Dynamic Analysis of an Open Piled Jetty
Subjected to Wave Loading

Master Thesis

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Preface

This report contains the results of a dynamic analysis of the first cruise jetty of Sint Maarten, when being subjected to wave loading. This research is part of a master thesis of the study Civil Engineering at the Delft University of Technology.

The research is accommodated by the TU Delft and Lievense Consulting Engineers. I would like to thank me graduation committee Prof.ir.A.C.W.M Vrouwenvelder, Dr.ir. J. van der Tempel, Prof.dr.A.Metrikine, ir.L.J.M.Houben, and ing.H.A.J. van den Elsen for their ideas, comments and time. Especially I would like to thank Prof.ir.A.C.W.M Vrouwenvelder for his support and detailed reviewing.

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This report consists out of 11 chapters. The first chapter describes the scope of the project. Chapter 2 until 8 present the modelled situation of the cruise jetty of Sint Maarten. Starting with the input of the dynamic model (chapter 3 and 4), describing the model (chapter 5), and presenting the output and interpretation (chapter 6, 7 and 8). Chapter 9 mentions the application of the findings for a new open piled jetty design. The last two chapters present the conclusions and recommendations of this research.

It should be noted that the content of this report does not reflect the official opinion of Delft University of Technology nor of Lievense Consulting Engineers. The report also does not show the opinion of the individual graduation committee members. This research is merely a student master thesis.

Den Haag, April 2013

Vroukje Bron
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Summary

At different locations in the Caribbean cruise jetties have been damaged during hurricanes. The specific cause of the damage in 2010 to the first jetty of Sint Maarten is not known. A hurricane with less high waves caused significantly more damage, than a hurricane with higher waves. This raised the question whether the dynamic behaviour of the jetty enlarges the stresses in the structure.

This research has the goal to elaborate circumstances under which the dynamic behaviour of the open piled jetty on Sint Maarten significantly enlarges the amplitude of vibration, when being subjected to wave loading. Another goal is to elaborate design components that have a negative influence on the dynamic behaviour and wave loading of an open piled jetty structure.

This is investigated by simulating the water surface and corresponding wave loading in the time domain and computing the dynamic response for the first jetty of Sint Maarten for different wave climates. The random sea is simulated from a wave spectrum. From this the pressure on the jetty deck is computed when the water surface is higher than the bottom of the jetty deck.

Wave loading on the jetty deck and piles is included in three directions in this research. The largest wave loads on the jetty occur when a wave hits the bottom of the jetty deck, causing a large vertical peak pressure. Many theories exist about this wave-in-deck loading. These researches mainly focus on the magnitude of the maximum load. For the dynamic analysis a formulation depending on time and location is required. The wave-in-deck loading used in this research is a simplistic empirical formulation depending on the undisturbed water surface elevation and undisturbed water particle velocities. The formulation is based on a physical scale model test performed for the situation of Sint Maarten.

The dynamic analysis is performed by modal analysis, making use of a finite element package. The model includes 3 degrees of freedom, and linear elastic behaviour is assumed. Computations are performed in the time domain, without non-linear interaction between the travelling waves and jetty structure. The response of the structure to the wave loading is analysed by looking at the variance spectrum of the displacement, dynamic amplification and displacement signal.

From the results it can be concluded that the dynamic behaviour of the jetty of Sint Maarten significantly enlarges the amplitude of vibration in horizontal direction for the investigated wave spectra. In vertical direction the maximum displacement decreases due to the dynamic behaviour. It is not possible to draw conclusions about wave spectra which were not considered in this research, because the dynamic reaction also depends on the shape of the wave load, approach angle of the wave and wave celerity. The largest wave spectrum investigated in this research causing a dynamic enlargement of the amplitude of vibration in horizontal direction has a significant wave height of 5.0 m. This indicates that wave spectra including high waves can cause a significant dynamic enlargement of the amplitude of vibration in horizontal direction.

For further research verification of the dynamic behaviour of an open piled jetty and improvement of the wave-in-deck pressure varying in place and time is recommended.
1. Introduction

1.1. Relevance of Research

All over the world jetties are being built, also in hurricane areas and in areas where they are exposed to ocean waves. This leads to high wave loads which sometimes cause damage to the jetty. Compared to offshore structures and industrial jetties it is not possible to place the deck of a mooring jetty far above the sea level, because the height of the jetty deck depends on the ships mooring to the jetty. This makes avoiding of waves hitting the bottom of the jetty deck (wave slamming) impossible.

Although open piled jetties are built all over the world, there are uncertainties in the design. One of them is the importance of a dynamic analysis for the design of a jetty. Currently jetty designs are based on static calculations, using a maximum wave load and mooring loads as load cases. Dynamic analysis are rare, because they are time consuming and expensive. Another uncertainty in the design of an exposed jetty is the wave loading. Research about wave slamming has lead to several theories. However no consensus about the wave-in-deck loading is reached. This leaves the designer with two large uncertainties.

Direct Motivation

The direct motivation for this research is the damage to a cruise jetty on Sint Maarten which occurred in 2010. The jetty was statically designed, and has been damaged during a hurricane. The mechanism and load case that have caused part of the jetty to fail are unknown. The damage did not occur during the largest waves that hit the jetty. Therefore it is thought that a dynamic effect might have caused the damage.

1.2. Goal

In this research a dynamic analysis is made of the open piled jetty on Sint Maarten. The goal is:

To identify circumstances for which the dynamic behaviour of the jetty of Sint Maarten significantly enlarges the amplitude of vibration when being subjected to wave loading, and to elaborate components that influence the dynamic behaviour and wave loading of open piled jetty structures.

Definition

Enlargement of the amplitude of vibration can be caused by the dynamic behaviour of the structure. Structures have shapes in which they vibrate at natural frequencies. When a load has a frequency close to one of these natural frequencies, and the shape of the load corresponds to the related mode shape, than the amplitude of vibration of the structure enlarges by its dynamic behaviour (Spijkers, et al., 2006).

The ratio between the dynamically calculated maximum displacement and the quasi static displacement is the dynamic amplification factor (DAF). This ratio is used as indicator for the degree of influence of the dynamic behaviour on the maximum displacement of the structure. A larger displacement also causes larger stresses, therefore the DAF also relates to the enlargement of the
stresses in the structure. A significant enlargement is defined as the dynamic amplification factor being larger than 2.

Scope of Research
This research focuses on the dynamic behaviour of open piled jetties exposed to wave loading. Other types of loading like mooring loads or earthquake loads are not considered. The open piled jetty platforms have a closed jetty deck, and are restricted in the distance between the mean sea level and the deck height. This makes avoiding of wave slamming impossible.

In this research a dynamic analysis is made of the first open piled jetty of Sint Maarten. From the findings of this dynamic analysis of the jetty of Sint Maarten, recommendations are extracted for general open piled jetty structures subjected by wave loading. The design of the first jetty of Sint Maarten is not checked in this research.

1.3. Method
The dynamic behaviour depends on the load frequency and shape. In this research the load is caused by the waves hitting the jetty deck and piles. The frequency of the load does not have to be equal to the frequency of the wave. The propagation speed and angle also influence the load frequency. It is therefore not sufficient to compare the wave frequencies to the natural frequencies of the jetty, in order to conclude whether the amplitude of the vibrations is enlarged by the dynamic behaviour. Therefore part of the jetty is modelled and the waves are simulated, to get a more reliable indication of the influence of the dynamic behaviour.

This analysis can be divided in several steps.

- Wave Climate
- Wave Loading
- Dynamic Calculations
- Sensitivity Analysis

1.4. Flowchart
For the first three steps (Wave Climate, Wave loading, Dynamic Calculation) a flow chart is made, which is shown in Figure 1. The dynamic analysis of the jetty of Sint Maarten starts with a wave spectrum, describing the waves near the jetty. After elaboration of the wave loading and performing dynamic calculations the response of the jetty is known. However from a displacement signal it is hard to see which frequencies are included in the response. In order to see in which shapes the jetty vibrates a variance spectrum is made of the displacements. This makes the dynamic analysis of Sint Maarten to start with a wave spectrum and end with a variance spectrum of the response.

Time Domain
To get from the wave spectrum to the variance spectrum two routes can be taken. The steps can be made in the time domain or in the frequency domain. Spectra are in the frequency domain, this means that their variable is the frequency. The quickest route is therefore by staying in the frequency domain for every step. However, this shortcut is only suitable after linearization of the wave loading, because the wave loading is non-linear. The second option is changing the wave spectrum into a
wave train and performing the calculations in the time domain. This second option is chosen. A disadvantage of this option is that it is more time consuming.

Flow Chart
In the flow chart the parts Wave Climate (Chapter 3), Wave loading (Chapter 4), Dynamic Calculation (Chapter 5 and 6) are schematized in five steps. The five grey blocks in the middle represent the large calculation steps.

1. First the significant wave height and wave period are translated to a wave spectrum. This wave spectrum is in the frequency domain.
2. From the wave spectrum a wave train is made, which is a function of time.
3. With this wave train the water particle velocities are determined and the wave loading on the deck and the piles is calculated.
4. The dynamic calculation starts by determining the natural frequencies and eigenvectors. This is done using the commercial software package called Scia Engineer. After that the dynamic analysis is performed by applying modal analysis. This results in the response of the structure as a function of time. In order to see which frequencies are dominant in this response a variance spectrum is made of the displacements.
5. With a structural calculation the stresses caused by the displacements are determined.

Figure 1 - Flow Chart
1.5. Bookmark

The first step in making the dynamic analysis is investigation of the wave climate near the jetty of Sint Maarten. This is done in the next chapter, chapter 2. Also an introduction on the jetty of Sint Maarten is given in this chapter. The second step is describing the wave climate with a wave spectrum, and simulation the water surface near the jetty. This is performed in chapter 3. The waves cause wave loads on the structure, in chapter 4 these loads are determined. After that the dynamic model is described in chapter 5. The results of the dynamic calculation are presented in chapter 6. A sensitivity analysis of these results is performed in chapter 7, to see how large the influence of model choices and input are on the results. In chapter 8 the results are compared to the observed damage at the jetty on Sint Maarten. From the findings of this dynamic analysis of the jetty of Sint Maarten, recommendations are extracted for general open piled jetty structures. These are presented in chapter 9. Chapter 10 summarizes the conclusions of this research, and chapter 11 the recommendations.
2. Sint Maarten and Jetty

2.1. Global Overview

Sint Maarten is situated in the north east of the Caribbean, as can be seen at Figure 2. It is a small island, located in the so called ‘hurricane belt’, as can be seen in Figure 2. The lines in the figure show the tracks of the hurricanes. Most of the storms start in the lower right of the figure, near the coast of Africa.

At the south side of the island a bay with two large cruise jetties is located. The oldest of the two jetties was built in 1999. The jetty is build for large cruise ships that come to visit Sint Maarten. This first jetty has had several problems since the beginning of construction. The jetty has been hit by multiple hurricanes, and some of them have caused severe damage to the jetty. This research focuses on the first (the oldest) jetty in the Great Bay of Sint Maarten.

2.2. Jetty

The jetty is 560 m long and the concrete deck is 20 m wide. The deck slabs are supported by beams, build on steel piles. The piles have a diameter of 914 mm and are placed in a grid of 6.0 m by 6.25 m. The jetty consists out of four parts, separated by expansion joints. The joints can transmit forces in vertical and horizontal direction (perpendicular to the jetty axis). In axial direction a spacing of 0.05 m is present. The distance between mean water level and the bottom of the jetty deck is 1.6 meter. During hurricanes this distance becomes even smaller, down to 1.1 m; therefore waves can easily hit the jetty deck. The jetty is shown in Figure 3. The geometry is described in more detail in appendix II.
The deck consists out of prefab slabs, with in situ concrete on top. The prefab slabs are placed on prefab beams. The beams are connected to the deck by reinforcement which goes into the in situ concrete, and into the piles. A concrete pile plug of 6 meter is located in the upper part of the pile. Between the prefab beam and the prefab slab, no direct connection is made. Figure 4 shows a schematization of the structure of the jetty and the phases in which it is constructed.

1. Placing prefab beam
2. Placing prefab slab
3. Pouring concrete in piles and holes prefab beams
4. Pouring concrete onto slabs (only at drawn position, see arrow)

Figure 4 - Schematization of the building sequences of the Jetty, drawing (Ballast Nedam, 1999)

Figure 5 shows how the jetty is constructed, and the different elements of the structure. The picture is taken during construction of the second jetty; the structure of the first (considered) jetty is the same. Only the dimensions are different, especially the thickness of the jetty deck is less for the first jetty.

Figure 5 - Left: Construction of Second Jetty on Sint Maarten (Lievense, 2005), Right: Construction of Jetty
2.3. Damage

The first jetty has been damaged multiple times. The registered occasions are listed below. This section chronologically describes these events.

Time line:

- **1999** Construction of First Jetty
  - Cracks in bottom of prefab slabs
- **1999** Hurricane Lenny
  - $Hs$: 5.2 m
  - Large damage, during construction
- **2000** Jetty in Operation
- **2008** Hurricane Omar
  - $Hs$: 5.9 m
  - Hardly any damage
- **2010** Hurricane Earl
  - $Hs < 6$ m
  - Large damage

The first jetty is designed for a significant wave height of 6.25 m, which corresponds to a return period of 100 years (WL Delft Hydraulics, 1998). The hurricanes that stroke the Great Bay have a smaller wave height: Lenny $Hs$: 5.2 m, Omar $Hs$: 5.9 m, Earl: less than 6 m. The wave loading is related to the wave height, a smaller wave is therefore not expected to cause damage. It has to be noted that no report of routine inspections are found. (Lievense, 2008) (Ballast Nedam Caribbean, Alkyon, 2000).

During construction the thickness of the prefab plates in the deck has been changed, because cracks occurred in the bottom of the prefab slab. It is not known how many of the slabs have been adapted.

**Hurricane Lenny**

During construction hurricane Lenny struck. The jetty was build from the abutment to the head of the jetty. The front part of the construction had less strength and these beams and slabs were soon washed away. Later, during the more rough part of the hurricane, also a slab in the finished part of the deck was pushed out of the deck as can be seen in Figure 6 (Arcadis, 2000). This deck part had an expansion joint at one side. The reinforcement partly broke off or was ripped out. Over the total deck cracks were visible and the sliding planes in the expansion joints were gone (St. Maarten Harbour Holding Company and Lievense, 2003).

![Figure 6 - Part of the Deck Pushed Out by Lenny, between grid 44 (expansion joint) and 45. (Arcadis, 2000)](image-url)
**Hurricane Omar**

In 2008 a category 3 hurricane called Omar passed Sint Maarten at a distance of 80 kilometres. The hurricane approached Sint Maarten from the south which is very uncommon, and caused large swell waves. The first jetty was hardly damaged. Only two of the four catwalks at the head of the jetty were washed away. (Lievense, 2008)

**Hurricane Earl**

In 2010 hurricane Earl passed Sint Maarten. No hindcast was made of the storm. Over the total length the jetty showed damage, especially at the expansion joints. Two large damage cases are shown below.

The expansion joint at grid 44 has the most visible damage, as can be seen in Figure 7. The concrete is partly gone, just as two of the 10 shear keys. The North side of the deck is also at this joint higher than the South side of the expansion joint. This is the same expansion joint as was damaged during hurricane Lenny.

![Figure 7 - Damage Expansion Joint, Grid 44, Hurricane Earl (Lievense, 2010)](image)

The second last grid, located about 8 m before the head of the jetty is cracked almost from the top to the bottom of the deck. Part of the concrete deck at the top has come off. Figure 8 shows this crack (Lievense, 2010)

![Figure 8 - Large Crack at Head of Jetty (Left: Picture of Crack. Right: Drawing of Top view of Head with Crack in Red)](image)
Damage as Verification
All three hurricanes caused large waves near the jetty. However, the damage to the jetty was totally different. The type of waves caused by the hurricane depend on the storm its propagation speed, direction, distance between the eye of the storm and the Great Bay and the position of the storm with respect to Sint Maarten. The difference in damage between Omar and Earl could be caused by a dynamic effect. The hurricanes approached Sint Maarten from a different side, causing different types of waves. Whether or not this difference caused a different dynamic behaviour is investigated in this research. The blown out blade during Earl is not expected to be caused by a dynamic effect, this is therefore not suitable for verification of the dynamic model.

Damage at Other Locations
The jetty of Sint Maarten is not the only structure being damaged by large waves. Also jetties at other Caribbean Islands have been damage. (Overbeek et al. 2001). Just as exposed bridges and offshore platforms (Cuomo et al. 2007).
3. Waves and Wave Climate

This chapter discusses the wave climate near Sint Maarten, and how this wave climate is simulated in this research. In order to make the dynamic analysis of Sint Maarten, the wave loading on the jetty has to be determined. The wave loading depends on the water particle velocity and wave heights. Which wave heights can occur near the jetty are determined by the wave climate in the Great Bay. Therefore first the wave climate is determined in this chapter, in section 3.2. After that the wave spectra used for simulation of the wave climate are discussed, in section 3.3. These wave spectra are used as input for the dynamic model. The last part, section 3.4 describes how the water surface elevation is simulated from these wave spectra. Before this chapter about waves starts, an introduction about waves is given and a couple of terms are defined, in section 3.1.

In this chapter the two steps shown in Figure 9 are discussed.

![Flow Chart](image)

**Figure 9 - Part of the Flow Chart from Chapter 1**

3.1. Introduction

**Definitions**

Waves can be defined as the vertical motion of the ocean surface (Holthuijsen, 2007). The wave field at a specific moment in time is a summation of different waves with different heights, lengths direction and velocities. This makes a chaotic looking sea surface. In order to describe this surface, the different waves (present in the summation) must be separated so they can be described. Also the time interval has to be shortened, because only for a short period in time the amplitude of a wave with a certain wave frequency can be assumed to be constant. This period is called a sea state.

The waves discussed in this research are gravity waves. For a wave the wave length $\lambda$, wave period $T$, and wave height $H$ are of importance. Figure 10 shows the properties of a regular wave.
Linear and Non-Linear Waves

The properties of a wave depend on the water depth, how the wave is generated and what happened since generation. This can lead to a linear or a non-linear wave. For linear waves the wave height consists out of two equal parts $A_c$ (distance between crest and the still water level (SWL)) and $A_t$ (distance between the trough and SWL) as shown in Figure 10.

Wave Theories

Different types of waves are described by different wave theories. Linear waves are described by the Airy wave theory, describing the wave as a sine function. Non-linear waves are described by the solitary wave theory, Cnoidal wave theory and a higher order of Stokes theories. Different indicators can be used to determine which type is most applicable to the wave. Part of the swell and wind waves occurring during passage of storms are indicated to best be described with second order Stokes waves (DNV, 2010). This means that the waves are non-linear.

Significant Wave Height

In the next section about the wave climate the wave height over a certain period is described with the significant wave height. This is the average of the one third highest waves that occurs during a period of observation. The wave period corresponding to the significant wave height, is the dominant wave period. This description of a wave climate during a sea state does not contain information about the rest of the wave frequencies and how the wave heights are distributed over these frequencies.

Wave Spectrum

A wave spectrum contains information about the distribution of the wave energy over the different wave frequencies in a sea state. From the wave spectrum it can be seen which wave frequencies are dominant in that wave climate.

Approach

For the dynamic analysis the water surface elevation at a certain point varying in time is needed. This formulation can be one of the wave theories describing a wave with a permanent wave height, length and period. However in reality the water surface elevation is a combination of different waves, with different wave lengths, heights and periods. The wave tops therefore do not occur with a constant time interval. This is very important for the wave loading on the deck, and influences the
Dynamic behaviour. When a regular wave is used to simulate the water surface this will lead to a load with a constant period. This might cause a much larger dynamic effect, than an irregular water surface. Therefore the water surface is described by a summation of waves, based on a wave spectrum, to get an irregular water surface. This approach requires a wave spectrum of the wave climate in the Great Bay. The next section describes this wave climate.

3.2. Wave Climate Sint Maarten

First the general wave climate in the Bay of the jetty is discussed, than the wave climate during passage of a hurricane is described. In the last part a conclusion is made about which waves are expected to occur near the jetty.

General Wave Climate
The jetty is located in the Great Bay, this is at the south side of the island, as can be seen in Figure 11. The dominant wind direction is North East because of the Coriolis force (Stewart, 2008). Due to this location at the South side of the island, the Great Bay is by nature protected against wind waves generated by the Trade Wind coming from the North East. This leads to a calm wave climate in the Great Bay for the most part of the year. About 97% of the time the significant wave height is less than 1.5 m (Alkyon, 1997).

About once a month swell waves come in to the Great Bay. These waves are generated at a distance from Sint Maarten by a storm. The swell waves travel over a long distance after being generated. These waves have a long wave length. The waves that travel into the Great Bay once a month have a wave height of not more than half a meter.

A small part of the year, the wave climate in Great Bay is not calm at all. In that situation a large storm or even hurricane passes close by Sint Maarten. High wind speeds than cause damage on the whole island and large wind waves in the Great Bay.
Water Depth Limitation
The maximum wave height in the Great Bay is limited by the water depth. The Great Bay is less deep than the waters offshore. The water depth decreases when approaching the Bay. This decrease in water depth causes a wave length decrease, wave celerity decrease and a wave height increase. This makes the waves steeper, which causes the higher waves to break and lose their energy. These high waves are therefore not present in the Bay. During the lifetime of the jetty, the water depth near the jetty has been increased, to facilitate larger ships.

Hurricane Wave Climate
Using the information about storms and waves a theoretical description can be made of the passage of a hurricane. It must be noted that the extreme wave climate in the Great Bay depends on many factors, like the storm its propagation speed, direction, distance between the eye of the storm and the Great Bay and the position of the storm with respect to Sint Maarten.

Swell and Wind Waves
Storms passing near Sint Maarten or storms passing at the South of Sint Maarten are expected to have an influence on the wave climate in the Great Bay. Two different types of waves are distinguished: swell waves and wind waves. Swell waves are generated outside of the Great Bay but travel over a long distance, sometimes into the Great Bay. Wind waves are generated by local wind in the Great Bay. Swell waves are long, with a long wave period and a narrow spreading in direction and frequency (narrow band). Because the propagation speed of a wave depends on its wave length and therefore frequency, the waves with a similar frequency enter the Bay at the same time. A storm passing Sint Maarten close by, generates wind waves in the Great Bay. These waves are influenced by the wind speed. The wind is turbulent, this causes small waves. Interaction between the waves leads to larger waves that travel faster. They have a shorter wave period and length than the swell waves and their direction spread is larger.

During Passage of a Hurricane
A storm can cause rough weather for several days. How many days the storm influences the wave climate depends on the storm its location, direction, and propagating speed. Hurricane Lenny caused high waves during 3 days and Hurricane Omar for 2. The wave climate differs during these days. A description of a storm passing near Sint Maarten, can be divided into four parts.

1. During the approach of a storm the first waves that arrive in the Great Bay will be swell waves, assuming the wind of the storm does not yet reach Sint Maarten. The wave period in the Great Bay will go up, because longer waves travel faster and over a longer distance.

2. Secondly the storm approaches closer to Sint Maarten. This causes the wind near Sint Maarten to change direction. The wind speed near Sint Maarten can still be small, like seen in the first days of the hindcast of Lenny and Omar. Small wind waves will now also be present in the Great Bay (if the wind is coming from the South) together with swell waves.

3. When the storm is really close to Sint Maarten, the wind speed will go up and large wind waves will be generated. Swell waves are not present, if the storm is very close. Another effect of a storm nearby is a lower atmospheric pressure. This causes a storm surge that raises the water level. The wind can also set up the water level. This can be seen in Figure 12 on the East side of Sint
Maarten. The dark blue colour circle West of Sint Maarten is caused by the eye of the storm (storm surge). This increase in water level makes a larger wave height possible, because the water depth in the Great Bay is the limitation to the wave height. This situation generates the largest waves in the Great Bay.

Figure 12 - Wind Setup and Storm Surge during Lenny, near Sint Maarten (Alkyon, 2000)

4. The storm travels away from Sint Maarten again. The water level will go down, the wind speed drops and the wind waves decrease in height. For Omar only a 1 m high wave remained, because the storm travelled with high speed towards the North East. Depending on the location of the storm, swell waves could reach the Great Bay.

These different events during the passing of a hurricane cause a wide range of wave frequencies and wave heights near the jetty. Also a combination of the wave types is possible, making a theoretical double peaked wave spectrum possible.

**Wave Climate Studies**

In order to get a more detailed description of the wave climate in the Great Bay, the wave climate studies and hindcast are used. Several wave climate studies have been done, for the design and feasibility study of the jetty in Great Bay. In chronological order:

- **1991** Delft Hydraulics studied the offshore wave conditions;
- **1997** Alkyon made a report about the normal and extreme wave conditions;
- **2005** Alkyon studied wave climate and environmental effects of the planned second jetty and other changes to the Great Bay.

The studies are based on measurements offshore. No buoys are located in the Great Bay. The offshore data is translated with a software package to the wave climate in the Great Bay. The waves with a return period of 100 years are presented in Table 1, also the information of two hindcasts of hurricanes are included.
Table 1 - Information about the Wave Climate in the Great Bay from Wave Climate Studies and Hindcasts

<table>
<thead>
<tr>
<th></th>
<th>$H_s$ [m]</th>
<th>$T_p$ [s]</th>
<th>Water Depth [m]</th>
<th>Direction [Degree north]</th>
<th>Return Period [year]</th>
</tr>
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<tbody>
<tr>
<td>1991</td>
<td>6.25</td>
<td>13.2</td>
<td>10.3</td>
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<td>1997</td>
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<td>10.3</td>
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<td>10</td>
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<tr>
<td>Hurricane Omar</td>
<td>5.9</td>
<td>10.2</td>
<td>12</td>
<td>200</td>
<td>100*</td>
</tr>
</tbody>
</table>

*The wave climate studies changed their estimation of the 100 year return period wave climate during the years. Also the range of wave heights which is expected to have a return period of 100 years, is very wide. This causes both hurricanes to have the same return period, despite of their different wave height.

Table 1 shows only two hurricanes. In the past years many more storms have past and about 6 hurricanes. Only the hurricanes from which information about the wave height is known are presented in the table.

The significant wave heights and dominant wave frequencies mentioned in the wave climate studies and hindcasts are transformed into wave spectra to see in which frequency range the waves occur. From theory it is expected that swell wave have a lower wave frequency than wind waves. The wave data are plotted in one graph with different colours for swell (red) and wind waves (blue) to see how large this difference is in the Great Bay. The result is shown in Figure 13. The largest wave spectra have a return period of 100 years.

![Figure 13 - Spectrum of Registered Wave in the Great Bay](image-url)
The legend of Figure 13 is:

- Black are extreme* waves, Red are swell waves. Blue are wind waves.
- Dotted lines show waves out of the wave climate study of Alkyon in 2005.
- Dashed lines show waves out of the wave climate study of Alkyon in 1997.
- The dash-dot lines shown wave registered during hurricane Omar.
- The continuous lines shown the wave spectra registered during hurricane Lenny.

*Extreme waves are called extreme, because there is no specific distinction made between wind and swell waves in the reports, and the extreme waves have a large return period or occur during the most extreme part of a storm.

It can be seen from Figure 13 that the wind waves are only present in the higher frequencies. The swell (red) waves are present over the whole graph. No clear difference in wave frequency shows from the graph between wind, swell, and extreme waves. From the graph it can be seen that a lot of different waves are possible over a wide frequency range. How the wave spectra are determined is described in section 3.3.

All Waves are Possible
From the information about Sint Maarten it is seen that many different wave frequencies and heights are possible. The bigger the distance between Sint Maarten and the storm, the longer the wave period and the smaller the wave height of a swell wave. The distance and intensity of storms differs strongly, which makes all different kind of swell waves possible. Also all wind speeds are possible; this makes many types of wind waves possible in the Great Bay. From the wave climate studies and hindcasts the significant wave height is estimated to be between the 0 and 7 meter, and the wave period varies between the 0 and 13 seconds. In the next section wave spectra are made based on these findings.

3.3. Wave Spectrum
With the wave climate determined in the previous section the wave spectra can be made. The water surface elevation is described by a summation of waves based on the wave spectrum. Therefore first the wave spectrum is determined.

The spectrum or significant wave height of a sea state can be derived from a set of data of an observation from the sea surface. However for the situation of Sint Maarten no such data is available near the jetty. Only the significant wave height and wave period are known. Therefore a spectrum is taken from literature and scaled to the significant wave height and period of Sint Maarten.

Hurricane Wave Spectra
The wave spectra describing large wave heights occur during hurricanes. For the wave climate during hurricanes no separate wave spectrum has been found in literature. Graphs of hurricane wave spectra show different kinds of shapes. A two peaked spectrum, a one peaked spectrum near the eye and a spectrum that looks like not having a dominant frequency (U.S. Army Corps of Engineers, 1984). No conclusions can be drawn from these wave spectra during hurricanes. Therefore general wave spectra are also assumed for the waves during hurricanes as well.
**JONSWAP and PM-spectrum**

In literature different wave spectra are mentioned. Common used wave spectra are the Pierson-Moskowitz spectrum (PM-spectrum), JONSWAP and TMA spectrum.

The PM-spectrum may be applied for sea states in deep water, with a zero mean and where not to steep waves pass. The PM-spectrum describes fully developed waves. For ‘younger’ waves the JONSWAP (Hasselmann, 1973) spectrum is more applicable. The waves described by JONSWAP have a smaller fetch length. This spectrum adds a part to the PM-spectrum which includes the fetch length. The peak of the spectrum is narrower than the peak of the PM-spectrum. For wind waves, which are expected to have a small fetch length during hurricanes, JONSWAP is expected to be more accurate. For shallow water the sea bottom will influence the wave spectrum. This can be taken into account with the TMA factor, which can be added to the JONSWAP spectrum. This factor changes the magnitude of the JONSWAP spectrum for the higher frequency. Most of the waves near the jetty act as being in intermediate water depth; the TMA factor is therefore not included.

It is chosen to use the JONSWAP spectrum for wind waves. For swell waves the PM-spectrum is used. Equation 3.1 shows the formulation of the JONSWAP spectrum. The shape of a JONSWAP spectrum can be seen in Figure 13 on page 22.

\[
S_{yy}(\omega) = \alpha_s \omega^2 e^{-5 \omega \gamma^s} \quad [m^2 s] \\
\alpha_s = \text{shape factor} \\
\omega = \text{angular frequency} \\
\gamma^s = \text{JONSWAP extension} \\
\]

The factor \(\alpha_s\) is the shape factor and \(\omega_0\) is the dominant wave frequency. These two parameters determine the shape of the spectrum, and are the only unknown in equation 3.1.

**Shape Factor of Wave Spectrum**

The shape factor and dominant wave frequency can be determined by the significant wave height and period, or by the fetch length and wind speed (CIRIA et al, 2007). The last method has as an advantage that no assumption about the probability density function of the wave height is required. However, the fetch length and wind speed are uncertain. On Sint Maarten the wind speed has been measured during hurricanes at the airport, but the fetch length is unknown. An assumption about the fetch length is not preferred, because the fetch length is very determining for the shape of the wave spectrum. Therefore the significant wave height and period are used to determine the shape factor and dominant frequency of the wave spectrum.

The dominant wave frequency is determined by the wave period. The shape factor can be found with the formula of equation 3.2 (Lecture Notes Random Vibrations, 2010).

\[
\omega_0^2 = \frac{4g \sqrt{\alpha_s}}{\sqrt{5H_s}} \\
\]  

[3.2]

This formula is derived using the assumption of the significant wave height being 4 times the variance; which is a property of the Rayleigh distribution. The amplitude of waves is often described by the probability density function of Rayleigh. This is only valid for waves that are not severely
deformed by shallow water or which have a high wave steepness. The use of a Rayleigh distribution during the passes of a hurricane is questionable according to Goodnight and Russell (1963).

**Sea State of 3 Hours**
The jetty is designed to withstand a 100 year return period storm. The wave spectra used in this research are therefore also based on waves with a return period of 100 years or lower. Storms near Sint Maarten can last for more than 24 hours. Within these 24 hours the wave types vary, therefore the wave climate occurring during one storm cannot be described by one sea state. A sea state is often taken as a period of 3 hours. The heaviest weather and largest waves only occur during a small part of the storm. It is assumed that this extreme period in which the maximum design waves occur is about 3 hours, therefore the extreme period of a storm can be described by one sea state. As sea state is therefore taken as a 3 hour period. In the wave climate reports of Sint Maarten and hindcasts hurricane waves up to a return period of 100 year are mentioned. The data points in the reports have an interval of 3 to 6 hours, which confirms the use of a 3 hour period for one sea state.

**One Directional Sea State**
Waves entering the Great Bay can come from several directions. This direction differs per sea state, and also during a sea state. The duration of one wave direction depends on the distance and propagation speed of the storm. When the storm is really close, the wind direction is expected to change rapidly. This also causes the wind waves to change direction. In other cases the wave direction is expected to be more stable. The time interval for which a wave direction is assumed to be stable in the calculation is during one sea state, a period of 3 hours. The direction of different sea states is varied, but the change of wave direction in a sea state is not included. This results in one directional sea states. In reality the wave direction is expected to change also during a sea state. This causes a distribution of the wave load over different directions. The wave load and frequency depend on the direction of the wave, but no under or over estimation is expected by the assumption of a one directional sea state.

**Approach Angle of Waves**
Swell waves are limited in the direction from which they can travel into the Great Bay. Wind waves can occur from different directions; however large wind waves are generated over a longer fetch length. This has to be outside of the Great Bay, therefore the extreme waves are limited in the direction as well.
The waves that reach the head of the jetty have direction between the 170° N and 280° N. The whole jetty is reached between the 180° N and 270° N. Waves travelling from these directions are not expected to keep their initial approach angle when entering the Great Bay. Refraction occurs due to water depth counters. This causes the waves to change direction, decreasing the angle between the wave and the jetty axis. In the wave climate studies of Sint Maarten the approach angles of waves are simulated using different offshore wave conditions with wave angles of 180° N, 240° N and 300° N. This results in different wave angles near the jetty, as is shown in Table 2. The jetty is orientated at 215° N. The wave directions correspond to an angle $\alpha$ between the 25° and -20°.

<table>
<thead>
<tr>
<th>Sources</th>
<th>Wave Direction near Jetty</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wave Study 1991</td>
<td>215° N</td>
<td>0</td>
</tr>
<tr>
<td>Wave Study 1997</td>
<td>190 - 220° N*</td>
<td>-5 to 25</td>
</tr>
<tr>
<td>Wave Study 2005</td>
<td>190 - 235° N</td>
<td>-20 to 25</td>
</tr>
<tr>
<td>Hindcast Lenny</td>
<td>210° N</td>
<td>5</td>
</tr>
<tr>
<td>Hindcast Omar</td>
<td>206° N</td>
<td>9</td>
</tr>
</tbody>
</table>

*In the wave climate study of Alkyon in 1997 a wider range of directions of waves is mentioned. The wave directions mentioned in Table 13, correspond to a wave climate with a significant wave height larger than 4.0 m.

The approach angle in this research is varied between 0, 10 and 25 degrees. The jetty is symmetrical; it therefore does not matter if the approach angle is positive or negative. Both lead to the same result. Hence only positive angles are used for the calculation. The probabilities of occurrence of the approach angles (0, 10, 25 degrees) are assumed to be equal.

**Maximum Wave Spectrum Frequency**

The JONSWAP and PM-wave spectra do not approach zero for large frequencies. However, the values of the wave spectrum for wave frequencies larger than 2.2 rad/s or 0.35 Hz, are very small and are therefore neglected. Wave train is made with frequency steps of $2\pi/T$ rad/s, with $T$ being the duration of the simulation (100 s).

**Air Gap**

The distance between the mean sea level and the bottom of the jetty deck is called the deck clearance or air gap. The air gap of the first jetty of Sint Maarten is 1.6 m. This has to be reduced during a hurricane with the wind setup, storm surge and tidal movement. This is assumed to be 0.5 m in this research. Figure 14 shows a schematization of these distances. For small wave spectra a smaller distance of 0.3 m is assumed, which is the tidal movement.
Dynamic Analysis of an Open Piled Jetty Subjected to Wave Loading

Simulated Wave Spectra
For a dynamic analysis not only the maximum waves are of importance. The whole frequency spectrum in which waves occur near the jetty is of interest. The dynamic behaviour is determined by the wave frequency as well; hence less high waves can cause larger dynamic responses, if their frequency is close to a natural frequency of the structure. Therefore different types of waves that can occur near the jetty are investigated in this research (also the more common waves).

Because of the wide range of wave frequencies and heights can occur, multiple wave spectra are used for the simulation of the waves during the dynamic analysis. These wave spectra are:

- A wave spectrum with the maximum wave height is used;
- A wave spectrum of a large wave, with a high dominant wave frequency;
- A wave spectrum with a very high wave frequency, that just hits the jetty deck;
- A wave spectrum near the lowest natural frequency;
- A wave spectrum with a dominant frequency equal to the lowest natural frequency.

The wave frequencies are much lower than the lowest natural frequency of the system. A larger dynamic reaction is expected, when the wave frequency is closer to a natural frequency. Therefore the higher wave frequencies are investigated. The fourth and fifth wave spectra do not hit the jetty deck. The next paragraphs describe these wave spectra in more detail.

Large Wave Spectrum
The spectrum with the largest area is made by combining the maximum wind and swell wave. This assumes that the maximum wind and swell wave occur simultaneously, as shown by the blue line in Figure 15. The two spectra are added, to get the superimposed spectrum. This assumes the sea and swell wave spectra to be independent of each other (correlation of zero). This may be assumed because the phase angle of every wave in the spectrum is randomly taken from a distribution. Therefore the spectra are independent. They are generated by the same storm, but the swell waves are generated long before the wind waves.
The maximum swell wave is taken from the wave climate study of Alkyon in 2005 (Hs: 7.1 m, Tp: 12.45 s). This wave has a return period of 100 years. The maximum wind wave is found in the hindcast of hurricane Lenny (Hs: 5.2 m Tp: 9.5 s). Lenny has an offshore return period of 1/25 years, but the wave conditions in the Great Bay were categorized with a return period of 100 years in 2000. The water depth in 2000 was less than 10 m near the jetty. This later enlarged to about 12 m. The simulation is done using a water depth of 12 m. The wind setup, storm surge and tidal movement are together set to be 0.5 m. Part of the waves is this wave spectrum are expected to break, due to the water depth.

**Large Wave Heights in High Frequencies**

For the second wave spectrum a wave climate with a much higher wave frequency is chosen, that still has high waves. The wave spectrum has a wave period of 7.0 seconds. It is the largest wave spectrum with a short wave period mentioned in the wave climate reports (Alkyon, 1997). The significant wave height is 5.0 meters. The exceedance probability is in 1997 mentioned to be about 1% per year.

**Wave Spectrum with Waves Just Hitting Jetty Deck**

Because the dynamic behaviour depends on the frequency of the load, the highest wave frequency is chosen of a wave that still hits the jetty deck. A wave spectrum with the significant wave height of 3.0 meters has its largest waves just hitting the jetty deck (if a setup of 0.3 m is assumed). A steep wave is assumed, to get the highest dominant frequency. The dominant wave period is 4.0 seconds.

The wind setup, storm surge and tide are together assumed to be 0.3 m. This is less high than for the previous described wave spectra with larger waves. This is assumed, because smaller waves occur when a storm is on a distance, or when a storm is small. In both situations the wind setup and storm surge are expected to be less high, than when a storm is closer or larger. Large waves described in the first two wave spectra only occur when a storm is large and close to Sint Maarten. This is expected to cause a larger wind setup and storm surge, therefore this is expected to be 0.5 m. The assumptions are based on data from the hindcast and wave climate studies of Sint Maarten.
Wave Spectra near a Natural Frequency
The fourth and the fifth wave spectra are determined by the natural frequency of the system. The fifth wave spectrum has a dominant wave frequency equal to the lowest natural frequency of 5.9 rad/s. The dominant wave period is therefore 1.1 s. This corresponds to a very low wave with a height of maximum 0.35 m. This height is taken as significant wave height. The waves in the wave spectrum do not hit the jetty deck.

The fourth wave spectrum has a dominant wave period of 1.9 seconds. This is in between the other wave periods. The corresponding wave height is 1.0 m, which is taken as the significant wave height. For both wave spectra the wind setup, storm surge and tide are assumed to be add up to 0.3 m.

3.4. Wave Simulation
The water surface elevation is described by a summation of waves. These individual waves in the summation are described by the linear wave theory, even though they are not expected to be linear. This is chosen because the formulation of the non-linear wave theories is more complicated, and these are not expected to have a large influence on the wave loading and therefore the dynamic reaction of the jetty when using a summation of waves as formulation for the water surface elevation in time.

