A Study on the Effect of Frequency in Delamination of CFRP

by

Aravind Premanand

in partial fulfillment of the requirements for the degree of

Master of Science
in Aerospace Engineering

at the Delft University of Technology,
to be defended publicly on Thursday December 14, 2017 at 10:00 AM.

Supervisor: Dr. ir. Rene Alderliesten
Committee Members: Dr. ir. Rene Alderliesten TU Delft
Dr. Calvin Rans TU Delft
Dr. ir. Julien van Campen TU Delft

An electronic version of this thesis is available at http://repository.tudelft.nl/.
Fatigue damage is considered to be one of the most critical phenomenon, contributing to the failure of an aircraft structure. Understanding the fatigue mechanism is essential for considering the different technical conditions that influence the fatigue life of structures. In recent years, fatigue tests are accelerated in order to reduce the testing time of materials/structures being developed, which in turn can reduce the time to market. However, an ideal fatigue experiment should involve testing the structure at its service conditions. Increasing the frequency of the fatigue test can alter the fatigue mechanisms that attribute to damage initiation and propagation. One possible solution to overcome this problem is to understand the effect of frequency on fatigue. By doing so, tests can be carried out at high frequencies and influence of frequency on damage progression can be taken into account during damage prediction. The scope of this thesis is narrowed down to carbon fibre reinforced plastics and aimed at understanding the effect of frequency on damage propagation through energy principles.

Fatigue can be seen as a material degradation process through which the applied work in the form of strain energy is dissipated into damage and other energy dissipation mechanisms. Experiments were performed for Mode I delamination on double cantilever beam specimens manufactured from carbon/epoxy laminate. Within a single fatigue cycle, energy dissipation (energy that is supplied during loading phase and not returned back during unloading phase) and crack growth are mutual. In other words, crack growth occurs with the consumption of dissipated energy and energy is dissipated in creating new crack surface. For simplification of calculation, energy loss and crack growth occurring in one cycle are averaged over that cycle. The averaged quantities $\frac{dU}{dN}$ and $\frac{da}{dN}$ are correlated with each other for different frequencies.

For the calculation of strain energy loss, strain energy at maximum displacement position is calculated from the area under load-displacement plots of the fatigue cycle. For the fatigue tests carried out at a frequency of 5 Hz, assumption that the load-displacement response is linear, holds good. For higher frequencies, the response becomes non-linear due to hysteresis effect. When strain energy was calculated for all the tests with the assumption of linear P-d response, no particular trend on the effect of frequency could be observed in $\frac{da}{dN}$ vs $\frac{dU}{dN}$ plots. Accounting for the non linearity in load-displacement response due to hysteresis provided light on the characterization of frequency effects. The size of hysteresis loop was approximated for different frequencies based on the observations during the experiments. It was also found that, more energy was dissipated to grow a unit crack (more crack growth resistance, $G^*$) for a higher frequency test.

Two possible mechanisms were investigated to find a suitable explanation for the observed increase in $G^*$ with the increase in frequency: heat dissipation and internal heat generation that would cause a rise in specimen temperature. Measurements from thermocouple and infra red camera showed that no significant temperature rise occurred in the specimen during fatigue. This confirmed that, for CFRP under mode I fatigue loading, the hysteresis energy due to high frequency fatigue testing did not heat up the specimen, but either got dissipated as heat at a rapid rate or by some other mechanism which is still not clear.

When results were compared with the conceptual model formulated in this thesis, it was found that increasing the frequency increases the available energy, $\frac{dU}{dN}$ and the crack growth resistance, $G^*$ such that crack growth rate may either increase or decrease (depending on the interaction between the two parameters). Further, a model based on quantitative measurements of heat dissipation at higher frequencies and determination of exact coordinates of the hysteresis loop evolution in load-displacement relationship of the fatigue cycles are recommended to fully understand the effect of test frequency in fatigue damage propagation.
This thesis marks the end of my Master’s programme at TU Delft. I would like to express my appreciation and gratitude for all the wonderful people who supported me during this period.

I would first like to thank my supervisor, Dr. ir. Rene Alderliesten for introducing me to the field of fatigue of structures and guiding me throughout my work. His suggestions have been instrumental in the successful completion of this thesis.

Many thanks to the technicians at the DASML laboratory. I would like to thank Berthil Grashof, Misja Huizinga, Gertjan Mulder for helping me to setup and operate the fatigue machine. Thanks to Victor Horbowiec and Fred Bosch for helping me with manufacturing of the specimens. I would also like to thank Johan Boender and Cees Paalvast for helping me find the appropriate tools and materials during my experiments. Thanks to Frans Oostrum for helping me with the SEM observations. I am also thankful to Marianne de Knecht-Overduin for providing me with access to ASML drive and to the DASML laboratory.

I would like to thank Lucas Amaral, John Alan Pascoe and Fabricio Ribeiro for their kind suggestions on setting up and conducting the fatigue experiments. I would also like to thank Dr. Irene Fernandez Villegas for her thoughts on the manufacturing of my specimens. Special thanks to Dr. Andrei Anisimov for providing me with the infra red camera. Thanks to the department secretary Gemma van der Windt for helping me out in the administrative matters.

I would also like to thank Dr. Calvin Rans and Dr. ir. Julien van Campen for their acceptance to be a part of my thesis committee.

This journey would not have been easy without my friends. I would like to thank all my friends across the globe for sharing all the wonderful moments.

Finally and most importantly, I would like to thank my parents and my brother for their thorough support during the entire period.

Aravind Premanand  
Delft, December 2017
Contents

List of Figures ix
List of Tables xi
Nomenclature xiv
Abbreviations xv

1 Introduction 1
1.1 Background ........................................... 1
1.2 Aim and scope of this thesis .............................. 1
1.3 Thesis outline ........................................ 2

2 Literature review 3
2.1 Introduction ............................................. 3
2.2 Fatigue phenomena in structures and materials .......... 3
2.3 Fatigue damage prediction models for fibre-reinforced composites .......................... 4
  2.3.1 Fatigue life models .................................. 4
  2.3.2 Residual stiffness models ............................ 4
  2.3.3 Residual strength models ............................. 5
  2.3.4 Progressive damage models .......................... 6
2.4 Fatigue delamination and characterization of delamination growth in composites ........ 6
  2.4.1 Classification of delamination characterization methods ............................................. 6
  2.4.2 Stress/strain models ................................ 7
  2.4.3 Fracture mechanics based models .................... 7
  2.4.4 Cohesive zone models ............................... 10
  2.4.5 Extended finite element models ...................... 11
2.5 Energy based models for fatigue damage characterization ............................................ 12
2.6 Conclusion ............................................. 16

3 Methodology 19
3.1 Introduction ............................................. 19
3.2 Conceptual Model ....................................... 19
3.3 Material description ................................... 20
3.4 Design and manufacturing of DCB specimens ............... 20
  3.4.1 Procedure for manufacturing CFRP laminate ............... 20
  3.4.2 Curing cycle of laminate ................................ 21
  3.4.3 Specimen arrangement in laminate ..................... 22
  3.4.4 Procedure for bonding Aluminium blocks ............... 24
3.5 Test matrix ........................................... 24
  3.5.1 A01 test series ..................................... 25
  3.5.2 A02 test series ..................................... 25
  3.5.3 A03 test series ..................................... 25
  3.5.4 B01 test series ..................................... 26
  3.5.5 B02 test series ..................................... 26
3.6 General information on the fatigue experiments ............. 27
  3.6.1 Data storage ....................................... 27
  3.6.2 Crack growth data .................................. 27
  3.6.3 Strain energy release rate ............................ 28
3.7 Calculation of strain energy dissipation .................................................. 28
  3.7.1 Strain energy calculation for linear P-d relationship .......................... 29
  3.7.2 Strain energy calculation for non-linear P-d relationship ................. 30
  3.7.3 Energy loss factor ........................................................................... 35
  3.7.4 Hysteresis loop energy .................................................................... 36
3.8 Calculation of crack growth resistance .................................................. 37
3.9 Conclusion .......................................................................................... 37

4 Results and discussion ........................................................................... 39
  4.1 Introduction .......................................................................................... 39
  4.2 Strain energy dissipation per cycle and fatigue crack growth rate ....... 39
  4.3 Energy dissipation and delamination under linear P-d response ........... 40
    4.3.1 Correlation between cyclic strain energy dissipation and fatigue crack growth rate ..................................................... 40
    4.3.2 Correlation between total strain energy dissipation and fatigue crack growth rate ..................................................... 41
    4.3.3 Crack growth resistance against crack growth ........................... 43
  4.4 Energy dissipation and delamination under non-linear P-d response .... 46
    4.4.1 Correlation between dU/dN and da/dN after energy correction .... 46
    4.4.2 Effect of frequency on crack growth resistance after energy correction ................................................................. 47
  4.5 Comparison with conceptual model ...................................................... 49
  4.6 Effect of fibre bridging on crack growth resistance at different frequencies .......................................................... 53
  4.7 Effect of stress ratio on crack growth resistance at different frequencies .......................................................... 55
  4.8 Fractography ....................................................................................... 56
  4.9 Conclusion .......................................................................................... 58

5 Conclusions and Recommendations ....................................................... 59
  5.1 Introduction .......................................................................................... 59
  5.2 Conclusions .......................................................................................... 59
    5.2.1 Characterization of fatigue delamination ...................................... 59
    5.2.2 Hysteresis behaviour during fatigue loading for CFRP ................. 59
    5.2.3 Effect of frequency on delamination and energy dissipation in CFRP .......................................................... 60
    5.2.4 Effect of fibre bridging and stress ratio ......................................... 60
    5.2.5 Limitations of current research ..................................................... 60
  5.3 Recommendations ............................................................................... 61
    5.3.1 Development of prediction model ................................................ 61
    5.3.2 Towards better experimental setup .............................................. 61
    5.3.3 Future research direction ............................................................. 62

Bibliography .............................................................................................. 63

A Curve Fit Parameters ............................................................................. 67

B Hysteresis Loop .................................................................................... 71
List of Figures

2.1 Modes of loading for delamination growth [1]. ........................................... 6
2.2 Finite element nodes at crack tip [2]. ......................................................... 8
2.3 A schematic of different possible load cycles for same $G_{max}$ (left panel) and different $\Delta G$ (right panel) values [3]. ................................................................. 9
2.4 A schematic of the setup inside climate chamber with gage section and temperature gradient along axial direction of the specimen [4]. ................................. 13
2.5 Experimental measurement of cooling rate [5]. ........................................ 14
2.6 A schematic of cyclic loading with time (left panel), load-displacement diagram for reversible case (middle panel) and load-displacement curve with hysteresis loop for irreversible case [6]. ........................................... 14
2.7 Fatigue crack growth conceptual model [3]. ............................................. 15
2.8 Crack growth rate as a function of energy dissipation per cycle. Energy dissipation is shown based on both $U_{cyc}$ (left panel) and $U_{tot}$ (right panel) [3]. .......... 16

3.1 Teflon layer for crack initiation in the specimen. ........................................... 21
3.2 Laminate setup for curing in autoclave. .................................................. 22
3.3 Top view of layup arrangement. ............................................................... 22
3.4 Front view of layup arrangement. ......................................................... 22
3.5 The curing cycle of the laminate. ............................................................ 23
3.6 A schematic of laminate with dimensions of specimens. ......................... 23
3.7 Example for the labelling of tests. ............................................................ 24
3.8 Setup for the experiments. ................................................................. 27
3.9 Crack length data and power fit curve against the number of cycles for 20 Hz frequency test. ................................................................. 28
3.10 A schematic of P-d curve from which energy dissipation is calculated. .......... 29
3.11 Cyclic energy data and power fit curve against the number of cycles for 20 Hz frequency test. ................................................................. 30
3.12 A schematic of force and displacement time plots for the 5 Hz frequency with $d_{min} = 0.4$ mm and $d_{max} = 4$ mm. ......................................................... 31
3.13 A schematic of the P-d hysteresis loop for the 5 Hz frequency test with $d_{min} = 0.4$ mm and $d_{max} = 4$ mm. ......................................................... 31
3.14 A schematic of force and displacement time plots for the 40 Hz frequency test with $d_{min} = 0.4$ mm and $d_{max} = 4$ mm. ......................................................... 32
3.15 A schematic of the P-d hysteresis loop for the 40 Hz frequency test for a single cycle with $d_{min} = 0.4$ mm and $d_{max} = 4$ mm. ......................................................... 32
3.16 The hysteresis loops for the 40 Hz test with $d_{min} = 0.4$ mm and $d_{max} = 4$ mm for a tracing time of 10.5 seconds. ................................................. 33
3.17 A schematic representation of P-d curve for 40 Hz during the start of the test (left panel) and at the end of test (right panel). ................................................. 33
3.18 A schematic of the work done on the specimen between two points [7]. ......... 35

4.1 Crack growth rate as a function of cyclic strain energy dissipation per cycle for A01 series (left panel) and A02 series (right panel). .......................... 40
4.2 Crack growth rate as a function of cyclic strain energy dissipation per cycle for B01 series (left panel) and B02 series (Right panel). ................................................. 41
4.3 Crack growth rate as a function of cyclic strain energy dissipation for A03 series. .... 41
4.4 Crack growth rate as a function of total strain energy dissipation per cycle for A01 series (left panel) and A02 series (Right panel). .......................... 42
List of Figures

4.5 Crack growth rate as a function of total strain energy dissipation per cycle for B01 series (left panel) and B02 series (Right panel). ......................................................... 42
4.6 Crack growth rate as a function of total strain energy dissipation for A03 series. ........ 43
4.7 Crack growth resistance plotted against crack growth for A01 test series. .................. 43
4.8 Crack growth resistance plotted against crack growth for 10, 30 and 40 Hz tests of A01 test series. ............................................................... 44
4.9 Crack growth resistance plotted against crack growth for A02 test series. ................. 44
4.10 Crack growth resistance plotted against crack growth for B01 series (left panel) and B02 series (right panel). ......................................................... 45
4.11 Crack growth resistance plotted against crack growth for A03 test series. .................. 45
4.12 da/dN Vs dU/dN before energy correction (left panel) and after energy correction (right panel) for A01 series. ......................................................... 46
4.13 da/dN Vs dU/dN before energy correction (left panel) and after energy correction (right panel) for A02 series. ......................................................... 46
4.14 da/dN Vs dU/dN after energy correction for B01 (left panel) and B02 (right panel) test series. ............................................................... 47
4.15 da/dN Vs dU/dN before energy correction (left panel) and after energy correction (right panel) for A03 series. ......................................................... 47
4.16 G* Vs a - a0 before energy correction (left panel) and after energy correction (right panel) for A01 test series. ......................................................... 48
4.17 G* Vs a - a0 before energy correction (left panel) and after energy correction (right panel) for A02 test series. ......................................................... 48
4.18 G* Vs a - a0 for B01 (left panel) and B02 (right panel) test series. .......................... 49
4.19 G* Vs a - a0 for A03 test series. ............................................................... 49
4.20 da/dN Vs dU/dN for B02 test series. ......................................................... 50
4.21 Temperature measurements Vs Number of cycles for B01 (left panel) and B02 (Right panel) series. ............................................................... 51
4.22 Image from Infrared camera during 30 Hz test. ..................................................... 51
4.23 Image from Infrared camera during 40 Hz test. ..................................................... 52
4.24 du/dN (left panel) and da/dN against a - a0 for B02 series. ....................................... 52
4.25 G* Vs a - a0 for different initial crack lengths at 5Hz frequency. .......................... 53
4.26 da/dN Vs dU/dN for different initial crack lengths at 5 Hz frequency. ....................... 54
4.27 G* Vs a-a0 without energy correction (left panel) and with energy (Right panel) correction at 10 Hz frequency for different initial crack lengths. .... 54
4.28 G* Vs a-a0 without energy correction (left panel) and with energy (Right panel) correction at 40 Hz frequency for different initial crack lengths. .... 55
4.29 G* Vs a-a0 at different stress ratios for dmax = 2mm (left panel) and dmax = 4mm (Right panel) for 5 Hz frequency tests. ......................................................... 55
4.30 G* Vs a-a0 at different stress ratios for dmax = 2mm (left panel) and dmax = 4mm (Right panel) for 10 Hz frequency tests. ......................................................... 56
4.31 G* Vs a-a0 at different stress ratios for dmax = 2mm (left panel) and dmax = 4mm (Right panel) for 40 Hz frequency tests. ......................................................... 56
4.32 Fracture surface of the bottom arm of specimen tested at 5 Hz frequency with mode I features magnified at 3000X with sharpen mask. .......................... 57
4.33 Striations in the fibre imprints of specimen tested at 20 H frequency viewed at 10.000X magnification with sharpen mask. ......................................................... 57
4.34 3000X magnification of fracture surface of specimen tested at 5 Hz (left panel) and 40 Hz frequency (right panel) with textured microflow. ......................... 58

B.1 A schematic of the decrease in area of loop and shift in the coordinate points. .......... 71
B.2 A schematic of the decrease in the length of major axes (shown with yellow lines) with the rotation of the ellipse in the clockwise direction. .......................... 72
# List of Tables

<table>
<thead>
<tr>
<th></th>
<th>Table Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1</td>
<td>Elastic property of M30SC/DT120 prepreg</td>
<td>20</td>
</tr>
<tr>
<td>3.2</td>
<td>Test matrix for A01 series.</td>
<td>25</td>
</tr>
<tr>
<td>3.3</td>
<td>Test matrix for A02 series.</td>
<td>25</td>
</tr>
<tr>
<td>3.4</td>
<td>Test matrix for A03 series.</td>
<td>26</td>
</tr>
<tr>
<td>3.5</td>
<td>Test matrix for B01 series.</td>
<td>26</td>
</tr>
<tr>
<td>3.6</td>
<td>Test matrix for B02 series.</td>
<td>26</td>
</tr>
<tr>
<td>3.7</td>
<td>Assumed eccentricity values of hysteresis loop ellipse for different frequencies.</td>
<td>34</td>
</tr>
<tr>
<td>4.1</td>
<td>da/dN values for various frequencies at dU/dN = 3.59×10^{-3} mJ/cycle for B02 test series.</td>
<td>50</td>
</tr>
<tr>
<td>4.2</td>
<td>Specifications of Flir A35 infrared camera.</td>
<td>51</td>
</tr>
<tr>
<td>A.1</td>
<td>Curve-fit parameters for the a versus N functions for all the tests.</td>
<td>67</td>
</tr>
<tr>
<td>A.2</td>
<td>Curve-fit parameters for the U\text{cyc} versus N functions for all the tests without energy correction ((\alpha, \beta, \gamma)) and with energy correction ((\alpha', \beta', \gamma')).</td>
<td>68</td>
</tr>
<tr>
<td>A.3</td>
<td>Curve-fit parameters for the U\text{tot} versus N functions for all the tests without energy correction ((\alpha, \beta, \gamma)) and with energy correction ((\alpha', \beta', \gamma')).</td>
<td>69</td>
</tr>
</tbody>
</table>
LATIN SYMBOLS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>Crack length</td>
<td>mm</td>
</tr>
<tr>
<td>A</td>
<td>Crack surface area</td>
<td>mm$^2$</td>
</tr>
<tr>
<td>A</td>
<td>Area under load-displacement plot</td>
<td>mm$^2$</td>
</tr>
<tr>
<td>$A_t$</td>
<td>Slope of curve generated using least square plot</td>
<td>(N/mm)$^3$</td>
</tr>
<tr>
<td>B</td>
<td>Width of the specimen</td>
<td>mm</td>
</tr>
<tr>
<td>c</td>
<td>Curve fit parameter</td>
<td>-</td>
</tr>
<tr>
<td>c</td>
<td>Specific heat</td>
<td>mJ</td>
</tr>
<tr>
<td>C</td>
<td>Compliance</td>
<td>mm/N</td>
</tr>
<tr>
<td>C</td>
<td>Curve fit parameter dependent on n</td>
<td>-</td>
</tr>
<tr>
<td>d</td>
<td>Displacement</td>
<td>mm</td>
</tr>
<tr>
<td>D</td>
<td>Damage parameter</td>
<td>-</td>
</tr>
<tr>
<td>e</td>
<td>Emissivity</td>
<td>-</td>
</tr>
<tr>
<td>$E^*$</td>
<td>Ratio of residual stiffness to failure stiffness</td>
<td>-</td>
</tr>
<tr>
<td>$E_k$</td>
<td>Kinetic energy</td>
<td>mJ</td>
</tr>
<tr>
<td>f</td>
<td>Test frequency</td>
<td>(Hz)</td>
</tr>
<tr>
<td>F</td>
<td>Strength</td>
<td>MPa</td>
</tr>
<tr>
<td>F</td>
<td>External work</td>
<td>mJ</td>
</tr>
<tr>
<td>G</td>
<td>Strain energy release rate</td>
<td>mJ/mm$^2$</td>
</tr>
<tr>
<td>$G'$</td>
<td>Energy dissipation per unit crack growth</td>
<td>mJ/mm$^2$</td>
</tr>
<tr>
<td>$\Delta G$</td>
<td>Strain energy release rate range</td>
<td>mJ/mm$^2$</td>
</tr>
<tr>
<td>h</td>
<td>Thickness of specimen</td>
<td>mm</td>
</tr>
<tr>
<td>h</td>
<td>Convective heat transfer coefficient</td>
<td>W/(m$^2$K)</td>
</tr>
<tr>
<td>H</td>
<td>Parameter</td>
<td>-</td>
</tr>
<tr>
<td>J</td>
<td>J integral</td>
<td>mJ/mm$^2$</td>
</tr>
<tr>
<td>k</td>
<td>Thermal conductivity</td>
<td>W/(mK)</td>
</tr>
<tr>
<td>$k_l$</td>
<td>Longitudinal thermal conductivity</td>
<td>W/(mK)</td>
</tr>
<tr>
<td>$k_t$</td>
<td>Transverse thermal conductivity</td>
<td>W/(mK)</td>
</tr>
<tr>
<td>$k_m$</td>
<td>Matrix thermal conductivity</td>
<td>W/(mK)</td>
</tr>
<tr>
<td>$k_f$</td>
<td>Fibre thermal conductivity</td>
<td>W/(mK)</td>
</tr>
<tr>
<td>K</td>
<td>Stress intensity factor</td>
<td>(MPa$\sqrt{mm}$)</td>
</tr>
<tr>
<td>$\Delta K$</td>
<td>Stress intensity factor range</td>
<td>(MPa$\sqrt{mm}$)</td>
</tr>
<tr>
<td>L</td>
<td>Length of specimen</td>
<td>mm</td>
</tr>
<tr>
<td>M</td>
<td>Ratio of maximum stress between fibre and loading directions</td>
<td>-</td>
</tr>
<tr>
<td>n</td>
<td>Curve fit parameter</td>
<td>-</td>
</tr>
<tr>
<td>n</td>
<td>Compliance calibration parameter</td>
<td>-</td>
</tr>
<tr>
<td>N</td>
<td>Number of cycle</td>
<td>-</td>
</tr>
<tr>
<td>P</td>
<td>Force</td>
<td>N</td>
</tr>
<tr>
<td>P</td>
<td>Power loss</td>
<td>W</td>
</tr>
<tr>
<td>$P_0$</td>
<td>Force at zero displacement</td>
<td>N</td>
</tr>
<tr>
<td>Q</td>
<td>Heat generated</td>
<td>mJ</td>
</tr>
<tr>
<td>R</td>
<td>Stress ratio</td>
<td>-</td>
</tr>
<tr>
<td>S</td>
<td>Applied stress</td>
<td>MPa</td>
</tr>
</tbody>
</table>
List of Tables

