

## **ENDOGENOUS RISK IN UNBALANCED BIDDING**

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#### **Abstract**

*Models of unbalanced bidding in unit price contracts (UPC) can be categorised into two types. The first category assists clients in detecting and contractors in optimising skew bidding. More theoretical oriented models of the second type focus on bidding behaviour in order to study market efficiency. These models predict corner solutions, i.e. zero prices, for unit prices of expected overestimated quantities. However, anecdotal evidence indicates a lack of zero prices in the actual contracts. A possible explanation for this discrepancy is risk-aversion of the contractor. However, none of the models of the latter category have incorporated risk as an endogenous variable. A model of such is presented in this paper.*

**Keywords:** Unbalanced bidding, risk, modelling

#### **INTRODUCTION**

Unit price contracting (UPC) is common in the construction industry throughout the world. This contracting form has the client – often a public entity – preparing and taking juridical responsibility for the design. The design consists of a bill of quantities with related descriptions of the work. Contractors then submit bids in forms of price vectors, one price for each quantity. The lowest vector product of prices and quantities, i.e. the lowest total price, is awarded the contract.

The simplicity of this contracting form is appealing, which can be one explanation for its frequent use. There are however problems with this contracting form. UPC has been accused of causing the low productivity in the sector. The logic behind this is that the contractor has low incentives to come up with innovations since the clients to a large extent provide rigid contracts on how the work is to be conducted (see e.g. Mandell and Nilsson, 2010).

Another argument against the use of UPC is that it opens up for unbalanced bidding, i.e., that bidders skew their bids in response to having superior information relative to the procurer. Research regarding this issue includes models for client detection and contractor optimisation of skewed bidding. Such models can provide practical guidance to practitioners but they cannot determine the extent of the problem from an efficiency perspective. Other models focus on predicting bidding behaviour in order to study market efficiency. Intuitively, the contractor's perception of risk may be an important part in unbalancing since the concept is based on uncertainty. However, to the best of our knowledge no model has incorporated the contractor's risk in bidding models.

The paper is further motivated by an apparent miss-match between the theoretical models in the literature and the empirical indications. In short, the existing theoretical models predict corner solutions that entail zero bids for overestimated quantities. However, very few zero bids are observed in data. There are potential explanations for this. An argument is made that the institutional setting in Sweden is such that the primary reason for not seeing zero bids in a Swedish data-set must be due to risk concerns among bidders. Thus, this paper sets out to incorporate the contractor's perception of risk as an endogenous variable in an unbalanced bidding model.

## UNBALANCED BIDDING

The idea of unbalanced bidding in UPC contracts is not a new concept in research or practice. Gates (1967) and Stark (1974) made some early and influential work on this phenomenon. Unbalanced bidding is sprung out of the fact that the contractor is better informed than the client. This is based on the contractor finding errors in the *ex ante* bill of quantities, which can be used to his advantage. In general terms, the best informed contractor can both win the tendering and enhance his profits, despite the fact that he may not be the most efficient contractor. Hence, the existence of unbalanced bidding entails an inefficient outcome on the market due to information rents.

There are basically two types of unbalanced bidding discussed in the literature; unbalanced bidding through "front loading" the bids and unbalanced bidding based on information rents regarding quantities. Hughes (1982) distinguished the former type of unbalancing as *finance cost/cash flow unbalancing* in contrast to the *error exploitation unbalancing*, described in the above example. Cash flow unbalancing involves the contractors marking up prices on quantities that are scheduled for early completion trading off quantities for late completion (Arditi and Chotibhongs, 2009; Skitmore and Cattell, 2011).

From an efficiency perspective, the problem with front loaded bids is that the client will make the payment to the contractor earlier than in a situation with full information about costs. Differences between contractors regarding information about the scheduling of the tasks to front load bids in different ways can however be questioned. Scheduling of a construction project has certain elements that cannot be changed. The pavement cannot be laid out before the construction of the sub base of the road is finalised.