Simulation of Wave Train
This makes the water surface elevation a summation of sine functions. Every sine represents a wave with a different wave height, frequency and phase angle. The water surface elevation is a random process. The phase angle is therefore chosen to be randomly taken from a uniform distribution. The water surface elevation that passes by an arbitrary location can be described as shown with equation 3.3.

\[ \eta(t) = \sum_{k=1}^{N} A_k \sin(\omega_k t + \varphi_k) \quad [m] \]  
\[ A_k = \sqrt{2\Delta \omega_k S_{\eta\eta}(\omega_k)} \quad [m] \]  

\[ \eta(t) = \text{vertical water surface elevation} \quad [m] \]
\[ A_k = \text{amplitude} \quad [m] \]
\[ \omega_k = \text{angular frequency} \quad [\text{rad/s}] \]
\[ \varphi_k = \text{phase angle} \quad [\text{rad}] \]
\[ S_{\eta\eta}(\omega_k) = \text{wave spectrum} \quad [m^2s] \]
\[ \Delta \omega_k = \text{angular frequency difference} \quad [\text{rad/s}] \]
\[ k = 1, 2, 3... \]
\[ t = \text{time} \quad [s] \]

The sine functions all have a different wave frequency \( \omega_k \), amplitude and a random phase angle \( \varphi_k \). The random phase angle is a sample of a uniform distribution between [0,2\pi]. The amplitude of each wave is determined by the area under the wave spectrum at its wave frequency. This is shown in Figure 16. In this way the wave train of one sea state is generated.
Figure 16 - Generation Random Wave Train from a Spectrum (Lecture Notes Random Vibrations, 2010)

The summation of all sine functions $\eta(t)$ is one realization of the wave train under the conditions of observation on which the spectrum is based. When the wave train $\eta(t)$ is generated for the second time another wave train is found. Just as doing a second observation at sea at the same location under the same conditions. Therefore when this method is used to describe the wave in this research, the simulation has to be made of a long period, or has to be repeated for several times to get a reliable result.

**Undisturbed Water Surface**
The simulated waves do not change shape because of differences in water depth along the jetty. This is expected in reality. Also the jetty has no influence on the propagation or shape of the waves. After a wave has hit the jetty, the shape of the wave is assumed to keep its initial shape. This is not expected to be realistic. However, including the change in wave shape after contact with the jetty deck is difficult. Reflection in the Great Bay is elaborated in appendix III. Reflection from the quay walls is included in the significant wave height. Reflection at the abutment is not included.

**Water Surface along the Jetty**
The wave train generated from the wave spectrum gives the vertical water surface elevation at one location. At the other locations near the jetty the water surface elevation is still unknown. To determine the water surface elevation over the total area around the jetty a formulation has to be made, depending on the x and y-coordinate. The coordinate system is defined in Figure 17. The origin of the coordinate system is located at the head of the jetty at the SWL (MSL including wind setup, storm surge and tidal movements).
It is chosen to place the generated wave train at the origin of the coordinate system (0,0,0). The water surface elevation along the rest of the jetty is not yet known, but is related to this point. The propagating direction of the wave is from outside of the Great Bay, towards the jetty. This direction is given by the $\{(\bar{x},\bar{y},\bar{z})\}$ coordinate system, shown in Figure 18. It is assumed that the wave approaches the jetty under an angle $\alpha$. 

![Figure 17 - Top and Side View Jetty Sint Maarten, introduction Coordinate System (Lieverse, 2010)](image)

**Figure 18 - Simulation Water Surface Elevation along the Jetty**
The water height in \( \overline{y} \) (perpendicular to the direction of the wave its propagation) is assumed to be equal to the elevation on the \( \overline{x} \) line. This is shown in Figure 18 by the blue dotted lines in the drawing (which have the same water height). This assumption makes the water height at \( \overline{x} = 0 \) m known. The wave height at 1 meter from the origin in (\( \overline{x} = 1 \) m) is dependent on the wave number of the waves. For the water surface elevation along \( \overline{x} \) direction, an assumption about the propagation velocity is needed to create a travelling wave. The celerity of a wave depends on the wave length. It is therefore expected that the propagation velocity differs between the different waves in the wave train (sine functions in the summation). The linear wave theory has the following dependence of the propagation of a wave with the wave number \( (k) \) on the wave frequency:

\[
k = \frac{\omega^2}{g}
\]  
[3.5]

This only holds for deep water waves. Not all waves in the wave train are expected to be deep water waves. When the propagation of the wave over \( \overline{x} \) is included this results in the following formulation for the water surface elevation along the jetty shown in equation 3.6. 3.7. Equation 3.7 also show the formulation transformed from the local \( \overline{x}, \overline{y} \) coordinate system to the global \( x,y \) coordinate system.

\[
\eta(t, \overline{x}) = \sum A_k \sin(\omega t + \phi_k - \frac{\omega_k^2}{g} \overline{x}) \quad \text{[m]} \quad [3.6]
\]
\[
\eta(t, x, y) = \sum A_k \sin(\omega t + \phi_k - \frac{\omega_k^2}{g} \cos(\alpha)x - \frac{\omega_k^2}{g} \sin(\alpha)y) \quad \text{[m]} \quad [3.7]
\]

**Water Particle Velocity and Acceleration**

The wave load depends on the water particle velocity. For a single wave the water particle velocity is given by the linear wave theory. For a wave as shown in equation 3.8, the corresponding water particle velocities are given by 3.9; the horizontal water particle velocity and 3.10 the vertical.

\[
\eta = \hat{\eta} \sin(\omega t - kx)
\]  
[3.8]
\[
v_x = \hat{\eta} \omega \frac{\cosh(kz + kd)}{\sinh(kd)} \sin(\omega t - kx)
\]  
[3.9]
\[
v_z = \hat{\eta} \omega \frac{\sinh(kz + kd)}{\sinh(kd)} \cos(\omega t - kx)
\]  
[3.10]

\[
v_x = \quad \text{Water particle velocity in x-direction} \quad \text{[m/s]}
\]
\[
v_z = \quad \text{Water particle velocity in z-direction} \quad \text{[m/s]}
\]
\[
z = \quad \text{Vertical coordinate} \quad \text{[m]}
\]
\[
d = \quad \text{Water depth} \quad \text{[m]}
\]

When this linear theory is applied to the wave train this results in equation 3.11, 3.12 and 3.12. With the transformation from the local to the global coordinate system, the horizontal water particle
velocity gets a x and a y part, because the water particle velocity in the propagation direction of the wave consists out of two components.

\[ v_y = \sin(\alpha) \sum_{k=1}^{n} A_k \omega_k \frac{\cosh(k_z + k_x d)}{\sinh(k_x d)} \sin\left(\omega t + \varphi_k - k_x \cos(\alpha)x - k_z \sin(\alpha) y\right) \]  \[3.11\]

\[ v_x = \cos(\alpha) \sum_{k=1}^{n} A_k \omega_k \frac{\cosh(k_z + k_x d)}{\sinh(k_x d)} \sinh\left(\omega t + \varphi_k - k_x \cos(\alpha)x - k_z \sin(\alpha) y\right) \]  \[3.12\]

\[ v_z = \sum_{k=1}^{n} A_k \omega_k \frac{\sinh(k_z + k_x d)}{\sinh(k_x d)} \cos\left(\omega t + \varphi_k - k_x \cos(\alpha)x - k_z \sin(\alpha) y\right) \]  \[3.13\]

The water particle accelerations are also used in the wave loading formulations. These are shown in equation 3.14, 3.15 and 3.16.

\[ \ddot{v}_y = \sin(\alpha) \sum_{k=1}^{n} A_k \omega_k^2 \frac{\cosh(k_z + k_x d)}{\sinh(k_x d)} \cos\left(\omega t + \varphi_k - k_x \cos(\alpha)x - k_z \sin(\alpha) y\right) \]  \[3.14\]

\[ \ddot{v}_x = \cos(\alpha) \sum_{k=1}^{n} A_k \omega_k^2 \frac{\cosh(k_z + k_x d)}{\sinh(k_x d)} \cos\left(\omega t + \varphi_k - k_x \cos(\alpha)x - k_z \sin(\alpha) y\right) \]  \[3.15\]

\[ \ddot{v}_z = -\sum_{k=1}^{n} A_k \omega_k^2 \frac{\sinh(k_z + k_x d)}{\sinh(k_x d)} \sin\left(\omega t + \varphi_k - k_x \cos(\alpha)x - k_z \sin(\alpha) y\right) \]  \[3.16\]

\[ \ddot{v}_x = \frac{\partial v_x}{\partial t} \quad \text{[m/s}^2\text{]} \]

\[ \ddot{v}_y = \frac{\partial v_y}{\partial t} \quad \text{[m/s}^2\text{]} \]

This water particle velocity and acceleration described in the previous equations are determined for undisturbed water. In these formulations it is assumed that the water particles after hitting the jetty deck do not change their direction because of the contact. The influence of the jetty structure on the wave is not included. The wave is assumed to keep its initial shape, velocity and acceleration. In reality this is not expected. In the situation of a wave hitting the bottom of the jetty deck, the structure prevents the water particles to flow upward. Preventing the next water particles to takes its place. This causes water to from underneath the jetty deck to squirt out on both sides of the width of the jetty.

**Vertical Stretching**

The linear wave theory is derived for small waves. The formulation for the water particle velocity is therefore only valid between the sea bottom and the waves equilibrium (\(d < z < 0\) m). The water particle velocity in the crest of the wave (\(0\) m < z) cannot be determined by substituting the z-coordinate in the equation. Substituting the location of the jetty deck (\(z = 1.1\) m) in the linear wave theory gives an unrealistic large water particle velocity. Therefore vertical stretching is used, the water particle velocity at \(z = 0\) m is used for the wave crest as well. This is also used at the location of the jetty deck, where the water particle velocity is needed to determine the wave loading on the deck.
**Scaling Factor**

The water particle velocity and acceleration in horizontal direction both contain the scaling factor shown in equation 3.17.

\[
\frac{\cosh(k_z + k_d d)}{\sinh(k_d d)}
\]  

[3.17]

This factor includes the oscillation of the water particles below the water surface to the formulation. The linear wave theory is derived with the scaling factor being one at the water surface (Holthuijsen, 2007). For the largest wave climate investigated in this research the scaling factor is larger than one. The type of waves in this wave spectrum do not comply with the restriction of application of the linear wave theory. This overestimation is resolved for the wave loading on the jetty deck by setting the scaling factor equal to one at the water surface (at \( z = 0 \) m). For the wave loading on the piles the scaling factor is included and is larger than one at \( z = 0 \) m for the largest wave spectrum. This causes an overestimation of the wave loading on the piles for this large wave spectrum.
3.5. Summary Wave and Wave Climate

The wave climate in the Great Bay of Sint Maarten is calm for the most part of the year. Near the jetty of Sint Maarten all different kinds of waves are possible, with a wave period between the 0 s and 13 s, and significant wave height between the 0 m and 6 m. The dynamic analysis is therefore made of different wave climates.

The water surface elevation is described by a summation of sine functions, based on a wave spectrum. It is chosen to use the JONSWAP spectrum for wind waves. For swell waves the PM-spectrum is used. The dominant wave frequency is determined by the peak period of the wave, and the shape factor is determined using the significant wave height.

Because of the wide range of wave frequencies and wave heights that can occur, multiple wave spectra are used for the simulation of the waves during the dynamic analysis. These wave spectra are:

- A wave spectrum with the maximum wave height is used;
- A wave spectrum of a large wave, with a high dominant wave frequency;
- A wave spectrum with a very high wave frequency, and height that just hits the jetty deck;
- A wave spectrum near the lowest natural frequency;
- A wave spectrum with a dominant frequency equal to the lowest natural frequency.

Each wave spectrum represents the wave climate during a sea state. A sea state is assumed to be a 3 hour period. One directional sea states are assumed.

As mentioned the water surface elevation is described by a summation of waves. Each individual wave in the summation is described by the linear wave theory. This is also used to determine the water particle velocities and accelerations.

The main assumptions made in this chapter are:

- The jetty structure has no influence on the propagation or shape of the waves;
- The water particle velocities and accelerations for undisturbed waters are used;
- The waves do not change shape because of differences in water depth along the jetty.

The steps described in this chapter are performed by the MATLAB script shown in appendix XI.
4. Wave Loading

The open piled jetty is subjected to two types of wave loading; wave loading on the piles and on the deck of the jetty. The wave load on the deck of the jetty is only present when waves are high enough to hit the jetty deck. When this happens this can lead to very large vertical peak pressures on the deck. The wave loading on the piles is present independent of the wave height.

In this chapter first a general idea is given about the wave loading on an open piled jetty, in section 4.1. Then both a formulation for the wave loading on the deck and on the piles is described. For the wave loading on the deck there is no general consensus about the formulation found in literature, therefore a summary about the literature is given in section 4.2. Eventually a simple formulation is derived for the simulation of the wave load on the deck, in section 4.3. In the last part, section 4.4, the wave loading on the piles is determined.

In this chapter the steps shown in Figure 19 are taken.

![Flow Chart](image)

**Figure 19 - Part of the Flow Chart Introduced in Chapter 1**

4.1. Introduction

**Loads on a Jetty Structure**

The cruise jetty of Sint Maarten is for the most part of the year in use. In that situation the loads on the jetty are caused by the mooring lines, people and goods on the jetty and small wave loads on the piles. The cruise ships mooring to the jetty are enormous compared to the size of the jetty. This can cause large mooring loads especially when about once a month small swell waves enter the Great Bay. During larger storms near Sint Maarten the jetty is not in use. During a hurricane the loads on the jetty are caused by the waves, wind and occasionally objects getting stuck between the piles or hitting the jetty after being picked up by the wind. Figure 20 shows the size of a ship mooring at the jetty of Sint Maarten and a sea container folded around a lamppost by a hurricane at Sint Maarten. In the left picture the coloured dots are people on the jetty.
Dynamic Analysis of an Open Piled Jetty Subjected to Wave Loading

Although it can be imagined from the pictures that both the mooring and the wave loads can be large. Only the wave loading on the jetty during normal and extreme conditions is considered in this research.

Wave Loading on a Jetty

Three types of waves can be distinguished that cause different types of loading, these are shown in Figure 21. The type of wave is determined by the wave height, wave length and storm surge, wind setup and tidal movement.

1. Normal waves: wave does not have contact with the jetty deck.
2. Wave partly hits the jetty deck.
3. Wave breaks on top of the jetty deck

Situation 1: Normal waves

For normal waves only wave loading on the piles is present. The force on the piles, generated by the water particle velocity is continuously present on every pile, and for every wave height. For all three situations this load occurs. This is the situation at Sint Maarten for 97% of the time (Alkyon, 1997).

Situation 2: Wave Partly Hits the Jetty Deck

When a storm is near, swell waves and wind waves can come into the Great Bay causing much larger waves. The height and length of these waves varies a lot, depending on the storm. Situation 2 occurs if the waves are high enough to hit the jetty deck. This situation causes a global and a local effect.

The local effect is present over a small area. When a wave hits the jetty deck, this causes a large vertical peak pressure over a very small area and for a small duration. This peak pressure is not large for the total structure to take in, but for the small area it is large, and high stresses are expected to occur (DNV, 2010).
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Figure 22 – Theoretical Maximum Wave Height in Proportion to Jetty, (Described as a single sine function $\lambda=110$ m, $H_{max}=13.5$, in reality wave would be non-linear)

To visualize the global effect it is easier to imagine a very large wave. An extreme wave can be meters higher than the top of the jetty deck, as shown in Figure 22. This can be seen as a large bulk of water approaching the jetty with a certain velocity. When it hits the jetty deck it causes part of the water mass to splash against the side and bottom of the jetty causing large loads on the structure. For large waves a part of the wave is on top of the jetty deck. After the first contact the wave still continuous and the jetty can be submerged in the wave for over 40 meters for multiple seconds. The water underneath the jetty pushes upward. The water particles in contact with the top, side and bottom of the jetty deck push the jetty in the direction of the water particle velocity. This causes a load in horizontal and vertical direction over a large area of the jetty deck. This part rolls further. During this large wave hitting the jetty deck, multiple wave loads are present on the jetty. These wave pressures are shown in the drawing of Figure 23.

Figure 23 - Types of Wave Loading present on Jetty During Inundation

Situation 3: Breaking Waves
For some of these large waves, part of the wave rolls further over the jetty deck. For this situation the lower section of this part is slowed down by the contact with the jetty deck, the top of the wave keeps its initial velocity which can cause the wave to break on top of the jetty deck. This is schematized with situation 3. This situation is not included in this research. Only situation 1 and 2 are included. The loads caused by waves breaking on top of the jetty deck could be large, for future research it is recommended to include these loads as well.
Wave In-Deck Loading

The vertical wave loads drawn in Figure 23 called overtopping, uplift and slamming are together called ‘wave-in-deck loading’. In literature also the term wave slamming is used for the three events together. In this research the term wave-in-deck loading is used, to prevent confusion. The next section goes further into the topic of wave-in-deck loading.

4.2. Wave-in-Deck Loading

This section describes the wave loading on the jetty deck. The events included in the term wave-in-deck loading occur in sequence. First the wave hits the jetty deck, causing a large peak pressure. After that a longer during upward pressure is present, followed by a small downward pressure. This shape of the wave-in-deck load is called the church roof, shown in Figure 25, the corresponding stages are schematized in Figure 24. This load type has been investigated many times, although no general consensus about the magnitude of the peak pressure has been reached.

![Figure 24 - Stages of Wave-in-Deck Loading](image)

The loading travels along the jetty following the wave crest. Therefore the forward velocity of the load is equal to the wave celerity adapted for the angle between the jetty and the propagation direction of the wave. In horizontal direction the wave-in-deck load acts in the direction of wave propagation.
Summary of the History on Wave-in-Deck Loading Theory
This section contains a summary of the literature on wave-in-deck loading. Including noticeable findings and which methods have been created to determine the wave-in-deck load. About some of the elements regarding wave-in-deck loading a consensus is reached, like the shape. After this summary the methods to determine wave-in-deck loading are compared in order to find a formulation to be used in the dynamic analysis of this research.

In 1963 El ghamry was the first of a number of researchers trying to derive the wave-in-deck loading analytically. After some years of trying no analytical formulation was found, and an empirical method seemed an easier method (Shih, 1992). A number of experiments have been done during this period. The results differ, even for the same conditions during the same experiment case (Cuomo, 2009). In spite of the variation a couple of empirical formulas have been derived and have been used for several years. Still now the empirical methods are used for engineering practise, because of their simple formulation. In 1995, Kaplan derived analytically a wave-in-deck load formulation, based on theories of a ship hull. This method is widely used since. The latest finding makes use of the computational fluid dynamics (CFD) techniques. A software package uses the CFD technique which simulates water particles. With the use of this technique the wave load on an object can be simulated with large precision. To sum up: the history of investigations has lead to several methods to calculate the wave-in-deck loading: experimentally based, analytical based and with CFD (Raaij van, 2008).

For different structures the wave-in-deck load is investigated. The largest fields of application are offshore platforms, jetties and bridges. The last topic has only been investigated for a small number of years. For offshore platforms investigations have been done for many years. Most researches, of all fields, focus on the maximum peak load only. In order to be used for a static design. Wave-in-deck formulations varying in time and location are very rare, but necessary for a dynamic calculation. The CFD method does give a formulation in with time history and varying in location, but it is time consuming and difficult to combine with a dynamic model, and it is therefore not used in this research. Therefore the older methods are elaborated, and a short list of the acquired knowledge
about the wave-in-deck loading is presented, to get more insight in the behaviour of the wave-in-deck load.

Some Noticeable Findings from Literature
Around 1930 it was already clear that breaking waves cause a larger pressure than standing waves. And that parts sticking out of a structure that could be hit by waves should be avoided.

(Suchithra, 1995) Transitional placed stiffeners and a grid of stiffeners under a horizontal plate, reduces the wave-in-deck loading. A smaller distance between the deck and the MSL leads to higher wave-in-deck forces. Although Rooij (2001) states it depends on the ratio between the wave height and the deck clearance, because a very small deck clearance could also lead to a decrease of the wave-in-deck loading.

(MSL, 2003) The Airy wave theory compared to higher order wave theories is expected to underestimate the horizontal loading and overestimate the vertical loading.

(Raaij van, 2008) Transitional placed beams beneath the deck result in a large peak load, about 5 times larger than the following uplift force.

(Cuomo, 2009) The wave loading on jetties by Cuomo is compared to the wave-in-deck loading on bridges. It is concluded that the load on the bridge deck with a beam grid underneath the deck is less. The effect of openings in the deck leads to a reduction of the vertical uplift load, but could lead to an enlargement of the downward vertical load.

(DNV, 2010) Slamming is the largest vertical force. Inertia acts downward, because the fluid acceleration in the crest is negative. A negative force can occur due to downward fluid velocity that causes low pressure. When large diameter piles are supporting the deck, diffraction has to be taken into account.

(Lobit, 2012) The wave theory used to describe the water surface elevation and movements of water particles is determinative for the wave-in-deck loading.

Also several experiments have been performed showing the dependence of the magnitude of wave-in-deck loading. From these experiments it is found that the magnitude of wave-in-deck loading depends on the wave height, wave period, approach angle of the wave, distance between the jetty deck and mean sea level, geometry of deck, shore connection, aeration, bottom profile and geometry of the harbour (Shih et al, 1992), (Ridderbos, 1999), (Rooij, 2001), (Ren et al, 2005), (Meng et al, 2010).

Methods to Calculate Wave-in-Deck Loading
Many years of research about wave-in-deck loading has led to different calculation methods. Experiments have led to different formulas. Also different analytical based formulas are presented to describe the slamming or wave-in-deck load phenomenon.

In general the wave loading theory for jetties can be split in three different calculation methods:

- Experiment based
- Analytical based (Momentum and Component approach)
- Computational Fluid Dynamics
The general form of the empirical based method for vertical wave-in-deck loading is shown in equation 4.1.

\[ P_z = C_{slam} \rho gh_s \]  \[4.1\]

- \( P_z \) = Slamming Wave load  \([N/m^2]\)
- \( C_{slam} \) = Slamming coefficient  \([-]\)
- \( \rho \) = Water density  \([kg/m^3]\)
- \( g \) = Gravitational acceleration  \([m/s^2]\)
- \( H_s \) = Significant wave height  \([m]\)

The slamming coefficient \( C_{slam} \) differs strongly (between 2 and 6) between the different guides and papers (Quist: Lievense, 2005).

The momentum method is an analytical method based on the principle of impact. The method assumes that the momentum of the water particles is totally lost at the moment of impact with the structure. The general formula used for the vertical wave-in-deck loading is shown below (MSL, 2003).

\[ F_z(t) = \int_{A(t)} \frac{dm}{dt} v_z dA \]  \[4.2\]

- \( F_z(t) \) = Slamming wave load  \([N]\)
- \( v_z(t) \) = Water particle velocity in z-direction  \([m/s]\)
- \( A \) = Contact area  \([m^2]\)
- \( m(t) \) = Mass per unit area  \([kg/m^2]\)

The momentum method does not show a church-roof shaped wave-in-deck load (Raaij van, 2008).

The component approach is based on the theory of Kaplan, 1992. The formula introduced by Kaplan for the vertical wave load on a flat horizontal deck, including buoyancy is shown in equation 4.3.

\[ F_z(t) = B(\rho \frac{\pi l^2 \eta^2}{8} + \rho \frac{\pi l \eta^2}{4} \frac{\partial l}{\partial t} + \rho \frac{1}{2} \eta \frac{\partial l}{\partial t} \eta |C_a| + l \rho g (\eta - z)) \]  \[4.3\]

- \( B \) = width of the plate  \([m]\)
- \( l \) = wetted length  \([m]\)
- \( \eta(t) \) = vertical water particle velocity  \([m/s]\)
- \( \eta(t) \) = vertical water elevation  \([m]\)
- \( z \) = z-coordinate of bottom deck  \([m]\)
Comparison of Methods

Although the CFD methods are expected to lead to the most accurate wave-in-deck loading, the technique is not used in this research, because it is time consuming and difficult to combine with a dynamic model.

For the situation of Sint Maarten, physical model tests have been performed. The slamming coefficients, for the empirical method, have been determined during this model test. The empirical method however does not take the actual water surface elevation into account. A wave spectrum includes different wave heights, but the peak load using the empirical method will be equal for every wave passing the jetty independent of the height. This method is therefore only suitable when using regular waves as wave climate. For a more realistic simulation of the wave-in-deck load a method is needed where the wave-in-deck load depends on the actual water surface elevation.

The momentum formulation does not result in a church-roof shaped wave-in-deck load. The church-roof is proven to be the shape of the wave-in-deck load. Therefore the momentum method is not used in this research.

The component method of Kaplan includes different wave loading types and leads to an accurate magnitude of the wave-in-deck loading (Raaij van, 2005). The formulation varies in time and depends on the wave height, but it is only derived as a load on an offshore platform. The formulation is not variable over the different locations over the deck.

For the dynamic analysis a wave-in-deck load varying in time and place is needed, because this influences the dynamic reaction of the jetty to the wave-in-deck pressure. None of the ‘simpler’ methods present this formulation, and a CFD technique is not preferred. A new wave-in-deck load formulation is therefore introduced varying in time and location, in the next section.

4.3. Formulation Wave-in-Deck Load for the Dynamic Analysis

In order to obtain a formulation for the wave loading on the jetty deck which depends on location and time, a more detailed look is given to the events that occur during a wave hitting the jetty deck. A simple formulation for the wave-in-deck pressure which can be used for the dynamic analysis is looked for in this section.

The difficulty on deriving an analytical wave-in-deck load formulation is determining the peak pressure (tower of the church-roof shape of the load). The duration of the peak pressure is found during experiments to be very small. The amount of water which is slowed down by the impact in this small moment of time is unknown. This problem is mentioned in literature to be the origin for empirical formulations. In this research the magnitude of the peak pressure is determined using the results from the physical model tests by WL Delft Hydraulics (1998).

Deriving the formulation for the vertical wave-in-deck load is done separate for the two parts of:

- Peak pressure (tower of the church-roof shape of the load);
- Slowly varying upward pressure (roof of the shape of the load).
After that the total formulation is presented, followed by the horizontal wave-in-deck load. When the formulations are known a few notes are pointed out about the modelling of the wave-in-deck load, and also the derived formulations are validated.

**General Description of Events during Wave-in-Deck Loading**

To derive a vertical wave-in-deck load, the jetty deck is approached as being a stiff fixed plate located at a distance above the water level. A large wave approaches the stiff plate. The situation is shown in Figure 26.

![Figure 26 - Wave Approaching Stiff Fixed Plate of B times L](image)

At a certain moment in time the trough of the wave is below the fixed plate ($t_1$). At that moment there is no wave load present on the jetty deck. After the trough, the water surface rises. Until the water level reaches the jetty deck ($t_2$). At that moment in time an impact force occurs on the plate, by a mass of water being slowed down by the contact with the jetty deck. It is assumed that the mass of water slows down to 0 m/s, in a very short moment of time. After this impact, the water level keeps rising, causing the water to flow around the structure. This causes drag and inertia as described by Morison’s equation for submerged structures. This situation is schematized in Figure 27.

![Figure 27 - Water Flowing Around the Plate](image)

The two events of impact and the flowing around the structure are separated. The impact is only present for a very small moment in time and is causing the peak pressure $P_{z,i}$. This is described in the next paragraph.

**Peak Pressure**

The peak pressure is determined by the amount of impact caused by the wave approaching the jetty. The mass of water which is slowed down is determined using the geometry of the water surface elevation. For a pervious deck the water would travel through the deck, because the deck is not previous in reality this amount of water hits the bottom of the jetty deck. It is assumed that this is the amount of water which causes the impact.
In Figure 28 a wave is shown at two moments in time. The initial shape of the wave is shown, and the water that hits the jetty deck is hatched.

The amount of water which in one time step of $\Delta t$ travels through a pervious deck, is taken as the mass of water which is slowed down by the contact with the jetty deck in a very short moment of time $\Delta t$.

The mass of water is described by equation 4.4.

$$m(t) = \rho \gamma(t) B$$  \hspace{1cm} [kg]  \hspace{1cm} [4.4]

$m$ = mass \hspace{1cm} [kg] \\
$\rho$ = density of water \hspace{1cm} [kg/m$^3$] \\
$\gamma$ = area of water hitting the deck \hspace{1cm} [m$^2$] \\
$B$ = width \hspace{1cm} [m] \\
$L_\gamma$ = length of $\gamma$ in x-direction \hspace{1cm} [m]

The mass of water hitting the jetty deck is assumed to be uniform. The water surface elevation and water particle velocity are assumed to be equal over the width of the jetty deck. Also the same water particle velocity is assumed over the width and height of the amount of water hitting the jetty deck.

The impact is mass times the velocity. The velocity is taken to be the vertical water particle velocity ($v_z$). This vertical water particle velocity is assumed to reduce to zero due to the impact. This results in an impact shown in equation 4.5.

$$I(t) = m v_z = \rho \gamma(t) B v_z(t)$$  \hspace{1cm} [Ns]  \hspace{1cm} [4.5]

The duration of the impact $\Delta t$ determines the mass of water being slowed down during $\Delta t$. The duration of impact is taken to be equal to the time step of the simulation $\Delta t$. Many moments of impacts follow each other, until the water surface does not increase anymore. The total force is independent of the duration of the impact. It is assumed that the force caused by one impact is constant for this very small moment of time $\Delta t$. The relation between impact and force is shown in equation 4.6.

$$I(t) = \int_{t}^{t+\Delta t} F_{z,t}(t) dt$$  \hspace{1cm} [Ns]  \hspace{1cm} [4.6]

$$F_{z,t}(t) = I(t) / \Delta t$$  \hspace{1cm} [N]  \hspace{1cm} [4.7]
\[ F_{z,j}(t) = \rho B v_z(t) \frac{\gamma(t)}{\Delta t} \]  
\[ I = \text{impact} \quad [\text{Ns}] \]
\[ F_{z,j} = \text{vertical wave-in-deck load due to impact} \quad [\text{N}] \]
\[ \Delta t = \text{timestep} \quad [\text{s}] \]
\[ v_z = \text{vertical water particle velocity} \quad [\text{m/s}] \]

This force is only present if the water surface elevation at the next moment in time is higher than the previous (\( \eta(t + \Delta t) > \eta(t) \)).

For the dynamic analysis, not the load but the pressure is of interest. The load is therefore divided by the contact area \( B \) times the length of \( \gamma \) in \( x \)-direction, called \( L_\gamma \). This results in \( \gamma / L_\gamma \) which is equal to the difference in water surface elevation between the two moments in time, as is shown in equation 4.9. When this is divided by \( \Delta t \) this is equal to the vertical water particle velocity for the linear wave theory at \( z = 0 \) m. The relations are shown in equation 4.9.

\[ \frac{\gamma(t)}{\Delta t \cdot L_\gamma} = \frac{\eta(t + \Delta t, x, y) - \eta(t, x, y)}{\Delta t} = v_z(t, x, y) \]  
[4.9]

However, it can be imagined that in reality the amount of water influenced by the initial contact between the wave and the bottom of the jetty deck is much larger. The relation of equation 4.9 assumes the water particles below the mass of water that hits the jetty deck not to be influenced at all. This is not expected to be realistic. In reality it can be imagined that the initial contact between the wave and the bottom of the jetty deck causes a pressure wave in the water, changing the direction of approaching water particles. This situation is schematized in the right drawing of Figure 29. The situation assumed by equation 4.9 is schematized by the left drawing.

**Figure 29 - Schematization of the Direction of Water Particles in Two Situations (Left: Eq. 4.9 situation; Right: More Realistic)**

How large the influenced mass of water and the change in velocity are during the initial contact is not known. This 3 dimensional problem is too complex to solve by a simple description as provided in this research. In order to include this unknown contribution to the impact an unknown factor \( \alpha \) is introduced.

\[ \frac{\gamma(t)}{\Delta t \cdot L_\gamma} = \alpha v_z(t, x, y) \]  
[4.10]
Equation 4.8 together with equation 4.10 result in a vertical pressure, proportional to the kinetic energy, as shown in equation 4.11. Equation 4.11 shows a simple formulation for the vertical wave-in-deck peak pressure caused by the initial contact between the wave and the jetty deck.

\[
P_{zj}(t, x, y) = \frac{F_{zj}(t, x, y)}{BL} = \frac{\rho B v_z(t, x, y) \gamma(t)}{BL} = \tilde{\alpha} \rho v_z^2(t, x, y) \tag{4.11}
\]

\[
P_{zj} = \text{vertical wave-in-deck pressure due to impact} \quad \text{[N/m}^2\text{]} \]
\[
\tilde{\alpha} = \text{unknown parameter} \quad [-]
\]

It is assumed that this only occurs during the first contact between the water surface elevation and the bottom of the jetty deck. A duration of 0.01 s is taken. The duration of the impact is assumed to be so short, that the variation in the water particle velocity in time can be neglected. Equation 4.11 can also be derived from a slightly different approach. This approach is shown in Appendix IV.

From equation 4.11 the density of water and the water particle velocity are known. Only the factor \(\tilde{\alpha}\) is unknown.

**Deriving a value for the unknown \(\tilde{\alpha}\) factor**

The factor \(\tilde{\alpha}\) of equation 4.11 is unknown. Its value is estimated from the results of the physical model tests performed for Sint Maarten. This leads to equation 4.12.

\[
P_{zj}(t, x, y) = \tilde{\alpha} \rho v_z^2(t, x, y) = C_{\text{slam}} \rho g H_s \tag{4.12}
\]

The slamming coefficient \(C_{\text{slam}}\) is determined during the physical model test of WL Delft Hydraulics (1998).

Deriving the value of \(\tilde{\alpha}\) is shown in Appendix V. The best estimation for \(\tilde{\alpha}\) is 16.0 for the situation of Sint Maarten. This is large compared to the coefficient mentioned in DNV (2010), which uses 2.5 to 5. It is assumed that \(\tilde{\alpha}\) is independent of the significant wave height and location. For the wave-in-deck pressure used in the dynamic analysis, \(\tilde{\alpha}\) is assumed to be a constant.

**Slowly Varying Upward Pressure**

When the structure is submerged the equation of Morison is used to derive the wave loads on the plate. Also buoyancy is present, in the event that a wave rolls over the plate. Equation 4.13 assumes that the part of the wave above the plate is shaved off, and causes the buoyancy.

\[
P_{z,b}(t, x, y) = \rho g (\eta(t, x, y) - z_{\text{bottom, deck}}) \tag{4.13}
\]

\[
P_{z,b} = \text{vertical wave-in-deck pressure due to buoyancy} \quad \text{[N/m}^2\text{]} \]
\[
z_{\text{bottom, deck}} = \text{location of bottom deck in z-direction} \quad \text{[m]} \]
\[
\eta(t, x, y) = \text{water surface elevation} \quad \text{[m]}
\]

The equation of Morison results in equation 4.14 for the pressure in vertical direction on the jetty deck.
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\[ P_{z,d}(t,x,y) = C_m \rho t_h \ddot{v}_z(t,x,y) + \frac{1}{2} C_d \rho v_z(t,x,y) |v_z(t,x,y)| \]  
[4.14]  

\[ v_z = \text{water particle velocity in } z\text{-direction} \]  
[m/s]  

\[ \ddot{v}_z = \frac{\partial v_z}{\partial t} \]  
[m/s²]  

\[ t_h = \text{thickness of deck} \]  
[m]  

\[ C_m = \text{hydrodynamic inertia coefficient} \]  
[-]  

\[ C_d = \text{hydrodynamic drag coefficient} \]  
[-]

The drag and inertia coefficient \((C_d \text{ and } C_m)\) are both assumed to be 2. The inertia coefficient is determined using a relation between the wave frequency and dimensions of the structure (Vrouwenvelder, 2010).

**Total Wave-in-Deck Formulation**

**Vertical Wave-in-Deck Pressure**

The formulation for the total vertical wave-in-deck pressure is found by combining equation 4.11, 4.13 and 4.14. This results in equation 4.15, which is the total formulation for the vertical wave-in-deck load. The vertical pressure on the bottom of the deck is only present if the water surface reaches the jetty deck, which is described by the first Heaviside step function. The second Heaviside step function is used to include the small duration of the peak pressure, and the restriction that the peak pressure only occurs during the first contact between the wave and the jetty deck.

\[ P_z(t,x,y) = H(\eta(t,x,y) - z_{\text{bottom_deck}})(\ddot{\rho}(z_{\text{bottom_deck}} - \eta(t - \Delta t, x, y)) + H(\eta(t,x,y) - \eta(t - \Delta t, x, y))(C_m \rho t_h \ddot{v}_z(t,x,y) + \frac{1}{2} C_d \rho v_z(t,x,y) |v_z(t,x,y)|) + \rho g(\eta(t,x,y) - z_{\text{bottom_deck}})) \]  
[4.15]  

\[ \Delta t = \text{time step of calculation} \]  
[s]  

\[ H = \text{Heaviside step function} \]

The time interval over which the peak pressure is present is in literature found to be 8 to 16 milliseconds. In this research a duration of 0.01 s is used, because a time step of 0.01 s is used for the simulations, which makes a shorter duration impossible.

It has to be noted that although the formulation shows terms related to physical laws, the formulation has multiple empirical factors. The peak pressure is based on the physical model tests, and has therefore no larger certainty than equation 4.16 in spite of its shape.

\[ P_{z,d} = C_{slam} \rho g H \]  
[4.16]
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Horizontal Wave-in-Deck Pressure

On the side of the jetty deck a load is present if a wave is high enough to hit the side of the jetty deck.

When the deck of the jetty is submerged by a wave the load on the side of the jetty deck can be determined using Morison’s equation. However, other than for submerged structures, the side of the jetty is not submerged permanently. The event of a wave front hitting the side of the jetty for the first time at a certain location has to be included in the formulation. This load is determined using impact. Derivation of the formulation can be done in the same way as for the vertical wave-in-deck load. This results in equation 4.17.

\[
P_y(t, x, y) = H(\eta(t, x, y) - z_{bottom\_deck}) \left( \dot{\alpha} \rho v_y(t, x, y)^2 H(z_{bottom\_deck} - \eta(t - \Delta t, x, y)) + C_n B \rho v_y(t, x, y) + \frac{1}{2} C_d \rho v_y(t, x, y) |v_y(t, x, y)| \right)
\]

\[
v_y = \text{Water particle velocity in y-direction} \quad [\text{m/s}]
\]

\[
\dot{v}_y = \frac{\partial v_y}{\partial t} \quad [\text{m/s}^2]
\]

The factor \(\dot{\alpha}\) is determined to be 38. However, the dominant horizontal wave load on the deck is the inertia term of the Morison’s equation. This is caused by the large width of the jetty deck (B). Morison’s equation is derived for slender submerged cylinders, which could lead to inaccuracy in the formulation for the different shape of the jetty deck. The inertia coefficient is determined to be 2 (Vrouwenvelder, 2010). This also causes the inertia term to be large. The impact causes for most water surface elevations hitting the side of the jetty deck only a small peak.

The horizontal wave pressure acts on the side of the jetty. The contact area varies. The height of the area depends on the wetted height of the side of the jetty. This is shown visually in Figure 30. The mathematical description is shown below. \(d_{wet}\) is the wetted height, at the side of the jetty deck.

\[
q_y(t, x, y) = d_{wet}(t, x, y) \cdot P_y(t, x, y)
\]

\[
\begin{cases}
\eta(t, x, y) < z_{bottom\_deck} & d_{wet}(t, x, y) = 0 \\
z_{bottom\_deck} < \eta(t, x, y) < z_{top\_deck} & d_{wet}(t, x, y) = \eta(t, x, y) - z_{bottom\_deck} \\
\eta(t, x, y) > z_{top\_deck} & d_{wet}(t, x, y) = 0.4 \text{ m}
\end{cases}
\]

Figure 30 - Side View of Jetty, Showing Wetted Height
Modelling of Wave-in-Deck Load

In the previous paragraph a wave-in-deck load formulation is derived. This formulation is used for the dynamic analysis. The modelling of the wave-in-deck load leaves out several factors, which do have an influence. In this section the modelling of the wave-in-deck load is described in consideration of:

- Beams;
- Suction;
- Breaking waves;
- Current action;
- Aeration.

**Beams**

Beneath the jetty deck beams are present perpendicular to the jetty axis. The wave loading on the side of these beams is included. This enlarges the horizontal wave loading on the jetty deck. The vertical wave-in-deck pressure caused by this beam is not included in the calculation. The beam is expected to cause an extra peak pressure, because of water particles getting stuck between the bottom of the jetty deck and the beam. This peak pressure caused by the beams occurs with a frequency dependent on the wave celerity, approach angle and distance between the beams. For the jetty of Sint Maarten this results in a frequency of about 10 rad/s. Assuming a wave celerity of 10 m/s. This is a frequency in between the natural frequencies of the system. For future research it is therefore recommended to include the vertical wave pressure caused by the beams beneath the jetty deck.

**Suction**

This load is caused by the difference in pressure between the bottom and the top of the deck. It occurs after the deck has been submerged. The water underneath the jetty deck loses contact, and air has to fill up its place. This causes suction underneath the jetty deck (Shih, 1992). No analytical formulation of this load is found in literature; therefore it is not included in the formulation.

**Vertical loading of breaking waves on top of the deck**

When a wave breaks on top of the jetty, a bulk water hits the top of the jetty deck. No formulations on wave loading due to breaking waves on a horizontal plate are known (HR Wallingford, 2005). This type of wave loading is not included in the formulation; however it could be a large load. For a future research the information can be taken from theories about other structures being subjected to breaking waves (like a breakwater).

**Current Action**

Current can increase the relative velocity of the water particles with respect to the structure. This is expected to influence the horizontal wave-in-deck loading of momentum, inertia and drag. Also loading on a submerged part of the piles is influenced by current. In the Great Bay of Sint Maarten the current is very low and therefore neglected in this research.

**Aeration**

The degree of air trapped in the water influences the wave load on the structure. Especially the magnitude of the vertical peak pressure depends on the aeration. During a hurricane the water is known to be white, which indicates a high aeration grade. Air in the water is expected to lead to less high wave-in-deck loads. Bea (2001) has introduced a formula to include aeration in the wave load
calculation. However, MSL (2003) sees the outcome of the formula as “not likely to be significant”. This formula is therefore not used.

