$ Ra $\lesssim 1.35 \times 10^{14}$ and Pr $\simeq 0.8$. For Ra $= 1.35 \times 10^{14}$, $\sigma^2(z)$ could be described well by a logarithmic dependence on the vertical position $z$ in a range of $z_1^* \lesssim z \lesssim z_2^*$ with $z_1^* \simeq 70\lambda_\theta$ and $z_2^* = 0.1L$. Here $\lambda_\theta \equiv L/(2Nu)$ is the thickness of a thin thermal sublayer adjacent to the horizontal plate where the heat flux (denoted by the Nusselt number $Nu$) is carried mostly by thermal diffusion. In the log layer, we found that the temperature spectra had a significant frequency range over which $P(f) \sim f^{-\alpha}$ with $\alpha$ close to 1. As Ra decreased, $\lambda_\theta$ increased so that the log layer became thinner. At Ra $= 2.05 \times 10^{11}$, $z_2^* \lesssim z_1^*$ and therefore there was no range for a log layer. Correspondingly, the temperature spectrum near the horizontal plate did not have the $f^{-1}$ scaling form either.

Details about the RBC sample and experimental procedures were reported in Refs. [1, 2]. In the present work, we installed 68 new thermistors to measure temperature fluctuations. These thermistors were positioned in 6 columns at various radial locations $r$ from 1.0 cm to 15.0 cm away from the side wall within the sample. The thermistor diameters were 0.36 mm. The vertical positions of the thermistors were distributed over a range of $0.013 \leq z/L \leq 0.990$, symmetrically about the mid-height of the sample. They were known with a precision of 1mm. The sample was carefully leveled relative to gravity to within $10^{-4}$ rad. For temperature spectral measurements we used an ac bridge and a lock-in amplifier for each thermistor. Each amplifier was operated at a working frequency in the range $f_0 \simeq 1 \pm 0.4$ kHz to measure temperatures at a rate of 40 Hz.

Figure 1 shows the results for the temperature variance profiles $\sigma^2(z)$ at the radial position $\xi = 0.064$ for different Ra. The vertical position $z$ is scaled by the length $\lambda_\theta \equiv L/(2Nu)$. Here $\lambda_\theta$ is the thickness of a thin thermal sublayer adjacent to the horizontal plate where the heat flux is carried mostly by thermal diffusion. This thermal sublayer in RBC plays a role similar to the viscous sublayer in wall-bounded shear flow. At the highest Ra $= 1.35 \times 10^{14}$, the data follow closely a logarithmic dependence on the vertical position $z$ in a range $z_1^* \lesssim z \lesssim z_2^*$ with $z_1^* \simeq 70\lambda_\theta$ and $z_2^* \simeq 0.1L$. When Ra decreases, $\lambda_\theta$ increases and the log-layer upper limit $z_2^*/\lambda_\theta$ decreases. As a result, the log-layer range becomes smaller. At Ra $= 2.05 \times 10^{11}$, $z_2^*/\lambda_\theta \lesssim z_1^*/\lambda_\theta$ and therefore there is no range for the log layer of $\sigma^2(z)$.

In Fig. 2 we show the compensated temperature frequency spectra $(f\tau_0) \times P(f\tau_0)$ as a function of the normalized frequency $f\tau_0$ measured at $z/L = 0.019$ and $\xi = 0.064$ for different Ra. Here $\tau_0$ is a characteristic time scale determined from the temperature auto-correlation function [3]. The two spectra, although measured at the same distance from the bottom plate, correspond to different $z/\lambda_\theta$ because of different Ra. For Ra $= 1.35 \times 10^{14}$ the measuring positions is inside the log layer with $z/\lambda_\theta \simeq 101$. In the low-frequency range $0.02 \lesssim f\tau_0 \lesssim 0.2$ the compensated spectrum has the scaling $P(f\tau_0) \sim (f\tau_0)^{-\alpha}$ with $\alpha \simeq 1$, as indicated by a plateau of $(f\tau_0) \times P(f\tau_0)$. This spectral scaling form and the corresponding logarithmic variance profile are consistent with previous measurements for $z/L \lesssim 0.1$ in a $\Gamma = 0.50$ sample with Ra above $1.63 \times 10^{13}$ [3]. For Ra $= 2.05 \times 10^{11}$ the measuring position corresponds to $z/\lambda_\theta \simeq 12.7$. Because there is no log layer as shown in Fig. 1 (c), the corresponding spectrum does not have the $f^{-1}$ scaling. These temperature variance profiles and the corresponding frequency spectra in turbulent RBC share many similarities with predictions for the variance profiles and the wave-number spectra of velocity fluctuations in the log layer of turbulent pipe flow [4, 5]