Hence, a rather mild assumption would be that front loading does not differ between the contractors and that it is private information about quantities that will be the base for unbalanced bidding. The remaining paper will not deal further with front loading, but will focus entirely on the first type, i.e., unbalanced bidding following from superior information about quantities actually required.

The inefficiency of this type of unbalanced bidding can be illustrated by a numerical example. Consider a setting where two contractors compete for a road investment contract that requires only two inputs, e.g. paving and provision of gravel. A bill of quantities including estimations for gravel and pavement is announced by the client, 100 m<sup>3</sup> and 150 m<sup>2</sup>. The contractors differ in the first being more efficient (i.e. having a lower marginal cost) but being less informed about the quantities actually required, i.e., the *ex post* quantities. Contractor 1 bids his marginal cost at prices 10. Assume that contractor 2 has a higher marginal cost than contractor 1 on both inputs but he has private information about the project and predicts that the gravel is underestimated and the pavement is overestimated by

the client. Given this expectation, contractor 2 raises his price on gravel and reduces the price pavement as seen in figure 1.

<b>Ex ante</b>	<b>Bill of quantities</b>	<b>Contractor 1's bid (Uninformed)</b>	<b>Contractor 2's bid (Informed)</b>
Provision of gravel	100 m <sup>3</sup>	10	12
Pavement (paving)	150 m <sup>2</sup>	10	8,5
<b>Total bid</b>		<b>2 500</b>	<b>2 475</b>

*Figure 1: Ex ante bill of quantities and bids*

As depicted in figure 1, contractor 2 submits the lowest total bid and wins the contract.

The project starts and contractor 2's prediction, that the quantities of gravel will be increased and pavement decreased, turns out to be correct. As seen in figure 2, contractor 2's skewing of prices based on his expectation of changing quantities made him win the contract and earn higher revenue.

<b>Ex post</b>	<b>Actual quantities</b>	<b>Contractor 1's revenue</b>	<b>Contractor 2's revenue</b>
Provision of gravel	110 m <sup>3</sup>	10	12
Pavement (paving)	145 m <sup>2</sup>	10	8,5
<b>Final cost for client</b>		<b>2 550</b>	<b>2 553</b>

*Figure 2: Ex post actual quantities and revenue*

Hence, due to unbalanced bidding the most efficient contractor did not win the contract. The client ended up paying an information rent to contractor 2 i.e. a higher cost than if the most efficient contractor 1 would have won.

## **EARLIER WORK ON UNBALANCED BIDDING**

This section addresses earlier work targeting unbalanced bidding, which consists both of theoretical modelling and empirical studies.

### **Theoretical models of unbalanced bidding**

There is a lack of exact definition of unbalanced bidding. Models are one way of making concepts more concrete. Two types of models have been developed regarding unbalanced bidding.

A first type focuses on assisting practitioners to detect possibilities for unbalanced bidding in the tendering stage and how to exploit these. The detection models supports clients' interests (see e.g. Arditi and Chotibhongs, 2009) and exploitations models assist contractors in optimising the skew (see e.g. Yizhe and Youjie (1992); Cattell et al (2008); Cattell et al (2010)).

The second types of models are not designed to provide direct practical guidance to unbalanced bidding. Rather, they focus on predicting and measuring the extent of the phenomena in order to determine efficiency effects. The present paper belongs to this latter category.

In particular, there are two prominent papers in this category. The first is Ewerhart and Fieseler (2003), who model a UPC contract where two tasks are required in order to complete a project; 1 unit of material and  $h$  hours of labour. The contractors have private information about their type, i.e. being fast or slow in order to finish the task. This information is not known by the client, hence contractors are better informed than the client. The UPC auction starts by the client announcing his estimate on the number of hours required to complete the project. This estimate will be underestimated for slow types and overestimated for fast types. Based on their type, the contractors submit bids i.e. price vectors of material and labour. Final payment is based on the vector product of prices and quantities, where the latter is actual hours put into the project and 1 unit of material.