**Validation of Wave-in-Deck Formulation**

The derived formulations of equation 4.24 for the vertical wave-in-deck pressure can be checked by remaking results from literature, and checking the results with the physical model tests. Also the magnitudes of the vertical wave-in-deck loading for the different considered wave spectra are compared. This is performed in this section.

**Remaking Results from Literature**

The first check, remaking the results of literature is described in Appendix VI. For one of the tests the formulation overestimated the peak load and for another it comes out to low.

**Physical Model Tests**

The analytical derived wave-in-deck pressures are compared to the physical model tests of WL Delft Hydraulics in 1998. During the physical model tests the significant wave height and peak period of a 100 year return period wave were chosen. With this wave a JONSWAP spectrum was made. The waves were simulated and the wave loading on the jetty was measured in vertical and horizontal directions at multiple locations along the jetty. This method has many similarities with the method used in this research. Only in this research the tests are not physically performed. The wave loading used for the design of the jetty, was the one with an exceedance probability of 0.4 % per wave. This value was filtered from the measured wave loads. In order to compare the wave load determined by this research to the one of the physical model test, the wave load with 0.4 % exceedance probability is taken as well.

During the physical model tests the pressure at certain locations along the jetty are measured. These pressure transducers are located at the bottom and the top side of the jetty deck. The locations along the jetty deck are shown in Figure 31.

![Figure 31 – Top View, Location of Pressure Transducers along the Jetty Deck during the Physical Model Tests by WL Delft Hydraulics (1998)](image)

The characteristics of the wave climate are equal to the one used for test 223 of WL Delft Hydraulics in 1998:

- $H_s$ is 5.8 m;
- $T_p$ is 13.2 s;
- Surge level of 1 m;
- Water depth of 11 m;
- Alfa is 20 degrees;
- Air gap of 1.9 m.

![Figure 32 - Schematization of Definition of Distances below Jetty Deck](image)
The definitions of the different distances are presented in Figure 32. The actual build situation (described in chapter 2) slightly differs from the situation during the physical model test. When the wave with the significant wave height of the wave spectrum is proposed by a single sine function, this has the proportions as shown in Figure 33.

![Figure 33 - Wave of Test 223 Proportional to Jetty, H= 5.8 m., T=13.2 s proposed as a regular wave](image)

In the simulation the water surface elevation is not described by a single sine function, but by a summation of sine functions. Also the wave is not expected to by linear, as it is schematized in Figure 33. The results are shown in Table 3.

<table>
<thead>
<tr>
<th>0.4 % exceedance probability</th>
<th>Physical model [kN/m²]</th>
<th>Numerical Method Average [kN/m²]</th>
<th>Numerical Method Standard Deviation [kN/m²]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upward (z)</td>
<td>209</td>
<td>211</td>
<td>67</td>
</tr>
<tr>
<td>Downward (-z)</td>
<td>40</td>
<td>16</td>
<td>3.8</td>
</tr>
<tr>
<td>Horizontal (y)</td>
<td>38</td>
<td>71</td>
<td>4.7</td>
</tr>
<tr>
<td>η</td>
<td>5.4 m</td>
<td>5.5 m</td>
<td></td>
</tr>
</tbody>
</table>

From Table 3 it can be seen that the average upward peak pressure is close to the results found by the physical model tests. This is caused by the peak pressure, being determined from the physical model tests. The downward pressure is much lower using the numerical formulation; this is because no formulation for suction is included in the numerical formulation. The numerical formulation results in a larger horizontal pressure than found by the physical model tests, this is caused by the dominant inertia term in the numerical formulation.

The numerical formulation is also checked by remaking the results from one of the other tests of the physical model tests. That test has a wave period of 11.7 s and a significant wave height of 6.5 m. The average peak pressure found by the physical model test is 242 kN/m². The numerical formulation, with α is 16, results in an average of 253 kN/m². This is quite close to each other.

**Church-Roof**

The shape of the vertical wave-in-deck load is known to be a church-roof. The wave load used for this research is therefore preferred to have this shape as well. The linear wave theory and a summation of sine functions are used to describe the water surface elevation, as described in the previous chapter.

The blue line in the upper graph of Figure 34 represents the wave, varying in time. The dotted line is the bottom of the jetty deck. The middle and lower graph of Figure 34 show the vertical wave-in-
deck load. The different terms in the formulation of equation 4.15 are shown with different colours in the middle graph.

- 1e term, Impact: Black
- 2e term, Inertia: Red
- 3e term, Drag: Blue
- 4e term, Buoyancy: Yellow
- Total Pressure: Green
The green line in Figure 34 is the vertical wave load varying in time. When the wave height exceeds the jetty deck, a peak load can be seen in the lower graphs. This peak is followed by a more slowly varying load. The shape of the vertical pressure caused by the second wave hitting the jetty deck is as described in literature. The peak pressure is about 5 times the slowly varying upward pressure. The slowly varying upward pressure is caused by buoyancy (yellow line). This is also found in literature. The first wave results in a less high peak pressure compared to the slowly varying pressure that follows. This can also be seen in the physical model tests, where not all waves cause a wave-in-deck pressure with the same shape.

From Figure 34 it can be seen that the peak value is much smaller than the peak pressure mentioned in Table 3 (120 kN/m² versus 211 kN/m²) this is because the mentioned peak pressure in Table 3 is the 0.4% exceedance probability pressure. The pressure in Figure 34 is an arbitrary taken moment in time and location, therefore the peak pressure has a larger exceedance probability. The 0.4% exceedance probability pressure mentioned in this chapter cannot be compared with the 0.1% exceedance probability per storm mentioned in the following chapters, because both are determined using different circumstances.

Figure 35 shows the shape of the vertical wave-in-deck pressure over the x-axis, which is equal to the jetty axis. The pressure is shown at three different moments in time, so the propagation of the wave load along the jetty can be seen. The wave shape is shown in the graph below. For the water surface elevation a single sine function is used, which leads to a wave that not changes its shape in time. This makes it easier to see the propagation of the wave-in-deck load.
Dynamic Analysis of an Open Piled Jetty Subjected to Wave Loading

Figure 35 - Below: Wave at three different moments in time. Upper Graph: Vertical Wave-in-deck pressure at the same moments in time

From Figure 35 it can be seen that the vertical pressure propagates along the jetty, as described in literature as well. The dotted horizontal line at 1.1 m represents the bottom of the jetty deck. 1.1 m is the air gap (1.6 m) reduced by the setup, wind surge and tidal movement which are together assumed to be 0.5 m.

The length in x-direction over which the peak pressure is present is about 1 m in the simulation. This corresponds to the findings of the physical model tests. The length of the slowly varying uplift is between the 10 m and 30 m, when using the summation of sine functions to describe the water surface elevation. During the physical model tests values between the 10m and 20 m were found. The wave loading in Figure 35 is present over a much longer length; this is caused by the extreme length of the wave when being represented by a single sine function. The length of the slowly varying uplift pressure used in the simulations is in the same range as found by the physical model tests.

Comparing Wave-in-Deck Loading of Considered Wave Spectra

In this research three wave spectra are used to simulate three different sea states near the jetty. These wave spectra are further described in section 3.3. This paragraph compares the 0.1 % exceedance probability of the vertical peak pressure of these three wave spectra.

Table 4 shows the vertical peak pressures with an exceedance probability of 0.1 %.

<table>
<thead>
<tr>
<th>Wave Spectrum</th>
<th>Hs [m]</th>
<th>Tp [s]</th>
<th>Pz [kN/m²]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Large Wave Spectrum</td>
<td>9.0</td>
<td>12</td>
<td>670</td>
</tr>
<tr>
<td>Large Waves in High Frequency</td>
<td>5.0</td>
<td>7.0</td>
<td>680</td>
</tr>
<tr>
<td>Waves Just Hitting Jetty Deck</td>
<td>3.0</td>
<td>4.0</td>
<td>697</td>
</tr>
<tr>
<td>Test 223 of Physical Model Tests</td>
<td>5.8</td>
<td>13.2</td>
<td>227</td>
</tr>
</tbody>
</table>

From Table 4 it can be seen that the wave spectra used in this research results in a much larger peak pressure than found by the physical model tests. Also the peak pressures of the investigated wave spectra do not differ much from each other, which is not expected from their difference in significant
wave height. This is caused by the linear wave theory and the smaller wave period of the smaller waves. The wave spectrum of test 223, of the physical model test (WL Delft Hydraulics, 1998) has a large wave period of 13.2 s. The empirical factor in the vertical peak pressure is derived for this wave spectrum. The wave spectra used in this investigation have a smaller wave period. The smaller wave period causes a larger angular frequency, which in the linear wave theory enlarges the water particle velocity and acceleration. It is uncertain whether these values are realistic. It is known that the simple wave-in-deck formulation used in this research does not include the complex phenomena (wave-deck interaction, turbulence, 3 dimensions) of wave slamming in reality. The formulation is therefore not suitable for investigation of the magnitude of the wave-in-deck loading.

4.4. Wave load on the piles

The wave loading on the deck is only present if the wave is higher than the bottom of the jetty deck. The wave load on the piles is continuously present. The wave load on the piles is determined using Morison’s equation. This equation is applicable for slender submerged structures. The equation (4.16) consists out of an inertia (equation 4.17) and drag part (equation 4.18). Both parts have to be integrated over the wetted height, shown in equation 4.19.

\[dF = dF_m + dF_d\]

\[dF_m(t,x,y,z) = C_m \rho_w \frac{\pi D^3}{4} \frac{\partial v_y(t,x,y,z)}{\partial t} \, dz\]

\[dF_d(t,x,y,z) = C_d \rho_w \frac{D}{2} v_y(t,x,y,z) \frac{\partial v_y(t,x,y,z)}{\partial z} \, dz\]

\[F(t,x,y,z) = \int_{-d}^{0} dF(t,x,y,z) \cdot dz\]

\[F = \text{load on a pile} \quad \text{[N]}\]

\[C_m = \text{hydodynamic inertia coefficient} \quad \text{[-]}\]

\[C_d = \text{hydodynamic drag coefficient} \quad \text{[-]}\]

\[D = \text{diameter of pile} \quad \text{[m]}\]

\[d = \text{water depth} \quad \text{[m]}\]

The drag and inertia coefficients \((C_m\) and \((C_d)\) are set to be 2. Their values depend on the wave frequency. This dependency is difficult to model in the chosen approach. The wave train is simulated from the wave spectrum. During this change from frequency to time domain, the information about the wave frequency at a certain time and place gets lost. This dependency is therefore not included in the model. The value of 2 is high, and therefore conservative.

The total wave load over the height of the pile can be found by integrating over the wetted length of the pile. It is assumed to integrate from the sea bottom at \(z= -d \) m to \(z = 0 \) m, which is the zero crossing of the wave height. The wave crest is above this line and the through below.

The drag and inertia part are integrated numerically. The integration of the inertia term can be done analytically because it is a linear equation. However the drag part is non-linear. The integration is done with a simple numerical integration rule; the midpoint rule. Only the water particle velocity is
variable over $z$. The water particle velocity does not change much over the vertical direction ($z$) in intermediate water. In deep water, which is assumed in the model, this difference is even smaller. Therefore the midpoint rule can be used.

Integration over the total wetted length gives the total load per pile. This resultant load has to be placed on a node. If it is placed at for instance half the height of the wetted length, this causes a different bending moment in the soil than when the original distributed load would have been used. Therefore the load is placed on two locations on the pile at -3 and -10 meter below the water surface, which is shown in Figure 36. This decreases the error in the bending moment to about 1 %, for a large wave near Sint Maarten.

![Figure 36 - Distributed Wave Load on Piles (Left) Placed on Two Locations over the Height of the Pile (Right)](image)

This method is applied on all the 100 piles of the modelled part of the jetty.

**Turbulence**

The description shown in this section shows the wave load on one pile. The load on the other piles is determined in the same way. The effect of turbulence in the water, or other disturbance because of piles being close to each other is not taken into account.
4.5. Summary Wave Loading

For wave-in-deck loading a formulation is made, in order to get a formulation depending on time and location. This formulation of the vertical wave-in-deck pressure is:

\[
P_z(t, x, y) = H(\eta(t, x, y) - z_{\text{bottom, deck}})(\ddot{\alpha} \rho \nu_z(t, x, y)^2 H(z_{\text{bottom, deck}} - \eta(t - \Delta t, x, y)) + H(\eta(t, x, y) - \eta(t - \Delta t, x, y))(C_m \rho t_v \nu_z(t, x, y) + \frac{1}{2} C_d \rho v_z(t, x, y)|v_z(t, x, y)|) + \rho g(\eta(t, x, y) - z_{\text{bottom, deck}}) \quad [\text{N/m}^2]
\]

Horizontal wave-in-deck loading is given by:

\[
P_y(t, x, y) = H(\eta(t, x, y) - z_{\text{bottom, deck}})(\ddot{\alpha} \rho \nu_y(t, x, y)^2 H(z_{\text{bottom, deck}} - \eta(t - \Delta t, x, y)) + C_m B \rho \nu_y(t, x, y) + \frac{1}{2} C_d \rho v_y(t, x, y)|v_y(t, x, y)|) \quad [\text{N/m}^2]
\]

The wave loading on the piles is determined using the equation of Morison. Diffraction and turbulence in the water are not included.

The steps described in this chapter are executed using MATLAB. The script is shown in appendix XI.
5. Dynamic Model of the Jetty

A dynamic model is made of the first jetty of Sint Maarten. A model is a simplified representation of reality. Which simplifications are made in the modelling of the jetty are discussed in this section.

Section 5.1 introduces which structural properties are included in the model, and which type of dynamic model is chosen. Also the principle of modal analysis is explained, and the numerical solver. Section 5.2 describes how Scia Engineer and MATLAB are combined in this research. After which section 5.3 mentions the most important model assumptions. The model choices made in Scia Engineer are discussed in section 5.4. The validation of the dynamic model is performed in section 5.5.

5.1. Introduction

Structural Properties
The jetty has different shapes in which it can vibrate at a natural frequency. These mode shapes are related to the bending stiffness, shear stiffness, mass and axial stiffness of the different structural elements. Which structural properties are preferred to be included in the model depends on their expected contribution to the dynamic behaviour. For instance: if the jetty would be schematized as a single degree of freedom system with the deck being the mass, than the bending stiffness of deck is neglected. However, this is not preferred for this jetty structure because the stiffness of the jetty is very large and bending of the deck is expected. It is preferred to include the following properties in the dynamic model:

- Mass;
- Bending stiffness;
- Shear stiffness of the deck;
- Axial stiffness of the piles.

Each of the structural properties is expected to have a significant influence on the dynamic behaviour.

Type of Dynamic Model
The mode shapes and their corresponding natural frequencies can be computed in different ways; with a continuous or discrete model. When using a continuous model the jetty deck would be modelled as an Euler-Bernoulli beam, and the piles are replaced by rotational and translational springs. For the wave load, depending on place and time, this results in a problem statement which is difficult to solve. Therefore it is chosen to use a discrete model in this research.

In the discrete model, the structure is cut up in small parts, called elements. An example of a simple discrete model is in shown in Figure 37. The black lines are the elements. Every element has a stiffness and mass in multiple directions. Which translational and rotational freedoms the element has are called its degree of freedom (DOF). The displacements (and rotations) are calculated at certain points on the edge of an element. These points are called nodes; the dots in Figure 37. The information about the displacement is therefore only known in the nodes, which makes the system discrete.
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Complex structural calculations are often done in this way by a computer program. Such a program often uses a type of Finite Element Method solver. One type of discrete dynamic analysis is called Modal Analysis. Modal Analysis can include all the properties of the material and structure, without the set of equations becoming more difficult to solve. Therefore this method is used for the dynamic analysis in this research. The principle of modal analysis is explained in the next section.

Principle of Modal Analysis

The dynamic calculation of the response of the jetty to the wave load is done using modal analysis. This method makes use of the structure being split in a finite number of elements. For every element an individual stiffness matrix exists. These matrices connect the nodal load to nodal displacements.

The stiffness matrix stores the stiffness of every element between two nodes for every DOF. For the total structure all these stiffness matrices are combined, resulting in stiffness matrix $K$ of $n \times n$ with $n$ being the number of nodes times the DOF per node. The elements also have a mass. These masses are combined in a mass matrix $M$. This is a diagonal matrix of $n \times n$.

The differential equation is shown in equation 5.1. The vector $w$ is the displacement vector. The vector contains entries for every number of nodes times the DOF per node, so is $n \times 1$. The double dotted $\ddot{w}$ represents the second derivative of the displacement with respect to time.

$$M\ddot{w} + Kw = 0 \quad [5.1]$$

As solution for the homogeneous system of equations, shown in equation in 5.1 the eigenmode in equation 5.2 is tried.

$$w(t) = \hat{x}\sin(\omega t + \varphi) \quad [5.2]$$

When this solution is substituted in the differential equation it follows that:

$$(-\omega^2 M + K)\ddot{x}\sin(\omega t + \varphi) = 0$$

$$(-\omega^2 M + K)\ddot{x} = 0 \quad [5.3]$$
Equation 5.3 can be written in matrix notation. This is shown in equation 5.4.

\[
\begin{pmatrix}
-\omega_1^2 & 0 & 0 & \cdots & 0 \\
0 & -\omega_2^2 & 0 & \cdots & 0 \\
0 & 0 & \ddots & \cdots & \vdots \\
0 & 0 & \cdots & -\omega_n^2 & 0 \\
0 & 0 & \cdots & 0 & m_n
\end{pmatrix}
\begin{pmatrix}
m_1 \\
m_2 \\
m_3 \\
m_n
\end{pmatrix}
+ \begin{pmatrix}
k_{11} & \cdots & k_{1n} \\
k_{21} & \cdots & k_{2n} \\
\vdots & \ddots & \vdots \\
k_{n1} & \cdots & k_{nn}
\end{pmatrix}
\begin{pmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3 \\
\dot{x}_n
\end{pmatrix}
= \begin{pmatrix}
0 \\
0 \\
\vdots \\
0
\end{pmatrix}
\]

[5.4]

From equation 5.3 and 5.4 it follows that:

\[
\det(-\omega^2 M + K) = 0
\]

[5.5]

This results in \( n \) natural frequencies \( \omega \) and eigenvectors. The total solution for the free vibration is given by a summation of the \( n \) mode shapes. The dimensionless free vibration shape is presented in the vector \( \hat{x} \). This vector contains the displacement at every DOF. So the vector has the length of \( n \).

\[
w(t) = \hat{x}_1 A_1 \sin(\omega_1 t + \phi_1) + \hat{x}_2 A_2 \sin(\omega_2 t + \phi_2) + \cdots + \hat{x}_n A_n \sin(\omega_n t + \phi_n)
\]

[5.6]

The amplitude \( A \) and the phase angle \( \phi \) are still unknown, and depend on the initial conditions. To calculate the forced vibration response of the dynamic system: \( u(t) \) is introduced. This is the uncoupled variable related to the physical DOF \( w \). The solution for the free vibration problem (given in equation 5.6) can also shortly be written in \( u(t) \) as shown in equation 5.7. \( \hat{x}_i \) is the eigenvector of mode shape \( i \).

\[
w(t) = \sum_{i=1}^{n} \hat{x}_i u_i(t)
\]

[5.7]

In order to solve the forced vibration problem, corresponding to the inhomogeneous equation 5.8, some extra parameters need to be introduced. Also damping is introduced in the system.

\[
M \ddot{w} + C \dot{w} + k w = F(t)
\]

[5.8]

The eigenvectors of all mode shapes are stored in a matrix called eigenmatrix \( E \), shown in equation 5.9. The natural frequencies are stored in matrix \( \Omega \), presented in equation 5.10.

\[
\begin{pmatrix}
\hat{x}_1 & \cdots & \hat{x}_n
\end{pmatrix} = E
\]

[5.9]

\[
\begin{pmatrix}
-\omega_1^2 & 0 & 0 & \cdots & 0 \\
0 & -\omega_2^2 & 0 & \cdots & 0 \\
0 & 0 & \ddots & \cdots & \vdots \\
0 & 0 & \cdots & -\omega_n^2 & 0 \\
0 & 0 & \cdots & 0 & m_n
\end{pmatrix} = \Omega^2
\]

[5.10]

When the mass matrix is multiplied by the eigenmatrix and the transposed eigenmatrix, the modal mass matrix is found, shown in equation 5.11. This matrix is diagonal because of the orthogonality condition. (Spijkers, et al., 2006)
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\[ E^T ME = \begin{pmatrix} \hat{\xi}^T M \hat{\xi} & \ldots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \ldots & \hat{\xi}^T M \hat{\xi} \end{pmatrix} \]  \[ [5.11] \]

\[ E^T KE = \begin{pmatrix} \hat{\xi}^T K \hat{\xi} & \ldots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \ldots & \hat{\xi}^T K \hat{\xi} \end{pmatrix} \]  \[ [5.12] \]

For the stiffness matrix \( K \) the same orthogonality holds, and the modal stiffness matrix is introduced in equation 5.12. Without damping \( \omega^2 = k/m \) so \( \omega^2 \cdot m = k \). This also holds for the modal mass and stiffness matrices because of their orthogonality. \( \Omega^2 E^T ME = E^T KE \)

With this information the differential equation can be rewritten from equation 5.13 to 5.16.

\[ M \ddot{w} + C \dot{w} + k w = F(t) \]  \[ [5.13] \]

\[ ME \ddot{u} + CE \dot{u} + kE u = F(t) \]  \[ [5.14] \]

\[ E^T ME \ddot{u} + E^T CE \dot{u} + E^T kE u = E^T F(t) \]  \[ [5.15] \]

\[ \ddot{u} + \frac{E^T CE}{E^T ME} \dot{u} + \Omega^2 u = \frac{E^T F(t)}{E^T ME} \]  \[ [5.16] \]

Equation 5.13 can be rewritten as 5.14 because \( w(t) = E \dot{u}(t) \).

As described above the orthogonality condition can be used to make the matrices diagonal. This leads to an uncoupled set of equations. When the same procedure is followed for the damping matrix no diagonal matrix emerges. This makes the use of uncoupled equations impossible, and the assumption of a synchronous motion per mode shape has to be changed in order to solve the system. This leads to a complex modal analysis. This method however, is much more complicated to solve. Therefore the damping matrix is often forced to be diagonal, in order to get an uncoupled system of equations. This is also done in this research. The damping is defined in equation 5.7.

\[ \frac{\hat{\xi}^T C \hat{\xi}}{\hat{\xi}^T M \hat{\xi}} = 2\zeta \omega_i \]  \[ [5.17] \]

This results in a set of \( n \) uncoupled equations. When the equations are separated equation 5.8 can be found.

\[ \hat{\xi}^T M \hat{\xi} \ddot{u}_i + \hat{\xi}^T M \hat{\xi} 2\zeta \omega_i \dot{u}_i + \hat{\xi}^T M \hat{\xi} \omega_i^2 u_i = \hat{\xi}^T F(t) \]  \[ i = 1, 2, 3, \ldots, n \]  \[ [5.18] \]

\( i \) is the considered mode shape. When this equation is divided by \( xMx \), the following uncoupled differential equation emerges:

\[ \ddot{u}_i + 2\zeta \omega_i \dot{u}_i + \omega_i^2 u_i = \frac{\hat{\xi}_i^T F(t)}{\hat{\xi}_i^T M \hat{\xi}_i} \]  \[ [5.19] \]

With \( u(t) \) being the variable of the uncoupled system. This is the equation of one mode shape, with \( i \) being the number of the mode shape. The differential equation can be solved with different
methods. It is chosen to solve the differential equation numerical, because this is a fast way of solving. The only note is the stability of the solver.

The damping in the equation differs per mode shape. In order to make use of the above described modal analysis the damping has to be determined per mode shape in order to keep a decoupled system of equations.

Numerical Solver
The differential equation of the uncoupled problem is determined to be equation 5.19, per mode shape of the system.

This is a second order differential equation. To solve this equation with a numerical method, the second order equation has to be changed into a system of first order equations. This is done with the introduction of $v_1$ and $v_2$.

\[ \begin{align*}
  u &= v_1 \\
  \dot{u} &= v_2 \\
  \ddot{u} &= v_3
\end{align*} \]  

[5.20]

The differential equation from 5.19 becomes 5.21, when substituting the equations from 5.20.

\[ \dot{v}_{2,j} + 2\zeta_j\omega_j v_{2,j} + \omega_j^2 v_{1,j} = \frac{\hat{X}^T F(t)}{\hat{X}^T M\hat{X}} \]  

[5.21]

The following system of first order equations can be made from equation 5.21:

\[ \begin{align*}
  \dot{v}_{2,j} &= -2\zeta_j\omega_j v_{2,j} - \omega_j^2 v_{1,j} + \frac{\hat{X}^T F(t)}{\hat{X}^T M\hat{X}} \\
  \dot{v}_{1,j} &= v_{2,j}
\end{align*} \]  

[5.22]

This system is solved with the backward Euler (Vuik, et al., 2006). The stability of the numerical solver is investigated in Appendix VII.

5.2. Method
The modal analysis is done with the help of two programs: Scia Engineer and MATLAB. Scia engineer is a finite element software package. MATLAB is a program in which numerical calculations and visualizations can be made, with a script written by the user. The natural frequencies and mode shapes are determined with Scia Engineer. Also the nodes are defined in Scia Engineer. The dynamic response is calculated in MATLAB. The output of Scia engineer functions as input for the MATLAB script, as can be seen in Figure 38. With the eigenvectors and natural frequencies the response of the structure to the wave load can be determined. This is done by using a calculation script written in MATLAB. The results are the response of every node in $x$, $y$ and $z$-direction.
5.3. Model Assumptions

3 DOF
The model only takes three translations into account; motion in x, y, and z-direction. These are expected to give a good indication of the dynamic behaviour of the jetty. Including all 6 DOF would have been more accurate, but it also enlarges the needed calculation time and memory.

1 Module of the Jetty
The jetty of Sint Maarten is about 650 meters long. The jetty consists of four modules, separated by expansion joints. Only one of the four modules is modelled, because the modules have the same geometry. Hence a similar dynamic behaviour is expected for each of the individual modules. The influence from the other modules on the model is included by placing boundary conditions on the head of the model. This makes it possible to model different situations.

Mode Shapes
The eigenvectors $\hat{\mathbf{x}}_i$ and the natural frequencies $\omega_i$ are determined with the help of Scia engineer. The eigenvectors contain information of every node (about 880) in every DOF (x, y, z). The number of mode shapes ($i$) which are considered determines the number of eigenvectors and natural frequencies. The number of considered mode shapes is chosen to be 20. These are the 20 lowest natural frequencies of the system. In reality the system has an infinite number of natural frequencies. Higher natural frequencies than mentioned in this report do exist. They are not included in this research because their influence is expected to be small.

Boundary Conditions
Different situations of modelling the expansion joint are used. The expansion joint is designed to allow movement in axial direction (x-direction) of about 0.05 m. The forces in vertical (z-direction) and horizontal direction perpendicular to the jetty axis (y-direction) are transmitted by the teeth in the expansion joint. This situation is modelled by fixing translations in z and y-direction at one side of the module, as schematized in Figure 39. This situation assumes the maximum displacement in x-direction to be less than 0.05 m.
When the neighbouring modules vibrate in the same direction, and the displacements are small, than movements in y and z-direction can be possible as well. Rotations are restrained by the teeth, but for small displacements over 152 meters the rotations are very small. They are expected to fit in the construction tolerances and holes made by earlier damage. During the lifetime of the jetty, the strength of one of the expansion joints decreased, until it failed in 2010. This situation is modelled as well, by not fixing either of the degrees of freedom. The results presented in this chapter are determined using this model. The boundary condition at the piles are discussed in the paragraph soil.

**Load Vector**

In the differential equation of the problem (shown in equation 5.23) the load vector is present. This vector contains all the loads at all the nodes in all 3 DOF (x,y,z). The load factor is called $F(t)$.

$$\ddot{u}_i + 2\zeta\omega_i \dot{u}_i + \omega_i^2 u_i = \frac{\hat{x}_i^T F(t)}{\hat{x}_i^T M \hat{x}_i}$$ \hspace{1cm} [5.23]

The load vector contains the vertical and horizontal wave pressure on the jetty deck, and the horizontal wave loads on the jetty piles. The loads are placed on the corresponding node. Therefore the load vector has a length of 3 DOF times the number of nodes (880). In the previous chapter the magnitude of the loads are determined. The loads are variable in location. To get the nodes from the coordinate system to the right node number, a list linking the node numbers to the coordinates is used. With the script shown in appendix XI the loads are placed on the right nodes.

In reality a load acts over the total contact area. Figure 40 shows the change from distributed load to concentrated loads in the nodes. It is shown for the horizontal wave load on the jetty deck.
The horizontal load is present at the nodes on the side of the deck and on the beams. The vertical pressure is only present at the nodes on the deck. This vertical wave pressure (z-direction) is in x-direction distributed over the nodes located every meter. Just as shown in the figure above for the horizontal load. Over the width of the deck (y-direction) the vertical wave pressure is distributed over 2 nodes only. This is shown in Figure 41 with the two vertical piles. These nodes are located at the side of the jetty deck (at y = 10 m and y= -10 m). Figure 41 shows a schematization of the locations where the distributed loads are placed in the nodes.
When a distributed load is modelled with concentrated loads in the nodes, placing half of the load at both nodes is not sufficient. The load in between the nodes would in reality have caused a bending moment on the element. However, because of the discretization of the load, this bending moment does not occur. Therefore these bending moments also have to be placed on the nodes. Figure 42 shows a schematization of this process for a static situation.

![Nodal Forces and Bending Moment Diagram](image)

Figure 42 - Equivalent Nodal Forces and Bending Moment for a Distributed Load

The magnitude of the bending moment at the nodes is determined by demanding the work to be equal for both situations, when displacing or rotating a node (Simone, 2010). In axial direction (x-direction) there are nodes every meter. The moments in the nodes will therefore be small. In horizontal, y-direction, there are only two nodes over the width of 20 meter of the jetty deck. These moments are therefore not expected to be negligible. However, only 3 DOF are used in this model. Rotations are not included in the dynamic model. Therefore no bending moment can be placed on the nodes. In a future research these moments are preferred to be included. Or the number of nodes is y-direction is recommended to be enlarged. By leaving out the bending moments, loading on the jetty deck is under estimated, as holds for this research.

**Initial Conditions**

The initial conditions are zero for the displacement and velocity in all three directions. In literature it is recommended for a dynamic analysis to use waves instead to set the starting conditions for the simulation. The first 12 seconds of the simulated displacements are therefore not included in the results and post-processing of the results. During the first seconds the initial conditions have an effect on the response, which is not expected in reality. The response is therefore only determined by the particular solution of the system.

**Time Delay**

In the modal analysis time delay of the vibration along the jetty is not included. If a wave hits the jetty at the head of the jetty, it is not expected that this pressure is immediately at the abutment of the jetty. For a pressure wave to travel through a structure takes time. The magnitude of this time delay depends on the stiffness and the length of the structure. In stiff structures pressure waves travel faster.

**Relative Velocity and Acceleration**

The water particles hit the jetty with a velocity. The magnitude of the load is related to the relative velocity and acceleration of the water particles with respect to the jetty structure. During a storm the jetty deck is expected to vibrate. However the effect of the velocity and acceleration of the jetty on the relative velocity and acceleration are neglected, for the simplicity of the model. The velocity of the movement of the jetty deck can become quite large. This will lead to an inaccuracy. The direction
of the velocity is expected to be both ways, the velocity will therefore vary between over and under approximated.

**Non-linear Geometry**

The wave load is determined at the initial location of the jetty. The displacements of the jetty are not considered when it comes to the location where the wave hits the jetty. Including this leads to geometrical non-linear problem. For large loads this is expected to have a significant influence.

### 5.4. Scia Engineer Model

The finite element program Scia engineer, computes the solution similar to the earlier described discrete model of the dynamic calculation. The program only calculates stresses and strains at the locations of the nodes. The user of the program needs to input the following items:

- Geometry of the Structure;
- Materials;
- Boundary Conditions;
- Added Mass;
- Calculation Type.

The user also chooses the type and size of elements and the number of mode shapes which are calculated. The number of DOF differs per element type. The error made by the discretization of the calculation depends on the element size. It is therefore important to take small elements and differ the size to see the error made in the calculation.

**Materials and Geometry**

The geometry of the jetty is given by the width of the deck \((B)\), the thickness of the jetty deck \((t_b)\) and the length of the modelled jetty \((L)\). Only one part of the jetty is modelled, to reduce the calculation time.

\[
B = 20 \quad [\text{m}]
\]
\[
t_b = 0.4 \quad [\text{m}]
\]
\[
\rho = 2400 \quad [\text{kg/m}^3]
\]
\[
E = 330 \cdot 10^8 \quad [\text{N/m}^2]
\]
\[
L = 152 \quad [\text{m}]
\]

*Figure 43 - Jetty Model in Scia Engineer*

In the piles a concrete pile plug is included in the finite element model, although this is not visible in Figure 43.

**FEM**

For the deck 2D plate elements are chosen. The elements have four nodes with each 6 DOF. For the piles 1D beam elements are used. These beam elements are like a line connecting two nodes. The element has 3 DOF per node.
The elements of the deck are 1 m wide, so nodes occur every meter. This is chosen for the distribution of the loads on the nodes, because loads can only be placed in nodes. Also the mode shape given by the eigenvector is given in discrete points, only in the nodes. To see bending of the deck, multiple elements are needed over 4.5 m. Elements of 1 m wide satisfy that requirement. The calculation in Scia engineer is done for smaller elements, because the defined elements are divided into a mesh. This mesh size is important for the calculation.

The load on the piles is placed at two points on the piles. Nodes are therefore needed at these locations $z = -3$ and -$10$ m.

**Added Mass**

Water around the piles and deck vibrates along with the structure. The mass of this water in contact with the structure therefore has to be added to the mass of the structure. The amount of water is uncertain. The calculation is therefore performed with and without added mass. It is assumed that the amount of water around the piles vibrating together with the piles is equal to the amount of water in the piles. This leads to the total added mass per pile of 

$$m_{\text{added mass}} = \rho_{\text{water}} \cdot A_{\text{pileplug}} \cdot 2 \text{ kg/m.}$$

Per pile the length of 13 m is used, which is the water depth. Water vibrating along with the jetty deck is not taken into account.

**Calculation**

The dynamic calculation done in Scia Engineer does not included loads. The program calculates the natural frequencies and the mode shapes of the structure. Scia Engineer makes use of lumped mass matrices. This means that the mass matrix is forced to be diagonal. The eigenvectors given by Scia Engineer are mass orthonormalized. This means that the eigenvectors are scaled to the mass matrix in the way that the modal mass matrix becomes an identity matrix. This scaling has no influence on the modal analysis, but is important to be aware of when doing other operations using the output of Scia engineer (Nemetschenk Scia, 2011).

The solver used by Scia engineer is a direct solver. The thick plate theory of Mindlin is used. This theory includes shear deformation. The calculation is done assuming linear elastic material behaviour.

**Damping**

In a structure different types of damping are present. A definition of damping, given in Spijkers, et al. (2006) is: “dissipation of mechanical vibration energy from the system.” This can be caused by heat production (friction) or loss of energy to the surrounding of the system.

The above described modal analysis uses diagonal matrices to get to a decoupled system of equations. This can only be used, if the damping matrix is forced to be diagonal. This is also done in this research. A sensitivity analysis is needed to see whether the simplification to a diagonal damping matrix is allowed. To get a diagonal damping matrix Rayleigh damping is used. This is a proportional damping.

Different types of damping are present in the jetty structure: material damping, aerodynamic damping, hydrodynamic damping, and soil damping. In this research the damping of the deck material is expected to be dominant. The damping is approximated as a viscous damping. For common used materials damping coefficients have been approximated for different situations.
Bachmann (1995) published damping coefficients of concrete. For un-cracked concrete and small stress intensity 0.01 is recommended as damping coefficient. For cracked concrete and higher stress intensity, the damping coefficient is higher, about 0.04.

The situation when waves hit the jetty deck occurs for large waves and causes large pressure peaks. The stress intensity is therefore expected to by high, as well as the amount of cracks in the concrete. Hence a damping coefficient close to 0.04 is used.

The damping coefficient differs per mode shape. For lower mode shapes the modal mass participation is higher than for higher mode shapes. This causes the critical damping to decrease, and the damping ratio to increase with a higher mode shape. (Chowdhury, et al, -) Assuming the same damping coefficient for every mode shape is not realistic. If a damping coefficient of 0.03 is assumed for the first mode shape and a damping ratio of 0.05 for the fifth mode shape, then the rest of the mode shapes can be calculated by linear interpolation. The relation of proportional damping is shown below:

\[ C = \alpha M + \beta K \]  

This proportional damping has a constant \( \alpha \) and \( \beta \) for all mode shapes. With \( \alpha \) is 0.023 rad/s and \( \beta \) 0.018 s/rad. This results in the following damping coefficients; listed in Table 5.

<table>
<thead>
<tr>
<th>Mode Shape</th>
<th>Natural Frequency [rad/s]</th>
<th>Damping Coefficient [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.8</td>
<td>0.03</td>
</tr>
<tr>
<td>2</td>
<td>5.9</td>
<td>0.03</td>
</tr>
<tr>
<td>3</td>
<td>6.0</td>
<td>0.04</td>
</tr>
<tr>
<td>4</td>
<td>16.5</td>
<td>0.05</td>
</tr>
<tr>
<td>5</td>
<td>38.5</td>
<td>0.05</td>
</tr>
<tr>
<td>6</td>
<td>43.4</td>
<td>0.05</td>
</tr>
<tr>
<td>7</td>
<td>43.4</td>
<td>0.05</td>
</tr>
<tr>
<td>8</td>
<td>43.5</td>
<td>0.05</td>
</tr>
<tr>
<td>9</td>
<td>43.5</td>
<td>0.05</td>
</tr>
<tr>
<td>10</td>
<td>43.7</td>
<td>0.05</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mode Shape</th>
<th>Natural Frequency [rad/s]</th>
<th>Damping Coefficient [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>44.0</td>
<td>0.05</td>
</tr>
<tr>
<td>12</td>
<td>44.4</td>
<td>0.05</td>
</tr>
<tr>
<td>13</td>
<td>44.9</td>
<td>0.05</td>
</tr>
<tr>
<td>14</td>
<td>45.5</td>
<td>0.05</td>
</tr>
<tr>
<td>15</td>
<td>46.2</td>
<td>0.05</td>
</tr>
<tr>
<td>16</td>
<td>47.2</td>
<td>0.05</td>
</tr>
<tr>
<td>17</td>
<td>48.3</td>
<td>0.05</td>
</tr>
<tr>
<td>18</td>
<td>49.2</td>
<td>0.05</td>
</tr>
<tr>
<td>19</td>
<td>49.3</td>
<td>0.05</td>
</tr>
<tr>
<td>20</td>
<td>49.4</td>
<td>0.05</td>
</tr>
</tbody>
</table>

**Soil**

The piles of the jetty are supported by soil. The piles are reaching until 26 meter beneath the sea bottom. Over this total by soil embedded length the load is transmitted from the pile to the soil. Soil is inhomogeneous and has a varying stiffness and damping over its depth and directions. Realistic modelling of soil is time consuming, and complex. Therefore the soil is modelled in Scia in a more simplified manner. Two simple methods are compared, because the dynamic behaviour of the jetty is highly influenced by the modelled stiffness of the soil. The soil is modelled by two different methods shown in Figure 44:

- The DOF at the end of the piles are fixed;
- Soil is modelled as springs over 2.5 m with different stiffness’s.
The piles are clamped in the soil at a depth of 5 m below the sea bottom. This is about 5 times the diameter of the pile, which is a rough design rule.

Six springs are used to model the soil. The magnitude of the springs is shown in Table 6. The springs are determined for the design of the second jetty in the Great Bay of Sint Maarten (Lievense, 2006).

Table 6 - Stiffness of Springs Modelling Soil

<table>
<thead>
<tr>
<th>Depth z [m]</th>
<th>X [MN/m]</th>
<th>Y [MN/m]</th>
<th>Z [MN/m]</th>
<th>Rx [MNm/ rad]</th>
<th>Ry [MNm/ rad]</th>
<th>Rz [MNm/ rad]</th>
</tr>
</thead>
<tbody>
<tr>
<td>-13.0 m</td>
<td>0.4</td>
<td>0.4</td>
<td>Free</td>
<td>Free</td>
<td>Free</td>
<td>Free</td>
</tr>
<tr>
<td>-13.5 m</td>
<td>2.1</td>
<td>2.1</td>
<td>Free</td>
<td>Free</td>
<td>Free</td>
<td>Free</td>
</tr>
<tr>
<td>-14.0 m</td>
<td>5.5</td>
<td>5.5</td>
<td>Free</td>
<td>Free</td>
<td>Free</td>
<td>Free</td>
</tr>
<tr>
<td>-14.5 m</td>
<td>12.6</td>
<td>12.6</td>
<td>Free</td>
<td>Free</td>
<td>Free</td>
<td>Free</td>
</tr>
<tr>
<td>-15.0 m</td>
<td>28</td>
<td>28</td>
<td>Free</td>
<td>Free</td>
<td>Free</td>
<td>Free</td>
</tr>
<tr>
<td>-15.3 m</td>
<td>Free</td>
<td>Free</td>
<td>134</td>
<td>483</td>
<td>483</td>
<td>Free</td>
</tr>
</tbody>
</table>

The stiffness’s of the springs are determined with a static calculation. The soil is expected to react differently to dynamic movements of the piles. This difference is not included in the modelling.