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>Temperature</td>
<td>°C</td>
</tr>
<tr>
<td>t</td>
<td>Time</td>
<td>s</td>
</tr>
<tr>
<td>u</td>
<td>Horizontal displacement</td>
<td>mm</td>
</tr>
<tr>
<td>U</td>
<td>Strain energy</td>
<td>mJ</td>
</tr>
<tr>
<td>U₀</td>
<td>Monotonic energy</td>
<td>mJ</td>
</tr>
<tr>
<td>Uᵉ</td>
<td>Damage energy</td>
<td>mJ</td>
</tr>
<tr>
<td>Uᵈ</td>
<td>Damage energy</td>
<td>mJ</td>
</tr>
<tr>
<td>Uₑ</td>
<td>Heat dissipation</td>
<td>mJ</td>
</tr>
<tr>
<td>Uᵢ</td>
<td>Internal energy</td>
<td>mJ</td>
</tr>
<tr>
<td>Uₑᵢ</td>
<td>Bridging energy</td>
<td>mJ</td>
</tr>
<tr>
<td>V</td>
<td>Volume of gage section</td>
<td>mm³</td>
</tr>
<tr>
<td>w</td>
<td>Width of specimen</td>
<td>mm</td>
</tr>
<tr>
<td>W</td>
<td>Energy consumed by crack growth</td>
<td>mJ</td>
</tr>
<tr>
<td>X, Y</td>
<td>X and Y co-ordinates</td>
<td>mm</td>
</tr>
<tr>
<td>z</td>
<td>Axial direction of specimen</td>
<td>mm</td>
</tr>
</tbody>
</table>

GREEK SYMBOLS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>β</td>
<td>Stephan-Boltzmann constant</td>
<td>W/(m²K⁴)</td>
</tr>
<tr>
<td>α, β, γ</td>
<td>Curve fitting parameters</td>
<td>-</td>
</tr>
<tr>
<td>α', β', γ'</td>
<td>Curve fitting parameters</td>
<td>-</td>
</tr>
<tr>
<td>σ</td>
<td>Stress</td>
<td>MPa</td>
</tr>
<tr>
<td>τ</td>
<td>Shear stress</td>
<td>MPa</td>
</tr>
<tr>
<td>θ</td>
<td>Angle between fibre and loading directions</td>
<td>radians</td>
</tr>
<tr>
<td>v</td>
<td>Vertical displacement</td>
<td>mm</td>
</tr>
<tr>
<td>γ</td>
<td>Mean stress sensitivity</td>
<td>-</td>
</tr>
<tr>
<td>δ</td>
<td>Displacement</td>
<td>mm</td>
</tr>
<tr>
<td>ε</td>
<td>Strain</td>
<td>-</td>
</tr>
<tr>
<td>Φ</td>
<td>Energy loss factor</td>
<td>-</td>
</tr>
</tbody>
</table>

SUBSCRIPTS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Fibre direction</td>
</tr>
<tr>
<td>c</td>
<td>Critical</td>
</tr>
<tr>
<td>cd</td>
<td>Conduction</td>
</tr>
<tr>
<td>cum</td>
<td>Cumulative</td>
</tr>
<tr>
<td>cv</td>
<td>Convection</td>
</tr>
<tr>
<td>cyc</td>
<td>Cyclic</td>
</tr>
<tr>
<td>eff</td>
<td>Effective</td>
</tr>
<tr>
<td>hys</td>
<td>Hysteresis</td>
</tr>
<tr>
<td>I, II, III</td>
<td>Mode I, II, III</td>
</tr>
<tr>
<td>max</td>
<td>Maximum</td>
</tr>
<tr>
<td>min</td>
<td>Minimum</td>
</tr>
<tr>
<td>mono</td>
<td>Monotonic</td>
</tr>
<tr>
<td>T</td>
<td>Transverse direction</td>
</tr>
<tr>
<td>tot</td>
<td>Total</td>
</tr>
<tr>
<td>rad</td>
<td>Radiation</td>
</tr>
<tr>
<td>rad</td>
<td>Radiation</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Description</td>
</tr>
<tr>
<td>--------------</td>
<td>-------------</td>
</tr>
<tr>
<td>AFT</td>
<td>Accelerated Fatigue Test</td>
</tr>
<tr>
<td>ASTM</td>
<td>American Society for Testing and Materials</td>
</tr>
<tr>
<td>CFRP</td>
<td>Carbon Fiber Reinforced Polymer</td>
</tr>
<tr>
<td>CZM</td>
<td>Cohesive Zone Models</td>
</tr>
<tr>
<td>DCB</td>
<td>Double Cantilever Beam</td>
</tr>
<tr>
<td>FEA</td>
<td>Finite Element Analysis</td>
</tr>
<tr>
<td>FEM</td>
<td>Finite Element Method</td>
</tr>
<tr>
<td>FCG</td>
<td>Fatigue Crack Growth</td>
</tr>
<tr>
<td>FRP</td>
<td>Fibre Reinforced Plastics</td>
</tr>
<tr>
<td>FRPC</td>
<td>Fibre Reinforced Polymer Composites</td>
</tr>
<tr>
<td>LEM</td>
<td>Linear Elastic Fracture Mechanics</td>
</tr>
<tr>
<td>SEM</td>
<td>Scanning Electron Microscope</td>
</tr>
<tr>
<td>SERR</td>
<td>Strain Energy Release Rate</td>
</tr>
<tr>
<td>SIF</td>
<td>Stress Intensity Factor</td>
</tr>
<tr>
<td>UD</td>
<td>Uni - Directional</td>
</tr>
<tr>
<td>VHCF</td>
<td>Very High Cycle Fatigue</td>
</tr>
<tr>
<td>XFEM</td>
<td>Extended Finite Element Models</td>
</tr>
</tbody>
</table>
1 Introduction

1.1. Background
Economic composite structures with long service life require detailed knowledge on their fatigue properties [8]. When the number of cycles of service life exceed more than $10^7$ cycles, the structures are classified to Very High Cycle Fatigue (VHCF) regime. VHCF experiments with $10^9$ cycles in a servo hydraulic testing machine with a frequency of 5 Hz would take 6.3 years. In order to conduct the tests over a shorter time period, the frequency of the test can be increased. However, accelerating the fatigue test by increasing the frequency could influence the damage propagation in unexpected ways. Heating and hysteresis effects associated with high frequency fatigue is considered to be a barrier for accelerated fatigue testing (AFT) method.

Our inability to understand the effect of frequency from AFTs has limited us to work with low frequency fatigue experiments. This limitation can be avoided if a correlation between the low frequency test results and AFT results could be developed [9]. When looking from energy perspective, heating up of specimen implies dissipation of energy by heat. This aspect may be exploited on purpose such that the effect of frequency on fatigue damage is understood.

1.2. Aim and scope of this thesis
Fatigue damage modeling is usually performed to predict the life of a structure. Most of the life prediction methodologies of composites are phenomenological and are based on statistical analysis of experimental data obtained from the fatigue tests. Fatigue models are usually classified into two groups. Namely, the empirical or semi-empirical models based on macroscopic measurements and mechanistic models accounting for microscopic damage mechanisms [10].

Macroscopic models are based on the experimental data predicting the fatigue life with applied stress. The microscopic models on the other hand are based on fracture mechanics. They have the potential to accurately describe physics behind the fatigue degradation. However, composites are inhomogeneous and damage mechanisms are more complex than the isotropic materials. Nevertheless, taking into consideration of all the factors that affect the fatigue life of a structure during damage prediction involves extensive experimentation as well as modeling.

From the energy perspective, any damage involves loss of the strain energy during each cycle. The idea of energy balance can provide a better understanding to the fatigue problem on a broader aspect. A good model should explain the correlation between the dissipated energy and damage growth. With sufficient experimental data being collected, the proposed model could be validated and developed further into a numerical model which can be implemented in a finite element method. Such an approach would give a clear understanding towards the fatigue phenomenon as well as be able to predict fatigue damage growth.

Thus the research objective of this thesis study is to develop an experimental approach to characterize the effect of frequency on fatigue damage of fibre reinforced composites using energy balance prin-
ciples. To narrow down the research towards a concrete objective, the study was limited to mode I delamination in carbon fibre reinforced plastics (CFRP).

Originally, the goal of the thesis project was to quantify the heat dissipation due to accelerated frequencies. However, during the literature study and while carrying out the fatigue experiments, several limitations were foreseen on the heat measurement techniques. Thus the primary objective of the thesis was decided to provide a qualitative understanding on the effect of frequency on energy dissipation and damage propagation. The secondary goal was to propose a model that takes the effect of frequency into account while predicting fatigue life. Thus the main question put forward for the thesis was:

**How does frequency influence the energy dissipation and delamination growth under mode I fatigue testing of CFRP?**

This question branched out to various sub-questions that were investigated during this thesis

- What are the possible energy dissipation mechanisms that directly or indirectly contribute to fatigue delamination?
- How can the total energy dissipation and damage growth be correlated with each other? How does frequency affect this correlation?
- Does CFRP undergo hysteresis heating due to high frequency?
- What is the effect of fibre bridging and stress ratio on the correlation between energy dissipation and crack growth. Are these effects frequency dependant?

**1.3. Thesis outline**

A review of the current damage prediction models for composite materials is provided in Chapter 2. The conceptual model, the necessary details of specimen material, manufacturing procedures, test matrices for the fatigue experiments and data analysis are described in Chapter 3. The results from the performed experiments are discussed in Chapter 4. Finally, Chapter 5 outlines the conclusions from the current thesis and recommendations for further research on developing a prediction model to account for the effect of frequency.
2. Literature review

2.1. Introduction

This chapter provides an overview of the previous studies on fatigue damage prediction for fibre-reinforced composites. In section 2.4, the different available models on characterization of fatigue delamination in composites is discussed. In section 2.5, approach to fatigue damage through energy dissipation is discussed.

2.2. Fatigue phenomena in structures and materials

Fatigue failure in metallic structures is a known technical problem. August Wohler recognized that a single load application, well below the static strength of a structure did not do any damage to the structure. But the same load, if repeated many times could induce complete failure \[11\]. Fatigue of structures is now recognized as a significant problem and as a result, extensive research and practical experience have been gained about fatigue of structures and the fatigue mechanism in materials.

An engineer’s acumen towards the phenomenon of fatigue is closely related with metallic materials which are homogeneous and isotropic and there has been a tendency to treat fibre composites as if they were metals. Metallic fatigue has been intensively studied for more than a century \[11, 12\] and design data have been accumulated for every known metal and alloy. Fatigue in metals often progresses with the initiation of a single crack that propagates until final catastrophic failure. Previously, it was common among the users of composite materials, that CFRPs did not suffer from fatigue. The actual scenario was that, because CFRPs were extremely stiff in the direction of fibre, the working strains in structures designed with conventional design stress levels were usually far too less to initiate any local damage and hence over-designed. Another issue is that unlike metals, composite materials are inhomogeneous and anisotropic. A stress system that may cause a small working strain in the fibre direction, may have strains high enough to damage the structure in the direction normal to the fibre. Moreover, the damage need not be a single macroscopic crack, and can include several mechanisms such as fibre breakage, matrix cracking, debonding, transverse ply cracking and delamination which may either act independently or interactively, depending on materials and test conditions. \[12\].

The essential parameters that influence the fatigue performance of composites are the types of fibre and matrix, type of fibre architecture (uni-directional, fabric, etc.), stacking sequence of the laminate, environmental conditions (temperature and moisture absorption) and loading conditions (stress ratio, frequency of fatigue cycle, etc.,) \[13\]. Thus, physical understanding of fatigue mechanism is essential when considering various parameters that affect fatigue life of a structure. Fatigue prediction methods can be only evaluated if fatigue as a phenomenon is clearly understood.
2.3. Fatigue damage prediction models for fibre-reinforced composites

A large number of existing fatigue models for composite laminates have been classified by authors into three major categories [13]:

1. Fatigue life models that use S-N curves or Goodman-type diagrams, but do not take into account of actual degradation mechanisms.

2. Phenomenological models for residual stiffness/strength.

3. Progressive damage models which use one or more damage variables related to measurable manifestation of damage such as matrix cracks and delamination.

Each of the three categories uses its own criterion for predicting the final failure and fatigue life of the composite structure.

2.3.1. Fatigue life models

In fatigue life models, usually, S-N curves are extracted and a fatigue failure criterion is proposed [13]. They fail to consider the damage accumulation and can only be used to perform safe-life assessment of a structure [14]. For instance, Hasim and Rotem (1973) [15] proposed one of the first fatigue failure criteria. The criterion distinguished between fibre and matrix failure given by,

\[
\sigma_A = \sigma_A^f \left( \frac{\sigma_T}{\sigma_T^f} \right)^2 + \left( \frac{\tau}{\tau^f} \right)^2 = 1
\]

where \( \sigma_A \) and \( \sigma_T \) are the stresses along the fibre and transverse fibre directions, \( \tau \) is the shear stress and \( \sigma_A^f \), \( \sigma_T^f \) & \( \tau^f \) are the ultimate tensile, transverse and shear stress respectively. The ultimate strengths are function of fatigue stress levels (\( \sigma_{Amax} \) and \( \sigma_{Amin} \)), number of cycles and frequency of cycles. The criterion is expressed in terms of three S-N curves which must be determined experimentally for uni-directional specimens under uni-axial load. So this criterion is only valid for laminates with unidirectional plies.

Jen and Lee (1998a, 1998b) [16, 17] modified the Tsai-Hill failure criterion for the plane stress multi-axial fatigue loading to predict fatigue strength and life in AS4 carbon/PEEK APC-2 laminates:

\[
M_{11}^2 \left( \frac{\sigma_{xx}}{\sigma_{11}} \right)^2 + M_{22}^2 \left( \frac{\sigma_{xx}}{\sigma_{22}} \right)^2 - M_{11} M_{22} \left( \frac{\sigma_{xx}}{\sigma_{11}} \right) + M_{12}^2 \left( \frac{\sigma_{xx}}{\sigma_{12}} \right) = 1
\]

where \( \sigma_{xx} \) is the maximum stress in the loading direction for an off-axis laminate, the 1-2 co-ordinate system is the stress system rotated to the principal material coordinates such that

\[
\sigma_{11} = \sigma_{xx} \cos^2 \theta \quad \sigma_{22} = \sigma_{xx} \sin^2 \theta \quad \sigma_{12} = -\sigma_{xx} \sin \theta \cos \theta
\]

\( \theta \) is the angle of inclination between loading direction and fibre direction, \( M_{11}, M_{12}, \) and \( M_{12} \) are the ratios of stresses given by

\[
M_{11} = \frac{(\sigma_{11})_{max}}{(\sigma_{xx})_{max}} \quad M_{22} = \frac{(\sigma_{22})_{max}}{(\sigma_{xx})_{max}} \quad M_{12} = \frac{(\sigma_{12})_{max}}{(\sigma_{xx})_{max}}
\]

where subscript ‘max’ denotes the maximum stress of the fatigue cycle. Fatigue strength and life of multi-directional laminates were predicted using the above equations and when compared with experimental results, it was found that, predicted curves were close for quasi-static and cross ply laminates. However, for \([\pm 45^\circ]\) a larger error was observed.

2.3.2. Residual stiffness models

Residual stiffness models describe the degradation of the elastic properties during fatigue loading. The loss of stiffness \( D \), for one-dimensional is given by \( D = 1 - E/E_0 \), where \( E_0 \) is the undamaged modulus [13]. Such models are considered to be phenomenological and not as progressive damage models, because \( dD/dN \) is expressed based on macroscopically observable properties and not from the actual damage mechanisms.
2.3. Fatigue damage prediction models for fibre-reinforced composites

Whitworth (1990) [18] proposed a residual stiffness model as a cumulative damage model where the damage parameter \( D \) is given by,

\[
D = \frac{H(1 - \dot{S})^{a}}{1 - \dot{S}^{a}} \left( \frac{n}{N} \right)^{m}
\]

(2.5)

where \( \dot{S} \) is the normalized applied stress range while \( a \) & \( H \) are parameters and \( n/N \) is the ratio of the applied fatigue cycles to the fatigue life \( N \). Whitworth (1998) [19] later proposed a new residual stiffness model given by

\[
\frac{dE^{*}(n)}{dn} = -\frac{a}{(n+1)[E^{*}(n)]^{m-1}}
\]

(2.6)

where \( E^{*} \) is the ratio of residual stiffness to the failure stiffness, \( E(n)/E(N) \), \( n \) is the number of applied loading cycles and \( a \) and \( m \) are parameters that depend on several test variables such as the applied stress, loading frequency and environmental conditions such as temperature, humidity, etc. The formulation of the model presented above is limited to specimens subjected to constant amplitude fatigue loading and it also assumes that the residual stiffness is a monotonically decreasing function with respect to the number of fatigue cycles.

2.3.3. Residual strength models

When composite specimens are subjected to high levels of stress, the residual strength of the specimen as a function of number of cycles stays constant during initial few cycles and decreases drastically when the applied number of cycle reaches close to failure. However, the loss of strength cannot be measured through non-destructive testing. Residual strength can also be evaluated through translation of a possible damage parameter with strength of the material. Two types of models can be distinguished under residual strength models: the sudden death and wear-out models. Sudden death models are generally suitable to describe low cycle fatigue where the loss of strength is drastic. On the other hand, for lower level of stresses, the residual strength of the laminate as a function of number of cycles degrades gradually and hence referred as wear-out models [13].

Daniel and Charwatwicz (1986) [20] studied damage accumulation and associated stiffness & residual strength reduction in cross-ply graphite/epoxy laminates under cyclic loading. They proposed a model based on change in residual strength given by

\[
\frac{F_{r} - \sigma_{a}}{F_{o} - \sigma_{a}} = g\left( \frac{n}{N} \right)
\]

(2.7)

where \( F_{r} \) is the residual strength after \( n \) cycles, \( F_{o} \) is the mean static strength (used as normalization factor), \( \sigma_{a} \) is the applied cyclic stress and \( N \) is the number of cycles to failure when \( \sigma_{a} \) is applied. \( g(n/N) \) is a function of normalized number of cycles. However, the determination of function \( g \) is not explained. Moreover, since the model fully relies on a good definition of the residual strength curve, it is considered unsatisfactory [13].

Whitworth (2000) [21] proposed a residual strength degradation model based on the previously proposed stiffness model in (Whitworth (1998) [19]). The relationship between failure stiffness \( E(N) \) and applied stress \( (S) \) is obtained as

\[
\frac{S}{S_{u}} = \frac{E(N)}{E(0)}
\]

(2.8)

where \( E(0) \) is the stiffness of undamaged specimen which can be determined from the relation \( E(0) = S_{u}/\epsilon_{u} \), \( S_{u} \) is the ultimate strength and \( \epsilon_{u} \) is the ultimate strain. To account for non-linearity between the stress-strain response the relation in Equation 2.8 was modified to

\[
\frac{S}{S_{u}} = c_{1}\left[ \frac{E(N)}{E(0)} \right]^{c_{2}}
\]

(2.9)

where \( c_{1} \) and \( c_{2} \) are parameters introduced to account for non linear effects. The final residual strength degradation expression was given by

\[
S_{R} = S_{u} - \frac{n}{N}(S_{u}^{*} - S^{*})
\]

(2.10)

where \( S_{R} \) is the residual strength and \( \gamma \) is a parameter.
2.3.4. **Progressive damage models**

Progressive models differ with the previously discussed models in a way that they consider one or more damage variables that describe the deterioration of the composite structure. As these models quantitatively account for progression of damage in composite materials, they are considered to be one of the most promising tools for fatigue studies. Delamination is considered to be a critical damage parameter and models based on characterization of delamination in composites is briefly explained in the following section.

2.4. **Fatigue delamination and characterization of delamination growth in composites**

"Delamination is deemed to be the most important damage mechanism from the strength and stiffness point of view" [22], degradation of which would lead to catastrophic failure of the component during its service life [23]. The threat of delamination arising due to service loads has been one of the major factors for the limitation of laminated composite materials in greater volume for primary structures in aircraft [12]. While other damage mechanisms such as matrix cracks can also occur, delamination results in larger stiffness loss and reduction in load bearing capability.