The usual prediction of unbalanced bidding applies; the slow types face an underestimated client prediction of hours and will mark up the price for labour and compensating the price for material downwards. This will result in a corner solution i.e. one sided bidding, where null bids will be submitted for labour by fast types and for material by slow types. The model predicts that the most inefficient types i.e. the slowest contractor, will win the bidding process. This is due to the fact that the slowest contractor will be able to exploit the gap between the estimated hours and the actual hours put down. The slow types profit function is increasing in hours. Ewerhart and Fieseler (2003) express this as the slow types are being subsidised. In the extended version of the model, it is shown that the subsidy to inefficient contractors will force the efficient ones to bid more aggressively, having a positive effect on client cost.

The second prominent article is Athey and Levin (2001), who model unbalanced bidding in timber auctions. These auctions differ from construction projects as the buyers of timber harvest rights are the bidders, and not the sellers as in a construction project. The model consists of two species of timber and allows the buyers to invest in private information. This is done by the buyers inspecting the tract before the auction in order to estimate the proportion of timber species. The auction starts with the Forest Service (seller) announcing their estimated proportion of timber species and total amount. In reverse order from the above model, the buyers will mark up the price on overestimated timber types and compensate through bidding a lower price on the underestimated type in order to secure a high bid *ex ante* and pay less *ex post*. Payment is made by the actual amount of timber and the *ex ante* prices. As in the above model, a risk neutral bidder will end up in a corner solution submitting the buyer's reservation price (in the extreme; zero).

The two models differ in respect to what there is asymmetric information about. In Ewerhart and Fieseler (2003) the contractors are better informed about their own ability while in Athey and Levin (2001) they are better informed about the project's characteristics. Even so, both papers predict corner solutions that entail zero price bidding on overestimated quantities.

## **Empirical studies on unbalanced bidding**

The empirical work on unbalanced bidding has focused on capturing the correlation between differences in prices and quantities respectively.

Athey and Levin (2001) have data from both oral (N=697) and sealed (N=63) timber auctions. The study has data on the sellers estimated volume and proportion of timber type, ex ante bids and ex post (actual) volume and proportion of timber type. This data enables the authors to calculate actual payments. Some control variables as number of bidders are also included. The overall conclusion is that private information is used in the bidding by the buyers. However, the information rents are to some extent offset by competition. This empirical result can be related to the theoretical argument put forward in Ewerhart and Fieseler (2003) about the inefficiency of unbalanced bidding being offset by strengthened competition by the efficient contractor.

Regarding sealed bidding, which is the most common auction form used in construction, the study shows that 65 percent of the winning bids are skewed in the expected direction. In other words, the direction in which the theory predicts, i.e. raising bids on overestimated type of timber. The empirical models regress the skew parameter, i.e. dollars spent on the overestimated species of wood, on the sellers' misestimate of the tract with some control variables. In accordance with theory the correlation is positive, which the authors interpret as the winning bid incorporating the superior information into the bid.

Bajari et al (2007) test three types of mark-ups in construction auctions, where one is the existence of unbalanced bidding. The data consist of 414 paving contracts from California between the years 1999 and 2000, which contains 1939 bids from 271 contractors. The actual ex post quantities and "Blue book" prices are also available. This data makes it possible to regress quantity overrun on the dependent variable percentage difference between the winning and the blue book price. In accordance to what the theory predicts the correlation is positive, indicating that on average a 10 percentage overrun on quantities will result in a mark-up of approximately 0.27 per cent.

## **THE LACK OF CORNER SOLUTIONS IN PRACTICE**

Both Ewerhart and Fieseler (2003) and Athey and Levin (2001) provide models that theoretically predict corner solutions in the form of bids equal to zero. It is then striking that the empirical studies discussed above, even though they find evidence indicating that unbalanced bidding exist, do not observe zero pricing. The same observation is done in a database under development of UPC for road investments in Sweden.

The lack of zero pricing can be explained by different factors. The most evident ones are

1. That the client or law prohibit zero pricing
2. A reputational mechanism of losing future work
3. That contractors are risk averse

There is however a disagreement regarding the explanatory power of these factors. Bajari et al (2007) argue that that bids with zero unit prices are very likely to be rejected and therefore not very common. Athey and Levin (2001) support the risk-aversion explanation, even though it is not formally modelled in their paper.