5.5. Validation

The dynamic model is validated by comparing results with the theoretical values. It is also checked if resonance occurs in the model.

SDOF

The dynamic model is checked by comparing the static displacement and resonance with the theoretical values for horizontal translations. The horizontal translation perpendicular to the jetty axis (y-direction) is chosen, as shown in Figure 45. This is done because the static displacement can easily be calculated, and the load activating this mode shape can easily be placed on the jetty. This is a harmonic load placed on the side of the jetty deck.
For the hand calculation the jetty is simplified into a SDOF system. The differences between the SDOF model and the modal analysis are:

- The piles also have a mass in the modal analysis;
- The mode shapes include 3 DOF in the modal analysis;
- The deck has a bending stiffness in the modal analysis;
- The stiffness of the piles.

**Static**

The static displacement calculated with the hand calculation assumes an infinite stiff deck. The stiffness of the piles for the hand calculation is determined as:

\[ k_{p\text{ile}} = \frac{12EI}{L^3} \]

This assumes the piles not to rotate at the deck and at 5 m below the sea bottom. In the Scia model the piles are also clamped below the sea bottom. However, at the jetty deck a rotational stiffness is present.

The static displacement obtained with the hand calculation is \( u = F/k \) 0.008 m, where the dynamic model gives 0.012 m for the same load with 0 rad/s. The difference is expected to be caused by the difference in modelling of the piles. When a static calculation is performed in Scia Engineer the a static displacement of 0.014 m is found.

**Resonance**

For a SDOF the transfer function is known for a harmonic load working on the centre of gravity of the mass. The DAF of the un-damped system goes to infinity for the theoretical situation of the load being present infinitely long and with the frequency of the load being equal to the natural frequency. The model made in this research should show a similar dynamic behaviour for a harmonic load working on the side of the jetty deck. For this check, the only difference with the dynamic analysis is that a harmonic load is placed on the side of the jetty deck, instead of the wave load. The frequency of the harmonic load is varied. The natural frequency of the mode shape of the horizontal translation in y-direction is about 6 rad/s. On the side of the jetty deck a load is placed of:
The transfer function of the modal analysis for a horizontal translation under a harmonic load is shown in Figure 47.

The DAF is only for a small frequency range larger than 2. The range is about 5 rad/s. Only for these frequencies the dynamic behaviour enlarges the amplitude of the vibration significantly. The system reacts statically to frequencies lower than these frequencies; this can be seen from the DAF being almost equal to 1.

The transfer function from the modal analysis shows a maximum DAF of not even 16 for the load frequency being equal to the natural frequency. This is very small compared to the theoretical value of infinity. This is caused by the small duration of the simulation of only 20 seconds. From Figure 48 it can be seen that the amplitude of the vibrations keeps growing. The DAF for a longer simulation is therefore expected to be larger. From Figure 49 it can be seen that the time step of the calculation has a large influence of the occurrence of resonance in the modal analysis.
Numerical Damping

The time step of the calculation has a large influence of the occurrence of resonance in the modal analysis, as can be seen from Figure 48 and Figure 49. This is caused by the numerical solver used to solve the set of differential equations. The time step needed to get the resonance in the system depends on the natural frequency. For a mode shape with a larger natural frequency a smaller time step is necessary to get the resonance in the system. This is a result of the formulation of the Euler Backward method, as shown in equation 5.25. The denominator of the fraction consist out of a part related to the damping ($\zeta$) and to the step width together with the natural frequency ($h^2\omega^2$). This part causes numerical damping. Without viscous damping the function is still damped, because the
natural frequency and step-width are both positive, and make the denominator larger than 1. The smaller $h^2 \omega_i^2$ the closer the denominator is to 1 and the less damping is introduced by the method.

\[
\frac{v_{n+1} - h\omega_n^2 v_n + h \frac{\phi^T F(t+1)}{\phi^T M\phi}}{1 + 2h\zeta_n \omega_n + h^2 \omega_n^2} \quad v_{n+1} = v_n + h \cdot v_{n+1} \quad [5.25]
\]

A larger natural frequency causes more damping in the solver. The mode shapes with a larger natural frequency belong to vertical vibrations. The needed step size is therefore also determined for the vertical direction. The above described investigation with a SDOF system for a horizontal movement is therefore repeated in vertical direction. In this direction a time step of less than 0.002 seconds is needed to have a growing amplitude. However for the simulations this time step is too small in order to keep an acceptable calculation time. For a time step of 0.01 s the DAF is 3.5 for the movement in vertical direction. This is also much larger than 1, and is expected to be noticed in the results. Therefore a time step of 0.01 s is used.

Other mode shapes are not investigated, because it is difficult to place the corresponding load on some of the mode shapes. This is also what is important for the dynamic reaction of the jetty to the wave load: does the shape of the wave load corresponds to one of the mode shapes. Together with the frequency in which it acts on the jetty it determines the dynamic reaction of the jetty.

**Simply Supported Beam**

The modal analysis is checked by comparing the result with a continuous beam calculation. This is done for a simple structure, a simply supported beam.

Instead of the jetty a simply supported beam is modelled in Scia Engineer. The simply supported beam is also modelled using a continuous model. The results of both analyses are compared. The results show two almost equal vibrations. Also the natural frequencies are close to each other. The calculations and results can be seen in appendix VIII.
6. Results

From the dynamic analysis of the first jetty on Sint Maarten the results are presented in three different ways: dynamic amplification factor, displacements and the variance spectrum of the displacement. The response of the jetty to five different wave spectra are compared in this chapter.

First in section 6.1 and 6.2 the mode shapes and natural frequencies of the system are presented. After that the indicators and products made of the response are discussed. The dynamic amplification factor (DAF) is discussed in section 6.3. The variance spectrum of the displacement is derived in section 6.4. Calculation of the stresses from the response is shown in section 6.5, and in section 6.6 the needed exceedance probability of the results is determined.

After that the results of three wave spectra are presented in more detail:

- Section 6.7 Results of Large Wave Spectrum;
- Section 6.8 Results of Large Wave Heights in High Frequencies;
- Section 6.9 Results of Waves just Hitting the Jetty Deck.

In section 6.10 two wave spectra with frequencies near the natural frequency are compared to the results from the previous three wave spectra. This is done in order to check the dynamic reaction to these waves, and to get the dynamic reaction of the jetty to all different wave frequencies that can occur near the jetty of Sint Maarten.

Section 6.11 discusses the found results, and section 6.12 describes a summary of this chapter.

In the previous chapters several model choices are discussed. The results presented in this chapter are made with the following model choices:

- Angle between wave and jetty of 0, 10, 25 degree;
- Expansion joint without boundary conditions (section 5.3);
- Soil modelled with springs (section 5.3);
- Simulation of 100 s, with a time step of 0.01 s;
- Damping is included.

6.1. Mode Shapes

20 mode shapes are included in the response. The different mode shapes of one module of 152 m are presented in this section. The mode shapes number 9 to 18 are presented in appendix IX.

1. The first mode shape is a translation in y-direction, with $\omega_1$ of about 5.8 rad/s.
2. Mode shape 2 is a translation in x-direction. With $\omega_2$ of about 5.9 rad/s.

3. Mode shape 3 is rotation in $xy$-plane. With $\omega_3$ of about 6.0 rad/s.

4. Mode shape 4, shows bending in the $xy$-plane, with $\omega_4$ of about 16 rad/s.
5. This mode shape, shows also bending in the $xy$-plane, with $\omega_5$ of about 38 rad/s.

6. Modes shape 6 shows bending in both the $zx$-plane and in the $yz$-plane. The jetty deck is not a small line, but it is a grey area because the middle of the deck is lower than the sides of the deck. This can be seen in the axial view on the right. This is further explained at the next mode shape. $\omega_6$ is about 43 rad/s.

7. Mode shape 7 shows bending in both $zx$-plane as in $yz$-plane. In Figure 50 a close up is given of the deformation. This shows the curvature of the jetty deck in horizontal direction. $\omega_7$ is about 43 rad/s.

![Figure 50 - Close up of Bending in $yz$-plane](image)
8. Mode shape 8 also shows bending in both the zx-plane and in the yz-plane. $\omega_8$ is about 43 rad/s.

Mode shape 7 to 18 all show bending in both the zx-plane and in the yz-plane. Mode shapes 9 to 18 are higher modes than the previous two modes. The number of thoughts and tops enlarges with the higher modes. The type of bending is equal, therefore these mode shapes are shown in appendix IX.

19. Mode shape 19 shows a new deformation. $\omega_{19}$ is about 49 rad/s.

20. Mode shape 20 shows a higher order of the same type of deformation as found in the previous mode shape. $\omega_{20}$ is about 49 rad/s.
In the figures showing the mode shapes the piles have a totally unrealistic shape. This is caused by the enlargement of the displacement and because only five nodes are present over the length of the 1D element. The figure only shows the displacements in the nodes, and connects the nodes by a straight line. Therefore the displacement in between the nodes is not accurately displayed.

6.2. Natural Frequencies

The natural frequencies corresponding to the modes shapes are shown in Table 7.

**Table 7 - Lowest 20 Natural Frequencies of the Jetty of Sint Maarten**

<table>
<thead>
<tr>
<th>Natural Frequencies</th>
<th>f [Hz]</th>
<th>$\omega$ [rad/s]</th>
<th>T [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.93</td>
<td>5.8</td>
<td>1.08</td>
</tr>
<tr>
<td>2</td>
<td>0.94</td>
<td>5.9</td>
<td>1.06</td>
</tr>
<tr>
<td>3</td>
<td>1.0</td>
<td>6.0</td>
<td>1.05</td>
</tr>
<tr>
<td>4</td>
<td>2.6</td>
<td>16.5</td>
<td>0.38</td>
</tr>
<tr>
<td>5</td>
<td>6.1</td>
<td>38.5</td>
<td>0.16</td>
</tr>
<tr>
<td>6</td>
<td>6.9</td>
<td>43.4</td>
<td>0.14</td>
</tr>
<tr>
<td>7</td>
<td>6.9</td>
<td>43.4</td>
<td>0.14</td>
</tr>
<tr>
<td>8</td>
<td>6.9</td>
<td>43.5</td>
<td>0.14</td>
</tr>
<tr>
<td>9</td>
<td>6.9</td>
<td>43.5</td>
<td>0.14</td>
</tr>
<tr>
<td>10</td>
<td>7.0</td>
<td>43.7</td>
<td>0.14</td>
</tr>
<tr>
<td>11</td>
<td>7.0</td>
<td>44.0</td>
<td>0.14</td>
</tr>
<tr>
<td>12</td>
<td>7.1</td>
<td>44.4</td>
<td>0.14</td>
</tr>
<tr>
<td>13</td>
<td>7.1</td>
<td>44.9</td>
<td>0.14</td>
</tr>
<tr>
<td>14</td>
<td>7.2</td>
<td>45.5</td>
<td>0.14</td>
</tr>
<tr>
<td>15</td>
<td>7.4</td>
<td>46.2</td>
<td>0.14</td>
</tr>
<tr>
<td>16</td>
<td>7.5</td>
<td>47.2</td>
<td>0.13</td>
</tr>
<tr>
<td>17</td>
<td>7.7</td>
<td>48.3</td>
<td>0.13</td>
</tr>
<tr>
<td>18</td>
<td>7.8</td>
<td>49.2</td>
<td>0.13</td>
</tr>
<tr>
<td>19</td>
<td>7.8</td>
<td>49.3</td>
<td>0.13</td>
</tr>
<tr>
<td>20</td>
<td>7.9</td>
<td>49.4</td>
<td>0.13</td>
</tr>
</tbody>
</table>
6.3. DAF

The DAF is taken as indicator, to see if the dynamic behaviour of the jetty of Sint Maarten enlarges the maximum displacement of the jetty deck. An enlargement of the maximum displacement will also cause larger stresses in the structure.

The ratio between the maximum dynamic displacement and the static displacement is called the dynamic amplification factor (DAF).

\[
DAF = \frac{w_{\text{dynamic, max}}}{w_{\text{static, max}}} \quad [6.1]
\]

The DAF used in this research is the maximum found dynamically calculated displacement at a node on the jetty deck at a moment in time, divided by the maximum found statically calculated displacement. The static displacement is derived for every time step by dividing the load of that times step by the stiffness. The maximum statically and dynamically displacement are not required to have occurred at the same location are moment in time. A more detailed explanation is presented in the following section.

The used dynamic displacement has to be a scalar, and not be depending on time. Therefore the amplitude of the response is used. However in the dynamic calculations performed in this research there is no analytical formulation of the response and the amplitude is unknown. Therefore the maximum displacement is taken from the response. This maximum displacement includes all mode shapes, because the displacement is a summation of the movements in all 20 considered mode shapes. Only the displacements of the nodes on the jetty deck are used.

The definition of the static displacement is the load divided by the stiffness, as shown in equation 6.2. For a modal analysis there is not one load or one stiffness, but there are several. The load vector is therefore divided by the modal stiffness matrix. This is equal to dividing the load vector by the modal mass matrix multiplied by the natural frequency squared. This system of equations is separated into a decoupled equation per mode (the \(i^{\text{th}}\) mode). This results in the formulation shown in equation 6.3.

\[
w_{\text{stat}} = \frac{F}{k} \quad [6.2]
\]

\[
w_{\text{stat},i} = \frac{\tilde{\xi}^T \tilde{F}(t)}{\tilde{\xi}^T \tilde{K} \tilde{\xi}} = \frac{\tilde{\xi}^T \tilde{F}(t)}{\tilde{\xi}^T \tilde{M} \tilde{\omega}^2} \cdot \frac{1}{\omega_i^2} \quad [6.3]
\]

\[
w_{\text{stat,max}} = \max \left( \sum_{i=1}^{m} \left| w_{\text{stat},i} \right| \right) \quad [6.4]
\]

The static displacement is a summation of all the modes together, as shown in equation 6.4. The static displacement used in the DAF is the maximum value. The absolute values are taken, because the mode shapes describe one shape of a vibration that contains both the positive and negative shape. Therefore the maximum displacement is found when the absolute values of the shapes are added. This method slightly overestimates the static displacement, which leads to a lower DAF.
When the absolute value is taken after summation over the mode shapes this results in a DAF of about 0.05 higher. This difference is not significant for this research.

6.4. Response Spectrum

To see in which shapes the jetty vibrates, the variance spectrum is made of the displacement. This spectrum shows which frequencies are present in the vibrations of the response. The larger the amplitude of a frequency in the displacement of the response of the system, the higher the peak in the spectrum is.

Figure 51 shows the step discussed in this section.

![Flow Chart](image)

Figure 51 - Part of the Flow Chart from Chapter 1.

The variance spectrum is made from the displacement. The response of the structure is given as a displacement varying in time. For the spectrum, this signal is transformed to being variable in frequency. This transformation can be done with a Fourier transformation. If the displacement in x-direction is called \( w(t) \), the Fourier transformation is given by equation 6.5 and 6.6. The variance spectrum is given by the combination of these two, shown in equation 6.7.

\[
S_x(\omega) = \frac{1}{\pi} \int_0^T w(t) e^{i \omega t} dt \quad [m \cdot s] \tag{6.5}
\]

\[
S_x^*(\omega) = \frac{1}{\pi} \int_0^T w(t) e^{-i \omega t} dt \quad [m \cdot s] \tag{6.6}
\]

\[
S_{xx}(\omega) = \frac{\pi}{T} S_x S_x^* \quad [m^2 \cdot s] \tag{6.7}
\]

This is a double Fourier Transform, shown in equation 6.7, is necessary because the simulation of the waves used for the calculation include a random phase angle. Or as described in chapter 3; the wave train is a summation of sine functions with a random phase angle. To get a wave spectrum independent of this random phase angle, two Fourier transforms have to be multiplied. The random phase angle is taken out of the sine function by introducing \( B_k \) and \( C_k \), as shown below. (Vrouwenvelder, 2010).

\[
\eta(t) = \sum_{k=1}^N A_k \sin(\omega_k t + \phi_k - k_x \bar{x}) = \sum_{k=1}^N \left( A_k \cos(\phi_k - k_x \bar{x}) \sin(\omega_k t) + A_k \sin(\phi_k - k_x \bar{x}) \cos(\omega_k t) \right)
\]

\[
B_k \quad C_k \tag{6.8}
\]

At the end of the calculation the sine and cosine with the random phase angle and wave number drop out of the equation. This results in a variance spectrum independent of the random phase angle and wave propagation speed.
6.5. Stress

The wave load acting on the jetty deck and piles cause stresses in the structure. In this section the stresses in the piles of the jetty are determined. Figure 52 shows the steps from the flowchart taken in this section.

![Flowchart](image)

**Figure 52 - Part of the Flowchart Introduced in Chapter 1**

The bending moment in the piles can be determined from the displacement in x and y-direction of the deck. For simplification only the displacement of the jetty deck is taken as variable to determine the stresses in the piles. The displacement of the jetty deck includes the wave loading on the jetty and the dynamic behaviour of the structure. An enlargement of the displacement at the jetty deck because of the dynamic behaviour of the structure also causes an enlargement of the stresses. This can easily be imagined with Figure 53.

![Pile Displacement](image)

**Figure 53 - Schematization of Pile with a Displacement at the Jetty Deck in respectively x, y, and z-direction**

The maximum stress in the pile is for a displacement in x, y, z-direction, marked in Figure 53 by the red dot. The red dot is just located above the springs, because the stresses in the pile at the location of the springs are not expected to be realistic. In reality the force is transferred to the soil over a much longer distance of the pile, which leads to a different stress distribution.

The maximum stress is calculated using the model of the jetty in Scia Engineer. A unit displacement is placed on the deck of the jetty at a pile row. The stress found by Scia Engineer shows the stress in the pile caused by a unit displacement. This can be used to determine the stress in the pile caused by the dynamic reaction, because a linear elastic behaviour is assumed. For large stresses this assumption is not valid. The stresses caused by the unit displacement of 1 mm are shown in Table 8. The table shows the stress in the piles for two different types of modelling of the soil. The soil can be modelled with the use of springs, as presented in Figure 53. Or the soil can be modelled by fixing the piles in all
degrees of freedom. Both situations are described in section 5.4, and lead to different stress in the piles.

<table>
<thead>
<tr>
<th></th>
<th>$W_x=1\text{mm}$</th>
<th>$W_y=1\text{mm}$</th>
<th>$W_z=1\text{mm}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{vm}$ in pile, soil as springs</td>
<td>1.3</td>
<td>1.3</td>
<td>3.1</td>
</tr>
<tr>
<td>$\sigma_{vm}$ in pile, soil fixes piles</td>
<td>1.4</td>
<td>1.4</td>
<td>17</td>
</tr>
</tbody>
</table>

The Von Mises stress ($\sigma_{vm}$) is based on the normal and shear stresses together. Scia Engineer does not show the stresses in $x$, $y$ and $z$-direction separately. With this data the stress in a pile caused by the dynamic reaction can be found by multiplying the displacement at a node with the values in Table 8. Only the nodes at the jetty deck located at a pile row are used.

### 6.6. Exceedance Probability of DAF

The response of the jetty is made for different wave spectra. Each wave spectrum is assumed to represent a sea state of about 3 hours. The simulations however cannot be done for 3 hours. The results made with shorter simulations have to be combined, to represent the result after a 3 hours period. In this section it is explained how this is performed.

The jetty is designed to survive a storm with a 100 year return period. It is assumed that the extreme part of the storm has a duration of about 3 hours, one sea state. For the situation of Sint Maarten the wave period is about 10 seconds. During 3 hours about 1000 waves from a 100 year return period wave spectrum hit the jetty. It is required that the jetty survives every of these 1000 waves. Therefore the DAF is needed with an exceedance probability of 0.1 %.

The simulation in this research has a duration of 100 s. The first 12 seconds are used to create the right initial conditions. The remaining 88 seconds are considered in the dynamic analysis. In this period about 10 waves hit the jetty deck. It is not possible to simulate 1000 waves, because of the large calculation time corresponding to 1000 waves. Hence several DAFs and a trend line are used to calculate the DAF with an exceedance probability of 0.1 %. From the about 10 waves that pass during a simulation the largest DAF is taken. The DAFs are independent random variables. However the probability density function of the DAF is not known. Therefore the extreme probability density function is also unknown. By repeating the simulation several times, a trend line can be made from the results. This trend line is used to get the DAF at an exceedance probability of 0.1 %. The method is explained with an example in Table 9. Figure 54 shows the corresponding plot of the example, with the trend line.

**Method**

The first simulation is done. About 10 waves pass, causing different DAF’s. The largest of these 10 DAF’s is taken. This DAF found from the first 10 waves is expected to have an exceedance probability of $1/10=10\%$. The largest DAF of the second simulation is also the largest from the about 10 waves that passed. However, when this second DAF is larger than the one from the first simulation, than this second DAF is the largest of 20 waves. Its exceedance probability than becomes $1/20=5\%$. In this way the calculation is proceeded.
The DAFs from the five simulations are placed from small to large. Their exceedance probability is respectively 10 %, 5 %, 3.3 %, 2.5 % and 2.0 %. The extreme value distribution of the independent random variables is known to be an asymptote, because of the central limit theorem (Stichting CUR, 2006). From the five points the DAF with 0.1 % exceedance probability is therefore approximated with a log trend line, shown in Figure 54.

<table>
<thead>
<tr>
<th>Number of Waves</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exceedance Probability</td>
<td>10 %</td>
<td>5.0 %</td>
<td>3.3 %</td>
<td>2.5 %</td>
<td>2.0 %</td>
</tr>
<tr>
<td>DAF Y</td>
<td>1.3</td>
<td>1.3</td>
<td>1.4</td>
<td>1.5</td>
<td>1.7</td>
</tr>
</tbody>
</table>

Table 9 - Example of Calculation of 0.1 % Exceedance Probability, from five Simulations

Sensitivity of the Method
A large DAF can be indicated by the trend line, caused by a wide range of the results. The largest found DAF during the five simulations can still be below 1, but if the smallest DAF is 0.3, the trend line indicated a DAF of 2.2 for 0.1 % exceedance probability. This can be seen in Figure 55. When the lowest number is taken out, the result becomes 1.4 instead of 2.2 (the red line instead of the blue line in Figure 55). This indicates that the trend line is very sensitive.

Figure 54 - Example of Exceedance Probability

Figure 55 - Example of Sensitivity of Exceedance Probability Trend line
Smaller Wave Periods

The wave spectrum used in the example is a large wave with a wave period of about 10 s and corresponds to a return period storm of 1/100 years. This wave spectrum is described in section 3.3 in paragraph Large Wave Spectrum. For smaller wave spectra investigated in this research, the wave periods are smaller. Therefore more waves occur during one sea state, and during a simulation of 100 s. Also the return period is different. In order to find the difference in dynamic reaction of the jetty to different wave spectra, the results with equal exceedance probability during one sea state are compared (0.1 % per sea state). For the wave spectra with smaller wave periods, the value with an exceedance probability of 0.1 % is not the maximum value expected to occur in a sea state neither in the lifetime of the jetty. However, for comparison of the dynamic behaviour the same exceedance probability per sea state is chosen. This value is determined using the same procedure as described in section ‘method’. Only the first and second rows of Table 9 are different, depending on the peak period of the wave spectrum.

6.7. Results of Large Wave Spectrum

In this section the results are presented of the dynamic analysis of the jetty of Sint Maarten being subjected to waves from the large wave spectrum. This wave spectrum is created by superimposing two wave spectra:

- $H_s$: 7.1 m, $T_p$: 12.45 s;
- $H_s$: 5.2 m $T_p$: 9.5 s.

The significant wave height of the superimposed wave spectrum is 9.0 m. The wave spectrum is described in chapter 3. In this wave spectrum the highest investigated waves occur. A wave included in this wave spectrum is shown in proportion to the jetty in Figure 56. The wave is proposed in Figure 56 as a linear wave described by single sine function, which is different from how it is simulated in this research (Chapter 3).

![Figure 56 - Wave of Large Wave Spectrum in Proportion to Jetty, $H_s$=7.0 m, $T_p$=12 s, $A_s$=114 m](image)

Displacement

After the modal analysis is done the vibrations of the jetty are known. The response of the jetty to the wave loads is known in every node, in $x$, $y$ and $z$-direction. The response is shown of node 391, which is located at $x$= 55 m, $y$= -10 m and $z$= 1.6 m. This is a node on the jetty deck. The location is shown with the red dot in Figure 56. In $x$-direction the displacements over the total length of the jetty are equal, therefore it does not matter which node is chosen for the vibrations in $x$-direction (axial direction). From the mode shapes including displacements in $y$-direction (perpendicular to jetty axis) it can be seen that nodes at the middle or head of the module are not preferred. From the mode shapes showing movements in vertical direction it can be seen that nodes located at a pile row...
show less vibrations. The location of node 391 satisfies all these restrictions. From a comparison between the variance spectra and the displacements of different nodes along the jetty it is found that node 391 is representative for the vibrations of the jetty deck.

$W_{x\text{, dyn}}$ is the maximum displacement found by the dynamic response by one of five simulations. The displacement is not extrapolated to a value with a 0.1% exceedance probability per storm. An indication of this 0.1% exceedance probability displacement can be found by multiplying the statically computed maximum displacement $W_{x\text{, stat}}$ by the DAF with an exceedance probability of 0.1%.

**X-direction**

The displacement in axial direction ($W_{x\text{, dyn}}$) of the jetty (x-direction) is shown in Figure 57.

![Figure 57 - Displacement in x-direction of the Large Wave Spectrum](image)

The displacement in axial direction has a maximum of about 0.1 m for the modelled situation. This is the maximum found by 5 simulations of 100 s. From one of the simulations the displacements are shown in Figure 57. This figure does not show the maximum displacement, because the maximum displacement does not occur at node 391 of which the displacement is shown.

The maximum displacement of 0.1 m is not expected to be realistic, because the linear wave theory is not suitable for this type of waves which leads to an overestimation of the horizontal displacements for this wave spectrum.

Only one part of the jetty (152 m) is modelled. Interaction between two neighbouring jetty parts is not included in the model. However for displacements exceeding 0.05 m in axial direction the different jetty parts meet and transmit forces from the one module to the other. This happens over the length the jetty until the force reaches the abutment, or the piles have transmitted the load to the soil. This makes large displacements in axial direction impossible. The interaction between the two jetty parts after a displacement of 0.05 m cannot easily be modelled with the method chosen in this research. The influence of the neighbouring jetty is investigated in chapter 7.
The displacement in axial direction looks smooth. When the displacement is compared to the maximum load in x-direction on the structure many similarities are found. This can be seen in Figure 58. The both graphs show the same pattern. This is because the displacement is dominated by the total load in x-direction on the piles. This also seems to determine the frequency of vibration. Which frequency dominates the vibration can be seen in Figure 59. The small graph at the right is an enlargement of the peak below 1 rad/s.

From the variance spectrum of the displacement in Figure 59 it can be seen that the dominant frequency in the response are the load frequencies. The natural frequencies located near 6 rad/s and around 40 rad/s are not visible at the plot of the variance spectrum of the displacement. This means that the amplitudes of the vibrations in the natural frequencies are much smaller than the one at the wave frequency. The dynamic behaviour of the structure is therefore not expected to lead to significant larger displacements in axial direction.

The DAF shows the ratio between the maximum displacement of the dynamic calculated vibrations and the static displacement. In x-direction the DAF found in this research for the first jetty of Sint Maarten is less than 1.0. This is the DAF with an exceedance probability of 0.1 %. This is in agreement
with the results found from the variance spectrum of the displacement. The static displacement is equal to the maximum displacement found with the dynamic calculation of 0.1 m. The maximum Von Misses stress occurring in the piles as a consequence of the displacement of the jetty deck in x-direction is $\sigma_{vm}$ 149 N/mm$^2$. This is the stress in the steel of the piles, more information about the stress is shown in section 6.5.

Summary of results of the large wave spectrum in x-direction:

- $DAFx$: up to 1.0
- $W_{x,dyn}$: 0.1 m
- $W_{x,stat}$: 0.1 m
- $\sigma_{vm}$: 149 N/mm$^2$

The displacement in horizontal directions are expected to overestimate reality for the large wave spectrum, because the linear wave theory is not suitably for this type of waves. This is further described in section 3.4.

**Y-direction**

Figure 60 shows the displacement perpendicular to the jetty axis (y-direction). The maximum found horizontal displacement perpendicular to the jetty axis (in y-direction) is about 0.06 m for this situation (found by one of the five simulations). The frequency of vibration is about 0.1 Hz. This is about the frequency of the waves. A higher frequency vibration with a small amplitude can be seen in Figure 60 for small periods of time (for instance between 90 s and 95 s).
The upper graph of Figure 60 shows the pressure at the side of the jetty deck. This pressure is only present when a wave hits the jetty deck. From Figure 60 it can be seen that the maximum displacement does not occur when the horizontal pressure on the side of the jetty deck reaches $x = 55$ m, which is the location of which the vibrations are displayed in the lower graph of Figure 60. This is because the jetty rotates and translates in the horizontal plane, which is determined by the total load in horizontal direction and the moment caused by the load. Figure 61 shows the distribution of the load over the length of the jetty deck at four moments in time.
From Figure 61 it can be seen that at 74 s no pressure acts at 55 m (the dotted line in the figures). However, the displacement in Figure 60 is at its maximum. This is caused by the load between the 0 and 50 m, which causes a rotation around the middle of the length of the jetty (middle drawing of Figure 62).

When the vibrations are seen from top view, the shapes in which the jetty module of 152 m vibrates become visible. The vibrations of the jetty deck seems to be a combination of the mode shapes with the lower natural frequencies. The translation in y-direction, combined with rotation in horizontal direction and a little bending over the width (y-direction) of the jetty deck, as shown in Figure 62.

Figure 63 shows the variance spectrum of the displacement perpendicular to the jetty axis (y-direction).

From the variance spectrum of the displacement it can be seen that the dominant frequencies in the response are the load frequencies. The natural frequencies located near 6 rad/s and around 40 rad/s are not visible in the plot of the variance spectrum of the displacement. The dynamic behaviour of the structure is therefore not expected to lead to significant larger displacements in y-direction.
The DAF confirms the findings in the variance spectrum of the displacement. In y-direction the DAF is less than 1.1 for the elaborated situation. This is the DAF with an exceedance probability of 0.1%.

The static displacement is almost equal to the maximum displacement found with the dynamic calculation. The maximum static displacement in y-direction is 0.06 m. The maximum Von Misses stress occurring in the piles as a consequence of the displacement of the jetty deck in y-direction is \( \sigma_{vm} = 82 \, \text{N/mm}^2 \).

Summary of results of the large wave spectrum in y-direction:

- \( \text{DAF}_y \): 1.0 to 1.1
- \( W_{y,\text{dyn}} \): 0.06 m
- \( W_{y,\text{stat}} \): 0.06 m
- \( \sigma_{vm} \): 82 N/mm\(^2\)

**Z-direction**

Figure 64 shows the displacement in vertical direction (z-direction) and the vertical pressure to the jetty deck at the same location. The maximum displacement in z-direction is 0.008 m. This displacement is small because the piles are loaded in axial direction in which they are very stiff. The vertical displacement of the jetty deck shows small oscillating parts at a high frequency (as can be seen at \( t = 8 \) to 11 seconds). This is expected to be caused by a very high frequency, which is expected to be a natural frequency. The natural frequencies including vertical movement are very high. It can be seen from the graph that the amplitude of vibration in the high frequency is much smaller than for the wave period of about 10 s.
When comparing the both graphs in Figure 64, it can be seen that the jetty deck moves upward when an upward pressure is present at the bottom of the jetty deck. A downward movement sometimes occurs, just before the upward movement. This is expected to be caused by the approaching wave, which causes an upward load just at the other side of a pile row. This causes the part of the jetty deck with the water contact to move upward, and the neighbouring part to move down.

Figure 65 shows the variance spectrum of the displacement in vertical direction. It can be seen from the graph that the largest peaks are at the wave frequencies. These are the dominant frequencies in the response.
The DAF is for the vertical direction for many of the simulation smaller than one. DAF’s around 0.7 are found. From the trend line a DAF with an exceedance probability of 0.1 % around 1.1 is indicated for the elaborated situation. The static displacement found by the simulations is therefore larger than the maximum displacement found with the dynamic calculation. The maximum static displacement in z-direction is 0.01 m. The maximum Von Misses stress occurring in the piles as a consequence of the displacement of the jetty deck in z-direction is $\sigma_{vm}$ 25 N/mm².

Summary of results of the large wave spectrum in z-direction:

- $\text{DAF}_z$: 0.6 to 1.1
- $W_{z\text{ dyn}}$: 0.008 m
- $W_{z\text{ stat}}$: 0.01 m
- $\sigma_{vm}$: 25 N/mm²
6.8. Results of Large Wave Heights in High Frequencies

For the second wave spectrum a wave climate with a much higher wave frequency is chosen, that still has high waves. The frequency of the waves hitting the jetty deck is expected to be important for the dynamic behaviour, therefore this wave spectrum is chosen. The wave spectrum is further described in chapter 3. The wave spectrum is based on:

- $H_s$: 5.0 m and $T_p$: 7.0 s.

A single wave with the wave height of 5.0 m from this wave spectrum is shown in proportion to the jetty in Figure 66. The wave is proposed in Figure 66 as a linear wave described by single sine function, which is different from how it is simulated in this research.

![Wave of Large Wave Height in High Spectrum in Proportion to Jetty, H= 5.0 m, T= 7.0 s, λ = 60 m](image)

**X-direction**

Figure 67 shows the displacement in axial (x-direction). The maximum displacement in axial direction is about 0.007 m. This is found by 5 independent simulations, done with this wave spectrum.

![Displacement in X-direction of Large Waves in High Wave Frequencies](image)

The frequencies of the vibrations have a period between the 5 s and 10 s. This is the frequency of the waves in the wave spectrum; the dominant wave period is 7 s. These wave frequencies can be found in Figure 67 causing the large amplitude vibration. Around this large period vibration small vibrations can be seen in a much higher frequency. The frequencies of these higher frequency vibrations can be found in the variance spectrum of the displacement in Figure 68.
It can be seen in Figure 68 that the frequency of the load is the dominant frequency. The higher frequency vibrations are near 5 rad/s, this is also where the first natural frequencies are located. The first natural frequencies cause small vibrations in the reaction of the jetty to the wave loading.

It can be seen that a peak just below 5 rad/s is present in Figure 68, however no natural frequency is present below 5 rad/s. This peak is expected to be caused by the time step of calculation, which is too large to accurate determine the frequency in the vibration. When a smaller time step is used only the peak near 6 rad/s occurs. In section 6.9 a smaller time step is used.

The DAF is between the 1.1 and 1.5. For a DAF with an exceedance probability of 0.1 % a maximum value of 2.1 is found. The maximum static displacement in x-direction is 0.006 m. This is much smaller than the displacement found in the previous section by the wave spectrum with a significant wave height of 9.0 m. This is caused by the linear wave theory used to describe the water particle velocities. The previous wave spectrum does not comply with the conditions to apply the linear wave theory. This causes an overestimation of the water particle velocity for the previous water particle velocity. The maximum Von Misses stress occurring in the piles as a consequence of the displacement of the jetty deck in x-direction is $\sigma_{vm}$ 8.9 N/mm$^2$.

Summary of results in x-direction:

- $\text{DAF}_{x}$: 1.0 to 2.1
- $W_{x,\text{dyn}}$: 0.007 m
- $W_{x,\text{stat}}$: 0.006 m
- $\sigma_{vm}$: 8.9 N/mm$^2$
Y-direction

Figure 69 shows the displacement perpendicular to the jetty axis (y-direction).

The maximum displacement in y-direction is about 0.02 m. The displacements in y-direction looks rough. A vibration with a large period of about 5 s to 10 s and a vibration with a very small period can be seen. The large period is caused by the waves. The small period vibrations are expected to be caused by the natural frequencies. The frequencies of the vibrations can be found in the variance spectrum of the displacement in Figure 70.

From Figure 70 it can be seen that the peak at the wave frequencies (near 1 rad/s) is almost equally large as the peak near 5 rad/s. The peaks around 5 rad/s are caused by the natural frequencies. The lowest natural frequencies are around 6 rad/s.
The DAF confirms the findings in the variance spectrum of the displacement. In y-direction the DAF is about 1.7 for the elaborated situation. The DAF with an exceedance probability of 0.1 % has a maximum of 2.3. The static displacement is smaller than the maximum displacement found with the dynamic calculation. The maximum static displacement in y-direction is 0.01 m. The maximum Von Misses stress occurring in the piles (section 6.5) as a consequence of the displacement of the jetty deck in y-direction is $\sigma_{vm}$ 19 N/mm$^2$.

Summary of results in y-direction:

- DAF$_y$: 1.3 to 2.3
- $W_{y,\text{dyn}}$: 0.02 m
- $W_{y,\text{stat}}$: 0.01 m
- $\sigma_{vm}$: 19 N/mm$^2$

Z-direction

In vertical direction the displacements are shown in Figure 71.

![Displacement in Z-direction](image)

**Figure 71 - Displacements in Z-direction, of Wave Spectrum with Large Waves in High Frequencies**

The maximum displacement in vertical direction is 0.003 m. The displacement does not look smooth, because of a high frequency vibration in the displacement. A vibration at this high frequency can be seen in the figure as a blue part, because of the small period of this vibration the lines in the graph overlay each other. This wave spectrum shows more high frequency parts than the displacement due to the wave loading by the Large Wave Spectrum of section 6.8. The amplitude of the high frequency vibrations is small. Which frequencies are included in the vibration can be seen in the variance spectrum of the displacement in Figure 72.
The large peaks in Figure 72 are located below 3 rad/s, this means that the peaks are not located at natural frequencies. There are also small peaks around 40 rad/s. Around this frequency natural frequencies are located, which have mode shapes including vertical displacements. The DAF in z-direction for the separated simulations of 100 s, is less than one. DAF’s around the 0.5 are found. The trend line shows a DAF of 1.0 for an exceedance probability of 0.1%. This is caused by the wide range in which the DAF’s are found for the separate simulations, as mentioned in section 6.6.

The static displacement is larger than the maximum displacement found with the dynamic calculation. The maximum static displacement in z-direction is 0.006 m. The maximum Von Misses stress occurring in the piles as a consequence of the displacement of the jetty deck in z-direction is $\sigma_{vm} = 11$ N/mm$^2$.

Summary of results in z-direction:

- $DAF_z$: 0.40 to 1.0
- $W_{z,\text{dyn}}$: 0.003 m
- $W_{z,\text{stat}}$: 0.006 m
- $\sigma_{vm}$: 11 N/mm$^2$
6.9. Results of Waves Just Hitting Jetty Deck

From the wave climate study it is found that all different types of waves occur in the Great Bay. Because the dynamic behaviour depends on the frequency of the load, the highest wave frequency is chosen of a wave that still hits the jetty deck, as can be seen in Figure 73. The wave spectrum is further described in chapter 3. The wave spectrum is based on:

- \( Hs \): 3.0 m and \( Tp = 4.0 \) s.

A wave with a wave height of 3.0 m from this wave spectrum is shown in proportion to the jetty in Figure 73. The wave is proposed in Figure 73 as a linear wave described by single sine function, which is different from how it is simulated in this research.

The wave load is expected to be much lower than for the large wave spectra. However the frequency is closer to the lowest natural frequency, which could lead to a larger displacement.

**X-direction**

Figure 74 shows the displacement in axial (x-direction). The frequency of the vibrations is very high. The dominant wave period is 4 s; this is hardly recognizable in Figure 74. The maximum displacement in x-direction is about 0.005 m. The response spectrum of the displacement shows which frequencies are present in the displacement in Figure 75.
Dynamic Analysis of an Open Piled Jetty Subjected to Wave Loading

Figure 75 - Variance Spectra of Displacement in X-direction of the Wave Spectrum with Waves that Just Hit the Jetty Deck

Figure 75 shows large peaks near 6 rad/s. This indicated that this frequency has a larger amplitude than the rest of the frequencies in the displacement. The wave frequency is about 1.5 rad/s. The peaks near 6 rad/s are caused by the natural frequencies of the system. The lowest natural frequencies are 5.8 rad/s, 5.9 rad/s and 6.0 rad/s. In this situation the dynamic behaviour enlarges the amplitude of vibration. The largest DAF found by five simulations of 100 s is 2.6. When using the log trend line, this extrapolates to a DAF of about 2.9 with 0.1 % exceedance probability.

The static displacement is smaller than the maximum displacement found with the dynamic calculation. The maximum static displacement in x-direction is 0.002 m. The maximum Von Misses stress occurring in the piles as a consequence of the displacement of the jetty deck in x-direction is \( \sigma_{vm} = 7 \) N/mm\(^2\).

Summary of results in x-direction:

- \( \text{DAFx} \): 1.8 to 2.9
- \( W_{x,dyn} \): 0.005 m
- \( W_{x,stat} \): 0.002 m
- \( \sigma_{vm} \): 7 N/mm\(^2\)

Y-direction

Figure 76 shows the displacements perpendicular to the jetty axis (y-direction). The frequency of the vibrations is very high. The wave period of about 4 s is hardly recognizable in Figure 76. The maximum displacement in y-direction is about 0.01 m. The variance spectrum of the displacement in Figure 77 shows which frequencies are present in the vibration.
Figure 76 - Displacement in Y-direction of the Wave Spectrum with Waves that Just Hit the Jetty Deck

Figure 77 - Variance Spectra of Displacement in Y-direction of the Wave Spectrum with Waves that Just Hit the Jetty Deck

Figure 77 shows a large peak near 6 rad/s. This indicates that this frequency has a larger amplitude than the rest of the frequencies in the displacement. The peak near 6 rad/s corresponds to the lowest natural frequencies of the system. A small peak is present around 1 rad/s, which is caused by the wave frequencies. The DAF in y-direction found after 5 simulations is 1.7. When extrapolating the results to an exceedance probability of 0.1 % per storm a DAF of 2.1 is found.