Delamination growth or a crack in a body in general, can occur in three different modes of loading that result in the displacement of the crack surfaces [1], namely, mode I (tensile loading), mode II (shear loading) and mode III (transverse shear loading) or a combination of the three (mixed-mode loading) as shown in figure 2.1.

![Figure 2.1: Modes of loading for delamination growth][1]

2.4.1. **Classification of delamination characterization methods**

Progressive damage models can provide physical explanation to the underlying damage mechanisms that lead to degradation of the structure. Pascoe et al. (2013) [14] critically reviewed the different prediction models for delamination growth in composites and adhesive bonds. The models were roughly classified into

1. Stress/strain based models that relate the delamination growth to the stress and strain of the material.
2. Fracture mechanics based models that relate delamination growth to fracture mechanics properties such as Stress Intensity Factor (SIF) or Strain Energy Release Rate (SERR).
3. Cohesive Zone Models (CZMs) which are used to model the interface between two layers using cohesive zone elements, where the damage parameter decreases with growth of damage.
4. Extended Finite Element Models (XFEM) where delaminations can be modeled at arbitrary points within the elements of the material.
2.4. Fatigue delamination and characterization of delamination growth in composites

2.4.2. Stress/strain models

Stress/strain methods are usually used for quasi-static analysis and there are very few methods that can be used for fatigue investigations [14]. The stress/strain methods, like fatigue life models are generally used to calculate the fatigue life and not the delamination growth. This is similar to the case of fatigue crack growth in metals where stress amplitude is used to determine the fatigue life of the specimen while the actual crack growth is predicted using stress intensity factor (SIF).

Poursartip and Chinatambi (1989) [24] proposed the following equation for delamination growth rate.

\[
\frac{da}{dN} = C \left( \frac{1 + R}{1 - R} \right)^m (\Delta \sigma)^n
\]  \hspace{1cm} (2.11)

where \( R \) is the stress ratio, \( \Delta \sigma \) is the stress amplitude. \( C, m \) and \( n \) are fitting parameters. They also correlated Equation 2.11 with SERR using the relation given by [14]

\[
G = \frac{P^2}{2B} \frac{dC}{da}
\]  \hspace{1cm} (2.12)

where \( G \) is the SERR, \( P \) is the applied load, \( B \) is the width of the specimen, \( C \) is the compliance and \( a \) is the crack length, giving \( da/dN \) in terms of \( G \),

\[
\frac{da}{dN} = C' \left( \frac{1 + R}{1 - R} \right)^{m'} (\Delta G)^{n'}
\]  \hspace{1cm} (2.13)

where \( C', m', n' \) are different curve fit parameters than that from Equation 2.11. In the final results, it was found that there was not much difference in the use of \( \delta \sigma \) or \( \delta G \). However this is not a generalized case because \( dC/da \) in their specimen was constant and this is not the case for all the geometries.

2.4.3. Fracture mechanics based models

Methods based on fracture mechanics concepts which relates crack growth to SIF or SERR have been widely used to investigate crack growth. In LEFM, the SIF parameter is used to determine the stress field in the vicinity of crack tip. This concept is used for both quasi-static as well as fatigue crack growth predictions. The two parameters (SIF and SERR) are similar to each other, but since the determination of SIF is complex at the crack tip in fibre reinforced plastics due to its inhomogeneity in the material, the use of SERR in composite materials is preferred.

Calculation of SERR

The SERR can be calculated analytically depending on the configuration of the specimen or with the use of a suitable finite element method [14]. While some methods are based on calculation of change in potential energy for different crack lengths, other methods are based on evaluation of change in stiffness matrix due to crack extension. But as these methods do not give separate values for \( K_I \) and \( K_{II} \) unless its already known that the loading produces only mode I or mode II response, Virtual Crack Closure Technique (VCCT) is generally used [2]. The method developed by Rybicki and Kanninen (1977) [2], allowed both \( K_I \) and \( K_{II} \) to be calculated in a single analysis.

VCCT is based on Irwin’s contention that when a crack extends by a small amount \( \Delta a \) (\( \Delta C \) in Figure 2.2), the energy that is absorbed in the process of extension is the work required to close the crack to its original length [2], [25]. In equation form, the SERR for mode I and mode II are given by,

\[
G_I = \lim_{\Delta a \to 0} \frac{1}{2 \Delta a} \tilde{F}_c (u_c - u_d)
\]  \hspace{1cm} (2.14)

\[
G_{II} = \lim_{\Delta a \to 0} \frac{1}{2 \Delta a} \tilde{T}_c (u_c - u_d)
\]  \hspace{1cm} (2.15)

where \( u_c \) and \( u_d \) are vertical displacements of nodes c and d, \( u_c \) and \( u_d \) is horizontal sliding of nodes c and d respectively as shown in Figure 2.2, \( \tilde{F}_c \) and \( \tilde{T}_c \) are the the forces in y and x directions. From the experiments, SERR can be calculated by measuring the change in compliance with respect to crack length, \( dC/da \). Looking back for the use of SERR for fatigue characterization, it was found that Griffith used energy rather than stress as the controlling parameter of crack growth [26]. Griffith recognized
that for a crack surface to be created, energy is required and thus the required amount of energy is equal to the surface energy of material multiplied by the area newly created by the crack surfaces.

As the crack growth requires energy to create crack surfaces, likewise, the crack extension will release strain energy. Thus Griffith proposed that the critical stress for which the crack would extend could be formulated from the energy balance between the released and consumed energy \[26\]. Interestingly, to formulate the above energy balance, Griffith made assumptions under which the crack can propagate, i.e. quasi static loading conditions with fixed grip.

Griffith’s concept was further developed by Irwin and Kies \[25\] in the form of energy balance

\[
\frac{dF}{dA} - \frac{dU}{dA} = \frac{dW}{dA} + \frac{dE_k}{dA} \tag{2.16}
\]

Where \(F\) is the work done by the external forces on the body, \(U\) is the strain energy inside the body, \(W\) is the energy consumed by damage (crack growth), \(E_k\) is the kinetic energy and \(A\) is the crack surface area. If the derivative of the kinetic energy with respect to the crack surface area is assumed to be zero, the derivatives of external work and the internal strain energy can be combined together into a single parameter called as Strain Energy Release Rate (SERR) \[1\].

\[
G = \frac{d(F - U)}{dA} = \frac{dW}{dA} \tag{2.17}
\]

Thus the SERR is the difference between the external work performed per unit of crack growth and the reduction in strain energy per unit of crack growth.

**Paris relation and its modifications**

Most of the fracture mechanics models for delamination growth are based on Paris relation \[14\]. Paris et al. (1961, 1964)\[27\], \[28\] formulated a relationship between the SIF and crack growth rate \(da/dN\). The basic forms of the Paris relations are given by,

\[
\frac{da}{dN} = c(\Delta K)^n \tag{2.18}
\]

\[
\frac{da}{dN} = c(K_{\text{max}})^n \tag{2.19}
\]

where \(c\) and \(n\) are curve-fitting parameters, \(\Delta K\) is the SIF range and \(K_{\text{max}}\) is the maximum SIF defined for a fatigue load cycle respectively.

Since the SIF used for metals is difficult to be calculated in the case of composites due to in-homogeneity,

---

![Figure 2.2: Finite element nodes at crack tip][1]
SERR is used as “fracture mechanics parameter” in delamination of composites. The SERR is the energy per unit crack surface area that is available for infinitesimal crack extension. The SERR is used as a parameter to measure the toughness of the material to resist the delamination. The Stress Intensity Factor, K and Strain Energy Release Rate, SERR are related by the equation 2.20.

\[ G = \frac{K^2}{E'} \]  

(2.20)

where

\[ E' = E \text{ for plane stress condition} \]
\[ E' = E/(1-\nu^2) \text{ for plane strain condition} \]

The basic forms of Paris relations in terms of SERR for fatigue growth in composites is given by

\[ \frac{da}{dN} = C(\Delta G)^n \]  

(2.21)

\[ \frac{da}{dN} = C(G_{\text{max}})^n \]  

(2.22)

where \( \Delta G \) is the SERR range and \( G_{\text{max}} \) is the maximum SERR defined for a fatigue load cycle respectively. Thus the general form of Paris relation modified by most of the fracture mechanics based models is given by

\[ \frac{da}{dN} = Cf(G)^n \]  

(2.23)

Modifications of Equation 2.23 have been proposed in order to account for various effects such as stress ratio effect or effect of crack closure.

**Stress ratio effect**

No clear agreement has been made on whether Equation 2.23 should be function of SERR range, \( \Delta G \) or Maximum SERR, \( G_{\text{max}} \). These two parameters have been alternatively used to correlate with fatigue crack growth. Looking at SERR range, some studies used \( G_{\text{max}} - G_{\text{min}} \) as SERR range \([29],[30]\) while others preferred using \( (\sqrt{G_{\text{max}}} - \sqrt{G_{\text{min}}}) \) \([31],[32]\). In the study performed on unidirectional Graphite/Epoxy laminates by Gustafson and Hojo (1987) \([29]\), \( da/dN \) was plotted against maximum SERR \( G_{\text{max}} \) and SERR range \( \Delta G = G_{\text{max}} - G_{\text{min}} \). Another study was performed by Mall et al. (1987) \([30]\) on adhesive joints and it was found that in both the studies, \( G_{\text{max}} \) as similitude parameter showed obvious stress ratio effect while the use \( \Delta G \) as similitude parameter provided ambiguous results. Thus with the case of \( G_{\text{max}} \), \( da/dN \) decreased with increasing stress ratio and the trend was entirely contrasting with the use of \( \Delta G \). So the stress ratio effect or R-effect seemed to imply some kind of material behaviour that is affected by the stress ratio R. However, if one looks at the fatigue cycle, it can be understood that \( G_{\text{max}} \) or \( \Delta G \) cannot uniquely define a fatigue cycle individually as shown in Figure 2.3.

![Figure 2.3: A schematic of different possible load cycles for same \( g_{\text{max}} \) (left panel) and different \( \Delta G \) (right panel) values [3].](image)
Gustafson and Hojo (1987) [29] and Hojo et al. (1994) [33] observed that for a certain value of \( \frac{da}{dN} \)

\[
\Delta K (1 - R) = \text{constant} \tag{2.24}
\]

and derived the following relationship

\[
\frac{da}{dN} = C \Delta K^{(1 - \gamma)n} K_{max}^n \tag{2.25}
\]

where \( \gamma \) is an empirical parameter which represents the sensitivity to mean stress, in other words, it accounts for the contribution of maximum and cyclic stresses. If written in the form of SERR, this equation is given by [34],

\[
\frac{da}{dN} = C' \sqrt{G}^{(1 - \gamma')n'} G_{max}^{n'} \tag{2.26}
\]

Khan (2013) [35] proposed a two parameter model. In his study, the model equation was obtained by superimposing monotonic and cyclic loading on micro features formed during delamination growth. The form of the model is given by

\[
\frac{da}{dN} = C_1 \Delta G^n + C_2 G_{max}^{n_2} \tag{2.27}
\]

The stress ratio effect vanished when two parameter models were used to correlate with the crack growth rate. However it can still be argued that the models are purely empirical and fail to provide the physical explanation on the stress ratio effect and the collapse of the curves when \( G_{max} \) is multiplied with \( \Delta G \) [36].

Inadequacy of LEFM models

"The different models successfully describe the delamination growth, but rarely consider the physics of the problem"[14]. The SERR used in fatigue crack growth (FCG) models are mostly modified from the Paris relationship. The Paris relationship which was originally formulated for relating crack growth rate as a function of SIF, which should be seen as the state of stress at the crack tip. However, SERR should be seen as an energy parameter. Several implications were made against the validity of \( G \) [3]. From the previous discussions \( G \) is the amount of energy released as a consequence of crack growth. So there should be no energy release if there is no crack growth. In contrast, \( G \) is treated as a virtual parameter that has a certain value regardless of whether there is a crack growth increment or not.

On the assumption of Griffith [26] that the SERR is calculated based on the fixed grip assumption, Yao (2015) [37] highlighted that the assumption is inapplicable for fatigue loading conditions. In the fixed grip condition, the strain energy is introduced into the specimen by the applied work. It is assumed that during the application of this work, no crack growth occurs and strain energy \( \Delta U \) is released upon the extension of crack \( \Delta a \). Considering a fatigue cycle with a certain maximum and minimum load, the SERR calculation at maximum and minimum load would relate to a certain crack extension \( \Delta a \). This would mean that the load is increased from the minimum to maximum without any crack increment and when the maximum load is reached, crack increment \( \Delta a \) occurs instantaneously after which unloading occurs without any further crack increment. But generally, the crack extension would take place in a continuous manner when the load increases from minimum to maximum. But this is contrasting to the assumption made by Irwin [25].

From the above models, it can be concluded that single parameter LEFM models do not provide any physical explanation on the correlation with \( \frac{da}{dN} \) as a fatigue cycle needs both \( G_{max} \) as well as stress ratio \( R \) to be uniquely defined. To have a model that is valid for different stress ratios such that the curves collapse together, two parameter model is necessary. But even then, the two parameter models provide no physical justification on how the stress ratio effect vanishes or why multiplication of two parameters should be done at the first place. Thus these models do not provide a clear explanation to fatigue crack growth and can be only used for damage predictions.

2.4.4. Cohesive zone models

Cohesive zone models are similar to VCCT discussed in the previous section. While implementing VCCT in a predictive model, re-meshing is required every time when the crack advances. This issue is eliminated in CZM approach as delamination growth is taken care by the cohesive zone elements. These
elements follow a specific traction-displacement relationship. When the traction reaches a certain critical value, say $\sigma_t$, the material damage initiates and material softening occurs. Once the damage is initiated, the traction decreases and material separation occurs when traction reaches zero [38].

Giuliese et al. (2014) [39] developed a CZM model for three dimensional fatigue debonding/delamination. The damage propagation was proposed to be a relation of SERR and process zone area $A_{cz}$ given by

$$\frac{dD}{dN} = \frac{1}{A_{cz}} C \Delta G^n$$

(2.28)

The evaluation of SERR, G for 3 D cracks, was done through calculation of J integral along the path corresponding to the top and bottom nodes of cohesive elements. The results were compared with $G_f$ calculated using VCCT and analytical solution and were found to be in good agreement.

In a recent study by Salih et al. (2017), a trapezoidal traction law was used to account for local plasticity in the model. Traction separation law usually provides the relationship between stress and cohesive length. The traction separation curve can be one of polynomial, exponential, bilinear or trapezoidal form and the area under the curve gives the fracture energy [38]. The model was proposed to provide the ability to capture fatigue crack growth which is dependant on the frequency of the fatigue cycle.

$$\sigma(\delta) = \sigma_{max}$$

\[
\begin{align*}
&\frac{\delta_{eff}}{\delta_{max}-\delta^p}, \quad \text{if } \delta_{eff} < 0 \\
&\frac{\delta_{eff}}{\delta_{max}-\delta^p} H(\delta_{eff}-\delta^p), \quad \text{if } 0 \leq \delta_{eff} \leq \delta_{max} \\
&1, \quad \text{if } \delta_{max} < \delta_{eff} \leq \delta_2 \\
&(1-D) \times \frac{\sigma_{cr}}{\sigma_{max}}, \quad \text{if } \delta_2 < \delta_{max} < \delta_{eff} < \delta_c \\
&0, \quad \text{if } \delta_{eff} \geq \delta_c
\end{align*}
\]

(2.29)

Where $\sigma_{cr}$ is the frequency dependant critical cohesion traction, $\sigma_c$ is the rate dependant critical cohesion traction, $\delta_2$ is displacement at which the element deterioration is assumed to start, $\delta_c$ is the rate-independent critical cohesive separation, $D$ is the damage variable given by the ratio of damaged area with undamaged area ($A_d/A_e$), $\sigma_{max}$ and $\delta_{max}$ is the stress and the separation point where unloading starts. The parameter $\delta^p$ records the remnant separation at the end of each cycle as separation does not reach zero on unloading. Finally, the $\delta_{eff}$ is given by,

$$\delta_{eff} = \delta^{yc} + \delta^p$$

(2.30)

though $\delta^{yc}$ and $\delta^p$ are the applied cyclic displacement and stored plastic displacement respectively. This model was implemented in ABAQUS solver and traction elements were associated with fatigue problem. It was found that crack growth rate reduced with increasing frequency [38].

Though CZM is able to predict crack growth, the framework is entirely phenomenological and is designed to capture the essence of fracture process [38]. Another interesting point is that fracture energy which is calculated based on the locus of the traction-law curve is solely dissipated by the processes of formation of new fracture surface even though the softening of the cohesive element needs to be accounted for mechanisms such as plastic deformation [14]. So it can be concluded that CZM suffers the same shortcomings as fracture mechanics on the lack of physical understanding underlying the delamination process. The mere difference between the two types of models is the calculation of SERR [14], [39].

2.4.5. Extended finite element models

The extended finite element models(XFEM) is a meshless finite element method that allows flexible modelling of crack growth. Unlike other methods like VCCT or CZM, XFEM allows the crack to grow through the element than just along the edge. This is done by incorporating enrichment functions into the finite element approximation [14], [40].

Until now, all the XFEM models that have been reported in the literature have only considered quasi-static delamination. By combining this approach with the of damage parameter in CZM such as in [38], it should be possible to model fatigue delamination using XFEM.
2.5. Energy based models for fatigue damage characterization

Several studies have been made on fatigue damage and its correlation with the corresponding energy or heat dissipation mechanisms \cite{41, 42, 43, 7, 44}. Bledzki et al. (1997) formulated a mathematical approximation of the accumulative dissipated energy for glass/epoxy material. The area enclosed by the hysteresis loop was said to correspond to the dissipated energy that one volumetric element of the material dissipates during one cycle, where the volumetric element is given by cross sectional area of the given sample multiplied by the gage length of the extensometer. In mathematical form, this was given by,

\[ W_v = \int \sigma d\epsilon \]  

(2.31)

This energy was assumed to be directly proportional to the accumulative dissipated energy and it was correlated to the number of cycles given by

\[ W_{v,cum} = C(N)^n \]  

(2.32)

where C and n are fit parameters and N represents the number of cycles. This model was implemented for multiple stresses and stress ratios and it was recommended that such a method can be used for the prediction of fatigue strength of the material. Rather than correlating this “accumulative dissipated energy” with damage, the linearity of the \( W_{v,cum} \) against N was attributed to damage progression. No further explanation on the relationship between \( W_{v,cum} \) and damage was provided.

Jacobsen et al. (1998) \cite{4} proposed that the energy dissipation during fatigue loading can be calculated by measuring the temperature of the specimen and calculating the heat loss. In practical cases, temperature field can exhibit large variation and heat can be dissipated through conduction, convection and radiation. Thus, a fair amount of physical constants needs to be known and the values of the constants may vary with temperature and micro-structural and damage. Moreover, the thermal boundary of the specimen must be clearly defined before the experiments so that heat loss can be calculated based on the applied boundary conditions. Jacobsen et al. (1998) \cite{4} proposed an approximate one-dimensional solution for heat loss and the equations are as follows.

\[ P_{loss} = P_{cd} + P_{cv} + P_{rad} \]  

(2.33)

where \( P \) denotes the heat dissipation (in terms of power) per unit volume and subscripts cd, cv and rad denotes conduction, convection and radiation respectively. Assuming the problem to be an one-dimensional case, the above equation is given by

\[ P_{loss} = \left[ h(T_s - T_a) + e\beta(T_s^4 - T_a^4) \right] \frac{A_{surf}}{V} + \frac{2kA_{cond}}{V} \frac{\Delta T}{\Delta z} \]  

(2.34)

where \( h \) is the coefficient of convective heat transfer, \( T_s \) and \( T_a \) are surface temperature of specimen and air temperature, e is the emissivity, \( \beta \) is the Stefan-Boltzmann constant (5.67 \times 10^{-8} \text{ W/m}^2 \text{K}^4), \( A_{surf} \) is the surface area of gage section, \( V \) is the volume of gage section, \( k \) is the thermal conductivity along the length of the specimen, \( A_{cond} \) is the cross sectional area of gage section, \( \Delta T/\Delta z \) is the axial temperature gradient of the specimen at the end of gage section with \( \Delta T = T_s - T_a \). The convective heat transfer coefficient can be approximated based on free convection condition, isothermal surface and laminar flow along a plate and is given by

\[ h = 1.42 \left( \frac{T_s - T_a}{L} \right)^{0.25} \]  

(2.35)

Figure 2.4 shows the representation of the test setup with the gage section and thermal gradient along the axial direction. The grips were cooled so as to simplify the problem to one-dimensional case. Now, this model was put-forth with several assumptions. The thermal conductivity, \( k \) is assumed to be constant with respect to temperature. The problem is considered to be a steady-state problem. However, in actual case, even with 1-D, the problem becomes transient. Also, the formulation does not provide much information on any damage mechanism or the energy that may be consumed because of damage. However, this method can be used as a first approximation and can be applied to cases
where damage is in the initiation phase. Naderi and Khonsari (2013) [45] further developed the model for glass/epoxy material by accounting for energy losses due to damage and internal heating. The total lost energy is given by the expression

\[ U = U_d + U_h + U_e \]  \hspace{1cm} (2.36)

The notations are renamed for uniformity and U is the total energy that is lost during fatigue, \( U_d \) is the energy consumed by damage mechanism, \( U_h \) is the dissipated heat energy similar to the power loss of Equation 2.33, \( U_e \) is the energy term responsible for rising the temperature of the specimen and given by

\[ U_e = \rho c \frac{dT}{dt} \]  \hspace{1cm} (2.37)

where \( \rho \) is the density, \( c \) is the specific heat, \( T \) is the temperature of the specimen and \( t \) represents the time. The U was calculated from the load-displacement data, \( U_h \) was calculated from Equation 2.34 and the thermal conductivity \( k \) in axial direction was calculated from longitudinal \( (k_l) \) and transverse \( (k_t) \) thermal conductivity given by

\[ k = k_l \cos^2 \theta + k_t \sin^2 \theta \]  \hspace{1cm} (2.38)

where \( \theta \) is the angle between the fibres and the axial direction (z direction in Figure 2.4). The longitudinal and transverse thermal conductivities are given by

\[ k_l = k_m (1 - v_f) + k_f v_f \]  \hspace{1cm} (2.39)

\[ k_t = \frac{k_m + k_f - v_f (k_m - k_f)}{k_m + k_f + v_f (k_m - k_f)} \]  \hspace{1cm} (2.40)

where \( k_m \) and \( k_f \) are epoxy (matrix) and glass (fibre) thermal conductivity respectively. \( v_f \) is the volume fraction. However, the calculation of \( \frac{dT}{dt} \) in Equation 2.37 is not clearly explained. The damage energy \( E_d \) is determined by subtracting \( U_h \) and \( U_e \) from U. This is questionable as energy can also be dissipated by other mechanisms which can be confirmed only by separately calculating the damage energy and see if Equation 2.36 is balanced.