The institutional setting in Sweden sheds further light on why there is a lack of zero bids in our Swedish data-set. A recent court order in Sweden challenges the first explanation for the lack of zero pricing. The case regarded a public UPC for road maintenance, which was rejected for being unbalanced with low prices (although separated zero) for one of the tasks (snow ploughing) in the contract. The rejection was appealed by the contractor with the court ruling in favour of the contractor (Förvaltningsrätten i Falun, 2010). There is a paragraph in the Public Procurement act saying that the client can reject a bid if it is too low. The paragraph refers to the overall total bid and not unique unit prices according to the verdict (Förvaltningsrätten i Falun, 2010). This can be interpreted as the law paragraph is not there to secure an efficient use of state funding but to guarantee delivery i.e. not risking standing without a contractor to take care of the state's responsibilities. This court order prevents Swedish clients from rejecting zero unit price bids and moreover inhibits client rejection as an explanatory variable for the lack of such bids on the Swedish market.

Regarding reputational mechanism, there is a law in Sweden (based on an EU directive i.e. this argument should apply to the EU) prohibiting the client to use repeated interaction and self-enforcing contracts (see e.g. Gibbons (2005) for relational contracts) The purpose of the Public Procurement Act is to hinder nepotism of government funded projects with the disadvantage of not being able to use repeated interaction as an incentive. As the law states that every new project should be objectively procured, the contractor cannot take credit for "a job well done" in prior projects. Regarding construction in general and road construction especially, public clients that are subject to this legislation represent a large part of the total amount of clients. The legislation can be used as an argument to downsize the effect that the reputational mechanism has as an explanatory variable for the lack of zero bidding. Even though repeated interaction effect cannot be completely ruled out, as tendering documents can be adjusted for a certain contractor, there is still a law against repeated interaction.

Hence, neither of the first two explanations above have much support in the institutional setting on the Swedish construction market. This leaves risk aversion as a prime candidate to explain the lack of zero pricing in Swedish UPCs.

None of the models above include risk as an endogenous factor in the models. The following section set out to do this.

### **THE MODEL: RISK AVERSION IN UNBALANCED BIDDING**

Assume that two tasks,  $A$  and  $B$ , are required to produce a product, e.g., a road. As above, we restrict our attention to Unit Price Contract Procurements under which the principal, e.g., the road administrator, specifies quantities for  $A$  and  $B$  respectively. There are a number of agents bidding in the procurement. Their bids consist of per unit prices on  $A$  and  $B$ , denoted  $P_A$  and  $P_B$ . The agent with the lowest total price,  $P_A A + P_B B$  wins.

Consider a stylized setting in which there are several agents bidding. These all have the same information as the principal, i.e., they have no reason to suspect that the actual ex post quantities will differ from the specified quantities  $A$  and  $B$ . We refer to these agents as

“uninformed”. We introduce one agent who is better informed about the quantities actually required,  $A_{true}$  and  $B_{true}$ , than the principal and the uninformed agents. To capture this, we assume that this “informed agent” knows that the *expected* quantities actually required is  $A + \alpha$  and  $B + \beta$ , respectively. It seems plausible that there, ex ante, is some uncertainty surrounding these quantities. Thus, let the required quantities be  $A_{true} = A + \alpha + \varepsilon$  and  $B_{true} = B + \beta + \eta$ , where  $\varepsilon$  and  $\eta$  are stochastic variables. For simplicity, let  $\varepsilon \sim U[\varepsilon_{low}, \varepsilon_{high}]$  and  $\eta \sim U[\eta_{low}, \eta_{high}]$ , where  $\varepsilon_{low} = -\varepsilon_{high}$  and  $\eta_{low} = -\eta_{high}$  such that the distributions are both symmetrical around zero.