The static displacement is smaller than the maximum displacement found with the dynamic calculation. The maximum static displacement in y-direction is 0.007 m. The maximum Von Misses stress occurring in the piles as a consequence of the displacement of the jetty deck in y-direction is \( \sigma_{vm} 15 \text{ N/mm}^2 \).
Summary of results in y-direction:

- $\text{DAF}_y$: 1.6 to 2.1
- $W_{y,\text{dyn}}$: 0.01 m
- $W_{y,\text{stat}}$: 0.007 m
- $\sigma_{vm}$: 15 N/mm²

Z-direction

Figure 78 shows the displacement in vertical direction (z-direction). The vibration shown in the graph shows a very high frequency. The frequency is too high to display in this figure; the lines in the graph overlay each other. This high frequency is expected to be a natural frequency. The longer period frequencies are about 4 s, which is the dominant wave frequency of the wave spectrum. Figure 79 shows the variance spectrum of the displacement.

![Figure 78 - Displacement in Z-direction of the Wave Spectrum with Waves that Just Hit the Jetty Deck](image)

![Figure 79 - Variance Spectra of Displacement in Z-direction of the Wave Spectrum with Waves that Just Hit the Jetty Deck](image)
Figure 79 shows the largest peaks at the wave frequencies. The graph also shows peaks at the frequencies between the 20 rad/s and 40 rad/s. These small peaks cannot be explained by the natural frequencies, because these frequencies do not occur in the natural frequencies of the system. The peaks could be located at the wrong frequency, because of a too large time step. The displacement signal is known in discrete points in time. These points have a distance of 0.04 seconds. The natural frequencies in vertical direction have a period of about 0.16 seconds. In order to recognize this frequency, several points (about 8, Sun et al., 2008) are needed per period. In this situation only 4 points are present. This could have caused a peak at a lower frequency, than the frequency which is present in the signal. No peak is seen around 6 rad/s, because the mode shapes corresponding to these natural frequencies only include movement in the horizontal plane. Between 40 rad/s and 50 rad/s larger peaks are seen in Figure 79, natural frequencies including vertical movements are located in this range. The DAF in z-direction found after 5 simulations are between the 0.3 and 0.5. When extrapolating the results to an exceedance probability of 0.1 % per storm this results in a DAF of 0.7.

The static displacement is larger than the maximum displacement found with the dynamic calculation. The maximum static displacement in z-direction is 0.005 m. The maximum Von Misses stress occurring in the piles as a consequence of the displacement of the jetty deck in z-direction is $\sigma_{vm}$ 8 N/mm$^2$.

Summary of results in z-direction:

- $DAF_z$: 0.3 to 0.7
- $W_{z,dyn}$: 0.002 m
- $W_{z,stat}$: 0.005 m
- $\sigma_{vm}$: 8 N/mm$^2$

### 6.10. Results of Waves near the Natural Frequency

Three different wave spectra have been elaborated that hit the jetty deck. In reality many different wave heights and periods occur. Therefore two other wave spectra are simulated as well, to see the dynamic behaviour for all wave frequencies that can occur. The waves in the two wave spectra do not hit the jetty deck. A comparison is therefore done in axial direction (x-direction), because all waves cause a load in axial direction. The wave is placed on the jetty with an angle of zero degrees. So the frequency of the load is equal to the wave frequency.

The two new wave spectra do not hit the jetty deck, but do have wave frequencies closer to the lowest natural frequency of the system. These two wave spectra are based on:

- $Hs=1.0$ m $T_p=1.9$ s setup, tidal movement, storm surge of 0.3 m;
- $Hs=0.35$ m $T_p=1.1$ s setup, tidal movement, storm surge of 0.3 m.

In chapter 3 the wave spectra are discussed in more detail. A linear wave with a wave height of 1.0 meter is shown in Figure 80, in proportion to the jetty.
DAF

The simulations are done for a wave approaching the jetty at the head, with an angle of 0 degrees. The wave load perpendicular to the jetty axis (y-direction) is therefore zero, and only the results in axial (x-direction) are presented. The results of five simulations per wave spectrum are shown in Table 10. The maximum displacement in x-direction at the jetty deck is shown in Table 10. Also the maximum DAF of the 5 simulations are shown in Table 10. The first three wave spectra correspond to the results presented in section 6.7, 6.8 and 6.9.

Table 10 - Comparison Results of Five Different Wave Spectra

<table>
<thead>
<tr>
<th>$H_s$ [m]</th>
<th>$T_p$ [s]</th>
<th>DAF X</th>
<th>Max Displacement in x-direction [m]</th>
<th>$F_x$ max [kN]</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.0</td>
<td>12</td>
<td>1.0</td>
<td>0.1</td>
<td>818</td>
</tr>
<tr>
<td>5.0</td>
<td>7.0</td>
<td>1.5</td>
<td>0.007</td>
<td>132</td>
</tr>
<tr>
<td>3.0</td>
<td>4.0</td>
<td>2.6</td>
<td>0.005</td>
<td>63</td>
</tr>
<tr>
<td>1.0</td>
<td>1.9</td>
<td>3.6</td>
<td>0.005</td>
<td>21</td>
</tr>
<tr>
<td>0.35</td>
<td>1.1</td>
<td>5.3</td>
<td>0.002</td>
<td>6</td>
</tr>
</tbody>
</table>

The maximum load in axial direction (x-direction) presented in Table 10 is the load on one pile. The maximum load on the jetty in this direction depends on the load on all piles. It can be seen from Table 10 that a larger wave height causes a larger wave load on a pile. The DAF gets bigger, when the wave load approaches a natural frequency. In Figure 81 the relations are shown in plots.
From Figure 81 it can be seen that although the DAF is much larger for the load with a frequency close to a natural frequency, the situation does not cause the largest stresses in the structure. The stress in the structure is related to the maximum displacement. This displacement is the largest for the largest wave spectrum.

It is noticeable from Figure 81 that the load on a pile is much larger for the largest wave spectrum. This is caused by the linear wave theory. This theory is only applicable under certain circumstances (Holthuijsen, 2007). The situation in which waves of the largest wave spectrum occur in the Great Bay do not compile with the conditions of the linear wave theory. This causes a too large water particle velocity, which results in a too large load on the piles. For the wave loading on the deck, this error is taken out (section 3.4), but not for the pressure on the piles which determines the load in
Figure 81. In reality the displacement and load on the pile are expected to be lower for the large wave spectrum. The large wave spectrum is still expected to lead to the largest load on the piles and displacement in horizontal direction.

6.11. Discussion

In this section the results from the previous sections are discussed. The results are compared and application of the results for other wave spectra on Sint Maarten is discussed.

Comparison of the Dynamic Behaviour of the Three Wave Spectra

The dynamic amplification factors of the three investigated wave spectra that hit the jetty deck are compared in this section.

Table 11 - DAF of Large wave spectrum, Hs= 9.0 m Tp=12 s (Section 6.7)

<table>
<thead>
<tr>
<th>Angle of 0</th>
<th>DAF X</th>
<th>DAF Y</th>
<th>DAF Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angle of 10</td>
<td>1.0</td>
<td>0.51</td>
<td>0.78</td>
</tr>
<tr>
<td>Angle of 25</td>
<td>1.0</td>
<td>0.51</td>
<td>0.78</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Angle of 0</th>
<th>0.1% Exceed. Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angle of 0</td>
<td>1.0</td>
</tr>
<tr>
<td>Angle of 10</td>
<td>1.0</td>
</tr>
<tr>
<td>Angle of 25</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Table 12 - DAF of Wave Spectrum with Large Wave Heights in High Wave Frequencies, Hs= 5.0 m Tp=7.0 s (Section 6.8)

<table>
<thead>
<tr>
<th>Angle of 0</th>
<th>DAF X</th>
<th>DAF Y</th>
<th>DAF Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angle of 10</td>
<td>1.0</td>
<td>0.51</td>
<td>0.78</td>
</tr>
<tr>
<td>Angle of 25</td>
<td>1.0</td>
<td>0.51</td>
<td>0.78</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Angle of 0</th>
<th>0.1% Exceed. Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angle of 0</td>
<td>1.0</td>
</tr>
<tr>
<td>Angle of 10</td>
<td>1.0</td>
</tr>
<tr>
<td>Angle of 25</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Table 13 - DAF of Wave Just Hitting the Jetty Deck, Hs= 3.0 m Tp=4.0 s. (Section 6.9)

<table>
<thead>
<tr>
<th>Angle of 0</th>
<th>DAF X</th>
<th>DAF Y</th>
<th>DAF Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angle of 10</td>
<td>1.0</td>
<td>0.51</td>
<td>0.78</td>
</tr>
<tr>
<td>Angle of 25</td>
<td>1.0</td>
<td>0.51</td>
<td>0.78</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Angle of 0</th>
<th>0.1% Exceed. Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angle of 0</td>
<td>1.0</td>
</tr>
<tr>
<td>Angle of 10</td>
<td>1.0</td>
</tr>
<tr>
<td>Angle of 25</td>
<td>1.0</td>
</tr>
</tbody>
</table>

The tables on the left side represent the largest value found by one of the five simulations done with that wave spectrum. This causes the exceedance probability of these tables to differ per wave spectrum. The DAF in y-direction, for an approach angle of zero differs strongly in the results. This is caused by the very small displacement in y-direction when the load in y-direction is zero.

The dynamic amplification factors of the horizontal (x and y) directions found by the first wave spectrum Table 11 are smaller, than the DAF’s presented in Table 12 and Table 13. Only in horizontal direction dynamic amplifications factors larger than 2 are found. For the second wave spectrum a DAF larger than 2 is found with an exceedance probability of 0.1 % per storm. For the third wave
spectrum a DAF larger than 2 is also found with a larger exceedance probability. A DAF larger than 2 indicates a significant enlargement of the amplitude of vibration by the dynamic behaviour of the jetty. This is indicated to occur only in horizontal direction for the second and third wave spectrum. It has to be noted that the value with a 0.1 % exceedance probability is uncertain, because the extrapolation is only based on five points and is very sensitive.

In vertical direction (z-direction), the dynamic amplification factors get smaller for smaller wave spectra (from Table 11 to Table 12 and Table 13). The DAF is also smaller than one, which means that in vertical direction the dynamic behaviour decreases the maximum displacement. This is caused by the small duration of the peak pressure.

Although the dynamic behaviour of the system significantly enlarges the amplitude of vibration in horizontal direction, the largest displacements are caused by the large wave spectrum. The wave spectra with wave frequencies closer to the natural frequencies cause a larger dynamic amplification, but these wave spectra also have lower significant wave heights which cause smaller loads. The static calculated displacement is therefore also smaller. When this is multiplied by the DAF, this does not lead to larger displacements than caused by the wave spectrum containing the most energy (large wave spectrum). The maximum displacements found by the simulations using the second and third wave spectrum are:

- In axial direction (x-direction) 0.007 m
- Perpendicular to jetty axis (y-direction) 0.02 m

These displacements are smaller than caused by the large wave spectrum. The maximum displacement found by the large wave spectrum is 0.1 m, this is expected to overestimate reality because of the too large water particle velocity simulated in this research. However, also without this overestimation the large wave spectrum is expected to lead to the largest displacement in horizontal direction.

The largest displacement is expected to lead to the maximum wave load case. This is for the investigated wave spectra not enlarged by the dynamic behaviour of the jetty. However, from the wave spectrum with a significant wave height of 5.0 m, and wave period of 7.0 s, it can be seen that also wave spectra including high waves can cause significant dynamic enlargement of the amplitude of vibration.

**Can a Conclusion be Made about All Wave Spectra?**

From section 6.10 it is expected that a larger dynamic enlargement occurs if the wave frequencies are closer to the lowest natural frequency. This can also explain the small DAF’s for the large wave spectrum of Table 11 and the DAF larger than 2 for the wave spectrum with a peak period of 4.0 seconds (the third wave spectrum). According to this theory the DAF’s of the second wave spectrum (Table 12) should be smaller than the DAF’s of the third wave spectrum (Table 13). However, from Table 12 and Table 13 it can be seen that in y-direction the largest DAF is indicated for the second wave spectrum.

This could be caused by the randomness of the water surface elevation, the extrapolation of the results, or the dynamic behaviour of the jetty to the wave loads could be more complicated than that of a SDOF (transfer function in section 5.5). The wave length for instance influences the shape of the
wave loading (distribution of the load in x and y-direction), which has an influence on the dynamic reaction of the structure. Different wave spectra contain different wave frequencies, wave lengths and wave height combinations. Which of these wave spectra causes a wave load that corresponds to one of the mode shapes cannot be predicted. Therefore no conclusions can be drawn about the wave spectra which are not simulated. An estimation of the dynamic behaviour of these not considered wave spectra is presented in section 8.3.

**Maximum DAF**
The maximum DAF mentioned in this chapter is 5.3, for small waves hitting the jetty with its natural frequency. This result is found by simulating a wave spectrum with the dominant wave frequency being the natural frequency. When the wave with this wave period of 1.1 s is simulated as a regular wave, the dynamic amplification is much larger. For a simulation of 15 s the DAF in x-direction is about 10. An almost regular wave pattern (very narrow wave spectrum) can only occur for a certain time interval. How large this time interval is for the waves with a period of 1.1 s is not known. However, it is expected that the DAF keeps enlarging when the duration of the regular wave pattern is longer than 15 s. This is caused by the constant period of the load.
6.12. Summary of Results of Dynamic Calculations

Dynamic Amplification Factor Results
A significant enlargement of the amplitude of vibration in horizontal direction due to the dynamic behaviour of the jetty can be caused by a wave spectrum with:

- \( H_s = 5.0 \) m \( T_p = 7.0 \) s (large waves in high frequencies, section 6.8)
- \( H_s = 3.0 \) m \( T_p = 4.0 \) s (just hitting the jetty deck, section 6.9)
- \( H_s = 1.0 \) m \( T_p = 1.9 \) s; (section 6.10)
- \( H_s = 0.35 \) m \( T_p = 1.1 \) s. (section 6.10)

In horizontal directions the largest DAF’s simulated are between the 1.7 and 5.3. This is estimated to lead to a DAF between the 2 and 6 with an exceedance probability of 0.1% per storm. In vertical direction no significant enlargement of the amplitude of vibration is found. The largest simulated DAF is 0.8 in z-direction. The DAF with an exceedance probability of 0.1 % per storm in z-direction is estimated to be 1.1. The jetty seems to enlarge the amplitude of vibrations more for waves with frequencies closer to the natural frequencies. The dynamic behaviour of the jetty however is too complicated to draw conclusions about wave spectra which have not been investigated.

Maximum Displacements
The maximum displacements of the jetty deck are found for the largest wave spectrum investigated in this research. This large wave spectrum does not cause a significant enlargement of the amplitude of vibration caused by dynamics. The maximum found displacements are:

- 0.1 m in axial direction (x-direction)*;
- 0.06 m in perpendicular to the jetty axis (y-direction);
- 8 mm in vertical direction (z-direction).

(*This is an overestimation because of the linear wave theory, see paragraph 3.4)

The largest displacements also cause the largest stresses in the structure. The large wave spectrum leads in this research to the maximum load case. However, from the wave spectrum with a significant wave height of 5.0 m, and wave period of 7.0 s, it can be seen that wave spectra including high waves can also cause en significant dynamic enlargement of the amplitude.
7. Sensitivity Analysis

The results presented in the previous chapter are computed for a certain situation, with a specific model and several assumptions are made. The influences of these assumptions on the dynamic reaction of the jetty to the wave loading are investigated in this chapter.

Appendix X shows the results for the different types of modelling. Section 7.1 presents a short overview of the sensitivities and how it effects the conclusions. The last section 7.2 describes a short summary of this chapter.

The following elements are varied:

- Modelling of expansion joint;
- Modelling of the soil;
- Cracked concrete deck;
- Damping;
- Added mass;
- Air gap;
- Duration of peak pressure;
- Length of jetty module;
- Randomness of water surface elevation;

7.1. Overview Sensitivity Analysis

Including added mass, adjustment of the length of the module of the jetty, and cracks in the concrete deck have a small influence on the dynamic behaviour of first jetty of Sint Maarten. Damping does have a large influence on the dynamic reaction, when the loading frequency is close to the natural frequency of the structure. Also the boundary conditions, behaviour of the soil and neighbouring jetty modules influence the dynamic behaviour. Table 14 shows rough the differences caused by different modelling of several elements.

Overview of Sensitivity

Most of the comparisons are performed with two wave climates: the large wave spectrum and the wave spectrum with waves just hitting the jetty deck. The sensitivity of the dynamic amplification is found to be small for the large wave spectrum ($H_s$ of 9.0 m). Even damping does not change the DAF for that wave spectrum. For vibrations of which the maximum displacement is influenced by the dynamic behaviour, the difference in modelling does have an influence. These sensitivities are shown in Table 14. The differences are on the DAF with an exceedance probability of 0.1%.
Table 14 - Effects of Differences in Modelling on DAF with 0.1 % Exceedance Probability

<table>
<thead>
<tr>
<th>Changes in the modelling</th>
<th>DAF X</th>
<th>DAF Y</th>
<th>DAF Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boundary condition at the expansion joint</td>
<td>+0.3</td>
<td>+/-0.8</td>
<td>+0.7</td>
</tr>
<tr>
<td>Soil modelled fixing piles instead of springs</td>
<td>+0.4</td>
<td>+0.6</td>
<td>+0.2</td>
</tr>
<tr>
<td>Cracked concrete deck (lower Young's-modulus)</td>
<td>+/-0.2</td>
<td>+/-0.2</td>
<td>+/-0.2</td>
</tr>
<tr>
<td>No damping included (not realistic situation)</td>
<td>+/-3</td>
<td>+/-0.7</td>
<td>0</td>
</tr>
<tr>
<td>Added mass</td>
<td>+/-0.1</td>
<td>+/-0.1</td>
<td>0</td>
</tr>
<tr>
<td>Air gap</td>
<td>0</td>
<td>0</td>
<td>+/-0.1</td>
</tr>
<tr>
<td>Duration of vertical peak pressure</td>
<td>0</td>
<td>0</td>
<td>+0.2</td>
</tr>
<tr>
<td>Length of jetty module</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Randomness of water surface elevation</td>
<td>+/-0.2</td>
<td>+/-0.2</td>
<td>+/-0.2</td>
</tr>
<tr>
<td>Physical model test base wave-in-deck formulation</td>
<td>+/-0.2</td>
<td>-0.4</td>
<td>+0.3</td>
</tr>
</tbody>
</table>

The total influence on the results cannot be determined by summing up all differences, because two adjustments to the model can also work against each other. The only known boundaries are the found differences.

**Effect on Conclusions**
From the results in chapter 6 it is concluded that the largest investigated wave spectrum does not result in a DAF larger than two. This conclusion is reinforced by the findings in the sensitivity analysis, because the investigated adjustments do not have a large influence on the DAF for this wave spectrum.

The smaller wave spectra \((Hs=5.0 \text{ m and } Hs=3.0 \text{ m})\) are expected to cause a DAF larger than two in horizontal direction \((x \text{ and } y)\). This can still be expected. However, the values of the DAF’s found in horizontal direction have a larger sensitivity. A range of about +/- 1 should be taken into account for the wave spectrum with a significant wave height of 3.0 m. For wave spectra causing a larger DAF \((Hs=1.0 \text{ m } Hs=0.35 \text{ m})\) a larger range is expected.

The wave spectrum with a significant wave height of 5.0 m only shows a DAF larger than two for a 0.1 % exceedance probability in chapter 6. The changes in the model on which the DAF reacts sensitive (modelling of soil, damping and expansion joint) are repeated using this wave spectrum. This did not lead to a DAF larger than two with a larger exceedance probability (maximum of 5 simulations). Only for the 0.1 % exceedance probability DAF’s larger than two are found. The conclusion from chapter 6 therefore remains unchanged.

The DAF’s found in vertical direction are smaller than one for all simulations performed. Only the 0.1 % exceedance probability DAF is indicated to be a bit larger than one for little situations. The sensitivity in vertical direction is smaller than for the horizontal directions. The conclusion of chapter 6, that in vertical direction no DAF larger than two is expected can remain.

**Stiffness**
It should be noted that the DAF does not show all effects of the different modelling on the structure. Stiffer construction of the piles, leads to a smaller dynamic reaction (smaller DAF) but to larger stresses in the piles.
7.2. Summary of Sensitivity Analysis

Including added mass, adjustment of the length of the module of the jetty, and cracks in the concrete deck have a small influence on the dynamic behaviour of first jetty of Sint Maarten. Damping and modelling of the expansion joint do have a large influence on the dynamic reaction.

The influences on the dynamic behaviour of the jetty due to changes in the dynamic model are small for wave spectra causing a small DAF. The conclusions drawn from the results in chapter 6 therefore remain unchanged.

The extensive sensitivity analysis is presented in appendix X.
8. Damage Sint Maarten

The motivation of this research is damage to the jetty of Sint Maarten. In this chapter the findings of this research are compared to the damage. Also the results of the dynamic analysis are interpreted with regard to the maximum load case and fatigue.

Especially the difference between the damage caused by hurricane Omar and Earl did conjecture that part of the damage was caused by a dynamic effect. In this chapter the wave climate during the extreme part of the storm of Omar and Earl are both simulated and the difference in dynamic behaviour of the jetty is compared. This is shown in section 8.1. Section 8.2 describes some noticeable findings about the first jetty of Sint Maarten. Section 8.3 describes an interpretation of the results of the dynamic analysis with regard to a maximum load case and fatigue.

8.1. Hurricane Earl versus Hurricane Omar

In August of 2010, hurricane Earl passed Sint Maarten. No hindcast was made of the storm. The damage was reported, because Hurricane Earl caused severe damage. In contrast to hurricane Omar, that passed 2 years earlier, which showed larger waves and did not cause damage to the jetty. This section investigates the difference in the dynamic behaviour between the two hurricanes.

Wave Climate
The situation before Earl is not known, because no reports of inspection are found of the jetty. For Earl this is not expected to be a problem, because the largest damage could not have been present without noticing. Also the freshness of cracks and damage can be seen, because the concrete colour is different. The situation before Omar is known, because pictures are taken a month before Omar hit. The jetty is expected to have been weakened at several locations, because of overloading before Earl and Omar hit.

It depends on the magnitude of the storm, the distance between the eye and Sint Maarten and the orientation of the storm how large the waves in the Great Bay become. The significant wave height predicted by NOAA at the offshore location of Sint Maarten was 6 m. This is less than the 8.3 m measured offshore Sint Maarten for Omar. The Great Bay is located at the South of Sint Maarten. Waves coming from the South can travel into the Great Bay. Therefore hurricane Omar is expected to have caused more swell waves in the Great Bay than Earl, because Omar approached Sint Maarten from the South. Earl on the other hand, was on a distance of only 40 km, which is less than the distance between the eye of the storm of Omar and the Great Bay. The wave climate during Earl is therefore expected to be dominated by wind waves, causing a rough wave climate for only a short period. Therefore shorter waves are expected to have occurred during Earl than during Omar, and the approach angle during Earl is also expected to be larger because of its different location with respect to the Great Bay.

Earl
The dynamic behaviour of the jetty during Earl is simulated with an approximated wave climate. The offshore wave height of 6 m is smaller than the theoretical maximum wave possible in the Great Bay. The wave height is therefore expected not to reduce a lot, because the influence of the sea bottom on the wave is small. A wave spectrum with a significant wave height of 5.0 m and a wave period of
7.0 s is used to simulate the waves during Earl. This wave is mentioned in a wave climate study of Alkyon in 1997. The waves are expected to have come from the south west and west. The angle between the waves and the jetty could have been -25 to -10 degrees. An angle of 25 degrees is chosen. The storm surge, tidal movement and setup are estimated to be 0.3 m together. The water depth was 13 m in 2010 at the head of the jetty.

Omar
The maximum significant wave height registered during Omar is 5.9 m. The wave periods are about 10 s. For the maximum significant wave height the period is 10.2 s. The setup, tidal movement and storm surge together are 0.3 m. An approach angle of 10 degrees is assumed.

DAF
Table 15 shows the dynamic amplification factors for the wave spectra representing hurricane Omar and Earl.

<table>
<thead>
<tr>
<th></th>
<th>DAF X</th>
<th>DAF Y</th>
<th>DAF Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Omar</td>
<td>1.0</td>
<td>1.1</td>
<td>1.0</td>
</tr>
<tr>
<td>Earl</td>
<td>1.1</td>
<td>1.3</td>
<td>0.64</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>0.1 % Exceed. Prob.</th>
<th>DAF X</th>
<th>DAF Y</th>
<th>DAF Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Omar</td>
<td></td>
<td>1.0</td>
<td>1.3</td>
<td>1.7</td>
</tr>
<tr>
<td>Earl</td>
<td></td>
<td>1.3</td>
<td>1.8</td>
<td>1.0</td>
</tr>
</tbody>
</table>

From Table 15 it can be seen that the dynamic amplification factors for hurricane Earl are a bit larger in horizontal direction (x and y). The DAF with the 0.1 % exceedance probability almost reaches 2 in y-direction for the assumed situation representing the wave climate during hurricane Earl. The angle between the waves and the jetty are also expected to be larger during Earl. This could have lead to a larger horizontal wave loading than expected. From the variance spectrum of the displacement in y-direction it can be seen that the frequency of vibration during Earl included the lower natural frequencies.

In vertical direction the maximum wave registered during Omar is indicated to cause a large dynamic amplification. However, the 0.1 % exceedance probability is very sensitive. The simulated DAF is not larger than 1.0 in vertical direction. This is much larger than the vertical DAF indicated for Earl.

Part of the damage during Earl occurred probably through vertical movement. In z-direction the DAF is smaller for the situation representing Earl than Omar. The difference in damage is therefore not expected to be caused by dynamic enlargement of the amplitude of vibration, for this investigated situation.

Wave Steepness
The difference in distance between hurricane Earl and Omar to Sint Maarten is expected to have lead to different wave climates. During Earl more wind waves are expected in the Great Bay than during Omar. This could have lead to larger vertical peak pressures during Earl, because wind waves are steeper than swell waves. A larger steepness of the wave can lead to a larger vertical wave-in-deck peak pressure (Rooij de, 2001). This could have caused the difference in damage caused by the two hurricanes.
**Approach Angle**

Also the approach angle of the wave to the jetty is expected to be different between the two hurricanes. A larger angle causes larger loads perpendicular to the jetty axis. The damage observed after hurricane Earl however, is not expected to be caused by horizontal loading. A larger approach angle is also expected to cause more reflection of waves, because of the location of the quall wall, which can be seen in appendix III. A larger reflection could have led to larger waves at the jetty, which cause larger vertical wave loading on the jetty. This could have influenced the difference in damage between the two hurricanes. The dynamic behaviour does not significantly differ for different approach angles.

**Air Gap**

The distance between the sea level and the bottom of the jetty deck is influenced by the wind setup, storm surge and tidal movement. This distance influences the wave-in-deck loading. From the available data no conclusions can be drawn about a difference in the wind setup, storm surge and tidal movement, although this could have influenced the wave-in-deck loading.

### 8.2. Noticeable Findings of First Jetty Sint Maarten

During this research several noticeable findings have been done about the first jetty of Sint Maarten. Some of them could have an influence on the occurrence of the damage.

The wave loading on the deck depends on the wave height. The wave height is restricted by the water depth. During the lifetime of the jetty the water depth has been increased, making larger waves with that larger wave loads possible.

Also no reports of routine inspections of the jetty are found. The condition of the jetty before large damage occurs is therefore not known.

**Expansion Joint**

The largest damage in 2010 occurred at the expansion joint which had already been damaged in 1999. The plate next to the expansion joint was blown out during construction in 1999. It is possible that by recovering this expansion joint, a weaker expansion joint was created than designed. A weakened expansion joint could be the reason for damage.

The situation with a weaker expansion joint is investigated in this research by allowing translations and rotations at the expansion joint. This did not lead to much larger maximum displacements, although a third mode shape with a frequency near 6 rad/s occurred. This however did not lead to a larger dynamic amplification.

**Sliding Planes**

After some of the hurricanes it was reported that the sliding planes at the expansion joints had moved or disappeared. These are rubber thin planes located between the teeth of the expansion joint, to prevent damage by friction between the teeth during movement.

During large waves the displacement in x-direction found in this research is more than 0.05 m, which is the distance between the teeth in the expansion joint. This causes the teeth to slide over the sliding planes, and to make contact with the neighbouring jetty module. It is possible that during these movements the sliding planes are displaced.
8.3. Interpretation of Results Regarding the Design

Maximum Load Case for the Design
From chapter 6 it can be seen that the dynamic behaviour of the jetty only causes a significant enlargement of the amplitude of vibration in the horizontal directions. In vertical direction the dynamic behaviour decrease the maximum displacement for the investigated situations. This is caused by the very short duration of the vertical peak of the wave-in-deck pressure.

Whether the maximum load case is significantly affected by the dynamic behaviour of the jetty depends on the wave loading and normative load case in horizontal direction (mooring loads or wave loads). The largest wave spectrum investigated in this research does not cause a significant dynamic enlargement of the amplitude of vibration (section 6.11). However, one of the wave spectra having a large wave heights ($H_s$ is 5.0 m) did show a DAF of 2 in horizontal direction, with an exceedance probability of 0.1 %. Which means that during a 2 hour sea state a DAF of 2 in horizontal direction is expected to occur once. This wave spectrum however did not lead to the maximum load case of this research. The largest investigated wave spectrum showed a larger maximum displacement, causing larger stresses in the structure. However, the wave loading used in this research is based on a very simplistic formulation and is expected to overestimate the wave loading. A more detailed described wave loading could lead to another normative wave spectrum.

Other wave spectra which have not been investigated in this research, but do have a large significant wave height could also cause a significant dynamic enlargement of the amplitude of vibration. Therefore it cannot be ruled out that another wave spectrum could lead to the maximum wave load case in horizontal direction, and be significantly enlarged by the dynamic behaviour of the structure.

If an estimation of this dynamic enlargement of the amplitude of vibration would have been needed for the design of the first jetty of Sint Maarten, it would be preferred to simulate more wave spectra. An estimation based on the results presented in this research leads to the following dynamic amplifications. Wave spectra with a peak wave period between the 8.0 s and 12 s are estimated to have a DAF not larger than 1.5. This largest value of dynamic amplification is expected to occur once during a period of 3 hours. The wave spectra with significant wave heights larger than 5.0 m have a return period of 100 years. This means that this DAF of 1.5 is expected to occur once during the lifetime of the jetty. In vertical direction the DAF is estimated to be about 1 for the wave spectra having wave periods longer than 7.0 s. Wave spectra with shorter wave periods are estimated to have a DAF smaller than 1 in vertical direction. Wave spectra with a wave period longer than 12 s are estimated to have a DAF of about 1 in all three directions.

Fatigue
The maximum wave is determined by the return period of the design wave of 100 years (as described in chapter 3). The rest of the wave spectra are based on all possible waves, with a lower wave height. These lower waves have a smaller return period, because a storm does not have to be extraordinary large and close to Sint Maarten in order to cause the wave climate. These lower waves also cause the larger values for the DAF’s in horizontal direction, as can be seen in section 6.10. The small return period together with a significant enlargement of the amplitude of vibrations because of the dynamic behaviour of the jetty could lead to a normative fatigue situation.
9. Application to General Jetty Design

Part of the goal of this research is elaborating components that have a negative influence on the dynamic behaviour and wave loading of an open piled jetty. During the dynamic analysis of the open piled jetty of Sint Maarten these components are identified.

These factors concerning the wave-in-deck loading are shown in section 9.1. The general aspects that influence the dynamic behaviour are mentioned in section 9.2. Also the natural frequencies of some adjusted open piled jetty designs are discussed. Section 9.3 mentions the required aspects for a dynamic analysis for the design of an open piled jetty.

9.1. Wave-in-Deck Loading

Wave-in-Deck Load from Literature

Wave loading on the deck of the jetty is influenced by many elements: wave height, wave period, approach angle of the wave, distance between the jetty deck and mean sea level, geometry of deck and geometry of the harbour. In literature the following relations are found:

General Idea about Vertical Wave-in-Deck Loading
The largest wave-in-deck load occurs when the wave first hits the deck. This peak load has a duration of about 8-16 milliseconds (Rooij de, 2010). A larger wave height causes a larger wave-in-deck pressure (Shih, 1992). Air intraption lowers this peak load, because air is compressible (Cuomo et al, 2009). This causes overestimation of the wave-in-deck load during model tests (WL Delft Hydraulics, 1998).

A smaller distance between the bottom of the deck and the mean sea level leads to larger wave-in-deck forces according to Suchithra (1995). Rooij (2001) and Meng (2010) state it depends on the ratio between the wave height and the deck clearance, because a very small deck clearance could also lead to a decrease of the wave-in-deck loading, dependant on the wave height.

A grid of beams beneath the jetty deck reduces the vertical peak pressure caused by a wave. This is caused by air getting trapped between the beams (Suchithra, 1995). However at the location of a beam underneath the jetty deck, perpendicular to the jetty axis, a peak load occurs, because water particles are trapped between the deck and the beam (Raaij van, 2008). Beams in axial direction underneath the jetty deck result in an increasing wave-in-deck pressure, because without these beams the water would have splashed to the side of the jetty deck from underneath. However, the beams trap the water which increases the pressure against the bottom of the deck. Beams perpendicular to the jetty axis decrease the wave-in-deck peak pressure.

Whether increasing steepness of a wave increases or decreases the wave-in-deck load depends on the geometry of the jetty and wave parameters. A larger steepness causes larger water particle velocities (Rooij de, 2001), which increases the wave-in-deck load. Meanwhile, a very low steepness can increase the wave-in-deck load, by increasing the contact area during initial contact and remaining fewer places for the water particles to flow to.
Research about wave-in-deck loading and slamming mainly focuses on the vertical wave-in-deck loading. To find information about the horizontal load caused by a wave hitting the side of jetty deck, a comparison can be made with submerged structures, break waters and offshore platforms.

The wave theory used to describe the water surface elevation and movements of water particles is determinative for the simulation of wave-in-deck loading (Lobit, 2012).

**Points of Attention in the Design**

**Abutment**

The abutment of the jetty is of importance for the wave-in-deck load on the jetty. At the abutment the wave hits against the quay wall underneath the jetty, causing a large horizontal pressure on the quay wall (Hofland, et al., 2010). The water is trapped between the bottom of the jetty deck and the quay wall, an upward pressure is therefore expected against the bottom of the jetty deck. Before the waves reach the abutment they are already influenced by the quay wall. The quay wall reflects the waves, which can lead to larger waves and therefore larger wave loads. Large waves could also be influenced by the water depth. For some jetties the water depth decreases toward the abutment. This leads to steeper waves, and can lead to breaking of the waves. Because of all these influences the abutment is a complicated area when it comes to wave loading on the jetty, which could lead to very large vertical wave-in-deck pressures.

**Water Depth during Lifetime**

Jetties are mostly located near the coast. The water depth near is coast is expected to be limited. This makes the water depth the limitation for the largest waves that can occur near the jetty. The larger the wave is, the larger the wave loading on the jetty. Therefore the water depth is of importance for the jetty design. During the lifetime of a jetty, the water depth can be increased. Larger ships require a large water depth. This change of water depth during the lifetime of a jetty is important for the jetty design, because it also influences the wave loading on the jetty.

**Change of Circumstances**

The wave climate is of importance for the design. The wave climate is influenced by different circumstances, for instance by the design of the harbour. The construction of a quay wall, can cause higher waves because of reflection (Alkyon, 2005).

**Combination**

Damage to a structure is not only influenced by the magnitude of the external loading. Also initial stresses in the structure and repetition have their influence.

Open piled jetty decks are mostly constructed using prefab concrete plates with poured concrete on top of that. This construction method causes initial stresses during the curing of the poured concrete. These initial stresses enlarge stresses in the concrete during external loading.

Damage can also be caused by repeating a high stress fluctuation multiple times, called fatigue. During a storm of about 6 hours, more than 2000 waves pass. During the design it is important to combine the external loading, dynamic effect and fatigue.
9.2. Dynamics

The dynamic behaviour of an open piled jetty depends on several aspects:

- Mass, stiffness, and structure of the jetty;
- Damping in the structure;
- Boundary conditions;
- Modelling of the soil;
- Wave-in-deck loading which depends on the geometry of the structure.

In this section the aspects damping, natural frequency are mentioned. Also the aspects needed for a full dynamic analysis of an open piled jetty subjected to wave loading are mentioned.

Damping

From the dynamic analysis of the first jetty of Sint Maarten it is found that damping has a large influence on the dynamic behaviour. It is therefore of importance to find the elements that enlarge the damping in the system.

Natural Frequencies

The natural frequencies of the structure are influenced by the mass, stiffness and geometry. Longer piles will lower the stiffness of the piles which lowers the natural frequencies. This is expected to lead to a larger dynamic amplification, because the natural frequencies become closer to the loading frequencies. However, a displacement in horizontal direction of the jetty deck also causes less high stresses in the piles, because of the less high stiffness of the piles.

Lowest Natural Frequencies

For the jetty of Sint Maarten the lowest natural frequencies correspond to translations and a rotation in the horizontal plane, which seem to be without bending of the jetty deck. These natural frequencies can be approximated by making a single degree of freedom system. However, it is not sufficient for a jetty with a different geometry to only model the system with one degree of freedom. This is not sufficient because it is not set that an open piled jetty with a different geometry has the same mode shapes to correspond with its lowest natural frequencies.

Different Open Piled Jetty Design

The influence of a change of the design of an open piled jetty on the natural frequencies is investigated in this section.

Thicker Deck

When the thickness of the concrete jetty deck is enlarged from 400 mm to 600 mm this changes the natural frequencies to become lower.

Less Stiff Piles

The diameter of all piles is reduced from 914 mm to 600 mm. This leads to less stiff piles and lower natural frequencies.

3 Piles

Every pile row underneath the jetty has four piles. One of these piles is taken out, to see the influence on the natural frequencies. Figure 82 shows the head view of the jetty for both situations.
**Half of the Number of Piles Rows**

Distance between the pile rows is 6.25 m. This is enlarged to 12.5 m. Figure 83 shows the side view of the jetty in both situations.

![Figure 83 - Side View Jetty, with Different Number of Pile Rows](image)

Table 16 shows the effect of the different geometry of the jetty on the natural frequencies.

<table>
<thead>
<tr>
<th></th>
<th>First Natural Frequency [rad/s]</th>
<th>20th Natural Frequency [rad/s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original Model (Chapter 2)</td>
<td>5.8</td>
<td>49</td>
</tr>
<tr>
<td>Thicker deck</td>
<td>5.4</td>
<td>46</td>
</tr>
<tr>
<td>Pile diameter of 600 mm</td>
<td>4.0</td>
<td>43</td>
</tr>
<tr>
<td>3 piles per pile row</td>
<td>5.2</td>
<td>48</td>
</tr>
<tr>
<td>Half of the number of pile rows</td>
<td>4.0</td>
<td>38</td>
</tr>
</tbody>
</table>

From Table 16 it can be seen that the diameter of the piles and the number of pile rows has a large influence on the natural frequencies of the structure. A thicker jetty deck lowers the natural frequencies because of the increase in mass in the system. This decrease of natural frequencies is only small. Taking out one of the pile of a pile row reduces the stiffness; however the effect on the natural frequencies is quite small. Although only the natural frequencies are presented in Table 16, the modes shapes are also expected to change. These differences are not shown, because the natural frequencies also show how large the influence is and whether the natural frequency enlarges or decreases by the change in geometry.

**Separated Dynamic Analyses Needed**

Lower natural frequencies are expected to lead to a larger dynamic amplification of the amplitude of vibration. However, this cannot be stated with certainty, because the dynamic reaction of a jetty depends on several more aspects. A difference in geometry also changes the stiffness of the element, and the proportion of this stiffness compared to the rest of the structure. This changes the force flow in the structure. For dynamic calculations the mode shapes are changed. It is therefore not enough to determine the effect of the different geometry on the natural frequency. A different mode shape has different waves that activate this shape. The dynamic analysis therefore has to be repeated for every geometry that has different mode shapes.
9.3. Requirements for a Dynamic Analysis

From this research conclusions can only be drawn about the investigated wave spectra for the first jetty of Sint Maarten. For a different design of an open piled jetty a separate dynamic analysis is needed. In order to perform a full dynamic analysis for the design of an open piled jetty the following aspects are of importance:

- Wave climate near the jetty.
- Modelling of the water surface elevation based on a wave spectrum.
  - Because regular waves enlarge the dynamic reaction of the jetty to the wave loading (Section 6.11).
- Detailed selection of wave theory for the simulation of the water particle velocity.
  - Because some wave theories overestimate or underestimate the water particle velocity (Raaij van, 2005) (MSL, 2003).
- Detailed wave-in-deck loading formulation varying in time and location.
  - Duration of peak pressure influences the dynamic reaction. (Chapter 7)
  - Shape of the wave-in-deck load influences the dynamic reaction. (Chapter 7)
  - The magnitude of the load is uncertain, an upper and lower bound method is recommended. (Chapter 3)
- Modelling of the soil has a large influence on the dynamic behaviour. (Chapter 7)
- Modelling of the boundary conditions has a large influence on the dynamic behaviour. (Ch. 7)
- Varying of several combinations of modelling and wave spectra.
  - Because the dynamic behaviour is complex, prediction of the dynamic behaviour is not recommended. Normative wave climates need to be simulated when computations are performed in the time domain.
10. Conclusion

Complex Dynamic Behaviour
No general conclusion about the dynamic behaviour for arbitrary open piled jetty structures exposed to wave loading can be drawn. In this research only the first jetty of Sint Maarten is considered. Therefore the conclusions only apply for that situation, because the dynamic behaviour depends on:

- Mass, stiffness, and structure of the jetty;
- Damping in the structure;
- Boundary conditions (expansion joint and soil);
- Wave-in-deck loading which depends on the geometry of the structure.