In another study by Naderi et al. (2012) [5], the \( \frac{dT}{dt} \) calculation was formulated by measuring the cooling rate after a sudden interruption of the fatigue test 2.5. Upon sudden interruption of the fatigue test at time \( t^* \) when surface temperature value is \( T^* \), the temperature of the specimen drops and the slope of the curve gives the cooling rate [5], this was done by fitting the curve for T Vs t using MATLAB and differentiating the fitted curve at \( t^* \).

In the study conducted by Lahuerta et al. (2012) [6], Lahuerta and Nijssen (2015) [7], and Lahuerta et al. (2015) [44] on glass/epoxy composite materials, the relation between energy loss and crack growth was quantified using a dimensionless parameter called energy loss factor. This is given by the...
ratio of energy loss per cycle and potential energy supplied to a cycle. So in an ideal case, the strain energy applied to the material during loading cycle is returned back during the unloading cycle (see middle panel in Figure 2.6). However, in practical cases, cyclic hysteresis occurs during loading and unloading process to a certain extent (see right panel in Figure 2.6). With hysteresis loop behaviour in force-displacement relationship, the energy applied during the loading cycle is not the same as the energy returned during the unloading cycle. This loss is quantified in terms of energy loss factor given by

$$\Phi = \frac{W_{1-2}^{\text{load-nonrev}} - W_{2-1}^{\text{unload-nonrev}}}{W_{1-2}^{\text{load-rev}}}$$  \hspace{1cm} (2.41)$$

Where 1-2, 2-1 define the loading and unloading paths of the fatigue cycle, $W_{1-2}^{\text{load-nonrev}}$ is the energy applied to the structure in loading from 1 to 2 in the non-reversible process, $W_{2-1}^{\text{unload-nonrev}}$ is the energy returned by the structure in unloading from 2 to 1 in the non reversible process and $W_{1-2}^{\text{load-rev}}$ is the loading of the structure in an ideal case (linear relationship between force and displacement).

For UD tension coupons, it was assumed that all the energy loss contributed to the heating of the specimen and energy required for creating new surface was ignored \cite{44}.

$$\dot{Q} = f \cdot \Phi \cdot W_{1-2}^{\text{load-rev}}$$  \hspace{1cm} (2.42)$$

where $\dot{Q}$ is the heat generated and $f$ is the frequency.
Assuming that
- the width and length of the specimen are infinite
- loss factor stays constant with time and space and
- strain energy stays constant with time

a model was then implemented in FEM as heat transfer problem with the mode-I geometry [7]. The loss factor was compared with the crack growth and it was found that the two quantities are strongly correlated. Moreover, da/dN was correlated with dΦ/dN, but any explanation for such a correlation was not provided. It is also worth noting that all the models discussed above, in this section used Glass/Epoxy material. It is stressed here that material plays a major role in modelling fatigue with energy dissipation as the behaviour of glass fibre is different from that of carbon fibre. Glass fibre for instance, does not conduct heat due to which the composite material can undergo local heating and hysteresis heating during fatigue. This is not however the case with carbon-fibre composites. Carbon fibres dissipate energy and the local heating is thus not prominent with CFRP when compared with Glass Fiber Reinforced Plastics (GFRP).

In recent years, several studies were carried out by correlating the energy dissipation per cycle with fatigue crack growth per cycle [3], [22], [23], [46]. This is done by considering a single fatigue cycle as the smallest time scale, i.e. the energy dissipation and crack growth rate are determined at a single point (at maximum displacement) in a cycle and this value is averaged to the entire cycle. So the energy dissipation and crack growth may vary within a cycle, but if the quantities are averaged and seen at a resolution of one cycle, an increasing quantity and an averaged quantity means the same. With this assumption, Pascoe (2014, 2015) [46], [3] attempted to make use of the first law of thermodynamics by considering that the finite amount of crack extension in a particular cycle is translated by the finite amount of energy that was available for that single cycle. So the amount of crack growth generated in one cycle follows from the total amount of energy that was dissipated in that cycle and the amount of energy that was required by the crack to grow. This can be represented in the form of an equation (for a unit width) [3]:

\[
\frac{da}{dN} = \frac{da}{d\Phi} \cdot \frac{d\Phi}{dN}
\]  

(2.43)

In the above equation, the dU/dN is the total amount of energy dissipated averaged for a single cycle. The inverse of da/dU is the energy dissipation required per unit of crack growth. A diagrammatic representation of the above equation is shown in Figure 2.7.

![Diagram](image)

Figure 2.7: Fatigue crack growth conceptual model[3].

Figure 2.7 can be interpreted that the crack growth rate relates to two parameters: the energy available for crack growth and the energy required per unit of crack growth. The available energy can be thought as the driving force and the energy required per unit of crack growth can be interpreted as the crack growth resistance. Both these parameters are dependant on the characteristics of the applied load.
Pascoe plotted $da/dN$ against $dU/dN$ (cyclic energy and total energy) for adhesive bonded DCB specimens under mode-I fatigue tests conducted at different stress ratios. It was observed that the curves collapsed to a small region unlike the trend observed when $da/dN$ was plotted against SERR parameters (See Figure 2.8) and stress ratio had no or little effect on the relationship between $da/dN$ and $dU/dN$. Physical explanation can be provided for the correlation between $da/dN$ and $dU/dN$ that the crack growth rate at an arbitrary cycle is dictated by the energy available at that cycle. This available energy is the energy that was lost during the cycle given by $dU/dN$. Alderliesten (2015) [36] recommended to dissect SERR in terms of dissipating mechanisms individually and proposed that a prediction model whose crack extension $da$ should be based on

$$\frac{da}{dN} = f(U_0 + U_1)$$

(2.44)

Where $U_0$ is the monotonic energy and $U_1$ is the cyclic energy supplied to the specimen.

2.6. Conclusion
An overview of important fatigue damage models for composites have been provided. Numerous amount of models have been proposed to predict the fatigue life and damage accumulation, especially, delamination growth due to fatigue loading in composites. Fatigue life models greatly depend on experimental input for each material, layup, loading and environmental conditions. Moreover, such models do not provide explanation about actual damage mechanism.

Residual stiffness models were based on degradation of elastic properties during fatigue loading. Unlike progressive damage models, the damage parameter was based on macroscopically observable properties and not from the actual damage mechanism. Residual strength models on the other hand do not allow for strength evaluation through non-destructive evaluation. Some models related the residual strength with damage parameter, but then, a new relation needs to be established between the residual strength and damage parameter.

A major portion of the models have been developed based on LEFM. Further, recent developments in CZM and XFEM models are also partly based on fracture mechanics. Most of the models are phenomenological and fail to provide proper explanation on the correlation between damage parameter and the driving mechanism.

Fatigue damage can be correlated with different energy mechanisms. When a structure is loaded, strain energy is applied. The same amount of energy is not returned back during the unloading cycle. The loss of energy can be used by several mechanisms such as damage, heat and for rise of specimen temperature. However the ratio at which the lost energy is consumed by different mechanisms depends on the material and other test conditions. Since major portion of the lost energy is consumed
for damage propagation, in creating new surfaces, the other two terms are usually ignored. However, in case of Glass/Epoxy material, self heating or hysteresis heating was observed and considerable part of the lost energy attributes to local heating of the specimen. Most of the proposed energy models were performed on Glass/epoxy material and very few literature was found with energy studies on CFRP specimens. Moreover, to make sure the problem is one-dimensional, flat specimens were tested with tension-tension and tension-compression fatigue tests were performed.

In recent studies, the energy loss per cycle $dU/dN$ was attempted to be correlated with the crack growth per cycle $da/dN$ for CFRP and adhesive bonded DCB specimens. It is proposed in such models that crack growth is a result of energy that is available for the crack and the crack growth resistance of the material. Further, these two parameters, the available energy and the crack growth resistance are translated from the load applied. It was found the effect of stress ratio was negligible and the relationship between $da/dN$ and $dU/dN$ collapsed to a narrow region exhibiting a power-law relationship.

Questions that still remain to be answered are how the frequency affects the relationship between crack growth rate and energy release rate and what are the mechanisms by which energy gets dissipated due to high frequency. In particular it must be investigated whether CFRP specimens undergo local or global heating at high frequency.
3 Methodology

3.1. Introduction
This chapter describes the procedures and techniques followed in this thesis research for manufacturing the specimens, conducting the fatigue experiments and calculation of the different energy parameters. It covers material description, manufacturing CFRP laminate for the test specimens, information on test equipment and data storage and the test matrix that was formulated before conducting the tests. Section 3.2 provides the conceptual model that was put forward in this thesis, the effect of frequency on the total strain energy loss and other possible mechanisms which consume the lost energy. Validation of this model will be however discussed in the next chapter.

Section 3.5 describes the series of tests that were carried out for characterizing the effect of frequency, fibre bridging and stress ratio on the correlation between crack growth rate and energy release rate. Section 3.6 provides information on how the raw data from the experiments were processed and used for analyzing fatigue crack growth & strain energy dissipation. Finally, in Sections 3.7 and 3.8, the calculation of strain energy loss and crack growth resistance are discussed.

3.2. Conceptual Model
From the first law of thermodynamics it can be put forth that the lost strain energy is the energy available for crack growth and other mechanisms that indirectly contribute to crack growth. The Energy dissipation could be in the form of damage, heat dissipation and internal energy which should rise the temperature of the specimen as given by Equation 2.36. For a single fatigue cycle, this can be written as:

\[ \frac{dU}{dN} = \frac{dU_d}{dN} + \frac{dU_h}{dN} + \frac{dU_e}{dN} \]  

(3.1)

So the left side of the equation is the energy that is lost by the specimen and available to be used for the crack growth at an arbitrary cycle, while on the right side, the consumption of the energy by three different mechanisms at the same cycle is described. In other words, whatever amount of energy that is lost in a cycle and is available for crack growth, can be used by these three mechanisms.

Two cases are possible for the trend of \( \frac{dU}{dN} \) at an arbitrary cycle when the frequency of the fatigue test is increased. Referring back to Equation 3.1, one possibility will be that \( \frac{dU}{dN} \) on the left side of the Equation 3.2 stays constant while the proportion in which the lost energy is consumed by the three mechanisms on the right side of the equation changes.

\[ \frac{dU}{dN}(f \uparrow) = \frac{dU_d}{dN} \uparrow + \frac{dU_h}{dN} \uparrow + \frac{dU_e}{dN} \uparrow \]  

(3.2)

the other possibility is, both \( \frac{dU}{dN} \) increases and the proportion in which the lost energy is consumed by three mechanisms, changes as shown in Equation 3.3.

\[ \frac{dU}{dN}(f \uparrow) = \frac{dU_d}{dN} \uparrow \downarrow + \frac{dU_h}{dN} \downarrow + \frac{dU_e}{dN} \downarrow \]  

(3.3)
Looking closely at the different components of the strain energy dissipation, usually, the major fraction of the lost energy is consumed by the crack for creating new surface \((dU_p/dN)\). The contribution of the other two terms \((dU_f/dN)\) and \((dU_l/dN)\) are limited in steady fatigue crack growth and hence ignored. However, these two terms depend on the material and geometry of the fatigue specimen. For instance, the glass fibre reinforced composites undergo intrinsic heating and increase in temperature was observed when tested under fatigue \([44]\). The infra-red measurements on the crack tip throughout the fatigue test could help in explaining the heating characteristics of the specimen. For UD carbon-fibre composites, if heat is generated, it can be dissipated quickly as fibres conduct heat. However, glass fibres do not conduct heat and this would locally heat up the crack edge or lead to a gradual increase in the temperature of the specimen.

### 3.3. Material description

For the experiments, a composite laminate for the unidirectional DCB specimens was manufactured by stacking 32 plies of CFRP M30SC/DT120 prepreg. It is a high strength and modulus carbon fibre/toughened thermosetting epoxy prepreg which was supplied by Delta-Tech S.p.a Italy and its elastic properties are summarized in Table 3.1.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitudinal Elastic Modulus (E_{11}) [GPa]</td>
<td>155</td>
</tr>
<tr>
<td>Transverse Elastic Modulus (E_{22}) [GPa]</td>
<td>7.8</td>
</tr>
<tr>
<td>In plane Shear modulus (G_{12}) [GPa]</td>
<td>5.5</td>
</tr>
<tr>
<td>In plane Poison ratio (\gamma_{12}) [-]</td>
<td>0.27</td>
</tr>
</tbody>
</table>

Table 3.1: Elastic property of M30SC/DT120 prepreg

### 3.4. Design and manufacturing of DCB specimens

A number of fatigue experiments were planned in-order to study the effect of frequency on the damage propagation in CFRP. Based on the total number of experiments that was planned to be conducted, the total number of specimens were decided. ASTM standard D-5528 \([47]\) was used as reference to design the specimen and decide its dimensions. From the dimension and the quantity of specimens (see Figure 3.6), the width and the length of the laminate was decided.

Since the thickness of the laminate was 5 mm, 32 plies were stacked, one over the other, in 0° direction. To avoid voids and defects in the laminate, following procedure was followed.

#### 3.4.1. Procedure for manufacturing CFRP laminate

The layup was done on an Aluminium tool which can be directly placed inside the Autolave for curing.

1. The Aluminium plate was first cleaned with sandpaper, then the grease and other forms of dust was removed using Acetone. Office tape was stuck on the border edges of the plate and Marbocote 220 (release agent) was applied throughout the plate for three times with 15 minutes interval.

2. Half an hour after the last layer of Marbocote 220 was applied, the prepreg layers were then carefully laid one by one. A small Squeegee pad was used to push the trapped air while laying one ply over the other.

3. After laying the first 4 plies, debulking process was carried out to make sure that the air pockets trapped between the layers were removed. This process was repeated for every four plies until all the 32 plies were stacked.
3.4. Design and manufacturing of DCB specimens

For debulking, the prepreg was covered with peel ply, a breather cloth and reusable vacuum bag and the arrangement was sealed with tacky tape. Vacuum was applied for 15 minutes.

4. After 16 layers, a teflon tape of 55 mm width was placed on either sides, perpendicular to the fibre orientation of the prepreg, to act as a crack initiator for the experiments (see Figure 3.1).

![Figure 3.1: Teflon layer for crack initiation in the specimen.](image)

5. After stacking the 32 layers of prepreg, a caul plate with marbocote applied on its face was laid directly above the prepreg.

The dimensions of the caul plate was such that it was slightly smaller than the dimensions of the prepreg. Above the caul plate, a peel ply and 3 layers of N10 breather material were laid to handle 6 bar pressure inside the autoclave. The peel ply above the plate was intended to isolate the uncovered portion of the prepreg from the breather so that the resin does not stick directly on the breather material.

6. Finally, the whole arrangement was covered by a vacuum bag and cured in the autoclave (see Figure 3.2) with curing cycle as prescribed in the following section.

A schematic representation of top view and front view of layup arrangement that is described in the above procedure can be seen in Figure 3.3 and 3.4 respectively.

3.4.2. Curing cycle of laminate

The laminate was cured in the autoclave at 6 bar pressure and with a temperature cure cycle as shown in Figure 3.5.

1. Increase temperature at a rate of $2^\circ$ C/min from room temperature (RT) to $120^\circ$ (approx. 50 minutes)

2. Dwell at $120^\circ$ for 90 minutes.

3. Lower the temperature at rate of $-4^\circ$ C/min from $120^\circ$ to $60^\circ$ (15 minutes).

Full vacuum was applied for the complete cure cycle. After curing, the laminate was removed from autoclave and vacuum was applied until the laminate cooled down to room temperature.
3.4.3. Specimen arrangement in laminate
The laminate was manufactured for DCB specimens of two different dimensions.

- 150 mm × 25 mm × 5 mm specimens.
- 200 mm × 25 mm × 5 mm specimens.
3.4. Design and manufacturing of DCB specimens

Figure 3.5: The curing cycle of the laminate.

The longer specimens (200 mm) were thought to be used for carrying up to three fatigue tests while the shorter ones (150) were thought to be used for carrying out either one or two runs. This will be explained in detail in Section 3.5.

Figure 3.6: A schematic of laminate with dimensions of specimens.

An extra length of 30 mm at the four edges and a 5 mm spacing between each specimen was taken into account to avoid cutting errors. A schematic of the laminate accommodating the specimens to be cut can be seen in Figure 3.6. The specimens were cut in the Proth cutting machine using diamond coated cutting edge. A pair of Aluminium blocks of dimensions 25mm × 20mm × 6mm were bonded on crack opening width edges of specimens through adhesive bonding method for load introduction on the specimen from the fatigue machine.
3.4.4. Procedure for bonding Aluminium blocks

Following procedure was followed in order to achieve proper bonding of the Aluminium blocks on the specimens.

1. The DCB specimens and the Aluminium block faces which were to be bonded, were sandpapered to achieve a rough surface for enhancing the adhesive to adhere well to the faces being bonded.

2. The holes in the Aluminium blocks were blown with air to ensure that there is no dust particle. Later the specimens were cleaned with PSFR liquid and Aluminium blocks were cleaned with acetone to remove the impurities and dust.

3. To avoid adhesive from flowing inside the hole, bolts (M4 X 10mm) were dipped in Marbocote 220 and dried. After drying the bolts were inserted into the holes so that the adhesive doesn’t stick to the bolt tips and the hole is free from adhesive.

4. The 7236 B/A two-part structural adhesive was used for bonding the blocks to the specimen. The base part and accelerator part were mixed with the ratio of 100:27 as prescribed by the manufacturer and stirred well until uniform colour was reached throughout the paste. Glass beads (200-300 microns) were added (five percent by weight of the total weight of paste) to the paste and mixed again in order to achieve uniform thickness of adhesive at the bondline.

5. The Adhesive was then applied to the portion of the faces in the specimens where the Aluminium blocks had to be bonded and to the faces of the Aluminium blocks. Two Aluminium blocks were stuck to the specimen and pressure was applied through means of large binder clips. These clips were also able to hold the blocks intact with the specimens without any misalignment.

6. The bonded specimens were then cured for 12 hours at room temperature and post cure was done at 70°C for 4 hours in oven.

One of the two sides of the specimen was coated with type-writer correction fluid to enhance visibility of delamination front during the test. Additionally, a grid paper was pasted on the same side of the specimen to aid the measuring of the delamination length during data analysis.

3.5. Test matrix

The objective of the fatigue experiments was to capture the effect of test frequency at different crack lengths and stress ratios on the crack growth rate and strain energy loss. So the tests were classified into five test series as explained in the following subsections.

Each test was named based on the specimen number, frequency and the test series in which the test was performed. Figure 3.7 provides an example on the labeling of the tests.

![Figure 3.7: Example for the labelling of tests.](image-url)
3.5. Test matrix

3.5.1. A01 test series
Five tests were performed under A01 series at different frequencies as shown in table 3.2. The initial crack length of the test specimens before the start of the tests lies in the same range for all the five tests. The maximum displacement and stress ratio for this test series are 2 mm and 0.1 respectively.

<table>
<thead>
<tr>
<th>Test Name</th>
<th>Frequency (Hz)</th>
<th>Initial crack length (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>01DCB05FA01</td>
<td>05</td>
<td>48.54</td>
</tr>
<tr>
<td>06DCB10FA01</td>
<td>10</td>
<td>45.82</td>
</tr>
<tr>
<td>09DCB20FA01</td>
<td>20</td>
<td>46.01</td>
</tr>
<tr>
<td>16DCB30FA01</td>
<td>30</td>
<td>44.26</td>
</tr>
<tr>
<td>13DCB40FA01</td>
<td>40</td>
<td>46.90</td>
</tr>
</tbody>
</table>

Table 3.2: Test matrix for A01 series.

3.5.2. A02 test series
All the tests in A02 series were performed on the same specimens of A01 test series. After the crack growth got fully retarded during fatigue in A01 test series, a crack of 3-5 mm was grown quasi-statically and A02 tests were conducted. For example, fatigue tests 01DCB05FA01 was conducted until the crack growth almost retarded at 57 mm. From 57 mm to 63.5 mm, the crack was grown quasi-statically and once the crack front reached 63.5 mm, fatigue test 01DCB05FA02 was conducted on the same specimen. Thus the second run (A02 tests) on any specimen, has the same test frequency as first run (A01 series).

<table>
<thead>
<tr>
<th>Test Name</th>
<th>Frequency (Hz)</th>
<th>Initial crack length (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>01DCB05FA02</td>
<td>05</td>
<td>64.10</td>
</tr>
<tr>
<td>06DCB10FA02</td>
<td>10</td>
<td>64.22</td>
</tr>
<tr>
<td>16DCB30FA02</td>
<td>30</td>
<td>63.91</td>
</tr>
<tr>
<td>13DCB40FA02</td>
<td>40</td>
<td>67.78</td>
</tr>
</tbody>
</table>

Table 3.3: Test matrix for A02 series.

Due to camera problem, the 20 Hz test of A02 series got interrupted. So in A02 test series, only four tests are presented in Table 3.3. The maximum displacement and stress ratio for this test series are 4 mm and 0.1 respectively.