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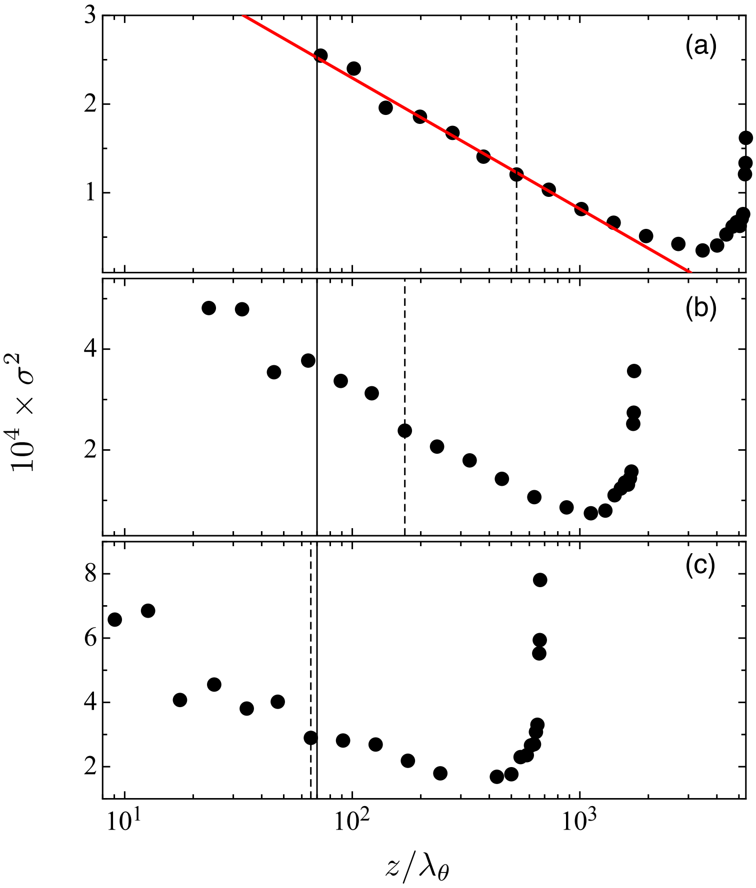


**Figure 1.** (a) Measured temperature variance $\sigma^2(z)$ as a function of the normalized vertical position $z/\lambda_\theta$ on a logarithmic horizontal scale for the three Rayleigh numbers (a) $Ra = 1.35 \times 10^{14}$, (b) $4.38 \times 10^{12}$, and (c) $2.05 \times 10^{11}$. The vertical solid lines are at $z/\lambda_\theta = 70$. Three vertical dashed lines represent $z/L = 0.1$. The red solid line in (a) is a fit to the data for $z/L < 0.1$ using the logarithmic function $\sigma^2(z,r) = M(r) * \ln(z/L) + N(r)$. All measurements were for the normalized radial location $\xi \equiv (R-r)/R = 0.064$.

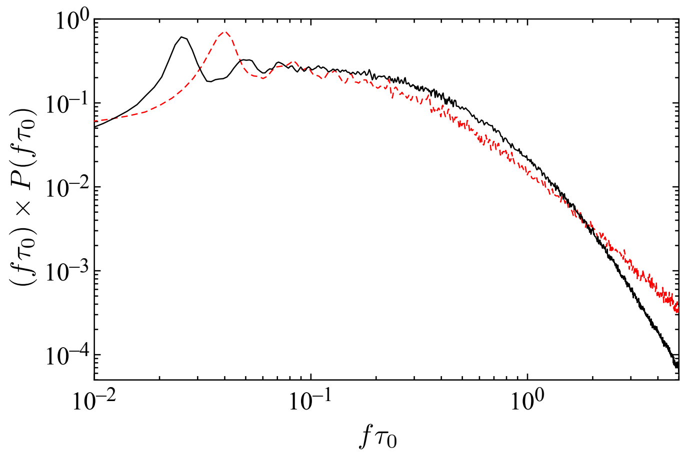


**Figure 2.** Normalized temperature spectra $(f\tau_0) \times P(f\tau_0)$ as a function of $f\tau_0$ at $z/L = 0.019$ for $Ra = 1.35 \times 10^{14}$ (black solid line) and $2.05 \times 10^{11}$ (red dashed line). All measurements were for the normalized radial location $\xi \equiv (R-r)/R = 0.064$.