Conclusions about the first jetty of Sint Maarten can only be drawn about the investigated wave spectra, because the dynamic reaction of the jetty to the wave loading depends on:

- Frequency and height of the waves;
- Approach angle of the waves;
- Shape of the load (distribution of the load of x and y-direction);
- Propagation velocity of the waves;
- Wave steepness influences the wave loading and whether the wave breaks.

Together this makes the dynamic behaviour of an open piled jetty subjected to wave loading complex which retains prediction of the dynamic behaviour for not considered situations.

Dynamic Behaviour of the Jetty of Sint Maarten
In this research a dynamic analysis is made of the first jetty of Sint Maarten. The dynamic calculations are solved in the time domain. Therefore only the dynamic behaviour of the jetty for certain situations is analysed. Conclusions can therefore only be drawn about the investigated situations:

- Five different wave spectra (based on $H_s$ and $T_p$ shown in Table 17);
- No breaking waves;
- Approach angle of 0, 10, 25 degrees;
- Only wave loading included;
- No beams beneath jetty deck.

The natural frequencies of the jetty of Sint Maarten are much higher (lowest is 5.8 rad/s) than the frequency of the waves with large wave heights (highest about 1 rad/s, with a significant wave height of 5 meter). Waves with a frequency equal to the natural frequency can occur, but have a low wave height (about 0.3 m). The lowest natural frequencies correspond to the mode shapes of horizontal translations and rotation in the horizontal plane. The dynamic reaction of the jetty to wave loading depends on the frequency and shape of the load. The frequency of the wave load on the deck depends on the wave frequency, propagation velocity, angle of approach and distance between the beams beneath the jetty deck. The vertical load peaks caused by the beams beneath the jetty deck are not included in this research, although the frequency of this vertical wave load is expected to be near the natural frequencies.
Results

The dynamic behaviour of the jetty causes a significant enlargement of the amplitude of vibration in the horizontal directions. In vertical direction the dynamic behaviour decreases the maximum displacement for most of the investigated situations.

The dynamic amplification factor of the first jetty of Sint Maarten for waves with a dominant wave period of 12 s, is for the investigated situation smaller than 1.2. The amplitude of the vibrations is not significantly enlarged by the dynamic behaviour of the structure in any of the three directions (x, y, z), in this situation.

For a storm with a dominant wave period of 7.0 s and waves with a significant wave height of 5.0 m, there is a probability of 0.1 % on a significant enlargement of the amplitude of vibration in the horizontal directions. During a sea state of 2 hours, about 1000 wave tops hit the jetty. The dynamic enlargement of about 2 can therefore be expected to occur once per 2 hours.

For the three investigated wave spectra with a wave frequency closer to the lowest natural frequency (about less than 1/4) the dynamic amplification factor is even larger than two, as can be seen in Table 17.

<table>
<thead>
<tr>
<th>$H_s$ [m]</th>
<th>$T_p$ [s]</th>
<th>$\omega_0/\omega_1$</th>
<th>DAF X</th>
<th>Max Displacement in x-direction [m]</th>
<th>Max Load on one Pile [kN]</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.0</td>
<td>12</td>
<td>1/11</td>
<td>1.0</td>
<td>0.1</td>
<td>818</td>
</tr>
<tr>
<td>5.0</td>
<td>7.0</td>
<td>1/6</td>
<td>1.5</td>
<td>0.007</td>
<td>132</td>
</tr>
<tr>
<td>3.0</td>
<td>4.0</td>
<td>1/4</td>
<td>2.6</td>
<td>0.005</td>
<td>63</td>
</tr>
<tr>
<td>1.0</td>
<td>1.9</td>
<td>1/2</td>
<td>3.6</td>
<td>0.005</td>
<td>21</td>
</tr>
<tr>
<td>0.35</td>
<td>1.1</td>
<td>1/1</td>
<td>5.3</td>
<td>0.002</td>
<td>6</td>
</tr>
</tbody>
</table>

For the three wave spectra with a frequency between the 4.0 s and 1.0 s the dynamic amplification factor is in the horizontal directions between the 2 and 6. For these waves the amplitude of vibration enlarges significantly, as a result of the dynamic behaviour of the jetty structure. The wave spectra with lower wave heights do not hit the jetty deck, therefore no load and dynamic amplification factor is present in vertical direction.

In vertical direction the dynamic behaviour decreases the maximum displacement of the jetty deck with factors between the 0.3 and 0.8 for most situations. This is caused by the small duration of the vertical peak pressure. A larger dynamic amplification is found for durations of the vertical peak pressure larger than 0.04 s.

The difference in dynamic amplification factor between the large waves with a significant wave height larger than 4 m, and the smaller waves is expected to be caused by the difference in wave frequency. The closer the frequency of the load is to a natural frequency, the larger the dynamic effect which is expected. This expectation cannot be used to draw conclusions about wave spectra which have not been investigated, because the dynamic behaviour of the jetty is too complicated.
Interpretation of the Results with Respect to the Design of an Open Piled Jetty

Whether the maximum load case is significantly affected by the dynamic behaviour of the jetty depends on the wave loading and normative load case in horizontal direction (mooring loads or wave loads). The largest wave spectrum simulated in this research causes the largest stresses in the structure of the investigated situations. This wave spectrum does not cause a significant dynamic enlargement of the amplitude of vibration. However, one of the wave spectra having large wave heights ($H_s$ is 5.0 m) did show a DAF of 2 in horizontal direction. Therefore it cannot be ruled out that another wave spectrum could lead to the maximum wave load case in horizontal direction, and be significantly enlarged by the dynamic behaviour of the structure. For wave spectra with a wave period between the 8 and 12 s a dynamic amplification is estimated of 1.5. A dynamic enlargement of the amplitude of vibrations for the largest investigated wave spectrum could occur due to breaking waves on top of the jetty deck or the vertical load caused by the beams beneath the jetty deck.

The dynamic behaviour of the smaller wave spectra could be of importance for the fatigue calculation. The closer the frequency of the load is to a natural frequency, the larger the dynamic effect which is seen in Table 17, column 4. The waves with a smaller wave period also have a lower wave height (less than 3 m). Therefore these waves have a larger probability of occurring near the jetty. This shorter return period together with the dynamic enlargement of the amplitude of vibration for the smaller wave spectra could be of importance for the determination of fatigue.

Damage to the Jetty of Sint Maarten

In this research two hurricanes that stroke the first jetty of Sint Maarten are simulated. The difference in dynamic behaviour is compared to the difference in observed damage to the jetty caused by these two hurricanes. The difference in dynamic behaviour between the two wave spectra representing the wave climates during the hurricanes is small. In horizontal direction the largest difference is dynamic reaction can be seen. However the damage is expected to be caused by vertical loading. The difference in damage to the jetty is therefore not expected to be significantly influenced by the difference in dynamic reaction of the jetty.

Influences on the Dynamic Behaviour of an Open Piled Jetty

For the situation of Sint Maarten it is found that: lowering of natural frequencies of the jetty, higher wave frequencies, and less damping enlarge the dynamic reaction. This is expected to apply for most open piled jetty designs, because the natural frequencies of the open piled jetty should be higher than the loading frequencies. About the magnitude of the dynamic reaction for other open piled jetty designs no conclusions can be drawn.

Requirements for a Dynamic Analysis of an Open Piled Jetty

Although no conclusions can be drawn about the dynamic reaction of other open piled jetty designs, conclusions can be made about the aspects which are important for a dynamic analysis of an open piled jetty. In order to perform a full dynamic analysis for the design of an open piled jetty subjected to wave loading the following aspects are of importance:
Dynamic Analysis of an Open Piled Jetty Subjected to Wave Loading

- Wave climate near the jetty.
- Modelling of the water surface elevation based on a wave spectrum.
- Detailed selection of wave theory for the simulation of the water particle velocity.
- Detailed wave-in-deck loading formulation varying in time and location.
  - Duration of peak pressure influences the dynamic reaction.
  - Shape of the wave-in-deck load influences the dynamic reaction.
  - The magnitude of the load is uncertain, an upper and lower bound is recommended.
- Modelling of the boundary conditions has a large influence on the dynamic behaviour.
- Varying of several combinations of modelling and wave spectra.

**Modelling of Wave-in-Deck Loading**

For the dynamic analysis a simple formulation of the wave-in-deck loading varying in time and location is derived in this research. The vertical wave-in-deck pressure shows the church-roof shape, as described in literature. This makes it suitable for a dynamic analysis. However it is found that a simplified wave-in-deck formulation cannot include the complexity and dependences of the wave-in-deck loading phenomenon which it has in reality.
11. Recommendations

This chapter describes the recommendations for future research. Recommendations for further research on the dynamic behaviour of an open piled jetty subjected to wave loading, are described in section 11.1 called improvements. Suggestions for future research on open piled jetties, in order to prevent damage during hurricanes to open piled jetties are presented in section 11.2.

11.1. Improvements

The recommendations of improvements can be split in four different subjects:

- Further verification of the dynamic model;
- Improvement of wave-in-deck loading;
- Improvement of wave simulation;
- Improvement of dynamic model.

Further verification of the Dynamic Model

The most important recommendation is to verify the dynamic model. The dynamic behaviour of an open piled jetty during a hurricane is uncertain. This behaviour can be modelled, but without verification, the reliability of this model is uncertain. In order to get a reliable result from a dynamic model, the model could be verified with real life measurements or with a different dynamic model which is verified.

Data for verification can be obtained by performing measurements during a hurricane on a jetty. During a hurricane the accelerations of the jetty deck could be measured. Also the wave height needs to be measured for verification. With this data the dynamic model can be verified. This can also be done by modelling the same case in a verified dynamic model, and comparing the results.

Improvement of Wave-in-Deck Loading

To simulate the dynamic reaction of an open piled jetty to wave loading, an accurate formulation of the wave-in-deck pressure is required. This load needs to depend on time and location. For further research it is recommended to use a more accurate wave-in-deck loading, by for instance using a computational fluid dynamic method. If measurements would be done during a hurricane for the dynamic behaviour, measuring the wave pressure at the bottom of the jetty deck would fill in the last big uncertainty.

In this research the load caused by waves breaking on top of the jetty deck is not included. This could lead to large pressures on the jetty deck. It is therefore recommended to include the wave pressure from breaking waves in a future research model.

At the location of a beam beneath the jetty deck perpendicular to the jetty axis, a high wave causes a vertical and horizontal peak pressure on the jetty deck. The distance between these peaks and on which time interval they occur depends on the distance between the beams. In this research the vertical load caused by the beams beneath the jetty deck is not included. For future research it is recommended to include this load, because this could lead to a load frequency close to the natural frequencies of the system. This could therefore cause the dynamic behaviour to significantly enlarge the amplitude of the vibrations.
**Improvement of Wave Simulation**

The simulation of waves in this research is done using undisturbed water surface elevations and water particle velocities. This is not expected to be realistic. Taking into account the influence of the jetty structure on the water particle velocities and wave heights is expected to lead to more realistic wave loading. Also including the decrease in water depth along the jetty, and the reflection of the quay wall on the wave height would lead to a more realistic water surface elevation especially for jetty modules closer to the abutment.

**Improvement of the Dynamic Model**

The vertical wave load on the deck of the jetty is divided over two nodes with a distance of 20 meter (the width of the deck). The bending moment caused by the pressure over this 20 meter, is not included in the model. This causes an under estimation of the load on the jetty. For further research it is recommended to use a 6 degrees of freedom model, or place extra nodes over the width of the jetty deck.

The non-linear displacements, velocities and accelerations are not included in this research. The contribution of the velocity and acceleration of the jetty to the relative velocity and acceleration are not included. Also the load on the jetty is determined, using the wave height at the initial location of the jetty. The displacements of the jetty are not considered when it comes to the location where the wave hits the jetty. Including this leads to geometrical non-linear problem. This can be included in the dynamic calculation in the time domain. Only for large loads this is expected to have a significant influence.

Instead of improving the research to the dynamic behaviour of the open piled jetty to wave loading, research could also focus on the enlargement of damping in the structure. It is found that damping has a large influence on the dynamic behaviour of the jetty.

### 11.2. Future Research to Prevent Damage to Jetty

The motivation for this research is the damage that occurred to the first jetty on Sint Maarten. This research investigates one of the possible causes; namely dynamics. Other possible causes are not investigated. The goal of the investigation to a possible cause for the damage, is to prevent damage to future jetties, which still have to be designed. This research only shows conclusions about dynamics for the first jetty of Sint Maarten, but from the investigation it is found that: the vertical wave-in-deck loading peak pressure is the largest wave loading to the open piled jetty. Further research to prevent damage to open piled jetties during hurricanes is recommended to focus on the prevention or decrease of this load. Suggestions are to investigate the influence of a grid of beams beneath the jetty deck, or the effect of holes in the jetty deck. For the investigation of holes in the jetty deck, it is recommended to investigate the wave load on top of the jetty deck caused by the water blown through the holes.
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Appendix I

List of Symbols

This appendix shows the symbols used in this report.
List of Symbols

\[
\begin{align*}
A_{\text{pil}} &= \text{cross-section of a pile} \quad [\text{m}^2] \\
A_k &= \text{amplitude} \quad [\text{m}] \\
B &= \text{width of jetty deck} \quad [\text{m}] \\
c &= \text{celerity} \quad [\text{m/s}] \\
C &= \text{damping matrix} \quad [\text{Ns/m}] \\
C_d &= \text{hydrodynamic drag coefficient} \quad [-] \\
C_m &= \text{hydrodynamic inertia coefficient} \quad [-] \\
C_{\text{slam}} &= \text{slamming coefficient} \quad [-] \\
D &= \text{diameter of pile} \quad [\text{m}] \\
d &= \text{water depth} \quad [\text{m}] \\
E &= \text{eigenvector} \quad [-] \\
F &= \text{load} \quad [\text{N}] \\
g &= \text{gravitational acceleration} \quad [\text{m/s}^2] \\
H_s &= \text{significant wave height} \quad [\text{m}] \\
k &= 1, 2, 3... \quad [-] \\
k_k &= \text{wave number} \quad [\text{rad/m}] \\
K &= \text{stiffness matrix} \quad [\text{N/m}] \\
L &= \text{length of jetty deck} \quad [\text{m}] \\
m &= \text{mass} \quad [\text{kg}] \\
M &= \text{mass matrix} \quad [\text{kg}] \\
P_z &= \text{verticale wave-in-deck load} \quad [\text{N/m}^2] \\
P_y &= \text{horizontal wave-in-deck load} \quad [\text{N/m}^2] \\
S_{\eta\eta}(\omega_k) &= \text{wave spectrum} \quad [\text{m}^2/\text{s}] \\
t &= \text{time} \quad [\text{s}] \\
\Delta t &= \text{time step of calculation} \quad [\text{s}] \\
t_h &= \text{thickness of deck} \quad [\text{m}] \\
v_y &= \text{water particle velocity in y-direction} \quad [\text{m/s}] \\
v_y &= \frac{\partial v_y}{\partial t} \quad [\text{m/s}^2] \\
v_z &= \text{water particle velocity in z-direction} \quad [\text{m/s}] \\
v_z &= \frac{\partial v_z}{\partial t} \quad [\text{m/s}^2]
\end{align*}
\]
$w_x$ = displacement in x-direction [m]
$w_y$ = displacement in y-direction [m]
$w_z$ = displacement in z-direction [m]
x = x-coordinate [m]
$\hat{x}$ = eigenvector [-]
y = y-coordinate [m]
z = z-coordinate [m]
$z_{\text{bottom\_deck}}$ = location of bottom deck in z-direction [m]

$\alpha$ = approach angle of wave [degree]
$\bar{\alpha}$ = empirical parameter [-]
$\alpha_s$ = shape factor [-]
$\gamma'$ = JONSWAP extension [-]
$\zeta$ = damping coefficient [-]
$\eta(t)$ = water surface elevation [m]
$\lambda$ = wave length [m]
$\rho$ = density of water [kg/m$^3$]
$\sigma$ = stress [N/m$^2$]
$\phi_\lambda$ = phase angle [rad]
$\omega$ = angular frequency [rad/s]
$\Delta\omega_\lambda$ = angular frequency difference [rad/s]
Appendix II

Jetty Sint Maarten

This appendix shows more detailed information about the first jetty of Sint Maarten.
Appendix - Dynamic Analysis of an Open Piled Jetty Subjected to Wave Loading

Geometry Jetty of the First Jetty of Sint Maarten.

Figure 1 - Geometry and Coordinate System of Jetty on Sint Maarten

Geometry
The geometry of the jetty is also shown in Table 1, the used coordinate system is shown in Figure 1. Appendix A and B show an overview of the jetty and the detail of the beam with the slabs and expansion joint.

Table 1 - Geometry First Jetty Sint Maarten

<table>
<thead>
<tr>
<th>Geometry</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geometry Note</td>
<td></td>
</tr>
<tr>
<td>Jetty</td>
<td></td>
</tr>
<tr>
<td>Length</td>
<td>560 m</td>
</tr>
<tr>
<td>Width</td>
<td>20 m</td>
</tr>
<tr>
<td>Orientation</td>
<td>215 degree North</td>
</tr>
<tr>
<td>Beam</td>
<td></td>
</tr>
<tr>
<td>Width (x)</td>
<td>1.75 m</td>
</tr>
<tr>
<td>Height (z)</td>
<td>400 mm</td>
</tr>
<tr>
<td>Length (y)</td>
<td>20 m</td>
</tr>
<tr>
<td>Bottom at (z)</td>
<td>+1.2 MSL</td>
</tr>
<tr>
<td>Deck</td>
<td></td>
</tr>
<tr>
<td>Prefab, thickness (z)</td>
<td>200 mm</td>
</tr>
<tr>
<td>Later changed to 250 mm</td>
<td></td>
</tr>
<tr>
<td>In situ, thickness (z)</td>
<td>200 mm</td>
</tr>
<tr>
<td>Later changed to 150 mm</td>
<td></td>
</tr>
<tr>
<td>Slab width (X)</td>
<td>4.5 m x</td>
</tr>
<tr>
<td>Slab length (y)</td>
<td>1.99 m</td>
</tr>
<tr>
<td>10 slaps over 20 meter</td>
<td></td>
</tr>
<tr>
<td>Deck level top</td>
<td>+ 2.0 m MSL</td>
</tr>
<tr>
<td>Expansion joints</td>
<td></td>
</tr>
<tr>
<td>Length Part I (x)</td>
<td>116 m</td>
</tr>
<tr>
<td>Part II and III (x)</td>
<td>150 m</td>
</tr>
<tr>
<td>Part IV (x)</td>
<td>144 m</td>
</tr>
<tr>
<td>Pile</td>
<td></td>
</tr>
<tr>
<td>Diameter 914 x thickness 16 mm</td>
<td>2 mm corrosion</td>
</tr>
<tr>
<td>Toe level</td>
<td>-41.5 m SMP</td>
</tr>
<tr>
<td>Design Load</td>
<td>1676 kN up</td>
</tr>
<tr>
<td>Grid (X *Y)</td>
<td>6.25 x 6.0 m</td>
</tr>
<tr>
<td>Water depth</td>
<td></td>
</tr>
<tr>
<td>-10.2 m SMP</td>
<td>Changed to - 11.8 m MSL</td>
</tr>
<tr>
<td>- 10.5 m MSL</td>
<td></td>
</tr>
</tbody>
</table>

Sint Maarten Peil (SMP) = - 0.3 m Mean Sea Level (MSL)

146
Tide
The tidal movement in the Great Bay is between the -0.3 m and +0.3 m MSL, because of its location of 18.3° North and 63.25° West (Lievense, 2005).
Appendix III

Reflection of Waves in the Great Bay

In this appendix the importance of wave reflection in the Great Bay is investigated.
Reflection of Waves in the Great Bay

The wave climate in the Great Bay of Sint Maarten is based on wave climate studies and the hindcast of hurricanes. Most of these reports, do not take the reflection of waves against the quay into account. In this appendix the importance of wave reflection is investigated.

Most of the wave climate studies only take the incoming wave energy into consideration, reflection of the waves by the quay walls and revetment are not included in the significant wave height. At the abutment of the jetty a revetment is located. Almost parallel to the jetty a vertical quay wall is present, as is shown in Figure 2.

One of the wave climate studies does include the reflection of the vertical quay wall. The reflection coefficient from a vertical quay wall is expected to be between the 0.9 and 1. This wave climate study shows the largest significant wave heights in the Great Bay. The results of this wave climate study are included in the wave spectra used in this research. Only the wave reflection from the revetment is not included. At the abutment of the jetty, a revetment is located. The layout of the revetment is shown in Figure 3.
The reflection coefficient from the revetment at the abutment of the jetty is determined using the following formulas: (Battjes, 1974) (Zanuttigh, 2006)

\[
K_r = \tanh(a\xi_0^b) \quad \text{[A.1]}
\]

\[
\xi_0 = \frac{\tan(\alpha_\beta)}{\sqrt{H/\lambda_0}} \quad \text{[A.2]}
\]

- \( K_r \) = reflection coefficient [-]
- \( \xi_0 \) = Iribarren parameter [-]
- \( a \) = coefficient [-]
- \( b \) = coefficient [-]
- \( \alpha_\beta \) = steepness of the revetment [-]
- \( H \) = wave height [m]
- \( \lambda_0 \) = wave length in deepwater [m]

The coefficients \( a \) and \( b \) are respectively 0.12 and 0.87 for the revetment at Sint Maarten (Zanuttigh, 2006). The steepness of the revetment is estimated to be 1:3. A wave height of 6.0 m is used and a length of 265 m. With these parameters the reflection coefficient of the revetment is 0.2. This means that only a small part of the wave is reflected at the abutment.

**Conclusion**
The reflection from the quay wall is included in the wave climate investigated in this research. The reflection at the abutment is not included in this research, and is indicated to have a reflection coefficient of 0.2.
Appendix IV

Alternative Method of Deriving Vertical Wave-in-Deck Peak Pressure

The vertical wave-in-deck peak pressure is derived in section 4.3. In this appendix and slightly different approach is used to derive the peak pressure, which leads to the same formulation.
Alternative Method of Deriving Vertical Wave-in-Deck Peak Pressure

The vertical wave-in-deck load is derived in section 4.3. A peak pressure occurs during the first contact between the wave and the bottom of the jetty deck. This peak pressure is derived in sections 4.3 and found to be proportional to the kinetic energy. In this appendix and slightly different approach is used to derive the peak pressure, which leads to the same formulation.

General Description of Events during Wave-in-Deck Loading
To derive a vertical wave-in-deck load, the jetty deck is approached as being a stiff fixed plate of B times L located at a distance above the water level. A large wave approaches the stiff plate in the direction of B. The wave length is much larger than the width B. The situation is shown in Figure 4.

At a certain moment in time the trough of the wave is below the fixed plate (t₁). At that moment there is no wave load present on the plate. After the trough, the water surface rises. Until the water level reaches the plate (t₂). At that moment in time an impact force occurs on the plate. The impact can be imagined to be caused by a water mass of B times L times a depth of α times B (shown with the box in Figure 6. It is assumed that the mass of water slows down to 0 m/s, in a very short moment of time, which causes the impact. After this impact, the water level keeps rising, causing the water to flow around the structure. The water flows around the plate, causing drag and inertia as described by Morison’s equation for submerged structures. This situation is schematized in Figure 5.

The two events of impact and the flowing around the structure are separated. The impact is only present for a very small moment in time and is causing the peak pressure Pₓ,i. This is described in the next paragraph.

Peak Pressure
The peak pressure in vertical direction is derived in this section. For a plate of B by L, the impact of the water mass hitting the plate is given by equation A.1. Figure 6 shows the volume of water being slowed down by the first contact with the plate. α is an unknown parameter.
The impact is mass times velocity. The velocity is taken to be the vertical water particle velocity ($v_z$).

The duration of the impact is assumed to be so short, that the variation in the water particle velocity in time can be neglected. This results in an impact shown in equation A.2.

$$I = m v = \alpha \rho B^2 L v_z$$  \[A.2\]

The duration of the impact $\Delta t$ is assumed to be related to distance B; the larger this distance the larger the time of impact. The inverse relation is assumed for the velocity. Apart from these assumed relationships the duration of the impact is unknown. Therefore an unknown factor $\beta$, is introduced. This makes the definition for the duration of impact:

$$\Delta t = \beta \frac{B}{v_z}$$  \[A.3\]

The impact is assumed to have a triangle shape. The load caused by the impact becomes as shown in equation 4.7. It has to be noted that this triangle shaped impact also causes a downward force, this downward force is neglected. After substituting equation A.2 and A.3 in equation A.4 this results in equation 4.8.

$$F_{\text{z,d}} = 2 I / \Delta t$$  \[A.4\]

$$F_{\text{z,d}} = 2 \frac{\alpha}{\beta} \rho B L v_z^2$$  \[A.5\]

For the dynamic analysis, not the load but the pressure is of interest. The load is therefore divided by the contact area of B times L. In this way the pressure at a certain location varying in time can be derived. The water particle velocity is assumed to be uniform over a square meter.

$$P_{\text{z,d}}(t, x, y) = \frac{F_{\text{z,d}}(t, x, y)}{B L} = 2 \frac{\alpha}{\beta} \rho v_z(t, x, y)^2$$  \[A.6\]

The water particle velocity depends on time and location (x and y-coordinate). Equation A.6 is the numerical formulation for the vertical peak pressure on the deck, which is present during a very short time interval of $\Delta t$. The density of water and the water particle velocity are known. Only the factor $\alpha/\beta$ is unknown.
Appendix V

Deriving $\alpha$ of the Wave-in-Deck Load

The formulation for the vertical wave-in-deck loading (equation 4.12) is derived in two steps. The first step is deriving the peak pressure (tower of the church-roof shape of the load). This peak pressure has an unknown $\alpha$ (equation 4.8). The value of this $\alpha$ factor is derived in this appendix.
Deriving a value for the unknown \( \alpha \) factor

The vertical wave-in-deck load is derived by first determining the peak pressure. This peak pressure varies in time and location as required for an accurate dynamic analysis. The formulation for the peak pressure is derived to be:

\[
P_{z,d}(t, x, y) = \alpha \rho v^2_c(t, x, y) \quad [\text{N/m}^2] \quad [A.1]
\]

The factor \( \alpha \) of equation \( A.1 \) is unknown. Its value is estimated from the results of the physical model tests performed for Sint Maarten.

For the situation of Sint Maarten physical model tests have been performed by WL Delft Hydraulics (1998). From the results slamming coefficients for an empirical formula for the wave-in-deck load are determined. This empirical formulation based on the physical model test depends on the density of water \( \rho \), gravitational acceleration \( g \) and the significant wave height \( H_s \). The formulation is shown in equation \( A.2 \). The value of \( \alpha \) is determined using equation \( A.3 \).

\[
P_{z,d} = C_{slam} \rho g H_s \quad [\text{N/m}^2] \quad [A.2]
\]

\[
P_{z,d} = \alpha \rho v^2_c = C_{slam} \rho g H_s \quad [A.3]
\]

The test results correspond to slamming coefficients between the 1.8 and 4.3 for the 0.4 % exceedance probability peak pressure of the vertical wave-in-deck loading. This range of coefficients is in between the values mentioned in literature (Rooij de, 2001). The results of the physical model tests of Sint Maarten are expected to be a good estimate for the magnitude of the wave-in-deck load at Sint Maarten. The value of \( \alpha \) is therefore determined from the results of the tests. A value for \( \alpha \) should be found for which the simulated peak pressure is close to the average of the results from the physical model tests.

This formula (equation \( A.2 \)) represents the 0.4% exceedance probability value. \( \alpha \) is determined by setting equation \( A.1 \) equal to equation \( A.2 \), shown in \( A.3 \). However, the empirical formulation depends on the significant wave height, and the derived formulation depends on the water particle velocity. In order to find the value of \( \alpha \) the water particle velocity should be rewritten into a formulation depending on the significant wave height. In this research the water particle velocity has the formulation as shown in equation \( A.4 \).

\[
v_c(t) = \sum_{k=1}^{N} A_k \omega_k \sinh(k \omega t + k \phi - k \phi) \quad [\text{m}]
\]

\[
A_k = \text{amplitude} \quad [\text{m}]
\]

\[
\omega_k = \text{angular frequency} \quad [\text{rad/s}]
\]

\[
\phi_k = \text{phase angle} \quad [\text{rad}]
\]

\[
k = 1, 2, 3... \quad [-]
\]

\[
t = \text{time} \quad [\text{s}]
\]

\[
k = \text{wavenumber} \quad [\text{rad/m}]
\]
The z-coordinate at the jetty deck is taken to be zero, because of vertical stretching of the Airy wave theory. This results in the scaling factor (hyperbolic sine fraction) to become one.

As described in chapter 3, the amplitude \( A_k \) is related to the wave spectrum, of which the shape is determined by the significant wave height. The relations are repeated in the next equations. The amplitude \( A_k \) is determined by the wave spectrum \( S_{\eta \eta} \) by equation A.5.

\[
A_k = \sqrt{2\Delta \omega S_{\eta \eta}(\omega_k)} \tag{A.5}
\]

\[
S_{\eta \eta}(\omega_k) = \text{wave spectrum} \quad [m^2/s]
\]

\[
\Delta \omega_k = \text{angular frequency difference}, \quad \frac{2\pi}{T} \quad [rad/s]
\]

With \( \omega_k \) being \( k \) times \( \frac{2\pi}{T} \) (T=100 s). For the wave spectrum a PM-spectrum is assumed, which is shown in equation A.6.

\[
S_{\eta \eta}(\omega) = \alpha_s g^2 \omega^{-5} e^{-1.25 \left( \frac{\omega}{\omega_s} \right)^4} \tag{A.6}
\]

The shape factor \( \alpha_s \) in the formulation depends on the wave frequency and significant wave height as described in equation A.7.

\[
\alpha_s = \left( \frac{\omega^2_s \sqrt{5} H_s}{4g} \right)^2 \tag{A.7}
\]

When the previous equations are implemented in each other equation A.4 becomes equation A.8.

\[
v_z(t) = \sum_{k=1}^{N} 2\Delta \omega \left( \frac{\omega^2_s \sqrt{5} H_s}{4g} \right)^2 g^2 \omega^{-5} e^{-1.25 \left( \frac{\omega}{\omega_s} \right)^4} \omega_k \cos(\omega_k t + \phi_k - k\bar{x}) \tag{A.8}
\]

From equation A.8 \( H_s \) is taken out of the root.

\[
v_z(t) = \sum_{k=1}^{N} \left( \frac{\omega^2_s \sqrt{5} H_s}{4g} \right)^2 \omega_k^4 g^2 \omega^{-5} e^{-1.25 \left( \frac{\omega}{\omega_s} \right)^4} \omega_k \cos(\omega_k t + \phi_k - k\bar{x}) = H_s \sum_{k=1}^{N} X_k \tag{A.9}
\]

\[
X_k = \frac{\omega^2_s \sqrt{5} H_s}{4g} \left( \frac{\omega^2_s \sqrt{5} H_s}{4g} \right)^2 \omega_k^4 g^2 \omega^{-5} e^{-1.25 \left( \frac{\omega}{\omega_s} \right)^4} \omega_k \cos(\omega_k t + \phi_k - k\bar{x}) \tag{A.10}
\]

The formulation of equation A.9 exists out of \( H_s \) multiplied with a long formulation. This long formulation 'behind' \( H_s \) is temporarily called \( \Sigma X_k \) for a shorter formulation. \( \Sigma X_k \) depends on location \( x \), the random phase angle \( \phi \) and time \( t \). However, \( \bar{\alpha} \) is assumed to be a constant, this is because not \( \Sigma X_k \) but \( \max_{0.4}(\Sigma X_k(x = 0 \text{ m})) \) is implemented in \( \bar{\alpha} \) which takes the dependence out. The next section explains what \( \max_{0.4}(\Sigma X_k(x = 0 \text{ m})) \) is.
For \( \Sigma X_k \) the 0.4% exceedance probability should be taken, because \( C_{\text{slam}} \) also corresponds to 0.4% exceedance probability. This can be done by simulating \( \Sigma X_k \) under comparable circumstance as used during the physical model tests. This is done by taking the maximum of \( \Sigma X_i \) during 5 s (\( \max(\Sigma X_i) \)), which is performed 250 times; the maximum value of these 250 \( \max(\Sigma X_i) \) is taken as the 0.4% exceedance probability value (\( \max_{0.4}(\Sigma X_i) \)). These steps are schematized below.

The location \( x \) where \( \Sigma X_k \) with 0.4% exceedance probability is determined is independent of the result. \( \Sigma X_k \) differs per location; however the probability distribution is equal. Therefore the same value for \( \max_{0.4}(\Sigma X_k(x = 0 m)) \) is expected for every location. Location \( x = 0 \) m is chosen. This results in:

\[
\max_{0.4} \left( \sum_{k=1}^{N} X_k(x = 0 \text{ m}) \right)
\]

1. The sum is taken of \( X_k \) over \( \omega_k \) at \( x = 0 \) m. This is done during a period of 5 s;
2. After which the maximum is taken (this is the maximum of the sum of \( X_k \) occurred during 5 s);
3. This repeated 250 times;
4. The maximum of these 250 is the value used in \( \bar{\alpha} \).

It has to be noticed that the dominant frequency \( \omega_0 \) is included in \( \max_{0.4}(\Sigma X_k(x = 0 m)) \) which is related to the significant wave height. In the section ‘is \( \bar{\alpha} \) a constant?’ this is further discussed.

With equation A.9 equation A.1 becomes:

\[
P_{z,i} = \bar{\alpha} \rho v_i^2 = \bar{\alpha} \rho H_i^2 \left( \max_{0.4}(\sum_{k=1}^{N} X_k(x = 0 \text{ m})) \right)^2
\]  

[A.11]

When this is set to be equal to the physical model test formulation a formulation for \( \bar{\alpha} \) is found. This is shown in equation A.12. Equation A.13 shows the numerical wave-in-deck load formulation, with \( \bar{\alpha} \) substituted in the formulation (equation 4.9).

\[
\bar{\alpha} = C_{\text{slam}} \frac{g}{\left( \max_{0.4}(\sum_{k=1}^{N} X_k(x = 0 \text{ m})) \right)^2} \frac{1}{H_s}
\]  

[A.12]

\[
P_{z,i}(t,x,y) = C_{\text{slam}} \frac{g}{\left( \max_{0.4}(\sum_{k=1}^{N} X_k(x = 0 \text{ m})) \right)^2} \frac{1}{H_s} \rho v_i^2(t,x,y)
\]  

[A.13]

For \( C_{\text{slam}} \) the average value found by the physical model tests is taken, which is 3.9 for the vertical peak pressure.

The random phase angle \( \varphi \) in the formulation of \( \Sigma X_k \) results in a different \( \max_{0.4}(\Sigma X_k(x = 0 m)) \) for every repetition. Values between the 0.6 s\(^{-1}\) and 0.7 s\(^{-1}\) are found for \( \max_{0.4}(\Sigma X_k(x = 0 m)) \). Which of these values should be taken is determined by implementing different values for \( \max_{0.4}(\Sigma X_k(x = 0 m)) \) in equation A.13, and comparing the result with the physical model tests. The average of the 0.4% exceedance probability peak pressure derived using the numerical formulation should be close to the average of the peak pressures found by the physical model tests. Table 2 shows the average of the 0.4% exceedance probability peak pressure for different values for \( \max_{0.4}(\Sigma X_k(x = 0 m)) \).
Table 2 - Peak Pressure using Physical Model Test and Different Numerical Formulations

<table>
<thead>
<tr>
<th>Peak Pressure 0.4% exceedance probability</th>
<th>Physical Model Tests</th>
<th>Numerical max0.4(ΣXₖ(x=0m)) = 0.60 α = 18</th>
<th>Numerical max0.4(ΣXₖ(x=0 m)) = 0.64 α = 16</th>
<th>Numerical max0.4(ΣXₖ(x=0m)) = 0.66 α = 15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average</td>
<td>211 kN/m²</td>
<td>245 kN/m²</td>
<td>209 kN/m²</td>
<td>176 kN/m²</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>24 kN/m²</td>
<td>73 kN/m²</td>
<td>67 kN/m²</td>
<td>28 kN/m²</td>
</tr>
</tbody>
</table>

The physical model tests show a slamming coefficient between the 1.8 and 4.3. For one of the tests (number 223, specifications of the test are given in section 4.3) this results in peak pressures between the 103 kN/m² and 252 kN/m², with an average of 211 kN/m². From Table 2 it can be seen that the max₀.₄(ΣXᵦ(x = 0 m)) of 0.64 results in a α of 16.0, and an average of the 0.4% exceedance probability peak pressure of 209 kN/m², which is very close to the average of 211 kN/m² of the physical model tests. It can also be seen that the standard deviation is much larger than the one of the physical model tests. However from the other simulations with the numerical wave-in-deck load formulation it can be seen that the variance strongly differs per set of simulations as well. From this α is concluded to be 16.0. This results in the following formulation for the vertical peak pressure:

\[ P_{vz}(t, x, y) = 16\rho v_z(t, x, y)^2 \quad \text{[N/m}^2\text{]} \quad \text{[A.14]} \]

*Is α a constant?*

In the previous section a value for α is derived. A general formulation for the peak wave-in-deck pressure arises. However, it is uncertain if this is a ‘general’ formulation valid for every situation, or that α depends on circumstances and the significant wave height. Therefore the dependencies in α and literature on this subject are discussed in this paragraph.

ΣXᵦ includes the dominant wave frequency ω₀ which is connected to the significant wave height Hₛ, and α has Hₛ in its formulation as well. Hₛ being in the formulation of α seems to make α depending on the significant wave height. However, the slamming coefficient is also derived for the significant wave height by which it is divided in α (equation A.12). So if the slamming coefficient would be linear depending on the significant wave height, this is corrected in α by dividing by Hₛ. In that situation dividing by Hₛ would not lead to a dependency of α on Hₛ. The dominant wave frequency is in the formulation of α (equation A.10) as well. The dominant frequency is connected to the significant wave height by the steepness of waves. (Part of this dependence on the dominant wave frequency can be taken out of α. This can be done by making α dependent of the peak period Tₛ. However, part of the dependence of the dominant frequency cannot easily be taken out of the equation because it is in the exponent.) The same reasoning applies for this dependency as for the significant wave height in α, because the slamming coefficient is empirically determined it remains uncertain what is included in this factor and what influences α.

Sun, 2011 concluded that the zeroth spectral moment of the wave-in-deck load strongly depends on the relative deck clearance. The relative deck clearance is defined as the ratio between the distance between the deck and the MSL and the wave height. This indicates that both the deck clearance and
the wave height have a strong influence on the wave-in-deck load. However, this relative clearance also influences the water particle velocity. It is therefore uncertain if this dependence on the relative deck clearance is already included in the numerical wave-in-deck load formulation (by $v_z$ in equation A.14), or that the relative deck clearance influences the $\alpha$ factor.

A conclusion about the dependence of the $\alpha$ factor could only be made based on experiments investigating this relation. Experiments for which the relation between the peak pressure and the water particle velocity $P_z/v_z^2$ is investigated have not been found in literature. Therefore, the dependence of $\alpha$ on circumstances and wave height remains uncertain.

For this research it is assumed that $\alpha$ is independent of the significant wave height and situation. The same $\alpha$ is used for the different wave spectra which are used in this research. The difference in peak pressure between the different water surface elevations hitting the jetty deck occurs due to the difference in water particle velocity.
Appendix VI

Validation Wave-in-Deck Load

The derived formulations for the vertical wave-in-deck loading (equation 4.12) are checked by remaking the results found in literature. The results and wave parameters are shown in this appendix. It is desirable for the wave load to have the shape of a church roof.
Remaking Results found in Literature

The situation for which the wave-in-deck load is calculated is for a wave rolling over the jetty. The part of the wave crests that rolls over the jetty deck does not break. Only the vertical wave load is calculated, assuming an approach angle of zero, which means that the wave approaches in the same line as the jetty axis. In this calculation movement of the jetty is not taken into account. A summary of the modelled situation:

Using Shih (2005) input parameters:

- \( H_s \): 0.22 m
- \( T_p \): 2.00 s
- \( \lambda \): 5.7 m
- \( d \): 1.2 m
- Deck clearance of 0.02 m (distance between bottom deck and SWL).

Results for equation 4.24 (with \( \alpha \) being 16) in a maximum force of 12 kN/m\(^2\). This is close to the maximum force found by Shih of about 13.6 kN/m\(^2\).

Using van Raaij (2005) input variables:

- \( H \): 24.3 m
- \( T_p \): 14.5 s
- \( \lambda \): 306 m
- Water depth 30 m
- Width of the deck 30 or 40 m

The distance between the deck and the SWL is not known. The calculation is done for 2, 4 and 8 meter inundation. These parameters result in a vertical load of respectively 12, 23 and 110 MN for one crest, using the numerical derived method (\( \alpha \) being 16). This is larger than the 7 and 9 MN found using the CFD method, in (Raaij van, 2005).