3.5.3. A03 test series
Only two tests were conducted in A03 series on the two long specimens (200mm × 25mm × 5mm) as a third fatigue test on each specimen. After the crack growth got fully retarded in A02 test series, a crack of 3-5 mm was grown quasi-statically and A03 tests were conducted as in Table 3.4. The maximum displacement and stress ratio for this test series are 7mm and 0.1 respectively.

It was thought that with tests conducted at different initial crack lengths, the state of the crack also changes. In other words, at different crack lengths, the amount of bridging fibres changes. For example, tests in A01 series will have the least fibre bridging effect at the crack fronts due to smaller crack length while for the case of A03 test series, the crack front will have fully bridged fibres as the crack length is in the range of 80-90 mm. So a comparison of tests at a certain frequency between A01, A02 and A03 series will provide explanation to the effect of bridging fibres on dU/dN.
3. Methodology

### 3.5.4. B01 test series

In B01 test series, five tests were performed at five different frequencies as shown in table 3.5. The maximum displacement and stress ratio for this test series are 2 mm and 0.5 respectively.

<table>
<thead>
<tr>
<th>Test Name</th>
<th>Frequency</th>
<th>Initial crack length</th>
</tr>
</thead>
<tbody>
<tr>
<td>05DCB05FB01</td>
<td>05</td>
<td>43.63</td>
</tr>
<tr>
<td>17DCB10FB01</td>
<td>10</td>
<td>44.66</td>
</tr>
<tr>
<td>12DCB20FB01</td>
<td>20</td>
<td>43.06</td>
</tr>
<tr>
<td>11DCB30FB01</td>
<td>30</td>
<td>51.99</td>
</tr>
<tr>
<td>14DCB40FB01</td>
<td>40</td>
<td>45.51</td>
</tr>
</tbody>
</table>

Table 3.5: Test matrix for B01 series.

The only difference between B01 and A01 test series is the minimum displacement of the fatigue cycle. So it was thought that a comparison between A01 and B01 tests at corresponding frequencies show the effect of stress ratio if any. For example, 13DCB40FA01 and 14DCB40FB01 tests have same test frequency, maximum displacement and similar initial crack lengths. So a comparison on the energy and crack growth rate between these two cycles can explain on the stress ratio effect which is the only independent variable between the two tests.

### 3.5.5. B02 test series

All the tests on B02 test series were performed on the same specimens of B01 test series (see Table 3.6). The maximum displacement and stress ratio for this test series are 4 mm and 0.5 respectively. Similar to A01 and B01 test series, only the minimum displacement of the fatigue cycle changes between the B02 and A02 test series.

<table>
<thead>
<tr>
<th>Test Name</th>
<th>Frequency</th>
<th>Initial crack length</th>
</tr>
</thead>
<tbody>
<tr>
<td>05DCB05FB02</td>
<td>05</td>
<td>62.63</td>
</tr>
<tr>
<td>17DCB10FB02</td>
<td>10</td>
<td>59.32</td>
</tr>
<tr>
<td>12DCB20FB02</td>
<td>20</td>
<td>62.13</td>
</tr>
<tr>
<td>11DCB30FB02</td>
<td>30</td>
<td>64.53</td>
</tr>
<tr>
<td>14DCB40FB02</td>
<td>40</td>
<td>66.13</td>
</tr>
</tbody>
</table>

Table 3.6: Test matrix for B02 series.
3.6. General information on the fatigue experiments

All the fatigue tests were performed on a MTS 10 kN hydraulic fatigue testing machine under displacement control mode (see Figure 3.8). In general, force and displacement control modes are possible for testing specimens under fatigue. Under displacement controlled fatigue testing, the compliance of the specimen will increase as the crack grows and the applied load reduces as a result of which, the crack growth rate also tends to reduce with the increase in the number of cycles. On contrary, in the force controlled fatigue tests, with the increase in specimen compliance, applied displacements increase resulting in acceleration of the crack growth rate which can sometimes lead to catastrophic failure when the maximum applied load is greater than the critical load that the specimen can carry for a given crack length.

3.6.1. Data storage

Prior to each fatigue test, the specimen was loaded quasi-statically to generate a natural crack tip with visual crack growth onset. The crack length was measured by means of a digital camera which was connected to the computer system and it aimed at the side of the specimen which was coated with the correction fluid. Photographs were taken by holding the specimen at maximum displacement for once in every 100 cycles for the first $10^3$ cycles and for once in every 1000 cycles afterwards. This was followed due to the fact that the crack growth rate is high at the start of the test and it keeps decreasing with the increase in the number of cycles. Each test was stopped when the crack growth almost retarded and no further growth was observed for several thousand cycles.

The crack length was measured from the distance between the load line and the crack tip. The crack opening displacement is the grip-to-grip displacement measured by the testing machine. This is assumed to be equivalent to the distance used in ASTM D5528 [3], [35]. The peak and valley values of force, displacement and the temperature at the top portion of the specimen, which was measured by means of a thermocouple, were recorded at the same cycle intervals at which the photographs were taken with the use of MTS controller software.

For 5 Hz, 30 Hz and 40 Hz tests in A01 test series (see Table 3.2), a FLIR a35 infra red camera was used to measure the temperature at the crack tip. Videos were taken at certain number of cycles, throughout the tests, so as to see if the temperature at the crack tip stays constant or changes during a fatigue cycle.

3.6.2. Crack growth data

The measured crack length data was matched with the corresponding fatigue cycle number at which the crack was photographed for all the fatigue tests. A three parameter power-law fit was used for all the crack growth data

$$a = aN^β + γ$$

(3.4)
where $\alpha$, $\beta$, and $\gamma$ are curve fit parameters (See Table A.1 for the fit coefficients for all the experiments). Figure 3.9 shows an example of the crack length data fitted against the number of fatigue cycles for a test that was run at 20 Hz frequency. The crack growth rate ($da/dN$) was then determined by taking the derivative of the power law fitting function.

### 3.6.3. Strain energy release rate

The SERR can be calculated using three data reduction techniques recommended by the ASTM standard [47], namely, the Modified Beam Theory (MBT) method, Compliance Calibration (CC) method, and Modified Compliance Calibration method. The SERR values determined by the different methods do not differ by more than 3.1% and none of the three were found to be superior than the others [47]. For the present work, the MCC method is used to determine SERR. The SERR according to MCC method is given by,

$$G_t = \frac{3P^2C^{2/3}}{2A_1bh}$$  \hspace{1cm} (3.5)

Where $C$ is the compliance of the DCB specimen, $b$ is the width of the specimen, $h$ is the thickness of the specimen and $A_1$ is the slope of the curve generated by the least square plot of delamination/crack length normalized by the specimen thickness ($a/h$) as a function of cube root of compliance $C^{2/3}$.

The SERR calculations were used only for the initial phase of the thesis in deciding the maximum displacement and corresponding initial crack length for each fatigue test.

The compliance was calculated by assuming a linear relationship between the two points in the P-d curve.

$$C = \frac{d_{max} - d_{min}}{P_{max} - P_{min}}$$  \hspace{1cm} (3.6)

### 3.7. Calculation of strain energy dissipation

The calculation of strain energy dissipation during fatigue loading was determined by different approaches by different authors. Pascoe (2015) [3] assumed a linear relationship between the maximum and minimum load in force-displacement plot. The strain energy calculation proposed by Pascoe [3] is discussed in Section 3.7.1. However, during the experiments, it was found that for higher frequencies (say $f > 5$ Hz), the fatigue loading of DCB specimens produced a hysteresis loop whose shape was similar to that of an ellipse. To account for the hysteresis effect on P-d relationship, the methodology proposed by Pascoe (2015) [3] is modified and explained in Section 3.7.2. Apart from the above two methodologies, Lauherta (2016) [7] proposed the use of “energy loss factor” which characterizes the ratio of loss energy and potential work with crack growth rate and this method is briefly discussed in Section 3.7.3. Literature was also found where the size of the hysteresis loop alone was calculated and plotted against the number of cycles [48]. The energy calculations proposed by Frunza and Diaconescu (2006) [48] is discussed in Section 3.7.4.
3.7. Calculation of strain energy dissipation

3.7.1. Strain energy calculation for linear P-d relationship

Pascoe (2015) [3] assumed that the relationship between the force and displacement in a fatigue cycle is linear. The strain energy is the area under the P-d curve and for a linear P-d relationship, the curve is a line (see Figure 3.10). The line between the the two points \((d_{\text{max}}, P_{\text{max}})\) and \((d_{\text{min}}, P_{\text{min}})\) was extrapolated and it was found that the line passed through the force axis (i.e. at \(d = 0\)) at a non zero-force \(P_0\). This was taken into account for calculating the energy values. Thus, the following equations were used to determine the energy dissipation values for all the fatigue tests.

\[
P_0 = P_{\text{min}} - C d_{\text{min}}
\]

\[
U_{\text{mono}} = \frac{1}{2} (P_{\text{min}} - P_0) d_{\text{min}} + P_0 d_{\text{min}}
\]

\[
U_{\text{cyc}} = \frac{1}{2} (P_{\text{max}} - P_{\text{min}})(d_{\text{max}} - d_{\text{min}}) + P_{\text{min}} (d_{\text{max}} - d_{\text{min}})
\]

\[
U_{\text{tot}} = U_{\text{mono}} + U_{\text{cyc}}
\]

These equations can be understood from Figure 3.10 which illustrates the schematic of a linear P-d curve for the fatigue tests. With the above set of equations, the strain energies \((U_{\text{cyc}}\) and \(U_{\text{tot}})\) can be calculated at the arbitrary cycles where the crack growth data and force-displacement data were measured. Thus the calculated strain energy corresponds to the strain energy that was applied to the specimen at the maximum displacement position for a fatigue cycle, say \(N\). The strain energy loss per cycle is then given by \(dU/dN\) (applies for both \(U_{\text{cyc}}\) and \(U_{\text{tot}}\)). To determine the energy dissipation, a three parameter power law fit was used for \(U_{\text{cyc}}\) and \(U_{\text{tot}}\) against \(N\)

\[
U = \alpha N^\beta + \gamma
\]

Figure 3.11 shows an example of the cyclic energy data fitted against the number of fatigue cycles for a test that was run at 20 Hz frequency. The strain energy per cycle \(dU/dN\) was then determined by taking the derivative of the fitted curve.

The above formulation on the calculation of applied strain energy assumes that the load-displacement behaviour is linear in a single fatigue cycle. Further, it is also assumed that damage growth within a single fatigue cycle is fairly limited and its effect on the load-displacement response can be considered
Methodology

Figure 3.11: Cyclic energy data and power fit curve against the number of cycles for 20 Hz frequency test.

to be negligible [37]. However, assuming linear relationship between force and displacement introduces error in the calculation of strain energy [49]. During a fatigue cycle, when the load is applied from minimum to maximum displacement, crack growth occurs. This causes loss in stiffness of the specimen and hence, the load drops resulting in a curved path rather than a straight line. Thus assuming a linear path introduces error in the calculation of strain energy. But as a rough approximation, this method should hold good in correlating the energy dissipation with crack growth rate. Previous studies [37], [46] on mode I delamination in CFRP and adhesive bonds were characterized using strain energy calculation proposed in [3].

3.7.2. Strain energy calculation for non-linear P-d relationship

Only for the fatigue tests conducted at 5 Hz frequency, the P-d curve was observed to be similar to that of a straight line. In contrast, with increasing frequencies, the P-d curve for a single fatigue cycle was found to be a closed loop due to hysteresis. The size of the loop was observed to decrease with the progression of fatigue cycles. In other words, the size of the loop decreased with the increase in the number of fatigue cycles. To account for the effect of hysteresis on P-d relationship, the methodology proposed by Pascoe in [3] was modified during data analysis, after conducting all the experiments.

The shape of the hysteresis loop was observed to be similar to that of an ellipse and area of the loop was dependent on the frequency, higher the frequency, larger the area of the loop. Thus, the area was the largest, for the fatigue tests carried out at 40 Hz and decreased with lower frequencies such that, a very thin loop, close to a line, was observed in 5 Hz fatigue tests. One possible reason that could explain the hysteresis loop for higher frequencies is force-displacement response of the material. For a displacement controlled test, the maximum and minimum displacements are given as inputs and the force at maximum and minimum displacements depend on the material stiffness. Now two things have to be considered here, first, the force response to the material is non-linear. This means, the force does not vary in a linear fashion with respect to the displacement. Secondly, the force and displacement are not inline with each other in the time plot and there is a phase lag between the force and displacement responses of material.

Frunza and Diaconescu (2006) [48] investigated the hysteresis and related energy dissipation in metals. It was proposed that inelasticity is always present during load-displacement process and as a result, phase shift occurs between strain and stress in the material. It was further explained that, in stress-strain co-ordinates, the geometrical locus of the operational point becomes a closed loop called hysteresis loop. Until stabilized, the hysteresis area may change and the loop would shift, such that the stress co-ordinates may either increase (drifting hysteresis) due to cyclic strain softening, or decrease (fading hysteresis) resulting in cyclic strain hardening [50].

To provide a possible explanation on the fatigue hysteresis, two loading cases with same maximum and minimum displacements but different frequencies are considered: a fatigue test with 5 Hz frequency and a test with 40 Hz frequency. Further, the schematic of the observations on P-d curve is discussed.
Case of 5 Hz frequency fatigue test

Figure 3.12 shows the schematic for force and displacement time plots of the 5Hz test for a fatigue cycle with \(d_{\text{min}} = 0.4\) mm and \(d_{\text{max}} = 4\) mm. It can be seen that the force and displacement sine-waves are in-phase with each other.

Figure 3.12: A schematic of force and displacement time plots for the 5 Hz frequency with \(d_{\text{min}} = 0.4\) mm and \(d_{\text{max}} = 4\) mm.

Figure 3.13 shows the schematic of the P-d curve of a single fatigue cycle for the same test. As can be seen, the area of the hysteresis loop is very small for 5 Hz frequency test and the difference in area between the upper curve of the ellipse A-C-B and the line A-B is very small that the error in calculation of \(dU/dN\) should also be small and can be ignored.

Figure 3.13: A schematic of the P-d hysteresis loop for the 5 Hz frequency test with \(d_{\text{min}} = 0.4\) mm and \(d_{\text{max}} = 4\) mm.
3. Methodology

Case of 40 Hz frequency fatigue test
Now consider the force and displacement time plots of 40 Hz test with $d_{\text{min}} = 0.4$ mm and $d_{\text{max}} = 4$ mm (see Figure 3.14). There is a phase shift between the force and displacement waves which means that, the maximum force is not in phase with maximum displacement.

![Force and displacement time plots](image1)

Figure 3.14: A schematic of force and displacement time plots for the 40 Hz frequency test with $d_{\text{min}} = 0.4$ mm and $d_{\text{max}} = 4$ mm.

A schematic of the P-d curve that was observed for the 40 Hz fatigue test with $d_{\text{min}} = 0.4$ mm and $d_{\text{max}} = 4$ mm is represented in Figure 3.15. From the figure it can be observed that at maximum displacement (4mm), the force is not maximum which is due to the phase shift that was observed in 3.15. Figure 3.15 is obtained by fitting an ellipse using MATALAB from five known co-ordinates of the hysteresis loop that was obtained from the photograph of the P-d curve from the experiment (See Figure 3.16).

![P-d hysteresis loop](image2)

Figure 3.15: A schematic of the P-d hysteresis loop for the 40 Hz frequency test for a single cycle with $d_{\text{min}} = 0.4$ mm and $d_{\text{max}} = 4$ mm.
3.7. Calculation of strain energy dissipation

For the 40 Hz test, with the assumption that P-d response is linear, the \( \frac{dU}{dN} \) is calculated for the area under the line A-B. The area between the curve A-C-B and the line A-B is missed and hence not considered. This can be interpreted as the energy that was supplied to the specimen during the test but was not taken into account for the calculation of the energy loss. The hysteresis loop in figure 3.15 was found to shrink and the ellipse tilted more towards horizontal axis as the test progressed (see Appendix B).

Correction for total energy loss per cycle \( \frac{dU}{dN} \)

Consider the fatigue test for 40 Hz frequency whose hysteresis loop at the start of the test looked like curve A-C-B-A in the left panel of Figure 3.17 and shrunk to the loop in the right panel of the same figure. Previously, with the assumption of linear relationship between load and displacement, for the calculation of total strain energy, the area under the line O-A-B was computed. As the test progresses the slope of the curve O-A-B shifts towards the horizontal axis and the area under the curve also gets reduced.

![Figure 3.17: A schematic representation of P-d curve for 40 Hz during the start of the test (left panel) and at the end of test (right panel).](image)

It was thought that the area that was missed by assuming the linear behaviour can be corrected by approximating the size of the hysteresis loop. This correction is possible by calculating the area between the curve A-C-B and line A-B and adding it back to the area determined with the linear case added back to the energy loss to check if the energy loss per cycle and crack growth rate provided any meaningful results. For the calculation of the area of the upper portion of the ellipse, several assumptions were made as obtaining the real values for co-ordinates of ellipse for each test was not possible.
1. Firstly, the hysteresis loop is assumed to be an ellipse. This is not the exact case as it was observed that the hysteresis loop had sharp edges at maximum and minimum displacements while the ellipse is smooth curve. So, the result may be an overestimation of the actual case.

2. Minor axis was calculated based on the eccentricity values and the eccentricity values for the five different frequency tests was assumed such that the 40 Hz test had an ellipse similar to that in Figure 3.15 and the hysteresis loop for 5 Hz frequency test almost converged to a straight line. The assumed eccentricity values can be found in Table 3.7.

3. For all the test series, it was assumed that the eccentricity would change only with respect to frequency and not with stress ratio and maximum displacement of the fatigue cycle. This means, it is assumed that the fatigue cycles with \( d_{max} = 4\text{mm} \) and \( R = 0.1 \) or 0.5 have the same eccentricity.

4. Once the missed out energy i.e., the area between the curve A-C-B and the line A-B was estimated, this area was assumed to decrease linearly with the number of cycles. One may argue that the area may decrease in any trend and not necessarily be linear with the number of cycles. But as a first approximation, it was thought that linear interpolation on the decrease of the missed out energy will be easier to calculate and to visualize the effect of frequency on the relationship between \( da/dn \) and \( dU/dn \).

5. As the slope of line A-B decreases towards the horizontal axis, the major axis of the ellipse would also rotate a little on the clockwise direction resulting in decrease of major axis length. This is because, with loss of stiffness in the specimen, only the applied load \( (P_{min} \) and \( P_{max} \) decreases while the displacements \( (d_{min} \) and \( d_{max} \) \) remain constant. Reduction in length of major axis results in decrease of area. But this reduction was not taken into account for energy calculation and major axis was calculated only for one time and energy was computed, this energy was assumed to decrease as if the area shrank linearly with constant major axis (See Appendix B).

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>Eccentricity [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>05</td>
<td>0.99999</td>
</tr>
<tr>
<td>10</td>
<td>0.99990</td>
</tr>
<tr>
<td>20</td>
<td>0.99900</td>
</tr>
<tr>
<td>30</td>
<td>0.99000</td>
</tr>
<tr>
<td>40</td>
<td>0.97000</td>
</tr>
</tbody>
</table>

Table 3.7: Assumed eccentricity values of hysteresis loop ellipse for different frequencies.

The length of semi-major axis \( X_{major} \) was calculated from the co-ordinates of minimum and maximum displacements of fatigue cycle is given by,

\[
X_{major} = \frac{1}{2} \sqrt{(d_{max} - d_{min})^2 + (P_{max} - P_{min})^2}
\] (3.12)

where \( d_{max} \) and \( d_{min} \) are the maximum and minimum displacement of the fatigue cycle, \( P_{max} \) and \( P_{min} \) are the maximum and minimum loads corresponding to the maximum and minimum displacements respectively. The semi-minor axis was then calculated based on the assumed eccentricity for each of the five different frequencies at which the tests were carried out. The semi-minor axis \( X_{minor} \) is given by

\[
X_{minor} = X_{major} \sqrt{1 - e^2}
\] (3.13)

Once the semi-major and semi-minor axes were calculated, the area of half ellipse was calculated as

\[
\text{Area}_{\text{HalfEllipse}} = \frac{1}{2} \pi (X_{major} X_{minor})
\] (3.14)

The calculated area gives the energy that needs to be corrected for calculation of total energy. So the
loss of this energy, call it hysteresis energy, was linearly interpolated such that it is zero at the end of the test because, it was observed that the hysteresis shrunk and converged towards a straight line at the end of the test. The interpolated hysteresis energy was then added to cyclic energy calculation in Equation 3.9. So the new cyclic energy is the summation of cyclic energy that considers the linear relationship between the maximum & minimum displacements of the fatigue cycle and the hysteresis energy. This is given by

\[ U'_{\text{cyc}} = \frac{1}{2} (P_{\text{max}} - P_{\text{min}}) (d_{\text{max}} - d_{\text{min}}) + P_{\text{min}} (d_{\text{max}} - d_{\text{min}}) + \frac{1}{2} \pi (X_{\text{major}} X_{\text{minor}}) \]  

(3.15)

where \( U'_{\text{cyc}} \) denotes the corrected fatigue-cyclic energy that takes into account of the hysteresis formed at high frequencies. The total strain energy \( U'_{\text{tot}} \) was then re-calculated with the corrected cyclic energy (see Equation 3.16) and power law fit was used for fitting the total energy data with the number of cycles. For the fitted parameters of all the experiments, see Appendix A. From the fitted curve, \( dU/dN \) was determined by differentiating the fitted curve with respect to the number of cycles.

\[ U'_{\text{tot}} = U_{\text{mono}} + U'_{\text{cyc}} \]  

(3.16)

Since the energy due to hysteresis was assumed to linearly decrease with the number of cycles, the fit of cyclic and total energies before correction (\( U_{\text{cyc}} \) and \( U_{\text{tot}} \)) and after correction (\( U'_{\text{cyc}} \) and \( U'_{\text{tot}} \)) against \( N \) varies only with the \( \gamma \) in Equation 3.11 (see Appendix A). Thus adding the energy due to hysteresis, only shifts the complete curve in Figure 3.11 upwards.