Conducting each task is assumed to be associated with a constant marginal cost. The informed agent’s total cost for task A is given by  $TC_A^I = C_A^I (A + \alpha + \varepsilon)$  and for B it is  $TC_B^I = C_B^I (B + \beta + \eta)$ . The net payment received is given by:

$$\pi = (P_A^I - C_A^I)(A + \alpha + \varepsilon) + (P_B^I - C_B^I)(B + \beta + \eta) \quad (1)$$

Where  $P_A^I$  and  $P_B^I$  denote the informed agent’s per unit bid for tasks A and B respectively. As will be shown, the informed agent may let its superior information influence the bid prices  $P_A$  and  $P_B$ . There are two restrictions limiting the degree to which the bids may be tweaked. Firstly, neither bid must be negative, i.e.,  $P_A^I \geq 0$  and  $P_B^I \geq 0$ . Secondly, the total bid, i.e.,  $P_A A + P_B B$ , must be (weakly) lower than the competing bidders’ to win. Let us denote the competing bids as  $P_A^U$  and  $P_B^U$  for task A and B respectively ( $U$  denotes that these are the uniform agents’ bids). We may then write a “bid restriction” which must be fulfilled for the informed agent to win the procurement as:

$$P_B^I \leq \frac{A(P_A^U - P_A^I)}{B} + P_B^U \quad (2)$$

We will subsequently assume that (2) is binding, as a strict inequality implies leaving unnecessary surplus to the procurer. Thereby, we may use (2) to substitute for the informed agent’s bid on B, thus allowing us to optimize over the bid on A only.

The agent may be risk averse. We capture this by letting the agent’s expected pay-off be  $E\{\pi\} - Rvar\{\pi\}$ , where  $R$  is a coefficient measuring risk aversion. When  $R = 0$  the agent is risk neutral and when  $R > 0$  he is risk averse. Using (1) and (a binding) (2) we may express the expected net payment as:

$$E\{\pi\} = (P_A^I - C_A^I)(A + \alpha) + \left(\frac{A(P_A^U - P_A^I)}{B} + P_B^U - C_B^I\right)(B + \beta) \quad (3)$$

and its variance,  $var\{\pi\}$ , as:

$$var\{\pi\} = \frac{1}{3}(\eta_{high}^2 \left(\frac{A(P_A^U - P_A^I)}{B} + P_B^U - C_B^I\right)^2 + \varepsilon_{high}^2 (P_A^I - C_A^I)^2) \quad (4)$$

The objective for the informed agent is to choose  $P_A^I$  and  $P_B^I$  so the expected pay-off is maximized. To facilitate the presentation, let us assume that all agents have the same marginal cost structure and that the competition among the uninformed agent is strong enough to drive their bids down to marginal cost. This assumption allows us to substitute  $P_A^U$  and  $P_B^U$  in (3) and (4) by  $C_A^I$  and  $C_B^I$ , respectively, which greatly simplifies the interpretation of the resulting outcome. The expected pay-off is then

$$(A + \alpha)(P_A^I - C_A^I) - \frac{A(B+\beta)(P_A^I - C_A^I)}{B} - \frac{R(C_A^I - P_A^I)^2 (A^2 \sigma_{\text{high}}^2 + B^2 \sigma_{\text{high}}^2)}{3B^2} \quad (5)$$

From (5) we may derive the following first order condition

$$P_A^{I*} = C_A^I + \frac{3B^2\alpha - 3AB\beta}{2R(A^2\sigma_{\text{high}}^2 + B^2\sigma_{\text{high}}^2)} \quad (6)$$

The first term in (6) is the agent's marginal cost. The second is a (possibly negative) mark-up that the agent will apply to his bid on task  $A$  as a consequence of his superior information. From (6) we can draw a series of conclusions regarding the agent's optimal bidding strategy.

The first observation is that the mark-up increases in  $\alpha$ . This is in line with the intuition behind strategic bidding. A positive  $\alpha$  implies that the (expected) true quantity is larger than the procured one. Thus, the agent would like to increase the asking price per unit of that quantity and thereby gain from the expected increase in required quantity.