<table>
<thead>
<tr>
<th>[MN]</th>
<th>Found by Raaij</th>
<th>Numerical based method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical load</td>
<td>8</td>
<td>12 to 110</td>
</tr>
</tbody>
</table>

The too large wave load could be caused by the linear wave theory. Raaij does not use the linear wave theory, because she states that this leads to a too large water particle velocity. This could be part of the reason for the too large wave in deck load.
Appendix VII

Validation of the Numerical Solver

The dynamic problem is solved using backward Euler method. The results are compared with the results found when using the Duhamel integral. Also the stability of the Backward Euler method is checked for a simulation of longer duration.
Validation of Numerical Solver

The differential equation of the uncoupled problem is determined to be equation 5.19, per mode shape of the system.

This is a second order differential equation. To solve this equation with a numerical method, the second order equation has to be changed into a system of first order equations. This is done with the introduction of $v_1$ and $v_2$.

\[
\begin{align*}
\ddot{u} &= v_1 \\
\dot{v}_1 &= v_2 \\
\dot{v}_2 &= \omega^2 v_1 - \frac{\omega^2}{\lambda^2} M \lambda x_0(t) + \frac{\omega^2}{\lambda^2} F(t)
\end{align*}
\]  

This results in the following system of first order equations:

\[
\begin{align*}
\dot{v}_{2,i} &= -2 \zeta_i \omega_i v_{1,i} - \omega_i^2 v_{1,i} + \frac{\omega_i^2}{\lambda_i^2} F(t) \\
\dot{v}_{1,i} &= v_{2,i}
\end{align*}
\]  

This system is solved with the backward Euler (Vuik, et al., 2006). To check if the backward Euler method stays stable for a calculation over a long period, and for a sharp load the results are compared with the results when using the Duhamel integral. Backward Euler is mentioned only to be adequate for ‘smooth’ loads. The vertical wave-in-deck loading is not expected to include in this category. The displacements calculated with the both methods should be the same. The only difference may be the numerical damping in the numerical solver.

The total solution for the forced system, including all considered mode shapes determined by Duhamel integral is shown in equation A.4.

\[
\begin{align*}
\ddot{w}(t) &= \sum_{i=1}^{n} \ddot{\hat{x}}_i u_i(t) \\
\ddot{w}(t) &= \sum_{i=1}^{n} \hat{x}_i C_i e^{-\zeta_i \omega_i} \sin(\omega_i (\sqrt{1 - \zeta_i^2} t + \phi_i) + \\
&+ \int_{t=0}^{t} \hat{x}_i M \hat{x}_i F(\tau) - \frac{1}{\omega_i \left(\sqrt{1 - \zeta_i^2}\right)} e^{-\zeta_i \omega_i (t - \tau)} \sin(\omega_i (\sqrt{1 - \zeta_i^2} (t - \tau)) d\tau)
\end{align*}
\]  

The first part of equation A.3 is the free vibration, containing the unknown amplitude and phase angle that depend on the initial conditions. The second part of the equation is determined by the load vector $F$, mass matrix, eigenvectors and natural frequencies. The formulation also includes damping.

In order to get a similar result from both solving methods the same wave is needed for the simulation. This can only be performed when using equal time steps for both solving methods. This is a disadvantage for the numerical solver, because a larger time step causes larger numerical damping. The Duhamel integral can only be simulated for a maximum duration of 40 seconds (using a time step of 0.02 s), because of the large number of calculation steps are needed to solve this integral, which causes a lot of calculation time and memory.
From Figure 7 and Figure 8 it can be seen that the rough displacements are equal for both solving methods. The displacements calculated with the Duhamel integral also has the first natural frequency visible in its displacement. The numerical solver has numerical damping because of its method of solving. The damping is related to the time step used in the calculation, this is 0.02 s. This causes the high frequency vibrations to decrease sooner, when using the numerical solving method. The calculation is also done using a smaller time step for the numerical solver, this result shows the high frequency vibrations to decrease more slowly. The amplitude of this high frequency vibration gets less in time using the Duhamel integral as well, this is caused by the damping in the system.
Stability of Numerical Solver

Some numerical solvers get unstable for longer simulations. For the backward Euler method this is not expected, because of the numerical damping. The three plots on the next page show:

- Water surface elevation;
- Vertical wave-in-deck pressure;
- Displacement in y-direction.

The calculation is performed for a period of 5 minutes. All plots are made at x = 55 m, which is of about one third of the jetty module length. The displacement in y-direction is the displacement perpendicular to the jetty axis.

From the plots it can be seen that the solver is stable also for sharp peaked loads over a longer duration. The large amplitudes of vibrations are caused by high wave peaks, and not by numerical instability even after 4 minutes.
Appendix - Dynamic Analysis of an Open Piled Jetty Subjected to Wave Loading

Graphs showing wave height, vertical pressure, and displacement in the y-direction at two different distances (150 m and 55 m) over time.
Appendix VIII

Validation of Dynamic Model with Simply supported Beam

The dynamic model is checked by comparing the results of a simply supported beam with a continuous bending beam calculation.
Simply Supported Beam

The dynamic problem is solved using model analysis. For a difficult structure like the jetty, the dynamic behaviour is difficult to check. The behaviour of a simply supported beam can be checked more easily. The results determined using modal analysis are compared with a results using a continuous bending beam model.

Equation of Motion

In order to do a calculation of a continuous bending beam, the equation of motion has to be derived. This is done using the constitutive and kinematic equations. The positive directions for the shear forces and bending moments are shown below on a very small part called $dx$.

The beam is modelled as bending beam. This means that the beam has a mass and a bending stiffness. Shear deformation is not included. The coordinate system is defined. The small part $dx$, is (enlarged) shown in the beam model.

The constitutive equation relates stress to strain. The kinematic relations show the relation between the curvature ($\kappa$), the bending moment and the rotation of the beam. When the different relations are combined with Newton’s second law, this leads to the following equation of motion for the bending beam.

$$ q_s(x,t) = \rho A \frac{d^2 w}{dt^2} + EI \frac{d^4 w}{dx^4} $$

The constitutive relations are related to stress and strain. Below a part of the beam is shown. At the end of this part the axial stress are shown. The bending moment is related to this axial stresses, by the constitutive equations, as shown in equations below.
\[ \sigma_z(z) = E\varepsilon(z) \]
\[ \varepsilon(z) = \kappa z \quad [A.6] \]
\[ M = \int \sigma_z(z) \cdot dA = \int E\varepsilon(z)z \cdot dA = \int E\kappa z^2 \cdot dA = EI\kappa \]

The kinematic relations show the relation between kappa and the rotation of the beam. It is assumed that the angle, called theta, is small. Therefore the following relation holds:

\[ \tan(\theta) = \sin(\theta) = \theta \]
\[ \cos(\theta) = 1 \]

In the figure the definitions of the different symbols are given. The horizontal line represents the initial position of the beam. The curved line represents the beam after deformation.

\[ \kappa = \frac{1}{\rho} \]
\[ d\theta \cdot \rho = ds \]
\[ \tan(-\theta) = \frac{dw}{dx} = -\theta \]
\[ \cos(\theta) = \frac{dx}{ds} \rightarrow dx = ds \]
\[ \kappa = \frac{d\theta}{ds} = \frac{d\theta}{dx} = -\frac{d^2w}{dx^2} \]
\[ \theta = -\frac{dw}{dx} \]
\[ \kappa = -\frac{d^2w}{dx^2} \quad [A.7] \]

With Newton’s Second Law the last relations are derived to form the equation of motion of the bending beam. It is assumed that inertia is set to zero. Also the bending moment due to the distributed load \( q_z \) on the small part \( dx \) is neglected.

\[ \sum F_z = m \cdot \frac{d^2w}{dt^2} = \rho A dx \cdot \frac{d^2w}{dt^2} = V + dV - V + q_z dx \]
\[ \rho A dx \cdot \frac{d^2w}{dt^2} - dV = q_z dx \]

\[ q_z = \rho A \cdot \frac{d^2w}{dt^2} - \frac{dV}{dx} \quad [A.8] \]
\[ \sum M = J \cdot \frac{d^2\theta}{dt^2} = \rho I dx \cdot \frac{d^2\theta}{dt^2} = M + dM - M - Vdx - \frac{1}{2} q_z dx^2 \]
Together with the assumptions this results in the simple relations shown in equation A.9. Together with equation A.6 and A.7 equation A.10 is derived.

\[ dM = Vdx \]

\[ V = \frac{dM}{dx} \quad [A.9] \]

\[ M = -EI \frac{d^2w}{dx^2} \]

\[ V = -EI \frac{d^3w}{dx^3} \quad [A.10] \]

When equation A.10 is substituted in equation A.8, equation A.11 is found. This is the equation of motion for the bending beam.

\[ q_0 = \rho A \frac{d^2w}{dt^2} - \frac{dV}{dx} = \rho A \frac{d^2w}{dt^2} + EI \frac{d^4w}{dx^4} \quad [A.11] \]

**Simply Supported Beam**

- **B** = 0.4 m
- **h** = 0.4 m
- **ρ** = 2400 kg/m³
- **E** = 330 \cdot 10^8 N/m²
- **I_{ss}** = 0.0021 m⁴
- **L** = 7 m

For the situation of the simply supported beam, a point load is present at \( x = 4 \) m.

For a bending beam with a point load the following equation of motion holds:

\[ 0 = \rho A \frac{d^2w^-}{dt^2} + EI \frac{d^4w^-}{dx^4} \quad 0 < x < 4 \]

\[ 0 = \rho A \frac{d^2w^+}{dt^2} + EI \frac{d^4w^+}{dx^4} \quad 4 < x < L \]

\[ P(t) = \hat{P} \sin(\Omega t) \]

Suppose the steady state response can be written in the form of \( w(x,t) = W(x) \sin(\Omega t) \). The variable time and place are separated in different functions, which together form the total response. After implementing this steady state response in the equation of motion this results in:
\[
\frac{d^4W(x)}{dx^4} - \beta^4 W(x) = 0 \quad \beta^2 = \frac{\rho A}{EI} \Omega^2 \quad 0 < x < L, \; x \neq 4
\]

The general solution for \( W(x) \) is:

\[
W^- (x) = A^- \cosh(\beta x) + B^- \sinh(\beta x) + C^- \cos(\beta x) + D^- \sin(\beta x)
\]

\[
W^+ (x) = A^+ \cosh(\beta x) + B^+ \sinh(\beta x) + C^+ \cos(\beta x) + D^+ \sin(\beta x)
\]

The boundary conditions are:

\[
W^-(0) = 0 \\
W^+(L) = 0 \\
M^-(0) = 0 = \frac{\partial^2 W^-}{\partial x^2} \\
M^+(0) = 0 = \frac{\partial^2 W^+}{\partial x^2}
\]

The interface conditions are:

\[
W^-(4) = W^+(4) \\
\frac{\partial W^-(4)}{\partial x} = \frac{\partial W^+(4)}{\partial x} \\
\frac{\partial^2 W^-(4)}{\partial x^2} = \frac{\partial^2 W^+(4)}{\partial x^2} \\
\left( \frac{\partial^3 W^-(4)}{\partial x^3} - \frac{\partial^3 W^+(4)}{\partial x^3} \right) = \frac{\hat{P}(t)}{EI}
\]

Together this leads to a set of 8 equations, which are used to solve the 8 unknowns \( A,B,C \) and \( D \) plus and minus. The set of equations can be written as a matrix \( M \), which should be equal to the force vector \( F \).
Solving this equation results in the 8 unknowns $A, B, C, D$ for plus and minus. The response of the bending beam in z-direction in steady state is given by:

$$w^-(x,t) = (A^+\cosh(\beta x) + B^-\sinh(\beta x) + C^+\cos(\beta x) + D^-\sin(\beta x))\sin(\Omega t)$$

This response at $x=4$ m for a steady state situation is plotted in Figure 9. For the following parameters of the load $P(t)$. The displacement in z-direction calculated using modal analysis is shown as the blue line in Figure 9. The red line is the displacement calculated using a bending beam.

$$\hat{P} = 10^5 \text{ N}$$
$$\Omega = 2 \text{ rad/s}$$

Figure 9 - Displacement Simply supported Beam Comparing Continuous (red) and Modal Analysis (blue)

From Figure 9 it can be seen that both methods give the same result for the simply supported beam.

The maximum displacement is almost 0.01 m. This is compared with the static calculation for a load placed in the middle of a beam.
\[ w = \frac{1}{48} \frac{\hat{P}L^3}{EI} = \frac{1}{48} \frac{10^4 \cdot 7^3}{330 \cdot 10^9 \cdot 2.1 \cdot 10^{-3}} = 0.01 \text{ m} \]

This results in a displacement almost equal to the results from the dynamic calculations.

For the calculation with the modal analysis the following parameters are used:

- 16 nodes are used. In x-direction having 1 m spacing;
- 3 DOF are included in the calculation;
- 5 mode shapes are used;
- The simulation is done for 20 seconds, with a time step of 0.1 s;

Figure 10 shows both the response spectra of the movement in z-direction at \( x = 4 \) meter. The two colours are difficult to see in the response spectrum graph, because the lines are exactly on top of each other.

![Figure 10 – Response Spectrum Simply supported Beam Comparing Continuous (red) and Modal Analysis (blue)](image)

This confirms the results from both dynamic models to result in the same displacement.
Appendix IX

Mode Shapes 9 to 18

The mode shapes from 7 to 18 all show bending in the zx and yz-plane. The mode shapes are shown in this appendix.
Mode Shapes 9 to 18

The mode shapes from 7 to 18 all show bending in the zx and yz-plane. The mode shapes 9 to 18 are shown in this appendix.

Modes shape 9, $\omega_9$ is about 44 rad/s.

Modes shape 10, $\omega_{10}$ is about 44 rad/s.
Modes shape 11, $\omega_{11}$ is about 44 rad/s.

Modes shape 12, $\omega_{12}$ is about 44 rad/s.

Modes shape 13, $\omega_{13}$ is about 45 rad/s.
Modes shape 14, $\omega_{14}$ is about 45 rad/s.

Modes shape 15, $\omega_{15}$ is about 46 rad/s.
Modes shape 16, $\omega_{16}$ is about 47 rad/s.

Modes shape 17, $\omega_{17}$ is about 48 rad/s.
Modes shape 18, $\omega_{18}$ is about 49 rad/s.
Appendix X
Sensitivity Analysis

The influences of the following model choices on the dynamic reaction of the jetty subjected to wave loading are investigated in this appendix:

- Modelling of expansion joint;
- Modelling of the soil;
- Cracked concrete deck;
- Damping;
- Added mass;
- Air gap;
- Duration of peak pressure;
- Length of jetty module;
- Randomness of water surface elevation;
Sensitivity Analysis

The results presented in chapter 6 are computed for a certain situation, with a specific model and several assumptions are made. The influences of these assumptions on the dynamic reaction of the jetty to the wave loading are investigated in this appendix.

Modelling of Expansion Joint

The expansion joint is modelled with boundary conditions at the head of a module. Four different types of boundary conditions are used to investigate the effect of the expansion joint on the dynamic behaviour:

- No BC;
- Translations fixed at one head of the module in z and y-direction;
- Translations fixed at both sides of the module in z and y-direction;
- Translations fixed at one head of the module in x, y and z-direction.

These four types of modelling are also shown in Figure 11.

The boundary conditions influence the mode shapes and the natural frequencies. The model without boundary conditions at the heads of the jetty module is only representative for small displacements. However, displacements of 0.1 m are found. For displacements larger than 0.05 m in x-direction the neighbouring jetty part is expected to influence the vibrations. Modelling this situation is not possible with the method used in this research. The mode shapes and natural frequencies are determined with Scia Engineer. Mode shapes only show the proportions between the different displacements along the jetty and other mode shapes. They do not have a dimension. A boundary condition that influences the structure after 0.05 m, can therefore not be included.

The expansion joint is designed to transfer forces in y and z - direction. Therefore a model is used fixing the head of the jetty in y and z-direction. For large displacements in x-direction the neighbouring jetty modules influence each other. This is situation is modelled by fixing displacements in x, y and z-direction.
Translational fixed in z and y-direction

By fixing the head of the jetty in y and z-direction the mode shapes change. Also the natural frequencies are influenced. The natural frequencies differ between the 6.4 rad/s and 55 rad/s. These natural frequencies are higher than for the model without boundary conditions (5.8 rad/s to 49 rad/s). Both situations are calculated and the difference in dynamic behaviour can be seen from Table 3 and Table 4.

For a module of the jetty which has an expansion joint on both sides, the situation can be modelled by placing boundary conditions in z and y-direction at both heads of this jetty module. The natural frequencies of this model are only a small bit higher than for the model restrained at one side (about 0.1 Hz). This situation is therefore not further taken into consideration, because the difference of influence on the dynamic behaviour is expected to be small as well.

Table 3 - DAF of Model Without Boundary Conditions

<table>
<thead>
<tr>
<th>Angle of</th>
<th>DAF X</th>
<th>DAF Y</th>
<th>DAF Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angle of 0</td>
<td>2.6</td>
<td>0</td>
<td>0.39</td>
</tr>
<tr>
<td>Angle of 10</td>
<td>1.8</td>
<td>1.7</td>
<td>0.52</td>
</tr>
<tr>
<td>Angle of 25</td>
<td>2.3</td>
<td>1.6</td>
<td>0.34</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Angle of</th>
<th>DAF X</th>
<th>DAF Y</th>
<th>DAF Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angle of 0</td>
<td>2.9</td>
<td>0</td>
<td>0.52</td>
</tr>
<tr>
<td>Angle of 10</td>
<td>2.3</td>
<td>2.1</td>
<td>0.70</td>
</tr>
<tr>
<td>Angle of 25</td>
<td>2.8</td>
<td>2.1</td>
<td>0.40</td>
</tr>
</tbody>
</table>

Table 4 - DAF of Model With Boundary Conditions in y and z-direction at One Side of the Head of the Module

<table>
<thead>
<tr>
<th>Angle of</th>
<th>DAF X</th>
<th>DAF Y</th>
<th>DAF Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angle of 0</td>
<td>2.6</td>
<td>0</td>
<td>0.50</td>
</tr>
<tr>
<td>Angle of 10</td>
<td>1.8</td>
<td>1.6</td>
<td>0.62</td>
</tr>
<tr>
<td>Angle of 25</td>
<td>1.8</td>
<td>1.4</td>
<td>0.41</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Angle of</th>
<th>DAF X</th>
<th>DAF Y</th>
<th>DAF Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angle of 0</td>
<td>3.4</td>
<td>0</td>
<td>1.0</td>
</tr>
<tr>
<td>Angle of 10</td>
<td>3.2</td>
<td>2.9</td>
<td>1.4</td>
</tr>
<tr>
<td>Angle of 25</td>
<td>2.8</td>
<td>2.0</td>
<td>0.58</td>
</tr>
</tbody>
</table>
The DAF’s shown in the left graph of Table 3 and Table 4 are in the same range. The differences for the DAF with a 0.1 % exceedance probability are larger. In x-direction the model including boundary conditions indicates a DAF of 3.4, which is 0.5 larger than the model without boundary conditions. In y-direction the difference is even larger 0.8. Also in z-direction the DAF is larger. The calculations are performed using the wave spectrum with waves just hitting the jetty deck ($H_s = 3.0 \text{ m}$ and $T_p = 4.0 \text{ m}$) in section 3.3, this spectrum is explained further. The calculations are also performed using the large wave spectrum. This did not show a significant difference in the dynamic amplification factor, between the with and without boundary condition model. Other wave spectra are not expected to cause larger differences between the two types of modelling.

The difference is a significant enlargement, which should be taken into account. However, contact at the expansion joint with the neighbouring module, does not mean that no movement is possible at that point. The neighbouring jetty module can also move along with the displacement. The DAF found by the model fixing the translation in y and z-direction is therefore considered to be an upper bound.

**Translations fixed in x, y and z-direction**

For large displacements in x-direction the neighbouring jetty modules influence each other. This is modelled by fixing the displacement in x-direction as well. However, this causes a much stiffer boundary than the expansion joint in reality is. In reality movement in x-direction is possible for 0.05 m. Also rotation at the head of the jetty is possible; however in the modelled situation the displacement is fixed over the total width of the jetty deck, which also restrains rotation in the horizontal plane at the head of the module.

Table 5 - DAF of Model Without Boundary Conditions

<table>
<thead>
<tr>
<th>Angle</th>
<th>DAF X</th>
<th>DAF Y</th>
<th>DAF Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2.6</td>
<td>0</td>
<td>0.39</td>
</tr>
<tr>
<td>10</td>
<td>1.8</td>
<td>1.7</td>
<td>0.52</td>
</tr>
<tr>
<td>25</td>
<td>2.3</td>
<td>1.6</td>
<td>0.34</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Angle</th>
<th>DAF X</th>
<th>DAF Y</th>
<th>DAF Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2.9</td>
<td>0</td>
<td>0.52</td>
</tr>
<tr>
<td>10</td>
<td>2.3</td>
<td>2.1</td>
<td>0.70</td>
</tr>
<tr>
<td>25</td>
<td>2.8</td>
<td>2.1</td>
<td>0.40</td>
</tr>
</tbody>
</table>

Table 6 - DAF of Model With Boundary Conditions in x, y, z-direction

<table>
<thead>
<tr>
<th>Angle</th>
<th>DAF X</th>
<th>DAF Y</th>
<th>DAF Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.76</td>
<td>0</td>
<td>0.49</td>
</tr>
<tr>
<td>10</td>
<td>0.63</td>
<td>1.4</td>
<td>0.38</td>
</tr>
<tr>
<td>25</td>
<td>0.94</td>
<td>1.3</td>
<td>0.37</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Angle</th>
<th>DAF X</th>
<th>DAF Y</th>
<th>DAF Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.8</td>
<td>0</td>
<td>0.92</td>
</tr>
<tr>
<td>10</td>
<td>1.4</td>
<td>1.9</td>
<td>0.61</td>
</tr>
<tr>
<td>25</td>
<td>1.7</td>
<td>1.6</td>
<td>0.56</td>
</tr>
</tbody>
</table>

A clear difference can be seen between the situation with and without boundary conditions in x, y and z-direction in Table 5 and Table 6. The dynamic amplification in x-direction has strongly decreased. Also in y and z-direction the DAF’s are smaller. This model is however not expected to be realistic; this decrease of the DAF is therefore not further considered.
Soil
The behaviour of soil is modelled in two ways; with springs and by fixing the pile in all directions. This influences the mode shapes and natural frequencies. Natural frequencies are higher when the soil is modelled fixing the piles in every direction (rotation and translation). The piles are clamped in the soil. The difference for the lower natural frequencies is 1 rad/s and for the higher natural frequencies almost 10 rad/s, as can be seen in Table 7.

Table 7 - Natural Frequencies of two Models: 1. Soil modelled with Springs; 2. Piles Clamped in Soil

<table>
<thead>
<tr>
<th>Natural Frequencies $\omega$</th>
<th>Soil modelled with Springs</th>
<th>Soil modelled fixing rotation and translation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_1$ [rad/s]</td>
<td>5.8</td>
<td>5.3</td>
</tr>
<tr>
<td>$\omega_{20}$ [rad/s]</td>
<td>49</td>
<td>58</td>
</tr>
</tbody>
</table>

The natural frequencies for the lower mode shapes are a bit higher when using springs. But for the higher mode shapes the natural frequencies are less high. This is because the mode shapes are different. The higher mode shapes show a vibration in z-direction (vertical). When the soil is modelled with springs, the piles can move vertically in the soil as can be seen in Figure 12. The displacements shown in the figures are enlarged. Figure 13 shows the mode shape when the piles are fixed for all DOF at 5 m below sea level.

![Figure 12 - Side View of Mode Shape 18, using the Spring Model for Soil](image1)

![Figure 13 - Side View of Mode Shape 18, with Piles being Clamped at 6.6m below the Sea Bottom](image2)

From the figures it can be seen that the piles can displace in vertical direction, when the soil is schematized with springs the dotted line is crossed. The different natural frequencies and mode shapes also have their influence on the dynamic behaviour of the jetty. Table 8 and Table 9 show the DAF using the different modelling of the soil.
Table 8 - DAF, Soil modelled with Springs, of Large Wave Spectrum

<table>
<thead>
<tr>
<th>Angle of</th>
<th>DAF X</th>
<th>DAF Y</th>
<th>DAF Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.0</td>
<td>0.0</td>
<td>0.78</td>
</tr>
<tr>
<td>10</td>
<td>1.0</td>
<td>1.0</td>
<td>0.79</td>
</tr>
<tr>
<td>25</td>
<td>1.0</td>
<td>1.0</td>
<td>0.80</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Angle of</th>
<th>DAF X</th>
<th>DAF Y</th>
<th>DAF Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.0</td>
<td>0.0</td>
<td>1.1</td>
</tr>
<tr>
<td>10</td>
<td>1.0</td>
<td>1.1</td>
<td>1.1</td>
</tr>
<tr>
<td>25</td>
<td>1.0</td>
<td>1.1</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Table 9 - DAF, Soil Modelled by Fixing Translations and Rotations in Every Direction, of Large Wave Spectrum

<table>
<thead>
<tr>
<th>Angle of</th>
<th>DAF X</th>
<th>DAF Y</th>
<th>DAF Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.0</td>
<td>0.0</td>
<td>0.53</td>
</tr>
<tr>
<td>10</td>
<td>1.0</td>
<td>1.0</td>
<td>0.53</td>
</tr>
<tr>
<td>25</td>
<td>1.0</td>
<td>1.0</td>
<td>0.42</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Angle of</th>
<th>DAF X</th>
<th>DAF Y</th>
<th>DAF Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.0</td>
<td>0.0</td>
<td>1.0</td>
</tr>
<tr>
<td>10</td>
<td>1.0</td>
<td>1.2</td>
<td>0.99</td>
</tr>
<tr>
<td>25</td>
<td>1.0</td>
<td>1.1</td>
<td>0.72</td>
</tr>
</tbody>
</table>

From the DAF it can be seen that the results in horizontal direction (x and y) do not differ much for both situations of modelling of the soil. The DAF in z-direction found with the clamped soil model shows very low numbers between 0.3 and 0.5. Modelling of the soil with springs shows different results, the largest DAF is 0.8 found by one of the five simulations. Modelling the soil by clamping the piles leads to lower DAF in z-direction of about 0.3. Not only the DAF is smaller, also the displacement in z-direction is much smaller using the clamped pile model. However, the stress in the piles is almost twice as large. This is because the piles are stiffer in vertical direction.

The two models are also compared for a wave spectrum with smaller waves. For the wave spectrum with waves just hitting the jetty deck different results are found. The results are presented in Table 10 and Table 11.

Table 10 - DAF, Soil modelled with Springs, of Wave Spectrum with Waves Just Hitting Jetty Deck

<table>
<thead>
<tr>
<th>Angle of</th>
<th>DAF X</th>
<th>DAF Y</th>
<th>DAF Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2.6</td>
<td>0.50</td>
<td>0.37</td>
</tr>
<tr>
<td>10</td>
<td>1.6</td>
<td>1.6</td>
<td>0.48</td>
</tr>
<tr>
<td>25</td>
<td>2.3</td>
<td>1.6</td>
<td>0.51</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Angle of</th>
<th>DAF X</th>
<th>DAF Y</th>
<th>DAF Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2.9</td>
<td>0.69</td>
<td>0.43</td>
</tr>
<tr>
<td>10</td>
<td>2.1</td>
<td>1.9</td>
<td>0.65</td>
</tr>
<tr>
<td>25</td>
<td>2.9</td>
<td>1.8</td>
<td>0.65</td>
</tr>
</tbody>
</table>

Table 11 - DAF, Piles Clamped in Soil, of Wave Spectrum with Waves Just Hitting Jetty Deck

<table>
<thead>
<tr>
<th>Angle of</th>
<th>DAF X</th>
<th>DAF Y</th>
<th>DAF Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2.8</td>
<td>1.0</td>
<td>0.50</td>
</tr>
<tr>
<td>10</td>
<td>2.0</td>
<td>2.0</td>
<td>0.62</td>
</tr>
<tr>
<td>25</td>
<td>2.3</td>
<td>1.6</td>
<td>0.54</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Angle of</th>
<th>DAF X</th>
<th>DAF Y</th>
<th>DAF Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3.2</td>
<td>1.7</td>
<td>0.62</td>
</tr>
<tr>
<td>10</td>
<td>2.5</td>
<td>2.5</td>
<td>0.86</td>
</tr>
<tr>
<td>25</td>
<td>2.8</td>
<td>1.9</td>
<td>0.75</td>
</tr>
</tbody>
</table>
From Table 10 and Table 11 it can be seen that the results in horizontal direction (x and y) are larger for the clamped pile situation. This could be caused by the lower natural frequencies and different mode shapes of the situation with the piles clamped in the soil. The results in z-direction do not differ much. Although the decrease which is found using the large wave spectrum, cannot be seen for this smaller wave spectrum. The displacement in vertical direction is smaller for the clamped situation, and the stresses in the piles are indicated to be about twice as large due to the vertical displacement.

The soil is also modelled by fixing the piles at 6 and 6.6 m below the sea bottom, instead of 5 m. The results show that the shorter the piles, the stiffer the piles and the higher the natural frequencies. This result is as expected. The mode shapes are hardly influenced. These models are therefore not further taken into consideration.

**Cracked Concrete Deck**

The jetty deck is modelled with Young’s-modulus of $330 \times 10^8$ N/m$^2$. This corresponds to the stiffness of un-cracked concrete. During the lifetime of the jetty the concrete has cracked at several locations. The hurricanes that pass cause enormous pressures to the jetty deck which cause the concrete to crack. These are local pressures, but after several years of service the multiple hurricanes and ships mooring to the jetty are expected to have cracked the concrete. A conservative assumption is made, that the jetty deck is cracked over the total length, width and height of one module. A stiffness of $200 \times 10^8$ N/m$^2$ is used. The natural frequencies of the jetty are lower when the jetty deck has a lower stiffness. The lowest natural frequency is 5.2 rad/s instead of 5.8 rad/s.

The Table 12 and Table 13 show the DAF found with the different stiffness of the jetty deck.

**Table 12 - DAF of Model with Un-Cracked Concrete**

<table>
<thead>
<tr>
<th>Angle</th>
<th>DAF X</th>
<th>DAF Y</th>
<th>DAF Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angle of 0</td>
<td>1.0</td>
<td>0.0</td>
<td>0.82</td>
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<tr>
<td>Angle of 10</td>
<td>1.0</td>
<td>1.1</td>
<td>0.75</td>
</tr>
<tr>
<td>Angle of 25</td>
<td>1.0</td>
<td>1.0</td>
<td>0.81</td>
</tr>
</tbody>
</table>

**Table 13 - DAF of Model with Cracked Concrete**

<table>
<thead>
<tr>
<th>Angle</th>
<th>DAF X</th>
<th>DAF Y</th>
<th>DAF Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angle of 0</td>
<td>1.0</td>
<td>0.0</td>
<td>0.72</td>
</tr>
<tr>
<td>Angle of 10</td>
<td>1.0</td>
<td>1.0</td>
<td>0.68</td>
</tr>
<tr>
<td>Angle of 25</td>
<td>1.0</td>
<td>1.0</td>
<td>0.83</td>
</tr>
</tbody>
</table>

The difference in dynamic behaviour is very small. Only in z-direction the result for 0.1 % exceedance probability differs 0.2. The calculations shown in the tables are done with the large wave spectrum. This wave spectrum is described in chapter 3. The same waves are used for the cracked and un-cracked deck calculation. Although hardly any difference is seen in the DAF, the maximum displacement is influenced. The cracked jetty deck has a lower bending stiffness, which causes larger displacements in x-direction. This occurs because the top of the pile (at the jetty deck) can rotate a
Appendix - Dynamic Analysis of an Open Piled Jetty Subjected to Wave Loading

bit easier with the lower stiffness of the jetty deck. This leads to larger stresses in the piles, because their stiffness is not changed. In y-direction this does not occur, because the beams beneath the jetty deck are in y-direction, which restrains bending of the top of the piles in this direction.

The cracked and un-cracked model are also compared for a wave spectrum with smaller waves. For the wave spectrum with waves just hitting the jetty deck (chapter 3), a bit larger difference can be seen. In x and y-direction the difference in DAF is 0.2. For part of the situations the DAF was 0.2 larger with the cracked concrete, for others it was 0.2 smaller. In z-direction a difference of 0.1 % is found. For other wave spectra the same differences are assumed between the cracked and un-cracked model.

**With or Without Damping**

The dynamic calculations are done assuming viscous damping of 0.03 to 0.05 for the damping coefficient. The damping matrix is forced to be diagonal, however the matrix is not expected to be diagonal in reality.

The magnitude of damping in the structure could be more or could be less than expected. To see the influence of damping on the dynamic behaviour of the structure the calculation is also done without any damping included in the structure. (Only numerical damping is present because of the backward Euler solving method, section 5.5). The situation without damping is a theoretical situation, because there will always be damping in the structure. The results are presented in Table 14 and Table 15.

**Table 14 - DAF of Un-Damped System of Large Wave Spectrum**

<table>
<thead>
<tr>
<th>Angle of 0</th>
<th>DAF X</th>
<th>DAF Y</th>
<th>DAF Z</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>0</td>
<td>0.78</td>
</tr>
<tr>
<td>Angle of 10</td>
<td>1</td>
<td>1.1</td>
<td>0.80</td>
</tr>
<tr>
<td>Angle of 25</td>
<td>1</td>
<td>1.0</td>
<td>0.80</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Angle of 0</th>
<th>DAF X</th>
<th>DAF Y</th>
<th>DAF Z</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.0</td>
<td>0</td>
<td>1.0</td>
</tr>
<tr>
<td>Angle of 10</td>
<td>1.0</td>
<td>1.4</td>
<td>1.0</td>
</tr>
<tr>
<td>Angle of 25</td>
<td>1.0</td>
<td>1.2</td>
<td>1.1</td>
</tr>
</tbody>
</table>

**Table 15 - DAF of Damped System, of Large Wave Spectrum**

<table>
<thead>
<tr>
<th>Angle of 0</th>
<th>DAF X</th>
<th>DAF Y</th>
<th>DAF Z</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.0</td>
<td>0.0</td>
<td>0.78</td>
</tr>
<tr>
<td>Angle of 10</td>
<td>1.0</td>
<td>1.0</td>
<td>0.79</td>
</tr>
<tr>
<td>Angle of 25</td>
<td>1.0</td>
<td>1.0</td>
<td>0.80</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Angle of 0</th>
<th>DAF X</th>
<th>DAF Y</th>
<th>DAF Z</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.0</td>
<td>0.0</td>
<td>1.1</td>
</tr>
<tr>
<td>Angle of 10</td>
<td>1.0</td>
<td>1.1</td>
<td>1.1</td>
</tr>
<tr>
<td>Angle of 25</td>
<td>1.0</td>
<td>1.1</td>
<td>1.0</td>
</tr>
</tbody>
</table>

From Table 14 and Table 15 it can be seen that the differences in DAF are very small. The damping hardly influences the DAF for this load case. The calculations are done with the large wave spectrum (described in chapter 3). The dynamic behaviour of the structure does hardly enlarge the amplitude of vibrations for these wave loading frequencies. This could be the reason for the small difference between the modelling with and without damping. Therefore the influence of damping is also checked, using the wave spectrum with a smaller wave period and the wave period equal to a natural frequency of the system.
The results using a damped and un-damped system for the wave spectrum with a significant wave height of 3.0 m and a wave period of 4.0 s are shown in Table 16 and Table 17.

Table 16 - DAF of Un-Damped System, Wave Spectrum Just Hitting the Jetty Deck $H_s=3.0$ m, $T_p=4.0$ s

<table>
<thead>
<tr>
<th>Angle of 0</th>
<th>DAF X</th>
<th>DAF Y</th>
<th>DAF Z</th>
<th>0.1 % Exceed. Prob.</th>
<th>DAF X</th>
<th>DAF Y</th>
<th>DAF Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angle of 0</td>
<td>2.9</td>
<td>0</td>
<td>0.39</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Angle of 10</td>
<td>1.9</td>
<td>2.1</td>
<td>0.49</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Angle of 25</td>
<td>2.9</td>
<td>2.1</td>
<td>0.53</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

From Table 16 and Table 17 it can be seen that the influence of damping is large in horizontal direction for this wave spectrum. The DAF in horizontal (x and y) directions are enlarged with 0.3 to 0.9. This is a large influence of the damping on the dynamic behaviour. In vertical direction the lack of damping hardly influences the DAF.

The results using a damped and un-damped system for the wave spectrum with a significant wave height of 0.35 m and a wave period of 1.1 s are shown in Table 18. For this wave spectrum the amplitude of vibration is enlarged by the dynamic behaviour. An approach angle of the wave of 0 degrees is used, therefore no load is present in y and z-direction.

Table 18 - DAF of Damped and Un-Damped System, of Wave Spectrum with Frequencies near Natural Frequency

<table>
<thead>
<tr>
<th>Damped</th>
<th>DAF X</th>
<th>DAF Y</th>
<th>DAF Z</th>
<th>0.1 % Exceed. Prob.</th>
<th>DAF X</th>
<th>DAF Y</th>
<th>DAF Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Damped</td>
<td>5.3</td>
<td>-</td>
<td>-</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Un-Damped</td>
<td>6.6</td>
<td>-</td>
<td>-</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

From the results in Table 18 it can be seen that the DAF in x-direction significantly enlarges when damping is not included. The DAF in x-direction for the un-damped system is 11 when the time step of the calculation is reduced. This reduces the numerical damping (as described in section 5.5).

A wave load with a frequency near a natural frequency can cause a large DAF. The degree of damping in a structure decreases this dynamic amplification factor. This reduces the dynamic enlargement of the amplitude of vibrations of a structure and with that the stresses. Damping is therefore important for the dynamic behaviour of the jetty.
Added Mass
When the jetty vibrates, not only the structure moves. Also water in and around the piles vibrates along with the structure. Water in contact with the jetty deck can also vibrate along with the structure. This water or marine growth, vibrating along enlarges the mass in the dynamic system. For a dynamic calculation this added mass should be included.

In the model used in this research added mass is not included. This is expected to lead to higher natural frequencies because of a lower mass is used in the system. To estimate the influence of the added mass, one calculation with added mass is done. It is assumed that only water in and around the piles vibrates along with the jetty. It is assumed that the amount of water around the piles vibrating together with the piles is equal to the amount of water in the piles. This leads to the total added mass per pile shown in equation A.1.

\[ m_{\text{added mass}} = 2 \rho_{\text{water}} A_{\text{pile}} \quad \text{[kg/m]} \]

\[ m_{\text{added mass}} = \text{added mass} \quad \text{[kg/m]} \]
\[ \rho_{\text{water}} = \text{density of water} \quad \text{[kg/m}^3]\]
\[ A_{\text{pile}} = \text{cross-section of a pile} \quad \text{[m}^2]\]

Per pile the length of 13 m is used, which is about the wetted length. Water vibrating along with the jetty deck is not taken into account.

The difference caused by the added mass is about 0.3 rad/s for the lower natural frequencies. For the higher natural frequencies the difference is about 9 rad/s. The added mass indeed lowers the natural frequencies. The mode shapes are not influenced. However because the natural frequencies differ the sequence of the mode shapes can be effected as well.

The influence of lower natural frequencies is investigated, by recalculation previous simulations with the only difference of lowering the natural frequencies of the structure. Table 19 shows the results from the dynamic analysis with lower natural frequencies.

<table>
<thead>
<tr>
<th>Max of 5 simulations</th>
<th>Lowest natural frequency [rad/s]</th>
<th>DAF X</th>
<th>DAF Y</th>
<th>DAF Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original</td>
<td>5.9</td>
<td>1.0</td>
<td>0.50</td>
<td>0.82</td>
</tr>
<tr>
<td>Natural frequencies -1 rad/s</td>
<td>4.9</td>
<td>1.0</td>
<td>0.40</td>
<td>0.82</td>
</tr>
<tr>
<td>Natural frequencies -2 rad/s</td>
<td>3.9</td>
<td>1.0</td>
<td>0.48</td>
<td>0.81</td>
</tr>
<tr>
<td>0.5 x Natural frequencies</td>
<td>3.0</td>
<td>1.0</td>
<td>0.50</td>
<td>0.81</td>
</tr>
</tbody>
</table>

These results are determined for a wave load caused by the wave climate with the large wave spectrum. The results are the maximum found DAF of five simulations of 100 s. The same wave load is used for all dynamic analysis; the difference in the results is therefore only caused by the difference in natural frequencies. The difference in natural frequency hardly influences the results from the dynamic analysis for this wave spectrum.
The differences are expected to be larger for wave spectra with wave frequencies closer to the lowest natural frequencies. Therefore the effect on the DAF for the wave spectrum that just hits the jetty deck is determined as well. The wave spectrum is explained in chapter 3. The results are shown in Table 20. The same waves are used for the simulations; therefore the difference in the results is caused by the difference in natural frequencies. The dominant wave frequency is $\omega_p 1.6$ rad/s. An approach angle of zero is used, therefore no load in y-direction is present and the DAF is not shown.