### 3.7.3. Energy loss factor

Lahuerta and Nijssen (2015) \[7\] proposed that the loss in hysteresis fatigue cycle is the area between the loading and unloading path. Further, the cyclic energy loss factor is the ratio between energy losses and potential work done from minimum displacement position to maximum displacement position given by Equation 3.17.

\[ \Phi_{\text{cyclic}}(N) = \frac{E_{\text{loss}}}{\int \Pi \, d\Pi} \]  

(3.17)

where \( E_{\text{loss}} \) is the work applied due to non-reversible process and \( \Pi \) is the potential work applied to the specimen. The potential work is defined here as the shortest distance between the minimum and maximum displacement position. It is also defined as the reversible path whose circular integral is zero.

\[ \oint d\Pi \frac{\Delta \Pi_{21}}{\Delta t} - \frac{\Delta \Pi_{12}}{\Delta t} = 0 \]  

(3.18)

This can be seen from Figure 3.18 where minimum and maximum displacement positions can be considered as state 1 and 2 respectively. Thus, if the total work done from point 1 to 2 given by \( W_{21} \)

![Figure 3.18: A schematic of the work done on the specimen between two points \[7\].](image)

and the reversible work is the shortest path between 1 to 2, then the energy loss factor is given by

\[ \Phi_{21} = \frac{E_{21}}{\Pi_{21}} = \frac{W_{21} - \Pi_{21}}{\Pi_{21}} \]  

(3.19)
Lahuerta and Nijssen (2015) proposed that the change in hysteresis loss factor per cycle \((\mathrm{d}\Phi/\mathrm{d}N)\) can be compared with \(\mathrm{da}/\mathrm{d}N\) for delamination characterization. Closely looking at the methodology proposed by Lahuerta and Nijssen (2015) and the previously discussed methodology on the calculation of total strain energy with hysteresis loop, one can say that both the methodologies are similar to each other. While one deals with total energy where the area under the line A-B and the area between the upper arc A-C-B and the line A-B are summed, the other deals with the ratio of the two areas.

### 3.7.4. Hysteresis loop energy

Frunza and Diaconescu (2006) proposed that the area enclosed by the loading and unloading curves in a load-displacement diagram is the hysteresis area, which equals to the amount of energy dissipated by the material upon one loading and unloading cycle. Doing so, the loaded and the unloaded energy are compared, the energy that was not returned during unloading is considered to be the hysteresis energy. The shape enclosed by the loading and unloading curves can be thought as an ellipse. Considering an ellipse as a general conic in a P-d plot, the equation of the second degree is given by [51] in \(x\) and \(y\) where \(x\) corresponds to displacement (d), \(y\) corresponds to load (P)

\[
F(x, y) = ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0
\]  
\((3.20)\)

Thus five co-ordinate points are needed to define a unique ellipse with five coefficients \((a, h, b, g, \text{ and } f)\). So, if five points are known (five measurements within a single fatigue cycle), an ellipse can be fit using direct least squares method [52]. To find the area of the ellipse, the conic equation has to be transformed to quadratic form. The \(x\) and \(y\) terms in equation 3.20 describe that the center of the ellipse is not at the origin, further the \(xy\) term describes that the ellipse is rotated and the major axis is at a certain angle with the \(x\) axis. The center of the ellipse has to be translated to the origin to obtain the quadratic form. Coordinates of the center can be obtained by differentiating the Equation 3.20 with respect to \(x\) and \(y\) and equating to zero [51]

\[
\frac{dF}{dx} = 0 \quad \frac{dF}{dy} = 0
\]  
\((3.21)\)

The center of ellipse \((x_1, y_1)\) can be substituted in Equation 3.20: \(x + x_1\) in place of \(x\) and \(y + y_1\) in place of \(y\). Doing so will reduce the equation to the form

\[
ax^2 + 2hxy + by^2 + c_1
\]  
\((3.22)\)

where \(c_1\) is given by

\[
c_1 = gx_1 + fy_1 + c
\]  
\((3.23)\)

The quadratic form of ellipse can be obtained by dividing equation 3.22 with \(-c_1\).

\[
\frac{a}{c_1}x^2 - \frac{2h}{c_1}xy - \frac{b}{c_1}y^2 = 1
\]  
\((3.24)\)

which is of the form

\[
Ax^2 + Hxy + By^2 = 1
\]  
\((3.25)\)

The area of the ellipse (or hysteresis area) is then given by

\[
A_{\text{hys}} = \frac{2\pi}{\sqrt{4AB - H^2}}
\]  
\((3.26)\)

This way the area and thus the hysteresis energy can be calculated at different fatigue cycles from which the loss of energy per cycle can be computed. Fatigue delamination can be then studied on the correlation between \(\mathrm{da}/\mathrm{d}N\) and \(\mathrm{dA}_{\text{hys}}/\mathrm{d}N\).

On comparing the four methodologies, it can be concluded that strain energy loss calculations based on linear P-d response can be used for fatigue characterization on the tests carried up to a frequency of 5Hz (as hysteresis was observed only for \(f > 5\) Hz). When working with frequencies greater than 5 Hz, the methodology outlined in 3.7.4 can be used, as hysteresis behaviour gets properly captured. For the calculation of hysteresis area, at least five co-ordinate points in the P-d plot for each fatigue
cycle are required. From the current experiments as only two points were stored, the methodology discussed in Section 3.7.2 was used. When comparing the calculations discussed in Section 3.7.2 and Section 3.7.3, it can be understood that while one method characterizes fatigue with the strain energy which is a summation of linear energy with additional energy due to hysteresis (see Equation 3.15), the other method uses the ratio of the additional energy to the linear energy (see Equation 3.17). It is thought that energy as physical quantity can provide more explanation than a parameter which is a ratio between two energies. Thus methodology discussed in Section 3.7.3 is not preferred.

### 3.8. Calculation of crack growth resistance

Pascoe (2015) [3] argued that the crack growth rate is a parameter that is a result of the interaction between the energy that was available for the crack growth (from the strain energy that was lost) and the resistance of the crack to grow. For a unit width, this equation is given by [3],

\[
\frac{da}{dN} = \frac{da}{dU} \frac{dU}{dN}
\]

(3.27)

where \(a\) is the crack length, \(N\) is the number of cycles and \(U\) is the total strain energy applied to the specimen. The reciprocal of the slope \(dU/da\) can be interpreted as the energy required per unit of crack growth or crack growth resistance \(G^*\) because it can be physically understood as the energy that needs to be dissipated to create a unit crack growth. Then, for a finite width \(w\), Equation 3.27 can be re-written as

\[
G^* = \frac{1}{w} \frac{dU}{da/dN} = \frac{dU}{dA}
\]

(3.28)

where \(A\) is the crack surface area. The crack growth resistance \(G^*\) has a time resolution of one cycle because it is determined by division of \(dU/dN\) and \(da/dN\) which are averaged for one cycle. Thus the amount of energy dissipated per unit of crack growth, can be calculated from the experimental data using Equation 3.28.

### 3.9. Conclusion

This chapter provides a description of the conceptual model, specimen manufacturing, experimental test setups, data storage techniques and fatigue tests conducted during this thesis research. Several series of fatigue tests were conducted to investigate the effect of frequency on energy loss and fatigue crack growth. Fatigue tests with different stress ratios and crack lengths were also conducted in order to identify the effect of fibre bridging and stress ratio on fatigue delamination growth in CFRP. Approaches for the calculation of strain energy were discussed and formulation of crack growth resistance was briefly described.
4

Results and discussion

4.1. Introduction
In this chapter, the results from the fatigue experiments described in Section 3.5 are discussed. In Section 4.3, the correlation between energy dissipation and delamination is compared for different test series with the approach outlined in Section 3.7.1. In Section 4.4, the results for the correlations between energy dissipation and delamination with non linear load-displacement response are discussed. Further, the results are then compared with the proposed conceptual model in Section 4.5. In Section 4.6 and Section 4.7, the effect of fibre bridging and stress ratio are briefly discussed. Results and observations from fracture surface of the specimens are presented in Section 4.8.

4.2. Strain energy dissipation per cycle and fatigue crack growth rate
During the tests, fatigue cycle was applied between a certain minimum and maximum displacement. In terms of energy, monotonous energy is applied to attain the minimum displacement of the fatigue cycle while the cyclic energy is applied between minimum and maximum displacement.

Both the monotonous and the cyclic energy were found to decrease with the increasing number of cycles. This is because, the fatigue tests are displacement controlled. With the loss of stiffness, the load applied at the maximum and minimum displacement also decreases. Delamination or crack growth can be attributed to the stiffness loss and decrease in load for a constant displacement can be interpreted as the loss in strain energy (because the area under P-d curve decreases). From thermodynamics point of view, crack extension implies loss of energy or in other words, energy is consumed by several mechanisms that contribute to the crack growth. So, dU/dN can be interpreted in two ways [3].

1. Energy that was available for the crack growth in that cycle.
2. Required amount of energy for the crack growth that took place in that cycle.

Either ways, it is clear that the energy loss per cycle and crack growth per cycle are strongly related to each other. In the previous studies [3], [37], da/dN was plotted as a function of dU_{tot}/dN for several fatigue experiments conducted at 5 Hz frequency, but with different loading conditions (stress ratios and maximum displacements). Results showed that, effect of stress ratio vanished and the trend between da/dN and dU/dN followed a power law relationship. Further, the correlation between the two variables (da/dN and dU/dN) was such that all the experimental data cramped down to a very narrow region (See the right panel in Figure 2.8) in the plot.

The collapse of the curves of different stress ratios to a single line in Figure 2.8 means that, irrespective of the stress ratios, all the curves have the same crack growth per cycle for a certain arbitrary energy loss per cycle. Similarly, for the present case, fatigue curves (or data points) for different frequencies in the da/dN Vs dU/dN plot were expected to yield meaningful correlations between crack growth and energy dissipation.
Initially, data analysis was performed for all the experiments, assuming that the relationship between load and displacement between minimum and maximum displacement positions to be linear. So the methodology outlined in Section 3.7.1 was followed and the results are discussed in Section 4.3. The effect of frequency on the correlation between crack growth rate and the strain energy dissipation did not follow any clear trend. Later, the observed hysteresis trend in the P-d plots was implemented with modification in the strain energy calculations which is discussed in Section 3.7.2 and the obtained results are discussed in Section 4.4.

### 4.3. Energy dissipation and delamination under linear P-d response

In section 4.3.1 the trend between the cyclic energy loss per cycle and the crack growth rate for the five different test series presented in Section 3.5 is discussed. In section 4.3.2, the trend between the total strain energy loss per cycle and crack growth rate is discussed. Later, in Section 4.3.3 the effect of frequency on the crack growth resistance (or the energy required for a unit crack growth) as a function of crack growth is discussed.

#### 4.3.1. Correlation between cyclic strain energy dissipation and fatigue crack growth rate

Figure 4.1 shows the crack growth rate, da/dN, plotted against the cyclic energy dissipated per cycle, \(dU_{\text{cyc}}/dN\), for A01 and A02 series. The \(dU/dN\) term is obtained by calculating the strain energy at certain number of cycles and denotes the change in energy per cycle. The negative sign in \(-dU/dN\) term represents the loss of energy per cycle.

For A01 test series, the data points (or curves) for different frequencies of tests seem to be spread out than that of the A02 test series. From the top portion of the plot in left panel of Figure 4.1, it can be seen that for a certain \(-dU_{\text{cyc}}/dN\) value, the da/dN of 5 Hz test is the lowest while that of 20 Hz test is the highest. However at the end of the test (bottom portion of the plot), da/dN for 5 Hz test is higher than da/dN of other frequency tests. Clearly no trend could be observed on the effect of frequency from the A01 test series.

In the A02 test series, the data points fall under a narrower region (compared with A01 series). On the bottom portion of the plot, it can be seen that 40 Hz test has higher da/dN than the other three tests which have similar crack growth rates.

Figure 4.2 shows the crack growth rate, da/dN, plotted against the cyclic energy dissipated per cycle, \(dU_{\text{cyc}}/dN\), for B01 and B02 series. In the B01 test series, on the bottom portion of the plot, a clear trend can be observed. For a certain value of \(-dU_{\text{cyc}}/dN\), the 5Hz test has the lowest da/dN and for the tests with higher frequencies, the da/dN is higher. This means, for a certain cyclic energy dissipation
per cycle, crack growth rate was higher at higher frequencies. In the B02 test series, no clear trend on the effect of frequency could be observed. However, like the A02 test series, the region of the data points can be found to be in a narrower region. For the A03 test series, interestingly, the data points for the 5 Hz and 20 Hz frequency tests overlap with each other (See Figure 4.3) throughout the tests, meaning that the crack growth rate for both the frequency tests were similar to each other for any value of \(-dU_{cyc}/dN\).

Figure 4.2: Crack growth rate as a function of cyclic strain energy dissipation per cycle for B01 series (left panel) and B02 series (Right panel).

Figure 4.3: Crack growth rate as a function of cyclic strain energy dissipation for A03 series.

The effect of frequency on the relation between cyclic energy loss per cycle and crack growth per cycle is less significant for all the test series except B01 series. It was also observed that the data points collapsed to a narrow band with the increase in initial crack lengths as the A01 series and B01 series (initial crack length \(a_0\) - 45mm) are observed to have a broader band than A02 and B02 series (\(a_0\) - 60mm) and A03 (\(a_0\) - 85mm) series of tests have overlapping data points.

The above plots only account for the change in cyclic energy loss per cycle with the crack growth rate. However since the monotonous energy also gets reduced as the crack progresses, the total strain energy loss per cycle is compared with the crack growth rate in Section 4.3.2.

4.3.2. Correlation between total strain energy dissipation and fatigue crack growth rate

For the total strain energy calculation, both the cyclic and monotonous energy losses were summed up and a relationship between the total strain energy loss per cycle and crack growth per cycle for different series was attempted. In all the 5 series of tests (Figure 4.4 to Figure 4.6), it was observed
that since there was only a small change in the magnitude of the monotonous energy, averaging the total energy per cycle did not make a huge difference on the trend of the curves when compared with the cyclic energy plots in Section 4.3.1. However, the $U_{\text{tot}}$ parameter will be more appropriate to be used for correlation with crack growth than $U_{\text{cycle}}$ as the former considers the net energy loss while the losses in $U_{\text{mono}}$ is not accounted in $U_{\text{cycle}}$. Hereafter for simplicity, $dU/dN$ represented in place of $dU_{\text{tot}}/dN$.

Figure 4.4: Crack growth rate as a function of total strain energy dissipation per cycle for A01 series (left panel) and A02 series (Right panel).

Figure 4.5: Crack growth rate as a function of total strain energy dissipation per cycle for B01 series (left panel) and B02 series (Right panel).

From Figure 4.4 to Figure 4.6, it can concluded that the $da/dN$ in all the tests decrease with the decrease in $dU/dN$ in a similar fashion. However, when having a closer look, the slopes of curves change with different frequencies and there is no clear trend on what happens if the frequency of the test is increased or decreased. For A01 test series, the curves for 5 Hz and 20 Hz are slightly rotated from the other curves. In case of A02 test series, only the fatigue curve of 40 Hz test seems to be rotated and the other curves overlap with each other. In B01 test series, fatigue curve of the 5 Hz test series is observed to be rotated. For B02 and A03 test series, all the fatigue curves seem to be oriented to the same direction. The rotation of curves from the rest of the data means that the trend of the corresponding test changes with the progression of test. For example, the 5 Hz test in A01 test series has the lowest $da/dN$ for an arbitrary $dU/dN$ at the start of the test. This trend completely changes at the end, where 5 Hz frequency test has the highest $da/dN$ for a certain $dU/dN$ among all the other tests in A01 series. Further, the slight rotation observed for the 5 Hz test in A01 and B01 are contrasting to each other. It can be observed to be in clockwise direction for A01 test series and anti-clockwise for B01 test series. This clearly indicates that frequency barely has any effect in the $da/dN$ Vs $dU/dN$ plots or the fatigue state for the different test series are entirely different. If the later is true, then the results between any two test series cannot be compared with each other.
4.3. Energy dissipation and delamination under linear P-d response

Recalling the plot with different stress ratios from Pascoe’s experiments in the right panel of Figure 2.8, the effect of stress ratio can be observed, but it is feeble. This is because, the influence of stress ratio on the material behaviour is less. With the data for different frequencies (Figure 4.4 to Figure 4.6) exhibiting a similar behaviour as seen in Figure 2.8, one can interpret that the influence of frequency on the material is also less. This means, for a certain crack growth rate, tests carried out at different frequencies, more or less dissipate the the same amount of energy per cycle.

It was thought that dU/da, the inverse of the slope of the curves in da/dN Vs dU/dN plots as outlined in Section 3.8 can give further understanding to the results on the frequency effects. This parameter can be interpreted as crack growth resistance in [3] or fatigue toughness [37].

4.3.3. Crack growth resistance against crack growth

Figure 4.7 shows the plot of crack growth resistance \( G^\ast \) plotted against the crack growth \( (a-a_0) \) for all the tests in A01 test series. The \( a-a_0 \) signifies the growth of the crack since the beginning of the fatigue test. So \( a-a_0 \) of 10 mm would mean that the crack has grown for 10 mm from an arbitrary initial crack length \( a_0 \). This is done because, different series of tests have different initial crack lengths \( a_0 \).

It can be seen from Figure 4.7 that the crack growth resistance decreases with the increase in crack length for the fatigue tests at all the frequencies except the 20 Hz test. In general, one can say that, the energy dissipated as well as the crack growth rate decrease such that the energy dissipated for
a unit crack growth keeps reducing with the growth of crack. In other words, as the crack length increases, less amount of energy is available for crack growth.

Looking closer at the Figure in 4.7, after the crack has grown upto 9mm, the $G^*$ value is the least for the 5Hz test with 0.04 mJ/mm$^2$ and the highest for 10 Hz frequency with a value of 0.17 mJ/mm$^2$. It can also be seen that for 20 Hz frequency test, the crack growth resistance is low at the start of the test and increases as the test progresses. This means, more energy was required to grow the crack with the increase in crack length for the 20 Hz. No explanation could be given for this particular trend. However, this trend (increase of $G^*$ with the crack growth) was observed only for 20 Hz in A01 test series out of all the tests that were conducted and thus appears to be an outlier.

If only the curves of 10 Hz, 30 Hz and 40Hz are considered (see Figure 4.8), it can be observed that the $G^*$ for all the three tests had the same value up till a crack length of 0.3mm and the curve shifts down with increasing frequency. So it can be interpreted that less energy was dissipated to grow the crack for 40 Hz test and when the frequency is decreased, more energy was required at the same arbitrary crack length. This could be possible only if there is a crack activation mechanism at higher frequencies such that less energy is required to grow the crack. In other words, the $G^*$ should be affected by some mechanism such that it decreases with increase in frequency.

For the A02 series (see Figure 4.9), it can be seen that the 40 Hz test had lower crack growth resistance compared to other tests and no other notable trend could be observed on the effect of frequency on the crack growth resistance.
Similarly the crack growth resistance was plotted for B01 (left panel in Figure 4.10), B02 (right panel in Figure 4.10) and A03 (see Figure 4.11) test series. The trend of the crack growth resistance curves was observed to be entirely different for B01 and B02 test (see Figure 4.10) series when compared with A01 (Figure 4.7) and A02 (Figure 4.9) test series. The 40 Hz frequency test series in B01 and B02 test series for instance, had higher crack growth resistance than the other frequency tests. One possible explanation for the inadequacy of capturing the effect of frequency could be the way the tests got carried out. During the time when the experiments were conducted at higher frequencies, say \( f > 5 \text{ Hz} \), the screws got loosened up due to high vibrations and it was impossible to conduct the tests without interruptions. One needs to test at least five specimens per test condition for obtaining a valid result \([47]\), however, such a condition would result in large number of fatigue tests which can be performed once a model is developed in order to validate and develop it further.

The other possibility could be the force-displacement behaviour between the maximum and minimum displacements. The \( P-d \) curves for each test greatly varied between each other, depending on the frequency at which the test was carried out. This variation could be a major contributor for the incapability of capturing the frequency effect on energy loss and crack growth correlations. In order to overcome this, the current energy loss calculation method was modified. Later, using the modified methodology (discussed in Section 3.7.2), the results were reanalyzed.
4. Results and discussion

4.4. Energy dissipation and delamination under non-linear P-d response

Since the energy calculated based on the method outlined in Section 3.7.2 is only a first approximation, only qualitative understanding could be obtained from the results discussed below.

4.4.1. Correlation between $\frac{dU}{dN}$ and $\frac{da}{dN}$ after energy correction

Figure 4.12 shows the comparison between $\frac{da}{dN}$ Vs $\frac{dU}{dN}$ plot that does not account for the hysteresis energy (left panel) and after adding the hysteresis energy term (right panel) for all the tests in A01 test series. A clear shift of the high-frequency test curves to the right, can be visualized. The amount by which the curves have shifted to the right depends on the assumed hysteresis area. However, one can qualitatively say that for a certain crack growth rate $\frac{da}{dN}$, the loss of total strain energy $\frac{dU}{dN}$ is more for higher frequency. Or in other words, for a certain value of $\frac{dU}{dN}$, $\frac{da}{dN}$ is the highest for the lowest frequency and decreases with the increase in frequency.

Figure 4.12: $\frac{da}{dN}$ Vs $\frac{dU}{dN}$ before energy correction (left panel) and after energy correction (right panel) for A01 series.