Second, the mark-up decreases in  $\beta$ . This follows from the bid restriction. If the agent increases his bid on task  $A$ , he must decrease the bid on some other task. As there are only two tasks in this model, this means reducing the bid on task  $B$ . A positive  $\beta$  implies that also the true task  $B$  quantity will, in expectation, exceed the procured one. In that case, decreasing the bid for task  $B$  results in an expected loss on task  $B$ . Thus, when  $\beta$  increases the bid on task  $A$  must decrease in optimum, ceteris paribus.

A related observation is found in that if  $B=0$ , i.e., only task  $A$  is procured, the second term in (6) is zero. That is, even if  $\alpha$  is positive, the agent is unable to bid strategically by increasing  $P_A^I$  above marginal cost, as there is no second task with which to compensate the total bid. On the other hand, if  $A=0$ , which implies that the principal only procures task  $B$ , but the agent believes that task  $A$  is needed, i.e.,  $\alpha$  is positive, he may place a bid above marginal cost for task  $A$ . In this, somewhat unrealistic case, neither  $B$  nor  $\beta$  puts any restriction on the bid simply because  $P_A^I$  will have no impact on the total bid (as it will be calculated at  $A=0$ ).

Third, and perhaps most interestingly, both the risk aversion coefficient,  $R$ , and the limits of the probability distributions appear with positive signs in the denominator of (6). The former implies that the optimal bid for  $P_A^I$  decreases in risk aversion and the latter that it decreases in the range of the stochastic variables, i.e. the risk. Both these suggest that there is a downside for the agent in bidding strategically following from that this behaviour exposes him to risk. Thus, there is a trade-off between expected net payment and risk exposure. The informed agent may use his superior information to increase the expected net payment by the means of unbalanced bidding, but in so doing the variance of the net payment will increase. The latter refrains a risk averse agent from skewing his bids too aggressively.

That this trade-off is central to the understanding of the agent's behaviour is illustrated by looking at the behaviour of a risk neutral agent. Equation (6) cannot handle risk neutral agents as it would result in a denominator equal to zero. By setting  $R = 0$  already in (5) and differentiating the resulting expression with respect to  $P_A^I$  yields

$$\frac{\partial R(\pi) | R=0}{\partial P_A^I} = \alpha - \frac{A\beta}{B} \quad (7)$$

Equation (7) shows the extreme incentives involved for an informed risk neutral agent. As long as  $\alpha > A\beta / B$ , the expected pay-off always increases in  $P_A^I$ . That is, we end up in a corner solution stating that  $P_A^I$  should be set as high as possible, i.e., a similar outcome as in Ewerhart and Fieseler (2003) and Athey and Levin (2001). The only restriction is through the bid restriction and that we do not allow for negative prices, i.e., what limits  $P_A^I$  is that  $F_B^I$  must be set to zero. When  $\alpha < A\beta / B$ , the opposite applies; we will end up in a corner solution only limited by that  $F_A^I$  may not be lower than zero. It is only when  $\alpha$  exactly equals  $A\beta / B$  that a corner solution is avoided. In that case the informed agent will bid his marginal costs.

## CONCLUSIONS

Most papers on unbalanced bidding conclude that risk neutral contractors will submit zero unit price bids on overestimated quantities. When making risk aversion endogenous, the model presented in this paper comes up with an internal solution. This result is more in line with the available data.

Apart from risk aversion, the inconsistency between theory and data can also be explained by a reputation mechanism or a ban on zero pricing. The institutional setting of the Swedish construction market gives little or no support to the latter explanations, leaving risk aversion as a prime candidate to explain the inconsistency. This entails that the explanatory value of a model for unbalanced bidding is improved by including the contractor's perception of risk.

The model captures the intuitively appealing characteristics that the contractor's risk perception may affect the extent of unbalanced bidding. It shows that unbalanced bidding increases the expected net payment from the contract. However, it also increases the contractor's risk exposure.

This is captured in the above model through a variable about an informed contractor's level of risk aversion. In line with earlier models in the literature, the model predicts that a risk neutral bidder will maximize skew bidding by submitting zero prices on the overestimated quantities. The model shows that the incentives for such behaviour are very strong. But by allowing the contractors to be risk averse, the model predicts an internal solution without zero pricing which is more in line with the data available.

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