Table 20 - DAF determined with Different Natural Frequencies, for Wave spectrum Just Hitting the Jetty Deck

<table>
<thead>
<tr>
<th>Max of 5 simulations</th>
<th>Lowest natural frequency [rad/s]</th>
<th>$\omega_p/\omega_1$</th>
<th>DAF X</th>
<th>DAF Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original</td>
<td>5.9</td>
<td>1/3.7</td>
<td>2.6</td>
<td>0.37</td>
</tr>
<tr>
<td>natural frequencies -1 rad/s</td>
<td>4.9</td>
<td>1/3.0</td>
<td>2.5</td>
<td>0.36</td>
</tr>
<tr>
<td>natural frequencies -2 rad/s</td>
<td>3.9</td>
<td>1/2.4</td>
<td>2.7</td>
<td>0.36</td>
</tr>
<tr>
<td>0.5 x Natural frequencies</td>
<td>3.0</td>
<td>1/1.8</td>
<td>4.3</td>
<td>0.27</td>
</tr>
</tbody>
</table>

From Table 20 it can be seen that the dynamic amplification in vertical (z-direction) is very small. This is expected to be caused by the high natural frequencies corresponding to mode shapes that included vertical motions. These mode shapes are not activated by this wave, because the natural frequency is still about 20 rad/s which is much larger than the wave frequencies. In x-direction the DAF is much larger when the natural frequencies are lowered. This is expected to be caused by the lowest natural frequency being much closer to the wave frequencies.

Table 20 shows that the DAF in x-direction hardly differs when the natural frequency is lowered by one or two rad/s. The DAF even decrease when the natural frequencies are lowered by one rad/s. This is expected to be caused by the moment on which the wave load hits the jetty. For a regular wave the DAF is expected to be the largest for the natural frequency of 4.9 rad/s, because the wave frequency is a round multiple of the lowest natural frequency. This can be seen in the third column of Table 20. The ratio between the dominant wave frequency $\omega_p$ and the lowest natural frequency $\omega_1$ is presented. This indicates that the wave load hits the structure at the same moment in its vibration. This is schematized in the upper graph of Figure 14. This can lead to an enlargement of the amplitude of vibration. When the load frequency is not a round multiple of the natural frequency (lower graph of Figure 14), this could lead to a decrease of the amplitude of vibrations, depending on the ratio between the frequencies. However, the water surface simulated using a wave spectrum does not cause a constant load period. Therefore a round multiple of the dominant wave frequency to the lowest natural frequency, does not result in the just described situation. However, the moment on which the wave load hits the structure in its vibration does still matter, and could explain the small difference in DAF in x-direction for the first two situations from Table 20.
Vibrations
Load $\omega_h/\omega_z=1/3.0$

Vibrations
Load $\omega_h/\omega_z=1/3.5$

**Figure 14 - Load with Two Different Periods in Relation to Displacement Signal**

In the sensitivity analysis only a difference in DAF of 0.1 is included, because half as low natural frequencies are not expected to occur due to added mass.

**Air Gap**

The distance between the mean sea level and the bottom of the jetty deck is called the deck clearance or air gap. This distance influences the wave-in-deck loading. In this paragraph it is investigated whether the dynamic reaction is influenced as well.

In order to see the influence of the air gap different simulations have been done with the same wave spectrum using different air gaps. The wave spectrum used during the physical model tests ($H_s=5.8 m$, $T_p=13.2 s$) are used for this investigation as well, because the magnitude of the wave-in-deck loading is the most reliable for this situation.

The deck clearance for cruise jetties depends on the ships mooring to the jetty. This is about 1.6 m for cruise ships. The difference in air gap therefore depends on the water setup, tidal movement and wind surge. In Sint Maarten this can theoretically vary between the 0.3 and 1.0 m. The results in chapter 6 are determined using in surge level of 0.5 m. The effect of a different air gap on the dynamic reaction of the jetty can be found in Table 21.

**Table 21 - Influence of Deck Clearance on Dynamic Reaction**

<table>
<thead>
<tr>
<th>$\Delta h/\eta$</th>
<th>Air Gap [m]</th>
<th>Setup [m]</th>
<th>$\Delta h$ [m]</th>
<th>$\eta_{max}$ [m]</th>
<th>DAF x</th>
<th>DAF y</th>
<th>DAF z</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.48</td>
<td>1.6</td>
<td>-0.3</td>
<td>1.9</td>
<td>4.0</td>
<td>1.0</td>
<td>0.99</td>
<td>0.70</td>
</tr>
<tr>
<td>0.24</td>
<td>1.6</td>
<td>0.5</td>
<td>1.1</td>
<td>4.5</td>
<td>1.0</td>
<td>1.1</td>
<td>0.83</td>
</tr>
<tr>
<td>0.14</td>
<td>1.6</td>
<td>1.0</td>
<td>0.6</td>
<td>4.3</td>
<td>1.0</td>
<td>0.97</td>
<td>0.81</td>
</tr>
</tbody>
</table>
From Table 21 it can be seen that the dynamic reaction is hardly influenced by a difference in air gap. The investigation is also done using an air gap of 0.1 m and 4.0 m, this did also not lead to different dynamic amplifications. The wave-in-deck loading is influenced by the difference in air gap, however because of the simplified wave-in-deck formulation used in this research it is uncertain whether the formulation of this research shows a realistic relation between the wave-in-deck loading and air gap.

The maximum found vertical peak pressure per simulation is plotted against the air gap. The results are shown in Figure 16.

![Wave-in-Deck Loading for Different Air Gaps](image)

**Figure 16 - Influence of Air Gap on Vertical Wave-in-Deck Loading**

From Figure 16 it can be seen that the vertical peak pressure is larger for smaller air gaps. This is determined using the wave-in-deck load formulations derived in chapter 4. This formulation does not include all dependencies of the wave-in-deck loading phenomenon. The values shown in Figure 16 are therefore uncertain. It is also uncertain whether the empirical factor in the formulation can be used for situations with a different air gap or significant wave height.

The physical model tests performed by WL Delft Hydraulics (1998) are performed using two different air gaps (0.9 and 1.9 m). The results of the test with a larger air gap show a significant lower vertical wave-in-deck loading.

In literature the influence of the deck clearance on the wave-in-deck loading is investigated as well. The ratio between the deck clearance and the water surface elevation is used as indicator. Meng (2010) concluded that the vertical wave-in-deck peak pressure increases until:

\[
\frac{\Delta h}{\eta} = 0.2
\]

\(\Delta h = \text{air gap} \quad \text{[m]}\)

\(\eta = \text{water surface elevation} \quad \text{[m]}\)

At 0.2 the peak of the vertical wave-in-deck loading is found. For smaller or larger ratios the wave-in-deck loading is expected to be lower. The results of WL Delft hydraulics have a ratio of 0.2 and 0.4. The vertical wave-in-deck loading for the test with a ration of 0.4 is lower. This complies with the findings of Meng (2010). The test with the lower wave-in-deck loading is also the test with a larger
deck clearance. The results in Figure 16 do not comply with this theory. This is expected to be caused by the simplicity of the wave-in-deck formulation.

**Duration of Peak pressure**

The duration of the vertical peak pressure is estimated based on the values found in literature. A value of 0.01 s is used, because of the time step of the computation a shorter duration is not possible. In literature different durations of the vertical wave-in-deck peak pressure are found:

- 0.008 s to 0.016 s (Rooij, 2001);
- smaller than 0.05 s (Shih, 1992);
- 0.02 s to 0.05 s (WL Delft Hydraulics, 1998)

Because the duration of the vertical wave-in-deck peak pressure is uncertain the influence of a longer duration of the peak pressure on the dynamic reaction of the jetty is investigated in the paragraph. Figure 17 shows two different durations of the vertical peak pressure (the plots are not based on the same water surface elevation).

![Figure 17 - Difference in Duration of Vertical Peak Pressure (Left: 0.5 s, Right: 0.01 s)](image)

Durations of 0.01 s, 0.02 s, 0.04 s, 0.05 s, 0.1 s and 0.5 s are investigated using the large wave spectrum ($H_s=9.0$ s, $T_p=12$ s). The results are presented in Table 22. Durations of 0.1 s and 0.5 s are not expected to be realistic for the duration of the vertical peak pressure, because these values are not mentioned in literature.

<table>
<thead>
<tr>
<th>Duration of $\text{P}_{\text{z,i}}$ [s]</th>
<th>Maximum DAF $z$</th>
<th>Average DAF $z$</th>
<th>Standard deviation</th>
<th>0.1 % exc. Prop. DAF $z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>0.80</td>
<td>0.73</td>
<td>0.05</td>
<td>1.0</td>
</tr>
<tr>
<td>0.02</td>
<td>0.79</td>
<td>0.74</td>
<td>0.04</td>
<td>1.0</td>
</tr>
<tr>
<td>0.04</td>
<td>0.87</td>
<td>0.78</td>
<td>0.07</td>
<td>1.1</td>
</tr>
<tr>
<td>0.05</td>
<td>0.96</td>
<td>0.87</td>
<td>0.05</td>
<td>1.2</td>
</tr>
<tr>
<td>0.1</td>
<td>0.97</td>
<td>0.81</td>
<td>0.08</td>
<td>1.4</td>
</tr>
<tr>
<td>0.5</td>
<td>0.98</td>
<td>0.91</td>
<td>0.06</td>
<td>1.2</td>
</tr>
</tbody>
</table>

The dynamic amplification factors in the horizontal directions are not influenced by the duration of the vertical peak pressure. These results are therefore not presented in Table 22. The second column shows the maximum value found by 5 simulations of 100 s.
From the results in Table 22 it can be seen that a DAF smaller than 0.8 is only found for a duration of 0.02 s, column two. From a duration of 0.05 s the dynamic amplification factors almost reach to 1. This is also expected for quasi-static loading. A larger duration of the peak pressure also shows the DAF to become more close to 1.

Whether the construction reacts quasi-statically depends on the ratio between the duration of the load and the natural frequencies. The first mode shapes do not have a vertical displacement. Therefore they are not activated by the vertical wave in deck loading. The fifth mode shape until the 20th mode shape do react on the vertical wave loading (also higher mode shapes can react on vertical loading, however these are not included in this computation). These mode shapes have a natural period between the 0.16 s to 0.13 s. It can be seen from the results that the maximum displacement is decreased by the dynamic behaviour up to a duration of the load of 0.04 s. This is about ¼ of the natural period.

The value 0.05 s can occur as duration of the vertical peak pressure according to the physical model test performed by WL Delft Hydraulics for Sint Maarten (1998). Other literature mentions shorter durations. If a duration of 0.05 s is expected, this means that a DAF in vertical direction of 1.0 can be simulated. The 0.1% exceedance probability is indicated to be even larger (column 5, Table 22), however this value is very sensitive.

**Length of Jetty**

The jetty consists out of four modules. Only one of the modules is modelled. However, not all modules have the same length. The length influences part of the natural frequencies. How large the influence is, is checked by modelling a module of 120 m, this is compared with the 152 meter long module. The difference in natural frequencies is about 0.2 Hz, for part of the natural frequencies. The mode shapes are not influenced by the shorter length of the module. This is a small difference. Therefore the dynamic behaviour of the jetty module is not expected to be significantly influenced by the length of the module. The results from the 152 m long jetty module are also expected to give a good indication for the other jetty modules.

**Random**

The DAF’s presented in the results are determined using 5 simulations of 100 seconds. The water surface elevation is a random process. When the 5 simulations are repeated, different waves will hit the jetty which might cause a different dynamic reaction. This difference is 0.2 for the DAF in x, y and z-direction.

The 0.1 % exceedance probability DAF is determined using a trend line. This method is known to be very sensitive (section 7.6). However, also the DAF with a 0.1 % exceedance probability shows a difference of 0.2 when the 5 simulations are repeated.

**Significant Wave Height versus Water Particle Velocity Based Wave-in-Deck Loading**

The wave-in-deck load formulation derived in chapter 4 is based on the physical model test by WL Delft Hydraulics (1998). This empirical wave-in-deck load depends on the water particle velocity, in order to include a dependence on time and location. From the test results also a different wave-in-deck formulation is made by Lievense (Quist), 2005. This formulation is shown in equation A.2 and depends on the significant wave height. Both formulations can be found in chapter 4 and appendix V.
\[ P_{z,j} = C_{slam} \rho g H_s \] \hspace{1cm} \text{[N/m}^2\text{]} \hspace{1cm} \text{(A.2)}

Pro and Cons of Both Methods

Figure 18 shows both wave-in-deck loads based on the different formulations. The water particle velocity based method is called \( v_z \) based formulation. The duration and the magnitude of the slowly varying vertical wave-in-deck load do not differ between the two methods. This is because the same conditions are used for the duration of the slowly varying wave-in-deck load for both formulations.

![Figure 18 - Vertical Wave-in-Deck Load Upper: \( v_z \) Based Formulation. Below: \( H_s \) based Wave Formulation](image)

From Figure 18 it can be seen that the formulation based on the significant wave height shows the same wave load for every wave in the wave spectrum that hits the jetty deck. This is not expected to be realistic. Therefore the water particle velocity based wave-in-deck load is used for the dynamic analysis. However, the magnitude of the peak pressure is uncertain when using the water particle velocity based method. The water particle velocity enlarges for waves with a short wave period. This can lead to an unrealistic large vertical peak pressures. The peak pressure caused by the wave spectrum with waves just hitting the jetty deck (\( H_s = 3.0 \text{ m} \) and \( T_p = 4.0 \text{ s} \)) are shown in Table 23 for both wave-in-deck load formulations.
Table 23 - Wave-in-Deck Load Compared for Vz and Hs Based Wave-in-Deck Load

<table>
<thead>
<tr>
<th>Approach angle</th>
<th>Vz based</th>
<th>Hs based</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Max Horizontal WID Load [kN/m]</td>
<td>Max Horizontal WID Load [kN/m]</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>53</td>
</tr>
<tr>
<td></td>
<td>10°</td>
<td>659</td>
</tr>
<tr>
<td></td>
<td>25°</td>
<td>135</td>
</tr>
</tbody>
</table>

From Table 23 it can be seen that the water particle velocity based wave load causes much larger loads on the jetty. The horizontal wave-in-deck load based on the physical model test does not depend on the approach angle of the wave.

Whether the difference in wave-in-deck pressure leads to a different dynamic effect can be seen from Table 24. Both tables show the maximum found DAF of five simulations.

Table 24 - DAF for Vz and Hs based Wave-in-Deck Load, of Wave Spectrum with Waves Just Hitting Jetty Deck

<table>
<thead>
<tr>
<th>Vz based</th>
<th>Hs based</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angle of 0</td>
<td>2.6</td>
</tr>
<tr>
<td>Angle of 10</td>
<td>1.8</td>
</tr>
<tr>
<td>Angle of 25</td>
<td>2.3</td>
</tr>
</tbody>
</table>

The comparison in dynamic behaviour is also investigated for a wave spectrum with larger waves (Large Wave Spectrum). The results are shown in Table 25.

Table 25 - DAF for Vz and Hs based Wave-in-Deck Load, of Large Wave Spectrum

<table>
<thead>
<tr>
<th>Vz based</th>
<th>Hs based</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angle of 0</td>
<td>1.0</td>
</tr>
<tr>
<td>Angle of 10</td>
<td>1.0</td>
</tr>
<tr>
<td>Angle of 25</td>
<td>1.0</td>
</tr>
</tbody>
</table>

The DAF’s in x-direction do not differ much from each other, about 0.2 for the wave spectrum with wave just hitting the jetty deck. For the large wave spectrum, no difference in DAF in x-direction is seen. In y-direction the DAF is larger for the water particle velocity based method for the small wave spectrum Table 24. For the large wave spectrum DAF in y-direction are larger for the significant wave height based method. No general consensus can be stated about this direction. In vertical (z-direction) a small difference can be seen for both wave spectra. The DAF in z-direction remains smaller than one for both methods.

For the investigations done in this research it is the aim to find circumstances under which the DAF is larger than 2, for the first jetty of Sint Maarten when being subjected to wave loading. The use of the water particle velocity based wave-in-deck loading is not expected to lead to a different result of DAF, than when the significant wave height based method would be used. Both methods show a DAF
larger than 2 in the same situations. Also the difference in DAF caused by the different methods is small.
Appendix XI

MATLAB Script

This appendix shows the MATLAB scripts. The total calculation is cut in parts, almost corresponding to the flow chart from chapter 1.

- Section 1.1: MotherFile, runs the other .m files in the right order
- Section 1.2: WaveData, Makes the wave spectrum from a significant wave height
- Section 1.3: WaveLoad, Makes the wave load in deck and piles
- Section 1.4: ScialImport: Transforms vectors to node sequence
- Section 1.5: ModalAnalysis: Modal analysis is performed.
1.1. Mother file

One script is used to run the different files in the right order. This also gives a first overview of the calculations, and the three scripts which are used.

```matlab
% File name: MotherFile
% Date: 30-08-2012, Delft

clc;
clear all;
%
%This Mother file runs the files in the good order

tic
%
% Makes wave spectrum of waves from Hindcast Hurricane Omar
OmarWaveData
%
% Makes Wave Spectrum which is used in the rest of the calculation
JONS_max=JONS_Omar(:,1);
%
% Makes Wave train (random sea) and Wave Loads on jetty deck en piles
WaveLoad
display(’load is determined’)
%
% Does the Modal Analysis and DAF
ModalAnalysis20MS
toc
```
1.2. Wave spectrum

Makes wave spectrum over waves found during Omar. Different wave spectra can be used to simulate the water surface elevation; every wave spectrum has its own WaveData.m script. In this appendix Omar is taken as example. One of the wave spectra is used for further calculation.

```matlab
% File name: OmarWaveData
% Date: 20-08-2012, Delft

% Omar[Hs;Tp]
Omar = [5.6 5.7 5.5 5.6 5.9 5.7 4.8 5.1 5.0 4.5 4.7 4.6 3.8 4.0 3.9;
        10.1 10.2 9.7 9.9 10.2 9.7 9.9 10.1 9.6 9.8 10 9.5 9.7 9.9 9.4]

% spectra are Snn(Omega)
omega0_Omar=2*pi./(Omar(2,:)); %[rad/s]
alfa_Omar=((omega0_Omar.^2*sqrt(5).*Omar(1,:))/(4*g)).^2;
N=T;
step_freq=1/T;
MF=0.35;
start_freq=0.05;
freq=[start_freq:step_freq:MF];
omega=freq'*2*pi;
for jj=1:15;
    PM_Omar(:,jj)=alfa_Omar(jj)*g^2.*omega.^(-5).*exp(-1.25*(omega0_Omar(jj)./omega).^4);
    for j=1:MF/step_freq-start_freq/step_freq+1;
        if omega(j)>omega0_Omar(jj);
            sigma_JONS_Omar(j,jj)=0.09;
        else
            sigma_JONS_Omar(j,jj)=0.07;
        end
    end
    JONS_Omar(:,jj)=PM_Omar(:,jj).*3.3.^sigma_JONS_Omar(:,jj);
end
```

One of the JONS_Omar or PM_Omar is renamed in JONS_max, by the MotherFile. This JONS_max is the wave spectrum used for the water surface simulation in the following files.
1.3. Wave Load

```matlab
% File name: WaveLoad
% Date: 10-09-2012, Delft
% V.A.G. Bron

% Determines Wave Load on deck and on Piles
% Variable over x and t.
% Analytical based Wave Load
% Airy Wave Theory
% run first a file to make a wavespectrum, jons_max is in script.

% Uses: JONS_max, omega, MF, step_freq, start_freq

% Input parameters
B=20; % width of deck in [m]
th=0.4; % Thickness of the jetty deck [m]
rho=1025; % [kg/m3]
setup=0.5; % Change per sea state [m]
airgap=1.6; % distance between MSL and bottom jetty deck
z_bottom=airgap-setup; % Z coordinate of bottom of jetty deck
g=9.81; % [m/s^2]
L_module=152; % Length of the jetty of one module [m]
d=12; % Water Depth [m]
D_pile=0.914; % Diameter pile [m]
Beamx=1.75; % Length of beam in x [m]
alfa=(50/180)*pi; % Approach Angle between jetty and wave

% Makes Time Vector
N=100;
step=0.01;
t = [0:step:N]';
deltat=0.01; % Duration of impact
alfatilda=16; % Empirical factor of vertical peak pressure
alfatilda_y=38;

% Make matrices right size
Phi_k=rand(MF/step_freq-start_freq/step_freq+1,1); % Random number between 0 and 1
Ak_1k=zeros(MF/step_freq-start_freq/step_freq+1,1);
PhiK=zeros(MF/step_freq-start_freq/step_freq+1,1);

for x=1:1:L_module;
    % Calculation for every meter in x direction
    y=0;
    % Horizontal coordinate, perpendicular to jetty axis
    z=0;
    % Z is Vertical axis (Vertical Streching)
    % Makes Wave train (Eta) with linear wave theory, and water particle velocity
    for k=1:MF/step_freq-start_freq/step_freq+1;
        % Wave frequencies index
        Ak_k(k)=sqrt(JONS_max(k,1)*2*pi*step_freq*2); % Amplitude
        PhiK(k)=Phi_k(k)*2*pi; % Random Phase angle
        WaveNumber(k)=omega(k)^2/g; % Wave number
        Eta_k(:,k,x)=Ak_k*sin(omega(k).*t+PhiK-WaveNumber*cos(alfa)*x-
          WaveNumber*sin(alfa)*y);
        vx_w_k(:,k,x)=Ak_k*omega(k).*[1]*sin(omega(k).*t+PhiK-
          WaveNumber*sin(alfa)*y); % in propagation direction of wave (Local coordinate Syst)
        vz_k(:,k,x)=Ak_k*omega(k).*[sinh(k*z+k*d)/sinh(k*d)]*cos(omega(k).*t+PhiK-
          WaveNumber*cos(alfa)*x-WaveNumber*sin(alfa)*y); % Because z=0 cosh(kd)/sinh(kd)=1
        dvz_k(:,k,x)=Ak_k*omega(k).^2*[sinh(k*z+k*d)/sinh(k*d)]*sin(omega(k).*t+PhiK-
          WaveNumber*cos(alfa)*x-WaveNumber*sin(alfa)*y); % 
        dvx_w_k_z(:,k,x)=Ak_k*omega(k).^2*[1]*cos(omega(k).*t+PhiK-
          WaveNumber*cos(alfa)*x-WaveNumber*sin(alfa)*y);
    end
end
```

*Next page file continuous*
% Water Particle acceleration. Sum over wave frequencies
Eta(:,x)=sum(Eta_k(:,x),2);
vx_w(:,x)=sum(vx_k(:,:,x),2);
vz(:,x)=sum(vz_k(:,:,x),2);
dvz(:,x)=sum(dvz_k(:,:,x),2);
dvx_w(:,x)=sum(dvx_w_k_z(:,:,x),2);

% change from local to global coordinate system
vx(:,x)=cos(alfa)*vx_w(:,x);
vy(:,x)=sin(alfa)*vx_w(:,x);
dvx(:,x)=cos(alfa)*dvx_w(:,x);
dvy(:,x)=sin(alfa)*dvx_w(:,x);

% Making criteria for which wave-in-deck exists (Heavisides)
Crit1=Eta(:,x)>airgap-setup; % 1 if wave hits jetty deck
Crit0=zeros(deltat/step,1);
Crit4=[Crit0; Crit1(
1:((1/step)*N+1)-deltat/step)];
Crit2=Crit1-Crit4; % includes duration of peak pressure
Crit3=vz(:,x)>0;

% Vertical Wave pressure Momentum, Inertia, Drag, Buoyancy [N/m^2] on jetty deck
Cd=2;
Cm=2;
Pzm(:,x)=alfatilda*rho*(vz(:,x)).^2.*Crit2.*Crit1;
Pzi(:,x)=Cm*rho*B*dvz(:,x);
Pzd(:,x)=rho*(1/2)*Cd*vz(:,x).*abs(vz(:,x));
Pzb(:,x)=rho*g*(Eta(:,x)-z_bottom).*Crit1;
Pz(:,x)=(Pzm(:,x)+Pzi(:,x).*Crit3(:,x)+Pzb(:,x)+Pzd(:,x).*Crit3(:,x)).*Crit1;

% Horizontal Wave pressure in y direction [N/m^2] on jetty deck
Cmy=2;
Cdy=2;
Pym(:,x)=alphabtilda_y*rho*vy(:,x).^2.*Crit2.*Crit1;
Pyi(:,x)=Cmy*rho*B*dvy(:,x);
Pyd(:,x)=rho*(1/2)*Cdy*vy(:,x).*abs(vy(:,x));
Py(:,x)=(Pym(:,x)+Pyi(:,x)+Pyd(:,x)).*Crit1;

% Wet height of the side of the jetty (called th: of thickness)
Crity1=Eta(:,x)>z_bottom+th; % One if wave is higher than top jetty deck, 0 if not
Crity0=Eta(:,x)>z_bottom;
Crity2=Crity0-Crity1;

wet_th(:,x)=Eta(:,x)-z_bottom % wetted height of jetty deck side
qy(:,x)=(Crity2.*wet_th(:,x)+Crity1*th).*Py(:,x); % N/m

end % ends the forloop of x

% Horizontal Wave load (y and x direction) on piles
step_z=0.5; % d*step_z needs to be roundnumber
Cm_pile=2;
Cd_pile=2;
hoh_pile_x=6.25; % distance between pile rows in x-direction hart op hart [m]
hoh_pile_y=6.0; % distance between pile rows in y-direction hart op hart [m]
mbbr_piles_y=4; % number of piles in y direction
mbbr_piles_x=24;

% Making Factors Right size
dvx_w_k_z=zeros(1*(step)+N+1,MB/step_freq-start_freq/step_freq+1,1+d/step_z);
vx_w_k=zeros(1*(step)+N+1,MB/step_freq-start_freq/step_freq+1,1+d/step_z);

Fx3_pile_var=zeros(1*(step)+N+1,mbbr_piles_y+1,mbbr_piles_x);
Fx10_pile_var=zeros(1*(step)+N+1,mbbr_piles_y+1,mbbr_piles_x);
Fy3_pile_var=zeros(1*(step)+N+1,mbbr_piles_x+1,mbbr_piles_y);
Fy10_pile_var=zeros(1*(step)+N+1,mbbr_piles_x+1,mbbr_piles_y);

*Next page file continuous*
The load on the piles is placed at \( z = -3 \) m and \( z = -10 \) m. On the node at \( z = -3 \) m the loads from \( z = 0 \) m until \( z = -6 \) m are placed. At \( z = -10 \) m the loads from \( z = -6.5 \) m until \( z = -d \) are placed. \( d = \text{waterdepth} \).

\( \text{Fy}_3 \text{\_pile\_var} \) is the load in y-direction. Placed at \( z = -3 \) m. Per pile.
1.4. Nodes from Scia Engineer

Before the loads defined in the previous script can be used in the modal analysis the load matrix has to change its form.

The loads on the deck have the shape of shown in the table. The left above corner corresponds to fy(0,0) and the right low corner corresponds to fy(360,152). Their size is [number of time steps x 152]. With 152 being the length of the jetty module in x-direction.

<table>
<thead>
<tr>
<th>Name</th>
<th>Coord X [m]</th>
<th>Coord Y [m]</th>
<th>Coord Z [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>N10</td>
<td>0</td>
<td>-10</td>
<td>1.6</td>
</tr>
<tr>
<td>N13</td>
<td>0</td>
<td>10</td>
<td>1.6</td>
</tr>
<tr>
<td>N74</td>
<td>6.25</td>
<td>-10</td>
<td>1.6</td>
</tr>
<tr>
<td>N75</td>
<td>6.25</td>
<td>10</td>
<td>1.6</td>
</tr>
<tr>
<td>N76</td>
<td>8</td>
<td>-10</td>
<td>1.6</td>
</tr>
<tr>
<td>N77</td>
<td>8</td>
<td>10</td>
<td>1.6</td>
</tr>
<tr>
<td>N78</td>
<td>12.5</td>
<td>-10</td>
<td>1.6</td>
</tr>
<tr>
<td>N79</td>
<td>14.25</td>
<td>-10</td>
<td>1.6</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>N880</td>
<td>152</td>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

However, the nodes are not placed in this order when extracted from Scia Engineer. They are listed on node number. Therefore the x-coordinates are not listed in sequence any more. This can be seen in the table below. This is a small part of the list of nodes with its coordinates, called SciaNodesCoor.

The following MATLAB script is the SciaImport file, which is used to make vectors that change the load matrices into the order of the nodes. This file is only used when a new geometry is placed in Scia engineer. The ‘transform vectors’ are copied into the ModalAnalysis20MS file. For calculations that use the same geometry, mode shapes and eigenvectors, the SciaImport file is only needed once.

The load vector includes all 3 DOF and nodes. This vector is made by placing the load vector with loads in x direction, on top of the vector with load in y-direction, as is shown below. Together the load vector becomes 3 x 880 long. Many other vectors, like the displacement have the same shape.
Appendix - Dynamic Analysis of an Open Piled Jetty Subjected to Wave Loading

% On which node should the corresponding load be?
% Soil modelled with Springs
% Nodes in piles at -3 and -10

% [ x y z]  
SciaNodesCoor=[Large matrix of which part is shown on previous page];  
Nmbr_Nodes=880;

for yy=1:Nmbr_Nodes
    % on which nodes qy much be placed
    if SciaNodesCoor(yy,2)<-9.99;
        qynodes_alfaplus(yy)=1;
    else
        qynodes_alfaplus(yy)=0;
    end
    if SciaNodesCoor(yy,2)>9.99;
        qynodes_alfamin(yy)=1;
    else
        qynodes_alfamin(yy)=0;
    end
    if abs(SciaNodesCoor(yy,1)-roundn(SciaNodesCoor(yy,1),0))<0.001;
        if abs(SciaNodesCoor(yy,3))>9.99;
            qznodes(yy)=1;
        else
            qznodes(yy)=0;
        end
    else
        qznodes(yy)=0;
    end
    if SciaNodesCoor(yy,3)<-2.9;
        if SciaNodesCoor(yy,3)>-3.1;
            pilesnodes3(yy)=1;
        else
            pilesnodes3(yy)=0;
        end
    end
    if SciaNodesCoor(yy,3)<-9.9;
        if SciaNodesCoor(yy,3)>-10.1;
            pilesnodes10(yy)=1;
        else
            pilesnodes10(yy)=0;
        end
    else
        pilesnodes10(yy)=0;
    end
end

qznodesx=SciaNodesCoor(:,1).*qznodes';
qznodesy=SciaNodesCoor(:,2).*qznodes';
qynodes_alfaplusx=SciaNodesCoor(:,1).*qynodes_alfaplus';
qynodes_alfaminx=SciaNodesCoor(:,1).*qynodes_alfamin';
pilesnodes3x(:,1)=SciaNodesCoor(:,1).*pilesnodes3';
pilesnodes3y(:,2)=SciaNodesCoor(:,2).*pilesnodes3';
pilesnodesxy10(:,1)=SciaNodesCoor(:,1).*pilesnodes10';
pilesnodesxy10(:,2)=SciaNodesCoor(:,2).*pilesnodes10';
1.5. Modal Analysis

In the ModalAnalysis file start with the copied vectors from the SciaImport file. With these transform vectors the loads are changed from the size [number of time steps x 152] to [Number of time steps x Number of Nodes].

```matlab
%% File name: ModalAnalysis20MS
%% Date: 5-12-2012, Delft
%% V.A.G. Bron

% Modal Analysis
% makes use of Scia output
% run first a file to make the loads, fzvariableEta

% Uses: time (N, step, t) Geometry (L_module, B, alfa, hoh_pile_x, hoh_pile_y ) Load (Pz,qy, Fy_pile_var)

% properties
rho_conc=2400; % density concrete [kg/m^3]
DOF=3; % Degrees of Freedom
damp1=0.03; % Damping of first mode shape
damp5=0.05; % Damping of fifth mode shape, rest is interpolated

% Change to Scia Nodes. Vectors beneath are build with SciaImport
qynodesx_alfaplus= [ *Many numbers therefore not copied*]
qynodesx_alfamin= qynodesx=
pilesnodesxy3= pilesnodesxy10=

% Makes Matrices right size end fills them with zeros
Empty=zeros((1/step)*N+1,1);
Fy3_piles_nodes=zeros((1/step)*N+1,Nmbr_Nodes);
Fx3_piles_nodes=zeros((1/step)*N+1,Nmbr_Nodes);
Fy10_piles_nodes=zeros((1/step)*N+1,Nmbr_Nodes);
Fx10_piles_nodes=zeros((1/step)*N+1,Nmbr_Nodes);
qynodesx_plus_r=roundn(qynodesx_alfaplus,0);
qynodesx_min_r=roundn(qynodesx_alfamin,0);

for yy=1:Nmbr_Nodes;
% number of nodes
% takes the upward pressure at x=x_node and puts it in the vector on node sequence
if qynodesx(yy)>0.5;
xnode_qz (yy)=qznodesx (yy)-tan(alfa)*qznodesy (yy);
xnode_qz_r (yy)=roundn (xnode_qz (yy),0);
if xnode_qz_r (yy)>0;
  if xnode_qz_r (yy)<L_module+0.1;
    qznodes2(:,yy)=Pz(:,xnode_qz_r (yy))*B/2;
  else
  end
else
  qznodes2(:,yy)=Empty;
end
else
  qznodes2(:,yy)=Empty;
end

if alfa>0
  if qynodesx_alfaplus(yy)>0.5;
    qynodes2(:,yy)=qy(:,qynodesx_plus_r (yy));
  else
    qynodes2(:,yy)=Empty;
  end
else
  if qynodesx_alfamin(yy)>0.5;
    qynodes2(:,yy)=qy(:,qynodesx_min_r (yy));
  else
    qynodes2(:,yy)=Empty;
  end
end
```

*Next page file continous*
if pilesnodesxy3(yy,1)>0.5;
pilesnodesx3(yy)=(pilesnodesxy3(yy,1)-0.875)/hoh_pile_x;
pilesnodesy3(yy)=(pilesnodesxy3(yy,2)+15)/hoh_pile_y;
else
pilesnodesx3(yy)=0;
pilesnodesy3(yy)=0;
end

if pilesnodesxy3(yy,1)>0.5;
Fy3_piles_nodes(:,yy)=Fy3_pile_var(:,pilesnodesx3(yy),pilesnodesy3(yy));
Fx3_piles_nodes(:,yy)=Fx3_pile_var(:,pilesnodesx3(yy),pilesnodesy3(yy));
%[N] [time x Nmbr_Nodes]
else
end

if pilesnodesxy10(yy,1)>0.5;
pilesnodesx10(yy)=(pilesnodesxy10(yy,1)-0.875)/hoh_pile_x;
pilesnodesy10(yy)=(pilesnodesxy10(yy,2)+15)/hoh_pile_y;
else
pilesnodesx10(yy)=0;
pilesnodesy10(yy)=0;
end

% Making the Forcevector
% |x|
% |y|
% |z|
Fx_vec=Fx3_piles_nodes'+Fx10_piles_nodes';
% [Numbr_Nodes X time]
Fy_vec=qynodes2'+Fy3_piles_nodes'+Fy10_piles_nodes';
Fz_vec=qznodes2';
Fxyz=vertcat(Fx_vec,Fy_vec,Fz_vec);
Fxyz_s=sparse(Fxyz);

% Import eigenvector out of Scia
% This one has: 20 mode shapes
EigenVector= *Many Numbers*
% eigenvector of modeshape i [Nmbr_nodes *DOF x Nmb r_Modes]
Nmbr_Modes=20;

% Eigenfrequency out of Scia
OmegaN=[5.84 5.88 5.96 16.46 38.5 43.3 43.4 43.6 43.7 44.0 44.4 44.8 45.5 46.2 47.2
48.3 49.2 49.3 49.4];
damp=((damp5-damp1)/(OmegaN(5)-OmegaN(1)))*(OmegaN-OmegaN(1))+damp1;

% Mass Matrix
EME=10^6*eye(Nmbr_Modes,Nmbr_Modes);    % Orthonormalized by Scia Engieneer

% Modal Analysis
% Making Matrices right Sizes
EigenVectorT_s=sparse(EigenVector');
W=zeros(DOF*Nmbr_Nodes,1);
x1=zeros(DOF*Nmbr_Nodes,1);

step_big=0.04;  
% Response only saved every 0.04 s, to reduce memory use
x_stat_mode=zeros(DOF*Nmbr_Nodes,N*(1/step_big));
X_mode=zeros(DOF*Nmbr_Nodes,N*(1/step_big));
x_p=zeros(DOF*Nmbr_Nodes,N*(1/step_big));
x_stat=zeros(DOF*Nmbr_Nodes,N*(1/step_big));
qznodes=[ *large*];  
% Is 1 for node on deck, 0 for pile node
qznodes=vertcat(qznodes',qznodes',qznodes');  
% [DOF*Nmbr_Nodes]
display('Start of Modal analysis')

*Next page file continous*
% Numerical integration Backward Euler. W is velocity, x₁ is displacement both in u(x=E*u).

for mm=1:Nmbr_Modes
    EigenVec_deck=sparse(qznodes.*EigenVector(:,mm));
    for j=2:N*(1/step)+1
        W=(W-step*OmegaN(mm)^2*x₁+step*(EigenVectorT_s(mm,:)*Fxyz_s(:,j-1)./
          EME(mm,mm)))/(1+step*2*damp(mm)*OmegaN(mm)+step^2*OmegaN(mm)^2);
        x₁=x₁+step*W;
        if ((j-1)*step/step_big)-roundn((j-1)*step/step_big,0)<10^-4;
            jj=roundn((j-1)*step/step_big,0);
            t_big(jj)=t(j);
            X_mode(:,jj)=EigenVec_deck.*x₁;
        else
            end
        end
    end
    W=zeros(DOF*Nmbr_Nodes,1); % initial condition velocity is zero
    x₁=zeros(DOF*Nmbr_Nodes,1); % initial condition displacement is zero
    x_p=sparse(x_p+X_mode); % sum over all mode shapes [ Nmbr_Nodes * DOF x time]
    x_stat=sparse(x_stat+abs(x_stat_mode)); %[ Nmbr_Nodes* DOF x times frames]
end

display('Modal Analysis is done')

% DAF  First 12 s not included
x_p_xmax=max(max(abs(x_p([1:Nmbr_Nodes],[12/step_big:N*(1/step_big)])))); % Wₓ,displacement
x_stat_x=max(max(abs(x_stat([1:Nmbr_Nodes],[12/step_big:N*(1/step_big)])))); % Wₓ,stat
DAF_x=x_p_xmax/x_stat_x
[xₓ,tx]=find(abs(x_p([1:Nmbr_Nodes],[12/step_big:N*(1/step_big)]))(x_p_xmax-0.0001)*x_p_xmax);

x_p_yamax=max(max(abs(x_p([Nmbr_Nodes+1:Nmbr_Nodes],[12/step_big:N*(1/step_big)])))); % Wᵧ,displacement
x_stat_y=max(max(abs(x_stat([Nmbr_Nodes+1:Nmbr_Nodes],[12/step_big:N*(1/step_big)])))); % Wᵧ,stat
DAF_y=x_p_yamax/x_stat_y
[xᵧ,ty]=find(abs(x_p([Nmbr_Nodes+1:Nmbr_Nodes],[12/step_big:N*(1/step_big)]))(x_p_yamax-0.0001)*x_p_yamax);

x_p_zmax=max(max(abs(x_p([2*Nmbr_Nodes:3*Nmbr_Nodes],[12/step_big:N*(1/step_big)])))); % Wz,displacement
x_stat_z=max(max(abs(x_stat([2*Nmbr_Nodes:3*Nmbr_Nodes],[12/step_big:N*(1/step_big)])))); % Wz,stat
DAF_z=x_p_zmax/x_stat_z
[xₚ,τz]=find(abs(x_p([2*Nmbr_Nodes:3*Nmbr_Nodes],[12/step_big:N*(1/step_big)]))(x_p_zmax-0.0001)*x_p_zmax);

% Velocity
v_p=zeros(DOF*Nmbr_Nodes,N*(1/step_big));
for j=2:N*(1/step_big)-1;
    v_p(:,j)=(x_p(:,j+1)-x_p(:,j-1))/(2*step_big);
end

% spectrum making
% x direction first
T=im=sqrt(-1); %
omegamax=50;
NN=roundn(omegamax*T/(2*pi),0);
for i=1:NN;
    omega_spec=(2*pi)/T;
    Sₓ(star)(i)=(1/pi)*sum(x_p(391,:).*exp(-i*omega_spec.*τ)^2);
    Sₓ(i)=(1/pi)*sum(x_p(391,:).*exp(i*omega_spec.*τ)^2);
    omega_statistics(1)=omega_spec;
end

% Next page file continuous*

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% y-direction
for i=1:NN;
    omega_spec=i*(2*pi/T);  % N661 on 148 m
    Sy(i)=(1/pi)*sum(x_p(880+391,:).*exp(im*omega_spec.*t_big))*step_big;
    Sy_star(i)=(1/pi)*sum(x_p(880+391,:).*exp(-im*omega_spec.*t_big))*step_big;
    S_y_y(i)=(pi/T)*Sy(i)*Sy_star(i);
    omegagraph(i)=omega_spec;
end

% z-direction
for i=1:NN;
    omega_spec=i*(2*pi/T);  % N394 on x=3 m.
    Sz(i)=(1/pi)*sum(x_p(2*880+391,:).*exp(im*omega_spec.*t_big))*step_big;
    Sz_star(i)=(1/pi)*sum(x_p(2*880+391,:).*exp(-im*omega_spec.*t_big))*step_big;
    S_z_z(i)=(pi/T)*Sz(i)*Sz_star(i);
    omegagraph(i)=omega_spec;
end