Similar trend of crack growth rate with respect to the total strain energy loss can be found in the right panel of figure 4.13 for A02 test series. It can also be observed in the right panel of Figure 4.13 that the curve of 30 Hz frequency shifts slightly to the right of the 40 Hz curve. One possible reason could be that the assumed hysteresis area for 30 Hz could be an overestimate or that of 40 Hz could be an underestimate when compared to the real case. It is also possible that for certain $\frac{da}{dN}$, the $\frac{dU}{dN}$ was higher for 30 Hz. One needs to repeat the two tests in order to confirm the suitable explanation.

Figure 4.13: $\frac{da}{dN}$ Vs $\frac{dU}{dN}$ before energy correction (left panel) and after energy correction (right panel) for A02 series.
4.4. Energy dissipation and delamination under non-linear P-d response

Figure 4.14: da/dN vs dU/dN after energy correction for B01 (left panel) and B02 (right panel) test series.

In B01 (left panel of Figure 4.14) and B02 (right panel of Figure 4.14) test series, again the effect of frequency is clear. For a certain loss of energy, crack growth rate is higher for lower frequency test and lower for high frequency test. Figure 4.15 shows the correlation between da/dN and dU/dN for the two tests that were conducted in A03 test series before and after correcting for the hysteresis energy. It can be concluded from plots in Figure 4.12 through Figure 4.15 that the missing hysteresis energy is indeed the reason why the effect of frequency could not be previously captured.

Figure 4.15: da/dN vs dU/dN before energy correction (left panel) and after energy correction (right panel) for A03 series.

With the corrected dU/dN and da/dN values for all the tests, the crack growth resistance was plotted against crack growth, the results are provided in the following section.

4.4.2. Effect of frequency on crack growth resistance after energy correction

The crack growth resistance defined in Equation 3.28 was used with the corrected dU/dN for all the test series and plotted against crack growth. From the two plots in the Figure 4.16 for the A01 test series, it can be observed that, the frequency curves shift upward with the increase in frequency meaning that higher frequency tests have higher crack growth resistance. Previously, without the energy correction, the region where all the curves fall was narrow (see left panel in Figure 4.16). After the energy correction, it can be found that the curves spread away from each other depending on the frequency. At the start of the crack, 5Hz test curve has the lowest G* while 40 Hz test has the highest value. This is mainly because, at high frequency more energy was supplied for the crack to grow, in the form of hysteresis energy while for 5 Hz test, the magnitude of hysteresis energy was very small. Interestingly, other than the 20 Hz and 30 Hz tests, for the other frequency tests in A01 series, crack growth resistance is either decreasing or almost constant with the progression of the test. For the 20 Hz test, the crack growth resistance increased with crack.
Results and discussion

Figure 4.16: $G^*$ Vs $a - a_0$ before energy correction (left panel) and after energy correction (right panel) for A01 test series.

length both before as well as after energy correction. However, for the case of 30 Hz test, the increase in $G^*$ with respect to crack growth could be due to the linear interpolation on the loss of hysteresis energy. If the hysteresis energy was decreasing in a power law relationship with $N$, then it would mean that more energy is dissipated at the beginning and very less energy would have been dissipated at the end. This can be also interpreted as higher crack growth resistance at the beginning of the test and lower crack growth resistance at the end of the test. In such a case, it is expected that, $G^*$ will be decreasing for all the frequencies including the 30 Hz frequency curve.

Figure 4.17: $G^*$ Vs $a - a_0$ before energy correction (left panel) and after energy correction (right panel) for A02 test series.

Similar trend of the frequencies was found for the A02 test series (right panel of Figure 4.17). For higher frequency tests, the frequency curves shifted upwards after energy correction. The 30 Hz test, also for the case of A02 test series can be found to have an increasing crack growth resistance with increase in crack length.

For the B01 (left panel of Figure 4.18) and B02 (right panel of Figure 4.18) test series after energy correction, the effect of frequency was very clear. At the start of the tests, the crack growth resistance for high frequencies is higher. For all the tests, both for B01 and B02 plots, the crack growth resistance decreased with the increase in crack length. Similar trend was observed for the 5 Hz and 20 Hz tests in A03 test series (See Figure 4.19). It has to be mentioned here that the hysteresis energy that was supplied to the specimen at higher frequencies was the outcome of material response, however, loss of that energy is not. In other words, for the case of 5Hz test, very small amount of hysteresis energy was supplied and it was lost during crack growth. However, for the case of 30Hz or 40Hz frequency tests, the total supplied energy was also comparatively large (because of hysteresis), and the supplied energy that was lost was also large which is why $G^*$ was higher for the case of high frequencies.
4.5. Comparison with conceptual model

Now that it is understood that the crack growth resistance is higher for higher frequencies, one can question the reason behind such a behaviour. Why does more energy gets dissipated to grow a crack to the same length for higher frequencies? To answer this, one needs to review Equation 3.3 that was discussed before. For ease, the equation is mentioned here.

\[
\frac{dU}{dN} \uparrow (f \uparrow) = \frac{dU_d}{dN} \uparrow + \frac{dU_h}{dN} \downarrow + \frac{dU_e}{dN} \downarrow \tag{4.1}
\]

Consider the \( \frac{da}{dN} \) Vs \( \frac{dU}{dN} \) for B02 (Figure 4.20) series for instance. For \( \frac{dU}{dN} = 3.59 \times 10^{-3} \text{mJ/cycle} \), the crack growth rate for different frequencies are given by the table 4.1. From the table, it can be inferred that \( \frac{da}{dN} \) decreases with the increase in frequency. So if the damage energy \( \frac{dU_d}{dN} \) is considered to be directly related to \( \frac{da}{dN} \), then for an increasing frequency at a constant \( \frac{dU_h}{dN} \), the damage energy decreases (\( \frac{dU_d}{dN} \)) because \( \frac{da}{dN} \) decreases (see Equation 4.2).

\[
\left( \frac{dU}{dN} \right)_{\text{constant}} (f \uparrow) = \frac{dU_d}{dN} \downarrow + \frac{dU_h}{dN} \downarrow + \frac{dU_e}{dN} \downarrow \tag{4.2}
\]

So if damage energy is decreased, it means that the total energy is redistributed to other components like heat dissipation or internal energy. It was also found that for high frequencies, noise was heard throughout the fatigue test. In other words, higher the frequency, louder the noise was heard. So one
may interpret that noise could also be a part of Equation 4.1. However, energy dissipation decreased with the propagation of test while noise was heard throughout the test. Hence, it cannot be true that the dissipated energy is spent for noise, otherwise, one should also expect the loudness of the noise to reduce with the progression of test as the dissipated energy decreases.

The next attempt would be to examine the heat dissipation term and internal energy terms separately. This was done with the use of FLIR A35 infra red camera and thermocouple based on the assumption that heat is related to temperature.

**Temperature Measurements**

Thermocouple was positioned either on the top or on the bottom face of the specimen in order to monitor the changes in the temperature of the body of the specimen. Measurements were carried out after pausing the test at maximum displacement for 2 seconds. From Figure 4.21 it can be observed that no significant change occurred during the test with temperature. Since the test was conducted at environmental conditions, one can argue on the effect of room temperature. However, at high frequency tests, if there was any increase in temperature, this will be captured by the thermocouple as cooling would take more time than the time taken for temperature measurement.

One may then question on the use of thermocouple which was positioned either to the top or bottom width face of the DCB specimen and not on the vicinity of the crack tip. Thermocouple could not be embedded, when the laminate was manufactured, between the 15th and 16th layer where delamination occurs. Such an idea would question the quality of the laminate that may be obtained. The tests were performed under the assumption that all the specimens have the same amount of fibre, matrix & voids and same test in a different specimen would produce the same result or with least amount error. Embedding thermocouple may introduce manufacturing defects that would attribute to more scatter in the data. Moreover, embedding thermocouple may affect the symmetry of DCB specimen and the crack may no longer grow in the same plane. So the only purpose of the thermocouple was to identify any global temperature rise in the specimen.
4.5. Comparison with conceptual model

Figure 4.21: Temperature measurements Vs Number of cycles for B01 (left panel) and B02 (Right panel) series.

The results plotted in Figure 4.21 can be either because there was no actual temperature rise in the specimen or the thermocouple was incapable of capturing the temperature rise. To find a proper explanation to the results observed in Figure 4.21, infrared camera was used. The specifications of FLIR A35 camera can be found in Table 4.2, provided by the manufacturer.

<table>
<thead>
<tr>
<th>Image Frequency</th>
<th>60 Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resolution</td>
<td>320 × 256 pixels</td>
</tr>
<tr>
<td>Thermal Sensitivity</td>
<td>0.05°C @ 30°C</td>
</tr>
<tr>
<td>Accuracy</td>
<td>± 5°C</td>
</tr>
<tr>
<td>Object temperature range</td>
<td>-40°C to 550°C</td>
</tr>
<tr>
<td>Operating temperature range</td>
<td>-15°C to 60°C</td>
</tr>
</tbody>
</table>

Table 4.2: Specifications of Flir A35 infra red camera.

Videos were taken targeting the crack tip at certain number of cycles using the IR camera so that images could be processed later from videos. Interestingly, no temperature gradient could be observed in any test at any frequency during any point of time (see Figure 4.22 and Figure 4.23), meaning that there was no specimen heating. Thus with the thermocouple and infra-red camera measurements, no specimen temperature rise or local heating was observed.

Figure 4.22: Image from Infra red camera during 30 Hz test.

For carbon fibre material under mode I loading, Equation 4.1 can be re-written by ignoring the \( \frac{dU_e}{dN} \) as

\[
\frac{dU}{dN} \uparrow (f \uparrow) = \frac{dU_d}{dN} \uparrow + \frac{dU_h}{dN} \uparrow \quad (4.3)
\]
which says that, with increase in frequency, dU/dN increases (see left panel of Figure 4.24), da/dN can either increase or decrease (see right panel of Figure 4.24) and heat dissipation increases. Influence of frequency on dU/dN could not be captured with the performed tests. So either heat dissipation increased with frequency increase at a much faster rate that current measurement technique was incapable to capture it or the lost energy was consumed by some other mechanism other than damage energy. Here, the dU/dN term cannot be ignored because the tests were conducted at room temperature. One has to conduct the tests in a climate chamber with controlled environmental temperature in order to confirm if heat was dissipated. Otherwise, there could be an entirely different possibility that energy was consumed by some other mechanism so as to balance Equation 4.3.

Now consider the energy loss per cycle and crack growth rate plotted against the crack growth in Figure 4.24 for B02 test series. Clearly, the trend of dU/dN Vs a - a_0 is not exactly the same as da/dN Vs a - a_0. This can be confirmed on having a closer look at the two plots. In the dU/dN against a - a_0 (left panel in 4.24), at a certain arbitrary crack length a - a_0, higher frequency tests dissipate higher energy per cycle than lower frequency tests. However, for the case of da/dN Vs a - a_0 (right panel in 4.24), no particular trend can be observed with different frequencies. Recalling back the Equation 3.27, the da/dN is dependant on two parameters: the resistance and the available energy. Referring to the right panel of Figure 4.18 and left panel of Figure 4.24 for the resistance and available energy per cycle plotted against crack growth, one can deduce that both the quantities are increasing with increase in frequency. The simultaneous increase in the two quantities is such that the trend of da/dN in the right panel of Figure 4.24 can either increase or decrease.

$$\left( \frac{da}{dN} \right)_{\nu} \text{m} = \frac{1}{w(G^* \nu^)} \left( \frac{dU}{dN} \right)$$  \hspace{1cm} (4.4)
So it can be concluded that with the increase in frequency, both crack growth resistance and available energy increase such that crack growth rate can either increase or decrease. To give an example, consider three cases where the frequency is increased:

1. \( \frac{dU}{dN} \) increases by 4 times and \( G^* \) increases by 3 times.
2. \( \frac{dU}{dN} \) increases by 5 times and \( G^* \) increases by 4 times.
3. \( \frac{dU}{dN} \) increases by 5 times and \( G^* \) increases by 3 times.

For case 1, the increase in \( da/dN \) will be 1.3 times. For case 2, \( da/dN \) should only increase by 1.25 (which is less than the case 1 even though both \( dU/dN \) and \( G^* \) are higher). For the case 3, the \( da/dN \) will increase by 1.67 times. So it can be said that the trend of \( da/dN \) is translated by how the two parameters (\( dU/dN \) and \( G^* \)) increase quantitatively.

4.6. Effect of fibre bridging on crack growth resistance at different frequencies

Figure 4.25 shows the crack growth resistance plotted against the crack growth for 5 Hz frequency tests. As can be seen from the plot, crack growth resistance increases with increasing initial crack lengths. This means, resistance to crack growth depends on the crack length. This increase in \( G^* \) can be attributed to fibre bridging. With smaller crack lengths, the bridging of the fibres is less, but as the length increases fibre bridging is fully developed. So the state of fatigue at the crack tip is no more the same for different crack lengths.

The increase in crack growth resistance with longer crack lengths, can be due to the fact that the part of energy available from \( dU/dN \) for crack growth becomes less for higher crack lengths. This is because, most of the available energy gets consumed for crack growth for shorter crack lengths while at higher crack lengths, the available energy is consumed by both crack growing surfaces as well as by bridging fibres which store energy temporarily [37]. Another reason that could be attributed to the increase in resistance with bridging fibres is that the bridging fibres in the wake of the crack tip will hold the surfaces of the crack and reduce the stress intensity factor around the tip of the crack [37]. One can also say that the intact bridging fibres will absorb a large amount of energy that is lost during extension of the crack.

The maximum displacements for different initial crack lengths in Figure 4.25 are different. So the total strain energy that was supplied during the fatigue tests was different for the three tests. However, from Figure 4.26, it can be seen that the three curves fall within a narrow region. Hence \( da/dN \) Vs \( dU/dN \) trend is the same for the three tests with a slight shift in the bottom portion of the curves. The test with shorter crack length has higher crack growth rate than the tests with longer crack lengths.
for the same dU/dN. This means, more energy that was available for crack growth is actually used for crack growth in case of shorter crack lengths while some part of energy is used by bridging fibres in case of longer crack lengths because of which crack growth rate is less.

For the case of 5 Hz, the trend did not change between $G^*$ and $a - a_0$ with the uncorrected and corrected $dU/dN$ in $G^*$ calculation. Interestingly this was not the case with other frequency tests that were compared for different crack lengths. The effect of bridging decreased with increasing crack lengths. In other words, the crack growth resistance decreased with increasing initial crack lengths. Figure 4.27 shows the crack growth resistance plotted against the crack growth without (left panel) and with (right panel) energy correction at 10 Hz frequency for two different initial crack lengths. Similar trend can be seen for 40 Hz frequency test in Figure 4.28.

For the 5 Hz test, the curves remain unchanged because no energy due to hysteresis was taken into account (as it was found to be negligible). However, for the other frequencies, the hysteresis curve was assumed to vary with frequency. This variation was implemented through the eccentricity of the ellipse (assumed hysteresis loop shape). Thus the length of minor axis of the loop is dependant on the assumed eccentricity and the major axis length (see Equation 3.13). Even though, for a given frequency, the eccentricity is the same, the minor axis length depends on the major axis length. So the 10 Hz and 40 Hz tests at different $a_0$ values, will have same respective eccentricity values but different minor axis lengths (because major axis lengths are different) and hence the size of the hysteresis loop will change. However, since no information on the dependency of hysteresis loop shape due stress ratio or the maximum displacement was recorded during the experiments, the explanation for the shift
4.7. Effect of stress ratio on crack growth resistance at different frequencies

Corresponding tests of A and B series with same frequencies were compared to see the effect of stress ratio on the crack growth resistance. The maximum displacement applied for both the cases were the same, however, the minimum displacement for A01 and A02 series was 0.2 mm and 0.4 mm while that of B01 and B02 series was 1 mm and 2 mm respectively. So the A01 tests results were compared with B01 test series at corresponding test frequencies. Similarly A02 test was compared with B02 test series.

Figure 4.29 shows the plots of $G^*$ against crack growth for $d_{\text{max}} = 2\text{mm}$ (left panel) and $d_{\text{max}} = 4\text{mm}$ (right panel) for 5 Hz frequency tests. In both the plots, the crack growth resistance can be found to be higher for the case of $R = 0.5$. This could be because $G^*$ is calculated from $\frac{dU}{dN}$ and $\frac{da}{dN}$ which are dependant on the applied load and the only difference between the two tests is the load cycle.

For higher frequencies, the trend was not consistent. Consider the case of 10 Hz for $d_{\text{max}} = 2\text{mm}$ and $d_{\text{max}}$ in Figure 4.30. For the case of $d_{\text{max}} = 2\text{mm}$, the crack growth resistance for $R = 0.1$ was found to be higher than the test with $R = 0.5$. However, for $d_{\text{max}} = 4\text{mm}$, it was found that both the stress ratios had the same crack growth resistance for most part of the test.
4. Results and discussion

In 40 Hz frequency tests (see Figure 4.31), for \( \text{d}_{\text{max}} = 2\text{mm} \) the tests at two stress ratios had the same crack growth resistance values for most part of the tests while for \( \text{d}_{\text{max}} = 4\text{mm} \), crack growth resistance for \( R = 0.5 \) was higher than \( R = 0.1 \) test. No clear explanation could be found for this behaviour. However, the correlation between \( \text{d}a/\text{dN} \) and \( \text{d}U/\text{dN} \) \( (\text{d}a/\text{dN} = a(\text{d}U/\text{dN})^\beta) \) for the tests with different stress ratios were found to have an exponent \( (\beta) \) of approximately 0.95. This is comparable to the results of Pascoe for adhesive bond where the exponent was found to be 0.86 \[3\]. But, such a comparison could only explain that the results are inline with the previous research. So, in order to clearly understand the effect of stress ratio at different frequencies on delamination and energy dissipation mechanisms, one has to perform more tests.

4.8. Fractography

Fractography was performed on the specimens in order to see if there was any difference observed in the features of the specimens tested under different frequencies. After all the fatigue tests were performed, a specimen from each frequency was picked, labelled and cut using a cutting machine. The two arms of the cut samples were then pulled apart and the fracture surfaces were sputter coated with gold before examining under SEM to ensure that the surfaces are conductive. When the arms were pulled apart, care was taken so that the surfaces were not ruptured, but in some cases, there were bridging fibres that had to be cut for getting the two fracture surfaces apart from each other.

Mode I fracture surfaces are usually rough textured, dark and spectrally reflective. Typical features of mode I fracture surfaces in epoxy systems are fibre tracks, scarp and textured micro-flows \[53\]. These features were observed during the SEM analysis of fracture surfaces (See Figure 4.32). Scars are one
of the most important phenomena of matrix fracture and occurs when multiple fractures initiate along the crack front. The textured micro-flow extend from fibres when the fracture surface extends along the fibres and spreads into surrounding matrix [53]. It was thought that failure process can be affected by frequency of fatigue cycle and local heating could have developed. Local heating usually promotes delamination and microbuckling of fibres [53]. So when observing the fracture surfaces of specimens whose test were conducted above 20 Hz, any sign for microbuckling was carefully examined.

Interestingly, striations were found on fibre imprints in the fracture surface of a specimen tested at 20 Hz (See figure 4.33). Striations appear as regular ripples or marks on the fracture surfaces and only during rare cases have striations been observed in mode I fatigue fracture surfaces [54]. This is because striations are usually identified in shear failures within fibre imprints.

Textured microflow was a common feature among specimens tested at different frequencies. However, this feature appeared mild in specimen tested at 5 Hz and the strongest in specimen tested at 40 Hz frequency (See Figure 4.34). Thus, the surface of specimen tested at 40 Hz was observed to be
rougher than the specimen tested at 5 Hz frequency.

So it could be only be concluded that the specimens tested at different frequencies were observed to have the same microscopic features. However, the fracture surface seems to be rougher for specimens tested at higher frequencies than that of the specimens tested at lower frequencies.

4.9. Conclusion

Results of the experiments performed to study the effect of frequency on mode I delamination of CFRP were presented. With the assumption that load displacement response is linear, it was found that for all the test series, the data points in the plot of $\frac{da}{dN}$ Vs $\frac{dU}{dN}$ fell over a narrow region and no particular trend could be found on the effect of test frequency. After the hysteresis energy was added to the previously determined total strain energy from experiments, the effect of frequency could be seen. The $\frac{da}{dN}$ Vs $\frac{dU}{dN}$ plots spread in such a way that for a certain value of $\frac{dU}{dN}$, crack growth rate was the highest for the lowest frequency test and vice-versa. This trend was observed with all the test series.

When the crack growth resistance $G^*$ was plotted against crack growth $a-a_0$, it was found that with growth of crack, the resistance decreased. For higher frequency tests, $G^*$ was higher compared to the lower frequency tests. This means, more energy is required to grow the crack at higher frequencies.

From the temperature measurements using thermocouple and IR camera, it was observed that no local heating occurred in the specimen even at high frequencies. The energy could have been dissipated as heat, but this could not be validated with the current experiments.

By comparing the results with the conceptual model, it was found that, both $\frac{dU}{dN}$ and $G^*$ increased with the increase in frequency such that $\frac{da}{dN}$ would either increase or decrease (depending on how the two parameters increase quantitatively).

Results plotted against different crack lengths for 5 Hz frequency showed that fibre bridging increased the resistance of the specimen. However, this trend changed with increasing frequencies. It was thought that the approximation of hysteresis energy could have influenced the trend. Precise calculation of the hysteresis energy is needed to make any further reasoning on the observed trends. Similarly, different trends were observed at different frequencies on the effect of stress ratio. No clear explanation could be made for the observed trends and it is recommended to perform more tests at different frequencies and stress ratios.

Fractography was conducted on the fracture surfaces of specimens tested at different frequencies. It was observed that the fracture surface of specimens tested at higher frequencies had a rougher surface than the fracture surface of the specimen tested at lower frequencies.
Conclusions and Recommendations

5.1. Introduction
The main objective of this thesis was focused on the effect of frequency on mode I fatigue delamination growth in CFRP. The characterization method was based on energy principles where fatigue crack growth per cycle \( da/dN \) was correlated to energy dissipation per cycle \( dU/dN \).

From the investigation presented in this thesis, several conclusions can be drawn, on the effect of frequency in fatigue delamination growth. These conclusions are briefly summarized in Section 5.2. Recommendations are given for the future research in Section 5.3. It focuses on how the experimental procedure can be improved and what would be the immediate step in the proposed research direction.

5.2. Conclusions
This section highlights the main conclusions of this thesis and briefly answers to the research questions formulated at the start of the thesis.

- Characterization of fatigue delamination.
- Hysteresis behaviour observed during fatigue loading for CFRP.
- Effect of frequency on delamination and energy dissipation.
- Effect of fibre bridging and stress ratio.
- Limitations of current research.

5.2.1. Characterization of fatigue delamination
While conducting accelerated fatigue tests (AFTs), effect of frequency on the fatigue damage should be considered. Only after understanding the effect of frequency, AFTs can be fully utilized.

When fatigue is applied on the specimen at a frequency of 5 Hz, the load-displacement response is linear. Assuming the same relationship (between load and displacement) for data obtained from high frequency tests fails to exhibit any particular trend on the correlation between \( da/dN \) and \( dU/dN \).

Higher test frequencies result in the formation of hysteresis loop in the load displacement diagram. At frequencies above 5 Hz, the P-d response is non-linear such that a closed loop is formed during fatigue loading-unloading. The size of the loop (area) increases with the increase in frequency.

5.2.2. Hysteresis behaviour during fatigue loading for CFRP
The hysteresis phenomenon can be attributed to the response of the material. When displacement signal is given to the material as a sinusoidal wave, the force response is usually in-phase to the displacement signal. However, at frequencies higher than 5 Hz, there is a phase-lag between force and displacement responses. Hence the relationship between force and displacement is no more linear.
Conclusions and Recommendations

This behaviour changes with different materials and it holds different effects on different materials. For CFRP, the hysteresis loop was observed to shrink in size with the progression of the test due to strain hardening. For some materials, hysteresis loop can also grow until it gets stabilized, due to strain softening phenomenon.

5.2.3. Effect of frequency on delamination and energy dissipation in CFRP

With displacement controlled tests, both \( \frac{dU}{dn} \) and \( \frac{d\alpha}{dn} \) decrease as the test progresses. When tests conducted at different frequencies are compared, it can be found that, more energy gets dissipated per cycle for higher frequency tests. Two other possible mechanisms can be considered, that could possibly consume the dissipated energy along with damage energy: energy dissipation through heat and energy that could rise the temperate of the specimen. It was observed that, for CFRP, energy dissipation was not possible through local heating. However it could not be strongly confirmed if energy dissipated as heat (through conduction, convection or radiation). Thermocouple positioned on the top or bottom width locations of the specimen showed that temperature rise was no more than 2°C for all the tests. On the other hand, infra red camera aimed at the crack tip of the specimen during fatigue showed that no local heating occurred due to hysteresis.

In terms of crack growth resistance, it can be deduced that, with the increase in test frequency more energy is dissipated to grow a unit crack. When the dissipated energy and delamination were separately analyzed, it was found that, only \( \frac{dU}{dn} \) followed a trend such that more energy was dissipated (or available for crack growth) during a cycle. No proper trend on \( \frac{d\alpha}{dn} \) could be observed when it was plotted against crack growth. This is because, with increase in frequency, both \( G^* \) and \( \frac{dU}{dn} \) increase in such a way that \( \frac{d\alpha}{dn} \) can either increase or decrease. So one can interpret that frequency affects the crack growth resistance and the available energy for crack growth. The interaction between the two (crack growth resistance and available energy) translates the crack growth rate.

On a general note, it can be concluded that, two tests carried out at different frequencies have different delamination rate. Delamination propagation observed at higher frequencies is influenced by the prior damage (damage that already exists in the structure) and hysteresis while delamination propagation at lower frequency is only an effect of damage. Thus, one needs to keep in mind, of the additional effects (strain softening, hysteresis heating) that arise due increase in frequency. Only then, damage at higher frequencies can be properly characterized.

5.2.4. Effect of fibre bridging and stress ratio

At a frequency of 5 Hz and at different initial crack lengths, the crack growth resistance increases with increasing initial crack lengths. This is because, at shorter crack length, the available energy is consumed by crack surfaces. However, for longer crack lengths, energy gets absorbed by the bridging fibres such that more energy is required to grow the same amount of crack (when compared to shorter crack length). This trend matches with the observations of Yao (2015) [37].

When \( \frac{d\alpha}{dn} \) Vs \( \frac{dU}{dn} \) was plotted for the tests with different stress ratios, it was found that stress ratio had little effect on the correlation. So for a certain \( \frac{dU}{dn} \), \( \frac{d\alpha}{dn} \) for the tests at different stress ratios were more or less the same. The curves followed a power-law relationship such that the coefficient of the power (\( \beta \) in \( \frac{d\alpha}{dn} = \alpha (\frac{dU}{dn})^\beta \)) was 0.95. This is comparable to the results of Pascoe (2015) [3] who observed a \( \beta \) value of 0.86 for adhesive bonds.

5.2.5. Limitations of current research

One major limitation in the current research was the inadequacy to capture the exact coordinates of the hysteresis loop for all the tests that were performed. During the fatigue experiments, data was stored only at the minimum and maximum point of the fatigue cycle. In order to capture the hysteresis loop, the locus of the hysteresis loop has to be known or in other words more experimental data needs to be known within one fatigue cycle. Thus, just two points for each cycle are insufficient.

The experiments were carried out at room temperature and temperature measurements were achieved using thermocouple and infra-red camera. It was observed that the specimens did not undergo hysteresis heating even at high frequencies and temperature did not vary more than 2°C, between the start and the end of the tests. In literature, Glass fibre/Epoxy specimens were observed to undergo
local heating due to fatigue. With CFRP, either such phenomenon is not prominent as the heat got dissipated or the measurement technique that was undertaken was inadequate to capture any temperature rise in the specimen. The correct explanation is unclear and it needs to be determined.

A proper trend could not be observed on the effect of fibre bridging or stress ratios at frequencies other than 5 Hz. One possible reason could be the way the hysteresis loop was estimated. The loop was assumed to be an ellipse whose eccentricity varied with frequency. However, stress ratio would also play a major role in the shape and size of the hysteresis loop. Pascoe et al. (2015) [49] found that assumption on linear behaviour introduces errors in the calculated value of strain energy. The error was found to be overestimated in case of lower stress ratio where the difference between $d_{\text{min}}$ and $d_{\text{max}}$ is larger. Thus, stress ratio would indeed have an effect on the hysteresis loop. Since, only a first approximation was done on estimation of the hysteresis loop, characterizing the effect of stress ratio on crack growth resistance at different frequencies was not possible.

5.3. Recommendations

With the current research, while certain questions were answered, there are a few more questions that still remain unanswered. In this section, recommendations are provided to further carry out the research in the right direction.

5.3.1. Development of prediction model

As discussed earlier, a good prediction model should also be able to provide an explanation to the observed fatigue phenomenon. The current research is focused on the effect of frequency on the relationship between $\frac{da}{dN}$ and $\frac{dU}{dN}$. From the crack growth resistance discussed in Section 4.3.3 and the conceptual model discussed in Sections 3.2 and 4.5, $\frac{da}{dN}$ can be written as function of crack growth resistance and available energy, both of which increase with increase in frequency.

$$
\left(\frac{da}{dN}\right)_f \uparrow \uparrow = \frac{1}{w(G' \uparrow)} \left(\frac{dU}{dN}\right) \uparrow
$$

(5.1)

where the increase in $\frac{dU}{dN}$ is attributed to the additional hysteresis energy that was supplied to the specimen at high frequencies and increase in $G'$ is attributed to heat dissipation or some other possible mechanism which consumes the available energy. At this point, it is recommended to find a suitable explanation for the increase in crack growth resistance $G'$ at higher frequencies.

In order to get better clarity with the effect of frequency on $G'$, experiments need to be carried inside climate chamber at controlled temperature such that any heat dissipation could be detected. If it is found that heat dissipation does not occur, it means that energy is consumed by some other mechanism. Such a mechanism could possibly be related to the material properties of CFRP such as material damping. Frequency effects are usually attributed to time-dependant, irreversible molecular motions in amorphous and other disordered region in the matrix of the specimen [9]. Such properties decide on the shift of hysteresis loop such that either cyclic hardening or cyclic softening takes place [48].

More experiments need to be carried out to have quantitative information on the effect of frequency on $G'$ and $\frac{dU}{dN}$ after which the prediction model proposed in Equation 5.1 can be further developed.

5.3.2. Towards better experimental setup

The maximum frequency at which the tests were carried out in current research was limited to 40 Hz. This was because, at such high vibrations, screws got loosened and it was impossible to run the tests without interruptions. Such interruptions were thought to contribute to the scatter in the test data. Careful measures have to be taken such that the tests are conducted smoothly, so that the data obtained from the experiments are more reliable.

Higher frequencies need to be achieved (in the order of $10^3$ Hz) so that the test time would significantly reduce. Once such an experimental setup is designed, fatigue delamination growth should be investigated using energy principles to quantify the heat dissipation at such high frequencies. Quantitative measurements on heat dissipation at higher frequencies can allow us to account for actual damage at an arbitrary loading frequency.
5.3.3. Future research direction

Before developing a numerical model, new experiments have to be designed and performed such that results provide material models for numerical analysis as well as explanation to the underlying fatigue mechanisms. The number of experiments was inadequate to characterize the effect of fibre bridging and stress ratio at different frequencies in the current research. So sufficient number of experiments need to be carried out to fully understand such effects.

The next step would be to repeat the experimental procedure for different materials and geometries. This could help us understand how the different experimental parameters affect the fatigue crack growth in a structure. Once, sufficient data is collected, numerical models could be developed. A good model should solve the multi-physics problem such that with the given loading condition and other input data, the model should predict temperature gradient as well as damage propagation in the structure.


[26] A. Griffith, he phenomena of rupture and flow in solids, Philosophical Transactions of the Royal Society of London Series A, Containing Papers of a Mathematical or Physical Character 221 163 (1921).


The crack lengths versus the number of cycles was fit according to the power law relationships given by,

\[ a = \alpha N^\beta + \gamma \]  

(A.1)

Where \( \alpha \), \( \beta \) and \( \gamma \) are fit coefficients. The obtained fit coefficients are shown in table A.1.

<table>
<thead>
<tr>
<th>Test</th>
<th>( \alpha )</th>
<th>( \beta )</th>
<th>( \gamma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>DCB05FA01</td>
<td>0.1056</td>
<td>0.3725</td>
<td>48.2129</td>
</tr>
<tr>
<td>DCB10FA01</td>
<td>-335.8640</td>
<td>-0.0046</td>
<td>375.9758</td>
</tr>
<tr>
<td>DCB20FA01</td>
<td>2.1726</td>
<td>0.1570</td>
<td>41.3012</td>
</tr>
<tr>
<td>DCB30FA01</td>
<td>2.0128</td>
<td>0.1741</td>
<td>39.9987</td>
</tr>
<tr>
<td>DCB40FA01</td>
<td>2.4768</td>
<td>0.1761</td>
<td>41.4188</td>
</tr>
<tr>
<td>DCB05FA02</td>
<td>56.8196</td>
<td>0.0295</td>
<td>-0.5373</td>
</tr>
<tr>
<td>DCB10FA02</td>
<td>11.9990</td>
<td>0.0803</td>
<td>50.1523</td>
</tr>
<tr>
<td>DCB20FA02</td>
<td>29.8733</td>
<td>0.0451</td>
<td>28.6147</td>
</tr>
<tr>
<td>DCB40FA02</td>
<td>3.2151</td>
<td>0.1745</td>
<td>60.9192</td>
</tr>
<tr>
<td>DCB05FA03</td>
<td>22.9210</td>
<td>0.0633</td>
<td>55.3546</td>
</tr>
<tr>
<td>DCB20FA03</td>
<td>9.6381</td>
<td>0.1031</td>
<td>71.2796</td>
</tr>
<tr>
<td>DCB05FB01</td>
<td>23.1200</td>
<td>0.0361</td>
<td>19.0101</td>
</tr>
<tr>
<td>DCB10FB01</td>
<td>1.2838</td>
<td>0.1737</td>
<td>41.8015</td>
</tr>
<tr>
<td>DCB20FB01</td>
<td>3.4885</td>
<td>0.1271</td>
<td>36.7104</td>
</tr>
<tr>
<td>DCB30FB01</td>
<td>4.7375</td>
<td>0.1090</td>
<td>45.9499</td>
</tr>
<tr>
<td>DCB40FB01</td>
<td>2.6979</td>
<td>0.1441</td>
<td>40.8337</td>
</tr>
<tr>
<td>DCB05FB02</td>
<td>12.3213</td>
<td>0.0589</td>
<td>48.9210</td>
</tr>
<tr>
<td>DCB10FB02</td>
<td>9.4839</td>
<td>0.0709</td>
<td>48.7755</td>
</tr>
<tr>
<td>DCB20FB02</td>
<td>5.1687</td>
<td>0.1073</td>
<td>54.1110</td>
</tr>
<tr>
<td>DCB30FB02</td>
<td>2.2445</td>
<td>0.1589</td>
<td>67.3317</td>
</tr>
<tr>
<td>DCB40FB02</td>
<td>2.1057</td>
<td>0.1568</td>
<td>62.9235</td>
</tr>
</tbody>
</table>

Table A.1: Curve-fit parameters for the a versus N functions for all the tests.
The cyclic and total strain energy, without and with energy correction, versus the number of cycles was also fit according to the three parameters power law relationship

\[ U = \alpha N^\beta + \gamma \]  

(A.2)

<table>
<thead>
<tr>
<th>Test</th>
<th>(\alpha)</th>
<th>(\beta)</th>
<th>(\gamma)</th>
<th>(\alpha')</th>
<th>(\beta')</th>
<th>(\gamma')</th>
</tr>
</thead>
<tbody>
<tr>
<td>DCB05FA01</td>
<td>229.8301</td>
<td>-0.0185</td>
<td>66.4047</td>
<td>-8.9559</td>
<td>0.1588</td>
<td>312.1631</td>
</tr>
<tr>
<td>DCB10FA01</td>
<td>189.7858</td>
<td>-0.0692</td>
<td>207.6125</td>
<td>1197.9023</td>
<td>-0.0222</td>
<td>-626.3926</td>
</tr>
<tr>
<td>DCB20FA01</td>
<td>-2.2009</td>
<td>0.2379</td>
<td>331.1980</td>
<td>-10.4119</td>
<td>0.2531</td>
<td>504.7191</td>
</tr>
<tr>
<td>DCB30FA01</td>
<td>-196.8872</td>
<td>0.0245</td>
<td>546.7724</td>
<td>-23.7637</td>
<td>0.3082</td>
<td>1260.4579</td>
</tr>
<tr>
<td>DCB40FA01</td>
<td>166.2951</td>
<td>-0.0523</td>
<td>175.0027</td>
<td>-223.7500</td>
<td>0.1522</td>
<td>1580.1044</td>
</tr>
<tr>
<td>DCB05FA02</td>
<td>451.5969</td>
<td>-0.0272</td>
<td>160.8439</td>
<td>1277.0034</td>
<td>-0.0106</td>
<td>-638.2483</td>
</tr>
<tr>
<td>DCB10FA02</td>
<td>412.6059</td>
<td>-0.0354</td>
<td>285.8849</td>
<td>-744.6299</td>
<td>0.0223</td>
<td>1525.8771</td>
</tr>
<tr>
<td>DCB30FA02</td>
<td>395.0732</td>
<td>-0.0435</td>
<td>311.6173</td>
<td>-277.4056</td>
<td>0.1140</td>
<td>1604.4579</td>
</tr>
<tr>
<td>DCB40FA02</td>
<td>620.2015</td>
<td>-0.0173</td>
<td>32.8167</td>
<td>-154.1717</td>
<td>0.1404</td>
<td>1301.2595</td>
</tr>
<tr>
<td>DCB05FA03</td>
<td>-579.4760</td>
<td>0.0215</td>
<td>1570.9168</td>
<td>-516.5693</td>
<td>0.0258</td>
<td>1524.9902</td>
</tr>
<tr>
<td>DCB20FA03</td>
<td>2091.7809</td>
<td>-0.0074</td>
<td>-981.9937</td>
<td>-126.9966</td>
<td>0.0970</td>
<td>1327.8880</td>
</tr>
<tr>
<td>DCB05FB01</td>
<td>126.8934</td>
<td>-0.0537</td>
<td>102.058</td>
<td>1392.9843</td>
<td>-0.0048</td>
<td>-1145.1478</td>
</tr>
<tr>
<td>DCB10FB01</td>
<td>-24.7385</td>
<td>0.0710</td>
<td>227.1396</td>
<td>-25.1127</td>
<td>0.1160</td>
<td>271.8320</td>
</tr>
<tr>
<td>DCB20FB01</td>
<td>-48.2541</td>
<td>0.049</td>
<td>247.5821</td>
<td>-29.7557</td>
<td>0.1422</td>
<td>326.2710</td>
</tr>
<tr>
<td>DCB30FB01</td>
<td>60.2732</td>
<td>-0.1041</td>
<td>155.4708</td>
<td>-323.3235</td>
<td>0.0507</td>
<td>756.2828</td>
</tr>
<tr>
<td>DCB40FB01</td>
<td>85.5394</td>
<td>-0.1591</td>
<td>168.8435</td>
<td>-323.7845</td>
<td>0.1071</td>
<td>1316.5873</td>
</tr>
<tr>
<td>DCB05FB02</td>
<td>789.3530</td>
<td>-0.0087</td>
<td>-362.9757</td>
<td>-387.9707</td>
<td>0.0165</td>
<td>821.5878</td>
</tr>
<tr>
<td>DCB10FB02</td>
<td>-239.4210</td>
<td>0.0213</td>
<td>642.3885</td>
<td>-83.0848</td>
<td>0.0647</td>
<td>511.9708</td>
</tr>
<tr>
<td>DCB20FB02</td>
<td>-2594.4008</td>
<td>0.0020</td>
<td>2985.5041</td>
<td>-210.5316</td>
<td>0.0400</td>
<td>666.0889</td>
</tr>
<tr>
<td>DCB30FB02</td>
<td>188.5091</td>
<td>-0.0345</td>
<td>214.6224</td>
<td>-267.5029</td>
<td>0.0502</td>
<td>824.6044</td>
</tr>
<tr>
<td>DCB40FB02</td>
<td>-354.3728</td>
<td>0.01574</td>
<td>775.3109</td>
<td>-189.6680</td>
<td>0.1076</td>
<td>1002.0462</td>
</tr>
</tbody>
</table>

Table A.2: Curve-fit parameters for the \(U_{\text{cy}}\) versus \(N\) functions for all the tests without energy correction \((\alpha, \beta, \gamma)\) and with energy correction \((\alpha', \beta', \gamma')\).
Table A.3: Curve-fit parameters for the $U_{\text{tot}}$ versus $N$ functions for all the tests without energy correction ($\alpha$, $\beta$, $\gamma$) and with energy correction ($\alpha'$, $\beta'$, $\gamma'$).
For the fading hysteresis loop, the shape is thought to be that of an ellipse. During fatigue, two events are considered to occur, simultaneously, on the shape on the hysteresis loop.

1. The decrease in the area of the loop.
2. The change in the coordinates of the two end points (at minimum and maximum displacements).

Figure B.1: A schematic of the decrease in area of loop and shift in the coordinate points.

Figure B.1 represents the schematic of the hysteresis behaviour, observed during the fatigue experiments. The darkest shade of loop represents the shape of loop at the start of the test. As the test progresses, the size of the loop shrinks and the loop can be observed to rotate. The lightest shade represents the shape of the loop towards the end of the experiment.

The rotation of the loop is because, as the crack propagates, the maximum load required to open the DCB specimen to maximum displacement decreases. Since the test is displacement controlled, the end points of all the ellipse have to lie on the maximum and minimum displacements. Thus the locus of the end points of the ellipse becomes the vertical lines ($d = d_{min}$ and $d = d_{max}$).

To compute the area of the loop, the minor and major axis lengths of the loop needs to be known at the corresponding fatigue cycles where area is calculated. The major axis length can be calculated from the length between $(d_{min}, P_{min})$ and $(d_{min}, P_{min})$. However, the calculation of minor axis is not
straight forward. The equation of the ellipse needs to be known. To determine the equation of ellipse, at least five co-ordinate points are required. Since the hysteresis loop is only assumed to be an ellipse and not exactly an ellipse, the five points can be used to fit an ellipse such that the locus of the ellipse passes through all the five points. Such an ellipse will give the closest possible shape for the hysteresis loop. The fitting of ellipse can be performed using least square method from which the coefficients in the general conic equation of ellipse can be determined. Once the coefficients are known, Equations 3.22 through 3.26 can be followed and the area of loop corresponding to the required fatigue cycle can be determined.

During the fatigue process, the shape of the hysteresis loop changes, such that lengths of both the major and minor axis decreases. The decrease in the minor axis length can be visualized from Figure B.1 as the width of the ellipse shrinks. Similarly, the decrease in the major axis length ($Y_{major,1} > Y_{major,N}$) with the increase in number of cycles can be seen from Figure B.2 where the major axes of the ellipse presented in B.1 are shown. Thus a proper approach to calculate the hysteresis energy drop would be to calculate the area, each time, at the corresponding fatigue cycle. However, since with the current experimental method, as only the two data points were stored (with the assumption that the load-response curve will be linear), the area of the hysteresis curve only during the start of the test was recorded. This area was assumed to linearly decrease with the number of cycles. The hysteresis area was then added to cyclic energy $U_{cyc}$.

Figure B.2: A schematic of the decrease in the length of major axes (shown with yellow lines) with the rotation of the ellipse in the clockwise direction.